

Western University  
Scholarship@Western

---

Department of Economics Research Reports

Economics Working Papers Archive

---

1971

# Non-Traded Goods and the Gains from Trade

Raveendra Batra

Follow this and additional works at: <https://ir.lib.uwo.ca/economicsresrpt>

 Part of the [Economics Commons](#)

---

## Citation of this paper:

Batra, Raveendra. "Non-Traded Goods and the Gains from Trade." Department of Economics Research Reports, 7110. London, ON: Department of Economics, University of Western Ontario (1971).

20327

ECONOMICS &

RESEARCH REPORT 7110

MAY 12 1971

NON-TRADED GOODS AND THE GAINS  
FROM TRADE

by

Raveendra Batra

May, 1971

NON-TRADED GOODS AND THE GAINS  
FROM TRADE

Previous analyses of the non-traded goods by writers such as Komiyama (1967), Kemp (1969) and Findlay (1971) among others have concentrated purely on the positive aspects of the theory of international trade. The normative or welfare implications of such goods in an open economy have not yet been explored.<sup>1</sup> The objective of this paper is to introduce a non-traded good in the usual two-good, two-factor, two-country model and examine the following theorems well known in the theory of gains from trade:<sup>2</sup>

1. Free trade is superior to no trade and for a small country which takes international prices as given, free trade is the optimal policy (Samuelson, 1939);<sup>3</sup>
2. An improvement in the terms of trade leads to an improvement in welfare, and conversely (Krueger and Sonnenshien (1967));<sup>4</sup>
3. Higher tariff is inferior to a lower tariff (Kemp, 1962);<sup>5</sup>
4. Production subsidy is superior to an equivalent tariff (Corden, 1957);
5. Consumption tax is superior to an equivalent tariff (Bhagwati and Srinivasan, 1969);<sup>6</sup>
6. For a large country possessing monopoly power in international trade, an optimal tariff is the optimum policy (Johnson, 1958 among others).

We shall refer to each one of these six results as the "standard" theorem. Section I lays down the general framework of our analysis; the implications of non-traded goods for the standard theorems are explored in Section II; finally the results of this study are summarized in Section III.

## I. Assumptions and the Model

Unless otherwise specified, the following assumptions will be maintained throughout the paper:

1. There is a small country which produces three commodities,  $X_1$ ,  $X_2$  and  $X_3$ , with the help of given supplies of two factors of production, capital (K) and labor (L). In its trading relations with the rest of the world, it takes international prices as given, imports the second commodity ( $X_2$ ) and exports the first commodity ( $X_1$ ). The third good,  $X_3$ , does not enter trade.
2. There is perfect competition in product as well as factor markets, and production functions are homogeneous of the first degree.
3. Factors are fully employed under all conditions.
4. Foreign trade is stable and inferior goods are absent.
5. The welfare of an economy can be represented by a well behaved social utility function which is assumed to possess the same properties as the individual indifference curves.
6. There is no satiation in consumption and no specialization in production, that is, all three commodities are always produced and consumed.

Let  $U$  stand for the total utility which a community derives from the consumption of the three goods, with their demands denoted by  $D_i$  ( $i=1,2,3$ ). The community's social welfare is then given by

$$(1) \quad U = U(D_1, D_2, D_3),$$

and

$$(2) \quad D_1 = X_1 - E_1$$

$$(3) \quad D_2 = X_2 + E_2$$

$$(4) \quad D_3 = X_3,$$

where  $E_1$  equals the export of the first commodity and  $E_2$  equals the import of the second commodity. Both  $E_1$  and  $E_2$  are defined to be non-negative. The balance of trade equilibrium requires that

$$(5) \quad P_1 E_1 = P_2 E_2,$$

where  $P_1$  and  $P_2$  are the foreign prices of the first and the second commodity, respectively.

For a small country, both  $P_1$  and  $P_2$  are given. The three linearly homogeneous production functions are:

$$(6) \quad X_1 = F_1(K_1, L_1);$$

$$(7) \quad X_2 = F_2(K_2, L_2);$$

$$(8) \quad X_3 = F_3(K_3, L_3).$$

Let  $a_i$  and  $b_i$ , respectively, be the marginal productivity of capital and labor in the  $i^{\text{th}}$  commodity. With perfect competition in the product as well as factor markets, the reward of each factor equals the value of its marginal product and is everywhere the same. In other words,

$$(9) \quad P_1 a_1 = P_2 a_2 = P_3 a_3$$

and

$$(10) \quad P_1 b_1 = P_2 b_2 = P_3 b_3,$$

where  $P_3$  is the price of the non-traded good.

