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**Modeling Stock Volatility with Trading Information**

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# Modeling Stock Volatility with Trading Information

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## Abstract

This paper studies volatility in individual stocks of the Toronto Stock Exchange (TSE), using a recently developed nonlinear approach, a stochastic threshold model. Trading information is embedded into the determination process of volatility in the stochastic threshold model with a generalized autoregressive conditional heteroskedastic variance (STGARCH). We use the number of price changes (quote changes) to approximate the trading information. This trading variable has significantly positive impact on stock volatility following a declining market and ambiguous impact on the stock volatility following a rising market; there is a higher probability to fall into a highly volatile state after a declining market than after a rising market. The GARCH-type persistence in volatility is reduced significantly in our nonlinear model for individual stocks with high persistence. The STGARCH model also gives satisfactory fitting in terms of alternative model selection criteria.

*Key words:* Volatility; Asymmetry; Trading variable; Information arrivals; Stochastic threshold

*JEL classification:* G12: C22

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## 1. Introduction

A great deal of literature exists on stock return volatility and the study about return volatility is at the heart of empirical finance. Generally speaking, time-varying stock volatility is usually described as a mixed distribution process, which is the so-called Mixed Distribution Hypothesis (MDH). For example, GARCH models assume mixed distributions to capture the volatility clustering and fat tails of the stock return distributions. From the information flow perspectives, return volatility process is heteroskedastic conditioned on the information arrival process. There might be three sources that cause the volatility to fluctuate over time, public information<sup>1</sup> including news release which is generally known to all the traders, private information which is only known to a subset of traders, and mispricing. When new information is assimilated in the market, prices adjust and volatility fluctuates. Consequently, it is important to consider the impact of information flow on the return volatility. However, information flow (mixed of public information, private information and mispricing) is not practically observable, and some proxies have to be used. It has been known that trading activities contain valuable information on the movement of stock returns. Com-

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<sup>1</sup>To consider the influence of public information, Engle and Ng (1993) examine the impact of news on volatility; Ederington and Lee (1993) consider how stock markets process information from news release.

monly used informational proxies include trading volume, average trading volume, transactions, quotes, quote changes (number of price changes), and executed order imbalance. Pervasive studies consider the role of trading variables in studying stock volatility. Karpoff (1987) provides a detailed survey on the theoretical and empirical studies on the relationship between price changes and trading volume. More recent studies include those by Anderson (1996), Lamoureux and Lastrapes (1994), Gallant, Rossi, and Tauchen (1992), and Lamoureux and Lastrapes (1990a). Anderson (1996) and Lamoureux and Lastrapes (1994) consider the co-movement between trading volume and stock volatility where both variables depend parametrically on an underlying information process in their papers. Gallant, Rossi, and Tauchen (1992) find positive correlation between conditional volatility and volume in the composite index of New York Stock Exchange using a semi-nonparametric approach. While most of the studies are on the index and aggregate volume, Lamoureux and Lastrapes (1990a) and Anderson (1996) conduct the study on actively traded individual stocks. They find that contemporaneous trading volume contributes greatly to the persistence in volatility.

Most of the previous models are based on the rationale that trading is generated due to asymmetric information and assume that the size of the trades (or volume) should reflect the degree of the information asymmetry on the security's value.

Jones, Kaul and Lipson (1994), however, shows that the frequency of trading contains more information than the size and that it is the occurrence of transactions generates volatility. The number of price changes (quote changes), which is a similar measure as transaction times, has not often been used as an informational proxy in the study of stock return volatility, possibly due to its unavailability. We fortunately have this variable available in our data. We find that the correlation between the lagged value of number of price changes and squared stock return is higher than the correlation between lagged transactions and squared stock return for most of the firms in our data. Therefore, we will use the number of price changes as our informational proxy.

The main purpose of this paper is to investigate the role of the past number of price changes in determining the future volatility. This is different from many studies, including Lamoureux and Lastrapes (1990a), Lamoureux and Lastrapes (1994) and Anderson (1996), in which they consider the contemporaneous relationship between some trading variable (trading volume) and stock volatility. As Blume, Easley and O'Hara (1994, p.160) argued, the actual market settings are not consistent with contemporaneous conditioning requirements, since traders who submit market orders do not know the price at which their order will execute until after the trade occurs. Since practitioners cannot care about the contempo-

aneous value at the every moment of decision making, in practice, it is always the observed information that is used to make new investment, though the co-movement of trading variable and stock volatility reserves a great deal of interest from the theoretic point of view. Nevertheless, the contemporaneous approach may introduce extra specification error through the specification of the process for a trading variable. Only when the specification for the evolvement of the trading variable is correct, shall it provide more efficient estimation for the parameters describing the stock volatility process. Under the proceeding consideration, we would rather focus on modeling the impact of the observed trading information on the stock volatility.

One important stylized fact on the behavior of stock volatility documented in previous research is the high persistence in volatility or volatility clustering. This is found by many researchers and is commonly captured by the ARCH model proposed by Engle (1982) and the GARCH model by Bollerslev (1986). However, over-high persistence is usually implied by alternative GARCH models (see Glosten, Jagannathan, and Runkle, 1993), as empirical studies suggest that the forecasting performance of GARCH models is not as good as what one might expect with the high level of persistence implied by these models. GARCH models are indeed linear in variance, which makes them unable to deal with the commonly

observed important nonlinear features in the volatility series. Diebold (1986), Lamoureux and Lastrapes (1990b), Hamilton and Susmel (1994) and many others suggested the notion that misspecification of the GARCH models – that is, not accounting for discrete shifts in the variance – can lead to misinterpretation of estimates of persistence in variance. To capture discrete shifts in time series, two classes of piece-wise linear models, switching-regime models (Hamilton, 1989) and threshold models (Tong, 1983), have been widely employed in the literature. There are two major differences between these two types of models from our concern. First, the switching-regime models are invented to pick up long-run structural changes (see Cai, 1994 for example). In the context of modelling the short-run fluctuations, threshold models are found to be more useful than the switching-regime models since the threshold models identify alternative structures of the market according to the recently realized threshold values/regimes rather than according to a Markov Chain process involving with state-dependence. Second, the threshold models deal with the asymmetry explicitly and market asymmetry is a common observation of many previous studies, for example, Christie (1982), and Schwert (1989). It is found that threshold models are able to capture the asymmetric patterns in stock volatility and to reduce the high persistence in volatility by considering the volatility process separately in a few regimes (Li and Li, 1996).



