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In the recent spurt on the two-sector, two-factor neoclassical growth models, the consequences of technical progress for relative stability have not been fully analyzed. The only studies that deal with this problem are those of Findlay [7], Takayama [10] and the present author [3][1]. In these papers it has been shown that technical progress leaves unchanged the conditions for the existence and the stability of the balanced growth path, defined by the constancy and the equality of the rates of growth of labor and capital only if (1) it occurs in the consumption-goods sector, and (2) it is Hicks- or Harrod-neutral.

Recognizing the limited significance of this result, Takayama goes a bit further and utilizes the concept of "equilibrium growth path," which he defines as one where capital stock grows faster than labor by a constant rate. He then shows that if (1) the consumption-goods sector is capital-intensive relative to the capital-goods sector, or (2) the elasticity of substitution in the capital-goods sector is at least equal to unity, the equilibrium growth path is stable even if technical progress occurs in the

[1]The results in [3], unlike those in [7] and [10], are derived in terms of a dynamic model of fixed coefficients. These results turn out to be special cases of our conclusions obtained in this paper where production coefficients are not kept constant.

capital-goods sector, provided it is Harrod-neutral. However, in the literature on two-sector growth models without technical progress, the balanced growth path has been shown to be stable under less restrictive conditions. For example Drandakis [6] and Amano [1] have proved that the two-sector growth model is relatively stable if the elasticity of substitution in _any_ sector equals or exceeds unity. In addition, the present author [4] has derived a still weaker set of sufficient conditions by showing that the relative stability requires the elasticity of substitution in any sector to be a positive _fraction_ greater than or equal to the relative share of labor in the other sector. One purpose of this paper is to show that for the equilibrium growth path to be stable, the set of sufficient conditions does not have to be as strong as that proposed by Takayama. Secondly, we show that the equilibrium growth path is stable even if technical progress in any sector is Hicks-neutral. To facilitate our analysis, we give a more general connotation to the equilibrium growth path by defining it as the path where the rate of growth of capital differs from the exogenously given rate of growth of labor by a constant rate, the difference being positive or negative. In this way, we show that the equilibrium growth path is stable even if capital stock grows _slower_ than labor by a constant rate.

I. The Model and Some Key Relations

It is assumed that the economy consists of two sectors of production, the consumption-goods sector (Y_2) and the capital-goods sector (Y_1), which, in the process of production, utilize two factors, capital (K) and labor (L). Perfect competition, constant returns to scale, diminishing returns along the isoquants, full employment of factors, inelastic factor supplies and perfect factor mobility are also assumed. Capital and labor are homogeneous and can be used in any sector. Workers consume all their income and capitalists save a part of their income.

The two production functions are:

(1) $Y_1 = F_1(K_1,L_1,t) = L_1 f_1(k_1,t)$

(2) $Y_2 = F_2(K_2,L_2,t) = L_2 f_2(k_2,t)$

where K_i, L_i are respectively the capital and labor inputs and $k_i = K_i/L_i$, is the capital/labor ratio in the ith sector ($i = 1,2$), and t is a shift parameter representing the technological level of the economy.

Let u_i and v_i respectively denote the marginal productivity of capital and labor in the ith sector. Then

$$v_i = f_i' = \frac{\partial f_i}{\partial k_i} \qquad (f_i' = f_i'(k_i))$$

$$u_i = f_i - k_i f_i'$$

It may be noted that all marginal productivities depend on factor-proportions and the shift parameter. It is assumed that $f_i' > 0$ and $f_i'' < 0$.

Under perfect competition, the price of each factor of production equals its marginal value product and is the same in both sectors. Let w stand for the wage-rate, r for the rental rate of capital and P_i for the price of the ith commodity. Factor rewards can then be expressed as:

(3) $w = P_i u_i = P_i(f_i - k_i f_i')$

(4) $r = P_i v_i = P_i f_i'$

(5) $\omega = \frac{w}{r} = \frac{u_i}{v_i} = \frac{f_i}{f_i'} - k_i$

With full employment

(6) $\rho_1 + \rho_2 = 1$

(7) $\rho_1 k_1 + \rho_2 k_2 = k$

where $\rho_i = L_i/L$ and $k = K/L$.

