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# Majority Voting on Tax Parameters - Some Further Results

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1. Introduction

In a recent paper [1], we explored the implications for progressivity and income redistribution of choosing the parameters of a linear tax function by simple majority vote. The analysis was carried out under the rather restrictive assumption that individual utility functions were of Cobb-Douglas form. Furthermore, although it was found that even with this assumption voters' preferences over tax rates may not be single-peaked, the discussion focussed on cases in which single-peakedness was assured. In this note, we extend the analysis in two directions. We investigate the robustness of the earlier results with respect to more general formulation of preferences. We then go on to characterize in some detail the nature of voting outcomes when preferences are not all single-peaked functions of tax rates. The notation used here is the same as in [1] and for compactness of exposition, we have generally omitted discussion of structure and detail which appears in that paper.

2. Generalization of Individual Behavior

Suppose that all individuals have identical, twice-differentiable quasi-concave utility functions  $U(C,L)$ , where  $C$  denotes consumption,  $L$  is the fraction of a unit time period spent working, and  $U_1 > 0$ ,  $U_2 < 0$ . An individual of ability  $n$  has before-tax income  $z_n = nL_n$  and pays a tax  $T(z) = k + tz$ , so that his budget constraint is given by

$$C_n = n(1-t)L_n - k.$$

Individual labour supply is then given by

$$L_n > 0 \quad \text{and} \quad U_2 + n(1-t)U_1 = 0$$

or  $L_n = 0 \quad \text{and} \quad U_2(-k,0) + n(1-t)U_1(-k,0) \leq 0.$

For a given pair of tax parameters, write the individual's work and consumption choices as  $L_n^*(k,t)$  and  $C_n^*(k,t)$ , respectively. We define

$$V_n(k,t) \equiv U[C_n^*(k,t), L_n^*(k,t)],$$

with

$$\frac{\partial V_n}{\partial k} = - \left. \frac{\partial U(C_n, L_n)}{\partial C} \right|_{C_n = C_n^*, L_n = L_n^*} < 0$$

$$\frac{\partial V_n}{\partial t} = - nL_n \left. \frac{\partial U(C_n, L_n)}{\partial C} \right|_{C_n = C_n^*, L_n = L_n^*}$$

$$= z_n \frac{\partial V_n}{\partial k} \leq 0.$$

The tax parameters are constrained to satisfy the revenue requirement

$$k + tZ(k,t) = G \tag{1}$$

where we have written<sup>1</sup>

$$Z = \int_{n_0}^N nL_n dF(n)$$

The tax possibility frontier (TPF),  $k = g(t)$  may be derived from (1). If  $Z$  is differentiable and the requirements of the implicit function theorem are satisfied, then the slope of the TPF,  $g'(t)$ , may be written as

$$g'(t) = - \frac{\partial(tZ)/\partial t}{1 + t \frac{\partial Z}{\partial k}} . \quad (2)$$

Note that this slope depends in part on the behavior of  $\partial(tZ)/\partial t$ , the response of tax revenue to changes in the marginal tax rate. In general, this response will depend on individual elasticities of labour supply and on the distribution of ability in a way that may be quite complicated.<sup>2</sup> It is easy to see that the numerator of (2) changes sign at least once, since for any value of  $k$ ,  $tZ = 0$  for  $t = 0$  and  $t = 1$  (since at  $t = 1$ ,  $L_n = 0$  for all  $n$ ). In general  $\partial(tZ)/\partial t$  may change sign a number of times, so that the TPF may have more than one turning point.

To relate this to individual behavior, we write

$$\tilde{V}_n(t) = V_n[g(t), t]$$

and

$$\frac{d\tilde{V}_n(t)}{dt} = g'(t) \frac{\partial V_n}{\partial k} + \frac{\partial V_n}{\partial t} ;$$

that is,

$$\frac{d\tilde{V}_n(t)}{dt} = \frac{\partial V_n}{\partial k} [z_n + g'(t)] \quad (3)$$

$\frac{\partial V_n}{\partial k}$  will be negative but the sign of the term in brackets depends on the nature of individual labour supply and, through  $g'(t)$ , on the distribution of abilities. The interaction of these factors may be quite complicated and the possible outcomes are diverse. It therefore seems unlikely that, except in fairly simple cases, single-peakedness of preferences over tax rates would result for all  $n$ .

