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Intermediate Products, the Transformation Curve and Optimal Trade Policy

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The purpose of this paper is to explore the implications of such intermediate products as are produced to be utilized solely as inputs by the final products, for the shape of the transformation curve and some standard theorems in the theory of gains from trade which have been traditionally derived in the absence of intermediate goods. Most of the previous analyses concerned with intermediate products in trade theory assume that intermediate and final goods are identical,[1] even though, as Yates [18] has shown, the bulk of the world trade occurs in intermediate products which serve purely as material inputs. The effects of such intermediate goods have been analyzed by Khang [7] in the context of a dynamic trade model and by Ruffin [14] in the context of effective protection. In this paper, we utilize that part of Ruffin's model where intermediate goods are assumed to be produced domestically. We show, contrary to the assumption made by Ruffin,[2] that the transformation curve may become convex to the origin in the presence of, what Khang calls, "pure" intermediate goods. This result is derived in section I, which also deals with the conditions that ensure the usual concavity of the transformation curve to the origin. In section II, the implications of non-traded intermediate goods are briefly analyzed for gains from trade. Section III is concerned with optimal trade policy under the assumption that the transformation curve is still concave to the origin in spite of the presence of "pure" intermediate goods. The paper is concluded with some remarks in section IV.

I. A Closed Economy Model with Intermediate Products

It is assumed that an economy consists of three commodities, two final goods (X_1 and X_2) and one intermediate good (X_3) which is produced only to serve as an input in the production of the final products. There are two primary factors of production, capital (K) and labor (L), which are utilized in the production of all three commodities. Full employment, perfect competition, constant returns to scale, diminishing returns to factor proportions, inelastic factor supplies, perfect factor mobility and non-reversibility of factor-intensities at all factor prices are also assumed. The three production functions are:

(1.1) $X_1 = F_1(K_1, L_1, \bar{X}_{31}) = L_1 f_1(k_1, \bar{x}_{31})$

(1.2) $X_2 = F_2(K_2, L_2, \bar{X}_{32}) = L_2 f_2(k_2, \bar{x}_{32})$

(1.3) $X_3 = F_3(K_3, L_3) = L_3 f_3(k_3)$

where K_i and L_i are respectively the capital and labor inputs and $k_i = K_i/L_i$ is the capital/labor ratio in the i^{th} sector ($i = 1,2,3$) and $\bar{X}_{3j}$ is the amount of X_3 utilized as material input in the j^{th} final product ($j = 1,2$) and $\bar{x}_{3j} = \bar{X}_{3j}/L_j$, where the bars indicate--what we assume hereafter--that the intermediate good is utilized by final products in fixed proportions. Let $a_j = (X_{3j}/X_j)$ denote the requirement of X_3, the intermediate product, per unit of the j^{th} final product ($j = 1,2$). The material input coefficients, a_1 and a_2 are assumed to be constant. Equation (1.3) can be written as:

(1.4) $X_3 = a_1X_1 + a_2X_2.$

Let V_i stand for the marginal product of capital and U_i for the marginal product of labor in the i^{th} commodity. Then

$$V_i = \frac{df_i}{dk_i} = f_i'$$

and (i = 1,2,3).

$$U_i = f_i - k_i f_i' \quad .$$

We assume that $f_i' > 0$ and $f_i'' < 0$.

With perfect competition in both product and factor markets, the price of each factor equals its marginal value-added product and is equal in al three industries. Let w stand for the wage rate, r for the rental of capital, p_2 for the price of the second commodity, X_2, in terms of the first, X_1, and p_3 for the price of the intermediate product in terms of the first commodity. Factor prices expressed in terms of the first commodity are given by:

(1.5) $r = f_1'(1-a_1p_3) = f_2'(p_2-a_2p_3) = p_3f_3'$

(1.6) $w = (f_1-k_1f_1')(1-a_1p_3) = (f_2-k_2f_2')(p_2-a_2p_3) = p_3(f_3-k_3f_3')$.

With full employment,

(1.7) $L_1 + L_2 + L_3 = \overline{L}$

(1.8) $L_1k_1 + L_2k_2 + L_3k_3 = \overline{K}$

where the bars indicate that factor supplies are fixed. With this last equation, the supply side of our model is complete. To close the model, we need demand equations, but since these will not be needed until section III , we do not present them here.

A. The Slope of the Transformation Curve: It has been shown by Vanek [15] and Warne [17] that the slope of the transformation curve equals the negative of the commodity-price ratio even if the final commodities also serve as intermediate goods. We will now show that this result also holds in our model where the material input is not identical with the final goods. Totally differentiating (1.1)-(1.4) we have:

(1.9) $dX_1 = U_1 dL_1 + V_1 dK_1$

(1.10) $dX_2 = U_2 dL_2 + V_2 dK_2$

(1.11) $dX_3 = U_3 dL_3 + V_3 dK_3$

(1.12) $dX_3 = a_1 dX_1 + a_2 dX_2$.

From (1.7) and (1.8),

(1.13) $dL_1 = -(dL_2 + dL_3)$

(1.14) $dK_1 = -(dK_2 + dK_3)$.

Substituting (1.13), (1.14) and (1.11) in (1.9) and using (1.5) and (1.6), we obtain:

$$dX_1(1 - a_1 p_3) = -[dX_2(p_2 - a_2 p_3) + p_3 dX_3].$$

Then substitution of (1.12) in this gives us,

(1.15) $$\frac{dX_1}{dX_2} = -p_2$$

which is nothing but the "standard" result that the slope of the transformation curve equals the negative of the commodity-price ratio.

B. The Shape of the Transformation Curve: It is well known that the shape of the transformation curves in models where (1.15) holds can be derived from the response of the outputs to their prices.[3] In particular, if the output of a commodity rises in response to a rise in its relative price, and conversely, the underlying transformation curve is concave to the origin. Otherwise, it is convex to the origin. Therefore, our task here is to find out the effect of a change in p_2 on the output of X_1 and X_2.

Differentiating (1.5) and (1.6) with respect to p_2 we obtain:

$$(1.16)\qquad \frac{dk_1}{dp_2} = \frac{f_2 f_3}{(1-a_1p_3)f_1'' B}$$

$$(1.17)\qquad \frac{dk_2}{dp_2} = \frac{(1-a_1p_3)f_1 f_3}{(p_2-a_2p_3)^2 f_2'' B}$$

$$(1.18)\qquad \frac{dk_3}{dp_2} = \frac{(1-a_1p_3)f_1 f_2}{p_3^2 f_3'' B}$$

$$(1.19)\qquad \frac{dp_3}{dp_2} = \frac{(1-a_1p_3)f_2(k_3-k_1)}{B}$$

where $B = f_3(k_2-k_1) + f_2(k_3-k_1)(a_2-p_2a_1)$.

