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WAGE AND PRICE FORMATION
IN SELECTED ECONOMETRIC MODELS

by
Ronald G. Bodkin

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WAGE AND PRICE FORMATION IN ECONOMETRIC MODELS*

A. Introduction

For a topic as broad as this one, some delimitation at the outset is needed. I have accordingly decided to limit myself to econometric models that have appeared in the last ten years, the decade of the 1960's. In addition, I am only going to discuss econometric models of Canada and its three principal trading partners--the United States, Japan, and the United Kingdom. A word in defense of both delimitations may be proffered.

The limitation as to geographical location is, I think, reasonably defensible. This is a conference on inflation in Canada; consequently, one would surely wish to look at Canadian history as modeled in the stylized fashion of econometric models of this economy. In addition, since an open economy like the Canadian can be strongly influenced by developments in its larger trading partners, it would seem of great interest to focus on models of these economies as well. This is not to imply that there are not price formation aspects of econometric models of countries other than Canada, the

* I should like to thank T. M. Brown, T. C. Liu, J. A. Sawyer, H. T. Shapiro, H. Tsurumi, and J. W. L. Winder for helpful comments on an earlier draft of this paper. Professors Sawyer and Shapiro were especially helpful in their role as official discussants of this paper at the conference. Professor Tsurumi very graciously ran several simulations of his model at my request; I am especially indebted to him for this. Professors Neil Swan and David Wilton, the organizers of this conference, first suggested that this paper be written; in addition, I am indebted to them for an enjoyable time at the conference. I should also like to thank my two long-suffering research assistants, Ingrid Shouldice and Nilu Vyas, for their assistance in doing the computations and drawing the charts. Finally, I should like to thank the Canada Council for financing in part this research through a grant. The usual caveat that the residual responsibility for errors of fact, logic, or interpretation rests with the author of the study is of course true for this paper also.

United States, Japan, and the United Kingdom from which we can draw important lessons.¹ However, limitations of space and energy preclude an exhaustive search.

The limitation as to time of appearance seems a bit more arbitrary; 1960 is an arbitrary point in time, with no second coming in that year to herald a transformation of an imperfect world into a new heaven and a new earth. As pointed out above, at times one must set arbitrary boundaries, due to the limitations of energy and other resources of the researcher. In the present case, the time limitation is in one sense less arbitrary, as I have reviewed or have participated in reviewing some of this literature elsewhere. ([4], Chapter 1; [5], Chapter 3.) Moreover, as a rough generalization, one can assert that the price formation aspects of earlier econometric models were relatively undeveloped. Sometimes the price level was taken as exogenous, which takes this variable out of the realm of economic theory. In some models the price level was considered an endogenous variable, but the theorizing on it was largely implicit and in any case it was difficult to see the exact mechanism by which various actors in the system set prices. Thus in Valavanis's model [31], there is a wage adjustment equation (or, as he termed it, a "wage bargain" equation) but no equation for explaining the price level explicitly. Since the general price level is in his model an endogenous variable, it must, however, get determined somewhere. Valavanis

¹Thus a model of prices and incomes, developed by Aukrust for the Norwegian economy [2], may well have some lessons for Canada. Aukrust discusses a model of a small country with some "exposed" industries which must set prices in accord with world prices and with other industries that are "sheltered" from the effects of international price competition. The model is used to simulate the effect of various settlements of highly centralized wage negotiations on prices and income shares in the economy.

himself asserted that it was "influenced" by the quantity of money which, nevertheless, had its primary impact in his liquidity preference equation. It is just this sort of vague theorizing that appears less than perfectly satisfactory by current standards.²

In any case, I have been unable to avoid making some arbitrary exclusions of models to be discussed, as will soon become clear to the reader. Thus, for example, in the U. S. section I have discussed neither the FRB-MIT model nor Gary Fromm's Data Resource Inc. model, both of which represent important contributions to econometric modelling. The only apology that I can offer for this short-coming is that, with limited resources, it is difficult to be both comprehensive and detailed.

In discussing a typical econometric model, I shall follow a definite pattern. In general, the wage equation will be discussed first, followed by a discussion of the principal price equation. Sectoral price equations, if present, will then be discussed. The various models surveyed below differ in the richness of the details of sectoral price level determination, and for some of the models sectoral details (e.g. the composition of demand pressures) may be an important part of the inflation story.

In the final portion of these descriptive surveys of the individual econometric models, I intend to examine the extent to which the model in question suggests that there may be a conflict between the goals of stable prices and full utilization of resources, particularly labour. At times this conflict will be illustrated by the calculation of a simplified trade-off

²Similarly, in two of his econometric models published in the early post-war period, Professor Klein interprets his wage adjustment equation as "completing" the system with respect to absolute wages and prices. That one equation should complete the system with respect to two variables seems a bit puzzling, unless the two variables are so similar that it is worth distinguishing them only on a formal basis. (For the references and for a brief additional discussion, see [4], pp. 26-28.) These models also illustrate the point that the theorizing about the price level in econometric models prior to the present decade tended to be less than perfectly explicit.

curve from a wage-price subsector of some of the econometric models under review. This trade-off curve calculation can generally be criticized on two grounds: first, it ignores sectoral aspects of inflation, which may well be important in particular instances; secondly, it may give somewhat different answers from those that one would obtain from a simulation of the full model. Indeed, one might well argue that a policy-maker should be interested primarily in the results of simulating the effects of different policies from an initial reference point, rather than in an abstract curve, usually based on steady state assumptions, in which the conflict appears (usually too simply) to be independent of the policy instruments utilized. Accordingly, the results from a simulation of the full model will be used wherever possible. It is my contention, however, that this admitted simplification is one of the useful abstractions of current macro-economic theory, like the consumption function or the aggregate production function, which illustrates a crucial issue in a clear manner and which at the same time does not do excessive violence to the "facts" of "real world" experience.³

Finally I should like to give a word of explanation about the general tone in which the price formation aspects of these various econometric models are discussed. In general, the discussion will be merely descriptive, with relatively few words of evaluation. However, where I judge that the model-builder (or builders) has (have) done exceptionally well or relatively poorly, this will be the subject of a comment by me. Thus no over-all evaluation of the individual econometric models will be given, with the exception of that of Andersen and Carlson [1], which is quite different from the others. An evaluation of the predominant method at present of modelling price formation

³Of course, it is possible that I am merely a prisoner of what Keynes once called the power of "the vested interest of established ideas." These issues will be clearer in the twenty-first century, if there are any individuals around at that time who care about these questions.

will be presented in the concluding section, Section F below.

B. Canada

1. University of Toronto Model: TRACE, 1969 [7]

The wage-price sector of this model was fitted to data on the Canadian economy for varying periods. The wage rate equation was fitted to the period 1949-1966, while the principal price equation was fitted to the years 1928-1940 and 1947-1966. For several of the sectoral price equations, the full period was used for estimation purposes; other sectoral equations were fitted to portions of the postwar period only. The method of parameter estimation was two stage least squares.

The wage adjustment equation, equation G. 1 of the model, is a standard form of this relationship, with the relative change in the average hourly wage rate in the business non-agricultural sector depending linearly on the relative change in consumer prices and non-linearly on the employment rate (linearly on its reciprocal). The second variable can be interpreted as measuring labour market demand pressures and the first variable can be viewed as being outside the direct adjustment mechanism of the labour market; the $\frac{1}{U}$ ratios suggest that the second variable (the reciprocal of the unemployment rate) is the more important explanatory variable. With the use of the approximation $\ln(1+x) \approx x$, valid when x is close to zero, the wage adjustment relationship of Choudhry, Kotowitz, Sawyer, and Winder can be written as follows:

$$(1) \quad \dot{W} = 0.01330 + 0.13955 \frac{1}{U} + 0.30923 \dot{P}_c,$$

where W is the average wage rate in the business non-agricultural sector, U is unemployment as a percentage of the labour force, P_c is the implicit deflator of consumption in the GNP accounts, and the dot operator ($\dot{}$) is the relative change in a variable, viz $\dot{X} = \frac{X - X_{-1}}{X_{-1}}$. (Here X is an arbitrary variable and X_{-1} is the value of this variable lagged one period.)

Note that, on the margin, the money wage rate in this sector rises by only an extra 0.3 of a percentage point for each additional percentage point rise in the rate of change of consumer prices.

As noted above, the central price level in this model is the price level of output in the business non-agricultural sector. Linearizing TRACE equation (G.4), we obtain:

$$(2) \quad \dot{P}_{Y_B} = 0.00446 + 0.48898 \dot{W} - 0.66673 \dot{A} \\ + 0.31051 \dot{P}_i + 0.28214 \dot{P}_{Y_B-1}$$

where P_{Y_B} is the price level of output in the business non-agricultural sector, A is output per man-hour in this sector, P_i is the implicit deflator of the imports of goods and services in the national income accounts, and other symbols in this equation, including the dot and lag operators, have been defined previously.⁴ All the coefficients in this equation are highly significant by conventional criteria, but it is interesting to note that the rate of change of sectoral productivity is the most important single explanatory variable, as judged by the t ratio. We may observe that this is a standard cost adjustment model for the determination of a sectoral price level, with a lagged adjustment term which presumably incorporates the effects of expectations. In this equation, there is no direct role for the pressure of demand, which is consistent with the authors' view of their equation as an explanation of supply price in this sector.

There are fifteen other sectoral price levels explained in this model, including four deflators for consumption expenditures or their components and

⁴I have omitted a significant dummy variable which attempts to capture the effects of price controls and their aftermath during the Korean War period.

four deflators for investment expenditures. In addition, it should be noted that exogenous price levels in this model include the price level of exports (and its components), import prices in U. S. dollars, and the implicit deflator for gross domestic product originating in agriculture. (It is unfortunate that export prices could not be made system-determined in the model, at least for the aggregate of exports as a whole.) Eight of these supplementary price equations are identities which equate the sectoral price deflator to the quotient of a nominal (current dollar) expenditure item divided by the corresponding item in real terms (constant dollars). It should be noted that the price level of imports of goods and services in Canadian dollars is endogenous to the model, as the exogenous import price level in U. S. dollars is multiplied by an index of the exchange rate. Most of the seven behavioural equations for the sectoral price levels simply relate the rate of change of the sectoral price level in question to the rate of change of the GNE deflator (or, in two cases, of the price level of the business non-agricultural sector) and to the rate of change of import prices, the varying importance of these two variables reflecting differences in the composition of inputs by sectors. However, there are variants; the residential rental cost index is principally explained by an autoregressive scheme, and the principal explanatory variable of the rate of change of the implicit deflator of gross investment in non-residential construction is the rate of change of the implicit deflator of gross investment in residential construction. In addition, it is interesting to note that demand pressure variables enter in five out of seven of the supplementary price level equations. In three cases, the demand variable is a variant of the authors' GAP variable (the ratio of real output to potential real output in the business non-agricultural sector). Two other demand variables employed are the approximate rate of change in the share of real output in the business non-agricultural sector going to real investment in non-residential

construction (this variable is a supplementary explanatory variable for the price level of gross investment in non-residential construction), and the lagged per capita stock of housing in the residential rental cost index equation. In all five cases, the demand pressure variables have coefficients with the theoretically appropriate sign, although generally the level of statistical significance of these coefficients is marginal.

One can illustrate the conflict that arises between the goals of full employment and stable prices by some manipulations of the wage adjustment and business non-agricultural price level equations, in order to calculate a simplified trade-off equation. These manipulations ignore sectoral details and unrealistically assume adjustment to a steady rate of inflation; nevertheless, they may be of some interest.⁵ First, we may eliminate the effects of lagged adjustment in equation (2), so that the steady state equation for the rate of change of the price level of output in the business non-agricultural sector is:

$$(3) \quad \dot{P}_{Y_B} = 0.00621 + 0.68116 \dot{W} - 0.92878 \dot{A} + 0.43254 \dot{P}_i .$$

Next, assume that sectoral detail can be safely ignored so that we may assume that the rate of change of consumer prices is equal to the rate of change of the price level of output in the business non-agricultural sector, viz.:

$$(4) \quad \dot{P}_{Y_B} = \dot{P}_c .$$

Now solve the simple subsystem of equations (1), (3), and (4) for the rate of change of the domestic price level, using the wage adjustment equation (1)

⁵Results of recent simulations of the TRACE model by Choudhry, Kotowitz, Sawyer, and Winder (which are to be presented at the second World Congress of Econometric Society in Cambridge in September) suggest two other ways in which the notion of a trade-off curve is an oversimplification. First, the curve may depend upon whether monetary or fiscal policy is being used to influence the unemployment rate. Secondly, the slope of the curve may depend upon whether the economy is on a regime of fixed or flexible exchange rates. Nevertheless, the notion of a trade-off curve survives, at least as a rough approximation; indeed, the TRACE authors use this term in their own discussion of their results.

to eliminate the rate of change of money wages in the business non-agricultural sector. The result is the familiar trade-off equation, which relates the rate of change of the domestic price level to the reciprocal of the unemployment rate, the rate of growth of productivity, and the rate of change of import prices (in Canadian dollars):

$$(5) \quad \dot{P}_{Y_B} = 0.01935 + 0.12042 \frac{1}{U} - 1.17661 \dot{A} + 0.54796 \dot{P}_i .$$

Equation (5) can be specified as a two-dimensional trade-off curve if the import change and the productivity change variables are parameterized. This is done in Table 1 and in Figure 1 below, in which the rate of growth of labour productivity in the business non-agricultural sector is set equal to either 2 or 3 per cent per year and the rate of change of import prices is set equal to either 0 or 3 per cent per year. These trade-off curves are instructive on several points. Observe that stable prices can be expected only under the most favourable circumstances at an 8 per cent rate of unemployment, while high employment (in the 3 to 4 per cent range) implies an expected rate of inflation between 1 1/2 and 5 per cent per year. As expected, stable import prices and high productivity growth mitigate the conflict between these objectives. The estimated trade-off curve (or the associated table or chart) suggests that an acceleration of productivity growth by 1 percentage point results in approximately a one percentage point retardation in the rate of inflation (at a given rate of unemployment), while foreign inflation appears to be "imported" into Canada with a marginal coefficient of 0.5. (In other words, a one percentage point increase in the rate of increase of the import deflator will, in the steady state, ultimately lead to approximately a 1/2 percentage point increase in the expected rate of rise of the domestic price level.)

TABLE 1

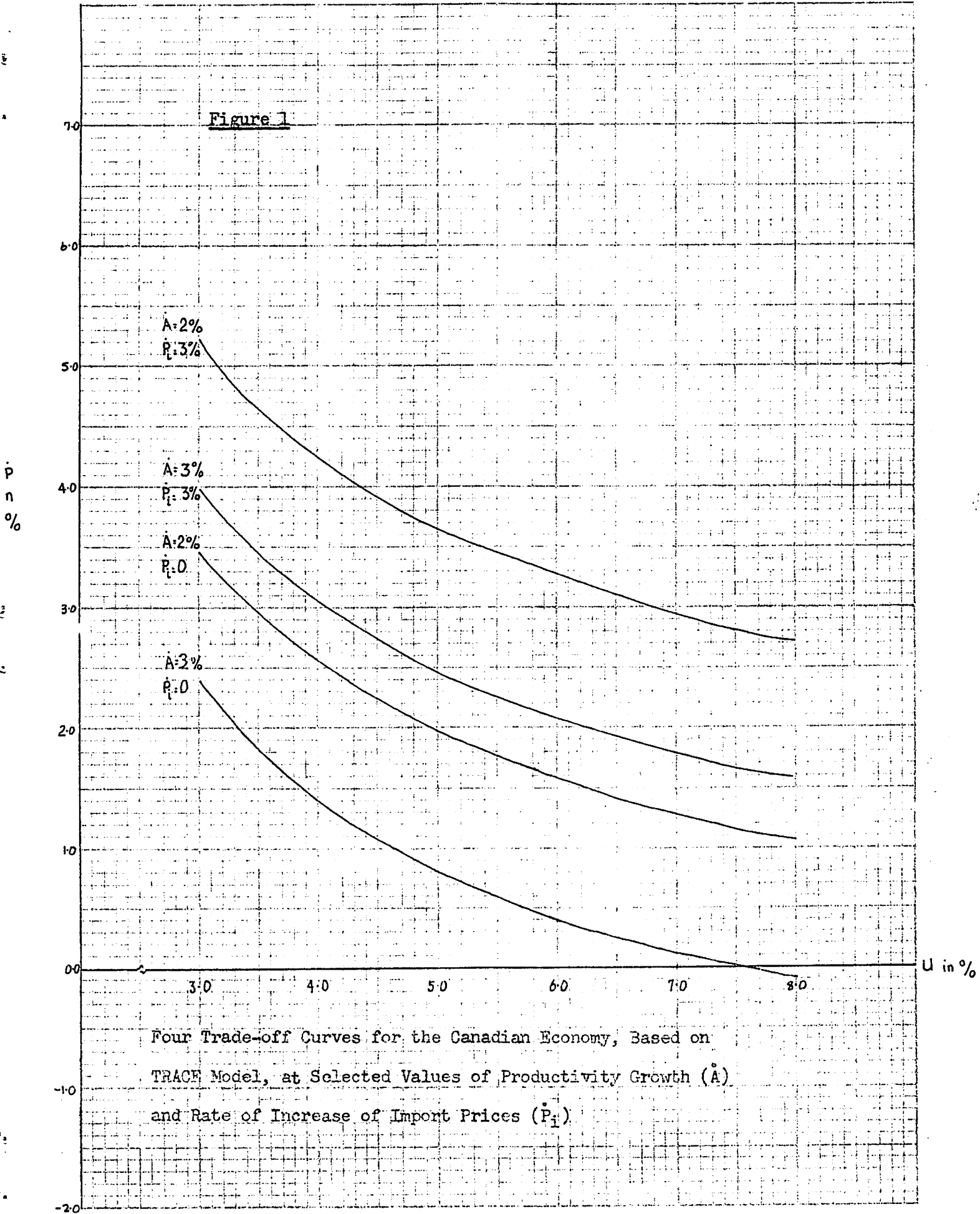
Expected Rates of Increase of Domestic Price Level (\dot{P}) at Varying Rates of Unemployment for Two Selected Values of Productivity Growth and the Rate of Change of Import Prices,

TRACE Model of Canadian Economy

Unemployment Rate (U) in Per Cent	Annual Percentage Rate of Increase of Domestic Price Level \dot{P} when:			
	Import Prices are Stable ($\dot{P}_i=0$)		Import Prices rise by 3% per year ($\dot{P}_i=3$)	
	and Productivity Growth (\dot{A})=2.0% per year	and Productivity Growth (\dot{A})=3.0% per year	and Productivity Growth (\dot{A})=2.0% per year	and Productivity Growth (\dot{A})=3.0% per year
3	3.60	2.42	5.24	4.06
3.5	3.02	1.85	4.67	3.49
4	2.59	1.42	4.24	3.06
4.5	2.26	1.08	3.90	2.73
5	1.99	0.81	3.63	2.46
5.5	1.77	0.59	3.42	2.24
6	1.59	0.41	3.23	2.06
7	1.30	0.12	2.95	1.77
8	1.09	-0.09	2.73	1.55

Source: See text, especially equation (5).

Figure 1



2. The Bank of Canada's RDX1

References: [17] , [18]

The Bank of Canada's RDX1 is a medium-sized quarterly model of the Canadian economy, with 50 behavioural equations and an additional 51 technical relationships and identities. The model was fitted (by both ordinary least squares and by a variant of two-stage least squares) to data from the first quarter of 1952 to the fourth quarter of 1965, although shorter runs of data are used in some of the individual equations. The behavioural equations are fitted to seasonally unadjusted data, with the seasonal adjustment procedure performed in the regression itself by means of dummy variables. The model stresses the high degree of openness of the Canadian economy, both on the real and the financial sides, to foreign influences, particularly those emanating from the United States. As one might expect from a model produced in a research department of a central bank, there is a reasonably well developed financial sector (at least in comparison with other macro-economic models in the Neo-Keynesian tradition).

