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Learning by Searching: An Analysis of Consumer Decision-Making

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LEARNING BY SEARCHING:
AN ANALYSIS OF CONSUMER DECISION-MAKING

by

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I. Consumer Behaviour With Imperfect Knowledge
of the Price Distribution

In a previous paper [7] it was assumed that the consumer was aware
(or believed to be aware) of the price distribution. This is, no doubt,
a very strong assumption--not only because it implies the possession of an
amount of information that no consumer would believe to have, but because
it was taken to mean that once the consumer accepts a distribution of prices
he sticks to it no matter what are the values of the prices observed in
successive searches. Instead, we would prefer to imagine a consumer with
some initial idea about the price distribution that is modified as he
keeps visiting different stores and observes the prices quoted. As before,
the consumer confronts the dilemma of stopping or continuing the search
after a price has been observed. But now, the "stopping states" T are
more difficult to characterize since each new observation brings additional
information about the price distribution. This means that the utility
gain of searching for another price has to include not only the potential
gain of buying at a lower price but also a measure indicating the benefits
of having a more complete knowledge of how prices are distributed. It also
means that, in general, after each search, the beliefs of the consumers on
the price distribution will be modified. This, in our formulation, is
represented by a new "prior" distribution which originates in the incidence
of the particular price observed on the previous prior beliefs. To take
into account this new dimension is not an easy task and it appears that a
general treatment of the decision problem arises serious analytical
difficulties. To proceed further on, it seems, therefore, convenient to

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restrict the breadth of the treatment. Consequently we make the following assumptions.

Assumption 1. Prices are distributed lognormally, $P \sim \Lambda(\mathfrak{M}^*, r^*)$, where $\mathfrak{M}^*$ and r^* are respectively the mean and precision of the random variable log P which is normally distributed.

Assumption 2. The consumer knows that prices are distributed lognormally, has only an uncertain knowledge of the mean of the distribution α, and behaves as if he knew r^* with certainty. [1]

Assumption 3. Specifically, the consumer believes at stage i (i=0,1,...) that the mean of the price distribution $\alpha \sim \Lambda(\mu_i^*, \tau_i^*)$.

Assumption 4. The consumer has an indirect utility function $\overline{V}(\overline{P}, Y) = V(P) + W(Y)$ where $V(P) = \log a - b \log P, \ b > 0$.

To give an idea of the appropriateness of the results obtained under these assumptions, a double checking is indispensable. On the one hand we have to question the empirical adequateness of the assumptions made. On the other hand, since the first question will only be answered in a casual way, we have to consider how sensitive are the conclusions obtained to a change in the assumptions. This is the question of the robustness of the analysis. How far-fetched is the assumption of a lognormal distribution of prices? First of all we have to be convinced that there are no a priori grounds for rejecting this hypothesis. A delicate feature of the lognormal distribution is that the aggregation of two random variables which are lognormally distributed is a random variable which is not lognormally distributed. Therefore the assumption of lognormality seems more plausible if we interpret "the price" quoted by a store, say a car dealer, not as the average price charged for a car but the price for a very specific model with specific

features. This point clarified it is obvious that, ultimately, the answer

to our question has to come from a study of price data. Still it might be

said that the fact that a lognormal distribution allows only for positive prices

is an appealing feature of such a distribution, while, on the other hand,

its long tail might be a disturbing factor. With respect to its appropriateness

to fit price data not much can be said, mainly because no thorough analysis

of nonspeculative prices has been, to my knowledge, undertaken. Still there

are some data available. For durable goods, the most oft-quoted appear in

the work of A. F. Jung [25], [26], and refer to the prices of cars in Chicago

and other cities. A quick look at those prices seem to suggest that a log-

normal distribution would give a good fit. The price histograms have a bell

shaped form with mean and median generally not coinciding. Similarly a

look at price histograms of some durable goods for the Minneapolis-St. Paul

area[2] suggest again a normal or lognormal fit. Although admitting that the

factors that play a role in the evolution of speculative price might be

quite different from those that determine a cross-section of nonspeculative

prices, both, the markets for speculative goods and nonspeculative goods

(especially if the market for nonspeculative goods has many participants

not well aware of the behaviour of each other), have in common that the

determination of prices involve a large number of independent variables. It

is then not completely idle to mention, in the context of this research, that

an important sector of the profession likes to treat the prices in speculative

markets as if they were lognormally distributed (see, e.g., M.F.M. Osborne [35],

S.S. Alexander [2], P. Clark [13]).[3] Let me now comment briefly on assumption 2.

It is assumed in it that although the consumer does not know the mean of the price

distribution with certainty, he believes as certain his knowledge of the

precision r^*. That is, who can doubt it, an empirically unwarranted assumption.

But a simple comparison of lemmas 4 and 5 will show that the simplifications resulting from this hypothesis are indeed very substantial. One is therefore tempted to say, using an expression attributed to Robert Solow,[4] "I know that the wheel is crooked, but there is no other game in town."

The next assumption refers to the a priori beliefs of the consumer about the price mean distribution. The assumption is that he believes that the price mean is distributed lognormally. There is no especial reason to pick this distribution in preference to any other continuous distribution with the property that $Pr(P \leq 0) = 0$. The election has been done on the basis of analytical convenience. It is important, therefore, to discuss the sensitivity of the results to a modification of this assumption. It appears to be suggestive at this point to bring for consideration the work of Edwards, Lindman and Savage [19] and what they denominate the principle of stable estimation. They show that under a rather mild set of assumptions, "two people with widely divergent prior opinions but reasonably open minds will be forced into arbitrary close agreement about future observations by a sufficient amount of data." This statement, which in our context we might prefer to call the dominance of the likelihood principle,[5] holds whenever, loosely speaking, "the prior density changes gently in the region favoured by the data and not itself too strongly favours other regions." (For a more rigourous statement see the article quoted.) Therefore if we accept that the consumer is not too wrong about where the mean is, a few observations may render the assumption of lognormality rather innocent.

Why assumption 4? Clearly for analytical convenience. But can we justify it otherwise? We do not think there are any grounds for defending it. We can only invoke its wide use and maybe, being pedantic, Weber-Fechner law that response is proportional to the logarithm of the stimulus. Clearly, the main drawback of this choice is its unboundedness, since although a Bernouilli utility function might be a good approximation for prices in

the intermediate range, it is difficult to take seriously (to quote Savage [45]) over extreme ranges.[6] Concerning specifically the consumer's decision problem, the most bothering trouble is created by the existence of long tails in the distribution of utilities. (Note that by virtue of our assumptions utilities are distributed normally.) In particular, a utility function that takes large values for small prices will multiply the effect of low aberrant observations (outliers) and therefore increase the number of searches. Let us observe, though, that this would be a much more serious problem in the non-adaptative case than in the learning model that we have used. It is unfortunate, anyway, that there appears to be little work done on the robustness of the decision rule to a change in the utility function. (On this matter see Lindley [30].)

