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INTERMEDIATE PRODUCTS AND THE PURE THEORY OF INTERNATIONAL TRADE: A NEO HECKSCHER-OHLIN FRAMEWORK*

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INTERMEDIATE PRODUCTS AND THE PURE THEORY OF INTERNATIONAL TRADE: A NEO HECKSCHER-OHLIN FRAMEWORK

Economists in general and trade theorists in particular have now come to recognize the importance of intermediate inputs, not merely of primary factors, in the process of production. This is well reflected in the recent studies by Vanek (1963), Melvin (1969), and Ruffin (1969) among others. The main feature of these models, with the singular exception of Ruffin, is to treat two commodities--traditionally taken to be consumption goods--as both final outputs and intermediate inputs for each other.[1] With such a minor difference, it is not surprising to find that the traditional theorems by Rybczynski, Stolper and Samuelson, Heckscher and Ohlin, etc., hold without any additional qualification in Vanek's model where the final goods also serve as intermediate products. Most of the traditional theorems depend crucially upon the factor-intensity relationship between the traded commodities, and so long as this relationship remains unaltered, as it does in Vanek's framework, these theorems will be valid in spite of the introduction of intermediate products. Citing Vanek's results, Kemp (1969) defends the neglect of intermediate products in earlier trade theory. In this paper we show that most of the traditional trade theorems may not hold without additional provisions if such intermediate products are introduced in the model as are produced solely to serve as inputs in the production of final goods. Furthermore, in a two good model where intermediate and final products are identical, one cannot explain the basis of trade in intermediate goods, because no such good exists in this model.[2] For this reason, there exists no theory at present which would explain why trade occurs in intermediate goods, even though, as Yates (1959) has shown, the bulk of international trade is in intermediate products--produced goods, like raw materials, spare parts, etc., which are solely used as inputs in the production of other products. Another purpose of this paper is thus to fill this gap and provide a theory explaining the basis of trade in intermediate products. In the traditional two final commodity trade model, we introduce a third good which is produced by primary factors only to serve as an input in the other two commodities. Such a model has been recently developed by Ruffin (1969) to analyze the theory of effective protection and by Batra and Singh (1970) to examine the growth behavior of a two-sector economy.

In Section I, we give a complete description of the model. The Rybczynski theorem is analyzed in Section II, the Stolper-Samuelson theorem in Section III, and the Heckscher-Ohlin theorem in Section IV which is also concerned with the basis of trade in intermediate products. The Findlay-Grubert theorem concerning the effect of neutral technical progress in a final good on the level of the two final outputs is examined in Section V. Section VI then deals with the consequences of technical progress in intermediate goods for the two outputs. The paper is concluded with some final remarks in Section VII.

I. The Model with Intermediate Products

It is assumed that an economy consists of three commodities, two final products (X_1 and X_2) and one intermediate good (X_3) which is produced solely to serve as an input in the production of the final products. There are two primary factors of production, capital (K) and labor (L) which enter into the production process of all three commodities, and remain fully employed. Perfect competition, diminishing returns to factor proportions, constant returns to scale, inelastic factor supplies, perfect factor mobility and non-reversibility of factor-intensities at all factor-price ratios are also assumed.

The thre production functions are:

(1) $X_1 = F_1(K_1, L_1, \overline{X}_{31}) = L_1 f_1(k_1, \bar{x}_{31})$

(2) $X_2 = F_2(K_2, L_2, \overline{X}_{32}) = L_2 f_2(k_2, \bar{x}_{32})$

(3) $X_3 = F_3(K_3, L_3) = L_3 f_3(k_3)$

where K_i and L_i are respectively the capital and labor inputs and $k_i = K_i/L_i$ is the capital/labor ratio in the i^{th} sector (i=1,2,3), and $\overline{X}_{3j}$ is the amount of X_3 utilized as material input in the j^{th} final product (j=1,2) and $\bar{x}_{3j} = \overline{X}_{3j}/L_j$, where the bars indicate that the material input enters into final products in fixed proportions. Let $a_j(=X_{3j}/X_j)$ denote the requirement of X_3, the intermediate product, per unit of the j^{th} final product (j=1,2). Following Samuelson (1953), Vanek (1963) and Ruffin (1969) and others who have worked with input-output models, a_j is assumed to be constant. Then

(4) $X_3 = D_3 = a_1X_1 + a_2X_2 + \alpha(p^*)$

where D_3 is the demand for the intermediate product, p* the price of X_3 in terms of X_1 and $\alpha(p^*)$ is some function of p* so that $\alpha' > 0$. In the absence of trade, $\alpha=0$; if intermediate goods are exported, the output of X_3 exceeds its domestic demand which equals $(a_1X_1+a_2X_2)$, and if these goods are imported, the output of X_3 falls short of its domestic demand. More on the behavior of $\alpha(p^*)$ will be said in subsequent sections. The marginal productivity of capital (V_i) is given by

$$V_i = \frac{df_i}{dk_i} = f_i' . \qquad (i=1,2,3).$$

The corresponding marginal productivity of labor (U_i) in each sector then equals

$$U_i = f_i - k_i f_i' .$$

We assume that $f_i' > 0$ and $f_i'' < 0$.

With perfect competition in both product and factor markets, the price of each factor equals its marginal <u>value-added</u> product and is the same in all three industries. Let w stand for the wage rate, r for the rental of capital, p for the price of the second commodity (X_2) in terms of the first (X_1) and ω for the w/r ratio. Factor prices expressed in terms of X_1 are then given by

(5) $$r = f_1'(1-a_1p^*) = f_2'(p-a_2p^*) = p^*f_3'$$

(6) $$w = (f_1-k_1f_1')(1-a_1p^*) = (f_2-k_2f_2')(p-a_2p^*) = p^*(f_3-k_3f_3')$$

(7) $$\omega = \frac{f_i}{f_i'} - k_i .$$

Without loss of generality, p and p* are initially assumed to be unity. With full employment,

(8) $$L_1 + L_2 + L_3 = \overline{L}$$

(9*) $$K_1 + K_2 + K_3 = \overline{K}, \quad \text{or}$$

(9) $$L_1k_1 + L_2k_2 + L_3k_3 = \overline{K}$$

where the bars indicate that factor supplies are inelastic. With this last equation, the supply side of our model is complete. We could also present the demand equations to complete the model, but since these equations will not be utilized in the model, we do not write them here.[3]

<u>The Slope of the Transformation Curve</u>: Vanek (1963) has shown that the slope of the production possibility curve equals the negative of the commodity-price ratio even when the final commodities also serve as intermediate products. We now show that in the model where intermediate and final goods are not identical, this equality holds only in the absence of trade in intermediate goods. If the latter are traded, then the marginal rate of transformation will be greater than the absolute value of the price ratio between the final products (p), as will be seen in section IV on the pattern of trade. Totally differentiating (1), (2) and (3), we have

(10) $dX_1 = U_1 dL_1 + V_1 dK_1$

(11) $dX_2 = U_2 dL_2 + V_2 dK_2$

(12) $dX_3 = U_3 dL_3 + V_3 dK_3$.

From (4), we have,

(13) $dX_3 = a_1 dX_1 + a_2 dX_2 + \alpha' dp^*$.

From (8) and (9*), we have,

(14) $dL_1 = -(dL_2 + dL_3)$

(15) $dK_2 = -(dK_2 + dK_3)$.

Substituting (14), (15), and (12) in (10) and using (5) and (6), we obtain:

$$dX_1(1-a_1 p^*) = -(p-a_2 p^*)\left[dX_2(p-a_2 p^*) + p^* dX_3\right].$$

Then substituting (13) in this gives us,

(16) $$\frac{dX_1}{dX_2} = -(p + p^*\alpha' \frac{dp^*}{dX_2}) .$$

In the absence of trade in intermediate goods, $\alpha' = 0$. If intermediate goods are traded, $\alpha' > 0$, and dp^*/dX_2 must be positive for the trade pattern to be determinate (see section (IV), so that the marginal rate of transformation $|dX_1/dX_2|$ is no longer equal to p, but is greater than p.

II. The Rybczynski Theorem

According to Rybczynski (1955) an increase in the supply of a factor of production promotes, at constant commodity prices, the expansion of the output of the commodity which uses the expanded factor intensively at the expense of the output of the other commodity. We now explore the consequences of a rise in the supply of a factor for the output of the final goods X_1 and X_2 in our model with intermediate products.[4] Suppose there is a rise in the supply of capital alone. If prices of the final goods are kept constant, the price of the intermediate good as well as the wage/rental ratio remains constant,[5] so that k_1, k_2 and k_3 will also be unchanged. Differentiating (1) and (2) with respect to K we have:

(2.1) $$\frac{dX_1}{dK} = f_1 \frac{dL_1}{dK}$$

(2.2) $$\frac{dX_2}{dK} = f_2 \frac{dL_2}{dK} .$$

Differentiating (3), (4), (8) and (9), we obtain:

(2.3) $$\frac{dL_1}{dK} = \frac{(a_2 f_2 + f_3)}{A}$$

(2.4) $$\frac{dL_2}{dK} = - \frac{(a_1 f_1 + f_3)}{A} ,$$

so that

$$(2.5) \quad \frac{dX_1}{dK} = \frac{f_1(a_2f_2 + f_3)}{A}$$

$$(2.6) \quad \frac{dX_2}{dK} = - \frac{f_2(a_1f_1 + f_3)}{A}$$

and

$$(2.7) \quad \frac{dX_3}{dK} = a_1 \frac{dX_1}{dK} + a_2 \frac{dX_2}{dK} = \frac{f_3(a_1f_1 - a_2f_2)}{A}$$

where $A = a_1f_1(k_3-k_2) + a_2f_2(k_1-k_3) + f_3(k_1-k_2)$.

It may be observed that the numerator of (2.5) is positive and that of (2.6) is negative, so that the signs of dX_1/dK and dX_2/dK depend on the sign of A. Examination of A reveals that if k_3 lies in between k_1 and k_2, then A has a definite sign. For example, if $k_1 > k_3 > k_2$, then $A > 0$, so that $dX_1/dK > 0$ and $dX_2/dK < 0$. On the other hand if $k_1 < k_3 < k_2$, $A < 0$ and $dX_1/dK < 0$ and $dX_2/dK > 0$

Theorem 2.1: Thus we may conclude that if the capital/labor ratio of the intermediate product lies in between the other two capital/labor ratios, the Rybczynski theorem holds: An increase in the stock of capital raises the output of X_1 and lowers that of X_2 if X_1 is capital-intensive relative to X_2 and vice versa, provided that commodity prices are constant.

