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The Game of Negotiations: Ordering Issues and Implementing Agreements *

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Abstract

We use a two-issue bargaining model with asymmetric information to study agent choice of how to structure bargaining. We uncover the settings in which different agenda structures are chosen in equilibrium, how the order in which issues are bargained over matters, and what impact the rules for implementing agreements have. If agreements are implemented as they are reached, "easy" issues are negotiated first and "hard" issues later; if agreements are implemented only after all issues are settled, then it is size that matters, with large issues settled first. All parties prefer the former rules of implementation to the latter.

Journal of Economic Literature Classification Number: C7

Notation

δ	delta
ω_t	omega subscripted with t
v_h	v subscripted by h
v_l	v subscripted by l
$\emptyset$	the empty set
τ	tau

1 Introduction

In most negotiations parties bargain over many issues, and the bargaining agenda and rules for implementing agreements are important elements of the negotiation process. Yet, bargaining models traditionally study the division of just a single pie.[1] To be sure, the abstraction of a single pie is applicable to bargaining over many issues; however, it is so only if the bargaining process is restricted exogenously to be one in which offers *must* be made on *all* issues simultaneously, acceptance *must* be on *all* elements of the offer (or none) and no allocations can be made until all issues are decided. By construction, this model cannot address questions of agenda setting or agreement implementation.

Interestingly, these elements of "bargaining structure" are ones that negotiators identify as crucial to a "successful" negotiation. To negotiators, structuring bargaining means making decisions on questions like: Should easy issues be negotiated first or hard ones? Will early concessions improve my later bargaining position or weaken it? Should agreements be implemented only after all matters are settled or after some are settled? A hint of why decisions on these matters might be relevant can be had from the advice negotiators give. Some argue that bargaining should begin with "easy" issues so that quick settlement can build trust and "bargaining momentum". Others counsel starting with the hard issues as a way of conveying a "tough bargaining stance".[2] Some see concessions as a useful way of obtaining later, corresponding concessions "in the interest of fairness". Others warn that concessions may cause an opponent to conclude that one's bargaining position is weak.[3] Piecemeal implementation schemes are seen by some to enhance efficiency by reducing hold-up problems.[4] Others argue that hold-up speeds settlement, making piecemeal

[1]For a survey of these models, see Rubinstein (1987). See also, Roth (1985).

[2]See, Ramundo (1992) p. 162, and Lewis (1981) p. 224, 226. It's also argued that, if agreement can't first be reached on the hard issues, there's no sense in dealing with the easy ones.

[3]See, for example, Churchman (1995) p. 8.

[4]Supporters of the line-item veto often use this argument.

implementation inefficient.

Whatever one makes of these arguments, the message from the negotiation literature is clear: understanding negotiated outcomes means understanding both what determines agent choice of how to structure bargaining and how the environment determines what are "successful" choices. Doing so means moving beyond a single-pie model. This paper is a first step in this process. It builds on a small but growing literature on multiple issue bargaining. Within this literature, it has been known for some time that the agenda (whether issues are bargained all-at-once or one-by-one) can matter to the bargained outcome (see Kalai (1977) and Ponsati and Watson (1997) in cooperative settings and Herrero (1989), Fershtman (1990, 2000) and Busch and Horstmann (1997b) in non-cooperative settings). Typically, though, this literature imposes an agenda exogenously and so doesn't address the questions at hand.

Recently, several authors have endogenized the agenda and provided environments in which agenda other than the single-pie agenda arise in equilibrium (Bac and Raff (1996) and Busch and Horstmann (1999a) endogenize the agenda in asymmetric information environments while Busch and Horstmann (1999b), Inderst (2000), and Lang and Rosenthal (1998) do so in complete information settings). While providing valuable insights into the rôle of the agenda (and what leads to agenda other than the single-pie one), these papers don't address in a systematic way the questions of why order should matter, what orderings are more or less advantageous to a given party (and on what does this ranking depend) and how implementation rules matter.

The model we develop here permits us both to endogenize the agenda and to study in a systematic way agent choices on how to structure bargaining. The model allows for issues with different sizes of surplus and of differing complexities ("hard" vs. "easy") and permits the bargaining parties to offer on a subset of the outstanding issues. In this way, it can address the question of agenda setting, the order in which issues are bargained and the role of concessions. The model also considers both a bargaining process in which agreements are implemented as reached and one in which implementation occurs only after all issues have been settled. Thus, the consequences of different implementation schemes can also be analyzed.

One of the challenges for this analysis is providing a workable definition of "hard" and "easy" issues. Here we take the terms "hard" and "easy" in the practitioner literature to refer to the expected time to agreement: easy issues are ones for which agreement can be reached quickly while hard issues involve the possibility of extended bargaining.[5] Under this definition and within the framework of non-cooperative bargaining, an easy issue can be defined as one for which the bargaining parties have complete information about all aspects of the bargaining setting and there is a unique equilibrium. The pie-splitting model of Rubinstein (1982) is an example. A hard issue can be defined as one for which there is potential delay in reaching an agreement, either because of incomplete information about the bargaining setting,[6] or because there are multiple equilibria.[7] In either of these cases, agreement may only occur after a sequence of offers has been made and rejected.

Here, we adopt the asymmetric information approach to modeling hard issues. Specifically, we assume that there are two issues/pies, one with a surplus whose value is known to both bargaining parties - the easy issue - and one whose value is private information to one of the parties - the hard issue. Two implementation schemes are considered: i) sequential implementation in which the surplus from a given issue is allocated once agreement is reached on that issue and ii) simultaneous implementation in which neither surplus is allocated until agreement is reached on both issues.[8]

The bargaining process itself is modeled as an offer-counter-offer process in which

[5]The practitioner literature does not define these terms, taking them as self-evident, apparently.

[6]See Rubinstein (1985), Grossman and Perry (1986), Fudenberg, Levine and Tirole (1985) or the survey by Wilson (1987).

[7]In two player games this could be due to strategic disagreement actions (as in Haller and Holden (1990), Fernandez and Glazer (1991), Busch and Wen (1995)) or money burning (Avery and Zemsky (1994) Busch *et al.* (1998)).

[8]Ponsati and Wattson (1997) refer to sequential implementation as "independent implementation", seeking to focus on the fact that the issues are decoupled. We believe that "sequential" better captures the sequentiality of bargaining together with the "independence" of agreements within this sequential process. The term "independent" is likely to cause confusion with a process in which issues may be discussed independently and simultaneously as in Jun (1989).

an offer can be made either on only one of the two issues or on both issues simultaneously. As long as both issues are unallocated, players can make either type of offer after any kind of history. Offers must be accepted or rejected in their entirety: an agent cannot accept one part of an offer and reject the other part.[9]

In this setting, we show that the order in which issues are bargained is determined by three things: the implementation rule, the informed player's valuation of the hard issue and the initial beliefs of the uninformed player. An issue-by-issue bargaining agenda arises when a low-valuation informed player faces an opponent who believes him to be likely high-valuation. The informed player uses this agenda to signal type. The implementation rule determines the order in which issues are bargained by determining how signaling is effectively achieved. Under sequential implementation the "easy" issue is negotiated first. The low valuation type makes a concession on this issue to signal type, thereby obtaining concessions on the hard issue via the updating of beliefs. The reverse order would not accomplish such signaling since the bargain on the easy issue is independent of beliefs under sequential implementation.

Under simultaneous implementation, to the extent that order matters, the issue with large surplus is negotiated first. Since payoffs are delayed until agreement has been reached on both issues, the informed party's delay cost/valuation remains relevant even if only the easy issue remains to be settled. Successful signaling by the low valuation type now requires a sufficiently large concession on the first issue that it cannot be undone by a corresponding concession by the uninformed on the second issue. Essentially, the uninformed player must obtain all of his payoff from that first issue and have all of the second issue go to the informed player. As a result, size, not easy or hard, is important.

A comparison of the two implementation schemes reveals that, as long as signaling occurs under either scheme, sequential implementation dominates. Under either scheme, a buyer type has the same utility. This outcome results from the fact that,

[9]For a perfect information model in which parts of offers may be accepted, see Weinberger (1997).

in a signaling equilibrium, the high-valuation type must be indifferent between revealing type and mimicking the low-valuation type. The uninformed strictly prefers sequential implementation because there is less consumption delay in the signaling equilibrium.