Summing up, we assume that the consumer believes that $P \sim \Lambda(\mathfrak{M}^*, r^*)$, and that at stage i of the search the price mean α is distributed in the following way: $\alpha \sim \Lambda(\mu_i^*, \tau_i^*)$. With this information the consumer will decide whether he continues the search, paying a cost c_{i+1}, or whether he stops it. As before, in Chapter III, we can describe the consumer knowledge and behaviour in terms of the distribution of utilities. And so we will say that the consumer believes that $V(P) \sim N(\mathfrak{M}, r)$ and that at stage i of the search the utility mean $\mathfrak{M}$ is distributed in the following way: $\mathfrak{M} \sim N(\mu_i, \tau_i)$. This is so since, if $P \sim \Lambda(\mathfrak{M}^*, r^*)$, then $aP^{-b} \sim \Lambda(a - b\mathfrak{M}^*, \frac{1}{b^2 \cdot r^*})$. (See Aitchison and Brown [1], Thm. 2.1.) Therefore,

$$V(P) = \log a - b \log P \sim N(\mathfrak{M}, r), \text{ where}$$

$$\mathfrak{M} = a - b\mathfrak{M}^* \quad \text{and} \quad r = \frac{r^*}{b^2}.$$

Now, since $\alpha = \exp(\mathfrak{M}^* + \frac{1}{2r^*}) \sim \Lambda(\mu_i^*, \tau_i^*)$, then

$$\log \alpha = \mathfrak{M}^* + \frac{1}{2r^*} \sim N(\mu_i^*, \tau_i^*) \quad \text{and} \quad \mathfrak{M}^* \sim N(\mu_i^* - \frac{1}{2r^*}, \tau_i^*).$$

Therefore $\mathfrak{M} \sim N(\mu_i, \tau_i)$, where $\mu_i = a - b(\mu_i^* - \frac{1}{2r^*})$ and $\tau_i = \frac{\tau_i^*}{b^2}$.

As in Chapter III, we shall specify whether recall is allowed, and whether the cost of search is constant or increasing from period to period. The consumer knowledge at period i, i=0,1,2,..., will be summarized by the parameters μ_i and τ_i, the mean and the precision,respectively,of the distribution of the utility mean which, like the distribution of the utility, is normal. The decision of the consumer will, in addition, take into account the cost of further search and the value of the last price observed p_i if no recall is allowed, or, alternatively, the sequence of values of the prices previously observed $p_1,...,p_i$, if recall is allowed. We will first consider the case of no recall.

No Recall

We assume, again, that given the initial income y, and being the cost of search $c_i > 0$, i=1,2,..., an optimal stopping rule exists and the expected utility of following the optimal rule is finite. Without recall, the relevant information for the consumer's decision at stage i is described by the triple $(V(p_i), \mu_i, \tau_i)$, where μ_i and τ_i summarize the past history of searches and the prior beliefs, in a way that will be made clear in the following discussion adapted from De Groot [15], section 13.6.

Suppose that the consumer believes at some stage 0, that $\mathcal{M} \sim N(\mu_0, \tau_0)$. If he now decides to go into another store and he observes the price $P_1 = p_1$, by lemma 4, the posterior distribution of $\mathcal{M}$ will be $N(\mu_1, \tau_1)$, where

$$\mu_1 = \frac{\mu_0 \tau_0 + rV(p_1)}{\tau_0 + r} \qquad\qquad [13]$$

$$\tau_1 = \tau_0 + r. \qquad\qquad [14]$$

In general, if he visits i additional stores, the posterior distribution of $\mathcal{M}$ will be $N(\mu_i, \tau_i)$, where

$$\mu_i = \frac{\tau_0}{\tau_0 + ir} \mu_0 + \frac{ir}{\tau_0 + ir} \overline{V(p)} \ ,$$

a linear combination of μ_0 and $\overline{V(p)}$, which is precisely a weighted average of the mean of the prior distribution of $\mathcal{M}$ and the value of the sample mean $\overline{V(p)}$. Note that the weights of μ_0 and $\overline{V(p)}$ are proportional, respectively, to the precision of the prior distribution of $\mathcal{M}$, τ_0, and the precision of the sample mean, ir. As the sample size increases the weight given to the sample mean increases, and the weight given to the prior decreases. On the other hand

$$\tau_i = \tau_0 + ir \ ,$$

and so the precision of the posterior distribution of $\mathcal{M}$ increases with each observation by the amount r, regardless of the prices observed. At stage 0, the marginal distribution of the utility of the next price is, by lemma 6,

$$V(P_1) \sim N(\mu_0, \frac{\tau_0 r}{\tau_0 + r}) \ ,$$

and the marginal distribution of the utility of the price observed i periods later is

$$V(P_i) \sim N(\mu_{i-1}, \frac{\tau_{i-1} r}{\tau_{i-1} + r}) \ .$$

Suppose that the consumer has just observed a price $P_i = p_i$ and that this observation, coupled with his previous information, leads him to believe that $\mathcal{M} \sim N(\mu_i, \tau_i)$. The triple $(V(p_i), \mu_i, \tau_i)$ describes the position of the consumer. Let us call $M(V(p_i), \mu_i, \tau_i)$ the expected utility, exclusive of the cost of previous search, from following the optimal procedure when

the consumer has just observed $P_i = p_i$. By an approach identical to the one taken when the consumer knew with certainty the price distribution, we can observe that

$$M(V(p_i), \mu_i, \tau_i) = \max\{V(p_i), E[M(V(P_{i+1}), \mu_{i+1}, \tau_{i+1}) - c_{i+1}]\} \quad i=1,2,\dots$$

[15]

Clearly, then, if we call

$$E[M(V(P_{i+1}), \mu_{i+1}, \tau_{i+1})] - c_{i+1} = \alpha_i, \quad i=1,2,\dots \quad [16]$$

The optimal rule is to stop whenever $V(p_i) \geq \alpha_i$ and to continue otherwise. Or, equivalently, to stop the search if $p_i \leq p_i^*$, where

$$p_i^* = V^{-1}(\alpha_i) . \quad [17]$$

Note, again, that α_i is the expected utility, at stage i, of taking one more observation by paying the search cost, and following the optimal rule afterwards. Combining [15] and [16] we obtain

$$\alpha_i = E[\max\{V(P_{i+1}), \alpha_{i+1}\}] - c_{i+1} .$$

Therefore, from lemma 1, we obtain

$$\alpha_i = \alpha_{i+1} + \int_{\alpha_{i+1}}^{\infty} [V(P_{i+1}) - \alpha_{i+1}] \, dF[V(P_{i+1})] - c_{i+1} , \quad [18]$$

and, calling $g(\alpha_{i+1}) = \int_{\alpha_{i+1}}^{\infty} [V(P_{i+1}) - \alpha_{i+1}] \, dF[V(P_{i+1})]$, by lemma 2, it is clear that

the sequence (α_i) is determined once one of its elements is known.

$$\alpha_i = \alpha_{i+1} + g(\alpha_{i+1}) - c_{i+1} , \quad i=1,2,\dots \quad [19]$$

Since

$$V(P_{i+1}) \sim N(\mu_i, \frac{\tau_i \cdot r}{\tau_i + r}) , \quad \text{calling} \quad \frac{\tau_i \cdot r}{\tau_i + r} = \Pi_i ,$$

$g(\alpha_{i+1}) = \Pi_i^{-\frac{1}{2}} \Psi [\Pi_i^{\frac{1}{2}} (\alpha_{i+1} - \mu_i)]$, by lemma 3 and the definition of Ψ given in Chapter III.