It is also found by Cao and Tsay (1992) that the threshold models yield better forecasts, compared to ARIMA and GARCH models.

As noticed by Li and Yang (1999), some limitations in Tong's threshold models reduce their usefulness in studying financial series. Li and Yang (1999) proposed a more flexible framework, namely the stochastic threshold model, which provides better fitting than the existing threshold models and the regime-switching models. Using this approach, we are able to investigate the impact of the number of price changes on stock volatility in a rising or declining market. Our empirical results at the end favor the stochastic threshold approach among many alternatives.

The rest of the paper is organized as the following. Section 2 specifies the adopted stochastic threshold model. Section 3 describes the data. We report the empirical results with comparison to alternative volatility models and discuss the economic implications in Section 4. Section 5 concludes.

## **2. Model Specification**

Models of stock volatility have been successful in capturing some of the well-known empirical attributes, including volatility clustering typically captured by GARCH models, asymmetric patterns by threshold models, sudden discrete changes by nonlinear models (mainly threshold models and regime-switching models), and

state-varying variance by the switching ARCH models of Hamilton and Susmel (1994).

A GARCH model of volatility is widely employed to allow conditional heteroskedasticity, in which current conditional variance is a linear function of the squared value of last innovation and the conditional variance of last period. A simple GARCH(1,1) model takes the form of

$$R_t = \phi_0 + \phi_1 R_{t-1} + u_t, \quad (2.1)$$

$$u_t = z_t \sqrt{h_t}, z_t \sim i.i.d.N(0, 1),$$

$$h_t = \alpha_0^2 + \alpha_1 u_{t-1}^2 + \alpha_2 h_{t-1}, \quad (2.2)$$

where  $\{R_t\}$  is the return series of one security, following a simple autoregressive process;  $\alpha_1 \geq 0$ ,  $\alpha_2 \geq 0$ , and  $\alpha_1 + \alpha_2 < 1$ . One possible way to consider the impact of a trading variable on volatility is to add it directly into the specification for the conditional variance, as by Lamoureux and Lastrapes (1990a). Here we consider using the lagged value instead of the contemporaneous value of a trading variable as we argued about earlier. The conditional variance now is as follows,

$$h_t = \alpha_0^2 + \alpha_1 u_{t-1}^2 + \alpha_2 h_{t-1} + \alpha_3 N_{t-1}, \quad (2.3)$$

where  $N_{t-1}$  is the number of price changes at time  $t-1$  and  $\alpha_3 \geq 0$ . Clearly, these models fail to capture discrete shifts and asymmetries in the volatility process. They also require a pre-test assumption of nonnegativity on  $\alpha_3$  to ensure nonnegative variances.

A threshold model is used to capture discrete changes and asymmetries. As proposed by Gouriéroux and Monfort (1992), we then have a threshold model with GARCH effects in the following form:

$$\begin{aligned}
 R_t &= \begin{cases} \phi_0 + \phi_1 R_{t-1} + u_{t1} & \text{if } R_{t-1} \geq 0 \\ \phi_0 + \phi_1 R_{t-1} + u_{t2} & \text{if } R_{t-1} < 0 \end{cases}, \\
 u_{t1} &= z_t \sqrt{h_t(\alpha_1)}, u_{t2} = z_t \sqrt{h_t(\alpha_2)}, z_t \sim i.i.d.N(0, 1), \\
 h_t &= \begin{cases} \alpha_{01}^2 + \alpha_1 u_{t-1}^2 + \alpha_2 h_{t-1} + \alpha_{31} N_{t-1} & \text{if } R_{t-1} \geq 0 \\ \alpha_{02}^2 + \alpha_1 u_{t-1}^2 + \alpha_2 h_{t-1} + \alpha_{32} N_{t-1} & \text{if } R_{t-1} < 0, \end{cases} \quad (2.4)
 \end{aligned}$$

where  $\alpha_1 \geq 0, \alpha_2 \geq 0, \alpha_{31} \geq 0, \alpha_{32} \geq 0, \alpha_1 = (\alpha_{01}^2, \alpha_1, \alpha_2, \alpha_{31}), \alpha_2 = (\alpha_{02}^2, \alpha_1, \alpha_2, \alpha_{32})$  and  $\alpha_1 + \alpha_2 < 1$ .

The above specified threshold model assumes that the current state of stock volatility deterministically depends on the observed information, particularly, the sign of the lagged return. It retains the pre-test assumption of nonnegativity on the coefficient of the number of price changes. To overcome these limitations and

to capture all the above desired features of volatility, we employ the stochastic threshold GARCH (STGARCH) model proposed by Li and Yang (1999), which allows stochastic dependence of the current state of stock volatility on the observed information and puts no restriction on the estimated impact of the number of price changes on volatility. The specific STGARCH model in this study is:

$$\begin{aligned}
 & \text{if } R_{t-1} \geq 0 \\
 R_t &= \begin{cases} \phi_0 + \phi_1 R_{t-1} + u_{t1} & \text{with probability } p_t \\ \phi_0 + \phi_1 R_{t-1} + u_{t2} & \text{with probability } 1 - p_t \end{cases} \\
 h_t^* &= \begin{cases} \alpha_{01}^2 + \alpha_1 u_{t-1}^2 + \alpha_2 h_{t-1} & \text{with probability } p_t \\ \alpha_{02}^2 + \alpha_1 u_{t-1}^2 + \alpha_2 h_{t-1} & \text{with probability } 1 - p_t, \end{cases} \quad (2.5)
 \end{aligned}$$

$$\begin{aligned}
 & \text{if } R_{t-1} < 0 \\
 R_t &= \begin{cases} \phi_0 + \phi_1 R_{t-1} + u_{t1} & \text{with probability } q_t \\ \phi_0 + \phi_1 R_{t-1} + u_{t2} & \text{with probability } 1 - q_t, \end{cases} \\
 h_t^* &= \begin{cases} \alpha_{01}^2 + \alpha_1 u_{t-1}^2 + \alpha_2 h_{t-1} & \text{with probability } q_t \\ \alpha_{02}^2 + \alpha_1 u_{t-1}^2 + \alpha_2 h_{t-1} & \text{with probability } 1 - q_t, \end{cases} \quad (2.6)
 \end{aligned}$$

