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I. INTRODUCTION

This paper analyzes the role of patents as information transfer mechanisms. One of the "stylized facts" that comes to us from the traditional Industrial Organization literature is that there is considerable variation across industries in the fraction of new products or processes patented by firms. Scherer (1983, 1967, 1965) suggests that cross-section studies of industry patenting activity reveal a clear difference in the "propensity to patent" new products or processes. He suggests, further, that this difference arises from differences across industries in the ability of the patent to appropriate rents for the patent holder.[1] Others (see for instance Machlup (1962) and Kahn (1962)) suggest that firms may tend not to patent new processes for fear that the patent may reveal useful information to competitors. Thus, industries in which process innovation predominates may tend to display a lower propensity to patent relative to other industries.

Surprisingly, traditional models of firm patenting activity would not predict this sort of behavior. Indeed, standard models would predict that firms should patent all new products or processes.[2] The explanation for this result lies in the joint assumptions that there is complete and perfect information and that there exists no viable competing product once a patent is obtained. Under these circumstances, it is clearly in the firm's interest to always obtain a patent. Equally clear, however, is the fact that it is the failure of such circumstances to exist which underlies the empirical evidence noted above.

In this paper, we present a model of firm patenting behavior under circumstances in which there exists incomplete information and a potentially viable competing product for the patented good. We show that under such conditions firms will display, in equilibrium, a propensity to patent which is directly dependent upon a number of industry specific variables. This result is due both to the existence of the competing good and the fact that the patent system plays a role in transferring information to potential competing firms. In this regard, we obtain a second, more interesting result concerning the exact nature of this information transfer mechanism. In particular, we show that in addition to the obvious ways in which the patent system might transfer information, it also serves a signalling function which is often overlooked in the informal discussions of patenting. Specifically, we show that even if competitors are immediately aware of a particular innovation, and the patent application provides no information at all about the nature of the innovation, the patenting behavior of the innovator can be used to signal information concerning the profitability of entry to potential competing firms.

The model itself is fairly simple and has its roots in some of the auction literature as well as in a model by Crawford and Sobel (1982). It is assumed that as part of the by-product of research and development, an innovator obtains private information concerning the profitability of the product itself or an imitation of the product which would not infringe upon patent rights. It is assumed costly for the competitor to obtain this information prior to actual production.[3] The case given most attention in the paper is one in which the private information takes the form of the innovator knowing the costs to the competitor of producing and marketing either a duplicate of the innovation or an imitation. One might think of this as depicting a situation in which the market for the innovation is fairly well-established but the innovation embodies a new technology (for instance xerography, synthetic rubber, artificial fibers, natural insulin, etc.). One could alternatively consider a case in which costs are common knowledge but the innovator possesses private information about revenues. This might be a situation in which the technology to produce the innovation is well-established but it is simply employed in a novel way (as with pocket calculators, home video games, etc.). This case is considered briefly, as well, in a concluding section.

In either case, by patenting the innovator can preclude duplication but not imitation. Based upon his information concerning the profitability of the product (and an imitation) and his understanding of the ways in which the patent system might transfer this information to his competitor, the innovator chooses a patenting behavior. The competitor, given any direct information which the patent system may convey, and realizing that the innovator's patenting decisions constitute optimal behavior, makes certain inferences concerning the profitability of the various options open to him. In the simplest of terms, the fact that the innovator obtains a patent should lead the competitor to infer something about the profitability of duplication relative to imitation. Based upon this information the competitor makes his decision to imitate, duplicate (if possible) or not participate.

The specifics of this situation are laid out in Section II. In Section III it is shown that optimal behavior for the innovator involves always patenting when imitation is relatively expensive and doing so occasionally when imitation is cheaper. Facing this strategy, reasonable assumptions yield the competitor's response as abstention when confronted with a patent and imitation otherwise. This behavior then generates a propensity to patent. Section IV derives the model's predictions about propensity to patent. Perhaps most interesting is that any change which makes imitation in the face of a patent more attractive actually reduces the equilibrium propensity to patent. The cause of this rather striking finding is found in the patent's role as an information transfer mechanism. Section V presents various extensions of the model, and some related issues. A phenomenon akin to "trade secrecy" (a lower propensity to patent) emerges as equilibrium behavior when patents reveal information which renders imitation less costly. Section VI summarizes.

II. THE MODEL

In this section the structure of the basic model studied in this paper is presented. An informal discussion is offered first.

There are two firms, referred to as the innovator and the competitor. The innovator has developed, and plans to produce and sell, a new product: the innovation.[4] To focus on the role of patents in information transfer apart from revelation of product design, etc., it is assumed that the characteristics of the innovation relevant to consumers are both common knowledge and exogenous.[5] The competitor can either produce the innovation (duplication) or a differentiated commodity: the <u>imitation</u>.[6] The consumption characteristics of the imitation are also known and fixed. Producing nothing (abstention) is also a possibility.

The innovator can patent the innovation. Having done so, the competitor's options involve imitating or not participating.

It is assumed that the innovator possesses an advantage in that, having made his discovery, he knows the costs of research and development, marketing and plant and equipment required for the competitor to produce either a duplicate or an imitation of the innovation. In particular, the innovator knows whether these costs are such that, in equilibrium, the competitor could make a profit at either of these activities. For simplicity, it is assumed that these costs represent exogenous fixed costs required at the outset to begin production; and that they are such that, in equilibrium, the competitor's profit from some activity are either strictly positive or strictly negative. Variable costs are assumed known by all and constant on a per unit basis.

One way to think about this information assumption is that the innovator is simply the winner in a particular patent race. A by-product of the successful research effort is information concerning the technology and associated costs required to develop and bring to market the new product or the specified imitation. This information is not available to the competitor who has either not been involved in the race or who was in the race but failed to develop a viable product.[7] In short, we simply assume that the product of research is not only a new product but specific information about the product as well.

Given this framework, the sequence of activities in the model proceeds as follows. The innovator, based upon a chosen patenting rule and his private information concerning the profitability (or unprofitability) of competing products decides in any situation whether or not to patent. Having observed the patenting decision, the competitor makes certain inferences concerning the profitability of entry and, based upon a chosen entry rule, settles on whether to duplicate (if there is no patent), imitate, or abstain from participating. Given the modes of operation chosen by the innovator and competitor, production occurs and the goods market operates according to a textbook quantity--choosing duopoly when both firms operate. When the competitor declines to participate, the innovator becomes a monopolist and proceeds as usual. Equilibrium is then determined such that, given both firms utilize all the information available to them, the innovator's patenting rule maximizes his profits given the entry rule of the competitor and vice versa.

