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202/3

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The Role of Speculation in
Competitive Price-Dynamics

Peter Howitt

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THE ROLE OF SPECULATION IN COMPETITIVE PRICE-DYNAMICS

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In his 1939 article, "Speculation and Economic Stability,"¹ Kaldor argued that the degree of price-stabilizing speculation in a market depends upon two elasticities: the elasticity of expected price with respect to current price, and the elasticity of speculative excess demand with respect to the difference between current and expected price. Speculation will generally stabilize the current price about the expected price, since a wide gap between the two gives rise to a counteracting pressure on the current price from the speculative excess demand. The smaller the first elasticity the smaller will be the fluctuations in the expected price resulting from fluctuations in underlying factors of supply or demand. The larger is the second elasticity the more closely will fluctuations in the current price reflect fluctuations in the expected price.

Kaldor's analysis, and much of the more recent literature on speculation and stability² addresses the question of whether or not speculation will serve its traditionally cited role of ironing out some of the fluctuations in prevailing market prices that would occur in its absence. The question of speculation and stability has been phrased somewhat differently in the literature on general competitive analysis, where it is asked whether or not the presence of speculation makes it more likely that the process of market adjustment will succeed in converging asymptotically upon an equilibrium.³ This literature has focused upon the role of the first of Kaldor's elasticities, with results that run parallel to those of Kaldor. Putting the question either way, we may roughly say that speculation exerts a stabilizing influence if the elasticity

of price expectations is less than unity, or if expectations are formed adaptively,⁴ it has no influence on stability if the elasticity is unity, and it exerts a destabilizing influence if the elasticity exceeds unity or if expectations are formed extrapolatively.⁵

The purpose of the present paper is to examine the importance of the second of Kaldor's elasticities for the asymptotic convergence of the market adjustment process. In the context of a single market it is clear that if this elasticity is large enough the price-adjustment process will be stable, provided that we don't have destabilizing expectation formation, for regardless of the slope of the non-speculative excess demand curve, a large enough elasticity of the speculative excess demand curve will imply a downward slope to the total excess demand curve. The central question of the present paper is whether or not the analogous result holds for the adjustment process in the multi-market economy of general competitive analysis.

The answer to this question is vital to the broader issue of the influence of speculation on stability, because the size of this elasticity can be interpreted as defining the existing degree of speculation. The characteristic feature of speculation that distinguishes it from similar activities, such as hedging or investing, that also are influenced by future price expectations, is that speculation is primarily motivated by the expectation of capital gain, not by the desire to consume or otherwise transform commodities, to avoid risk, or to realize any other kind of yield associated with holding commodities. Speculation is prevalent, therefore, in commodities with relatively low storage costs, and is undertaken by traders who are relatively undeterred by the risk of capital losses.⁶ The degree of speculation will generally be greater the less important are those factors of storage cost and risk.

But it is also true that the elasticities of the speculators' demands will be greater the less important are these two factors. In the limiting case where they are entirely absent the smallest decline in current relative to expected price will induce the speculator to increase his demand for that commodity up to the limit of his funds.

Thus we may interpret the question of the present paper as being whether or not the presence of enough speculation implies the stability of a competitive equilibrium. Our approach will be to consider a pure exchange economy and investigate the effects upon the dynamic behavior of prices that result from introducing into the economy an agent whose trading plans are almost entirely motivated by the single objective of maximizing expected capital gains. The main result of this simple conceptual experiment is that the Walras-Samuelson tâtonnement process becomes locally asymptotically stable with an infinitely rapid speed of convergence.

This result has important implications for fundamental matters of theory construction. One of the major outstanding problems in general competitive analysis is how to provide a more satisfying explanation of how prices are adjusted in response to disequilibrium without invoking the ethereal Walrasian auctioneer. The present analysis suggests a possible solution to this problem. Despite this powerful result on tâtonnement-stability, the stability of the market adjustment process as a whole will depend also upon the dynamic process of expectation-formation. It turns out that when we take this other process into account, the dynamic behavior of prices with enough speculation will be almost identical to the behavior of the tâtonnement process with no speculation at all. This leads us to an interpretation of the tâtonnement process in which the auctioneer is replaced by a maximizing agent who finds it in his self interest not only to adjust prices but also to allow the other traders'

plans to be realized when the system is out of equilibrium.

