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by

C. Scott Clark

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C. Scott Clark\*

Most economists are familiar with the observed short-run behavior of productivity as measured by output per man or output per manhour. It is generally observed that productivity tends to rise most rapidly when output is recovering towards capacity and to fall or rise less rapidly as output declines from capacity. According to the law of diminishing marginal productivity, however, the opposite behavior should be observed. That is, with capital fixed in the short run, there should be decreasing returns to labor as output increases towards capacity and increasing returns to labor as output declines from capacity. This apparent contradiction between reality and theory has not been confined solely to studies of how output fluctuates with employment. Econometric studies of short-run employment behavior have consistently obtained estimates of the equilibrium output-employment elasticity which suggest increasing returns to labor services [2], [4], [5], [19], [20].

In recent years a number of hypotheses have been put forward in an attempt to reconcile this apparent paradox [6], [11], [20]. One possible explanation, which recently has begun to attract an increasing amount of support is referred to as the labor-hoarding theory. This theory operates on the assumption that there are important costs associated with changing the labor force. Given these adjustment costs a minimum cost employment policy for a firm may be one in which the firm employs more labor during periods of declining

output than is required to produce the desired level of output. If this hypothesis is true, then during periods of declining output the actual labor used in the production process may in fact be declining in proportion or more than in proportion to changes in the output even though measured employment does not. At the same time, once output begins to recover, the increase in measured employment will be less than proportionate to the increase in output since the firm can now use in its production the excess labor that was accumulated during the previous periods of declining output. The fact that measured employment may not be a very good measure of the true production function input could very well account for the contradictory results obtained by the earlier productivity and short-run employment studies.

The purpose of this paper is to construct a model of short-run employment behavior in durable goods industries based on an assumption of labor-hoarding. The paper is composed of four sections. In section I the labor-hoarding model is outlined and the necessary and sufficient conditions for a minimum cost employment policy are established. Using the cost minimization conditions a programming algorithm is derived which permits us to obtain estimates of the maximum length of time it would be profitable for firms to hoard excess labor, referred to as the labor-hoarding lag. Two models are constructed. The first model assumes there is no production lag in durable goods industries, while the second model allows for a production lag in the determination of required employment. In section II the data are described. Section III presents the estimates of the labor-hoarding lags. Several hypotheses are also tested. Section IV contains the summary and conclusions.

I. THE MODELS

1. We assume that the objective of the firm is to produce output  $Q_1, Q_2, \dots, Q_T$  over  $T$  periods at least cost. We also assume that the production function of the firm is such that a fixed number of workers is required per unit of output. That is,

$$(1) \quad L_t \cong \lambda Q_t$$

where  $L_t$  is the number of workers employed by the firm during period  $t$ ,  $Q_t$  is the firm's output, and  $\lambda$  the production coefficient or labor-output ratio.

We postulate that there are three costs associated with using labor: a wage,  $w$ , paid each period; a hiring cost,  $h$ , paid each time the labor force is enlarged; and a lay-off cost,  $f$ , paid each time the labor force is reduced.<sup>1</sup> The firm's total labor cost over  $T$  periods is thus,<sup>2</sup>

$$(2) \quad C = w(L_1 + L_2 + \dots + L_T) + h([L_1 - L_0]^+ + [L_2 - L_1]^+ + \dots + [L_T - L_{T-1}]^+) + f([L_0 - L_1]^+ + [L_1 - L_2]^+ + \dots + [L_{T-1} - L_T]^+)$$

where  $[X]^+ = 0$  if  $X < 0$   
 $= X$  if  $X \geq 0$ .

Given (1) and (2) the necessary and sufficient conditions for a cost minimization employment policy can be derived as follows. In general, a firm will hold no more labor than is required to produce current output, except that workers will not be laid off for a temporary drop in output. The maximum length of time that it is profitable for a firm to keep a worker on the payroll without having him work is given by the ratio of the layoff plus hiring cost to the wage cost. That is

$$(3) \quad t^* = \frac{h+f}{w} .$$

In other words, a minimum cost employment policy for a firm is one in which the firm will never have more than  $t^*$  successive periods of positive excess labor. At the same time, in order for employment costs to be minimized, a layoff by a firm will never be followed by a hire within  $t^*$  periods, since in that case it would be cheaper for the firm to eliminate both the layoff and the hire and carry excess labor. These two conditions are necessary and sufficient for a cost minimization employment policy.

An easy algorithm for estimating the labor-hoarding lag,  $t^*$ , and the production coefficient,  $\lambda$ , is suggested by the two cost minimization conditions. We begin by taking as our initial guess of the firm's employment policy, the policy of holding no excess labor. In this case we have from the production function that,

$$(4) \quad L_t = \lambda Q_t .$$

Given data on output, and assuming a value for  $\lambda$ , we calculate the employment series that would occur if no excess labor were held. We then assume a value for  $t^*$  and consider the earliest layoff in the no excess labor employment series. If a hire occurs within  $t^*$  periods after the layoff, then the layoff and the hire are cancelled and the labor force increased for every intermediate period. This process is repeated over the entire sample until no hire occurs within  $t^*$  periods of any layoff. At every iteration the second cost minimization condition is satisfied so that the process will reach a minimum cost policy in a finite number of iterations. We select the maximum likelihood estimates of  $t^*$  and  $\lambda$  from the set of all possible combinations by comparing the sum of squared residuals between the observed employment series and the estimated employment series based on our cost minimization conditions.

The production function adopted in this paper is the simplest of all those available, that of fixed coefficients. This implies that capital and labor are not substitutable, and therefore that variation in relative factor prices are not important in determining factor intensities. The large body of empirical work on production functions indicates, however, that manufacturing industries are characterized by factor substitutability. This would appear to indicate that the assumption of constant factor proportions for determining required employment in (4) is inappropriate. However, this is not necessarily so. It is not a contradiction to believe the empirical evidence which indicates that the true production technology is characterized by factor substitutability and at the same time still believe that, for the purpose of deducing factor demand from output demand, a fixed coefficient technology provides a satisfactory approximation to the true technology. Factor substitutability may not be important for the purpose of deducing factor demand if relative factor prices do not change very much.

Ray Fair in his study of the short-run demand for workers concluded that the "postulate of no short-run 'substitution possibilities' between workers and machines may not be an unreasonable approximation of reality, but no direct empirical evidence is given here to support it." [6, p. 53] However, our approximation of the true technology by a fixed coefficient production function has received some empirical support in the case of demand for investment goods in studies done by D. Jorgenson and M. Nadiri [8] and by Jorgenson and C. Siebert [9]. In the former study, alternative investment models were compared for fifteen two-digit SIC manufacturing industries, while in the latter study investment models were compared for fifteen large manufacturing companies selected from 14 different OBE-SEC industry groups. These studies concluded that on the basis of the standard error of the regression, the investment model allowing for

substitutability<sup>3</sup> performed better than the model excluding substitutability,<sup>4</sup> for 67 percent of the industries, and 80 percent of the companies compared. This is an impressive record, but a careful examination of the actual differences between the standard errors reveals that of the total 27 industries and companies for which the investment model allowing for substitutability was judged better, 21 had standard errors which were only .0001 to .0050 billion dollars smaller than the standard errors of the investment model excluding substitutability. When considering gross investment measured in billions of dollars, differences of 100 thousand to 50 million dollars have hardly any economic significance.

The results obtained by these two careful studies of the demand for investment goods provide strong justification for using constant factor proportions as an approximation to the true technology for the purpose of deducing factor demand from output demand.

There are of course some more practical reasons for using the fixed coefficient approximation. First, because of the estimation technique used, it is important to keep the number of structural parameters to be estimated to a minimum. Second, in this paper, data for three-digit SIC industries are used and there are, to date, no capital data available at this level of disaggregation.