The details of the model and the bargaining process are set out in Section 2. Section 3 provides our results on the bargaining equilibrium. Section 4 contains a discussion of the results and some concluding remarks. Proofs are collected in an Appendix.

2 Model Description and Notation

A buyer, B, and a seller, S, bargain over the price of two distinct, indivisible goods, X and Y. The seller's valuation (cost) for each good is common knowledge and normalized to zero. The buyer values good X at \$1; this valuation is common knowledge. The buyer's valuation for good Y, $\$V$, is private information for the buyer. It is common knowledge that $V \in \{v_l, v_h\}$, with $v_h > 1 > v_l$ and $1 > v_h - v_l$.[10] The seller's prior that $V = v_h$ is given by $\omega_0 \in (0,1)$; seller beliefs in period t of the game are denoted ω_t. The seller's beliefs are updated after receiving an offer from the buyer. The buyer and seller are risk neutral and both prefer agreement earlier rather than later. These features are captured by the standard assumptions that utilities are time separable and linear in money, with future dollars discounted by the (common) discount factor $\delta \in (0,1)$.

Bargaining is via alternating offers, with one offer per discrete time period $t = 1, 2, 3, \ldots$. The buyer is assumed to make the first offer. An offer at time t, P_t, is a price (pair of prices) to be paid for the transfer of the underlying good(s). As long

[10]The fact that $v_h > 1 > v_l$ means that Y is not guaranteed to provide a larger surplus than X. This makes bargaining on Y "hard" both because the surplus is uncertain *per se* and because it may be of greater or lesser economic importance than X. In this way, there is less ambiguity as to which issue is hard. The assumption that $v_h - v_l < 1$ guarantees that the returns to the v_h type from being perceived as the v_l type are not so large that he would be willing to turn over all of the surplus from X to the seller if doing so would convince the seller he is the v_l type. In this sense, this assumption guarantees that X is not a trivial issue to the buyer.

as agreement has been reached on neither good, offers can be made either on just one of the two goods or on both goods together. An offer on X alone is denoted by $P = p_x \in [0,1]$, an offer on Y alone by $P = p_y \in [0, v_h]$, and an offer on both X and Y by $P = (p_x, p_y)$.[11] Having received an offer, an agent can either accept it, denoted A, or reject it, R. Offers on both goods must either be accepted or rejected in their entirety: it is not possible for only one of the prices in an offer of (p_x, p_y) to be accepted. No restrictions are imposed on the type of offer as long as both issues are still outstanding. In this way, the order in which issues are bargained over and agreements are reached is determined endogenously as part of the bargaining equilibrium rather than imposed exogenously as part of the game tree. If a single price offer on some good is accepted, however, this agreement becomes binding and not renegotiable: further offers on this good are precluded.[12] The game ends as soon as an agreement on both goods exists.

A history in this game is a sequence of offers, P_t and accept/reject decisions, D_t. It is useful to distinguish among histories at the first and second information set within a period, denoted by subscripts $i \in \{1, 2\}$, as well as by which, if any, issues have been settled already, denoted by superscripts $j \in \{\emptyset, x, y\}$. So, for example, $H_1^{\emptyset}(t)$ denotes the set of all sequences $\{P_s, D_s\}_{s=1}^{t-1}$ with $D_s = R$ for all s; i.e., $H_1^{\emptyset}(t)$ denotes the set of all histories at the start of period t in which no offers have yet been accepted. Similarly, $H_2^y(t)$ is the set of period t histories with an offer on Y accepted some time in the past and the offer in period t made. The null history of the game is $H_1^{\emptyset}(0) = \emptyset$.

Strategies for B and S are maps from histories into price offers or accept/reject decisions. A pure strategy for B, f_B, is a sequence of functions $\{f_B(t)\}_{t=1}^{\infty}$ with $f_B(t)$:

[11] Since we consider unknown surplus size, share offers are precluded. The assumption of non-negative prices is equivalent to the common assumption of non-negative shares.

[12] This assumption is implicit in our definition of the offers, p_x, p_y. Within collective bargaining, a re-opening of a previously settled issue by one party is usually deemed to be "bad-faith bargaining". Our results suggest, more generally, that it may be in both parties' interests to commit to such a rule. The full analysis of a general model with renegotiation, while interesting, is beyond the scope of this paper.

$H_1^j(t) \times \{v_l, v_h\} \mapsto P^j$ if t is odd and $f_B(t) : H_2^j(t) \times \{v_l, v_h\} \mapsto D$ if t is even, where P^j is the set of feasible price offers given previous agreement of type j. Similarly, a strategy for S, f_S, is a sequence of functions $\{f_S(t)\}_{t=1}^{\infty}$ with $f_S(t) : H_1^j(t) \mapsto P^j$ if t is even and $f_S(t) : H_2^j(t) \mapsto D$ if t is odd.

Any two strategies (f_B, f_S) lead to an outcome of the game. An outcome can either be i) an agreement on X at time t of p_x and an agreement on Y at time τ of p_y; ii) an agreement on $Y(X)$ at time t of $p_x(p_y)$ and no agreement on $X(Y)$; iii) no agreement on either X or Y. Payoffs in each of outcomes i) and ii) depend on the rules by which agreements are implemented, while for outcome iii) the payoffs for B and S are assumed to be zero. Two implementation rules are considered. In one, implementation is sequential, allowing for exchange as soon as agreement is reached on a particular good and regardless of whether agreement is ever reached on the other good. In the other, implementation is simultaneous, so that all exchange takes place only after (and only if) agreement has been reached on *all* issues. We use the convention that $t(\tau) = \infty$ if no agreement is reached on $X(Y)$).

Definition 1 (Sequential Implementation)
Under **sequential implementation** *exchange of a given good takes place at the time of agreement on a price for that good. Agents' utilities from the strategy pair* (f_B, f_S) *leading to agreements* (p_x, t) *and* (p_y, τ) *are:*

$$\begin{aligned} U_B(f_B, f_S) = u_B((p_x, t), (p_y, \tau)) &= \delta^{t-1}(1 - p_x) + \delta^{\tau-1}(V - p_y), \\ U_S(f_B, f_S) = u_S((p_x, t), (p_y, \tau)) &= \delta^{t-1}p_x + \delta^{\tau-1}p_y. \end{aligned}$$

Definition 2 (Simultaneous Implementation)
Under **simultaneous implementation** *exchange of no good takes place unless agreement has been reached on the prices of all goods. Agents' utilities from the strategy pair* (f_B, f_S) *leading to agreements* (p_x, t) *and* (p_y, τ) *are:*

$$\begin{aligned} U_B(f_B, f_S) &= u_B((p_x, t), (p_y, \tau)) = \delta^{\max\{t,\tau\}-1}(1 - p_x + V - p_y), \\ U_S(f_B, f_S) &= u_S((p_x, t), (p_y, \tau)) = \delta^{\max\{t,\tau\}-1}(p_x + p_y). \end{aligned}$$

As is well known, bargaining games of this sort have a large number of Perfect Bayesian Equilibria. In what follows, we focus attention on a subset of these equilibria. This subset is defined by a set of three belief restrictions similar to those imposed in Rubinstein (1985). While we believe these restrictions are plausible, their main function is to guarantee that the equilibria displayed subsequently are robust in the sense of not being supported either by some carefully constructed combination of "strange" off-equilibrium path beliefs or by some complicated structure of punishment paths that exploit a multiplicity of equilibria.[13]

The first restriction is identical to one used in Rubinstein (1985) and relates to the behavior of beliefs at zero and one. It is:

Assumption 1 *If* $\omega_t = 0(1)$ *then* $\omega_{t+k} = 0(1)\ \forall k = 1, 2, \ldots$

