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Abstract

This paper presents a model of auctions in which sellers may fail to sell an object in spite of receiving a bid above the announced reserve price. In this sense, sellers use a secret reserve price. Such behavior is seen frequently in auctions and yet would be sub-optimal in most existing auction models. Here, sellers have information about the object that cannot credibly be communicated to buyers. Sellers with more favorable information about the object's value would prefer a higher reserve price; however, any such reserve price would be mimicked by a seller with less favorable information. As a consequence, a seller's only option is to have a secret reserve price, rejecting bids that are too low and auctioning the object in the future. The model predicts that re-auctioned items, on average, sell for higher prices and that prices of re-auctioned items rise as the delay in re-auctioning increases.

1 Introduction

Auction models typically assume that the seller of an object announces a minimum acceptable bid, known as the reserve price, prior to any bidding by buyers. This price represents a commitment by the seller to sell if and only if the highest bid exceeds the reserve price. A standard result in the auction literature is that, if the seller can commit to an announced reserve price, it is always in his interest to do so: the reserve price serves to increase the bids of buyers and so raises the seller's expected revenue from the auction. The implication is that the issue for reserve-price setting is not whether the seller *wants* to set a reserve price – it is optimal to do that – but rather whether the seller is *able to commit* to this price *ex ante.*

Interestingly, a feature of actual auctions is that the reserve price often is *not* announced *ex ante,* even when it would appear that the seller is able to commit. Referring to English auctions, Ashenfelter (1989) writes:

> Auctioneers are very secretive about whether and at what level a reserve price may have been set, and there is a real art in getting the bidding started on each item without revealing the reserve price... If you sit through an auction you will find that every item is hammered down and treated as if it were sold. Only after the auction does the auctioneer reveal whether and at what price an item may have actually been sold. In short, the auctioneers do not reveal the reserve price and they make it as difficult as they can for bidders to infer it.

Hendricks and Porter (1988) and Porter (1995) note a similar phenomenon in sealed-bid auctions for off-shore oil and gas leases in the U.S.. In these auctions, leases are often not sold even though the highest bid exceeds the announced reserve price. Porter reports that, over the period 1970-1979, 12.7% of all highest bids that exceeded the reserve price were nevertheless rejected. Apparently, government decisions were being based not on the announced minimum acceptable bid, but on some higher, unannounced reserve price. Porter also notes that, among all tracts with rejected high bids over the period 1954-1979, 46.8% were subsequently re-auctioned with an average delay time to re-auction of 2.7 years. The mean value of the high bid on these tracts increased 2.5 times upon re-auction.

These observations, as Ashenfelter pointed out, raise two questions. First, why are sellers *not* announcing their reserve prices if it is in their interests to do so? And second, when sellers are not announcing their reserve prices, why do they typically use delay in sale as an alternative means of raising the ultimate sale price? Both Ashenfelter and Porter suggest that secret reserve prices might be used as a means of deterring collusive bidding behavior. While this explanation has some appeal, it is not apparent why sellers would find a secret reserve price preferable to a high announced reserve price that would directly reduce the rents available to the cartel (as in Graham and Marshall (1986)). Ashenfelter also suggests that a "search" motive could account for the seller's refusal to sell to the highest bidder. The delay in sale is then presumed to provide the seller with an opportunity to search for a group of new, higher-valuation buyers. Vincent (1995), finally, offers an explanation of secret reserve prices as a way for the seller to increase bidder participation at the auction (and so expected sales revenue), low-valuation bidders being unwilling to bid when there is a high announced reserve price. Vincent does not seek to explain re-sale behavior by sellers when the high bid is rejected.

In this paper, we propose an alternative explanation for the twin observations of secret reserve prices and re-sale behavior. In contrast to the above explanations, we view these outcomes as the result of both attempts by the seller to communicate information and learning on the part of the buyers. The setting for our analysis is a common-value auction model in which the seller is assumed to possess information about the value of the object that cannot be directly transmitted to the buyers. In such a situation, one might imagine the seller trying to signal his information via the announced reserve price. However, if this announced price were to signal the value of the object, the seller would always prefer to set a reserve price associated with a highly-valued object rather than reveal its value to be low. Therefore, equilibrium must result in a seller with information that the value of the object is low announcing the same reserve price as would be announced were the object of some greater value.

In cases in which the seller attaches some intrinsic value to owning the object, the above pooling behavior is sufficient to explain the use of secret reserve prices. In such cases, a seller with a high personal valuation for the object may well reject a bid above the announced reserve price that that same seller would accept were the object of lesser value to him. The setting we examine, however, is one in which the seller attaches no intrinsic value to owning the object, only valuing it for its re-sale possibilities (as would be the case for oil and gas leases). A refusal to sell can then only be optimal if the seller with a high value object expects to be able to sell it subsequently for a higher price. In a common value auction, it is possible that, while the seller's information reveals a high value for the object, buyers' information indicates that the object is of low value and so buyers submit low bids. If the seller expects buyers to learn more about the object's value over time, then it may be in the interest of a seller with a highly-valued object to refuse to sell now and wait for better information to come to light. Buyer learning, then, provides a motive for the seller's refusal to sell and subsequent re-auctioning.

In this setting, we find that the seller's inability to condition his reserve price announcement on information concerning the object's value, combined with buyer learning about this value, means that the seller will operate with a secret reserve price. That is, given the possibility of generating greater revenue by re-auctioning the object later when buyers have acquired more information, the seller will find it profitable to refuse to sell below some (unnannounced) price. This price, which is increasing in the object's value, is the seller's secret reserve price.[1] The refusal to sell, in this case, allows a seller with a more highly-valued object to signal this fact and so use this signal in combination with buyer learning to obtain higher bids in the future. Signaling via delay is effective because learning by the buyers makes imitation unprofitable for the seller with an object of lower value.

Our model, then, provides an explanation for the government's decision to reject bids above the announced reserve price (use of a secret reserve price) in offshore oil and gas lease auctions. It also helps to explain other features of oil and gas lease auctions. For instance, Porter notes that the announced reserve price is the same for all tracts in a given sale, being set at either $15/acre or $25/acre. McAfee and Vincent (1992) argue that this announced

1 Porter notes that "the government's rejection decision takes into account its *private estimate of the value of the tract* (italics added; Porter (1995) p. 4).

price is far too low, the optimal reserve price being around $200/acre. This pattern of announced reserve prices, however, is exactly what our model would predict: the marginal tracts drive reserve prices for all tracts and so $15/acre could be the equilibrium announced reserve price. Porter also notes that sale prices on re-auctioned tracts were on average 2.5 times that of the rejected high bid on initial auction but that this re-auction price is still low relative to accepted high bids on initial auctions. As will be seen, this pattern of sale prices is also consistent with our model.

We note, finally, that the presence of a secret reserve price means that buyers face a "double" winner's curse in the initial auction of an object. There is the usual winner's curse associated with the fact that the highest bidder received a higher signal of the object's value than did all other bidders. In addition, there is a possible second winner's curse due to the fact that the objects that are sold initially are ones for which the buyers' information is optimistic relative to the seller's. This second winner's curse leads the buyers to shave their bids more at the initial auction than they would at a one-shot auction.

The remainder of the paper provides more details on all of these issues. In the next section, the specifics of the model are presented. In Section 3, the buyers' and seller's optimization problems are formulated and the conditions that define the equilibrium are derived. Section 4 discusses the implications of the model for auction behavior while Section 5 provides some concluding remarks. Proofs of the results are collected in the Appendix.

2 The Model

A seller uses a second-price, sealed-bid auction to sell an indivisible object. There are n potential buyers for the object where the value of n is fixed exogenously and known to all agents. The object has no intrinsic value to the seller. The value of the object is the same to each buyer and is denoted by V. Buyers are assumed to be uninformed about this value but to have common prior beliefs given by the probability distribution $F(v)$, with $v \in [\underline{v}, \overline{v}]$. The seller, in contrast, is assumed to know the object's value (that is, the realization v of the random variable V).[2] Prior to bidding, each buyer obtains noisy information about the value of the object, summarized by a scalar random variable, X_i, referred to as the buyer's *signal.* The vector of first-period signals is denoted by $X = (X_1, \ldots, X_n)$. The signal-generating process is described by a conditional probability distribution of the form $G_{X_i}(\cdot \mid v)$, with the signal for each buyer, i, being an independent draw from $G_{X_i}(\cdot \mid v)$. It is assumed that the family of conditional densities $\{g_{X_i}(\cdot \mid v)\}$ associated with the signal-generating process possesses the strict monotone likelihood ratio property (MLRP): for all $x > x'$, all $v > v'$, $g_{X_i}(x \mid v) g_{X_i}(x' \mid v') > g_{X_i}(x \mid v') g_{X_i}(x' \mid v)$. This condition captures the notion that the signal is informative regarding V; roughly speaking, it guarantees that a larger value of the signal is more likely the larger is the value of V.

The sequence of events in the model is as follows. At time $t = 1$, a seller with an object of

[2] The assumption that the value of the object is known by the seller is made to simplify the exposition. The substance of our results would not be altered were it assumed that the seller observed a signal correlated with V rather than the value itself, as long as this signal were more informative than the buyers' signals. The case for the superiority of the information held by the seller is apparent in the case of oil and gas leases, where the government has access to the seismic information collected by *all* bidders on *all* tracts being leased at the auction. A bidding firm, on the other hand, only has its own seismic readings on the tracts of interest to the firm.

value, $V = v$, announces an auction and a reserve price, $r(v)$. The reserve price commits the seller to accepting no bid below r but does not commit the seller to selling if the highest bid is above the reserve price. Each buyer, i, receives a signal of the value, $x_i \in [\underline{x}_i, \overline{x}_i]$, drawn from the conditional distribution $G_{X_i}(\cdot \mid v)$. Given the signal and any information contained in the seller's choice of reserve price, each buyer chooses a bid, $b_{i,1}(x_i, r)$. Having received the bids, the seller chooses either to sell the object to the highest bidder at the second-highest bidder's bid (assuming that the bid is above the reserve price) or to reject all bids and not sell the object this period. In the former case, the highest bidder obtains the object and the game ends. In the latter case, the game proceeds to a second period. In either case, all bids are revealed at the end of the first auction.