Under full employment,

$$(11) \quad L_1 + L_2 + L_3 = L$$

and

$$(12) \quad K_1 + K_2 + K_3 = K .$$

This completes the specification of our model allowing for the presence of the non-traded goods.<sup>7</sup> The method of analysis followed by us throughout the paper consists in obtaining expressions for a change in the level of welfare when a slight deviation occurs in the conditions prevailing in the initial equilibrium. The sign of the change in welfare then determines whether or not the displacement from the initial equilibrium is beneficial to social welfare. However, if this expression is zero, then the initial situation is optimal.

Before proceeding directly to our analysis, the derivation of the relationship among the outputs of the three commodities is in order. Differentiating (6) and (7) totally and dividing through by  $dX_2$ , we have

$$(13) \quad \frac{dX_1}{dX_2} = \frac{a_1 dK_1 + b_1 dL_1}{a_2 dK_2 + b_2 dL_2} ,$$

which, by using (9) and (10) and from the fact that under full employment equations (11) and (12)

$$dL_1 + dL_2 + dL_3 = 0$$

and

$$dK_1 + dK_2 + dK_3 = 0 ,$$

can be written as

$$(14) \quad \frac{dX_1}{dX_2} = \frac{P_2}{P_1} \left[ \frac{a_2 (dK_2 + dK_3) + b_2 (dL_2 + dL_3)}{a_2 dK_2 + b_2 dL_2} \right]$$

$$= - \frac{P_2}{P_1} \left[ 1 + \frac{a_2 dK_3 + b_2 dL_3}{a_2 dK_2 + b_2 dL_2} \right]$$

In the analogous manner,

$$(15) \quad \frac{dX_3}{dX_2} = \frac{P_2 \left[ a_2 \frac{dK_3}{dL_3} + b_2 \right]}{P_3 \left[ a_2 \frac{dK_2}{dL_2} + b_2 \right]},$$

which when substituted in (14) furnishes,

$$(16^*) \quad \frac{dX_1}{dX_2} = - \frac{P_2}{P_1} \left[ 1 + \frac{P_3}{P_2} \frac{dX_3}{dX_2} \right].$$

Equation (16\*) describes the marginal rate of transformation between the first and the second commodity. In the absence of the non-traded good, (16\*) reduces to

$$\frac{dX_1}{dX_2} = - \frac{P_2}{P_1},$$

which is the familiar result that the marginal rate of transformation equals the negative of the commodity-price ratio. At times, in the derivation of subsequent equations, we will use the alternative form of (16\*), namely,

$$(16) \quad P_1 dX_1 + P_2 dX_2 + P_3 dX_3 = 0.$$

A. Optimality of Free Trade: Totally differentiating the utility function given by (1), we get

$$dU = U_1 (dD_1 + \frac{U_2}{U_1} dD_2 + \frac{U_3}{U_1} dD_3),$$

where  $U_i = \partial U / \partial D_i$  is the marginal utility of the  $i^{\text{th}}$  commodity. Under conditions of consumer equilibrium, the marginal rate of substitution between any pair of commodities equals the ratio between their prices, i.e.,  $(U_2/U_1) = (P_2/P_1)$  and  $(U_3/U_1) = (P_3/P_1)$ . Therefore,

$$(17) \quad dU = U_1 \left[ dD_1 + \frac{P_2}{P_1} dD_2 + \frac{P_3}{P_1} dD_3 \right].$$

Differentiating (2)-(5) totally, we obtain

$$(2^*) \quad dD_1 = dX_1 - dE_1$$

$$(3^*) \quad dD_2 = dX_2 + dE_2$$

$$(4^*) \quad dD_3 = dX_3$$

$$(5^*) \quad dE_1 = \frac{P_2}{P_1} dE_2.$$

Substituting (2\*)-(5\*) in (17) then yields

$$dU = U_1 (P_1 dX_1 + P_2 dX_2 + P_3 dX_3) / P_1,$$

which from (16) equals zero. In other words, free trade is the optimal policy for a small country. This is a straightforward generalization of Samuelson's theorem to a model which allows for the presence of non-traded goods.<sup>8</sup>

B. Changes in the Terms of Trade: Let us now examine the implications of the presence of the non-traded good for the Krueger-Sonnenschien theorem that an exogenous improvement (deterioration) in the terms of trade leads to a rise (decline) in welfare.