For the next two restrictions, some additional notation is required. Let $f = (f_B, f_S)$ be a Perfect Bayesian Equilibrium strategy pair. Consider any period, t, with history $h_1^j(t) \in H_1^j(t)$, $j = \emptyset, x$, such that $0 < \omega_{t-1} < 1$. For t odd, let $P_t(V) = f_B(h_1^j(t), V)$ be the buyer's equilibrium price offer and $\widehat{P}_t \neq P_t(V)$ be a deviating price offer. For t even, let $f_B(h_2^j(t), V) = A$ be the buyer's equilibrium accept decision and let a deviation be "reject" at t followed by the price offer $\widehat{P}_{t+1}$ at $t+1$. Let the buyer's expected utility at date t along the equilibrium continuation path be given by $U_B(f, h_1^j(t)) = U_B(f_B|_{(h_1^j(t), P_t(V))}, f_S|_{(h_1^j(t), P_t(V))})$, t odd and $U_B(f, h_2^j(t)) = U_B(f_B|_{(h_2^j(t), A)}, f_S|_{(h_2^j(t), A)})$, t even. Define $\widehat{U}_B(\widehat{f}, [h_1^j(t), \widehat{P}_t], \omega_t, t)$ as the buyer's expected utility at date t, t odd, under any Perfect Bayesian Equilibrium continuation, $\widehat{f}$, after history $[h_1^j(t), \widehat{P}_t]$ and with associated beliefs satisfying Assumption 1 and being given by ω_t at t. Define $\widehat{U}_B(\widehat{f}, [h_2^j(t), R, \widehat{P}_{t+1}], \omega_{t+1}, t)$ similarly for t even. In the current setting, the utility for the buyer in any continuation is maximal when

[13] As to the former, because our belief restrictions are stronger than those imposed by Cho-Kreps (1987) or Cho (1987), the equilibria we identify would also be equilibria under these weaker belief restrictions. Regarding the latter, Rubinstein (1985) can be used to show that there is a unique equilibrium to our game whenever it is an "as if" single pie setting. This uniqueness property is valuable for our subsequent uniqueness results.

$\omega_t(\omega_{t+1})$ equals zero. Consequently, we define a deviation for the buyer of type v_k as bad at t odd if $U_B(f, h_1^j(t)) \geq \widehat{U}_B(\widehat{f}, [h_1^j(t), \widehat{P}_t], \omega_t = 0, t)$ for $V = v_k$. A similar definition holds for t even.

The remaining two restrictions impose conditions on beliefs for deviating offers of the types described above. These restrictions are:

Assumption 2 *Consider any t and history $h_i^j(t)$ such that $0 < \omega_{t-1} < 1$. If the deviation $\widehat{P}_t$ is bad for the buyer of type $v_h(v_l)$ and not bad for the buyer of type $v_l(v_h)$, then $\omega_t = 0(1)$ after such a deviation. The same applies for the deviation $R, \widehat{P}_{t+1}$*

Assumption 3 *Consider any t and history $h_i^j(t)$ such that $0 < \omega_{t-1} < 1$. Suppose that, for a deviation $[R, \widehat{P}_{t+1}]$, $\widehat{U}_B(\widehat{f}, [h_2^j(t), R, \widehat{P}_t], \omega_{t+1} = \omega_{t-1}, t) > U_B(f, h_1^j(t))$ for both V and for any Perfect Bayesian Equilibrium, $\widehat{f}$, such that i) the associated beliefs satisfy Assumptions 1 and 2 and ii) the time to final agreement is no shorter for either type and strictly longer for at least one type. Then belief updating must be such that $\omega_{t+1} \leq \omega_{t-1} = \omega_t$. If the same conditions hold for deviation $\widehat{P}_t$, then $\omega_t \leq \omega_{t-1}$.*

The second belief restriction is similar to that in Admati and Perry (1987) and, in our context is the analogue of the intuitive criterion of Cho (1987) and Cho and Kreps (1987).[14] The last assumption restricts the amount of seller optimism that is allowed in the updating of beliefs. It requires that delay cannot convince the seller that he is more likely facing the impatient (high valuation buyer). This restriction is the analogue of Rubinstein's (1985) assumption B-2.

We also impose a restriction on the values of the key parameters of the environment, (δ, v_h, v_l). This restriction, again, is one found in Rubinstein (1985). It serves to rule out signaling via pure delay when bargaining is restricted to be only

[14]Analogous to Admati and Perry, Assumption 2 is actually more stringent than the one in Cho (1987). We also note that, for those uncomfortable with Assumption 1, this assumption and Assumption 2 could be replaced by (A2) in Admati and Perry (1987) without affecting the results. The use of the current set of assumptions simply allow us to re-produce the results from Rubinstein (1985) without further proof.

joint-offer bargaining. We maintain this assumption, in part, to demonstrate that an issue-by-issue agenda can serve a signaling role in situations in which signaling is not otherwise possible.[15] The restriction is:

Assumption 4 $\delta(v_h - v_l) > (1 - \delta^2)(1 + v_h)$

The essence of this assumption is that, when $V = v_l$, the buyer cannot expect to pay the full-information price $(\delta(1+v_l)/(1+\delta))$ simply by delaying agreement. Also, the seller cannot expect to screen via delay simply by making offers that give the different buyer types their full-information prices.

Finally, we also assume as in Rubinstein (1985) that neither buyer nor seller makes an offer that is surely rejected. This restriction embodies the notion of "bargaining in good faith", a requirement commonly contained in labor laws governing collective bargaining. The term "equilibrium" in what follows refers to a pure strategy Perfect Bayesian Equilibrium that satisfies this restriction as well as Assumptions 1-3 in settings for which Assumption 4 is satisfied.

3 Bargaining With an Agenda

Before proceeding to an analysis of the equilibrium with an endogenous agenda, we provide, as a benchmark, the equilibrium for this game when all offers are restricted to be joint offers. In this case, the bargaining problem is a standard one-sided, incomplete information game in which the surplus is either $S_h := (1 + v_h)$ or $S_l := (1 + v_l)$. The equilibrium for this problem is identical to that in Rubinstein (1985) or Grossman and Perry (1986). We re-state the result here as:

Proposition 1 (Rubinstein/Grossman and Perry) *In the joint offers bargaining game with priors ω_0:*

[15] Bac and Raff (1996), for example, do not maintain this assumption and subsequently confuse signaling with cost savings under sequential implementation (see Busch and Horstmann (1997a).)

(i) (screening) If $\omega \geq \omega^* = S_l/S_h$, both buyer types make the same offer of $p = \delta\omega S_h/(1+\delta)$ which the seller accepts. In subgames in which the seller offers, the offer is $p = (1-\delta)S_h + \delta^2\omega S_h/(1+\delta)$. This offer is accepted by the v_h type and rejected by the v_l type.

(ii) (no screening) If $\omega \leq \omega^*$, both buyer types make an offer of $p = \delta S_l/(1+\delta)$, which the seller accepts. In subgames in which the seller offers, the offer is $p = S_l/(1+\delta)$ which both buyers accept.

One key feature of this equilibrium is that buyer types always pool on their offers. As will be shown below, this feature of equilibrium doesn't persist when partial offers are possible. The precise structure of the equilibrium will depend on the implementation rule. We consider each case in turn, starting with sequential implementation.

3.1 Sequential Implementation

In order to characterize properties of the equilibria of this game, we proceed by establishing a sequence of lemmas which restrict the possible outcomes along any equilibrium path. These lemmas show that: i) there are no equilibria in which the buyer offers first on Y, the hard issue, and then X; ii) the low valuation type can signal by offering first on X, the easy issue, with signaling accomplished through a large price concession on X; iii) for large enough values of ω_t, there are no equilibria in which either the seller or the high valuation type buyer make partial offers. Together these results establish that, when the seller believes the buyer to be likely a high valuation type and with sequential implementation, an issue-by-issue bargaining agenda can be observed, the order of issues can only be easy then hard and a concession on the easy issue is used as a means of signaling that the buyer is the low valuation type.

We begin the process of establishing these results by examining buyer strategies. The first lemma rules out equilibria in which the buyer's strategy involves both types pooling on a sequence of partial offers beginning with Y. This lemma and Lemma 2 to follow establish items i) and ii) above. In all of the lemmas to follow, it is assumed that ω_t is strictly between zero and one.

Lemma 1 *There exist no equilibrium strategy pairs, $f^* = (f_B^*, f_S^*)$, with the property that, if no agreement has been reached on either issue at t, both buyer types make the same price offer on Y alone at t; i.e., it cannot be that $f_B^*(h_1^\emptyset(t), v_h) = f_B^*(h_1^\emptyset(t), v_l) = P_t$ with $P_t = p_y$.*

The reason that pooling cannot occur here is that, unless the offer p_y is very small, it pays the v_l type to deviate to an offer on X only. This offer, while profitable for the v_l type, results in a lower payoff to the v_h type even if posterior beliefs are set to 0 after the deviation. While (A4) implies that delay alone is not enough to separate types, a partial offer on X allows the v_l type to make a price concession that, combined with delay on Y, makes separation possible. Such a deviation is successful even for $p_y < \delta v_l/(1+\delta)$, the lowest possible price offer on Y only acceptable to the seller.