The key aspect of this problem is the information which the patent system conveys. The innovator can, if he chooses, patent all the time and prevent duplication. However, the competitor, utilizing all the information available to him and understanding that the innovator's patenting policy constitutes optimal behavior, will make certain inferences about the innovator's private information given this (or, in fact, any) patenting policy. Thus, depending on the inferences the competitor makes, it may not be in the innovator's interest to patent all the time. This could occur, for instance, if the innovator choosing the rule "always patent" led the competitor to infer that imitation was on average, profitable and so chose the rule "always imitate" while choosing a different rule led the competitor to infer that duplication was the profitable strategy and so the competitor chose to abstain if confronted with a patent.[8] The problem thus faced by the innovator is how much and what kind of information to convey to the competitor by his patenting policy. The interesting strategic element is that the competitor understands the innovator's motives and vice versa.

The formal model of this problem is comprised of several elements. First, the options available to each agent must be specified.[9]

The innovator can choose either to patent or not.[10] Let

$$P = \begin{cases} 1 & \text{if the innovator chooses to patent,} \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

p denotes a particular value of P.

The competitor can abstain, imitate or duplicate. Accordingly, describe the competitor's behavior by

$$B = \begin{cases} a & \text{if abstention is chosen} \\ i & \text{if imitation is chosen} \\ d & \text{if duplication is chosen} \end{cases} \tag{2}$$

Letting b represent a particular value of B, $B = b$ is decided upon after observing $P = p$.

The net returns to each of these actions depend on, among other things, the fixed costs. Let $C = (C^I, C^D)$ be a vector valued random variable where $C^I(C^D)$ is the fixed cost associated with imitation (duplication); c will denote a realization of C. The innovator, as part of the costs of R+D pays a sunk cost F as a result of which he obtains the actual values of both C^D and C^I, prior to deciding on $P = p$.[11] The competitor must pay $C^I(C^D)$ if he chooses $B = i$ $(B = d)$. However, he must make his decision without knowing C, observing only the innovator's choice $P = p$.

Prior to the development of the innovation C is not known with certainty by anyone, although the (exogenous) probability distribution of C is common knowledge. It is assumed that imitation (duplication) is either cheap or costly: $C^I = \underline{c}^I$ or $\bar{c}^I$, $\underline{c}^I < \bar{c}^I$ $(C^D = \underline{c}^D$ or $\bar{c}^D$, $\underline{c}^D < \bar{c}^D)$. The probabilities of these events are as follows[12]

$$\begin{aligned} \alpha^L &\equiv \Pr[C = (\underline{c}^I, \underline{c}^D)], \\ \alpha^D &\equiv \Pr[C = (\bar{c}^I, \underline{c}^D)], \\ \alpha^I &\equiv \Pr[C = (\underline{c}^I, \bar{c}^D)], \\ \text{and} \quad \alpha^H &\equiv \Pr[C = (\bar{c}^I, \bar{c}^D)]. \end{aligned} \tag{3}$$

Much algebra is saved and the essential character of the model retained if $\alpha^H = 0$ is specified. Whether this is a sensible restriction is examined in Section V(c).

The rest of the return side is obtained as follows. If the competitor chooses $B = a$, the innovator earns monopoly revenues (net of variable cost) $R^M > 0$, the competitor receiving 0. If imitation (duplication) is the choice, both agents obtain $R^I > 0$ ($R^D > 0$).[13] R^M, R^I and R^D are known from the monopoly or duopoly game following the choice of $P = p$ and $B = b$. It is assumed that

$$R^M > R^I > R^D.^{14} \qquad (4)$$

(4) is of course not the outcome of the game for every possible specification of demand, but is both intuitive and clearly the leading case.

Realized profits depend on both returns and fixed costs. For example, the competitor's profit, having chosen imitation, if imitation is inexpensive is $\pi^I = R^I - \underline{c}^I$. In general, profits are random variables denoted by π^M, π^I and π^D. Where it is necessary to distinguish between the profits of the innovator and competitor, the innovator's (competitor's) profits will be subscripted with a 1 (2). In keeping with (4), it is assumed that when both duplication and imitation are relatively less costly, both are profitable but imitation is the more profitable of the two

$$R^I - \underline{c}^I > R^D - \underline{c}^D > 0. \qquad (5)$$

Furthermore, when both are relatively costly, both are unprofitable. (It is not necessary to assume imitation is still more profitable.)[15]

$$R^I - \bar{c}^I < 0 \tag{6}$$

and

$$R^D - \bar{c}^D < 0. \tag{7}$$

Next, in order for this problem to be of interest at all, it must be the case that even if the patent did not reveal any information, the competitor would expect it to be profitable to duplicate the product. This is simply the analogue to the standard notion that, without a patent, competitors would enter with a duplicate product and appropriate the innovator's profits. Using $E^{\alpha}(\cdot)$ to denote expectations computed using the unconditional probabilities α^L, α^D and α^I,

$$E^{\alpha}(\pi^D) = (\alpha^L + \alpha^D)(R^D - \underline{c}^D) + \alpha^I(R^D - \bar{c}^D) \geq 0.$$

Actually, a somewhat stronger assumption is made: duplication would be profitable on average even if it were known that $C = (\underline{c}^I, \underline{c}^D)$ has not occurred. That is

$$E^{\alpha}[\pi^D | C \neq (\underline{c}^I, \underline{c}^D)] \doteq \alpha^D(R^D - \underline{c}^D) + \alpha^I(R^D - \bar{c}^D) \geq 0. \tag{8}$$

Along the same lines, it is assumed that the unconditional expected profit associated with imitation is positive:

$$E^{\alpha}(\pi^I) = (\alpha^L + \alpha^I)(R^I - \underline{c}^I) + \alpha^D(R^I - \bar{c}^I) > 0. \tag{9}$$

The impact of both (8) and (9) is to allow the analysis to focus on the most interesting equilibrium, and to avoid the taxonomy of equilibria usually encountered in games of incomplete information. Specifically, (8) rules out a trivially different equilibrium, the nature of which is briefly discussed in Section V. (9) eliminates the case where the innovator has such a strong advantage that he can invariably induce abstention by the competitor, and so always obtain the monopoly profit. Just how this is achieved in the absence of (9) is left to Section V.