Differentiating (1.1)-(1.4) and (1.7) and (1.8) with respect to p_2 and using (1.16)-(1.18), we obtain:

$$(1.20)\qquad \frac{dX_1}{dp_2} = - \frac{p_2(1-a_1p_3)}{A.B}\lambda$$

$$(1.21)\qquad \frac{dX_2}{dp_2} = \frac{(1-a_1p_3)}{A.B}\lambda$$

$$(1.22)\qquad \frac{dX_3}{dp_2} = \frac{(1-a_1p_3)(a_2-p_2a_1)}{A.B.}\lambda$$

where $\lambda = \dfrac{L_1 f_2^2 f_3^2}{(1-a_1p_3)^3 f_1''} + \dfrac{L_2 f_1^2 f_3^2}{(p_2-a_2p_3)^3 f_2''} + \dfrac{L_3 f_1^2 f_2^2}{p_3^3 f_3''}$

and

$$A = a_1 f_1 (k_3 - k_2) + a_2 f_2 (k_1 - k_3) + f_3 (k_1 - k_2).$$

If the output response to commodity prices is to be "normal", then $dX_1/dp_2 < 0$ and $dX_2/dp_2 > 0$. From (1.20) and (1.21) it is clear that the signs of dX_1/dp_2 and dX_2/dp_2 depend on the signs of λ and A.B. In view of $f_i'' < 0$, $\lambda < 0$. Therefore $dX_1/dp_2 < 0$ and $dX_2/dp_2 > 0$ only if A.B. < 0. If there are no intermediate products, $a_1 = a_2 = 0$ and A.B. $= -f_3^2 (k_1 - k_2)^2 < 0$, so that the response of the outputs of final products to changes in their prices is normal. In the presence of intermediate products, an examination of A and B reveals that none of them may have a definite sign. However, in the following cases both A and B have definite signs:

i) $k_1 \gtrless k_3 \gtrless k_2$, $a_2 > p_2 a_1$

ii) $k_1 \gtrless k_2 \gtrless k_3$, $a_2 > p_2 a_1$ and $a_2 f_2 > a_1 f_1$

iii) $k_2 \gtrless k_1 \gtrless k_3$, $a_2 < p_2 a_1$ and $a_1 f_1 < a_2 f_2$.

In case (i), suppose $k_1 > k_3 > k_2$. Then $A > 0$ and if $a_2 > p_2 a_1$, $B < 0$, and both A and B have definite signs. If $k_1 < k_3 < k_2$, $A < 0$, but with $a_2 > p_2 a_1$, $B > 0$. It may be observed, however, that when both A and B have definite signs, their signs are opposite so that $A.B < 0$. We have already shown that if $A.B < 0$, the output response to changes in prices is normal and the transformation curve is concave to the origin. The following theorem is then immediate.

Theorem 1.1: If the capital/labor ratio of the intermediate product lies between the capital/labor ratios of the final products, and the final commodity which is treated as *numeraire* (commodity 1) has a lower material input coefficient in value terms than the other final commodity, the transformation curve is concave to the origin.

The proof of this theorem follows from the discussion of case (i) where k_3 lies between k_1 and k_2 and a_1, the material input coefficient of the first commodity which is our numeraire, is lower than a_2/p_2 ($=X_{32}/p_2X_2$), the ratio of the amount the intermediate good used in X_2 and the value of the output of X_2 in terms of the first commodity.

In case (ii), suppose $k_1 > k_2 > k_3$. Here $(k_1-k_2) > 0$, and with $a_2f_2 > a_1f_1$, one can see that $a_2f_2(k_1-k_3) > a_1f_1(k_2-k_3)$, because $(k_1-k_3) > (k_2-k_3)$. Hence $A > 0$. Moreover, with $a_2 > p_2a_1$, $B < 0$. Similarly, with $k_1 < k_2 < k_3$, $a_2 > p_2a_1$ and $a_2f_2 < a_1f_1$, $A < 0$, $B > 0$ and $A.B < 0$. A similar pattern holds in case (iii). One can see that here again $A.B < 0$. We can now derive the following theorem:

Theorem 1.2: If the final commodity, whose capital/labor ratio lies between the capital/labor ratios of the other final commodity and the intermediate product, not only has a higher material input coefficient in value terms but is also more intensive in the use of the intermediate good, then the output response to changes in their prices is normal and the transformation curve is concave to the origin.

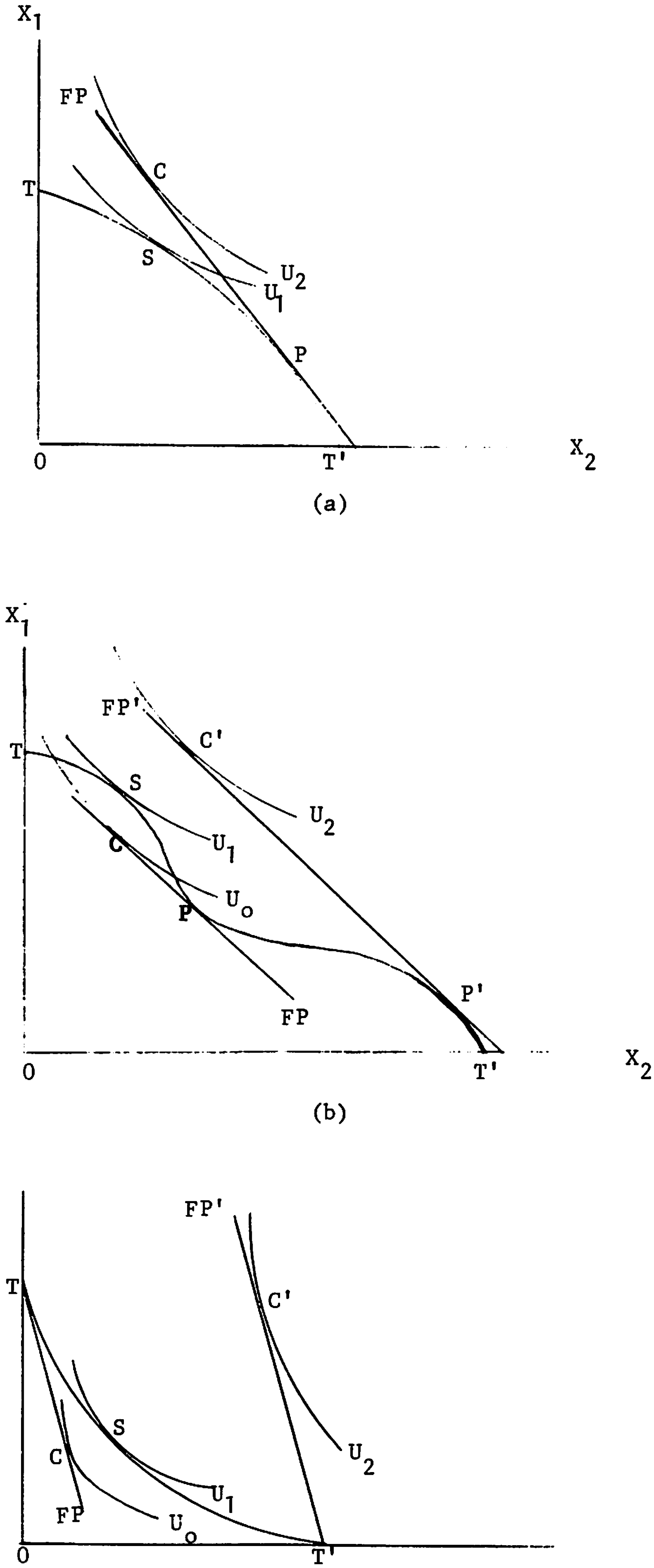
The notion of intensity in the use of the intermediate good is implied in the comparison between a_2f_2 and a_1f_1, because $a_jf_j = X_{3j}/L_j$ ($j=1,2$).

