1982

A Costing Model for Canadian Airlines

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7069

ECONOMICS AND LAW WORKSHOP

82-04

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by

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4032 SSC 4:00 p.m. Feb. 4

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Preliminary draft

Chapter 4

A Costing Model for Canadian Airlines

Much of the previous discussion centers on the behaviour of costs as the amount of output, the number of competitors, and the number and mix of products change. We also noted that the concepts of marginal costs, economies of scale, economies of scope, and joint (or common) costs will play key roles in evaluating the efficiency implications of the airline regulations, and in determining the incidence and burden of cross-subsidization.

A standard argument in the discussion of airline costs and regulation is that economies of scale are exhausted after three to five aircraft. This is a crucial issue, because if this argument is in fact true, then there is not a strong economic rationale for regulating the airlines on dense routes. However, the airline cost models estimated to date including Straszheim [1969] and Keeler [1972] are inadequate for investigating the crucial issues such as this because they aggregate airline's heterogeneous multiple outputs into a single output, and more seriously, their models are not capable of completely describing the airline production technology due to ad hoc specification.

To investigate the following issues, indicated in chapter 1, we wish to use a cost function which is capable of handling the multi-product nature of airline production and of identifying the structure of airline production technology completely:

a) Fare level and structure which would prevail under alternative regulatory scenarios: This requires the estimation of both marginal and average costs over outputs and a comparison with the existing fare structure.

b) Economic efficiency of publicly owned carriers relative to privately owned: This requires estimation of a cost function and the calculation of marginal costs for crown-owned and privately-owned carriers and a comparison of costs and fares on different types of routes with varying degrees of intra-mode competition.

c) Economies of scale, scope and traffic density: These can be identified directly from the estimated cost function.

d) Price responsiveness of demand for factor of production--particularly fuel: This can be derived from the estimated cost function, since it fully describes the industry's production technology.

I Characterization of Airline Cost Function

The major purpose of this chapter is, therefore, to specify the airline cost function consistently with the neo-classical production theory and to estimate it from the yearly data of 7 cross-sectional Class I and II airlines in Canada over the time period 1970-78. Structure of airline production technology such as question of efficiency of joint production of heterogeneous services, scale economies, economies of scope, existence of a consistent output aggregator (equivalently weak separability of outputs in the cost function), homotheticity and log-linearity of the cost function, are also empirically tested.

Statistics Canada and the airline industry's publications normally report airline outputs and operating statistics separately for each of the three major categories: Scheduled passenger service, scheduled freight service, and Charter (passenger and freight) service. Any attempt to use a single output measure by aggregating these three heterogeneous services would be bound to show a biased information on the structure of costs because each class has unique cost implications. Furthermore, even within a given output class, cost of the output produced by an airline may not be directly comparable with those of other airlines because each airline operates in its unique network and market environments, and thus produces the service with unique dimensions such as average distance transported, average size of aircrafts used, and load factors which are normally endogenous to the airline's network and market variables. Attempts to study airline costs to date have failed to use a cost model which takes into account the nature of multiple output technology of airline production and the dependence of airline's production (or choice) possibility sets on the network and market characteristics. They suffer also from ad hoc specification of the cost function which fails to completely describe the airline production technology.

In this study, the 'duality' relation between cost function and production technology is utilized to specify the airline cost function which is dual to the airline's multi-product/multi-input transformation function. Diewert's contribution [Diewert, 1971], expanded by many others including the articles in Fuss and McFadden [1978], made it possible to use a 'flexible' cost function to test many hypotheses concerning structure of the production technology without imposing them as maintained hypotheses. Jacobsen [1968], Shephard [1970] and McFadden [1978] have generalized the single output production function to the multiple output case and have established a duality between the (multi-output) production correspondence (or equivalently the product transformation function) and the multi-output cost function, $C(Y,W)$, which gives the minimum cost of producing an arbitrary output vector Y with given input price vector W. The duality theory implies that if the firm minimizes costs and input prices are exogenous, and if the product transformation function, $T(Y,W)=0$, where Y and X are output and input vectors, respectively, satisfies the usual regularity conditions (i.e., strictly convex isoquants), there exists a dual multi-product cost function $C(Y,W)$ which is just as good a representation of the firm's production technology as the product transformation function and also satisfies the following regularity properties (McFadden [1978], Varian [1978]):

(1)

1. C is nonnegative, differentiable, nondecreasing, linearly homogeneous and concave in W for each fixed nonnegative output vector Y.
2. C is strictly positive for non-zero output vector Y and is strictly increasing in Y.

Note that the implicit form of product transformation function, $T(Y,W)$, is impossible to estimate empirically without imposing a stringent *a priori* restriction on its structure; normally weak separability of the outputs from the inputs in the transformation function, implying a symmetric separability between output and input aggregators. Whether or not there exists a consistent output aggregator should be determined empirically rather than imposing it as a maintained hypotheses without an empirical test.[1] Invoking the duality between cost and transformation functions, however, one can estimate a dual cost function satisfying the above regularity conditions (1) and use it directly to deduce structure of the multi-product airline production technology.

Before we specify the airline cost function in a specific functional form, it is essential to discuss the special requirements for an airline cost function to realistically describe the production technology. The cost function should be capable of accommodating the following special natures of firm-specific airline production technology:

(i) Since the dimensions of airline outputs such as average stage length, average seat capacity of aircrafts used and load factors are largely dictated by the network and markets environments in which the airline operates and yet have implications on its operating costs, it is essential to include these variables in the cost function.

(ii) Since the airline production technology, particularly the productive capability of aircrafts such as speed, size and fuel consumption, has changed over time, time variable should be included in the cost function.

Equation (2) may be the simplest form that might reasonably satisfy the above requirements:[2]

(2) $C = C[\phi^1(Y_1,q_1),\ \phi^2(Y_2,q_2),\ldots,\phi^m(Y_m,q_m),\overset{*}{W}]$

where

Y_i = output of class $\underline{i}$, $i=1,2,..,m$,

$q_i = [q_{i1}, q_{i2}, .., q_{ir_i}]$ $r_i \times 1$ vector of dimensions describing the nature of the $\underline{i}$th output class,

$W^* = [W_1^*, W_2^*, .., W_n^*]$

$W_i^* = W_i \cdot e^{\alpha_i \cdot t}$ representing non-neutrally factor-augmenting technological changes over time.

W_i^* = adjusted price of input $\underline{i}$,

W_i = observed price of input $\underline{i}$,

t = time variable indicating the year.