One problem still remains concerning the assumed fixed coefficient technology. A brief examination of the employment-output data for the industries indicates that for most of the industries there has been a downward trend in the labor-output ratio. Having assumed away capital-labor substitution there are two other possible explanations for this behavior. First, it is quite possible that the production function is one of increasing returns to scale



rather than constant returns to scale. That is

$$(5) \quad L_t \cong \lambda Q_t^\eta, \quad \eta > 1 .$$

Fair, in a careful examination of his data, concluded that "there is not enough evidence from these results to determine which is the most realistic assumption about short-run returns to scale to make." [6, p. 45] A second possible explanation is that there has been labor-augmenting technical change during the sample period. Allowing for this possibility, the technology approximation becomes

$$(6) \quad L_t \cong \lambda(t) Q_t .$$

We assume that technological change takes place smoothly at a constant rate  $m$ , so that,

$$(7) \quad \lambda(t) = \lambda_0 e^{-mt} .$$

Equations (6) and (7) simply say that a given level of output can now be obtained from a given capital input and a labor input,  $L_t$ , measured in men that decreases over time. Within this model a decreasing number of men represents an increasing number of labor efficiency units so that a fixed coefficient production relation is maintained between capital and labor efficiency units.

2. The labor-hoarding lag,  $t^*$ , will be estimated using data for five three-digit SIC durable goods industries. By using three-digit industry data it is possible to avoid some of the problems that might be caused by aggregation. It is very likely that many of the diverse relationships that exist between industries would be concealed if aggregate data were used. The use of three-digit industry data, however, does have some costs associated with it. Most important, inventories by stage of fabrication are not available for three-digit industries. Consequently, for the model in section 1 it is necessary to use shipments as a measure of output. Since most of the durable goods

industries used in this paper<sup>5</sup> probably produce primarily to order, the assumption of no finished goods inventories is not unreasonable. It is difficult to determine, however, what change in our output measure would occur if we were able to include the change in work-in-process inventories. It is quite possible that a decline in shipments could be completely offset by a build-up in work-in-process inventories so that output as measured by shipment plus the change in work-in-process inventories would show no decline whereas output as measured by shipments would. This possibility would be particularly true for industries with fairly long production lags. If production of a good takes several periods then the amount of labor required in any period will depend not only on the goods which reach completion and are shipped, but also on those goods still in the production process at different stages of completion (i.e., work-in-process inventories). To assume that required labor services depend only on shipments in the current period implies that there is no production lag.

There is sufficient evidence, [1], [3], to indicate that in the durable goods industries production does take time. Consequently, in order to compensate for the possible inadequacy of shipments as a measure of output in this type of industry we will develop a second labor-hoarding model based on the assumption of a production lag. The model is developed for an industry which produces  $q+1$  different goods. Each good is distinguished by its production time, defined as the time between initiation of production and the completion of the final product.

We define the following variables:

$X_{t,j}$  = The value of production initiations of good  $j$   
in period  $t$

$L_{t,j}$  = The number of workers required during period  $t$   
for the production of good  $j$ .

As before we assume that a fixed coefficient technology provides a satisfactory approximation to the true industry production function. More specifically, we assume that for good  $j$  a fixed number of workers is required per unit of production initiations of good  $j$ .

Consider now the demand for workers in period  $t$  to be used in the production of good  $j$ . In period  $t$  there will be some of good  $j$  reaching the final stage of production. These will be goods whose production was initiated in period  $t-j$  (i.e.,  $X_{t-j,j}$ ). These goods will require some labor services. At the same time, there will be some of good  $j$  which will reach the final stage of production in period  $t+1$ . These goods whose production was initiated in period  $t-j+1$  will also require some labor input. Thus, for each production of good  $j$  initiated in the current period and over the past  $j$  periods there will be some labor input required in period  $t$ . Employment required for the production of good  $j$  in period  $t$  can then be written as a distributed lag function of production initiations of good  $j$ :

$$(8) \quad L_{t,j} = \sum_{\tau=0}^j \beta_{\tau}^{L,j} X_{t-\tau,j} .$$

For  $j=0$ , the lag function would collapse to include only  $X_{t,0}$ :

$$(9) \quad L_{t,0} = \beta_0^{L,0} X_{t,0}$$

For  $j=1$ , we would have the relation

$$(10) \quad L_{t,1} = \beta_0^{L,1} X_{t,1} + \beta_1^{L,1} X_{t-1,1} .$$

Since there are  $q+1$  goods there would be  $q+1$  distributed lag labor requirement functions. Using a lag operator  $Z$ , equation (8) can be rewritten as,

$$(11) \quad L_{t,j} = \sum_{\tau=0}^j \beta_{\tau}^{L,j} Z^{\tau} X_{t,j} .$$

Letting  $\beta^{L,j}(Z) = \sum_{\tau=0}^j \beta_{\tau}^{L,j} Z^{\tau}$  we have the final labor requirement function for good  $j$ :

$$(12) \quad L_{t,j} = \beta^{L,j}(Z) X_{t,j}$$

Total labor required by the industry in period  $t$  is the sum of the labor requirements for each of the  $q+1$  goods:

$$(13) \quad L_t = \sum_{j=0}^q \beta^{L,j}(Z) X_{t,j}$$

We now make the aggregation assumption that the mix of production initiations (i.e.,  $X_{t,0}, X_{t,1}, \dots, X_{t,q}$ ) is constant for all  $t$ . That is

$$(14) \quad X_{t,j} = \beta_j^P X_t \quad (j = 0, 1, \dots, q)$$

where  $X_t$  is the value of total production initiations in period  $t$ , and the  $\beta_j^P$  measures the relative importance of the production initiations of each good. By definition,

$$(15) \quad \sum_{j=0}^q \beta_j^P = 1 .$$

Substituting the aggregation assumption into (13) gives

$$(16) \quad L_t = \sum_{j=0}^q \beta^{L,j}(Z) \beta_j^P X_t .$$

Letting  $\beta^L(Z) = \sum_{j=0}^q \beta^{L,j}(Z) \beta_j^P$  we have the final distributed lag relation between

total labor requirements and the value of total production initiations:

$$(17) \quad L_t = \beta^L(Z) X_t .$$

In the labor requirement equation, (17), the key variable production initiations is not observable. To complete the model for empirical testing we must relate  $X_t$  to some observable variable. In this paper  $X_t$  is equated to the value of new orders received during period  $t$ . The labor requirement

function then becomes

$$(18) \quad L_t = \beta^L(Z)N_t .$$

The lag function,  $\beta^L(Z)$ , is assumed to be stationary. The stationarity assumption is explicit in our aggregation assumption. The choice of new orders as a measure of production initiations in any period may create situations, however, in which this assumption is not valid. If an industry is operating at less than full capacity then any new orders received during a period can be initiated into production almost immediately. On the other hand, when capacity is being fully utilized new orders will not be initiated immediately but instead will be placed in a queue to await production some-time in the future. Consequently, in addition to the work-time lag there is now a wait-time lag. The latter lag component will vary over time depending on the degree of capacity utilization. In order for the stationarity assumption to be valid we must assume that there is always sufficient excess capacity in the industries so that the wait-time lag is zero.<sup>6</sup> There are other reasons why the function may not be stationary. Even if there is excess capacity so that there is no queue, the work-time lag may change. First, there may be technological improvements resulting in a shortening of the work-time required for production. Secondly, there may be a cyclical response in work time. As demand conditions improve firms may be able to shorten their work time by increasing their work speed. However, if demand continues to rise at a fairly rapid rate there may be a relative deterioration in the earlier improvement as less efficient capital and labor must be employed. Finally, there may be a change in the relative importance of the various commodities in total demand.