Should all the bids be rejected at $t = 1$, then the seller re-auctions the object at $t = 2$. Before this second auction, each buyer obtains a new signal, $y_i \in [\underline{y}_i, \overline{y}_i]$. The signal-generating process in the second period shares the basic features of the first-period one: the signals $Y_1, \dots Y_n$, drawn from a distribution, $G_{Y_i}(\cdot \mid v)$, are conditionally independent and the corresponding family of densities, $\{g_{Y_i}(\cdot \mid v)\}$, satisfies MLRP.[3] In addition, the densities of first- and second-period signals are related by the fact that the vector of first-period signals, $X_1, \dots, X_n$, is a "garbling" of any one of the second period signals, Y_i. To be precise, it is assumed that the joint density of $(V, X_1, \dots, X_n, Y_1, \dots, Y_n)$ can be written as $\alpha(V, Y_1, \dots, Y_n) \cdot \beta(X_1, \dots, X_n \mid Y_i)$, for any $i = 1, \dots, n$.[4] This assumption is meant to

[3] In what follows we use $G_{\cdot}(\cdot)$ to designate distribution functions on signals, with the subscript indicating the random variable(s) being considered. We use $H_{\cdot}(\cdot)$ to denote a distribution involving both the value of the object and signals, with the random variables of interest again being given by the subscript. The expression $F_t(\cdot)$ designates the beliefs of the buyers regarding the value of the object if it sells in period t.
[4] This concept is used in communication theory (see Kullbach (1978)), and is called "garbling" by Milgrom and Weber (1982).

capture the notion that the signal received in the second period is at least as informative regarding the value of the object as the complete sample of first-period signals. Such would be the case if, for instance, the distribution of the first-period signals were the same as the distribution of a random sample generated by adding n realizations of a white noise variable, Z, to one of the signals Y_k. This specification of the information structure incorporates into the model the notion that buyers learn about the value of the object over time; that is, buyers obtain not only more information in the second period but also more accurate information.

Based on this new information and the fact that the seller refused to sell at $t = 1$, each buyer places a bid at the second auction. The bid for buyer i at $t = 2$ is denoted by $b_{i,2}(r, b_1, y_i)$ where b_1 is the vector of first-period bids. At this point, since it is assumed that there are no future periods, the seller sells the object to the highest bidder at the second-highest bidder's bid as long as this bid exceeds the reserve price.

All buyers and the seller are assumed risk neutral. Buyers choose bids each period to maximize the present value of their expected surplus. The seller determines a reserve price and re-auction decision to maximize expected sales revenue. All agents discount future returns at a common rate, δ, with $0 \leq \delta \leq 1$. Equilibrium strategies are those that constitute a Perfect Bayesian Equilibrium.

3 Strategies and Equilibrium Conditions

The presentation of equilibrium behavior is divided into two parts, corresponding to the two main issues outlined in the Introduction: i) the equilibrium in *announced* reserve prices and

ii) the seller's equilibrium *secret* reserve price and re-auction policy. This latter issue, of necessity, will require a presentation of the buyers' equilibrium bidding strategies.

3.1 Announced Reserve Prices

It is fairly immediate that any equilibrium announced reserve price strategy must involve some pooling on v. To see why, suppose, by way of contradiction, that announced reserve prices completely revealed the value of the object. Then, buyers could correctly infer v from the announced reserve price and would bid as if they had perfect information; that is, each buyer would submit a bid of v. Clearly, a seller with a low v could profitably deviate from the proposed reserve price schedule by announcing the reserve price that would have been chosen by a seller with a high v. Therefore, this cannot constitute an equilibrium. We summarize this result below as:

Lemma 1 *In equilibrium, the announced reserve price* $r : [\underline{v}, \overline{v}] \mapsto [0, \overline{v}]$ *cannot be strictly monotonic in the seller's type.*

As for the form that pooling takes, one possibility is that all seller types announce the same reserve price. For such complete pooling to obtain, it is sufficient for the buyers to believe that, if another reserve price were announced, then the seller is of type $\underline{v}$ with probability one. The buyers would then all bid $\underline{v}$, making such a deviation unprofitable for any seller type. This type of equilibrium is the one examined in what follows, with all seller types adopting a common announced reserve price of $\underline{v}$. Consideration of this case is at least partly motivated by the evidence for oil and gas leases auctions presented in the

Introduction. In that context, a single reserve price is adopted for all tracts in any given sale. Setting the announced reserve price to the lowest type is a technical convenience; since buyers would never bid below $\underline{v}$, it ensures that the reserve price is not binding.

There are other potential equilibrium announced reserve price strategies involving only partial pooling; i.e., the same reserve price is set by seller types from a strict subset of $[\underline{v}, \overline{v}]$ but reserve prices differ across subsets of seller types. If each subset of seller types announcing a common reserve price is a connected set, then the announced reserve price for any set of seller types must exceed the infimum of that set of types, except for the set containing $\underline{v}$.[5] For this case, the analysis of buyer and seller behavior presented below can be applied to each connected subset of seller types simply by weighting the seller's expected profit by the probability that the high bid exceeds the announced reserve price and by adjusting buyer bidding functions so that the reserve price serves as a binding lower bound on bids. If the subsets of seller types adopting a common reserve price is not connected, then, in additon, the support of the distribution of V would have to be modified to account for this feature of the equilibrium.

3.2 Secret Reserve Prices and the Re-auction Decision

That some pooling on announced reserve price must always occur in equilibrium, coupled with the fact that the object has no intrinsic value to the seller, means that there may be circumstances in which the seller can gain by refusing to sell to the highest bidder at $t = 1$

[5] Indeed, if this were not the case, sellers of types $[v^1, v^2] \subset [\underline{v}, \overline{v}]$ announcing a reserve price $r^1 \leq v^1$ could expect a high bid in excess of v^1 with probability one. This, in turn, would imply that a seller with an object of value $v < v^1$ would prefer to announce a reserve price of r^1 rather than another reserve price which would yield a high bid *below* v^1 with certainty.

and re-auctioning the object at $t = 2$. Clearly, such will be the case whenever the seller's discounted expected profits should he wait and re-auction the object exceed the proposed price for the object in the first period (the second highest bid). The place to begin an analysis of the seller's decision to re-auction, therefore, is in the second period with the buyers' bidding strategy. Doing so allows us to determine the expected profits when the seller waits and re-auctions the object and the circumstances under which a refusal to sell at $t = 1$ occurs. The seller's re-sale decision, in turn, allows us to characterize the buyers' bidding behavior at $t = 1$.

3.2.1 Second Period Bidding Behavior

Suppose, then, that the seller has refused to sell at $t = 1$ and now must re-auction the object at $t = 2$. Since the seller has no future opportunities for re-sale and attaches no intrinsic value to the object itself, the highest bid at $t = 2$ will be accepted. This auction, therefore, is described by the standard second-price, common-value auction model. As has been shown elsewhere (see Migrom and Weber (1982) or Wilson (1992)), the equilibrium strategy is for each buyer to bid the expected value of the object, given all the information available to him. As in the standard case, the information available to buyer i includes his signal y_i, and the fact that, should he submit the winning bid, all other buyers value the object less than he does. In addition, from the first auction, each buyer i can infer the vector of first-period signals, $x = (x_1, \ldots, x_n)$, from the announcement of the first-period bids and the first-period bidding function (which will be shown to be monotonic). Finally, the seller's refusal to sell may convey information about the value of the object.

A consequence of the assumption that X is a garbling of each Y_i is that each buyer's value, and so the equilibrium bidding function, is independent of X at $t = 2$. Intuitively, the reason for this result is that X being a garbling of each Y_i means that the first-period signals provide no additional information, either about the object's value or the highest signal among bidders other than i, beyond that available through Y_i. As a consequence, each buyer's expected rents (the difference between the expected value of the object and the expected value of the second highest bid) are independent of first-period signals and so, therefore, is each buyer's bid. This result on second-period bidding behavior is given in the following lemma.

Lemma 2 *Buyers' bidding behavior in the second period is independent of the vector of first-period signals, x.*

An implication of Lemma 2 is that the additional (i.e., beyond Y_i) information provided by the first auction outcome is the information about the value of the object conveyed by the seller's refusal to sell. Let $F_2(v)$, defined on the support $[v^L, v^H] \subseteq [\underline{v}, \overline{v}]$, be the probability distribution for the value of the object that updates the prior probability distribution $F(v)$ on the basis of the seller's equilibrium re-auction strategy and the fact that the seller refused to sell to the highest bidder at $t = 1$. This distribution represents the beliefs that buyers hold regarding the value of the object before each gets his private signal; it is the distribution that buyers will use, in conjunction with second-period signals, to calculate the expected value of the object. The strategic situation faced by each buyer at $t = 2$ is then captured by a one-shot common-value auction where the prior probability distribution on V is given by $F_2(v)$ and where each buyer observes one of n conditionally independent signals $Y_1, \ldots, Y_n$.