Let s denote the propensity to save of the capitalists. The saving-investment equilibrium requires that

(8) $P_1 Y_1 = srK$

where $0 < s < 1$. It is assumed that s is constant. Let σ_i be the elasticity of factor substitution in the ith sector. Then

(9) $\sigma_i = \dfrac{\partial k_i}{\partial \omega} \cdot \dfrac{\omega}{k_i} > 0.$

Let β_i denote the relative share of labor in the ith sector. Then

(10) $\beta_i = \dfrac{L_i u_i}{Y_i} = 1 - \dfrac{K_i v_i}{Y_i} = \dfrac{(f_i - k_i f_i')f_i'}{f_i f_i'} = \dfrac{\omega}{\omega + k_i}.$

From the equations of production functions and equation (5), it is clear that

(11) $k_i = k_i(\omega, t)$

From the Euler theorem with linear homogeneous functions:

(12) $Y_i = u_i L_i + v_i K_i$, or

(13) $P_i Y_i = w L_i + r K_i$

We now transform some of the production relations into variables of relative rates of change in order to obtain certain key relations which will be utilized in the subsequent analysis. Let the asterisk refer to the relative rate of change of a variable (e.g., $k^* = dk/k$). Differentiating equations (1) and (2), we have:

(14) $Y_i^* = \beta_i L_i^* + (1 - \beta_i)K_i^* + \alpha_i$

where α_i denotes the rate of growth of the ith output when the factor-input in the industry is kept constant[2] ($\alpha_i = \dfrac{1}{Y_i}\dfrac{\partial Y_i}{\partial t} dt$). Differentiating equation (5), we have:

(15) $\omega^* = w^* - r^* = u_i^* - v_i^*$

Differentiating equation (11) we have:

(16) $k_i^* = \sigma_i \omega^* + \lambda_i$

where $\lambda_i = \dfrac{1}{k_i}\dfrac{\partial k_i}{\partial t} dt$ is the relative rate of change in the capital/labor ratio in the ith sector if there were only a change in the shift parameter. The partial derivative notation in obtaining λ_i means that the factor prices

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[2]Equation (14) is obtained by first differentiating equations (1) and (2) and then utilizing equation (10).

are kept constant. In other words, λ_i is the Hicks-measure of technical progress.[3] Its sign indicates the nature of Hicks-technical progress.

If $\lambda_i = 0$, technical progress is neutral; if $\lambda_i > 0$, it is capital-using; if $\lambda_i < 0$, it is capital-saving. Differentiating equations (12) and (13) and substituting equation (10) we obtain the following two equations:

(17) $\quad Y_i^* = \beta_i(u_i^* + L_i^*) + (1 - \beta_i)(K_i^* + v_i^*)$

(18) $\quad P_i^* + Y_i^* = \varepsilon_i(w^* + L_i^*) + (1 - \beta_i)(K_i^* + r^*)$

By combining equations (14), (15) and (17), we obtain:

(19) $\quad \omega^* = \dfrac{\alpha_i - v_i^*}{\beta_i}$

From equations (14), (15) and (18), we have:

(20) $\quad P_i^* = \varkappa_i\omega^* - \alpha_i + r^*$

We are now in a position to obtain a relationship between the Hicks-measure of technical progress and the Harrod-measure of technical progress. According to Harrod technical progress is neutral, capital-using or capital-saving if, at the same rate of profit (v_i), the value of the capital/output ratio remains constant, increases or decreases. Let x_i be the value of the capital/output ratio in the ith sector. Then

(21) $\quad x_i = \dfrac{P_1 K_i}{P_i Y_i}$, or

(22) $\quad x_i^* = (P_i^* - P_i^*) + (K_i^* - Y_i^*)$

If the rate of profit is kept constant, then $v_i^* = 0$, so that from equation (19),

(23) $\quad \omega^* = \dfrac{\alpha_i}{\beta_i}$

From equation (16),

(24)[4] $\quad K_i^* = \sigma_i\omega^* + L_i^* + \lambda_i$

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[3] Hicks originally defined the nature of technical progress in terms of the change in the marginal rate of substitution at the old capital/labor ratio. It is well known that this definition is equivalent to that given in the text. This is also the classification used by Takayama [10].

[4] This is because $k_i^* = (K_i/L_i)^* = K_i^* - L_i^*$.

