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I. Introduction

Ever since the path-breaking work of W. A. Lewis (1954) economists have been interested in the analysis of growth and resource allocation in low-income economies with non-competitive wage levels in the "modern" sector. Early work on allocation problems in a two-sector economy of this kind included Everett Hagen's paper on tariff policy (1958) and more recently, John Harris and Michael Todaro (1970) have analyzed a model in which a non-competitive wage in the manufacturing sector causes rural-urban migration and unemployment. The policy conclusions they derived, which were based on a discussion of wage subsidies and forcible restriction of migration, have been criticized and extended in a paper by Jagdish Bhagwati and T. N. Srinivasan (1974) who demonstrated the possibility of achieving an efficient ("first-best") resource allocation through various combinations of taxes and subsidies, without the necessity of migration restriction.

The practical value of the different conclusions derived from these models depends, as usual, on the realism of the underlying assumptions. On the one hand, in order to find the "best" tax policy for a particular country, one must obviously restrict the analysis to those policies which are feasible, administratively and otherwise, in that country. On the other hand, one must generally also extend the analysis to <u>all</u> feasible policy combinations.

The (explicit and implicit) assumptions made in this respect by the authors referred to above, are somewhat arbitrary. The analysis by Hagen considers only a tariff on the manufactured good, even though the revenue from the tariff presumably could be used for some form of subsidy which could improve welfare. Harris and Todaro analyze a wage subsidy, implicitly assuming that revenue is available to finance it; but in that case, it would be natural to assume that part of government revenue could be used for some other type of subsidy as well. Bhagwati and Srinivasan place no restrictions at all on the feasible set of tax-subsidy policies, or on the ability of the government to raise revenue through lump-sum taxation with no effects on production or consumption.[1]

The purpose of the present paper is to consider the problem of formulating an optimal second-best tax policy in an economy with domestic distortions, subject to an explicit government budget constraint, and under various assumptions regarding the feasibility of different taxes and subsidies. The main finding is that the optimal tax-subsidy combinations are quite sensitive to changes in these assumptions. I also derive the conditions under which policies similar to those recommended by the above authors will be optimal.

The organization of the paper is as follows. In section II, I analyze the general problem of optimal indirect taxes on production, consumption and international trade in a small, open economy, under various restrictions concerning the government's ability to raise revenue through lump-sum taxation and on the controllability of various taxes. The methodology in this section relies heavily on that described in the paper by Partha Dasgupta and Joseph Stiglitz (1974). In section III, I consider the application of the general principles to a two-sector model of an open economy; the model is similar to the one constructed by Harris and Todaro, i.e., resource allocation is

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[1] In a similar vein, Bhagwati and Ramaswami (1963) also criticized and extended Hagen's 1958 analysis, showing that "a suitable tax-cum-subsidy on domestic production" to correct a domestic wage distortion would be superior to a tariff. Again, they placed no restrictions on the feasible set of tax-subsidy policies.

distorted by an institutionally given minimum wage in the manufacturing sector, fixed at a level high enough to cause rural-urban migration and unemployment in the way they describe. Section IV, finally, contains some qualifications and concluding comments.

II. The General Problem

To analyze the problem of an optimal tax policy under varying restrictions, we proceed as follows. Following Harris and Todaro, we will confine the analysis to the short run, and the capital stock in each producing sector is assumed to be fixed so that the output levels are uniquely determined by the amount of labour in each sector. We will assume that employers take wages and output prices as given and maximize profits; hence the amount of labour employed in each sector will be a function of output prices and wage rates. Consumers, on the other hand, are also supposed to take commodity prices as given, and are assumed to maximize utility subject to their budget constraints. In general, we will not assume that wage rates and output prices are competitively determined, but will instead posit the existence of various types of market imperfections and distortions such as minimum wage rates, non-controllable producer subsidies in certain sectors, etc.

The problem of optimal second-best taxation is then analyzed as follows. We consider an open economy in which all goods are tradable at fixed international prices, and assume that the government has the ability to impose producer taxes/subsidies and consumption taxes or subsidies on some or all commodities. With fixed international prices, this is equivalent to assuming that the government can control the corresponding producer and consumer prices. The government is also supposed to be able to pay employment subsidies in some sectors, and may be able to raise certain amounts of revenue through lump-sum taxation. The solution to the optimal tax problem will consist in finding the values of the various controllable taxes and subsidies which maximize aggregate consumer welfare, subject to the government budget constraint and the balance-of-trade constraint.

The problem is first analyzed in general terms, for an arbitrary number of sectors and without specifying the exact nature of the market imperfections existing in the economy; the methodology is quite similar to that employed by Dasgupta and Stiglitz.

The general model is specified as follows. We consider an economy with m commodities, all of which are traded. By appropriate choice of units, the world price of each commodity is set at unity. Real output at world prices, Y*, is thus given by:

$$Y^* = \sum_{i=1}^{m} Q_i \tag{1}$$

where Q_i is the output level in the i^{th} sector. As noted above, the level of output in each sector is determined by the producer price q_i and the wage rate paid by employers, which is influenced by the subsidy s_i paid for employment of labour in the i^{th} sector, L_i. This means that in general, each Q_i will be a function of the q_i's and s_i's, so that real output Y^* can also be taken as a function of those control variables. We will also consider the case where the wage rate in some sectors is influenced by the consumer prices p_i;[1] in this case, some of the Q_i and hence Y* will be functions of the p_i as well.

[1]This case would arise if the wage rates in some sectors were set through bargaining or legislation, and cost-of-living considerations were taken into account in the wage-setting process.

Total disposable income at world prices will be given by:

$$Y = Y^* + \Sigma(q_i - 1)\ Q_i + \Sigma\ s_i L_i - B, \qquad (2)$$

i.e., it is equal to the value of real output at world prices plus producer subsidies plus subsidies to labour, minus B, the amount of revenue raised by the government in the form of lump-sum taxes.

The government's budget constraint is given by:

$$G = \Sigma\ (p_i - 1)\ C_i - \Sigma\ (q_i - 1)\ Q_i - \Sigma\ s_i L_i + B = 0 \qquad (3)$$

where C_i is the total consumption (at world prices) of the i^{th} commodity and p_i is the price paid by consumers, so that $(p_i - 1)$ is recognized as the consumption tax on the i^{th} commodity. Equation (3) thus states that total net subsidies to output and labour must be paid for by revenues from consumption taxes and lump-sum taxation.

The j^{th} consuming unit maximizes his utility from consumption of the m commodities subject to his budget constraint, i.e., the consumption levels C_{ij} are determined as the solution to the problem:

$$\text{Maximize } U_j(C_{1j}, C_{2j}, \ldots, C_{mj})$$

$$\text{subject to: } \sum_i p_i\ C_{ij} = Y_j$$

where Y_j is the disposable income of the j^{th} consumer. Since each consumer satisfies his budget constraint, we can aggregate the constraints over all of them, so that for the economy as a whole we will have:

$$\sum_i p_i\ C_i = Y \qquad (4)$$

The economy is also subject to a balance-of-payments constraint given by:

$$Z = \Sigma\ (Q_i - C_i) = Y^* - \Sigma\ C_i = 0, \qquad (5)$$

i.e., real consumption at world prices has to equal real output at world prices. It is easy to show, however, that the constraints (3) - (5) are not independent, so that, for example, if (3) and (4) are satisfied, the balance-of-payments constraint will also be satisfied. An implication of this, which will be further discussed below, is that the optimal taxation problem can be formulated using either restriction and that the shadow prices of government revenue and foreign exchange need not be separately determined.

In general, the problem of designing an optimal tax structure will now be defined as that of finding those values of the controllable taxes and subsidies which maximize aggregate welfare in the economy subject to the government budget constraint and the balance-of-payments constraint.

Aggregate welfare will of course depend on the utility levels of individual consuming units. We shall abstract from distribution problems by assuming that consuming units have identical utility functions and also have the same income level. The latter assumption may be particularly unrealistic. While it is true that a certain amount of real income redistribution takes place in low-income countries through direct taxation, through selective distribution of some public goods, and also privately through the extended family system, substantial after tax real income differentials are likely to remain between, say, rural and urban families in LDCs, or between urban workers and capitalists, and it would in principle be desirable to take the effects of indirect taxes on income distribution into account. The main reason for excluding them from the present analysis is the resulting simplification of the formal analysis: when identical utility functions are assumed, the individual functions can be meaningfully aggregated into a welfare function for the economy as a whole with total consumption of each of the commodities as its arguments. Aggregate demand for each commodity can then be determined by maximizing this welfare function subject to the budget constraint (4). As is well known, since the C_i's derived as a solution to this problem are functions of the p_i's and Y, the maximum attainable level of utility can be written as $W = W(p_1,p_2,\ldots,p_m,Y)$ where the function W is known as an indirect utility function.

The optimal taxation problem can now be written as:

$$\text{Maximize} \quad W(p_1,p_2,\ldots,p_m,Y)$$

$$\text{subject to } G = 0$$

where Y and G have been defined in (2) and (3), and where the set of control variables K includes some or all of the p_i's, q_i's and s_i's.