The effect on the output of X_3, the intermediate good, depends not only on the sign of A, but also on $a_1f_1 (= \bar{x}_{31})$ and $a_2f_2 (= \bar{x}_{32})$. If $A > 0$, so that the output of X_1 rises as a result of capital accumulation, $dX_3/dK \gtreqless 0$ if $a_1f_1 \gtreqless a_2f_2$. If $a_1f_1 > a_2f_2$, that is, if X_1 uses the intermediate product more intensively than X_2, the output of X_3 will rise; otherwise it will decline. Analogous conclusions can be derived if $A < 0$ and the output of X_2, instead of X_1, rises.

If k_3 does not lie in between k_1 and k_3, then the Rybczynski theorem may not hold. If $a_1f_1 = a_2f_2$ then, of course, the theorem always holds. But this is a trivial case, because here the output of X_3 is constant.

The difficulty arises when $a_1f_1 \neq a_2f_2$ and k_3 lies outside the confine of k_1 and k_2. There are two possibilities: i) $k_1 \gtrless k_2 \gtrless k_3$ and ii) $k_2 \gtrless k_1 \gtrless k_3$. It turns out that in case (i) the Rybczynski theorem holds if $a_2f_2 > a_1f_1$, whereas in case (ii), its validity requires $a_1f_1 > a_2f_2$. Take, for example, the possibility where $k_1 > k_2 > k_3$. Here $(k_1-k_2) > 0$ and if $a_2f_2 > a_1f_1$, one can see that $a_2f_2(k_1-k_3) > a_1f_1(k_2-k_3)$, because $(k_1-k_3) > (k_2-k_3)$. Hence $A > 0$, and $dX_1/dK > 0$ and $dX_2/dK < 0$. Therefore, the Rybczynski theorem is valid. Similarly, it can be proved that the Rybczynski theorem will also hold in case (ii). These results lead to the following theorem:

Theorem 2.2: If the commodity whose capital/labor ratio lies in between the capital/labor ratio of the intermediate product and that of the other final commodity is relatively intensive in the use of the intermediate good, the Rybczynski theorem holds.

In theorem (2.2) intensity in the use of the intermediate good is measured by $a_jf_j = \bar{X}_{3j}/L_j$ $(j=1,2)$. It is evident that if conditions implied in theorem 2.2 are not satisfied, then the Rybczynski theorem may not hold.

Theorems 2.1 and 2.2 can also be proved diagrammatically by recourse to Melvin's geometry (1968). Consider Figure 1 where 0_3, 0_1 and 0_2, represent, respectively, the origins for X_3, X_1 and X_2; $0_1E\,0_2$ is the contract curve between the final products X_1 and X_2, 0_1E is the equilibrium output of X_1 and 0_2E the equilibrium output of X_2 and these together utilize 0_30_1 output of the intermediate good. The capital/labor ratio in X_1 and X_2 is respectively

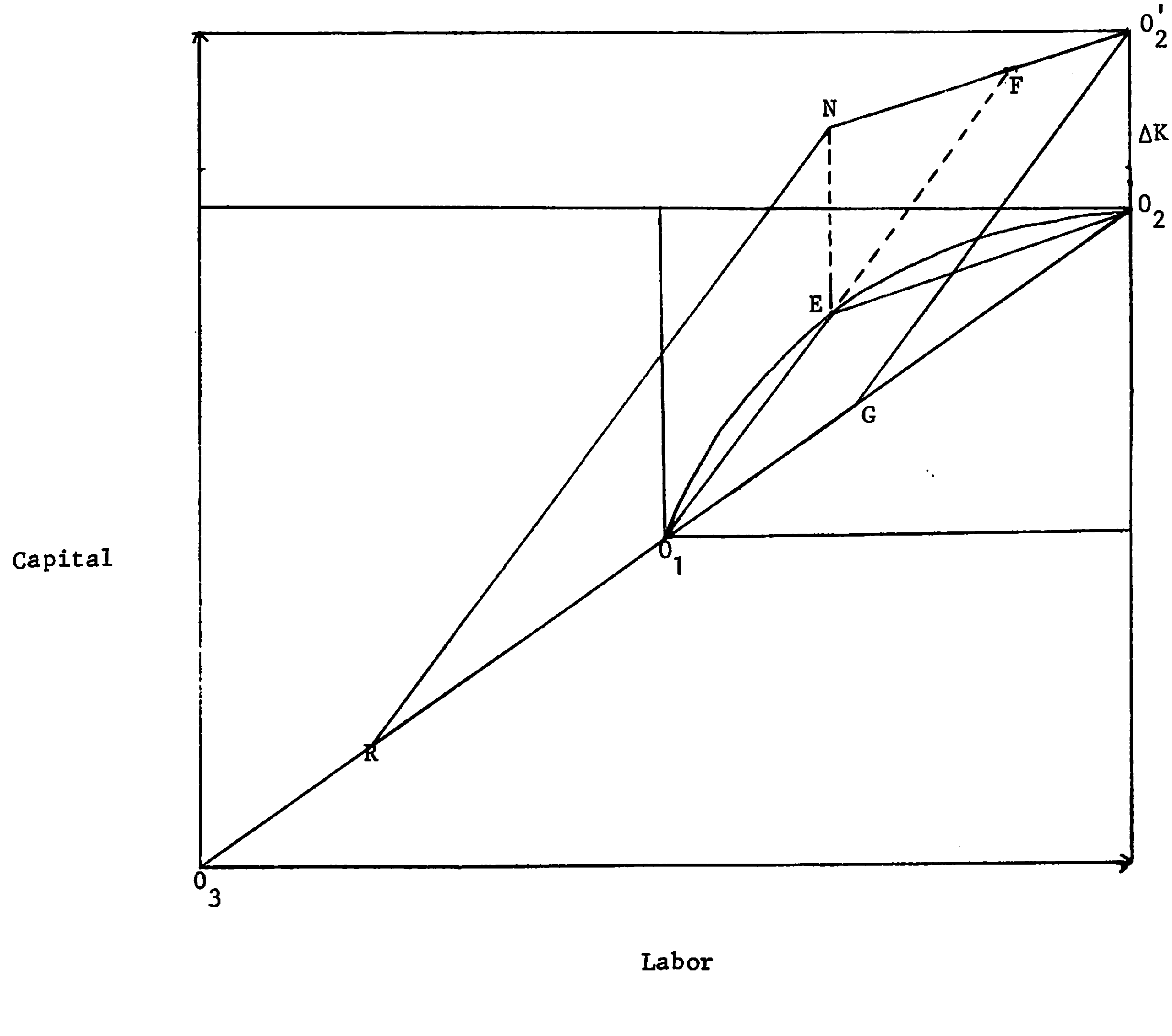

Figure 1

given by the slope of the lines 0_1E and 0_2E, whereas the capital/labor ratio in X_3 is given by the slope of $0_3 0_1$ which, for the sake of diagrammatic simplicity and visual clarity, is shown identical with the diagonal $0_1 0_2$.[6] The diagram clearly shows that the capital/labor ratio of the intermediate product lies in between the capital/labor ratios of X_1 and X_2 and that X_1 is capital-intensive relative to X_2.

Now suppose there is a rise in the stock of capital so that the 0_2 origin shifts to $0_2'$, showing a rise in capital supply by $0_2 0_2'$. To begin with, suppose that the final products are equally intensive in the use of intermediate goods, i.e., $a_1 f_1 = a_2 f_2$. In this case the output of X_3 remains unchanged at $0_3 0_1$ even if the other two outputs change, so that the 0_1 origin also does not change. With the capital/labor ratio remaining constant in all three commodities due to constancy of commodity prices, the new production equilibrium is given by F, a point obtained by extending 0_1E to intersect $0_2'F$, which is parallel to 0_2E. The Rybczynski theorem clearly holds because the output of X_1, the capital-intensive commodity has risen to 0_1F, whereas the output of X_2, the labor-intensive commodity, has declined to $0_2'F$. Now suppose that $a_1 f_1 \neq a_2 f_2$. First, take the case where $a_1 f_1 > a_2 f_2$. With the output of X_1 rising and that of X_2 declining, the output of X_3 now must rise. Draw $0_2'G$ parallel to 0_1F. Since the output of X_3 must rise to satisfy the higher input requirement from X_1, the origin for X_1 moves upwards along the diagonal $0_1 0_2$ whose slope, incidentally, equals the capital/labor ratio in X_3. However, the new origin for X_1 must lie between 0_1 and G, because at G, the output of X_2 ceases to exist and beyond G full employment cannot be maintained without changing capital/labor ratios. Hence the new X_1 origin must lie between 0_1 and G. Now consider the second case where $a_1f_1 < a_2f_2$. Here the output of X_3 must decline, because the expanding industry requires relatively less of X_3 than the contracting industry. Draw EN parallel to $0_20_2'$, and then draw NR parallel to 0_1F. The new origin for X_1 will now lie between 0_1 and R, because if the new X_1 origin is given by R, the production equilibrium is given by N, which shows the unchanged output for X_2, (as $0_2'N = 0_2E$), and higher output for X_1 (as $RN > 0_1E$); this position is clearly inconsistent with a lower output for X_3. If the new origin lies at any point between 0_3 and R, the same inconsistency arises again. It is only when the new X_1 origin lies between R and 0_1 that this inconsistency disappears. Thus, we conclude that whatever the relationship between a_1f_1 and a_2f_2, the new origin for X_1 must lie between R and G, which means that the output of X_1 must rise and that of X_2 must decline as a result of an increase in the capital stock, so that the Rybczynski theorem necessarily holds. If we draw from any point between R and G a line parallel to 0_1F, we can see that the new output of X_1 is higher than 0_1E, its original output, and the new output of X_2 is lower than its original output, 0_2E. The Rybczynski theorem is valid in spite of the presence of intermediate goods, and this occurs because, as stated earlier, k_3 lies in between k_1 and k_2.

Figure 2 depicts the case where k_3 does not lie in between k_1 and k_2, so that the Rybczynski theorem may not hold. If Figure 2, 0_30_1 is drawn very flat so that $k_1 > k_2 > k_3$. The new production point as a result of the rise in capital stock by $0_20_2'$ is again given by F, provided that $a_1f_1 = a_2f_2$, so that the output of X_3 remains unchanged at 0_30_1. If $a_1f_1 < a_2f_2$, the output of X_3 must decline so that the new origin for X_1 will lie between R and 0_1, where R is obtained in the same way as in Figure 1.