Then combining equation (14) with equation (24) we have:

(25) $K_i^* - Y_i^* = \beta_i(\sigma_i \omega^* + \lambda_i) - \alpha_i$

From equation (20) we obtain:

(26) $P_1^* - P_i^* = (\beta_1 - \beta_i)\omega^* - (\alpha_1 - \alpha_i).$

Finally by substituting equations (23), (25) and (26) in equation (22) we obtain:

(27)[5] $x_i^* = \beta_i \left[\dfrac{\alpha_1}{\beta_1} (\sigma_i - 1) + \lambda_i \right] = \beta_i H_i,$

(27a) where $H_i = \left[\dfrac{\alpha_1}{\beta_1}(\sigma_i - 1) + \lambda_i \right]$

From equation (27a), it is clear that technical progress in the Harrod-sense is neutral, capital-using or capital-saving, according as $H_i = 0$,

$H_i > 0$ or $H_i < 0$. Note that if there is no technical progress in the capital goods sector, α_1/β_1 equals zero, so that $H_2 \gtrless 0$ depends on $\lambda_2 \gtrless 0$. In other words, in the absence of any improvement in Y_1, the Hicks and the Harrod classifications of technical improvement in Y_2 are equivalent. Again it may be noted that if $\alpha_1/\beta_1 > 0$, then $\lambda_1 = 0$ implies $H_1 = 0$ only if $\sigma_1 = 1$. In other words, the Hicks and the Harrod classifications of neutral technical progress in the capital-goods sector are equivalent only if the elasticity of substitution in the capital-goods sector equals unity.

With a number of results at our disposal, we can now proceed towards the analysis of relative stability in the presence of technical improvements.

II. Existence and Stability of the Short Run Equilibrium

In the short run capital stock, labor supply, and the level of technology are given. From equations (8) and (5), we obtain:

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[5]This formula can be found in the apendices of Amano [2] and Takayama [10].

(28) $\quad \rho_1 = \dfrac{sk}{\omega + k_1}$

Then substituting equations (6) and (7) into equation (28) yields

(29) $\quad k = \dfrac{(\omega + k_1)k_2}{\omega + (1 - s)k_1 + sk_2} = \dfrac{a}{b}$

Differentiating equation (29) and substituting equation (16), we obtain:

(30) $\quad k* = \dfrac{k_2}{ab} \; [(g + c\sigma_1 + c\sigma_2)\omega* + (c\lambda_1 + e\lambda_2)]$

where

$\quad g = s(k_2 - k_1)\omega$

$\quad c = s(\omega + k_2)k_1$

$\quad e = (\omega + k_1)\,[\omega + (1 - s)k_1]$

Let $g + c\sigma_1 + e\sigma_2 = D$ and $c\lambda_1 + e\lambda_2 = B$, and $\dfrac{k_2}{ab} = \dfrac{1}{R} > 0$. Then equation (30)

can be written as:

(31) $\quad Rk* = \omega*D + B$

As noted before, B is zero in the short run, and since $R > 0$, the

relationship between k* and $\omega*$ depends on the sign of D. It is evident

that k* and $\omega*$ will have the same sign only if $D > 0$. Similarly, since

$c\sigma_1$ and $e\sigma_2$, two of the three constituents of D, are positive, the relation-

ship between k* and $\omega*$ will be unique only if

(i) $\quad g > 0$, or

(ii) $\quad c\sigma_1 + g > 0$, or

(iii) $\quad e\sigma_2 + g > 0$.

In case (i) $g > 0$ means that $k_2 > k_1$. In other words, if the consumption-

goods sector is at least as capital-intensive as the capital-goods sector, k*

and $\omega*$ have the same sign. This condition in the literature on two-sector

growth models is known as the capital-intensity condition, and we may conclude

that if this condition is satisfied, ω is uniquely determined from k. Once

ω is determined, the rest of the variables in the model can be easily shown

to have positive and determinate values. Thus the capital-intensity condition is sufficient for the existence of the short-run competitive equilibrium.

In case (ii), $c\sigma_1 + g \geq 0$ means that

$$(32) \quad \sigma_1 \geq [\frac{\omega}{\omega + k_2}] [\frac{k_1 - k_2}{k_1}]$$

From equation (10), we know that $\frac{\omega}{\omega + k_2} = \beta_2$.
Therefore inequality (32) becomes

$$(33) \quad \sigma_1 \geq \beta_2 N < 1$$

where $N = \frac{(k_1 - k_2)}{k_1} < 1$.

From inequality (33), it is clear that if σ_1 is at least equal to β_2, the relative share of labor in the consumption-goods sector, inequality (ii) will be satisfied and k* and ω* will have the same sign.

