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Effective Protection in General Equilibrium: A Geometrical Analysis

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EFFECTIVE PROTECTION IN GENERAL EQUILIBRIUM: A GEOMETRICAL ANALYSIS*

by

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Introduction

The purpose of this paper is to develop a simple geometrical apparatus which can be used to derive the recent results obtained mathematically by Jones [10], Mayer [11] and Batra and Casas [5] concerning the general equilibrium implications of effective protection for resource allocation and factor rewards. The general equilibrium analyses of effective protection themselves are in short supply relative to the output of the partial equilibrium analyses, and the exposition in the former has been couched almost entirely in terms of laborious and intricate mathematical tools.[1] The concept of effective protection is unquestionably superior to that of nominal protection, for the latter, being a special case of the former, conveys much less information. However, the comprehension and popularity of a concept, especially with the students, augments proportionately with the availability of the simple diagrammatical tools. Although the latter are available in abundance with the theory of nominal protection, the theory of effective protection, perhaps because of its much shorter history, enjoys no such prosperity.

The purpose of this paper is to fill this gap and develop an appropriate diagrammatic device. The main tool of our analysis will be the well known activity analysis diagram used by Batra [1] [2] in the analysis of the two-sector, two-factor general equilibrium model.

I. Assumptions and the Model

Unless otherwise specified, we assume throughout our analysis that there are three domestically produced commodities: X_2, the importable good, X_1, the exportable good, and M, the non-traded intermediate product, which is produced solely to be used as an input in the production of the importable good. A fourth commodity, Q, is imported from abroad and used as an input again in X_2 but not X_1. The material inputs could also be utilized in X_1, but this would unnecessarily complicate the analysis without adding anything to the exposition. There is perfect competition in all markets, production functions exhibit constant returns to scale and diminishing marginal rates of substitution, and two primary factors, labor (L) and capital (K), are inelastically supplied and fully employed. World prices of all traded commodities are constant, but domestic prices of the importables as well as the non-traded input may be altered due to the imposition of tariffs.

Let C_{ij} represent the amount of the i^{th} factor per unit of the j^{th} product (i = L,K,M,Q; and j = 1,2,M); for example $C_{L1} = L_1/X_1$. With full employment,

$$C_{L1} X_1 + C_{L2} X_2 + C_{LM} M = L \qquad (1.1)$$

and

$$C_{K1} X_1 + C_{K2} X_2 + C_{KM} M = K \,. \qquad (1.2)$$

Since $M \equiv C_{M2} X_2$, these two equations can be written as

$$C_{L1} X_1 + (C_{L2} + C_{LM}C_{M2})X_2 = L \qquad (1.1a)$$

$$C_{K1} X_1 + (C_{K2} + C_{KM}C_{M2})X_2 = K \,. \qquad (1.2a)$$

With perfect competition in product markets, unit costs reflect product prices. Let r stand for the reward of capital, w for the wage rate and P_j for the price of the j^{th} commodity (j = 1,2,M,Q). Then

$$C_{L1}w + C_{K1}r = P_1 \tag{1.3}$$

$$C_{L2}w + C_{K2}r + C_{M2}P_M + C_{Q2}P_Q = P_2 \tag{1.4}$$

and

$$C_{LM}w + C_{KM}r = P_M . \tag{1.5}$$

Substituting (1.5) in (1.4), we obtain

$$(C_{L2} + C_{LM}C_{M2})w + (C_{K2} + C_{KM}C_{M2})r = P_2 - C_{Q2}P_Q . \tag{1.4a}$$

If production coefficients are fixed, (1.1)-(1.5) describe the production side of the system. However, in the general case of variable coefficients, additional equations must be introduced in order to determine the input-output coefficients (C_{ij}'s). These are determined by the requirement that, with linearly homogeneous production functions, each C_{ij} is itself a linear homogeneous function of input prices. In general, we may write

$$C_{i2} = C_{i2}(w, r, P_M, P_Q) , \tag{1.6}$$

$$C_{i1} = C_{i1}(w, r) , \tag{1.7}$$

and

$$C_{iM} = C_{iM}(w, r) . \tag{1.8}$$

The production side of the model described by the system of equations (1.1)-(1.8) can be solved as follows: First, the three equations (1.3)-(1.5) containing three parameters, P_1, P_2 and P_Q, can be solved to obtain three unknowns, w, r, and P_M, in terms of the international prices of the traded goods and the input/output coefficients. We assume that this solution is unique. With w, r, and P_M so determined and with P_Q exogenously given, C_{ij}'s can be solved from (1.6)-(1.8). These values in turn can be substituted in (1.1a) and (1.2a) to derive expressions for X_1 and X_2 in terms of the given supplies of K and L; finally C_{M2} and X_2 can be used to obtain M. Hence the set of equations (1.1)-(1.8) specify a completely determinate production side of the model. The demand side will not be presented since the focus of the paper is on resource allocation and factor price changes brought about by changes in the domestic prices of the two importables, X_2 and Q, through the imposition of tariffs.

Factor-intensities in the final products are defined by columns of the C matrix which is formed by incorporating the C_{ij}'s specifying the full employment equations (1.1a)-(1.2a):

$$C = \begin{bmatrix} C_{L1} & C_{L2} + C_{LM}C_{M2} \\ C_{K1} & C_{K2} + C_{KM}C_{M2} \end{bmatrix}.$$

As regards the price equations (1.3) and (1.4a), the factor-intensities are specified by the <u>rows</u> of the C matrix, even though it does not contain C_{Q2}; the reason for this asymmetry is attributable to the fact that although C_{Q2} enters into the determination of P_2 it absorbs no proportion of domestic primary factors, because Q is produced abroad. Since it is the proportion of the domestic primary factors used in the production of each commodity that determines its factor-intensity, the use of Q by X_2 causes no modification to the factor-intensities.

At this stage it is necessary to introduce a distinction between the net and the gross capital/labor ratio in the second commodity, for X_2 is using not only K_2 and L_2 directly but by utilizing M as an input, it is also using, although indirectly, K_M and L_M embodied in the production of M. Let k_n denote the net capital/labor ratio and k_g the gross capital/labor ratio in the second commodity. Then

$$k_n = \frac{K_2}{L_2}$$

and

$$k_g = \frac{K_2 + K_M}{L_2 + L_M} = \frac{\frac{K_2}{X_2} + \frac{K_M}{M} \cdot \frac{M}{X_2}}{\frac{L_2}{X_2} + \frac{L_M}{M} \cdot \frac{M}{X_2}} = \frac{C_{K2} + C_{KM}C_{M2}}{C_{L2} + C_{LM}C_{M2}} .$$

Note that no distinction between the gross and the net capital/labor ratio need be made in the case of the first commodity. It may now be observed that the determinant of C is positive if X_2 is capital-intensive relative to X_1 in the gross sense, that is, $k_g > k_1$, where $k_1 = C_{K1}/C_{L1}$ is the capital/labor ratio in the first commodity, because

$$|C| = C_{L1}(C_{K2} + C_{KM}C_{M2}) - C_{K1}(C_{L2} + C_{LM}C_{M2}) > 0$$

if

$$\frac{C_{K2} + C_{KM}C_{M2}}{C_{L2} + C_{LM}C_{M2}} = k_g > \frac{C_{K1}}{C_{L1}} = k_1 .$$

Without loss of generality, we assume hereafter that X_2 is the capital-intensive commodity and X_1 the labor-intensive commodity in the gross or the "true" sense, so that $|C| > 0$. However, this assumption does not rule out the possibility that X_2 may be actually labor-intensive relative to X_1 in the net sense, that is, k_n may be less than k_1 even if $k_g > k_1$, for $|C|$ can be written as:

$$|C| = C_{L1} \left[C_{L2}(k_n - k_1) + C_{M2}C_{LM}(k_M - k_1) \right] , \qquad (1.9)$$

where $k_M = K_M/L_M$. Clearly $|C|$ may be positive, which implies $k_g > k_1$, even if $k_n < k_1$, provided $k_M > k_1$. In other words, factor-intensities defined in the gross sense need not be identical to those defined in the net sense. A sufficient condition for the gross and the net factor-intensity rankings to be identical is that X_1 is the most labor-intensive of all domestically produced goods, so that $k_1 < k_n$ and $k_1 < k_M$.

