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Åke G. Blomqvist

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TARIFF REVENUE AND OPTIMAL CAPITAL ACCUMULATION
IN LESS DEVELOPED COUNTRIES.

A.G. Blomqvist*

I. Introduction

The problem of the optimal division over time of an economy's output between consumption and capital accumulation has been studied by many writers, beginning with the classic article by Ramsey [1928]. The basic problem has been elaborated upon in many ways, for example by incorporating several productive sectors, by allowing for international trade and/or borrowing, by postulating technical progress in various forms, etc. (see the essays in Shell [1967]). Some attempts have also been made to apply this type of analysis to the problem of formulating optimal policies for development of low-income countries (Goodwin [1961], Stoleru [1965], Uzawa [1966]). The contribution by Goodwin is particularly interesting in this context because of his explicit numerical illustrations of optimal policies under plausible assumptions concerning the various parameters and initial conditions; he generally finds that optimal development involves a very high degree of initial austerity with consumption remaining constant at a low level throughout most of the planning period and rising rapidly towards its end; savings, on the other hand rise rapidly in the beginning and fall towards the end of the program. Similarly, Stoleru's numerical illustration shows very low initial levels of consumption.

* The author is a Visiting Lecturer at the University of Ghana under an exchange program with the University of Western Ontario. The program is financially supported by the Canadian International Development Agency.

In many cases, the analysis has been carried out on the (explicit or implicit) assumption that the economy was centrally planned, so that the policy-makers have had direct control over the allocation decisions necessary to implement the optimal policy. Alternatively, it has been shown that under certain conditions, monetary and fiscal policies exist which make it possible to indirectly achieve the optimum. Generally, however, the problem has been formulated in such a way that no conflict has arisen between the efficiency with which the economy's resources are used at any given time, and the achievement of inter-temporal optimality.

In many countries, however, especially low-income ones, it is only possible for the policy-makers to reallocate a significant proportion of real output to capital formation through the imposition of various types of taxes which do have considerable effects on the "instantaneous" efficiency of resource allocation and thereby cause welfare losses over time. For example, as has been noted by Vanek [1971], in many small, primary-producing economies, the export sector is heavily dominant in the monetized part of the economy, and substantial public savings may not be possible unless taxes are levied on international trade.¹ The potential welfare losses caused by such taxes are well known.

But if the rate of capital formation can only be controlled through taxation which imposes an "excess burden" in the form of such efficiency losses, the formulation of a program of optimal accumulation ought to take this fact into account. For example, if tariff revenue

is used to finance capital formation, an acceleration of investment through an increase in tariff rates will entail a cost not only in the form of reduced consumption but also in the form of an increased excess burden, and it is possible that this will lead to an optimal program which is quite different from what it would have been if it were possible to finance investment without incurring such a burden. Conversely, the results regarding optimal tariff rates derived from static analysis will become invalid in some respects if tariffs are necessary to finance development.

The purpose of the present paper is to consider the simplest type of optimal savings problem (as described, e.g., by Dorfman [1969], pp. 824-827), but to modify the assumptions in the sense of letting tariff revenue be the only source of development financing over which the policy-makers have control once savings from other sources have reached an upper limit. Particular attention will be given to the question how the existence of an excess burden will influence the policy problem, and it will be found that in general, the optimal distribution of the burden of capital formation between present and future generations is strongly affected by the degree to which such a burden is present. In some cases, this means that the policy of heavy restraint on initial consumption recommended in the analyses of Goodwin and Stoleru must be considerably modified.

Vanek [1971] has also analyzed the problem of an optimal tariff policy when the revenue is used for capital formation. While our method of analysis is similar to his in many respects, his basic

conclusion, in contrast to ours, is that the excess-burden effect is generally of relatively minor importance, and that the optimal tariff is fairly close to that level which would maximize revenue. Vanek's results depend strongly on some rather special restrictions that he imposed on the problem, however. First, he considered the case in which a tariff was imposed in the initial time period but was then abolished for all subsequent periods. This neglects the problem of the optimal time pattern of tariffs and savings, and rules out the possibility of a relatively moderate tariff over a long time period as an alternative to an initially high but rapidly declining tariff; our analysis indicates that the former alternative is more likely to be optimal when the excess burden is significant. Second, Vanek assumed constant marginal utility of income (or consumption). As a consequence, the optimal tariff which he derives is independent of the amount of savings available from non-tariff sources.² This is a counter-intuitive result which disappears when the assumption is relaxed.