In case (iii), $e\sigma_2 + g \geq 0$ means that

$$(34) \quad \sigma_2 \geq \beta_1 [\frac{(k_1 - k_2)s}{1 + (1 - s)k_1}]$$

It is clear that, for certain values of s, the condition that the elasticity of substitution in the consumption-goods sector be greater than or equal to the relative share of labor in the capital-goods sector satisfies inequality (34).[6] If inequality (34) is satisfied, D is greater than zero and k* and ω* have the same sign. Once we obtain sufficient conditions for D > 0, ω is uniquely determined from the given level of k in the short-run, and once ω is uniquely determined, the rest of the variables, as stated before, are

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[6]It can be easily seen that inequality (34) will be satisfied when

$$\sigma_2 \geq \beta_1 \text{ if } s < \frac{1 + k_1}{2k_1 - k_2}.$$

In [4] we have shown that this restriction is unnecessary if workers also save a part of their income. In any case, it can be shown that $\sigma_2 \geq 1$ is a sufficient condition for inequality (34) to be satisfied.

also uniquely determined.[7]

III. Existence and Stability of the Long-Run Equilibrium

Given that the competitive equilibrium is ~~stable~~ unique in each time period, the stability of the long-run equilibrium is determined by the behavior of the rate of growth of capital over time. Specifically, a growth model is said to be relatively stable if, when capital-stock is growing faster than labor, the system operates in such a way as to lower the rate of growth of capital, and vice-versa. If, in the limit, the rate of growth of capital, asymptotically approaches the exogenously determined growth rate of labor, the growth path so attained is called the balanced growth path. In this path capital and labor grow at the same constant rate and the capital/labor ratio remains constant over time. If the rate of growth of capital becomes constant before actually equalling the growth rate of labor, the growth path so attained has been called the equilibrium growth path. In this path labor and capital grow at constant but unequal rates and the capital/labor ratio may rise or decline over time. The process of capital accumulation in this model is described by equation (8), so that

$$P_1 Y_1 = P_1 dK = srK, \text{ or}$$

$$(35) \quad \frac{dK}{K} = G = sv_1$$

Here we assume that capital is everlasting so that there is no depreciation factor to worry about.[8] Let n be the constant rate of growth of labor. Then the growth process is described by:

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[7]See Uzawa [8].

[8]Depreciation factor is ignored only for the sake of simplicity. In fact, depreciation by "suddendeath" can be easily introduced in our model.

(36) $k^* = G - L^* = sv_1 - n.$

In the balanced growth path $G = sv_1 = n$ or $k^* = 0$, but in the equilibrium

growth path, $G = n + \mu$ and $k^* \neq 0$, where μ is a constant not equal to zero.

In this way the balanced growth path becomes a special case of the

equilibrium growth path when $\mu = 0$. Differentiating equation (35), we

have:

(37) $G^* = v_1^*$

Substituting ω^* from equation (19) into equation (31) yields:

(38) $v_1^* = \alpha_1 - \frac{\beta_4}{D} [Rk^* - B]$

whence

(39) $G^* = \alpha_1 - \frac{\beta_4}{D} [Rk^* - B]$

where

$$R = \frac{ab}{k_2} > 0, \quad D = g + c\sigma_1 + c\sigma_2, \quad \text{and}$$

$$B = c\lambda_1 + e\lambda_2.$$

As suggested before, relative stability requires that if capital stock is

growing faster than labor, the adjustment mechanism should be such as to

lower the rate of growth of capital, and conversely. In other words, if

$k^* \gtrless 0$, $G^* \lessgtr 0$. From equation (39), it is clear that, if technical progress

is absent, $G^* \lessgtr 0$ when $k^* \gtrless 0$ only if $D > 0$. For then $\alpha_1 = B = 0$. In the

previous section, we have obtained the sufficient conditions under which

$D > 0$. Thus we may state our proposition I in this manner.

Proposition I: If technical progress is absent, then the sufficient

conditions for relative stability in our model are: (1) the capital-

intensity condition holds, (2) $\sigma_1 \geqslant \beta_2$, or (3) $\sigma_2 \geqslant \beta_1$.[9]

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[9]Given, of course, that the restriction shown in note 7 holds. In what
follows, we will assume that this restriction holds. Otherwise the reader
may prefer a more restrictive condition of $\sigma_2 \geqslant 1$. For the sake of brevity
these three separate conditions will be represented by the condition $D > 0$.

