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Commercial Banks and the Deposit Supply Equation

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In 1967, J. H. Kareken observed that the traditional money supply equation used in monetary economics was not a proper supply equation in the true micro-economic tradition.[1] He further reasoned that since it was usual to assume that the rate payable on demand deposits was fixed at zero, the demand equation for these deposits plus the specified deposit rate were sufficient, along with the condition of equilibrium, to fully determine the equilibrium values for the market. Under these circumstances it made little difference that the supply equation for deposits was so neglected since it was not really used in any event.

However, in Canada, the deposit rate for a significant portion of the deposits that are usually included in the money supply is not zero. Therefore, if this deposit rate is to be market determined, it is no longer an acceptable procedure to use the previous short-cut approach to determine the money supply equation within the Canadian banking system. A desired supply equation for bank deposits is derived in this article and some of the implications that this equation has for open market operations and various specified interest rates, when it is embedded in a fuller model of the banking system, is discussed.

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[1] J. H. Kareken, "Commercial Banks and the Supply of Money: A Market-Determined Deposit Rate," Federal Reserve Bulletin, October, 1967, pp. 1699-1712. More recently, B. Pesek has given an alternative treatment to many of the same issues surrounding this approach to a deposit supply function. For example, see Boris Pesek, "Bank's Supply Function and the Equilibrium Quantity of Money," The Canadian Journal of Economics, August, 1970, pp. 357-385.

I. The Model[2]

In order to simplify the discussion it is assumed that the asset side of the commercial bank balance sheet is composed of commercial bank loans to private individuals and corporations, government securities, vault cash, and commercial bank deposits at the central bank. In addition, both a primary reserve requirement and a secondary reserve requirement are assumed to exist. There is a rate of return paid for government securities, which comprise part of the secondary reserve requirement, but no return is paid on primary reserves. It is further assumed that both loans and deposit liabilities of the commercial banks are subject to increasing costs.[3] Formally, the model can be summarized as:

(1) $R + S + L \equiv D$

(2) $R \geq kD \quad (0 < k < 1)$

(3) $S \geq sD \quad (0 < s < 1)$

(4) $\Pi_L = r_L L - aL^2$

(5) $\Pi_R = 0$

(6) $\Pi_S = r_s \cdot S$

(7) $C = r_d \cdot D + b \cdot D^2$

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[2]The approach initially employed in this discussion of the model is similar, although not identical, to the approach employed by Kareken, *op. cit.*

[3]Kareken, *op. cit.*, makes similar assumptions. Pesek, *op. cit.*, cites a number of references to actual bank cost studies but notes that most of them appear to have some grave shortcomings. However, G. J. Benston in "Branch Banking and Economies of Scale," *Journal of Finance*, May, 1965, pp. 312-331, reports that he found some evidence of increasing costs for both deposits and loans in the branch banking system. The solution to the model would not be altered in any essential manner if deposits were assumed to be subject to constant costs, with only loans having increasing costs. The assumption of increasing costs for loans appears reasonable if one takes into account the possibility of much larger losses for bad debts, etc., that are likely to accompany increased loans, especially if the credit quality of the loan applicants is deteriorating as loan volume increases.

where

R = the primary reserves of the commercial banks, held in the form of currency, or deposits at the central bank,

S = commercial bank secondary reserves, consisting of government securities,

L = the supply of loans by commercial banks,

D = the supply of deposit liabilities by the commercial banks,

k = the required primary reserve ratio,

s = the required secondary reserve ratio,

Π_L = the net return obtained from commercial bank loans, expressed in dollars,

Π_R = the net return obtained from primary reserves, expressed in dollars,

Π_S = the net return obtained from secondary reserves, expressed in dollars,

C = the net cost of deposit liabilities, incurred by commercial banks, expressed in dollars,

r_L = the commercial bank loan rate,

r_S = the interest rate payable on government bonds.

From the foregoing equations it is apparent that the total net return from commercial bank assets, Π, is represented by

$$\Pi = r_S \cdot S + r_L \cdot L - aL^2 .$$

If there was no constraint imposed upon the amount of loans that could be supplied by the commercial bank, as a result of insufficient deposits, it would pay the commercial bank to expand its loans to the point,[4]

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[4]Since $\Pi = r_S \cdot S + r_L \cdot L - aL^2$ and $S = D - R - L$,

$$\frac{\partial \Pi}{\partial L} = - r_s + r_L - 2aL = 0$$

or

$$L^* = \frac{r_L - r_s}{2a} .$$

$$L^* = \frac{r_L - r_s}{2a} .$$

The minimum quantity of deposits that the commercial bank must have on hand to support a quantity, L*, in loans is given by,

$$D^* = \frac{1}{1-k-s} L^*$$

Thus for a quantity of actual deposits, D, less than D*, the commercial bank should set

$$L = (1-k-s)D$$

and $$S = sD .$$

Then the net return function Π would be

$$\Pi = r_s \cdot sD + r_L (1-k-s)D - a (1-k-s)^2 D^2 .$$

Similarly, if D > D*, the optimum allocation of assets will be given by

$$R = kD$$

$$L = L^*$$

and $S = D - R - L^*$, so the net return function will be

$$\Pi = r_s(1-k)D - r_s L^* + r_L L^* - aL^{*2} .$$

Equation 7 represents the cost of obtaining deposits. The marginal cost of obtaining these deposits is given by

$$\frac{\partial C}{\partial D} = rd + 2bD . \qquad (8)$$

For profit maximization, the commercial banks should have the deposit levels indicated below.[5]

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[5]Since, for $D \le D^*$, $\frac{\partial \Pi}{\partial D} = r_s \cdot s + r_L(1-k-s) - 2a(1-k-s)^2 D$,

then when $\frac{\partial C}{\partial D} = \frac{\partial \Pi}{\partial D}$,

$$rd + 2bD = r_s \cdot s + r_L(1-k-s) - 2a(1-k-s)^2 D$$

or

(a) If $D \leq D^*$

$$D = \frac{r_s \cdot s + r_L(1-k-s) - r_d}{[2b + 2a(1-k-s)^2]} \tag{9}$$

(b) If $D > D^*$,

$$D = \frac{r_s(1-k) - r_d}{2b} \tag{10}$$

When the loan rate, r_L, is zero, D^* will be negative.[6] Also, when r_L equals zero, D (where $D > D^*$) would be positive. Graphically, the situation is represented in Figure 1.[7]