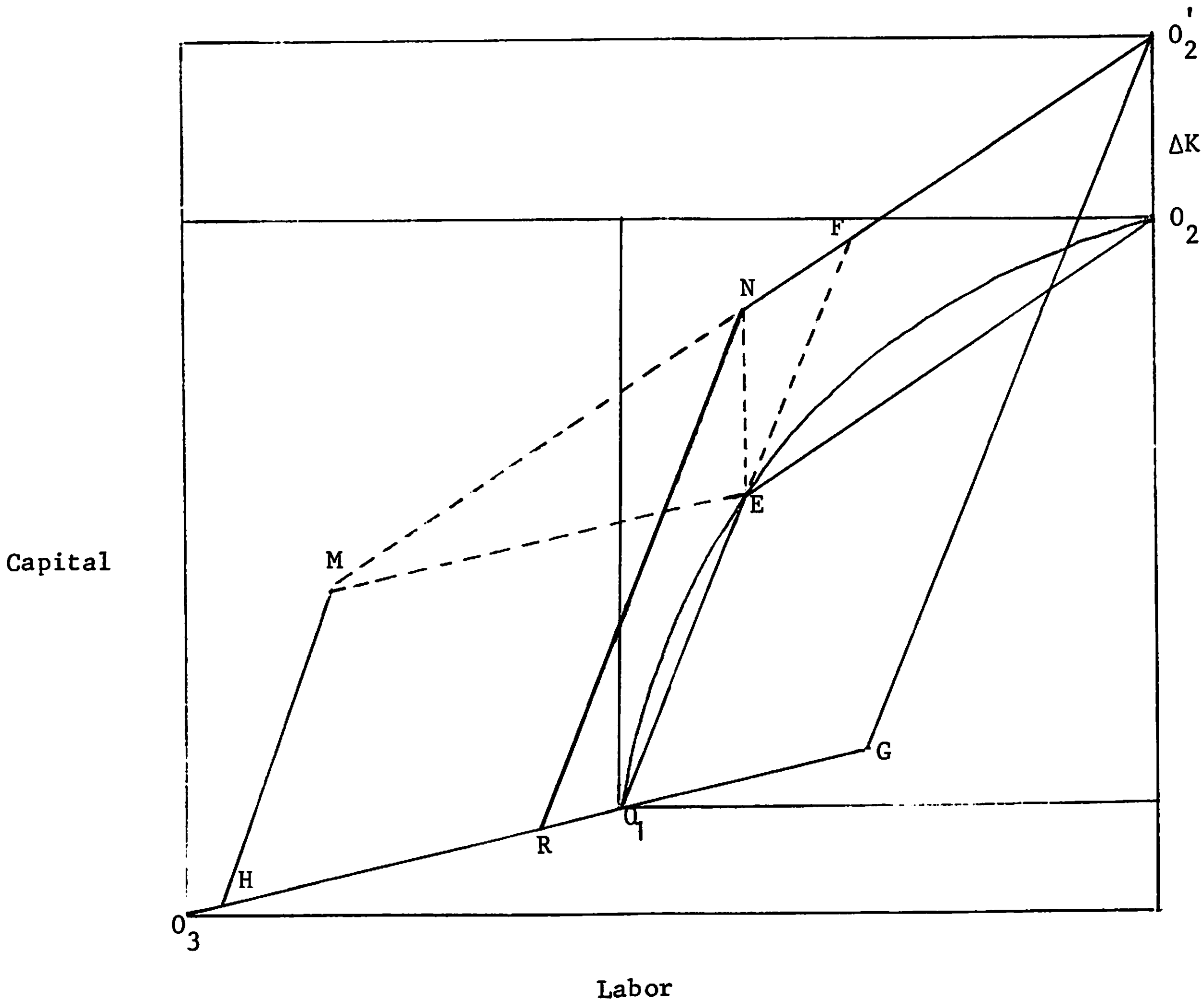

Figure 2

However, if $a_1 f_1 > a_2 f_2$, there are two possibilities. From previous reasoning we know that the new X_1 origin should lie between 0_1 and G. However, it could lie between 0_3 and H as well, where H is obtained by first drawing EM parallel to $0_3 0_1$ and then MH parallel to $0_1 E$. If the new X_1 origin lies on H, the new equilibrium production point is given by M which displays a higher output for X_2 and the same output for X_1, so that the situation is inconsistent with the reduced output for X_3. However, if the new X_1 origin lies between 0_3 and H, the resultant equilibrium point of production will show a higher output for X_2 but a lower output for X_1. Clearly, this situation is consistent with a reduced output of X_3, because $a_1 f_1 > a_2 f_2$. In other words, given that X_1 is more intensive in the use of the intermediate good than X_2, expansion of X_2 requires a lesser amount of X_3 than what is released by the contraction of X_1. Thus we conclude that if $a_1 f_1 < a_2 f_2$, the Rybczynski theorem is valid in spite of the fact that k_3 does not lie between k_1 and k_2, because here the new X_1 origin must lie between R and 0_1. However, if $a_1 f_1 > a_2 f_2$, the new X_1 origin may lie between 0_1 and G or between 0_3 and H. In the former case, the Rybczynski theorem holds, whereas in the latter, it does not.[7]

III. The Stolper-Samuelson Theorem

According to the Stolper-Samuelson theorem (1941), a rise in the relative price of a commodity raises the real reward of the primary factor employed more intensively by it and lowers the real reward of the other primary factor, and conversely. We will now examine the implications of a change in the price ratio between the final products for factor rewards in our model incorporating intermediate goods.[8]

Differentiating (5) and (6) totally with respect to p, and remembering that p=p*=1 initially, we obtain:

(3.1) $$\frac{dk_1}{dp} = \frac{f_2 f_3}{(1-a_1)f_1'' B}$$

(3.2) $$\frac{dk_2}{dp} = \frac{(1-a_1)f_1 f_3}{(1-a_2)^2 f_2'' B}$$

(3.3) $$\frac{dk_3}{dp} = \frac{(1-a_1)f_1 f_2}{f_3'' B}$$

(3.4) $$\frac{dp^*}{dp} = \frac{(1-a_1)f_2(k_3-k_1)}{B}$$

(3.5) $$\frac{dr}{dp} = \frac{f_2(1-a_1)(a_1 f_1 + f_3)}{B}$$

(3.6) $$\frac{dw}{dp} = \frac{-f_2(1-a_1)(k_1 f_3 + a_1 k_3 f_1)}{B}$$

where $B = f_3(k_2-k_1) + f_2(k_3-k_1)(a_2-a_1)$.

For the Stolper-Samuelson theorem to hold, $dr/dp < 0$ and $dw/dp > 0$ if $k_1 > k_2$ and vice versa. The numerator of (3.5) is positive and of (3.6) negative. Therefore, the sign of dr/dp and dw/dp depends on the sign of B. If $a_2=a_1$, $B \lesseqgtr 0$ if $k_1 \gtrless k_2$, so that $dr/dp \lesseqgtr 0$ and $dw/dp \gtrless 0$. In this trivial case, the Stolper-Samuelson theorem holds. However, if $a_2 \neq a_1$, then the theorem holds if B has the same sign as (k_2-k_1). In the following cases B and (k_2-k_1) necessarily have the same sign:

i) $k_1 \gtrless k_3 \gtrless k_2$ and $a_2 > a_1$

ii) $k_1 \gtrless k_2 \gtrless k_3$ and $a_2 > a_1$

iii) $k_2 \gtrless k_1 \gtrless k_3$ and $a_2 < a_1$.

In case (i), with $a_2 > a_1$, $B > 0$ if $k_2 > k_1$, so that a rise in the relative price of the second commodity raises the real reward of capital, used intensively by it, and lowers the real reward of labor used unintensively by it, and the Stolper-Samuelson theorem holds. If $k_1 < k_2$, then $B < 0$ and the Stolper-Samuelson theorem is again valid.

In cases (ii) and (iii), B has the same sign as (k_2-k_1) provided the commodity, whose capital/labor ratio lies between the other two capital/labor ratios, has a higher intermediate input coefficient (a_j) than the other final commodity. From this we can derive the following theorems with some degree of generality.

Theorem 3.1: The Stolper-Samuelson theorem necessarily holds if a commodity, whose capital/labor ratio lies in between the capital/labor ratios of the other final commodity and the intermediate good, also uses greater quantity of the intermediate good per unit of output than the other commodity.

Theorem 3.2: If, however, the sign of B and (k_2-k_1) are not identical, the Stolper-Samuelson theorem does not hold.

The explanation for theorem (3.2) is provided by equations (3.1)-(3.3). Suppose $k_2 > k_1$, but $B < 0$, because, say, $k_3 > k_1$ and $a_1 > a_2$. It is clear from (3.1)-(3.3) that in this case a rise in the relative price of the second commodity raises the capital/labor ratio in all three industries (because $f_i'' < 0$ and $1-a_j > 0$). A rise in the capital/labor ratio in turn results in a decline in the marginal physical productivity of capital and a rise in the marginal physical productivity of labor in all of the industries. Hence the real rental declines and the real wage rises unambiguously even though the second commodity, whose price increased, is capital-intensive relative to the other commodity.

IV. The Pattern of Trade

It is fashionable nowadays to explain the pattern of trade between two countries in terms of the Heckscher-Ohlin theorem according to which a country exports the commodity which uses intensively its relatively abundant factor, and imports the commodity which is intensive in the use of its relatively scarce factor. Relative factor abundance between countries has been traditionally defined in two ways. Suppose there are two countries, a home (H) country and a foreign (F) country. Then according to the price definition of factor abundance, H is abundant in capital and scarce in labor relative to F if

$$\omega_h > \omega_f$$

where ω is as before the wage/rental ratio and the two subscripts denote the countries. The same relationship between H and F in terms of the physical definition is defined by

$$k_h > k_f$$

where $k = K/L$. If the Heckscher-Ohlin theorem is to be proved, we have to add the assumption of international identity of production functions to the list of assumptions already made in section I. Additionally, in the case of the physical definition, we need to assume that consumption patterns are also similar internationally. We will now examine the Heckscher-Ohlin theorem in terms of our model with intermediate goods.