A. Hicks-Technical Progress in the Consumption-Goods Sector:

Let us now examine the conditions for relative stability in the presence of technical progress. First, consider the case in which Hicks-technical progress occurs in the conseumption-goods sector only. Here $\alpha_1 = \lambda_1 = 0$. If $\lambda_2 = 0$, that is, if technical progress in the consumption-goods sector is Hicks-neutral, one can see from equation (39) that the conditions for relative stability remain unchanged.

Proposition II: If Hicks-neutral technical progress occurs in the consumption-goods sector, the necessary and sufficient condition for relative stability is that $D > 0$. The growth process starting from any arbitrary capital/labor ratio approaches the balanced-growth path as is evident from the fact that $G* = 0$ only when $k* = 0$.

If, however, $\lambda_2 \neq 0$, that is, if Hicks-technical progress in Y_2 is non-neutral, the analysis becomes more complicated. Let equaltion (39) be written as:

$$(40)^{10} \quad G* = \alpha_1 - \frac{\beta_1}{D} [(g + c + e)k* - (c\lambda_1 + e\lambda_2)]$$

From equation (40) it is clear that when $\alpha_1 = \lambda_1 = 0$, but $\lambda_2 \neq 0$, then $G* = 0$ only when

$$(41) \quad k* = E\lambda_2, \text{ where } E = \frac{e}{g + c + e} < 1.$$

Equation (41) defines an equilibrium-growth path, because when $k* = E\lambda_2$, the

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[10]By simple manipulation it can be shown that $g + c + e = R$. Because

$$g + c + e = \omega[sk_2 + (1 - s)k_1 + \omega] + k_1[sk_2 + (1 - s)k_1 + \omega]$$

$$= [\omega + k_1] [sk_2 + (1 - s)k_1 + \omega] = ab/k_2 = R.$$

Substituting $(g + c + e)$ for R in equation (39) yields equation (40).

rate of growth of capital becomes constant. In this path, capital and labor

grow at constant but unequal rates, and the capital/labor ratio declines

or rises over time depending on whether $\lambda_2 < 0$ or $\lambda_2 > 0$. Moreover, since

$E < 1$, a necessary condition[11] for the existence of the equilibrium-growth

path is that $k^* < |\lambda_2|$.

Let this equilibrium-growth path exist. Our next task is to see

whether or not the growth process starting at any arbitrary capital/labor

ratio actually converges to the path defined by equation (41). For this

purpose, it is necessary to observe the behavior of G when k^* diverges from

its equilibrium-growth path value. If at any point of time, k^* lies above

its equilibrium-growth-path value, the system must operate in a way that

G declines, and vice-versa if the model is to be relatively stable. Let

(42) $k^* = E\lambda_2 + Z$

Where $Z \neq 0$. If at the start of the growth process k^* lies above its value

defined by the equilibrium-growth path, Z is positive; in the opposite

case it is negative. Substituting equation (42) in equation (40), we

have, remembering that $\alpha_1 = \lambda_1 = 0$:

(43) $G^* = -\dfrac{\beta_1}{D} ZR$

where $R = g + c + e > 0$. It may be observed from equation (43) that the

necessary and sufficient condition for $G^* \underset{>}{<} 0$ when $Z \overset{>}{<} 0$ is that $D > 0$.

Proposition III: If Hicks-technical progress in the consumption-goods

sector is non-neutral, the necessary and sufficient condition for the

growth process to converge to the equilibrium-growth path is that $D > 0$.

In this path, capital and labor grow at constant but unequal rates. If

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[11] $E = \dfrac{e}{g + c + e} = \dfrac{1}{1 + \dfrac{sk_2}{\omega + (1 - s)k_1}} < 1.$

Hicks-technical progress in the consumption-goods sector is capital-using, i.e., if $\lambda_2 > 0$, the capital stock grows faster than labor by a constant rate; if it is capital-saving i.e., if $\lambda_2 < 0$, the capital stock grows slower than labor by a constant rate.