$\phi^i(Y_i, q_i)$ = dimension-adjusted output of class $\underline{i}$, $i = 1,2,..,m$.

The way in which effects of the firm-specific network and market characteristics are treated in the cost function (2) is moderately restrictive because the dimension-adjusted output functions, $\phi^i(Y_i, q_i)$, imply that effects on the dimension-adjusted outputs of variations in the output dimensions dictated by the network and market environments is independent of time t and relative factor prices W. The value of $\phi^i(Y_i, q_i)$ serves as the measure of output class $\underline{i}$ in this form of cost function, and can be regarded as a hedonic measure of output, adjusted for variations in dimensions of output dictated by the network and market characteristics. This is customary to make that $\phi^i(\cdot)$ be homogeneous of degree one in the actual output quantity Y_i because the dimension-adjusted output $\phi^i(\cdot)$ should be proportional to the actual output at a given dimension.

Therefore, it can be written as:

(3) $\phi^i(Y_i,q_i) = Y_i \cdot q^i(q_{i1},q_{i2},..,q_{ir})$

where

the form of the hedonic output-adjustment function $q^i(\cdot)$ is, in general, unrestricted.

The introduction of hedonic functions, $q^i(\cdot)$, into the cost function raises questions about identifiability as Rosen [1974] pointed out that hedonic functions represent the reduced forms of the market equilibrium determined by the interface between demand and supply (or cost) for the qualities in the continuum of quality space. However, since the extents of the Canadian airline price regulation and route licensing make the arguments of the hedonic functions, $q^i(\cdot)$, beyond the control of an individual airline, it seems possible to treat those variables as being exogenous to the firm. Therefore, it is possible to interpret the hedonic coefficients in the cost function as representing technology dictated by the exogenously given network and market characteristics of airlines.

As the means to take into account of technological changes, primarily for the interest of reducing the number of parameters to estimate, and for ease of interpretation of the nature of technical changes, we used the method of adjusting each input price by an exponential function of time as used in Oum [1979]; i.e., $W_i = W_i \cdot e^{\alpha_i \cdot t}$.

II Specification of the Cost Function

For the purpose of estimation, the cost function (2) and the hedonic functions in (3) should be postulated in specific functional forms. To test hypotheses concerning structure of airline production technology, it is desirable to specify the cost function, $C(\phi,\overset{*}{W})$, in a 'flexible' functional form which does not impose any a priori restrictions on the first and second order derivatives of the cost function and thus is capable of providing a valid quadradic approximation to an arbitrary differentiable cost function.[3] In recent years, considerable effort has been devoted to developing so-called flexible functions which satisfy our requirements. Diewert [1971] developed the generalized Leontief functional form for the cost function and the generalized linear functional form for the production function. Hall [1973] suggested to combine these two forms into a functional form for the multi-product case. He called the resulting functional form the generalized Leontief-generalized linear or "hybrid Diewert" for short.

Christensen/Jorgenson/Lau [1971, 1973] proposed 'translog' function, and Burges [1974] used 'translog' form to specify the multi-product cost function for the first time, followed by a few others including Fuss and Waverman [1978] and Caves/Christensen/Tretheway [1978]. There are a few other 'flexible' functional forms occasionally referred to in economic literature including generalized Cobb-Dougals function [Diewert, 1973], square-root quadratic (SRQ) function, quadratic function [Lau, 1974], etc.

Recently, Caves/Christensen/Thretheway [1978] have shown that: (i) The quadratic function is not an attractive candidate for the multi-product cost function because it is not possible to impose parametric restrictions for the linear homogeneity (in input prices); (ii) For multi-purpose cost function, translog form is preferred to 'hybrid Diewert' form primarily on the grounds that it requires lesser parameters to estimate and yet allows to test as a wide range of hypotheses concerning structure of the technology as 'hybrid Diewert' form. For the same reasons, the translog function used in this study to approximate the airline multi-product cost function (2)

To use 'translog' function for a quadratic approximation of the multi-product cost function (2), it is essential to have information on the dimension-adjusted output measures, ϕ^i, $i = 1,2,..,m$, which are unobservable. However, the hedonic output measures $\phi^i = Y_i \cdot g^i(q_{i1}, q_{i2}, .., q_{ir})$ will be quantified as soon as the hedonic output-adjustment functions $q^i(\cdot)$ are determined. We use a log-linear function for a first-order approximation to an arbitrary hedonic output-adjustment function $g^i(\cdot)$ as follows:

$$\text{(4)} \qquad \ln \phi^i = \ln Y_i \cdot \prod_{\ell=1}^{\gamma_i} q_{i\ell}^{\beta_{i\ell}}$$

$$= \ln Y_i + \sum_{\ell=1}^{\gamma_i} \beta_{i\ell} \ln q_{i\ell}$$

$$i=1,2,..,m$$

It is interesting to note that our specification of the hedonic output function in (4) is also consistent with the exact interpretation of the translog function as the multi-product cost function because Blackorby/Primont/Russell [1977] and Denny and Fuss [1977] have shown that if the overall cost or production function is 'translog' form, then the microaggregators imbedded in the translog function must be log-linear form.[4] Since ϕ^i are unknown, we imbed the hedonic functions themselves in the macro translog function, and estimate the parameters of both the cost function and the hedonic functions simultaneously.

Our translog m-output/n-input airline cost function is specified as:

$$(5) \quad \ln C(\phi,W,t) = a_o + \sum_i^m a_i \ln\phi^i + \sum_i^n b_i \ln W_i^* + \frac{1}{2}\sum_i^m\sum_j^m a_{ij}\ln\phi^i\ln\phi^j + \frac{1}{2}\sum_i^n\sum_j^n b_{ij}\ln W_i^*\ln W_j^* + \sum_i^n\sum_j^m c_{ij}\ln W_i^*\ln\phi^j$$

where

$\phi^i = Y_i \cdot \prod_{\ell=1}^{\gamma_i} q_{i\ell}^{\beta_{i\ell}}$ is the hedonic (dimension-adjusted) output function for class $\underline{i}$,

Y_i; observed output of class $\underline{i}$,

$W_i^* = W_i \cdot e^{\alpha_i t}$ adjusted price of input $\underline{i}$,

W_i; price of input $\underline{i}$,

t; time variable,

a_o, a_i's, a_{ij}'s, b_{ij}'s, c_{ij}'s, are the parameters of the translog function,

$\beta_{i\ell}$'s are the parameters of the hedonic output function for output class $\underline{i}$,

α_i's are the parameters of factor price adjustment function representing non-neutrally factor-augmenting technical change over time.

