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Shmuel Sharir

Yoram Weiss

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TWO RULES OF AGGREGATION OF
WORK EFFORTS AND LEISURE TIMES

Shmuel Sharir*
Yoram Weiss**

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* Tel-Aviv University and the University of Western Ontario

** Tel-Aviv University and the National Bureau of Economic Research

ABSTRACT

In this note it is shown that there are two meaningful rules for aggregating work efforts and leisure time. One is when all wage rates change by the same rate and the other when they change by the same magnitude. In the first case the aggregate formed is expressed in value terms, while in the second case it is expressed in physical (time) units. One important implication of the discussion is that static demand theory cannot predict, as is usually alleged, that a negative income tax will necessarily have an adverse effect on labor supply by a family.

Two Rules of Aggregation of Work Efforts and Leisure Times

1. Introduction and Conclusion

Situations might arise where a researcher has or wants to aggregate work efforts or leisure time for the purpose of estimating a supply of, or demand for, these commodities. For example, when investigating the effect of the negative income tax, Hollister (1974) is interested in the family's labor supply response. When more than one family member works, aggregation of work efforts is necessary. Sloan and Richupan (1975) estimate nurses' supply of labor using micro data, and to the extent that nurses were multiple jobholders or worked in both daytime and nighttime shifts, weekdays and weekend shifts, etc., the problem of aggregation of work efforts arises again. Or, Fair (1969) estimates the demand for hours of work and for workers in the case of production workers; but this group of workers probably includes various grades, which are not perfect substitutes from the firm's point of view, so that aggregation of work efforts must take place again. In theoretical investigations of the supply of and/or demand for work efforts (or leisure time) aggregation is a regular practice because the researchers want to reduce the problem dealt with to manageable proportions.

One obvious requirement from an aggregate is that it must be meaningful. Put differently, the researcher must be satisfied that the units of the various work efforts (leisure time or consumption goods) to be aggregated, are identical from the point of view that he considers to be important. For example, one could view the total hours of work of a multiple jobholder, or total hours of work of a family with more than one participating member, as

a meaningful aggregate. Moreover, theory will even suggest the relevant variables that determine such an aggregate. It could be shown that in the above problems these are the various wage rates (of the different jobs or the different family members), non-labor income and tastes. But, in general, theory will not yield any prediction concerning the effect of these explanatory variables on the quantity of the aggregate that has been formed.¹

In this note we are interested in finding rules of aggregation so that the aggregate could be viewed as a "composite good," to which the usual predictions of static demand (or supply) theory could be applied. In particular, is Hicks's (1957, 312-13) rule of aggregation--that relative prices should not change--the only possible one, and what types of aggregates are formed under each possible rule.

The conclusion reached in the note is that there are two rules of aggregation. One, when relative wage rates (prices) are constant, work efforts (leisure times or consumption goods) could be aggregated into a "composite good," the quantity of which is measured in value (dollar) terms. The second, when absolute differences among the wage rates (prices) are constant, work efforts (etc.) could be aggregated into a "composite good," the quantity of which is measured in physical (e.g., time) terms.²

While most of the discussion will be carried out for work efforts, we shall discuss briefly its application for leisure time.

There are various ways to prove the composite good theorem--Hicks (1957, 312-13), Liviatan (1968, 291-304) and Lancaster (1968, 125-27). We find it useful to employ the "generalized substitution effect" approach, which has been employed by Lancaster.

2. Aggregation of the Supply of Work Efforts

2.1 The Individual: One Type of Time

Assume that the individual works in n (market) work activities, spending H_i hours in the i^{th} activity and receiving an hourly wage rate which is denoted by w_i^0 . The individual also spends time in m leisure activities (defined as those time activities with a zero wage rate). The time spent in the j^{th} leisure activity is denoted by L_j . The individual also buys q consumption goods. The quantity bought from the k^{th} good is X_k and its price (per unit) is P_k^0 .

The individual wants to maximize his utility

$$(1) \quad U = U(X_1, \dots, X_q, H_1, \dots, H_n, L_1, \dots, L_m) .$$

Subject to the money income, time and non-negativity constraints, respectively,

$$(2) \quad \sum_{k=1}^q P_k^0 X_k = \sum_{i=1}^n w_i^0 H_i + A$$

$$(3) \quad \sum_{i=1}^n H_i + \sum_{j=1}^m L_j = T$$

$$(4) \quad X_1, \dots, X_q, H_1, \dots, H_n, L_1, \dots, L_m \geq 0$$

where A is non-wage income and T is total time available for work and leisure purposes.