The wage-price sector of this model (as interpreted by me) consists of one behavioural wage equation and six equations for various price levels. In addition, there are four technical relationships or identities, in which private wages per employee, unit labour costs, the inventory-sales ratio, and the trend value of this ratio are defined (or estimated). The only exogenous price-wage variables are the average quarterly wage in the government sector, the implicit price index of goods imports, the implicit price index of service imports, and the price of world exports (in Canadian dollars); this seems a very reasonable specification. Finally, we may note that the parameter estimates of the behavioural equations of the wage-price sector based on ordinary least squares are very close to the corresponding parameter estimates from the two-

stage least squares technique; accordingly, the discussion below will not distinguish among these two sets of parameter estimates.

The wage adjustment equation explains the percentage change (over corresponding quarters of adjacent years) of the average hourly rate of compensation in the private sector. The explanatory variables in this relationship are the rate of change of the implicit price index for the consumption of non-durables, the reciprocal of the square of the unemployment rate, after-tax corporate profits per unit of output, and (with a negative influence) the rate of change of hourly wages (the dependent variable) four quarters previously. All of the explanatory variables except the final one are averages of current values and the values during the preceding three quarters. This is a standard wage adjustment relationship in which demand pressures play a role through the unemployment rate variable, which according to the t statistic is the most important single explanatory variable in this relationship, and in which non-market forces influence wage changes through the consumer price change variable (which shows virtually a unitary influence, in contrast to the TRACE model), the level of profits per unit of output, and the influence of the wage rate change four quarters before the present period. (The statistical significance of this final variable is quite marginal.)

The principal level of this model is the implicit price deflator for private GNE (less farm inventories); in addition, the implicit deflator for consumer non-durables is explained by a very similar relationship. These dependent variables are explained in the level form; the explanatory variables are seasonal dummies (which show significant seasonal effects), the weekly wage rate, a distributed lag in unit labour costs (roughly, wage costs per unit of output) ranging from the current quarter to four quarters previously, and a distributed lag in the implicit deflator for goods imports ranging

from two quarters previously to eight quarters previously. In addition, in the case of the implicit deflator of private GNE (but not for the consumption non-durables deflator), the pressures of demand directly in the formation of the prices of final goods and services are captured by a distributed lag in a four quarter average of the deviations from trend of the inventory-sales ratio. In all cases, the coefficients of the distributed lag variable decline smoothly from the most recent value included to the most remote value. The current weekly wage variable has an insignificant coefficient in the private GNE deflator equation and a marginally significant coefficient in the consumption non-durables deflator equation, suggesting that the effects of this variable are largely captured by the unit labour cost variable. Price formation appears to be heavily cost-oriented, if one judges by the high t ratios for the unit labour cost variables in both of these relationships and the import price variables in the consumption non-durables deflator equation. But demand pressures appear to play some role in the formation of the private GNE deflator; the negative signs on the inventory-sales ratio variables suggest that an excess of inventories (in relation to sales and after the trend in this ratio is taken into account) depresses prices, which is the expected direction of effect.

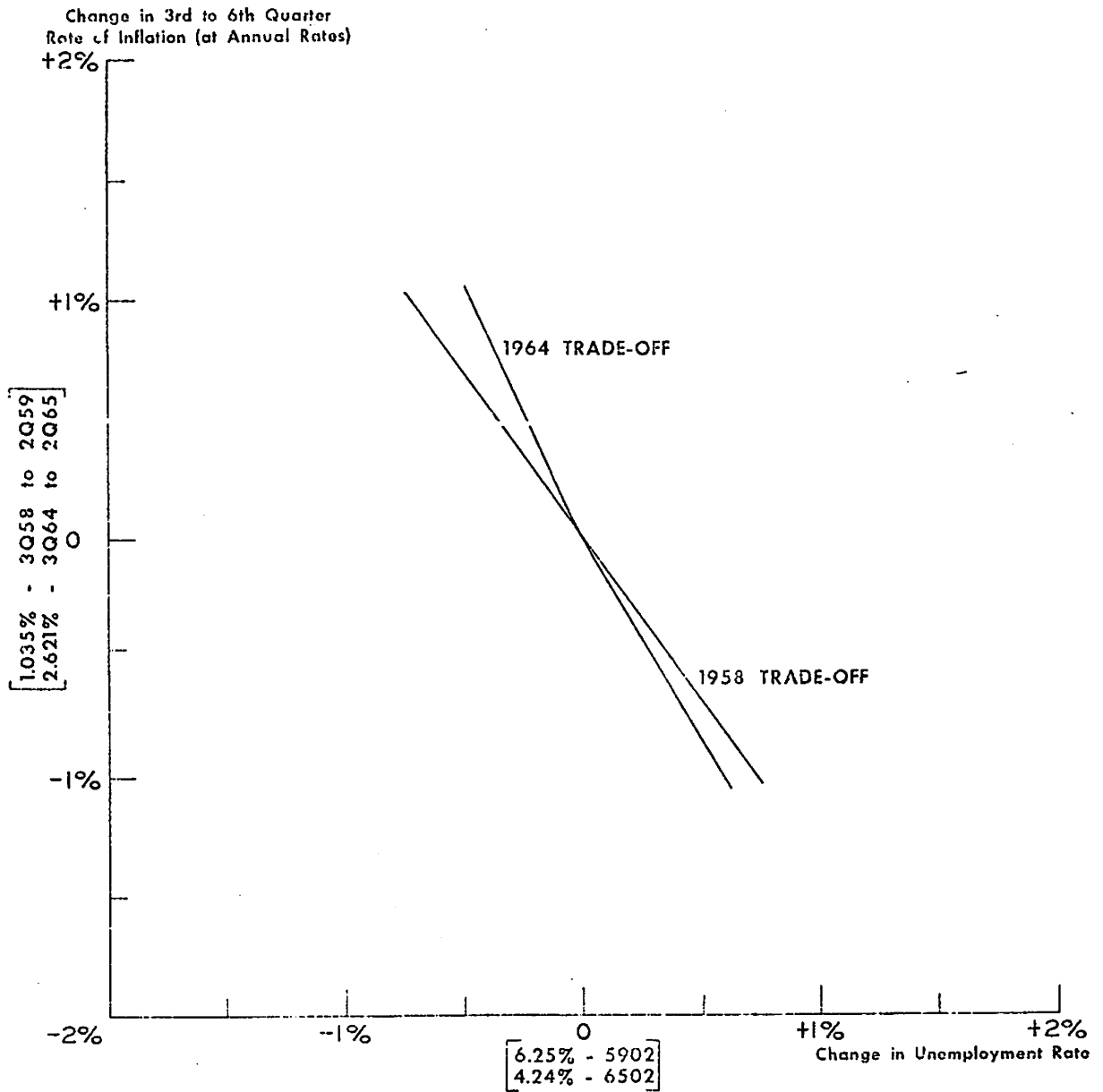
The four remaining price equations may be summarized very briefly. The implicit deflator for consumer durables is explained as an absolute first difference over adjacent quarters; this implicit deflator is tied directly to the implicit deflator for private GNE, aside from seasonality effects. The implicit deflator for goods exports depends partially on home prices and slightly more strongly (as judged either by the marginal coefficients [0.4 in comparison to 0.3] or the t ratios) on the price index of world exports in Canadian dollars. The implicit deflator for the export of services depends

(aside from a third quarter seasonal low) primarily on the implicit deflator for private GNE and secondarily on the implicit deflator for goods exports, which Helliwell, Officer, Shapiro, and Stewart say is an artifact. Finally, the price of houses (which is utilized as part of the explanation of the residential construction subsector) is explained, apart from seasonality effects, principally by the implicit deflator of private GNE for the preceding quarter; however, this price level also depends positively on per family real "permanent income" (a distributed lag of real disposable income over the present and past seven quarters, with geometrically declining weights) and negatively on the per family level of the stock of houses. This equation could be regarded as a partially reduced form equation describing price formation in the housing market; the permanent income variable would reflect demand influences, while the per family housing stock variable would reflect supply influences, and both of these variables do exert the theoretically expected direction of effect.

In this framework, it is easy to see how a trade-off between high employment and stable prices can arise, although the lag structure is such that it will probably take some time for all the effects to work through the system. High demand will reduce the unemployment rate and accelerate the rate of rise of per-hour wages; if corporate profits per unit of output improve, wage inflation will be given an additional fillip. Rising wages will eventually raise prices through their effects on unit labour costs; in addition, high demand may deplete inventories (in relation to sales and after the trend in this ratio is taken into account), thus providing an independent impetus for a price rise, in the case of the implicit deflator of private GNE. While one could derive a steady-state trade-off curve, under simplifying assumptions, there is no need to do so. In the later study [18], Helliwell, Officer, Shapiro, and Stewart have derived a sort of trade-off curve, which is based on the initial conditions at a given point in time. This

Figure 2

TRADE-OFFS BETWEEN INFLATION AND UNEMPLOYMENT
USING POLICY MIXES THAT KEEP
FOREIGN EXCHANGE RESERVES CONSTANT



Source: [18], p. 28.

diagram, which is reproduced with the permission of the authors, shows deviations in simulation values from control values for the two policy objectives, given the development of exogenous variables in the model (except for the policy instruments) and the history of the endogenous variables. As the title of the diagram notes prominently, the curves are constructed on the assumption that international reserves are to be held constant. The authors of RDX1 also point out that the 1964 trade-off curve is steeper than the 1958 curve, suggesting that the inflation cost of an additional reduction of unemployment is greater when the starting level for the rate of unemployment is lower. Moreover, the 1958 curve is virtually a straight line with the range drawn while the 1964 curve is slightly convex to the origin, "illustrating the increasing non-linearity of the unemployment-to-price-level trade-off at higher levels of capacity utilization."

3. Government of Canada Econometric Models

References: [6], [23], and [26]

This sequence of annual econometric models of the Canadian economy started in the Department of Reconstruction and Supply in 1947 with the advice and guidance of Professor Lawrence R. Klein. With the disappearance of this department, the model was transferred to the Economics Branch of the Department of Trade and Commerce, where it remained until 1964. In recent years, the model has been housed in the Department of Finance. The three references above are the main sources of information on these models readily available to an outsider; published discussion of the econometric models utilized by the Government of Canada has been fairly scanty.

In terms of size, these models display a trend from medium-small to medium-large. Brown's model [6] has 40 equations, of which only 8 are behavioural equations with stochastic disturbances. Kuiper's version of the

model [23] has 90 equations, of which 70 are identities or technical relationships. May's model [26], which is intermediate in time, is also intermediate in size, although it is much closer to Kuiper's version in terms of the number of equations. All three models were fitted to annual data for both the prewar and postwar years; the sample periods employed were 1926-1956 in [6], 1927-1961 in [26], and 1927-1968 in [23]. (In all three cases, the war years 1942-1945 were excluded.) The methods of parameter estimation included ordinary least squares in all three papers, with Brown supplementing this technique with a truncated form of the full information maximum likelihood method and May indicating that two stage least squares techniques were utilized in his variant of these models. In his paper, which is a preliminary version, Kuiper promises to re-estimate the parameters with the use of simultaneous equation techniques.

In [6], the dependent variable of the wage equation is the level of average hourly earnings in the private sector, while May and Kuiper split off the agricultural portion of the economy and explained the level of average hourly earnings of paid workers in the private non-farm economy. For the studies of Brown and May, one can interpret the wage equation as basically indicating a non-linear influence of demand in the labour market on wage changes. In [23], the development of the wage rate is, statistically speaking, mainly a matter of trend extrapolation. However, Kuiper finds that the level (or, one could interpret the dependent variable as the absolute change) of the wage rate is significantly and positively related to changes in the GNE deflator and changes in output (GDP) in the private non-agricultural sector, as well as being positively and insignificantly related to the reciprocal of the unemployment rate. (The changes in non-agricultural private real GDP could be interpreted as a demand variable, and so the negative but insignificant influence of the unemployment rate on the

wage rate of this sector should not be regarded as indicating that this variable does not respond to market forces.)

The determination of the absolute price level appears to go through one complete cycle through the three papers, from an implicit determination through an explicit equation and back again to implicit determination. In [6], there is no explicit behavioural equation explaining the deflator of GNE, although this variable is endogenous to the system and indeed appears on the left-hand side of one of the accounting identities. Brown interprets this variable as being determined by approximate market-clearing (on average, through the calendar year) between aggregate demand and aggregate supply emanating from the production relationships. In [26] May gives an explicit equation for the explanation of the implicit deflator of GNE; indicating that predecessor models which had been closed by production functions displayed "sometimes exaggerated price sensitivity." This equation relates the price level to its value in the preceding period, to the change in the wage level in the private nonfarm sector, and to the absolute wage level in the preceding period. Thus cost elements appear to predominate. However, demand pressures are allowed to play some role in that the GNE deflator also depends negatively on the lagged inventory-sales ratio for the economy. In [23] the GNE deflator again appears to be implicitly determined within the system, though it is difficult to tell from the fairly sparse discussion of Kuiper's preliminary working paper. There is, once again, an explicit accounting identity for the implicit deflator of GNE, as the ratio of the current dollar to constant dollar value of this magnitude. Again, the system appears to determine this magnitude by an approximate balancing of aggregate supply and aggregate demand, and indeed this interpretation is consistent with Kuiper's simplified flow chart on p.4-4 of his paper.

The sequence of the three models presented here shows an increased development with regard to the explanation of sectoral price levels. Brown not only takes the two price levels of imports as exogenous, which is certainly reasonable in the context of relatively small economy like the Canadian, but also the price level of total consumption expenditures, which is more questionable.⁶ May breaks out a number of sectoral price levels, which are "anchored" to the implicit deflator of GNE. Kuiper has five behavioural equations explaining sectoral price levels; in the typical one, the sectoral price index in question is strongly related to its own lagged value, with a coefficient of nearly unity in several cases, and then marginally to the change in the wage rate in the private non-farm sector and to the change in the implicit deflator for imports.

In these models, the notion of the trade-off or the conflict between the goals of full employment and stable prices is again present. In the case of all three wage equations, high demand in the labour market will bid up the wage rate of workers in the private (or private non-farm) sector, with subsequent pressures (indirect, in the case of Brown's and Kuiper's models) on the price level of final output. In all three cases, there is a role for the direct effect of demand pressures on the price level of final output, either through the balancing of aggregate supply and aggregate demand at a higher price level in the case of Brown's and Kuiper's models or because there is a demand variable present in the case of May's model. In discussing his model, Brown noted that the forecast for 1958 was for an increase in GNE deflator of 0.8 per cent, despite the fact that unemployment was expected to average 7.4 per cent over the course of the year. (In the event, this price index actually

⁶As Christ's discussion at the Chapel Hill conference has indicated, in practice the model may be utilized more sensibly than the formal specification would suggest. There is an auxiliary equation relating the implicit deflator for consumption expenditures to the GNE deflator, and the same treatment is accorded certain other price level variables also.

went up 1.8 per cent, despite a somewhat higher level of unemployment.) Brown argues that, if aggregate demand had been pushed to the level that would have assured full employment, then the price level would have risen by (he guesses) 3 or 4 per cent, a trade-off that he would have regarded as acceptable.⁷ Brown interprets this conflict as arising from the market power of large firms, unions, and professional associations and suggests that such a conflict would not exist in "the impersonal demand and supply forces of perfectly competitive markets."

4. Tsurumi's Four-Sector Model [30]

The discussion of this model is a good lead-in to major Section C below, as Tsurumi stresses the wage-price links of the Canadian economy with its much larger neighbour. (Indeed, one application of the model is to the interesting issue of the effects of wage parity with the U. S. on the Canadian economy.) The model is fitted to annual data from 1947-1967 by a technique known as Modified Sargan Two Stage Least Squares, which generalizes the standard two stage least squares procedure to allow for a first order autoregressive scheme in the stochastic disturbances. The model is of medium to large dimensions, with 60 behavioural equations possessing stochastic disturbances and 37 identities or technical relationships. The model has sectoral details; the four productive sectors distinguished are: agriculture, fishing, and forestry, abbreviated in the discussion below as agriculture; mining and manufacturing, below termed manufacturing; construction (no abbreviation necessary); and utilities, transportation, trade, finance, public administration, and other services, termed services.

⁷ It is unfortunate that this simulation (together with several others that his conference discussants suggested) was not actually performed. However, in evaluating this study, one must recognize that Brown was working with resources that were exceedingly modest by current standards.

The wage equations in three of the four sectors are modified Phillips or wage adjustment relationships, with the level (not the rate of change) of the sectoral wage on an annual basis related to the economy-wide unemployment rate. The manufacturing wage rate is negatively (but not significantly) related to the unemployment rate and positively related to output per man in the sector and to the U. S. manufacturing wage, the latter variable having a marginal coefficient of 0.46 and being the most important single variable (as judged by the t ratio) in this relationship. A very similar relationship holds for the construction sector, where the level of the sectoral wage is negatively (but insignificantly) related to the unemployment rate and positively related to sectoral output per man and the U. S. construction sector's wage, the marginal coefficient on this variable being 1.24 (but insignificantly different from unity). In the service sector, the only significant explanatory variable is the corresponding sectoral wage rate in the U. S., with a marginal coefficient of 0.6324 (significantly less than unity), although Canadian service sector wages are weakly and insignificantly related to output per man in this sector in the expected positive direction. Finally, wages in agriculture appear to be independent of direct influences from the corresponding U. S. sector, being negatively related to the unemployment rate and positively related to the consumer price index and output per man in the sector; all three explanatory variables are statistically significant with the consumer price index being the most important single explanatory variable (as judged by the t ratio). To recapitulate, the level of sectoral productivity has a positive impact on the sectoral wage in all four sectors (and is statistically significant in all sectors except services): the wage level in the corresponding U. S. sector has a positive and highly significant impact in all sectors except agriculture; the unemployment rate exerts a negative impact on the

sectoral wage in three out of the four sectors but is significant (by conventional criteria) only in agriculture. Tsurumi interprets the strong pull of U.S. wages on corresponding Canadian wages as reflecting the power of Canadian unions, which is not unreasonable, although the presence of an easily crossed border (especially for the sample period) might well be expected to play a role, even in an unorganized labour market.

The four basic price indexes in this model are the four sectoral price levels. In all four sectors, the sectoral labour costs per unit of output is a significant explanatory variable of the sectoral price index (marginally so in the case of the service sector); moreover, as judged by the t ratio, this variable is the most important explanatory variable for two of the four sectors (manufacturing and construction, where pricing is generally believed to be more cost-orientated). Another explanatory variable common to all four sectoral price equations is the ratio of actual output in the sector to capacity output; this variable can be interpreted as capturing the effects of demand pressures. In all four cases the coefficient on this variable has the expected positive sign; however, only in the case of agriculture (where it is the more important explanatory variable) is this variable statistically significant. For agriculture, there appear to be no direct price links with the U. S. economy, and the sectoral price equation stands as described. For both the manufacturing and service sectors, an important explanatory variable is the corresponding sectoral price index in the U. S., with a coefficient roughly equal to $2/3$ in both cases. In the construction sector, the manufacturing price level plays the role of the corresponding U. S. sectoral price index, although this variable has a low coefficient (0.15) and is only marginally significant. Tsurumi interprets the presence of price links as reflecting the

existence of "price synchronization" by the Canadian subsidiaries of large U. S. corporations, although one might equally well point up the large volume of imports into Canada from the U. S. which exert a direct influence on Canadian prices, either through competition with Canadian goods (final goods) or by becoming incorporated into the Canadian cost structure (intermediate goods).

The other seven endogenous price levels in the model are determined directly or indirectly from the four sectoral price levels. Four of these supplementary price levels (those of nondurable goods, durable goods, investment goods, and residential construction) are determined by regression, while the remaining three (export, consumer goods and services, and GNE deflators) are explained by accounting identities, which in this case can be interpreted simply as weighted averages (with variable weights). As is customary, import, U. S., and world prices are taken as exogenous, as are the levels of U. S. wage rates in the construction, services, and manufacturing and mining sectors of that economy.

Again, there appears to be a conflict between the goals of high employment and stable prices. High employment reduces the unemployment rate, thus raising wage rates in all sectors except services. In turn, higher wage rates lead to higher sectoral prices, through their effects on unit labour costs. In addition, high demand will raise sectoral output closer to its capacity level, thus generating additional pressures in all four sectors, especially agriculture. However, high demand also has the effect of raising labour productivity in all sectors except agriculture, due to increasing returns to scale in the production functions of the other three sectors. This in turn should reduce price pressures through their effects on unit labour costs, although the effect of improved productivity is not unambiguous since this phenomenon also gives an additional fillip to the sectoral wage level,

in all four sectors. In his extrapolations for the period 1968-1975, Tsurumi notes (merely by examining the short term fluctuations) an apparent conflict between the goals of rapid growth and high employment, on the one hand, and a slow rate of inflation of the other.