Since the translog function is used as a quadratic approximation, the following symmetry conditions must be imposed:

$$(6) \quad a_{ij} = a_{ji},\ b_{ij} = b_{ji}, \qquad \text{for all } i \neq j.$$

The share of expenditure on the $\underline{i}$th input, s_i, can be obtained by applying the Shephard's lemma as the following:

$$(7) \quad s_i = \frac{\partial \ell n C(\cdot)}{\partial \ell n W_i^*}$$

$$= b_i + \sum_j^n b_{ij} \ell n W_j^* + \sum_j^m c_{ij} \ell n \phi^j$$

$$i=1,2,..n$$

Linear homogeneity of the cost function in the input prices and summing up conditions of the expenditure shares, $\sum_i^n s_i = 1$, impose the following further restrictions on the parameters:

$$(8) \quad \sum_i^n b_i = 1, \; \sum_j^n b_{ij} = 0, \; \sum_i^n c_{ij} = 0$$

One of the purposes in estimating the cost function developed above is to quantify summary measures which describe the nature of the production technology, provide estimates of the responsiveness of the demand for a factor with respect to both its own price and other factor prices, and which provide a measure of the degree of factor substitutability. For our translog cost function, the Allen partial elasticities of substitution (APES) between inputs i and j, σ_{ij}, and the compensated price elasticities[5] of input demand, E_{ij} can be computed using the result of Uzawa [1962] as follows:

$$(9a) \quad \sigma_{ij} = \frac{C\, c_{ij}}{c_i c_j} = \begin{cases} \dfrac{b_{ij}}{s_i \cdot s_j} + 1 & i \neq j \\[2ex] \dfrac{b_{ii} - s_i}{s_i^2} + 1 & i = j \end{cases}$$

where

$$c_i = \frac{\partial C(\cdot)}{\partial w_i} \qquad c_{ij} = \frac{\partial C(\cdot)}{\partial w_i \partial w_j}$$

$$\text{(b)}\quad E_{ij} = \left(\frac{\partial x_i}{\partial w_j}\right)_{\text{isoquant}} \cdot \frac{w_j}{x_i} = \begin{cases} \dfrac{b_{ij}+s_i \cdot s_j}{s_i} = \sigma_{ij}s_j & i \neq j \\[2ex] \dfrac{b_{ii}+s_i^2-s_i}{s_i} = \sigma_{ii}s_i & i=j \end{cases}$$

Furthermore, Allen [1938] has derived the following relationships between the partial elasticities of substitution and the price elasticities of Marshallian (ordinary) demand, F_{ij},

$$\text{(9c)}\quad F_{ij} = \frac{d\ln x_i}{d\ln w_j} = s_j(\sigma_{ij}+\eta) \qquad i,j=1,2,..,n$$

where $\eta = \frac{d\ln Y}{d\ln P}$ is own price elasticity of the Marshallian demand for the (aggregate) output.

III. Data Construction

In this study, we decided to examine the three outputs and three inputs which are listed below.

Also listed are the basic measures of the outputs (Y_i) and the three variables (q_{ik}) for each output explaining the dimensions of the output class dictated largely by the network and market characteristics:

Outputs and dimensional variables:

1) Y_1 : scheduled (unit toll) revenue passenger-miles,

q_{11}: passenger load-factor,

q_{12}: average distance of passenger travel,

q_{13}: average number of available seats per aircraft departure;

2) Y_2 : scheduled (unit toll) revenue freight ton-miles including goods, mail and express services,

q_{21}: total weight load-factor,

q_{22}: average length of haul for revenue freight,

q_{23}: average available tons per aircraft departure;

3) Y_3 : non-scheduled (charter) revenue ton-miles including passenger and freight services,

q_{31}: average stage length of charter services,

q_{32}: average revenue tons per departure,

q_{33}: The percentage of charter passenger ton-miles to total ton-miles.

Input Prices:

1) W_1: price of labour,

2) w_2: rental price of capital including flight and non-flight capitals,

3) w_3: fuel price.

The data on outputs, dimensions of the outputs, expenditures and inputs were taken from the Statistics Canada publications (Catalogue Nos. 51-001, 51-002, 51-004, and 51-206). The data are developed for the two Class I (trunk) carriers, Air Canada and Canadian Pacific Airways, and the four regional carriers, Pacific Western Airlines (PWA), Transair, Quebecair and Eastern Provincial Airways (EPA). Additional data on individual airlines' yearly aircraft stocks were obtained from Air Canada airline annual reports and aviation statistics.[6] Since the output for scheduled freight service (y_2) involves three sub-categories, goods, express cargo, and mail services an aggregate output measure was derived through the Divisia procedure. Similarly for the non-scheduled service output (y_3), the same Divisia procedure was used to aggregate non-scheduled passenger and freight services.

The input price and quantity indices for labour, fuel, and capital were calculated using a Divisia aggregation procedure.

The fuel input was composed of turbo oil, turbo fuel, gasoline, and other oil. Quantity and expenditure data were taken from Statistics Canada information. Each category was adjusted by a BTU factor so each were expressed in common units. A single measure of real fuel input was then constructed as:

$$(10) \quad \ell n\ F_k - \ell n\ F_\ell = \sum_{i=1}^{4} \frac{W_{ik} + \bar{W}_i}{2} \ \ell n\ (F_{i,k}/F_i^*) - \sum_{i=1}^{4} \frac{W_{i\ell} + \bar{W}_i}{2} \ \ell n\ (F_{i\ell}/F_i^*)$$

where

F_R : aggregate fuel input for observation R (specific firm for a given year)

F_{ik} : quantity of fuel type i for observation k

W_{iR} : expenditure share of fuel type i for observation k

$\bar{W}_i$: arithmetic mean of the shares of expenditures on fuel category i

F_i^* : geometric mean of the quantity used of fuel category i.

The price and quantity index for labour was constructed using the method described above. We used six categories of labour; pilots and co-pilots, other flight personnel, general management, maintenance labour, aircraft and servicing labour and other employees.[7]

The third input, capital, is an aggregate of flight capital, non-flight capital and materials. The real stock of flight and non-flight equipment was constructed based on the perpetual inventory method [Christensen and Jorgensen, 1969];

(11) $K_{it} = I_{it} + (1-\mu_i)\, K_{i,\, t-1}$

where K_{it} = the real capital stock of category i in year t

I_{it} = the real (constant dollar) value of net investment on category i at time t

μ_i = $2/n_i$ where n_i is the economic life of asset type i

The capital rental price (P_k) is calculated as

P_{ki} : $P_{Ai}(R + S_i)$

P_{ki} : rental price of capital good type i

P_{Ai} : asset price

R : rate of interest

S_i : depreciation rate of asset type i.