Following Samuelson (1965, 107-16), we shall derive the "generalized substitution effect" for the problem (1) - (4). Let us substitute for, say, L_m , from (3) into (1), and change the place of wage income in (2). The problem then becomes that of maximizing

$$(1') \quad V = V(X_1, \dots, X_q, H_1, \dots, H_n, L_1, \dots, L_{m-1})$$

subject to

$$(2') \quad \sum_{k=1}^q P_k^0 X_k - \sum_{i=1}^n w_i^0 H_i = A$$

and the non-negativity constraint on the X_k 's, H_i 's and L_j 's. If we assume that the utility function is strictly quasi-concave there will be a unique solution to the problem. (The consumption set is convex.) Let us assume that this solution, $X_1^0, \dots, X_q^0, H_1^0, \dots, H_n^0, L_1^0, \dots, L_{m-1}^0$,³ is internal (or a regular corner one). That solution fulfills the requirement that the "cost" of achieving the obtained level of utility--given the P_k^0 's and w_i^0 's--are minimized. Put differently, any other bundle yielding the same level of utility will require a higher non-labor income, A .

Consider now another set of prices, P_k^1 's, and wage rates, w_i^1 's. Suppose that the "cost" of achieving the previous level of utility--given the new prices and wage rates--are minimized for the bundle $X_1^1, \dots, X_q^1, H_1^1, \dots, H_n^1, L_1^1, \dots, L_{m-1}^1$. Since both bundles yield the same level of utility, neither is preferred to the other, and it must be that

$$(5) \quad \sum_{k=1}^q P_k^0 X_k^0 - \sum_{i=1}^n w_i^0 H_i^0 \leq \sum_{k=1}^q P_k^0 X_k^1 - \sum_{i=1}^n w_i^0 H_i^1$$

$$(6) \quad \sum_{k=1}^q P_k^1 X_k^1 - \sum_{i=1}^n w_i^1 H_i^1 \leq \sum_{k=1}^q P_k^1 X_k^0 - \sum_{i=1}^n w_i^1 H_i^0$$

If we define $Y^1 = Y^0 + \Delta Y$, for $Y = X_k$'s, P_k 's, H_i 's, w_i 's, and Δ denotes a change in the variable, we could write, instead of (5) and (6), respectively,

$$(5') \quad \sum_{k=1}^q P_k^0 \Delta X_k - \sum_{i=1}^n w_i^0 \Delta H_i \geq 0$$

$$(6') \quad \sum_{k=1}^q (P_k^0 + \Delta P_k) \Delta X_k - \sum_{i=1}^n (w_i^0 + \Delta w_i) \Delta H_i \leq 0$$

Subtracting (5') from (6') yields the "generalized substitution effect"

$$(7) \quad \sum_{k=1}^q \Delta P_k \Delta X_k - \sum_{i=1}^n \Delta w_i \Delta H_i \leq 0$$
⁴

Suppose at first that good prices are unchanged (i.e., all $\Delta P_k = 0$), but that the wage rates change, i.e., $\Delta w_i \neq 0$. If all wage rates change in

the same direction and at the same proportion, α , (so that $\Delta w_i = \alpha w_i^0$),

(7) will become

$$(7') \quad -\alpha \sum_{i=1}^n w_i^0 \Delta H_i \leq 0.$$

This implies that total work effort valued at the original wage rates (w_i^0 's) will change (if at all) in the same direction as the change in wages. Put differently, if one aggregated work effort in value (dollar) terms, using the original wage rates to form $C = \sum_{i=1}^n w_i^0 H_i$, C could be viewed as a "composite good," the price change of which is α . It should be emphasized that we could write $\Delta w_i / w_i^1 = \beta$ for any i (so that $\Delta w_i = \beta w_i^1$) and the "composite good" will become $C = \sum_{i=1}^n w_i^1 H_i$. Thus, either the original or the new set of wage rates could be used as weights in forming the "composite good."

Suppose now that all the wage rates change by the same absolute magnitude, D, then (7) becomes

$$(7'') \quad -D \sum_{i=1}^n \Delta H_i \leq 0.$$

This suggests that total work effort measured by physical (time) units will change (if at all) in the same direction as the change in wages. And if one aggregates work effort in terms of time units, $H = \sum_{i=1}^n H_i$, then H could be viewed as a "composite good," the price change of which is D.