One can, of course, employ an econometric model to simulate the behaviour of an economy under a variety of assumptions regarding the development of the exogenous variables. This is precisely what I asked Professor Tsurumi to do with his model, and the results of this and the succeeding paragraph are based on these simulations, for which I am most appreciative. In his paper [30], Tsurumi made one set of extrapolations of the endogenous variables in his model for the period 1968-1975, under a unique set of assumptions regarding the development of the exogenous variables in his model. Thus he assumed that real government expenditures would trend upward (at a rate between 1 and 2 per cent per annum), that the discount rate would follow its upward trend of the decade of the sixties, and that other exogenous variables (including U. S. sectoral prices and wages) would follow recent trends. Let us term this set of extrapolations the control simulation; these projections suggested that, for the period 1968-1975, the growth rate of real GNE would be 3.28 per cent per year and the GNE deflator would rise at an annual rate of 0.77 per cent per year, while the economy-wide rate of unemployment would average 5.7 per cent of the labour force over this period. Thus this control simulation already suggests a certain incompatibility between the goals of full employment and stable prices, in the external environment of recent years. In Experiment One, the growth rate of real government expenditures (both the current or consumption portion and also government investment) was set at the much higher level of 5 per cent per year; as one would expect, this additional stimulation increased the growth rate of real GNE over the 1968-1975

period to 4.35 per cent per year (in comparison to 3.28 per cent per year). Accompanying the faster growth rate was a slightly faster rate of increase of the GNE deflator (0.86 instead of 0.77 per cent per year) and a slightly lower rate of unemployment (5.2 per cent of the labour force instead of 5.7 per cent). The comparison between these two simulations suggests that, in the unemployment range studied, the medium-term trade-off between inflation and unemployment is fairly flat, with only a little more inflation needed to be incurred to reduce the unemployment rate by half a percentage point. In Experiment Two, a mild monetary stimulus was applied; the discount rate was held at its 1967 value (4.3 per cent per year) instead of being allowed to rise along its recent historical trend, as was the case for the control simulation. This change in the time path of this exogenous variable stimulated slightly the growth rate of real GNE over the 1968-1975 extrapolation period (this growth rate became 3.31 per cent per year instead of 3.28 per cent per year), due principally to the stimulus afforded residential construction. However, it is interesting to note that the development of the GNE deflator, on average over this period, was unaffected (it still rose at a rate of 0.77 per cent per annum), while the rate of unemployment continued to average 5.7 per cent of the labour force. This simulation reinforces slightly one's faith in the trade-off curve as an analytical device for summarizing the information contained in an econometric model of this type.

Finally, Professor Tsurumi performed a different sort of simulation which indicates the dependence, at least in part, of Canadian prices and wages on developments in the United States. In Experiment Three, U.S. price indexes in the manufacturing and construction sectors were assumed to increase at an annual rate of 5 per cent, instead of the much less rapid development of these price indexes assumed in the control simulation. The result could be described as an upward shift of the trade-off curve; growth of real GNE

was much lower, at 1.69 per cent per year, while the unemployment rate was much higher (at 10.43 per cent of the labour force) and the GNE deflator rose much more rapidly (at a rate of 2.80 per annum). This simulation illustrates dramatically what the entire model suggests in a qualitative fashion: that, because of the important wage and price links with the United States economy, a fair proportion of Canadian inflation is imported from Canada's large neighbour to the South. For reasons adduced by both Tsurumi and me, this conclusion should hardly strike the reader as surprising, provided that he "buys" Professor Tsurumi's model (or one like it) as a realistic description of the Canadian economy.

C. United States

1. Introductory Remarks

As Cooper's work [9] indicates, the performance of various quarterly econometric models in predicting various kinds of price levels one quarter into the future is not strong, as judged by comparison with a naive model. There are a few exceptions to this rule, but as a generalization this assertion is valid. This result is not altogether surprising, as it is generally believed that pricing behaviour exhibits a fair degree of inertia in the very short run (e.g., within the calendar quarter).

Indeed, two models, Friend and Taubman [13] and Suits [28,29], which stress the forecasting aspects of econometric modelling, take the price level as exogenous to their model. This suggests that they feel that their judgmental forecasts of the price level (where this is necessary) will be stronger than a formalized procedure, which can be modelled. Presumably these judgmental forecasts will utilize a sophisticated "naive" model.

2. Klein, A Postwar Quarterly Model [19]

This model, a lineal descendant of the Klein-Goldberger model of the U. S. economy, is a small-medium quarterly model with 29 structural equations and 5 identities. The equations of the model are fitted to seasonally

adjusted data for the 44 quarters between 1948 I and 1958 IV, by the method of limited information maximum likelihood estimation (or, occasionally, by the method of two stage least squares).

The wage adjustment equation, in which the absolute change over corresponding quarters of adjacent years in the economy-wide annual wage rate for a standardized man-year is the dependent variable, is a variant of a standard form for this relationship. This dependent variable is positively related to a four-quarter average of the absolute change in the private GNP deflator (for the current and the immediately preceding three quarters), positively related to a time trend (which may merely reflect the fact that a given absolute wage increase has less relative impact as the wage level trends upward), and negatively related to a four quarter moving average (centred one and a half quarters prior to the current date) of the absolute level of unemployment; all parameter estimates are statistically significant (by classical tests). As explained in Section B above, one can interpret the unemployment variable as representing the influence of excess demand (or supply) in the labour market, while the change in the price level variable represents the pressure of a non-labour-market force (in the direct sense) on money wage changes. (As equation (6) below indicates, money wages tend to be adjusted so that the marginal impact of an additional increase in prices is almost -- but not quite -- to maintain real wages.) The time trend variable, to the extent that it is not a statistical artifact from using absolute wage and price changes and the absolute level of unemployment, would represent an additional source of non-market pressures on the wage level.

The determination of the absolute price level in the model is more problematical. Cooper [9] says that the private GNP deflator is explained as the ratio of current GNP from the demand side and real GNP from the production side. However, Klein himself says that his equation (12), which is a generalization of a constant labour share equation in the private sector

of the economy, can be transformed into a mark-up of price over unit labour costs. Hence this central price level, in Klein's view (and mine), basically has a cost orientation in terms of the direct explanation, although obviously demand elements play a role indirectly, through the simultaneity of the system.⁸

Sectoral price levels for consumer durables, for consumer nondurables, for consumer services, for plant and equipment, for residential construction and for exports are also explained in the model, while the implicit deflator for imports is exogenous. In the case of the implicit deflators for consumer services and for residential construction, the principal or sole explanatory variable is the wage level while for the other four implicit deflators the principal or sole explanatory variable is the implicit deflator of private GNP. In the case of both the implicit deflators for consumer durables and nondurables, the dependent variable is related to the level of unfilled manufacturers' orders in the sector with the expected positive sign of the coefficient of this variable suggesting the influence of demand pressures. In the case of consumer services, the sectoral price level is negatively and perversely related to the proportion of total consumption that takes the form of consumer services.⁹ For the export

⁸ Later variants of this model ([24] and [12]) do allow for an influence of demand pressures directly on the price level of final output.

⁹ In the O.B.E. econometric model [24] discussed below, Liebenberg, Hirsch, and Popkin re-estimate a similar equation for the implicit deflator for consumer services excluding housing services and find that the coefficient on the services proportion of real consumption expenditures (except housing services) is positive and significant (by ordinary standards). Thus the demand pressures would appear to play a normal role in the services markets, at least in a nearly "normal" period (1953-I through 1964-IV). The perverse results obtained by Professor Klein might be explicable either in terms of the peculiarities of the 1948-1953 period, which may have been characterized by continuing postwar adjustment and then Korean War disturbances, or in terms of the atypicalness of the behaviour of the implicit deflator for housing services.

price level, an exogenous index of the world price of competing exports is an explanatory variable with a small positive influence (the marginal coefficient is 0.07), which is statistically insignificant. (Canadian readers will note the contrast with their economy, as the results of RDX1 confirm. A similar variable in the RDX1 equation explaining the implicit deflator of goods exports had a marginal coefficient of 0.40, which was highly significant, by usual standards.)

Again, we may discuss the conflict between high employment and price level stability present in the model. This conflict arises mainly through high demand reducing unemployment, which then accelerates the rate of rise of wages. In turn, these higher wages are marked up into higher prices of final goods and services, even after allowing for normal productivity increases.

For the fourth quarter of 1960, $t=60$, and if we take the labour force to be 70 (millions), the average annual wage in 1959 IV to be \$4500, and the GNP deflator to be roughly 1.10 in the middle of 1959, then Klein's wage adjustment equation (13) can be cast into relative terms (ignoring lags) as follows:

$$(6) \quad \dot{W} = \frac{W - W_{-4}}{W_{-4}} = 0.0584 - 0.5942 U + 0.7380 \dot{P} ,$$

where W is the average annual wage, \dot{P} is the annual rate of change in the private GNP deflator, and U is unemployment as a proportion of the labour force.

Assuming a simplified mark-up pricing scheme, we may write:

$$(7) \quad \dot{P} = \dot{W} - \dot{A} ,$$

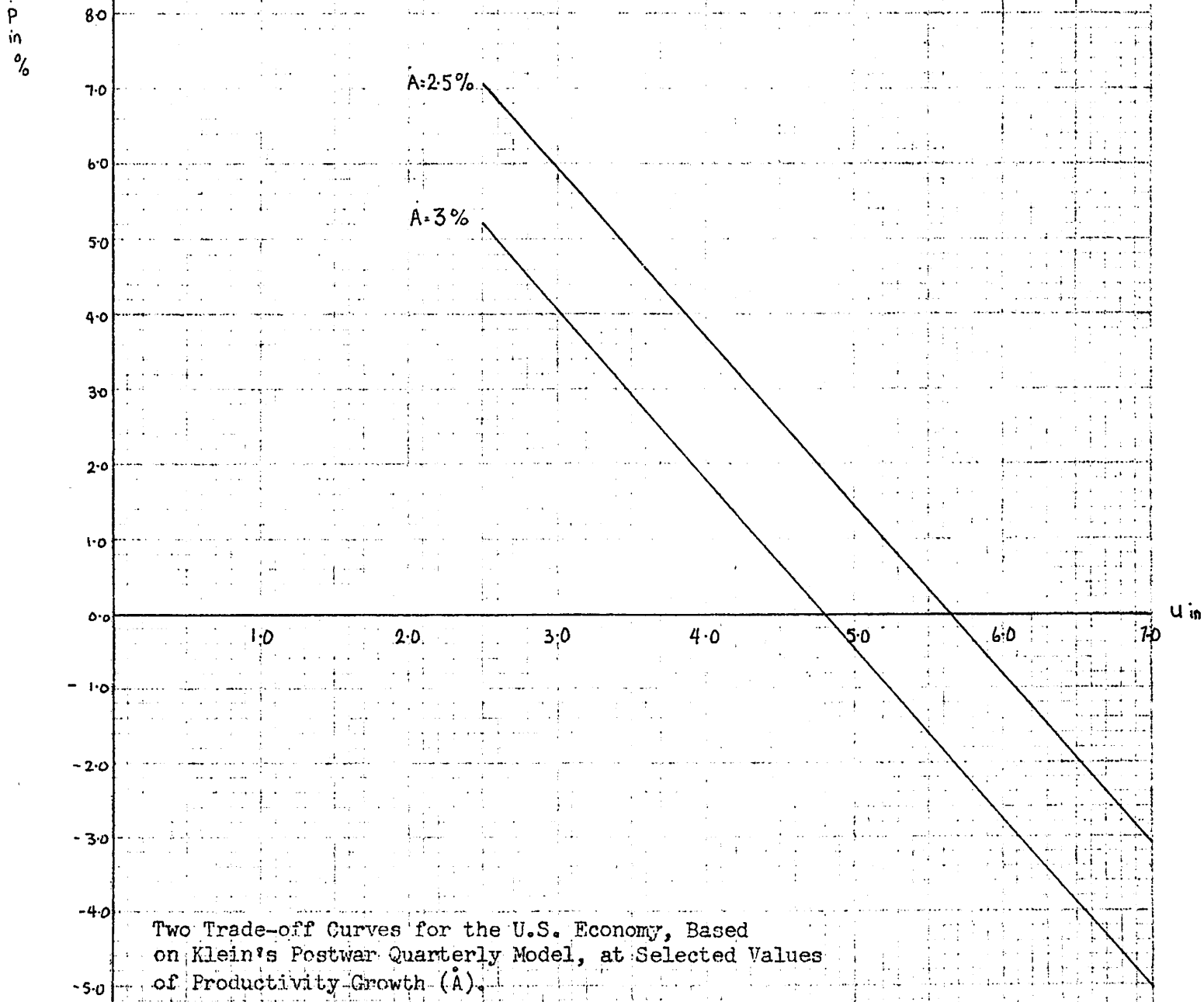
TABLE 2

Predicted Rates of Change of Private GNP Deflator (\dot{P}) at Varying Rates of Unemployment for Two Selected Values of Productivity Growth, Klein's Post-war Quarterly Model of the U.S. Economy

Unemployment Rate (U) in Per Cent	Annual Percentage Rate of Increase of Private GNP Deflator (\dot{P}) when:	
	Productivity Growth (\dot{A}) is 2.5 per cent per year	Productivity Growth (\dot{A}) is 3.0 per cent per year
2.5	7.08	5.2
3.0	5.95	4.0
3.5	4.81	2.9
4.0	3.68	1.8
4.5	2.54	.6
5.0	1.41	-.5
6.0	-.86	-2.7
7.0	-3.31	-5.0

Source: See the text, especially equation (8).

FIGURE 3



Two Trade-off Curves for the U.S. Economy, Based on Klein's Postwar Quarterly Model, at Selected Values of Productivity Growth (\dot{A}).

where \dot{A} is the annual rate of change of output per man-hour.¹⁰

Substitution of equation (6) into equation (7) yields the trade-off relationship:

$$(8) \quad \dot{P} = 0.2229 - 2.2679 U - 3.8168 \dot{A}$$

If values for the \dot{A} variable are specified, we obtain a two-dimensional trade-off curve. Such a curve is tabulated below for values of \dot{A} equal to 2.5 and 3 per cent per annum and also graphed in Figure 3.

The table and chart are largely self-explanatory, although two comments may be profered.

Note how a high rate of growth of productivity mitigates conflicts between the goals of high employment and stable prices. Also, although there appears to be a definite conflict between these two goals (even with the faster rate of productivity growth assumed), given any reasonable definition of full employment, note how a moderately small recession appears to do the trick in stabilizing the expected level of prices. This is largely due to the linear way in which the rate of change of wages depends upon the unemployment rate, which does not appear to be the most appropriate formulation, in present-day views.

3. Office of Business Economics Model [24]

This model was fitted to seasonally adjusted data from 1953 I through 1964 IV by the technique of two stage least squares, with principal components of the predetermined variables used in first stage regressions. The model is a lineal descendant of Klein's Postwar Quarterly Moded [19], as the authors

¹⁰This is not the place to justify such a macro-economic price level equation, which is used in place of Professor Klein's equation (12). Interested readers are referred to Chapter 2 of [5] for an extended discussion of this issue; the conclusion of that discussion is that such a relationship is valid in the closed economy case, at least as a first approximation.

(Liebenberg, Hirsch, and Popkin) note. The O.B.E. is slightly larger than its parent, with 36 structural equations and 13 identities. It may be noted that the authors themselves are fairly dissatisfied with the wage-price sector of their model. They feel that this sector does not simulate well within the sample period and that their forecasts for 1965, the first post-sample year, are improved by taking prices as exogenous. (However, in my view, this last assertion is debatable.)

In the wage adjustment relationship, the percentage change over corresponding quarters of adjacent years in the annual wage rate (for a standardized man-year) in the private sector is explained by four explanatory variables. The dependent variable is positively related to a four quarter average of the unemployment rate centered one and a half quarters previously to the current period, which implies that the wage rate change is negatively related to the unemployment rate itself, in an appropriate, non-linear fashion. The rate of change of wages is negatively related to the value of the dependent variable, lagged four quarters; as the authors point out, this suggests that current wage changes are moderated by high values of this magnitude in the preceding year. The rate of wage changes is also positively related to two variables which may be interpreted as non-competitive influences that particularly play a role in collective bargaining: the percentage change in consumer prices (with a short-run marginal coefficient of 0.88, which is not significantly different from unity)¹¹ and the change (between corresponding quarters of adjacent years) of non-wage personal income.

¹¹However, the long run (or "steady state") value of this parameter is somewhat lower, at 0.67, as equation (9) below indicates.

The principal price level is the implicit deflator of gross private output excluding housing services. For this variable, Klein's implicit markup equation has been made explicit, as the principal explanatory variable (as judged by the t ratio) is wage costs per unit of output, averaged over the current and the preceding two quarters. A significant constant term and a significantly positive coefficient on a time trend variable suggest the influence of fixed costs (which presumably rise over time). The role of demand pressures is captured by the positive influence on this deflator of the two quarter change in private final sales; this explanatory variable is weighted by a capacity utilization measure "in order to reflect increased sensitivity of prices to demand pressures when output is near capacity."¹²

¹²The O.B.E. equation for the implicit deflator of private gross output excluding housing services may be written (when the two-quarter change in private final sales is zero) as follows:

$$(i) \quad P = \alpha_0 + \alpha_1 \frac{WN}{X} + \alpha_3 t,$$

where the α_i 's are parameter values (all with a positive sign), WN is the total private wage bill, P is the implicit deflator of private gross output excluding housing services, X is real private gross output excluding housing services, and t is a time trend. This equation can easily be inverted so that it becomes a labour share (in private gross output) relationship; the result is:

$$(ii) \quad \frac{WN}{PX} = \frac{1}{\alpha_1} - \frac{\alpha_0 + \alpha_3 t}{\alpha_1 P}.$$

Hence it is easily seen that the labour share falls with rising values of the time trend (if prices are steady) and rises with a rising price level for a given point in time. The net effect on the shares in private gross output will in general be the resultant of these two competing pulls, ignoring the demand pressures variable and the stochastic disturbance. If this equation is a valid description of the American economy, it vividly suggests why guidepost policies are not popular with trade-union leaders.

There are six separate equations for the implicit deflators of personal consumption expenditures on durables (other than automobiles and parts), on consumer nondurables, on consumer services (except housing), on non-farm residential structures, on plant and equipment, and for total consumption expenditures. The implicit price deflator for exports vanishes from explicit consideration by the device of taking the current dollar value of exports as exogenously determined. Exogenous sectoral price levels are the implicit deflators for imports, for automobiles and parts (for personal consumption) and for housing services and the wholesale price index for durable goods. While it is understandable that the first price index should be exogenous, ideally the remaining three price levels should be system-determined. The first five O.B.E. sectoral price equations are very similar to their counterparts in [19], except that an autoregressive scheme is introduced for the implicit deflators of non-automobile durables, of consumer nondurables, and of plant and equipment, in which the sectoral deflator is principally related to its own lagged value, with the change in the implicit deflator for private output then exerting an additional influence in the positive direction. As pointed out in footnote 9 above, it is interesting to note that, in the O.B.E. equation for the implicit deflator of non-housing consumer services, the proportion of total consumption taking the form of services has the expected positive influence. A role for demand pressures appears also in the equation explaining the implicit deflator for non-automobile consumer durables, in which this variable is positively and significantly related to the accumulated backlog of unfilled manufacturer's orders for durable goods, deflated by the wholesale price index for durable goods. Finally, the implicit deflator for total consumption expenditures is "explained" by an identity relating current and constant dollar

magnitudes, which as pointed out previously can be regarded as a weighted average (with variable weights) of the component price deflators.