A. The Price Definition

First, take the case of the price definition. Differentiating equation (7) with respect to p, we have:

$$(4.1)\quad \frac{d\omega}{dp} = -\frac{f_1 f_1''}{f_1'^2}\frac{dk_1}{dp}$$

Substituting dk_1/dp from (3.1) we obtain, with $p = p^* = 1$ initially:

$$(4.2)\quad \frac{d\omega}{dp} = -\frac{f_1 f_2 f_3}{B(1-a_1)f_1'^2}$$

where as before $B = f_3(k_2-k_1) + f_2(k_3-k_1)(a_2-a_1)$. In order to examine the pattern of trade, we also need the relation between p and p* which is given by equation (3.4), and is rewritten to facilitate our exposition.

$$(3.4)\quad \frac{dp^*}{dp} = \frac{(1-a_1)f_2(k_3-k_1)}{B}$$

Given that the numerator is negative in (4.2) and positive in (3.4), the signs of $d\omega/dp$ and dp^*/dp depend on the sign of B. A monotonic relationship between ω and p and p* and p exists only if B has a definite sign at all levels of p. If $a_2 = a_1$, $B = f_3(k_2-k_1)$ so that $B \gtreqless 0$ if $k_2 \gtreqless k_1$. Consider, for example, the case where $k_1 > k_2$ so that $B < 0$, $d\omega/dp > 0$ and, assuming for the time being that $k_1 > k_3$, $dp^*/dp > 0$. Under the assumption that the material input coefficients of the final commodities are the same (i.e., $a_2 = a_1$), these relations are diagrammatically depicted in Figure 3, where RS shows the positive relationship between p and p*, and TU shows the positive relationship between p and ω. Let us further assume that production functions are similar internationally not only for the two final commodities, but also for the intermediate product, so that TU and RS apply to both H and F. With $\omega_h > \omega_f$,

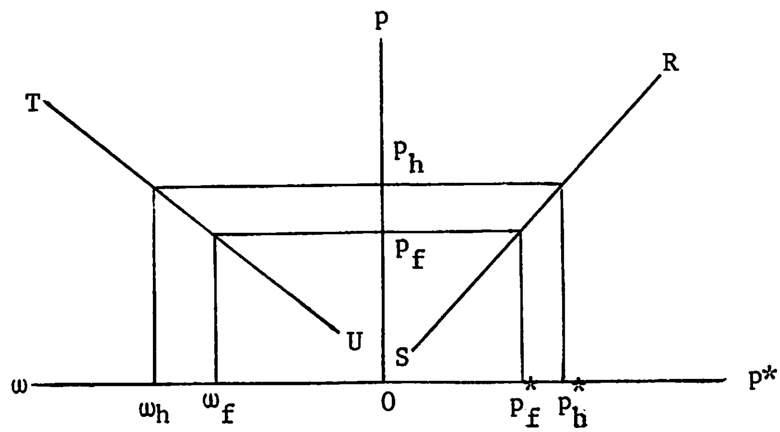

Figure 3

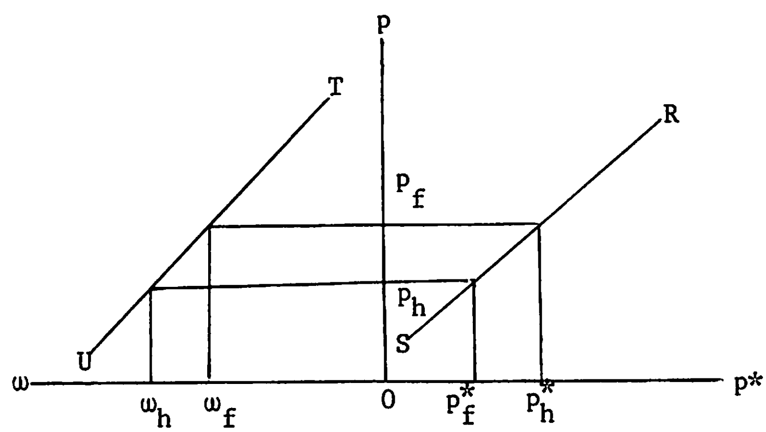

Figure 4

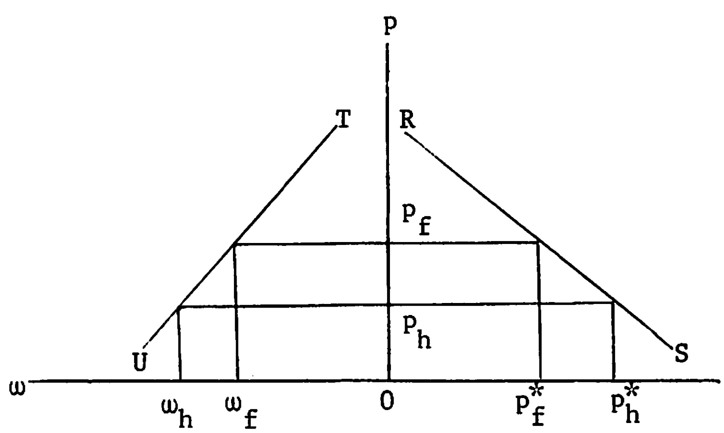

Figure 5

i) $p_h > p_f$

and

ii) $p_h^* > p_f^*$.

It is clear from (i) that H will export X_1 and import X_2 from F. Since H is capital-abundant relative to F and since X_1 is capital-intensive relative to X_2, it can be seen that the Heckscher-Ohlin theorem holds. From (ii), H will export X_1 and import X_3. Again since X_1 is capital-intensive relative to X_3, the trade pattern between a final good and an intermediate good also conforms to the Heckscher-Ohlin dictum. Thus a country exports its most capital-intensive product and imports the other two labor-intensive products including the intermediate good.

Take another case where $k_1 < k_3 < k_2$, and $a_2 = a_1$. Here $d\omega/dp < 0$ and $dp^*/dp > 0$. With the help of Figure 4, we can show that with $\omega_h > \omega_f$,

i) $p_h < p_f$

ii) $p_h^* < p_f^*$,

so that H will export X_2 and X_3 in exchange for X_1 from F. In this case the capital abundant country imports the labor-intensive commodity and exports the other two. This conclusion is different from the one derived in the case where $k_1 > k_3 > k_2$, but still one could say that the Heckscher-Ohlin theorem is valid.

So far we have discussed the case where (k_3-k_1) has the same sign as (k_2-k_1), so that $dp^*/dp > 0$. What if these signs differ and $dp^*/dp < 0$? This would happen, for example, if $k_3 < k_1 < k_2$, still assuming that $a_2 = a_1$. Here $B > 0$ and $d\omega/dp < 0$. These relations are depicted in

Figure 5 which shows that now

i) $p_h < p_f$

ii) $p_h^* > f_h^*$.

Here arises a conflict. From (i), H exports X_2 and imports X_1, whereas from (ii), H exports X_1 and imports X_3. In this case the pattern of trade is indeterminate. From our discussion so far we can derive the following theorems:

Theorem 4.1: If the capital/labor ratio of the intermediate good lies in between the capital/labor ratios of the final products, the Heckscher-Ohlin theorem holds, given that the material input coefficients in the final products are the same.

In general, we are able to predict the pattern of trade between the final products, but trade in intermediate goods may go either way.

Theorem 4.2: If the signs of (k_3-k_1) and (k_2-k_1) are not the same, the pattern of trade is indeterminate in the absence of additional restrictions.[9]

From theorem 4.1 follows another theorem:

Theorem 4.3: Given that there exists a basis for trade in final goods, there will normally exist a basis for trade in intermediate goods also.

So far we have discussed cases where trade follows the pattern set by the Heckscher-Ohlin theorem, but where the direction of trade in intermediate products cannot be predicted. However, if we relax our assumption that $a_2 = a_1$, we can derive conditions under which trade in intermediate goods also follows a definite direction, provided that the Heckscher-Ohlin theorem holds.

When $a_2 \neq a_1$, there arises the possibility that the relationships between p and ω on the one hand, and p and p*, on the other, may no longer be monotonic, which, in turn, gives rise to the possibility of multiple equilibria.[10] Under these conditions, the Heckscher-Ohlin theorem may not be valid. Furthermore, we have already seen, as in Figure 5, that the pattern of trade is indeterminate if dp*/dp has a negative sign. Therefore if we rule out the possibility of multiple equilibria and of the negative relationship between p and p*, we can not only establish the validity of the Heckscher-Ohline theorem, but also derive a neat theorem about trade in intermediate goods. In our discussion of the Stolper-Samuelson theorem in Section III, three cases were presented in which B had a definite sign. These cases are rewritten below:

i) $k_1 \lesseqgtr k_3 \lesseqgtr k_2$, and $a_2 > a_1$

ii) $k_1 \lesseqgtr k_2 \lesseqgtr k_3$, and $a_2 > a_1$

iii) $k_2 \lesseqgtr k_1 \lesseqgtr k_3$, and $a_1 > a_2$

As stated before, in all these cases, B has the same sign as (k_2-k_1). Let us first take case (i), which has two subcases: a) $k_1 < k_3 < k_2$, b) $k_1 > k_3 > k_2$, along with $a_2 > a_1$. In (a) $B > 0$, $dp^*/dp > 0$ and $d\omega/dp < 0$. In (b), $B < 0$, $dp^*/dp > 0$ and $d\omega/dp > 0$. In (a), the relationships are diagrammatically depicted in Figure 4, whereas in (b), they are depicted by Figure 3. We already know that in both diagrams, the Heckscher-Ohlin theorem holds and the pattern of trade is determinate. In case of Figure 4, H exports both X_2 and X_3 and imports X_1, whereas with Figure 3, H exports X_1

and imports X_2 and X_3 from F. One may observe a certain pattern in the conclusions derived from Figures 3 and 4. In both cases, the home country exports the capital-intensive final commodity. In the case of Figure 3, H imports X_2 but also imports X_3 because X_2, its importable final commodity, has a higher material input coefficient than X_1, its exportable commodity. In the case of Figure 4, H imports X_1, and exports X_2, but since its importable commodity, X_1, has a lower material input coefficient than X_2, its exportable commodity, it also exports X_3. Thus we may derive the following theorem.

Theorem 4.4: A country imports (exports) the intermediate goods if its imports (exports) utilize a greater quantity of intermediate goods per unit of output than its exports (imports).

Although theorem 4.4 has been derived from case (i), the interested reader can verify to the validity of the theorem in case (ii) also. However in case (iii), the Heckscher-Ohlin theorem can be shown to be valid, but if intermediate goods also enter trade, the trade pattern is indeterminate, because in this case p and p* will be inversely related.

So far we have ruled out the possibility of multiple equilibria. If we do have multiple equilibria, then we can write the following theorem.