The three components of the capital stock variable contained the following sub-components:

(a) Non-flight capital

i) ramp equipment

ii) communications and meterological equipment

iii) maintenance and engineering equipment

iv) surface transport vehicles and equipment

v) furniture fixtures and office equipment

vi) miscellaneous ground equipment

b) building and other improvements

c) flight capital

i) airframes

ii) aircraft engines

iii) flight equipment, spare parts and assemblies

d) materials: total expenses less expenses on labour, fuel, flight and non-flight capital.

The price index for flight capital was taken as the price index for engineering construction (1971 = 100) of the air transport industry [cat. 13-568 Statistics Canada]. The asset price (P_A) for building and other improvements was taken as the price index for building construction (1971 = 100) of the air transport industry, while for non-flight capital the price index for machinery and equipment of the air transport industry was selected. For materials the price was taken as the GNE deflation [cat. 13-201, Statistics Canada].

The rate of interest R, is represented by the McLeod, Young, and Weir bond rate [Bank of Canada Review].

The aggregate price and quantity indices for capital was obtained by the Divisia index utilizing (10).

IV Alternative Structures of the Production Technology

Overall Returns to Scale

Marginal cost and economies of scale play important roles in evaluating impacts of the price and entry control. In the absence of economies of scale and regulations, airlines are expected to price at its marginal cost (including the opportunity value of equity capital) of providing service in each market. On the other hand, the marginal cost pricing would be noncompensatory to airlines if economies of scale prevails at the current output level. Therefore, it is important to examine behavior of the marginal costs and identify nature of the scale economies in airline industry.

The marginal cost of producing $\underline{i}^{th}$ (adjusted) output ϕ^i can be obtained from the translog function in (5) as follows:

$$(12) \quad MC(\phi^i) = \frac{\partial C(\cdot)}{\partial \phi^i} = \frac{\partial \ln C(\cdot)}{\partial \ln \phi^i} \frac{C}{\phi^i}$$

$$= \frac{C}{\phi^i} [a_i + \sum_j^m a_{ij} \ln \phi^j + \sum_k^n C_{ki} \ln w_k^*]$$

For a single product case, the elasticity of <u>total</u> cost with respect to output (E_{cy}) indicates the direction of scale economies; i.e., $E_{cy} > E_{cy} = 1$, $E_{cy} < 1$ imply decreasing, constant, and increasing returns to scale, respectively. For our multi-product cost function, the elasticity of total cost with respect to $\underline{i}^{th}$ output ϕ^i can be written as:

$$(13)\quad E_{c\phi^i} = \frac{\partial \ell nC}{\partial \ell n\phi^i} = a_i + \sum_j^m a_{ij}\,\ell n\phi^j + \sum_k^n C_{ki}\,\ell nw_k^* \qquad i = 1,2,..,m$$

However, Fuss and Waverman [1978] have presented lengthy discussions on the reasons why the product-specific cost elasticity $E_{c\phi^i}$ is not an appropriate indicator for product-specific scale economies, and why there is no unambiguous measure of product-specific scale returns to scale under a (true) joint production technology (i.e., when joint and/or common costs exist). Therefore, we are left with only the overall measure of returns to scale. To obtain a measure of 'overall' returns to scale, we totally differentiate the multi-product cost function as follows:

$$(14)\quad d\ell nC = \sum_i^m \frac{\partial \ell nC(\cdot)}{\partial \ell n\phi^i}\,\frac{d\phi^i}{\phi^i}$$

As Fuss and Waverman [1978] pointed out, this measure also has ambiguity as an indicator of 'overall' scale economy because, depending on demand relationships, the firm may want to increase its outputs by differential proportions. For the sake of obtaining an overall scale economy measure, however, we impose the assumption that outputs change in the same proportion, i.e.,

$$\frac{d\phi^i}{\phi^i} = d\ell n\phi^i = \lambda, \quad i=1,2,...,m,$$

and obtain equation (15):

(15) $$\frac{d\ln C}{d\ln\phi^i} = \frac{d\ln C}{\lambda} = \sum_i^m \frac{\partial\ln C(\cdot)}{\partial\ln\phi^i}$$

In this overall cost elasticity measure, input quantities are allowed to follow the least-cost expansion path, which in general does not involve proportionate changes in input quantities. Notice that $\frac{d\ln C}{\lambda} > 1, = 0,$ and < 1, imply the decreasing, constant, and increasing overall returns to scale, respectively, as outputs are varied in the same proportion λ. Using Baumol's [1977] characterization, these three cases represent strictly increasing, constant, and strictly declining "ray" average costs (SDRAC), respectively.

In terms of our translog cost function (5), equation (15) can be written as follows:

(15a) $$\frac{d\ln C}{\lambda} = \sum_i^m (a_i + \sum_j^m a_{ij}\ln\phi^j + \sum_k^n c_{ki}\ln W_k^*)$$

Since this overall returns to scale measure depends on the data, it is in general impossible to impose parametric restrictions for the overall <u>constant</u> returns to scale. However, there are two ways to identify the overall returns to scale: Firstly, one can estimate the unrestricted translog cost function, and compute the point estimate of overall scale economy measure (15a) at each data point. Secondly, one can scale the data to set all output ϕ^j, j=1,2,...,m, and input measures W_i^*, i=1,2,..,n, at an observation to 1, impose the following parametric restrictions for the overall constant returns to scale, and test this null hypothesis against the unrestricted returns to scale at the point of approximation [1,1,...,1].

$$\sum_{i}^{m} a_i = 1$$

This second approach is equivalent to testing the local <u>ray</u> constant returns to scale and the local <u>ray</u> constant average costs at the point of approximation. We decided to use both of the approaches. For the second approach, entire data are scaled such that the 1977 CP Air data becomes the point of approximation.

The effect of changes in the adjustment factors (which include network characteristics), q_{ij}, on total cost is

$$\frac{\partial C(\cdot)}{\partial q_{ij}} = (a_i + \sum^{m} a_{ij} \ell n\ Y_j + \sum_{k} c_{ki} \ell n\ W_k^*) \quad \frac{C(\cdot)}{q_{ij}}$$

while the effect of a change in q_{ij} on marginal cost is:

$$\frac{\partial MC(Y_i)}{\partial q_{ij}} = \frac{C(\cdot)}{Y_i \cdot q_{ij}} \left[(a_i + \sum_{j} a_{ij}\ \ell n\ Y_i + \sum c_{ki} W_k^*)^2 + a_{ij} \beta_{ij} \right]$$

Thus one can determine how changes in route length, load factors and aircraft size can shift the cost function. This is an important point since the 'optimal size of firm' is defined of a given values of these adjustment factors and just as the long run cost function is the envelope of the cost function over firm sizes, so too can one obtain a locus of optimal firm sizes over their adjustment characteristics.