Section I describes the exchange economy; section II introduces the speculator, section III presents the result on tâtonnement-stability, and section IV discusses the implications for theory construction.

I

The starting point of the analysis is a model of a pure exchange, stationary, competitive economy with no futures markets. Although our analysis will be conducted in continuous time it is helpful to visualize the model in terms of discrete market-periods. At the beginning of each period each trader receives an endowment of commodities and brings them to a marketplace where an auctioneer calls out a price vector. The traders then compute their excess demands and submit them to the auctioneer who collates them and arranges for trades to be executed according to some rule for allocating rationing in the event that not all excess demand sums to zero. These trades having been executed the traders leave the market place and undertake their consumption activities. In each subsequent period activities proceed in exactly the same way except that the prices called out by the auctioneer will be somewhat higher (lower) than in the previous period for commodities that were in excess demand (supply) during the previous period. The commodities are all pure flows, so that no carry-over is possible from one period to the next, and each trader's endowment vector is constant from one period to the next.

Let the traders be indexed by $\alpha=1, \dots, m$, and the commodities by $i=1, \dots, n$. Let $w^\alpha = (w_1^\alpha, \dots, w_n^\alpha)$ and $x^\alpha = (x_1^\alpha, \dots, x_n^\alpha)$ be the vectors of endowments and excess demands for the α th trader. Let p_i represent the auctioneer's announced price for commodity i , measured as a rate of exchange for the n th

commodity, so that we always have $p_n=1$. The vector $p=(p_1, \dots, p_{n-1})$ denotes these relative prices. Let $u^\alpha(\cdot)$ be the α^{th} trader's utility function. The individual excess demands are defined as the solutions to the problems:

$$(1) \quad \text{Max}_{\{x^\alpha\}} u^\alpha(w^\alpha + x^\alpha) \quad \text{subject to} \quad w^\alpha + x^\alpha \geq 0 \quad \text{and} \quad \sum_{i=1}^n p_i x_i = 0; \quad \alpha=1, \dots, m$$

which we assume may be represented by the functions:

$$(2) \quad x^\alpha = x^\alpha(p) = (x_1^\alpha(p), \dots, x_n^\alpha(p)); \quad \alpha=1, \dots, m$$

which are continuously differentiable for all $p > 0$.⁷ From these we may define the market excess demand functions:

$$(3) \quad x_i(p) = \sum_{\alpha=1}^m x_i^\alpha(p); \quad i=1, \dots, n$$

We assume that there exists a unique equilibrium price vector, \hat{p} , defined by the conditions:

$$(4) \quad x_i(p) = 0; \quad i=1, \dots, n; \quad \text{and} \quad p > 0.$$

The auctioneer's adjustment procedure is the tâtonnement process, which may be represented by the system of differential equations:

$$(5) \quad \frac{dp_i}{dt} = g_i(x_i(p)); \quad i=1, \dots, n-1; \quad t \geq 0$$

where the adjustment functions $g_i(\cdot)$ are assumed to be continuously differentiable, with:

$$(6) \quad g_i(0) = 0, \quad \text{and} \quad g_i'(\cdot) \geq g > 0; \quad i=1, \dots, n-1$$

Previous investigations of the stability properties of (5) have been summarized by Arrow and Hahn (1971) and Negishi (1962). In the present paper we shall be concerned with the particular question of local asymptotic stability,

which obtains if all of the characteristic roots of the matrix:⁸

$$(7) \quad A = \left[g_i'(0) \cdot \frac{\partial x_i(p)}{\partial p_j} \right] \Big|_{p=\hat{p}} ; \quad i, j=1, \dots, n-1$$

have negative real parts. In general we cannot say without further assumptions whether or not this stability property holds.

II

There is no room for speculation in the economy that we have just described, because of the assumption that no commodities can be carried over between periods. So let us introduce into the economy a trader with storage facilities. This trader will naturally engage in some degree of speculative activity in the sense that his trading plans will be influenced by the prospect of capital gains or losses on his carryover. But he will not necessarily be a speculator in the commonly used sense of the term, unless his trading plans are almost entirely motivated by these prospects.