Theorem 4.5: In the presence of multiple equilibria, which may arise even if all the assumptions of the traditional trade model without intermediate goods are satisfied, both the Heckscher-Ohlin theorem and theorem 4.4 may not hold.

B. The Physical Definition

The proof of the Heckscher-Ohlin theorem in terms of the physical definition of relative factor abundance requires, as stated earlier, an additional assumption of international similarity of consumption patterns. It is well known that, once this assumption is made, the validity of the Heckscher-Ohlin theorem depends on the validity of the Rybczynski theorem. If the latter holds, the former also holds, and vice versa. From our discussion in section II, we know that a sufficient condition for the validity of the Rybczynski theorem in our model with intermediate products is that the capital/labor ratio of the intermediate product lies in between the capital/labor ratios of the final products (see theorem 2.1), or that the commodity with capital/labor ratio lying in between the other two capital/labor ratios is also more intensive in the use of the intermediate product (see theorem 2.2). It follows therefore that theorems 2.1 and 2.2 provide sufficient conditions for the proof of the Heckscher-Ohlin theorem.

Theorem 4.6: Thus if $k_1 \gtrless k_3 \gtrless k_2$, $k_1 \gtrless k_2 \gtrless k_3 (a_2 f_2 > a_1 f_1)$, or $k_2 \gtrless k_1 \gtrless k_3 (a_1 f_1 > a_2 f_2)$, the Heckscher-Ohlin theorem is valid in terms of the physical definition of relative factor abundance.

Theorem 4.7: If k_3 does not lie in between k_1 and k_2, the Heckscher-Ohlin theorem in terms of the physical definition may not hold.

What about our theorem 4.4 concerning trade in intermediate goods? From theorem 4.6 and our earlier discussion in this section (under price definition), it can be shown that theorem 4.4 will hold in terms of the physical definition if i) $k_1 \gtrless k_3 \gtrless k_2$ and $a_2 > a_1$ and ii) $k_1 \gtrless k_2 \gtrless k_3$ and $a_2 f_2 > a_1 f_1$ (along with $a_2 > a_1$), because these cases ensure the positive relationship between p and p* and also the validity of the Heckscher-Ohlin theorem in terms of the physical definition.

Of greater interest is the case where our theorem concerning trade in intermediate goods may hold in spite of the invalidity of the Heckscher-Ohlin theorem in terms of the physical definition. Take, for instance, the case where $k_2 > k_1 > k_3$, $a_2f_2 > a_1f_1$, and $a_2 > a_1$. Here the Heckscher-Ohlin theorem may not hold simply because the Rybczynski theorem may not hold. Consider the right hand quadrant of Figure 3 where RS shows the positive relationship between p and p*. Let $p_h > p_f$ in spite of the fact that the home country is relatively capital-abundant and $k_2 > k_1 > k_3$. Here H will export X_1, the labor-intensive commodity and import X_2, the capital-intensive commodity--clearly a negation of the Heckscher-Ohlin theorem. However, $p_h^* > p_f^*$, so that H will also import the intermediate good which enters more importantly in the production of X_2 than X_1. Theorem 4.4 still holds even though the Heckscher-Ohlin theorem in terms of the physical definition does not. The following theorem is immediate.

Theorem 4.8: Given that p* and p are positively related, a country imports the intermediate commodity if it enters more importantly in the production of its final commodity imports than in its final commodity exports, and vice versa, even if the Heckscher-Ohlin theorem does not hold in terms of the physical definition of relative factor abundance between countries.

C. Equilibrium in International Trade

Given that trade takes place in intermediates as well as final commodities, what is the nature of equilibrium in free trade? In general, the nature of free trade equilibrium in the presence of intermediate products will be different from that established in their absence, unless, of course, intermediate products are produced solely to be used in domestic production and do not enter trade. If intermediates are exported, the transformation curve tends to shrink towards the origin because the export of a fraction of the total output of the intermediate good is equivalent, from the point of view of the whole economy, to the export of the labour and capital utilized in its production and this, in effect, reduces the available factor supplies.[11] By similar reasoning, an import of intermediates results in an outward movement of the transformation curve. These possibilities are described in Figures 6 and 7. In Figure 6, TT' is the transformation curve for final goods in the home country[12] and S the point of self-sufficiency equilibrium where a social indifference curve, U_0, touches the transformation curve. The autarky commodity price ratio between final goods (not shown in the diagram) is given by the slope of TT' or U_0 at point S. If X_2 is exported, the relative price of p will rise in the free trade equilibrium and, from the fact that p* and p are assumed to have a positive relationship, there will also be a rise in p*, which, in turn, implies that X_3 will also be exported. Therefore, a movement along TT' towards the right of S, which implies a rise in p, means that X_3 will be exported resulting, thereby in the shrinking of the transformation curve from ST' to SG'. Similarly, a leftward movement from S along ST implies a decline in p and hence p*, so that X_1 and X_3 will be imported and the transformation curve will tend to shift outward to SG. Thus TT' is the transformation curve in the absence of trade in intermediate products and, with S representing the point of self-sufficiency equilibrium, GSG' is the transformation curve when intermediates are also traded. Figure 6 depicts the case where X_2 and X_3 are exported and X_1 is imported. Let us now go back to equation (16) in Section I, which can be written as follows:

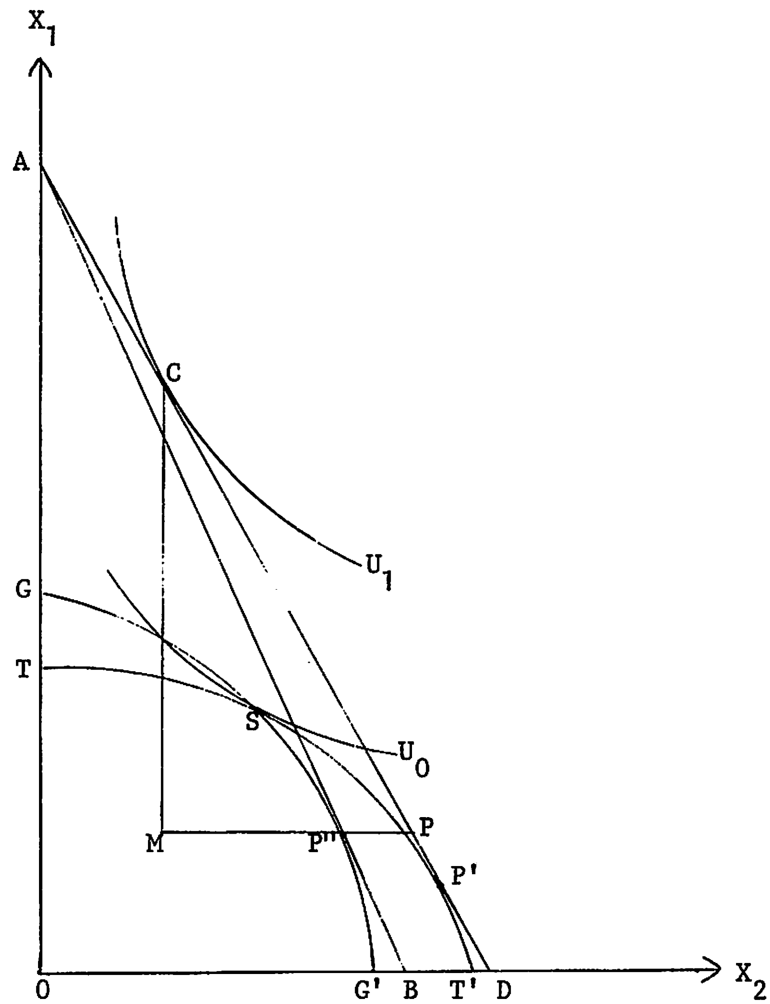

Figure 6

(16) $$\frac{dX_1}{dX_2} = -\left(p + p^*\alpha' \frac{dp^*}{dp}\frac{dp}{dX_2}\right)$$

where $dp/dX_2 > 0$. As pointed out earlier, $\alpha'=0$ in the absence of trade in intermediate products and $\frac{dX_1}{dX_2} = -p$. The production point moves to P' and the consumption point to C as a result of free trade, with AD indicating the international relative price of the second commodity, and U_1 indicating the level of social welfare. However, when intermediate goods are exported, then from 16, remembering that $\alpha'(dp^*/dp)(dp/dX_2) > 0$,

$$\left|\frac{dX_1}{dX_2}\right| > p .$$

As for the consumption equilibrium, the marginal rate of substitution still equals p. In Figure 6, the slope of AB, tangential to GSG at P", furnishes the marginal rate of transformation which exceeds the slope of AD by $\alpha'(dp^*/dp)(dp/dX_2)$. The actual production point, when intermediate goods are exported, is given by P". Assuming that the consumption point is still given by C, MC of X_1 is imported, MP" of X_2 and PP" of X_3 are exported.[13]

The case where intermediate products are imported is depicted in Figure 7. With the introduction of trade, production shifts from S to P", consumption from S to C, and welfare from U_0 to U_1; MP" of X_1 is exported, part of it (MP) in exchange for the import of MC of X_2 and the other part, PP", in exchange for the import of X_3. It may be noted that the marginal rate of transformation, (given by the slope of AB), again exceeds the relative price of the second commodity (indicated by the slope of AD), as indeed the case should be in terms of equation 16.