Therefore

$$\alpha_i = \alpha_{i+1} + \Pi_i^{-\frac{1}{2}} \Psi [\Pi_i^{\frac{1}{2}} (\alpha_{i+1} - \mu_i)] - c_{i+1} .$$

Using lemma 7,

$$\alpha_i = \Pi_i^{-\frac{1}{2}} \Psi [\Pi_i^{\frac{1}{2}} (\mu_i - \alpha_{i+1})] + \mu_i - c_{i+1} . \qquad [20]$$

Little can be said about α_i . The a priori mean μ_i enters the right hand side of the above equality as an argument of Ψ and in α_{i+1} . But, at least it can be verified that the entire sequence (α_i) is determined recursively from any of its elements. This is so since Ψ can be inverted to yield

$$\alpha_{i+1} = \mu_i - \Pi_i^{-\frac{1}{2}} \Psi^{-1} [\Pi_i^{\frac{1}{2}} (\alpha_i - \mu_i + c_{i+1})].$$

Suppose now that present and future utilities are reduced by an amount k. Since the utility of the presently observed price is reduced by k, the utility of stopping the process of search now is reduced by k. Similarly, since the utility of every future observed price is reduced by k, the utility from continuing the process is reduced by k. Therefore,

$$M[V(p_i), \mu_i, \tau_i] - k = M[V(p_i) - k, \mu_i - k, \tau_i] .$$

Or, in particular,

$$M[V(p_i), \mu_i, \tau_i] = M[V(p_i) - \mu_i, 0, \tau_i] + \mu_i . \qquad [21]$$

It can be shown now that[7]

$$M(z_i, 0, \tau_i) = \max\{z_i, E[Y_{i+1}, 0, \tau_{i+1}] - c_{i+1}\} \qquad [22]$$

where $z_i = V(p_i) - \mu_i$, $\tau_{i+1} = \tau_i + r$,

$$Y_{i+1} = \frac{\tau_i r}{\tau_{i+1}} Z_{i+1} \quad \text{and} \quad Z_{i+1} \sim N(0, \frac{\tau_i r}{\tau_{i+1}}), \quad \text{and}$$
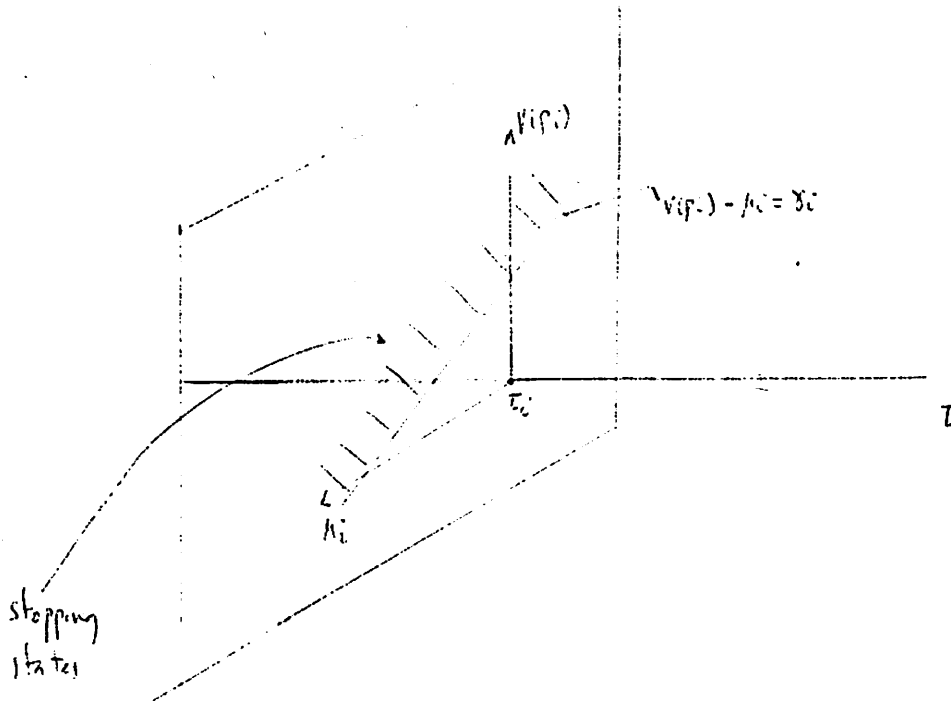
therefore

$$Y_{i+1} \sim N(0, \frac{\tau_{i+1}}{\tau_i r}) .$$

Calling $\gamma_i = E[M(Y_{i+1}, 0, \tau_{i+1})] - c_{i+1}$,   [23]

it follows that the optimal rule for a consumer who has just observed $P_i = p_i$, and therefore believes that $\mathfrak{M} \sim N(\mu_i, \tau_i)$, is to buy at this price whenever

$$V(p_i) \geq \gamma_i + \mu_i ,$$ [24]

and to look for another price otherwise.

In that way the expected utility of searching optimally for another price can be expressed as the sum of μ_i and another term γ_i independent of τ_i. From [22] and [23]

$$\gamma_i = E[\max\{Y_{t+1}, \gamma(\tau_{i+1})\}] - c_{i+1}.$$

Proceeding as before for α_i,

$$\gamma_i = \gamma_{i+1} + g(\gamma_{i+1}) - c_{i+1}$$

and therefore,

$$\gamma_i = \gamma_{i+1} + \left[\frac{\tau_i}{\tau_{i+1}r}\right]^{-\frac{1}{2}} \psi \left\{\left[\frac{\tau_i}{\tau_{i+1}r}\right]^{-\frac{1}{2}} \cdot \gamma_{i+1}\right\} - c_{i+1},$$

or

$$\gamma_{i+1} = -\left[\frac{\tau_i}{\tau_{i+1}r}\right]^{\frac{1}{2}} \psi^{-1}\left\{\left[\frac{\tau_i}{\tau_{i+1}r}\right]^{-\frac{1}{2}} \left[\gamma_i + c_{i+1}\right]\right\}.$$

And we see that the whole sequence of (γ_i) can be obtained from any of its elements.

Note that in the present case, when the consumer does not know with certainty the price distribution, no simplification is gained by assuming that the cost of search is constant from period to period.

Before proceeding, it seems convenient to have a closer look at the optimal rule obtained. How can this result be interpreted? The consumer is comparing the utility of buying at the last observed price with an expression which is the sum of the expected utility mean and γ_i. γ_i is in itself the algebraic sum of two terms. The cost of search and another term $E\{M(Y_{i+1}, 0, \tau_{i+1})\}$, which can be interpreted in the following way.[8] Suppose that an individual is sampling without recall and observes a sequence of independent but not identically distributed utility random variables $Y_1, Y_2, \ldots,$ where $Y_{i+1} \sim N(0, \frac{\tau_i + r}{\tau_i r})$, $i = 0, 1, 2, \ldots$. Suppose that at some

stage i, $Y_i = y_i$ is observed. Then the distribution of the next observation Y_{i+1} is precisely that of Y_{i+1} in [22]. And so, we can call $E\{M(Y_{i+1}, 0, \tau_{i+r})\} - c_{i+1}$ the optimal expected utility among the procedures that specify that at stage i, one more observation should be taken in the auxiliary search problem described.

We can say, therefore, that the optimal procedure for the consumer is to buy if the observed utility minus the expected utility mean is greater than the optimal expected utility of searching at least once more in the auxiliary problem.

What kind of results are we interested in? On the one hand we want to take advantage of the analytical framework constructed above to do some comparative statics. In particular we want to know how the optimal behaviour of the consumer is affected by changes in his prior beliefs about the parameters of the distributions and by changes in the cost of search. On the other hand our main interest consists in paving the way for an eventual description of the evolution of the prices in the market, once the sellers' decisions have been taken into consideration. To this end we would like to determine (a) what sort of market demand function results from the consumers behaving optimally, (b) whether there are any grounds for anticipating any monotonicity in the evolution of the reservation prices, as in Chapter III, and, as we also found in that chapter, (c) whether we have a "stabilizing" property of the sort that the larger being the price "dispersion," the larger will be the expected number of searches.[9]

An answer can be given right away to question (a). It is quite clear that at any period the demand function will be well behaved, the expected demand being a non-increasing function of the price charged by the seller. This is so since we have what Rothschild calls the reservation price property,

that for every state of information there is a <u>unique</u> reservation price.[10]

But in the adaptative case it is also clear that the seller cannot be certain that an increase in price will reduce the probability of a sale or that a reduction in price will increase it, due to the fact that as the state of information changes from period to period so does the reservation price.[11]

Turning now to comparative statics, the first question that we want to ask is whether we can observe a consumer's preference for concentration or dispersion of the price distribution. That is, whether the expected utility when following the optimal rule is greater or lower as prices are more "dispersed." The following proposition answers this question.

