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Recent articles[1] on common-property natural resource management have generally illustrated optimal exploitation paths, leading to positive stock steady-states or extinction, in phase space using the derived resource stock "price" or dual variables as the dependent variable on the vertical axis.

In this article the optimal path for a simple model will be illustrated in more traditional diagrams using the geometric procedures developed by Liviatan (1970) and discrete maximization procedures[2] and compared with a golden rule maximum sustained yield path.

An analysis will be presented which can be represented on a standard two period diagram and which is consistent with the multiperiod dynamic analysis.

Following this, various steady state solutions will be represented, in particular a Maximum Sustainable Yield state and an Optimal state. A pure consumption model will be used which, while containing most of the essential properties of the problem, avoids a much more involved analysis.

The Growth Model

Following Clark (1973, pp. 952-53) define N_k as the breeding population of resource at the beginning of the k^{th} breeding season, C_k as the consumption or harvest of that season and $f(\cdot)$ as the biological growth rate of the breeding stock.

Then clearly

$$(1) \qquad N_{k+1} = f(N_k - C_k) \qquad 0 \le C_k \le N_k$$

$$= f(S_k) \qquad S_k \geqq 0$$

where S_k represents the breeding stock (after harvesting).[3]

The biological growth model with human predation can then be illustrated as in Figure 1 below.[4]

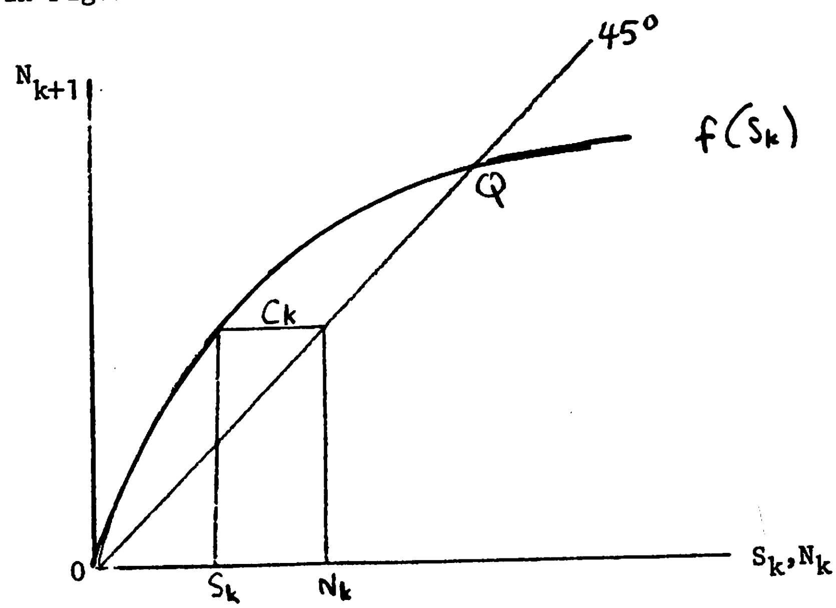

FIGURE 1

The graph of $f(S_k)$ intersects the 45° line at the origin and at Q .

A Two Period Analysis

Suppose the stock initially has a value N_o, then consumption C_o is to be chosen, and the resulting stock N_1 will result from breeding according to (1)

$\therefore$ (2) $$N_1 = f(N_o - C_o)$$

Obviously C_o must be chosen to be less than N_o, the existing stock, if extinction is to be avoided.

Assume $f'(S_o)$ is always positive.[5] This is the (usually made) assumption of positive net reproduction.

The effect of present consumption may be to expand, reduce or leave the resource stock intact.

Equation (2) represents a trade off between present consumption and resource stock growth. It is easily shown that the graph of this trade off represented by $A_o B_o$ in Figure 2 is concave to the origin[6] because of the concavity of f(S). For high levels of present consumption, one unit of consumption curtailment will lead to greater expansion in resource stock than will one unit of curtailment at low levels of C_o.