The Lagrangean function for this problem may be written as:

$$F = W(p\ ,p\ ,\ldots,p\ ,Y) + \lambda G. \tag{6}$$

We first consider the optimal set of producer prices. Differentiating (6) with respect to $q_j \in K$ and equating to zero, yields:

$$\frac{\partial F}{\partial q_j} = \frac{\partial Y}{\partial q_j} W_y + \lambda \frac{\partial G}{\partial q_j} = 0 , \qquad q_j \in K \tag{7}$$

where $W_y = \partial W/\partial Y$ is the marginal utility of income. Defining

$$T = \Sigma\ (p_i - 1)\, C_i \tag{8}$$

$$P = -\ \Sigma\ (q_i - 1)\ \ Q_i\ -\ \Sigma\ s_i L_i$$

as net revenue from consumer and producer taxation, respectively, and noting that $\partial P/\partial q_j = \partial Y^*/\partial q_j - \partial Y/\partial q_j$ from (2) above, we obtain, after rearrangement:

$$\frac{\partial F}{\partial q_j} = \frac{\partial Y}{\partial q_j} \left(W_y - \lambda\ \left(1 - \frac{\partial T}{\partial Y}\right)\right) + \lambda \frac{\partial Y^*}{\partial q_j} = 0, \qquad q_j \in K \tag{9}$$

Using (8) and (4), we can also show that:

$$\frac{\partial T}{\partial Y} = \Sigma\ (p_i - 1)\ \frac{\partial C_i}{\partial Y} = 1 - \Sigma\ \frac{\partial C_i}{\partial Y}\ , \tag{10}$$

which, after substitution into (9) yields

$$\frac{\partial F}{\partial q_j} = \frac{\partial Y}{\partial q_j} \cdot W_y - \lambda \left[\Sigma\ \frac{\partial C_i}{\partial Y}\right] \frac{\partial Y}{\partial q_j} + \lambda\ \frac{\partial Y^*}{\partial q_j} = \frac{\partial Y}{\partial q_j} \cdot W_y + \lambda\ \frac{\partial Z}{\partial q_j} = 0, \qquad q_j \varepsilon K \tag{11}$$

where we have used (5). A comparison between (7) and (11) illustrates the equivalence between the shadow prices of foreign exchange and government revenue, which was noted above. Thus equation (7) states that at its optimum value, the change in utility caused by a small change in disposable income when a producer price is changed must be exactly offset by the resulting small change in government revenue valued at the shadow price λ; equivalently, (11) implies that at the optimum the change in utility caused by the change in disposable income must be exactly offset by the change in the trade balance, valued at the shadow price of foreign exchange, which is also λ.

The optimal values for the controllable set of labour subsidies are found in an exactly analogous manner. Differentiating the Lagrangean with respect to s_j and equating to zero gives:

$$\frac{\partial F}{\partial s_j} = \frac{\partial Y}{\partial s_j}\ W_y + \lambda\ \frac{\partial G}{\partial s_j} = \frac{\partial Y}{\partial s_j} \left[W_y - \lambda\ (1 - \frac{\partial T}{\partial Y})\right] + \lambda\ \frac{\partial Y^*}{\partial s_j} = 0, \quad s_j \varepsilon K \tag{12}$$

with the same interpretation as for the previous case.

Turning now to the determination of the optimal level of consumption taxes, the first-order conditions corresponding to the controllable consumer prices, p_j, are:

$$\frac{\partial F}{\partial p_j} = \frac{\partial W}{\partial p_j} + \lambda \frac{\partial G}{\partial p_j} = 0, \qquad p_j \varepsilon K \tag{13}$$

We consider first the case where the wage rates paid in the producing sectors are not influenced by consumer prices, so that the p_j will only influence the C_i but not the Q_i. Utilizing the Slutsky equation, we may then write $\partial G/\partial p_j$ as:

$$\frac{\partial G}{\partial p_j} = C_j + \Sigma\ (p_i - 1) \frac{\partial C_i}{\partial p_j} = C_j - C_j\ \Sigma\ (p_i - 1) \frac{\partial C_i}{\partial Y} + \Sigma\ (p_i - 1) \left[\frac{\partial C_i}{\partial p_j}\right]_{\bar{u}} \tag{14}$$

where the last partial derivative is the slope of the income-compensated demand curve. Further, noting that $\partial W/\partial p_j = - C_j W_y$, we may rewrite (13), using (14), as:

$$\frac{\partial F}{\partial p_j} = - C_j \left[W_y - \lambda\ (1 - \frac{\partial T}{\partial Y})\right] + \lambda\ \Sigma\ (p_i - 1) \left[\frac{\partial C_i}{\partial p_j}\right]_{\bar{u}} = 0, \tag{15}$$

$$p_j \varepsilon K$$

As argued above, when consumer prices do influence wage rates in some sectors, the Q_i and hence Y and Y^* will also depend on the p_j. Using manipulations similar to the above, (13) may then be rewritten:

$$\frac{\partial F}{\partial p_j} = (\frac{\partial Y}{\partial p_j} - C_j) \left[W_y - \lambda\ (1 - \frac{\partial T}{\partial Y})\right] + \lambda \left[\frac{\partial Y^*}{\partial p_j} + \Sigma\ (p_i - 1)\left(\frac{\partial C_i}{\partial p_j}\right)_{\bar{u}}\right] = 0$$

$$p_j \varepsilon K \tag{16}$$

Finally, we note that in the case where there is no restriction on the government's ability to raise revenue through lump-sum taxation, the variable B will also be a controllable policy instrument. For the optimal value of B, we have:

$$\frac{\partial F}{\partial B} = \frac{\partial Y}{\partial B} W_y + \lambda \frac{\partial G}{\partial B} = \frac{\partial Y}{\partial B} \left[W_y - \lambda \left(1 - \frac{\partial T}{\partial Y}\right) \right] = 0 \qquad (17)$$

In interpreting these conditions, consider first the case when there is no constraint on lump-sum revenue. We first observe that a lump-sum tax, by definition, has no effect on production and no substitution effect on consumption. A small change in the amount of lump-sum revenue does, however, have an income effect on the trade balance and hence on the amount of revenue that has to be raised from other taxes in order to offset it; this effect is given by $(\partial Y/\partial B)(1 - \partial T/\partial Y)$, as shown above. Equation (17) therefore states that at the optimum, the change in welfare caused by the effect of a change in lump-sum revenue on disposable income, $(\partial Y/\partial B) \cdot W_y$, has to be exactly offset by the opportunity cost of the extra tax revenue that has to be raised from other sources, given by $\lambda(\partial Y/\partial B)(1 - \partial T/\partial Y)$.

Suppose, on the other hand, that there is an upper limit $\bar{B}$ on the amount of revenue that can be raised through lump-sum taxation.[1] If the constraint is binding at the optimum, we will have $\partial F/\partial B > 0$; from (17), noting the fact that $(\partial Y/\partial B) = -1$, this is seen to imply that $\lambda(1 - \partial T/\partial Y) > W_y$. The negative expression $\psi = W_y - \lambda(1 - \partial T/\partial Y)$ then measures the loss in welfare that would result from a small decrease in lump-sum taxation and the consequent need to raise tax revenue through indirect taxation which will have an "excess burden." We will call ψ the "marginal excess burden" of indirect taxation; it is implied by the previous discussion that with no constraint on lump-sum revenue this burden is zero.

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[1] The possibility of financing government deficits through borrowing is also assumed to be limited. An explicit analysis of optimal amounts of borrowing is difficult to handle within the present static framework; in any case, the narrow capital market in LDCs probably means that assuming a limit on the amounts that can be raised this way, is quite realistic.

Consider now the conditions determining the optimal levels of producer prices and employment subsidies, i.e., equations (9) and (12). A change in either kind of variable will have both a production effect (i.e., an effect on the level of real income at world prices) and an effect on disposable income. The latter effect, as is seen from the conditions, is exactly equivalent to the impact of a change in lump-sum taxation: the change in disposable income directly changes the level of welfare but also affects the balance-of-trade and hence necessitates a change in government revenue. In the same way as argued above, the opportunity cost of this government revenue may exceed the change in utility, i.e., there may be a non-zero marginal excess burden corresponding to the income effect. In that case, equations (9) and (12) state that at the optimum, the level of production or employment subsidies should be such that their marginal impact on the balance-of-trade and hence on government revenue through the production effect is precisely offset by the marginal excess burden corresponding to the effect on nominal income. An interesting implication is that whenever the marginal excess burden of the income effect is zero, (9) and (12) reduce to:

$$\frac{\partial Y^*}{\partial q_j} = 0, \quad \frac{\partial Y^*}{\partial s_j} = 0, \qquad q_j, \ s_j \in K \tag{18}$$

These conditions, of course, imply that whenever tax revenue can be raised without an excess burden, optimal production and employment subsidies should be such as to maximize real income at world prices; this is entirely in agreement with one's economic intuition.