Proposition 4.1

At any stage of the search i, a utility-mean preserving increase in the price dispersion, that is, a decrease in β, with an unchanged distribution of m, <u>decreases</u> the expected utility of following the optimal rule and increases the reservation price.

Proof

Recall first that $\left.\dfrac{dr}{d\beta}\right|_{\mu,\tau \text{ constant}} > 0$ and that $\dfrac{dr^*}{dr} > 0$. Therefore we will prove the proposition if we can show that a decrease in r, with μ and τ constant, leads to a decrease in the expected utility of following the optimal rule. Consider now two different precisions r^A and r^B, such that $r^A > r^B$. When the precision is r^A, the maximum expected utility is (from [24]) $\mu_0 + \gamma_0^A$ where $\gamma_0^A = E[M(Y_1, 0, \tau_1^A)] - c_1$ (from [23]) and $\tau_1^A = \tau_0 + r^A$. Similarly the maximum expected utility when the dispersion is r^B is $\mu_0 + \gamma_0^B$, where $\gamma_0^B = E[M(Y_1^B, 0, \tau_1^B)] - c_1$ and $\tau_1^B = \tau_0 + r^B$. We want to show, therefore that $\gamma^A > \gamma^B$. If the distribution of the random variable Y_1^j, j=A,B, did not change in subsequent periods in the auxiliary search problem described above, that is, if $Y_i^j \sim N(0, \dfrac{\tau_0 + r^j}{\tau_0 + r^j})$, j=A,B, i=1,2,..., then, since $r^A > r^B$, $\dfrac{\tau_0 + r^A}{\tau_0 \cdot r^A} < \dfrac{\tau_0 + r^B}{\tau_0 \cdot r^B}$.

And so, by proposition 3.8, we could say that the expected utility of searching optimally when the random variable is $Y_i^A = Y_1^A$, $i=1,2,\ldots$, α^A, is greater than the expected utility of searching optimally when the random variable is $Y_i^B = Y_1^B$, $i=1,2,\ldots$, α^B. That is $\alpha^A > \alpha^B$. But the distribution of Y_i^j $i=1,2,\ldots$, $j=A,B$, is different for different i's according to the rules established above, or, specifically, the precision of Y_i^j $i=1,2,\ldots$, $j=A,B$, is larger, the larger is i according to $\tau_{i+1}^j = \tau_i^j + r^j$, $i=0,1,\ldots$, $j=A,B$. Also, the following inequality will be satisfied for all i, $i=0,1,\ldots$: $\dfrac{\tau_i^A + r^A}{\tau_i^A \cdot r^A} < \dfrac{\tau_i^B + r^B}{\tau_i^B \cdot r^B}$. That is, at any stage i, the precision of Y_i^B will be greater than the precision of Y_i^A.

In addition, $\dfrac{\tau_i^B + r^B}{\tau_i^B \cdot r^B} - \dfrac{\tau_i^A + r^A}{\tau_i^A \cdot r^A} > \dfrac{\tau_{i-1}^B + r^B}{\tau_i^B \cdot r^B} - \dfrac{\tau_{i-1}^A + r^A}{\tau_i^A \cdot r^A}$.

Therefore, <u>a fortiori</u>, $\gamma^A > \gamma^B$.

This result is in flagrant opposition to the one obtained when the search did not provide with additional information on the price distribution. <u>It indicates that the case of no learning is not, at least under our assumptions and consequently not in general, a good approximation to the case on which search involves in itself an updating process of the previous beliefs.</u>[12] That this result obtains, depends on the fact that, in the Bayesian adaptative rule postulated, the degree of dispersion of the price distribution affects the rate at which the degree of uncertainty about the utility mean distribution changes from period to period.[13]

To statisticians, a natural way of specifying a lognormal distribution is by means of the parameters μ^* and τ^*, the mean and precision respectively of the associated normal distribution. We might be interested, then, in the relation between the consumer's search behaviour and a change in those parameters. The following proposition answers the case in which τ^* is constant but μ^* changes.

Proposition 4.2

Suppose that at some stage of the search i, two consumers A and B have the same beliefs about the distribution of utilities except that A believes that the expected utility mean is μ_i^A and B believes that the expected utility mean is μ_i^B, where $\mu_i^A < \mu_i^B$. If both follow the optimal procedure, the expected utility of pursuing the search optimally is smaller for consumer A than for consumer B, or, equivalently, the reservation price is larger for consumer A than for consumer B. But the subjective estimate of the average number of additional searches is the same for both consumers.

Proof

The expected utility of continuing searching at least one more time is $\gamma_i^j + \mu_i^j$ $j=A,B$, and therefore the result is immediate. On the other hand,

$$E(N_i) = \nu^{-1} \quad \text{where} \quad \nu = \int_{\gamma_i^j + \mu_i^j}^{\infty} dF^j[V(P_{i+1})] \ , \quad j=A,B.$$

From our point of view, it seems, anyway, more interesting to characterize the price distribution by its own mean and precision. Therefore, we would like to establish a result analogous to the previous one, for two consumers who believe that the price mean distribution has the same precision but different mean. Unfortunately this does not seem to be possible without further constraints or, at least, without a more complete characterization of function γ. This is so since if the means are different but the degree of uncertainty with which they are known is equal, then $\mu_A < \mu_B$ but $\tau_A > \tau_B$. [14]

We can still show that for "similar" price mean distributions (and the same price distribution), different beliefs about the expected price mean yield unequivocally different search patterns. By "similar" in this context we mean lognormal distributions with the same degree of skewness. Skewness is a measure of non-symmetry which is defined by the ratio of the third moment

about the mean and the third power of the standard deviation. In the case

of the distribution $\Lambda(\mu^*, \tau^*)$, the coefficient of skewness $\Delta = \eta^3 + 3\eta$

where $\eta = \exp\{1/\tau^*\} - 1$.

Proposition 4.3

Suppose that two consumers A and B have the same utility function,

the same beliefs about the price distribution, and believe that the dis-

tribution of the price mean has the same degree of skewness, but consumer A

believes that the expected price mean is $E(\alpha^A)$ while consumer B believes

that the expected price mean is $E(\alpha^B)$, where $E(\alpha^A) > E(\alpha^B)$. Then, if both

follow the optimal procedure, the expected utility of searching at least

once more is smaller for consumer A than for consumer B or, equivalently,

the reservation price is larger for consumer A than for consumer B.

Proof

$$E(\alpha^A) = \exp\{\mu^{*A} + \frac{1}{2\tau^{*A}}\}, \quad E(\alpha^B) = \exp\{\mu^{*B} + \frac{1}{2\tau^{*B}}\} .$$

Since both A and B believe that the price mean distribution has the same

degree of skewness, $\tau^{*A} = \tau^{*B}$ and $\tau^A = \tau^B$. Then $E(\alpha^A) > E(\alpha^B)$ $\Rightarrow$

$\Rightarrow \mu^{*A} > \mu^{*B} \Rightarrow \mu^A > \mu^B$ and therefore $\gamma^A + \mu^A < \gamma^B + \mu^B$.