where  $u_{ti} = z_t \sqrt{h_t(\alpha_i)}$ ,  $z_t \sim i.i.d.N(0, 1)$ ,  $h_t(\alpha_i) \equiv h_{ti} = \alpha_{0i}^2 + \alpha_1 u_{t-1}^2 + \alpha_2 h_{t-1}$ , for  $i = 1, 2$ ; and  $\alpha_{01} > 0, \alpha_{02} > 0, \alpha_1 \geq 0, \alpha_2 \geq 0, \alpha_1 + \alpha_2 < 1$  and  $\alpha_{01} > \alpha_{02}$ . The definition for  $h_t^*, h_{t-1}$  and  $u_{t-1}$  will be explained later. In the above model, there

are two threshold regimes, determined by the sign of lagged return  $R_{t-1}$ . Within each threshold regime, there are two states for the market, state 1 is highly volatile and state 2 is low volatile. The states are characterized by two sets of parameters  $\theta_1 = (\phi_0, \phi_1, \alpha_{01}, \alpha_1, \alpha_2)$  and  $\theta_2 = (\phi_0, \phi_1, \alpha_{02}, \alpha_1, \alpha_2)$ . The probabilities of falling into different states are different among the threshold regimes. At time period  $t$ , state 1 occurs with probability  $p_t$  when following a positive stock return ( or in regime  $R_{t-1} \geq 0$ , a rising market) and it occurs with probability  $q_t$  when following a negative stock return ( or in regime  $R_{t-1} < 0$ , a declining market); state 2 occurs with probabilities  $1 - p_t$  and  $1 - q_t$ , respectively, under different threshold regimes. It is worthwhile noting that, in the above setting, we only consider one single AR(1) process for the mean of stock return series, or equivalently, we assume the mean processes of return are the same under different states. We do so with the following consideration. Although it is often found that the trading variables contain useful information for the study on the stock volatility, it is not clear how they could be helpful in describing the stock return process itself. Consequently, many studies on market volatility focus only on the volatility process.

Since the conditional variance is with GARCH specification and has probability to be in different dynamics, we face the similar path-dependence problem as the regime-switching model with GARCH variance in our STGARCH model. To

overcome the path-dependence problem we follow the approach proposed by Gray (1996). Thus in our model,  $h_t^*$  denotes the true but unobserved (path-dependent) variance,  $h_{t-1}$  is the expected variance conditional on the information set at time  $t - 1$ , and  $u_{t-1}$  is the expected error term conditional on the information set at time  $t - 1$ . In particular, under conditional normality within each state, for the conditional variance  $h_{t-1}$ , we have

if  $R_{t-2} \geq 0$

$$\begin{aligned} h_{t-1} &= p_{t-1}(\mu_{t-1,1}^2 + h_{t-1,1}) + (1 - p_{t-1})(\mu_{t-1,2}^2 + h_{t-1,2}) \\ &\quad - (p_{t-1}\mu_{t-1,1} + (1 - p_{t-1})\mu_{t-1,2})^2 \\ &= p_{t-1}h_{t-1,1} + (1 - p_{t-1})h_{t-1,2} \end{aligned}$$

if  $R_{t-2} < 0$

$$\begin{aligned} h_{t-1} &= q_{t-1}(\mu_{t-1,1}^2 + h_{t-1,1}) + (1 - q_{t-1})(\mu_{t-1,2}^2 + h_{t-1,2}) \\ &\quad - (q_{t-1}\mu_{t-1,1} + (1 - q_{t-1})\mu_{t-1,2})^2 \\ &= q_{t-1}h_{t-1,1} + (1 - q_{t-1})h_{t-1,2} \end{aligned}$$

where  $\mu_{t-1,1} = \mu_{t-1,2} = \phi_0 + \phi_1 R_{t-1}$  in our model.

The probabilities are time varying. They take a specification of a standard logistic function form (see, for example, Diebold, Lee and Weinbach, 1994).  $p_t$  is the probability of falling into a highly volatile market when following a *positive* lagged return (or a rising market), at time  $t$ .  $q_t$  is the probability of falling into a highly volatile market when following a *negative* lagged return (or a declining market), at time  $t$ . The probabilities are determined by recently observed information, the number of price changes of last period,  $N_{t-1}$ .

$$\begin{aligned}
 p_t &\equiv p(N_{t-1}) = \frac{\exp(\beta_{0p} + \beta_{1p}N_{t-1})}{1 + \exp(\beta_{0p} + \beta_{1p}N_{t-1})} \\
 q_t &\equiv q(N_{t-1}) = \frac{\exp(\beta_{0q} + \beta_{1q}N_{t-1})}{1 + \exp(\beta_{0q} + \beta_{1q}N_{t-1})}.
 \end{aligned}
 \tag{2.7}$$

We can see clearly that there is no restriction on the sign of the parameters,  $\beta_{1p}$  and  $\beta_{1q}$ , which are the coefficients for the impact of the informational proxy,  $N_{t-1}$ , on the stock volatility. Previous studies usually restrict the impact to be nonnegative in the model specification.

Given that the structural parameters characterizing the mean process of the stock return are the same for the two states, we can intuitively explain our model as follows. At each period, there is probability  $p_t$  and  $1 - p_t$  to have innovation  $u_{t1}$  and  $u_{t2}$ , respectively, if following the  $R_{t-1} \geq 0$  regime; and there is probability  $q_t$

and  $1 - q_t$  to have innovation  $u_{t1}$  and  $u_{t2}$ , respectively, if following the  $R_{t-1} < 0$  regime. In other words, for the stock return part, the states of the market are fully identified by different innovations,  $u_{t1}$  and  $u_{t2}$ . Again as in Gray (1996), we work out that our  $u_{ti}$  ( $i = 1, 2$ ) are defined by one single equation

$$u_{t-1,i} = R_{t-1} - (\phi_0 + \phi_1 R_{t-2}),$$

for  $i = 1, 2$ . Since we do not distinguish the mean process through the coefficients any more, we can see that the expected error  $u_{t-1} = R_{t-1} - (\phi_0 + \phi_1 R_{t-2})$  under any state. We notice that although we can obtain the same realization of  $u_{t-1}$  under different states, we have no full information about which of the processes that  $u_{t-1}$  is generated.