This proposition gains further clarification from Fig. 1 which is actually an adaptation from Fig. 5 in [3]. The rates of growth of capital and labor are measured along the vertical axis and time along the horizontal axis. The exogenously determined rate of growth of labor is given by on. If at the start of the growth process at time t_0, G > n, than G declines over time, and conversely, till at time t_5 it comes to equal n provided, of course that D > 0, and technical progress is either absent or is Hicks-neutral in Y_2. The time-path followed by G is either G_1An^* or G_0An^*. The balanced-growth path is given by nn^*, whereas the equilibrium growth path is given by cc* if $\lambda_2 > 0$ or by FF* if $\lambda_2 < 0$. If at the start of the growth process G = G_1 > 0c, then given that D > 0, G declines over time and comes to approach the equilibrium growth path either at t_4 or t_6 and thereafter continues to follow the equilibrium-growth path. The time-path followed by G, if $\lambda_2 > 0$, is G_1Bc^*; if $\lambda_2 < 0$, it is G_1HF^*. In the former case, capital-stock grows faster than labor by a constant rate BJ and the capital/labor ratio also grows at this rate; in the latter case, capital stock grows slower than labor by NH and the capital/labor ratio also declines at this rate. If, on the other hand, G at the start of the growth process, lies below any of the two equilibrium-growth paths, the time-path to be followed by it can be determined analogously.

B. Hicks-Technical Progress in the Capital-Goods Sector:

If technical progress occurs in the capital-goods sector or both sectors then $\alpha_1 > 0$, so that from equation (39), G* = 0 only if
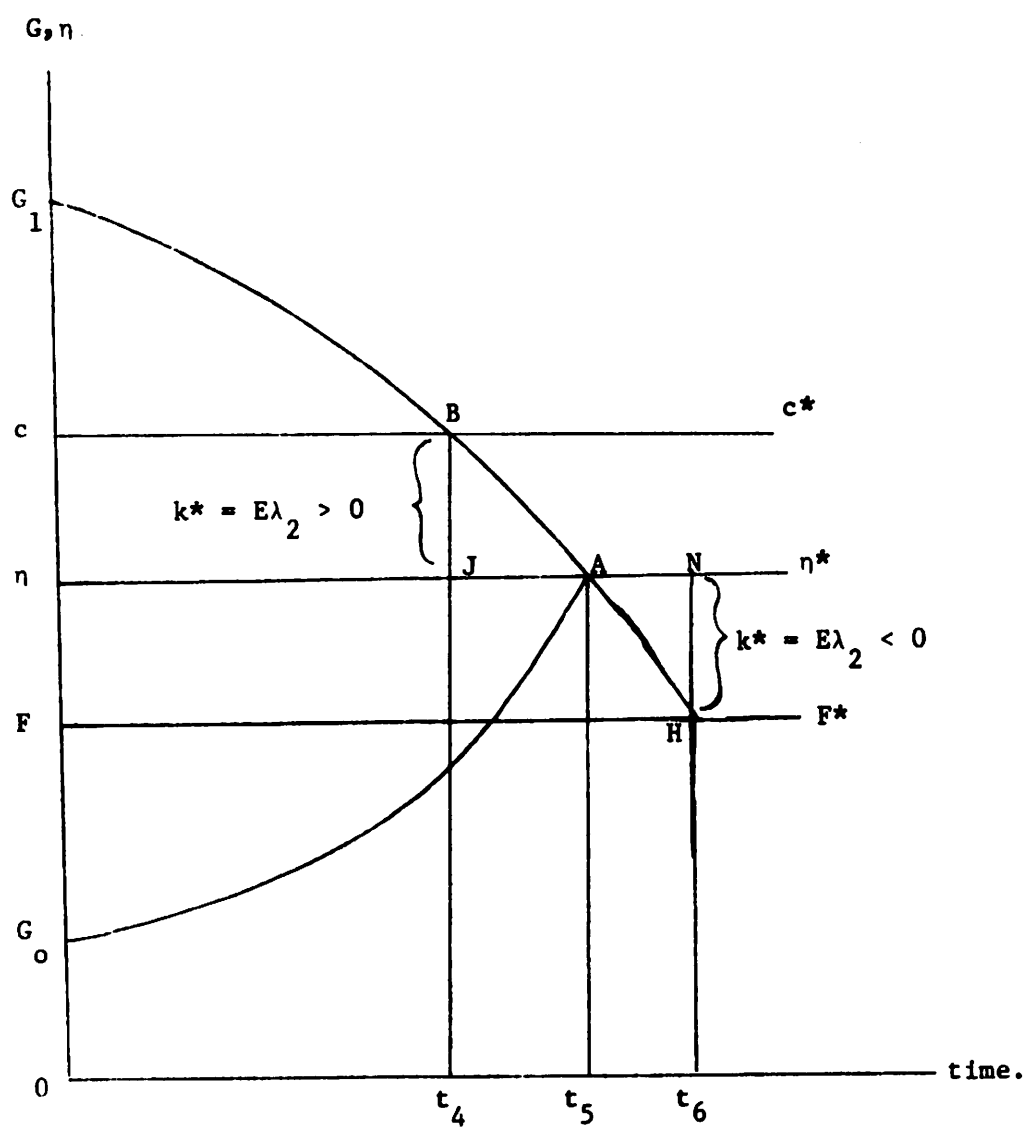
Figure 1

(44) $k* = \frac{1}{R} [\alpha_1 \frac{D}{\beta_1} + B]$

where $R > 0$ and $B = c\lambda_1 + e\lambda_2$.