Before the problems faced by the agents can be specified, a small amount of additional notation is required.

Optimal behavior on the part of the innovator may involve randomization; that is, for some realizations $C=c$ the innovator will patent with some probability less than unity. The function describing this relationship is called the patenting rule, denoted $\Delta(c)$. Specifically

$$\Delta(c) = \Pr[P=1|C=c].$$

It is convenient to define

$$\delta^L = \Delta[(\underline{c}^I, \underline{c}^D)],$$
$$\delta^D = \Delta[(\bar{c}^I, \underline{c}^D)],$$

and

$$\delta^I = \Delta[(\underline{c}^I, \bar{c}^D)].$$

Next, to say that the competitor understands the innovator's motives comes to stating that while the competitor does not know $C=c$, he is aware of the relationship $\Delta(c)$. He can thus use this, in conjunction with $P=p$, to update his views about how likely the various cost configurations are. This conditional probability distribution is[16]

$$\Phi^\Delta(c|p) \equiv \Pr[C=c|P=p, \Delta(c)].$$

Furthermore, let

$$\phi^L(p) = \Phi[(\underline{c}^I, \underline{c}^D)\,|\,p],$$

$$\phi^D(p) = \Phi[(\bar{c}^I, \underline{c}^D)\,|\,p],$$

and

$$\phi^I(p) = \Phi[(\underline{c}^I, \bar{c}^D)\,|\,p].$$

Exceptions computed utilizing Φ will be expressed by $E^{\Phi}(\cdot)$.

Finally, like the innovator, it may be optimal for the competitor to randomize. The function describing the relationship between the probability of the various choices open to the competitor and the observation $P = p$ is called the <u>response function</u>:

$$\Psi^{\Delta}(b\,|\,p) \equiv \Pr[B = b\,|\,P = p,\ \Delta(c)].$$

In addition, it is notationally helpful to let $\psi^b(p)$ denote $\Psi^{\Delta}(b|p)$ $(b=a,i,d)$.

Given this framework the decision problem facing each agent can be described. But before doing so, two points should be emphasized. First, the agents choose entire <u>rules</u> $\Delta(c)$ and $\Psi^{\Delta}(b|p)$. This is to distinguish from the more familiar situation wherein points (e.g. particular quantities) are chosen. Second, though the information structure has a sequential (hence asymmetric) structure, the choices of rules are made simultaneously. In particular, while the innovator may be able to influence $\psi^a(p)$ (for example) by manipulating $\Delta(c)$, he must take the manner in which this effect operates as given.[17]

The innovator's problem is to choose $\Delta(c)$ so as to maximize his expected profits, given the competitor's rule. $\Delta(c)$ solves

$$\max\ \psi^a \pi^M + \psi^i \pi^I + \psi^d \pi^D.$$

Similarly, the competitor's problem is to choose $\Psi^{\Delta}(b|p)$ to maximize expected profits:

$$\max \psi^{a}(p) \cdot 0 + \psi^{i}(p)E^{\Phi}(\pi^{I}) + \psi^{d}(p)E^{\Phi}(\pi^{b}).$$

A preliminary result which is helpful below is as follows:

PROPOSITION 1: <u>The competitor will never randomize. That is, for all p, there is some behavior $\hat{b}$ such that</u>

$$\Psi^{\Delta}(b|p) = \begin{cases} 1 & \underline{\text{for}}\ b = \hat{b} \\ 0 & b \neq \hat{b}. \end{cases}$$

<u>Proof</u>: See Appendix.

The intuition for this result is straightforward. The competitor takes $\Delta(c)$ as fixed. Given this, the expected profits from abstention, imitation and duplication do not depend on how likely it is that each would be chosen. The best the competitor can do is thus to invariably pick the largest of 0, $E^{\Phi}(\psi^{I})$, and $E^{\Phi}(\psi^{\Delta})$; that is, not to randomize.

III. EQUILIBRIUM

The equilibrium concept employed in this analysis is the now familiar Bayesian-Nash equilibrium. Specifically, the pair of rules $\langle\tilde{\Delta}(c), \tilde{\Psi}^{\Delta}(b|p)\rangle$ is an equilibrium if and only if $\tilde{\Delta}(c)$ solves the innovator's problem when he is confronted with $\tilde{\Psi}^{\Delta}(b|p)$, and $\tilde{\Psi}^{\Delta}(b|p)$ solves the competitor's problem when he faces $\tilde{\Delta}(c)$.

The main result in the paper is the following.

PROPOSITION 2: <u>The model has a unique equilibrium given by</u>

$$\text{(i)} \quad \tilde{\delta}^D = 1, \; \tilde{\delta}^L = \tilde{\delta}^I = \left[\frac{\alpha^D}{\alpha^I + \alpha^L}\right]\left[\frac{\bar{c}^I - R^I}{R^I - \underline{c}^I}\right];$$

and

$$\text{(ii)} \quad \tilde{\Psi}^a(1) = 1, \; \tilde{\Psi}^i(o) = 1; \; \tilde{\Psi}^b(p) = 0 \; \underline{\text{otherwise}}.$$

<u>Proof</u>: See Appendix.

The innovator always patents when duplication is cheap relative to imitation, $c = (\bar{c}^I, \underline{c}^D)$, and patents sometimes when imitation is cheap, $c = (\underline{c}^I, \underline{c}^D)$ <u>or</u> $c = (\underline{c}^I, \bar{c}^D)$. The competitor responds by abstaining when a patent is observed and imitating otherwise. Though the proof is fairly involved, the basic reasoning underlying this result is straightforward. From (8) and (9), were the innovator to always patent, the competitor would always enter with a competing imitation. Given this, the innovator could increase his expected profits by choosing to patent only when imitation were costly (i.e. $c = (\bar{c}^I, \underline{c}^D)$) and not patent when imitation were cheap. In this case, the patent system reveals all of the private information relevant to competitor decision making; and the competitor thus abstains on observing a patent but enters and imitates if no patent is observed. Clearly, however, were the innovator to patent not only when imitation were costly but some small fraction of the time when imitation were cheap, it could increase its profits. Then, when duplication is possible (i.e. no patent), the competitor correctly infers that imitation is certainly cheap: $C^I = \underline{c}^I$. Given this, our assumptions imply imitation is preferred to duplication. Further, since imitation is assumed to yield positive expected profits in the absence of this information, a fortiori positive profits are expected given it, and the competitor imitates.