Trade-off considerations arise in the usual fashion. Demand pressures reduce the rate of unemployment, thereby accelerating the rate of rise of wages, which are then marked up into higher prices. (In addition, if profits spurt, an additional fillip is provided to wage increases.) High demand has some additional direct effects in the equation for the implicit deflator of private output except housing services and in the subsidiary equations explaining the deflators of consumer services except housing and of non-automobile consumer durables. However, there is something of an offset to these pressures in the fact that high demand raises labour productivity (via the operation of a short-run production function in which the elasticity of output with respect to labour input is greater than unity), and this in turn mitigates the rise in unit labour costs. Again we can compute a simplified trade-off curve. . If we set the change in non-wage personal income equal to zero, ignore the lags in the unemployment and price change variables, and wash out the lagged wage change variable by assuming that wage rates rise at a constant rate over time so that we may calculate a steady state relationship, the O.B.E. wage adjustment equation becomes:

$$(9) \quad \dot{W} = -1.14 + 20.6 \frac{1}{U} + 0.669 \dot{P}_c ,$$

where \dot{W} is the annual percentage change in the private sector wage rate, U is unemployment as a percentage of the labour force, and \dot{P}_c is the annual percentage rate of change of the implicit deflator for total consumption expenditures.¹³ Once again, we may utilize a simplified markup equation to explain

¹³We have converted the O.B.E. proportionate rates of change to percentage rates of change; also the moving total on unemployment as a proportion of the

the annual rate of change of the private GNP (excluding housing services) deflator:

$$(7) \quad \dot{P} = \dot{W} - \dot{A} ,$$

where again \dot{A} represents the annual percentage rate of growth of output per man-hour in the private sector. If we ignore sectoral complications, we may assume that the rate of change of the consumption deflator is equal to the rate of change of the private GNP deflator; in symbols,

$$(10) \quad \dot{P} = \dot{P}_c .$$

A trade-off relationship can now be easily derived from equations (9), (7), and (10); the result is:

$$(11) \quad \dot{P} = -3.46 + 62 \frac{1}{U} - 3.02 \dot{A} .$$

With assumed rates of productivity growth, this trade-off relationship can again be represented as a two-dimensional curve, which is done in Table 3 and Figure 4 below. Again, there appears to be a conflict between the goals of full employment and price level stability, which, however, can be mitigated in part by more rapid productivity growth. Note also that the curves suggest that a little bit of recession still appears to be a fairly hopeful policy for price level stability. This probably reflects the fact that the wage adjustment curve is fitted to data for the 1953-1964 period; the evidence from the 1965-1969 post-sample period would probably be less optimistic on this point.

civilian labour force has been changed to be a moving average concept. No change in the moving total on quarterly proportionate changes in the consumer expenditures deflator variable is required, as a moving total for quarterly proportionate changes is (approximately) the same as an annual proportionate change variable.

TABLE 3

Predicted Rates of Change of Private GNP Deflator
Excluding Housing Services (\dot{P}) at Varying Rates of
Unemployment for Two Selected Values of Productivity
Growth, O.B.E. Econometric Model of the U.S. Economy

Annual Percentage Increase of \dot{P} When:

<u>Unemployment Rate</u> (per cent)	<u>Productivity Growth</u> (\dot{A}) is 2.5 per cent per year:	<u>Productivity Growth</u> (\dot{A}) is 3.0 per cent per year:
2.5	13.79	12.28
3.0	9.65	8.15
3.5	6.70	5.19
4.0	4.49	2.98
4.5	2.77	1.26
5.0	1.39	-0.12
6.0	-0.68	-2.19
7.0	-2.15	-3.66

Source: See the text, especially equation (11).

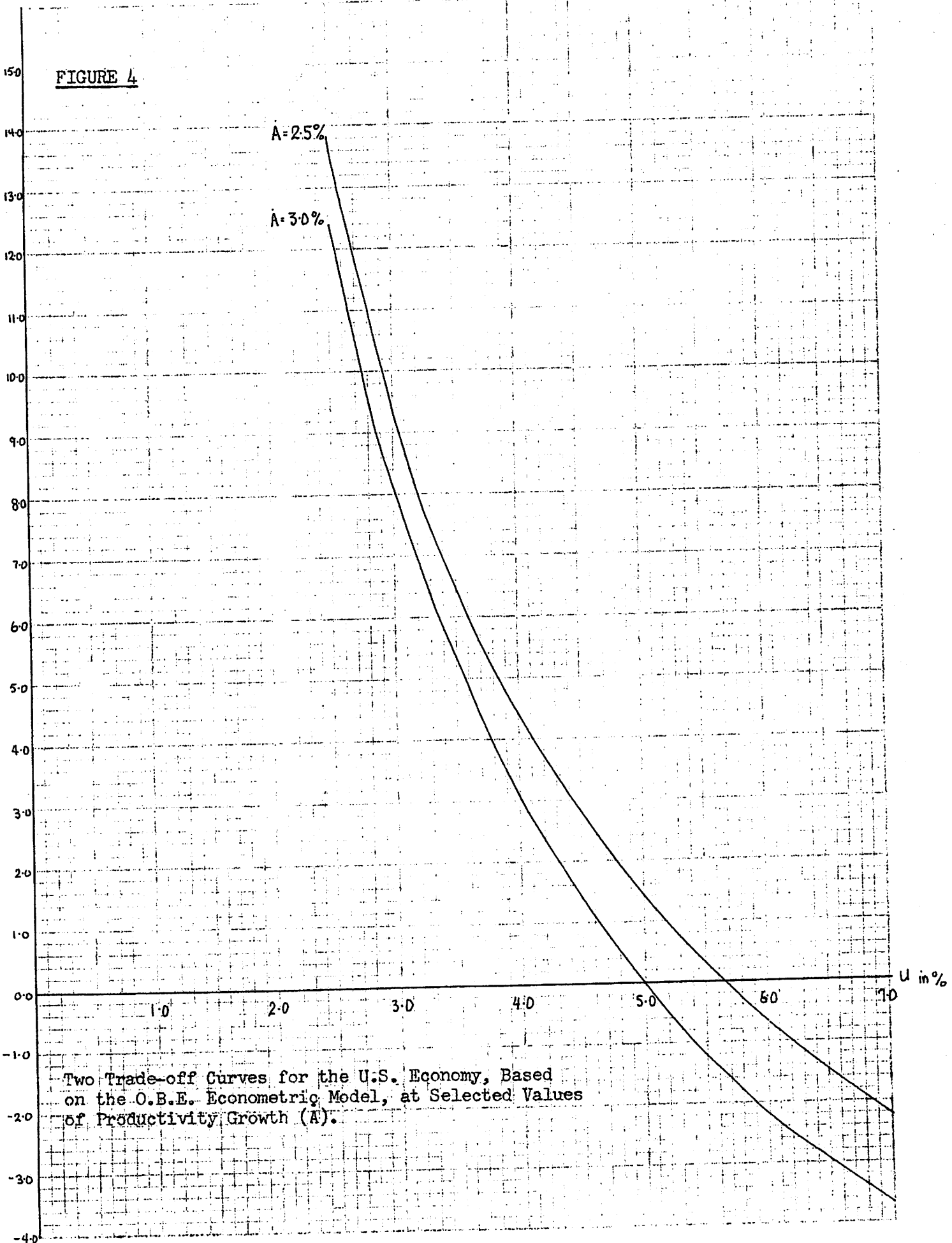


FIGURE 4

Two Trade-off Curves for the U.S. Economy, Based on the O.B.E. Econometric Model, at Selected Values of Productivity Growth (A).

4. The Wharton Econometric Forecasting Model [12]

This model has two basic antecedents--Klein's Postwar Quarterly Model [19] and Michael Evans' own postwar quarterly model of the U.S. economy. The model is fitted to data on the U.S. economy from the first quarter of 1948 through the fourth quarter of 1964, although shorter periods are used for several of the relationships. The method of parameter estimation is a variant of the two stage least squares method, with the technique of principal components utilized to generate the instrumental variables which are substituted in the second stage for the right-hand-side endogenous variables. The model itself is of medium-large dimensions--there are 47 behavioural equations and 29 identities. (Parenthetically, I can't help remarking that econometric models of modern economies show quantitative growth rates that rival, if not exceed, the growth of the economies being modelled.) There is a rudimentary disaggregation of production into five sectors: manufacturing; non-manufacturing, non-farm, non-rent private; rent (housing services); farm; and government. (Output originating in the last two sectors is exogenous to the model.) As interpreted by me, the wage-price submodel of the Wharton model consists of 12 equations: two wage adjustment equations, one principal price level equation (for the implicit deflator of gross output originating in the manufacturing sector), and 9 subsidiary price equations, of which 6 are more or less behavioural equations and 3 are simply identities relating the ratios of current-dollar and constant dollar magnitudes.

The two wage adjustment equations explain the absolute change, over corresponding quarters of adjacent years, in the money wage for a standardized man-year in both the manufacturing and the non-manufacturing, non-farm private sectors. For manufacturing wage changes, the demand pressures explanatory

variable is an average for the present and the three preceding quarters of the gap between the over-all unemployment rate for the civilian labour force and the unemployment rate for males, aged 25-34. When this gap widens (i.e., when the rate of unemployment for the prime or desirable group falls relative to the over-all average), it is assumed that market forces lead to an acceleration of wage changes (because of a drying up of the pool of the most desirable type of worker), other factors remaining unchanged. Since, in the range of 3.8 to 7 per cent unemployment rates, there is a negative relationship between this gap (over-all unemployment rate minus prime group unemployment rate) and the over-all rate, we have the familiar negative effect (for given values of the other determinants) of variations in the rate of unemployment on expected wage changes.¹⁴ The other two explanatory variables of manufacturing wage changes are a consumption deflator change variable, which has a positive effect, and the lagged value of the dependent variable, which indicates that wage adjustments may overshoot and hence that last year's wage change may exert a negative influence on the current wage change. Both of these variables represent, it may be argued, influences that are not strictly labour market forces.¹⁵ In the other wage adjustment relationship, the absolute change

¹⁴ Below an over-all unemployment rate of roughly 3.8 per cent, there is a positive relationship between the over-all rate and the gap between the over-all rate and the prime group rate, as less desirable groups of workers find their rates of unemployment dropping to frictional levels. In turn, this implies that wage change pressures from labour market forces will be less at a 3 per cent over-all unemployment rate than at a 4 per cent rate, which seems difficult to believe. Probably the best explanation is that the model should be used only in less than full employment ranges of the over-all unemployment rate and that behaviour during the Korean War period, when the over-all rate dipped below 3.8 per cent, may not be typical of other full employment or over-full employment periods.

¹⁵ As a minor quibble, we may note that wage changes and price changes in these wage adjustment relationships are expressed in absolute terms, whereas unemployment (for both the prime group and in total) is expressed as a rate. For comparability with the work of others, for the ease of comparisons, and possibly because it may be truer to the economy's structure, it would seem desirable to express these changes in percentage terms.

between corresponding quarters of adjacent years in the money wage for a standardized man-year in the non-manufacturing, non-farm private sector is explained by three explanatory variables. This wage change is positively related to the wage change in the manufacturing sector, with a marginal coefficient of 0.3; negatively related to the wage change in this sector four quarters previously, thus showing the same tendency for wage adjustments to overshoot as in manufacturing; and positively (but with marginal statistical significance) related to the consumption price change variable.

The key price level in this model, as noted above, is the implicit deflator of gross output originating in the manufacturing sector. The level of this deflator is positively (and significantly, by conventional criteria) related to four explanatory variables: manufacturing wage costs per unit of real output; the rate of capacity utilization in the manufacturing sector; a Korean War dummy variable; and the average over the immediately preceding four quarters of the dependent variable, the implicit deflator of manufacturing gross output. Evans and Klein explain the significance of the first two variables in terms of a profit maximization model, slightly modified from the usual textbook formulation, but (as they note) the model seems equally consistent with mark-up pricing practices adjusted to variations in the rate of capacity utilization. The Korean War dummy shows the effect of unusual speculative influences on manufacturing prices, and it may be argued that the role of expectations is equally seen in the fourth explanatory variable, the average of the sectoral deflator over the preceding four quarters. This variable can also be interpreted as indicating a lack of instantaneous adjustment in price formation in this sector.

Before examining the subsidiary price equations, it is worth noting that there are seven price deflators which are taken as exogenous to the model.

Four of these are international prices, the implicit deflators of three categories of imports and the price level of world trade.¹⁶ For domestic price levels, three are exogenous: the implicit deflator for government purchases, the index of prices received by farmers, and a price index of rents. The last two variables would ideally be endogenous in a more complete model, although the treatment of the index of prices received by farmers is consistent with the treatment of this sector, which is known to present many problems to econometricians, as exogenous.

There are behavioural subsidiary price equations for the following price levels: the implicit deflator for automobiles, the implicit deflator for consumer durables except automobiles, the implicit deflator for consumption of non-durables and services, the implicit deflator for exports, the implicit deflator for non-farm residential construction, and the implicit deflator for non-residential fixed business investment. All six of these subsidiary price equations are estimated in first difference form. For the first four (the deflators of the components of consumption and of exports), the current change and the lagged change in the implicit deflator of gross output originating in manufacturing are explanatory variables, except for the automobiles deflator equation, where only the lagged change in the manufacturing deflator is utilized. The equation explaining the changes in the deflator for the consumption of non-durables and services also has the change in farm prices included as an explanatory variable; if one judges by the t ratio, this would appear to be the key explanatory variable. From a Canadian view-point, the export price equation is notable for what it does not contain, namely the changes in the

¹⁶ Although the documentation on this latter variable is a bit skimpy, this is an index of the prices of goods and services moving in world trade, with the weights presumably proportional to the importance of the item in total (non-communist?) world trade.

price level of world trade. Presumably the absence of this variable reflects its statistical unimportance, in view of the wide experimentation with the model equations that the authors report elsewhere (on p. 71 of the monograph); thus one must conclude that U.S. export prices are insensitive to world prices, contrary to the Canadian case. The changes in the two investment goods deflators are related to current changes in the GNP deflator (whose genesis is explained in the next paragraph) and to a single demand-pressures variable, non-farm plant and equipment investment (in real terms) as a proportion of real GNP in the case of the implicit deflator for non-residential fixed business investment and the two-quarter change in real non-farm fixed investment (i.e., the sum of non-farm residential construction and non-farm plant and equipment investment in constant dollars), in the case of the implicit deflator for non-farm residential construction. It may be noted that Evans and Klein do not consider their subsidiary price equations too satisfactory, but argue that they do not analyze enough productive sectors to explain price formation directly on a sectoral basis, as is done in the case of the Brookings model. (See also the discussion in Section 5 below.)

Finally, the price submodel is closed by three identities stating the equality of the implicit deflators of GNP, of total consumption expenditures, and of the gross product originating in the non-farm, non-manufacturing private sector to the ratios of the appropriate current dollar to constant dollar magnitudes. Thus, the implicit deflator of total consumption is equal to current dollar total consumption divided by total consumption in 1958 dollars, and similar equalities hold for the other two implicit deflators. As pointed out before, the two identities "explaining" the implicit deflators of GNP and of total consumption have a ready interpretation as weighted averages

(with variable weights) of the implicit deflators of the components of the aggregate under consideration. However, this interpretation cannot readily be given to the equation "explaining" the implicit deflator of gross product originating in the non-farm, non-manufacturing private sector, and it seems best to recognize, as do Evans and Klein, that this deflator is "the one price level which must be estimated as a residual if the aggregate demand and aggregate supply identities are to hold in both current and constant dollars."¹⁷

In the Wharton model, there will exist (ignoring sectoral qualifications) a trade-off curve between the rate of change in prices and the level of the unemployment rate, at least in a range of the latter between 4 and 8 per cent of the labour force. An increment in aggregate demand will reduce the over-all unemployment rate and increase the gap between the prime group unemployment rate and the over-all unemployment rate, thus accelerating the rate of rise of manufacturing wages. In turn, increases in labour cost per unit of output in this sector will raise the implicit deflator of gross output originating in the manufacturing sector. Demand pressures acting directly on final price levels will also raise prices in the manufacturing sector and, in addition, capital goods prices. These points emerge clearly from a study of Evans and Klein's simulations of the U.S. economy over the period 1966-1975. In the control simulation, the exogenous variables were adjusted to keep the unemployment rate at approximately 4 per cent of the labour force; in consequence, real output was projected to grow at a 4 per cent rate and the GNP deflator at a 2 per cent rate, thus suggesting an inconsistency between the goals of full employment and stable prices. In addition, Evans and Klein examined the effects of five types of fiscal policy stimulation (2 on the expenditures side and 3 types of tax cuts), of an exogenous increase in export

¹⁷ [12], p. 33.

demand, and of two types of monetary policy stimulations. The usual pattern was one of faster price rises and lower unemployment rates, in comparison to the control solutions, although the slopes of these dynamic trade-off curves were sensitive to the broad category of stimulus utilized.¹⁸ Finally, it is worth noting that the lags in the wage-price relationships are generally fairly long, so that conventional monetary and fiscal policy is likely to have its full or nearly full impacts on output and employment well before price levels respond--a conclusion that seems corroborated by recent experience with the U.S. economy.

One final comment regarding the forecasting accuracy of the price level equations of the Wharton model may be appended. Evans and Klein pride themselves (and largely rightly so, in my view) on the forecasting accuracy of their model as a whole. However, in forecasting one quarter ahead, their price level predictions are only roughly as good as that of an extrapolative naive model. Is this the whole story on prediction? Not really; as one lengthens the horizon of the forecasts, the price level forecast becomes better, and for forecasts four quarters (one year) into the future, the

¹⁸ Broadly speaking, one could say that the slopes of the fiscal policy trade-off curves (in the neighbourhood of the control solution) were approximately the same, regardless of which of two types of expenditures policy or which of two types of tax cuts were utilized. (Excise tax reductions present some difficulties if one attempts to classify them in a similar manner; the initial effect of a reduction in excise taxes is to lower the price level; over time, the induced effects of a reduction of excise taxes lead to an erosion of the initial reduction, though not (according to the model and the secondary assumptions) its elimination after 40 quarters.) Because exports are in elastic supply, an export stimulus leads to far less induced price level increases, making this "most preferred as the method for stimulating the economy, if it can be accomplished." (Shades of mercantilism!) Finally, although a monetary stimulus leads to comparable increases in real output and prices (as compared to the control solution) after normalizing for the size of the stimulus, in the long run such a stimulus reduces employment or increases unemployment because such a stimulus also induces capital-labour substitution. Thus there would appear to be some kernel of truth, in our Evans-Klein world, to the monetarist view that there is no trade-off curve in the long run. Unlike the usual monetarist argument, we can specify an exact time dimension for the Evans-Klein model: such a reversal (stimulus increasing instead of reducing unemployment) would appear to take 4-5 years (16 to 20 quarters).

forecasts of the implicit deflator of GNP are better, by a substantial margin, than those of two "reasonable" naive models.

5. The Brookings Model

The Brookings Model has been widely discussed in recent years; the reader may be referred to the two model volumes edited by Duesenberry, Fromm, Klein, and Kuh [10] and [11], to the Fromm-Taubman simulation volume [15], to Griliches' critical review article [16], and to Fromm and Klein's spirited rejoinder [14] to Griliches' criticisms. The discussion in this section will largely be based on the version of the model which appears in the Fromm-Taubman simulation volume [15], which as far as I know is the latest published statement of the full model. This version of the model was fitted by ordinary least squares to U.S. data for the period from the first quarter of 1948 through the final quarter of 1960. This version of the model contains 216 equations, of which 37 may be considered the wage-price sector of the model. The basic wage and price equations explain wage-price behaviour in six productive sectors of the economy: durable manufacturing, non-durable manufacturing, trade, contract construction, regulated services, and others (mining and non-regulated services). There are also a large number (25 to be precise) of subsidiary price equations translating sectoral price equations into the familiar implicit deflators.

The six sectoral wage equations are wage adjustment equations in which the dependent variable is the proportionate rate of change, between corresponding quarters of adjacent years, of hourly compensation per employee. The explanatory variables are four in number: a four quarter average of the proportionate change in the consumer price index, with an average lag of $1\frac{1}{2}$ quarters; a four quarter average of profits per unit of output, with an average lag of $2\frac{1}{2}$ quarters; the reciprocal of a weighted average rate of

unemployment, with an average lag of 2 quarters; and the value of the dependent variable lagged four quarters, the proportionate wage change of the corresponding quarter of the previous year. As we have argued previously, the unemployment rate variable would represent the influence of market forces on wage adjustments in these producing sectors, while the other three variables would represent influences that cannot strictly be explained in terms of purely competitive market forces. It is interesting to note that the consumer price change variable has the expected positive sign in all six cases, with a coefficient less than unity always, which in turn suggests less than full escalation for changing consumer prices (at given values of the other determinants of wage changes). The profits variable has the expected positive sign in only four cases out of six, while the reciprocal of the unemployment rate has the expected positive sign in five cases out of six. The lagged wage changes always have a negative coefficient whose absolute value is less than unity.¹⁹ It is worth noting that these wage adjustment equations suggest considerable inflationary potential to the economy, as the chapter by Schultze and Tryon in the original Brookings volume [10] pointed out. It may also be noted that Griliches criticized specifically the inclusion of the lagged wage change variable, arguing that the results obtained could well be spurious because of the negative intercorrelation because current and lagged wage changes due simply to the way in which the variable is constructed. Fromm and

¹⁹ Although no standard errors of the estimated coefficients are given, perusal of similar equations in Chapter 18 of [10] suggests that, as a general rule, the coefficients with the expected signs are statistically significant and those with perverse signs are insignificant, by usual criteria.