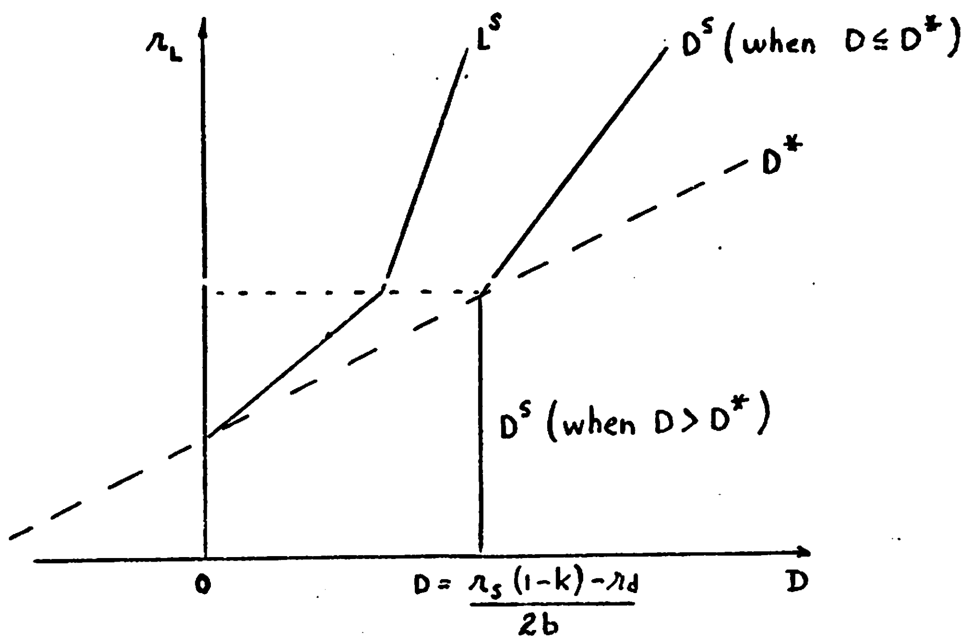

Figure 1

$$D = \frac{r_s \cdot s + r_L(1-k-s) - r_d}{[2b + 2a(1-k-s)^2]} .$$

For $D > D^*$, $\frac{\partial \Pi}{\partial D} = r_s(1-k)$, so $D = \frac{r_s(1-k) - r_d}{2b}$.

[6]Since

$$D^* = \frac{1}{(1-k-s)}\left(\frac{r_L - r_s}{2a}\right)$$

when $r_L = 0$, $D^* = \frac{-r_s}{(1-k-s)2a}$.

[7]The graph is drawn on the assumption that the yield on government bonds, r_s, is independent of the loan rate, r_L. Otherwise, an increase in r_L, leading to an

In Figure 1, the desired amount of deposits is represented by a constant amount D up to the point where D equals D*; after this intersection point, the desired deposit equation is represented by equation (9) of this paper (where $D \leq D^*$). In a similar fashion, the desired supply of loans is given by $L^* = (1-k-s)D^*$ up to the point where $D=D^*$, and then by $L = (1-k-s)D$ after this point.[8]

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increase in r_s, would result in a positively sloped curve for deposits (where $D > D^*$) instead of the vertically sloped curve indicated in Figure 1.

[8]Note that

$$L^* = \frac{r_L - r_s}{2a}, \quad \text{so } \frac{\partial r_L}{\partial L^*} = 2a. \quad \text{Also, } L = (1-k-s)D,$$

when $D \leq D^*$. Therefore the supply of loans can be represented as

$$L = (1-k-s)\left[\frac{r_s \cdot s + r_L(1-k-s) - r_s}{2b + 2a(1-k-s)^2}\right]$$

so $\dfrac{\partial r_L}{\partial L} = \dfrac{2b}{(1-k-s)^2} + 2a.$ Clearly if $k > 0$, which is the only range of values for which k would make sense, $\dfrac{\partial r_L}{\partial L} > \dfrac{\partial r_L}{\partial L^*}$; thus, the change in slope of the desired supply of loans function has the shape indicated in Figure 1.

From the preceding equations it follows that the function D* will shift left in Figure 1 when the values of r_s or a are increased, or when the values of k or s are decreased. Similarly, the function D will shift right when r_s, k, or s, is increased, or shift left when r_d, a, or b, is increased. The function L^S, will shift in two directions; when $L^S = L*$, an increase in r_s will shift this portion of the function to the left. Simultaneously, the portion of $L^S = L$ will shift to the right. To illustrate, an increase in r_s would shift the functions as indicated in Figure 2.