Before proceeding it is interesting to note the relation between the labor requirement function, (18), and the labor requirement function (4) which assumes no production lag. If we adopt the assumption that there is no production lag then there will be only one good produced in the industry (i.e., good 0). Using the notation of this section we would have,

$$(19) \quad X_{t,0} = X_t .$$

Equating production initiations to new orders gives

$$(20) \quad X_t = N_t .$$

Since we are assuming that there is no waiting lag, every new order in period  $t$  becomes a shipment in period  $t$ . We have then that

$$(21) \quad N_t = S_t$$

The labor requirement function is then simply,

$$(22) \quad L_t = \beta_o^{L,0} S_t$$

which is the form of the labor requirement function without technological change used in section 1.

We can now complete the new labor-hoarding model. We assume that the objective of the firm is to fill all new orders  $N_1, N_2, \dots, N_T$ , over  $T$  periods at least cost. Based on the assumption of a production lag and a fixed coefficient technology the number of workers employed in period  $t$  is given as

$$(22) \quad L_t \cong \beta^L(Z) N_t .$$

The cost function, (2), used in the first model is also applicable in this model. Since only the labor requirement function has been changed the necessary and sufficient conditions for a cost minimization employment policy are essentially the same as those in section 1. In general, a firm will hold no more labor than is required to complete the production of those goods in the final stages of processing,

$$(23) \quad N_{t,0}, N_{t-1,1}, \dots, N_{t-q,q}$$

and also labor required to maintain work on goods which will reach completion in periods  $t+1, t+2, \dots, t+q$ :

$$(24) \quad \begin{array}{c} N_{t,1} \\ N_{t,2}, N_{t-1,2} \\ \cdot \\ \cdot \\ \cdot \\ N_{t,q}, N_{t-1,q}, \dots, N_{t-q+1,q} \end{array}$$

On the other hand, workers will not be laid off for a temporary drop in new orders. The maximum length of time that is profitable for a firm to keep a worker on the payroll without having him work is given by the ratio of the layoff plus hiring cost to the wage cost:

$$(25) \quad t^* = \frac{h+f}{W} .$$

A minimum cost employment policy for a firm is one in which the firm will never have more than  $t^*$  successive periods of positive excess labor. At the same time, in order for employment costs to be minimized, a layoff by a firm will never be followed by a hire within  $t^*$  periods, since in that case it would be cheaper for the firm to eliminate both the layoff and the hire and carry excess labor. These two conditions are necessary and sufficient for cost minimization.

To estimate the labor-hoarding lag,  $t^*$ , we employ the algorithm described in section 1. Three different specifications of  $\beta^L(Z)$  will be used. To begin with we adopt the simple specification that the lag coefficients correspond to a rectangular lag function. This can be expressed as

$$(26) \quad \begin{array}{l} \beta_{\tau} = \alpha \quad \text{if } 0 \leq \tau \leq q \\ \phantom{\beta_{\tau}} = 0 \quad \text{otherwise.} \end{array}$$

The labor requirement function then becomes

$$(27) \quad L_t = \alpha \sum_{\tau=0}^q N_{t-\tau}$$

leaving only three parameters to be estimated:  $\alpha$ ,  $q$ , and  $t^*$ .

Assuming a value for  $\alpha$ , for a given  $q$ , we obtain an estimate of the no excess labor series from (27). With this series we are then able to estimate the labor-hoarding lag,  $t^*$ , using the cost minimization algorithm. An initial estimate of  $\alpha$  can be obtained by dividing the observed employment series by  $\sum_{\tau=0}^q N_{t-\tau}$ . Once having obtained the best estimates of  $\alpha$  and  $t^*$ , for each  $q$ , we select as our best estimates of all three parameters that equation which has the smallest variance corrected for degrees of freedom.

The second function to be used in this study is the inverted V or symmetric triangle. Using this specification the lag coefficients are given by

$$(28) \quad \beta_{\tau} = \alpha_1 \tau \quad 0 \leq \tau \leq \alpha_2$$

$$= \alpha_1 \alpha_2 - \alpha_1 (\tau - \alpha_2) \quad \alpha_2 < \tau \leq 2\alpha_2$$

where  $\alpha_1$  is the slope of the triangle and  $2\alpha_2$  its length. Substituting this specification into the labor requirement function gives

$$(29) \quad L_t = \alpha_1 \sum_{\tau=0}^{\alpha_2} \tau N_{t-\tau} + \sum_{\tau=\alpha_2+1}^{2\alpha_2} [\alpha_1 \alpha_2 - \alpha_1 (\tau - \alpha_2)] N_{t-2\alpha_2}$$

Since the triangle is symmetric this is equivalent to

$$(30) \quad L_t = \alpha_1 \sum_{\tau=0}^{\alpha_2-1} \tau (N_{t-\tau} + N_{t-2\alpha_2+\tau}) + \alpha_1 \alpha_2 N_{t-\alpha_2}$$

In the case of the triangle there are only three parameters to estimate: the slope  $\alpha_1$ , the mid-point of the triangle,  $\alpha_2$ , and  $t^*$ .



The estimation of the model with a triangular lag function is similar to that with the rectangle. Given a value for  $\alpha_2$ , and assuming a value for  $\alpha_1$ , we obtain an estimate of the no excess labor series from (30). The labor-hoarding lag,  $t^*$ , is then estimated as before. We select as our best estimates of  $\alpha_1$ ,  $\alpha_2$  and  $t^*$  that equation with the smallest variance corrected for degrees of freedom. An initial estimate of the slope  $\alpha_1$  can be obtained by dividing observed employment by

$$(31) \quad \frac{\alpha_2^{-1}}{\sum_{\tau=0}^{\alpha_2^{-1}-1} \tau (N_{t-\tau} + N_{t-2\alpha_2 + \tau}) + \alpha_2 N_{t-\alpha_2}} .$$

The third distributed lag function to be used in our labor requirement equation is the common geometric function. Under this specification the coefficients are given by

$$(32) \quad \beta_{\tau} = \alpha_1 \alpha_2^{\tau} \quad 0 \leq \alpha_2 < 1 .$$

The labor requirement function becomes

$$(33) \quad L_t = \alpha_1 \sum_{\tau=0}^{\infty} \alpha_2^{\tau} N_{t-\tau} .$$

Using a Koyck transformation we obtain the labor requirement equation to be used in estimation.

$$(34) \quad L_t = \alpha_1 N_t + \alpha_2 L_{t-1} .$$

With the geometric lag function there are three parameters to be estimated;  $\alpha_1$ ,  $\alpha_2$ , and  $t^*$ . Initial estimates of  $\alpha_1$  and  $\alpha_2$  can be obtained in the following manner. First, we know that  $\alpha_2$  must be less than one and equal to or greater than zero. Assuming a value for  $\alpha_2$  within these bounds we obtain an estimate of the range of  $\alpha_1$  by dividing observed employment by

$$(35) \quad \sum_{\tau=0}^K \alpha_2^{\tau} N_{t-\tau}$$

There is, however, one additional estimation problem. It is necessary in the case of the geometric lag function to have an estimate of the number of non-idle workers held in period zero (i.e.,  $L_0$ ). In other words, it is necessary to consider different initial values for  $L_0$ , and for each value obtain the best estimates of  $\alpha_1$ ,  $\alpha_2$  and  $t^*$ . We select as our best estimates of  $L_0$ ,  $\alpha_1$ ,  $\alpha_2$  and  $t^*$ , that equation which has the smallest variance corrected for degrees of freedom.