2.2 The Individual or the Family: Various Types of Time

The discussion above could easily be generalized to the case of several types of time from the individual's or the family's point of view. Let us assume that there are s such types of time--weekends vs. weekdays for the individual, husband's time vs. wife's time in the family context. H_1, \dots, H_s and L_1, \dots, L_s will then replace H_1, \dots, H_n and L_1, \dots, L_m of the previous section; and instead of one time constraint there will be s such constraints,

which take the form

$$(3^*) \quad H_r + L_r = T_r \quad r=1, \dots, s$$

substituting from (3*) into (1) (appropriately defined) for L_r , the family's (individual's) problem becomes that of maximizing

$$(1^*) \quad V = V(X_1, \dots, X_q, H_1, \dots, H_s)$$

subject to

$$(2^*) \quad \sum_{k=1}^q P_k^0 X_k - \sum_{r=1}^s w_r^0 H_r = A .$$

Following the previous argumentation, equations similar to (7), (7') and (7'') could be derived.

If the wage rates of different members of the family change by the same magnitude, one could aggregate their hours of work, and view them as a "composite good." If, however, their wage rates change by the same proportion, it is the aggregation of the value (in dollars) of their work efforts--using one set of wage rates as weights--which could be treated as a "composite good." Of course, one could still aggregate the hours of work of the various family members even in that case. But that aggregate will not form a "composite good;" i.e., one cannot assign a "wage rate" to the aggregate, so that its effect--holding utility constant--could be predicted on a priori grounds.

2.3 Implication: The Effect of the NIT on Family Labor Supply

It is a widely held belief that static demand theory predicts that a negative income tax (NIT) will have a negative effect on hours of work supplied by the family (e.g., Rees (1974, 160)). The discussion above suggests that this is not the case for either the family or for a multiple jobholder. A negative income tax whose rate is to reduce all the wage rates (of family members or jobs) to $w_i^0(1 - t_0)$, where w_i^0 is the wage rate that the jobs pay (i.e., before the negative tax). Thus, all the wage rates decline (from the family's point of view) by the same

proportion. According to (7') one could view $C = \sum w_i^0 H_i$ as a "composite good." This turns out to be exactly the family's (or the multiple jobholder's) earnings. The substitution effect is to reduce the quantity of C (earnings). The income effect of the NIT depends on whether C is an inferior or a normal good. If it is inferior, we can predict that the NIT will reduce the family's earnings. No specific prediction could be derived, however, for total hours of work, $H = \sum H_i$, if more than one job is held by the family. This analysis provides a theoretical justification for Hollister's (1974, 223) intuition, which led him to claim that the effect of the NIT is more appropriately measured according to its effect on family earnings rather than on its total hours of work.

3. Aggregation of the Demand for Work Efforts

Turning to the theory of the firm, it is well known that minimization of costs of a given output--which any profit maximizing firm must achieve--yields the "generalized substitution effect" too (e.g., Samuelson (1965, 73-74)). If X_k is interpreted now as the quantity demanded from the k^{th} type of capital, and H_i as the amount of the i^{th} work effort (type of workers) demanded, inequality (7)--except that the minus is replaced by a plus--will hold for the firm. The rest of the discussion will then follow.

That discussion has obvious implications to empirical studies of demand for labor. Usually data are available in an aggregate form, e.g., for all production workers. Suppose that there are n types of workers (occupations) and that the firm employs T_i workers from the i^{th} type of workers. If only wage rates change, and assuming they change by the same magnitude for each type of labor, (7'') could be written as

$$(\tilde{7}'') \quad \Delta w \sum_{i=1}^n \sum_{t=1}^{T_i} \Delta H_{ti} = T \Delta w \left(\sum_{i=1}^n \sum_{t=1}^{T_i} \Delta H_{ti} \right) / T \geq 0$$

when $\sum_{i=1}^n T_i = T$. Thus, if the researcher has good reasons to believe that wage rates (for all types of workers) change by the same magnitude among firms or industries, (i.e., the distributions of wage rates are "spread preserving"), he could view total hours of work of all (production) workers, or hours per (production) worker, as a "composite input." The price of that input could be represented by any central value of the distribution of wage rates within a firm or industry--mean, median or mode--that is available.

This discussion could yield a theoretical justification for Rosen's (1969) estimation of the demand for hours per worker using micro data, aggregated to the industry level. His demand function involves the mean hours per worker (standardized for the occupational mix) and the mean hourly earnings. This procedure is sound according to our discussion if the distribution of the wage rate over occupations within an industry shifts from one industry to another by a fixed amount. It is possible, however, that a better approximation to reality would have been to assume that the distribution of the wage rate shifts by a certain proportion, α . In that case (7') is relevant, and it becomes

$$(\tilde{7}') \quad \alpha \sum_{i=1}^n w_i^a \sum_{t=1}^{T_i} \Delta H_{ti} = n \alpha \left(\sum_{i=1}^n w_i^a \sum_{t=1}^{T_i} \Delta H_{ti} \right) / n \geq 0$$

where w_i^a 's are the wage rates of a given industry, which is chosen to serve as the standard. The "composite input" in that case is the total standardized labor cost or the standardized labor cost per worker.⁵ The change in the price of the "composite input" is α , or the ratio of the mean wage rate between any industry and that of the industry which serves as a standard.