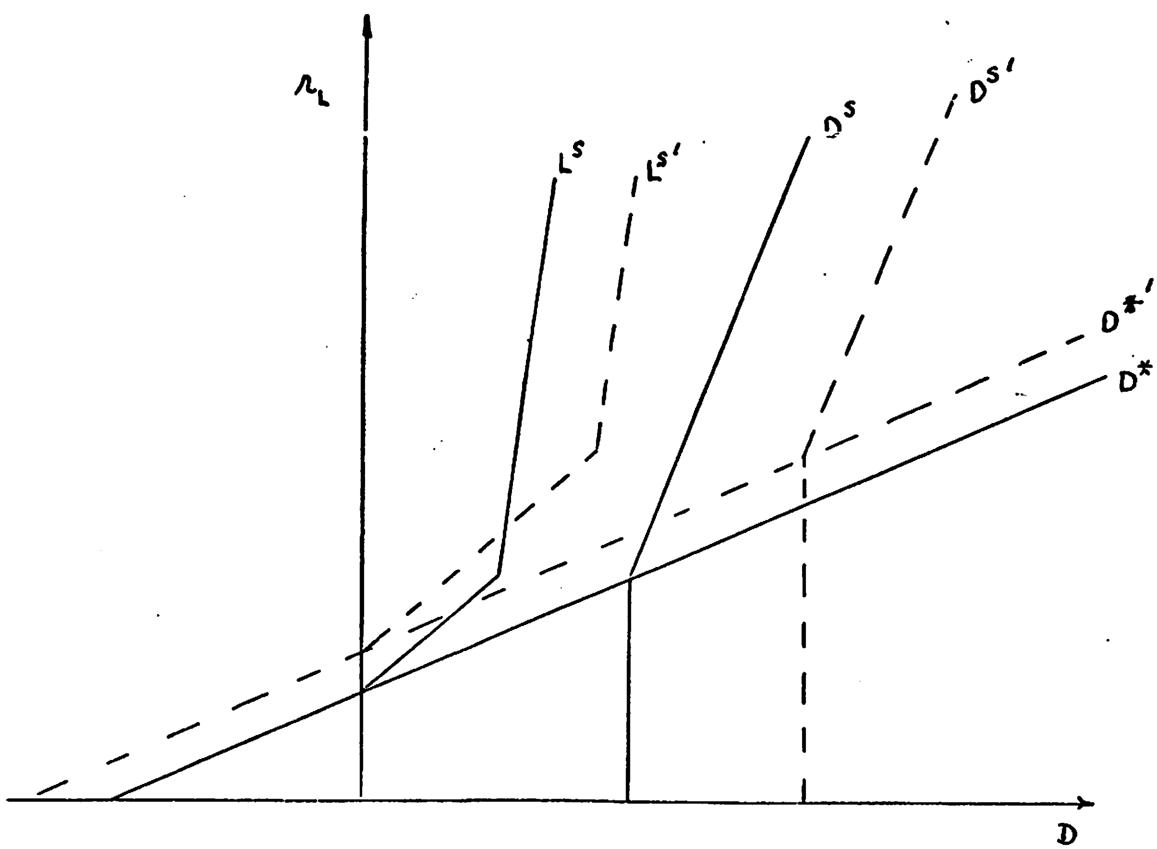

Figure 2

II. Reserve Requirements

Up to this point, no regard has been given to the constraints imposed upon commercial bank deposits by the primary and secondary reserve requirements. In Canada these requirements may be represented as

$$R \geq kD,$$

and

$$S \geq sD.$$ [9]

When $L \leq L^*$, excess secondary reserves and excess primary reserves would be zero under the condition of profit maximization. Thus, the restraint imposed upon deposits would be

$$D^S = \frac{H-C^*}{k}$$ [10]

where H = high powered money

C^* = the minimum amount of currency in circulation outside the commercial banks that the public would be prepared to hold.

The minimum amount of currency, C^*, is likely to be a function of the interest rates payable on assets which are substitutes for currency and upon the level of national income. However, for our purposes the actual determinants need not be spelled out here.

In addition, when this primary reserve requirement constraint is effective,

$$L^S = (1-k-s)D^S \ .$$

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[9] Actually, more correctly, the secondary reserve requirement should be represented as $S + (R-kD) \geq (k+s)D$, since excess primary reserves are considered part of the secondary reserves of commercial banks. However, in the subsequent discussion the omission of this distinction does not alter the argument.

[10] If excess primary reserves equal zero,

$$kD^S = R$$
$$= H - C^*$$

so

$$D^S = \frac{H-C^*}{k} \ .$$

Thus, depending upon the value of H-C*, a constraint is imposed on D^S, represented as in Figure 3.

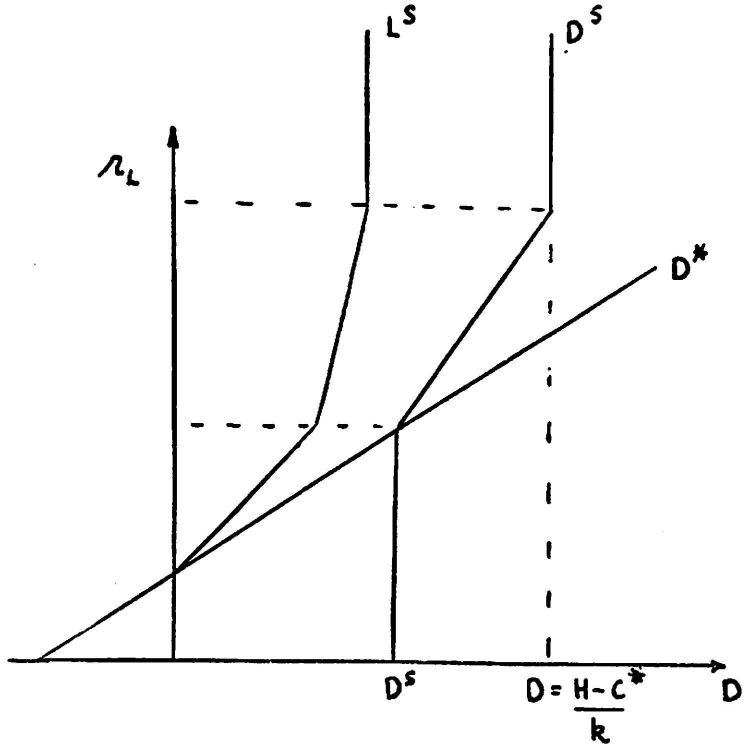

Figure 3

Clearly, if $D = \frac{H-C^*}{k}$ is less than $D = \frac{r_s(1-k) - r_d}{2b}$, the constraint imposed by the reserve requirement will be effective throughout the whole range of values of the interest rate, r_L. Assuming that this is not the case, imposing demand curves for deposits and loans on the diagram in Figure 3, gives the graph portrayed in Figure 4, where an equilibrium situation is shown.

The demand curve for deposits is shown with a negative slope because it is reasoned that as r_L is decreased, more loans are demanded, which leads to a higher level of deposits.