The organization of the paper is as follows. In section II, we specify a simple neoclassical model in which all sources of capital formation except tariff revenues have a fixed and given upper limit. We then formulate and solve an optimal control problem in order to find a tariff policy over a given time period which leads to an optimal path of capital accumulation and consumption. The solution is analyzed and interpreted in economic terms in section III, and in section IV we discuss the characteristics of the optimal time paths of tariffs

and savings under varying assumptions. Section V contains a brief summary and some qualifications.

II. The Model

We consider a country which is engaged in international trade; for simplicity, it is assumed that it is small enough so that it cannot influence its terms of trade, which are assumed to remain constant throughout the planning period. The units are so chosen that the ex-tariff price of imported goods in terms of home produced goods is equal to unity. There is no international borrowing and lending, so that exports and imports must balance. Production is assumed to take place using only labour and capital as inputs. For convenience, we will abstract from depreciation by postulating that capital is infinitely long-lived.³ There are constant returns to scale, and potential real per capita income (i.e., the level of per capita income at free trade) is assumed to depend uniquely on the economy-wide ratio of capital to labour, k . If a uniform tariff is imposed on imports, actual real per capita income will be less than potential as a consequence of the welfare loss due to the divergence between foreign and domestic relative prices. Letting MM in Fig. 1 be the equilibrium demand curve for imports⁴ this welfare loss can be approximated by the area of the triangle BEC when the tariff is τ . Actual real income per capita, y , hence can be seen to depend both on the capital/labour ratio and on the tariff rate. Real consumption is given simply by income less savings and it is assumed that there

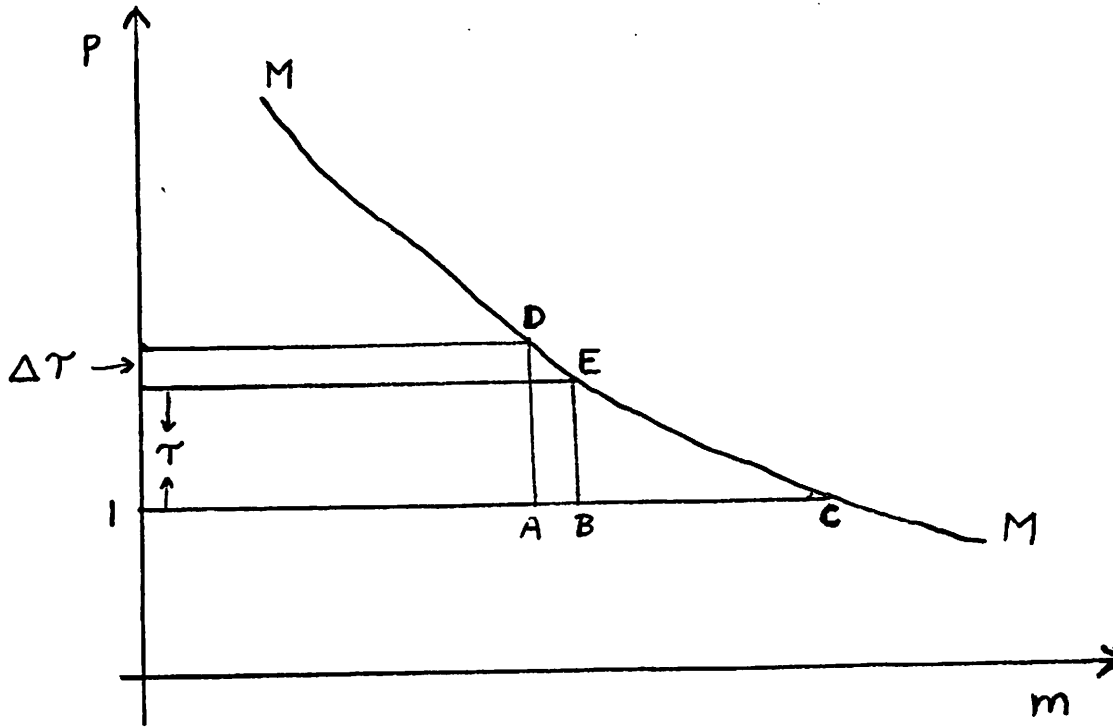


Fig. 1

exists a cardinal utility function $u(c)$ giving utility per capita as a function of consumption per capita. We postulate:

$$u'(c) > 0, u''(c) < 0.$$

Total savings per capita, s , are subdivided into two components, i.e., tariff revenue τm , where m is imports per capita, and savings from other sources, σ . The latter component consists of private voluntary savings and public savings through non-tariff taxation; we assume that no excess burden arises as a result of this taxation. We further specify that σ has an upper limit, $\bar{\sigma}$. i.e., there is a point beyond which the supply of private savings and the yield from non-tariff taxation become completely inelastic.