Suppose for the moment that all preferences were, in fact, single-peaked in  $t$ . Would the majority voting outcomes in general be qualitatively similar to those in the Cobb-Douglas case? In particular, it is interesting to know whether the ranking of tax rates by individuals of different abilities remains the same as in the special case--with low tax rates preferred by highly-skilled persons and higher tax rates by those of relatively low ability. To see this, let

$$\tilde{V}'_n \equiv \frac{d\tilde{V}_n(t)}{dt} .$$

Then

$$\frac{d\tilde{V}'_n}{dn} = \left[ z_n + g'(t) \right] \frac{d\left(\frac{\partial V_n}{\partial k}\right)}{dn} + \frac{\partial V_n}{\partial k} \frac{dz_n}{dn}$$

Suppose that at  $t = t^1$ ,  $\tilde{V}'_n = 0$  for  $n = n_1$ . From (3),

$$[z_{n_1} + g'(t^1)] = 0$$

so that, at  $n = n_1$ ,

$$\frac{d\tilde{V}'_n}{dn} = \left( \frac{\partial V_n}{\partial k} \right) \left( \frac{dz_n}{dn} \right) \tag{4}$$

If  $z_n$  is non-decreasing in  $n$ , (4) indicates that for  $n$  in the neighbourhood of  $n_1$ ,  $\tilde{V}'_n \leq 0$  if  $n > n_1$  and  $\tilde{V}'_n \geq 0$  if  $n < n_1$ .

With single-peakedness of all preferences, the above result suggests that if  $t^*(n)$  denotes  $n$ -man's most-preferred tax rate, then (with  $z_n$  non-decreasing in  $n$ )  $t^*(n)$  is non-increasing in  $n$ . Over the range of increasing  $z_n$ ,  $t^*(n)$  will be decreasing in  $n$ . Voting results will then be qualitatively similar to those discussed in detail in [1], even under the more general formulation of utility functions.

3. "Local" voting equilibria<sup>3</sup>

The voting process implicit in our analysis so far may be characterized as one in which each voter evaluates a given tax rate against all other feasible values of  $t$ . The majority voting equilibrium we have been dealing with is thus a "global" one: the equilibrium tax rate is one that cannot be defeated in a majority vote by any other feasible tax rate. It is well known that such a global voting equilibrium need not exist when some voters have preferences that are not single-peaked. We have seen that non-single-peakedness is quite likely in the general case, so that the existence of a "global" voting equilibrium cannot be assured.

Rather than postulating that voting takes place over the entire feasible range, however, we may suppose that only small variations about a particular value of  $t$  are being considered by the electorate. Such a voting process seems reasonably applicable to political systems where typically only small changes in the status quo are contemplated, and where voters may be quite unfamiliar with alternatives significantly different from the prevailing situation.

If a majority of voters prefers an incremental move away from a given tax rate, then this tax rate is not a majority equilibrium. On the other hand, if the given tax rate is preferred by a majority of voters to tax rates in its "neighbourhood", then the current rate may be said to be a "local" equilibrium. More formally, a feasible tax rate  $t^m$  is a local majority voting equilibrium if and only if there exists a neighbourhood  $(t^m - \epsilon, t^m + \epsilon)$  of  $t^m$  (with  $\epsilon > 0$ ) which contains no feasible tax rate preferred by a simple majority.

Kramer and Klevorick [2] have shown that under a fairly weak assumption about voter preferences a local majority voting equilibrium exists. In the

present context, the required assumption is:

For all  $n$ ,  $\tilde{V}_n$  has only a finite number of proper relative maxima<sup>4</sup> on the closed interval  $[\tilde{t}, 1]$ , where  $\tilde{t}$  is the lowest permissible tax rate.