So far in our discussion of this model we have not considered technological change. We now assume that technological change takes place smoothly at a constant rate,  $m$ , so that employment is now given by

$$(36) \quad L_t \geq \lambda_0 e^{-mt} \beta^L(Z) N_t .$$

This results in one additional parameter to be estimated for each of the three distributed lag functions.

With technological change and a rectangular lag function the labor requirement equation becomes

$$(37) \quad L_t = \alpha e^{-mt} \sum_{\tau=0}^q N_{t-\tau} .$$

In the case of the triangular lag function, using (30), we have

$$(38) \quad L_t = \alpha_1 e^{-mt} \left[ \sum_{\tau=0}^{\alpha_2-1} \tau (N_{t-\tau} + N_{t-2\alpha_2+\tau}) + \alpha_2 N_{t-\alpha_2} \right] .$$

For the geometric lag function the inclusion of technological change given

$$(39) \quad L_t = \alpha_1 e^{-mt} \sum_{\tau=0}^{\infty} \alpha_2^{\tau} N_{t-\tau} .$$

Employing a Koyck transformation, gives

$$(40) \quad L_t = \alpha_1 e^{-mt} N_t + \alpha_2 L_{t-1} .$$

The introduction of technological change in this manner allows the lag functions in the labor requirement equation to be semi-nonstationary. For example, in the case of the rectangle the inclusion of technological change implies that the height of the rectangle is decreasing over time while in the case of the triangle it implies that the slope is decreasing over time. In both cases, however, the length of the production lag remains constant. For the geometric lag function the inclusion of technological change implies that the height of the function is declining.

In section III estimates of the labor-hoarding lag are presented based on equations (4), (6), (37), (38), and (40). Before proceeding to the empirical results some brief comments on the roles of foresight and labor utilization, applicable to both models, are in order. The interpretation of  $t^*$  as the maximum length of time it is profitable for a firm to hoard idle workers implies that the firm has knowledge of its future output or new orders and consequently, from the production function, the path of required employment. Without such knowledge the firm would be unable to know exactly when it should begin hoarding labor or how much labor it should hoard. For the durable goods industries used in this study the assumption of perfect foresight can be defended on two grounds. First, durable goods industries produce primarily to order and the lag between orders and production is sufficiently long to enable the firm to use its orders data in planning layoffs and hiring. Secondly, on the basis of past experience firms have sufficiently good knowledge of the seasonal behavior of orders and shipments to permit accurate employment planning.

The second assumption worth noting is the role of labor utilization in both models. In constructing the labor-hoarding models we have not allowed for substitution between the stock variable, workers, and the flow variable,

the number of hours worked per worker. In our models an increase or decrease in shipments, and new orders, necessitates an increase or decrease in required workers. This excludes the possibility of simply changing the utilization of the existing stock of workers (i.e., the number of hours worked per worker) in response to a change in shipments or new orders. The task of constructing a minimum cost employment model based on a theory of labor-hoarding which allows for substitution between men and hours is left to future work.

## II. DATA

The overhead labor lag,  $t^*$ , is estimated for the following three-digit SIC durable goods industries:

- 331 - Blast furnaces and steel mills<sup>7</sup>;
- 351 - Engines and turbines;
- 353 - Construction, mining, and material handling equipment;
- 354 - Metalworking machinery;
- 361 - Electrical distribution equipment.

The shipments and new orders data are monthly unseasonally adjusted data obtained from the monthly survey of manufacturers published by the Department of Commerce, [19], [20]. The data were not seasonally adjusted since the purpose of this paper is to examine the short-run behavior of employment. The sample used in this study covers the period January 1960 to January 1969. This includes one recession, 1960-1961; the moderate expansion beginning late in 1961 and culminating in the more rapid expansion of 1965 and early 1966; the mini-recession of 1967 followed by a period of moderate growth until the beginning of 1969.

To obtain a constant dollar shipments and new orders series, wholesale

price indices published by the Bureau of Labor Statistics were used. Unfortunately, the industry classifications used by the Bureau of Labor Statistics are not the same as the SIC classifications used by the Bureau of the Census for its shipments data. It was not until 1967 that the Bureau of Labor Statistics began to publish wholesale price indices on an SIC basis. As a result, it was necessary to combine BLS price indices which would represent as much as possible the SIC industry.

Employment data are available for SIC industry classifications, [24]. There were some differences, however, between the industry classifications used by the Department of Labor and the classifications used by the Department of Commerce. The differences applied only to industry 354, metalworking machinery, and industry 361, electrical distribution equipment. Fortunately, however, the employment data are available for four-digit industry classifications so that it was possible to derive employment series for industries 354 and 361 which are comparable to the industry classifications on which the shipments and new orders data are based. Both total employment and employment of production workers are available for our sample period. An estimate of employment of non-production workers is obtained as the differences between total employment and employment of production workers.

### III. Empirical Results

Estimates of the labor-hoarding lag,  $t^*$ , are presented for the model without a production lag based on (4) and (6) and for the model with a production lag based on (37), (38) and (40). Several hypotheses are also tested. In each model the hypothesis of no labor-hoarding (i.e.,  $t^*=0$ ) and the hypothesis that production workers and non-production workers are the same are tested. Also since the model without a production lag is a constrained form

of the model with a production lag, it is possible to test the hypothesis that the production lag is not important in explaining the demand for employment in the durable goods industries.

Before turning to the estimates we should note an important problem concerning the selection of appropriate sample periods for estimation. Since our estimation algorithm is based on the cost minimization condition that there should be no hire within  $t^*$  periods of a layoff, it is necessary to choose as our beginning month one in which shipments, in the case of the first model, and  $\beta^L(Z)N_t$ , in the case of the second model, are at a peak or declining. If we were to select as our beginning month one in which either shipments or  $\beta^L(Z)N_t$  were rising, we would be unable to apply the cost minimization conditions since we would not have information as to the amount of layoffs in earlier months. At the same time, the terminal month must be one in which shipments or  $\beta^L(Z)N_t$  are at a peak or rising. In this case if we were to select as our terminal month one in which either shipments or  $\beta^L(Z)N_t$  were declining, we would be unable to apply the cost minimization conditions since we would not have information about subsequent hirings.<sup>8</sup> Finally, it is necessary to exclude from the sample any months in which the behavior of employment, shipments, or new orders are considerably different from their behavior in immediately prior or subsequent months (e.g., due to strikes or possible errors in data).

Table 1 contains the estimates of the labor-hoarding lag,  $t^*$ , for the model without a production lag, with and without technological change. As expected the model with labor-augmenting technical change performed substantially better, on the basis of the sum of squared residuals, than the model without technological change. In the case of total employment, estimates of  $t^*$  in the model with technological change ranged from a low of 5

TABLE 1 ESTIMATES OF LABOR-HOARDING LAG,  $t^*$ , FOR THE MODEL WITHOUT A PRODUCTION LAG

Industry	Without Technological Change		With Technological Change		$\lambda_0$	m	$t^*$
	$S^2$	$\lambda$	$S^2$	$t^*$			
Total Employment							
331 Blast Furnaces and Steel Mills	361,365.0	.3044	92,990.0	12	.34	.003	14
351 Engines and Turbines	24,575.26	.3551	3,496.97	6	.48	.005	5
353 Construction, mining, and material handling equipment	8,187.53	.4738	2,962.33	12	.53	.002	9
354 Metalworking Machinery	27,479.8	.5931	2,221.69	7	.76	.004	6
361 Electrical Distribution Equipment	13,302.8	.3064	7,531.29	5	.32	.001	7
Production Workers							
331 Blast Furnaces and Steel Mills	254,943.0	.2468	87,960.92	12	.28	.003	14
351 Engines and Turbines	9,123.93	.2397	2,239.63	6	.31	.004	5
353 Construction, mining, and material handling equipment	3,519.08	.3211	1,614.09	9	.34	.001	9
354 Metalworking Machinery	13,724.89	.4176	1,258.60	7	.54	.004	5
361 Electrical Distribution Equipment	3,858.27	.2067	3,032.13	5	.22	.001	5
Non-Production Workers							
331 Blast Furnaces and Steel Mills	14,221.94	.0577	5615.49	12	.06	.002	14
351 Engines and Turbines	4,101.97	.1154	171.37	6	.16	.007	9
353 Construction, mining, and material handling equipment	2,200.87	.1557	662.69	11	.17	.002	11
354 Metalworking Machinery	2,555.81	.1754	350.89	7	.22	.004	8
361 Electrical Distribution Equipment	3,198.68	.0997	1,077.25	5	.11	.002	7