In the case where employment and production do not depend on consumer prices, the first-order conditions (15) corresponding to the optimal values of those consumer prices $p_j \in K$ which are assumed controllable through consumption taxes can be interpreted along somewhat similar lines. A consumption tax does then not affect disposable income, but it has an income effect which affects welfare directly and which also affects the balance-of-trade and hence required tax revenue, in the same way as before. It also has a substitution effect which influences trade and tax revenue. Condition (15), therefore, states that at the optimum, the substitution effect should be such as to exactly offset the marginal excess burden of the income effect.[1] We may note in passing that this is equivalent to the generalization of Ramsey's rule for optimal indirect taxation derived by Dasgupta and Stiglitz (1974, p. 17).

As before, we are particularly interested in the case where the shadow price of foreign exchange is such that $\psi = 0$, i.e., when there is no marginal excess burden corresponding to the income effect. In this case, (15) reduces to the condition

$$\Sigma\ (p_i - 1) \left[\frac{\partial C_i}{dp_j}\right]_{\bar{u}} = 0 \qquad p_j \varepsilon K \qquad (19)$$

As pointed out in footnote 1 above, the left-hand side of (19) can be regarded as the marginal loss of consumers' surplus resulting from a consumption tax on the j^{th} commodity. Condition (19) therefore states that when $\psi = 0$, the marginal loss of consumers' surplus should be zero for each consumption tax at the optimum.

Consider now the case where all consumption taxes are controllable, i.e., where $p_j \varepsilon K$, j=1...m. It is easily seen that (19) will then be satisfied

[1] In partial equilibrium analysis, the excess burden of a consumption tax on a single commodity in an open economy could be identified with the integral of the distance between the income-compensated demand curve and the (horizontal) supply curve between the quantities consumed with and without the tax. This integral (the area of "the little triangle") corresponds to the loss in consumers' surplus resulting from the tax. The expression $\Sigma(p_i-1)(\partial C_i/\partial p_j)_{\bar{u}}$ can be regarded as the marginal loss in consumer surplus, or the marginal excess burden of the tax in a general equilibrium context. Therefore, an alternative interpretation of (15) is to say that the marginal excess burden of each individual tax should exactly offset the general marginal excess burden resulting from the extra revenue needed to offset the income effect. Equations (9) and (12) may be given similar interpretations.

by zero taxes on all commodities, i.e., $p_i = 1$, $j=1\ldots m$. It is, however, important to note that a proportional tax on all commodities would also satisfy (19). If a uniform tax t is levied on all commodities, we have $p_i - 1 = t\, p_i$, $i=1\ldots m$, so that the left-hand side of (19) can be rewritten:

$$\Sigma\, (p_i - 1) \left[\frac{\partial C_i}{\partial p_i}\right]_{\bar{u}} = t\, \Sigma\, p_i \left[\frac{\partial C_i}{\partial p_j}\right]_{\bar{u}}; \tag{20}$$

By the properties of the Slutsky matrix, however, this expression is zero. An implication of this is that when all consumer prices are independently controllable, the optimal producer prices and employment subsidies will always be set so as to maximize real income at world prices, regardless of the resulting requirements for government revenue and whether or not there is a limit on lump-sum taxes, since the government can raise any amount of revenue without an excess burden, through proportional consumption taxes.

In the situation where consumer prices have an influence on employment and income as well as on consumption, these rules must be modified. With no constraint on lump-sum revenue, the optimum conditions become:

$$\frac{\partial Y^*}{\partial p_j} + \Sigma\, (p_i - 1)\left[\frac{\partial C_i}{\partial p_j}\right]_{\bar{u}} = 0 \qquad p_j \in K \tag{21}$$

which simply states that at the optimum,consumer prices must be set in such a way that at the margin, the effect on real income at world prices must exactly offset the marginal loss in welfare resulting from the deadweight loss in consumption when the domestic price differs from the international price.

We finally analyze a situation of particular relevance to low income countries, namely where a substantial proportion of government revenue is derived from taxes on international trade. In a sense, the rules for optimal taxation of trade have already been implicitly covered in the analysis above, since they can be regarded as combinations of consumption and production taxes and subsidies. Therefore, the optimal structure of trade taxes is already implied by the previous analysis for the case where production and consumption taxes can be independently controlled. In many cases, however, it is administratively difficult to impose separate consumption and production taxes, so that for some commodities trade taxes may be the only feasible alternative. The rule for the optimal tax rate can then be derived by imposing the restriction $p_j = q_j$ for those commodities and maximizing the welfare function subject to these restrictions as well as the government budget constraint. The result, combining equations (9) and (16) above, is:

$$\left(\frac{\partial Y}{\partial q_j} + \frac{\partial Y}{\partial p_j} - C_j\right)\cdot \Psi + \lambda\left(\frac{\partial Y^*}{\partial q_j} + \frac{\partial Y^*}{\partial p_j} + \Sigma\ (p_i - 1)\left[\frac{\partial C_i}{\partial p_j}\right]_{\bar{u}}\right) = 0, \qquad p_j = q_j\ \varepsilon\ K \tag{21}$$

where, of course, $\partial Y/\partial p_j$ and $\partial Y^*/\partial p_j$ may be zero if consumer prices do not influence employment and output. Equation (21) can be interpreted as saying that at the optimum, the marginal excess burden of the combined income effects of the consumption tax and production subsidy of, say, an import tariff, has to exactly offset the combined production and substitution effects on the trade balance and tax revenue. We note that in this case it will generally not be true that a zero marginal excess burden will imply maximization of real income at world prices as the optimal policy. Instead, optimality requires that the combined marginal effect on real income of the producer subsidy and consumer tax implied by a tariff exactly balances the marginal welfare reduction through the deadweight loss in consumption.

Taken by themselves, the conditions for optimal taxation derived above are too general to be very useful; the actual levels of optimal taxes and subsidies will vary greatly depending on the specific form of distortions existing in the economy and on the set of taxes and subsidies which can be controlled by the policymakers. One interesting point which does emerge, however, concerns the role of the lump-sum constraint in the optimal policy. Before turning to the application of the general results to the specific case of a two-sector model à la Harris and Todaro, we briefly summarize the results in this regard.

First, with a non-binding lump-sum constraint, implications of tax and subsidy policies for government revenue become irrelevant,[1] and the optimal tax rules can be formulated with reference to "real" effects only. For example, if producer and consumer prices are independently controllable, and production is independent of consumer prices, producer prices should be set so as to maximize real income at world prices and consumer prices so as to maximize consumer surplus given the international terms of trade.

When the lump-sum constraint is binding, the implications for government revenue of the various tax and subsidy policies do become relevant. This situation is characterized by the fact that <u>any</u> increment in government revenue will have to come from an increase in tax revenue (or a decrease in a subsidy) which will result in an extra deadweight loss in consumption or a decrease in real income at world prices. The optimal level for a particular tax or subsidy will then be such that at the margin, the "real" effect of a

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[1]Note that the value of the Lagrange multiplier λ corresponding to the shadow price for government revenue becomes irrelevant in the optimal tax formulae whenever $\psi = 0$. The government budget constraint will, of course, continue to be part of the optimum conditions, but its only role will be to determine the value of B, the amount of lump-sum revenue to be raised by the government.

change in the tax or subsidy rate would be exactly offset by the welfare-reducing effect of raising the extra government revenue necessary to finance it; equivalently, this means that at the optimum, the welfare-increasing effect of an extra unit of government revenue would be the same when applied to any individual tax decrease or subsidy increase.

Not much can be said in general about the conditions under which the lump-sum constraint will or will not be binding; this will, of course, depend on the maximum amount the government can raise from lump-sum taxation,[1] as well as on the nature of the distortions in the economy. It will also depend on the set of taxes and subsidies which are controllable. For example, it was demonstrated above that in the case where all consumer prices are independently controllable and do not influence production decisions, the lump-sum constraint will never be binding. The type of situation in which the constraint would be likely to be binding, on the other hand, would be if there existed a distortion on the production side such that maximization of real income would require a large subsidy and if revenue (other than lump-sum revenue) could only be raised by taxing consumption of some commodities so that a welfare loss in consumption would arise due to differences between international and domestic relative prices. It is difficult to say much beyond this sort of general statement, however, and we now turn to an analysis of the specific case of the Harris-Todaro model.

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[1]In some cases it might be appropriate to specify $\bar{B}$ as being negative; it would then represent maximum lump-sum revenue net of "irrevocable" government expenditures such as repayment of foreign borrowing, or government expenditures on capital formation if these are taken as fixed *a priori*.