When a patent is observed, the competitor abstains (even if it turns out imitation is cheap). This occurs because the patent is more likely when imitation is most costly. (Indeed note that when imitation is expected to be cheap, $\alpha^D \to 0$, the existence of a patent virtually assures imitation has turned out to be expensive; $\delta^L = \delta^I \to 0$.) In fact, $\tilde{\delta}^L$ and $\tilde{\delta}^I$ are chosen so that given the observation of a patent, the competitor expects zero profit from entry.

This is the correct choice of δ^L and δ^I for the following reason. δ^L and δ^I could be made smaller. The effect of this is to lower the fraction of the time patenting occurs, thereby raising the frequency of imitation. As the innovator prefers the competitor to abstain rather than imitate, lowering δ^L and δ^I does not pay. Similarly, δ^L and δ^I could be raised. While this raises the frequency of patenting, it reduces the information content of the patent enough to make expected profits from imitation of a patented good positive. (This is why it is not possible for the innovator to invariably induce abstention.) Thus $\tilde{\delta}^L$ and $\tilde{\delta}^I$ are low enough to deter imitation when a patent occurs, but as high as possible subject to this deterrence being successful.

Finally, $\tilde{\delta}^D = 1$ is the correct choice. Suppose $\bar{\delta}^D < 1$ were chosen. By raising $\bar{\delta}^D$, the fraction of the time patenting (hence abstention by the competitor) occurs rises.. Also the information content of the patent is improved, serving to promote abstention by the competitor. Thus raising $\bar{\delta}^D$ is unambiguously profitable for the innovator.

One of the implications of Proposition 2 is that within any industry, one should expect firms to display, in equilibrium, a propensity to patent. In particular, Proposition 2 indicates that patenting always occurs when imitation is expensive, an event obtained with probability α^D. Also, a patent is chosen with probability $\tilde{\delta}^L = \tilde{\delta}^D$ when imitation is cheap, an event arising with frequency $\alpha^L + \alpha^I$. Accordingly, the (unconditional) propensity to patent is (after manipulation)

$$\nu \equiv \alpha^D\left(1 + \frac{\bar{c}^I - R^I}{R^I - \underline{c}^I}\right). \tag{10}$$

Since R^I and C are specific to the industry in question, this value should be expected to vary across industries. The predictions which may be obtained from (10) are examined in the next section.

As a final note, the reader should be aware that the equilibrium in Proposition 2 is unique in a particular sense. $\tilde{\delta}^D = 1$, $\tilde{\psi}^a(1) = 1$ and $\tilde{\psi}^i(o) = 1$, in conjunction with any δ^L and δ^I yielding $E^{\Phi}(\pi^I|1) = 0$ (also satisfying $0 \leq \delta^L \leq 1$ and $0 \leq \delta^I \leq 1$), is also an equilibrium. That is, $\tilde{\delta}^L$ and $\tilde{\delta}^I$ are not uniquely determined. This occurs because imitation is the dominant competitor response to a no-patent situation. Hence, whether $C^D = \underline{c}^D$ or $\bar{c}^D$ _given_ $C^I = \underline{c}^I$ is of no consequence; distinguishing between δ^L and δ^I serves only to communicate useless information. It is easily checked that none of the entities in the model (e.g., ν, etc.), except δ^L and δ^I themselves, depend on the normalization $\tilde{\delta}^L = \tilde{\delta}^I$. Thus the restriction involves no loss of generality. The equilibrium in Proposition 2 is "behaviorally" unique.

IV. PREDICTIONS

The predictions of the theory come in two varieties.

First of all there are the "gross" predictions: innovating firms will not patent all inventions; non-patented goods will be imitated, patented goods will be neither imitated nor duplicated. This latter prediction might strike the reader as being somewhat odd. For one could invariably suggest an example of some patented product for which there exists a good which could reasonably be called an "imitation". Such anecdotal evidence, however, is scarcely a proper test of the theory. Since the non-existent imitations are unobservable, a proper test of the theory must involve those elements which are observable, like the propensity to patent. The second type of predictions involve just this variable, and are discussed below.

This model makes a number of predictions concerning the propensity to patent: ν. First, ν depends positively on α^D. This is simply because patenting is always optimal when duplication is cheap relative to imitation, and α^D is the probability of that event. Also, since $\alpha^D = 1 - \alpha^L - \alpha^I$, ν declines with an increase in the probability that imitation is cheap. One should expect, therefore, that for industries in which new products are likely to be cheap to duplicate but expensive to imitate, there should be a higher propensity to patent relative to those industries for which imitation is likely to be cheap.

Second, ν varies inversely with R^I (actually R_2^I).[18] δ^L ($= \delta^I$) is chosen to set the expected profit from imitation of a patented good to zero. Thus when R^I rises, expected costs of imitation must rise. This is accomplished by patenting less often when imitation is cheap; that is, lower δ^L, and so lowering ν.

In a similar fashion, any parameter which raises the attractiveness of imitation in response to a patent (increases in α^L or α^I, reductions in α^D, $\underline{c}^I$, $\bar{c}^I$) reduces δ^L and ν. Thus, for instance, industries in which imitations are relatively cheaply produced should display less patenting behavior than those in which imitation is more expensive.

These predictions have a counterintuitive flavor; even though the innovator seeks to deter imitation, parameter changes which make imitation in the face of a patent more attractive result in less patenting (imitation-deterring behavior) and more imitation. The reason is just that the decision to patent must credibly convey stronger information under such parameter changes. The patent, as a communication device, must be used less frequently to remain effective.

V. EXTENSIONS

The basic analysis presented in the preceding sections can be extended in numerous directions. This section briefly pursues several of these modifications and considers relaxation of some of the restrictions imposed above.