### 3. Data Description

The data used in this study is the daily data of the individual stocks listed in the Toronto Stock Exchange 35 Index (TSE35). The Toronto Stock Exchange (TSE) is Canada's largest stock exchange. In 1995, the TSE accounted for more than 81% of the value of the equity traded on Canadian exchange. The TSE35 was developed by the TSE in 1987. The 35 stocks listed in the Index are some



of Canada's largest publicly traded corporations and among the more heavily traded issued on the TSE. It was designed to track the performance of the TSE300 composite index and has direct representation from all of the TSE300 industry groups, excluding Real Estate. The TSE35 daily trading data used in this study was extracted from the Canadian Financial Market Research Center (CFMRC), produced by Western Business School, the University of Western Ontario.

The stock returns used in this study are the daily stock returns from TSE35 for the period from January 1988 to December 1995 inclusive. The period is chosen to avoid the impact of 1987 stock crash. A return in the CFMRC database is the fully adjusted daily return calculated as if the security was purchased at the close yesterday and sold at the close today. Let  $P_t$  be today's closing price,  $D_t$  be the cash or cash equivalent dividend paid today (that is, today is the ex-dividend date) and  $S_t$  be the stock split factor for a stock dividend or split today. The return is

$$R_t = \{(P_t + D_t) * S_t - P_{t-1}\} / P_{t-1}.$$

If there is no cash dividend,  $D_t = 0$  and if there is no stock dividend or split,  $S_t = 1$ . In that case,  $R_t = \{P_t - P_{t-1}\} / P_{t-1}$ , which is the definition for the returns used in our study. We know that this definition for stock return is an approximation

to the equally commonly used definition of the log difference of stock prices.

We choose the stocks which are listed in TSE35 every year from the starting sample year 1988 to the ending sample 1995. We exclude those listed since 1988 but with stock splits/dividends, since the effect of stock split/dividend on stock return and volatility can be very different. Therefore, we have observations on 17 qualified stocks, with variables including returns, closing prices, open prices, high, low, trading volume, transactions, quotes and the number of price changes (quote changes), on a daily base. The sample consists of 2015 observations for each firm.

Table 1 presents summary statistics for the return series for the 17 firms. The returns used are in percentage in all the calculations. The sample mean of stock return is small and around zero for all the firms. The skewness is positive for 12 firms, with values close to zero, and it is negative for the rest of 5 firms. The kurtosis is substantially larger than 3 for all the 17 firms, which is consistent with the observation of fat-tailed distribution of returns. The excess kurtosis also implies that it is not appropriate to assume normal distribution for the stock returns and the GARCH type heteroskedasticity is not enough to capture the extreme kurtosis found in our data. In our specification, we have a mixed normal distribution for the stock return, which will be able to accommodate the exhibited excess kurtosis in the data.

We calculated the correlations between the squared return and lagged trading variables<sup>2</sup>. Firstly, the correlations are not positive for some firms, which implies that it might be inappropriate to pre-assume the effect of trading variable to be positive as in the GARCH specification by Lamoureux and Lastrapes (1990a). In our specification, there is no such restriction on the sign of the impact of trading variables, which has important implications for our following empirical findings. Secondly, we find that the number of price changes (quote changes) is the one with the highest correlation in most firms and trading volume, the most commonly used trading variable in the literature, actually shows the weakest correlation to squared stock return among these four trading variables. In our empirical study, we use the number of price changes<sup>3</sup> (quote changes) as the informational proxy.

#### 4. Empirical Results

This section presents the empirical results on the study of individual stocks of TSE. We compare the alternative models using various model selection criteria. Then we report our findings on the role of the number of price changes on stock

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<sup>2</sup>Not presented in the paper. Available upon request.

<sup>3</sup>Though it is generally claimed in the literature that time trend commonly appears in trading volume, very weak time trending is found in only three (BCE, CM and TRP) of the 17 firms in the series of the number of price changes in our data. We thus do not apply any detrending to the series of the number of price changes that we use.

volatility and attempt to give some explanations about our results.

#### 4.1. Model Selection

To illustrate our results, we study various specifications for the stock volatility process and compare our STGARCH model with a constant-variance model, a simple GARCH model, a GARCH model with the number of price changes ( $\text{GARCH}_N$ ) and a threshold GARCH model with the number of price changes ( $\text{TGARCH}_N$ ).

In Table 3 we report the various model selection statistics, including the maximal value of the log-likelihood function, the Akaike Information Criterion (AIC) and the Schwarz's criterion. Our model has the biggest maximal value of the log-likelihood function for all the 17 firms. It is clear that our model gives best fitting results for all the firms even when both sample size and number of parameters are taken into account. Table 3 also reports the performance of in-sample forecasting, which are measured by mean squared error (MSE) and mean absolute error (MAE). Our STGARCH model provides better in-sample forecast in most cases. In general, we conclude that our model fits the data best in terms of various model selection criterions and in-sample forecasting performance.

## 4.2. Findings

We reinforce the empirical finding on the important role of trading variable in determining the volatility process. The estimation results for various volatility models are presented in Table 2. In particular, we find that the coefficients of the number of price changes,  $\beta_{1p}$  and  $\beta_{1q}$ , are significantly different from zero in most cases. We find, however, that in the  $GARCH_N$  model and  $TGARCH_N$  model, the impact of the trading variable is very close to zero and insignificant for most of the firms. Nevertheless, whenever the trading variable is not found to be significant in our model, it is not found to be significant in any of the alternatives.