If total savings are greater than $\bar{\sigma}$, the only way in which they can be controlled by policy-makers is therefore by variations in the tariff rate and hence tariff revenue. When total savings are less than $\bar{\sigma}$, on the other hand, they can be controlled by variations in non-tariff taxation or in government current expenditure. Tariff revenue per capita depends on the tariff rate and on the level of imports per capita, which in turn also is partially determined by the capital/labour ratio, both because of its effect on per capita income and because relative factor proportions influence comparative advantage and hence the extent of trade.

Given the assumptions above, it is now possible to write consumption per capita as a function of k , τ and σ :

$$(1) \quad c = c(k, \tau) = y(k, \tau) - \sigma - \tau m(k, \tau)$$

The time rate of change of the capital/labour ratio will depend on savings and on the (constant) rate of population growth, n ⁵:

$$(2) \quad \dot{k} = s - nk = \sigma + \tau m(k, \tau) - nk$$

The policy-makers wish to maximize the total discounted utility enjoyed by all persons living during the time period 0 to T ,⁶ with the discount factor being the social rate of time preference, denoted by δ ; it is further assumed that the terminal capital/labour ratio must be at least equal to some minimum value k_T . The policy instruments for which optimal time paths must be found are τ and σ , where the latter cannot be greater than $\bar{\sigma}$. If one chooses the scale so that the initial population is equal to 1, the problem may be formally stated as that of finding functions $\tau(t)$, $\sigma(t)$, $\sigma \leq \bar{\sigma}$,

for $0 \leq t \leq T$, which maximize

$$(3) \int_0^T e^{(n-\delta)t} u(c) dt,$$

subject to (1) and (2) and with $k(0) = k_0$ and $k(T) = k_T$.

This may be regarded as a problem in optimal control theory, and we may apply the Maximum Principle (see Dorfman [1959] and references there) to the analysis of its solution. We first form the Hamiltonian corresponding to this problem; it is:

$$(4) H = e^{(n-\delta)t} u(c) + \lambda \dot{k}$$

where $\lambda(t)$ is an auxiliary dynamic multiplier which corresponds to the marginal value of a unit of capital at time t .

Let us first suppose that σ remains at its maximum value through-out the time period. In that case, the policy problem reduces to that of finding an optimal path $\tau(t)$ for the tariff rate over the period 0 to T. According to the Maximum Principle, for $\tau(t)$ to be an optimal path, it is necessary that

$$(5) \frac{\partial H}{\partial \tau} = e^{(n-\delta)t} u'(c) \frac{\partial c}{\partial \tau} + \lambda \frac{\partial \dot{k}}{\partial \tau} = 0,$$

and that

$$(6) \frac{\partial H}{\partial k} = e^{(n-\delta)t} u'(c) \frac{\partial c}{\partial k} + \lambda \frac{\partial \dot{k}}{\partial k} = -\dot{\lambda}.$$

Assuming that the second-order conditions are fulfilled⁷, the problem of finding the optimal time path for τ now reduces to that of finding an initial value $\lambda(0)$ such that the terminal value $k(T) = k_T$, with (1), (2), (5) and (6) being satisfied for all t .

If we do not assume that σ is equal to its maximum value throughout, it must also be regarded as a control variable, and (5) must be supplemented by

$$(7) \quad \frac{\partial H}{\partial \sigma} = e^{(n-\delta)t} u'(c) \frac{\partial c}{\partial \sigma} + \lambda \frac{\partial \dot{k}}{\partial \sigma} \geq 0$$

and

$$(8) \quad \frac{\partial H}{\partial \sigma} \cdot (\sigma - \bar{\sigma}) = 0$$

In other words, (7) must hold with equality whenever σ is below its maximum value, but may otherwise be an inequality. From (1) and (2), is easily seen that

$$(9) \quad \frac{\partial c}{\partial \sigma} = -1; \quad \frac{\partial \dot{k}}{\partial \sigma} = 1$$

Hence, (7) states that the marginal value of a unit of investment at time t must be greater than or equal to the marginal utility of consumption at time t , with equality whenever non-tariff savings are less than their maximum value. As will be shown below, (8) ensures that whenever the optimal rate of capital accumulation is small enough to be financed out of non-tariff savings, the optimal tariff will be equal to zero. In view of the excess burden associated with tariff revenue, this certainly agrees with one's economic intuition.