To define the buyers' bidding strategies, we need only define the expected value of the object for the above auction. Using superscripts to denote ordered samples of n bids or signals, and using the combination of a "^" and a superscript to denote an ordered sample of $n-1$ observations where the bid or signal from the agent under consideration has been omitted, we have $\hat{Y}^1 > \hat{Y}^2 > \ldots > \hat{Y}^{n-1}$ as the ordered sample of second-period signals for all bidders other than i. The expected value of the object is then given by:

$$w_2(y_i, u, F_2(\cdot)) = E[V \mid Y_i = y_i,\ \hat{Y}^1 = u,\ V \sim F_2(v)] \tag{1}$$

which is the expected value of the object for i when his signal has value y_i, the highest signal among all other bidders is u and the common beliefs on the value of the object are given by $F_2(\cdot)$. Then i's strategy in a symmetric equilibrium is to bid $b_2^*(y_i) = w_2(y_i, y_i, F_2(\cdot))$. Because the family of densities associated with the signal-generating process $\{g_{Y_i|v}(\cdot \mid v)\}$ satisfy MLRP, $w_2(\cdot, \cdot, F_2(\cdot))$ is increasing in its first argument so that the buyer with the highest signal is awarded the object. The price is the value of the object to the buyer with the second-highest signal, yielding second-period seller revenue of $w_2(y^2, y^2, F_2(\cdot)) = b_2^2$.

3.2.2 The Seller's Re-Auction Decision

Knowing that second-period sales revenue is defined as above, the seller can decide whether to sell to the highest bidder at $t = 1$ once bids at $t = 1$ have been submitted. Letting

$p(b_1^2 \mid v)$ be the probability that b_1^2 is accepted by a seller of type v, the seller's problem is

$$\max_{p(b_1^2|v)} \; p(b_1^2 \mid v)\, b_1^2 + (1 - p(b_1^2 \mid v))\, \delta \int_{y^2} w_2(y^2, y^2, F_2(\cdot)) g_{Y^2|V}(y^2 \mid v) dy^2 \tag{2}$$

The expected revenue from the auction at $t = 1$ is the second highest bid, b_1^2, weighted by the probability that the seller accepts this price. The expected revenue from the auction at $t = 2$ is the expected price weighted by the probability that the seller has refused to sell at $t = 1$ and by the discount factor δ. Defining $K(y^2, F_2(\cdot)) = \int_{y^2} w_2(y^2, y^2, F_2(\cdot)) g_{Y^2|V}(y^2 \mid v) dy^2$, then the first-order condition of the maximization problem in (2) yields

$$p(b_1^2 \mid v) = \begin{cases} 0, & \text{when } b_1^2 < \delta K(y^2, F_2(\cdot)) \\ \in [0,1], & \text{when } b_1^2 = \delta K(y^2, F_2(\cdot)) \\ 1, & \text{when } b_1^2 > \delta K(y^2, F_2(\cdot)) \end{cases}$$

These conditions say that, for given second period beliefs $F_2(\cdot)$, the seller accepts (refuses) the price from the auction at $t = 1$ whenever its value exceeds (is less than) the discounted expected price from the auction at $t = 2$. When these two values are just equal, the seller is indifferent between accepting and rejecting the first-period price. Because the family of densities $\{g_{Y^2|V}(\cdot \mid v)\}$ satisfies MLRP whenever $\{g_{Y_i|V}(\cdot \mid v)\}$ does, if there exists a value of v for which $b_1^2 = \delta K(y^2, F_2(\cdot))$, this value is unique.

We are now able to characterize the seller's sell/re-auction strategy. It is given below as:

Proposition 1 *Suppose that in the first-period auction each buyer uses the increasing bidding function $b_1^*(\cdot)$. Given the buyers' beliefs in the second auction, $F_2(\cdot)$, the bidding strategy in the second auction, $b_2^*(\cdot)$,and a second highest bid in the first auction, $b_1^*(x^2)$, the seller* *accepts (refuses) the first auction price whenever v is smaller (greater) than $v^*(x^2)$:*

$$p(b_1^2 \mid v) = \begin{cases} 0 & \textit{when } v > v^*(x^2) \\ \in [0,1] & \textit{when } v = v^*(x^2) \\ 1 & \textit{when } v < v^*(x^2) \end{cases}$$

An implication of this proposition is that buyers must recognize that, if the object is sold at $t = 1$ at price $b_1^*(x^2)$, then its value is no higher than v^*. Similarly, if the object fails to sell at this price, then its value is at least v^*. It is in this sense that refusal to sell acts as a signal of value. Second-period beliefs after a refusal to sell are given by $F_2(v) = \Pr(V \leq v \mid V \geq v^*) = \dfrac{F(v) - F(v^*)}{1 - F(v^*)}$. Further, because v^* completely determines second-period beliefs, given any prior $F(v)$, a buyer's second-period valuation, $w_2(s, t, F_2(\cdot))$, can be more conveniently expressed as $W_2(s, t, v^*)$.

To see the intuition behind this result, consider a first-auction price of $b_1^*(x^2)$. By definition, a seller with object of value v^* is just indifferent between accepting and rejecting the bid. Now consider a seller of type $v > v^*$. Because the family of densities $\{g_{Y_i|V}(\cdot \mid v)\}$ satisfies MLRP, the densities $\{g_{Y^2|V}(\cdot \mid v)\}$ do as well. As a consequence, the conditional distribution of the second highest signal, Y^2, puts more weight on higher values of the signal when the value of the object is v than when it is v^*. This fact, combined with the fact that the second-auction bidding function is increasing in the buyer's signal, means that a seller of type v can expect on average higher bids in the second period than a seller of type v^*. This immediately implies that a seller of type $v > v^*$ has a higher expected revenue in the second auction and so should reject the first auction bid. A completely analogous reasoning can be used to argue that a seller of type $\tilde{v} < v^*$ would accept the first auction price.

Proposition 1 can be used to define the secret reserve price policy for the seller. A seller with an object of value $v^*(x^2)$ is just indifferent between selling and not in the first auction, given that the second-highest signal and bid were x^2 and $b_1^*(x^2)$ respectively. One can show that, given the equilibrium second-period beliefs held by the buyers, $F_2(\cdot)$, the function $v^*(\cdot)$ is increasing whenever the first-period bidding function $b_1^*(\cdot)$ is increasing. This means that it is possible to define an inverse function, $\chi(v)$, giving, for any value of the object, the second highest signal that would make a seller of type v indifferent between accepting and rejecting the first-period price. The secret reserve price for a seller with object of value v is then given by $b_1^*(\chi(v))$; that is, by the bid that an agent having the signal $\chi(v)$ would submit. Because the function $v^*(\cdot)$ is increasing, its inverse is also. This fact, combined with the fact that the first-period bidding function is increasing, means that the secret reserve price increases with the value of the object. The result is summarized in the following proposition.

Proposition 2 *In equilibrium, a seller of type v has a secret reserve price of $s(v) = b_1^*(\chi(v))$, where $v^*(\chi(v)) = v$, and where $s(v)$is an increasing function of v.*

Clearly, for Propositions 1 and 2 to be relevant, it must be that the bidding function employed by the buyers at $t = 1$ is increasing and that there are circumstances for which the value v^* is interior to the interval $[\underline{v}, \overline{v}]$. If no such v^* exists, then either the object will never be sold at $t = 1$, or it will always be sold at $t = 1$. In the former case there is essentially no auction at $t = 1$; in the latter case a secret reserve price is never used.

We turn, then, to the consideration of buyer bidding behavior at $t = 1$ and the circumstances under which an interior value of v^* exists. Recall that at the start of the first auction, each buyer receives a private signal X_i. Further, buyers know the (uninformative) announced reserve price, $r = \underline{v}$, and the prior distribution of the value of the object, $F(\cdot)$. To determine the bidding strategy for buyer i, we assume that all other bidders are using the bidding function $b_1^*(x_k)$, $k = 1, \ldots, n$; $k \neq i$, and that this bidding function is increasing. We later show that $b_1^*(\cdot)$ is indeed the equilibrium bidding strategy and that it is increasing in x_k.

3.2.3 First Period Bidding Behavior

To proceed, note that, for any given bid $b_{i,1}$ for buyer i, there are four relevant outcomes to be considered. If the seller awards the object to the highest bidder in the first auction, either bidder i has the highest bid, $b_{i,1} = b_1^1$, (outcome 1) or not (outcome 2). These are the usual outcomes of a one-shot auction. If the seller decides to re-auction in the second period, bidder i's expected rents only accrue to him in the second period. Given that the second period bidding behaviour is independent of the first period signals, bidder i's bid in the *first* period only affects his *second* period rents when his bid is used to shape the second period beliefs formed on the basis of the seller's re-sale strategy. The set of seller types that find it profitable to re-auction is a function of the second-highest first-period bid, b_1^2. Therefore, the other two relevant outcomes are that the seller decides to re-auction, and that bidder i has submitted the second highest bid, $b_{i,1} = b_1^2$, (outcome 3) or not (outcome 4).

If the seller decides to sell the object in the first period, bidder i only earns positive rents when he submits the highest bid (outcome 1). This outcome occurs when: first, buyer i's bid, $b_{i,1}$, exceeds the bid submitted by the buyer holding the highest signal among all buyers but i, $b_1^*(\hat{x}^1)$; and second, when the seller would not find it profitable to re-auction in the second period, $v < v^*(\hat{x}^1)$. Letting:

$$w_1(x_i, t, F_1(\cdot)) \equiv W_1(x_i, t, v^*) = E\left[V \mid X_i = x_i,\, \widehat{X}^1 = t,\, V \sim F_1(v)\right] \tag{3}$$

where $F_1(v) = \Pr(V \leq v \mid V \leq v^*) = \dfrac{F(v)}{F(v^*)}$, then i's expected rents can be written as follows:

$$E\left[\left(V - b_1^*(\widehat{X}^1)\right) I\{b_{i,1} > b_1^*(\widehat{X}^1),\, V \leq V^*(\widehat{X}^1)\} \mid X_i = x_i\right]$$

$$= \int_{x_i}^{(b_1^*)^{-1}(b_{i,1})} \left[W_1(x_i, t, v^*(t)) - b_1^*(t)\right] F_{V|X_i\widehat{X}^1}(v^*(t) \mid x_i, t)\, g_{\widehat{X}^1|X_i}(t \mid x_i)\, dt \tag{4}$$

These rents give the usual expression for rents in a one-shot common value auction when the beliefs of the buyer are $F_1(\cdot)$, but weighted by the probability that these rents are realized. This probability is the probability that $v \leq v^*(\hat{x}^1)$.