Since  $X_1$  has been assumed to be the exportable good and  $X_2$  the importable good, an exogenous improvement (deterioration) in the terms of trade can be represented by a rise (fall) in  $P_1$  alone or a fall (rise) in  $P_2$  alone. Let us then examine the implications for welfare of a small change in  $P_1$  only, keeping  $P_2$  as constant. Differentiating (1)-(5) totally with respect to  $P_1$  and following essentially the same procedure



as that used above, we obtain

$$(18) \quad \frac{1}{U_1} \frac{dU}{dP_1} = \frac{E_1}{P_1},$$

which is positive. In other words, the Krueger-Sonnenschien theorem continues to be valid in the presence of non-traded goods.

C. Higher Versus Lower Tariff: Next we turn to Kemp's theorem that a rise in tariff results in a loss in welfare. Assume that a small tariff already exists on the imports of the second commodity, so that

$$P_t = P_2(1+t),$$

where  $P_t$  is the domestic, tariff-inclusive price of the importable commodity and  $t$  is the rate of non-prohibitive tariff. With  $P_2$  constant for the small country under consideration,

$$(19) \quad \frac{dP_t}{dt} = P_2 > 0.$$

Differentiating (1)-(5) with respect to  $t$  and remembering that now

$(U_2/U_1) = (P_t/P_1)$  and  $(U_3/U_1) = (P_3/P_1)$ , we obtain

$$(20) \quad \begin{aligned} \frac{1}{U_1} \frac{dU}{dt} &= \frac{dX_1}{dt} - \frac{P_2}{P_1} \cdot \frac{dE_2}{dt} + \frac{P_t}{P_1} \left[ \frac{dX_2}{dt} + \frac{dE_2}{dt} \right] + \frac{P_3}{P_1} \cdot \frac{dD_3}{dt} \\ &= \frac{dX_1}{dt} + \frac{P_t}{P_1} \cdot \frac{dX_2}{dt} + \frac{P_3}{P_1} \cdot \frac{dD_3}{dt} + \frac{tP_2}{P_1} \cdot \frac{dE_2}{dt} \quad (\text{because } P_t = P_2(1+t)) \end{aligned}$$

With the imposition of the tariff,  $P_2$  is replaced by  $P_t$  in (16), so that

$$P_1 dX_1 + P_t dX_2 + P_3 dX_3 = 0.$$

Dividing through this by  $P_1 dt$  and substituting in (19), we obtain

$$(20) \quad \frac{1}{U_1} \cdot \frac{dU}{dt} = \frac{tP_2}{P_1} \cdot \frac{dE_2}{dt}.$$

In the absence of non-traded goods,  $dE_2/dt < 0$ , that is, a rise in the rate of tariff leads to a decline in the level of imports, and  $(1/U_1)(dU/dt) < 0$ , so that a rise in tariff is detrimental to the level of welfare. However, when a non-traded good is introduced, the sign of  $(dE_2/dt)$  becomes uncertain. For

$$\frac{dE_2}{dt} = \frac{dE_2}{dP_t} \frac{dP_t}{dt} = P_2 \frac{dE_2}{dP_t} = P_2 \left[ \frac{-dD_2}{dP_t} - \frac{dX_2}{dP_t} \right].$$

Now  $dD_2/dP_t < 0$  and, in the absence of the non-traded good,  $dX_2/dP_t > 0$ , so that  $dE_2/dP_t < 0$ . However, when a non-traded good is present, the sign of  $dX_2/dP_t$ , as shown by Komiya (1967, pp. 136-9), is ambiguous even if all goods are assumed to be gross substitutes, because now a change in the domestic price of the importable good as a result of the rise in tariff also causes a change in the price of the non-traded good which may be positive or negative, depending on the factor-proportions in the three industries. This latter factor tends to make the sign of  $dE_2/dP_t$  unpredictable, and, if  $D_3$  is large relative to  $D_1$ ,  $dE_2/dP_t$  may be positive, in which case the rise in the tariff rate will lead to a gain in welfare. Thus we conclude that a higher tariff may be superior to a lower tariff and Kemp's theorem may not hold in the presence of non-traded goods.<sup>9</sup>