One striking finding is that  $\beta_{1q}$  is found to be positive for all the firms, while  $\beta_{1p}$  is positive in some cases and negative in some other cases. Consequently, we can conclude that, when  $R_{t-1} < 0$  which indicates a falling market, the number of price changes has positive and more significant impact on return volatility; when  $R_{t-1} \geq 0$  which indicates a rising market, the number of price changes has ambiguous impact on the volatility of the stock market. Although this finding seems to be surprising, it appears to be consistent with the previous finding that bad news has greater impact on the market than good news (Campbell and Hentschel, 1992 and Braun, Nelson and Sunier, 1996). We can interpret this well-known finding in at least two ways: (1) the impact of good news on *current* market is less

significant than bad news; (2) the impact of good news on *future* market is less persistent than bad news. Either of the above two situations will suggest that the number of price changes can have much greater impact on stock volatility when following a declining market than following a rising market. Based on the market microstructure theory, the trading variable, the number of price changes ( $N_{t-1}$ ), can be decomposed into an informed component ( $IN_{t-1}$ ) and a noise component ( $NN_{t-1}$ ), i.e.,

$$N_{t-1} = IN_{t-1} + NN_{t-1}.$$

Noise trading randomly arrives to the market and is usually assumed to be governed by a stochastic process with a constant arrival intensity per day. The systematic variation in the number of price changes is due solely to fluctuations in the informed part. The informed component,  $IN_{t-1}$ , measures the informativeness of the informational proxy, the number of price changes in our case. Bad news tends to drive negative returns and it has greater impact on the market than good news. Following a declining market, the informativeness of the informational proxy is higher than following a rising market, when there is proportionally more noisy component in the informational proxy. It implies that the number of price changes,  $N_{t-1}$ , is a better informational proxy in a declining market than in a rising

market. That is, in a declining market, the number of price changes has significant and positive impact on the stock volatility, while in a rising market, it has overall ambiguous impact, since there is a higher proportion of noisy component in the number of price changes. In other words, we find that the information-sensitive proportion in the number of price changes is higher when following a falling market than when following a rising market.<sup>4</sup> Even if the information-sensitive proportions are the same in a falling market and in a rising market, the impact of the number of price changes on future volatility could be ambiguous in a rising market simply since good news has less persistent impact on the future market. This result together with the results from model selection analysis suggests that the STGARCH model is a more appropriate approach to model the impact of the number of price changes on volatility than the alternatives.

Recall that  $p$  is the probability of falling into a highly volatile market when following a rising market and  $q$  is the probability of falling into a highly volatile market when following a declining market. In most of the cases, we find that  $|\beta_{1q}| > |\beta_{1p}|$ . This implies that the number of price changes affects the volatility in a higher magnitude when following a declining market than when following a

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<sup>4</sup>Anderson (1996) shows that there is 30 to 60 percent of trading volume is information-insensitive on average.

rising market.

Additionally, we can see that in Table 4 the persistence in the conditional variance is reduced dramatically after accounting for both the impact of information contained in trading activities and the nonlinear features in volatility. This implies that there are various sources for the high persistence in the volatility implied by the GARCH models: (1) one might be the misspecification of GARCH model which ignores the effect of discrete shifts in the economy; (2) the other source could be the persistence in the trading variable itself, which is observed through persistence in the volatility, when the impact of trading variable is not properly captured. We can conclude that our consideration on discrete changes and on the impact of the number of price changes explains most of the reduction in the persistence in volatility.

## 5. Conclusion

This paper studies the volatility of returns in individual stocks within a new nonlinear framework, incorporating the impact of information arrivals as suggested by the theory of market microstructure. We find that the number of price changes has greater impact on the volatility in a falling market than in a rising market. Asymmetric patterns appear in the volatility of stock returns in a way particularly



suggested by our model that on average there is a higher probability of falling into a highly volatile market when following a declining than when following a rising market. Trading information and discrete shifts in volatility series both attribute greatly to the documented high persistence in the conditional variance implied by GARCH models.

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Table 1. Summary statistics of return series (in percentage)

	mean	st.dev.	skewness	kurtosis
BCE	0.039	0.757	0.050	6.093
BNS	0.071	1.322	0.123	4.219
CM	0.061	1.109	0.098	3.934
CP	0.030	1.403	0.208	4.642
ECO	0.001	2.657	0.249	5.697
IMO	0.017	1.002	0.127	4.940
LDM	0.014	1.998	-0.200	12.17
MB	0.008	1.346	0.210	4.685
MCL	0.024	1.300	-0.576	8.913
NA	0.038	1.568	-0.041	4.550
NOR	0.032	1.411	0.183	5.092
NTL	0.067	1.682	-1.224	19.77
NVA	0.035	1.767	0.237	4.585
PDG	0.053	2.061	0.108	4.291
POW	0.041	1.188	0.126	4.964
RGO	0.048	2.141	0.327	5.059
TRP	0.034	1.165	-1.216	17.30

Table 2. Estimation results for selected parameters ( $n = 2014$ )

(\* : significant at 5% level)

BCE

	Constant	GARCH	GARCH <sub>N</sub>	TGARCH <sub>N</sub>	STGARCH
$\alpha_{01}^2$	0.7541*	0.4100*	0.3717*	0.4255*	1.3381*
$\alpha_{02}^2$				$0.3 \times 10^{-5}$	0.3429*
$\alpha_1$		0.1885*	0.1518*	0.1416*	0.1153*
$\alpha_2$		0.5232*	0.3349*	0.4366*	0.3886*
$\alpha_{31}$			0.0444*	0.0142	
$\alpha_{32}$				0.0725*	
$\beta_{0p}$					-2.9955*
$\beta_{0q}$					-3.5121*
$\beta_{1p}$					0.1984*
$\beta_{1q}$					0.3262*

**BNS**

	Constant	GARCH	GARCH <sub>N</sub>	TGARCH <sub>N</sub>	STGARCH
$\alpha_{01}^2$	1.3163*	0.1607*	0.1607*	0.0580	2.0349*
$\alpha_{02}^2$				0.2127*	0.6(10 <sup>-4</sup> )
$\alpha_1$		0.0325*	0.0325*	0.0262*	0.0650*
$\alpha_2$		0.9523*	0.9523*	0.9625*	0.8435*
$\alpha_{31}$			0.0000	0.0000	
$\alpha_{32}$				0.0000	
$\beta_{0p}$					-2.3631*
$\beta_{0q}$					-3.7136*
$\beta_{1p}$					-0.5269*
$\beta_{1q}$					0.2623*