Klein replied that there may well be good structural reasons, such as long term contracts, for such an association.²⁰

There are six corresponding price level equations, in which the implicit deflator for sectoral gross product originating is explained in level form. The explanatory variables are typically the normal level of unit labour costs (unit wages divided by labour productivity averaged over the current and preceding eleven quarters), the discrepancy between current unit labour costs and normal unit labour costs, and, in the two manufacturing sectors and trade, the discrepancy of the inventory-output ratio from a twelve-quarter moving average of this ratio.²¹ Price levels in all six sectors are of course positively related to both normal unit labour costs and to the discrepancy between actual and normal unit labour costs. In principle, the normal unit cost variable should have a larger numerical impact than the discrepancy between actual and normal unit costs, although this hypothesis is corroborated in only three out of the six regressions calculated. The difference between the inventory-output ratio in the sector in question and its moving average serves as a demand-pressures variable;

²⁰In evaluating this point of controversy, I should accord some truth to Griliches' argument but should prefer to put it slightly differently; if there is a measurement error in the wage series, this measurement error will induce a negative coefficient for lagged wage changes even if the true structural coefficient is in fact zero. Thus this factor would tend to bias upward (in numerical value) the estimates of the coefficient of lagged wage changes, even if the influences pointed up by Fromm and Klein are indeed present. Parenthetically, I should remark that many of Griliches' criticisms (in this section of his review article) seem to me wide of the mark and the reply by Fromm and Klein seems to me (on these issues) a fair enough rejoinder.

²¹Two variants deserve brief comment. For non-durable manufacturing, an additional explanatory variable is a farm price index, while for the regulated industries, the two cost variables are the sum of normal unit labour costs plus normal unit capital costs and, as well, the discrepancy between actual unit labour and capital costs and the total of the corresponding normal unit costs.

when inventories are high (in relation to output, relative to past values of this ratio), this should lead to downward pressures on the sectoral price level. This hypothesis is corroborated in two out of the three regressions, namely in the two manufacturing sectors. Nevertheless, the Brookings model gives the reader the distinct impression that price formation is highly cost-oriented for the six sectors under study.

The remainder of the wage-price sector may be described very briefly. Exogenous price levels include two unit value indexes of import prices, a unit value index of world exports excluding U.S. components, and a farm price index. There are 9 price conversion equations, which explain the implicit deflators of durables other than automobiles, of automobiles, of non-durables other than food and beverages, of food and beverages, of consumer services, of plant and equipment expenditures, of new nonfarm private residential construction, of exports of goods and services, and of government purchases of goods and services; these equations generally utilize the implicit deflators of the producing sectors as explanatory variables. The most interesting feature of these equations to me is the fact that export prices are explained solely in terms of domestic sectoral price levels, with no allowance for an impact of world prices. There are also 9 autoregressive price correction equations, which suggests that price formation does not take place instantaneously but only with some lags. Finally, there are 7 subsidiary price level equations explaining three wholesale price indexes, the consumer price index, and the implicit deflators of goods in the GNP accounts, of total consumption expenditures, and of total GNP itself. (The latter two price levels are in the form of identities equating the implicit deflators to the ratios of the appropriate current dollar to constant dollar magnitudes and can of course be interpreted as simply weighted average relationships with variable weights.)

In this model, it is easy to see that inflation may arise as the economy approaches full utilization of resources. High demand will reduce the economy-wide rate of unemployment; in consequence wage rates will rise more rapidly than normal. If profit margins widen in consequence of stronger demand, an additional fillip to the acceleration of wage rates is provided. Because of the highly cost-oriented nature of pricing in the producing sectors, these wage increases will, to the extent that they exceed productivity gains, result in higher sectoral price levels, which then feed through the entire system. In addition, there may be a direct impact on price formation in manufacturing, as inventories become depleted in relation to output, as judged by recent levels of this ratio. Fromm and Taubman discuss the possibility of a trade-off curve (which they term a "Phillips Curve") arising from interrelationships like these ([15], pp. 101-106). However, the short run simulations of Fromm and Taubman show no such inflationary consequences of stimulative monetary and fiscal policy, except for the increase in government employment, in a situation in which there was a great deal of slack in the economy. (The control solution was a "realistic" simulation of the U.S. economy over the period 1960 III through 1962 IV, a period of fairly large slack in the utilization of resources.) Indeed, the simulations suggested that the main immediate effect of stimulating the economy in such a situation was to reduce the price level (relative to control values), due to raising average productivity through the short run production relations and reducing unit labour costs. (See [15], pp. 28 and 29, and the associated Table 2.3, especially pp. 38 and 40.) However, the simulations also suggest that the deflationary impact peaks out after four to six quarters have elapsed, and then the inflationary forces take over. Because of the lags in the wage

and price equations, it is not surprising that such results do occur, particularly in light of the large amount of slack that is assumed in the control solution.

6. Ta-Chung Liu's "An Exploratory Quarterly Econometric Model of Effective Demand in the Postwar U.S. Economy" [25]

This is an interesting aggregative model fitted to quarterly data on the U.S. economy running from the third quarter of 1947 through the fourth quarter of 1959. The model is of medium size, with 19 behavioural equations and 17 identities or definitional relationships. In general, the parameters of the model are estimated by the techniques of ordinary least squares and a variant of the two stage least squares method. Exceptions to this statement occur in the inventory-price submodel, where standard methods cannot be applied because of the unavailability of data on hypothetical constructs like the equilibrium stock of inventories. (Estimation of the price equation is briefly discussed below.) The inventory-price submodel has four behavioural equations and two identities;²² this submodel will be the focus of our discussion in this section. We may note that this model is limited in its objectives principally to studying aggregate demand patterns; there are no production nor employment equations, which means that aggregate supply is treated in fairly cursory fashion. Thus the path of the money wage rate (average hourly gross earnings in manufacturing) is taken as exogenous, which gives away most of the game as far as a trade-off theorist is concerned.

The inventory-price submodel may be briefly described. The price equation relates the percentage change at an annual rate in the GNP deflator (\dot{P})

²²In the presentation of Liu's text, there are three identities, not two. However, the construct of the desired stock of inventories is readily eliminated, as one can work simply with desired inventory investment. When this is done, as in the appendix to this article, there remain only two identities in the submodel.

to its own lagged values for the preceding two quarters, to the percentage change at an annual rate to the manufacturing wage rate in the current quarter (\dot{W}), and to the discrepancy between desired and actual inventory investment. In symbols, we have:

$$(12) \quad \dot{P}_t = \alpha_0 + \alpha_1 \dot{P}_{t-1} + \alpha_2 \dot{P}_{t-2} + \alpha_3 \dot{W}_t + \alpha_4 (v_t^d - v_t) + u_t,$$

where the α_i 's are parameters, u_t is a stochastic disturbance, v_t is actual inventory investment, v_t^d is desired inventory investment, and the remaining variables have been defined previously. The explanatory variables of this price level equation can be given a ready interpretation; the lagged price changes can be regarded as reflecting the effects of cyclical expectations as price movements tend to develop a momentum of their own. The wage change variable would, of course, indicate the presence of cost pressures, while the discrepancy between desired and actual inventory investment (which is the same as the discrepancy between the desired and actual stocks of inventories) would indicate the direct influence of demand pressures on the adjustment of the prices of final goods and services. The constant term in equation (12) could be regarded as an estimate of the effects of productivity growth on price level changes, at given values of the other determinants, on average over the period of estimation.

The other three behavioural equations in the submodel may be briefly described also. There is a structural equation which relates the equilibrium stock of inventories (which should not be confused with the desired stock of inventories) positively to sales of privately produced final commodities and to real business liquid balances, and negatively to the "real" rate of interest. (Elsewhere, the real rate of interest is defined to be the nominal rate on prime commercial paper minus the annual rate of change of the GNP deflator in the previous quarter.) The next structural equation

is an adjustment equation in which desired inventory investment is related to the discrepancy between the equilibrium stock of inventories and the beginning of the period level of this stock and also to recent inventory investment in the preceding two quarters. Liu interprets these latter two variables as representing the effects of the costs of changing orders and as possibly capturing some of the expectational phenomena so important in these relationships. The fourth structural equation of the submodel is an output decision equation relating real output of privately produced final commodities positively to the discrepancy between desired and actual inventory investment, to price level changes, and to its own lagged value and negatively to the rate of change of wages. The submodel is completed by two identities: the first identity defines inventory investment as the difference between the end of the quarter and the beginning of the quarter stock of inventories; the second states that output is equal to final sales plus inventory investment.²³

The estimation of the inventory-price submodel is difficult, as two unobservable hypothetical constructs (desired inventory investment and the equilibrium stock of inventories) appear as variables in the equations. The method of estimating the parameters of the price equation may be briefly described; for details, the reader is referred to Liu's article [25], especially pp. 313-316. The reduced form equation for actual inventory investment in the submodel involves only observable variables; consequently, the parameters of this regression can be calculated. In turn, this reduced form equation yields estimates of the parameters of the equilibrium inventory stock equation,

²³ Actually, farm inventory investment is split off from the rest of inventory investment and made exogenous in both the submodel and the full model. This technical point need not trouble us in the broad descriptions of this paragraph.

if additionally one assumes that the average and marginal inventory-sales coefficients were constant over the period in question. In turn, one can compute a series for the equilibrium stock of inventories, which, along with three of the parameters from the reduced form equation for actual inventory investment, allows Liu to compute a series that is proportional to desired inventory investment. In turn, Liu can finally compute the parameters of the price level equation by running the following regression:

$$(13) \quad \dot{P}_t = 0.340 \frac{\hat{v}_t^d}{c_1} - 0.208 v_t + 0.254 \dot{P}_{t-1} + 0.169 \dot{P}_{t-2} \\ + 0.317 \dot{W}_t - 19.74 , \\ (0.168) \quad (0.156) \quad (0.158) \quad (0.130) \quad (0.124)$$

where the numbers in parentheses are the associated standard errors, \hat{v}_t^d is an estimate for the series of desired inventory investment, and c_1 is the adjustment parameter in the equation in which desired inventory investment is related to the discrepancy between equilibrium and actual stocks of inventories (c_1 is this coefficient) and to the two lagged values of inventory investment. The theory is corroborated by the fact that all of the variables have appropriately signed coefficients, even though the coefficients on lagged price changes are not statistically significant by usual standards. The demand pressures effect receives especial confirmation by the fact that the coefficient on the series that is proportional to estimated desired inventory investment is statistically significant; although the coefficient on actual inventory investment is not statistically significant, it at least has an appropriate sign. Finally, the estimate of the constant term, which as noted above can be interpreted as a productivity effect, must have been reported incorrectly. Taken at face value, it suggests that if actual inventory investment is equal to desired inventory investment and if the rate of change

of wages and the past rates of change of the price level are zero, then the GNP deflator will fall at a rate of 19.74 per cent per year! This is ridiculous, and there must be a reporting error somewhere.²⁴ From this regression, one can solve for the parameter c_1 and so get estimates of the parameters of equation (12).

Finally, we may note that simulations from the model display some conflict between the goals of full employment and price level stability. Indeed, Liu notes presciently in a footnote that the results of the simulations and in particular the lag of price level changes behind real GNP movements "should perhaps qualify current [the article was published in July, 1963, and written some time before that] optimism about price stability." I would argue that even more of a conflict between the goals of price level stability and full employment would have appeared if Liu had allowed the rate of change of money wages to be endogenous to the system.²⁵ (In the

²⁴A telephone conversation with Professor Liu failed to resolve this difficulty, as Professor Liu was unable to produce the worksheets from which the results in the article were reproduced. Apparently old econometric models, like other old objets d'art, should be admired for their esthetic properties but not examined too closely. My own guess is that either one decimal point has been slipped in the constant term (so that it should be -1.974), or that two decimal points have been slipped (so that the constant term should be -0.1974) and that the original regression was computed from quarterly rates of change of prices and wages instead of annual rates of change, contrary to what the article reports. From some tentative calculations with trade-off curves based in part on Liu's price level equation, I should tend to guess that the second alternative is correct.

²⁵One can of course consider a small subsystem which combines Liu's price level equation (or a steady state version of it) with two other equations: a wage adjustment equation and an additional equation relating the discrepancy between desired and actual inventory investment to the economy-wide rate of unemployment. In the first version of this paper, this was done, utilizing both the O.B.E. wage adjustment [24] and a wage adjustment equation estimated by Perry [27]. Although the results were interesting, they are not worth reproducing here in light of the uncertainty regarding the actual magnitude of the constant term in Liu's price equation. In any case, this paper is already quite long.

simulations, the rate of increase of money wages is assumed to be 4 per cent per year throughout the simulation period, a magnitude that looks quite modest by standards at the end of the decade of the 1960's.)

7. Andersen and Carlson's Monetarist Model [1]

This quarterly model is in sharp contrast with all of the other models of the U.S. economy reviewed in this section. The authors admit that the model is formulated with a number of monetarist hypotheses in mind: the long run neutrality of the growth of money on the growth of real output, the much greater impact (at least after several quarters) of variations in the money stock relative to fiscal policy changes, the view that "the economy is basically stable and not necessarily subject to recurring periods of severe recession and inflation." The model itself is a small scale one, with four stochastic equations and four identities or definitional relationships. The four behavioural equations are estimated by the method of ordinary least squares, with the lag structure estimated by means of the Almon lag technique. The sample period runs from the first quarter of 1955 through the fourth quarter of 1969, with the exception of the total spending equation, the parameters of which are estimated from data beginning in the first quarter of 1953.

Before examining the price level equation of this model in some detail, it is worth noticing two additional features of this model. First, the model has only three exogenous variables: changes in the money supply, changes in high-employment federal government expenditures (which represent federal expenditures adjusted to remove the influence of varying levels of economic activity on unemployment benefit payments), and the level of potential (full employment) output. Secondly, the price level equation is central to the workings of the model, as this equation separates the increase in total money GNP (from the total spending equation, which relates this increase to a

distributed lag in money stock increases and to a distributed lag in high-employment federal government expenditures) into price level and real output components, via a total spending identity. Once the increase in real output has been determined, the current rate of unemployment can be determined from a third structural equation, the unemployment rate equation, which relates this rate to the levels of the potential GNP gap (the difference between potential and actual GNP as a percentage of potential GNP) of the current and the immediately preceding quarters. The fourth structural equation, an interest rate equation, is used in the price level equation in an interesting but indirect manner, to be explained below.

How then is the price level determined in this model? The price level variable explained by this equation is the dollar change in GNP in current prices due to price level changes, which is the absolute change in the GNP deflator multiplied by the level of real output in the preceding quarter, if my algebra is correct. This variable is related linearly to a distributed lag in a demand pressures variable and to an anticipated price change variable. The demand pressures variable is defined to be the increase in total nominal GNP between the current and the immediately preceding quarter minus the difference between current potential GNP (in real terms) and real GNP in the preceding quarter.²⁶ The anticipated price change variable is an estimate

²⁶At first glance, this would appear to be a queer demand pressures variable, with the increase in spending (an approximation to the increase in ex ante demand) in nominal terms and the gap between potential output in the current period and actual output in the preceding period (an approximation to the increase in ex ante supply) in real terms. Actually, as an appendix to the article explains, Anderson and Carlson view the role of an increase in aggregate demand as making the economy travel up an aggregate supply curve (such as that derived in [4], Chapter 2); the GNP gap part of this variable then adjusts for the well-known result that the slope of the aggregate supply function is steeper, the closer one approaches to the full-employment level of output. Thus this demand pressures variable would appear to be a satisfactory one, if one regards the appropriate formulation of demand pressures to be one in nominal terms.

of the price effect on the dollar change in total spending due to anticipated price changes; it is obtained by multiplying money GNP of two quarters previously by a distributed lag in rates of change of the GNP deflator for seventeen quarters in the past, commencing with the immediately preceding quarter. The rates of change of the GNP deflator for these seventeen quarters are scaled by an index of the unemployment rate, on the view that high unemployment rates tend to scale down the importance of a particular quarter's rate of inflation in the representative formation of expectations regarding future inflation.²⁷ The weights for this distributed lag in scaled rates of past price level changes are obtained from the long-term interest rate equation, on the view that price level expectations can be seen most clearly in the market for long-term bonds.

The results of the estimation of the parameters of the price level equation may be briefly discussed. The anticipated price change variable is highly significant as an explanatory variable, with a marginal coefficient of 0.86, suggesting that price level changes are quite sensitive to anticipations of this phenomenon. The demand pressures effect is formulated as an Almon distributed lag in the demand pressures variable, using observations on this explanatory variable running from the current quarter to five quarters previously. The weights on the individual values of this variable decline with the passage of time, with the sum of the weights and the first four individual weights being statistically significant, by conventional standards.

²⁷Formulating the price anticipations variable in this manner would almost appear to guarantee the existence of a short run trade-off curve between the rate of change of the price level and the level of the unemployment rate. That this is in fact so is seen in Andersen and Carlson's short run simulations, which are discussed briefly below.

Finally, it is interesting (and possibly important) to note that the constant term is positive and statistically significant, by classical standards; the point estimate implies that, on average over the sample period, rising prices will add \$2.70 billions to the quarterly increase in nominal GNP (at an annual rate), even when the values of the demand pressures and the anticipated price change explanatory variables are zero. The economic significance of this result is further discussed below.

Before looking at some model properties, it is interesting to note that, in 19 pages of text and appendix, the authors do not mention the term "wages," "money wages," or "wage rate" once. This virtual neglect of wages and labour costs is, of course, typical of the monetarist approach to problems of inflation. However, Andersen and Carlson are not as extreme as other members of the monetarist school. In their Appendix A, in which their price equation is discussed, Andersen and Carlson interpret their anticipated price change variable as an influence on price level changes other than demand pressures. In terms of their aggregate demand and aggregate supply model, this variable would capture leftward shifts in the aggregate supply schedule, which in turn could reflect "the influence of past prices on current pricing policies of firms and factors of production." The phrase, "pricing policies ...of factors of production," would presumably include money wage movements.

Andersen and Carlson perform two sets of simulations within the sample period that are of special interest. First, they examine the ability of the model to track the key variables (rates of change of nominal GNP, real GNP, and the implicit deflator of GNP) over the entire sample period, 1955 I through 1969 IV. While the model's ability to track these key variables is regarded by them as generally satisfactory (and I should not disagree in general),

one feature of the implicit price deflator simulations should be emphasized. For the period running from late 1960 through much of 1964, the model predicts negative rates of change of the implicit deflator (actual decreases in this price level variable), while the actual increases in the GNP deflator during this subperiod ranged from 0 to slightly over 2 per cent per year, with absolutely stable prices (as measured by a zero value of the rate of change of the GNP deflator) being attained in only one quarter. This discrepancy between the model and the economy would lead one to question the empirical applicability of the long run simulations of the model, which are briefly discussed in the next paragraph. Andersen and Carlson also simulate the eight quarters of the 1963-1964 subperiod, comparing their results with those of similar simulations of the Wharton Econometric Forecasting Model [12]. By the criterion of either the average absolute error or the root mean squared error, the ability of the Wharton model to explain variations is roughly twice as great as that of Andersen and Carlson's monetarist model. However, before one concludes that the monetarist explanation of the price level is definitely inferior, a caveat is in order: this monetarist model is much smaller than the Wharton model, with far fewer exogenous variables. Consequently, it is possible that the superior performance of the Wharton model merely reflects its much greater detail of specification, along with the exogeneity of some price levels in the Wharton model (e.g. international prices, the implicit deflator for government purchases, and the level of prices received by farmers).