III. Optimal Taxes in the Harris-Todaro Model

We turn now to an application of the methodology discussed above to the case of a two-sector model of a low-income country with non-competitive wage levels in the "modern" sector and open unemployment as a result of rural-urban migration, similar to that analyzed by Harris and Todaro. As before, the analysis is confined to the short run, so that capital and land in both the urban (manufacturing) sector and in agriculture are considered fixed, and output in either sector is written solely as a function of the sectoral labour force:

$$Q_m = f(L_m) \qquad \partial f/\partial L_m > 0, \quad \partial^2 f/\partial L_m^2 < 0 \tag{22}$$

$$Q_a = h(L_a) \qquad \partial h/\partial L_a > 0, \quad \partial^2 h/\partial L_a^2 < 0 \tag{23}$$

Competitive behavior and profit maximization are assumed to prevail in the market for manufactured goods, which implies:

$$q_m \cdot f_L = w_m - s_m, \tag{24}$$

where $f_L = \partial f/\partial L_m$ is the marginal physical product of labour, and w_m is the prevailing wage rate in the manufacturing sector; q and s have the same meaning as in the previous section.

Following Harris and Todaro, rural-urban migration is assumed to take place whenever the value of the marginal product of labour in agriculture falls short of the <u>expected</u> wage rate in the manufacturing sector. The expected wage is defined as the actual wage multiplied by the ratio of employed to total[1] urban labour, on the assumption that this ratio measures the probability for an urban worker to be employed at any given time. The condition for migration equilibrium can then be written, after rearrangement:

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[1]I.e., employed plus unemployed.

$$q_a h_L \cdot (L-L_a)=w_m \cdot L_m, \tag{25}$$

where $h_L = \partial h/\partial L_a$. We further have the condition,

$$L_a + L_m \leq L \tag{26}$$

where L is the total labour force, assumed exogenously given and constant.

At this point in the analysis, Harris and Todaro introduced the assumption of a non-competitive element in the urban labour market, resulting in a minimum level below which the urban wage rate could not fall. This level was assumed fixed in terms of the manufactured good. We will take a somewhat more general approach by letting the wage rate be an institutionally determined function of the consumer prices of agricultural and consumer goods; as noted above, in a situation where the non-competitive element in the labour market results from the existence of relatively strong urban labour unions and/or politically motivated minimum-wage legislation, one would clearly expect a general cost-of-living index reflecting the level of consumer prices to be important in wage determination. We thus write:

$$w_m \geq w(p_a, p_m) \tag{27}$$

Special attention will also be given to the case where the minimum wage is fixed in nominal terms, however, i.e., where $w_m \geq \bar{w}$.

As in the previous section, we will regard producer and consumer prices either as being fixed by virtue of the economy engaging in international trade at given world prices, or as being indirectly controlled by consumption or production taxes of various kinds. The subsidy to manufacturing labour, s_m, clearly is a policy variable; we are ruling out a subsidy to agricultural labour as being administratively infeasible.

Given the levels of consumer and producer prices, one may note that equations (22) - (25) together with either (27) or (26) holding as an equality, will uniquely determine labour allocation and production levels in the economy, with (26) as an equality corresponding to full employment. The case of interest here of course occurs when, in the absence of specific policies, (27) would hold with equality and some urban unemployment would exist.[1]

We now turn to the problem of analyzing the problem of an optimal tax policy in this model, under certain restrictions on the types of taxes regarded as being controllable. In particular, we will focus attention on the situation, characteristic of many low-income countries, in which it is difficult to control separately taxes or subsidies on consumption and production, so that commodity taxation is confined to import tariffs or export taxes or subsidies. As noted above, we will also assume that a labour subsidy is possible only in the manufacturing sector but is administratively infeasible in agriculture. Again, this seems a realistic assumption in view of the widely dispersed small-scale agriculture, in which labour is primarily supplied by the self-employed farmers and their families, in most LDC's.

1. Employment Subsidy Alone

Consider first the Harris-Todaro case, where s_m, the labour subsidy in manufacturing, is the only policy instrument. Suppose first that there is no constraint on the amount of lump-sum revenue. The optimal policy then simply entails maximization of real income at world prices, $Y^* = Q_m + Q_a$, with the first-order condition:

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[1]We will also assume that in all cases, except the ones where the first-best solution is attainable, residual unemployment will exist even with optimal tax and subsidy policies. Extension of the analysis to the case where the optimal policy produces full employment (which can formally be done by adding inequality (26) to the maximization problems and using Kuhn-Tucker theory) would pose no particular analytical difficulties, but would render an already long paper longer (and messier).

$$\frac{\partial Y^*}{\partial s_m} = f_L \frac{\partial L_m}{\partial s_m} + h_L \frac{\partial L_a}{\partial s_m} = 0 \tag{28}$$

which implies:

$$\frac{f_L}{h_L} = \frac{w_m - s_m}{q_m} \cdot \frac{q_a}{w_m} \frac{L_u}{L_m} = - \frac{\partial L_a/\partial s_m}{\partial L_m/\partial s_m} \tag{29}$$

where we have used (24) and (25), and $L_u \equiv L - L_a$. In other words, at the optimal level of subsidy, the ratio of the marginal products of labour should be equal to the ratio of the marginal changes in the agricultural and manufacturing labour force, respectively, when the subsidy is varied. When there is unemployment in the economy, the values of $\partial L_m/\partial s_m$ and $\partial L_a/\partial s_m$ are found from total differentiation of (24) and (25), to be given by:

$$\frac{\partial L_m}{\partial s_m} = - \frac{L_m}{w_m - s_m} \cdot \eta_m \tag{30}$$

$$\frac{\partial L_a}{\partial s_m}\left(\frac{L_u}{L_a} \frac{1}{\eta_a} - 1 \right) = \frac{\partial L_m}{\partial s_m} \cdot \frac{L_u}{L_m} \tag{31}$$

where η_a and η_m are the elasticities of the demand for labour with respect to the real wages in agriculture and manufacturing respectively. Equation (31) may be interpreted as follows. Consider first the case where the demand for labour in agriculture is infinitely elastic, so that the agricultural wage rate always remains constant. In that case (31) reduces to $\partial L_a/\partial s_m = -(L_u/L_m) \cdot (\partial L_m/\partial s_m)$; in other words, for every worker added to the manufacturing labour force, $(L_u/L_m) \geq 1$ workers will migrate from agriculture in order for the probability of finding a job in the urban areas to remain unchanged. When the demand for labour in agriculture is less than infinitely elastic, migration from agriculture resulting from urban job creation will be reduced below this level. This reduction is represented by the term $(L_u/L_a\eta_a)$, i.e., it will be inversely proportional to the elasticity of labour demand and proportional to L_u/L_a; the latter ratio, in turn, is a measure of the relative effect that a given proportional change in the agricultural labour force will have on the total urban labour force and hence on the probability of finding a job in the urban area: the greater is L_u/L_a, the smaller is this effect. We can now write $\partial L_a/\partial s_m$ as

$$\frac{\partial L_a}{\partial s_m} = \frac{\partial L_m}{\partial s_m} \cdot \frac{L_u}{L_m} \cdot \theta \tag{32}$$

where

$$\theta = \left(\frac{L_u}{L_a}\frac{1}{\eta_a} - 1\right)^{-1} \tag{33}$$

We have $\theta < 0$, $|\theta| \leq 1$.[1] Using this definition, (29) may be rewritten as:

$$\frac{w_m - s_m}{w_m} \cdot \frac{q_a}{q_m} + \theta = 0 \tag{34}$$

It follows that, when producer prices are equal to world prices (i.e., $q_a = q_m = 1$), the optimal subsidy is found by solving

$$\frac{w_m - s_m}{w_m} = -\theta \leq 1 \tag{35}$$

which extends to an open economy the Harris-Todaro result that welfare can

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[1]One may note that θ is a measure of the impact that a labour subsidy will have on the rate of urban unemployment, since by definition, $(\partial L_a/\partial s_m) = -\partial L_u/\partial s_m$. Thus, the smaller is θ in absolute value, the greater is the reduction in the urban unemployment rate.

be improved through a non-zero subsidy to manufacturing labour. We note, however, that when the demand for agricultural labour is infinitely elastic, a zero subsidy is optimal, since we then have $\theta = -1$.

We now analyze the case where there is a constraint on the government's ability to raise lump-sum revenue. If the constraint is not binding, (28) will still hold; if it is binding, the simplest way of finding the solution will be directly from the budget constraint, which in this case, with $q_m = q_a = 1$, will simply take the form:

$$s_m L_m - \bar{B} = 0; \tag{36}$$

this equality can then be substituted back into the rest of the system. For future reference, however, we are also interested in the relative values of the shadow prices of government revenue and the multiplier associated with the lump-sum constraint.[1] We can analyze these by noting that at the optimal value of the subsidy, the optimality condition (12) will hold, i.e., we will have:

$$\Psi \frac{\partial Y}{\partial s_m} + \lambda \frac{\partial Y^*}{\partial s_m} = 0 , \tag{37}$$

where $\Psi = W_y - \lambda(1 - (\partial T/\partial Y)) < 0$, and where Y is given by

$$Y = q_m Q_m + q_a Q_a + s_m L_m - B \tag{38}$$[2]

The derivative $\partial Y/\partial s_m$ can be expressed as

$$\frac{\partial Y}{\partial s_m} = q_m f_L \frac{\partial L_m}{\partial s_m} + q_a h_L \frac{\partial L_a}{\partial s_m} + s_m \frac{\partial L_m}{\partial s_m} + L_m \tag{39}$$

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[1] It is easy to show that this multiplier, which would result from adding the constraint $B \leq \bar{B}$ to the maximization problem in the previous section, will equal the expression $-\Psi$ discussed there.