It is worth pointing out that the presence of imported inputs makes no difference to the ranking of commodities in terms of their factor-intensities, but the presence of non-traded material inputs does. In the absence of the latter, the possibility of the conflict between the gross and net factor-intensity rankings does not exist as may be observed from the technology matrix by setting C_{M2} equal to zero. In what follows we assume that gross and net factor-intensity rankings are identical, that is, X_2 is capital-intensive relative to X_1 in both the gross and the net sense.

II. The Diagrammatical Apparatus

The full employment equations (1.1a) and (1.2a) and the price equations (1.3) and (1.4a) are depicted diagrammatically in Figures 1 and 2. Consider Figure 1 where X_2 is measured along the vertical axis and X_1 along the horizontal axis. The given supplies of capital and labor together with the input coefficients furnish the lines EF and GH which are drawn by joining the points that determine the maximum output of one commodity, with zero output of the other. Let R_{L2} and R_{K2} be the gross labor and capital output coefficients. For example, $R_{L2} = C_{L2} + C_{LM}C_{M2}$. Then $OE = L/R_{L2}$ is the maximum output of X_2 (with zero output of X_1) and is obtained by employing the entire supply of labor in X_2.[2] Along EF, labor would be fully employed, along GH capital would be, and at their intersection at J, there would be full employment of both factors. The diagram is drawn in such a way as to reflect our assumption that X_2 is capital-intensive relative to X_1. This can be verified by comparing the slopes of EF and GH, so that $(C_{L1}/R_{L2}) > (C_{K1}/R_{K2})$ or $(R_{K2}/R_{L2}) > (C_{K1}/C_{L1})$. At the production point given by J, the output of X_1 equals OB, that of X_2 equals JB and the ratio between the two outputs, i.e., X_2/X_1, is given by the slope of OJ.

Figure 2 on the other hand furnishes a geometrical statement of equations (1.3) and (1.4a); the wage rate is measured along the vertical axis