D. Tariffs Versus Subsidies: We now turn to the standard theorem established by Corden (1957) that a tariff is inferior to an equivalent production subsidy. Equivalence is defined here in terms of the equality between the protective effect generated by the two policies on the production of the importable commodity. Suppose  $s$  stands for the rate of production subsidy to the importable commodity, so that

$P_s = P_2(1+s)$  and  $(dP_s/ds) = P_2$ , where  $P_s$  is the subsidy-inclusive price to the producer of the importable commodity. Then the equivalence between the tariff and subsidy is defined by

$$s=t, \quad dP_s = dP_t \quad \text{and} \quad \frac{dX_2}{dt} = \frac{dX_2}{ds} ;$$

that is to say, we compare those rates of the tariff and the production subsidy which result in an equal change in the output of the second commodity.

In order to compare the effects of tariffs and subsidies on welfare, we now need the expression for  $dU/ds$ . Differentiating (1)-(5) with respect to  $s$ , we obtain

$$(21) \quad \frac{1}{U_1} \frac{dU}{ds} = \frac{dX_1}{ds} - \frac{P_2}{P_1} \cdot \frac{dE_2}{ds} + \frac{P_2}{P_1} \left[ \frac{dX_2}{ds} + \frac{dE_2}{ds} \right] + \frac{P_3}{P_1} \cdot \frac{dX_3}{ds}$$

$$= \frac{dX_1}{ds} + \frac{P_2}{P_1} \frac{dX_2}{ds} + \frac{P_3}{P_1} \frac{dX_3}{ds} ,$$

where  $(U_2/U_1)$  still equals  $(P_2/P_1)$  because the award of the production subsidy to the importable good raises its price to its producers without causing any change in its price to its consumers. However, (16) changes to

$$P_1 dX_1 + P_s dX_2 + P_3 dX_3 = 0.$$

Dividing this by  $P_1 ds$  and substituting in (21), we have

$$(22) \quad \frac{1}{U_1} \cdot \frac{dU}{ds} = \frac{dX_2}{ds} \left[ \frac{P_2 - P_s}{P_1} \right] = - \frac{sP_2}{P_1} \cdot \frac{dX_2}{ds} .$$

In order to evaluate the sign of  $(dX_2/ds)$ , we borrow from Komiya (1967, p. 137) the expression for the effect of a change in the price of the importable good on its output. This is given by

$$\frac{dX_2}{dP_2} = - \Pi_2 \frac{dX_3}{dX_2} + S ,$$

where  $\Pi_2 = \frac{f_2(k_3 - k_1)}{f_3(k_2 - k_1)}$ ,  $f_i$  is the positive average product of labor in the  $i^{\text{th}}$  commodity,  $k_i$  is the capital/labor ratio in the  $i^{\text{th}}$  commodity ( $i=1,2,3$ ), and

$$S = - \left[ \frac{f_2^2 L_1}{f_1''(k_2 - k_1)^2} - \frac{f_1^2 L_2}{f_2''(k_2 - k_1)^2} - \frac{f_1^2 f_2^2 L_3}{f_3^2 f_3''(k_2 - k_1)^2} \right]$$

represents the substitution term in production and is positive, because  $f_i'' < 0$ . In the presence of the tariff,  $dP_2$  is replaced by  $dP_t$  on both the production and the consumption side, but with the production subsidy,  $dP_2$  is replaced by  $dP_s$  on the production side only. Therefore,

$$(22) \quad \frac{dX_2}{dP_t} = - \Pi_2 \frac{dX_3}{dP_t} + S = - \Pi_2 \frac{dD_3}{dP_t} + S,$$