[2] It is easily seen that this is equivalent to (2) above with $i = m, a$; $s_a = 0$; and $Y^* = Q_m + Q_a$.

Using (39), (28), (32), (30), and (25), manipulating and noting that $q_a = q_m = 1$, we may rewrite (37) as:

$$-\frac{\lambda}{\Psi} = \left[1 + \frac{1 - \frac{w_m - s_m}{w_m}\left(\frac{1}{\eta_m} + 1\right)}{\frac{w_m - s_m}{w_m} + \theta}\right] \tag{40}$$

By assumption, when the lump-sum constraint is binding, we will have $\partial Y^*/\partial s_m > 0$, which will imply that the denominator of the fraction within brackets is positive; it is also easy to show that the numerator will be positive as well since $\eta_m < 0$. One may now analyze the conditions under which the shadow price ($-\Psi$) of a unit of lump-sum revenue will be high relative to λ (i.e., the conditions under which the right-hand side of (40) is small).[1] For given w_m and s_m, it will be higher the smaller is θ in absolute value, i.e., the greater the reduction in unemployment when urban jobs are created; it will also be higher the greater (in absolute value) is η_m, the elasticity of labour demand in the manufacturing sector. The smaller the value of $\bar{B}$, the maximum amount of lump-sum revenue, the smaller will be s_m relative to w_m, and the larger will be Ψ relative to λ.[2]

Before continuing, we note that the above analysis is equally applicable to the case where the nominal wage rate is assumed fixed and to the case where it depends on the consumer prices of the two goods, since the producer and consumer prices are not regarded as policy instruments but remain fixed by international trade.

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[1]Or, put differently, the conditions under which an extra unit of government revenue applied to an employment subsidy would be high in terms of increased real income and welfare.

[2]These results may be formally established by differentiating (40) with respect to the various parameters.

2. Tariff on the Manufactured Good

We turn now to the case, suggested by the analysis in Hagen (1958), where the single policy instrument is a tariff on the imports of the manufactured good. We again start by considering the case where there is no constraint on the amount of revenue that can be raised by lump-sum taxation, and where $w_m = \bar{w}$. From the analysis in the previous section, the optimum condition for this case is:

$$\frac{\partial Y^*}{\partial q_m} + (p_m - 1)\left(\frac{\partial C_m}{\partial p_m}\right)_{\bar{u}} = 0, \qquad p_m = q_m \tag{41}$$

or

$$f_L \frac{\partial L_m}{\partial q_m} + h_L \frac{\partial L_a}{\partial q_m} + (p_m - 1)\left(\frac{\partial C_m}{\partial p_m}\right)_{\bar{u}} = 0, \qquad p_m = q_m . \tag{42}$$

Evaluating $\partial L_m/\partial q_m$, $\partial L_a/\partial q_m$, we find:

$$\frac{\partial L_m}{\partial q_m} = \frac{-L_m \eta_m}{q_m}, \qquad \frac{\partial L_a}{\partial q_m} = \frac{\partial L_m}{\partial q_m} \cdot \frac{L_u}{L_m} \cdot \theta \tag{43}$$

Using these results, (24) and (25), and defining $\phi_m \equiv (\partial C_m/\partial p_m)_{\bar{u}} \cdot (p_m/C_m) < 0$, expression (43) can be shown to be equivalent to

$$-\left(\frac{w_m - s_m}{w_m} \cdot \frac{q_a}{q_m} + \theta\right) + \left(\frac{w_m - s_m}{w_m}\right) t_m \frac{\phi_m}{\eta_m} \frac{C_m}{f_L L_m} = 0 \tag{44}$$

where $t_m \equiv p_m - 1$ is the tariff rate.

Under present assumptions we have $q_a = 1$ and $s_m = 0$. Using these conditions, it is easy to show that whenever the demand for agricultural labour is less than infinitely elastic with respect to the real wage, the optimal tariff is positive.[1] The proportional tariff rate is given by

[1] We are assuming here and throughout the rest of the paper that the manufactured good is always imported.

$$\frac{t_m}{q_m} = \frac{f_L L_m}{C_m} \cdot \frac{\eta_m}{\phi_m}\left(\frac{1}{q_m} + \theta\right) \tag{45}$$

Whenever $|\eta_a| < \infty$, we have shown that $0 > \theta > -1$. A non-positive tariff, i.e., $q_m \leq 1$, would then imply that the right-hand side is strictly positive which would contradict $t_m \leq 0$ on the left-hand side, so that in fact we must have $t_m > 0$, $q_m > 1$. We may further note that the tariff will be higher the greater the share of labour in manufacturing output and the greater in absolute value the demand elasticity for manufacturing labour; the effect of a tariff in the form of a deadweight loss in consumption is reflected by the fact that the optimal tariff is inversely proportional to the ratio of consumption to domestic production of manufactured goods and to the absolute value of the price elasticity of demand for consumption goods. Other things equal, the optimal tariff will also be higher the smaller in absolute value is θ, or in other words, the greater the reduction in the urban unemployment rate when the manufacturing labour force increases.

It is furthermore possible to show from (45) that in comparison with the case of a labour subsidy without a lump-sum revenue constraint, the case where a tariff only can be used will involve a smaller manufacturing labour force. From (35), it is evident that when the labour subsidy is used, the value of the real wage at the optimum equals $w_m|\theta|$. For the case of the tariff only, it equals $w_m \cdot (1/q_m)$; but from (44) with $s_m = 0$ and $q_a = 1$, it is evident (since the second term is positive) that at the optimum, one has $(1/q_m) > |\theta|$, so that the real wage is higher and the manufacturing labour force is smaller than in the subsidy-only case. This is intuitively reasonable: the opportunity cost of labour transfer to manufacturing through a tariff is not only the loss of agricultural output caused by migration, but also the loss in consumer surplus caused by the reduction in consumption of manufactured goods as a result of a tariff.[1]

The situation becomes considerably more complicated when we assume that the nominal wage rate is not fixed, but instead depends on the consumer prices p_m and p_a. To consider this case, we first define the elasticity of the nominal wage with respect to p_m, as:

$$\epsilon_m = \frac{p_m}{w_m} \cdot \frac{\partial w_m}{\partial p_m} \tag{46}$$

One may note that this formulation includes as a special case the basic assumption made by Harris and Todaro, i.e., that the wage rate is fixed in terms of manufactured goods; we then simply set $\epsilon_m = 1$.

The condition for an optimal tariff level for this case becomes

$$\frac{\partial Y^*}{\partial q_m} + \frac{\partial Y^*}{\partial p_m} + (p_m - 1)\left(\frac{\partial C_m}{\partial p_m}\right)_{\bar{u}} = 0, \qquad p_m = q_m \tag{47}$$

or

$$f_L\left(\frac{\partial L_m}{\partial q_m} + \frac{\partial L_m}{\partial p_m}\right) + h_L\left(\frac{\partial L_a}{\partial q_m} + \frac{\partial L_a}{\partial p_m}\right) + (p_m - 1)\left(\frac{\partial C_m}{\partial p_m}\right)_{\bar{u}} = 0 \tag{48}$$

Utilizing the fact that under the present assumptions, $q_m = p_m$ and $s_m = 0$, it is first easy to show that $\partial L_m/\partial p_m = -\epsilon_m(\partial L_m/\partial q_m)$, and the expressions $(\partial L_m/\partial q_m)$ and $(\partial L_a/\partial q_m)$ have been previously evaluated. To get an expression for $(\partial L_a/\partial p_m)$, one again uses total differentiation of (24) and (25), taking into account that a change in p_m will lead to rural-urban migration not only by affecting L_m and hence the probability of finding a job in the urban areas,

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[1] Since we have proved that the optimal policy involves a positive tariff, the problem of a binding lump-sum constraint will not arise here as long as $\bar{B} \geq 0$.

but also by changing the nominal wage in manufacturing employment. After some manipulation, and again using the assumption $s_m = 0$, we find:

$$\frac{\partial L_a}{\partial p_m} = \frac{L_u}{L_m} \theta \left(\frac{1}{\eta_m} + 1\right)^{1} \frac{\partial L_m}{\partial p_m} \tag{49}$$

One may note that $\partial L_a/\partial p_m$ may be $\gtreqless 0$ depending on whether $|\eta_m| \gtreqless 1$.

Using the above results, and noting that $(\partial L_m/\partial q_m) = (-\eta_m L_m/q_m)$, one now obtains, after some manipulation:

$$\frac{t_m}{q_m} = \frac{\eta_m}{\phi_m} \cdot \frac{f_L L_m}{C_m} \left[(1 - \epsilon_m)\left(\frac{1}{q_m} + \theta\right) - \frac{1}{q_m} \frac{\epsilon_m \theta}{\eta_m}\right] \tag{50}$$

It is easily confirmed that (50) collapses into (45) when $\epsilon_m = 0$.