The above suggests both that the v_l type can reveal his valuation through a sequence of partial offers and that, if he does so, they must be offers beginning with X. The following lemma confirms this intuition and describes the features of such strategies.

Lemma 2 *If there is an equilibrium, f^*, such that the buyer types separate via price offers at date t, then it must be that:*

i) no agreement has been reached on either issue prior to t: $h_1(t) \in H_1^\emptyset(t)$,

ii) the v_h type makes a joint offer of $\delta(1+v_h)/(1+\delta)$ which the seller accepts: $f_B^*(h_1^\emptyset(t), v_h) = (p_x, p_y)$ with $p_x + p_y = \delta(1+v_h)/(1+\delta)$,

iii) the v_l type makes a partial offer on X of $p_x^l \in [\delta/(1+\delta), 1]$: $f_B^*(h_1^\emptyset(t), v_l) = p_x^l$. The value of p_x^l is uniquely determined by the equality $\frac{1+v_h}{1+\delta} = 1 - p_x^l + \delta\left(v_h - \frac{v_l}{1+\delta}\right)$.

iv) The seller accepts the partial offer and then offers a price, $p_y = v_l/(1+\delta)$ on Y at $t+1$: with history $h_1^x(t+1) = (h_1^\emptyset(t), p_x^l, A)$, $f_S^*(h_1^x(t+1)) = v_l/(1+\delta)$.

The equation that defines p_x^l gives the smallest price offer on X alone for which separation can occur. The important features of p_x^l are that: i) the seller prefers a strategy of accepting p_x^l and offering $v_l/(1+\delta)$ on Y over one of rejecting p_x^l and countering with a joint offer of $(1+v_l)/(1+\delta)$ $(p_x^l > \delta/(1+\delta))$; ii) the v_h type is indifferent between a joint offer of $\delta(1+v_h)/(1+\delta)$ and mimicking the v_l type. The v_l type signals both by inducing delay and making a price concession on X. Because signaling is only relevant for the bargain over Y, Y bargaining must occur second.

It follows from Lemmas 1 and 2 that there can be no equilibria, either pooling or separating, in which the buyer's strategy involves partial offers beginning with Y (item i) above). Further, when $V = v_l$, the buyer can signal type but only by making partial offers beginning with X (item ii) above). In brief, the buyer never bargains on hard issues first but may choose to bargain sequentially starting with the easy issue as a means of signaling bargaining strength. The reader should note that these results rely only on the belief restrictions given by (A1) and (A2).

A final possibility is that buyer types pool on a joint offer, as in Proposition 1. An argument analogous to that for Lemma 1 shows that this outcome can arise only if the price offer is quite low. For higher prices, the v_l type gains by a deviation to partial offers beginning with X.

Lemma 3 *If there is an equilibrium, f^*, such that the buyer types pool on a joint offer at date t — $f_B(h_1^0(t), v_h) = f_B(h_1^0(t), v_l) = P_t$ with $P_t = (p_x, p_y)$ — then this offer must be such that $p_x + p_y \leq \overline{p} = (1-\delta)v_l + \delta(1+v_l)/(1+\delta)$.*

The price $\overline{p}$ corresponds to the pooling price offer of Proposition 1 when beliefs for the seller are $\omega^P = \omega^* + [(1-\delta^2)v_l]/[\delta(1+v_h)] < 1$ (by (A4)). The implication of the lemma is that, for beliefs $\omega_t \geq \omega^P$, the buyer strategy of Proposition 1 is not part of an equilibrium. In fact, there can be no equilibria in which buyer types pool on a joint offer at a price above $\overline{p}$.

Turning to the seller, results similar to those for the buyer can be shown to hold when the seller makes offers that both buyer types accept in equilibrium. Specifically:

Lemma 4 *If there is an equilibrium, f^*, such that, at t even either i) no agreement has been reached on either issue and the seller makes a joint offer that both types accept: $f_S^*(h_1^\emptyset(t)) = P_t = (p_x, p_y)$, $f_B^*((h_1^\emptyset(t), P_t), V) = A$ both V ii) no agreement has been reached on either issue and the seller makes a partial offer on Y that both types accept $f_S^*(h_1^\emptyset(t)) = P_t = (p_y)$, $f_B^*((h_1^\emptyset(t), P_t), V) = A$ both V, iii) an agreement has been reached on X only and the seller makes an offer on Y that both types accept: $f_S^*(h_1^x(t)) = P_t = (p_y)$, $f_B^*((h_1^x(t), P_t), V) = A$ both V, then*

i) in case i) $p_x + p_y \leq (1 + v_l) / (1 + \delta)$;

ii) in case ii) $p_y \leq (1 + v_l - \delta^2) / (1 + \delta)$;

iii) in case iii) $p_y \leq v_l / (1 + \delta)$.

As with pooling offers by the buyer, offers by the seller that induce no information revelation must involve low prices. Otherwise, rather than accepting the offer, the v_l type buyer can reject and counter with an offer that signals type (i.e., an offer that the v_l type prefers but the v_h type does not) and that the seller accepts. Since these prices yield the seller utility of at most $(1 + v_l)/(1 + \delta)$, he offers them only if ω_t is small. Then, since joint offers yield the same utility as partial offers beginning with Y, the seller has no strict incentive to utilize the latter.

If ω_t is large enough, the seller also uses only joint offers. In this case, though, the offers are used to screen buyer types (as in Proposition 1). This result is given below.

Lemma 5 *Let $\underline{v} = v_h^2 / (1 + 2v_h)$. If a strategy pair, f^* is an equilibrium, then, whenever i) agreement has been reached on neither issue — $h_1(t) \in H_1^\emptyset(t)$, and ii) $\omega_t \geq \omega^P$, the seller's strategy must specify joint offers only: $f_S(h_1^\emptyset(t)) = P_t = (p_x, p_y)$.*

The essence of the proof here is as follows. The seller has a strict disincentive for using partial offers in the sense that, whatever he can accomplish with partial offers (in terms of screening or pooling the buyer types) he can also accomplish with joint offers yielding him strictly higher utility. The only way that partial offers by the seller can be supported, then, is if a deviation to an equivalent joint offer is rejected by the buyer. For large enough values of ω_t (and for v_l not too small), there are no continuations that the buyer prefers to accepting the joint offer. As a result, rejecting the offer is not equilibrium behavior.

Together, these results imply that, for large enough values of ω_t, there can be no equilibria in which either the buyer or the seller use partial offers beginning with Y. We show finally that there is, in fact, a unique equilibrium in this case, involving partial offers by the v_l type buyer beginning with X. This equilibrium is the separating one described in lemma 2 above.

Proposition 2 *If $v_l \geq \underline{v}$ and given Assumptions 1-4, there exists a unique equilibrium to the bargaining game with sequential implementation for all initial beliefs $\omega_0 \geq \omega^P$. This equilibrium is a separating one and is defined by:*

$f_B(h_1^{\emptyset}(1), v_h) = P_1^h = (p_x + p_y) = \delta(1 + v_h)/(1 + \delta);$

$f_B(h_1^{\emptyset}(1), v_l) = P_1^l = p_x = \delta(1 + v_h - v_l)/(1 + \delta) - (1 - \delta)v_h.$

For the seller, $f_S((h_1^{\emptyset}(1), P_1^h)) = f_S((h_1^{\emptyset}(1), P_1^l)) = A.$

In the case of the offer P_1^l, the seller responds next period with an offer on Y of $v_l/(1 + \delta)$ which the buyer accepts.

There are several points worth noting here. First, an issue-by-issue agenda is a valuable signaling device when the seller believes that the buyer is in a weak bargaining position (ω_0 large) when in fact he is not ($V = v_l$). Signaling is accomplished by the v_l type making a price concession (offering a high price) on X only.