**CM**

	Constant	GARCH	GARCH <sub>N</sub>	TGARCH <sub>N</sub>	STGARCH
$\alpha_{01}^2$	1.0982*	0.2703*	0.2703*	0.2148	1.3476*
$\alpha_{02}^2$				0.3282*	0.3(10 <sup>-5</sup> )
$\alpha_1$		0.0553*	0.0553*	0.0511*	0.0814*
$\alpha_2$		0.8839*	0.8839*	0.8896*	0.7354*
$\alpha_{31}$			0.0000	0.0000	
$\alpha_{32}$				0.0000	
$\beta_{0p}$					-1.7165*
$\beta_{0q}$					-2.0207*
$\beta_{1p}$					-0.3296*
$\beta_{1q}$					0.1621

**CP**

	Constant	GARCH	GARCH <sub>N</sub>	TGARCH <sub>N</sub>	STGARCH
$\alpha_{01}^2$	1.3964*	0.3348*	0.2957*	0.5292*	1.8952*
$\alpha_{02}^2$				0.0001	0.7200*
$\alpha_1$		0.0349*	0.0296*	0.0371*	0.0450*
$\alpha_2$		0.9069*	0.9056*	0.8476*	0.3344
$\alpha_{31}$			0.0142	0.0000	
$\alpha_{32}$				0.0538	
$\beta_{0p}$					-1.3923*
$\beta_{0q}$					-2.8246*
$\beta_{1p}$					0.1420
$\beta_{1q}$					0.4255*

**ECO**

	Constant	GARCH	GARCH <sub>N</sub>	TGARCH <sub>N</sub>	STGARCH
$\alpha_{01}^2$	2.6557*	0.1658*	0.1658*	0.2266	4.6669*
$\alpha_{02}^2$				0.0006	0.0012
$\alpha_1$		0.0235*	0.0235*	0.0232*	0.0578*
$\alpha_2$		0.9725*	0.9725*	0.9727*	0.7686*
$\alpha_{31}$			0.0000	0.0000	
$\alpha_{32}$				0.0000	
$\beta_{0p}$					-3.0972*
$\beta_{0q}$					-3.6443*
$\beta_{1p}$					0.2637
$\beta_{1q}$					0.4937

**IMO**

	Constant	GARCH	GARCH <sub>N</sub>	TGARCH <sub>N</sub>	STGARCH
$\alpha_{01}^2$	0.9900*	0.4758*	0.4813*	0.5258*	1.1584*
$\alpha_{02}^2$				0.4515*	0.2765
$\alpha_1$		0.1127*	0.1148*	0.1207*	0.1145*
$\alpha_2$		0.6564*	0.6187*	0.5922*	0.3024*
$\alpha_{31}$			0.0277	0.0057	
$\alpha_{32}$				0.0730	
$\beta_{0p}$					-0.8216*
$\beta_{0q}$					-1.1321*
$\beta_{1p}$					0.3556
$\beta_{1q}$					0.5818

**LDM**

	Constant	GARCH	GARCH <sub>N</sub>	TGARCH <sub>N</sub>	STGARCH
$\alpha_{01}^2$	1.9955*	0.4040*	0.8269*	0.0006	4.3796*
$\alpha_{02}^2$				0.5829*	0.4413
$\alpha_1$		0.0709*	0.1179*	0.0476*	0.0903*
$\alpha_2$		0.8906*	0.5399*	0.9010*	0.5760*
$\alpha_{31}$			0.3180*	0.0389	
$\alpha_{32}$				0.0287	
$\beta_{0p}$					-3.3393*
$\beta_{0q}$					-3.4465*
$\beta_{1p}$					0.2199*
$\beta_{1q}$					0.3036*



**MB**

	Constant	GARCH	GARCH <sub>N</sub>	TGARCH <sub>N</sub>	STGARCH
$\alpha_{01}^2$	1.3424*	0.4449*	0.4667*	0.4781*	1.7230*
$\alpha_{02}^2$				0.6137*	0.1(10 <sup>-5</sup> )
$\alpha_1$		0.0681*	0.0664*	0.0609*	0.0678*
$\alpha_2$		0.8222*	0.7683*	0.7290*	0.5871*
$\alpha_{31}$			0.0837	0.0537	
$\alpha_{32}$				0.1618	
$\beta_{0p}$					-1.7484*
$\beta_{0q}$					-1.3871*
$\beta_{1p}$					0.1741
$\beta_{1q}$					0.3848

**MCL**

	Constant	GARCH	GARCH <sub>N</sub>	TGARCH <sub>N</sub>	STGARCH
$\alpha_{01}^2$	1.2928*	0.9892*	0.9892*	0.9982*	2.0279*
$\alpha_{02}^2$				0.8825*	0.5210*
$\alpha_1$		0.1673*	0.1673*	0.1745*	0.0901*
$\alpha_2$		0.2562*	0.2562*	0.2334*	0.3272*
$\alpha_{31}$			0.0000	0.0979	
$\alpha_{32}$				0.1014	
$\beta_{0p}$					-1.4044*
$\beta_{0q}$					-1.7229*
$\beta_{1p}$					-0.0343
$\beta_{1q}$					0.1051

**NA**

	Constant	GARCH	GARCH <sub>N</sub>	TGARCH <sub>N</sub>	STGARCH
$\alpha_{01}^2$	1.5607*	0.4975*	0.4975*	0.6784*	2.0802*
$\alpha_{02}^2$				0.3769*	0.7×10 <sup>-4</sup>
$\alpha_1$		0.0778*	0.0778*	0.0952*	0.0804*
$\alpha_2$		0.8210*	0.8210*	0.7233*	0.6515*
$\alpha_{31}$			0.0000	0.0000	
$\alpha_{32}$				0.2902	
$\beta_{0p}$					-1.5699*
$\beta_{0q}$					-2.4055*
$\beta_{1p}$					-0.1977
$\beta_{1q}$					0.6323*

NOR

	Constant	GARCH	GARCH <sub>N</sub>	TGARCH <sub>N</sub>	STGARCH
$\alpha_{01}^2$	1.4032*	0.5857*	0.5799*	0.5958*	1.6393*
$\alpha_{02}^2$				0.5339*	0.0003
$\alpha_1$		0.1168*	0.1169*	0.1173*	0.1078*
$\alpha_2$		0.7082*	0.7044*	0.7079*	0.6011*
$\alpha_{31}$			0.0102	0.0005	
$\alpha_{32}$				0.0282	
$\beta_{0p}$					-1.5356*
$\beta_{0q}$					-1.3277*
$\beta_{1p}$					0.1202
$\beta_{1q}$					0.0277