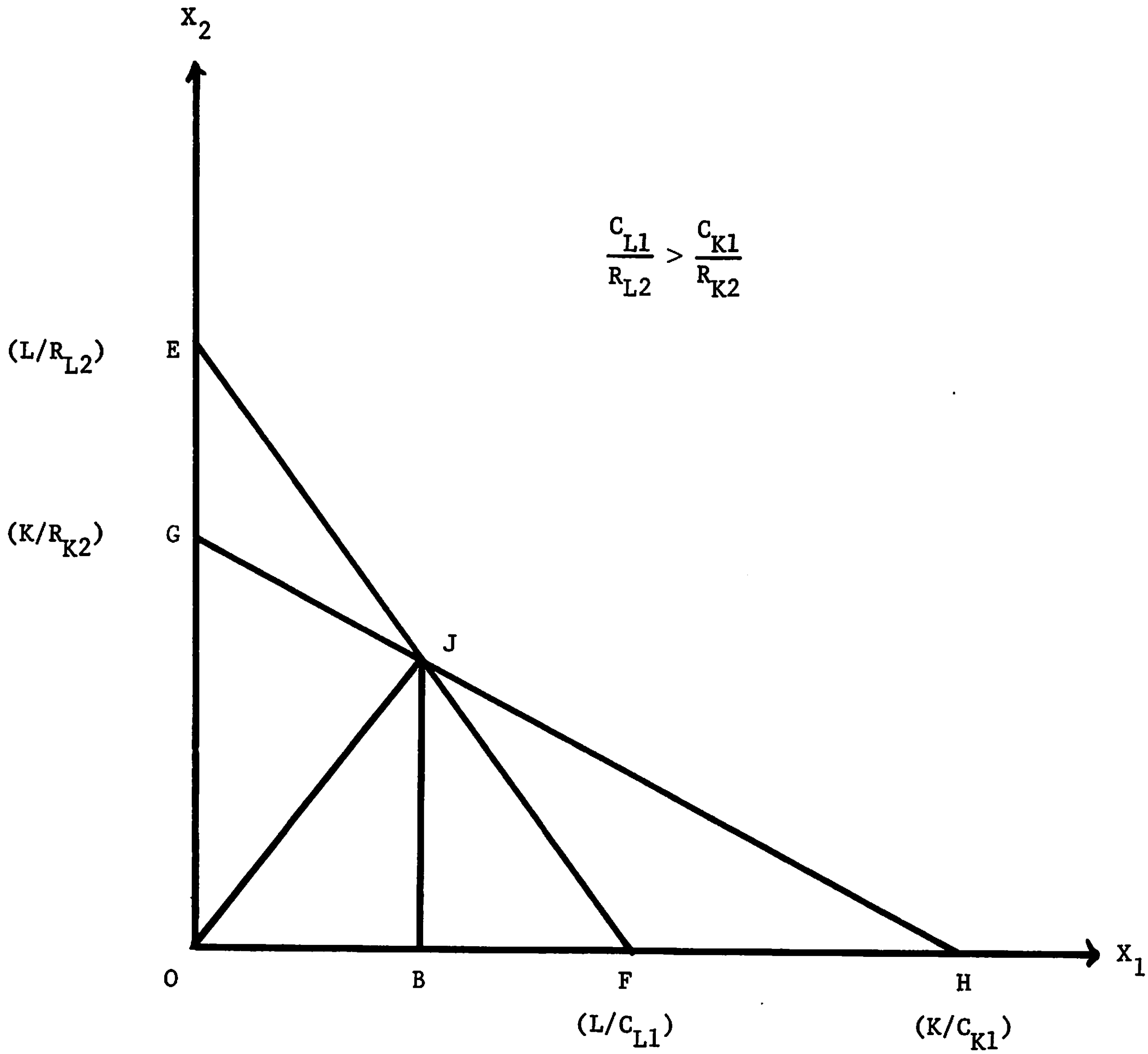

Figure 1

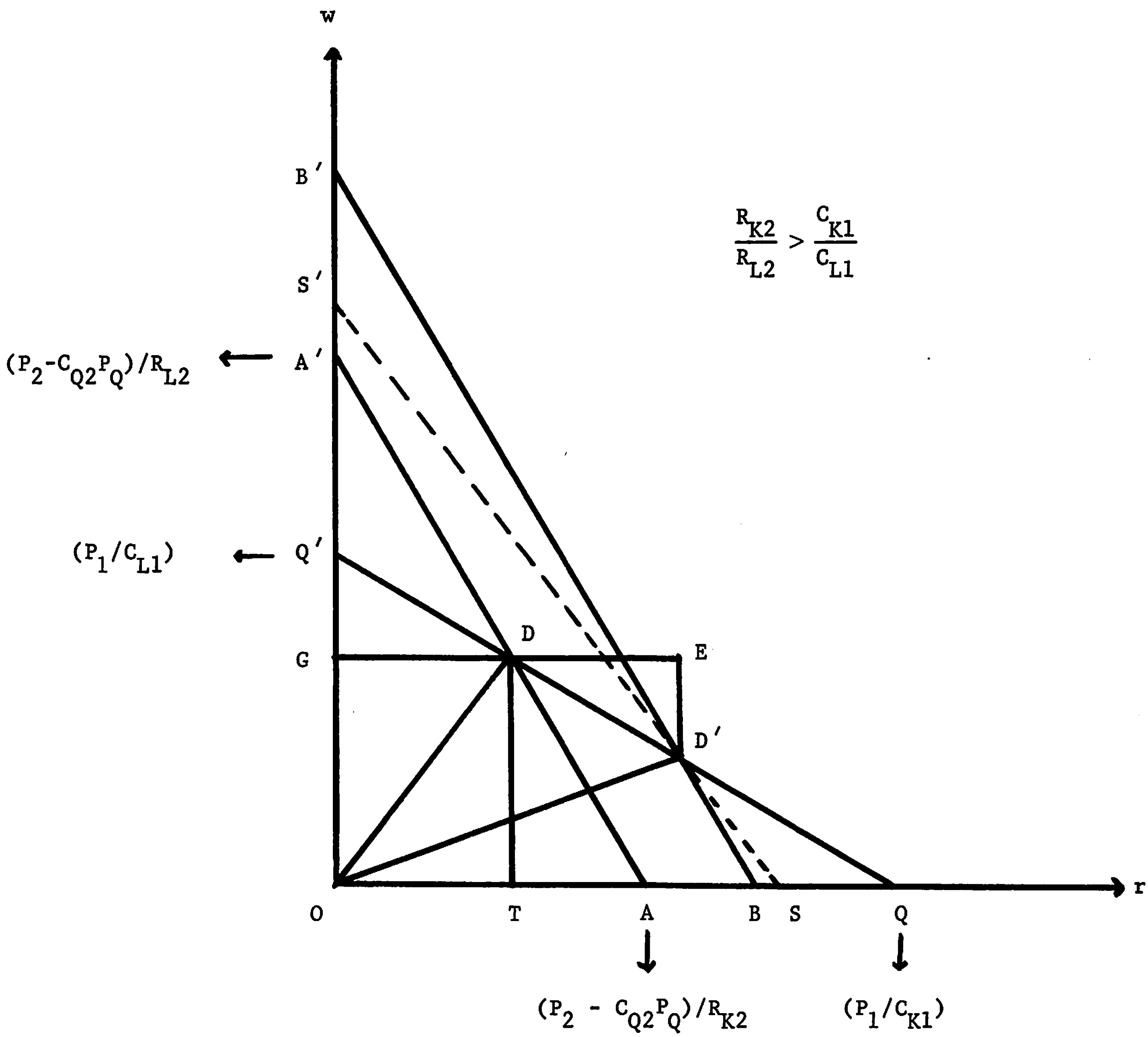
w
B′
S′
A′
$(P_2 - C_{Q2}P_Q)/R_{L2}$
(P_1/C_{L1})
Q′
G
D
E
D′
$\frac{R_{K2}}{R_{L2}} > \frac{C_{K1}}{C_{L1}}$
O
T
A
B
S
Q
r
$(P_2 - C_{Q2}P_Q)/R_{K2}$
(P_1/C_{K1})

Figure 2

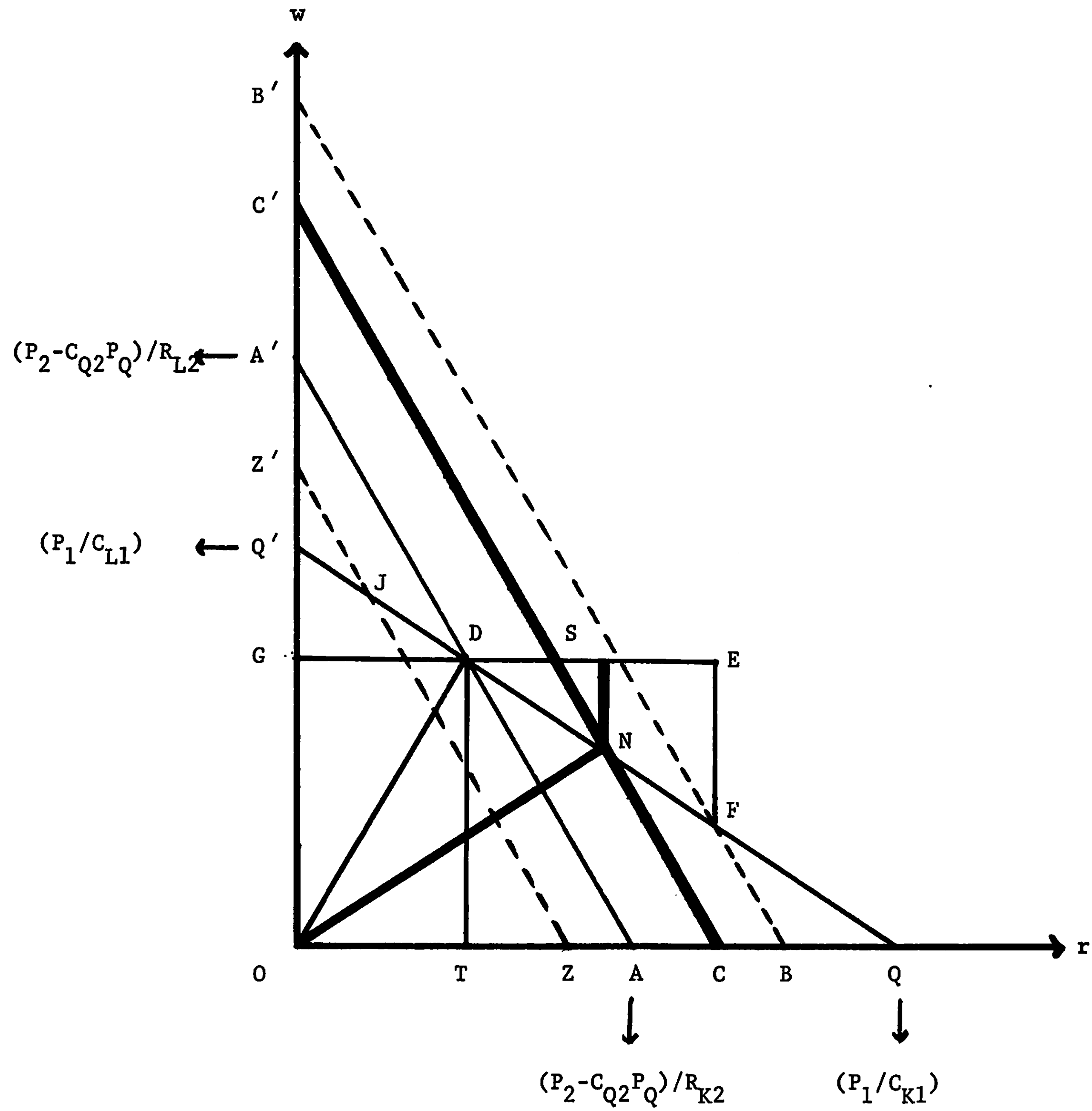
w
B′
C′
$(P_2-C_{Q2}P_Q)/R_{L2}$
A′
Z′
(P_1/C_{L1})
Q′
J
D
S
G
E
N
F
r
O
T
Z
A
C
B
Q
$(P_2-C_{Q2}P_Q)/R_{K2}$
(P_1/C_{K1})

Figure 3

and the return of capital along the horizontal axis. The initial level of market prices, P_1, P_2, and P_Q, together with the input-output coefficients determine the lines AA′ and QQ′. Each line is obtained by joining the points that represent the maximum reward of one factor and zero reward of the other. For example, $OQ' = P_1/C_{L1}$ represents the maximum level of w that will be paid to labor, assuming that the latter was the only factor involved in the production of X_1; $OQ = P_1/C_{K1}$ can be given an analogous interpretation. If no labor was employed in the production of X_1, the reward of capital would equal OQ. Along QQ′, the line joining the two points Q and Q′, P_1 equals the unit cost of production. The line AA′ is obtained in a similar manner. Here it is P_2 that equals the unit production cost along any point on AA′.[3] The point where the prices of both final goods equal the unit cost is given by D where AA′ and QQ′ intersect. At point D, the wage rate is DT, the reward of capital is OT =DG, and the wage/rental ratio is given by the slop of OD, which equals DT/OT. The construction in Figure 2 also reflects our assumption that X_2 is capital-intensive relative to X_1, as can be seen from the fact that the slope of AA′, equal to R_{K2}/R_{L2}, exceeds that of QQ′, equal to C_{K1}/C_{L1}.