Using (50), we may now consider the case where the manufacturing wage is fixed in terms of manufactured goods: the somewhat surprising result is that the optimal tariff will then be negative! Equation (50) becomes:

$$\frac{t_m}{q_m} = (-1) \frac{f_L L_m}{C_m} \cdot \frac{\theta}{\phi_m} < 0 , \tag{51}$$

recalling that $\theta < 0$, $\phi_m < 0$. An intuitive explanation of this result is as follows. If the manufacturing wage is fixed in terms of the manufactured good, a tariff will not affect the real wage in terms of this good, and hence will not affect employment in the manufacturing sector. It will, however, increase the urban wage relative to labour earnings in the rural areas and a positive tariff will hence lead to increased migration, and therefore to increased urban unemployment and decreased agricultural output. A negative

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[1]When $s_m \neq 0$, the expression in parenthesis becomes

$$\left(\frac{w_m - s_m}{w_m} \cdot \frac{1}{\eta_m} + 1\right) . \tag{49a}$$

tariff (i.e., an import subsidy) will have the converse effect; in other words, it will decrease unemployment and increase real income at world prices. One may finally observe that even with $\epsilon_m < 1$, the optimal tariff may still be negative; this is more likely to happen the closer is ϵ_m to unity, the greater is θ in absolute value (i.e., the smaller the decrease in urban unemployment as the wage rate rises), and the smaller (in absolute value) is the elasticity of demand for labour in the manufacturing sector.

Summarizing the results for this subsection, we first conclude that for the case where the urban wage rate is fixed in nominal terms, the result of Hagen that a tariff on the manufactured good will improve welfare when the wage rate in the manufacturing sector is in excess of the competitive level, generalizes to a model where migration and urban unemployment is allowed. When the wage rate depends on the consumer price of the manufactured good, however, a positive tariff may reduce welfare if the elasticity of the wage rate with respect to the price of the manufactured good is high enough, and an import subsidy may be optimal instead.[1]

3. Tariff and Employment Subsidy

We now analyze the case which combines the previous ones, i.e., where it is assumed that the available policy instruments include both a subsidy to labour and a tariff on manufactured goods. Consider first the situation where there is no lump-sum revenue constraint, and also again assume that the wage rate is fixed in nominal terms. From a comparison of (34) and (44), it is evident that optimality will require $t_m = 0$ (or, $p_m = q_m = 1$), so that

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[1]If the lump-sum constraint is binding, this policy may of course not be possible, since a negative tariff revenue is generated. We will not explicitly analyze this case, however.

the optimal policy is a labour subsidy alone given by (35); this clearly is in agreement with economic intuition. The more interesting case arises when lump-sum revenue is limited. A natural policy alternative would then be to impose a positive tariff in order to raise revenue to be used for the financing of a labour subsidy. We now demonstrate that such a policy will generally be optimal and will lead to a different solution and to a higher labour force in manufacturing than would be optimal when an employment subsidy is used in isolation. The optimum conditions for a labour subsidy and a tariff on the manufactured good are given by (12) and (21); in the present case they are written as:[1]

$$\Psi \frac{\partial Y}{\partial s_m} + \lambda \frac{\partial Y^*}{\partial s_m} = 0 \tag{52}$$

$$\Psi \left(\frac{\partial Y}{\partial q_m} - C_m \right) + \lambda \left(\frac{\partial Y^*}{\partial q_m} + (p_m - 1) \left(\frac{\partial C_m}{\partial p_m} \right)_{\bar{u}} \right) = 0, \tag{53}$$

$$q_m = p_m$$

The term $\partial Y/\partial s_m$ is given by (39); $\partial Y/\partial q_m$ can be written:

$$\frac{\partial Y}{\partial q_m} = Q_m + s_m \frac{\partial L_m}{\partial q_m} + q_a h_L \frac{\partial L_a}{\partial q_m} + q_m f_L \frac{\partial L_m}{\partial q_m} \tag{54}$$

The terms $\partial Y^*/\partial s_m$ and $\partial Y^*/\partial q_m$ may be evaluated from previous results.

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[1]Note that the terms $\partial Y/\partial p_m$ and $\partial Y^*/\partial p_m$ in (21) will be zero since we are assuming here that the manufacturing wage rate is not influenced by consumer prices.

It is not difficult to show that[1]

$$\left(\frac{\partial L_m}{\partial q_m} \Big/ \frac{\partial L_m}{\partial s_m}\right) = \left(\frac{\partial L_a}{\partial q_m} \Big/ \frac{\partial L_a}{\partial s_m}\right) = \left(\frac{\partial Y^*}{\partial q_m} \Big/ \frac{\partial Y^*}{\partial s_m}\right) = \frac{w_m - s_m}{q_m} = f_L \; . \tag{55}$$

Multiplying (52) by f_L and subtracting from (53), we obtain the following simple condition for the optimal tariff:

$$\Psi\left[(Q_m - f_L L_m) - C_m\right] + \lambda \frac{t_m C_m}{p_m} \phi_m = 0 \tag{56}$$

or

$$\frac{t_m}{p_m} = -\frac{\Psi}{\lambda} \cdot \frac{1}{\phi_m} \cdot \frac{[(Q_m - f_L L_m) - C_m]}{C_m} \tag{57}$$

which can easily be seen to be positive, as asserted.[2]

This formula may be intuitively interpreted as follows. The expression within square brackets in the numerator on the right-hand side measures the difference between the cost to the government of bringing about a given increase in manufacturing employment through an employment subsidy, on the one hand, and the (negative) cost of accomplishing the same increase through a tariff, on the other. Other things equal, the greater this difference (in absolute value), the higher the tariff at the optimum. The absolute value of the demand elasticity for manufactured goods, which appears in the denominator, is proportional to the welfare cost, in the form of a deadweight loss in consumption, of raising a given amount of revenue; hence, the smaller the elasticity, the lower the welfare cost implicit in financing an employment

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[1]Compare (30), (32), (43), (28) and (41).

[2]Recall again that we are assuming $C_m > Q_m$.

subsidy through a tariff, which would tend to raise its optimal level. These interpretations are based on a given value of Ψ/λ; however, as the tariff is raised and the proceeds used to subsidize manufacturing employment, it is easy to show from (40) that the absolute value of Ψ/λ will decrease,[1] so that the greater the degree to which the above conditions are fulfilled, the closer the optimal solution to that which would result if there were no lump-sum revenue constraint.

It is also easy to show that the labour force in manufacturing at the optimum with the two policy tools is greater than it would be if the employment subsidy were the only one. The government budget constraint is

$$\overline{B} - s_m L_m - t_m(Q_m - C_m) = 0 \tag{58}$$

which, with $t_m > 0$, implies $s_m L_m > \overline{B}$. Now, denote as L_m^o and s_m^o the optimal values of L_m and s_m when the employment subsidy is the only policy instrument. Suppose now that at the optimum, we have $L_m \leq L_m^o$, contrary to the assertion above. Since we have $s_m^o L_m^o = \overline{B}$, the inequality $s_m L_m > \overline{B}$ implies $s_m > s_m^o$ whenever $L_m \leq L_m^o$. This, together with the fact that a positive tariff implies $q_m > 1$, means that the real wage $(w_m - s_m)/(q_m)$ is lower than $w_m - s_m^o$, the real wage when the employment subsidy alone is used, which contradicts the hypothesis $L_m \leq L_m^o$, given a downward sloping demand curve for manufacturing labour.

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[1]When $q_m \neq 1$, (40) becomes

$$-\frac{\lambda}{\Psi} = \left[1 + \frac{1 - \frac{w_m - s_m}{w_m}\left(\frac{1}{\eta_m} + \frac{1}{q_m}\right)}{\frac{1}{q_m}\frac{w_m - s_m}{w_m} + \theta}\right] \tag{40a}$$

from which it can be established that $\partial(-\Psi/\lambda)/\partial q_m < 0$.