So far we have discussed the effect on searching of changes in the

consumer's beliefs. A different question refers to how searching is affected

by a change not in the beliefs themselves, but in the degree of certainty with

which these beliefs are held. The following proposition tries to answer this

question.

Proposition 4.4

A decrease of the consumer's degree of uncertainty about the utility

mean, that is, an increase in τ , leads, with the utility mean unchanged,

to an increase in the expected utility of pursuing the search (and equiv-

alently to a lower reservation price).[15]

Proof

Following the same procedure that in the proof of proposition 4.1 we can show that γ increases with τ. And then, from [24], the result follows.

This result can be referred as the <u>consumer's preference for certainty</u>.[16]

In the same context we can show that a consumer who searches the market for a low price will prefer, other things being equal, to have a certain knowledge of the price distribution. The following proposition makes this statement precise.

Proposition 4.5

Let A and B be two consumers who decide to search <u>without recall</u> the market, have the same logarithmic utility function and incur in the same search costs. But consumer A knows with certainty that $P \sim \Lambda(\mu^*, r^*)$ while consumer B believes that $P \sim \Lambda(m^*, r^*)$ and, not knowing with certainty m^*, believes that $m^* \sim \Lambda(\mu^*, \tau^*)$, where $\mu^* < \infty$, $0 < r^* < \infty$, $0 < \tau^* < \infty$. Let us call U^A consumer A's expected utility of following the optional stopping rule and U^B consumer B's expected utility of following the optional stopping rule. Then $U^A > U^B$.

Proof

From [11**] $U^A = \mu_i + \hat{\gamma}_i$.

From [24], $U^B = \mu_i + \gamma_i$.

From lemma 8

$$\lim_{\tau \to \infty} \gamma_i = \hat{\gamma}_i ,$$

and since γ is strictly increasing in τ by Proposition 4.4 and $(\tau^* < \infty) \Leftrightarrow (\tau < \infty)$, the inequality is strict.

From the optimal stopping rule given in [24] it is clear that if γ is sufficiently high, prices whose utility is greater than the expected utility mean minus the cost of search <u>can</u> be rejected. And, more surprisingly, that the consumer <u>may prefer</u> to visit at least one additional store (and, thereby, pay the additional cost of search) to buying at a price whose utility is greater than the expected utility mean. That this is a possibility is made clear in the next lemma. But this can only happen for a cost of search sufficiently low and then the result is after all not that strange. To keep searching allows the consumer to try his luck. He might hit a very low price and if a high price is observed he can always reject it. Therefore this lack of symmetry in his attitude towards high and low prices may be considered an inducement for further search. And, one should not forget, in addition, that to keep searching brings a further benefit in terms of additional information.

What might appear as extremely intriguing is the fact that the more accurate are the consumer's beliefs about the price mean (the larger τ) the greater is the expected utility of search (see proposition 4.4). And especially since after the results of the last chapter, we have come to expect that a higher expected utility is associated with a longer expected search. But, then, what is the sense of maintaining that the more accurate the consumer's beliefs the greater is the expected number of searches? Because with greater accuracy, the lower is the subjective probability of finding a very low price with further search. In addition, if we think of the value of information as the difference between the expected utility when the consumer is perfectly informed about the price distribution and the expected utility when he is not perfectly informed, we are facing a case in which the value of information decreases as the uncertainty diminishes.[17] Therefore the

claim that the expected number of searches increase with more accurate information seems utterly nonsensical. What has gone wrong? One may conjecture that for the case of an unknown distribution a higher expected utility of optimal search is not associated with a greater expected number of searches. But this is not true in general, and there are situations, as the following discussion shows, in which a higher expected utility implies a greater expected number of searches. Suppose two alternative cases:

$$\text{(A)} \quad \tau_i = \tau_i^A \quad \text{and} \quad \text{(B)} \quad \tau_i = \tau_i^B, \quad \text{where} \quad \tau_A^i > \tau_B^i.$$

Then in case (A) the expected utility is $\mu_i + \gamma_i^A$ and in case (B) $\mu_i + \gamma_i^B$, where, by proposition 4.4, $\gamma_i^A > \gamma_i^B$. In case (A) the marginal distribution of the utility of the next price is by lemma 6: $V(P_{i+1}) \sim N(\mu_i, \dfrac{\tau_i^A \cdot r}{\tau_i^A + r})$, while in case (B) $V(P_{i+1}) \sim N(\mu_i, \dfrac{\tau_i^B \cdot r}{\tau_i^B + r})$. The expected number of searches in case (A) is ν_A^{-1}, where $\nu_A = \displaystyle\int_{\mu_i + \gamma_i^A}^{\infty} dF^A[V(P_{i+1})]$ and in case (B) ν_B^{-1}, where $\nu_B = \displaystyle\int_{\mu_i + \gamma_i^B}^{\infty} dF^B[V(P_{i+1})]$. Since the precision of the distribution of $V(P_{i+1})$ is greater in case (A) than in case (B), then if $\gamma_i^A > 0$, $\nu_A < \nu_B$. Since Y is a positive function, it is clear that for sufficiently low c_{i+1}, γ_i can be made positive. This being the case we have to acknowledge our puzzlement and we will exorcise the result to less threatening grounds by calling it a paradox.

We have seen that it is conceivable that a consumer observing a price whose utility is above the a priori expected utility mean will consider that his chances of finding an even lower price are enhanced.[18] But this, as we show in the following proposition, can only happen for costs of search sufficiently low.

Proposition 4.7

At any stage i, a necessary condition for continuing the search when the utility associated with the price observed is equal or greater than the expected utility mean is that the cost of one additional observation c_{i+1} satisfies $c_{i+1} \leq (2\pi r)^{-\frac{1}{2}}$. Or, contraposing the statement, if $c_{i+1} \geq (2\pi r)^{-\frac{1}{2}}$, then, any price whose utility is equal or greater than the expected price mean will be taken.

To prove the proposition we first state the following lemma.

Lemma

γ_i will be negative <u>for all</u> $0 < r < \infty$, $0 < \tau_i < \infty$ if $c_{i+1} \geq (2\pi r)^{-\frac{1}{2}}$.

Proof of the Lemma

In the non-adaptive case with constant cost of search c,

$$\alpha^* = \mu + r^{-\frac{1}{2}} \Psi^{-1}[c\, r^{\frac{1}{2}}] , \quad \text{(see [11])}.$$

Now $\Psi(0) = (2\pi)^{-\frac{1}{2}}$ and, therefore, $\Psi^{-1}[(2\pi)^{-\frac{1}{2}}] = 0$, and since Ψ^{-1} is a monotone decreasing function,

$$r^{\frac{1}{2}}c \geq (2\pi)^{-\frac{1}{2}} \Leftrightarrow r^{-\frac{1}{2}} \Psi[c\, r^{\frac{1}{2}}] < 0 .$$

If we suppose now that costs of search increase from period to period, and we make $c_{i+1} = c$, $\mu_i = \mu$ and $\tau_i = \tau$, then $\alpha_i < \alpha^*$ where $\alpha_i = \mu_i + \hat{\gamma}_i$, i.e., $\hat{\gamma}_i < r^{-\frac{1}{2}} \Psi[c\, r^{\frac{1}{2}}]$.

Therefore

$$c_{i+1} \geq (2\pi r)^{-\frac{1}{2}} \Rightarrow \hat{\gamma}_i < 0 .$$

From Proposition 4.5, $\hat{\gamma}_i > \gamma_i$ for $\tau < \infty$. Therefore $c_{i+1} \geq (2\pi r)^{-\frac{1}{2}} \Rightarrow \gamma_i < 0$.