NIL

	Constant	GARCH	GARCH <sub>N</sub>	TGARCH <sub>N</sub>	STGARCH
$\alpha_{01}^2$	1.6616*	1.6337*	1.5586*	1.6656*	4.2156*
$\alpha_{02}^2$				1.3622*	1.0395*
$\alpha_1$		0.0349	0.0225	0.0318	0.0732*
$\alpha_2$		0.2(10 <sup>-4</sup> )	0.2(10 <sup>-4</sup> )	0.1(10 <sup>-4</sup> )	0.1717
$\alpha_{31}$			0.1226	0.0292	
$\alpha_{32}$				0.2761*	
$\beta_{0p}$					-2.9611*
$\beta_{0q}$					-3.0026*
$\beta_{1p}$					0.1380
$\beta_{1q}$					0.0745

NVA

	Constant	GARCH	GARCH <sub>N</sub>	TGARCH <sub>N</sub>	STGARCH
$\alpha_{01}^2$	1.7613*	0.6440*	0.6440*	1.5911*	2.6869*
$\alpha_{02}^2$				1.3397*	0.0004
$\alpha_1$		0.0636*	0.0636*	0.1047*	0.0723*
$\alpha_2$		0.8037*	0.8037*	0.1358	0.7141*
$\alpha_{31}$			0.0000	0.0000	
$\alpha_{32}$				0.1129	
$\beta_{0p}$					-2.3540*
$\beta_{0q}$					-2.2663*
$\beta_{1p}$					0.0217
$\beta_{1q}$					0.0033

**PDG**

	Constant	GARCH	GARCH <sub>N</sub>	TGARCH <sub>N</sub>	STGARCH
$\alpha_{01}^2$	2.0589*	0.2359*	0.2746*	0.2942*	2.4480*
$\alpha_{02}^2$				0.2628	0.0025
$\alpha_1$		0.0243*	0.0242*	0.0248*	0.0373*
$\alpha_2$		0.9624*	0.9505*	0.9489*	0.5957*
$\alpha_{31}$			0.0118	0.0087	
$\alpha_{32}$				0.0171	
$\beta_{0p}$					-1.4618*
$\beta_{0q}$					-1.2931*
$\beta_{1p}$					0.1074
$\beta_{1q}$					0.1246

**POW**

	Constant	GARCH	GARCH <sub>N</sub>	TGARCH <sub>N</sub>	STGARCH
$\alpha_{01}^2$	1.1859*	0.1067*	0.0679	0.6(10 <sup>-4</sup> )	2.1454*
$\alpha_{02}^2$				0.1461*	0.2(10 <sup>-5</sup> )
$\alpha_1$		0.0254*	0.0234*	0.0212*	0.0671*
$\alpha_2$		0.9662*	0.9626*	0.9631*	0.8250*
$\alpha_{31}$			0.0260*	0.0146	
$\alpha_{32}$				0.0440	
$\beta_{0p}$					-3.7325*
$\beta_{0q}$					-3.6103*
$\beta_{1p}$					0.3627
$\beta_{1q}$					0.5555

**RGO**

	Constant	GARCH	GARCH <sub>N</sub>	TGARCH <sub>N</sub>	STGARCH
$\alpha_{01}^2$	2.1316*	0.3324*	0.3223*	0.4349*	3.1880*
$\alpha_{02}^2$				0.0003	1.1894*
$\alpha_1$		0.0358*	0.0349*	0.0414*	0.0467
$\alpha_2$		0.9394*	0.9339*	0.9194*	0.2394
$\alpha_{31}$			0.0581	0.0425	
$\alpha_{32}$				0.1518	
$\beta_{0p}$					-2.1314*
$\beta_{0q}$					-2.0248*
$\beta_{1p}$					1.3139*
$\beta_{1q}$					0.8957

TRP					
	Constant	GARCH	GARCH <sub>N</sub>	TGARCH <sub>N</sub>	STGARCH
$\alpha_{01}^2$	1.1605*	0.5584*	0.5584*	0.4(10 <sup>-4</sup> )	3.7938*
$\alpha_{02}^2$				0.1072*	0.0113
$\alpha_1$		0.2227*	0.2227*	0.0150*	0.0685*
$\alpha_2$		0.5789*	0.5790*	0.9819*	0.6711*
$\alpha_{31}$			0.0000	0.0000	
$\alpha_{32}$				0.0000	
$\beta_{0p}$					-3.2090*
$\beta_{0q}$					-4.4092*
$\beta_{1p}$					-0.4351*
$\beta_{1q}$					0.5552*

Table 3. Summary statistics of various specifications

	Constant	GARCH	GARCH <sub>N</sub>	TGARCH <sub>N</sub>	STGARCH
#parameters	3	5	6	8	10
<b>BCE</b>					
Log-likelihood	-2289	-2227	-2209	-2201	-2152
AIC	-2292	-2232	-2215	-2209	-2162
Schwarz	-2294	-2235	-2219	-2214	-2169
MSE	1.6522	1.6139	1.5919	1.5908	1.5920
MAE	0.6196	0.6037	0.5954	0.5979	0.5973
<b>BNS</b>					
Log-likelihood	-3411	-3370	-3370	-3369	-3349
AIC	-3414	-3375	-3376	-3377	-3359
Schwarz	-3416	-3378	-3380	-3382	-3366
MSE	9.5448	9.2803	9.2803	9.2721	9.2537
MAE	1.8239	1.7860	1.7860	1.7836	1.7700
<b>CM</b>					
Log-likelihood	-3046	-3016	-3016	-3014	-2996
AIC	-3049	-3021	-3022	-3022	-3006
Schwarz	-3051	-3024	-3026	-3027	-3013
MSE	4.2530	4.1553	4.1553	4.1499	4.1556
MAE	1.2637	1.2477	1.2477	1.2460	1.2390
<b>CP</b>					
Log-likelihood	-3530	-3516	-3514	-3511	-3480
AIC	-3533	-3521	-3520	-3519	-3490
Schwarz	-3535	-3524	-3524	-3524	-3497
MSE	13.637	13.541	13.523	13.489	13.564
MAE	2.0506	2.0261	2.0252	2.0208	2.0334
<b>ECO</b>					
Log-likelihood	-4825	-4730	-4730	-4730	-4709
AIC	-4828	-4735	-4736	-4738	-4719
Schwarz	-4830	-4738	-4740	-4743	-4726
MSE	231.02	220.65	220.65	220.57	224.13
MAE	7.8850	7.5916	7.5916	7.5864	7.4948
<b>IMO</b>					
Log-likelihood	-2838	-2805	-2805	-2805	-2759
AIC	-2841	-2810	-2811	-2813	-2769
Schwarz	-2843	-2813	-2815	-2818	-2776
MSE	3.5794	3.5139	3.5111	3.5149	3.5848
MAE	1.0712	1.0552	1.0549	1.0556	1.0552