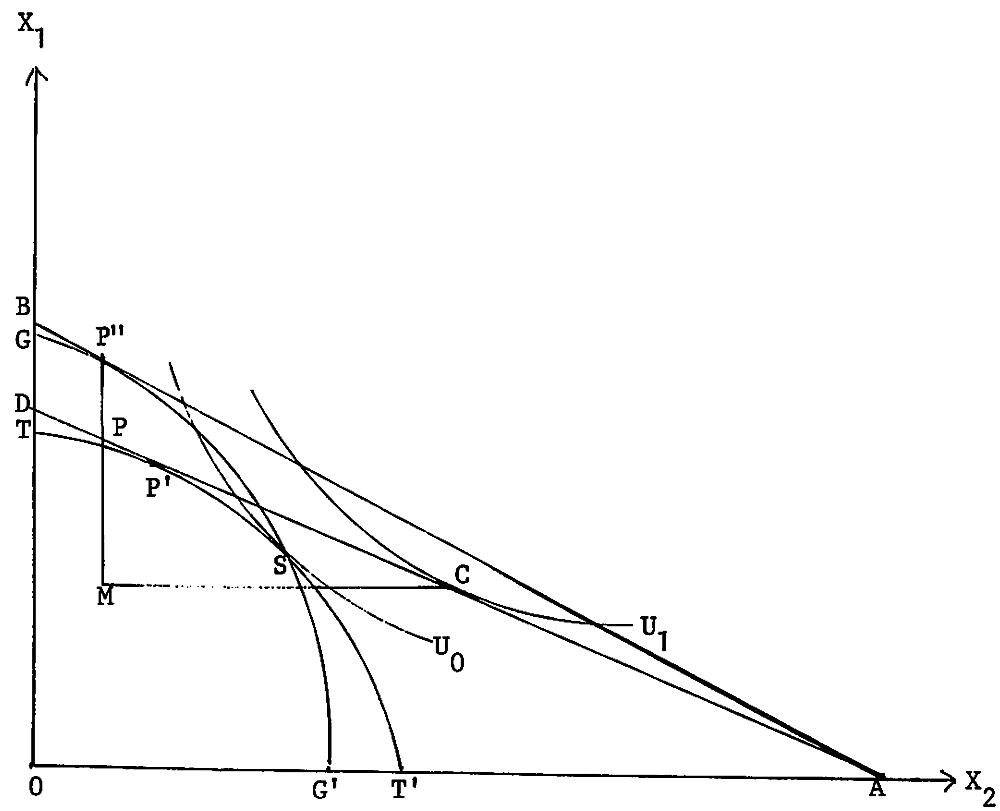

Figure 7

D. Factor Price Equalization

There still remains the question of whether factor prices will be equalized internationally in our model with intermediate products. Under free trade with no transport costs, prices for all commodities including intermediate products will be equalized. Given the additional assumption that ω and p on the one hand and p and p* on the other have monotonic relationships, and that production functions are similar internationally for all commodities, there exists only one ω in both countries. Since in perfect competition the capital/labor ratio in a commodity depends only on the factor-price ratio, capital/labor ratios in all commodities will be the same in both countries. This means that the marginal productivity of each factor is also the same in both countries. The free trade equilibrium is thus characterized by the following relations:

$$p_h = p_f$$

$$p_h^* = p_f^*$$

$$(k_i)_h = (k_i)_f \qquad (i = 1,2,3).$$

$$w_h = w_f$$

and

$$r_h = r_f \; .$$

However, in the presence of multiple equilibria, there may not be a unique value of ω for a given free trade commodity-price ratio. In this case factor prices may not be equalized between the two countries.

V. Technical Progress in Final Commodities

Using a two final commodity model, Findlay and Grubert (1959) have shown that Hicks-neutral technical progress in any commodity raises, at unchanged commodity prices, the output of the progressive commodity and lowers the output of the other commodity. We will now examine the Findlay-Grubert theorem in terms of our model with intermediate products. Suppose Hicks-neutral technical progress occurs in, say, the first commodity and is represented by $d\lambda$ where λ is the agent of technological change, which initially equals unity. Productions relations are now modified in the following way:

(5.1) $X_1 = \lambda L_1 f_1(k_1, \bar{x}_{31})$

(2) $X_2 = L_2 f_2(k_2, \bar{x}_{32})$

(3) $X_3 = L_3 f_3(k_3)$

For the sake of simplicity of mathematical calculations, we here assume that intermediate goods are not traded, so that α=0 and (4) becomes

(5.2) $X_3 = a_1 X_1 + a_2 X_2$,

where in the presence of technical progress $a_1 = \frac{\lambda \bar{X}_{31}}{X_1}$ and $a_2 = \frac{\bar{X}_{32}}{X_2}$ as before. The new value of a_1 raises an important definitional question which may be resolved very briefly. Take the case where intermediate goods are absent, so that $X_1 = \lambda L_1 f_1(k_1)$. Neutral technical progress in this case raises the marginal productivity of both factors in the same proportion. In other words, the same amount of X_1 is now produced by smaller quantities of K_1 and L_1. Therefore a rise in λ lowers the capital/output ratio (K_1/X_1) as well as the labor/output ratio (L_1/X_1) by the same proportion. Hence when there are three factors of production including the intermediate good, a rise in λ also lowers the intermediate input/output ratio $(\bar{X}_{31}/X_1)$, so that the material-input coefficient for X_1, a_1, has now to be divided by λ.

Factor rewards are now given by:

$$(5.3)\quad r = \lambda(1-a_1p^*)f_1' = (p-a_2p^*)f_2' = p^*f_3'$$

$$(5.4)\quad w = \lambda(1-a_1p^*)(f_1-k_1f_1') = (p-a_2p^*)(f_2-k_2f_2') = p^*(f_3-k_3f_3').$$

The rest of the equations remain unchanged. Differentiating (5.3) and (5.4) with respect to λ, keeping p constant, and with $p=p^* = \lambda=1$ initially, we obtain:

$$(5.5)\quad \frac{dk_1}{d\lambda} = -\frac{f_2f_3}{f_1''B}$$

$$(5.6)\quad \frac{dk_2}{d\lambda} = -\frac{(1-a_1)f_1f_3}{(1-a_2)^2B}$$

$$(5.7)\quad \frac{dk_3}{d\lambda} = -\frac{(1-a)f_1f_2}{f_3''B}\,.$$

Differentiating equations (5.1), (5.2), the full employment equations (8) and (9), and substituting equations (5.5)-(5.7), we obtain:

$$(5.8)\quad \frac{dX_1}{d\lambda} = \frac{1}{A.B}\left[\frac{L_1f_2^2f_3^2}{f_1''(1-a_1)} + \frac{(1-a_1)L_2f_1^2f_3^2}{f_2''(1-a_2)^3} + \frac{(1-a_1)L_3f_1^2f_2^2}{f_3''}\right] + L_1f_1$$

$$(5.9)\quad \frac{dX_2}{d\lambda} = \frac{1}{A.B}\left[\frac{L_1f_2^2f_3^2}{f_1''(1-a_1)} + \frac{(1-a_1)L_2f_1^2f_3^2}{f_2''(1-a_2)^3} + \frac{(1-a_1)L_3f_1^2f_2^2}{f_3''}\right]$$

where as before

$$A = a_1f_1(k_3-k_2) + a_2f_2(k_1-k_3) + k_3(k_1-k_2), \quad \text{and}$$

$$B = f_3(k_2-k_1) + f_2(k_3-k_1)(a_2-a_1).$$

It is clear that the signs of $dX_1/d\lambda$ and $dX_2/d\lambda$ depend on the sign of A.B.

In all those cases where A.B has a negative sign, the Findlay-Grubert theorem, which requires $dX_1/d\lambda > 0$ and $dX_2/d\lambda < 0$, is valid. From (5.8), the second term is positive, so whether $dX_1/d\lambda > 0$ depends on the first term which, in view of $f_1'' < 0$ will be positive only if $A.B < 0$. Similarly, in (5.9) $dX_2/d\lambda < 0$ only if $A.B < 0$, which requires that A and B have opposite signs. It turns out that whenever A and B have definite signs, their signs are opposite. These cases are given as follows:

i) $k_1 \gtrless k_3 \gtrless k_2$; $a_2 > a_1$

ii) $k_1 \gtrless k_2 \lesseqgtr k_3$; $a_2 > a_1$ and $a_2f_2 > a_1f_1$

iii) $k_2 \gtrless k_1 \gtrless k_3$; $a_1 > a_2$ and $a_1f_1 > a_2f_2$.

The signs of A and B are quite apparent in case (i). In case (ii), suppose $k_1 > k_2 > k_3$. With $a_2 > a_1$, $B < 0$; given that $a_2f_2 > a_1f_1$ and the fact that $(k_1-k_3) > (k_2-k_3)$, $A > 0$; therefore $A.B < 0$. Similarly in case (ii) when $k_1 < k_2 < k_3$, $B > 0$, $A < 0$ and $A.B < 0$. The sign of A.B can be analogously found to be negative in case (iii). We can thus derive the following theorems.

<u>Theorem 5.1</u>: If the capital/labor ratio of the intermediate good lies in between the capital/labor ratios of the two final goods and if the second commodity has a higher material input coefficient than the first commodity, the Findlay-Grubert theorem holds. Hicks neutral technical progress in any of the final commodities raises, at constant terms of trade, the output of the progressive commodity and lowers the output of the other commodity.

<u>Theorem 5.2</u>: If the final commodity, whose capital/labor ratio lies between the capital/labor ratio of the other final commodity and that of the intermediate

good, has not only a higher material input coefficient than the other final commodity, but is also more intensive in the use of the intermediate good, the Findlay-Grubert theorem is valid.

Theorem 5.3: If A and B do not have definite signs, the Findlay-Grubert theorem may not hold.

VI. Technical Progress in the Intermediate Product

So far we have assumed that technical progress occurs in a final commodity. In this section, we explore the implications of Hicks neutral technical progress in the intermediate product for the output of the two final commodities. The implications of technical change in the intermediate good are more intricate than may appear to be superficially. Such a technical change has the effect of lowering the unit cost of production in all three commodities, the extent of the unit cost reduction in the two final commodities depending on the extent of technical change in the intermediate product and the two material input coefficients (a_j). This is equivalent to technical change in both the final commodities in proportion to a_j, and one may argue that it could be treated in the same way as the Findlay-Grubert theorem. However, a technical improvement in the intermediate good also tends to raise the supply of the material input, because the previous output of this product can now be produced by smaller quantities of capital and labor. The rise in the supply of material input in turn has its own repercussions on the output of final products. Thus in the case of technical progress in the intermediate good we have elements of both the Findlay-Grubert theorem and the Rybczynski theorem. This point is highlighted further in the remainder of this section. The production function for X_3 now becomes

(6.1) $X_3 = \beta\ L_3 f_3(k_3)$

and factor rewards are given by

(6.2) $(1-a_1 p^*)f_1' = (p-a_2 p^*)f_2' = \beta\ p^* f_3'$

(6.3) $(1-a_1 p^*)(f_1-k_1 f_1') = (p-a_2 p^*)(f_2-k_2 f_2') = \beta\ p^*(f_3-k_3 f_3')$,

where β stands for technical change in the intermediate product and initially equals unity, and $a_j = \overline{X}_{3j}/X_j$ (i=1,2) as before. Other production functions and full employment equations remain the same as in (1) and (2) and (8) and (9), respectively.