It is interesting to discover that this ray overall cost elasticity measure in (15) is exactly the reciprocal of the following measure of economies of scale (5) originally due to Panzar and Willig [1977: Theorem 2]:

$$(16)\quad S = \frac{C(\phi, \overset{*}{w})}{\sum_i \phi^i c_i(\phi, \overset{*}{w})} = 1/\sum_i \frac{\partial \ \ell nC(\cdot)}{\partial \ \ell n\phi^i} = 1/\sum_i \ (a_i + \sum_j a_{ij}\ell nY_i + \sum_k c_R \ell nW_R^*)$$

where $c_i(y,\overset{*}{w}) = \dfrac{\partial \ C(\phi,\overset{*}{w})}{\partial \ \phi^i}$

Panzar and Willig [1977] show that the technology exhibits economies, diseconomies, and locally constant returns to scale at $(\overset{*}{x},\phi)$ if and only if $S > 1$, $S < 1$, and $S = 1$, respectively, and these are necessary and sufficient conditions for the revenues from marginal cost pricing to, respectively, fall short of, exceed and exactly cover production costs.

Willig's contribution for the two product case has recently been extended for J products by Mintz [1980]. He shows that economies of scale can be expressed in terms of <u>local</u> measures of product specific scale economies and two types of economies of scope. 'Strong' scope economies exist if costs are reduced when all products are jointly produced rather than being produced by separate firms. 'Incremental' scope economies exist if costs are reduced from the joint production of all products rather than producing <u>one</u> of the products seaparately. Mintz demonstrates that a sufficient condition for economies of scale is if 'incremental' scope economies dominate 'strong' scope economies; i.e. if producing all products jointly is less costly than producing some but not all products separately.

Strong economies of scope is defined as:

$$(16a) \quad S_1 = \sum_i \frac{C(\phi^*, w) - C(\phi, w)}{C(\phi, w)}$$

where $C(\phi^*, w)$ represents the output vector $[0, 0, \phi^*, 0, \ldots, 0]$.

and incremental economies of scope is defined as:

$$(16b) \quad S_2 = \frac{C(\phi^*, w) - C(\phi^{**}, w) - C(\phi, w)}{C(\phi, w)}$$

where $C(\phi^{**}, w)$ represents the costs of producing all outputs except ϕ^* ; $[\phi_1, \phi_2, \ldots, 0, \ldots, \phi_n]$

Multiproduct economies of scale can then be defined as [Mintz, 1980].

$$(16c) \quad \hat{S} = \frac{\sum_i \frac{\partial C(\phi, w)}{\partial \phi^i} \phi^i \cdot \dfrac{C(\phi_j^{**}, w) - C(\phi^i, w)}{\frac{\partial C(\cdot)}{\partial \phi_i} \cdot \phi_i}}{(1 - \sum_i S_2 + S_1)}$$

Utilizing (16a-c) we are able to calculate for each firm the cost savings (increases) if one product from the n products produced is dropped (added). Furthermore, (16c) provides a local measure of scale economies, and it is equal to (16).

The determination of (local) scale economies is important in evaluating regulation and the feasibility of of marginal cost pricing in a more competitive environment. If scale economies exist, pricing at marginal cost will result in revenues falling short of costs resulting in a subsidy; a rearrangement of (16) provides a measure of this required subsidy as $C(\phi,w)[1-1/s]$. Finally, the measures of S_2 over different combinations of outputs provides a measure of the additional cost revenues from each product line with these calculated costs provides a measure of the degree of cross-subsidy between product lines.

Economies of Scope and Subadditivity of the Cost Function

In a multiple output production process, particularly one which is regulated as are airlines, one must be able to determine any potential advantages or disadvantages when a new service is to be introduced. This is important since it affects the costs of the overall operation and the pricing of the new product or service should be such as to avoid any cross-subsidy. In the case of regulated industries, the behavior of costs as the scope of operations changes provides some insights as to whether regulation is required at all and whether regulated industries will, if the new output is in the non-regulated sector, use their protected status to exert power in the market for the new output. For example, should airlines be able to offer a range of schedule <u>and</u> charter service for passengers and cargo, should they be allowed to offer express package/mail service, or should they be allowed into the package tour business, air hotels, etc.; such as Air Canada is now allowed under its new act established by Bill C-3 in 1977. Finally, if they are, how should the CTC regulate these activities if at all? Intuitively, all these regulatory questions require to determine whether the airline production can be characterized by a true joint production technology or rather by economies of specialization.

Conceptually, we wish to investigate whether a single firm multi-product production technology is more or less efficient than a multi-firm single product production technology. Recently, in his seminal work, Baumol [1977] has introduced the concept of "subadditive" cost function as a means to evaluate this question. A cost function C is strictly <u>subadditive</u>[8] at a specific output vector Y^* if, for division of Y^* into any number, say <u>k</u>, output vectors $Y^1, Y^2, \ldots, Y^k$, the cost of sum of the <u>k</u> output vectors is less than the sum of the costs of producing them separately, i.e.,

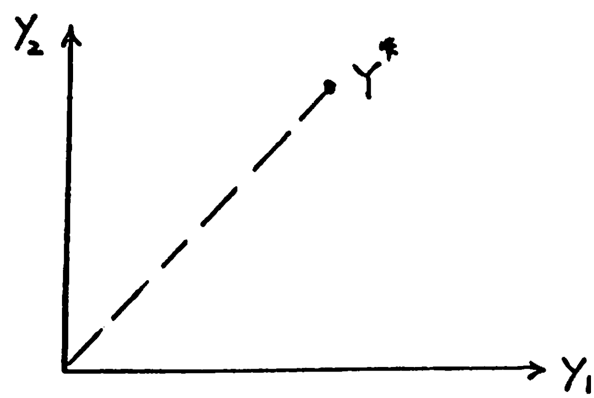
$$C(Y^1 + \ldots + Y^k) < C(Y^1) + \ldots + C(Y^k). \tag{18}$$

where Y^i is a vector of outputs, $i=1,2,\ldots,k$.
This means that it is always cheaper to have a single firm produce the total output vector Y^* than splitting them to more than one firm in any fashion. Subadditivity may be examined either at each <u>specific output</u> vector if we are con-

cerned with one or more output vectors, or <u>globally</u> if one is interested in whether it holds at all possible output vectors. Clearly sub-additivity is the basis upon which to determine if cost savings exist by combining outputs under one producer.