Should the seller decide to sell only upon re-auction, then bidder i only has positive rents if he submits the highest bid in the second period. These rents depend on bidder i's bid in the first period, $b_{i,1}$ whenever this bid influences the second period beliefs, $F_2(v) = \Pr(V \leq v \mid V \geq v^*((b_1^*)^{-1}(b_1^2)))$. If bidder i submits the second highest bid at $t = 1$, (outcome 3), then his bid determines directly the lower bound of the support of second-period beliefs about the value of the object. If i submits any other bid, this bid still influences expected rents if it constrains the possible values that the second highest bid could take. Such occurs if i submits the highest bid in the first period (which places an upper bound on b_1^2) or the third highest bid (which means that $b_1^2 \geq b_{i,1}$). These are the only two events from outcome 4, $b_{i,1} \neq b_1^2$, for which the expected rents will be a function of i's bid.

Letting Γ be the median of $\{\widehat{X}^1, \widehat{X}^2, ((b_1^*)^{-1}(b_{i,1}))\}$, the rents from outcomes 3 and 4 result from the following expectation:

$$\begin{aligned}
\mathbf{E} & \left[\left(V - W_2(\hat{Y}^1, \hat{Y}^1, v^*(\Gamma))\right) \mathbf{I}\{ b_{i,2} > W_2(\hat{Y}^1, \hat{Y}^1, v^*(\Gamma)),\, V \geq v^*(\Gamma) \} \mid X_i = x_i \right] \\
&= \mathbf{E} \left\{ \mathbf{E} \left[(V - W_2(\hat{Y}^1, \hat{Y}^1, v^*(\Gamma))) \right.\right. \\
&\qquad \left.\left. \cdot\, \mathbf{I}\{ b_{i,2} > W_2(\hat{Y}^1, \hat{Y}^1, v^*(\Gamma)),\, V \geq v^*(\Gamma) \} \mid \hat{Y}^1, Y_i, \widehat{X}^1, \widehat{X}^2 \right] \mid X_i = x_i \right\} \\
&= \mathbf{E} \left\{ \mathbf{E} \left[M(\hat{Y}^1, \Gamma) \mid \hat{Y}^1, Y_i, \widehat{X}^1, \widehat{X}^2 \right] \mid X_i = x_i \right\}
\end{aligned}$$

where:

$$M(\hat{Y}^1, \Gamma) = (V - W_2(\hat{Y}^1, \hat{Y}^1, v^*(\Gamma))) \cdot \mathbf{I}\{ b_{i,2} > W_2(\hat{Y}^1, \hat{Y}^1, v^*(\Gamma)),\, V \geq v^*(\Gamma) \}$$

The inside expectation above divides into three terms, each corresponding to one of three mutually exclusive events: the buyer submits the second-highest bid, the highest bid and the third-highest bid, respectively, in the first period:

$$\begin{aligned}
& \mathbf{E} \left[M(\hat{Y}^1, \Gamma) \cdot \mathbf{I}\{ \widehat{X}^1 > (b_1^*)^{-1}(b_{i,1}) > \widehat{X}^2 \} \mid \hat{Y}^1, Y_i, \widehat{X}^1, \widehat{X}^2 \right] \\
+ & \mathbf{E} \left[M(\hat{Y}^1, \Gamma) \cdot \mathbf{I}\{ (b_1^*)^{-1}(b_{i,1}) > \widehat{X}^1 > \widehat{X}^2 \} \mid \hat{Y}^1, Y_i, \widehat{X}^1, \widehat{X}^2 \right] \\
+ & \mathbf{E} \left[M(\hat{Y}^1, \Gamma) \cdot \mathbf{I}\{ \widehat{X}^1 > \widehat{X}^2 > (b_1^*)^{-1}(b_{i,1}) \} \mid \hat{Y}^1, Y_i, \widehat{X}^1, \widehat{X}^2 \right] \qquad (5)
\end{aligned}$$

Finally, let:

$$\begin{aligned}
& R_2(Y_i, \hat{Y}^1, v^*(\Gamma)) \\
& \quad = \left[W_2(Y_i, \hat{Y}^1, v^*(\Gamma)) - W_2(\hat{Y}^1, \hat{Y}^1, v^*(\Gamma)) \right] \cdot \left[1 - F_{V|Y_i \hat{Y}^1}(v^*(\Gamma) \mid \hat{Y}^1, Y_i) \right] \qquad (6)
\end{aligned}$$

Recalling that $W_2(y_k, y_k, v^*)$ is the equilibrium bidding function in the second period and that $W_2(y_i, \widehat{y}^1, v^*)$ is the expected value of the object for buyer i, given his signal y_i and the highest of the other signals $\widehat{y}^1$, this equation expresses the second period rents for bidder i, weighted by the probability that the value of the object is such that the seller decides to proceed to a second auction. Then, using $R_2(Y_i, \widehat{Y}^1, v^*(\Gamma))$ defined in (6) and equation (5), the expected rents for outcomes 3 and 4 can be expressed as follows.

$$
\begin{aligned}
&\int_{(b_1^*)^{-1}(b_{i,1})}^{\overline{x_i}} \int_{\underline{x}_i}^{(b_1^*)^{-1}(b_{i,1})} \int_{\underline{y}_i}^{\overline{y_i}} \int_{\underline{y}_i}^{(b_2^*)^{-1}(b_{i,2})} R_2\left(y_i, \widehat{y}^1, v^*((b_1^*)^{-1}(b_{i,1}))\right) \\
&\qquad \cdot\, g_{Y_i\widehat{Y}^1\widehat{X}^1\widehat{X}^2|X_i}(y_i, \widehat{y}^1, \widehat{x}^1, \widehat{x}^2 \mid x_i)\, d\widehat{y}^1\, dy_i\, d\widehat{x}^2\, d\widehat{x}^1 \\
&+ \int_{\underline{x}_i}^{(b_1^*)^{-1}(b_{i,1})} \int_{(b_1^*)^{-1}(b_{i,1})}^{\overline{x_i}} \int_{\underline{y}_i}^{\overline{y_i}} \int_{\underline{y}_i}^{(b_2^*)^{-1}(b_{i,2})} R_2(y_i, \widehat{y}^1, v^*(\widehat{x}^1)) \\
&\qquad \cdot\, g_{Y_i\widehat{Y}^1\widehat{X}^1\widehat{X}^2|X_i}(y_i, \widehat{y}^1, \widehat{x}^1, \widehat{x}^2 \mid x_i)\, d\widehat{y}^1\, dy_i\, d\widehat{x}^2\, d\widehat{x}^1 \\
&+ \int_{(b_1^*)^{-1}(b_{i,1})}^{\overline{x_i}} \int_{x^2}^{\overline{x_i}} \int_{\underline{y}_i}^{\overline{y_i}} \int_{\underline{y}_i}^{(b_2^*)^{-1}(b_{i,2})} R_2(y_i, \widehat{y}^1, v^*(\widehat{x}^1)) \\
&\qquad \cdot\, g_{Y_i\widehat{Y}^1\widehat{X}^1\widehat{X}^2|X_i}(y_i, \widehat{y}^1, \widehat{x}^1, \widehat{x}^2 \mid x_i)\, d\widehat{y}^1\, dy_i\, d\widehat{x}^1\, d\widehat{x}^2 \qquad (7)
\end{aligned}
$$

Total discounted expected rents for buyer i are given by the sum of (4) and (7), with the expression in (7) weighted by the factor δ/n. The first-period bidding function is determined by differentiation with respect to $b_{i,1}$ of total rents. Using that, in equilibrium, $(b_1^*)^{-1}(b_{i,1}) = x_i$, the first order condition gives the equilibrium bidding function as:

$$
\begin{aligned}
b_1^*(x_i) \quad = \quad & W_1(x_i, x_i, v^*(x_i)) \quad + \quad \left\{\frac{\delta}{n}\right\} \qquad (8) \\
& \cdot\, \frac{\int_{x_i}^{\overline{x_i}} \int_{\underline{x}_i}^{x_i} \int_{\underline{y}_i}^{\overline{y_i}} \int_{\underline{y}_i}^{(b_2^*)^{-1}(b_{2,i})} \frac{\partial R_2(y_i, \widehat{y}^1, v^*(x_i))}{\partial v^*} \frac{\partial v^*}{\partial x_i}\, g_{Y_iY^1\widehat{X}^1\widehat{X}^2|V}(y_i, \widehat{y}^1, \widehat{x}^1, \widehat{x}^2 \mid v)\, d\widehat{y}^1 dy_i d\widehat{x}^2 d\widehat{x}^1}{F_{V|X_i\widehat{X}^1}(v^*(x_i) \mid x_i, x_i)\, g_{\widehat{X}^1|X_i}(x_i \mid x_i)}
\end{aligned}
$$

It can be shown that, if v^* is interior, the expression $\frac{\partial R_2}{\partial v^*} < 0$, implying that $b_1^*(x_i) < W_1(x_i, x_i, v^*)$, the equilibrium bid in a one-shot auction with V distributed on the interval $[\underline{v}, v^*(x_i)]$ according to the function $F_1(\cdot)$. That is, when a secret reserve price is used, bidders shade their bid below the value they would bid were they certain that an object with a value of at most v^* were to be awarded. This shading of bids with respect to the relevant one-shot context results from the fact that, while raising the first-period bid creates information regarding the value of the object should the auction proceed to a second period, this information is public and so cannot be exploited by bidder i. In essence, the extra information causes all other buyers to bid higher in the second-period, thereby dissipating any rents i might obtain from the improved information in the second period.