(Table 3: Continued)

	Constant	GARCH	GARCH <sub>N</sub>	TGARCH <sub>N</sub>	STGARCH
#parameters	3	5	6	8	10
<b>LDM</b>					
Log-likelihood	-4249	-4165	-4157	-4153	-4040
AIC	-4252	-4170	-4163	-4161	-4050
Schwarz	-4254	-4173	-4167	-4166	-4057
MSE	177.03	176.48	174.83	174.78	174.55
MAE	4.5410	4.5221	4.4381	4.4527	4.4780
<b>MB</b>					
Log-likelihood	-3450	-3428	-3424	-3419	-3381
AIC	-3453	-3433	-3430	-3427	-3391
Schwarz	-3455	-3436	-3434	-3432	-3398
MSE	11.852	11.747	11.716	11.654	11.715
MAE	1.9232	1.9263	1.9283	1.9170	1.9168
<b>MCL</b>					
Log-likelihood	-3375	-3343	-3343	-3340	-3260
AIC	-3378	-3348	-3349	-3348	-3270
Schwarz	-3380	-3351	-3353	-3353	-3277
MSE	22.037	21.979	21.979	22.083	21.906
MAE	1.8701	1.8689	1.8689	1.8720	1.8538
<b>NA</b>					
Log-likelihood	-3754	-3715	-3715	-3713	-3684
AIC	-3757	-3720	-3721	-3721	-3694
Schwarz	-3759	-3723	-3725	-3726	-3701
MSE	21.980	21.482	21.482	21.448	21.569
MAE	2.5828	2.5280	2.5280	2.5230	2.5085
<b>NOR</b>					
Log-likelihood	-3540	-3491	-3491	-3491	-3466
AIC	-3543	-3496	-3497	-3499	-3476
Schwarz	-3545	-3499	-3501	-3504	-3483
MSE	16.063	15.486	15.486	15.486	15.484
MAE	2.1117	2.0809	2.0811	2.0821	2.0700
<b>NTL</b>					
Log-likelihood	-3880	-3878	-3876	-3871	-3698
AIC	-3883	-3883	-3882	-3879	-3708
Schwarz	-3885	-3886	-3886	-3884	-3715
MSE	148.06	148.03	148.02	148.13	148.79
MAE	3.1566	3.1505	3.1421	3.1560	3.1703

(Table 3: Continued)

	Constant	GARCH	GARCH <sub>N</sub>	TGARCH <sub>N</sub>	STGARCH
#parameters	3	5	6	8	10
<b>NVA</b>					
Log-likelihood	-3998	-3981	-3981	-3985	-3947
AIC	-4001	-3986	-3897	-3993	-3957
Schwarz	-4003	-3989	-3991	-3998	-3964
MSE	35.125	34.936	34.936	34.942	35.079
MAE	3.2314	3.1953	3.1953	3.2005	3.1893
<b>PDG</b>					
Log-likelihood	-4312	-4287	-4285	-4285	-4263
AIC	-4315	-4292	-4291	-4293	-4273
Schwarz	-4317	-4295	-4295	-4298	-4280
MSE	58.804	57.985	57.912	57.908	58.457
MAE	4.4569	4.4048	4.3898	4.3902	4.4137
<b>POW</b>					
Log-likelihood	-3201	-3140	-3136	-3134	-3118
AIC	-3204	-3145	-3142	-3142	-3128
Schwarz	-3206	-3148	-3146	-3147	-3135
MSE	8.0277	7.8354	7.8201	7.8047	7.9666
MAE	1.5243	1.4726	1.4676	1.4654	1.4639
<b>RGO</b>					
Log-likelihood	-4382	-4343	-4342	-4342	-4302
AIC	-4385	-4348	-4348	-4350	-4312
Schwarz	-4387	-4351	-4352	-4355	-4319
MSE	85.390	84.002	83.918	83.852	85.846
MAE	4.9214	4.8043	4.8025	4.8053	4.8723
<b>TRP</b>					
Log-likelihood	-3158	-3107	-3107	-3088	-2964
AIC	-3161	-3112	-3113	-3096	-2974
Schwarz	-3163	-3115	-3117	-3101	-2981
MSE	29.470	31.318	31.318	29.276	29.334
MAE	1.5408	1.6212	1.6212	1.5218	1.5438

Table 4: Persistence in various specifications ( $\alpha_1 + \alpha_2$ )

	GARCH	GARCH <sub>N</sub>	TGARCH <sub>N</sub>	STGARCH
BCE	0.7117	0.4867	0.5782	0.5039
BNS	0.9848	0.9848	0.9887	0.9085
CM	0.9392	0.9392	0.9407	0.8168
CP	0.9418	0.9352	0.8747	0.3794
ECO	0.9960	0.9960	0.9959	0.8264
IMO	0.7691	0.7335	0.7129	0.4169
LDM	0.9615	0.6578	0.9486	0.6663
MB	0.8903	0.8347	0.7899	0.6549
MCL	0.4235	0.4235	0.4079	0.5576
NA	0.8988	0.8988	0.8185	0.7319
NOR	0.8250	0.8213	0.8252	0.7089
NTL	0.0349	0.0225	0.0318	0.2449
NVA	0.8673	0.8673	0.2405	0.7864
PDG	0.9867	0.9747	0.9737	0.6330
POW	0.9916	0.9860	0.9843	0.8921
RGO	0.9752	0.9688	0.9608	0.2761
TRP	0.8016	0.8017	0.9969	0.7396