Second, there are no equilibria in which the buyer adopts an issue-by-issue agenda beginning with Y. Combined with the first observation, we have that, for sequential implementation, the prescription is that a strong informed type should always bargain on (and make a concession on) the easy issue first to signal bargaining strength. The uninformed and the informed weak type should always makes joint offers. This prescription holds when the uninformed believes his opponent to be weak.

Note finally that, for values of ω_0 sufficiently small, there is also a unique equilibrium involving joint offers of $(1+v_l)/(1+\delta)$ and $\delta(1+v_l)/(1+\delta)$ by the seller and buyer respectively. For intermediate values of ω_0, $\omega_0 \in [\omega^S, \omega^P)$ for instance, there are multiple equilibria.[16] The separating equilibrium of Result 2 continues to be an equilibrium as is the screening equilibrium of Result 1. Because of these two equilibria, it may be possible to construct a third equilibrium in which the seller screens using partial offers beginning with Y. This outcome would be supported by a "threat" to switch to the Result 1 equilibrium should the seller deviate to a joint screening offer. If strategies were additionally restricted to stationary Markov strategies, then no equilibrium involving partial offers by the seller would be supportable.

3.2 Simultaneous Implementation

The key to understanding the differences in bargaining strategy between simultaneous and sequential implementation is understanding how these two rules affect partial agreements. By delaying consumption of one good until agreement is reached on both, simultaneous implementation changes the nature of partial agreements in two crucial ways. First, even if Y is settled before X, the buyer's delay cost continues to depend on the value of V. This fact means that the distinction between "hard" and "easy" is no longer relevant in determining the order of issues. In essence, bargaining on either issue is "hard" given that the information asymmetry persists until agreement is achieved on both.

Second, a low price offer on the first good only represents a "concession" by the buyer if it cannot be undone (in utility terms) by a corresponding seller concession on the second.[17] Put differently, successful signaling now requires larger concessions on the first issue. This requirement means that the order of issues still may matter but that now it is the size of surplus that is relevant. Specifically, when the necessary

[16]The value of ω^S is defined in the appendix in the proof of Proposition 2.

[17]This point is made in Busch and Horstmann (1999a) in the context of a bargaining model in which one agent's discount factor is the source of private information.

concession for signaling is greater than the total surplus from one of the issues, this issue cannot be agreed upon first.

To proceed formally, assume that a price for one of the goods, p_1, has been offered by the buyer and agreed to by the seller and that this price reveals that $V = v_l$. Then the follow-up offer by the seller, p_2^s, and the reply by the buyer, p_2^b, must be such that

$$\begin{aligned} 1 + v_l - p_1 - p_2^s &\geq \delta(1 + v_l - p_1 - p_2^b), \\ p_1 + p_2^b &\geq \delta(p_1 + p_2^s). \end{aligned}$$

These two inequalities imply that $p_2^s = (1+v_l)/(1+\delta) - p_1$, the full information price for the v_l type less the already agreed upon price p_1. Were offers unrestricted, the outcome would be the full-information price, one period delayed. This price cannot be part of a signaling equilibrium, however, by (A4). Therefore, if signaling is to occur, it must be that the non-negativity constraint binds ($p_2^b = 0$), implying that $p_2^s = (1 + v_l - p_1)(1 - \delta)$. This outcome occurs if $p_1 \geq \delta(1 + v_l)/(1 + \delta)$; that is, if the price already agreed upon is at least as large as the uninformed player's full-information payoff when bargaining against v_l. For this price to be feasible, the issue bargained on initially must have surplus of at least this value. This constraint is the one that determines the minimum size of the initial issue.

In the interest of brevity, and since the basic ideas have been demonstrated in the previous section, we will not provide a similar list of lemmas for this case.[18] The reader can verify that, as before, the v_l type has an incentive to deviate to a partial offer from a joint offer if the joint offer is sufficiently large relative to his full information price. Signaling arises for large enough values of ω_t, with the v_h type making a joint offer that is his full information price and the v_l type making a sequence of partial offers involving a concession on the first issue. As previously, the seller always has an incentive to make joint offers and any offer that is accepted by both buyer types

[18]The essentials of the proofs for the analogues of lemmas 2 and 3 can be found in Busch and Horstmann (1999a). The proof for the analogue of lemma 4 is as in Rubinstein (1985). There is no analogue of lemma 1 for this case. Rather, the possibilities associated with buyer pooling on the first issue are covered by the other lemmas.

yields a low total payment to the seller.

The result for simultaneous implementation is:

Proposition 3 *Under Assumptions 1-4 and with simultaneous implementation, there exists an essentially unique equilibrium to the bargaining game for all initial beliefs* $\omega_0 > \omega^{P_2} = \omega^*(1+\delta(1-\delta))/\delta$. *This equilibrium is a separating one and is defined by:*

i) $f_B(h_1^{\emptyset}(1), v_h) = P_1^h = p_x + p_y = \delta(1+v_h)/(1+\delta)$;

ii) $f_B(h_1^{\emptyset}(1), v_l) = P_1^l = p_1 = (1+v_l) + [(v_h - v_l)/\delta] - [(1+v_h)/(\delta^2(1+\delta))]$.

iii) For the seller, $f_S((h_1^{\emptyset}(1), P_1^h)) = f_S((h_1^{\emptyset}(1), P_1^l)) = A$. *In the case of the offer* P_1^l, $f_S((h_1^{\emptyset}(1), P_1^l, A)) = P_2^S = (1-\delta)\left[\frac{1+v_h}{\delta^2(1+\delta)} - \frac{v_h - v_l}{\delta}\right]$ *and* $f_B((h_1^{\emptyset}(1), P_1^l, A, P_2^S), v_l) = A$. *The identity of the issue on which the first offer is made is indeterminate if* $p_1 < v_l$.

As with sequential implementation, signaling occurs when ω_0 is large and is accomplished through an initial high price offer by the v_l type on one of the goods. If both goods generate surplus greater than p_1, the order in which goods are bargained is indeterminate — the v_l type can offer first on either X or Y. Should Y have surplus smaller than p_1, then the v_l type has a strict preference for an agenda that involves bargaining on X first. In this case, though, it's not because X is easy and Y hard but simply because Y's surplus is too small.

Note also that, unlike sequential implementation, there is no minimum size restriction imposed on v_l. The reason is that, for fixed p_x, p_y, buyer types are indifferent between the agenda X then Y and the agenda Y then X with simultaneous implementation. With sequential implementation, by contrast, the v_l type prefers the former agenda while the v_h type prefers the latter. The restriction on v_l arises because of this conflict under sequential implementation.

4 Discussion and Concluding Remarks

We have shown that, independent of the implementation rule, the choice of agenda can transmit information and that concessions on early issues are the means by which signaling is accomplished. This signaling function is relevant when an agent is believed to be in a weak bargaining position when in fact he is not. The willingness to delay agreement embodied in a piecewise agenda coupled with a concession on the first issue signals strength.

Our results also show that, when issue-by-issue bargaining is used, the order of issues is relevant. If agreements are implemented sequentially, then "easy" issues go first. If implementation of agreements occurs only at the conclusion of bargaining, no "natural" order exists if the surpluses from both issues are sufficiently large. If, however, the surplus from one issue is small, then signaling can only occur by bargaining on the large surplus first. Thus, the size of the surplus determines the order of issues when implementation is simultaneous, with the large surplus going first when order matters.

The ability to signal in this latter case does rely on the non-negative price restriction (the signaling result for the former case does not). If negative prices could be agreed upon but total payments were limited to the value of the items, then signaling could still occur but would require initial price agreements much larger than the value of both issues. If there were no constraint on valid price offers (and players have outside wealth), then signaling could not occur at all. Since the seller prefers the signaling equilibrium to the pooling/screening equilibrium in the absence of agenda signaling, this fact might explain a restriction to non-negative price offers: it is in at least one party's interest.[19]

In the language of the negotiation literature, our analysis suggests that: i) Concessions on early issues serve to improve one's later bargaining position — concessions

[19] Compare the seller's payoff under Prop^n for large ω to that under Prop^n 3: $\delta\omega S_h/(1+\delta) < \omega(\delta S_h/(1+\delta)) + (1-\omega)\delta((\delta(1+\delta)-1)S_h)/(1+\delta)$ if $\omega < \omega + (1-\omega)(\delta(1+\delta)-1))$, which is true.

signal bargaining strength. ii) One should bargain on "easy" issues first if implementation is sequential. Doing so builds "bargaining momentum" in that the bargain on the hard issue is transformed into an easy bargain via the signaling. iii) One should bargain on issues with large surplus first under simultaneous implementation. Whether these issues are hard or easy is irrelevant. iv) When bargaining is issue-by-issue, sequential implementation should be adopted over simultaneous implementation — the former represents a Pareto improvement over the latter.