It is interesting to examine the trade-off question in the context of a model like this. As noted above, the formulation of the price anticipations variable in the way in which it was done almost guarantees the presence

of a short run trade-off curve. This may be seen in Andersen and Carlson's short run simulations of the U.S. economy with their model, commencing in the fourth quarter of 1969 and running to the fourth quarter of 1971. If the money stock is held constant over this period, then by the fourth quarter of 1971 the unemployment rate is predicted to be 7.7 per cent of the labour force and the GNP deflator is expected to rise at the rate of 1.9 per cent per annum, given the history of the U.S. economy over the second half of the decade of the 1960's. By contrast, if the money stock is allowed to grow at the rate of 6 per cent per year over these eight quarters, then the unemployment rate in the fourth quarter of 1971 is predicted to be only 5.7 per cent, while the annual rate of rise of the GNP deflator would be expected to be 3.8 per cent. It is in the long term, however, that dramatic differences with neo-Keynesian econometric models appear. Based on their thirty year simulations (reported but not shown in the article), Andersen and Carlson argue that there is no long run incompatibility between high employment and price stability. They report that these long run simulations show the rate of growth of output moving toward its potential (full employment) rate, unemployment settling at an equilibrium rate, and the price level growing at a rate determined by the rate of growth of the money stock. While a perusal of Andersen and Carlson's supplementary charts (which they were kind enough to send me) indicate that this is a valid interpretation of their model,²⁸

²⁸At first glance, it seems difficult to reconcile the results of these simulations with the significant positive constant term in Andersen and Carlson's price level equation. However, when one recalls that the constant term refers to a constant arithmetic contribution of rising prices, it becomes believable that this arithmetic magnitude would become swallowed in a growth context. It is worth noting, however, that Andersen and Carlson's alternative price level equations do not possess the property that the unemployment rate will stabilize (within the horizon period) at the same level under varying rates of growth of the money stock.

the question remains as to how valid a conclusion this finding is for the "real" U.S. economy. Does it assume a degree of downward flexibility in the GNP deflator that may perhaps be unwarranted in view of the model's failure to track the GNP deflator over the 1960-1964 subperiod, which was mentioned above?²⁹ In any case, is it worth paying the price of unemployment in terms of foregone output and social tensions for 15 years, 30 years, or however long a period of time may be necessary in order to attain a neo-classical nirvana of the sort described by Andersen and Carlson?³⁰ The critical nature of this final comment should not be interpreted as my overall evaluation of this model; Andersen and Carlson have done a very nice job in putting together a quantitative, operational alternative to the Neo-Keynesian models which today dominate the field of econometric modelling of the economy. Time, and not this writer, shall yield the definitive judgment in this debate.

²⁹ Thus, speaking personally, I just don't believe that a continued 2 per cent rate of growth of the money stock would eventually lead to high employment, a 4 per cent rate of growth of output, and a 2 per cent decline of the price level! In my view, the wage and price setting institutions of the U.S. economy don't possess that sort of flexibility, and no model (estimated outside this range of experience) is going to convince me otherwise.

³⁰ Andersen and Carlson have kindly sent me two sets of long term simulations of their model, which apparently differ slightly in some unspecified way. One set suggests a convergence of the unemployment rate at a value of 4.8% in 1989 (19 years after the beginning of the simulations in 1970). The second set suggests a convergence of the unemployment rate at a 4.3% level, but not until the year 2005 (35 years after the beginning of the simulations)! For what they are worth, Andersen and Carlson's very long run simulations suggest that eventually (i.e., after the year 2020) the unemployment rate might actually be expected to be lower, with the lower rate of growth of the money stock.

D. Japan

1. The Klein-Shinkai Model [22]

This is a medium-small model of the Japanese economy, with 15 structural equations and 7 identities. The equations of the model were estimated from annual data on the Japanese economy for the two disjoint periods 1930-1936 and 1951-1958, by means of the technique of two stage least squares (with selected variables for the first stage regressions). The wage-price sector of this model consists of two equations, a wage adjustment relationship explaining the relative change in the nonagricultural wage rate and a price markup relationship explaining the level of the implicit deflator of GNP. Exogenous price variables in this model are five in number, of which the only one directly relevant for the determination of the absolute price level is an index of import prices.³¹

The wage adjustment relationship is of standard form, indicating that wages are sensitive to both labour market demand pressures (as measured by the unemployment rate) and to pressures for adjusting money wages to changing price levels. Klein and Shinkai estimate the following wage adjustment equation (where the parameters have been adjusted to make all variables in percentage terms):

$$(14) \quad \dot{W} = 26.0 - 16.98 U + 0.746 \dot{P} .$$

Here \dot{W} is the annual percentage change in the nonagricultural money wage rate, \dot{P} is the annual percentage change in the GNP deflator, and U is

³¹The four other exogenous price variables enter only in ratio form, viz. the ratio of the world export price index to the Japanese export price index in the export demand equation and the ratio of the domestic price of rice to the imported rice price in the food import demand equation.

the level of unemployment as a proportion of the labour force. The level of statistical significance of the unemployment rate variable is quite marginal, with a t ratio of 1.57, in contrast to the two other econometric models of the Japanese economy reviewed in this major section.

The price level equation relates the implicit deflator of GNP to non-agricultural labour costs per unit of output, to the level of import costs per unit of output (total nominal value of imports divided by real net national product), and to a prewar-postwar dummy variable (D). As Klein and Shinkai explain, this is a generalization of a mark-up pricing hypothesis applied to an open economy, with the elements of prime cost represented by both labour costs and the costs of (largely intermediate) imports. Similar relationships have been estimated for the Canadian economy in both the TRACE model [7] and the Bank's model [17] and for the British economy in the Klein-Ball-Hazlewood-Vandome model [20]. The original price level equation, as estimated by Klein and Shinkai, is:

$$(15) \quad P = 0.370 + 0.995 \frac{W}{A} + 0.73 P_i \frac{M}{Y} + 124.4 D,$$

where P is the price level (the implicit deflator of GNP); A is non-agricultural output per unit of non-agricultural employment, which we shall interpret as a labour productivity measure; P_i is the import price level; M is the level of imports in real terms; Y is real net national product; and the other two symbols have been previously defined. All the regression coefficients of this equation are highly significant, by conventional standards. Differencing equation (15), we obtain:

$$(16) \quad P - P_{-1} = 0.995 \left[\frac{W}{A} - \left(\frac{W}{A} \right)_{-1} \right] + 0.73 \left[P_i \frac{M}{Y} - \left(P_i \frac{M}{Y} \right)_{-1} \right],$$

where the operator "-1" lags the variable in question one year. In turn, we

can write this equation in percentage rate of change form as follows:

$$(17) \quad \dot{P} = \frac{\left(\frac{W}{A}\right)^{-1}}{P_{-1}} 0.995 \dot{\left(\frac{W}{A}\right)} + \frac{\left(P_i \frac{M}{Y}\right)^{-1}}{P_{-1}} 0.73 \left(P_i \frac{M}{Y}\right) \dot{\quad},$$

where the dot operator indicates the percentage rate of change of the variable in question between two adjacent years, e.g. $\dot{P} = \frac{P - P_{-1}}{P_{-1}} \cdot 100$. Substituting 1958 values for the shares of labour costs and import costs in the GNP deflator and using the logarithmic derivative formula as an approximation for the percentage rate of change of a product and a quotient, we obtain the form of the price level equation utilized for trade-off calculations:

$$(18) \quad \dot{P} = 0.5659 (\dot{W} - \dot{A}) + 0.0927 \left[\dot{P}_i + \left(\frac{M}{Y}\right) \right]^{32}.$$

We are now ready to calculate the estimated trade-off curve. Substituting equation (14) into equation (18) and solving for \dot{P} , we obtain:

$$(19) \quad \dot{P} = 25.45 - 16.63 U - 0.9792 \dot{A} + 0.1605 \dot{P}_i + 0.1605 \left(\frac{M}{Y}\right) \dot{\quad}.$$

In order to reduce this to a two-dimensional curve, we need estimates of the rates of change of labour productivity and import requirements per unit of output.

³²In discussing their results, Klein and Shinkai asserted that as measured by the elasticity concept, the Japanese economy appeared to be highly sensitive to the influence of variations in import costs, in comparison with results for the U.K. or Dutch economies. However, by the interpretation of this paper, the influence of a variation in import costs is fairly marginal; a 1 per cent per year rise in import costs results in only a slightly higher rate of inflation in Japan, one that is only 0.09 percentage points greater than it otherwise would have been. This is much smaller than the estimate from the TRACE econometric model for the Canadian economy (0.43) or Dicks-Mireaux's estimate (0.20) for the British economy, which is employed below. The difference between this interpretation and Klein and Shinkai's own interpretation is the use of the elasticity measure; since imports are quite small in relation to net national product in Japan, this would tend to make the elasticity measure large but to reduce the share of import costs in the GNP deflator.

For the postwar period in the sample to which the model was fitted, we have estimated a rate of growth of labour productivity of 2.88 per cent per annum and a rate of growth of import requirements per unit of output of 8.05 per cent per annum. With these values substituted for \dot{A} and $\left(\frac{\dot{M}}{P}\right)$ respectively in equation (19), the trade-off equation may be written as follows:

$$(20) \quad \dot{P} = 23.92 - 16.63 U + 0.1605 \dot{P}_i .$$

In turn, we may represent equation (20) either in tabular form or as a two-dimensional curve, once we select a value for the rate of change of the exogenous import price index. This is done in Table 4 and Figure 5, respectively, where we have used 0 and 3 per cent per annum as alternative values for the rate of change of import prices. I have followed the judgment of Klein and Shinkai by taking a 5:1 ratio in adjusting the Japanese rate of unemployment to equivalent North American ratios. Looking at Table 4 and Figure 5, one can see immediately that there would appear to be a conflict between the goals of full employment and stable prices; at a 1 per cent rate of unemployment (equivalent to a level of unemployment equal to four or five per cent of the labour force, by North American standards), prices are predicted to rise at a rate over 7 per cent per year, while an unemployment rate greater than 1.4 per cent (equivalent to a rate above 7 per cent, in North American terms) is required to hold the price level stable, according to the model. It may also be noted that, by Canadian or even British standards, Japan would appear to be relatively insulated from the influence of foreign inflation, as judged by the chart or (equivalently) by the low coefficient (0.16) on the rate of change of import prices in equation (20). It might also be noted that the anterior trade-off relationship, equation (19), suggests that rapid domestic productivity growth does have a strong influence in mitigating the conflict between the objectives of stable prices and high employment.

TABLE 4

Predicted Rates of Change of Net National Product Deflator, at Varying Rates of Unemployment with Two Selected Values of the Rate of Change of an Index of Import Prices, Klein and Shinkai Econometric Model of Japan.

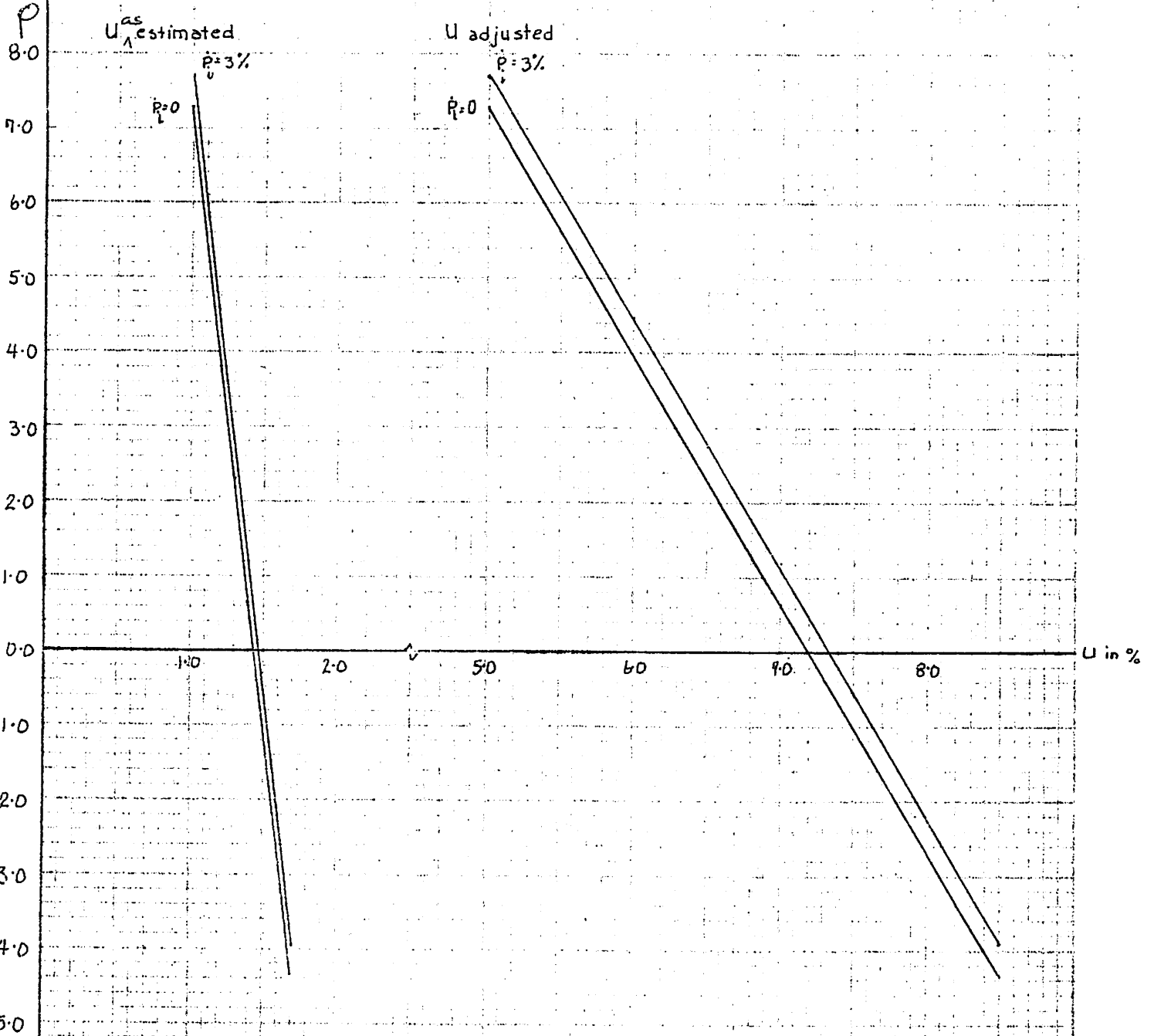
Original Unemployment Rate (Per Cent of Labour Force)	Unemployment Rate Adjusted to North American Standards*	Expected Annual Increase of Net National Product Deflator When:	
		$\dot{P}_i = 0$	$\dot{P}_i = 3.0\%$
1.0	5.0	7.29	7.77
1.1	5.5	5.63	6.11
1.2	6.0	3.97	4.45
1.3	6.5	2.30	2.78
1.4	7.0	0.64	1.12
1.5	7.5	-1.02	-0.54
1.7	8.5	-4.35	-3.87

* Unemployment rate adjusted to a North American Standard on the basis of a 5:1 ratio, based on the judgment of Klein and Shinkai.

Source: See the text, especially equation (20).

FIGURE 5

KLEIN & SHINKAI Model



Two Trade-off Curves for the Japanese Economy, Based on the Klein-Shinkai Econometric Model, ~~based on~~ ^{based on} two Assumed Values of the Rate of Change of the Index of Import Prices (\dot{P}_1).

2. Watanabe's Two Equation Model [33]

The inclusion of this interesting study probably requires some justification. A number of other interesting pieces of research have not been reviewed in this paper, on the grounds that we were specifically concerned with the wage and price mechanisms of econometric models. Although one can view Watanabe's work as a two equation econometric model, there are other very small simultaneous equation models of the wage-price sector of these four countries that have not been summarized in this paper.³³ Perhaps the most ready justification for including this study is that it furnishes a check on the Klein-Shinkai trade-off curve, which was computed in the preceding section. In addition, the two equations of Watanabe's model are very similar to two of the nine wage and price equations of the Tatemoto-Uchida-Watanabe model reviewed in the following section. In any case, I shall eschew consistency, that hobgoblin of small minds, and proceed.³⁴

Watanabe's wage adjustment equation is reasonably straightforward. Letting \dot{W}_t be the annual percentage change in manufacturing average wage earnings over corresponding quarters of adjacent years, U_t be the percentage of total employees who received unemployment insurance in quarter t ,³⁵ and

³³Two very fine small scale econometric models of this sort which come to mind are Vanderkamp's three equation model of wage and price formation in the Canadian economy [32] and L. A. Dicks-Mireaux's two equation model for the U.K. economy [9a]. (Perhaps I have not reviewed Vanderkamp's study because several colleagues and I have already summarized this study in [5], pp. 63-67, 69, and 86-88.)

³⁴Unfortunately, eschewing consistency probably does not guarantee that one is a great mind; there are lots of small minds who cannot be charged with the "fault" of consistency.

³⁵For various reasons, Watanabe does not like the official labour force statistics on unemployment and prefers to work with this measure.

P_t the consumer price index at time t , Watanabe estimates the following relationship by ordinary least squares from quarterly data on the Japanese economy from 1956-IV through 1962-IV:

$$(21) \quad \dot{W}_t = 16.73 - 2.97 U_{t-1} + 1.36 \left[\frac{P_{t-1} - P_{t-2}}{P_{t-2}} \cdot 100 \right].$$

By usual standards, both of these explanatory variables are statistically significant. By the t ratio measure, the lagged unemployment rate variable, which Watanabe interprets as expressing the role of demand pressures in the labour market, is the more significant explanatory variable. The consumer price change variable is interpreted by Watanabe as being "a strategic indicator of union power," which is used to press for cost of living adjustments. Two subsidiary findings of Watanabe's study are of interest: first, including the consumer price change variable without a lag (and estimating by the method of two stage least squares, to avoid simultaneous equation inconsistencies) gave inferior results. Secondly, formulating the role of the unemployment variable in a reciprocal form gave slightly inferior results, over the range of data studied. For purposes of computing a trade-off curve we shall want to reformulate slightly Watanabe's wage adjustment equation. Let \dot{P}_t be the annual percentage rate of change of the consumer price index, for quarter t , over corresponding quarters of adjacent years. Then, if one assumes that the consumer price index has been changing at a uniform rate over the past four quarters, one can rewrite equation (21) as follows:

$$(22) \quad \begin{aligned} \dot{W}_t &= 16.73 - 2.97 U_{t-1} + 1.36 \left(\frac{\dot{P}_{t-1}}{4} \right) \\ &= 16.73 - 2.97 U_{t-1} + 0.34 \dot{P}_{t-1} . \end{aligned}$$

The form of the price level equation selected as "best" in Watanabe's article was as follows:

$$(23) \quad \dot{P}_t = -2.72 - 0.18 \dot{A}_t + 0.70 \dot{W}_t,$$

where \dot{A}_t is the annual percentage rate of change, over corresponding quarters of adjacent years, in an index of labour productivity in manufacturing industries. This equation was not estimated by ordinary least squares techniques; instead, the predicted values of the dependent value of equation (21) were utilized as a regressor in place of the \dot{W}_t variable. (This procedure, which is analogous to the two-stage least squares technique, obviates simultaneous equation biases which can arise if the universe disturbance terms of equations (21) and (23) are intercorrelated.) According to the t ratios, both variables are highly significant by usual tests, although the wage change variable is the slightly more important explanatory variable. Watanabe's price level equation can be regarded as a variant of a labour cost mark-up equation, although he resists this interpretation. Instead, he prefers to interpret it as the reduced form of price determination in competitive markets for final goods and services, with the productivity change variable representing supply-side shifts and the wage change variable representing demand-side shifts.³⁶

³⁶Watanabe tests as well as he can the issue of whether the influence of the wage change variable emanates from the demand side or the cost side, presenting some additional results which support, in his view, the demand side hypothesis. He admits, however, that this conclusion is tentative and debateable. In my view, there is a paradox that is still unresolved in Watanabe's interpretation. Even according to the competitive hypothesis, the supply curve will shift if the factor price of a key input changes. If the supply side is responsive to one element of labour cost (namely output per man-hour), why isn't it at least equally responsive to a more visible element of labour cost, namely the money wage rate?