(a) Stochastic Revenue or Profit

As presented in Section II, the innovator has the advantage of knowing the fixed costs of producing either a duplicate of the innovation or an imitation of it. It is easy to check that the preceding analysis applies directly to the case in which only the innovator is certain about the revenue associated with each course of action, say because part of the innovation process involves market research, while fixed costs are common knowledge. This reinterpretation merely involves replacing $\underline{c}^I$ and $\bar{c}^I$ ($\underline{c}^D$ and $\bar{c}^D$) by a constant $c^I(C^D)$, and letting revenues be the vector valued random variable $R = (R^I,R^D)$, where $R^I = \underline{r}^I$ or $\bar{r}^I > \underline{r}^I$ ($R^D = \underline{r}^D$ or $\bar{r}^D$, $\bar{r}^D > \underline{r}^D$). Assumptions (3)-(9) can be translated appropriately.[19]

It is, of course, tempting to extend this claim to random profits generally. This can be done provided profits from each course of action can be described by a vector random variable, the components of which can take on but two values (e.g., $\Pi^I = \underline{\pi}^I$ or $\bar{\pi}^I$, $\bar{\pi}^I > \underline{\pi}^I$, etc.). However, note that this setup cannot concurrently accommodate stochastic revenues and fixed costs described above unless revenues and costs are perfectly correlated.

(b) Raising Costs for the Competitor

An assumption of this model is that the type of product which the innovator obtains upon winning the patent race cannot be influenced by any action of the innovator within the R + D process. One might suspect, however, that the innovator does have some leeway in determining the cost characteristics of the innovation he obtains. What is of particular interest is an analysis of the innovator's incentives to try to choose his product design so as to make imitation more costly.[20]

To consider this issue, suppose that the innovator can devote resources to raising the probability that the cost of imitation is high; that is, the innovator can devote resources to raising α^D, subject to the constraint that $\alpha^D = 1 - \alpha^I - \alpha^L$. If one assumes that (i) a minimal level of α^D (i.e., one that satisfies (8) and (9)) is freely available; and (ii) increments to α^D can be achieved at increasing marginal cost; and (iii) the chosen value of α^D is common knowledge (so the innovator's informational advantage remains as before), it is easily shown that the basic character of the equilibrium in Proposition 2 is undisturbed. This occurs simply because given α^D, the game is unchanged. Given this, minor manipulation yields the innovator's expected profit as (recall R_1^I is the innovator's net revenue under imitation)

$$R^M\nu + R_1^I(1-\nu) - F - \chi(\alpha^D), \tag{11}$$

where $\chi(\alpha^D)$ is expenditures on increasing α^D above the requisite minimal level; $\chi' \geq 0$, $\chi'' > 0$. From (10), raising α^D augments ν, thereby increasing the probability of receiving R^M as opposed to R_1^I. The optimal α^D balances these marginal returns against marginal cost χ'.

The predictions are as follows. First consider parameters entering (11) via $\nu\,(R_2^I,\ \bar{c}^I$ and $\underline{c}^I)$. Increases in R_2^I, as well as decreases in $\bar{c}^I$ or $\underline{c}^I$, reduce ν for fixed α^D. Such changes reduce marginal returns, lowering the optimal α^D. Since ν is proportional to α^D, the full effect (α^D varying) of these parameters on ν is the same as the impact for given α^D. For these parameters: (i) the results of Section IV stand qualitatively unaltered; and (ii) parameter changes making imitation more profitable when the competitor is confronted with a patent <u>reduce</u> the level of activities devoted to raising the probability that imitation is more costly.

An increase in R^M raises marginal returns directly (not via ν); optimal α^D rises accordingly. Note that whereas in Section IV ν was not a function of R^M, ν rises with R^M here. An increase in R_1^I (R_2^I constant) operates in just the opposite fashion.

(c) <u>Relaxing $\alpha^H = 0$</u>

In Section II, the assumption $\alpha^H = 0$ was made. Given the discussion of the logic underlying the equilibrium with $\alpha^H = 0$, it is straightforward to show that $\alpha^H > 0$ does not influence its basic features.

First of all, even with $\alpha^H = 0$, under the assumptions made, imitation dominates duplication when there is no patent. Allowing $\alpha^H > 0$ only serves to exaggerate this dominance by raising the probability that duplication is expensive. Duplication can again be ignored.

Next, by choosing $\delta^D = 1$ and $\delta^L = \delta^I < 1$, the innovator is essentially causing the competitor to infer that imitation is likely to be relatively costly when a patent is observed. Even if $\alpha^D = 0$ and $\alpha^H > 0$ is assumed, since the relevant difference is only in terms of the <u>innovator's</u> cost, the competitor's problem is essentially unchanged and the innovator will again operate in the same fashion setting $\delta^H = 1$ with both δ^L and δ^I less than unity. $\alpha^H = 0$ is therefore a useful simplification.

(d) <u>Relaxation of (8) or (9)</u>

When either (8) or (9) does not hold, equilibria other than that examined in Proposition 2 may occur.

First, suppose (8) fails. This is enough to guarantee that duplication is dominated by abstinence when there is no patent. Accordingly, the patent has no role whatsoever, except as an information transfer mechanism. It follows that $P^* \equiv 1 - P$ can convey the same information as P, and hence that (using "$\hat{}$" to denote equilibrium rules when (8) fails, and "$\tilde{}$" for rules from Proposition 2):

$$\hat{\delta}^D = 0, \; \hat{\delta}^L = \hat{\delta}^I = 1 - \tilde{\delta}^I = 1 - \tilde{\delta}^L;$$
$$\hat{\psi}(0) = 1 \text{ and } \hat{\psi}^i(1) = 1$$

is also an equilibrium. The existence of this "mirror image" equilibrium is not a matter of concern simply because (8) is required if patents are to be of interest in the first instance.

When (9) fails, it is easy to check that the rule $\delta^D = \delta^I = \delta^L = 1$, $\psi^a(0) = \psi^a(1) = 1$ is an equilibrium; that is, always patenting completely deters competitors. The logic is just that always patenting prevents duplication and reveals no information; hence $E^{\Phi}(\pi^1) = E^{\alpha}(\pi^1) < 0$ and the competitor always prefers abstinence to imitation. Further, it is possible to show that when (9) fails, there is a continuum of equilibria described by the (demonstrably non-empty) set of δ^D, δ^L and δ^I satisfying

$$E^{\Phi}(\pi^I | o) \le 0,$$

$$E^{\Phi}(\pi^I | 1) \le 0,$$

and

$$E^{\Phi}(\pi^D | o) \le 0.$$

The logic here is just that if imitation is a poor option, the innovator can use the patent to deter duplication without fear or signalling that imitation might be profitable. The deck is stacked in the innovator's favor. The same is true when both (8) and (9) fail.