Baumol [1977, proposition 12] has shown that a cost function C is <u>strictly sub-additive</u> at output vector Y^* if the ray average costs are strictly declining and (non-strict) trans-ray convexity holds along any one hyperplane passing through Y^*:

a) strictly declining ray average costs (SDRAC) along the ray from origin to Y^*:



<u>Figure 4:1</u>

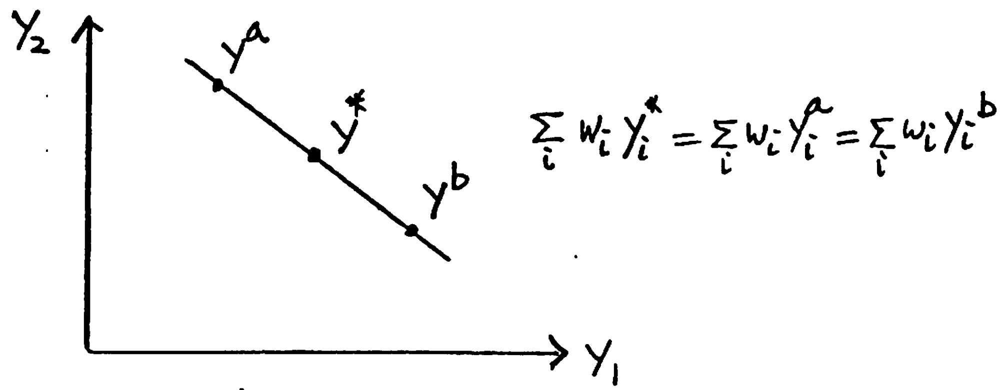
On a ray from the origin, the average costs along this ray are strictly decreasing if

$$\frac{C(\beta Y^*)}{\beta} > \frac{C(\delta Y^*)}{\delta} \quad \text{for} \quad \delta > \beta > 0$$

Note that since we are moving along the ray, output proportions are held constant. But this is precisely the way in which we constructed the measure of ray overall scale economies in equation (16). Indeed, the test for SDRAC is equivalent to testing for the ray overall returns to scale.

b) transray convexity through Y^*, i.e., (non-strict) convexity along any one hyperplane, say

$\sum_i w_i Y_i$, $w_i > 0$, passing through the point Y^*:



<u>Figure 4:2</u>

Referring to Figure 4:2, a cost function C(Y) is <u>transray convex</u> through Y^* if there exists any set of positive constants $w = [w_1, w_2 2$ such that, for any two points, say y^a and y^b, on the transray $w \cdot y = w \cdot y^*$, we have

$$C[kY^a + (1-k)y^b] \leq kC(Y^a) + (1-k)C(Y^b)$$

for any k, $0 < k < 1$.

Transray convexity concerns with the properties of the cost function when the product mix changes. Intuitively, it means inter-product complementarity. Indeed, Baumol [1977] has noted that <u>transray</u> convexity is an equivalent notion as Panzar-Willig's [1977] "economies of scope", while Panzar and Willig

[1978, proposition 2] have shown that cost complementaries are sufficient for economies of scope. Economies of scope means that it is more efficient to produce m outputs together than by m separate production processes: i.e.,

$$C(Y_1,\ldots,Y_m) < C(Y_1) + \ldots + C(Y_m).$$

The result of Panzar and Willig [1978] suggests that it is possible to test sufficiency condition for economies of scope (equivalently, transray convexity) by examining the cost complementarities among the products a firm produces.

For our translog cost function, the condition for the cost complementaries is:

(16)

$$\frac{\partial^2 C}{\partial\phi^i \partial\phi^j} = \frac{C}{\phi^i\phi^j}\left[\frac{\partial \ln C}{\partial \ln\phi^j} \cdot \frac{\partial \ln C}{\partial \ln\phi^j} + \frac{\partial^2 \ln C}{\partial \ln\phi^i \partial \ln\phi^j}\right]$$

$$= \frac{C}{\phi^i \cdot \phi^j}\left[\left(a_i + \sum_{\ell}^{m} a_{i\ell} \ln\phi^\ell + \sum_{k}^{n} c_{ki} \ln W_k^*\right)\right.$$

$$\left. \cdot \left(a_j + \sum_{\ell}^{m} a_{j\ell} \ln\phi^\ell + \sum_{k}^{n} c_{kj} \ln W_k^*\right) + a_{ij}\right] < 0$$

Since equation (16) depends on the data, we scale the data so that the point of approximation (CP Air 1977 data) becomes

$$\phi^i = W_j = 1,\ i=1,\ldots,m, \text{ and } j=1,2,\ldots,n\ .$$

With this scaling the condition becomes:

$$(17)\quad \left.\frac{\partial^2 C}{\partial\phi^i\partial\phi^j}\right|_{\phi^i = W_j = 1} = a_i \cdot a_j + a_{ij} < 0$$

$$i \neq j,\ i,j=1,2,..,m\ ,$$

Therefore, one can test for the lack of total economies of scope at the point of approximation by imposing

$$(18)\quad a_i \cdot a_j + a_{ij} = 0$$

In sum, a sufficient condition for subadditive cost function is: (i) strictly declining ray average costs, i.e., $\frac{d\ell nC}{\lambda} < 1$ defined in equation (16) for all $\lambda > 0$, and (ii) product complementarities as described in equation (16), everywhere in the output space of our concern. Unfortunately, a test for these conditions requires a global description of shape of the entire cost function from the origin up to the output in question, thus calling for the data that may lie well beyond the range of empirical observations. In the absence of a known exact testing procedure for the sub-additivity, we decided to use the local test[9] in the neighborhood of the point of approximation, [1,...,1].

The above discussion suggests that a simultaneous rejection of the overall ray constant returns to scale (condition (17)) and the lack of product complementarities at the point of approximation (condition (18)), is sufficient but not a necessary condition for the local subadditivity at the point of approximation. This test will be conducted in the empirical section.

Other Special Structures of Production Technology

The following three special structures of production technology have received much attention in many previous empirical studies:

(i) Input-output separable structure

(ii) Homothetic structure

(iii) Cobb-Douglas structure

These three special structures[10] will be tested against the general non-homothetic structure represented by the translog cost function (5).