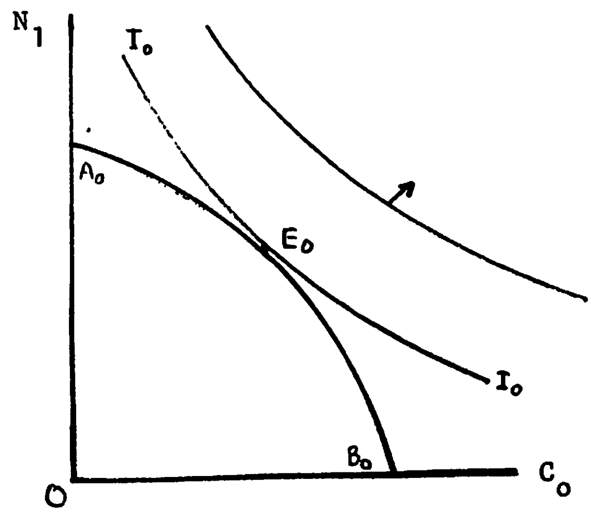

FIGURE 2

Consider next the problem of a planner choosing among points on $A_o B_o$.

Intertemporal Optimization

Assume a discrete objective function: $U = \sum_{t=o}^{\infty} a^t U(C_t)$ where 'a' is a constant social discount factor, $0 < a < 1$ and it is assumed that $U' > 0$, $U'' < 0$ for $C_t < 0$ and $\lim_{C_t \to o} U'(C_t) = \infty$.

The related maximization problem is to maximize U with respect to C_o, $C_1, \ldots,$ subject to $N_{t+1} = f(N_t - C_t)$ and $N_{(o)} = N_o$.

Using the Kuhn-Tucker-Lagrange conditions[7] one can derive the following necessary conditions for a maximum:

$$(3) \qquad \frac{U'(C_t)}{aU'(C_{t+1})} = f'(N_t - C_t), \qquad t=0,1,\ldots,$$

which is the usual equality (intertemporally) between marginal rates of substitution and the slope of the transformation curve at each time. This equality will be represented graphically.

The choice that must be made is between the utility of present consumption $U(C_o)$, and the 'value' in (discounted marginal utility terms) of the extra future stock resulting from consumption curtailment. This last value can be expressed as

$$(4) \qquad aU'(C_1)\cdot(N_1 - N_o)$$

Hence we may write as an "indifference curve" the expression

$$(5) \qquad U(C_o) + aU'(C_1)(N_1 - N_o) = \text{constant}$$

This results in a social rate of substitution[8] between consumption today and resource stock tomorrow as given by (3).

In Figure 2 the point E_o of tangency between $A_o B_o$ and $I_o I_o$ gives optimal values C_o^* and N_1^*.

We note in passing that if the above maximizing problem has a steady state denoted by $C_t = \bar{C}$, $N_t = \bar{N}$ then from (3).

$$(6) \qquad f'(\bar{N} - \bar{C}) = \frac{1}{a} = f'(\bar{S}).$$[9]

Further comments about Figure 2 and the related analysis can be made.

The maximizing problem produces an iterative or recursive solution,[10] and so can be expressed as a sequence of two period problems and represented geometrically.

The state variable of the problem is the stock of resource. And so once N_o is given the solution is derived iteratively. In particular $\bar{C}_o$

is obtained as the optimal value of C_o and $\bar{N}_1 = f(N_o - \bar{C}_o)$. It is readily shown[11] that

(7) $$0 < \frac{d\bar{C}_o}{dN_o} < 1, \text{ and}$$

(8) $$\frac{d\bar{N}_1}{dN_o} < 0.$$

Equation (7) expresses the fact that if somehow today's resource stock is increased, so also will be the optimal value of current consumption, but the increase in $\bar{C}_o$ will never be as large as the increase in N_o. It will always be optimal to devote some of the increase in N_o to 'breeding.' This point is made in (8) as well.