In the three cases discussed above, both A and B have definite but opposite signs and $A.B < 0$. However, this need not necessarily be the case. For example, in case (i) if $a_2 < p_2a_1$, then A still has a definite sign and B does not, and if B has the same sign as A, $A.B > 0$. In this case the outputs of the final products will respond "perversely" to changes in their prices, so that the transformation curve will become convex to the origin. Another interesting possibility is that for some values of p_2, $A.B < 0$, but for other values, $A.B > 0$. Here the transformation curve will have local convexity towards the origin. The following theorem may then be derived:

Theorem 1.3: If $A.B > 0$ for all levels of p_2, the transformation curve is globally convex to the origin; if A.B is positive for some values of p_2 and negative for other values, the transformation curve will be locally convex to the origin.

Thus the transformation curve, contrary to the traditional belief, may not be concave to the origin in our model with intermediate goods. It may have any of the shapes described in Figure 1. The output response of the final products with respect to changes in their prices and hence the shape of the transformation curve can also be determined diagrammatically by recourse to Melvin's geometry [12]. Consider Figure 2 where 0_3, 0_2 and 0_1 represent, respectively, the origins for X_3, X_2 and X_1; 0_2E0_1 is the contract curve between the final products X_1 and X_2, 0_2E is the equilibrium output of X_2 and 0_1E the equilibrium output of X_1, and these together utilize 0_30_2 output of the intermediate good. The capital/labor ratios in X_2 and X_1 are respectively given by the slopes of the lines 0_2E and 0_1E, whereas the capital/labor ratio in X_3 is given by the slope of 0_30_2. Now suppose there is a rise in the price of X_2, so that p_2 rises.

From equations (1.16)-(1.18) we know that dk_1/dp_2, dk_2/dp_2 and dk_3/dp_2 have the same signs, which means that a change in p_2 changes the capital/labor ratios in all three commodities in the same direction. Suppose that B has the same sign as (k_2-k_1).[4] If $k_2 > k_1$, as is the case in Figure 2, then $B > 0$. From equations (1.16)-(1.18), it is clear that $dk_i/dp_2 < 0$. In other words, a rise in p_2 will lower the capital/labor ratio in all three commodities. In terms of Figure 2, the production point will move along the contract curve towards the origin 0_1. For the time being, suppose that the


X1
FP
C
T
S
U2
U1
P
0
T'
X2
(a)
X1
FP'
C'
T
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U2
U1
C
Uo
P
P'
FP
0
T'
X2
(b)
FP'
T
C'
S
U2
C
U1
FP
Uo
0
T'