Table 4-1 summarizes the properties of the five special structures of technology and the corresponding cost functions discussed so far. The corresponding restrictions imposed on the parameters of the translog function (5) are also listed in the table.

Table 4-1, Alternative Structures of Technology

Hypotheses	Form of cost function	Restrictions on translog parameters	Number of independent restrictions in our 3-output/3-input cost function
(1) General non-homothetic structure	$C=C(\phi,w)$	Standard: symmetry, linear homogeneity and adding-up of shares in (6) and (8)	0
(2) Input-Output separable structure	$C=C[0(\phi),w]$	$c_{ki}=\rho_k \cdot a_i$ $i=1,2,..,m$ $k=1,2,..,n$	3
(3)* Homothetic structure	$C=0(\phi)\cdot I(w)$	$c_{ij}=0$, $i=1,2,..,n$ $j=1,2,..,m$	6
(4)* Log-linear structure	$C=A\prod_i^m \phi^{i\,a_i} \cdot \prod_k^n W_k^{b_k}$	$a_{ij}=b_{ij}=c_{ij}=0$ for all i,j	15
(5)** Non-joint technology	lack of economies of scope	$a_{ij}=-a_i \cdot a_j$ $i \neq j$ $i,j=1,2,..,m$	6
(6)** Overall constant returns to scale		$\sum_i^m a_i = 1$	1

* We use the translog function as the exact form of our cost function when these hypotheses are tested. When we interprete the translog function as a quadratic approximation, the restrictions on parameters get complicated as seen in Oum and Gillen[1979].

** As described previously, these restrictions are valid only at the point of approximation, i.e., $\phi^i=W_j=1$ for all i and j. We scaled the data such that $\phi^i=W_j=1$ at year 1977.

V Total Factor Productivity

Many people have argued that firms protected by government regulations are less anxious to improve the efficiency of their operations and thus productivity, and that crown corporations whose prime objective is not profit maximization are normally less efficient than privately-owned firms. To examine productivity implications of the airline regulations and of ownership type, we propose to measure the productivity growth of each of the seven Canadian airlines included in this study utilizing the cost functions specified in previous section. Productivity measurement also provides many insights into the structure of airline production technology. In this study we have formally incorporated network and market characteristics into the multiple-output multiple-input cost functions. These factors may influence relative factor productivities. Since routes are regulated, being awarded by the CTC, we will be able to determine the impact of this form of regulation upon factor productivity. In addition, since the number, types, and amounts of inputs differ between trunk and regional air carriers, one is able to evaluate the government policy to restrict regional carriers to specific geographic areas, and thereby market densities and route lengths.

Total factor productivity stems from two sources: A scale (or output) effect if there are non-constant returns to scale in production since factor productivity can change as the scale of the operation changes; secondly, technical change, for example, the introduction of new generation aircraft, can change the production function. There may also be interactive effects since technical change in shifting production function also shifts the derived demand functions of factor inputs which may contribute to some of the growth in factor inputs [Nadiri and Schankerman; 1979].

The conventional approach to a productivity study has utilized the residual method of computing the Törnqvist-Divisia index of the total factor productivity. For example, Harris and Dhuvarajan [1978] Caves/Christensen/Tretheway [1979] in their productivity studies of Canadian and U.S. airlines, respectively, computed the Divisia indices to measure total (or partial) factor productivities. It is well known (see, for example, Diwert [1979] and Denny/Fuss/Waverman [1979] that, in the context of production theory, the usage of conventional Divisia index of total factor productivity assumes constant returns to scale, marginal cost pricing, the cost-minimizing behavior and non-existence of regulations.[11] Clearly, except for the cost-minimizing behavior, most of these conditions are unlikely to be satisfied in the Canadian airline industry. Therefore, usage of the Törnqvist-Divisia index without adjustment for these would give us a biased information on the rate of growth in total factor productivity of the Canadian airlines. A further complication arises in airline productivity study due to the multiple output production. In the remainder of this section where these special problems in measuring airline productivity are treated, we follow closely with the discussions made in Denny/Fuss/Waverman [1979], Berndt and Khaled [1978] and Ohta [1974].[12]

Total factor productivity (TFP) is a measure of aggregate output per unit of aggregate factor input. Therefore, whenever one computes TFP, one assumes implicitly the existence of consistent input and output aggregators. TFP is normally measured by determining the rate of growth of total factor productivity ($T\dot{F}P$) as the residual of the rate of growth of aggregate output ($\dot{Y}$) less the rate of growth of aggregate input ($\dot{X}$) as follows:

$$T\dot{F}P = \dot{Y} - \dot{X} \tag{19a}$$

$$\dot{Y} = \sum_j \frac{(p_j y_j)}{R} \dot{y}_j \qquad \dot{X} = \sum_i \frac{(w_i x_i)}{C} \dot{x}_i \tag{19b}$$

where

$R = \sum_j p_j y_j$ is total revenue

$C = \sum_i w_i x_i$ is total input cost

p_j, y_j: price and quantity of output $\underline{j}$,

w_i, x_i: price and quantity of input $\underline{i}$,

a dot $(\cdot)$ represents the instantaneous rate of growth; for instance, $\dot{x}_i = \frac{\partial x_i}{\partial t}/x_i$.

What follows are the discussions on the reasons why this method would give us a biased measure of TFC and how it should be modified to resolve those problems.

The duality between cost function and the production technology discussed earlier allow us to examine the growth in factor productivity in terms of the cost function. By totally differentiating our cost function (2) with respect to time t, we obtain the following expression:

$$\frac{dC(\cdot)}{dt} = \sum_i^m \frac{\partial C}{\partial y_i}\frac{\partial y_i}{\partial t} + \sum_j^n \frac{\partial C}{\partial w_j}\frac{\partial w_j}{\partial t} + \frac{\partial C}{\partial t}$$

By dividing this equation through by C and rearranging the terms, we get:

$$(20) \qquad \dot{C} = \sum_j^m \varepsilon_{cj}\,\dot{y}_j + \sum_i^n \frac{x_i w_i}{C}\,\dot{w}_i + \dot{T}$$

where

$\dot{T} = \frac{\partial C}{\partial t}/C$ is the instantaneous rate of change in cost function due to technical change,

$\varepsilon_{cj} = \frac{\partial C}{\partial y_j}\frac{y_j}{C}$ cost elasticity w.r.t. output y_j .