Let the new trader be indexed by $\alpha=0$. Let $z = (z_1, \dots, z_n)$ denote his inventory holdings at the beginning of the market period. Let $q = (q_1, \dots, q_{n-1})$ denote the expected values of future relative prices, according to his subjective probability distribution. We may say that he is a pure speculator if his excess demands are derived from the decision problem:

$$(8) \quad \text{Max}_{\{x^0\}} \sum_{i=1}^n q_i (x_i^0 + w_i^0 + z_i) \text{ subject to } x^0 + w^0 + z \geq 0, \quad \sum_{i=1}^n p_i x_i^0 = 0$$

The following example of an intertemporal choice problem shows how the absence of risk and storage costs would lead a utility-maximizing trader to become a pure speculator. Suppose the trader's planning horizon consists of two periods. In period one he faces the relative prices p and expects with

certainty to face the relative prices q in period two. There are no costs of storage. To avoid unnecessary complications suppose that he plans to undertake no consumption until the second period. Let $c = (c_1, \dots, c_n)$ be his planned consumption in period two. Given the excess demands x^0 in period one the total carry over to period two will be $x^0 + w^0 + z$, to which the endowment, w^0 , will be added to determine his endowment at the start of trading in period two. So, given any value of x^0 his planned consumption will solve the problem:

$$(9) \quad \text{Max}_{\{c\}} v(c) \text{ subject to } \sum_{i=1}^n q_i (x_i^0 + z_i + 2w_i^0 - c_i) = 0$$

where $v(\cdot)$ is the second period utility function. The optimized value of (9) can clearly be expressed as a function of total expected wealth and expected relative prices:

$$(10) \quad \phi\left(\sum_{i=1}^n q_i (x_i^0 + z_i + 2w_i^0), q\right).$$

He will choose x^0 so as to maximize ϕ , subject to $\sum_{i=1}^n p_i x_i^0 = 0$, $x^0 + w^0 + z \geq 0$

But, regarded as a function of x^0 , ϕ is just a monotonic transformation of the objective function in (8), so that the choice of x^0 is identical to the choice determined by (8).

With the addition of this new trader the excess demands to which the auctioneer will be reacting will be $\bar{x}_i = \sum_{\alpha=0}^m x_i$; $i=1, \dots, n-1$. But if the new trader is a pure speculator we cannot write \bar{x}_i as a function of p because the problem (8) will not always possess a unique solution. In particular, when $q=p$ the pure speculator will be indifferent between all values of x^0 that satisfy the constraints of (8). In order to overcome this analytical problem, let us consider a sequence of new traders, indexed by $a=1, 2, \dots, \infty$, that become more and more like a pure speculator, but each of whom has well defined excess demand functions. We shall call the a^{th} new trader the a^{th} speculator. This sequence defines a sequence of economies, the a^{th} of which consists of the economy described in section I with the addition of the a^{th} speculator as the $m+1^{\text{st}}$ trader. We are interested in seeing whether or not there is some point in the sequence beyond which all economies have a locally asymptotically stable equilibrium.

The a^{th} speculator's excess demands are derived from the decision problem:

$$(11) \quad \text{Max}_{\{x^{oa}\}} u^{oa}(x^{oa}, q, z) \quad \text{subject to } x^{oa} + w^0 + z \geq 0, \quad \text{and } \sum_{i=1}^n p_i x_i^{oa} = 0.$$

The solution to which can be written as the functions:

$$(12) \quad x_i^{oa} = x_i^{oa}(p, q, z); \quad i=1, \dots, n.$$

which we assume are continuously differentiable for all p, q, z such that $p > 0$, $q > 0$, and $x^{oa}(p, q, z) + w^0 + z > 0$. The existence of the non-negativity constraints in (11) is what generally prevents these functions from being continuously differentiable when the last of these conditions is not satisfied.