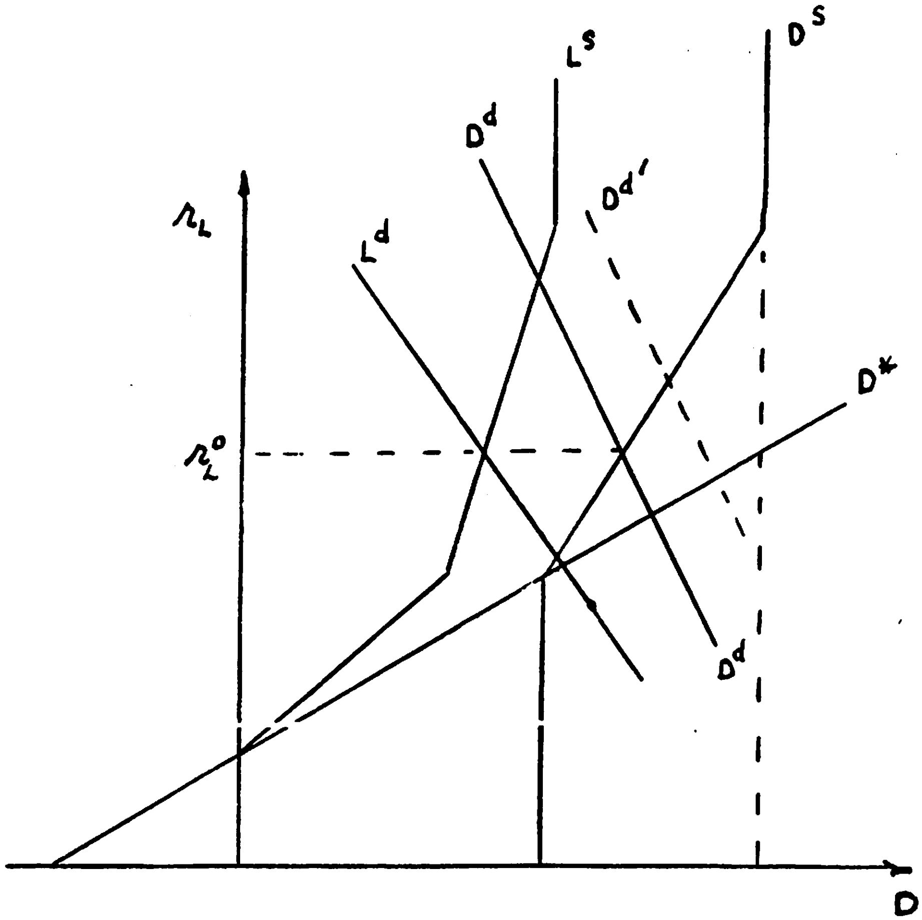

Figure 4

In Figure 4, if the deposit demand was shown by the dashed line $D^{d'}$, the markets would not be in equilibrium. At a loan rate of r_L^o, there would be a tendency for r_d to decrease which would cause the deposit supply curve to shift right, except where the reserve requirement constraint was effective; D* would remain unchanged. The public's demand curve for deposits would presumably shift left with the reduction in the deposit rate, r_d. If it was assumed that the demand for loans and the rate of interest on government securities were unaffected by these actions, the new equilibrium would be as indicated in Figure 5 where the solid line curves represent the original situation.

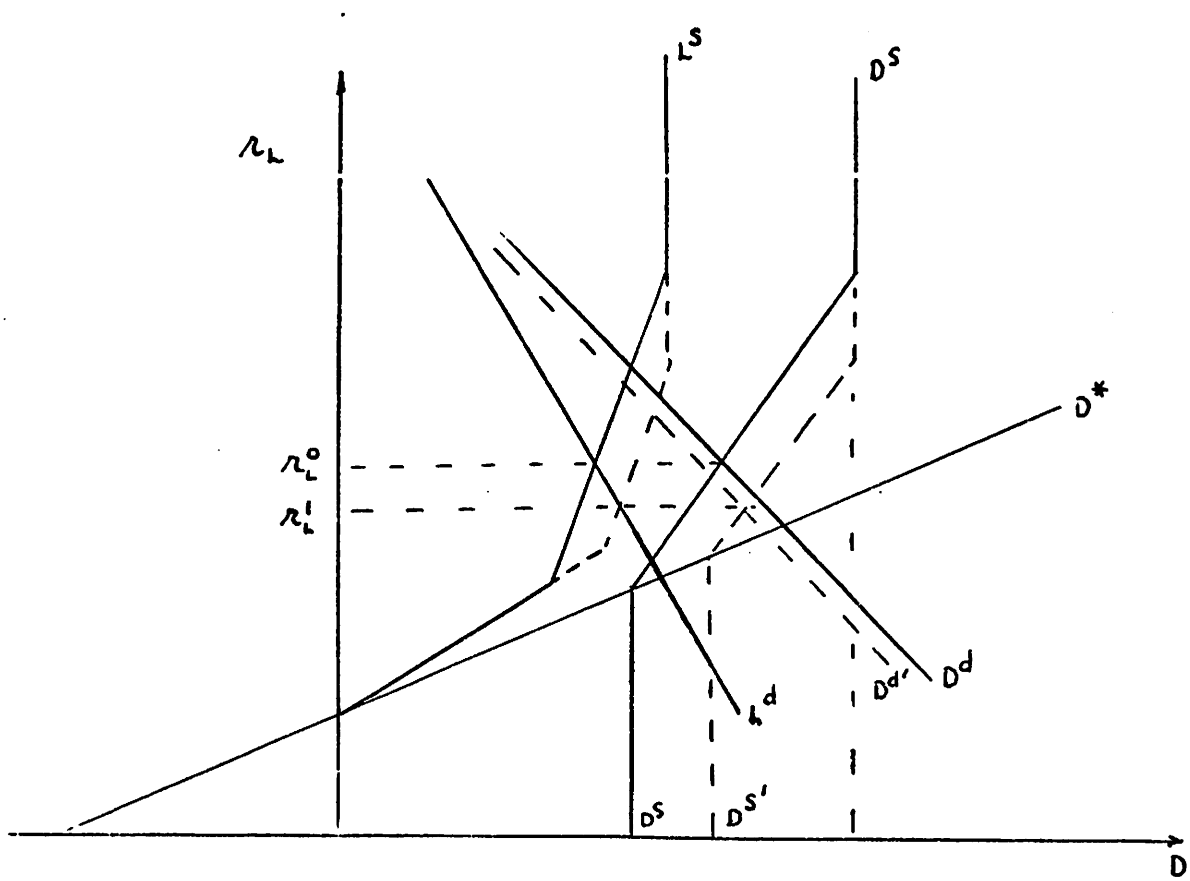

Figure 5

Note that only a portion of the loan supply equation shifts in response to the reduction in the deposit rate, r_d, with the new equilibrium shown at the new loan rate, r_L'. If the assumption was made that the government bond rate, r_s, also moved down in sympathy with the deposit rate, r_d, as some substitution takes place between bank deposits and government securities, the movements of the various curves would be modified somewhat. However, the essential characteristic that the new equilibrium would be at a loan rate, r_L, lower than the original level, r_L^o, would remain unaltered.

In a similar manner, if there was an excess supply of deposits at the old equilibrium rate, r_L^o, the curves would adjust to eliminate this disequilibrium in the deposit market.

Also, note that the causality between deposits and required reserves in this model is opposite to the traditional money multiplier approach. In the latter model it is assumed that, given a specified amount of reserves, bank deposits will expand until there are either no excess reserves or until these excess reserves reached some level desired by the commercial bank. However, in the present model, the amount of reserves held by the commercial bank is determined by the amount of deposits desired. If the amount of deposits desired by the commercial bank increases, the commercial bank must obtain the additional reserves required to support these deposits; usually this will be accomplished through changes in interest rates on assets that are substitutes for currency. If interest rate changes do not yield additional reserves, the attempt by the commercial bank to increase deposits will be frustrated.