Proof of the Proposition

If $c \geq (2\pi r)^{-\frac{1}{5}}$ and $V(p_i) \geq \mu_i$, then $V(p_i) > \gamma_i + \mu_i$. Therefore p_i will be accepted.

We are familiar by now with the concept of a reservation price which, defined for each state of information, is used in determining the optimal search behavior of the consumer. One can anticipate that the evolution from period to period of the consumer's reservation prices will be crucial to the market prices' dynamics. We should be, therefore, interested in establishing under what conditions we can claim definite evolutive patterns for the reservation prices. In particular we want to verify the monoton-icity of the evolution of the reservation prices. Diamond [17], for instance, assumes that the reservation prices of the consumers staying in the market increase, obtaining, not surprisingly, that the market prices converge to a monopoly price. But his assumption is, in general, unwarranted. Suppose, for instance that at stage i the consumer observes a price $P_i = p_i$ such that $V(p_i) > \mu_{i-1}$. Then, from [13], $\mu_i > \mu_{i-1}$. On the other hand, from lemma 8, $\gamma_i > \gamma_{i-1} - (c_{i+1} - c_i)$. Therefore, unless the increase in the cost of search from one period to another is sufficiently high we will have $p_i^* = V^{-1}[\gamma_i + \mu_i] < V^{-1}[\gamma_{i-1} + \mu_{i-1}] = p_{i-1}^*$. In particular, this inequality is clearly satisfied if the cost of search is constant. On the other hand, to have a monotonically decreasing sequence of reservation prices, the fol-lowing necessary and sufficient condition must be satisfied at every stage i of the search:

$$\gamma_i - \gamma_{i-1} > \frac{r}{\tau_{i-1} + r} \left[\mu_{i-1} - V(P_i) \right] ,$$

obtained from $\gamma_i + \mu_i > \gamma_{i-1} + \mu_{i-1}$ and [13].

If the cost of search increases from period to period, then γ_i does not have to be necessarily greater than γ_{i+1}. Therefore if the difference between c_i and c_{i+1} is sufficiently large, the left hand side will become negative. Now, given that the probability of the right hand side being negative is .5, since $V(P_i) \sim N(\mu_{i-1}, \cdot)$, the probability of having a monotonically decreasing sequence of reservation prices becomes very small as the number of searches increase, the convergence to zero being faster the larger is the difference between the cost of search from period to period. Only in the most favourable case of constant cost of search, the probability of having a monotonically decreasing sequence of reservation prices could be high for a certain number of periods, but would eventually become very small if the number of searches increased sufficiently.

We are forced therefore to end this section with a pessimistic note. It does not seem that an optimal behaviour of the consumer brings about a discernible pattern of the reservation prices evolution.

Recall

When the price distribution was known, we observed that, with recall, we were in what is called the monotone case, and the optimal rule was easily found. Unfortunately, when the search brings new information that is used to update the consumer's beliefs, this is not necessarily the case. It may be remembered that in Chapter III, section 2, we defined α_i as the unique solution of $E\{\max[V(P_{i+1}) - \alpha_i, 0]\} = c_{i+1}$. Since, in addition, $\{VP_i\}$ was a sequence of iid random variables, we were entitled to say that $c_{i+1} \geq c_i \Rightarrow \alpha_i \leq \alpha_{i-1}$, and from this and the fact that $m_i \geq m_{i-1}$, we could infer that we were in the monotone case. But now, since $V(P_i)$ and $V(P_{i+1})$ are differently distributed it will not be true, <u>in general</u>, that $c_{i+1} \geq c_i \Rightarrow \alpha_i \leq \alpha_{i-1}$. Still, it can be shown (see De Groot [15], section 13.7, lemma 3) that for costs of search

greater than a specific value (which depends on the precision of the mean distribution), we are in the monotone case. If this is so, the optimal rule will be to stop at stage N, where N is the first $i \geq 1$ such that $x_i \geq E(X_{i+1})$.[19] Following the same steps as is Chapter II section 2, it can be shown that the optimal rule says to stop at the first i at which $p_i < p_i^*$, i=1,2,..., where p_i^* is the unique solution of $g[V(p_i^*)] = c_{i+1}$. Now, taking into account the assumptions made in this chapter about the price distribution and the utility function, we can write an equation analogous to [11*] except for the fact that the precision of the utility distribution is r and the mean and precision of the utility mean distribution are μ_i and τ_i, (defined from μ_0 and τ_0 by the rules stated in [13] and [14]). This equation is

$$V(P_i^*) = \mu_i + \delta_i^{-\frac{1}{2}} \Psi^{-1}[\delta_i^{\frac{1}{2}} \cdot c_{i+1}] \qquad [25]$$

where $\delta_i = \dfrac{\tau_i \cdot r}{\tau_i + r}$.

Given [25] we can perform the same sort of "comparative analysis" as we did before. We will not spend any time in it, though, except to show that with recall, the principle of preference for certainty does not hold. To this end we do not need to assume that search costs are at each stage above the minimum that guarantees that we are dealing with the monotone case. In this broader context, we know that the optimal rule will be, at stage i, to continue searching if $x_i < E(X_{i+1})$. But, of course, if $x_i \geq E(X_{i+1})$ nothing can be said about the optimal decision.[20] It is clear therefore, that at any stage i, the expected utility of continuing the search will be at least as large as $E(X_{i+1})$ or, equivalently, that

$$V(p_i^*) \geq \mu_i + \delta_i^{-\frac{1}{2}} \Psi^{-1} [\delta_i^{-\frac{1}{2}} \cdot c_{i+1}] . \qquad [26]$$

Now we can establish the following proposition.

Proposition 5.1

Let A and B be two consumers who decide to search <u>with recall</u> the market, and have the same loglinear utility function and incur in the same search costs. But consumer A knows with certainty that $P \sim \Lambda(\mu^*, r^*)$ while consumer B believes that $P \sim \Lambda(\mathfrak{m}^*, r^*)$ and, not knowing with certainty $\mathfrak{m}^*$, believes that $\mathfrak{m}^* \sim \Lambda(\mu^*, \tau^*)$, where $\mu^* < \infty$, $0 < \tau^* < \infty$, $0 < r^* < \infty$. Let us call U^A consumer A's expected utility of following the optimal rule, and U^B consumer B's expected utility of following the optimal stopping rule. Then

$$U^A < U^B .$$

Proof

From $[11^*]$ $U^A = \mu + r^{-\frac{1}{2}} \Psi^{-1} [r^{\frac{1}{2}} \cdot c_1] .$

From $[26]$ $U^B \geq \mu + \delta^{-\frac{1}{2}} \Psi^{-1} [\delta^{\frac{1}{2}} \cdot c_1] .$

The fact that $\delta = \frac{\tau r}{\tau + r} < r$, and Ψ^{-1} is strictly decreasing, completes the proof.

This result is in sharp contrast with the one obtained when recall was not allowed. And yet both results seem to be plausible. We do not have to part company with our intuition to believe that the consumer will prefer to have a more accurate information unless he has a way to get insured against misconceptions. Such insurance does not exist when recall is not allowed. When a price is rejected the consumer cannot go back to it even if further observations convince him that he is very unlikely to find a price as good as the one not accepted. But, when recall is allowed, if the consumer becomes convinced that he was too optimistic when he rejected a price, he can always go back to the store that quoted it and buy at the previously rejected price. He is therefore protected against missing a good opportunity and he is more

willing to pay the cost of further search and bet for a better price. Note though that this does not mean that the consumer prefers to be kept in the dark, or, to put it another way, that the consumer is willing to pay for not receiving additional information. It has to be understood as indicating that the consumer always prefers to get information suggesting that he is wide off the mark to information pointing at the precision of his beliefs.