To see this final point, consider initial beliefs such that the signaling equilibrium (Results 2 and 3) occurs under either rule. The informed agent, in this case, is indifferent between simultaneous and sequential implementation. When $V = v_h$, the informed obtains his full-information payoff regardless of the form of implementation. If $V = v_l$, the payoff is such that the v_h type is just indifferent between taking his equilibrium payoff and mimicking the v_l type. The only difference between the two agents' evaluations of any given pair of prices arises from the price for good Y, consumption of which is delayed by one period under either implementation procedure.[20] The uninformed agent strictly prefers sequential implementation since it allows him to allocate less total surplus to the informed while still achieving the required indifference for the v_h type. This is due to the ability to "consume as you go" under sequential implementation.

5 Appendix

Proof of Lemma 1: Suppose there were an equilibrium strategy pair f^* such that $f_B^*(h_1^\emptyset(t), v_h) = f_B^*(h_1^\emptyset(t), v_l) = P_t$ with $P_t = p_y$ and $f_S^*(h_1^\emptyset(t), P_t) = A$. In this case, the continuation outcome is unique and involves an offer by the seller on X of $1/(1+\delta)$ at $t+1$ which is accepted. As result of this latter fact, any equilibrium offer, p_y, is

[20]The payoffs to the high valuation agent under sequential and simultaneous implementation are $1 - p_x + \delta(v_h - p_y)$ and $\delta(1 + v_h - p_1 - p_2)$. They can be rewritten as $1 - p_x + \delta(v_l - p_y) + \delta(v_h - v_l)$ and $\delta(1 + v_l - p_1 - p_2) + \delta(v_h - v_l)$ respectively, which differ from the low valuation agent's payoffs by the constant amount $\delta(v_h - v_l)$.

bounded below by $\underline{p}_y = \delta v_l/(1+\delta)$. Any lower offer could be rejected by the seller and countered with the offer $(1+v_l)/(1+\delta)$ on both issues, which will be accepted by both informed.

Now consider a deviation by the v_l type using a sequence of partial offers starting with X. The highest possible continuation payoff to the informed arises if the uninformed makes the offer $v_l/(1+\delta)$ on Y. Thus, if there exists a $p_x \in [0,1]$ that is accepted by the seller and is such that

$$v_h - p_y + \frac{\delta^2}{1+\delta} \geq 1 - p_x + \delta\left(v_h - \frac{v_l}{1+\delta}\right) \quad (1)$$

and $$v_l - p_y + \frac{\delta^2}{1+\delta} < 1 - p_x + \delta\left(v_l - \frac{v_l}{1+\delta}\right) \quad (2)$$

beliefs will be updated to $\omega_{t+1} = 0$ by (A2) and stay at zero by (A1) for all continuation paths. The seller accepts the offer p_x, if it yields higher utility than rejecting and countering with the price $(1+v_l)/(1+\delta)$. This requires $p_x \geq \delta/(1+\delta)$. The smallest p_y such that the v_l type finds this deviation profitable is given by the p_y for which (2) holds as an equality when $p_x = \delta/(1+\delta)$; this is $p_y = \delta v_l/(1+\delta) + (1-\delta)(v_l - 1) < \underline{p}_y$ since $v_l < 1$.

Since $p_x < 1$, the largest possible value of p_y for which the deviation is possible is given if (1) holds as an equality when $p_x = 1$. This yields $p_y \leq v_h/(1+\delta) - \delta(\delta(v_h - 1) + v_l)/(1+\delta)$ which is clearly less than the largest pooling offer on Y. Thus the deviation is always possible. qed

Proof of Lemma 2: Suppose that $f_B^*(h_1^\theta(t), v_h) = P_t, f_B^*(h_1^\theta(t), v_l) = p_x \neq P_t$ and that P_t is not a joint offer with $p_x + p_y = \delta v_h/(1+\delta)$. Since $\omega_{t+1} = 1$ after observing P_t, the seller will accept only joint offers $p \geq \delta(1+v_h)/(1+\delta)$, or partial offers $p_x \geq \delta/(1+\delta)$ and $p_y \geq \delta v_h/(1+\delta)$. Since, for these values

$$1 + v_h - \delta\frac{1+v_h}{1+\delta} > \max\left\{1 - p_x + \delta\left(v_h - \frac{v_h}{1+d}\right),\ v_h - p_y + \delta\left(1 - \frac{1}{1+\delta}\right)\right\},$$

the v_h type will offer $\delta(1+v_h)/(1+\delta)$ in equilibrium.

For the v_h type to follow this strategy, it must be better than following the v_l type's strategy. This requires that $1 - p_x + \delta\,(v_h - v_l/(1+\delta)) \leq (1+v_h)/(1+\delta)$, or $p_x \geq \delta(1+v_h-v_l)/(1+\delta)-(1-\delta)v_h$. For any such offers on X the seller will set $\omega_{t+1}=0$. The p_x for which strict equality holds is the only value that survives refinement via (A2). For the seller to accept p_x (rather than reject and counter-offer $(1+v_l)/(1+\delta)$ on both,) $p_x \in [\delta/(1+\delta),1]$. Further $p_x \leq 1$ if $\delta(v_h-v_l) < 1+(1-\delta^2)v_h$, which it is given $v_h - v_l < 1$. qed

Proof of Lemma 3: Consider a joint offer $p=(p_x,p_y)$ and a deviation by the v_l type using a sequence of partial offers. Repeating the arguments in the proof of Lemma 1, with p substituted for p_y, one finds that the smallest p such that the v_l type finds deviation profitable is given by $p=\delta(1+v_l)/(1+\delta)+(1-\delta)v_l$. The largest possible value of p in any pooling equilibrium is $(1+v_h)/(1+\delta)$. Thus, for all p such that $\delta(1+v_l)/(1+\delta)+(1-\delta)v_l \leq p \leq (1+v_h)/(1+\delta)$, the deviation to p_x is successful for the v_l type as long there exists a $p_x \leq 1$ that satisfies the analogue of equation (1). Such is the case as long as $\delta(v_h-v_l) < 1+(1-\delta)v_l$. This inequality is satisfied given $v_h - v_l < 1$. qed

Proof of Lemma 4: This result follows from Rubinstein's (1985) Prop^n 5. qed

Proof of Lemma 5: The proof consists of three parts. Part i) shows that there is no equilibrium in which the seller screens with a partial offer beginning with X. Part ii) shows that, for $\omega_t \geq \omega^P$ and v_l not too small, there is no equilibrium in which the seller screens with a joint offer while the buyer types pool. Part iii) shows that, if the seller makes a partial offer and the buyer types pool, the seller has an incentive to deviate to a joint offer. Combined, these three parts then prove the lemma.

i) no screening with an offer on X only :

Let U_s^s denote the present value of the seller's utility under the proposed equilibrium should the buyer be of type v_h, neither issue is settled and it's the seller's turn to offer. U_s^b denotes the seller's equilibrium utility when it's the buyer's turn to offer, neither issue has been settled and the buyer is using a pooling strategy. In both cases, these values are the present values of equilibrium price paths.