We finally consider the situation where the nominal wage rate is influenced by the consumer prices, and where both an employment subsidy and a tariff on manufactured goods are feasible. For this case also, the results become considerably more complex than for that when the wage rate is assumed independent of consumer prices. The first-order condition (52) is unchanged, but (53) becomes:

$$\Psi\left(\frac{\partial Y}{\partial q_m} + \frac{\partial Y}{\partial p_m} - C_m\right) + \lambda\left(\frac{\partial Y^*}{\partial q_m} + \frac{\partial Y^*}{\partial p_m} + (p_m - 1)\left(\frac{\partial C_m}{\partial p_m}\right)_{\bar{u}}\right) = 0 \qquad (59)$$
$$p_m = q_m$$

To simplify this expression, one first expresses the terms $\partial Y/\partial q_m$, $\partial Y/\partial p_m$, $\partial Y^*/\partial q_m$, and $\partial Y^*/\partial p_m$ in terms of the partial derivatives of L_a and L_m with respect to q_m and p_m. It is furthermore easy to show that we will now have

$$\frac{\partial L_m}{\partial p_m} = -\hat{\epsilon}_m \cdot \frac{\partial L_m}{\partial q_m} \qquad (60)$$

where $\hat{\epsilon}_m = \epsilon_m(w_m/(w_m - s_m))$. One then obtains:

$$\Psi\left[\frac{\partial L_m}{\partial q_m}(1-\hat{\epsilon}_m)(q_m f_L + h_L\,\theta\,\frac{L_u}{L_m} + s_m) - \frac{\partial L_m}{\partial q_m}\frac{\epsilon_m}{\eta_m} h_L\,\theta\,\frac{L_u}{L_m} + Q_m - C_m\right] \qquad (61)$$
$$+ \lambda\left[\frac{\partial L_m}{\partial q_m}(1-\hat{\epsilon}_m)(f_L + h_L\frac{L_u}{L_m}\theta) - \frac{\partial L_m}{\partial q_m}\frac{\epsilon_m}{\eta_m} h_L\,\frac{L_u}{L_m}\,\theta + (p_m - 1)\left(\frac{\partial C_m}{\partial p_m}\right)_{\bar{u}}\right] = 0$$
$$p_m = q_m$$

where we have used (43), (48), (54) and (60). Expressing (52) in similar form, multiplying it by $f_L(1-\hat{\epsilon}_m)$ and subtracting from (61) one obtains after some simplification:

$$\psi\,(Q_m - f_L\, L_m(1-\hat{\epsilon}_m(1+\theta)) - C_m) + \lambda(f_L L_m \hat{\epsilon}_m\, \theta + \frac{t_m C_m}{p_m}\, \phi_m) = 0 \quad (62)$$

$$q_m = p_m$$

or

$$\frac{t_m}{p_m} = -\frac{\psi}{\lambda}\frac{1}{\phi_m}\left[\frac{Q_m - f_L L_m(1-\hat{\epsilon}_m(1+\theta)) - C_m}{C_m}\right] - \left(\frac{f_L L_m}{C_m}\right)\hat{\epsilon}_m \frac{\theta}{\phi_m}, \quad (63)$$

$$q_m = p_m$$

We note that when $\epsilon_m = \hat{\epsilon}_m = 0$, this formula collapses into (51). A further observation pertains to the case where the lump-sum constraint is not binding, i.e., when $\psi = 0$ and the first term on the right-hand side drops out; the optimal tariff will then be negative, i.e., we will have an import subsidy. It is interesting to note that this is true even though it is assumed possible to directly subsidize manufacturing employment in this case; by contrast, we showed that in the case where the wage rate was fixed in nominal terms and the lump-sum constraint was non-binding, the optimal tariff rate was zero if a direct employment subsidy was possible. The reason for this difference can be intuitively explained as follows. As was shown above, when the subsidy is set at the optimal level where its marginal effect on real income is zero, the tariff rate will also have a zero marginal effect on real income if the nominal wage is independent of the tariff. A non-zero tariff, however, will have a non-zero marginal deadweight loss because it affects consumer prices; hence the optimal tariff in this situation is zero. When the nominal wage does depend on consumer prices, and therefore on the tariff, however, the marginal effect of a tariff on rural-urban migration, agricultural output, and hence on real income at world prices, will not be proportional to that of an employment subsidy. The reason is that a subsidy will influence migration only via its effect on the manufacturing labour force and therefore on the probability of finding a job in the urban sector, but a tariff will influence migration via its effect on the nominal wage rate as well. At a zero tariff, with the employment subsidy at its optimal level, the marginal effect of the tariff on real income will be non-zero, but the marginal deadweight loss in consumption will be zero. Hence, a zero tariff cannot be optimal: instead the optimal tariff has to be such that these two effects just offset each other when the employment subsidy is at its optimal level, which, as was just seen, will require a negative tariff rate, i.e., an import subsidy.

Returning to the case where the lump-sum constraint is binding, it is possible to show (by comparing (57) and (63)) that the optimal tariff in the present case will necessarily be lower than in the case where the nominal wage is fixed. Other things equal, the difference will be greater the greater is ϵ_m, the elasticity of the wage rate with respect to the price of manufactured goods, and the greater (in absolute value) is θ, i.e., the smaller the reduction in unemployment in response to job creation in the manufacturing sector. These, of course, are also the conditions making it more likely that the optimal policy for this situation may involve an import subsidy.

Turning finally to a comparison between the allocation of labour for the case where both policy instruments can be used and that when an employment subsidy only is assumed feasible, it is no longer true (as it was in the case of a fixed nominal wage) that the former situation <u>always</u> involves a larger labour force in the manufacturing sector. One can show, however, that the manufacturing labour force will be larger whenever the optimal tariff is positive. Suppose to the contrary that $L_m \leq L_m^o$, where L_m and L_m^o represent the optimal values with both instruments and with the subsidy alone, respectively. Since $t_m > 0$ and $\epsilon_m > 0$ imply $w_m > w_m^o$, migration equilibrium can be seen (referring to (25)) to imply that $L_a < L_a^o$ as well. This situation, migration via its effect on the nominal wage rate as well. At a zero tariff, with the employment subsidy at its optimal level, the marginal effect of the tariff on real income will be non-zero, but the marginal deadweight loss in consumption will be zero. Hence, a zero tariff cannot be optimal: instead the optimal tariff has to be such that these two effects just offset each other when the employment subsidy is at its optimal level, which, as was just seen, will require a negative tariff rate, i.e., an import subsidy.

Returning to the case where the lump-sum constraint is binding, it is possible to show (by comparing (57) and (63)) that the optimal tariff in the present case will necessarily be lower than in the case where the nominal wage is fixed. Other things equal, the difference will be greater the greater is e_m, the elasticity of the wage rate with respect to the price of manufactured goods, and the greater (in absolute value) is θ, i.e., the smaller the reduction in unemployment in response to job creation in the manufacturing sector. These, of course, are also the conditions making it more likely that the optimal policy for this situation may involve an import subsidy.

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[1] This means that a positive tariff will only be optimal when the resulting revenue is sufficient to offset the increase in the real wage through an increased employment subsidy measured in real terms.

This concludes the analysis of the different cases arising from the assumption that a tariff or subsidy can be levied on the manufacturing good alone. In the next section we briefly consider the modifications in the optimal policies when it is assumed that international trade in the agricultural good may also be taxed or subsidized.

4. Tariffs on Both Goods

Suppose first that the wage rate is fixed in nominal terms. It is not difficult to demonstrate that the Bhagwati and Srinivasan result, i.e., that an appropriate choice of taxes will make it possible to reach the first-best solution represented by the perfectly competitive resource allocation and consumption pattern, will then apply, even though the choice of policy instruments is restricted to taxation of international trade in the present analysis. This result holds whether or not one assumes that it is also possible to pay employment subsidies in manufacturing, and whether or not there is a constraint on the amount of lump-sum revenue that the government can raise.

To show this, we first note that the competitive solution is characterized by the following conditions: full employment of labour, relative prices paid by consumers being equal to relative world prices,[1] and equality between the marginal products of labour (at world prices) in manufacturing and agriculture. In the notation of the present model:

$$L_a + L_m = \bar{L}; \quad p_a = p_m; \quad f_L = h_L. \tag{64}$$

The restriction that taxes and subsidies can be levied only on international trade, together with the second condition above, also implies $q_a = q_m = q$. Using this condition, noting that at full employment we have $L_u = L_m$, and

[1]Or equivalently, a uniform consumption tax on all commodities.

assuming the employment subsidy $s_m = 0$, the condition for migration equilibrium and the equation giving employment in the manufacturing sector can be written, respectively:

$$q\, h_L(L_a) = w_m$$
$$q\, f_L(L_m) = w_m \tag{65}$$

Since these conditions, given w_m, yield L_a and L_m as increasing functions of q,[1] it will always be possible to find a q such that $L_a + L_m = \bar{L}$. Since this solution fulfills the conditions (64), it represents the first-best solution, as stated.

When $s_m = 0$, the government budget constraint becomes:

$$q(C_m - Q_m) + q(C_a - Q_a) + \bar{B} \geq 0 \tag{66}$$

Since trade is balanced at world prices, we have $C_m - Q_m = -(C_a - Q_a)$, so that the budget constraint will be satisfied for any $\bar{B} \geq 0$. It can be noted that in this situation, since trade taxes alone are sufficient to reach the first-best solution, there is no reason to subsidize employment in manufacturing; all that is required when the wage rate is fixed in nominal terms is, in effect, a devaluation of the domestic currency, as is implied by a uniform import tariff and export subsidy.