We may combine the wage and price equations (equations (22) and (23)) in order to derive a trade-off equation, which shows quantitatively the conflict between the goals of full employment and stable prices. Such a calculated trade-off equation is independent of the issue of whether the wage change variable should be interpreted as a demand-side or cost-side variable. Indeed, Watanabe's own discussion of these two relationships suggests the existence of a conflict between stable prices and full employment for the Japanese economy although he stopped short (perhaps wisely) of computing the explicit curve.³⁷ Substituting equation (22) into equation (23), we obtain:

$$(24) \quad \dot{P}_t = 8.991 - 0.18 \dot{A}_t - 2.079 U_{t-1} + 0.238 \dot{P}_{t-1} .$$

The steady-state particular solution of this first order difference equation is:

$$(25) \quad \dot{P}_t = 11.7992 - 0.2362 \dot{A}_t - 2.7283 U_{t-1} .$$

This is the final trade-off equation. Once the magnitude of the productivity change variable \dot{A}_t has been specified, we can reduce this trade-off equation to a two-dimensional curve. Two values for \dot{A}_t were in fact utilized: a fairly modest level of 4 per cent per year and the sample mean over the period, 8.1 per cent per annum. The results have been tabulated in Table 5 and plotted in Figure 6.³⁸

³⁷ However, the trade-off relationship is an algebraic implication with which it is difficult to argue, if one accepts the premises. Thus the immediate trade-off curve, equation (24), is no worse (nor no better) than the two relationships from which it is derived. Of course, one is entitled to object to the calculation of a steady-state relationship, such as equation (25) above.

³⁸ Watanabe's unemployment rate has been put on an equivalent North American basis by a two step procedure. First, Watanabe's measure of the unemployment rate was calculated to average 2.33 times as much as the figures reported by Klein and Shinkai, for the four years for which overlapping data were available from the two articles. Secondly, the Watanabe rates were divided by this factor and multiplied by 5.0, to get a final correction factor (2.15) which could then be applied to Watanabe's measure to get equivalent North American rates of unemployment.

TABLE 5

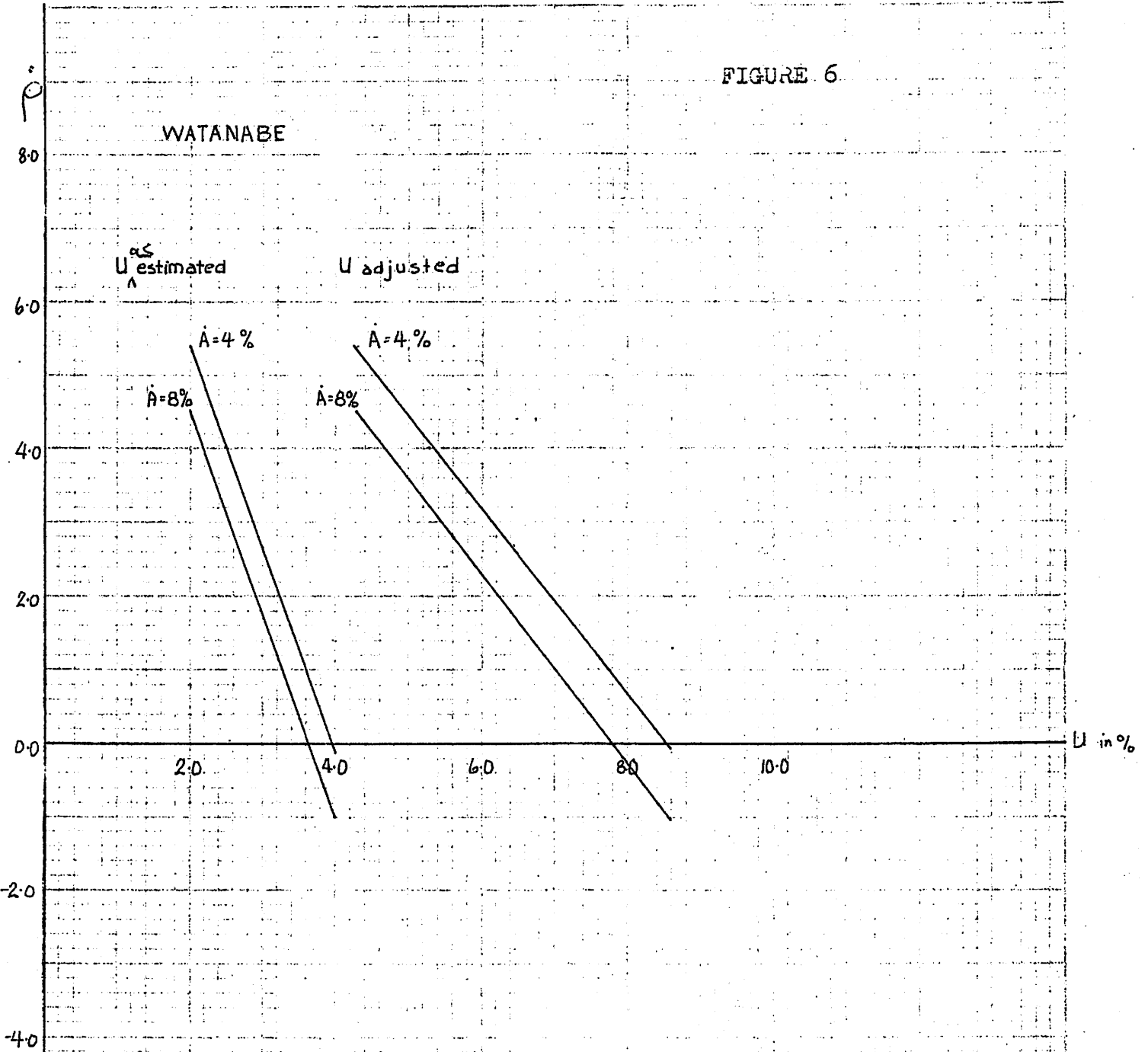
Predicted Rates of Change of Consumer Price Index, at Varying Rates of Unemployment with Two Selected Values of Productivity Growth Rate (\dot{A}_t), Watanabe's Model of Japanese Wage-Price Sector

Original Unemployment Rate (Per Cent)	Unemployment Rate Adjusted to North American Standards*	Expected Rate of Growth of Consumer Price Index When:	
		$\dot{A}_t = 4.0$	$\dot{A}_t = 8.1\%$
2.0	4.30	5.40	4.45
2.5	5.38	4.03	3.09
3.0	6.45	2.67	1.72
3.5	7.53	1.31	0.36
4.0	8.60	-0.06	-1.00

* For method of adjustment, see the text, especially footnote 38.

Source: See the text, especially equation (25).

FIGURE 6



Two Trade-off Curves for the Japanese Economy,
Based on Watanabe's Study [33], with Two Assumed
Values of the Rate of Growth of Labour Productivity
(\dot{A}).

The results of this exercise may deserve some brief comment. Watanabe would appear to be quite correct in arguing that his results suggest an incompatibility between full employment and stable prices. Prices are not expected to remain stable until the rate of unemployment reaches approximately 8 per cent of the labour force, on an adjusted North American basis; moreover, even at a 5 per cent (equivalent North American) rate of unemployment, one can expect the consumer price index to rise by roughly 4 per cent per annum. The degree of incompatibility is of the same order as that shown by Klein and Shinkai, although the Klein-Shinkai trade-off curve is steeper: thus the expected rate of inflation at a 5 per cent (equivalent North American) unemployment rate is over 7 per cent per annum, while completely stable prices require an unemployment rate that is over 7 per cent, on an adjusted North American basis. (It should be recalled that the Klein-Shinkai price level is the implicit deflator of NNP, which of course is a slightly different concept from that of the consumer price index.) Watanabe's trade-off curves (or, equivalently, equation (25)) indicate a modest role for more rapid productivity growth in mitigating the conflict between stable prices and full employment.³⁹

³⁹If one derives Watanabe's trade-off curve from the wage adjustment equation in which the unemployment rate appears in reciprocal form, then the results are very similar, over the range of the unemployment rate experienced during the sample period. For example, the expected rate of inflation at a 2 per cent rate of unemployment (Watanabe's original measure of the unemployment rate) is either 5.6 or 4.6 per cent per year (depending on the productivity growth rate assumed), instead of the 5.4 or 4.4 estimates from Table 5. Similarly, the expected rate of change of consumer prices at a 4 per cent rate of unemployment (Watanabe's measure) is either 0.3 per cent per annum or -0.7 per cent per annum, again depending upon the productivity growth assumptions. This compares with the estimates from Table 5 of -0.1 and -1.0, respectively.

3. The Tatemoto-Uchida-Watanabe Medium-Term Model: [8] and [30a]

This model is a medium-size model of the Japanese economy, fitted to the 18 half-years between the first half of 1954 and the second half of 1962. Where necessary, seasonal adjustment is performed right in the estimation of the structural equations of the model, by means of a single dummy variable employed as a regressor. In the journal article version [30a], there are 26 structural equations and 17 identities in the full model. The methods of parameter estimation are ordinary least squares (for all 26 structural equations), and limited information and two stage least squares estimates for the 19 equations in which there are right-hand side endogenous variables. The wage-price sector of this model consists of one wage equation, five behavioural price equations, and one identity. The only exogenous price levels in the model are the import price index (which has the principal price level significance) and the index of the world price of manufactured goods (which enters only in ratio form in the export demand equation).

The wage adjustment equation explains the percentage change, over corresponding half-years of adjacent calendar years, in the money wage rate (unstandardized for hours worked) of all employees in the economy. (This wage rate is thought of as applying mainly to the progressive, industrial sector of the Japanese economy.) The explanatory variables are the rate of unemployment and the current rate of change, over corresponding half-years of adjacent calendar years, of the consumer price index. The interpretation of these two explanatory variables probably needs no additional elucidation by this point in this paper. Both explanatory variables are significant, although the consumer price change variable appears to be slightly more important, as judged by the t ratios. Also, an interesting implication of the Tatemoto-Uchida-Watanabe estimates of the wage adjustment equation is that the estimates of the

coefficient of the consumer price change variable are insignificantly different from unity. In turn, this implies that, for the Japanese economy, variations in the rate of change of money wages are fully adjusted for variations in the rate of change of consumer prices or that (as the authors point out) the market adjustment mechanism could just as well be formulated in terms of real wages. It should be noted that this result is not in strong agreement with either that of Klein-Shinkai or especially with that of Watanabe's study reviewed in the preceding section.

There are price level equations to explain the consumer price index, the wholesale price index (which is used for the valuation of inventory investment), the investment goods price index, the implicit deflator of private housing construction, the export price index, and the implicit deflator of GNP. Most of the five structural equations have an interesting admixture of demand and cost elements. The consumer price index basically explains the annual percentage (over corresponding half-years of adjacent years) in terms of the annual percentage change in the money wage rate (which has a marginal coefficient of 0.60 to 0.66) and the annual percentage change in a productivity variable, real GNP per employed member of the labour force (with a marginal coefficient of -0.16 to -0.25). Both explanatory variables are statistically significant, although the wage change variable is far more important, as judged by the t ratio. Although I should interpret this as almost exclusively a cost pressures equation, Tatemoto-Uchida-Watanabe, following Watanabe [33], interpret the wage change variable as an indicator of demand pressures.⁴⁰ The wholesale price index,

⁴⁰It is interesting to note that the wage adjustment equation and the consumer price change equation form a small subsystem from which one can derive a trade-off relationship which relates the rate of change of consumer prices to the level of the unemployment rate and to the rate of change of real GNP per employed member of the labour force (labour productivity). Although this equation will not be presented explicitly in this discussion, some of its implications are discussed briefly below.

explained in level form, is positively related to the value of imported materials and fuel per unit of output in the mining and manufacturing industries (which is obviously a cost variable) and is negatively related to the interest rate, which the authors regard as an indicator of demand pressures. (The authors view high interest rates as symptomatic of difficulties in financing and monetary tightness which will tend to make current inventories excessive and give rise to adjustments in wholesale prices.) According to the t ratios, both explanatory variables are statistically significant, with the level of imported materials unit costs being much more important as an explanatory variable. The investment goods price index is explained in level form; in [30a] it is positively and significantly related to the wholesale price index of the preceding half-year and negatively and significantly related to the current interest rate.⁴¹ Hence the prices of investment goods would appear to be explained by the same melange of demand pressures and cost forces as most of the other price indexes. The implicit deflator of private housing construction is explained as an annual rate of change over corresponding half-years of adjacent calendar years; the explanatory variables are the annual rate of change of the investment goods price index and the annual rate of change of the money wage rate. Hence the explanation of this price level would appear to run entirely in terms of cost pressures. Judged by both the t ratios and also the marginal coefficients (1.4 vs. 0.5), the annual rate of change of the money wage rate would appear to be the more important explanatory variable in this equation. The export

⁴¹In [8], there is an alternative explanation of the price index of investment goods. Here, this price level is explained by the value of all imports per unit of real GNP, by total wage payments per unit of real GNP, and (negatively) by the interest rate. This version doesn't appear to be as good an explanation, either in terms of the degree of autocorrelation of the residuals or in terms of the tightness of the fit.

price index equation is interesting for what it leaves out; not included as an explanatory variable is some measure of the world price of competing exports. This price level is explained as an annual rate of change over corresponding half-years of adjacent calendar years. The rate of change of the export price index is significantly related to the annual percentage change in the wholesale price index, with a marginal coefficient of roughly unity, and negatively and significantly related to the deviation from trend in the ratio of the stock of inventories to real GNP. As the latter variable measures demand pressures because it is a reasonable proxy for the theoretical construct of excess supply, we again have an admixture of demand and cost elements. Judged by the t ratios, the cost variable (the percentage change in the wholesale price index) would appear to be the more important explanatory variable.⁴² Finally, the implicit deflator of gross national product can be written as the ratio of the appropriate current dollar to constant dollar magnitudes. Thus this deflator can be interpreted as a weighted average (with variable weights) of the preceding five price indexes, together with the import price index (which directly has a negative weight in this calculation).⁴³

⁴²Of course, if the prices of export goods were determined in moderately competitive markets, there should be some influence of the prices of competing goods from other exporting countries, through the well known relationships of substitution.

⁴³One interesting relationship that I have not discussed in the text is the authors' wage share relationship, in which the share of employees in nominal GNP is related (negatively but with marginal statistical significance) to past growth in fixed capital formation, which is viewed as a carrier of non-neutral technological change. As the authors point out, in other contexts this equation might have a significance for the general price level (the implicit deflator of GNP) but that variable is taken care of by the ratio of nominal to real GNP as explained above. Instead, this relationship plays the role of a short run production function in neo-Keynesian analysis: given real GNP (and also, in this case, the money wage rate and the implicit deflator of GNP), one can determine the number of employees required. Since one can view a constant wage share as arising from the maximization of profits subject to a Cobb-Douglas production function (in competitive factor and product markets), this interpretation does not seem unreasonable.

Before looking at the issue of the possible conflict between objectives, it is worth noting the authors' own evaluation of their wage-price sector. Tatemoto, Uchida, and Watanabe single out this sector, along with their inventory investment and interest rate equations, as the weakest part of their model. In their forecast for the first post sample year, 1963, they found that the model estimated reasonably accurately (within two per cent of the actual figures) the consumer price index, the wholesale price index, the export price index, and the money wage rate, although the investment goods price index and the implicit deflator of private housing construction were badly over-estimated. However, when one looks at the ability of the model to "track" the values of the variables within the sample period, a different picture emerges. The graphs of the actual series against the model's predicted magnitudes suggest for the consumer price index and the money wage rate, particularly the former, that the time path of these variables are not particularly well replicated by the model. In particular, the model fails to capture the sharp acceleration in the rate of change of the consumer price index, which began in early 1961 and which has largely continued unabated throughout the decade of the 1960's. Indeed, Tatemoto, Uchida, and Watanabe admit that these simulations may be an indication of "significant structural change in the price mechanism of the Japanese economy," which a translation from the jargon of the econometrician would probably render explicit the point that the theories of price determination in this model (like those in the other models reviewed in this paper) are only at best partial and incomplete theories.

Finally, to return to the unifying theme of this paper, there appear to be definite conflicts between the goals of full employment and stable (consumer) price levels in this model. Indeed, a glance at the data in the authors' appendix will corroborate the view that, whatever misspecifications may be present

in the wage-price sector of this model, this conflict is not merely a vague suggestion from the model but appears to be present in the data for the Japanese economy also. The patterns of the influences are something like the following. High demand for the output of the Japanese economy will raise employment (even taking account of the induced shift out of agriculture) and so reduce the rate of unemployment. In turn, this will lead to a faster rate of rise of the money wage rate, which then spreads through the system. (In the model, the rate of change of the money wage rate directly affects the consumer price index and the implicit deflator of private housing construction.) There will be an added fillip to accelerated wage rate changes from the induced rise in consumer prices, while the rise in the money wage rate, to the extent that it raises the real wage rate (in terms of the GNP deflator) should lead to an induced increase in real GNP per employee, which will tend to offset somewhat the inflationary pressures. Finally, high demand may reduce the ratio of the stock of inventories to real GNP (relative to trend), giving a spur to export prices. It may be noted that the development of the wholesale price index, the investment goods price index, and the export price index (particularly the first two) appear to be largely independent of developments elsewhere in the economy, as the wholesale price index and also the investment goods price index affect, but are not affected by, the other price levels in the system. Thus, in discussing inflation in the Japanese context, it would appear to be important to specify the exact price level under discussion.⁴⁴

⁴⁴The trade-off relationship, discussed in footnote 40 above, suggests a slightly more optimistic picture for the Japanese economy than that of Klein-Shinkai or Watanabe. Assuming that real GNP per employed member of the labour force continues to grow at the 9 per cent per annum rate characteristic of the sample period, one can calculate that a 1 per cent rate of unemployment (roughly equivalent to a 5 per cent rate in North America, according to Klein-Shinkai) would be expected to give rise to a rate of change of consumer prices of 3 per cent per year, while a completely stable level of prices would require an unemployment rate of

One additional topic may be considered. The model gives the impression that price level developments in Japan are not strongly influenced by world price level developments, as gauged by the import price index. Thus this price level influences only the wholesale price index directly; although there are indirect influences on the investment goods price index, the implicit deflator for private housing construction, and the export price index, there is not even an indirect influence on the consumer price index, and private and government consumption comprise the bulk of real GNP (roughly 60 per cent, in the last sample year). The view that the import price level is not an important influence on the consumer price index or the implicit deflator of real GNP is corroborated by the existence of a downward trend in the import price index and an upward trend in the latter two variables, over the sample period.

E. The United Kingdom

1. The Klein-Ball-Hazlewood-Vandome Model: [21], [20], and [9a]

This quarterly econometric model of the United Kingdom economy has 31 structural equations and 6 identities. The structural equations were fitted to postwar data on the United Kingdom economy running from the first quarter of 1948 through the fourth quarter of 1956 by the method of limited information maximum likelihood or (in the case in which the right-hand side variables were all predetermined) by ordinary least squares techniques. One interesting feature of the estimation procedure was the use of seasonally unadjusted variables, with seasonal adjustment performed right in the process of estimation

$1\frac{1}{4}$ per cent of the labour force (roughly $6\frac{1}{4}$ per cent, in equivalent North American rates). Of course, these trade-offs could be expected to worsen if the rate of growth of labour productivity were to fall back from such a fast pace.

itself, by means of three seasonal dummy variables. A nice overview of the model itself is given by Professor Nerlove in his review article [9a]; following Nerlove, I shall refer to the authors collectively as KBHV.

The wage-price sector of the model, as I interpret it, consists of two wage equations, one principal price level equation, and 7 subsidiary price level equations, of which one is an identity. The exogenous price levels in the model are the price index of total imports, the price indexes of three of the components of imports, and the index of export prices of competing countries. (There are two other exogenous price levels, the index of domestic prices of manufactured goods in the dollar area and the index of world domestic food prices, which enter in relative form in the export demand equations and which therefore do not have direct price level significance.) Professors Klein and Ball published a study [20] in which they summarized the wage-price sector of the larger econometric model (aside from the subsidiary price level equations), arguing strongly that the interrelated structure of the model indicated the complex nature of postwar inflation and that it was naive and inaccurate to ascribe such inflation to any one source.⁴⁵

The two wage equations are a wage adjustment equation, in which the dependent variable is the absolute difference, between corresponding quarters of adjacent years, of an index of weekly wage rates in British industry, and a wage drift equation, which explains the spread between an index of average weekly earnings and the index of weekly wage rates. The absolute change in the wage rate index is negatively related to a four-quarter moving average

⁴⁵ I should probably warn the reader that I've had, singly or in combination, two previous passes at summarizing the wage and price relationships of this model. See [4], pp. 38-39 and 47-49, and [5], pp. 42-45 and 78-79. I shall draw on this latter summarization in the measurement below of a simple trade-off curve for the U.K. economy.