Differentiating with respect to β, keeping p constant and remembering that initially $p=p^* = \beta=1$, we obtain

(6.4) $$\frac{dk_1}{d\beta} = \frac{-f_2 f_3(a_1-a_2)}{f_1''(1-a_1)B}$$

(6.5) $$\frac{dk_2}{d\beta} = \frac{-(1-a_1)f_1 f_3(a_1-a_2)}{f_2''(1-a_2)^2 B}$$

(6.6) $$\frac{dk_3}{d\beta} = \frac{-(1-a_1)f_1 f_2(a_1-a_2)}{f_3'' B}$$

Differentiating equations (1) and (2) and (8) and (9) and substituting (6-4)-(6-6), we obtain:

(6.7) $$\frac{dX_1}{d\beta} = \frac{(1-a_1)(a_1-a_2)}{A.B}\left[\frac{L_1 f_2^2 f_3^2}{(1-a_1)^3 f_1''} + \frac{L_2 f_1^2 f_3^2}{(1-a_2)^3 f_2''} + \frac{L_3 f_1^2 f_2^2}{f_3''}\right] + \frac{L_3 f_1 f_3(k_3-k_2)}{A}$$

$$(6.8)\quad \frac{dX_2}{d\beta} = \frac{-(1-a_1)(a_1-a_2)}{A.B}\left[\frac{L_1 f_2^2 f_3^2}{(1-a_1)^3 f_1''} + \frac{L_2 f_1^2 f_3^2}{(1-a_2)^3 f_2''} + \frac{L_3 f_1^2 f_2^2}{f_3''}\right] - \frac{L_3 f_2 f_3 (k_3-k_1)}{A} .$$

$$(6.9)\quad \frac{dX_3}{d\beta} = \frac{(1-a_1)(a_1-a_2)^2}{A.B}\left[\frac{L_1 f_2^2 f_3^2}{(1-a_1)^3 f_1''} + \frac{L_2 f_1^2 f_3^2}{(1-a_2)^3 f_2''} + \frac{L_3 f_1^2 f_2^2}{f_3''}\right]$$

$$+ \frac{L_3 f_3}{A}\left[a_1 f_1 (k_3-k_2) + a_2 f_2 (k_1-k_3)\right] .$$

Let "cost-effect" denote the effect of technical progress in the intermediate product on the unit costs of the two final goods, and let "factor-supply effect" denote the effect of such a technical change on the total supply of the material input. The cost effect on the final commodities depends on a_2 and a_1. If $a_2=a_1$, unit costs in both commodities decline by the same proportion. If $a_1 > a_2$, the cost effect is equivalent to a higher rate of technical progress in X_1 than X_2, and vice versa. Consider the case where $a_1 > a_2$, so that the unit cost in X_1 declines relative to the unit cost in X_2. Let us further assume the absence of multiple equilibria so that B has the same sign as (k_2-k_1).[14] If final commodity prices are to be kept constant, then a decline in the unit cost of X_1 relative to X_2 must raise the price of the primary factor intensively employed in X_1 and lower the price of the factor employed unintensively in X_1. Suppose $k_2 > k_1$ and $B > 0$. In this case, the rental will decline and the wage rate will rise, which in turn will raise the capital/labor ratio in all three industries. That this is what happens is clear from equations (6.4)-(6.6) where $dk_i/d\beta > 0$. Similarly if $k_2 < k_1$, $dk_i/d\beta < 0$. Now given that both A and B have definite signs,[15] the rise (decline) in the capital/labor ratio in all industries will raise (lower) the output of the labor-intensive industry, X_1, and lower (raise) the output of the capital-intensive industry, X_2. This is the "cost effect" of technical change in intermediate product on the two final outputs and is given by the first term in (6.7) and (6.8). We know, from our discussion in the previous section, the conditions under which A and B have definite, but opposite, signs. Therefore, given that $A.B < 0$ and that $a_1 > a_2$, the "cost effect", conforming to the Findlay-Grubert case, leads to a rise in the output of X_1 and to a decline in the output of X_2.

The second term in (6.7) and (6.8) arises from the "factor-supply effect" of the technical improvement in intermediate product and may be referred to as the Rybczynski effect. If the output of the intermediate good is kept constant, then, depending on the extent of technical progress in X_3, capital and labor are released from X_3 in the proportion k_3. Therefore, if k_3 lies in between k_1 and k_2, the output of both final commodities will rise, at constant commodity prices, from the factor supply effect.[16] If $k_1 > k_3 > k_2$, then $A > 0$ and the second term in both (6.7) and (6.8) is positive. If $k_1 < k_3 < k_2$ then $A < 0$, but both the terms in (6.7) and (6.8) are still positive. However, if k_3 does not lie in between k_1 and k_2, the implications of the factor supply effect become more difficult to interpret. We have assumed until now that $a_1 > a_2$. In addition suppose we assume that $a_1 f_1 > a_2 f_2$. That is to say, we assume that if a commodity has a higher material input coefficient it is also more intensive in the use of the intermediate good than the other commodity. Then if k_1 lies in between k_3 and k_2, the output of X_1 rises and that of X_2 declines from the "factor supply effect." However, if k_2 lies in between k_1 and k_3 and $a_1 f_1 > a_2 f_2$, the sign of A is determinate, and so is the factor supply effect.

The overall effect of the technical progress in the intermediate good on the two final outputs is the sum of the "cost effect" and the "factor supply effect," which is determinate only if both effects have the same sign. On the basis of the discussion in this section so far, we can derive the following theorems:

<u>Theorem 6.1</u>: If the capital/labor ratio of the intermediate product lies in between the capital/labor ratio of the final products, then technical progress in the intermediate good raises the output of the commodity which has the higher material input coefficient, given that the terms of trade are constant; the effect on the other commodity is indeterminate.

<u>Theorem 6.2</u>: If the capital/labor ratio of the commodity with the higher material input coefficient lies in between the other two capital/labor ratios, neutral technical progress in the intermediate good raises, at constant terms of trade, the output of that commodity and lowers the output of the other commodity, provided the final commodity, with the higher material input coefficient, is also more intensive in the use of the intermediate good.

There remains the question of what happens to the output of the intermediate good. It can be easily seen from (6.9) that, given the definite but opposite signs of A and B, the output of X_3 will always rise. The first term in (6.9) is always positive; furthermore if $A > 0$, the numerator of the second term is positive; if $A < 0$, this numerator is negative. The following theorem is then immediate.

<u>Theorem 6.3</u>: Given the absence of multiple equilibria, the output of the intermediate good rises as a result of neutral technical progress in it, given that

the final commodity prices are kept constant.

Thus we may conclude from these three theorems that neutral technical progress in the intermediate products at constant final commodity prices raises its own output as well as the output of the commodity with the higher material input coefficient and higher intensity in the use of intermediate good. The output of the other commodity may rise, decline, or remain unchanged.

VII. Concluding Remarks

Introducing an intermediate product in the traditional two-factor two-commodity constant returns to scale model, we have shown in the foregoing analysis that a strong possibility of multiple equilibria arises even in the absence of reversible factor-intensities, in which case none of the traditional theorems like the Rybczynski theorem, the Stolper-Samuelson theorem, the Heckscher-Ohlin theorem, the Factor-Price Equalization theorem, the Findlay-Grubert theorem, etc., may be valid. However, if we rule out the possibility of multiple equivilibria, all these theorems hold without additional provisions.[17] If we assume that the second commodity has a higher material input coefficient than the first commodity, then a country imports or exports the intermediate good if the second commodity is imported or exported. The incidence of technical progress in the intermediate good includes the elements of the Findlay-Grubert theorem as well as the Rybczynski theorem, because it results in i) differential unit-cost reduction in the two final commodities (provided $a_1 \neq a_2$), which is equivalent to technical progress in the two industries at different rates, and ii) an increase in the supply of the intermediate good which is a factor of production.

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Footnotes

[1]For other studies in trade theory which treat intermediate goods identically as final goods, see Samuelson (1953), McKinnon (1966) and Warne (1970). For studies where intermediate goods are solely produced as material inputs, see Khang (1969) who presents a dynamic model of trade between one final good and an intermediate good.

[2]It is true that in Vanek's model a part of the final good imported may be used as an intermediate good in domestic production. However, the primary reason for the import of the final good would still be the fact that domestic consumers switch over to the consumption of cheaper imports. In any case, Vanek's model contains no framework for analyzing the basis of trade for such material inputs produced only to be used in the production of final goods. Clearly there is need for an alternative theory because most of the world trade takes place in such intermediate goods.

[3]For a model that provides demand equations in the presence of intermediate goods, traded or non-traded, see Ruffin (1969).

[4]For other extensions of the Rybczynski theorem, see Batra (1970), Kemp (1969) and Jones (1968).

[5]For rigorous proof concerning the relationship between the final good prices and the price of the intermediate good, see Section IV on the pattern of trade.

[6]The diagrammatical illustration does not depend on this configuration. The reader can verify to this fact by drawing another diagram but making k_3 lie in between k_1 and k_2.

[7]It may be observed that conditions described in Figure 2 (where $k_1 > k_2 > k_3$ and $a_1 f_1 > a_2 f_2$) do not accord with those specified in theorem 2.2. That is why the Rybczynski theorem may not hold.

[8]For other extensions of the Stolper-Samuelson theorem, see Minabe (1967), Batra (1968), Kemp (1969) and Jones (1968).

[9]The pattern of trade could be predicted in such a case by placing restrictions on the trade of intermediate goods. One could, for example, specify that the domestic supply of intermediate goods always equals domestic demand, or even place a prohibitive tariff (or tax) on the import (or export) of such goods, in which case the trade pattern in final goods will follow the Heckscher-Ohlin dictum.

[10] In the traditional two good model without intermediate products, multiple equilibria can arise only in the presence of reversible factor-intensities. See Johnson (1958). However, when intermediate products are introduced, our analysis shows the possibility of such equilibria even when factor-intensities are assumed to be non-reversible. This introduces an important difference to the traditional Heckscher-Ohlin model in that many of the traditional conclusions may not hold even with irreversible factor-intensities.

[11] This does not mean that the national income falls because the export of the intermediate product is made in exchange for the import of a final good. Therefore, if something goes out, another comes in. However, this raises an interesting question of whether the introduction of trade in intermediate goods tends to raise or lower the gains from trade. Evidently, this is outside the scope of our paper, and we make no attempt to provide an answer.

[12] In Figure 6, the transformation curve is given the usual shape of concavity to the origin. Although we have ruled out this possibility, the presence of intermediate goods may make the transformation curve convex to the origin.