III. Interpreting the solution

In this section, we turn to a characterization in economic terms of the mathematical solution to the optimal control problem. Again, we first treat the case when σ is at its maximum value, and then discuss the modifications that result when this assumption is relaxed.

To interpret (5) we must first evaluate the partial derivatives $\partial c/\partial \tau$ and $\partial k/\partial \tau$. From (1), we have:

$$(10) \quad \frac{\partial c}{\partial \tau} = \frac{\partial y}{\partial \tau} - m - \tau \frac{\partial m}{\partial \tau}$$

Now, $\partial y/\partial \tau$ is the decrease in per capita income resulting from an increase in the tariff rate. From Fig. 1, it can be seen that when the tariff is increased by $\Delta \tau$, the increase in the excess burden and hence the decrease in income is approximately equal to the area of the shaded strip ADEB. This strip has an average height approximately equal to $(\tau + \frac{1}{2}\Delta \tau)$, and its base is given by $\Delta \tau(\partial m/\partial \tau)$. Finding $\partial y/\partial \tau$ by dividing the area by $\Delta \tau$ and letting $\Delta \tau$ go to zero, we find that $\partial y/\partial \tau = \tau(\partial m/\partial \tau)$. This result may be inserted into (10) to yield:

$$(11) \quad \frac{\partial c}{\partial \tau} = -m$$

We further have, using (2),

$$(12) \quad \frac{\partial k}{\partial \tau} = m + \tau \frac{\partial m}{\partial \tau} = m \left(1 + \frac{\tau \epsilon}{1 + \tau} \right)$$

where we have introduced ϵ , the price elasticity of the equilibrium demand for imports;⁸ it is given by:

$$(13) \quad \epsilon = \frac{p}{m} \frac{\partial m}{\partial p} = \frac{1 + \tau}{m} \cdot \frac{\partial m}{\partial \tau},$$

recalling the fact that the ex-tariff price of imports has been assumed to be 1, so that the tariff-inclusive price p equals $(1 + \tau)$.

After division by m and rearranging terms, we may now rewrite

(5) as

$$(14) \quad e^{(n-\delta)t} u'(c) = \lambda \left(1 + \frac{\tau \epsilon}{1 + \tau} \right)$$

or

$$(15) \quad \mu = \lambda \theta$$

where $\mu = e^{(n-\delta)t} u'(c)$ is the marginal utility of consumption at time t , and $\theta = \left(1 + \frac{\tau \epsilon}{1 + \tau} \right)$ may be called the excess-burden factor. It is easily seen that in fact $\theta = \frac{\partial k}{\partial c}$, i.e., θ indicates the number of units of capital formation that can be obtained through the sacrifice of a marginal unit of consumption. Since $\epsilon < 0$, we will have $\theta < 1$ whenever the tariff rate is positive. In other words, because of the excess-burden effect, a marginal unit of investment will cost more than one unit of consumption. Equation (15) may thus be interpreted as saying that the marginal value of a unit of capital must be equal to its marginal cost in utility terms, i.e. the discounted marginal utility of consumption multiplied by $1/\theta$, the marginal cost of a unit of investment in terms of foregone consumption units.

It is furthermore easy to see that when τ is set so as to maximize tariff revenue⁹ we will have $\theta = 0$; but from (5), it is clear that along the optimal path, this will only happen if $\lambda \rightarrow \infty$. As long as λ remains finite, the optimal tariff will always be less than the revenue-maximizing one, which implies $\theta > 0$. It is also seen that, given the tariff level, θ will be smaller the greater is ϵ in absolute value. Hence, given the marginal utility of consumption and the marginal value of a unit of capital, the optimal tariff will be lower the greater is ϵ in absolute value. Since the "marginal excess burden" of a tariff increase varies directly with ϵ , this is a plausible conclusion, and is similar to the results derived by Vanek [1971].