In the no recall case we talked of a consumer's preference for certainty. We might now talk of the consumer's preference for uncertainty, or better still, of the consumer's preference for insured uncertainty. It turns out, therefore, that, in order to be able to make predictions about the consumer's behavior when the price distribution is not known with certainty, it is crucial to distinguish between the two polar cases of recall and no recall. For example, from our results it follows that a better spread of information will reduce the maximum expected utility if recall is allowed. But a better spread of information in a market without recall will increase the maximum expected utility. One has to conclude, therefore, that any statement on optimal consumer behavior that fails to distinguish between those two cases should be looked upon with extreme scepticism, if the consumer is supposed to learn from the search. And since most practical cases fall between the two extremes, one should be very cautious when applying the results obtained in this chapter.

II. Conclusion

Observing the work done under the prism of our ultimate interest in describing the evolution of a market in which the agents have limited information about some crucial parameters, we have to stress two results, both obtained when there is no learning on the part of the consumer (see [7]). One indicates that as the price dispersion increases, the expected number of searches increases, and this, one might anticipate, will reduce the price spread through its impact on the price setters' decisions. The other result refers to the monotone evolution of the reservation prices as the search proceeds. Unfortunately, and this is quite important, the first result cannot be guaranteed, and the second can be shown to be false, when the consumer is not certain about the price distribution and learns about it as he searches (the adaptive case).

Not only do some results not carry over to the adaptive case, but the exact opposites are obtained when we move from the non-adaptive to the adaptive case. And so, for example, we observe how in the first case the consumer shows a preference for price dispersion, while in the second he shows a preference for price concentration. We cannot but conclude, therefore, that the non-adaptive case is not in general a good approximation to the adaptive one.

And then, to give a last gloomy touch to the bleak panorama, we are not even safe in the adaptive case. It appears that the distinction between the situations with and without recall is in itself crucial. For example, we observe that when recall is allowed the consumer has a preference for uncertainty, while his preference is for certainty when recall is not allowed. But then, one wonders how many real examples of clear cut cases of search with recall or,for that matter, without recall can one think of. No matter how vivid our imagination, we have to yield to the evidence that most real life instances are impure mixtures of the two cases, about which our results cannot shed much light! Caution, then, seems to be the password to enter the realms of the search problems.

Lemmata

Lemma 1

Let X be a random variable with a d.f. $F(x)$ for which the mean exists.
Then, for any s $\in(-\infty, \infty)$,

$$E[\max(X, s)] = s + \int_s^\infty (x - s) \, dF(x).$$

Proof

$$E[\max(X, s)] = s \int_{-\infty}^s dF(x) + \int_s^\infty x \, dF(x) = s[1 - \int_s^\infty dF(x)]$$

$$+ \int_s^\infty x \, dF(x) = s + \int_s^\infty (x - s) \, dF(x).$$

Lemma 2

Let F be a distribution function on the real line for which a mean exists.
Then $g(s) = \int_s^\infty (x - s) \, dF(x)$ $-\infty < s < \infty$, is a positive strictly decreasing
function of s, for any value of s such that $g(s) \neq 0$.

Proof

Using Leibniz' formula (see, e.g., Bartle [5] p. 307),

$$g'(s) = - \int_s^\infty dF(x) = -[F(x)]_s^\infty$$

which is negative as long as $\Pr(s \leq x < \infty) \neq 0$, that is, as long as
$g(s) \neq 0$.

Lemma 3

Let F be the d.f. of a random variable x which is normally distributed with

mean μ and precision τ.

Then $\int_{s}^{\infty} (x-s)\, dF(x) = \tau^{-\frac{1}{2}} \int_{\tau^{\frac{1}{2}}(s-\mu)}^{\infty} (z - \tau^{\frac{1}{2}}(s-\mu))\, \psi(z)\, dz$, where z

is a standard normal random variable and $\psi(z)$ is its p.d.f.

Proof

Applying the change of variable theorem for integral equations (see, e.g.,

Bartle [5] p.305) and the change of variable theorem for random variables

(see, e.g., Hoel [24], App. B) and calling $z = (x-\mu)\tau^{\frac{1}{2}}$, the result is immediate.

Lemma 4

Let $X_1, \ldots, X_n$ be a random sample from a normal distribution with an unknown

value of the mean $\mathcal{M}$ and a specified value of the precision $r(r > 0)$. Suppose

that the prior distribution of $\mathcal{M}$ is a normal distribution with mean μ and

precision τ such that $-\infty < \mu < \infty$ and $\tau > 0$. Then the posterior distribu-

tion of $\mathcal{M}$ when $X_i = x_i$ $(i=1,2,\ldots,n)$ is a normal distribution with mean μ'

and precision $\tau + n r$, where

$$\mu' = \frac{\tau\mu + n r \bar{x}}{\tau + n r}.$$

See DeGroot [15], p. 167.

Lemma 5

Suppose that $X_1, \ldots, X_n$ is a random sample from a normal distribution with

an unknown value of the mean $\mathcal{M}$ and an unknown value of the precision R.

Suppose also that the prior joint distribution of $\mathcal{M}$ and R is as follows:

The conditional distribution of $\mathcal{M}$ when $R = r(r > 0)$ is a normal distribution

with mean μ and precision τr such that $-\infty < \mu < \infty$ and $\tau > 0$, and the

marginal distribution of R is a gamma distribution with parameters α and β such that $\alpha > 0$ and $\beta > 0$. Then the posterior joint distribution of $\mathfrak{M}$ and R when $X_i = x_i$ (i=1,...,n) is as follows: The conditional distribution of M when R = r is a normal distribution with mean μ' and precision $(\tau + n)r$, where

$$\mu' = \frac{\tau\mu + n\bar{x}}{\tau + n}$$

and the marginal distribution of R is a gamma distribution with parameters $\alpha + (n/2)$ and β', where

$$\beta' = \beta + \tfrac{1}{2} \sum_{i=1}^{n} (x_i - \bar{x})^2 + \frac{\tau n(\bar{x} - \mu)^2}{2(\tau + n)} .$$

See DeGroot [15] p. 169.

Lemma 6

Let X and Y be two random variables. Suppose that the d.f. of $(X|Y=y)$ for $-\infty < y < \infty$ is normal with mean y and precision τ_1 and the marginal distribution of Y is normal with mean μ and precision τ_2 . Then the marginal distribution of X is normal with mean μ and precision $\dfrac{\tau_1 \tau_2}{\tau_1 + \tau_2}$.

Proof

Call $\sigma_1^2 = 1/\tau_1$ and $\sigma_2^2 = 1/\tau_2$.

$$E(X) = \int_{-\infty}^{\infty} E(X|y)\, dF_Y(y) = \int_{-\infty}^{\infty} y\, dF_Y(y) = \mu$$

On the other hand,

Var X = E(Var $(X|Y)$ + var$(E(X|Y))$ (See, e.g., Lindgren [27] p. 119).

Therefore,

$$\text{Var } X = \sigma_1^2 + \sigma_2^2 \quad \text{and} \quad \text{Pre } X = \frac{\tau_1 \tau_2}{\tau_1 + \tau_2} .$$

To show that X is normal the appropriate integrals have to be computed.

Lemma 7

$$\int_{-s}^{\infty} (x+s) \, h(x) \, dx = s + \int_{s}^{\infty} (x-s) \, h(x) \, dx \text{ for any } -\infty < s < \infty,$$

where $h(\cdot)$ is the p.d.f. of a standard normal distribution.