(c)
Figure 1

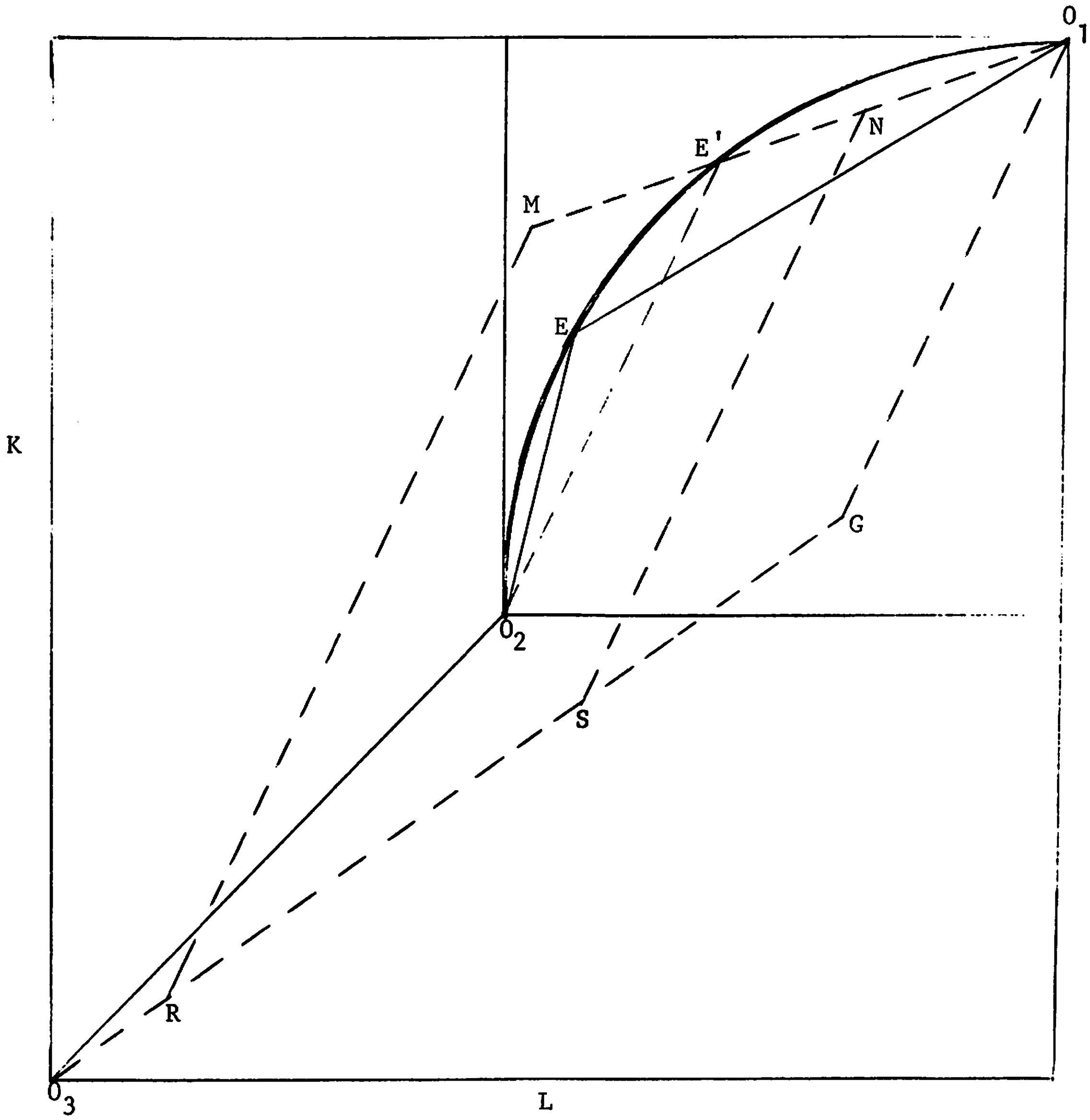

Figure 2

capital/labor ratio and the output of the intermediate product remain unchanged, and the production point on the contract curve is given by E' which shows that the capital/labor ratios in X_2 and X_1 have declined to the slopes of $0_2E'$ and $0_1E'$, respectively. Now suppose the capital/labor ratio in X_3 has declined to the slope of 0_3S, and let S be the point where the output of X_3 has remained unchanged (i.e., $0_3S = 0_30_2$). This is possible if $a_2 = p_2a_1$, as is evident from (1.22). If the output of X_3 remains unchanged at S, the origin for X_2 will now shift to point S, and there will be a new contract curve for the final products between the points S and 0_1. The new production point will lie on the new contract curve, but the capital/labor ratios in X_2 and X_1 will still equal the slopes of $0_2E'$ and $0_1E'$, respectively. Such a production point is given by N which is obtained by drawing SN, which intersects $0_1E'$ at N, parallel to $0_2E'$. Thus if the output of X_3 remains unchanged, because $a_2 = p_2a_1$, the output of X_2 is given by SN and that of X_1 by 0_1N. Since $SN > 0_2E$ and $0_1N < 0_1E$, the output of X_2 has risen and that of X_1 declined as a result of the rise in the relative price of X_2, p_2. If $a_2 \neq p_2a_1$, the output of X_3 will not remain unchanged after p_2 has changed. If the output of X_3 rises in the new situation, the new origin for X_2 will move along 0_3S towards G and lie between S and G, because at point G, the output of X_1 will be zero and the economy will be completely specialized in X_2. On the other hand, if the output of X_3 declines as a result of the rise in p_2, the new X_2 origin will move along 0_3S towards 0_3. Suppose the length 0_1M equals 0_1E, the original output of X_1. From M draw MR parallel to $0_2E'$. Then the new X_2 origin must lie between S and R. This is because if the new X_2 origin lies at point R, the output of $X_1 (=0_1M =0_1E)$ remains unchanged, whereas the output of $X_2 (=MR > 0_2E)$ rises. This result is clearly inconsistent with the reduced output of X_3. If the new X_2 origin lies anywhere between 0_3 and R, this inconsistency remains. It is only when the new origin of X_2 lies between R and S that this inconsistency disappears.

Whatever the level of output of X_3 in the new situation, the new X_2 origin will lie anywhere between R and G, which means that the new output of X_2 will be higher and the new output of X_1 lower than before. In other words, the response of the output of the final products in terms of Figure 2 is shown to be normal, so that the transformation curve will be globally concave to the origin.

The same is not true in Figure 3 where, as before, the origins for X_3, X_2 and X_1 before the change in p_2 are given by 0_3, 0_2 and 0_1, respectively; 0_2E and 0_1E are the outputs of X_2 and X_1, and 0_30_2 is the output of X_3. In Figure 3, k_2 still exceeds k_1, but k_3, unlike the case in Figure 2, is now lower. This is because 0_30_2 in Figure 3 is flatter than 0_30_2 in Figure 2. As p_2 increases, the new origin for X_2 is again given by S, if the output of X_3 remains unchanged. If the output of X_3 does not remain the same, the new X_2 origin will again lie between S and G if the output of X_3 rises. However, if the output of X_3 declines in the new situation, there are two possibilities. The new X_2 origin may lie between R and S, or even at points on 0_3S closer to 0_3. This is because, although at point R and at some other points on 0_3R to the left of R, the lower output of X_3 is inconsistent with the same output of X_1 (as at R where $0_1M = 0_1E$) and a higher output of X_2, or with the higher outputs of both X_1 and X_2, yet there are some points on 0_3S to the left of R where the output of X_1 is higher but that of X_2 is lower than the two outputs in the pre-price change situation. This means that there are some points on 0_3S to the left of R which give consistent configurations for output levels for all three commodities. Point H is one such point, and if the new X_2