The data needed for the estimation of demand for the "composite input" suggested here, are identical to that used by Rosen for his demand function.

One could suggest criteria on how to decide which of the two rules of aggregation describes the real world better.⁶ But, in the present case one might question the very need for aggregation over occupations. The same data could be used to estimate a series of demands for hours per worker for the various occupations separately. The aggregation itself does not seem to serve any useful purpose--except if there were not enough observations for occupations by industry--while the choice of the wrong rule of aggregation (if any is the right one) will yield a misspecified demand function. Clearly, if one has all the information, he is better off proceeding without resorting to aggregation.

4. Aggregation of Leisure Times for an Individual and the Family

Suppose that an individual works in one job, spending there H hours and receiving a wage rate, W^0 . He also spends his leisure time on m activities. The time constraint for him is, then,

$$(3'') \quad H + \sum_{j=1}^m L_j = T$$

substituting from (3'') into both the utility function (1), and the money constraint (2) (appropriately formulated) yields the maximization of

$$(1'') \quad V = V(X_1, \dots, X_q, L_1, \dots, L_m)$$

subject to

$$(2'') \quad \sum_{k=1}^q P_k^0 X_k + W^0 \sum_{j=1}^m L_j = W^0 T + A$$

It is abundantly clear that in this case, leisure times could always be aggregated in physical (time) terms into a "composite good", $L = \sum_{j=1}^m L_j$, the price of which is the wage rate.

It should be emphasized that the above result concerning the aggregation of various leisure times is obtained even if they are not perfect substitutes in consumption. Moreover, it is upheld under the relevant rule of aggregation even if there are different wage rates for different jobs. For example, let W_1^0 and W_2^0 be, in the family context, the husband's and wife's wage rate, respectively. There are two time constraints, as in (3*), in that case. Their substitution into (1) and (2) (appropriately formulated) will yield the problem of maximizing

$$(1^{**}) \quad V = V(X_1, \dots, X_q, L_1, L_2)$$

subject to

$$(2^{**}) \quad \sum_{k=1}^q P_k^0 X_k + W_1^0 L_1 + W_2^0 L_2 = W_1^0 T_1 + W_2^0 T_2 + A$$

Assuming fixed goods prices the "generalized substitution effect" will be

$$(7^{**}) \quad \Delta W_1 \Delta L_1 + \Delta W_2 \Delta L_2 \leq 0$$

And if $\Delta W_1 = \Delta W_2 = \Delta W$ one could view $L = L_1 + L_2$ as a "composite good", while if $\Delta W_1 / W_1^a = \Delta W_2 / W_2^a = \alpha$, ($a = 0, 1$), the "composite good" becomes $G = W_1^a L_1 + W_2^a L_2$.⁷

Footnotes

¹For additional comments on the term 'a meaningful aggregate' see Green (1964, Ch. 1).

²For a discussion on the role of absolute and relative wage differentials in determining the supply of effort to a given work activity, see Sharir and Weiss (1974).

³Due to the time constraint (3), L_m is determined too.

⁴If the utility function is twice differentiable anywhere the inequality will hold, and the new solution to the maximization problem is not identical to the old one. Note that equation (7) describes all the information that we could possibly give on the adjustment behavior. The changes in expenditure on consumption goods and in wage income can be deduced from it. As we shall see below, for the changes in leisure time (or work time) to be deduced from it, a certain condition must be met (see equation (7')).

⁵The standardized labor cost equals to the actual labor cost only for the industry whose wage rates serve as weights.

⁶A necessary (but not sufficient) condition for aggregation of physical units, such as hours of work, is that the standard deviation of the wage rates is equal for all the industries. A necessary (but not sufficient) condition for aggregation of value units is that the coefficient of variation of the wage rates increases with the level of the mean wage rate.

⁷The discussion above applies, of course, to consumption goods as well. We have only to assume in (7) that $\Delta P_k \neq 0$ while $\Delta W_i = 0$. If all prices change by the same proportion we obtain the well known result, that $Y = \sum_{k=1}^q P_k^a X_k$ ($a = 0, 1$), could be viewed as a "composite good." If, however, all prices change by the same magnitude, it is $X = \sum_{k=1}^q X_k$ that could be treated as a "composite good." The latter, however, has a meaningful interpretation only when the X_k 's are different brands of the same good.

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