III Open Market Operations

Consider an open market purchase of government bonds; the value, H-C*, would increase, assuming that the increase in high powered money is not all accounted for by a change in C*. If r_s is bid down as a result of this open market operation, D^S will shift left, D^d would presumably shift right (if it responded to r_s), and L^d would either remain unchanged or shift left. Also, D* would shift right. Clearly, initially, there will be an excess demand for deposits at the old loan rate, r_L, so r_d will have a tendency to decrease, causing D^S to shift right, D^d to shift left, and the L^S function to shift right. To the extent that

$$\left|\frac{\partial D^S}{\partial r_d}\right| > \left|\frac{\partial D^S}{\partial r_s}\right| \text{ and } \left|\frac{\partial D^d}{\partial r_d}\right| > \left|\frac{\partial D^d}{\partial r_s}\right|$$

, the new equilibrium would be at a lower value for r_L, r_d, and r_s, in conformity with normal expectations.[11] Graphically, the new equilibrium position in relation to the old equilibrium position is so portrayed in Figure 6. An open market sale can be traced out in a similar manner. Alternatively, the foregoing model can be portrayed

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[11] Actually these conditions are more restrictive than required. If the condition

$$\left|\frac{\partial D^d}{\partial r_d}\right| + \left|\frac{\partial D^s}{\partial r_d}\right| > \left|\frac{\partial D^d}{\partial r_s}\right| + \left|\frac{\partial D^S}{\partial r_s}\right|$$

holds, it would be sufficient to result in a lower value of r_L, r_d, and r_s. Since it appears reasonable to expect the effect of own-rate of interest to exceed the effect of other rates on deposits, it appears reasonable to conclude that the net effect of the open market purchase would be as shown in Figure 6.

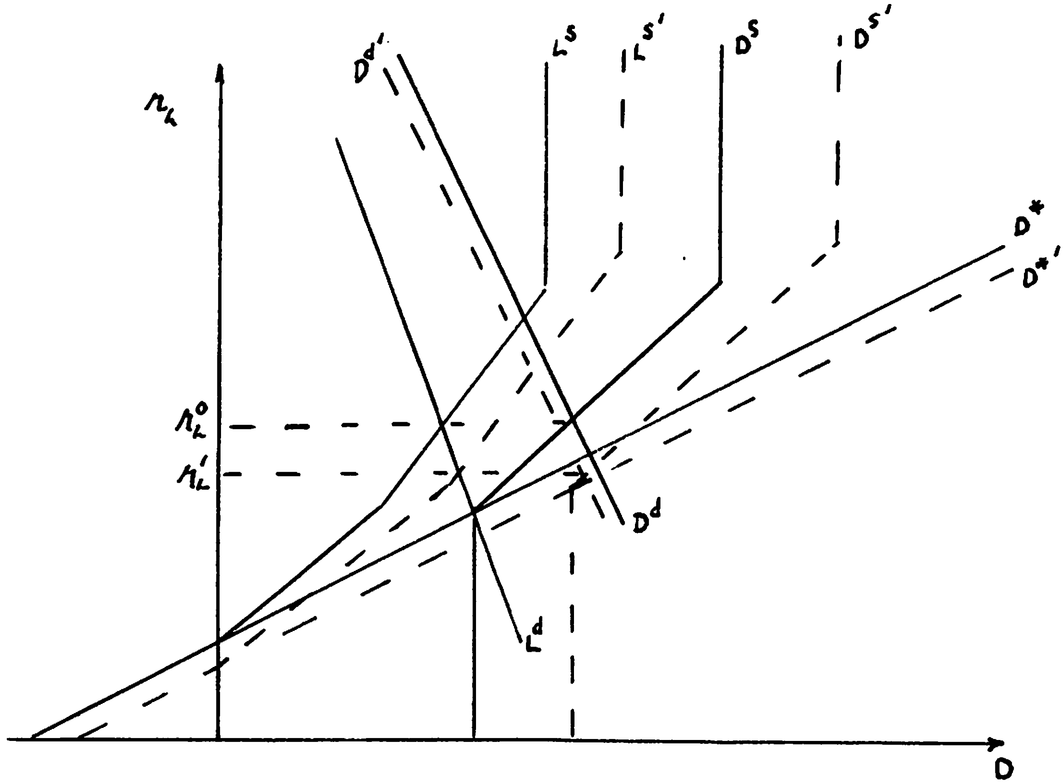

Figure 6

in the graph space (D, r_d) as shown in Figure 7.[12] The private sectors' demands for deposits and loans have been drawn to indicate an equilibrium at the deposit rate r_d^o.

For a decrease in r_s or r_L, D^S shifts to the left and D^d will shift to the right, if at all, under these circumstances. L^S will shift left for a decrease in either r_s or r_L and L^d would presumably shift right for

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[12] For example, when $D \leq D^*$, $\frac{\partial r_d}{\partial d} = -(2b + 2a(1-k-s)^2)$. Similarly, when $D > D^*$, $\frac{\partial r_d}{\partial D} = -2b$. Thus $\left|\frac{\partial r_d}{\partial D}\Big|_{D \leq D^*}\right|$ is $> \left|\frac{\partial r_d}{\partial D}\Big|_{D > D^*}\right|$, so the D^S curve has the slopes shown in Figure 7.