Note also that first-period bidding reflects two winner's curses. As in a standard common-value auction, being awarded the object means that the bidder received the highest signal and thus held the most optimistic estimate about the value of the object. In addition, winning at this auction means that the seller did not find it profitable to wait to re-auction the object. This fact implies that the signal obtained by the second highest bidder exceeded $\chi(v)$ and that the highest bidder's signal was even higher. Thus, the buyer information was sufficiently optimistic that it was in the seller's interest to forgo the opportunity of additional buyer learning and the signaling value of refusing to sell. In constructing a first-period bid, the buyer must take account of this additional information contained in winning the object.

Finally, one can check that the equilibrium bidding function implied by (8) is an increasing function. To do so, one need only substitute into (8) the value implied by the seller's first-order condition for the expression $\dfrac{\partial v^*}{\partial x_i}$. This substitution yields an ordinary differential equation defining $b_1^*(x_i)$. The solution for the bidding function may take any one of three possible forms. In one, the solution is such that $v^* = \overline{v}$ for all possible values of x_i. This outcome would occur, for instance, were δ very close to 0 so that it never paid the seller to re-auction at $t = 2$. In this case, $b_i^*(x_i) = W_1(x_i, x_i, \overline{v})$ and so the bidding function is increasing in x_i. A second possible solution results in $v^* = \underline{v}$ for all possible x_i. This outcome would occur were $\delta = 1$. In this case, $b_1^*(x_i) = \underline{v}$ and so the bidding function is weakly increasing in x_i. The final possibility is that the solution results in a bidding function such that, for some values of x_i, v^* is interior to the interval $[\underline{v}, \overline{v}]$. In this case, it can be shown that $b_1^{*\prime}(x_i) > 0$ for all interior v^*. This fact, in conjunction with the fact that v^* is an increasing function of b, implies that the bidding function is increasing for all x_i greater than the smallest value of x_i such that v^* is interior.[6] For all x_i less than this value, $b_i^*(x_i) = \underline{v}$ while for all x_i greater than this value $b_1^*(x_i) \geq \underline{v}$ and increasing in x_i. Thus, in all cases, $b_1^*(x_i)$ is an increasing function. These results are summarized below.

Proposition 3 *If $v^* > \underline{v}$, then the equilibrium first-period bidding function $b_1^*(y_i)$ is strictly increasing. The bidding function is increasing at when $v^* = \underline{v}$.*

To this point, we have been assuming that the seller employs a secret reserve price policy. For such to be the case, not only must the first-period bidding function be increasing, but v^* must be strictly interior to the interval $[\underline{v}, \overline{v}]$ (see Proposition 2). As the above discussion

[6] Since v^* is an increasing function of b, either v^* will remain interior for larger values of x_i, in which case $b_1^{*\prime} > 0$, or $v^* = \overline{v}$, in which case $b_1^*(x_i) = W_1(x_i, x_i, v^*)$. Moreover, once $v^* = \overline{v}$, it will remain there for all larger x_i. Thus, the sets of signals for which v^* is either interior or equal to $\overline{v}$ are each connected sets and the union is a connected set.

indicates, this latter condition is not always satisfied. In particular, when δ is either very large or very small, v^* will be equal either to $\underline{v}$ or $\overline{v}$ respectively. In the former case, no sales will ever be observed at $t = 1$ while in the latter no re-auctioning will ever occur. It is also the case that an equilibrium in which $v^* = \overline{v}$ can be supported even when δ is not small by the appropriate specification of beliefs. In particular, such an equilibrium can be supported by beliefs requiring that any refusal to sell at $t = 1$ signals a value of $\underline{v}$ with probability 1. With these beliefs, it would never pay the seller to refuse to sell at $t = 1$ and so the equilibrium is supported.

If one rules out such beliefs and imposes, instead, that beliefs when $v^* = \overline{v}$ are the limits of beliefs as v^* approaches $\overline{v}$, then one can provide conditions under which v^* lies strictly between $\underline{v}$ and $\overline{v}$: the seller adopts a secret reserve price policy. This condition is a joint condition on δ and x_i, specifying features of the signal-generating process and time to re-auction such that the seller uses a secret reserve price. The condition is given below in Lemma 3.

Lemma 3 *For any $x^2 \in [\underline{x_i}, \overline{x_i}]$ such that:*

$$\frac{W_1(x^2, x^2, \overline{v})}{\overline{v}} \quad < \quad \delta \quad < \quad \frac{\underline{v}}{\int_{y^2} W_2(y^2, y^2, \underline{v}) g_{Y^2|V}(y^2 \mid v)\, dy^2}$$

then $v^* \in (\underline{v}, \overline{v})$ and the seller uses a secret reserve price policy.

Clearly, $\delta = 0$ or 1 violates the above condition and so, as noted previously, a secret reserve price policy is not employed. For intermediate values of δ, the condition is likely satisfied if the second-period signal generating process is very informative for low values of the object. In this case, expected second-period revenues for a seller with object of value $\underline{v}$ (the denominator of the upper limit of δ) will be close to $\underline{v}$ and so the upper limit will be close to 1. Intuitively, if the second-period signal is informative for low values of v, it is in the seller's interest, when v is low, to sell in the first period when there is less information rather than waiting. This means that delay will be a useful way of signaling for sellers with objects of higher value and so they will employ a secret reserve price (being unable to signal through the announced reserve price).

4 Implications of Secret Reserve Price Policies

What are the implications for observed auction behavior when the seller employs a secret reserve price policy? First, note that in such equilibria buyers bid at most the estimate of the value of the object in any period. In the initial auction this value could not possibly exceed v^* if the bid is accepted; in the re-auction this value must surely exceed v^*. It follows, therefore, that if there are two auctions in equilibrium, the bids in the second auction must exceed the ones in the first. It is worth noting that this result is consistent with the observations of Hendricks, Porter and Spady (1988) and Porter (1995) for the sealed-bid auctions of drainage and development leases on federal lands in the U.S..

Second, consider what happens if δ is decreased. As there is no natural interpretation of a period length, a reduction in δ can be interpreted as an increase in delay between the initial auction and the re-auction. This increased delay might be due to an increase in time until buyers learn additional information about the object. Alternatively, one might think of this delay as having been the result of a change in re-auction policy by the seller.[7] In any event, Proposition 1 implies that an increase in delay to re-auction (a decrease in δ) results in an increase in the value of v^* for any given bid, $b_1^*(x^2)$. The implication of this is that, for any given (potential) selling price, the object is more likely to be sold at the intial auction as time to re-sale increases. Further, if the object is not sold, the sale price will, on average, be higher the longer the delay to re-auctiuon (the lower is δ).

More interestingly, the same is true when one conditions on first-period *signal* rather than the *bid.* That is, even allowing the first-period *bidding function* to adjust to a decrease in δ, v^* increases for any given signal (the function $v^*(x^2)$ decreases with δ). This point can easily be illustrated via a diagram. Consider Figure 1. There, the locus labelled **S** gives first-period bids and associated values of v^*, the seller type just indifferent between accepting the given bid and rejecting it. From Proposition 1, this locus is upward sloping. The locus labelled **FOC** gives the first-period bid and v^* pairs that satisfy the first-order condition (8). In the neighborhood of equilibrium, this locus may be either downward sloping, as in Figure 1A, or upward sloping, as in Figure 1B; if it is upward sloping, however, second-order conditions imply that it is less steep than the locus **S**. The intersection of these two loci gives the equilibrium values of v^* and first-period bid for any given signal. Now, consider the impact of a decrease in δ. From equation (A4), the locus **FOC** is independent of δ, while, from above, the locus **S** shifts right as δ decreases. The result is that, regardless of the slope of the locus **FOC**, v^* increases. Thus, for any given first-period signal, an increase in length

[7] Under this interpretation, the change in policy must not be related to the seller's information regarding the value of the object. If it were, then buyers would want to draw inferences about the seller's information from the delay to re-auction. This latter problem is a subject of on-going research.

of time to re-auction leads to an (expected) increase in second-period sale prices.

This feature of the equilibrium provides a potential means of testing this model relative to a search model of auction behavior. In particular, consider an alternative interpretation of observed seller behavior involving a private values model in which the seller's refusal to sell is simply the result of his searching for a new set of buyers (new draw of private values). Under both this interpretation and our model, increased delay in time to re-auction would result in more intial sales. However, under the search interpretation there should be no correlation between exogenous increases in delay and ultimate sale price. There should also be no correlation between number of auction attempts and sale price either. Under our model sale price should be positively correlated with both delay time and number of re-auctions.