$S^2$  = sum of squared residuals

$\lambda$  = Labor-output ratio;  $\lambda_0$  = beginning period Labor-output ratio

$t^*$  = Labor-hoarding lag in months

m = rate of labor-augmenting technical change.

months in industry 351 to a high of 14 months in industry 331. For industries 353, 354 and 361 the estimates of  $t^*$  were 9, 6, and 7 months, respectively. The introduction of technological change lowered the estimates of  $t^*$  in industries 351, 353 and 354 by one, three, and one month, respectively; and increased the estimates by two months in both industry 331 and industry 361.

For employment of production workers, estimates of  $t^*$  in the model with technological change ranged from a low of 5 months in industries 351 and 361 to a high of 14 months in industry 331. For industries 353 and 354 the estimates of  $t^*$  were 9 and 5 months, respectively. In three of the industries, 331, 351 and 353, the estimates of the labor-hoarding lag  $t^*$  for production workers were the same as the estimates of  $t^*$  for total employment. For industry 354, metalworking machinery, the estimate of  $t^*$  for production workers was one month less than the estimate for total employment, and for industry 361, electrical distribution equipment, the estimate of  $t^*$  for production workers was two months less. The introduction of technological change into the model increased the estimate of  $t^*$  for industry 331, lowered the estimates for industries 351 and 354, and did not change the estimates for industries 353 and 361.

In the case of non-production workers we would expect the estimates of the labor-hoarding lag,  $t^*$ , to be longer than the estimates for production workers. This group includes professional and technical personnel and it might be argued, that because of the cost of orienting these workers to the operations of the firm and also because of their relative scarcity, firms will be less willing to lay off these workers for temporary drops in output. Our estimates of  $t^*$  tend to support this hypothesis. In no industry is the estimate of  $t^*$  for non-production workers less than that for production workers. In



industries 351, 353, 354, and 361 the estimates of  $t^*$  for non-production workers were 4, 2, 3, and 2 months longer, respectively, than the corresponding estimates for production workers. For industry 331 the estimate of  $t^*$  was the same as the estimates for production workers. Finally, in the case of non-production workers, the introduction of technological change increased the estimates of  $t^*$  in every industry except industry 353 where the estimate remained the same.

We turn now to the estimates of the labor-hoarding lag for the model with a production lag. Table 2 contains the best estimates of  $t^*$ ,  $\alpha$ ,  $m$  and  $q$  for the model with a rectangular lag function. In the case of total employment estimates of the labor-hoarding lag ranged from a low of 3 months in industry 331 to a high of 15 months in industry 351. For industries 353, 361, and 354 the estimates were 8, 9, and 12 months, respectively. For employment of production workers the estimates of  $t^*$  ranged from a low of 6 months in industry 331 to a high of 17 months in industry 353. Industry 351 had the third highest estimate with 16 months. For industries 354 and 361 the estimates were 14 and 12 months, respectively. Finally, in the case of employment of non-production workers the estimates of the labor-hoarding lag ranged from a low of 9 months in industry 331 to a high of 17 months in industry 353. Industries 351 and 361 both had estimates of 15 months, while for industry 354, the estimated labor-hoarding lag was 14 months. In only one industry, industry 351, was the estimated labor-hoarding lag for non-production workers less than the labor-hoarding lag for production workers. For industries 353 and 354 the estimated labor-hoarding lags were identical for both types of employment, while for industries 331 and 361 the labor-hoarding lag for non-production workers exceeded the labor-hoarding lag for production workers.

TABLE 2 - ESTIMATES OF THE LABOR-HOARDING LAG,  $t^*$ , USING A RECTANGULAR LAG FUNCTION

Industry	$t^*$	$\alpha$	q	m	$S^2$	$s^2$
Total Employment						
331 Blast Furnaces and Steel Mills	3	.040	10	.004	163,259.18	1,855.22
351 Engines and Turbines	15	.161	2	.006	1,023.45	12.18
353 Construction, Mining and Material Handling Equipment	8	.039	14	.002	2,756.46	32.43
354 Metalworking Machinery	12	.051	14	.004	10,971.68	129.08
361 Electrical Distribution Equipment	9	.029	12	.004	815.88	11.03
Production Workers						
331 Blast Furnaces and Steel Mills	6	.029	11	.004	90,378.10	1,038.10
351 Engines and Turbines	16	.079	3	.005	931.74	11.23
353 Construction, Mining and Material Handling Equipment	17	.022	12	.002	820.67	9.89
354 Metalworking Machinery	14	.039	12	.003	5,141.14	59.09
361 Electrical Distribution Equipment	12	.020	11	.003	607.72	8.10
Non-Production Workers						
331 Blast Furnaces and Steel Mills	9	.0058	13	.004	7,621.70	89.67
351 Engines and Turbines	15	.032	5	.009	47.23	.58
353 Construction, Mining and Material Handling Equipment	17	.019	9	.002	1,758.90	19.54
354 Metalworking Machinery	14	.019	11	.004	1,959.37	22.27
361 Electrical Distribution Equipment	15	.0090	13	.005	59.26	.81

$t^*$  = labor-hoarding lag in months  
 $\alpha$  = height of the rectangle  
 $q$  = length of the lag function minus one in months  
 $m$  = rate of labor-augmenting technical change  
 $S^2$  = sum of squared residuals  
 $s^2$  =  $S^2/(N-Deletions-q-3)$

We turn now to the best estimates of the production lag in the case of the rectangular lag function. These are obtained by adding 1 to the best estimates of  $q$  in Table 2. Doing this we find that in the case of total employment, estimates of the production lag ranged from a low of 3 months in the case of industry 351 to a high of 15 months in the case of industries 353 and 354. Industry 331 had an estimated production lag of 11 months while for industry 361 the estimated production lag was 13 months. In the case of employment of production workers industry 351 again had the lowest estimated production lag with only 4 months, followed by industries 331 and 361 both with estimates of 12 months, and finally by industries 353 and 354 both with estimated production lags of 13 months. In the case of employment of non-production workers the estimated production lags ranged from a low of 6 months in industry 351 to a high of 14 months in industries 331 and 361. Industries 353 and 354 had estimates of 10 and 12 months, respectively. It is interesting to note that in the case of industry 351, engines and turbines, the estimates of the production lag were relatively short. This provides some tentative evidence for rejecting the hypothesis of a capacity constraint in that industry. For industry 354, however, the estimates ranged from a low of 12 months for non-production workers to a high of 15 months for total employment. Although these estimates do not seem unreasonably long, it is impossible to determine whether or not they include the effects of a capacity constraint.