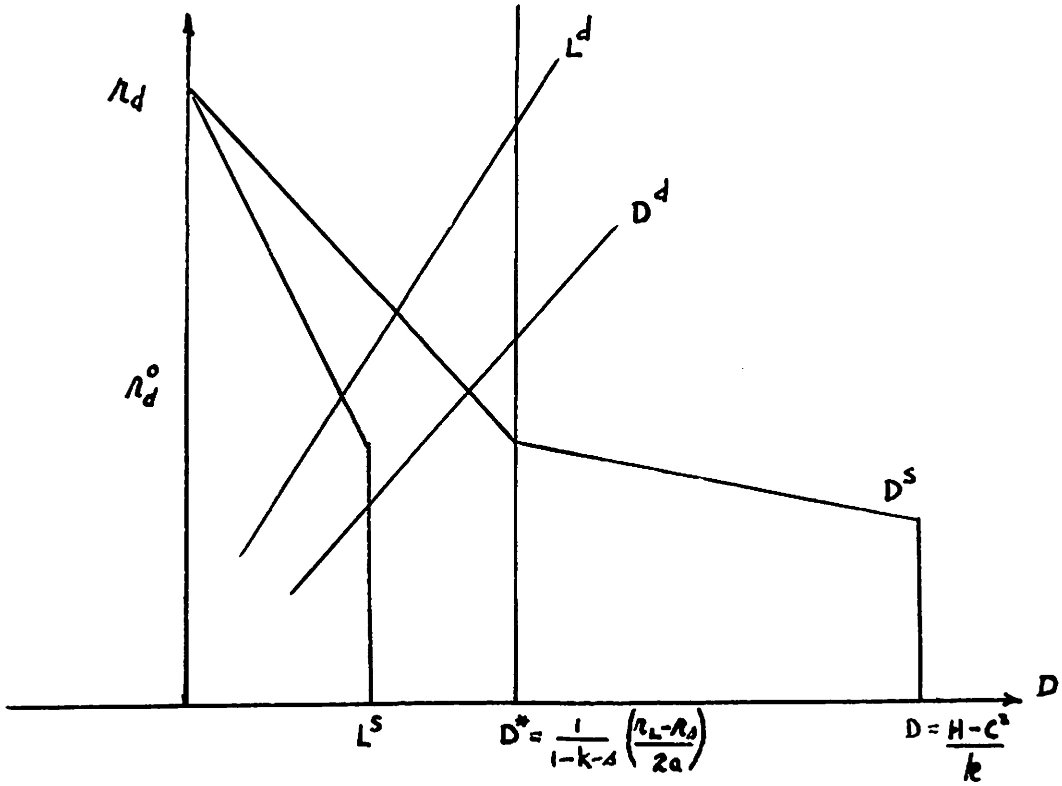

Figure 7

a decrease in r_L and, if anything, left for a decrease in r_s.

Thus, for the previous open market purchase the initial movement in the curves, in response to a decrease in r_s, would shift D^d right, D^S left, L^S left and L^d left. D* will initially shift right. After these initial shifts in the relevant curves, under reasonable assumptions, there will be an excess supply of loans in the loan market at the new deposit rate level, r_d, which equates the demand for deposits with the supply of deposits.[13]

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[13] As in the diagram which utilized the loan rate, r_L on the vertical axis, it is possible, without additional restrictions on the shifts in the curves, to have an excess demand in the loan market after the initial shift in the curves. However, as in the preceding case, this would lead to results that differ from one's normal expectations of the results of an open market operation.

Consequently, if the interest rates are market determined, r_L will decrease thereby shifting D* left, D^S left, L^S left, and L^D right until both the market for loans and the market for deposits are brought into equality for a single deposit rate, r_d. The new equilibrium position, in relation to the equilibrium which existed before the open market purchase, is shown in Figure 8.

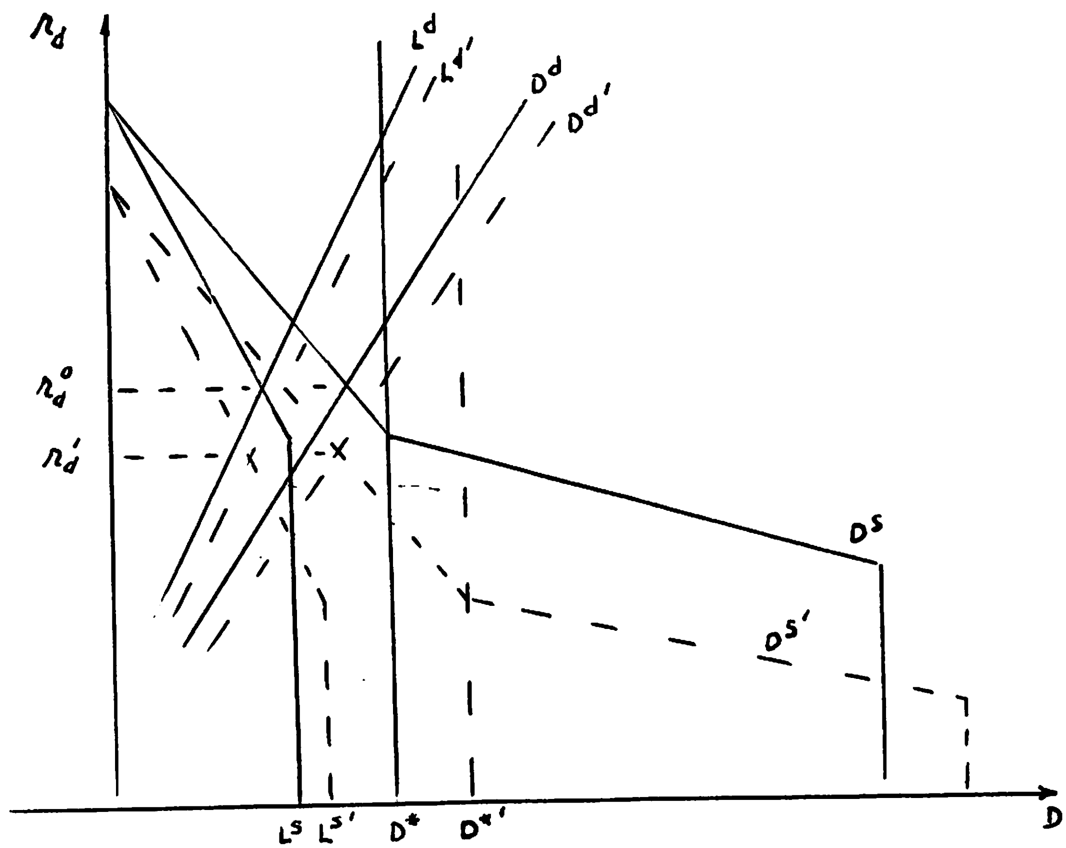

Figure 8

IV Imperfect Markets

The foregoing analysis has been predicated upon the assumption that the relevant rates freely adjust to the market pressures of excess demand or excess supply to bring about an equilibrium situation. This model has been developed on the basis of profit maximization; therefore, it is appropriate to ask whether, under oligopolistic conditions, such as prevail in the Canadian banking system, it would be expedient for the commercial banks to set either their loan rate, r_L, or their deposit rate, r_d, at some value other than those values dictated by the solution in the foregoing model.