and

$$(23) \quad \frac{dX_2}{dP_s} = - \Pi_2 \frac{dD_3}{dP_2} + S.$$

In the case of the tariff, suppose  $dD_3/dP_t > 0$ , that is, the importable and the non-traded goods are gross substitutes. Then if  $\Pi_2 > 0$ , which for instance occurs if  $k_1 \geq k_3 \geq k_2$ ,  $(dX_2/dP_t)$  may be negative if  $S$  is relatively small. Similarly, if  $dD_3/dP_t < 0$  and  $\Pi_2 < 0$ ,  $(dX_2/dP_t)$  may again be negative. In any case, the sign of  $dX_2/dP_t$ , as stated earlier, is uncertain. With subsidies, however, the story is different, because here the price to the consumers (i.e.,  $P_2$ ) remains unchanged, so that here  $dD_3/dP_2 = 0$  and, in effect,  $L_3 = 0$ . Hence

$$(24) \quad \frac{dX_2}{dP_s} = S^* > 0,$$

where

$$0 < S^* = - \left[ \frac{f_2^2 L_1}{f_1'' (k_2 - k_1)^2} - \frac{f_1^2 L_2}{f_2'' (k_2 - k_1)^2} \right] < S .$$

With  $dX_2/dP_s > 0$ , it is clear from (22) that  $(1/U_1)(dU/dP_s) < 0$ . In other words, a rise in the production subsidy is welfare-decreasing.

On the contrary, a rise in tariff rate may be welfare-increasing.

In other words, Corden's theorem that the production subsidy to the importable good is superior to the equivalent tariff may not be tenable when a non-traded good is introduced, simply because their effects on the output of the importable commodity are least likely to be equivalent. Further comprehension of Corden's theorem may be gained by subtracting (22) from (20) and obtaining

$$(25) \quad G = \frac{1}{U_1} \left[ \frac{dU}{dt} - \frac{dU}{ds} \right] = \frac{P_2}{P_1} \left[ t \frac{dE_2}{dt} + s \frac{dX_2}{ds} \right]$$

$$= \frac{P_2^2}{P_1} \left[ t \left\{ \frac{dD_2}{dP_t} - \frac{dX_2}{dP_t} \right\} + s \frac{dX_2}{dP_s} \right],$$

by using the fact that  $(dP_2/dt) = (dP_2/ds) = P_2$ . For Corden's theorem to be valid,  $G < 0$ . In the two-good model, with  $s=t$  and  $dP_s = dP_t$ , the output of the importable good changes (actually rises) in the same proportion (i.e.,  $dX_2/dP_t = dX_2/dP_s$ ), so that (25) reduces to

$$G = \frac{t P_2^2}{P_1} \cdot \frac{dD_2}{dP_t},$$

which is negative because  $dD_2/dP_t < 0$ . This is a simple proof of Corden's theorem. In order to appreciate the implications of the non-traded goods,

we substitute (22) and (23) in (25) and derive,

$$(26) \quad G = \frac{P_2}{P_1} \left[ t \left\{ \frac{dD_2}{dt} + \Pi_2 \frac{dD_3}{dP_t} - S \right\} + sS^* \right].$$

It may be immediately seen now that a sufficient condition for the validity of Corden's theorem, or for  $G < 0$ , is that  $\Pi_2 < 0$  if the importable and the non-traded goods are gross substitutes (so that  $dD_3/dP_t > 0$ ), for  $dD_2/dp_t < 0$  and with  $t=s$ ,  $s(S^*-S) < 0$ . However, if  $\Pi_2 > 0$  when  $dD_3/dP_t > 0$ ,  $G$  can be positive or even zero. Even if  $dX_2/dP_t > 0$ , it will be only by chance that  $(dX_2/dP_t) = (dX_2/dP_s)$ . Therefore even if  $(dX_2/dP_t) > 0$ ,  $G$  may still be positive if  $(dX_2/dP_t) < (dX_2/dP_s)$ , as is evident from (25).

These results can be logically explained with the aid of the familiar concepts introduced by Johnson (1962), of the "specialization gain," which is positively related to the domestic production of the exportable good, and the "exchange gain," which is positively related to the domestic consumption of the importable good. In a two-good model, both the specialization and exchange gains decline as a result of an increase in trade restriction, but, although the decline in specialization gain is the same with both the tariff and the equivalent production subsidy to the importable good, the decline in the exchange gain is larger with the tariff, because the subsidy, unlike the tariff, does not cause a rise in the price of the importable commodity to its consumers. In the presence of the non-traded good, however, the production subsidy necessarily leads to a decline in the specialization gain, whereas the tariff may or may not, for with the tariff, the output of the exportable good may actually rise. Therefore, even though the fall in the exchange gain is still

larger with the tariff, it is possible that the fall in the specialization gain may be larger with the subsidy. Hence the total decline in welfare under the tariff, which is the outcome of both these effects, may be larger than, smaller than, or equal to that caused by the introduction of the same rate of subsidy.