(e) A Welfare Result

Along with all of the usual considerations that arise in any problem with patents an additional issue arises in this model with the introduction of incomplete information. Specifically, it is of interest to consider whether the patent system conveys an efficient amount of information. A natural conjecture is that, in equilibrium, it does not; and that requiring full revelation would be welfare improving. The basis for this conjecture is simply the fact that, in equilibrium, the innovator is able to prevent entry and reap monopoly profits in some situations in which, with full revelation, entry and imitation would occur.

Unfortunately, while this argument has intuitive appeal, it is not obviously correct as long as imitation involves a positive fixed cost C. If there were no fixed cost, then usual social surplus arguments could be used to show that full revelation would be strictly welfare improving. With fixed costs, however, the value of lower prices and increased variety must be weighed against the incursion of the fixed cost needed to acquire these benefits. This is the usual conflict arising in product differentiation problems. Whether the surplus arising from the revelation of the additional information more than offsets the costs created by the new information depends upon the usual group of factors (demand elasticities, size of the fixed cost, etc.).[21]

(f) Trade Secrecy

Thus far patenting has revealed information only to the extent that competitors obtained an improved estimate of fixed production costs via inferences based on the innovator's optimal patenting decision. Naturally patents may also make it less costly to construct a plant to produce an imitation. For example, the patent may reveal that certain production processes work better than others. For these additional reasons, innovators may not wish to patent; that is, trade secrecy could develop. Does the model predict this?

Suppose that if the innovator patents, the imitation costs are $(1-\beta)C^I$ (still random), where β is exogenously specified and such that $0 < \beta < 1$. Further, assume that the existence of a patent does not ensure imitation is always profitable:

$$R^I - (1-\beta)\bar{c}^I < 0. \qquad (12)$$

(12) keeps the analysis within the spirit of that presented in Section II.

Then, proceeding as above, the equilibrium is

$$\delta^D = 1, \ \delta^I = \delta^L = \frac{\alpha^D}{\alpha^L + \beta^I}\left[\frac{(1-\beta)\bar{c}^I - R^I}{R^I - (1-\beta)\underline{c}^I}\right],$$
$$\psi^a(1) = 1, \ \psi^i(0) = 1.$$

That is, the equilibrium is just as in Section II, except that the common value of δ^I and δ^L is lower. This implies the propensity to patent (ν) is lower, or trade secrecy.

The explanation for this is that were the innovator to continue patenting at the same rate as when $\beta = 0$ (i.e. when the patent system conveyed no direct cost information), the competitor would obtain enough cost reducing information to make it worthwhile, on average, to imitate in the face of a patent. Given this, the innovator must reduce δ^L (= δ^I) in order to signal more forcefully that imitation is expensive. Thus, the innovator uses the signalling aspect of the patent system to offset the direct cost information transmission aspect of the system. The result is a lower propensity to patent and a resultant increase in the innovator's resorting to trade secrecy.

VI. SUMMARY

This paper has considered a model of patenting behavior when patents reveal information important to competitors. It was shown that the Bayesian Nash equilibrium in such a model involves the innovating firm always patenting when imitation is costly, and sometimes patenting (i.e., randomly doing so) when imitation is cheap. The competitor's optimal response to this is to abstain from production (including production of a non-patented differentiated product) when a good is patented, and production of the differentiated product (the imitation) when there is no patent.

This equilibrium behavior implies a propensity to patent about which several predictions can be made. Most interesting is that any factors making imitation a more attractive strategy in the face of a patent by the innovator make it <u>less</u> likely that a patent will be chosen. This occurs because under such a change, the decision to patent must convey stronger information to competitors; that is, patenting must be a more surprising event.

The model was extended in several directions. First it was shown that the model could be reinterpreted as a stochastic revenue model, and, in a limited way, in a random profit setup. The analysis is somewhat more general than first appears. It was also shown that the intuition that requiring full revelation of the innovating firm's private information would be welfare improving is not generally correct unless there are no sunk costs associated with production of an imitation. Further, when patenting actually reduces costs for competitors, trade secrecy (in the sense of a reduced propensity to patent) emerges as equilibrium behavior. Finally, when it is possible to make imitation more costly, revenues will be expended to do so. But the amount of such expenditure declines with the competitor's expected profitability when confronted by a patent.

FOOTNOTES

[1]See Scherer (1983, 1965). Also see Scherer, Herzstein, Dreyfoos _et al_ (1959) in which it is shown that firms deprived of patent rights through anti-trust judgements reduce their patenting activity.

[2]This is even true in a model by Tandon (1982) in which there is mandatory licensing of all patented goods. Tandon analyzes how such a requirement affects a firm's level of R & D, but assumes its patenting behavior is unaffected.

[3]This asymmetry idea is not new in the patent literature. See Scherer (1973), p. 385 for a discussion.

[4]Though we do not pursue the "pre-game", it is straightforward to include the decision of whether to attempt innovation given that the situation following innovation is as described in the text.

[5]This does not imply that the technology required to produce the product is common knowledge.

[6]The imitation can be treated as a differentiated commodity, the characteristics of which are optimally chosen in a Hotelling fashion.

[7]In fact, we assume that the cost to the competitor of obtaining this information is infinite. The model could easily be adjusted to deal with finite information costs.

[8]This concept is not a new one. It is the basis of the paper by Crawford and Sobel (1982). It is also one familiar to any poker player. In general, choosing to always bid high leads your opponents to infer that you are an easy mark and is not a profitable strategy. Similarly, bidding high only when you have a sure winner reveals all your information to your opponents and also is not the best strategy. In fact, bluffing plays an important part in the game, and it is generally profitable to bid high <u>some of the time</u> when you have been dealt a poor hand.

[9]As what happens in the post-patent monopoly or duopoly situation, is obvious, the strategies and payoffs therein are suppressed.

[10]Patenting is costless. When there is a patent fee, the analysis becomes cumbersome. However, when the cost of patenting is a direct benefit to competitors, much can be said. See Section V(f).

[11]For simplicity, it is assumed that the innovator is committed to production once the invention has been made.

[12]The mnemonics here are L for "low cost generally", D for "duplication is low cost", I for "imitation is low cost", and H for "high cost generally".

[13]$R_1^I \neq R_2^I$ is a straightforward extension.