The consequence of an increase in the loan rate, r_L, is illustrated in Figure 7.[14] Initially it was assumed that the equilibrium position in Figure 7 was given by the co-ordinates (r_L^o, D_o) with the amount of loans outstanding shown as L_o. Now, if the commercial banks raise their loan rate, r_L, to r_L', loans will be reduced to L_1, given by the intersection of the horizontal line at the loan rate, r_L', and the demand for loans equation, L^d. Initially, as a result of the increased loan rate, the commercial banks will have a desire to supply more deposits; this disequilibrium in the deposit market will lead to an increase in the deposit rate, r_d, which in turn causes the deposit supply and demand curves and the loan supply curve to shift in the manner shown in Figure 9. Thus, the new equilibrium position is shown where the deposits market is now in equilibrium at D_1.

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[14] In the interests of simplicity, it has been assumed that the change in the loan rate, r_L, does not cause a subsequent change in the government bond rate, r_s. The essential conclusions of the discussion are unaltered by this assumption.

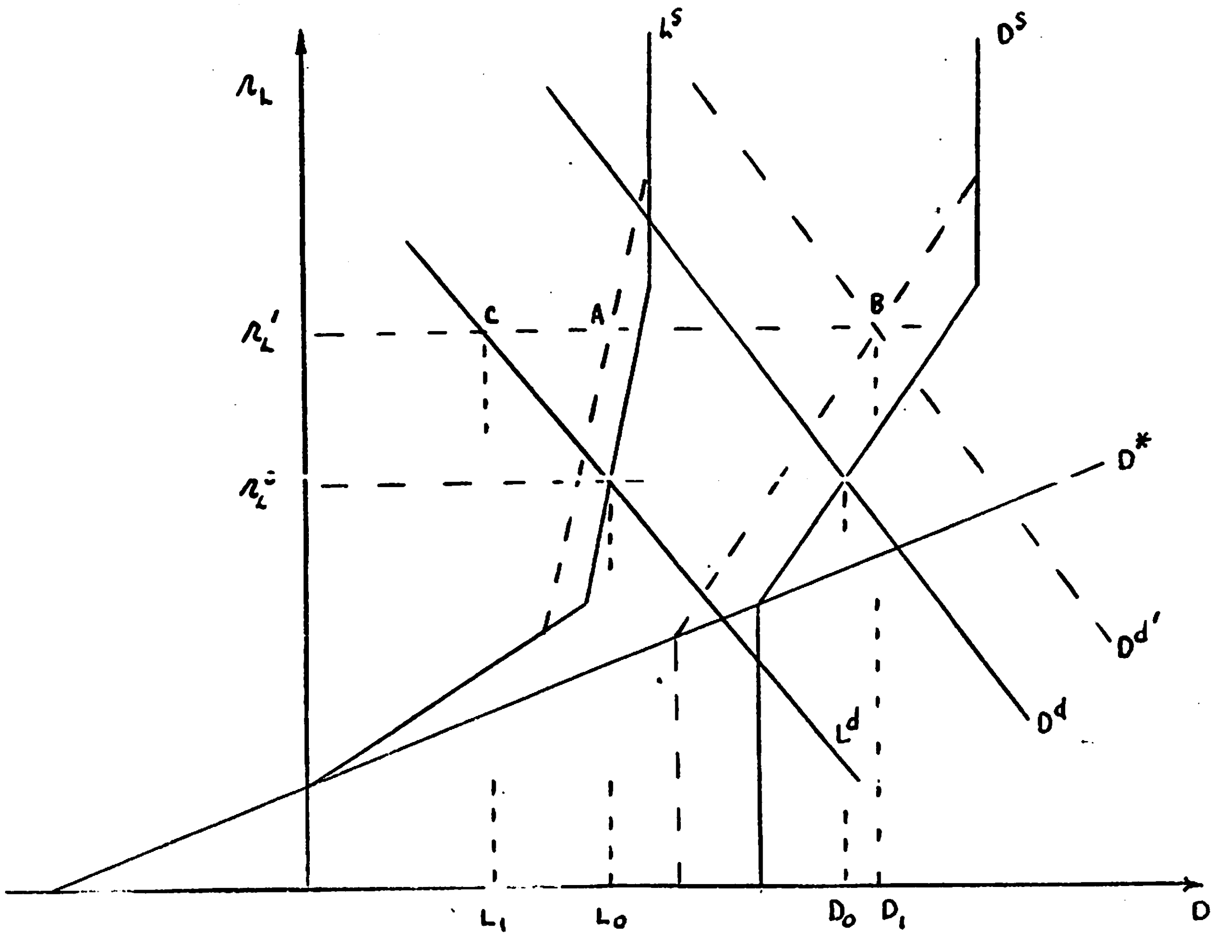


Figure 9

The horizontal distance between A and B, in the figure, represents the amount of primary and secondary reserves required. In this situation, the horizontal distance CA represents the demand for excess secondary reserves. Depending on the magnitude of the shifts of the deposit demand and supply curves, the equilibrium quantity of deposits could have occurred on the vertical section of the D^S curve where the primary reserve requirement constraint would have been the effective limiting factor. If the equilibrium quantity of deposits does not occur on the vertical section of the D^S curve, it may be in the commercial banks' interests to again raise their loan and deposit rates. The actual equation required to determine whether a further increase in rates is warranted does not reduce to a concise form: for those who are interested, it is developed in the appendix to this paper.

V Conclusions

A desired deposit supply equation for commercial banks has been derived utilizing the assumption of profit maximization. In Canada, where an interest rate is paid on chequable savings deposits, a deposit supply equation is needed to solve for the equilibrium quantity of deposits and the deposit rate, r_d, in the absence of market imperfections. In addition, this model implies a different direction of causality between deposit liabilities and required reserves from that assumed in the more traditional money multiplier approach.

The preceding analysis indicated that both the deposit supply equation and the loan supply equation are likely to be non-linear functions. Also, if the commercial banks do not have the market power to establish the loan rate, r_L, and the deposit rate, r_d, the model indicates that these commercial banks would have a tendency to carry excess secondary reserves only within the range of values of the loan rate, r_L, where $D > D^*$. However, where the commercial banks do have the necessary monopoly power to establish loan and deposit rates, the model indicates that there is a tendency to carry excess secondary reserves over a much wider range of values of loan rates.

When the deposit supply equation is included in a model for the commercial banks the effects of various monetary policy actions can be analyzed. In the case of an open market purchase of government bonds the model indicated that, under reasonable assumptions, the commercial bank loan rate, r_L, the government securities rate, r_s, and the commercial bank deposit rate, r_d, could all be expected to fall. At the same time, both loans and deposits would become larger, in conformity with normal expectations.