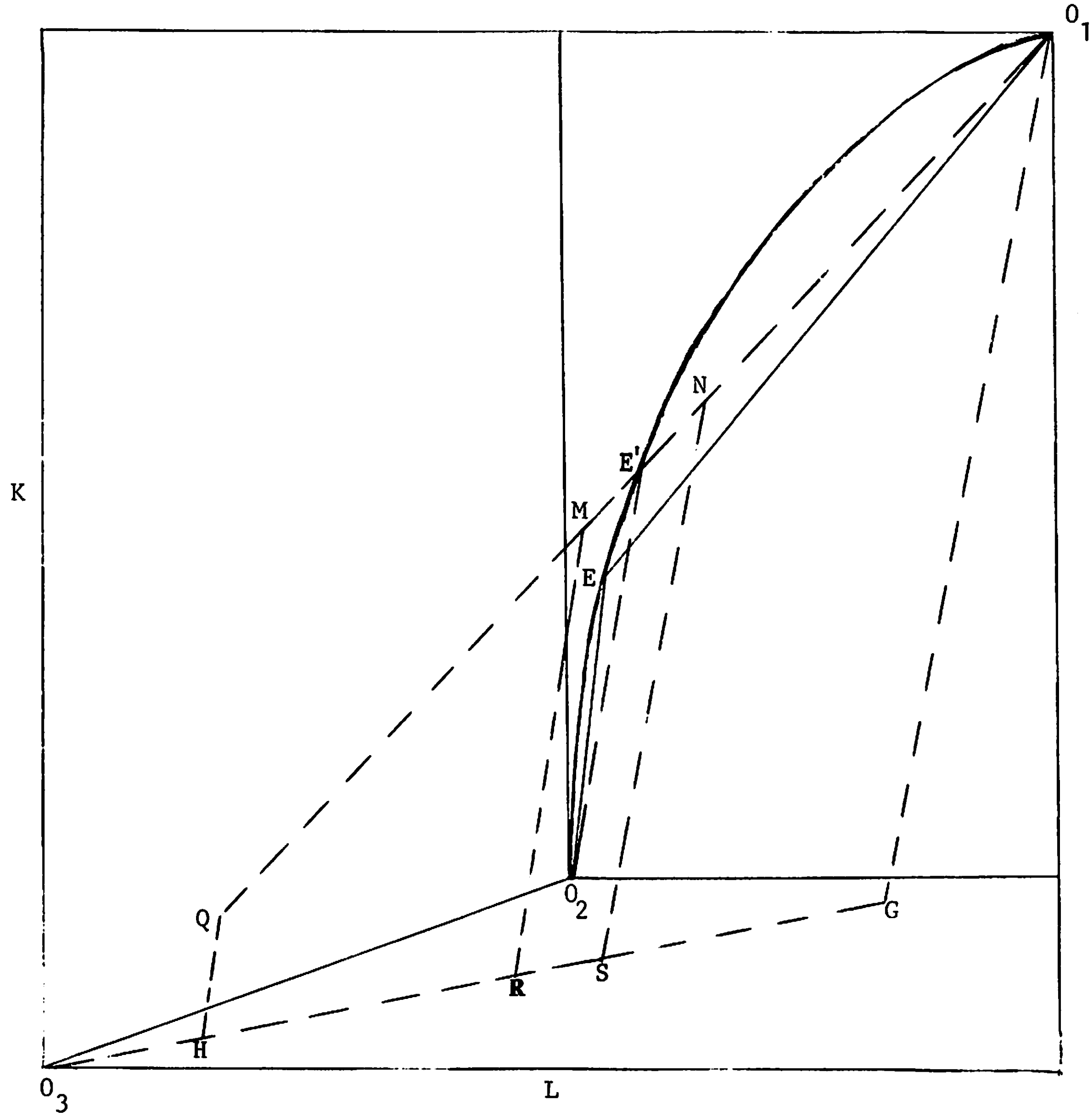

Figure 3

origin lies at H, the new output of X_2(=HQ)[5] is lower than its previous output (=0_2E), whereas the new output of X_1(=0_1Q) is higher than its previous output (=0_1E). In other words, the output response of final products to a rise in p_2 has been shown to be perverse, which means that the transformation curve may be locally or globally convex to the origin

C. Other Properties of the Model: So far we have shown that in our model with intermediate products, the transformation curve may locally or globally become convex to the origin. Another interesting result concerns the relationship between p_3 and p_2, which is described by (1.19), where it is clear that dp_3/dp_2 may be positive, negative, or even zero. The relative price of the intermediate product p_3, can be treated in two ways: First, it can be treated as the price of a factor employed in the final commodities, and, therefore, should behave like prices of the primary factors in response to changes in prices of the final products; second, p_3 is the relative price of the intermediate product which is produced with the help of the primary factors, so that p_3 should also change as a result of changes in the prices of the primary factors. Due to this dual role of p_3, the effect of a change in p_2 on p_3 may have any sign, even if the implications for the change in primary factor prices are determinate. This can be seen as follows:

Differentiating (1.5) and (1.6) and utilizing any of dk_i/dp_2, we obtain:

$$(1.23) \qquad \frac{dr}{dp_2} = \frac{f_2(1-a_1p_3)(a_1f_1+f_3)}{B}$$

$$(1.24) \qquad \frac{dw}{dp_2} = \frac{f_2(1-a_1p_3)(k_1f_3+a_1f_1k_3)}{B} .$$

Suppose that B has a definite sign, which means that it has the same sign as (k_2-k_1). Then if $k_2 > k_1$, $B > 0$, $dr/dp_2 > 0$ and $dw/dp_2 < 0$. In other words, a rise in the relative price of the second commodity raises the real reward of capital, the primary factor utilized intensively by it, and lowers the real reward of labor, the other primary factor, and conversely. This is, of course, the famous Stolper-Samuelson theorem. However, dp_3/dp_2 from (1.19) is positive if $k_3 > k_1$ and negative if $k_3 < k_1$. In other words, the effect of a change in p_2 on p_3 may go in any direction independent of its effect on r and w. The relationship between p_3 and p_2 will be utilized in the subsequent section on optimal trade policy.

II. Intermediate Goods Treated as Non-Traded Goods

Consider now the implications of the introduction of international trade in final products for gains from trade. If the transformation curve is globally concave to the origin, free trade is necessarily superior to no trade. This is depicted in Figure 1a, where S is the point of self-sufficiency equilibrium on a transformation curve given by TT'. The introduction of trade at the foreign price ratio, FP, results in production at P, consumption at C and welfare at U_2. Since U_2 lies above U_1, the autarky level of welfare, free trade is necessarily superior to no trade.

If the transformation curve is either locally (Figure 1b) or globally (Figure 1c) concave to the origin, the analysis of gains from trade in our model with intermediate goods is similar to that in the traditional model with increasing returns to scale.[6] Here free trade may or may not be superior to no trade. In Figure 1b, free trade at the foreign-price ratio, FP (parallel to FP') may lead to lower welfare, U_o, or to higher welfare, U_2. The same result holds in Figure 1c where the transformation curve is globally convex to the origin. Because of this convexity, the production point in free trade may be given by either T or T' (see Melvin [13], p. 393). In the former case, free trade is inferior and in the latter case superior to no trade.[7]

III. Optimal Trade Policy with Trade in Intermediate Products

If the transformation curve is not globally concave to the origin, the optimal policy is not clearly defined. To avoid this indeterminacy, we assume in this section that the transformation curve is strictly concave to the origin. Furthermore, we assume that intermediate goods are also traded. Let U be the level of welfare attained in the home country. This utility or social welfare function, viewed as a concave Scitovsky index of welfare, is a function of the amounts of the two final commodities consumed by the home country:[8]

(3.1) $U = U(D_1, D_2)$

(3.2) $D_1 = X_1 + E_1$

(3.3) $D_2 = X_2 + E_2$

where D_j is the home consumption and E_j the home excess demand for the j^{th} final product (j=1,2). If there is trade in intermediate products also, then

(3.4) $X_3 = D_3 - E_3$

where $D_3 = a_1X_1 + a_2X_2$ and E_3 is the home excess demand for the intermediate product. Let the asterisk denote the symbols for the foreign country.

Then international trade market equilibrium requires that

(3.5) $E_i + E_i^* = 0, \quad i=1,2,3$

so that (3.2), (3.3) and (3.4) can be written as:

(3.6) $D_1 = X_1 - E_1^*$

(3.7) $D_2 = X_2 - E_2^*$

(3.8) $X_3 = D_3 + E_3^*$

The foreign country is, furthermore, subject to the budget constraint:

(3.9) $E_1^* + p_2^* E_2^* + p_3^* E_3^* = 0$

where p_2^* and p_3^* denote the world prices of the second and the third commodities in terms of the first. It is assumed that $E_1^* < 0$ and $E_2^* > 0$, so that the foreign country exports the first commodity and imports the second commodity; E_3^* may be positive or negative depending on whether the intermediate product is imported or exported by the foreign country.

When the intermediate good also enters trade, the marginal rate of transformation no longer equals the commodity-price ratio. This can be seen by differentiating (1.1), (1.2) and (3.8) and then following the same procedure as in section I. Here

(3.10) $dX_1 = - (p_2 dX_2 + p_3 dE_3^*)$

or

(3.11) $\frac{dX_1}{dX_2} = - (p_2 + p_3 \frac{dE_3^*}{dX_2})$

It is clear from (3.11) that the marginal rate of transformation is no longer equal to $-p_2$, as was the case in section I, where E_3^* was assumed to be zero.