From (3) it will be seen that if the TPF has a finite number of turning points, the above "finite-peakedness" assumption will be satisfied. (For reasonably smooth, "well-behaved" skill distribution functions, we would expect the TPF to have the required property.)

It should be noted that although existence of a local majority voting equilibrium is assured under the "finite-peakedness" assumption, uniqueness of such an equilibrium does not follow--there may be several local majority voting equilibria. Again, the precise character of these local equilibria may be quite diverse and little more can be said about them without more concrete specification of individual behavior.

As an example, we conclude this section with a discussion of the possible voting outcomes for the Cobb-Douglas specification. From [1], we recall that in that case, we may distinguish between a "well-behaved" region and a possibly "badly-behaved" one. The "well-behaved" region consists of the interval  $t \in [\tilde{t}, t_0)$ , where  $t_0$  is the tax rate at which the least-skilled person drops out of the labour force. On this interval, all preferences are single-peaked. Any "irregularities" that occur lie in the region with  $t \geq t_0$ . In the proposition below, we summarize a number of results which give more precise definition to the structure of this voting model.

Lemma: For all feasible  $t$  and  $n_0 \leq n_1 < n_2 \leq N$ ,

$$\frac{d \tilde{V}_{n_1}}{dt} > \frac{d \tilde{V}_{n_2}}{dt}, \text{ unless } n_1\text{-man and } n_2\text{-man are both idle,}$$



in which case  $\frac{d \tilde{V}_{n_1}}{dt} = \frac{d \tilde{V}_{n_2}}{dt}$ .

Proof: See Section 5.2 of [1].

Proposition Denote by  $M$  the set of local majority voting equilibria. If the "finite-peakedness" assumption is satisfied for all individuals, then

- (a) there exists a local majority voting equilibrium;
- (b) if  $t^u \equiv \sup (M)$  and  $t^L \equiv \inf (M)$ ,  $t^u$  and  $t^L$  are themselves local majority voting equilibria;
- (c)  $t^u < 1$ ;
- (d) if  $t^W$  is the value of  $t$  that maximizes  $\tilde{V}_{\hat{n}}$  on the interval  $[\tilde{t}, t_0]$ , where  $\hat{n}$  is the median ability level, then  $t^L \geq t^W$ .

Proof: Parts (a) and (b): The conditions of the Kramer-Klevorick theorems on local equilibria are satisfied and these results follow.

Part (c): At  $t=1$ , every voter would prefer a lower tax rate, so that  $t=1$  will be defeated by tax rates slightly less than one. Thus,  $t=1$  cannot be a local majority voting equilibrium. Since  $t^u$  is a local majority voting equilibrium, it follows that  $t^u < 1$ .

Part (d): If  $t^W = \tilde{t}$ , the result follows immediately. For  $t^W > \tilde{t}$ , suppose that  $t^L < t^W \leq t_0$ . On the interval  $[\tilde{t}, t_0)$ , all voters have single-peaked preferences. It follows from this and the Lemma that  $t^L$  will be defeated

by tax rates slightly greater than  $t^L$ , so that  $t^L < t^W$  cannot be a local majority voting equilibrium. This contradicts the result of Part (b). Therefore,  $t^L \geq t^W$ .

[End of Proof]

Suppose that  $\left. \frac{d\tilde{V}_{\hat{n}}}{dt} \right|_{t=t_0}$  is nonnegative. Single-peakedness of

preferences on the interval  $[\tilde{t}, t_0)$  rules out "flats" in  $\tilde{V}_{\hat{n}}$ , so that  $d\tilde{V}_{\hat{n}}/dt$  is positive for tax rates in this interval. From the Lemma,

$$\frac{d\tilde{V}_{\hat{n}}}{dt} > 0, \quad n \leq \hat{n}$$

for  $t \in [\tilde{t}, t_0)$ . Therefore, for any tax rate  $t' \in [\tilde{t}, t_0)$ , a majority of voters prefers tax rates slightly higher than  $t'$ . If voting is not restricted to tax rates such that everyone works, then in this case,  $t^L \geq t_0$  -- that is, no tax rate in the interval  $[\tilde{t}, t_0)$  will be a local majority voting equilibrium.