In order to let the sequence of speculators approach the position of a pure speculator, assume that:

$$(13) \quad \left\{ \begin{array}{l} \text{For all } q > 0 \text{ and } z \geq 0, u^{oa}(x, q, z) \rightarrow \sum_{i=1}^n q_i (x_i + w_i^0 + z_i) \\ \text{as } a \rightarrow \infty, \text{ uniformly for all } x \text{ such that} \\ x + w^0 + z \geq 0. \end{array} \right.$$

In the a^{th} economy the tâtonnement process can be represented by the system:

$$(14) \quad \frac{dp_i}{dt} = g_i(x_i(p) + x_i^{oa}(p, q, z)); \quad i=1, \dots, n-1, \quad t \geq 0.$$

A complete description of the dynamic behavior of the a^{th} economy also requires equations to tell us how q and z change over time. Let us write these provisionally as:

$$(15) \quad \frac{dq_j}{dt} = h_j^a(p, q, z); \quad j=1, \dots, n-1, \quad t \geq 0$$

$$(16) \quad \frac{dz_k}{dt} = f_k^a(p, q, z); \quad k=1, \dots, n, \quad t \geq 0.$$

In section IV we shall be considering the behavior of this entire $3n - 2$ dimensional system. Until then we shall be concerned only with the tâtonnement process (14). Let us now assume that we are given fixed values of q and z , which we shall continue to take as given until section IV. Assume that for each economy there exists a unique equilibrium, p^a , defined by the conditions:

$$(17) \quad x_i(p) + x_i^{oa}(p, q, z) = 0; \quad p > 0.$$

Assume furthermore that there exists a unique equilibrium to the limiting economy with a pure speculator, p^0 , defined by the conditions:

$$(18) \quad -(x_1(p), \dots, x_n(p)) \in \xi(p, q, z), \quad p > 0$$

where $\xi(\cdot)$ denotes the set of solutions to the pure speculator's decision problem (8). Now assume that:

$$(19) \quad p^a \rightarrow p^0 \text{ as } a \rightarrow \infty,$$

and

$$(20) \quad z + w^0 > 0, \quad q > 0.$$

Then we may establish the following result:

Theorem 1: There exists an $\epsilon > 0$ such that $p^a \rightarrow q$ as $a \rightarrow \infty$, if $|\hat{p}-q| < \epsilon$.

Proof: From (19) we just need to show that $p^0 = q$. It follows from (8) and

(18) that $p^0 = q$ if (a) $-x_i(q) + z_i + w_i^0 \geq 0$; $i=1, \dots, n$, and

(b) $\sum_{i=1}^n q_i x_i(q) = 0$. By the assumed continuity of the $x_i(\cdot)$'s and the

definition of \hat{p} , we may choose an $\epsilon > 0$ such that $|\hat{p}-q| < \epsilon$ implies

$\max_i x_i(q) < \min_i (z_i + w_i^0)$. This implies (a), and the individual budget

constraints in (1) imply (b).

This result confirms the intuitive notion that a speculator with sufficient funds will dominate the market. In this case his expected price cannot be too far from what would be the equilibrium price in his absence or he will run out of some inventories.

III

In this section we shall demonstrate the basic formal result of the paper--that under one mild regularity restriction there will be some point in the sequence beyond which the equilibrium of every economy is locally asymptotically stable, with a speed of adjustment that approaches infinity. That is, the presence of "almost pure speculation" implies an indefinitely rapid convergence of the tâtonnement process. The basic idea behind this proposition is quite simple. The almost pure speculator has almost flat indifference curves, implying an almost unlimited responsiveness to price changes. The regularity condition implies that his substitution-effects, but not his income-effects, become indefinitely large. They eventually swamp all of the income-effects in the economy. Since the tâtonnement

process can only be unstable if income-effects are too large relative to substitution-effects, the equilibrium in this sequence must eventually become stable. As the substitution-effects continue to grow in magnitude so does the speed of adjustment.

This result is noteworthy in at least four different respects. First, it sheds new light on the nature of stabilizing speculation. If pure speculation produces instantaneous convergence it is reasonable to infer that not-so-pure speculation at least contributes an element of stability to an economy. It does this by magnifying the role of substitution-effects without adding in the same way to income-effects.