Table 3 contains the best estimates of  $t^*$ ,  $\alpha_1$ ,  $\alpha_2$ , and  $m$  for the model with a triangular lag function. For total employment estimates of the labor-hoarding lag,  $t^*$ , ranged from a low of 3 months in industry 354 to a high of

TABLE 3 - ESTIMATES OF THE LABOR-HOARDING LAG,  $t^*$ , USING A TRIANGULAR LAG FUNCTION

Industry	$t^*$	$\alpha_1$	$\alpha_2$	m	$S^2$	$s^2$
				Total Employment		
331 Blast Furnaces and Steel Mills (a)	7	.0066	8	.004	239,771.55	2,755.99
351 Engines and Turbines	17	.055	3	.006	1,902.72	22.65
353 Construction, Mining and Material Handling Equipment (a)	17	.057	3	.001	6,198.07	65.94
354 Metalworking Machinery (a)	3	.0091	9	.003	12,341.57	143.51
361 Electrical Distribution Equipment	17	.0076	7	.004	344.45	4.92
				Production Workers		
331 Blast Furnaces and Steel Mills	7	.0054	8	.004	160,468.03	1,844.47
351 Engines and Turbines	16	.014	5	.006	1,358.57	16.34
353 Construction, Mining and Material Handling Equipment	7	.0049	9	.002	1,059.57	12.77
354 Metalworking Machinery (a)	4	.0063	9	.003	5,214.80	61.35
361 Electrical Distribution Equipment	14	.0067	6	.003	224.18	3.11
				Non-Production Workers		
331 Blast Furnaces and Steel Mills (a)	16	.0012	8	.004	10,689.76	122.87
351 Engines and Turbines	19	.019	3	.008	77.32	.92
353 Construction, Mining and Material Handling Equipment	16	.043	2	.002	1,944.81	20.91
354 Metalworking Machinery (a)	1	.0025	9	.003	1,821.87	21.43
361 Electrical Distribution Equipment	20	.0050	5	.005	51.35	.79

$t^*$  = labor-hoarding lag in months

$\alpha_1$  = slope of triangle

$\alpha_2$  = mid-point of triangle in months

m = rate of labor-augmenting technical change

$S^2$  = sum of squared residuals

$s^2$  =  $S^2/N$ -Deletions- $(2\alpha_2-1)$ -3)

(a) These estimates are not based on complete convergence of the variance

17 months in industries 351, 353, and 361. Industry 331 had an estimated labor-hoarding lag of 7 months. For production workers industry 354 again had the lowest estimate of  $t^*$  with only 4 months. The highest estimates of  $t^*$  occurred in industries 351 and 361, with 16 months and 14 months, respectively. Industries 331 and 353 both had estimated labor-hoarding lags of 7 months. In the case of non-production workers industry 354 once again had the smallest labor-hoarding lag with a surprisingly low estimate of only one month. For the remaining industries the estimated labor-hoarding lags for non-production workers exceeded the estimates for production workers. Industry 361 had the highest estimate of 20 months followed by industry 351 with 19 months. Industries 331 and 353 both had estimates of 16 months.

We turn now to the best estimates of the production lag as measured by the length of the triangular lag function. The length of the production lag is given by twice the estimated mid-point of the triangle (i.e.,  $2\alpha_2$ ).<sup>9</sup> We see from Table 3 that in the case of total employment estimates of the production lag ranged from a low of 6 months in industries 351 and 353, to a high of 18 months in the case of industry 354. Industry 331 had the second longest production lag with 16 months followed by industry 361 with 14 months. For production workers estimates of the production lag ranged from a low of 10 months in industry 351, to a high of 18 months in industries 353 and 354. Industry 331 had an estimate of 16 months followed by industry 361 with 12 months. Finally, for employment of non-production workers estimates of the production lag ranged from a low of 4 months in industry 353 to a high of 18 months in industry 354. Industry 351 had the second lowest estimate with 6 months followed by industry 361 with an estimate of 10 months and industry 331 with an estimate of 16 months.

The estimates of the parameters for five of the employment series given in Table 3 are not based on complete convergence in the estimated variance. This was true for all three employment series for industry 354, and for total employment and employment of non-production workers in the case of industry 331. In each of these five employment series it was found that for large values of  $\alpha_2$  the estimated minimum cost employment series underestimated observed employment at the beginning of the samples for a number of months. Consequently, as  $\alpha_2$  was increased and the sample size reduced, the sums of squared residuals continued to decline. The estimates of the labor-hoarding lag, however, did not appear to be very sensitive in these series to increases in  $\alpha_2$  beyond 9 or 10 months. It was decided, therefore, to stop estimation at this point with the result that although the estimates of  $t^*$  are fairly close to their maximum likelihood estimates, the estimates of the production lag probably contain some downward bias.

The low estimates of the production lag for industry 351 obtained for all three employment series lends additional support to the evidence obtained in the case of the rectangular lag function, for rejecting the hypothesis of a capacity constraint in that industry. For industry 354, identical production lag estimates of 18 months were obtained for all three employment series. As indicated above these estimates probably underestimate the maximum likelihood estimate of the production lag. It is impossible, therefore, to determine whether or not they include any of the effects of a capacity constraint.

The final distributed lag function considered in our labor requirement equation is the geometric lag function. The best estimates of the labor-hoarding lag,  $t^*$ , and the parameters,  $\alpha_1$ ,  $\alpha_2$ , and  $m$ , are given in Table 4.<sup>10</sup> For total employment estimates of the labor-hoarding lag ranged from a low of only 2 months in industry 331 to a high of 15 months in industry 353. Industry

TABLE 4 - ESTIMATES OF THE LABOR-HOARDING LAG,  $t^*$ , USING A GEOMETRIC LAG FUNCTION

Industry	$t^*$	$\alpha_1$	$\alpha_2$	m	$S^2$	$s^2$
				Total Employment		
331 Blast Furnaces and Steel Mills	2	.045	.90	.004	64,612.46	680.13
351 Engines and Turbines	14	.31	.31	.005	1,354.46	15.94
353 Construction, Mining and Material Handling Equipment	15	.15	.70	.0004	3,461.45	36.44
354 Metalworking Machinery	12	.030	.96	.002	849.95	8.76
361 Electrical Distribution Equipment	7	.28	.20	.004	1,717.13	19.97
				Production Workers		
331 Blast Furnaces and Steel Mills	6	.037	.89	.003	60,016.35	631.75
351 Engines and Turbines	14	.19	.30	.004	961.66	11.31
353 Construction, Mining and Material Handling Equipment	2	.038	.90	.001	1,301.98	13.99
354 Metalworking Machinery	19	.030	.94	.002	645.48	6.80
361 Electrical Distribution Equipment	10	.16	.29	.003	793.59	9.23
				Non-Production Workers		
331 Blast Furnaces and Steel Mills	22	.0075	.90	.003	2,652.26	27.92
351 Engines and Turbines	13	.11	.29	.008	54.59	.64
353 Construction, Mining and Material Handling Equipment	16	.013	.93	.002	357.94	3.85
354 Metalworking Machinery	22	.009	.96	.003	176.20	1.86
361 Electrical Distribution Equipment	9	.094	.20	.005	132.14	1.54

$t^*$  = labor-hoarding lag in months  
 $\alpha_1$  = height of the function for  $t=0$  and  $\tau=0$   
 $\alpha_2$  = rate of decline of the geometric function  
 m = rate of labor-augmenting technical change  
 $S^2$  = sum of squared residuals  
 $s^2$  =  $S^2 / (N - \text{Deletions} - 3)$

351 had the second highest estimate with 14 months followed by industry 354 with 12 months and industry 361 with 7 months. For employment of production workers estimates of the labor-hoarding lag ranged from a low of 2 months in industry 353 to a high of 19 months in industry 354. Industry 351 had the second highest estimate with 14 months followed by industries 361 and 331 with 10 months and 6 months, respectively. Finally, in the case of employment of non-production workers the estimates of the labor-hoarding lag exceeded the estimates for production workers in four of the five industries. For industry 351 the estimated labor-hoarding lag for non-production workers was 13 months, only one month less than the estimate for production workers. Industries 331 and 354 had the highest estimates both with 22 months, followed by industry 353 with an estimate of 16 months, and lastly by industry 361 with an estimated labor-hoarding lag of 9 months.