To be an equilibrium the v_h type must be indifferent between accepting the seller's screening offer and rejecting and countering with the proposed pooling offer. Otherwise, by (A2) the v_l type could reject and demand a price below the proposed equilibrium which the seller would accept. The v_l type strictly prefers to reject the screening offer. If the seller screens with a partial offer beginning with X, it must then be that the price offers satisfy the conditions: $1 + \delta v_h - U_s^s = \delta + \delta v_h - \delta U_s^b$ and $1 + \delta v_l - U_s^s < \delta + \delta v_l - \delta U_s^b$. These conditions cannot both hold simultaneously, however.

ii) no screening with joint offers:

Lemmas 1 and 3 establish that, for $\omega_t \geq \omega^P$, the only possible equilibrium with seller screening using a joint offer must have the buyer making a pooling offer beginning with X. As above, any equilibrium of this sort must satisfy the condition $1+v_h-U_s^s = \delta\left(1+\delta v_h - U_s^b\right)$; equivalently, $U_s^s = \delta U_s^b + (1+(1+\delta)v_h)(1-\delta)$. The lowest discounted price stream that the buyer could extract in any such equilibrium must satisfy the condition $U_s^b = \delta[\omega_t U_s^s + \delta(1-\omega_t)U_s^b]$. This implies $U_s^b = (\delta\omega_t U_s^s)/(1-\delta^2(1-\omega_t))$. Substituting for U_s^b in the above yields $U_s^s = [1-\delta^2(1-\omega_t)][1+v_h(1+\delta)]/(1+\delta)$. This offer yields utility for the v_h type buyer of $U_b^h = \delta + \delta^2(1-\omega_t)v_h - (\delta^2\omega_t)/(1+\delta)$. For these strategies to comprise an equilibrium, $U_b^h \geq \delta(1+v_h)/(1+\delta)$. Such will be the case only if $1-\omega_t \geq v_h/(\delta[1+v_h(1+\delta)])$. This value will be less than ω^P if

$$1 - \frac{1+v_l}{1+v_h} \leq \frac{(1-\delta^2)v_h}{\delta(1+v_h)} + \frac{v_h}{\delta(1+v_h(1+\delta))}.$$

Since the RHS of this inequality is decreasing in δ, this condition is guaranteed to be satisfied if it holds for $\delta = 1$, which is the case if $v_h(v_h - 2v_l) \leq v_l$. This condition is satisfied given $v_l \geq \underline{v}$.

iii) Seller has incentive to deviate from a partial offer

From the above and following from lemma 4, the seller either screens with a partial offer beginning with Y or pools with a partial offer beginning with X. In both cases, the seller has an incentive to deviate to a joint offer. Consider, first, the screening case. The seller's offer must be such that the v_h type is indifferent between accepting and rejecting while the v_l type strictly prefers to reject. This condition will be $v_h+\delta-U^s_s = \delta+\delta v_h-\delta U^b_s$, $v_l+\delta-U^s_s < \delta+\delta v_l-\delta U^b_s$ if the buyer makes a joint offer and $v_h+\delta-U^s_s = \delta+\delta^2 v_h-\delta U^b_s$, $v_l+\delta-U^s_s < \delta+\delta^2 v_l-\delta U^b_s$ if the buyer makes a partial offer.

Consider the case in which the buyer makes a joint offer and imagine a deviation by the seller to a joint offer, $\widehat{P}$, that yields the v_h type the same utility as the partial screening offer. This offer is defined by $v_h+\delta-U^s_s = v_h+1-\widehat{P}$; or $\widehat{P}=U^s_s+1-\delta$. Because the v_h type continues to be indifferent, the v_l type must strictly prefer to reject this offer. The seller strictly prefers this offer to the partial screening offer. As a consequence, the seller can offer $\widehat{P}+\varepsilon$, ε positive, such that the v_h type buyer prefers to accept rather than reject and offer U^b_s, the v_l type buyer prefers to reject and the seller is better off. This deviation is possible for all U^b_s; in particular, it is possible for the U^b_s that yields the buyer the maximum possible payoff among all such screening equilibria.

Analogous arguments hold for the case of screening when the buyer makes a partial offer as well as for the case of the seller pooling on a partial offer beginning with X. Thus, if any of these outcomes are to be supported as equilibria, it must be that the deviation by the seller to $\widehat{P}$ is met by a switch to some other equilibrium path involving patterns of offers different from those in the proposed equilibrium (for instance, if the seller is deviating from a partial offer screening path, the deviation must be met by a switch to a partial offer pooling path).

The proof of the lemma can now be completed. Consider a proposed equilibrium in which the seller screens with a partial offer beginning with Y. From the above, the seller can deviate to a joint offer, $\widehat{P}$, which, if accepted by the v_h type yields higher utility for the seller than the proposed path. For the deviation not to be successful, then, the v_h type must reject the offer. A rejection can only be in the v_h type's interest if the utility received from rejection in the continuation equilibrium is greater than $1+v_h-\widehat{P}$. Consider, then, possible continuations after a rejection by the buyer. A separating outcome yields the v_h type the lowest utility among all possible equilibria and so he will not reject $\widehat{P}$ if rejection is followed by a separating outcome. Given a rejection must leave ω_t unchanged, the above results imply that the only other possibilities have the seller making a partial offer also. Each of these also provides the seller with an incentive to deviate and so can only support the desired outcome if the sequence of prices, $\widehat{P}$ is strictly declining. This sequence violates the fact that $\widehat{P} \geq (1+v_l)/(1+\delta)$ and so no such equilibrium can be supported. qed

Proof of Proposition 2: The expected payoff of the seller under the equilibrium separating strategies is $\delta\left[\frac{\delta(1+v_h)}{1+\delta} - (1-\delta)v_h\right] + \omega_t\left[(1-\delta)(1+v_h(1+\delta))\right] \equiv U_S^E$. Consider the seller's payoff under possible deviations.

If the seller deviates to an offer that both buyers accept, then from Lemma 4, offers on Y alone or joint offers on X and Y yield a payoff of $(1+v_l)/(1+\delta)$. This deviation is profitable, then, only if $\omega_t \leq 1 - \frac{v_h - v_l}{(1-\delta^2)(1+v_h(1+\delta))} = \omega^{S1}$. Some algebraic manipulation shows that, under (A4), $\omega^* = \frac{1+v_l}{1+v_h} \geq 1 - \frac{v_h - v_l}{(1-\delta^2)(1+v_h(1+\delta))} = \omega^{S1}$. Since $\omega_t \geq \omega^*$ the deviation is not profitable.

Next the seller could deviate to just an offer on X which both types accept. This deviation would have to be followed by the screening equilibrium on Y of Proposition 1 at the original beliefs. For the seller, such a deviation is profitable if $\widehat{p}_x + \delta\frac{\delta\omega_t v_h}{1+\delta} \geq U_S^E$. The v_h type would accept $\widehat{p}_x$ if $1 - \widehat{p}_x + \delta\left(v_h - \frac{\delta\omega_t v_h}{1+\delta}\right) \geq \delta\frac{1+v_h}{1+\delta}$ while the v_l type accepts if $1 - \widehat{p}_x + \delta\left(v_l - \frac{\delta\omega_t v_h}{1+\delta}\right) \geq \delta\frac{1+v_h}{1+\delta} - \delta^2(v_h - v_l)$. These conditions are satisfied if $\widehat{p}_x \leq 1 + \delta v_l - \frac{\delta^2\omega_t v_h}{1+\delta} - \delta\frac{1+v_h}{1+\delta} + \delta^2(v_h - v_l)$. A $\widehat{p}_x$ satisfying this condition can only make the seller better off if $\omega_t \leq \frac{1+v_l}{1+v_h(1+\delta)} = \omega^{S2} < \omega^*$.

Finally, the seller could deviate to screen on Y first, but it is easy to verify that screening on both issues simultaneously (as in the proposed equilibrium strategies) has a higher payoff.

As far as the buyers are concerned, both types of buyer could deviate to a joint offer. Since such a deviation can be followed by seller beliefs of $\omega_{t+1} = 1$, it will be rejected unless the price is equal to the high valuation player's full information share.

Since the v_h type already obtains this amount in equilibrium and the v_l type more than this, no such deviation is profitable.

Next, consider a deviation by both types to a pooling offer on X. Given that the proposed equilibrium strategy for the v_l type yields the lowest possible price for Y of any equilibrium, this type is willing to undertake the deviation only if $p_x < \delta(1+(v_h - v_l))/(1+\delta) - (1-\delta)v_h$. Further, total payment (appropriately discounted) must be strictly less under the deviation than under the proposed equilibrium. The seller will only accept this deviating offer if the payoff from doing so is greater than that from rejecting and countering with the proposed separating offer given priors $\omega_{t+1} = \omega_t$ (A3). The value of total payments to the seller from accepting the equilibrium offer, p_x, is $\delta\frac{1+(v_h-v_l)}{1+\delta} - (1-\delta)v_h + \frac{\delta v_l}{1+\delta}$. The expected value of total payments, discounted one period, under the seller's equilibrium separating offer with prior ω_t is δU_S^E. Simple manipulation shows that the later is larger if $\omega_t > 1 - (v_h/(\delta[1+v_h(1+\delta)]))$. The RHS of this inequality will be less than ω^* as long as $v_h(v_h - 2v_l) < v_l$. This condition is satisfied for $v_l \geq \underline{v}$. Thus, the seller will not accept the proposed deviating offer.