In other words, even if Hicks-technical progress occurs in the capital-goods sector or both sectors, there exists an equilibrium-growth path defined by equation (44), where capital and labor grow at constant but unequal rates and the capital/labor ratio increases or declines over time at a constant rate, depending on the sign of the expression in square bracketts in equation (44).[12] Let k* at any point of time be given by:

(45) $k* = \frac{1}{R} [\alpha_1 \frac{D}{\beta_1} + B] + Z$

where $Z \neq 0$. Substituting equation (45) in equation (44) again yields equation (43), namely

(43) $G* = - \frac{\beta_1}{D} ZR$

Again it is evident from equation (43) that G* and Z move in the opposite direction only if $D > 0$. In other words, if $Z > 0$, that is, if k* lies above its value defined by equation (44), G* declines, and vice-versa. Therefore, given that $D > 0$, the growth process starting at any arbitrary capital/labor ratio converges to the equilibrium growth path defined by equation (44).

Proposition IV: If Hicks-technical progress occurs in the capital-goods sector or both sectors, then the necessary and sufficient condition for the uniqueness and stability of the equilibrium growth path defined by equation (44) is again that $D > 0$. If technical progress in the capital-goods sector or both sectors is Hicks-neutral, then, in the equilibrium

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[12]Whether or not k* is constant is not apparent from equation (44). But since k* is the difference between G and η, and since these two are constant, k* must also be constant in the equilibrium-growth path.

path, capital stock grows faster than labor by a constant rate.[13]

Indeed, a sufficient condition for the capital stock to grow faster than labor in the equilibrium-growth path is that $B \geq 0$. If, however, $B < 0$, there arises a possibility that k^* may be zero or negative, depending on whether

$$(46) \quad \frac{\alpha_1}{\beta_1} < \frac{-B}{D}$$

If technical progress occurs in Y_1 alone, then a necessary condition for (46) to be satisfied is that $B = c\lambda_1 < 0$.

Proposition V: Given that $\alpha_1/\beta_1 = -c\lambda_1/D$, and given also that $D > 0$, the necessary condition for the existence of the balanced-growth path where $k^* = 0$ is that $\lambda_1 < 0$, that is, technical progress in the capital-goods sector alone is Hicks-capital saving.[14]

If, however, $\alpha_1/\beta_1 < -c\lambda_1/D$ and $D > 0$, the equilibrium-growth path will lie below the balanced-growth path and capital stock will grow slower than labor by a constant rate.

C. Harrod-Technical Progress in the Consumption-Goods Sector:

It has already been shown in section I that the definition of Harrod-technical progress in the consumption-goods sector is equivalent to that of Hicks-technical progress. Thus we may simply conclude that propositions II and III concerning the effects of Hicks-neutral and non-neutral technical progress in the consumption-goods sector alone also apply to the case of

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[13]This is because, here $B = 0$ and k^* from equation (44) is positive.

[14]Even if $\lambda_1 = 0$, the balanced growth path may exist if $B = e\lambda_2 < 0$. In other words, the balanced-growth path may exist even if technical progress is Y_1 is Hicks-neutral, provided that technical progress in y_2 is Hicks-capital saving. Thus, it is interesting to note that there is at least one case where the balanced-growth path exists and is stable even if technical progress in the capital-goods sector is Hicks-neutral.

Harrod-technical progress in the same sector.

D. **Harrod-Technical Progress in the Capital-Goods Sector:**

The implications of Harrod-technical progress for relative stability can be explored simply by substituting λ_1 from equation (27a) in equation (40). Doing this substitution and some readjustment yield:

(47) $\quad G^* = -\frac{\beta_1}{D} \left[(k^* - \frac{\alpha_1}{\beta_1}) R - (cH_1 + eH_2) \right]$

where $R = g + c + e > 0$, $D = g + c\sigma_1 + e\sigma_2$ and H_1 is the Harrod measure of technical progress in the ith sector.