Let us now consider the modifications which must be made when we allow σ to be less than its maximum value. Equation (7) may be taken to state that we must then have (again using μ for the discounted marginal utility of consumption):

$$(16) \quad \mu \leq \lambda$$

with equality whenever $\sigma < \bar{\sigma}$. Now whenever we have positive tariff rate and $\epsilon < 0$, we will have $\theta < 1$, so that (15) implies that (16) is then satisfied with inequality. Hence we must always have $\sigma = \bar{\sigma}$ whenever $\tau > 0$. On the other hand, equality in (16) requires $\theta = 1$ in (15), which will only happen when $\tau = 0$, so that we can only have $\sigma < \bar{\sigma}$ when the tariff rate is zero. As observed above, this result certainly makes sense intuitively: tariff revenue is an economically inferior source of finance for capital formation because it has an excess burden, and should be used only when other sources of financing have been exhausted. A comparison of (15) and (16) also yields the intuitively obvious result that the optimal tariff rate will never be negative.

We now turn to an interpretation of (6), where the left hand side specifies the rate at which the price of capital should decline along the optimal path. Dividing both sides by λ , using (15), and evaluating $\partial c/\partial k$ and $\partial \dot{k}/\partial k$ from (1) and (2), one obtains:

$$(17) \quad -\frac{\dot{\lambda}}{\lambda} = \theta(y_k - \tau m_k) + \tau m_k - n$$

where $y_k = \partial y/\partial k$ and $m_k = \partial m/\partial k$.

When $\theta = 1$, (17) will reduce to the familiar expression:

$$(18) \quad - \frac{\dot{\lambda}}{\lambda} = y_k - n$$

which states that the proportional rate of decline of the value of capital must be equal to the marginal product of capital minus the rate of population growth. This is identical with the corresponding result in the standard form of the optimal accumulation problem for a closed economy (see Dorfman [1969], p. 825). When $\epsilon < 0$ and $\tau > 0$, however, we will have $\theta < 1$, so that the proportional rate of decline of λ is smaller than indicated in (18). An intuitive justification of this can be seen as follows. As Dorfman has pointed out, $-\dot{\lambda}$ can be regarded as the social marginal cost of holding one unit of capital over a unit interval;¹⁰ along the optimal path, this cost must be equal to the contribution that this capital makes to social welfare. This contribution is given by the value of the marginal addition to capital formation, plus the value of the marginal addition to consumption of the extra unit of capital. Since the marginal value of one unit of consumption corresponds to less than the value of one unit of capital because of the excess burden factor, however, the consumption increase must be weighted by the factor $\theta < 1$, so that the total contribution of a unit of capital to welfare and hence also the cost of holding it along the optimal path, must be less than it would be in the absence of an excess-burden effect (i.e., than it would be if θ were = 1).

Differentiating (12) logarithmically with respect to time, one obtains:

$$(19) \quad \frac{\dot{\mu}}{\mu} = \frac{\dot{\lambda}}{\lambda} + \frac{\dot{\theta}}{\theta}$$

Again if θ is uniformly equal to 1, this corresponds to the result from the standard problem: the proportional rate of decline of the value

of a unit of capital along the optimal path should be the same as the proportional rate of decline in the marginal utility of consumption over time. Without an excess burden, the "utility" cost of acquiring a unit of capital is indeed equal to the marginal utility of consumption, so that this result corresponds to the fact that the rate of change in the value of capital must be equal to the rate of change in its marginal cost in terms of utility. The same general principle holds when $\theta < 1$, with the modification that the marginal "utility" cost of a unit of capital is now equal to the marginal utility of consumption multiplied by $1/\theta$, the number of units of consumption which must be sacrificed in order to acquire one unit of capital. The proportional rate of change in this cost is equal to the rate of change in the marginal utility of consumption plus the proportional rate of change of $1/\theta$. This is indeed the result that emerges when one rearranges terms in (15).

IV. The Time Paths of τ and σ .

We now wish to use equation (19) to characterize the solution to the optimization problem explicitly in terms of the time paths of τ and σ . Consider first those portions of the optimal path where $\tau > 0$; as argued above, we will have $\sigma = \bar{\sigma}$ whenever this is the case. The time path of τ can then be derived as follows. Recalling the definition of μ , differentiating (1) with respect to time (holding σ constant) and using (11) one finds:

$$(20) \quad \frac{\dot{\mu}}{\mu} = (n-\delta) + \frac{u''(c)}{u'(c)} \dot{c} = (n-\delta) + \frac{u''(c)}{u'(c)} [k(y_k - \tau m_k) - m\tau \cdot \frac{\dot{\tau}}{\tau}]$$