The description of Figures 1 and 2 show the determination of the structure of production in our model with three domestically produced goods and four factors of production, one of which is imported from abroad. Figure 2 determines factor prices and commodity prices, whereas Figure 1 aids in determining the output levels of the final goods. Quite obviously, both these diagrams must be consistent with each other. Here it is perhaps appropriate to assert that the geometrical apparatus presented above is very simple relative to the complexity of the model. The traditional diagrammatic tools like the box diagram and the isoquant diagrams are not adequate to cope with the problems that arise in models incorporating intermediate goods.

III. Effective Protection and Real Factor Rewards

The concept of the effective rate of protection (ERP) conferred on an importable commodity takes into account not only its own nominal tariff but also the nominal tariff on imported intermediate inputs. To begin with, let us assume that a nominal tariff alone is imposed on the imports of the second commodity so that the domestic price of the second commodity increases proportionately by the amount of the tariff. For example,

$$\frac{\Delta P_2}{P_2} = P_2^* = \frac{P_2(1 + t_2) - P_2}{P_2} = t_2 .$$

In other words, the proportionate rise in the price of the second good equals the level of the tariff. Now let us see what this implies for factor rewards in terms of Figure 2. As a starting point, let us assume that production coefficients are fixed. Later on we will see that this assumption is not at all relevant to the results derived in this section; but for the time being we maintain it in the interest of exposition. With R_{L2} and R_{K2} thus given by assumption, a rise in P_2 will involve outward shift of AA′ to say BB′ in Figure 2, such that BB′ is parallel to AA′, so that the capital/labor ratio in the second industry is unchanged. With R_{K2} unchanged, letting $P_2' = P_2(1 + t_2)$

$$AB = OB - OA = \frac{1}{R_{K2}} [(P_2' - C_{Q2}P_Q) - (P_2 - C_{Q2}P_Q)] = \frac{\Delta P_2}{R_{K2}} .$$

Therefore

$$\frac{AB}{OA} = \frac{\Delta P_2}{R_{K2}} \cdot \frac{R_{K2}}{(P_2 - C_{Q2}P_Q)} = \frac{P_2^*}{1 - Q_{Q2}} ,$$

where $Q_{Q2} = C_{Q2}P_Q/P_2$ equals the relative share of the imported input in the total production cost of the second commodity and is by definition less than unity. In other words, $t_2 = P_2^* = (1 - Q_{Q2})AB/OA$. Since P_1 is unchanged and since C_{K1} and C_{L1} are unchanged by assumption, the position of QQ′ is unaltered.

The outward shift of AA′ to BB′ induced by the introduction of the tariff implies that the point where the prices of both final goods equal their unit costs shifts from D to D′, so that the reward of capital rises from DG to DE, the wage rate declines by ED′ and the wage/rental ratio declines to the lower slope of OD′. Since the proportionate rise in the rate of return on capital, equal to DE/GD, is greater than the corresponding rise in P_2, equal to $(AB/OA)(1 - Q_{Q2})$, the introduction of the tariff unambiguously raises the reward of capital, the factor employed intensively by the capital-intensive good, X_2.[4] Since w has declined, protection has lowered unambiguously the real reward of labor, the factor employed unintensively by the second good. This is a simple demonstration of the Stolper-Samuelson theorem in our model incorporating traded and non-traded material inputs.

Coming now to our assumption of constant production coefficients, we begin with the observation that first Jones [9] and subsequently many others including Jones [10], Mayer and Batra and Casas [3] [5] have shown that the effects of small changes in commodity prices on factor prices are not influenced by whether the production coefficients are fixed or variable. The reason for this lies in our assumption of linearly homogeneous production functions, because of which the "secondary" changes in input-output coefficients arising from variations in factor prices that are caused by commodity price changes tend to exert a negligible effect on the original changes in factor prices. This can be illustrated very simply in terms of Figure 2. Suppose as w/r declines as a result of the rise in P_2, R_{L2} and C_{L1} rise and R_{K2} and C_{K1} decline.[5] First, let C_{L1} and C_{K1} be unchanged, so that with P_1 unaltered, the position of QQ′ is also unaltered. Now a rise in R_{L2}, with P_2 unchanged at the new level, will tend to shift B′ along the vertical axis in Figure 2 towards A′. This shifting by itself would tend to raise w and lower r. For example, suppose B′ shifts to S′ and if R_{K2} is kept unchanged for the time being, one could draw BS′ that would intersect QQ′ between D and D′, showing that w would rise and r decline from their levels prevalent at D′. However, a decline in R_{K2} would create exactly the opposite effect. It would tend to shift B along the horizontal axis towards Q, and in the process lower w and raise r. This decline in R_{K2} then would tend to neutralize the effects of a rise in R_{L2} on w and r. In the limiting case of small variations, the position of D′ would not be altered. For example, if B shifts to S, then SS′ intersects QQ′ at D′, and w and r are unchanged. A similar explication applies to the influence on w and r exerted by variations in C_{K1} and C_{L1}. The end result is that the effect on factor prices of variations in input coefficients which initially arose from changes in factor prices is rather small and for all practical purposes can be ignored. A categorical and convincing statement verifying to the verity of this conclusion has been made by Jones [9, p. 560].

> ...the relationship between changes in factor prices and changes in commodity prices is identical in the variable and fixed coefficients cases, an example of the Wong-Viner envelope theorem. With costs per unit of output being minimized, the change in costs resulting from a small change in factor prices is the same whether or not factor proportions are altered. The saving in cost from such alterations is a second order small. (His italics)

Thus we conclude that the use of Figure 2 is most appropriate for analyzing the implications of variations in factor prices for changes in factor rewards, and our procedure of keeping R_{i2} and C_{i1} (i = L,K) constant, while shifting AA′ to reflect any change in $(P_2 - C_{Q2}P_Q)$ is not open to any objection even if input coefficients are variable.