Let us now proceed to obtain the conditions under which the level of welfare will be maximized. Assume that no satiation in consumption takes place. Then differentiating (3.1) we have

$$(3.12) \qquad \frac{dU}{U_1} = dD_1 + \frac{U_2}{U_1} dD_2$$

where $U_j = \frac{\partial U}{\partial D_j} \quad j = 1,2.$

With the domestic-price ratio equal to the marginal rate of substitution, $U_2/U_1 = p_2$; whereas dU/U_1 is an index of changes in real national income (dy). Therefore,

$$(3.13) \qquad dy = dD_1 + p_2 dD_2.$$

Differentiating (3.6) and (3.7) and substituting in (3.13), we have:

$$(3.14) \qquad dy = (dX_1 + p_2 dX_2) - (dE_1^* + p_2 dE_2^*)$$

$$= (dX_1 + p_2 dX_2) - (dE_1^* + p_2^* dE_2^*) + (p_2^* - p_2) dE_2^* .$$

From (3.10) $dX_1 + p_2 dX_2 = p_3 dE_3^*$ and from (3.9)

$$- (dE_1^* + p_2^* dE_2^*) = E_2^* dp_2^* + p_3^* dE_3^* + E_3^* dp_3^* .$$

Substituting these in (3.14), we obtain:

$$(3.15) \qquad dy = E_2^* dp_2^* + E_3^* dp_3^* + (p_2^* - p_2) dE_2^* + (p_3^* - p_3) dE_3^*.$$

From this last equation, we can derive optimal policies depending on whether the country under consideration is a small country or a large country enjoying monopoly power in international trade.

A. <u>The Small Country Case</u>: First, take the case where the home country is small and is a price taker. World prices in this case are fixed, so that $dp_2^* = dp_3^* = 0$. For an interior maximum we require that dy = 0. From (3.15) it is clear that with $dp_2^* = dp_3^* = 0$, $dy = 0$ only if $p_3^* = p_2$ and $p_3^* = p_3$. In other words, welfare maximization requires that the foreign prices of all traded goods be equalized to the domestic prices. Hence the optimal policy is one of laissez-faire for trade in all products. This is the standard result and the introduction of intermediate products necessitates no additional qualification provided, of course, the transformation curve is concave to the origin.

B. Monopoly Power in Trade: If the home country enjoys monopoly power in international trade, then p_2^* and p_3^* are no longer fixed, but dependent on E_2^* and E_3^*. As before, the interior maximum requires that $dy = 0$, and it is evident from (3.15) that $dy = 0$ only if $p_2^* \neq p_2$ and $p_3^* \neq p_3$. Hence a tariff by the home country on the first commodity which introduces inequalities between p_2^* and p_2 on the one hand and p_3^* and p_3 on the other, becomes an optimal policy. If, in addition, the home country imports the intermediate product, a tariff may have to be imposed on this product also. From (3.15), we can write:

$$\frac{dy}{dp_2^*} = E_2^*\left(1 + \frac{p_2^* - p_2}{p_2^*}\eta_2^*\right) + E_3^*\left(1 + \frac{p_3^* - p_3}{p_3^*}\eta_3^*\right)\frac{dp_3^*}{dp_2^*} \tag{3.16}$$

where $\eta_2^* = \frac{dE_2^*}{dp_2^*} \cdot \frac{p_2^*}{E_2^*}$ is the foreign elasticity of demand for imports of the second commodity, and $\eta_3^* = \frac{dE_3^*}{dp_3^*} \cdot \frac{p_3^*}{E_3^*}$ is the foreign elasticity of demand (supply) for imports (exports) of the intermediate product. It is assumed that $\eta_2^* < 0$, whereas $\eta_3^* \lesseqgtr 0$ depending on whether $E_3^* \gtreqless 0$. Let t_1 denote the tariff imposed by the home country on the first commodity, so that

$$p_2(1+t_1) = p_2^* \text{ , and } p_3(1+t_1) = p_3^* .$$

Substituting these in (3.16) and equating dy^*/dp_2^* to zero, we have:

$$(3.17) \qquad (1 + \frac{t_1}{1+t_1} \eta_2^*) + \frac{E_3^*}{E_2^*} \cdot \frac{dp_3^*}{dp_2^*} (1 + \frac{t_1}{1+t_1} \eta_3^*) = 0,$$

whence the optimum tariff, t_o, can be shown to be

$$(3.18) \qquad t_o = \frac{-(1 + \alpha_3^* \beta_3^*)}{(1+\eta_2^*) + \alpha_3^* \beta_3^* (1+\eta_3^*)}$$

where $\alpha_3^* = \frac{p_3^* E_3^*}{p_2^* E_2^*}$ and $\beta_3^* = \frac{dp_3^*}{dp_2^*} \frac{p_2^*}{p_3^*}$ is the elasticity of the world relative price of the intermediate product with respect to the world relative price of the second commodity. If the foreign country imports the intermediate product, $\alpha_3^* > 0$; if it exports the intermediate product, $\alpha_3^* < 0$. β_3^* may be positive or negative depending on the sign of dp_3^*/dp_2^* which can be determined from (1.19) in section I.

Let us start with the case where the intermediate product is not traded. Here $\alpha_3^* = 0$, so that the optimum tariff reduces to

$$t_o = - \frac{1}{1+\eta_2^*}$$

This is the traditional optimal tariff formula, and it may be observed that the optimum tariff is positive only if $|\eta_2^*| > 1$, i.e., only if the foreign import demand is elastic. This is the necessary condition for a positive optimum tariff to exist in the traditional model without intermediate goods, and the result remains unchanged even if the intermediate good is introduced in the model, provided it does not enter trade. However, if the intermediate product is traded, then $\alpha_3^* \neq 0$. First, consider the case where $\alpha_3^* > 0$, so that the foreign country also imports the intermediate product. Here $\eta_3^* < 0$. If $\beta_3^* > 0$, then the numerator of (3.18) is negative. Therefore if $|\eta_3^*| \leq 1$, $|\eta_2^*| > 1$ is still a necessary condition for the optimum tariff to be positive. Furthermore, even if $|\eta_2^*| > 0$, the optimum tariff may be negative if $|\eta_3^*| < 1$. Hence a sufficient condition for optimum tariff to be positive is that both foreign import demands are elastic.

In the absence of trade in the intermediate good, a tariff on the importables results in an improvement in the terms of trade (i.e., p_2^* rises), which tends to raise welfare. Welfare is maximized when the slope of the foreign offer curve equals the home country's internal commodity-price ratio. Since the latter is positive, the former has to be positive, and as is well known, a positive slope of the foreign offer curve implies an elastic foreign import demand (see Vanek [16], Ch. 16). However when the foreign country imports the intermediate good also, there are two terms of trade involved, and a tariff by the home country on its final commodity importables, improves them both (i.e., p_2^* rises and p_3^* rises when $\beta_3^* > 0$). The result is symmetric. For optimum tariff to be positive, we require that both foreign import demands be elastic.