The rate of change of the excess burden factor θ also depends

on the time path of the tariff rate. Differentiating θ logarithmically with respect to time yields:

$$(21) \quad \frac{\dot{\theta}}{\theta} = \frac{1}{\theta} \frac{d\theta}{d\tau} \dot{\tau} = \left(\frac{\epsilon}{\theta(1+\tau)^2} \right) \dot{\tau} = \frac{\theta - 1}{\theta(1+\tau)} \cdot \frac{\dot{\tau}}{\tau}$$

Substituting (17), (20) and (21) into (19) and rearranging terms, we finally obtain the following equation:

$$(22) \quad -\delta + \frac{u''(c)}{u'(c)} [\dot{k}(y_k - \tau m_k) - \tau m \frac{\dot{\tau}}{\tau}] - \frac{\theta - 1}{\theta(1+\tau)} \cdot \frac{\dot{\tau}}{\tau} = \\ = -\theta(y_k - \tau m_k) - \tau m_k$$

During those phases of the optimal program, when $\tau = 0$, we will have $\theta = 1$. The right-hand side of (22) then reduces to $-y_k$, and the third term on the left-hand side drops out. To find the optimal path of σ at such times, we differentiate (1) with respect to time, holding τ constant at zero, and substitute for \dot{c} in (20). Equation (22) then becomes:

$$(23) \quad -\delta + \frac{u''(c)}{u'(c)} [\dot{k} y_k - \dot{\sigma}] = -y_k$$

It may be worth observing that during these phases, the behavior of consumption and capital formation is identical to what it would be in the solution of the standard optimal savings problem for a closed economy.

We may now use (22) for a characterization of the optimal time path of τ during those phases when it is greater than zero. For simplicity, we will carry out the analysis under the assumption that the values of y_k , m_k , and the elasticity of the marginal utility

of consumption $u''(c)/u'(c)$, are all constant throughout the program. We also postulate that $y_k > \delta$, i.e., that the marginal productivity of capital is greater than the social rate of time preference.¹¹

To bring out the influence of the excess burden factor as clearly as possible, we will first analyze the optimal tariff policy when $\epsilon \rightarrow 0$, and then consider how it changes when higher absolute values of ϵ are assumed. When $\epsilon \rightarrow 0$, the value of θ will approach one, and the time paths of consumption and capital formation will again become identical to those that would be optimal if savings could be directly controlled. Equation (22) reduces to

$$(24) \quad -\delta + \frac{u''(c)}{u'(c)} [k(y_k - \tau m_k) - m\tau \frac{\dot{\tau}}{\tau}] = -y_k$$

By hypothesis, $y_k > \delta$, so that the second term on the L.H.S. must be negative. With the elasticity u''/u' being negative, this means that the expression within square brackets (which is simply the rate of growth of consumption) must be positive. It has to be larger than the lower is u''/u' in absolute value. The first term within the square brackets reflects the rate at which consumption would grow if τ were held constant. Given y_k , this rate will be higher the higher are total savings, the smaller is the term τm_k ,¹² and the lower is the value of m_k , i.e., the increase in imports as the capital/labour ratio increases.¹³

Hence it is seen that the warranted proportionate rate of increase in the tariff rate will be greater the greater the extent to which these conditions are true, the larger is u''/u' in absolute value, and the smaller is total tariff revenue. The rate of increase will also be greater the higher is the value of the social rate of time preference.

One may interpret the foregoing discussion in somewhat more intuitive terms: the higher is the social rate of time preference and the higher the elasticity of the marginal utility of consumption, the slower will be the rate of increase in consumption along the optimal path. In order to slow down this rate of increase, the tariff rate will have to increase faster the greater the rate of increase in the capital/labour ratio, and the higher the marginal propensity to consume at a constant tariff rate. We may further observe that a slower rate of growth of consumption will permit a higher initial consumption, given the initial and terminal conditions on the capital stock. Thus, the greater the extent to which the conditions above are fulfilled, the more likely it is that an optimal policy will involve a relatively low tariff initially. It is easy to demonstrate that it may even involve an initial phase of free trade followed by a phase of rising tariffs, if non-tariff savings are high enough.

A comparison between (23) and (24) shows that the analysis of the time pattern of σ during phases when $\tau = 0$ is very similar to the case just discussed. This is to be expected, since there is no excess burden in either case. The only difference stems from the fact that in the latter case, we are discussing a tax rate which is levied on a changing tax base, whereas in the former case, total savings are controlled directly. In view of the close similarity, we omit a detailed discussion of this case.