[14]This ranking is compatible with a number of models in which product specification is a choice variable, and which show that firms will choose products with distinct specifications. See, for instance, Novshek (1980), Salop (1979), Shaked and Sutton (1982). The reader should be careful not to infer from this, however, that the competitor will always

choose imitation. The decision to duplicate or imitate depends on both revenues and the random fixed costs.

[15]Note that though it turns out that imitation generally dominates duplication in equilibrium, the latter is more profitable for $C = (\bar{c}^I, \underline{c}^D)$.

Also, while invention is taken to be costless here, an inventor cost is easily accommodated provided such costs are internally financed. For an analysis of the revelation properties of external financing, see Bhattacharya and Ritter (1983).

[16]The updating is of the Bayes variety.

[17]An interesting avenue not pursued here is to suppose that the separation between the patent choice and the likelihood of an innovation were not possible. This could occur if, say, because producing patentable innovations was a useful signal of innovative ability, inventors approached firms with a high propensity to patent more frequently.

[18]A plethora of other predictions are easily obtained by parameterization of R^I, R^M, $\underline{c}^I$, etc. For example, the impact of changing factor prices or demand elasticities is readily obtained. Effectively, all that need be decided is how the parameter change affects the expected profitability of imitation when confronted by patent.

[19]That R^M becomes stochastic does not influence anything so long as an assumption analogous to (4) holds.

[20]Analysis of attempts to change the cost of duplication, presumably leaving the innovator's costs unaltered, is less interesting because duplication does not occur in equilibrium.

[21]For a discussion of the general product variety problem, see Spence (1976).

Appendix

A.1 PROOF OF PROPOSITION 1

Given $P = p$ and $\Delta(C)$, $\psi^a \cdot 0 + \psi^i E^{\Phi}(\pi^I) + \psi^a E^{\Phi}(\pi^D)$ is linear in the ψ^b. For $E^{\Phi}(\pi^I) \neq E^{\Phi}(\pi^D) \neq 0$, and for each $P = p$, $\psi^b = 1$ for some b and $\psi^b = 0$ for all others.

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A.2 PROOF OF PROPOSITION 2

The common value of $\delta^I = \delta^L$, say $\bar{\delta}$, yields $E^{\Phi}(\pi^I|1) = 0$ for the competitor. Also $(0 <)\ E^{\Phi}(\pi^I|0) = R^I - \underline{c}^I > \alpha^I(R^D - \bar{c}^D) + \alpha^L(R^D - \underline{c}^D) = E^{\Phi}(\pi^D|0)$. So $\psi^i(1) = 1$ is optimal given $\Delta(c)$.

Now consider the innovator facing the proposed $\Psi^{\Delta}(\cdot|\bar{p})$. $E^{\alpha}(\pi) = [\alpha^D + \bar{\delta}(\alpha^L + \alpha^I)]R^M + (1 - \bar{\delta})(\alpha^L + \alpha^I)R^I > 0$. Since this expression is a weighted average of the two largest possible payoffs for the innovator, he can do better only by one of:

(a) raising the probability of achieving R^M while retaining $\psi^d(p) = 0\ \forall\ P = p$;

or (b) allowing for some duplication (this will involve $\psi^a(1) = 1$, $\psi^d(0) = 1$, for it is easy to check that this dominates either $\Psi^i(p) = 1\ \forall\ P$, or $\Psi^i(1) = 1$, $\Psi^d(0) = 1$.

Focus on case (a). First, any change in $\Delta(c)$ resulting in $\psi^i(p) = 1\ \forall\ p$ obviously reduces the probability of receiving R^M. Thus no such change could be optimal. Next, $\psi^a(p) = 1\ \forall\ p$ is obviously not possible. So consider the "mixed" cases. First, suppose the competitor retains $\Psi^a(1) = 1$ and $\Psi^i(0) = 1$. Consider the auxiliary problem $(0 \leq \delta^D \leq 1, \ldots)$

$$\max \ \delta^L\alpha^L + \delta^I\alpha^I + \delta^D\alpha^D$$

s.t. $(\delta^L\alpha^L + \delta^I\alpha^I)(R^I - \underline{c}^I) + \delta^D\alpha^D(R^I - \bar{c}^I) \le 0 \qquad (E^{\Phi}(\pi^I|1) \le 0$

and $[1 - \delta^L)\alpha^L + (1 - \delta^D)\alpha^D](R^D - \underline{c}^D) + (1 - \delta^I)\alpha^I(R^D - \bar{c}^D)$

$$\le [(1 - \delta^L)\alpha^L + (1 - \delta^I)\alpha^I](R^I - \underline{c}^I + (1 - \delta^D)\alpha^D(R^I - \bar{c}^I)$$

$$(E^{\Phi}(\pi^D|0) \le E^{\Phi}(\pi^I|0))$$

First solve the problem ignoring the second constraint. The first constraint is loosened by raising δ^D, and the maximand is rising in δ^D. So $\delta^D = 1$ is optimal. As the maximand is rising in $\delta^L\alpha^L + \delta^I\alpha^I$, the best that can be achieved is to set $\delta^L\alpha^L + \delta^I\alpha^I$ to satisfy the first constraint with equality. (That α^L and α^I are not too small to cause $\delta^L = \delta^I = 1$ to yield the constraint not binding is ruled out by $R^I - E^{\alpha}(C^I) > 0$. That is, $(\alpha^L + \alpha^I)(R^I - \underline{c}^I) < \alpha^D(R^I - \bar{c}^I) \Leftrightarrow E^{\alpha}(\pi^I) < 0.$) Next, for $\delta^D = 1$, (4) implies the second constraint is not binding for the $\Delta(c)$ solving the first problem. Thus this $\Delta(c)$ solves the full problem. Finally, since only the sum $\delta^L\alpha^L + \delta^I\alpha^I$ is determined, and $\delta^L = \delta^I = 1$ reverses the first inequality, $\delta^L = \delta^I \equiv \bar{\delta}$ may be specified.