Next we consider the effects of a change in P_Q alone on factor prices, keeping P_1 and P_2 unchanged. The focus of analysis shifts to Figure 3 which collects a part of the information gathered in Figure 2. Thus AA' and QQ' as before represent unit cost lines for X_2 and X_1 respectively; BB' represents a shift in P_2 resulting from the introduction of a tariff on the second commodity if there was no tariff on the intermediate input, such that

$$P_2^* = (1 - Q_{Q2})AB/OA \ .$$

Let us disregard BB' for the time being and assume that a tariff (t_Q) is imposed on the imported intermediate good so that P_Q rises domestically. Following the procedure outlined above, this can be depicted by a downward shift of AA' to ZZ' such that ZZ' is parallel to AA'. Under the procedure that keeps R_{L2} and R_{K2} "temporarily" unaltered, and denoting $P_Q' = P_Q\ (1 + t_Q)$

$$AZ = OA - OZ = \frac{1}{R_{K2}}\left[(P_2 - C_{Q2}P_Q) - (P_2 - C_{Q2}P_Q')\right]$$

$$= \frac{1}{R_{K2}}\left[C_{Q2}\ (P_Q' - P_Q)\right] = \frac{C_{Q2}\Delta P_Q}{R_{K2}}\ , \quad \text{so that}$$

$$\frac{AZ}{OA} = \frac{C_{Q2}\Delta P_Q}{R_{K2}} \cdot \frac{R_{K2}}{(P_2 - C_{Q2}P_Q)} = \frac{Q_{Q2}P_Q^*}{(1 - Q_{Q2})}\ ,$$

where $P_Q^* = (\Delta P_Q/P_Q)$ equals t_Q. The consequences of the introduction of t_Q alone for factor rewards are now straight forward. The equilibrium representing the equality of prices of the final goods to their respective unit costs shifts from D to J in Figure 3, w rises and r declines. Since P_1 and P_2 are unchanged, we conclude that the imposition of the tariff on the imported input alone, lowers the real reward of the primary factor which is employed intensively by the final commodity using the imported input and raises the real reward of the other primary factor.

As suggested before, the ERP given an imported good takes into consideration the nominal tariffs on the final as well as the intermediate imported good. If t_2 and t_Q were introduced simultaneously, then with reference to Figure 3,

$$\begin{aligned} ERP &= \frac{AB}{OA} - \frac{AZ}{OA} = \frac{AC}{OA} \\ &= \frac{P_2^* - Q_{Q2}P_Q^*}{1 - Q_{Q2}} = \frac{t_2 - Q_{Q2}t_Q}{1 - Q_{Q2}}, \end{aligned}$$

where BC = AZ.[6] In other words, line CC′ represents the introduction of ERP for the second commodity (CC′ is parallel to AA′, BB′ and ZZ′) in contrast to BB′ representing the imposition of only t_2 and ZZ′, which represents the introduction of t_Q alone. The concept of the negative ERP frequently encountered in the literature can also be visualized very simply from Figure 3. If CC′ lies above AA′, ERP is positive and $t_2 > Q_{Q2}t_Q$; if below AA′ ERP is negative and $t_2 < Q_{Q2}t_Q$. Since $Q_{Q2} < 1$, a necessary condition for a negative ERP is that $t_Q > t_2$. Figure 3 depicts the case of the positive ERP and the point where both P_1 and P_2 equal the unit cost in the presence of the ERP is given by N, which shows that the effect of the introduction of the ERP, equal to AC/OA, is to lower w by NS, raise r by DS and lower the w/r ratio to the slope of the ray ON. Thus effective protection conferred on a commodity leads to a rise in the relative reward of the primary factor which is utilized intensively by the protected commodity. Whether effective protection also generates categorical effects on the real factor rewards in terms of both commodities, however, is no longer certain. Since P_1 is unchanged, a rise in r and a decline in w amount to a rise in the real reward of capital and a decline in the real wage rate in terms of the first commodity. The lower w and a higher P_2 also imply the lower real wage rate in terms of the second commodity. The complication arises in the case of the real reward of capital in terms of the second commodity, for it is no longer clear whether the proportionate rise in r would necessarily exceed the corresponding rise in P_2. However, a sufficient condition is available under which this result can be shown to exist. Let r* be the proportionate change in r. From Figure 3, it is evident that

$$r^* = \frac{DS}{GD} > \frac{AC}{OA} = ERP = \frac{t_2 - Q_{Q2}t_Q}{1 - Q_{Q2}}$$

$$= \frac{P_2^* - Q_{Q2}P_Q}{1 - Q_{Q2}} .$$

Now ERP $\geq t_2 = P_2^*$ if $t_2 \geq t_Q$, which implies that $r^* > P_2^*$ for $t_2 \geq t_Q$. In other words, a rise in the real reward of capital in terms of the second commodity is ensured if the nominal tariff granted the final good exceeds or equals the nominal tariff imposed on the imported input.

All this is directly relevant to the validity of the Stolper-Samuelson theorem in our model incorporating intermediate products. The following theorem can be immediately derived:

Theorem 1:

In the presence of nominal tariffs on the imports of the final as well as intermediate products, the Stolper-Samuelson theorem may not hold, provided the output tariff is less than the input tariff, even through the ERP on the final product itself is positive.

Thus it is interesting to note that the Stolper-Samuelson theorem which until now has been couched in terms of the nominal tariffs may not be valid in terms of the effective tariff. Of greater interest, however, is the fact that the ERP always raises the relative reward of the primary factor intensively employed by the "effectively" protected commodity. Thus we are back in the pre-Stolper-Samuelson world, for the main contribution of Stolper and Samuelson was to demonstrate that (nominal) protection moves the real and the relative reward of a factor in the same direction. With effective protection, however, this may no longer be true, and we confront the same "index number" problem, which prior to the contribution by Stolper and Samuelson, is believed to have hindered the achievement of unequivocal conclusions concerning the effects of protection on income distribution.

IV. The ERP and Resource Allocation

The results derived in the previous section have direct relevance to the question of the implications of the ERP for the allocation of resources. In the two-factor, two-commodity model without imported inputs, nominal protection always results in the shift of resources away from the unprotected commodity towards the protected commodity and in the process raises the output of the latter at the expense of the output of the former. Does this result continue to hold when nominal protection gives way to effective protection? The answer depends wholly on how factor prices change in response to the introduction of the ERP.

We have shown in the preceeding section that the ERP conferred on the capital-intensive commodity, X_2, raises the reward of capital (though not necessarily its real reward) and lowers the wage rate. If the supply of the imported input were fixed, with given supplies of capital and labor, this would ensure a rise in the output of X_2, a decline in the output of X_1 and a rise in the X_2/X_1 ratio. However, when the world prices are given and an intermediate good is imported at these prices, the supply of the imported input is not constant, especially when substitution between the domestic and imported inputs is permitted. Is it then possible that X_2/X_1 may actually decline even though effective protection was provided to X_2? In this section, we first show that such a "perverse" result is within the realm of possibility and then derive the conditions that are necessary for the existence of this outcome.

A. The Non-Traded Material Input Does Not Exist:

To begin with, we analyze the simpler case where no intermediate product is produced domestically. This is the model utilized by Jones [10]. Our task is considerably simplified because the set of equations (1.1), (1.2), (1.3) and (1.4a) reduces to the following set:

$$C_{L1}X_1 + C_{L2}X_2 = L \ , \tag{1.10}$$

$$C_{K1}X_1 + C_{K2}X_2 = K \ , \tag{1.11}$$

$$C_{L1}w + C_{K1}r = P_1 \ , \text{ and} \tag{1.12}$$

$$C_{L2}w + C_{K2}r = P_2 - C_{Q2}P_Q \ . \tag{1.13}$$

Figure 1 is also then replaced by Figure 4 where R_{L2} and R_{K2} are superseded by C_{L2} and C_{K2} respectively. If there were no imported inputs, then C_{ij} would be related only to w and r and a rise in r and a decline in w consequent upon the grant of protection to X_2 would lead to a rise in C_{L1} and C_{L2} and a decline in C_{K1} and C_{K2}. This follows from the assumption of linearly homogeneous production functions. Let the prime on an input coefficient denote its value in the new situation. Then

$$C'_{Lj} > C_{Lj} \text{ and } C'_{Kj} < C_{Kj} \ (j = 1,2) \ .$$

With given supplies of K and L, all the four points E, F, G and H in Figure 4 would move along the axes either up or down. For example, with $C'_{L2} > C_{L2}$, point E would move down to say, E$'$. Similarly, G would shift to G$'$, F to F$'$ and H to H$'$, and the new full employment equilibrium would be established at J$'$ where the dotted lines E$'$F$'$ and G$'$H$'$, representing, respectively, the labor and capital constraint lines in the new situation, intersect. It may be observed that X_2 and (X_2/X_1) ratio have risen and X_1 declined in the new situation. This is how nominal protection in the usual two-good, two-factor model raises the output of the protected commodity at the expense of the