Next consider the case where $\beta_3^* < 0$. When $\alpha_3^* > 0$, $\alpha_3^*\beta_3^* < 0$. Here then arises the possibility that the optimum tariff may be zero. This possibility from (3.18) requires that $\alpha_3^*\beta_3^* = -1$. The economic explanation for this result is that when $\beta_3^* < 0$, a tariff by the home country improves its terms of trade with respect to the second commodity, but worsens them with respect to the third. The former effect tends to raise welfare, whereas the latter effect tends to lower it. The final result will depend on the relative strength of these two forces working in the opposite direction. When $\alpha_3^*\beta_3^* = -1$, the relative strength of the two forces is equal, because

$$\alpha_3^*\beta_3^* = \frac{p_3^*E_3^*}{p_2^*E_2^*} \cdot \frac{dp_3^*}{p_3^*} \cdot \frac{p_2^*}{dp_2^*} = \frac{E_3^*}{E_2^*}\frac{dp_3^*}{dp_2^*} .$$

When $\beta_3^* = -1$, p_3^* declines in the same proportion as the rise in p_2^*, and since $-E_3^*dp_3^* = E_2^*dp_2^*$, a rise in the value of home exports of the second commodity is exactly balanced by the decline in the value of home exports of the third commodity. Thus when $\alpha_3^*\beta_3^* = -1$, the home country neither benefits nor loses from the imposition of the tariffs, so that t_o is zero.[9]

However, when $|\alpha_3^*\beta_3^*| < 1$, the numerator of (3.18) is negative, a sufficient condition for $t_o > 0$ is that $|\eta_2^*| > 1$ and $|\eta_3^*| \leq 1$. In this case, it is no longer necessary that both foreign demands be elastic for $t_o > 0$. If, on the other hand, $|\alpha_3^*\beta_3^*| > 1$, the numerator of (3.18) is positive. A sufficient condition for t_o to be positive in this case is that $|\eta_2^*| < 1$ and $|\eta_3^*| \geq 1$. Here, again both foreign import demands need not be elastic for the home country's optimum tariff to be positive. Thus we conclude that when $|\alpha_3^*\beta_3^*| \neq 1$, a sufficient condition for the optimum tariff to be positive is that at least one of the foreign import demands is elastic.

So far we have assumed that the foreign country imports the intermediate product. Let us now consider the case where the foreign country exports the intermediate product. Here $\eta_3^* > 0$. It is evident from (3.18) that if $\beta_3^* < 0$, the foreign import demand should be very elastic for t_o to be positive. Because $\alpha_3^* < 0$, $\beta_3^* < 0$, and $\alpha_3^*\beta_3^* > 0$, so that the numerator of (3.18) is negative. In the denominator, $\alpha_3^*\beta_3^*(1+\eta_3^*)$ is positive so that for the denominator to be negative we require that $|\eta_2^*| > [1 + \alpha_3^*\beta_3^*(1+\eta_3^*)] > 1$. Hence now the foreign import demand has to be even more elastic. On the other hand, if $\beta_3^* > 0$, t_o may again be zero provided, of course, $\alpha_3^*\beta_3^* = -1$. If $|\alpha_3^*\beta_3^*| < 1$, so that the numerator of (3.18) is negative, $t_o > 0$ even if $|\eta_2^*| < 1$. In other words, the home country's optimum tariff on its final commodity imports may be positive even if the foreign import demand is inelastic or even zero. If $|\alpha_3^*\beta_3^*| > 1$, then t_o can never be positive. Here the numerator is positive, but the denominator is necessarily negative, even if $\eta_2^* = 0$. Here then arises the need for a tariff on the imports of the intermediate product.

A. Home Country's Tariff on Both Importables

So far we have assumed that the home country imposes a tariff only on its imports of the final commodity. Suppose it also imposes a tariff on its imports of the intermediate good. Here

$$p_2 = p_2^*(1+t_1), \quad \text{and}$$

$$p_3(1+t_1) = p_3^*(1+t_3)$$

where t_3 is the home country's tariff on the intermediate product. Substituting these in (3.16) and denoting t_o^* fot t_1 at the optimum position, we obtain:

$$(3.19) \qquad t_o^* = t_o + \frac{\alpha_3^*\beta_3^*(1 + t_3\ \eta_3^*)}{(1+\eta_2^*) + \alpha_3^*\beta_3^*(1+\eta_3^*)} .$$

It may be observed from (3.19) that t_o^*, the optimum tariff obtained in the presence of a tariff on the imports of the intermediate good, may be greater or less than t_o, the optimum tariff obtained in the absence of a tariff on the intermediate product, depending on whether the second expression in the right hand side of (3.19) is positive or negative. If $\alpha_3^*\beta_3^* = -1$, we know from our previous analysis that $t_o = 0$. However, if $|\eta_2^*| > \eta_3^*$, t_o^* is positive. If $\alpha_3^*\beta_3^*$ is positive, then $t_o^* > t_o$ if $|\eta_2^*| \leq 1$. Other interesting results may be similarly derived by placing restrictions on $\alpha_3^*\beta_3^*$ and η_2^*.[10]

C. A Tariff only on the Intermediate Product: Suppose for some administrative or political reasons, the home country cannot impose a tariff on its imports of the final commodity. The country may still be able to reap benefits from its monopoly power in trade, by imposing a tariff on the imports of the intermediate product. In this case $p_2^* = p_2$ and $p_3 = p_3^*(1+t_3)$. Substituting these in (3.16) and replacing t_3 by t_3^* in the optimum position, we obtain:

$$(3.20) \qquad t_3^* = \frac{1 + \alpha_3^*\beta_3^*}{\alpha_3^*\beta_3^*\eta_3^*} .$$

Since $\eta_3^* > 0$, $t_3^* > 0$ if $\alpha_3^*\beta_3^* > 0$, which, with $\alpha_3^* < 0$, requires that $\beta_3^* < 0$. In other words, the optimum tariff on the intermediate product alone is positive if an improvement in the home country's terms of trade with respect to the intermediate good as a result of the tariff (so that p_3^* declines) also results in an improvement in its terms of trade with respect to the second commodity (so that p_2^* rises, which makes $\beta_3^* < 0$).[11] Again if $\alpha_3^*\beta_3^* = -1$, $t_3^* = 0$. If $|\alpha_3^*\beta_3^*| > 1$, t_3^* is still positive. Thus for a positive t_3^*, we require either a positive $\alpha_3^*\beta_3^*$ or a negative $\alpha_3^*\beta_3^*$ with absolute value greater than unity.

Conclusions

Introducing an intermediate product in the traditional two-country, two-commodity, two-factor model we have derived the following conclusions:

1. The transformation curve may become locally or globally convex to the origin if, in addition to the primary factors, the final products also utilize the material input in the process of production.

2. Given the possibility of the convexity of the transformation curve towards the origin, free trade may or may not be superior to no trade. This result stands in sharp contrast to that derived in the traditional models of intermediate goods, where the latter are assumed to be identical with the traded final goods.