We now turn to an assessment of the influence of the excess burden factor on the optimal policy by relaxing the assumption that $\epsilon \rightarrow 0$, so that θ becomes less than one whenever the tariff is greater than zero.

The first thing to note is that, as long as $(y_k - \tau m_k) > 0$, this will decrease the absolute value of the right-hand side of (22); it is easy to show that this increases the likelihood of the optimal policy involving a rising tariff level. Furthermore, it influences the speed with which the tariff will be changing along the optimal path, by adding a term reflecting the excess burden to the coefficient of $\dot{\tau}/\tau$ on the left-hand side of (22). As noted before, when the elasticity is greater than one in absolute value, there will also be an upper limit to τ along the optimal path, since it will never be greater than the value which would maximize tariff revenue.¹⁴ It is also seen in (22) that as the tariff approaches its upper limit, we will have $\theta \rightarrow 0$, so that the coefficient of $\dot{\tau}/\tau$ approaches infinity, and the optimal rate of change of τ will go to zero. This may significantly influence the optimal time pattern of the tariff policy. Suppose for example that if ϵ were zero and there were no excess burden, the optimal policy would have involved an initially low and rising tariff level. If on the other hand the elasticity were high in absolute value, the revenue-maximizing tariff rate would be relatively low and the tariff rate might have to stop increasing considerably sooner than would be the case if ϵ were zero. Since this would imply lower levels of saving during some phases of the program it is likely that the initial tariff level and the initial level of savings would have to be higher in the high-elasticity case. Another way of interpreting this result is to note that in the absence of an excess burden, or with only a low one, an optimal policy would involve transferring a substantial share of the financing of capital

formation to future generations by means of a rising tariff level. When the elasticity is high in absolute value, however, and the excess burden is significant, the extent of this transfer is likely to be smaller. In other words, the presence of the excess burden reduces the degree to which the policy-makers can redistribute the burden of capital formation over time in order to finance it at the lowest possible cost in terms of utility.

Similar considerations can be applied to the case where the optimal policy without an excess burden would involve an initially high but falling tariff level. If an excess burden is introduced, the warranted proportional rate of decline would become smaller, both because the right-hand side of (22) would become smaller in absolute value,¹⁵ and because the coefficient of $\dot{\tau}/\tau$ on the left-hand side would become larger. The optimal level of savings would therefore decline more slowly in the presence of an excess burden (both because the tariff would decline more slowly, and because a given proportional reduction in the tariff would reduce savings by a smaller amount the larger the elasticity in absolute value). Hence, given the initial and terminal conditions, the initial levels of the tariff rate and total savings would be likely to be lower in the high-elasticity case.¹⁶ In this case, without a significant excess burden, the optimal policy would involve a high degree of redistribution of welfare toward future generations by means of an initially high but declining tariff rate; again, however, the presence of a significant excess burden reduces the extent to which this should be done under an optimal policy. As noted in the introduction, a type of policy which involves heavy initial restraint and subsequent rapid growth of consumption has sometimes

been recommended for low-income countries, e.g., in the work of Goodwin and Stoleru. The present analysis indicates that such a policy may be far from optimal when tariff revenue is a significant source of financing of capital formation.

V. Summary and Qualifications

In many low-income economies taxes on foreign trade constitute a very large share of total government revenue, in spite of the well-known fact that such taxes will generally lead to losses in economic welfare. One reason for this is the fact that the policy-makers are anxious to increase the rate of capital formation through public savings, and it may be difficult to raise large amounts of tax revenue from other sources, or to stimulate private savings. The policy-makers may therefore be willing to accept certain efficiency losses in order to increase capital formation. Existing analysis of optimal rates of capital formation has typically disregarded the possibility that the financing of capital formation may lead to such losses, however. In the present paper, we have considered the case where the rate of capital accumulation is controlled by the tariff rate, on the assumption that tariff revenue is used for development financing. It was found that the optimal tariff policy over time depended heavily on the elasticity of demand for imports, and that the solution to the optimal accumulation problem differed in many respects from the "traditional" solution, due to the existence of the excess burden. In particular, it was argued that when the demand elasticity was high, the influence of the excess burden was particularly important, and the policy of a degree of high initial austerity with

subsequent rapid growth of consumption often prescribed by traditional analysis was much less likely to be optimal when the excess burden was taken into account.