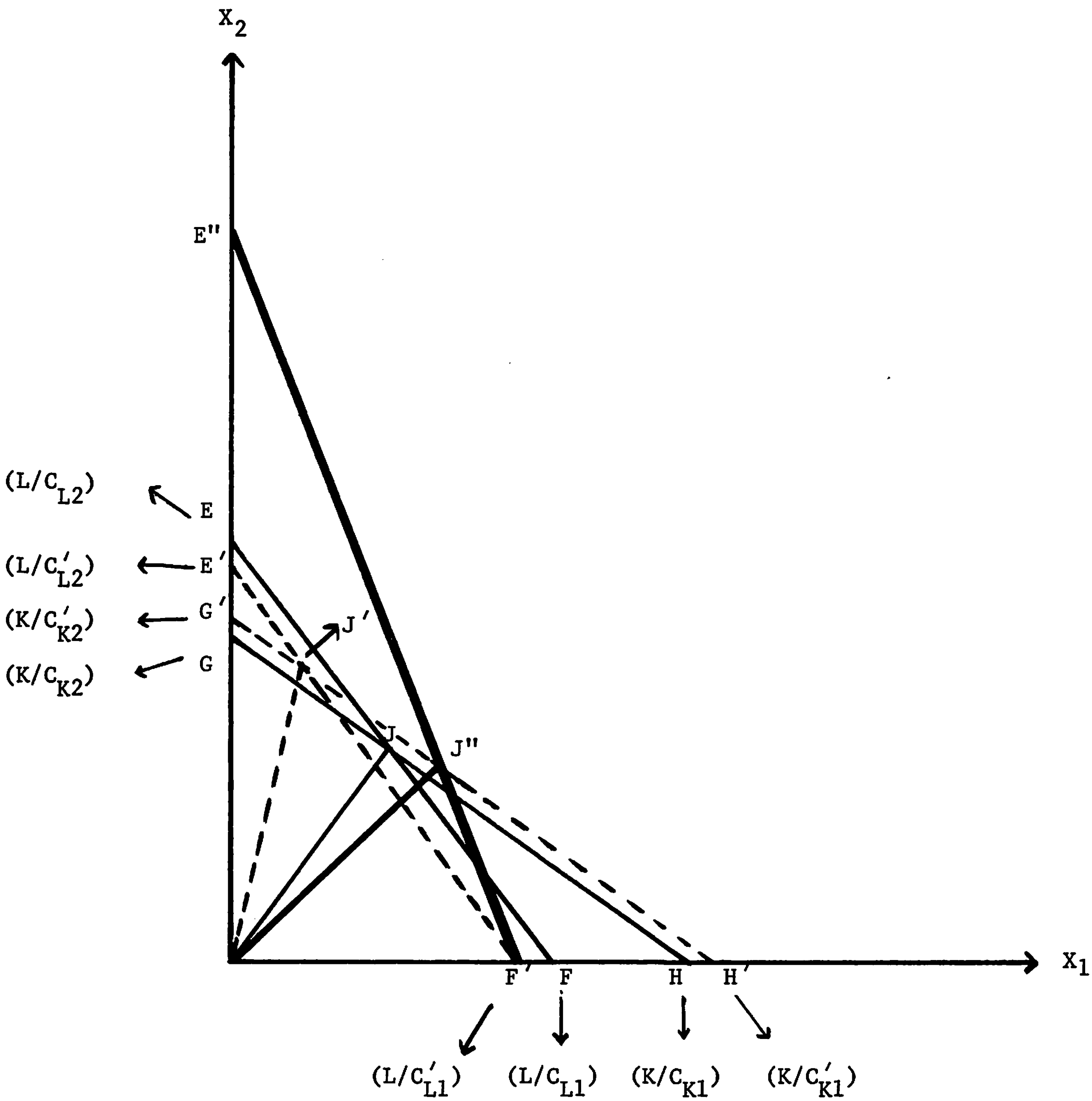

Figure 4

output of the unprotected commodity.

Let us now suppose that an intermediate good (Q) is also imported in addition to X_2. If the effective protection is granted only by introducing a nominal tariff on X_2, the conditions of the model do not undergo a substantial change; P_Q remains constant and the only factor prices to change are w and r, so that the results are very much similar to those derived above from Figure 4. Consider the other extreme case where effective protection is granted to X_2 by subsidizing the import of the intermediate good, so that P_Q declines and $(P_2^* - Q_{Q2}P_Q^*) > 0$ because $P_2^* = 0$ and $P_Q^* < 0$. Following Jones [10] we assume that all factors are gross substitutes. This assumption is not crucial to our analysis and it underlines the fact that the "perverse" results derived below do not depend on the complementarity between any two factors. This type of effective protection would also result in a rise in r and a decline in w. Since the first industry does not use the intermediate product, C_{L1} would rise and C_{K1} would decline as before, so that F in Figure 4 would shift backward, say again, to F' and H would shift outward, say, to H'. The outcome is not so clear in the second industry where input coefficients now depend on three factor prices, w, r and P_Q. Here the change in C_{L2} and C_{K2} is determined by the changes in w and r relative to the change in P_Q. Since $r^* > 0$, $w^* < 0$ and $P_Q^* < 0$, both (r/w) and (r/P_Q) rise, so that substitution must occur against the amount of capital employed in the second commodity and in favour of labour and Q. In other words, C_{K2} must decline. In addition, if $P_Q^* > w^*$, that is, if the proportionate decline in the wage rate exceeds the corresponding decline in P_Q so that both (P_Q/w) and (r/w) rise, substitution takes place against the imported input and in favour of labor, so that C_{L2} must rise. Thus if

$$r^* > P_Q^* > w^* , \qquad (1.14)$$

C_{K2} declines and C_{L2} rises, which in turn ensures the outward shift of G to, say, G′ and a backward shift of E to, say, E′. In other words, the direction of shifts in the input-output coefficients is the same as was the case when effective protection involved the imposition of a tariff on the imports of X_2 alone. Thus if condition (1.14) is satisfied, the ERP granted to X_2 by subsidizing the imports of Q leads to a rise in X_2 relative to X_1. However, if $P_Q^* < w^* < r^*$, that is if the decline in w falls short of the decline in P_Q so that (w/P_Q) rises, substitution in the second industry may take place against labor and in favour of the imported input, so that C_{L2} may actually decline. Thus if

$$r^* > w^* > P_Q^* , \tag{1.15}$$

C_{K2} declines as before, but C_{L2} may also decline which means that point E shifts outward along the vertical axis and if this shift is sufficiently large, (X_2/X_1) may actually decline.[7] This possibility has been depicted in Figure 4 where E shifts to E" and the solid line E"F′, which now represents the labor-constraint line, intersects the capital-constraint line G′H′ at J" to show that (X_2/X_1) ratio has declined from the slope of OJ to that of OJ".