The model also has implications for observed bids and values of (initially) unsold objects. Porter notes that, in the case of oil leases, the "mean high bid on tracts that were sold ($6.07 million) is more than seven times that on tracts with rejected high bids ($0.82 million)". From these data Porter concludes that the unsold tracts were "marginal tracts". These observations actually accord with our model. In particular, Proposition 1 implies that low initial bids are typically rejected while high initial bids are typically accepted. Further, when low initial bids are accepted, they are accepted by seller's having little to gain, either through buyer learning or signaling, from re-auction. Finally, since the bidding function is increasing in the signal, low bids are associated with low signals and these are more likely when the value of the object is low. In total, then, the model predicts that i) accepted intial bids will be high on average, ii) rejected intial bids will be low and iii) the sellers rejecting intial bids are likely to have lower valued objects (low signals are more likely when v is low) but still objects of such value that the increase in price resulting from re-auctioning makes this decision profitable. This last implication suggests that, upon re-auction, price should rise significantly but, because the intrinsic value of the re-auctioned object is still low, not to average levels of objects sold intially (where the intrinsic value is, on average, high). Based on our model, then, it is not surprising that the high bid on rejected tracts was quite low and that, even after re-sale, the high bid is low relative to that on tracts sold intially (bids on re-auctioned tracts increasing approximately 2.5 times). This outcome is what would be expected given the selection process that is generating the data.

5 Conclusion

This paper has provided a model to explain a seller's use of a secret reserve price policy in a common-value auction. Because the seller holds information on the value of the object to be sold that cannot be directly and credibly transmitted to the buyers, he sets an announced reserve price that, in equilibrium, may prove to be too low. As a consequence, it may be in the seller's interest to refuse to sell to the highest bidder at the auction and wait to re-sell the object later. In the presence of buyer learning about the object's value, refusal to sell becomes a means by which the seller can signal his information to the buyers. The result is that the seller adopts a secret reserve price policy. The model makes several predictions about the pattern of bids at initial and re-sale auctions and these predictions seem to accord well with observed bids in offshore oil and gas leases.

6 Appendix

This appendix contains proofs of all lemmas and propositions in the text.

Proof of Lemma 1. If $r(v)$ is strictly monotonic, then there exists an inverse mapping $v(r)$ which is also strictly monotonic. If the announced reserve price schedule $r(v)$ were an equilibrium strategy, then upon observing a reserve price r, buyers would infer that the seller is of type $v(r)$ with probability one. Each buyer would submit a bid equal to $v(r)$ and a seller of type v would obtain sales revenue of v. In particular, the announced reserve price strategy under consideration requires a seller of type $\tilde{v} \in [\underline{v}, \overline{v})$ to announce $r(\tilde{v})$. However, the seller can profitably deviate by announcing $r(\overline{v})$ since he would then obtain sales revenue of $\overline{v} > \tilde{v}$. **QED.**

Proof of Lemma 2. Suppose that all buyers but $i = 1$ use the bidding function $b_2^*(y_k)$. This function has buyer k's second-period signal y_k $(k = 2, \ldots, n)$ as its only argument. The reserve price $r = \underline{v}$ is not a binding lower bound on the range of possible bids, and it does not convey any information to the buyers, so it can be omitted without loss of generality. The vector of first-period signals, $x = (x_1, \ldots x_n)$, is also omitted. We will show that this implies that the expected rents for buyer 1 are independent of x. Therefore, buyer 1's optimal bid is independent of x also, and the assumption that the bidding function $b_2^*(y_k)$ is independent of x will be verified in equilibrium.

The expected rents for buyer 1 are:

$$E\left[\left(V - b_2^*(\hat{Y}^1)\right)\, I\left\{b_{1,2} > b_2^*(\hat{Y}^1)\right\} \quad \middle| \quad Y_1 = y_1, X = x\right]$$

where $\widehat{Y}^1$ designates the first order statistic of the sample of second-period signals for all buyers excluding buyer 1, and $b_{1,2}$ is buyer 1's bid at $t = 2$. Given the information conveyed by the seller's decision to re-auction, buyers update the prior distribution on V to $F_2(v) = \dfrac{F(v) - F(v^*)}{1 - F(v^*)}$ (see Proposition 1 for details). The joint density of the value and the signals from both periods will be updated to $\widetilde{h}_{VXY}(V, X, Y) = \frac{1}{1 - F(v^*)} h_{VXY}(VXY)$. This implies that the updated density $\widetilde{h}_{VXY}(V, X, Y)$ still has the multiplicative form $\widetilde{\alpha}(V, Y)\beta(X|Y_i)$ for $i = 1, \dots, n$ as a consequence of the garbling assumption. This implies that buyer 1's expected rents can be written as follows:

$$\int_{v^L}^{v^H} \int_{\underline{y}_i}^{((b_2^*)^{-1})(b_{1,2})} (v - b_2^*(t))\, \widetilde{h}_{V\widehat{Y}^1|XY_1}(v, t \mid x_1, \dots, x_n, y_1)\; dt dv$$

For these rents to be independent of x, it is sufficient to show that:

$$\widetilde{h}_{V\widehat{Y}^1|XY_1}(v, \widehat{y}^1 \mid x_1, \dots, x_n, y_1) = \widetilde{h}_{V\widehat{Y}^1|Y_1}(v, \widehat{y}^1 \mid y_1)$$

where:

$$\widetilde{h}_{V\widehat{Y}^1|XY_1}(v, \widehat{y}^1 \mid x_1, \dots, x_n, y_1) = \frac{\widetilde{h}_{V\widehat{Y}^1XY_1}(v, \widehat{y}^1, x_1, \dots, x_n, y_1)}{\widetilde{g}_{XY_1}(x_1, \dots, x_n, y_1)}$$

Given that

$$\widetilde{h}_{VXY}(v, x_1, \dots, x_n, y_1, \dots, y_n) = \widetilde{\alpha}(v, y_1, \dots, y_n)\beta(x_1, \dots, x_n \mid y_1) \qquad (v, x, y) \in \Sigma$$

then:

$$\widetilde{h}_{VXY\widehat{Y}^1}(v, x, y, \widehat{y}^1) = \begin{cases} \widetilde{\alpha}(v, y)\beta(x \mid y_1) & \text{when } (v, x, y) \in \Sigma \text{ and } \widehat{y}^1 = \max_{j \neq 1} y_j \\ 0, & \text{otherwise.} \end{cases}$$

which implies that:

$$\widetilde{h}_{V\widehat{Y}^1|XY_1}(v, \widehat{y}^1 \mid x, y_1) = \frac{\beta(x \mid y_1)\widetilde{\alpha}(v, y)\, I\{\widehat{y}^1 = \max_{j \neq 1} y_j\}}{\beta(x \mid y_1) \underset{\substack{v, y_2 \dots y_n \in \Sigma \\ \widehat{y}^1 = \max_{j \neq 1} y_j}}{\int \cdots \int} \widetilde{\alpha}(v, y)\; dy_n \dots dy_2 dv}$$

$$\tilde{h}_{V\hat{Y}^1|Y_1}(v,\hat{y}^1 \mid y_1) = \frac{\left[\int_{x_1 \ldots x_n} \cdots \int \beta(x \mid y_1)\, dx_n \ldots dx_1\right] \tilde{\alpha}(v,y) I\{\hat{y}^1 = \max_{j \neq 1} y_j\}}{\left[\int_{x_1 \ldots x_n} \cdots \int \beta(x \mid y_1)\, dx_n \ldots dx_1\right] \int \cdots \int_{\substack{v,y_2 \ldots y_n \in \Sigma \\ \hat{y}^1 = \max_{j\neq 1} y_j}} \tilde{\alpha}(v,y)\, dy_n \ldots dy_2 dv}$$

QED

Proof of Proposition 1 Let $Q \subseteq [\underline{v}, \overline{v}]$ be the set of types indifferent between accepting and rejecting b_1^2:

$$v \in Q \quad \Leftrightarrow \quad v \in [\underline{v}, \overline{v}] \text{ and } b_1^2 = \delta \int_{y^2} w_2(y^2, y^2, F_2(\cdot)) g_{Y^2|V}(y^2 \mid v) dy^2 \tag{A1}$$

If $Q \neq \emptyset$, then Q is a singleton. Indeed, if we suppose instead that $\hat{v} \in Q$ and $\tilde{v} \in Q$, and, without loss of generality that $\hat{v} > \tilde{v}$, then this leads to the following contradiction:

$$\begin{aligned} b_1^2 &= \delta \int_{y^2} w_2(y^2, y^2, F_2(\cdot)) g_{Y^2|V}(y^2 \mid \hat{v}) dy^2 \\ &> \delta \int_{y^2} w_2(y^2, y^2, F_2(\cdot)) g_{Y^2|V}(y^2 \mid \tilde{v}) dy^2 \end{aligned}$$

The inequality follows immediately from the fact that $w_2(y^2, y^2, F_2(\cdot))$ is increasing in its first argument, and from the fact that whenever the family of densities $\{g_{Y^2|V}(\cdot \mid v)\}$ satisfies the strict monotone likelihood ratio property, it is also true that $G_{Y^2|V}(\cdot \mid \hat{v})$ dominates $G_{Y^2|V}(\cdot \mid \tilde{v})$ in the sense of first order stochastic dominance (see Milgrom (1981)).[8]

Thus, if $Q \neq \emptyset$, there is just one type for which (A1) is satisfied. This implies that (A1) implicitly defines the seller type who is indifferent between accepting and rejecting the first-period bid as a function of the first-period bid, $v^*(b_1^2)$. If the first-period bidding function is increasing, then the same equation also defines $v^*(x^2) = v^*((b_1^*)^{-1}(b_1^2))$. The reformulation of the first order conditions is then immediate. **QED**

[8] We also assume that the range of y^2 is independent of the realization of V.