E. Tariffs Versus Consumption Taxes: According to Bhagwati and Srinivasan (1969), a consumption tax on the importable good is superior to the tariff when the community's objective is to restrict the consumption of the importable good to a certain level. In other words, the consumption tax is superior to the equivalent tariff, where equivalence may here be defined in terms of equal reduction in the consumption of the importable good from its level in the initial situation. Let  $c$  denote the rate of consumption tax on the second commodity, so that  $P_c = P_2(1+c)$  and  $(dP_c/dc) = P_2$ , where  $P_c$  stands for the consumption tax-inclusive price to the consumers of the importable good. The difference between the tariff and the consumption tax is that the tariff raises the domestic price of the importable good to both its producers and consumers, whereas the consumption tax raises its price only to the consumers.

Differentiating (1)-(5) totally with respect to  $c$ , we obtain

$$(27) \quad \frac{1}{U_1} \cdot \frac{dU}{dc} = \frac{dX_1}{dc} - \frac{P_2}{P_1} \cdot \frac{dE_2}{dc} + \frac{P_c}{P_1} \left[ \frac{dX_2}{dc} + \frac{dE_2}{dc} \right] + \frac{P_3}{P_1} \cdot \frac{dX_3}{dc} .$$

Dividing (16) by  $P_1 dc$  and substituting in (27), we obtain

$$(28) \quad \frac{1}{U_1} \cdot \frac{dU}{dc} = \frac{(P_c - P_2)}{P_1} \left[ \frac{dX_2}{dc} + \frac{dE_2}{dc} \right] = \frac{cP_2}{P_1} \cdot \frac{dD_2}{dc} = \frac{cP_2^2}{P_1} \cdot \frac{dD_2}{dP_c} .$$

Since  $(dD_2/dc) = (P_2 dD_2/dP_c) < 0$ , it is clear that a small increase in the consumption tax on the importable good is welfare-decreasing.<sup>10</sup> Again since a small rise in the tariff rate may be welfare-increasing, we conclude that the consumption tax may be inferior to the equivalent tariff, and the Bhagwati-Srinivasan (1969) theorem may not be valid in the presence of the non-traded goods.

Subtracting (28) from (20), with obtain, with  $c=t$ ,

$$(29^*) \quad H = \frac{1}{U_1} \left[ \frac{dU}{dt} - \frac{dU}{dc} \right] = \frac{tP_2}{P_1} \left[ \frac{dE_2}{dt} - \frac{dD_2}{dc} \right] = \frac{tP_2^2}{P_1} \left[ \frac{dE_2}{dP_t} - \frac{dD_2}{dP_c} \right]$$

by using  $(dP_t/dt) = (dP_c/dc) = P_2$ . Since  $(dE_2/dP_t)$  equals  $(dD_2/dP_t) - (dX_2/dP_t)$ , (29\*) can be written as

$$H = \frac{tP_2^2}{P_1} \left[ \frac{dD_2}{dP_t} - \frac{dX_2}{dP_t} - \frac{dD_2}{dP_c} \right].$$

Under the definition of equivalence  $(dD_2/dP_t) = (dD_2/dP_c)$ , so that

$$(29) \quad H = - \frac{tP_2^2}{P_1} \cdot \frac{dX_2}{dP_t}.$$

For the validity of the standard theorem by Bhagwati and Srinivasan, we require that  $H < 0$ , which is necessarily true in the two-good model because there  $(dX_2/dP_t) > 0$ . However, in the three-good model the sign of  $(dX_2/dP_t)$  is uncertain. Hence the necessary and sufficient condition for the Bhagwati-Srinivasan theorem to hold in the presence of the non-traded goods is that  $(dX_2/dP_t)$  has the "normal" positive sign.