There are two necessary conditions for the existence of the perverse output response to effective protection. First, condition (1.15), which provides the basis for C_{L2} to move in the "wrong" direction must be satisfied. Second, the intermediate product must be a better substitute of labor than it is of capital. This is because, the change in C_{L2} is governed by the sign of not only $(w^* - P_Q^*)$ which is positive but also of $(w^* - r^*)$ which is negative. The latter tends to raise C_{L2} whereas the former tends to lower it. Therefore, if C_{L2} is to decline on balance, the imported intermediate product must be a better substitute of labor than it is of capital. In the diagram point J", when compared to J, shows that not only the relative output of X_2, but also its absolute output has declined. However, this is by no means necessary. An interesting property of this model is that both X_2 and X_1 may rise as a result of the effective protection and yet the (X_2/X_1) ratio may decline. This would, for example, be the case if E"F were to intersect G′H′ slightly to the left of J", but to the right of J.

Until now we have considered two polar cases, one where effective protection involves only the imposition of the tariff on the imports of the final product and the other where effective protection is provided by subsidizing the imports of the intermediate input. The output response is normal in the former case simply because no change in P_Q occurs, whereas with the latter case the output response may be perverse precisely because of the change in P_Q. For the same reason, if the imported input is used in fixed proportions, the perverse output response is ruled out.

In the general case, where effective protection is conferred by means of both the input and the output tariff such that P_2^*, P_Q^* and $(P_2^* - Q_{Q2}P_Q^*)$ are all positive, the output response may again be perverse because of the change in P_Q, provided (i) the imported input is used in variable proportions, (ii) P_Q^*, though less than P_2^*/Q_{Q2}, is sufficiently large so that effective protection is provided mainly by raising the input tariff relative to the output tariff, and (iii) P_Q^* does not lie between r^* and w^*, that is, condition (1.14) is not satisfied. The latter alone is of course sufficient to rule out the perverse result. Moreover the necessary condition for the perverse result when both P_2^* and P_Q^* are positive is not given by condition (1.15) for with $P_Q^* > 0$ and $w^* < 0$, this condition cannot be fulfilled. It is now the following condition, namely,

$$P_Q^* > r^* > w^* \ , \qquad (1.16)$$

that is crucial for the existence of the perverse result.[8] As before, C_{K1} declines and C_{L1} rises and as a result H rises to H′ and F declines to F′ in Figure 5 which is drawn on the same principles as Figure 4. Since both P_Q^* and r^* are greater than w^*, both (r/w) and

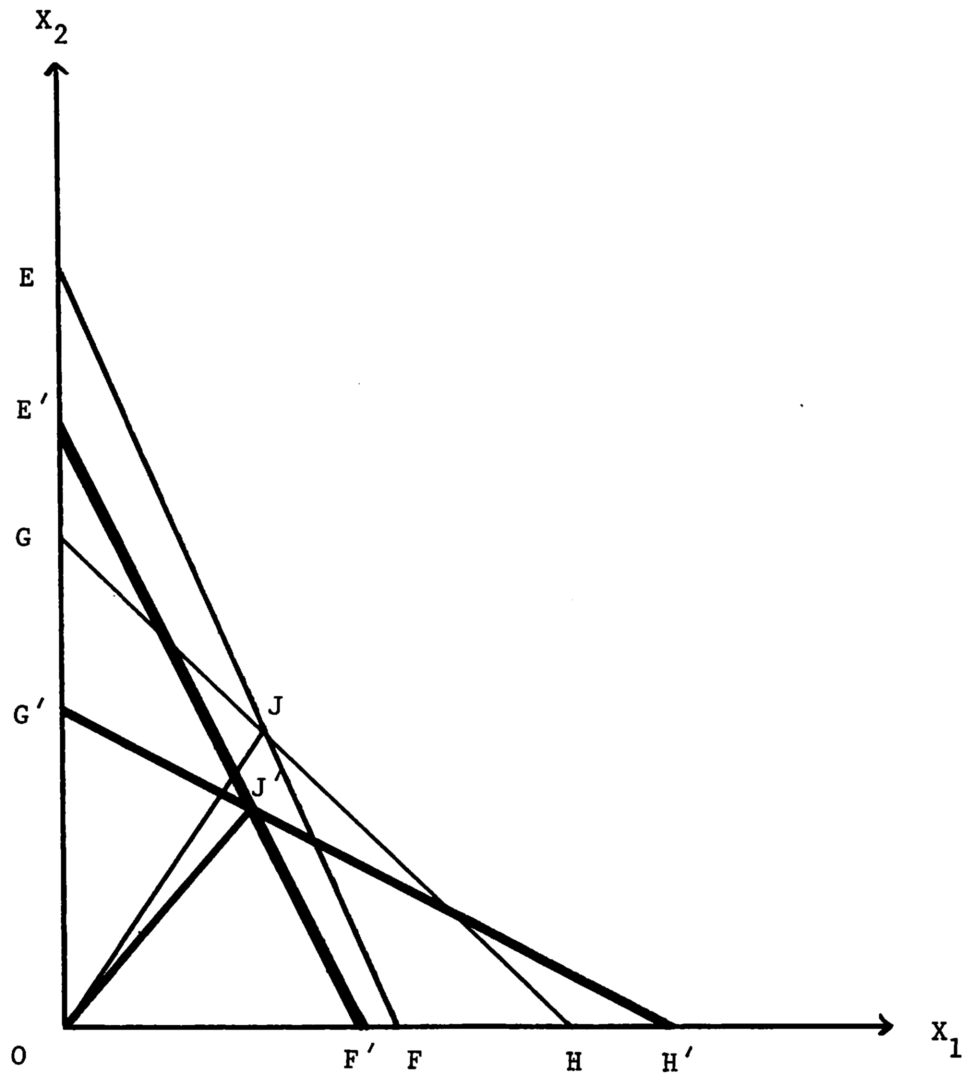

Figure 5

(P_Q/w) and hence C_{L2} rise, so that E shifts down to E′ in Figure 5. However, if $P_Q^* > r^*$, (P_Q/r) also rises and so may C_{K2}; thus it is now C_{K2} (instead of C_{L2}) that may shift in the "wrong" direction, and if this shift is sufficiently large, it is possible that the (X_2/X_1) ratio may actually decline. This is depicted in Figure 5, where G moves down to G′, G′H′ intersects E′F′ at J′, and, as a consequence, the output ratio (X_2/X_1) declines to the slope of OJ′. Based on the previous line of reasoning, the necessary condition, in addition to (1.16), requires that the imported input be a better substitute of capital than it is of labor. It is interesting to note that with this particular type of effective protection the absolute output of both goods may decline. All this discussion leads to the following theorem:

Theorem 2:

If P_Q^* does not lie between w* and r* and if the imported input is more substitutive with one primary factor than it is with the other, effective protection conferred on the imports of a final good may actually lead to a decline in its output relative to the output of the other commodity. This possibility does not occur if effective protection involves the imposition of the output tariff alone.