We now totally differentiate the identity of total cost $C = \sum_i w_i x_i$ with respect to time t and divide it through by C to obtain:

$$(21) \qquad \dot{C} = \sum_j \frac{w_i x_i}{C}\,\dot{w}_i + \sum_i \frac{x_i w_i}{C}\,\dot{x}_i$$

Substitution of (21) into (20) gives:

$$(22)\quad -\dot{T} = \sum_j \varepsilon_{cj}\dot{y}_j - \sum_i \frac{x_i w_i}{C}\dot{x}_i = \sum_j \varepsilon_{cj}\dot{y}_j - \dot{X}$$

Therefore, given information on the rates of change in outputs and inputs and the cost elasticities with respect to outputs, one can calculate shifts in the cost function due to pure technological change, T. Suppose now firms violate the marginal cost pricing: i.e., $p_j \neq MC(y_j)$. Then the following expression represents the discrepancy in the measured growth of output due to the departures from marginal cost pricing principle:

$$(23a)\quad \dot{y}^p - \dot{y}^c$$

$$(23b)\quad \dot{y}^p = \sum_j \frac{p_j y_j}{R} \cdot \dot{y}_j$$

$$(23c)\quad \dot{y}^c = (\sum_k \varepsilon_{ck})^{-1}(\sum_j \varepsilon_{cj} \cdot \dot{y}_j)$$

where

$\dot{y}^p$ is the growth rate of the aggregate output in the Divisia productivity index,

$\dot{y}^c$ is the growth rate of the aggregate output computed by using the cost elasticities as weights for aggregation.

Using (23c), equation (22) can be rewritten as:

$$(24)\quad -\dot{T} = (\sum_K \varepsilon_{CK})\, \dot{y}^c - \dot{X}$$

$$= (\sum_K \bar{\varepsilon}_{CK} - 1)\, \dot{y}^c + (\dot{y}^c - \dot{y}^p) + (\dot{y}^p - \dot{X})$$

Replacing the expression for the rate of change in total factor productivity $\dot{TFP} = \dot{y}^p - \dot{X}$ in equation (24), we obtain:

$$(25)\quad \dot{TFP} = [-\dot{T}] + [(1-\sum_K \varepsilon_{CK})\, \dot{y}^c] + [\dot{y}^p - \dot{y}^c]$$

Note that equation (25) decomposes the conventional Törnqvist-Divisia measure of the rate of growth of total factor productivity into three components: (i) the component due to pure technological change $(-\dot{T})$, (ii) the component due to non-constant returns to scale in production, $(1 - \sum_K \varepsilon_{CK})\, \dot{y}^c$, and (iii) the component due to the firm's departures from the marginal cost pricing, $(\dot{y}^p - \dot{y}^c)$. Therefore, the conventional measure of the total factor productivity (TFP) may under-/over-estimate the real change in total factor productivity due to technological change when constant returns to scale and/or marginal cost pricing are violated.

To adjust the conventional Törnqvist-Divisia measure of total factor productivity, it is necessary to compute the following ray cost elasticity from our translog cost function (5) as well as $\dot{y}^p$ and $\dot{y}^c$.

$$(26)\quad \sum_{i}^{m} \varepsilon_{ci} = \Sigma \frac{\partial\, \ell nC(\cdot)}{\partial\, \ell n\phi^{i}} \frac{\partial\, \ell n\phi^{i}}{\partial y_{i}}$$

$$= \sum_{i}^{m} (a_{i} + \sum_{j}^{m} a_{ij}\, \ell n\phi^{j} + \sum_{k}^{n} c_{ki}\, \ell nw_{k}^{*})$$

With this information, we can always measure:

$$(27)\qquad -\dot{T} = T\dot{F}P + (\sum_{K} \varepsilon_{CK} - 1)\, \dot{y}^{c} + (\dot{y}^{c} - \dot{y}^{P})$$

We have argued earlier (chapter 2) that extensive cross-subsidy between products and routes exist in the case of airlines. Prices will clearly deviate from marginal costs. In order to evaluate the effect of this deviation on factor productivity measurement, assuming constant returns to scale, Denny, Fuss, and Waverman [1979] decompose $T\dot{F}P$ as follows:

$$(28)\quad T\dot{F}P = -\dot{B} + \sum_{j} [\frac{(P_{j} - mc_{j})}{C} y_{j}]\, \dot{y}_{j} + \sum_{j} [(P_{j}y_{j})\, (\frac{1}{R} - \frac{1}{C})]\, \dot{y}_{j}$$

If cross-subsidy exists, $P_{j} \gtrless mc_{j}$ depending on whether the product (or route) is a recepient or provider of the subsidy. If revenues (R) > costs (c), $T\dot{F}P < -\dot{B}$. Thus, utilizing (27) and (28) one is able to calculate TFP and its difference from $\dot{B}$, once marginal costs are determined from the cost function (12). The total factor productivities and its three components will be computed in the empirical section.

Footnotes

1. In their recent empirical study on The Bell Canada, Fuss and Waverman [1978] rejected the existence of a consistent output aggregators in the Canadian telecommunication industry.

2. Spady and Friedlaender [1978] used a similar form as this for their single-product cost function for U.S. Trucking firms.

3. See Blackorby/Primont/Russell [1977] for the discussions on the "flexibility" of a function and a class of functions known as general quadratic flexible forms.

4. Spady and Friedlaender [1978] have used 'translog' approximation to their output hedonic function which is imbedded in a translog approximation to the macro cost function. Their specification would be invalid if the translog function is used as the exact form of the macro cost function.

5. The detailed derivations are available in Oum [1979b, Appendix 7A].

6. Nordair in the 60's was not classified as a regional carrier and data were, therefore, not available from Statistics Canada. Despite the generous cooperation of Nordair we were unable to construct a complete data set compatable with the other airlines and therefore excluded Nordair from the analysis.

7. Since Statistics Canada reports only the number of employees in each category and not hours worked, we must assume that there are no differences in the number of hours worked per year for category or do not vary across time or airline.

8. See Baumol [1977] and Baumol and Braunstein [1977] for the distinction between global and output-specific subadditivity of a cost function.

9. Fuss and Waverman [1978] used the same procedure for the first time.

10. For the detailed discussions and derivations on these special structures of production and cost (or utility and indirect utility) functions and the corresponding restrictions on the parameters of translog function, see Denny and Fuss [1977], Jorgenson and Lau [1974] and Oum and Gillen [1979].

11. Denny, Fuss, and Waverman [1979] also point out the assumption of no regulatory constraints; this refers to direct-rate of return, for example, rather than indirect constraints.

12. Nadiri and Schankerman work in terms of the production function and deal only with the single output case.

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