If  $\left. \frac{d\tilde{V}_{\hat{n}}}{dt} \right|_{t=t_0}$  is negative, then it is readily seen that  $t^W < t_0$ . Now,

$t^W$  is the median voter's "most-preferred" tax rate when everybody works and for  $t \in [\tilde{t}, t_0)$  all voters have single peaked preferences. Therefore,  $t^W$  is preferred to any tax rate  $t'' \in [\tilde{t}, t_0)$  by a majority of voters. Clearly,  $t^W$  is a local majority voting equilibrium, so that  $t^L = t^W$ . On the interval  $(t^W, t_0)$ ,  $\frac{d\tilde{V}_{\hat{n}}}{dt} < 0$ ; the Lemma implies that

$$\frac{d\tilde{V}_n}{dt} < 0, \quad n \geq \hat{n}$$

for  $t \in (t^W, t_0)$ . Therefore, for any tax rate  $t_1 \in (t^W, t_0)$ , a majority of voters

prefers tax rates slightly lower than  $t_1$ , so that  $t_1$  cannot be a local majority voting equilibrium. The tax rate  $t^W$  is thus the only local majority voting equilibrium on the interval  $[\tilde{t}, t_0)$ .

In summary, when not all voters' preferences are single-peaked over the entire feasible range of tax rates but the "finite-peakedness" assumption holds,

- (a) there will be at most one local majority voting equilibrium over the range of feasible tax rates such that everyone works;
- (b) there may be a number of local majority voting equilibria over the range of feasible tax rates such that some people are idle. If there is no local majority voting equilibrium for  $t \in [\tilde{t}, t_0)$ , then there will be at least one such equilibrium for  $t \in [t_0, 1]$ .

#### 4. Conclusion

Generalization of individual preferences indicates that single-peakedness may not be very likely, though should it occur, the majority voting outcomes can be shown to have the qualitative properties found for the simple Cobb-Douglas case. Without single-peakedness, there may not exist a "global" majority voting equilibrium. Nevertheless, a "local" majority voting equilibrium, relevant to a somewhat more narrowly restricted but quite plausible voting process, will exist under fairly weak assumptions about individual preferences. Though such a local equilibrium need not be unique, it is possible, as in the Cobb-Douglas case, to characterize the equilibria as falling in particular intervals within the set of feasible tax rates.

Footnotes

\* Helpful comments by A. Klevorick and W. Nordhaus on an earlier version are much appreciated.

<sup>1</sup>  $F(n)$  is the distribution function of  $n$ , with  $\int_{n_0}^N dF(n) = 1$ .

Assumptions about production are the same here as in [1].

$$\frac{\partial(tZ)}{\partial t} = Z + t \frac{\partial Z}{\partial t} = \int_{n_0}^N z_n (1 + E_n) dF(n), \quad \text{where } E_n = \frac{\partial L_n}{\partial t} \frac{t}{L_n},$$

the elasticity of individual labour supply with respect to a change in the tax rate.

<sup>3</sup> In this section, we draw on the terminology and results of a recent paper by Kramer and Klevorick [2].

<sup>4</sup> A continuous function  $\varphi(X)$  defined on an interval has a proper relative maximum at a point  $x^0$  in the interval if and only if it has a relative maximum at  $x^0$  and does not have a relative minimum at  $x^0$ .

References

- [1] T. Romer, "Individual Welfare, Majority Voting, and the Properties of a Linear Income Tax," Journal of Public Economics, 1975.
- [2] G. Kramer and A. Klevorick, "Existence of a 'Local' Co-Operative Equilibrium in a Class of Voting Games," Review of Economic Studies, Oct. 1974.