Finally, the buyers could deviate to a pooling offer on Y first. Since the v_l type prefers the order X then Y over Y then X at constant total payment, such a deviation is profitable for this type only if total payment falls. It has already been shown, however, that the seller rejects an offered total payment less than that associated with the v_l type's equilibrium strategy and so any such deviation is also rejected. This then concludes the proof that the above strategies constitute an equilibrium.

It remains to verify that no other equilibria exist. Lemma 2 states that any separating/signaling equilibrium must employ the above strategies, so any other equilibria would have to have the buyers pooling on their offers. By lemma 1 no equilibria exist in which they pool on just a Y offer. Lemma 3 shows that if they pool on a joint offer in equilibrium then ω_t must be less than ω^P. The only remaining possibility is pooling by the buyers on an offer on X only, but in the second part of the proof of lemma 5 it was shown that such an equilibrium cannot exist. qed

Partial Proof of Proposition 3: We first derive the ω^{P2} cutoff for this case. Next we demonstrate that the strategies constitute an equilibrium. Finally, we show uniqueness. Before beginning, note that $p_2 = (1-\delta)(1+v_l-p_1)$, as per the discussion preceding the result. It follows that the total price to be paid will be $p_1 + p_2 = (1-\delta)(1+v_l) + \delta p_1$.

i) value of ω^{P2}

Consider the arguments of lemma 3 applied to this case. From a joint offer P the v_l type can deviate to a partial offer if

$$1 + v_h - P \geq \delta(1 + v_h - \delta p_1 - (1-\delta)(1 + v_l)) \tag{3}$$

$$1 + v_l - P \leq \delta(1+v_l-\delta p_1 - (1-\delta)(1+v_l)) = \delta^2(1+v_l-p_1) \tag{4}$$

Recall that $p_1 > \delta(1 + v_l)/(1 + \delta)$ is required in order to have the necessary corner solution in the second bargain. The lowest P for which a deviation is available and profitable is therefore given when (4) holds as an equality with $p_1 = \delta(1+v_l)/(1+\delta)$: $P = (1+v_l)(1-\delta^2)+\delta^3(1+v_l)/(1+\delta) = (1+\delta-\delta^2)(1+v_l)/(1+\delta)$. From Proposition 1, the screening joint price offer is $\delta\omega_t(1 + v_h)/(1 + \delta)$. Thus, for any ω_t such that $(\delta\omega_t(1 + v_h))/(1 + \delta) \geq (1 + \delta - \delta^2)(1 + v_l)/(1 + \delta) \implies \omega_t \geq \omega^{P2} = \frac{1+v_l}{1+v_h}\frac{1+\delta-\delta^2}{\delta} = \omega^*(1+\delta-\delta^2)/\delta$, a deviation exists which destroys pooling on joint offers. (A4) implies that $\omega^{P2} < 1$, while $\delta < 1$ implies that $\omega^* < \omega^{P2}$.

ii) proof of equilibrium

The payoffs to the players under the equilibrium strategies of the proposition are: v_h type: $\frac{1+v_h}{1+\delta}$; v_l type: $\frac{1+v_h}{1+\delta} - \delta(v_h - v_l)$; Seller: $\frac{\delta(1+v_h)(\delta(1+\delta)-1)}{1+\delta} + \omega_t(1 - \delta^2)(1 + v_h)$. Here the seller's offer strategy if no price has yet been agreed upon and it is his turn is to offer $(1 + v_h)/(1 + \delta)$, which only the v_h type accepts. Note that the v_h type is indifferent between his and the v_l type's equilibrium offer if: $\frac{1+v_h}{1+\delta} = \delta(1 + v_h - p_1 - p_2) = \delta(1 + v_h - \delta p_1 - (1 + v_l)(1 - \delta))$. Solving for p_1 we obtain the expression in the result, $p_1 = (1 + v_l) + \frac{v_h-v_l}{\delta} - \frac{1+v_h}{\delta^2(1+\delta)}$. The v_l type strictly prefers his offer strategy to that of the v_h type if $1 + v_l - \delta\frac{1+v_h}{1+\delta} = \frac{1+v_l}{1+\delta} - \frac{\delta(v_h-v_l)}{1+\delta} <$

$\frac{1+v_h}{1+\delta} - \delta(v_h - v_l), \Longrightarrow \quad \delta^2(v_h - v_l)/(1+\delta) < (v_h - v_l)/(1+\delta)$, which it clearly is. This also implies that the updating of beliefs to $\omega_1 = 0$ after the partial offer p_1 is consistent with (A2), and thus that the offer p_2 as computed before is the appropriate continuation.

As in Proposition 2, no joint deviation by both informed types is possible. If they were to deviate to a joint offer $p_x + p_y$ preferred by both, the seller is justified in setting $\omega_{t+1} = 1$. Hence any joint price below $\delta(1 + v_h)/(1 + \delta)$ is rejected.

Next, consider a deviation by both buyer types to a pooling offer on a single good. The price in this case must be below the proposed equilibrium price, p_1, if the deviation is to be profitable for the v_l type. As in the proof of Proposition 2, this offer is rejected by the seller as long as the total payment under the proposed equilibrium offer for the v_l type is less than the expected payment, appropriately discounted and given priors ω_t, under the sellers proposed equilibrium strategy. That is, the deviating offer is rejected if $1 + v_l + \frac{v_h - v_l}{\delta} - \frac{1+v_h}{\delta^2(1+\delta)} + (1-\delta)\left[\frac{1+v_h}{\delta^2(1+\delta)} - \frac{v_h - v_l}{\delta}\right] < \frac{\delta(1+v_h)(\delta(1+\delta)-1)}{1+\delta} + \omega_t(1-\delta^2)(1+v_h)$. Some manipulation yields that this inequality is satisfied if $(\delta + \delta^2 - 1)\frac{1-\delta^2}{\delta} < \omega_t(1 - \delta(\delta + \delta^2 - 1)$. As the RHS of this inequality is increasing in ω_t, it will be satisfied for all relevant beliefs if it is satisfied at $\omega_t = \omega^*$. Substitution for this value of ω_t yields the inequality $(1 + v_h)/(\delta(1+\delta)) > v_h - v_l$ which is guaranteed by the assumption that $v_h - v_l < 1$. Therefore, the seller rejects the deviating offer.

As for the seller, observe that if the seller screens the buyers on only one issue this is equivalent to screening them on both due to the joint implementation. Screening with joint offers also generates the largest payoff of all screening equilibria for the seller, so he would not deviate. On the other hand, if the seller were to deviate to a sequence of offers which both buyers accept (no screening), then the highest payoff he can obtain from such a strategy is $(1 + v_l)/(1 + \delta)$ (by implication from Rubinstein's Proposition 5 (1985).) Thus, the seller will only deviate to such a sequence if $(1 + v_l)/(1+\delta) > \frac{\delta(1+v_h)(\delta(1+\delta)-1)}{1+\delta} + \omega_t(1-\delta^2)(1+v_h)$. This requires $\omega_t < \frac{\omega^* + (\delta - \delta^2(1+\delta))}{(1+\delta)(1-\delta^2)} = \omega^S$.

Manipulation verifies that $\omega^S < \omega^* < \omega^{P2}$ under (A4).

iii) uniqueness

Finally to uniqueness. It is easy to verify that any separating strategies must be of the type in the proposition, since the v_h type must pay his full information price by the standard arguments. Therefore we only need to check for any pooling equilibria (that is, equilibria in which both types make the same offer which is accepted.) We already argued above that pooling on a joint offer in conjunction with screening by the seller will break down above ω^{P2}. If the buyers where to pool on a partial offer that is accepted, then this would be followed by screening on the remaining issue (note that with one issue remaining there do not exist signaling equilibria in the continuation paths.) As in the proof of Lemma 5, the v_h type has an incentive to deviate to a joint offer that reveals type rather than following the pooling strategy. Thus no equilibria of this sort exist in the region of initial beliefs. qed

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