F. The Optimum Tariff: Let us now relax the assumption that the country in question possesses no monopoly power in trade. Suppose  $P = P_2/P_1$



stands for the international terms of trade. Then in the presence of monopoly power in trade,  $dP$  is no longer equal to zero. Let  $P^*$  be the ratio between the domestic price of the importable good and that of the exportable good, and  $P_3^*$  be the ratio between the domestic price of the non-traded good and that of the exportable good. Differentiating (1)-(5) totally, we obtain

$$\begin{aligned}
 (30) \quad \frac{dU}{U_1} &= [dX_1 - dE_1 + P^*(dX_2 + dE_2) + P_3^* dX_3] \\
 &= [(dX_1 + P^* dX_2 + P_3^* dX_3) + \left\{\frac{P^*-P}{P}\right\}dE_1 - \frac{P^*}{P} E_2 dP] . \\
 &= \left[ \frac{(P^*-P)}{P^*} \frac{dE_1}{dP} \cdot \frac{P}{E_1} - 1 \right] \frac{P^*}{P} E_2 dP ,
 \end{aligned}$$

by using (16) and the balance of trade equilibrium equation which shows that  $E_2 = E_1/P$ .

The interior maximum requires that  $dU/U_1 = 0$ . For a small country  $dP = 0$ , so that this condition is satisfied if  $P^*$  is equated to  $P$  and a policy of free trade is followed. However, when  $dP \neq 0$ , equating  $P^*$  to  $P$  is not sufficient for the interior maximum. For  $P^*$  to be different from  $P$ , a tariff (or an export tax) must be introduced, so that

$$P^* = P(1+t) \quad \text{and} \quad \frac{P^*-P}{P^*} = \frac{t}{1+t} .$$

Substituting this in (30) and equating it to zero, we obtain the expression for the optimum tariff ( $t_0$ ):

$$t_0 = \frac{1}{\eta_1 - 1} ,$$

where  $\eta_1 = \frac{P}{E_1} \cdot \frac{dE_1}{dP}$  is the foreign elasticity of demand for imports.

In the two-good model,  $\eta_1$  is positive, and for optimum tariff to exist (or be positive), it is necessary that  $\eta_1 > 1$ . This is the well-known result derived by Johnson (1958) among many others. However, in the presence of the non-traded good,  $\eta_1$  may possess any sign. In other words, in the presence of non-traded goods, the optimum policy may require the award of an optimum subsidy to imports rather than the imposition of the tariff, provided, of course, that  $\eta_1 < 1$ . In any case there may not exist any positive tariff that will maximize welfare. In the absence of non-traded goods, the imposition of the tariff leads to an improvement in the terms of trade if the country in question enjoys monopoly power in trade. However, when a non-traded good is introduced, "one cannot exclude the possibility of the terms of trade deteriorating rather than improving as a result of a raise in the tariff rate..." (Komiya, 1967, p. 149). Such a situation necessitates a policy opposite to the imposition of the tariff, namely, the award of the optimum trade subsidy. It is worth stating here that this possibility perhaps arises from the instability of foreign trade equilibrium in the presence of the non-traded good.

### III. Conclusions

In the foregoing analysis, we have demonstrated how some of the standard theorems in the theory of gains from trade must be modified when a third, non-traded good is incorporated in the conventional two-commodity, two-factor, two-country model of international trade. It

has been established that Samuelson's theorem (1939) concerning the optimality of free trade and the Krueger-Sonnenschien theorem (1967) concerning the effects of changes in the terms of trade are fully valid in our two-factor, three-commodity framework under the same set of assumptions as are needed in the standard two-by-two model. However, Kemp's theorem (1962) concerning the effects of higher versus lower tariffs, Corden's theorem (1957) concerning tariffs versus subsidies and Bhagwati and Srinivasan theorem (1969) concerning consumption taxes versus tariffs may not be valid in the three-good case. Furthermore, there may not exist any tariff that will maximize welfare when the country possesses monopoly power in trade. A trade subsidy may be the optimal policy instead. The likelihood of the invalidity of some of these standard theorems rises with the rise in the domestic consumption of the non-traded good relative to the consumption of the traded goods.

## FOOTNOTES

<sup>1</sup>In any country there are several commodities which do not enter international trade at all. Prohibitive tariffs or transportation costs, the nature of the commodities (e.g., immovables like housing, etc.) are some reasons among many others which explain why some commodities are non-traded.

<sup>2</sup>The theory of gains from trade has been advancing gradually but steadily over the last thirty years, beginning with the seminal contribution by Samuelson (1939). In more recent years, the primary focus has been on the exploration of the implications of factor market distortions for welfare in an open economy. For these latter developments, see Bhagwati and Ramaswami (1963) and Batra and Pattanaik (1970,1971).