Second, this result provides us with one of the few existing easy-to-interpret conditions upon tastes or opportunities that imply tâtonnement-stability. Most of the conditions derived so far in the literature, such as gross substitutability or diagonal dominance, have been imposed directly upon the excess demand functions. Their generality and scope have been hard to interpret in terms of the more basic properties of an economy.⁹

Third, it sheds new light on the role of substitutability in determining the stability of the tâtonnement process. It is well known that this process is stable if income-effects are small enough relative to substitution-effects, or if excess demand functions display gross substitutability. The present result implies that it is also stable not only in the case of enough speculation but more generally in any situation where some trader's indifference surfaces are flat enough. Flatness of indifference surfaces obviously implies large substitution-effects, but it does not necessarily rule out income-effects that are also large. For this we need the crucial regularity assumption referred to above. Nor does flatness of a trader's indifference surfaces imply that his excess demand functions will display gross

substitutability. This can be seen by means of the following example. Suppose that $u^{oa}(x, q, z) = (1 - \frac{1}{a}) \sum_{j=1}^n q_j(z_j + x_j) + \frac{1}{a} u^o(x, q, z)$, where $u^o(\)$ is a function such that the matrix $\left[\frac{\partial u^o(\)}{\partial x_i \partial x_j} \right]; i, j=1, \dots, n$ is negative definite. For large enough a , the sign of $\frac{\partial x_i^{oa}(\)}{\partial p_j}$ will be that of $\frac{B_{ji}}{B}$, where B is the bordered Hessian of $u^o(\)$, and B_{ji} the i - j cofactor in B . But there is nothing in general requiring that this term be positive for $i \neq j$.

Fourth, the adjustment speeds, $g_i'(0)$, are any positive numbers. Thus the present result implies stability regardless of the speed of adjustment. This is a strong concept of stability which is sometimes very difficult to demonstrate.¹⁰

Before proceeding with the formal statement and proof of the theorem, let us state the crucial regularity condition. Define

$$u_{ij}^a \equiv \frac{\partial^2}{\partial x_i \partial x_j} u^{oa}(x, q, z) \Big|_{x=x^{oa}(p^a, q, z)}. \quad \text{The regularity condition is:}$$

$$(21) \quad \left\{ \begin{array}{l} \text{There exists a sequence of positive real numbers } \{\delta^a\}, \text{ and} \\ \text{a utility function } u^o(\) \text{ with a Hessian } [u_{ij}^o] \text{ that is} \\ \text{negative definite under the constraint } \sum_{i=1}^n q_i \eta_i = 0; \text{ where} \\ \delta^a \rightarrow 0, \text{ and } [u_{ij}^a / \delta^a] \rightarrow [u_{ij}^o], \text{ as } a \rightarrow \infty. \end{array} \right.$$

That the u_{ij}^a 's approach zero is obviously a weak assumption, since $u^{oa}(\)$ uniformly approaches a function with second derivatives all equal to zero. The uniqueness of the solution to (11) implies that $[u_{ij}^a]$ will be negative semi-definite under the constraint $\sum_{i=1}^n p_i^a \eta_i = 0$. To assume further that $[u_{ij}^a]$ is negative definite under this constraint is equivalent to assuming that the function $u^a(\)$ has positive Gaussian curvature at the solution values (12),¹¹ which Mas-Colell (1974) has shown is a regularity condition that will almost always be satisfied under standard convexity and continuity

assumptions upon preferences. The regularity condition (21) can be interpreted roughly as requiring that the $u^a(\cdot)$'s display this positive Gaussian curvature, and that the u_{ij}^a 's approach zero at the same speed.

Theorem 2: Suppose that the premises of Theorem 1 are satisfied. Then there exists an integer \bar{a} such that $a \geq \bar{a}$ implies local asymptotic stability for the a^{th} economy. Furthermore the speed of convergence approaches infinity as $a \rightarrow \infty$.

Proof: Define $x_i^{\text{oa}} \equiv x_i^{\text{oa}}(p^a, q, z)$; $i=1, \dots, n$. It follows from the definition of ϵ in Theorem 1 that there exists an a_1 such that $a \geq a_1$ implies $x^{\text{oa}} + z + w^0 > 0$, and hence the right hand side of (14) is continuously differentiable at $p = p^a$. Thus, local asymptotic stability depends upon the characteristic roots of the matrix:

$$(22) \quad GC^a = G(A^a + S^a + Y^a)$$

where G is the diagonal matrix with $g_i'(0)$; $i=1, \dots, n-1$, on the diagonal, $A^a = \left[\frac{\partial x_i(p)}{\partial p_j} \right]_{p=p^a}$; $i, j=1, \dots, n-1$, and S^a and Y^a are the matrixes of the speculator's substitution- and income-effects:

$$S^a \equiv [\lambda^a B_{ji}^a / B^a], \quad Y^a \equiv [x_j^{\text{oa}} B_{n+1,i}^a / B^a]; \quad i, j=1, \dots, n-1; \text{ with}$$

$$B^a = \begin{bmatrix} u_{ij}^a & p_i^a \\ p_j^a & 0 \end{bmatrix}; \quad i, j=1, \dots, n, \quad B_{rs}^a = \text{the } r\text{-s cofactor of } B^a, \text{ and } \lambda^a = \frac{\partial u^{\text{oa}}(x, q, z)}{\partial x_n} \Bigg|_{x=x^{\text{oa}}}$$

It follows from (17), Theorem 1, and the continuity of the $x_i(\cdot)$'s that:

$$(23) \quad x_i^{\text{oa}} \rightarrow -x_i(q); \quad i=1, \dots, n.$$

From (13), (23), and the first-order conditions of (8), we get:

$$(24) \quad \lambda^a \rightarrow 1.$$

Define B as the bordered Hessian of u^0 , evaluated at $x = -(x_1(q), \dots, x_n(q))$.

It follows from (21) and (23) that:

$$(25) \quad Y^a = \left[x_j^{oa} \frac{\begin{vmatrix} u_{ij}^a / \delta^a & q_i \\ q_j & 0 \end{vmatrix}_{n+1,i}}{\begin{vmatrix} u_{ij}^a / \delta^a & q_i \\ q_j & 0 \end{vmatrix}} \right] \rightarrow \left[-x_j(q) B_{n+1,i}/B \right]$$

and from (21) and (24) that:

$$(26) \quad \delta^a S^a = \left[\lambda^a \frac{\begin{vmatrix} u_{ij}^a / \delta^a & q_i \\ q_j & 0 \end{vmatrix}_{ji}}{\begin{vmatrix} u_{ij}^a / \delta^a & q_i \\ q_j & 0 \end{vmatrix}} \right] \rightarrow [B_{ji}/B] \equiv S$$

It then follows from (21), (22), (25), (26), and the continuity of A^a that:

$$(27) \quad \delta^a GC^a \rightarrow GS.$$

The usual argument that establishes the negative semi-definiteness of the full (n-dimensional) Slutsky matrix implies that S is negative definite.¹²

It then follows from a result of Arrow and McManus (1958) that all of the characteristic roots of GS have negative real parts. This, together with (22), (27), and the fact that the set of characteristic roots of a matrix is a continuous function of the elements of the matrix, implies that the real part of each of the characteristic roots of GC^a approaches minus infinity. This proves the theorem.

IV

Now let us consider the entire dynamical system consisting of (14) ~ (16). As long as q is close enough to \hat{p} , Theorems 1 and 2 imply that the time paths of p and q will be almost identical; any discrepancy between the two will result in an almost instantaneous convergence of p upon q through the tâtonnement process. Thus the dynamics of the system can be

approximated by the equations:

$$(28) \quad \frac{dp_i}{dt} = h_i^0(p, p, z); \quad i=1, \dots, n-1 \quad t \geq 0$$

$$(29) \quad \frac{dz_k}{dt} = f_k^0(p, p, z); \quad k=1, \dots, n; \quad t \geq 0$$

where h_i^0 (f_k^0) is the limiting function of the h_i^a 's (f_k^a 's) with pure speculation.

The function f_k^0 represents the rate of change of the pure speculator's stocks of good k when all markets are clearing at prices equal to his expected prices. Let us assume that he engages in no consumption or production activities. Then this rate of change will be his net current trade of the commodity, which in turn will be equal to its non-speculative excess supply:

$$(30) \quad f_k^0(p, p, z) = -x_k(p); \quad k=1, \dots, n.$$

It is another matter altogether to say anything conclusive about the expectation-function $h^0(p, q, z)$. We cannot invoke adaptive expectations, because as long as $p=q$ there is no discrepancy for the speculator to adapt to. Nevertheless it is possible for him to make forecasting errors. For if $q \neq \hat{p}$, his expectations will only be fulfilled until his inventories run out, which some of them will do if he does not revise q . After that he will cease to dominate the market and the price will begin changing independently of his expectations.