Still referring to case (a), the other possibility is $\psi^a(0) = 1$, $\psi^i(1) = 1$. The auxiliary problem is

$$\max(1 - \delta^L)\alpha^L + (1 - \delta^I)\alpha^I + (1 - \delta^D)\alpha^D$$

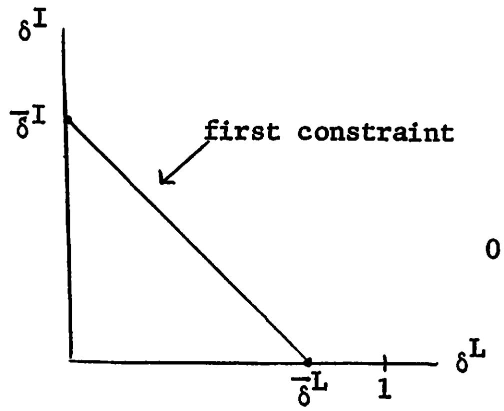
s.t. $[(1 - \delta^L)\alpha^L + (1 - \delta^I)\alpha^I](R^I - \underline{c}^I) + (1 - \delta^D)\alpha^D(R^I - \bar{c}^I) \le 0 \ (E^{\Phi}(\pi^I|0)$

and $[(1 - \delta^L)\alpha^L + (1 - \delta^D)\alpha^D](R^D - \underline{c}^D) + (1 - \delta^I)\alpha^I(R^D - \bar{c}^D) \le 0 \ (E^{\Phi}(\pi^D|0) \le 0$

$(E^{\Phi}(\pi^I|1) \ge 0$ implied when first constraint satisfied.)

Again ignore the second constraint. Since the problem is identical to that just worked out, except "patent" is replaced by "no patent", the innovator cannot do better, $\delta^D = 0$ is optimal in this problem. The first constraint can be written (as an equality)

$$(1 - \delta^L)\frac{\alpha^L}{\alpha^L+\alpha^I} + (1 - \delta^I)\frac{\alpha^I}{\alpha^L+\alpha^I} = \frac{\alpha^D}{\alpha^L+\alpha^I}\,\frac{\bar{c}^I - R^I}{R^I - \underline{c}^I} \quad (< 1 \text{ by } E^{\alpha}(\pi^I) > 0)$$



$0 < \bar{\delta}^I < 1,\ 0 < \bar{\delta}^L < 1$

Consider the LHS of second constraint for $\delta^I = \delta^I$, $\delta^L = 0 = \delta^D$

$$(\alpha^L + \alpha^D)(R^D - \underline{c}^D) + (1 - \bar{\delta}^I)\alpha^I(R^D - \bar{c}^D)$$

$$> (\alpha^L + \alpha^D)(R^D - \underline{c}^D) + \alpha^I(R^D - \bar{c}^D)$$

$$> \alpha^D(R^D - \underline{c}^D) + \alpha^I(R^D - \bar{c}^D) \qquad (\alpha^L > 0)$$

$$\geq 0 \text{ by } (8)$$

Consider the LHS of second constraint for $\delta^I = 0 = \delta^D$, $\delta^L = \bar{\delta}^L$.

$$[(1 - \bar{\delta}^L)\alpha^L + \alpha^D](R^D - \underline{c}^D) + \alpha^I(R^D - \bar{c}^D) \qquad (8)$$

$$> \alpha^D(R^D - \underline{c}^D) + \alpha^I(R^D - \bar{c}^D) \geq 0$$

Therefore no δ^L, δ^I pair satisfying the first constraint as an equality can satisfy the second constraint. Thus the maximal value of the maximand must be lower than under the previous problem.

Now turn to case (b). $\psi^d(0) = 1$, $\psi^a(1) = 1$. The auxiliary problem is

$$\max\ \delta^L\alpha^L + \delta^D\alpha^D + \delta^I\alpha^I$$

$$\text{s.t.}\quad (\delta^L\alpha^\delta + \delta^I\alpha^L)(R^I - \underline{c}^I) + \delta^D\alpha^D(R^I - \bar{c}^I) \le 0$$

$$\text{and}\quad [(1 - \delta^L)\alpha^L + (1 - \delta^D)\alpha^D](R^D - \underline{c}^D) + (1 - \delta^I)\alpha^I(R^D - \bar{c}^D)$$

$$\ge [(1 - \delta^L)\alpha^L + (1 - \delta^I)\alpha^I](R^I - \underline{c}^I) + (1 + \delta^D)\alpha^D(R^I - \bar{c}^I)$$

Again ignoring the second constraint, $\delta^D = 1$ follows, yielding at best the same value of $\delta^L\alpha^L + \delta^D\alpha^D + \delta^I\alpha^I$ as before. But this means that expected profit involves R^M no more often, and R^D ($< R^I$) at least as often as under option (a). Thus expected profit is lower than under part (a). ||

REFERENCES

Bhattacharya, S. and J. R. Ritter. "Innovation and Communication: Signalling with Partial Disclosure," R.E.Stud. 50 (April 1983), 331-346.

Crawford, V. P. and J. Sobel. "Strategic Information Transmission," Econometrica 50 (November 1982), 1431-1452.

Kahn, A. E. "The Role of Patents" in J. P. Miller (ed.), Competition, Cartels and their Regulation (Amsterdam: North Holland, 1962), p. 317.

Machlup, F. "An Economic Review of the Patent System" in J. P. Miller (ed.), Competition, Cartels and their Regulation (Amsterdam: North Holland, 1962), p. 76.

Novshek, W. "Equilibrium in Simple Spatial (or Differentiated Product) Models," Journal of Economic Theory 22 (1980), 313-326.

Salop, S. C. "Monopolistic Competition with Outside Goods," Bell Journal of Economics (Spring 1979), 141-156.

Scherer, F. M. "The Propensity to Patent," International Journal of Industrial Organization 1 (March 1983), 107-128.

________. Industrial Market Structure and Economic Performance (Chicago: Rand McNally, 1973).

________. "Market Structure and the Employment of Scientists and Engineers," American Economic Review 57 (June 1967), 524-530.

________. "Firm Size, Market Structure, Opportunity and the Output of Patented Inventions," American Economic Review 55 (December 1965), 1097-1125.

Scherer, F. M., S. E. Herzstein, A. W. Dreyfoos *et al*. *Patents and the Corporation*, 2nd edition (Boston: 1959), 137-146, 153-155.

Shaked, A. and J. Sutton. "Relaxing Price Competition through Product Differentiation," *R.E.Stud.* 49 (January 1982), 3-14.

Spence, A. M. "Product Differentiation and Welfare," *American Economic Review* 66 (May 1976), 407-414.

Tandon, P. "Optimal Patents with Compulsory Licensing," *Journal of Political Economy* 90 (June 1982), 470-486.