[13] The new consumption point need not be the same as the one obtained in the absence of trade in intermediate goods. However, we have assumed the two points to be identical firstly for diagrammatic simplicity, and secondly to avoid the question of whether the introduction of trade in intermediate goods modifies the gains from trade, a question which is, as stated earlier, beyond the scope of this paper. This type of diagram has also been used by Ruffin (1969).

As far as the export of intermediate good is concerned, one can see that it must be equal to PP" in terms of the first commodity. In Figure 6,

$$\frac{dX_1}{dX_2} = \frac{MC}{MP''} \quad \text{and} \quad p = \frac{MC}{MP'' + PP''} .$$

From (16), the export of the intermediate good equals

$$\frac{dX_1}{dX_2} - p = \frac{MC}{MP}(MP'' + PP'' - MP'') = p(PP'')$$

which is nothing but the export of X_3 in terms of X_1.

[14] See section V. It can be shown that for the absence of multiple equilibria, both A and B, not B alone, must have definite signs.

[15] So that there are no multiple equilibria.

[16] This is equivalent to a rise in the supply of both factors of production. It has been shown by Guha (1963), Kemp (1964), and Batra (1969) that if the incremental capital/labor ratio of the entire economy lies in between the capital/labor ratios in the two final commodities, then the output of both commodities rise at constant commodity prices. Our analysis confirms this thesis.

[17] For the absence of multiple equilibria, we require that both A and B, rather than B alone, have definite signs. This point is apparent from our analyses in sections V and VI, although not in other sections. However, it can be easily shown that $A.B < 0$ is a sufficient condition for the absence of multiple equilibria as well as the outward convexity of the transformation curve.

MATHEMATICAL APPENDIX

Some of the equations which appear in the text without proof will be derived here, remembering that in the text $p = p^* = 1$ initially.

I

By totally differentiating equations (5) and (6) with respect to p - taking the members of each equation two by two -, and rewriting in matrix form, we have

$$\text{(A-1)} \quad \begin{bmatrix} (1-p^*a_1)f_1'' & -(p-p^*a_2)f_2'' & 0 & a_2f_2'-a_1f_1' \\ (1-p^*a_1)f_1'' & 0 & -p^*f_3'' & -a_1f_1'-f_3' \\ -(1-p^*a_1)k_1f_1'' & (p-p^*a_2)k_2f_2'' & 0 & a_2(f_2-k_2f_2')-a_1(f_1-k_1f_1') \\ -(1-p^*a_1)k_1f_1'' & 0 & p^*k_3f_3'' & -a_1(f_1-k_1f_1')-(f_3-k_3f_3') \end{bmatrix} \begin{bmatrix} dk_1/dp \\ dk_2/dp \\ dk_3/dp \\ dp^*/dp \end{bmatrix} = \begin{bmatrix} f_2' \\ 0 \\ (f_2-k_2f_2') \\ 0 \end{bmatrix}$$

Solving the above system gives equations (3.1)-(3.4). Equations (3.5) and (3.6) are found by differentiating (5) and (6), to obtain

$$\text{(A.2)} \quad \frac{dr}{dp} = p^*f_3'' \frac{dk_3}{dp} + f_3' \frac{dp^*}{dp}$$

$$\text{(A.3)} \quad \frac{dw}{dp} = - p^*k_3f_3'' \frac{dk_3}{dp} + (f_3-k_3f_3') \frac{dp^*}{dp}$$

and substituting from (3.3) and (3.4)

II

If we denote the matrix of coefficients of the system (A-1) by [L], then, differentiation of (5.3) and (5.4) with respect to λ gives:

$$\text{(A.4)} \qquad [L]\begin{bmatrix} dk_1/d\lambda \\ dk_2/d\lambda \\ dk_3/d\lambda \\ dp^*/d\lambda \end{bmatrix} = -(1-p^*a_1)\begin{bmatrix} f_1' \\ f_1' \\ (f_1-k_1f_1') \\ (f_1-k_1f_1') \end{bmatrix}$$

Solving (A.4) for $dk_i/d\lambda$ $(i=1,2,3)$ gives equations (5.5)-(5.7).

If we differentiate (3), (8), (9), and (5.2), we get

$$\text{(A.5)} \qquad \begin{bmatrix} 1 & 1 & 1 \\ k_1 & k_2 & k_3 \\ a_1f_1 & a_2f_2 & -f_3 \end{bmatrix}\begin{bmatrix} dL_1/d\lambda \\ dL_2/d\lambda \\ dL_3/d\lambda \end{bmatrix} = -\begin{bmatrix} 0 \\ L_1\dfrac{dk_1}{d\lambda} + L_2\dfrac{dk_2}{d\lambda} + L_3\dfrac{dk_3}{d\lambda} \\ a_1L_1f_1'\dfrac{dk_1}{d\lambda} + a_2L_2f_2'\dfrac{dk_2}{d\lambda} - L_3f_3'\dfrac{dk_3}{d\lambda} \end{bmatrix}$$

Substituting from (5.5)-(5.7), and solving:

$$\text{(A.6)} \qquad \frac{dL_1}{d\lambda} = \frac{p^2(1-p^*a_1)}{A.B}\left\{\frac{L_1f_2f_3\left[f_3+a_2f_2+a_1f_1'(k_3-k_2)\right]}{p(1-p^*a_1)f_1''} + \frac{L_2f_1f_3^2}{(p-p^*a_2)^3\,f_2''} + \frac{L_3f_1f_2^2}{p^{*3}\,f_3''}\right\} - \frac{a_1\,L_1\,f_1\,(k_3-k_2)}{A}$$

$$\text{(A.7)} \qquad \frac{dL_2}{d\lambda} = \frac{-p(1-p^*a_1)}{A.B}\left\{\frac{L_1f_2f_3^2}{(1-p^*a_1)f_1''} + \frac{L_2f_1f_3\left[f_3+a_1f_1+a_2f_2'(k_3-k_1)\right]}{(p-p^*a_2)^2\,f_2''} + \frac{L_3f_1^2f_2}{p^{*3}\,f_3''}\right\} + \frac{a_1L_1f_1(k_3-k_1)}{A}$$

From (5.1) and (5.2)

$$\text{(A.8)} \quad \frac{dX_1}{d\lambda} = L_1 f_1' \frac{dk_1}{d\lambda} + f_1 \frac{dL_1}{d\lambda} + L_1 f_1$$

$$\text{(A.9)} \quad \frac{dX_2}{d\lambda} = L_2 f_2' \frac{dk_2}{d\lambda} + f_2 \frac{dL_2}{d\lambda}$$

Substituting (5.5) and (A.6) into (A.8), and (5.6) and (A.7) into (A.9) gives (5.8) and (5.9).

III

Total differentiation of (6.2) and (6.3) with respect to β gives:

$$\text{(A.10)} \qquad [L] \begin{bmatrix} dk_1/d\beta \\ dk_2/d\beta \\ dk_3/d\beta \\ dp^*/d\beta \end{bmatrix} = \begin{bmatrix} 0 \\ p^* f_3' \\ 0 \\ p^*(f_3 - k_3 f_3') \end{bmatrix}$$

from which we may derive (6.4)-(6.6).

Also, by differentiating (8), (9), (5.2), and (6.1), we have

$$\text{(A.11)} \quad \begin{bmatrix} 1 & 1 & 1 \\ k_1 & k_2 & k_3 \\ a_1 f_1 & a_2 f_2 & -f_3 \end{bmatrix} \begin{bmatrix} dL_1/d\beta \\ dL_2/d\beta \\ dL_3/d\beta \end{bmatrix} = - \begin{bmatrix} 0 \\ L_1 \frac{dk_1}{d\beta} + L_2 \frac{dk_2}{d\beta} + L_3 \frac{dk_3}{d\beta} \\ a_1 L_1 f_1' \frac{dk_1}{d\beta} + a_2 L_2 f_2' \frac{dk_2}{d\beta} - L_3 f_3' \frac{dk_3}{d\beta} - L_3 f_3 \end{bmatrix}$$

Substituting from (6.4)-(6.6) and solving:

$$\text{(A.12)}\quad \frac{dL_1}{d\beta} = \frac{-p(1-p^*a_1)(a_2-pa_1)}{A.B}\left\{\frac{L_1f_2f_3\left[f_3+a_2f_2+a_1f_1'(k_3-k_2)\right]}{(1-p^*a_1)^2f_1''} + \frac{pL_2f_1f_3^2}{(p-p^*a_2)^3f_2''} + \frac{pL_3f_1f_2^2}{p^{*3}f_3''}\right\}$$

$$+ \frac{L_3f_3(k_3-k_2)}{A}$$

$$\text{(A.13)}\quad \frac{dL_2}{d\beta} = \frac{p^*(1-p^*a_1)(a_2-pa_1)}{A.B}\left\{\frac{L_1f_2f_3^2}{(1-p^*a_1)^3f_1''} + \frac{L_2f_1f_3\left[f_3+a_1f_1+a_2f_2'(k_3-k_1)\right]}{(p-p^*a_2)^2f_2''} + \frac{L_3f_1^2f_2}{p^{*3}f_3''}\right\}$$

$$- \frac{L_3f_3(k_3-k_1)}{A}$$

$$\text{(A.14)}\quad \frac{dL_3}{d\beta} = \frac{p^*(1-p^*a_1)(a_2-pa_1)}{A.B}\left\{\frac{L_1f_2^2f_3(a_2-pa_1)}{(1-p^*a_1)^3f_1''} + \frac{L_2f_1^2f_3(a_2-pa_1)}{(p-p^*a_2)^3f_2''}\right.$$

$$\left. + \frac{L_3f_1f_2\left[a_2f_2-a_1f_1+f_3'(k_2-k_1)\right]}{p^{*2}f_3''}\right\} + \frac{L_3f_3(k_2-k_1)}{A}$$

From equations (1), (2), and (6.1)

$$\text{(A.15)}\quad \frac{dX_1}{d\beta} = L_1f_1'\frac{dk_1}{d\beta} + f_1\frac{dL_1}{d\beta}$$

(A.16) $$\frac{dX_2}{d\beta} = L_2 f_2' \frac{dk_2}{d\beta} + f_2 \frac{dL_2}{d\beta}$$

(A.17) $$\frac{dX_3}{d\beta} = L_3 f_3' \frac{dk_3}{d\beta} + f_3 \frac{dL_3}{d\beta} + L_3 f_3$$

Substituting (6.4) and (A.12) into (A.15), (6.5) and (A.13) into (A.16), and (6.6) and (A.14) into (A.17), we obtain equations (6.7)-(6.9).