Thus one could argue that it would make sense for the speculator to revise q_i upward whenever $p_i = q_i < \hat{p}_i$ and vice versa in the opposite case. If he knew the exact value of \hat{p} it would be a rational self-fulfilling expectation for him to set $q = \hat{p}$. But of course if this much information were available to him we would have no market adjustment problem to worry about; the auctioneer could just announce the price vector \hat{p} once and for all and then retire. Let us suppose that his only information consists of the current

excess demands, $x_i(q)$, which he can observe simply by looking at the current rate of change of his inventories. On the basis of this information he will try to infer which q_i 's are below and which above the corresponding \hat{p}_i 's. But this is exactly the problem facing the Walrasian auctioneer in section I, the only difference being that the auctioneer is acting directly upon p rather than q . When he observes $x_i(p) > 0$ the reason why he raises p_i is that he infers from this information that $p_i < \hat{p}_i$. If this is a reasonable inference for the auctioneer it is reasonable for the speculator. When he observes $x_i(q) > 0$ he ought to infer that $q_i < \hat{p}_i$. But we have just argued that when he thinks that $q_i < \hat{p}_i$ he will likely raise q_i . Thus we are led to the conclusion that, just as the auctioneer raises p_i when $x_i(p) > 0$, so the speculator will raise q_i when $x_i(q) > 0$. In other words, the expectation-function will take the form:

$$(31) \quad h_i^0(p, p, z) = g_i(x_i(p)); i=1, \dots, n-1.$$

In this case the dynamics of the system would be determined by the system:

$$(32) \quad \frac{dp_i}{dt} = g_i(x_i(p)); i=1, \dots, n-1; \quad t \geq 0$$

$$(33) \quad \frac{dz_k}{dt} = -x_k(p); k=1, \dots, n; \quad t \geq 0$$

for values of p close enough to \hat{p} . The local dynamic behavior of prices in the economy with pure speculation would be identical to that in the economy with no speculation. In particular, the overall system is locally asymptotically stable with pure speculation if and only if it is with no speculation.

Thus we are led to the paradoxical conclusion that pure speculation renders the price adjustment process infinitely stable yet it has no affect on the dynamic behavior of prices. The pure speculator's sensitivity to price changes makes the auctioneer's work so easy that we no longer observe

any of the latter's gropings. The pure speculator in effect drives the auctioneer out of business. The system behaves as if the speculator, rather than taking prices as given by the auctioneer, sets the prices himself, at the values q that he expects to clear all markets, but having replaced the auctioneer he then proceeds to behave almost exactly the way the auctioneer had been behaving. Given the same problems of inference he adjusts prices in exactly the same way, with one major difference. The speculator holds inventories that he is willing to see run up or down to allow the others' trading plans to be fulfilled when the system is out of equilibrium, whereas the auctioneer is forced to ration the other traders in disequilibrium.

This equivalence relationship provides a potentially valuable insight into one of the major outstanding problems in economic theory--how to account for competitive price adjustment in the absence of the ethereal Walrasian auctioneer. Several recent articles¹³ on this subject have suggested for various reasons that one ought to model the process of price adjustment as being conducted by inventory holding traders who find it in their self-interest to set prices, to allow other traders to carry out their desired trades at those prices and to adjust the prices in response to any perceived disequilibrium. The present results provide another justification for this new approach to modeling price adjustment by showing that it is formally equivalent to introducing a pure speculator into an economy that is initially dependent upon the auctioneer.

But the equivalence relationship cuts both ways. It also provides a new interpretation of the tâtonnement process. The standard interpretation, according to which the process represents the gropings of the auctioneer, poses two major problems. The first is that the auctioneer has no apparent incentive for performing this task. The second problem is that either we

make the strong assumption of recontracting, in which case everyone waits for the auctioneer to arrive at \hat{p} before any trading occurs, or we will have trading out of equilibrium with wide spread rationing which is difficult to square with casual observations of how markets function and which is difficult to handle analytically. But if we interpret the process as representing the gropings of the pure speculator, he has a simple and plausible objective of maximizing expected capital gains, and he will allow trading to take place out of equilibrium without any rationing having to occur unless his expectations are so far off base that his inventories run out.