Proof of Proposition 2 Given the acceptance rule of the seller, the second-period beliefs of the buyers are completely characterized by the priors $F(\cdot)$ and the range of possible values of V. Suppose that second-period beliefs are defined on the range $[v_B, \overline{v}]$ and let v_T be the seller's true type. We can now define:

$$K(y^2, F_2(\cdot)) \equiv J(v_B, v_T)$$

$$W_2(a, b, v_B) \equiv w_2(a, b, F_2(\cdot))$$

Let $\widehat{v} > \widetilde{v}$. Then

$$J(\widehat{v}, \widehat{v}) = \int_{y^2} W_2(y^2, y^2, \widehat{v}) g_{Y^2|V}(y^2 \mid \widehat{v})\, dy^2$$
$$> J(\widehat{v}, \widetilde{v}) = \int_{y^2} W_2(y^2, y^2, \widehat{v}) g_{Y^2|V}(y^2 \mid \widetilde{v})\, dy^2$$
$$> J(\widetilde{v}, \widetilde{v}) = \int{}^2_y W_2(y^2, y^2, \widetilde{v}) g_{Y^2|V}(y^2 \mid \widetilde{v}) dy^2$$

where the first inequality follows from stochastic dominance, and the second follows from the fact that, straightforwardly, the second-period price, $W_2(y^2, y^2, v_B)$, is increasing in v_B. These inequalities have two implications. First, $J(v_B, v_T)$ is increasing in both its arguments. Second, given the equilibrium second-period beliefs for the buyers, v^* is the solution to:

$$b_1^2(x^2) \quad - \quad \delta\, J(v^*, v^*) = 0 \tag{4}$$

and (4) implicitly defines the function $v^*(\cdot)$. Assuming differentiability of the required functions yields:

$$\frac{\partial v^*}{\partial x^2} \quad = \quad - \left[\frac{\dfrac{\partial b_1^*}{\partial x_i}}{-\delta \left(\dfrac{\partial}{\partial v^*} J(v^*, v^*) \right)} \right] \quad > \quad 0$$

This implies that $\frac{\partial \chi}{\partial v} > 0$. Since $\frac{\partial b_1^*}{\partial x_i} > 0$, then $\frac{\partial s}{\partial v} > 0$. **QED.**

Proof of Proposition 3

The expression for the rents can be derived in the following way. First, consider the rents resulting from outcomes 3 and 4. To see how (7) is obtained from (5), consider the first term of (5).

$$E\ \Big[(V - W_2(\widehat{Y}^1, \widehat{Y}^1, v^*((b_1^*)^{-1}(b_{i,1}))))$$
$$\cdot\ I\{\, b_{i,2} > W_2(\widehat{Y}^1, \widehat{Y}^1, v^*((b_1^*)^{-1}(b_{i,1}))),\ V \geq v^*((b_1^*)^{-1}(b_{i,1})), \}$$
$$\cdot\ I\{\widehat{X}^1 > (b_1^*)^{-1}(b_{i,1}) > \widehat{X}^2, \}\ \mid\ \widehat{Y}^1, Y_i, \widehat{X}^1, \widehat{X}^2\Big]$$

$$= \int\int\int\int \Big[\int_{v^*(b_1^{*-1}(b_{i,1}))}^{\overline{v}} [v - W_2(\widehat{y}^1, \widehat{y}^1, v^*(b_1^{*-1}(b_{i,1}))]\, f_{V|Y_i\widehat{Y}^1\widehat{X}^1\widehat{X}^2}(v \mid y_i, \widehat{y}^1, \widehat{x}^1, \widehat{x}^2)\, dv\Big]$$
$$\cdot\ g_{Y_i\widehat{Y}^1\widehat{X}^1\widehat{X}^2|X_i}(y_i, \widehat{y}^1, \widehat{x}^1, \widehat{x}^2 \mid x_i)\, d\widehat{y}^1\, dy_i\, d\widehat{x}^2\, d\widehat{x}^1 \qquad \text{(A2)}$$

where the bounds on the first four integrals are omitted for notational convenience. As a consequence of the "garbling" assumption and of the definition of $W_2(\cdot\,,\,\cdot\,, v^*)$, it is true that:

$$\int_{v^*(b_1^{*-1}(b_{i,1}))}^{\overline{v}} [v - W_2(\widehat{y}^1, \widehat{y}^1, v^*(b_1^{*-1}(b_{i,1}))]\, f_{V|Y_i\widehat{Y}^1\widehat{X}^1\widehat{X}^2}(v \mid y_i, \widehat{y}^1, \widehat{x}^1, \widehat{x}^2)\, dv$$

$$= [1 - F_{V|Y_i\widehat{Y}^1}(v^*(b_1^{*-1}(b_{i,1})) \mid y_i, \widehat{y}^1)] \qquad \text{(A3)}$$
$$\cdot\ \int_{v^*(b_1^{*-1}(b_{i,1}))}^{\overline{v}} [v - W_2(\widehat{y}^1, \widehat{y}^1, v^*(b_1^{*-1}(b_{i,1})))] \frac{f_{V|Y_i\widehat{Y}^1}(v \mid y_i, \widehat{y}^1)}{1 - F_{V|Y_i\widehat{Y}^1}(v^*(b_1^{*-1}(b_{i,1})) \mid y_i, \widehat{y}^1)}\, dv$$

Replacing (A3) in (A2) yields the first term in (7). The other terms of the expression for the rents for outcomes 3 and 4 can be obtained in a similar way.

The total rents for bidder i are then given by the sum of the expression in (4) and the expression in (7), where the latter is discounted by δ and multiplied by $\frac{1}{n}$. Assuming first that there is an interior solution for v^*, the first order condition for the problem, together with the equilibrium condition $(b_1^*)^{-1}(b_{i,1}) = x_i$, yields (8) after re-arranging terms. Substitution into (8) of:

$$\frac{\partial v^*}{\partial x^2} = -\left[\frac{\frac{\partial b_1^*}{\partial x_i}}{-\delta\left(\frac{\partial}{\partial v^*}J(v^*,v^*)\right)}\right]$$

yields, after re-arranging terms:

$$\frac{\partial b_1^*(x_i)}{\partial x_i} = [\, b_1^*(x_i) - W_1(x_i, x_i, v^*(x_i))\,]$$

$$\cdot \frac{n\, g_{\widehat{X}^1|X_i}(x_i \mid x_i)\, F_{V|X_i\widehat{X}^1}(v^*(x_i) \mid x_i, x_i)\, J(v^*(x_i), v^*(x_i))}{\int\int\int\int \left[\frac{\partial}{\partial v^*}\left(R_2(y_i, \widehat{y}^1, v^*(x_i))\right)\right] g_{Y_i\widehat{Y}^1\widehat{X}^1\widehat{X}^2|X_i}(y_i, \widehat{y}^1, \widehat{x}^1, \widehat{x}^2 \mid x_i)\, d\widehat{y}^1 dy_i d\widehat{x^1} d\widehat{x^2}} \qquad \text{(A4)}$$

To show that the bidding function is increasing in the case where v^* is interior, we will show that $\frac{\partial R_2}{\partial v^*} < 0$. From (8), this implies that $[b_1^*(x_i) - W_1(x_i, x_i, v^*(x_i))] < 0$ and the result then follows immediately from (A4).

Claim $\quad \frac{\partial R_2(y_i, \widehat{y}^1, v^*)}{\partial v^*} < 0$

Proof The distribution and density in what follows are $F_{V|Y_i\widehat{Y}^1}(\cdot \mid \cdot,\cdot)$ and $f_{V|Y_i\widehat{Y}^1}(\cdot \mid \cdot,\cdot)$, respectively. The subscripts are omitted below for notational convenience.

$$\frac{\partial}{\partial v^*}\left\{\left[\frac{\int_{v^*}^{\overline{v}} v f(v \mid y_i, z) dv}{1 - F(v^* \mid y_i, z)} - \frac{\int_{v^*}^{\overline{v}} v f(v \mid z, z) dv}{1 - F(v^* \mid z, z)}\right][1 - F(v^* \mid y_i, z)]\right\}$$

$$= -v^* f(v^* \mid y_i, z) + v^* f(v^* \mid z, z)\frac{1 - F(v^* \mid y_i, z)}{1 - F(v^* \mid z, z)}$$

$$- \left[\int_{v^*}^{\overline{v}} v f(v^* \mid z, z) dv\right] \cdot \left\{\frac{-f(v^* \mid y_i, z)[1 - F(v^* \mid z, z)] + f(v^* \mid z, z)[1 - F(v^* \mid y_i, z)]}{[1 - F(v^* \mid z, z)]^2}\right\}$$

$$= [1 - F(v^* \mid y_i, z)]\,(v^* - W_2(z, z, v^*)) \cdot \left\{ \frac{f(v^* \mid z, z)}{1 - F(v^* \mid z, z)} - \frac{f(v^* \mid y_i, z)}{1 - F(v^* \mid y_i, z)} \right\}$$

Since:

$$W_2(z, z, v^*) > v^* \qquad \text{and} \qquad 1 - F(v^* \mid y_i, z) > 0$$

Then,

$$\frac{\partial R_2}{\partial v^*} < 0$$

$$\iff \frac{f(v^* \mid z, z)}{1 - F(v^* \mid z, z)} = f_{V|Y_i\widehat{Y}^1}(v \mid z, z, V \geq v^*) > \frac{f(v^* \mid y_i, z)}{1 - F(v^* \mid y_i, z)} = f_{V|Y_i\widehat{Y}^1}(v \mid y_i, z, V \geq v^*)$$