We turn now to the more interesting case where the wage rate depends on the prices of the two goods. To simplify the analysis, we will assume that the function describing this relationship is homogeneous, with the degree of homogeneity being given by $(\epsilon_a + \epsilon_m)$, the sum of the elasticities of the

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[1]Recall that $\partial^2 f/\partial L_m^2 < 0$, $\partial^2 h/\partial L_a^2 < 0$.

wage rate with respect to the consumer prices of agricultural and manufactured goods respectively.[1] We first observe that if the degree of homogeneity is less than one, the previous result will still hold, i.e., the first-best solution is still attainable through a uniform tariff and export subsidy. Conditions (65) become:

$$q^{(1-\epsilon_a-\epsilon_m)}\, h_L = w_o$$

$$q^{(1-\epsilon_a-\epsilon_m)}\, f_L = w_o \qquad (67)$$

where w_o is the wage rate prevailing when $q = 1$. Since by assumption $(1-\epsilon_a-\epsilon_m) > 0$, L_a and L_m will still be increasing functions of q, so that the first-best solution is attainable for some q. It is immediately obvious, however, that if the wage function is homogeneous of degree one in the two prices, the first-best solution cannot be attained if there is initial unemployment. In particular, this applies to the cases where the wage rate is fixed in terms of the manufactured good ($\epsilon_m = 1$, $\epsilon_a = 0$) or the agricultural good ($\epsilon_m = 0$, $\epsilon_a = 1$), i.e., in the cases analyzed by Harris-Todaro and Bhagwati-Srinivasan. The introduction of an employment subsidy in the manufacturing sector does not change this.[2]

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[1]If these elasticities are constant, the function is of the log-linear form $w_m = w_o p_a^{\epsilon_a} p_m^{\epsilon_m}$, which seems as good a functional form as any.

[2]It is possible to attain full employment at uniform trade taxes with a wage subsidy, but the marginal products of labour at world prices in the two sectors will not be the same.

Given that the first-best solution is not attainable, we turn to a brief discussion of the second-best tax and subsidy policy. We first note that when the nominal wage rate is a linearly homogenous function of the commodity prices, the real wage rate, measured in terms of either commodity, will depend only on the relative commodity prices. Economic intuition suggests that in such a situation, the entire pattern of resource allocation and consumption will depend only on relative and not on absolute prices. While this is indeed true in the present model, it must nevertheless be kept in mind that absolute prices might still play a role via the government budget constraint. In particular, it now becomes a matter of crucial importance how the constraint on government lump-sum revenue, if any, is defined.

Suppose first that $\bar{B}$, the maximum amount of lump-sum revenue, is given in nominal terms, and that $\bar{B} > 0$. It is then possible to show that in the present situation, the lump-sum revenue constraint can never be binding. Intuitively, this is explained by the fact that the "real" value of $\bar{B}$ can be made as large as desired by changing absolute prices but without disturbing relative prices or the real value of the employment subsidy. Since resource allocation and consumption depend only on relative prices, we may then choose $q_a = 1$, so that the analysis for this case is identical to that in the previous section with $\Psi = 0$.

Suppose on the other hand that the level of $\bar{B}$ is fixed in terms of either one of the two commodities. One can then show that the optimal tax and subsidy policy will be entirely determined by relative prices. If $\bar{B}$ is defined in terms of the agricultural good, we may again set $q_a = 1$, and the analysis becomes identical to that in the previous sections, with $\Psi = 0$. If it is defined in terms of the manufactured good, we may still choose $q_a = 1$, but the government budget constraint and the level of nominal income must be redefined by substitution of the term $q_m B^*$ for $\bar{B}$, where B^* is the constant maximum level of lump-sum revenue in terms of the manufactured good. This will also add another term to those expressions involving $\partial Y/\partial q_m$ in (54) and (61) in the previous sections, and the optimal tariffs or export subsidies will generally differ depending on the units in which the maximum lump-sum revenue is fixed. Apart from this modification, however, the results will be identical to those derived in the previous sections and we will not repeat them here.

IV. Conclusions and Qualifications

The purpose of the present paper has been to analyze the problem of optimal taxation in an open dual economy with a noncompetitive wage rate in the modern sector and rural-urban migration resulting in urban unemployment, in situations where certain restrictions applied to the policy instruments which the authorities could use. In particular, it was assumed that the policymakers were limited to using either an employment subsidy to manufacturing labour or taxes or subsidies on international trade, or a combination of these instruments, and that there was a constraint (which might or might not be binding) on the amount of revenue that could be raised by non-distortionary (lump-sum) taxation. Separate taxes or subsidies on agricultural production or consumption, or on labour employed in agriculture, on the other hand, were specifically excluded, in recognition of the often noted difficulties in assessing and collecting taxes or administering subsidy schemes in the agricultural sector in low-income countries.

The extension to an open economy of the analysis by Harris and Todaro of the case where an employment subsidy to manufacturing labour is the only policy instrument, apart from lump-sum taxation, was relatively straightforward and yielded results similar to those derived by them for a closed economy. An explicit expression for the optimal subsidy level in terms of the ratio of urban to rural labour and the elasticity of demand for agricultural labour was given. For the case where lump-sum revenue was limited, the conditions were analyzed under which the presence of this limitation would be likely to yield a significantly different solution and lower welfare than would be possible in its absence.

The case where commercial policy alone is used to counteract the effects of a noncompetitive manufacturing wage was originally analyzed by Hagen in the context of a model which assumed full employment; he demonstrated that a positive import tariff on manufactured goods would, under general conditions, improve welfare. In the context of the present model, this was also shown to be true for the case where the wage was fixed in nominal terms, or in terms of the agricultural good. If, however, the consumer price of manufactured goods influences the noncompetitive wage (for example, if it is partially determined by bargaining or legislation which takes the cost-of-living into account), it was shown that the optimal tariff could be either positive or negative. In fact, we proved that if the wage were fixed in terms of the manufactured good, an import subsidy (a negative tariff) would necessarily be optimal. Explicit expressions were given for the optimal tariff rates which were shown to depend on the wage and price elasticities of the demand for labour in manufacturing and of the consumption demand for manufactured output respectively, as well as on the same parameters which determined the optimal subsidy in the previous case.

The most interesting but also the most complicated cases arise in the situation where both an employment subsidy and taxes on international trade are assumed feasible. The results here can be summarized as follows. When there is no lump-sum revenue constraint, the optimal policy will involve a positive employment subsidy but a zero or negative tariff on manufactured-goods imports, the latter when the manufacturing wage depends on the consumption price of manufactured goods. When there is a lump-sum revenue constraint, the optimal tariff may be either negative or positive, the former being more likely the larger the elasticity of the manufacturing wage with respect to the price of manufactured goods. When the wage rate was fixed in nominal terms, the optimum manufacturing labour force was shown to be larger when both trade taxes and an employment subsidy could be used than with the latter alone. When the wage level depended on the price of the manufactured good, however, it was possible for the former situation to involve a smaller manufacturing labour force than would be optimal with the subsidy alone, but the agricultural labour force would be greater. As before, explicit formulae for the optimal values of taxes and subsidies in terms of the various demand elasticities were derived and interpreted. It was finally shown that the above results were essentially the same whether or not it was assumed possible to impose trade taxes on both commodities rather than on manufacturing goods only, except when the manufacturing wage rate was effectively fixed in nominal terms in which case a de facto devaluation of the currency via equivalent import tariffs and export subsidies would make the first-best solution attainable; in other cases such as when the wage rate is fixed in terms of either commodity, the first-best solution was shown to be unattainable when only trade taxes and a subsidy to manufacturing labour were allowed. This is of interest in the light of the Bhagwati-Srinivasan result that the first-best solution in the Harris-Todaro model will always be attainable when employment subsidies are assumed feasible in either sector and when production and consumption taxes can be separately controlled.

Numerous qualifications must of course be made if the above results are to be useful as starting points for actual policy formulation in low-income countries. First, the analysis is entirely in static terms, and the capital stocks in the two sectors are taken as given. In an actual policy-making situation, the analysis could be extended to take account of differential effects of various taxes on the profits and thus on the rates of investment in the two sectors. In the cases where the effects of the lump-sum revenue constraint were analyzed, it was assumed that the maximum amount of such revenue was rigidly fixed. A more realistic assumption would be that there exists a whole spectrum of taxes which have varying degrees of distortionary impact, and that revenue from sources other than trade taxes can be expanded at the cost of increasing amounts of distortion. Furthermore, there is the general question whether a two-sector open-economy model is too aggregative to yield useful policy conclusions in low-income countries; for example, it might be desirable to introduce a sector producing non-traded commodities. Finally, not much empirical evidence is available to support (or reject) the realism of the Harris-Todaro assumptions regarding the determinants of rural-urban migration, in particular with respect to the equilibrating role of the urban unemployment rate.

Despite all of this, however, the fact remains that the Harris-Todaro model has come to be seriously considered as a more relevant general framework for short-term policy analysis than most existing models of low-income countries, because it at least attempts to incorporate migration and urban unemployment which represent some of the most significant concerns of governments in these countries at the present time. Theoretical and empirical analysis of appropriate tax policies to deal with these problems, in a way which explicitly recognizes the administrative and other constraints under which the policymakers in low-income countries must operate, but which at the same time tries to incorporate more than a single policy instrument, therefore appears important. We hope that the present paper will be useful in stimulating further work in this direction.

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