There is a question with this new interpretation of whether or not the pure speculator can completely replace the auctioneer and still be attempting to maximize expected capital gains. As long as he was taking prices as given his behavior was optimal, but if we now suppose that he is setting prices rather than taking them we may ask if it is still in his self-interest to behave in the same way. In order to answer this question we need to impose some constraints on his behavior in the form of competition. In general this is an exceedingly difficult problem to analyze, but there are simple cases where something can be said. In particular, if we replace the speculator by a large group of small, identical speculators who form consistent expectations of each others' behavior, then they will behave as we have described. The rest of this section is a sketch of how this could be demonstrated in the simple case of only two goods.

Suppose that each of a large group of identical speculators believes that the market clearing price, \hat{p} , has the value q . Each of them wants to maximize his expected capital gains. Furthermore each of them knows that all of his competitors will be acting in the same way with the same beliefs. In order to determine his optimal price-setting decision he

also has to form some expectation about the prices that will be set by his competitors. Since his competitors are identical to each other he will assume that they will all charge the same price, p^e . So the individual speculator will choose his own price, p , in the light of q and p^e , so as to maximize his expected capital gain. Let this choice be represented by the function:

$$(34) \quad p = p(p^e, q).$$

We can characterize the nature of this function as follows. Suppose that $p^e < q$. Then the individual speculator will believe this to be a temporary situation, because his competitors would eventually run out of inventories at that price, so the price will sooner or later have to rise. Thus it will be to his advantage to set $p^e < p < q$, for then he will be able to buy all he wants (assuming perfect information of prices by the price takers and atomistic competition among the price setters). This running up of inventories will allow him a large capital gain when the market price eventually approaches q . The analogous result holds for $p^e > q$. When $p^e = q$ the same reasoning implies that he will set $p = q$. So we may infer that:

$$(35) \quad \left\{ \begin{array}{ll} p^e < p(p^e, q) < q & \text{if } p^e < q \\ p^e = p(p^e, q) = q & \text{if } p^e = q \\ p^e > p(p^e, q) > q & \text{if } p^e > q \end{array} \right.$$

We have already discussed the process by which the expectation, q , might be formed. In order to analyze the determination of the other expectation, p^e , let us suppose that the speculator forms a consistent expectation: he will not hold any expectation, p^e , which he believes, if it

were shared by his competitors would induce them to set a price $p \neq p^e$. In other words, since all speculators are identical, he constrains his expectations to satisfy:

$$(36) \quad p^e = p(p^e, q).$$

It follows immediately from (35) and (36) that all the speculators will set $p=q$. We can indeed throw away the auctioneer.

Footnotes

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¹Kaldor (1939), esp. 8-10.

²See, for example, Friedman (1953), Baumol (1957), Telser (1959). Much of the literature has been summarized by Sohmen (1969), ch. 3.

³The question was first put this way, although somewhat obliquely, by Hicks (1939), with notable subsequent contributions by Enthoven and Arrow (1956), Arrow and Nerlove (1958), and Arrow and Hurwicz (1962). A summary and elaboration of these results is presented by Arrow and Hahn (1971), pp. 309-315.

⁴See Cagan (1956), Arrow and Nerlove (1958).

⁵See Arrow and McManus (1958).

⁶These matters are discussed by Cootner (1968) and Kaldor (1939): 3-5. The problem of distinguishing speculators from hedgers is discussed by Hirschleifer (1975).

⁷The notation $p > 0$ indicates $p_i > 0$, $i=1, \dots, n-1$.

⁸The expression within the brackets denotes the element in the i^{th} row and j^{th} column of the matrix.

⁹An interesting attempt to make such an interpretation was made by Fisher (1972).

¹⁰See Arrow (1971).

¹¹For a definition of Gaussian curvature, see Debreu (1972). That this is equivalent to the positive definiteness condition can be inferred from the theorem proved by Mann (1943).

¹²See Samuelson (1947): 111-15.

¹³See Clower (1975), Clower and Leijonhufvud (1975), and Clower and Howitt (1975).

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