Market Failure

At point E_o the central planner has managed to capture in term (4) an important consideration that the market for this unappropriated resource will not. That is, the incorporation of the value to society of the resource stock in the future.

It is generally concluded in the resource literature that the present generation will be myopic in the sense that even if they perceive the fact that high present consumption will mean less future consumption they will nonetheless overconsume today because they are not able individually to do anything about it. That is, because the resource is common-property, and costless to harvest, it will not benefit anyone to curtail present consumption so someone else can consume more in the future.

Steady States

It is noticed that Figure 2 represents a two period analysis with given N_o and which determines N_1^*. This value N_1^* is the starting point of another two period analysis at time t_1. The curve $A_o B_o$ will shift to a new location A_1B_1 and the social optimum E_1 will be on a different 'indifference' curve. Over time we will have a Resource Stock Consumption Curve OE_1E_2 ... as illustrated in Figure 3.

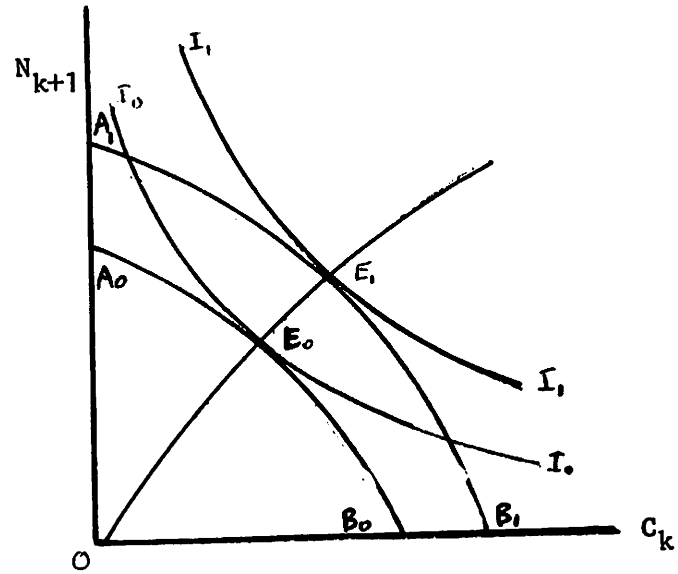

FIGURE 3

By equations (7) and (8) this curve OE_1E_2 ... is seen to be upward sloping.

A steady state will be represented by a point on the Resource Stock Consumption Curve E_∞ such that $N_{k+1} = N_k = \bar{N}$ and $C_{k+1} = C_k = \bar{C}$. The resulting (short run) transformation curve $A_\infty B_\infty$ will be stationary over time.

To find the point E_∞ let us refer back to Figure 1. In Figure 1, the horizontal displacement between $f(S_k)$ and the 45° line represents consumption level C_k. This follows from careful consideration of equation (1) along with Figure 1. And so the <u>biological</u> relationship between C_k and N_{k+1} can be represented as in Figure 4 by curve OM.

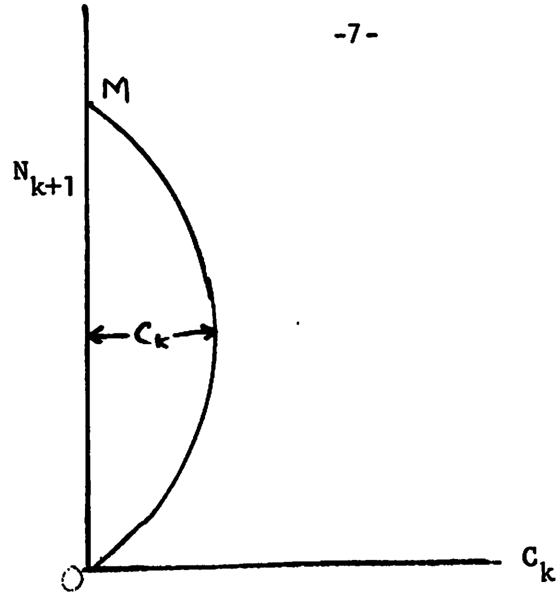

FIGURE 4

Now the intersection of OE_1E_2 ... and curve OM will be E_∞ , the point of biological and economic equilibrium. This is represented in Figure 5.