It is easy to show that, given that the family of densities $\{g_{Y_i|V}(\cdot \mid v)\}$ satisfy MLRP, so does the family of densities $\{f_{V|Y_i\widehat{Y}^1}(\cdot \mid y_i, \widehat{y}^1, V \geq v^*)\}$, which we denote as $f^*(\cdot \mid y_i, y^1)$ in what follows. MLRP means that for any $(v_1, v_2, s_1, s_2, t_1, t_2)$ such that $\overline{v} \geq v_1 > v_2 \geq$, $(s_1, t_1) \geq (s_2, t_2)$ and either $s_1 > s_2$ or $t_1 > t_2$:

$$\frac{f^*(v_1 \mid s_1, t_1)}{f^*(v_1 \mid s_2, t_2)} > \frac{f^*(v_2 \mid s_1, t_1)}{f^*(v_2 \mid s_2, t_2)}$$

$$\Rightarrow \frac{f^*(v \mid s_1, t_1)}{f^*(v \mid s_2, t_2)} > \frac{f^*(v^* \mid s_1, t_1)}{f^*(v^* \mid s_2, t_2)} \qquad \forall\, v \in (v^*, \overline{v}]$$

$$\Rightarrow \frac{f^*(v \mid z, z)}{f^*(v \mid y_i, z)} > \frac{f^*(v^* \mid z, z)}{f^*(v^* \mid y_i, z)} \qquad \forall\, v \in (v^*, \overline{v}]$$

$$\Rightarrow \frac{f^*(v^* \mid z, z)}{f^*(v^* \mid y_i, z)} < 1$$

otherwise, we would have:

$$\frac{f^*(v \mid z, z)}{f^*(v \mid y_i, z)} > \frac{f^*(v^* \mid z, z)}{f^*(v^* \mid y_i, z)} > 1 \qquad \forall\, v \in (v^*, \overline{v}]$$

but this leads to an immediate contradiction since:

$$\int_{v^*}^{\overline{v}} f^*(v \mid z, z)\, dv = \int_{v^*}^{\overline{v}} f^*(v \mid y_i, z)\, dv = 1$$

So the claim is established, and in the case where the solution to v^* is interior, the bidding function is strictly increasing. There are two other cases to consider. If $v^*(x^2) \geq \overline{v}$ for all x^2, then the seller never proceeds to a second auction. In that case, the bidders are faced with a one-shot auction in the first period, with beliefs $F(\cdot)$ and $v \in [\underline{v}, \overline{v}]$. In the usual way, the equilibrium bidding function would be $W_1(x_i, x_i, \overline{v})$, which is strictly increasing in its first argument since $\{g_{X_i|V}(\cdot \mid v)\}$ satisfy MLRP. If $v^*(x^2) \leq \underline{v}$ for all x^2, then all seller types prefer to wait for buyers to acquire additional information before auctioning the object. In that case, it is a best reply for buyers to all bid $\underline{v} = r$ regardless of the signal that each obtained. The bidding function is then weakly increasing. **QED**

Proof of Lemma 3

Suppose that $v^*(x^2) \leq \underline{v}$ for all $x^2 \in [\underline{x_i}, \overline{x_i}]$. If this were an equilibrium, then all seller types would reject the first-period bid, buyers would use $b_1^*(x_i) = \underline{v}$ in the first period (see proposition 3) and they would use $b_2^*(y_i) = W_2(y_i, y_i, \underline{v})$ in the second period which is the bidding function for the one-shot auction. This implies that a seller of type $\underline{v}$ can expect a revenue of:

$$\sigma_2(\underline{v}) = \int_{\underline{y_i}}^{\overline{y_i}} W_2(y^2, y^2, \underline{v}) g_{Y^2|V}(y^2 \mid \underline{v})\, dy^2$$

For the proposed outcome to be an equilibrium, it would have to be that:

$$\underline{v} \leq \delta\sigma(\underline{v})$$

otherwise, a seller of type $\underline{v}$ (and, as a consequence of MLRP, all other seller types as well) would accept the first period bid. Therefore, if $\underline{v} > \delta\sigma(\underline{v})$ for some $\delta \in [0, 1]$, then it is not possible for $v^*(x^2) \leq \underline{v}$ for all $x^2 \in [\underline{x_i}, \overline{x_i}]$ in equilibrium.

Suppose next that $v^*(x^2) \geq \overline{v}$ for all $x^2 \in [\underline{x_i}, \overline{x_i}]$. If this were an equilibrium, then all seller types would accept the first-period bid, buyers would use $b_1^*(x_i) = W_1(x_i, x_i, \overline{v})$ (the one-shot auction bidding function) in the first period (see proposition 3) and, should there be a second period, they would use $b_2^*(y_i) = \overline{v}$ as their bidding function. This implies that a seller of type $\overline{v}$ has a revenue of $W_1(x^2, x^2, \overline{v})$ in the first period. For the proposed outcome to be an equilibrium, it must be true that:

$$W_1(x^2, x^2, \overline{v}) \quad \geq \quad \delta \overline{v}$$

Otherwise, it would pay a seller of type $\overline{v}$ (and thus all other sellers) to re-auction the object in the second period. Therefore, if there exists some $(x^2, \delta) \in [\underline{x_i}, \overline{x_i}] \times [0, 1]$ such that $W_1(x^2, x^2, \overline{v}) < \delta \overline{v}$, then it is not possible for $v^*(x^2) \geq \overline{v}$ for all $x^2 \in [\underline{x_i}, \overline{x_i}]$ in equilibrium.

Therefore, whenever:

$$\frac{W_1(x^2, x^2, \overline{v})}{\overline{v}} \quad < \quad \delta \quad < \quad \frac{\underline{v}}{\int_{y^2} W_2(y^2, y^2, \underline{v}) g_{Y^2|V}(y^2 \mid v)\, dy^2}$$

is satisfied for some $x^2 \in [\underline{x_i}, \overline{x_i}]$, then v^* will be interior. **QED**

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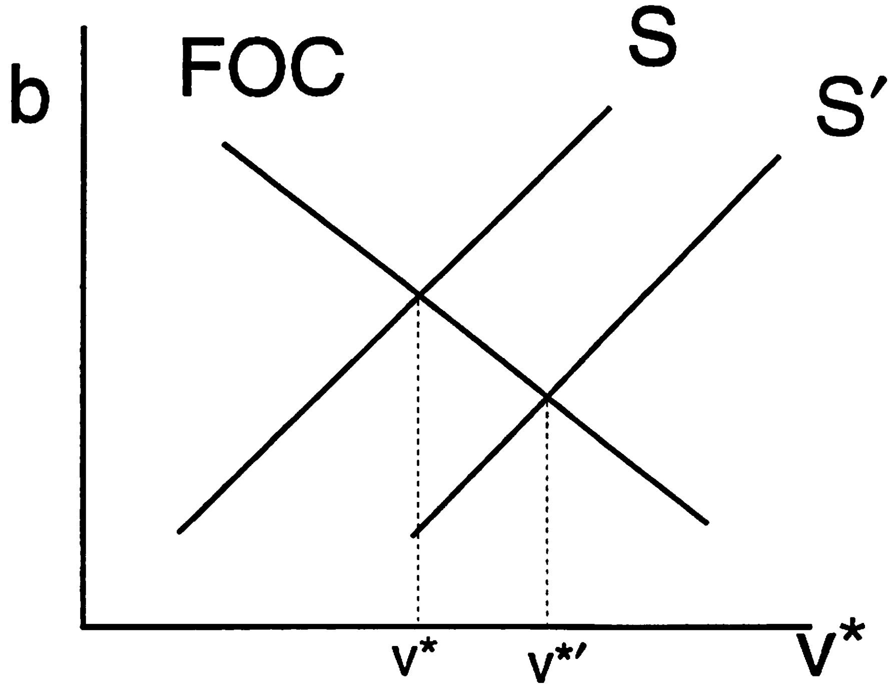

Figure 1A

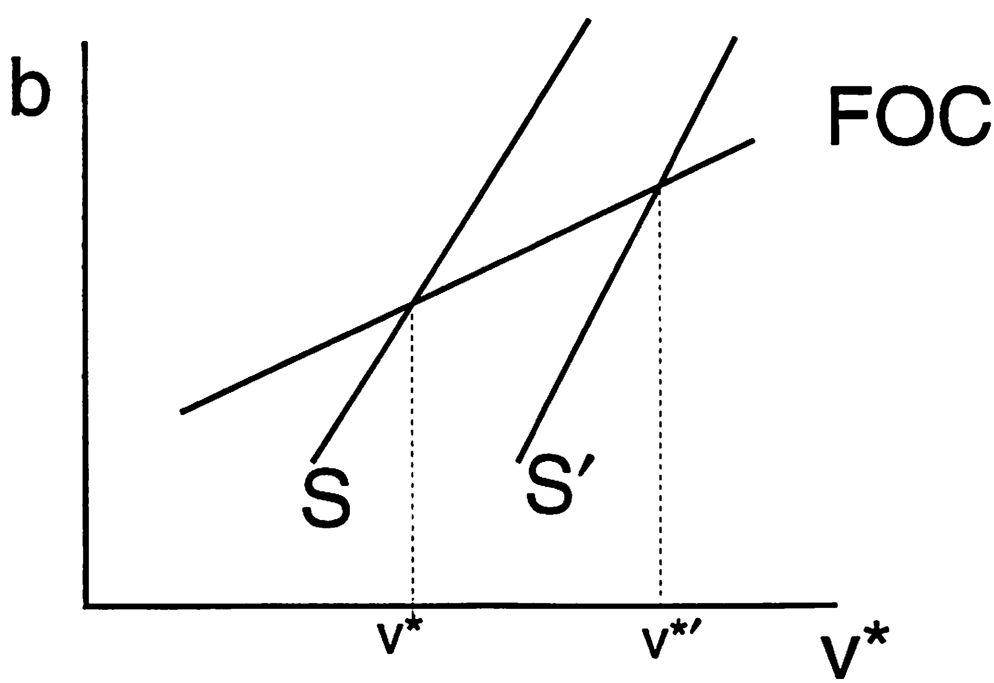

Figure 1B