It may be observed from equation (47) that $G^* = 0$ only if

(48) $\quad k^* = \frac{\alpha_1}{\beta_1} + \frac{cH_1 + eH_2}{R}$

In other words, there again exists an equilibrium growth path defined by equation (48) where capital and labor grow at constant but unequal rates. Consider first the case where technical progress in Y_1 alone is Harrod-neutral. Here $H_1 = H_2 = 0$, but $\alpha_1 > 0$, so that $k^* = \alpha_1/\beta_1$. Let $k^* = \alpha_1/\beta_1 + Z$. Substituting this in equation (47) and remembering that $H_1 = H_2 = 0$, we obtain

(43) $\quad G^* = -\frac{\beta_1}{D} ZR$

Again we see from equation (43) that the necessary and sufficient condition for G^* and Z to move in the opposite direction is that $D > 0$.

Proposition VI: If Harrod-neutrality occurs in the capital-goods sector alone, then the necessary and sufficient condition for the growth process starting at any capital/labor ratio to converge to the equilibrium-growth path defined by equation (48) is that $D > 0$. In this path capital stock grows faster than labor by a constant rate equal to α_1/β_1, and the rate

of profit remains constant.[15]

It is evident from equation (48) that proposition VI remains valid even if Harrod-technical progress in Y_1 is non-neutral, or if it is neutral in both sectors or has any other combination in the two sectors so long as $cH_1 + eH_2 = 0$.

Again there arises a possibility that $k* = 0$ if $(cH_1 + eH_2) < 0$. Thus we may derive:

Proposition VII: There exists a case where the growth process converges to the balanced-growth path when $D > 0$, provided that $\alpha_1/\beta_1 = -(cH_1 + eH_2)/R$. In this path, capital and labor grow at a constant and equal rate inspite of Harrod-technical progress in any one or both sectors.

Again if $\alpha_1/\beta_1 < -(cH_1 + eH_2)/R$, the equilibrium growth path, when $D > 0$, lies below the balanced-growth path.

IV. Concluding Remarks

It is intersting to see the similarity between the implications for relative stability of Hicks and Harrod technical progress. To begin with, the two-sector growth model is relatively stable without any additional restriction only if Hicks or Harrod neutrality occurs in the consumption-goods sector alone. If Hicks or Harrod neutrality occurs in the capital-goods sector or in both sectors, the balanced-growth path does not exist in either case. The equilibrium growth path exists in both cases and the conditions for the relative stability of the model are also the same. Indeed, if the elasticity of factor substitution in each sector equals unity, the equilibrium growth paths with Hicks and Harrod neutrality occurring in Y_1

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[15]Amano [2] and Takayama [10] have interpreted α_1/β_1 as the rate of growth of output per man in the capital-goods sector when the capital/output ratio is kept constant.

alone, or in both sectors coincide.[16] It is well-known that the implications

of Hicks and Harrod-neutrality for economic stability in a one sector model

are the same if the elasticity of substitution equals unity [See, Uzawa 9].

Our result that the implications of Hicks and Harrod-neutrality in the

capital-goods sector alone for relative stability are the same only if

$\sigma_1 = \sigma_2 = 1$ may then be considered as the two-sector model counterpart of

the well-known result in the one sector model.

Finally we mention a result derived by Burmeister [5]. He concludes that

the economy approaches the equilibrium-growth path in the presence of

Harrod-technical progress ~~only~~ if it is neutral and occurs at the same

rate in both sectors, provided, of course, that the model without technical

progress is stable. We have, however, shown in the preceeding section

that the growth process converges to the equilibrium-growth, appropriately

defined, even if (i) Harrod-neutrality occurs only in the capital-goods

sectors, (ii) it occurs in both sectors at unequal rates, and (iii)

Harrod-technical progress in one or both sectors is non-neutral.

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[16]From equation (44)

$k* = [\frac{\alpha_1}{\beta_1} D + B] \frac{1}{R}$ where $D = g + c\sigma_1 + e\sigma_2$ and $R = g + c + e$. It is

evident that if $B = 0$ when $\lambda_1 = \lambda_2 = 0$, $k* = \frac{\alpha_1}{\beta_1}$ if $\sigma_1 = \sigma_2 = 1$. It may

be seen that this is also the value of $k*$ in equation (48) if $H_1 = H_2 = 0$.

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