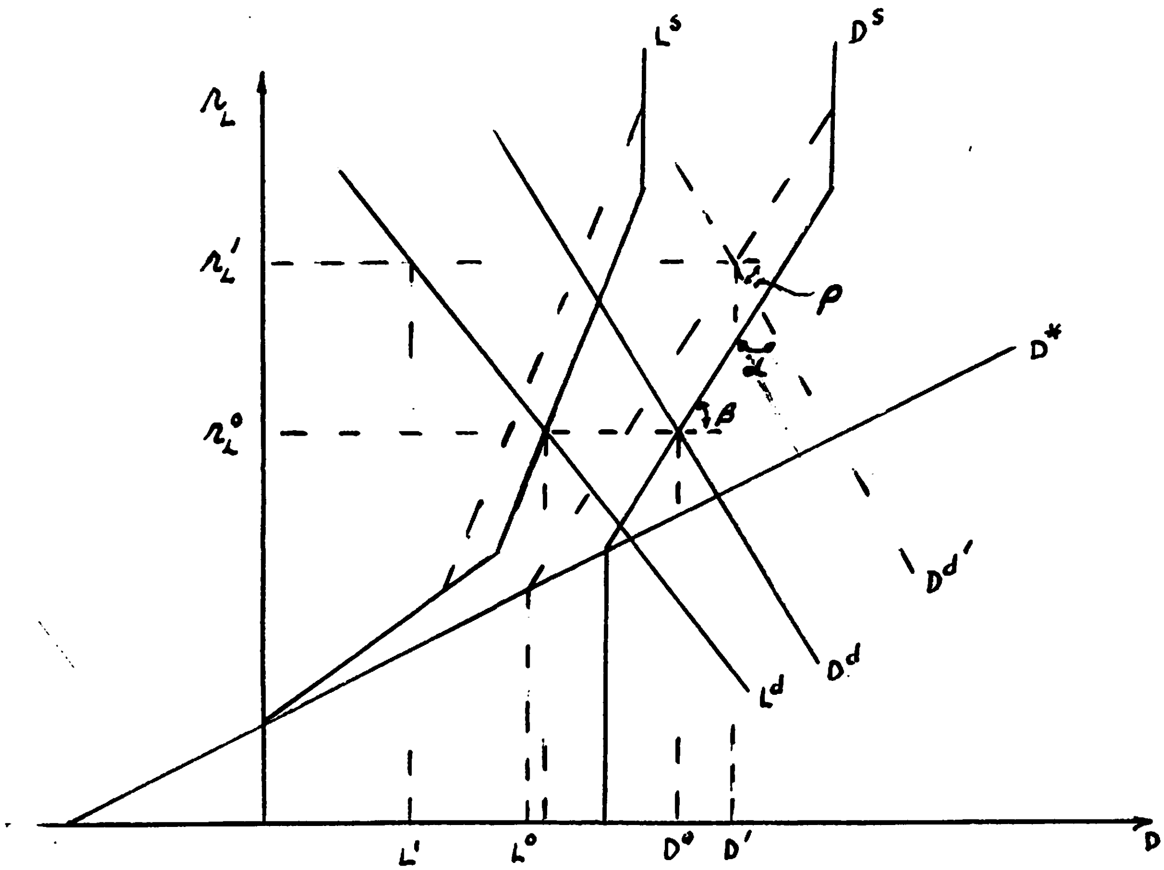
The preceding analysis also indicates that there appears to be only a limited range of values for the loan rate, r_L, the deposit rate, r_d, and the government security rate, r_s, over which the primary reserve requirement acts as an effective constraint on the desired deposit supply of commercial banks. Unless it can be illustrated that the values of the foregoing rates are normally within these specified ranges, the use of a multiplier approach to determine the money supply equation is clearly inappropriate.

However, in addition, if the commercial banks possess the monopoly power which enables them to establish their own loan rate and deposit rates, the use of a desired supply equation such as derived in the foregoing model would also yield an inappropriate equilibrium solution for the various loan and deposit variables. In this latter instance, it would be necessary to use an equation similar to the one described in the appendix to determine the exact solution to this model.

In summary, this analysis has indicated that, although under various specified conditions such approaches as the use of a money multiplier or even the derivation of a desired supply of deposits equation may yield correct solutions for the money supply equation, there are many circumstances in which the use of these approaches is inappropriate. This paper has attempted to illustrate some of the many possible variations to the solution of this difficult problem of establishing an appropriate deposit supply function for the commercial banks. In addition, the analysis has resulted in a model of the commercial bank sector which allows the assets and the liabilities of these intermediaries to be simultaneously determined.

Appendix A

1. Figure 1 illustrates the situation of the old equilibrium solution and the new equilibrium solution obtained by the commercial banks raising the loan rate, r_L, and the deposit rate, r_d. The old equilibrium values are indicated by L^o, D^o, r_L^o) and the new values are given by (L', D', r_L').



The shift in the deposit curve, D^d, due to the change in the deposit rate is given by $\frac{\partial D^d}{\partial r_d}$; similarly, the shift in the supply of deposits equation, D^s, is given by $\frac{\partial D^S}{\partial r_d}$. The net change in deposits given by the shifts in these two curves can be summarized as

$$D_1 - D_0 = [\frac{\partial D^d}{\partial r_d} \cdot \cos \beta - \frac{\partial D^S}{\partial r_d} \cdot \cos \rho] \cos \alpha ,$$

where β, ρ, and α are the angles denoted in Figure 1. Since the slopes of the demand and supply curves for deposits are known, the values of these angles are also known.

Therefore, the commercial banks could improve their profit position in relation to the profit implied by the competitive solution in the model as long as the condition below prevails:

$$\Delta r_L \cdot L_1 + r_s \Delta L + r_s \cdot s \cdot \Delta D > r_L^o \Delta L + \Delta r_d \cdot D_o + \Delta r_d \cdot \Delta D$$

$$+ \Delta D \cdot r_d^o + 2bD_o \Delta D + b(\Delta D)^2 ,$$

where $\Delta D = D_1 - D_0$, and

$\Delta L = L_0 - L_1$.

Alternatively, designating the elasticity of demand for loans, e_L as $\frac{\Delta L}{\Delta r_L} \cdot \frac{r_L'}{L}$, the condition for profit maximization could be rephrased to state that commercial banks should raise their loan and deposit rates until the following condition no longer prevails

$$\Delta L\left(\frac{r_L'}{e_L} + r_s - r_L^o\right) > (\Delta r_d + r_d^o + 2bD_o - r_s \cdot s)\Delta D + \Delta r_d D_o + b(\Delta D)^2 ,$$

where ΔL and ΔD are as previously defined.