The point E_∞ represents an optimal steady state derived iteratively through a sequence of two stage maximizations--each picking an E_i.

It is useful to compare E_∞ with point G, where the tangent at OM is vertical. N^*, the steady state stock associated with G exceeds $\bar{N}$.[12] N^* is referred to as the Maximum Sustained Yield Resource stock and it has been shown to be larger than optimal. This is due to discounting. If the discount parameter 'a' were unity E_∞ would coincide with G.

It is of general interest in a more elaborate model whether maximum yield programs are optimal. It is shown[13] that they generally are not. This is due not only to the presence of discounting but also to production costs and the stock externality.

Concluding Comments

This paper has illustrated optimal resource exploitation in standard diagrams eliminating the use of the artificial dual price variable in representation. It has not dealt specifically with the dynamic path although this can be visualized in Figure 3. Notice that the model will allow for

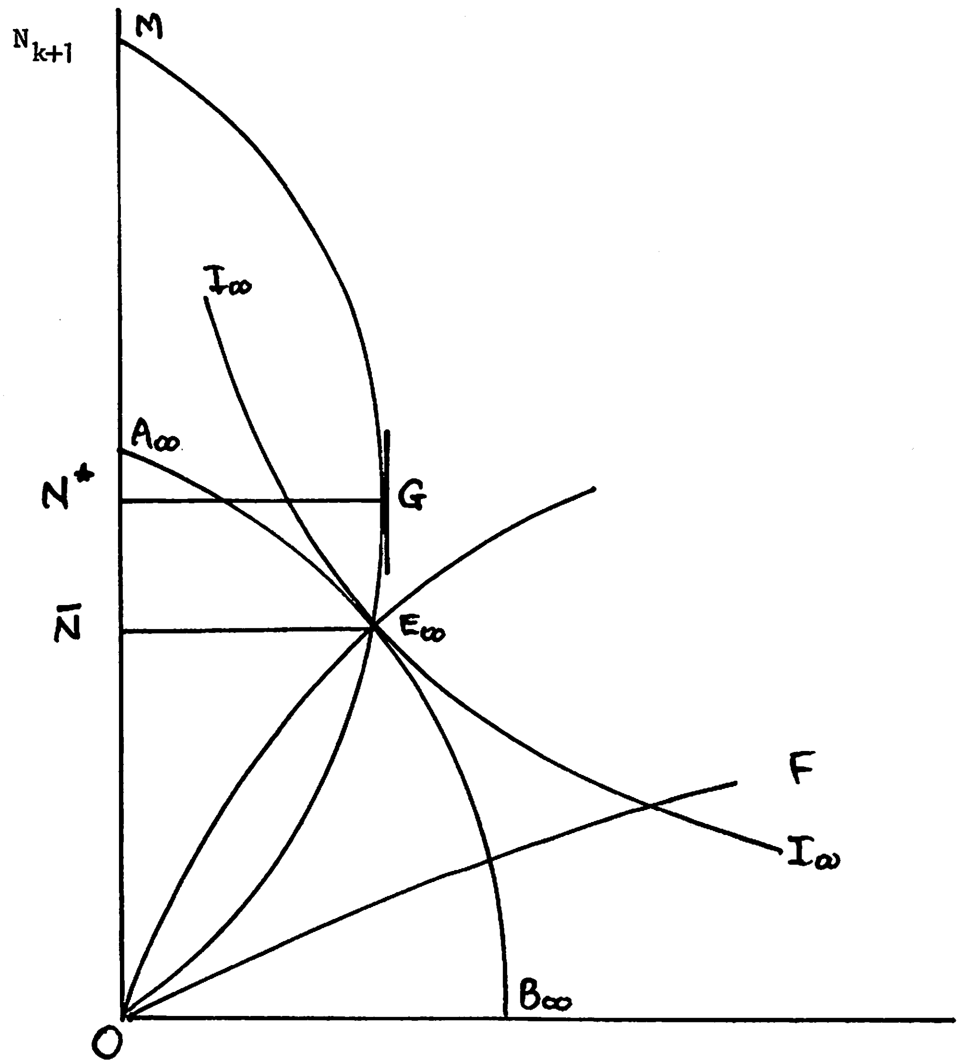

FIGURE 5

extinction if for example the Resource Stock Consumption Curve is as illustrated by OF in Figure 5.

No mention is made of markets, but one could easily establish fishing quotas to lead the economy optimally from any initial state N_o, to the terminal steady state $\bar{N}$. These quotas would be the values C_k^* from Figure 3.

Footnotes

[1]Examples: Smith (1968, 1974), Plourde (1970), Quirk and Smith (1969), Vousden (1973), Neher (1974).

[2]See Benavie (1970).

[3]A quadratic $f(S_k)$ can be used. That is, where explicit representation is convenient it will be specified that

$$(1^*) \qquad f(S_k) = (1+\lambda)S_k - \varepsilon S_k^2 \text{ where}$$

λ, ε represent positive parameters provided as data from the ecological system. For details concerning (1*) see Gause (1934), Lotka (1956), Volterra (1931). The assumption is that for the unexploited resource $\dot{N} = \lambda N - \varepsilon N^2$ as in Plourde (1970).