Several qualifications of the preceding analysis are in order. In the first place, the analysis of the optimal tariff policy in this paper only considers the objective of raising revenue for capital formation. But as Vanek [1971] has noted, taxes on international trade have a multiplicity of objectives: if a country has some degree of monopoly power over its exports, a non-zero level of trade taxation will be optimal; a country may wish to promote industrialization through protection of certain domestic industries, etc. In actual policy-making, any such objectives must of course be taken into account together with the capital formation objective in formulating an optimal trade policy.

Secondly, it was assumed that the amounts of capital formation that could be financed from sources other than tariff revenue, had a given and fixed upper limit. This clearly is an oversimplification: policy-makers in low-income countries are generally faced with a large number of ways in which they can raise finance for capital formation, including measures to strengthen the income tax system (often they yield might be considerably increased simply by more rigid enforcement of existing tax laws), increasing taxation on domestic production, heavier taxation of foreign investors, etc. Another important policy variable which directly influences public savings is the rate of current government expenditure. All these ways of raising public savings, however, may entail losses in the form of an excess burden similar to those for tariffs, or may involve heavy collection costs. To find a truly optimal policy, the marginal

cost of raising revenue from all these sources should be equalized, and the appropriate degree of reliance on trade taxes would depend on the costs in all these categories.

Finally, the general argument may be made that complex optimization models such as the one described in this paper are of limited value as guides to decision-making in low-income countries, in view of the short planning horizon of the policy-makers, and in view of the constraints (political, administrative, etc.) under which their decisions must be made. While there is considerable merit in this viewpoint, it is nevertheless true that the necessity of raising revenue for purposes of stimulating capital formation and development is often cited by the policy-makers themselves as a reason for imposing relatively high levels of trade taxes. It therefore appears appropriate to investigate the conditions under which economic analysis would lead one to conclude that such a policy would be an optimal one.

Footnotes.

1. For a description of the large share of trade taxes in total government revenue in some low-income countries, see, e.g., Due [1963].
2. It also has the peculiar implication that when the elasticity of demand for imports is constant and less than one in absolute value, the optimal policy would involve collection of all income as tariff revenue to be devoted to investment!
3. I.e., the curve showing equilibrium levels of imports as a function of their relative price, taking into account the adjustments both in consumption and resource allocation which will take place when the tariff changes.
4. This is equivalent to measuring output, savings, and the return to capital net of depreciation. The analysis can be modified to take into account uniform proportional rate of depreciation (as is done in Dorfman [1969]) without altering the substance of the results.
5. We recall again that we have abstracted from depreciation, i.e., s represents net savings.
6. We follow Dorfman [1969] in using this form of the objective function which Samuelson [1959] has called the "Benthamite criterion."
7. More precisely, (5) must define the maximum value of H with respect to τ for all t . We will not deal explicitly with the second-order conditions in the following analysis, but it is not difficult to show, using the results in section III, that as long as $u''(c) < 0$ and the price elasticity of demand for imports is negative, they will generally be fulfilled for both equations (5) and (7).

8. I.e., the relative price elasticity taking into account adjustments both in consumption and production.
9. Since $\partial(m\tau)/\partial\tau = m\theta$, maximizing tariff revenue implies $\theta = 0$.
10. Or alternatively, as the loss incurred by postponing the acquisition of a marginal unit of capital by one time unit.
11. This assumption, we believe, is likely to reflect conditions in poor, capital-scarce economy where the policy-makers have a strong concern with growth.
12. We may write $\dot{k}(y_k - \tau m_k) = \dot{k}y_k(1 - \tau(m_k/y_k))$ where the term m_k/y_k can be interpreted as a "marginal propensity to import"; it must be recalled that m_k reflects both the "income effect" of k and the "import substitution" effect. If the latter out-weighs the former, m_k may be negative. The expression within parenthesis can also be interpreted as the marginal propensity to consume at a given tariff rate. If m_k is negative, it will be greater than one.
13. Or the warranted rate of proportional decrease will be smaller (in absolute value).
14. When $|\epsilon| < 1$, there is no such upper limit since no finite tariff exists which maximizes revenue.
15. Still assuming $(y_k - \tau m_k) > 0$.
16. When $|\epsilon| > 1$, the initial tariff level can of course be no higher than the revenue-maximizing level, however strong the factors favouring an initially high level of savings.

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