<sup>3</sup>This theorem may not hold if factor markets are distorted. See Bhagwati and Ramaswami (1963).

<sup>4</sup>The presence of inter-industry wage-differentials is destructive of the full validity of this theorem. See Batra and Pattanaik (1970) and Batra and Scully (1971).

<sup>5</sup>This theorem may not hold if either, as demonstrated by Bhagwati (1968) the importables are inferior goods, or, as shown by Batra and Pattanaik (1970), factor markets are distorted.

<sup>6</sup>Bhagwati and Srinivasan (1969) present their results in terms of deriving optimality conditions in the presence of "non-economic" objectives which are guided by principles other than economic rationality. If the objective is to restrict imports below the free trade level and thus raise the domestic availability of the importable good, then a consumption tax on the importables is superior to the equivalent tariff which restricts imports to the desired level.

<sup>7</sup>In order to understand the causality of the determination of equilibrium in this model with the non-traded good, see Komiya (1967).

<sup>8</sup>It is worth noting that we are not ignoring the income distributional aspects of a transition from no trade to free trade. These considerations are implicitly taken into account by the suitable choice of the social welfare function.

<sup>9</sup>Bhagwati (1968) has recently established that an increase in the tariff rate may be welfare-increasing if importables are subject to a tariff initially, provided there is inferiority in the consumption of the importable good. This results follows very simply from (20). For when the importable good is an inferior good, the sign of  $dE_2/dt$  is uncertain even in the absence of the non-traded good. Therefore, a rise in the tariff rate may have unpredictable effect on social welfare. Note also that this result is untenable if initially the rate of tariff is zero, so that with  $t=0$ ,  $(1/U_1)(dU/dt) = 0$  from (20).

<sup>10</sup> Another theorem derived by Bhagwati (1968) in the absence of non-traded goods that a rise in the consumption tax on importables is welfare-increasing, provided the importables are inferior goods can also be proved from (28). With  $(dD_2/dP_c) > 0$  when the importables are inferior goods,  $(1/U)(dU/dc) > 0$  which, means a higher consumption tax is beneficial to welfare.

## REFERENCES

- Batra, R. and P. K. Pattanaik, "Domestic Distortions and the Gains from Trade," Economic Journal, September 1970, 638-49.
- Batra, R. and P. K. Pattanaik, "Factor Market Distortions, the Terms of Trade and Welfare," American Economic Review, September 1971.
- Batra, R. and J. W. Scully, "The Theory of Wage-Differentials: Welfare and Immiscrizing Growth," Journal of International Economics, June 1971.
- Bhagwati, J., "The Gains From Trade Once Again," Oxford Economic Papers, July 1968, 137-48.
- Bhagwati, J. and V. K. Ramaswami, "Domestic Distortions, Tariffs, and the Theory of Optimum Subsidy," Journal of Political Economy, February 1963, 44-50.
- Bhagwati, J. and T. N. Srinivasan, "Optimal Intervention to Achieve Non-Economic Objectives," Review of Economic Studies, January 1969, 27-38.
- Corden, W. M., "Tariffs, Subsidies and the Terms of Trade," Economica, August 1957.
- Findlay, R., "Factor Proportions and Comparative Advantage in the Long Run," Journal of Political Economy, January 1971, 27-34.
- Johnson, H. G., International Trade and Economic Growth, (George Allen and Unwin: London, 1958), Ch. 2.
- Johnson, H. G., Money, Trade and Economic Growth, (Harvard University Press: Cambridge, 1962), Ch. 2.
- Kemp, M. C., "The Gain from Trade," Economic Journal, December 1962, 303-19.
- Kemp, M. C., The Pure Theory of International Trade and Investment (Prentice Hall: Englewood Cliffs, 1969), Ch. 6.
- Komiya, R., "Non-Traded Goods and the Pure Theory of International Trade," International Economic Review, June 1967, 132-52.
- Krueger, A. O. and H. Sonnenschien, "The Terms of Trade, the Gains from Trade and Price Divergence," International Economic Review, February 1967, 121-27.
- Samuelson, P. A., "The Gains from International Trade," Canadian Journal of Economics and Political Science, May 1939, 195-205.