Since the geometric function is not a finite function we cannot say much about the length of the production lag in this case. It is true, however, that the higher the value of  $\alpha_2$ , the longer the tail of the distribution. In this respect we note from Table 4 that in seven of the fifteen employment series estimated we obtained estimates of  $\alpha_2$  equal to or greater than .9. These occurred in industries 331, 353, and 354. In industries 351 and 361 estimates of  $\alpha_2$  were close to either .2 or .3. The low estimate of  $\alpha_2$  in all three employment series for industry 351 provides additional support to the evidence found earlier in the case of the rectangle and triangle that a capacity constraint did not exist in this industry during the sample period.

A summary of the estimates of  $t^*$  for each employment series for each distributed lag function is given in Table 5. It is quite evident from this table that the estimates of the labor-hoarding lag are highly sensitive to the specification of the distributed lag function. In the case of total employment



TABLE 5 - COMPARISON OF THE ESTIMATED LABOR-HOARDING LAG, t\*  
FOR EACH OF THE THREE DISTRIBUTED LAG FUNCTIONS

Industry	Rectangle	Triangle	Geometric
Total Employment			
331	3	7	2
351	15	17	14
353	8	17	15
354	12	3	12
361	9	17	7
Production Workers			
331	6	7	6
351	16	16	14
353	17	7	2
354	14	4	19
361	12	14	10
Non-Production Workers			
331	9	16	22
351	15	19	13
353	17	16	16
354	14	1	22
361	15	20	9

Source: Tables 2, 3 and 4.

industry 361 exhibited the greatest variability with a low estimate of 7 months obtained from the model with a geometric lag function and a high estimate of 17 months in the case of the triangle. Industries 353 and 354 also exhibited a fairly wide range in estimates. In the case of industry 353 estimates ranged from a low of 8 months from the rectangle to a high of 17 months from the triangle, while for industry 354 the lowest estimate of 3 months was obtained from the triangle and the highest estimate of 12 months from the rectangle and geometric. For industry 331 estimates of  $t^*$  ranged from a low of only 2 months for the model with a geometric lag function to a high of 7 months in the case of the model with a triangular lag function. The least variability in the estimates of the labor-hoarding lag occurred in industry 351. In this industry the estimates were all fairly high ranging from a low of 14 months in the case of the geometric to a high of 17 months in the case of the triangle.

For production workers the greatest stability in the estimates of  $t^*$  again occurred in industries 331 and 351. In the case of industry 331 estimates of 6 months were obtained from the models with rectangular and geometric lag functions while an estimate of 7 months was obtained from the triangular lag function. For industry 351 identical estimates of 16 months were obtained from the rectangle and triangle and an estimate of 14 months from the model with a geometric function. Industry 361 ranked third in terms of least variability of estimates with a range of 10 to 14 months. Industries 353 and 354 again exhibited the most variability in estimates of  $t^*$  with ranges of 2 to 17 months and 4 to 19 months, respectively.

Finally, in the case of non-production workers we find that the greatest difference in estimates occurred in industry 354. In this industry the labor-hoarding lag for non-production workers was estimated to be 1 month in the case of the triangle and 22 months in the case of the geometric lag function. The

second greatest difference occurred in industry 331 with a range of 9 to 22 months, followed by industry 361 with a range of 9 to 20 months. Industries 351 and 353 had the least variability across functions with ranges of 13 to 19 months and 16 to 17 months, respectively.

We now ask whether our estimates of  $t^*$  for both the model without a production lag and the models with a production lag provide strong enough evidence for rejecting the hypothesis that there is no labor-hoarding (i.e.,  $t^*=0$ ). A test of this hypothesis can be made by constructing the statistic

$$(41) \quad Z = \frac{S_0^2 - S_1^2}{s^2}$$

which is asymptotically  $\chi^2(1)$ .  $S_0^2$  is the sum of squared residuals under the null hypothesis that  $t^*=0$ ;  $S_1^2$  is the unconstrained sum of squared residuals; and,  $s^2$  is an estimate of the variance from the unconstrained equation. Table 6 contains estimates of  $Z$  for each of the models. For the model without a production lag the values of  $Z$  provide strong evidence for rejecting the hypothesis that  $t^*=0$  in every equation. For the model with a production lag, in only three of the forty-five equations estimated are the values of  $Z$  such that we cannot reject the hypothesis of no labor-hoarding. In the case of the rectangular lag function the hypothesis is rejected in all fifteen equations. For the triangular lag function the hypothesis is accepted in the case of production workers and non-production workers in industry 354 but rejected in all other equations. Finally, for the geometric distributed lag function the hypothesis is accepted in the case of total employment in industry 331 and rejected in all other employment series.

Since we have estimated the employment model for total employment, and its two components employment of production workers and employment of non-production workers, it is possible to test the hypothesis that production

TABLE 6 - TEST OF THE NULL HYPOTHESIS  $t^*=0$

Industry	Employment Series	No Production Lag	Assuming a Production Lag		
		Z	Z		
			Rectangle	Triangle	Geometric
331	TE	1,843	12	43	0
	PW	1,081	81	39	8
	NPW	1,397	33	82	560
351	TE	1,166	1,822	768	1,931
	PW	707	608	243	1,341
	NPW	4,539	2,241	4,352	500
353	TE	2,675	47	843	1,303
	PW	1,909	18,741	1,739	127
	NPW	1,991	195	543	417
354	TE	14,148	13	0	20
	PW	1,061	16	0	111
	NPW	1,192	25	22	6
361	TE	477	341	297	1,100
	PW	418	68	259	1,031
	NPW	405	363	903	1,813

TE: total employment  
 PW: production workers  
 NPW: non-production workers

The critical value of  $\chi^2(1)$  at the .05 level is 3.84.

workers and non-production workers are the same. This test can be made by computing the statistic

$$(42) \quad Z = \frac{S_{TE}^2 - (S_{PW}^2 + S_{NPW}^2)}{s_{PW}^2 + s_{NPW}^2}$$

which is asymptotically  $\chi^2(3)$  for the model without a production lag and  $\chi^2(4)$  for the model with a production lag.<sup>11</sup>  $S_{TE}^2$  is the sum of squared residuals from the total employment equation, representing the null hypothesis that production workers and non-production workers are the same;  $S_{PW}^2$  and  $S_{NPW}^2$  are the sum of square residuals from the unconstrained equations for production workers and non-production workers; and,  $s_{PW}^2$  and  $s_{NPW}^2$  are estimates of the variances from these equations.

Table 7 contains the estimates of Z for each model. In the case of the model without a production lag the values of Z for four of the five industries provide strong evidence for rejecting the hypothesis at the 5 percent significance level. For the model with a production lag the null hypothesis is rejected in eleven of the fifteen equations. In the case of the triangular lag function the hypothesis is rejected in all five industries. In the case of the rectangle it is accepted in industries 351 and 353, while in the case of the geometric it is accepted in industries 331 and 354.

The final hypothesis that we can test is the hypothesis that the production lag is not important in explaining the short-run demand for employment in the durable goods industries. A test of this hypothesis can be made by using the test statistic (41) which will be asymptotically  $\chi^2(1)$ .<sup>12</sup>  $S_0^2$  will now be the sum of squared residuals under the null hypothesis that there is no production lag;  $S_1^2$  will be the sum of squared residuals from the unconstrained model which includes a production lag; and  $s^2$ , is the estimate of the variance adjusted for degrees of freedom from the unconstrained equation.