Proof

$$\int_{-s}^{\infty} (x+s) \, h(x) \, dx = \int_{-s}^{\infty} x \, h(x) \, dx + \int_{-s}^{\infty} s \, h(x) \, dx \, .$$

The first term of the sum is equal to

$$\int_{-s}^{s} x \, h(x) \, dx + \int_{s}^{\infty} x \, h(x) \, dx = \int_{s}^{\infty} x \, h(x) \, dx \, .$$

The second term of the sum is equal to

$$s \int_{-s}^{\infty} h(x) \, dx = s \int_{-\infty}^{s} h(x) \, dx = s[1 - \int_{s}^{\infty} h(x) \, dx] = s - \int_{s}^{\infty} s \, h(x) \, dx \, .$$

Lemma 8

$E[M(Y_{i+1}, 0, \tau_{i+1})] = \gamma_i + c_{i+1}$, $i=0,1,\dots$, is a continuous, strictly increasing function of τ_{i+1}, $(\tau_{i+1} > 0)$.

Furthermore

$$\lim_{\tau_{i+1} \to \infty} \gamma_i = \hat{\gamma}_i \, .$$

See DeGroot [15], p. 339 for an outline of the proof. Note that if the cost of search is constant, since $\tau_{i+1} > \tau_i$, then $\gamma_{i+1} > \gamma_i$.

Footnotes

[1] Recall that $\alpha = \exp(\eta^* + \frac{1}{2r^*})$. β, the precision of the price

distribution, is $\beta = \frac{1}{\alpha^2 \eta^2}$, where $\eta^2 = \exp(1/r^*) - 1$. See Aitchison and

Brown [1]. Note that the certain knowledge of r^* does not imply the certain knowledge of β, the precision of the price distribution.

[2] Data provided by Professor Scott Maynes. I should point out that the statement in the text should be made with some caution since the price data have not yet been, as Professor Scott Maynes made me observe, carefully scrutinized. In any case, it is interesting to point out that they seem to indicate that the spread between the minimum and maximum price observed is much larger than suggested by Telser [46]. Let me add finally that a log-normal fit seems adequate for the prices of life insurance policies as quoted by Belth [6] in Tables 10 and 13.

[3] Assuming that prices are distributed lognormally, we are accepting that prices can conceivably be very close to zero. This is something that most empirical data do not seem to grant. Therefore, one would prefer to constraint the prices to be above some threshold $\xi > 0$. This being so, a "three parameter distribution" might be assumed where not the random variable p itself, but the random variable $p' = p - \xi$, is lognormally distributed. In a similar vein, it would appear to be more realistic to constraint the prices between two bounds $\xi < p < \lambda$, $\xi > 0$, $0 < \lambda < \infty$. In that case a "four parameter distribution" could be conveniently assumed, where the random variable $p'' = p - \xi / \lambda - p$ would be distributed lognormally. (On these matters see Aitchison and Brown [1].) Unfortunately by assuming that p' or p'', is lognormally distributed many disturbing difficulties would arise, since $V(p)$ would not have an easily tractable distribution.

[4] By a former MIT student.

[5] This term has been used by Box and Tiao [8].

[6] The unboundedness assumption is even more serious from purely logical grounds. This is so since the expected utility hypothesis can be made only if the utility function is bounded (see Arrow [4]). To bypass this difficulty we can suppose that for the price of the i-th good there exists an $\epsilon_i > 0$ such that $\Pr(P_i < \epsilon_i) = 0$, $i=1,2,\ldots m$. But even simpler is to rule out St. Petersburg cases.

[7] DeGroot [15], pp. 337-38, gives the following proof. (The notation has been changed to suit our needs.) From [13], [15] and [21] it follows that

$$M(z_i, 0, \tau_i) = \max[z_i, E(M(Z_{i+1}, \frac{rZ_{i+1}}{\tau_i + r}, \tau_{i+1})) - c_{i+1}]$$

$$= \max[z_i, E(M(\frac{\tau_i Z_{i+1}}{\tau_i + r}, 0, \tau_{i+1}) - \frac{rZ_{i+1}}{\tau_i + r}) - c_{i+1}]$$

and equation [22] follows.

[8] This interpretation is taken from DeGroot [15], section 13.6.

[9] We have no reason to call this property "stabilizing." And yet we might presume that any convincing description of the market should yield that increased search reduces the price dispersion.

[10] On the so called "reservation price property" and its relation with a downward sloping demand curve see Rothschild [40].

[11] Rothschild [41] has to make the assumption, and he shows his dissatisfaction, that the seller cannot be certain of the effect of a change in the price charged on the probability of a sale. But in the light of our results this assumption is justified.

[12] For a different conclusion see Rothschild [40].

[13] Or, using Rothschild [40] terminology, we can say that the "subjective uncertainty" is related to the "objective uncertainty" in a clearly determined way. The smaller is the objective uncertainty, the greater is the decrease in the "subjective uncertainty" as the search proceeds.

[14] Let the price mean for consumer A be $\alpha_A \sim \Lambda(\mu_A^*, \tau_A^*)$ and for consumer B be $\alpha_B \sim \Lambda(\mu_B^*, \tau_B^*)$. $E(\alpha_A) = \exp\{\mu_A^* + \frac{1}{2\tau_A^*}\}$ and $E(\alpha_B) = \exp\{\mu_B^* + \frac{1}{2\tau_B^*}\}$.

We assume that both consumers believe that the price mean distribution has the same precision:

$$Pre(\alpha_A) = Pre(\alpha_B) = \frac{\exp\{-2\mu_A^* - 1/\tau_A^*\}}{\exp\{1/\tau_A^*\} - 1} = \frac{\exp\{-2\mu_B^* - 1/\tau_B^*\}}{\exp\{1/\tau_B^*\} - 1}$$

$$= \frac{1}{E(\alpha_A)} \cdot \frac{1}{\exp\{1/\tau_A^*\} - 1} = \frac{1}{E(\alpha_B)} \cdot \frac{1}{\exp\{1/\tau_B^*\} - 1} .$$

Now,

$$E(\alpha_A) > E(\alpha_B) \Rightarrow \frac{1}{\exp\{1/\tau_A^*\} - 1} > \frac{1}{\exp\{1/\tau_B^*\} - 1} \Rightarrow \tau_A^* > \tau_B^*.$$

Since $\tau_i = \tau_i^* / b^2$, (i=A,B), $\tau_A > \tau_B$.

On the other hand $\tau_A^* > \tau_B^* \Rightarrow \mu_A^* > \mu_B^*$,

since $E(\alpha_A) > E(\alpha_B)$, but since $\mu_i = a - b(\mu_i^* - \frac{1}{2r^*})$ (i=A,B),

$\mu_A < \mu_B$.

Note that a result analogous to the one obtained in Chapter III would hold if the price and price mean distributions were normal and the utility function linear on the price (see [7]).

[15] It is not clear whether a decrease in the degree of uncertainty about the price mean leads, when the _price_ mean does not change, to an increase in the expected utility of the optimal search.

[16] This result and the following should be contrasted with the proposition obtained below where, allowing the consumer the recall privilege, he shows a preference for uncertainty.

[17] This is so since the expected utility, when information is complete, is precisely the limit of the expected utility, with uncertainty of the price distribution, when this uncertainty is reduced (see lemma 8). Note that it is not always true, as one might be inclined to think, that a higher degree of uncertainty implies that information has a higher value. On this matter see Gould [22].

[18] Diamond [17] points out the possibility of this behaviour, although he observes that it is more likely in the early stages of search. But we find precisely the opposite. A new side of the paradox?

[19] See Chapter III, section 2.

[20] See note 28.

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