[4]This is Figure 1 of Clark (1973) and Figure 1 of Liviatan (1970). For the specification of $f(S_k)$ given in (1*) above Q represents the value $S = \lambda/\varepsilon$. Here $f'(o) = (1+\lambda)$, $f'(\lambda/\varepsilon) = (1-\lambda)$ which is assumed to be positive.

[5]As above $f'(S_o)$ will be contained in the interval $(1-\lambda, 1+\lambda)$ for specification (1*).

[6]$$\frac{dN_1}{dC_o} = -f'(S_o) < 0 \quad \text{and} \quad \frac{d^2N_1}{dC_o^2} = f''(S_o) < 0 .$$

[7]Or see Benavie (1970, p. 427).

[8]Implicit differentiation of (5) would also give this result, as

$$MRS = -\frac{dN_1}{dC_o} = \frac{U'(C_o)}{aU'(C_1)} .$$

[9]It will be useful later to compare steady state values of S using equation (1*). Call the steady state associated with equation (3) $\bar{S}$ and the maximum sustained yield steady-state S^*. Since $\frac{1}{a} > 1$ and $f'(S)$ changes monotonically from $(1+\lambda)$ to $(1-\lambda)$ over the interval $S \varepsilon [0, \frac{\lambda}{\varepsilon}]$ then $\bar{S}$ will be to the left of S^*. That is, the optimal steady state value of the breeding stock is less than the MSY value. This is consistent with the results in Plourde (1970). Note that if $1/a > 1-\lambda$ the resource is left unharvested in the steady state.

[10]See for example Burt and Cummings (1970).

[11]See Liviatan (1970, pp. 305-307) for an analysis.

$$\frac{d\bar{N}_1}{dN_o} = f' > 0$$

[12]At E_∞ $f'(\bar{S}) = \frac{1}{a} > 1$ where $f'(S^*) = 1$ at G. For the specification given in equation (1*), fn. 1 $\frac{dC(N)}{dN} = 1 - \frac{1}{f'(S)} = 0$ for $f(S^*) = 1$

$\therefore$ for $f(S) = (1+\lambda)S - \mathcal{E}S^2$ $\quad S^* = \frac{\lambda}{2\mathcal{E}}$.

[13]See Plourde (1971), Quirk and Smith (1969), Brown (1974), and Neher (1974).

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