B. The Non-Traded Material Input Exists:

When a non-traded intermediate product is introduced, the model and the explanatory logic become more complicated. If we allow substitution among all factors of production, we cannot show any more that effective protection provided by means of the output tariff alone will necessarily lead to a rise in the output of the protected commodity, as was demonstrated in the model where the non-traded intermediate product did not exist. The logic behind this result is more or less the same as given above, where it is the change in P_Q effected by the input tariff that was responsible for the perverse output response. If P_Q was unchanged, the latter result could not occur. In the presence of the domestically produced intermediate product (M), the price of M, P_M, is subject to a change as a result of a rise in P_2 irrespective of whether P_Q changed or remained constant. For a change in P_2 leads to a change in w and r and from equation (1.5) we can see that the change in prices of primary factors will normally result in a change in P_M. Thus in the model allowing for the existence of traded and non-traded inputs, P_M^* plays the same role as P_Q^* with one difference, namely, the change in P_M, unlike P_Q, is endogenous, so that even when P_Q^* is zero, as is the case in the presence of the output tariff alone, a change in P_M resulting from the rise in P_2 could give rise to the same possibilities as the change in P_Q did in Jones' model presented above.

In the second industry we now have to deal with gross production coefficients in X_2. For normal results, we now require a rise in R_{L2} and a decline in R_{K2} as the (w/r) ratio declines consequent upon the imposition of the output tariff. As usual C_{K1} declines and C_{L1} rises, because P_M does not influence the input choice in the first industry. For the same reason, C_{LM} rises and C_{KM} declines. Since $r^* > 0$ and $w^* < 0$ and since changes in both w and r affect the change in P_M, it is obvious that

$$r^* > P_M^* > w^* ,$$

which implies that substitution in the second industry takes place in favour of labor and away from capital, so that C_{L2} rises and C_{K2} declines. These changes serve to raise $R_{L2} = (C_{L2} + C_{LM}C_{M2})$ and lower $R_{K2} = (C_{K2} + C_{KM}C_{M2})$, as is required for the normal results. As regards C_{M2}, there are two factors working in the oppositive direction that influence the sign of the change in C_{M2}. Since $r^* > P_M^*$, C_{M2} tends to rise, but with $P_M^* > w^*$, C_{M2} tends to decline. Therefore, if the non-traded material input is more substitutive with labor than it is with capital, C_{M2} may on balance decline.

Such a shift in C_{M2} will be strengthened if $P_M^* > 0$. Now a decline in C_{M2} reinforces the original decline in R_{K2}, but tends to weaken the original rise in R_{L2}, and if C_{M2} falls sufficiently, R_{L2} may eventually decline. Such shifts in the production coefficients can be introduced in Figure 1. The interested reader will find that for some level of the decline in R_{L2}, the output ratio (X_2/X_1) may also decline, thereby creating the paradox. The following theorem is then immediate:

Theorem 3:

In the presence of traded and non-traded material inputs, the imposition of the output tariff alone may lead to the paradox of a decline in the relative output of the protected commodity.

Even if C_{M2} was to rise, the perverse result could still occur, because this factor tends to raise R_{K2}, and if the impact is sufficiently pronounced, R_{K2} could actually rise, which would again run into conflict with the conditions sufficient to rule out the paradox.

When effective protection is provided by means of output as well as input tariffs the picture becomes fuzzier, but the explanation advanced above still applies. Specifically, the presence of non-traded material inputs may impair or strengthen the results derived in their absence. But the source of the paradox always lies in the presence of the imported input which makes the effective factor supply variable, irrespective of the existence or the non-existence of the non-traded material input. If there is no imported input in the model, Batra and Casas [4] have shown that the output of a final good always responds positively to the rise in its relative price.

V. Conclusions

The main purpose of this paper was to devise a simple geometrical apparatus for analyzing the implications of effective protection for primary-factor prices and the allocation of resources in a general equilibrium model. The development of the geometrical techniques is essential if the concept of effective protection is to compete successfully for popularity with the concept of nominal protection which is undoubtedly inferior to the former. This purpose seems to have been amply served. The diagrams presented in the foregoing are quite simple especially in relation to the complexity of the model of two final goods and at least three factors including the intermediate good.

There are two principal conclusions that spring out of our analysis. First, effective protection, unlike nominal protection, may not lead to an unambiguous rise in the real reward of the factor employed intensively by the protected commodity, although its relative reward will certainly rise. This creates the well-known index number problem which is sufficiently resolved if the output tariff is at least as high as the input tariff.

Second, effective protection may actually lower the relative output of the protected commodity, provided production coefficients are variable. If non-traded intermediate goods are absent, this paradox never occurs if protection involves only the output tariff. However, the latter is not sufficient to rule out the paradox when both traded and non-traded intermediate inputs exist.

FOOTNOTES

[1]Two articles by Corden [7] [8] provide an exception to this rule. However, Corden is interested in different type of questions.

[2]Actually, the entire supply of labor is employed in producing X_2 as well as M. But since M is used back in the production of X_2, the entire labor supply at point E is in effect employed in X_2. The output of X_1 is zero at this point.

[3]Actually line AA′ shows that $(P_2 - C_{Q2}P_Q)$ equals the left hand side of equation (1.4a) which falls short of the unit cost. But although, $(P_2 - C_{Q2}P_Q)$ does not equal unit cost, P_2 does. Hence along any point on AA′, P_2 equals the unit cost which includes the cost of the imported input.

[4]$DE/GD > AB/OA$ because $DE > AB$ and $GD < OA$. Similarly, since $Q_{Q2} < 1$, $DE/GD > (AB/OA)(1 - Q_{Q2})$. Since P_1 is constant, this implies that r rises in terms of both commodity prices. For the same reason w declines in terms of both prices.

[5]This is normally the case in the absence of imported inputs. Later on we will show that R_{L2} may actually decline and R_{K2} rise in the presence of the imported input. However, C_{K1} must decline and C_{L1} must rise, because X_1 does not use the imported input. In any case, the direction of the change in R_{K2} and R_{L2} is immaterial to the remainder of the argument in this section.

[6]This is the ERP formula used by Corden [6] in a partial equilibrium approach and by Jones [10] in a general equilibrium framework. The formula, however, ignores the substitution possibilities between the primary factors

and the imported input. This does not mean that C_{Q2} is assumed fixed. All it means is that the change in C_{Q2} is not taken into account while defining the ERP. This is the procedure used by Jones [10] and perhaps the only justification for this lies in the simplicity and convenience afforded by this formula. The results derived in this paper are subject to this limitation, as are the results derived by Jones [10], Mayer [11] and Batra and Casas [5]. Corden [6, p. 234] provides some vindication for this method by arguing that the objective of the ERP is to indicate the direction of the resource shifts and not the readjustment of resource use.

[7]The reader may be wondering whether it is at all possible for w* to exceed P_Q^*. The satisfactory resolution of this question requires reverting to Figure 3. If $P_2^* = 0$, then ERP = (AC/OA) = $-\,[Q_{Q2}/(1-Q_{Q2})]\,P_Q^*$, so that

$$P_Q^* = -\,(AC/OA)[(1 - Q_{Q2})/Q_{Q2}] = -\,(A'C'/OA')[(1 - Q_{Q2})/Q_{Q2}].$$

Now w* = - SN/DT.

In comparing w* and P_Q^*, two points should be emphasized. First, $(1 - Q_{Q2})/Q_{Q2}$ is positive but may be greater or less than unity depending on the size of Q_{Q2}. Second, it is not unambiguously clear whether SN/DT is greater or less than (A'C'/OA'), for even if DT is smaller than OA', SN is not greater than A'C'. Hence $w^* \gtrless P_Q^*$.

[8]The remarks made in footnote 7 are relevant here as well. Going back to Figure 3, $r^* = \frac{DE}{GD} > \frac{AC}{OA}$ = ERP, whereas $P_Q^* = \frac{BC}{OA}[(1 - Q_{Q2})/Q_{Q2}]$. Now $AC \gtrless BC$ and $[(1 - Q_{Q2})/Q_{Q2}] \gtrless 1$. Hence $r^* \gtrless P_Q^*$.

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