3. If we assume that the transformation curve is concave to the origin, then the traditional result concerning gains from free trade holds. Furthermore, if there is monopoly power in international trade, welfare maximization requires an optimum tariff. The formula for the optimum is the same as the traditional one, if intermediate products are not traded, so that an elastic foreign import demand is a necessary condition for optimum tariff to be positive. If trade is allowed in intermediate products, the optimum tariff may be positive even if the foreign import demand of the final product is inelastic.

4. If in addition to a final commodity, the intermediate good is imported, there may be two optimum tariffs: one on the final product imports and the other on the imports of the intermediate product. Both tariffs are, of course, interrelated.

Footnotes

[1]See, for example, Vanek [15], Mckinnon [10], and Melvin [11] among others.

[2]See Ruffin [14], p. 267. Ruffin's model has also been utilized by Batra and Singh [2] to analyze the stability properties of a two-sector growth model.

[3]However, where $dX_1/dX_2 \neq -p$, the shape of the transformation curve may not be dependent on the response of the outputs to their prices. For example, in the presence of an inter-industry wage-differential where $dX_1/dX_2 \neq -p$, Kemp and Herberg [8] and Bhagwati and Srinavasan [6] have shown that even if the output response to price-change is "perverse," the transformation curve need not be convex to the origin; conversely, even if the output response to price change is "normal," the transformation curve may be convex to the origin. For other analyses of the wage-differential, see Bhagwati and Ramaswami [5], Batra and Pattanaik [1], and Batra and Scully [4].

[4]It can be seen that if B has the same sign as (k_2-k_1), it always has a definite sign. This, however, does not mean that A also has a definite sign.

[5]HQ is drawn parallel to $0_2E'$ in Figure 3.

[6]See Mathews [9] and Melvin [13].

[7]The results in section III have been derived under the assumption of the absence of trade in intermediate products. This has been done for the sake of geometrical simplicity. For, when intermediate goods are traded, the marginal rate of transformation, as shown in section III, is no longer equal to the commodity-price ratio. Furthermore, the autarky and the trade transformation curves differ when intermediate goods are also traded. This is because trade in intermediate goods is equivalent to the exchange of the primary factors employed in the production of such goods. This effectively changes the available factor supplies and in turn makes the autarky transformation curve different from the one obtained in trade. No such complication is encountered when intermediate goods do not enter trade. For details on this point, see Batra and Casas [3].

[8]It may be noted that the income distribution effects of the optimal policies have not been ignored, but are implicit in the selection of the social welfare function.

[9]If $\alpha_3^*\beta_3^* = -1$, but $\beta_3^* \neq 1$ and $\alpha_3^* \neq 1$, the result and the reasoning behind it remain the same.

[10] For example, we can derive the optimum tariff on the imports of the intermediate product from (3.19). A conclusion of some interest is that the optimum tariff on any of the two importables is not independent of the tariff on the other product.

[11] Throughout our analysis, we have assumed that a tariff on any one of the importables improves the home country's relevant terms of trade. This, however, may not be always trye. But a detailed analysis of this problem is not within the scope of this paper.

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Mathematical Appendix

Some of the equations which appear in section I of the text without proof will be derived here. Totally differentiating equations (1.5) and (1.6) with respect to p_2, taking the members of each equation two by two, and writing in matrix form, we have

$$\text{(A.1)}\quad \begin{bmatrix} (1-a_1p_3)f_1'' & -(p_2-a_2p_3)f_2'' & 0 & a_2f_2' - a_1f_1' \\ (1-a_1p_3)f_1'' & 0 & -p_3f_3'' & -a_1f_1' - f_3' \\ -(1-a_1p_3)k_1f_1'' & (p_2-a_2p_3)k_2f_2'' & 0 & a_2(f_2-k_2f_2')-a_1(f_1-k_1f_1') \\ -(1-a_1p_3)k_1f_1'' & 0 & p_3k_3f_3'' & -a_1(f_1-k_1f_1')-(f_3-k_3f_3') \end{bmatrix} \begin{bmatrix} dk_1/dp_2 \\ dk_2/dp_2 \\ dk_3/dp_2 \\ dp_3/dp_2 \end{bmatrix} = \begin{bmatrix} f_2' \\ 0 \\ f_2-k_2f_2' \\ 0 \end{bmatrix}$$

Solving the system of equations (A.1) then gives equations (1.16) - (1.19). Differentiating (1.3), (1.4), (1.7) and (1.8) with respect to p_2, we obtain:

$$\text{(A.2)}\quad \begin{bmatrix} 1 & 1 & 1 \\ k_1 & k_2 & k_3 \\ a_1f_1 & a_2f_2 & -f_3 \end{bmatrix} \begin{bmatrix} dL_1/dp_2 \\ dL_2/dp_2 \\ dL_3/dp_2 \end{bmatrix} = - \begin{bmatrix} 0 \\ L_1\dfrac{dk_1}{dp_2} + L_2\dfrac{dk_2}{dp_2} + L_3\dfrac{dk_3}{dp_2} \\ a_1L_1f_1'\dfrac{dk_1}{dp_2} + a_2L_2f_2'\dfrac{dk_2}{dp_2} - L_3f_3'\dfrac{dk_3}{dp_2} \end{bmatrix}$$

Substituting from (1.16) - (1.18), and solving, we have:

(A.3) $$\frac{dL_1}{dp_2} = \frac{-(1-a_1p_3)}{A.B}\left[\frac{L_1f_2f_3\left\{f_3+a_2f_2+a_1f_1'(k_3-k_2)\right\}}{(1-a_1p_3)^2f_1''} + \frac{p_2L_2f_1f_3^2}{(p_2-a_2p_3)^3f_2''} + \frac{p_2L_3f_1f_2^2}{p_3^3f_3''}\right]$$

(A.4) $$\frac{dL_2}{dp_2} = \frac{(1-a_1p_3)}{A.B}\left[\frac{L_1f_2f_3^2}{(1-a_1p_3)^3f_1''} + \frac{L_2f_1f_3\left\{f_3+a_1f_1+a_2f_2'(k_3-k_1)\right\}}{(p_2-a_2p_3)^2f_2''} + \frac{L_3f_1^2f_2}{p_3^3f_3''}\right]$$

Differentiating (1.1), (1.2) and (1.4) with respect to p_2, we have:

(A.5) $$\frac{dX_1}{dp_2} = L_1f_1'\frac{dk_1}{dp_2} + f_1\frac{dL_1}{dp_2}$$

(A.6) $$\frac{dX_2}{dp_2} = L_2f_2'\frac{dk_2}{dp_2} + f_2\frac{dL_2}{dp_2}$$

(A.7) $$\frac{dX_3}{dp_2} = a_1\frac{dX_1}{dp_2} + a_2\frac{dX_2}{dp_2}\ .$$

Substituting (1.16), (1.17), (A.3) and (A.4) in (A.5) and (A.6) we can derive equations (1.20) and (1.21). Then substituting (1.20) and (1.21) in (A.7) gives us (1.22).