TABLE 7 - TEST OF THE NULL HYPOTHESIS THAT PRODUCTION WORKERS AND NON-PRODUCTION WORKERS ARE THE SAME

Industry	No Production Lag	Assuming a Production Lag		
	Z		Z	
		Rectangle	Triangle	Geometric
331	0	58	35	3
351	45	4	27	37
353	26	6	73	100
354	39	42	65	3
361	86	17	18	72

The critical values of  $\chi^2(3)$  and  $\chi^2(4)$  at the .05 significance level are 7.81 and 9.49, respectively.

Table 8 contains the estimates of the labor-hoarding lags for the model without a production lag and the best estimates of the labor-hoarding lags for the model with a production lag, as well as the estimated values of the test statistic. The values of Z provide strong evidence for rejecting the null hypothesis in fourteen of the fifteen equations. Only for total employment in industry 353 is the hypothesis not rejected. In this one equation it would probably be better to suspend judgement rather than accept the null hypothesis, until further testing of the model could be made. This is particularly true since for production workers and non-production workers in this industry strong evidence was found for rejecting the null hypothesis. Finally, in comparing the estimates of  $t^*$  for the two models, we see that in twelve of the fifteen equations, the best estimates of the labor-hoarding lag from the model with a production lag were substantially larger than the model without a production lag. This is not surprising since the new orders series displayed substantially larger fluctuations than the shipments series. At the same time by using a



weighted average of part new orders, month to month fluctuations would tend to be smoothed out, and longer fluctuations would be given a greater relative weight in the estimation.

#### IV. SUMMARY AND CONCLUSIONS

The purpose of this paper has been to construct a model of short-run employment behavior in durable goods industries based on a theory of labor-hoarding. This theory operates on the assumption that there are important costs associated with changing the labor force. Given these labor force adjustment costs an optimal or minimum cost employment policy for a firm may be one in which the firm employs more labor during periods of declining output than is required to produce the desired level of output. Using the necessary and sufficient conditions for cost minimization, a programming algorithm was constructed which permitted us to obtain estimates of the maximum length of time it would be profitable for a firm to hoard idle workers, referred to as the labor-hoarding lag. Two models were constructed, the first model assumed that there was no production lag in durable goods industries, while the second model allowed for a production lag in the determination of required employment.

The results from both models provide strong evidence in support of the hypothesis that durable goods industries do hoard labor. Estimates of the labor-hoarding lag ranged from a low of 5 months to a high of 14 months in the case of the first model, and from a low of 2 months to a high of 22 months in the case of the second model. In addition, the performance of the second model in explaining the short-run behavior of employment in these industries was definitely superior to that of the first model. It was found, however, that the estimates of the labor-hoarding lag were fairly sensitive to the specification of the lag function.



The importance of the overall results of this study are two-fold. First, the results of this study clearly support the intuition of earlier investigators that firms do hoard labor. Such a phenomena provides one explanation for the contradictory results found by other investigators in empirical employment studies, regarding output-employment elasticities. If firms do hoard idle workers then observed employment will not be a good measure of the true production function input. Second, and perhaps more important, a willingness on the part of firms to hoard labor will have important consequences on the ability of monetary and fiscal policies to affect economic activity. Authorities in attempting to contract demand, output and employment must cope not only with a fairly long production lag but, on the basis of the evidence found in this paper, must also cope with a willingness by firms to hoard idle workers for periods as long as eighteen months.

FOOTNOTES

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1. Hiring costs are usually defined as including recruiting costs, orientation costs, recalling costs, training costs, and so forth. Lay-off costs will include severance pay, that part of part training costs that will not be recouped if the worker is laid off, and also unobservable costs such as bad worker morale, bad community relations, and union conflicts.

2. We could have included a discount rate in our cost function. Assuming a constant discount rate,  $r$ , total discounted labor cost is

$$C = \sum_{t=1}^T \frac{wL_t}{(1+r)^t} + \sum_{t=1}^{T-1} \frac{h \cdot [L_t - L_{t-1}]^+}{(1+r)^t} + \sum_{t=1}^T \frac{f \cdot [L_{t-1} - L_t]^+}{(1+r)^t}$$

The inclusion of the discount rate does not affect the derivation of the labor-hoarding lag  $t^*$ . We would now have for any  $t$  that,

$$\frac{(h+f)/(1+r)^t}{w/(1+r)^t} = \frac{h+f}{w} = t^*$$

That is to say, when considering the ratio of two nominal values discounted at the same rate we need only consider the ratio of the two nominal values.

3. The Jorgenson-Nadiri Siebert investment model assumes a Cobb-Douglas production function and includes a price for capital services in the investment equation.

4. For the industry comparisons, the model referred to is Eisner's investment equation which includes output and profits, while for the company comparisons the model referred to is a pure accelerator including only output.

5. The possible exceptions are the blast furnaces and steel mills industry and the electrical distribution equipment industry.

6. This problem has arisen in other empirical work dealing with the distributed lag relationship between shipments and new orders. A number of investigators in this area attempted to allow for nonstationarity by using an estimation method known as variable weighted distributed lags [18], [21]. This approach to the problem is somewhat ad hoc in that models are not usually based on a convincing underlying behavioral hypothesis. A second alternative is to allow for varying capacity utilization through the specification of production initiations. For example, we might let

$\chi_t = \text{Min}(C_t, Q_t)$  where  $C_t$  is capacity and  $Q_t$  the queue of unfilled orders for which production has not been initiated at the beginning of period  $t$ . This model implies a switching between two functions; when there is excess capacity production initiations are equal to  $Q_t$ ; when there is a capacity constraint production initiations are then equal to  $C_t$ .

7. The name of this industry classification is misleading. Unlike the other industries, the name of industry 331 does not refer to the output but to the means used to produce output. The output of this industry consists of ferrous and non-ferrous alloy and finished and semi-finished steel products.
8. This procedure for selecting initial and terminal months does not eliminate completely the possibility of error. If we choose as our beginning month, one in which shipments or  $\beta^L(Z)N_t$  were declining, it might be the case that they were also declining in previous months. Also if we choose as our terminal month one in which shipments or  $\beta^L(Z)N_t$  were rising, then it is quite possible that they continued to rise after that period. The result in both cases would be that our minimum cost employment series would underestimate the actual employment series during these months.
9. The triangular lag function given in (38) is constrained to zero at both ends of the distribution (i.e., for  $\tau=0$  and  $\tau=2\alpha_2, \beta_\tau=0$ ). The result is that in estimation, new orders in period  $t$  and period  $t=2\alpha_2$  do not influence the amount of labor required in period  $t$ .
10. In estimation it was found that the estimates of the parameters were not very sensitive to the choice of  $L_0$  in the vicinity of the beginning value of the observed employment series. For this reason  $L_0$  was fixed at a value very close to the beginning value of the observed employment series.
11. For the model with a production lag we are assuming that there are 4 parameters identical (i.e.,  $\alpha_1, q, t^*,$  and  $m$  in the case of the triangle;  $\alpha_1, \alpha_2, t^*,$  and  $m$  in the case of the triangle; and  $\alpha_1, \alpha_2, t^*,$  and  $m$  in the case of the geometric lag function.) For the model without a production lag three parameters are assumed equal  $\alpha, m,$  and  $t^*$ .
12. If the production lag is not important, then this is equivalent to saying that  $q=0$  in the case of the rectangle; that  $\alpha_1=0$  in the case of the triangle; and, that  $\alpha_2=0$  in the case of the geometric function.

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