(with an average lag of one and a half quarters) of the absolute amount of unemployment, a four-quarter moving average (also with an average lag of one and a half quarters) of the absolute change (over corresponding quarters of adjacent years) in the consumer price level, and a "pushfulness" dummy variable. All of the explanatory variables (except for the seasonal dummies) are highly significant by conventional criteria. The level of unemployment variable represents, of course, the direct influence of market forces inducing excess supply in the labour market to moderate the rate of wage rise. Although Klein and Ball assert that there is little evidence of non-linearity in this relationship during the sample period, a later study by Ball [3a] suggests that the relationship may indeed be non-linear in the unemployment level once one goes beyond the fairly low levels of unemployment experienced in the immediate post-war period. The change in the price index of total consumption suggests that trade unions attempt to obtain compensating or nearly compensating adjustments in money wages for increases in the prices of consumer goods. The pushfulness dummy, which had a value of zero during a Labour government (1948-1951) and unity during a Conservative government (1952-1956), is the most interesting explanatory variable in this relationship. Taken at face value, it suggests that wage rates increased by an additional 2.90 index points under a Conservative government than under a Labour government, at given values of the other determinants of wage rate increases. The size and statistical significance of the coefficient of this explanatory variable suggests an important non-market element, namely trade union power, in the determination of wage rates in British industry.

Turning to the wage drift equation, we may note that the spread between wage earnings and wage rates is positively and significantly related to the

current value of an index of weekly hours worked and to the current value of an index of labour productivity in British industry. In that one regards hours worked as "a very sensitive indicator of the level of demand," as do Klein and Ball, the wage drift relationship suggests an additional source of pressure on wage costs under high aggregate demand conditions.⁴⁶

In the principal price level equation, which KBHV term an equation for the mark-up of prices over costs, the price index of total consumption is explained in level form. This variable is positively related to the index of average weekly wage earnings, with a marginal coefficient of 0.42, and to the level of import prices of two quarters previously, with a marginal coefficient of 0.22. Both of these variables are highly significant by usual tests, with the t ratios confirming the impression from the estimated marginal coefficients that the domestic wage costs are the more important explanatory variable. Aside from significant seasonality, the only other influence (in this equation) on the consumer price level is the ratio of indirect taxes net of subsidies to total consumption expenditures. While this variable had the expected positive direction of effect, it was quite insignificant from a statistical viewpoint. Klein and Ball offer two explanations of this result: first, they point out that the rate of net indirect taxation did not vary much over the sample period, so that perhaps the unimportance of this variable is merely a function of the historical period selected; secondly, it is possible that indirect tax rates, being selectively directed, may influence only the structure but not the level of consumer prices.⁴⁷

⁴⁶ In the larger system, there is an equation to explain weekly hours worked, which relates this variable positively to total industrial production, as well as to seasonal dummies.

⁴⁷ This suggestion today seems almost facetious, based on what we have learned from simulations of U.S. econometric models, such as the Brookings model or the Wharton model.

The 7 subsidiary price level equations, which explain in level form four components of the price index of total consumption (the indexes of food, drink, and tobacco; of consumer durables; of other consumer goods; and of consumer services), two export price indexes (those of total exports and of exports of food, drink, and tobacco), and the price index of fixed assets (investment goods), may be briefly discussed. The price index of food, drink, and tobacco is positively (but insignificantly) related to the price index of imported food, lagged two quarters, with a marginal coefficient of 0.11, and to a food deration dummy; the only significant explanatory variable in this relationship is net subsidies on food and agriculture, which has the expected effect of holding down the domestic price level of food. The price index of consumer durables is positively and significantly related to the general consumer price level and to the price index of imported non-ferrous metals, lagged two quarters; the latter explanatory variable has the higher t ratio although its marginal coefficient (0.15) is smaller than that of the domestic price level (0.34). The price index of other consumer goods is positively and significantly related to the general consumption price level, with a marginal coefficient of 0.78, and to the price index of imported basic metals, with a marginal coefficient of 0.06; judged by either the t ratios or the marginal coefficients, the former explanatory variable is the more important one. There is no behavioural equation for the price index of consumer services; however, since there is a behavioural equation for the general consumption price level, one can interpret the identity equating the general consumption price level to a fixed weight average of its four components as completing the system with regard to the price index of consumption services. The price index of investment goods is positively and significantly related to the wage earnings index, with a marginal

coefficient of 0.33, and to the price index of imported non-ferrous metals, lagged two quarters, with a marginal coefficient of 0.12; judged by either the criterion of the marginal coefficient or the t ratio, the wage earnings explanatory variable would appear to be the more important explanatory variable.⁴⁸

The price index of total exports depends on both the domestic consumption price level and (slightly more importantly, according to the t ratios) the index of export prices of competing countries, lagged two quarters; the marginal coefficients are 0.45 and 0.53, respectively. Thus, the KBHV model of the U.K. economy contrasts sharply with the models of the U.S. and Japanese economies reviewed in this paper on this particular point, while the picture of the structure of export pricing is very close to that of the Bank of Canada's model of the Canadian economy. Finally, the subsidiary price level equations of the KBHV model are completed by an equation which relates the price index of food, drink, and tobacco exports to the price index of total exports, with a marginal coefficient of 0.75 (highly significant, by the usual tests). In summary, six out of seven of these subsidiary price levels are tied directly or indirectly to the general price or wage level, which is the most important explanatory variable in the majority of cases. (The price index of domestic food, drink, and tobacco seems to possess its own determinants.) In all cases, the behavioural equations explaining the subsidiary price levels display a secondary determinant (or secondary determinants), which show individual influences peculiar to the development of the subsidiary price level, such as the prices of the exports of competing countries in the case of the price index of British exports.

⁴⁸The beginning-of-the period stocks of durable materials relative to the volume of production was used as an explanatory variable in some experimental estimates of this equation and of the equation for the price index of consumer durables. This variable had the expected negative direction of effect in both cases (with borderline statistical significance) and so might be interpreted as a proxy for the direct influence of excess supply on price formation in these two sectors.

We may comment briefly on the conflict between the objectives of full employment and stable prices, as indicated by the KBHV model. This point was of some interest to KBHV themselves, who devoted 16 pages of text (largely a reprinting of [20]) and a 15 page appendix to this issue. Qualitatively, the influences work as follows. High demand will reduce the unemployment rate, leading to a large absolute change in the money wage rate. High demand will also raise average hours worked and so increase the spread between money wage earnings and money wage rates. The higher money wage earnings are then marked up into higher prices of consumption goods; there is, however, no influence (in the model) of excess demand directly on the price level of final consumption goods and services. The rise in the price index of total consumption produces a secondary increase in the money wage rate, but the values of the coefficients are such that the system is dynamically stable (or at least non-explosive). The discussion of Appendix III of [21] formalizes the verbal discussion of this paragraph into the algebraic analysis of the wage-price subsystem of model, comprised of four structural equations in one case and five in the second case.⁴⁹ The result of this analysis is a seventh order, linear non-homogeneous difference equation with constant coefficients in the price index of total consumption; the non-homogeneous portion of this equation is comprised of the impacts of the absolute change in import prices (over corresponding quarters of adjacent years), lagged two quarters; the political restraint dummy variable; the absolute change in industrial production; and a constant term in one

⁴⁹Nerlove [9a] also breaks off a wage-price subsystem, which in his case is comprised of 8 structural equations. Nerlove utilizes his subsystem to discuss, in a fairly noncommittal fashion, the impact effects of two alternative anti-inflationary policies (wage restraint vs. a restriction of the volume of imports).

case and a four quarter average of the unemployment level in the other case.⁵⁰ The price level difference equation would appear to suggest (at least) four conclusions. First, there would appear to be a long lag in the response of the price level to any policies; initial conditions would appear to be of great importance and a large part of any current price level movement might well be "inherited" from recent past history, under some circumstances. Secondly, the British economy would appear to be fairly open to the influence of conditions in world markets; the two coefficients of the absolute change in import prices (which measure the impact effect of this variable) are 0.22 and 0.24. Thirdly, wage pushfulness has deleterious effects on the price level, as one might expect. Finally, there would appear to be a trade-off between more rapid growth of the industrial production index and/or a lower level of unemployment, on the one side, and a more rapid rate of increase of the price level, on the other side. Indeed, this conflict is the principal focus of this appendix. KBHV conclude, after due qualifications, that "stabilizing the price level by reducing the level of demand may be an extremely costly business, apart from the point that such a procedure hardly represents a long-run solution to the problem of inflation."

These ideas may be illustrated by a simple two equation subsystem. As a group of us have shown in [5], p. 79, the KBHV wage rate adjustment equation can be approximated in the following manner (provided one ignores lags and incorporates the effect of the pushfulness dummy):

$$(26) \quad \dot{W} = 9.014 - 3.335 U + 0.7721 \dot{P} ,$$

⁵⁰ For what it's worth, this subsystem dynamic equation appears to track the price level very accurately. This is hardly surprising, in view of the fact that KBHV used actual values of the lagged values of the price index of total consumption.

where U is unemployment as a percentage of the labour force, \dot{W} is the annual percentage rate of increase of weekly wage rates in British industry, and \dot{P} is the annual percentage rate of increase of the price index of total consumption. We can simplify the price structure part of the model by utilizing a price level equation fitted by L. A. Dicks-Mireaux [9a]:

$$(27) \quad \dot{P}' = 1.95 + 0.35 \dot{W}' + 0.20 \dot{F} - 0.52 \dot{A} ,$$

where \dot{P}' is the annual percentage rate of change of an index of final prices, \dot{W}' is the annual percentage rate of change of average wages and salaries per employed person, \dot{F} is the annual rate of change of import prices (with a three month lag built into this variable), and \dot{A} is the annual rate of growth of output per man in the British economy. If we ignore statistical differences in coverage (and also the issue of wage drift, for the rate of change of the money wage variable), we can derive the following trade-off relationship:

$$(28) \quad \dot{P} = 6.9949 - 1.5995 U + 0.2740 \dot{F} - 0.7125 \dot{A} .$$

If we specify a rate of growth of productivity and a rate of change of import prices, we may reduce this equation to a two-dimensional trade-off curve. This has been done in Table 6 and Figure 7, where we have assumed that \dot{A} is 2.0 (close to its average over the 1953-1959 subperiod in Dicks-Mireaux's sample data), and \dot{F} is set equal to 0 and to 3.0 respectively, to illustrate the difference between a relatively stable and a relatively inflationary external environment and one that is relatively stable. The column of unemployment rates adjusted to North American levels is based on a factor of 1.51, which was obtained from a study by R. J. Myers for the Joint Economic Committee of the U.S. Congress.⁵¹ The usual lessons appear: more rapid productivity growth

⁵¹For a slightly more complete discussion, see [5], pp. 35-37.

TABLE 6

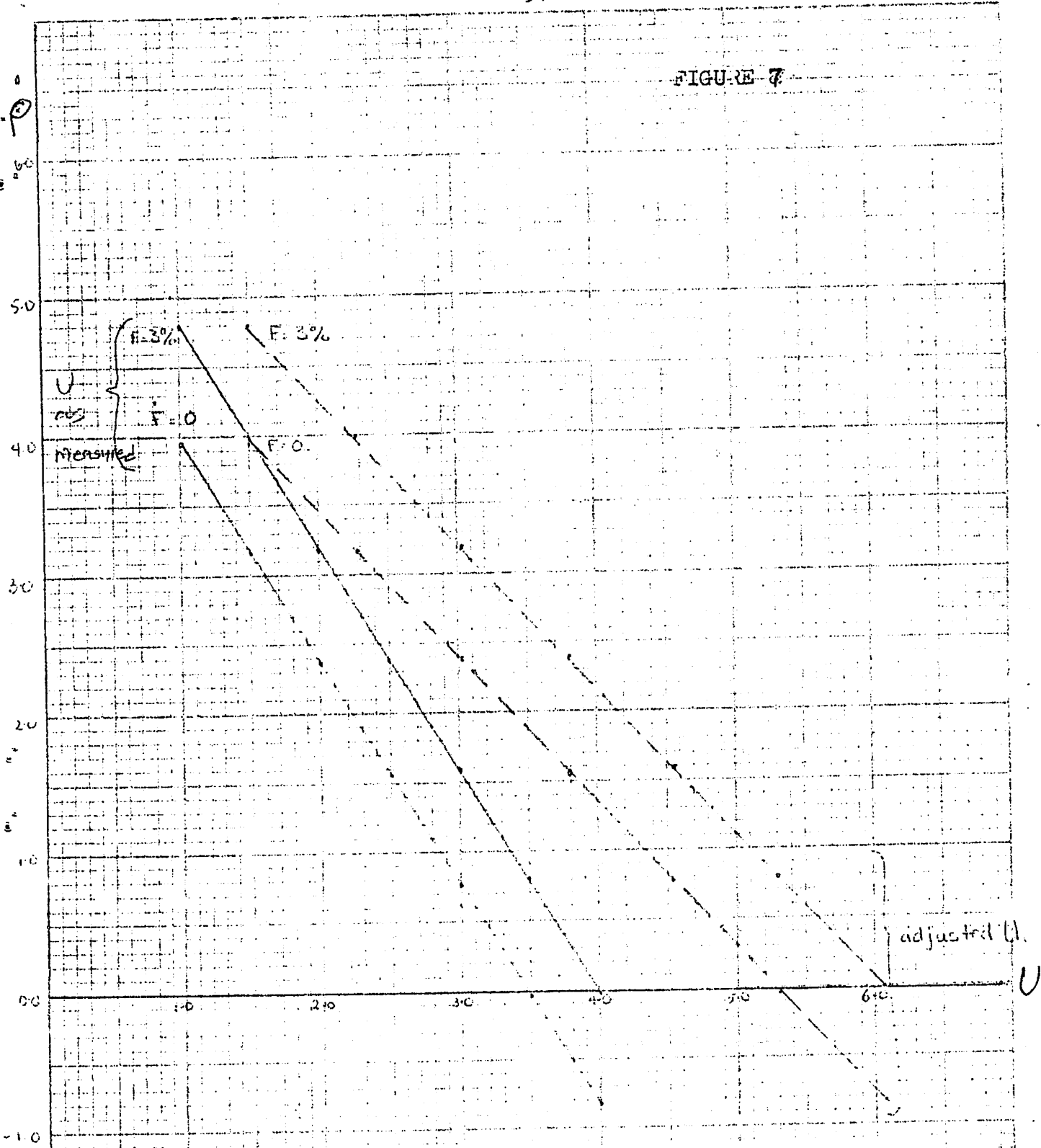
Predicted Rates of Change of a Price Index of Total Consumption, at Varying Rates of Unemployment, with Two Selected Rates of Change of Import Prices, (\dot{F}), for the United Kingdom Economy

Original Unemployment Rate	Adjusted Unemployment Rate*	Expected Rate of Change of Consumption Price Index When:	
		$\dot{F} = 0$	$\dot{F} = 3$
1.0	1.51	3.97	4.79
1.5	2.27	3.17	3.99
2.0	3.02	2.37	3.19
2.5	3.78	1.57	2.39
3.0	4.53	0.77	1.59
3.5	5.29	-0.03	0.79
4.0	6.04	-0.83	-0.01

* Adjusted by multiplication by a factor of 1.51.

Sources: See the text, especially equation (28).

FIGURE 7



Two Trade-off Curves for the U.K. Economy,
Based on Studies by Klein, Ball, Hazlewood,
and Vandome [21] and by Dicks-Mireaux [9a],
with Two Assumed Rates of Change of the
Index of Import Prices (\dot{F}).

will mitigate the conflict between the objectives of full employment and stable prices, while more rapid inflation in the countries which are the source of British imports will exacerbate this conflict. The conflict appears, on the basis of these relationships, to be a real one; completely stable prices require an unemployment rate equivalent to a North American rate of 5.3 to 6 per cent of the labour force, while the equivalent of a 3 per cent rate of unemployment would appear to be associated with an expected rate of increase of the domestic price level equal to 2.4 to 3.2 per cent per annum.⁵² This conflict of objectives would appear to be present in even a casual reading of postwar British economic history, mixed in, as it has been, with the issue of balance of payments deficits and the resulting "stop-go" policy management of the economy.

2. The Ball-Burns Quarterly Model of the U.K. Economy [3]

This model may be regarded as a development of the KBHV model of the United Kingdom; despite differences in scale (and in several other regards), the spirit of the analysis is reasonably similar. The scale of this model is most comparable to that of the Brookings model, with 178 estimated equations and (at least) 18 identities. The model has 14 producing sectors and 10 categories of final consumption goods, which probably gives a better idea of the size of the model than an actual count of the number of equations. The model was estimated from quarterly data⁵³ generally running from 1956 through 1966 by the methods of ordinary least squares and two stage least squares; as in the KBHV

⁵²The linearity of the trade-off curve reflects, of course, the linearity of the wage adjustment relationship which, as indicated above, post-sample evidence has tended to call into question.

⁵³Several of the equations, however, are estimated from annual data in cases in which estimation from quarterly data was presumably not feasible.

model, the data were left in seasonally unadjusted form, with seasonal adjustment performed directly in the estimation procedure, by means of dummy variables. A 14 by 14 input-output table for the United Kingdom economy for the year 1963 was also utilized in the estimation of the price equations of the model.

In broad overview, the wage-price sector of this model is as follows. There are wage equations for ten out of the fourteen producing sectors of the model. The resulting rates of wage payments then give rise to unit labour costs in these ten sectors, which then percolate through a four stage system to be one of the principal determinants of the final consumer price indexes for eight out of the ten categories of consumption goods and services. (Presumably unit labour costs are taken as exogenous in the remaining four producing sectors, as are the final consumer price indexes for housing and for fuel and light, which Ball and Burns regard as subject to political whim.) There is then a fixed weight averaging relationship which ties the price index for total consumption to the price indexes of its ten components. Other exogenous variables in the wage-price part of the model are the indexes of indirect taxes paid (less subsidies received) for the various categories of consumption goods and services, and (presumably) the price indexes of various categories of imports, both of final goods and services and of intermediate products.⁵⁴

The dependent variable in the ten wage equations is current wage and salary payments per employee divided by the value of the total consumption price index

⁵⁴The slight hesitancy in this assertion is due to the fact that, in their appendix, Ball and Burns list the unit import costs for the producing sectors as endogenous instead of as exogenous. This would appear to be a typographical mistake. There is no mechanism (as far as I can tell) for determining world prices of British imports within the model; in principle one would need an econometric model of world trade to do this.

for the preceding quarter. Thus the model is neo-classical in its approach to wage determination in the steady state, although in motion the previous period's consumer price index is predetermined and the wage equations essentially determine the per employee rate of money wage payments, for the current quarter. The explanatory variables in these relationships are three seasonal dummies, the sectoral level of real output per employee (the level of sectoral productivity), the economy-wide unemployment rate, and the lagged value of the dependent variable. The level of sectoral "real" wages (interpreted as above) is positively related to sectoral productivity in all ten cases, with a statistically significant regression coefficient in eight out of the ten relationships. The unemployment rate variable has the expected negative sign in only four cases out of ten; this variable is, however, never significant by standard tests. However, the influence of the unemployment rate on money wage determination is present once removed; there are ten sectoral productivity equations, which relate output per employee in the producing sectors positively to a time trend (which presumably captures such effects as technological progress and capital accumulation) and negatively to the unemployment rate (which presumably captures the effects of short run variations in productivity due to utilization factors). (The unemployment rate variable has a significantly negative coefficient in eight out of the ten productivity relationships; it has an insignificant coefficient in the other two regressions.) Finally, the lagged dependent variable always has a significant positive coefficient, which ranges in value over the ten relationships from 0.4 to 0.9. Ball and Burns interpret the importance of this variable in terms of the Koyck distributed lag model; alternatively, one could regard its importance as corroborating the view that the determination of the appropriate change in wages is the principal issue in wage determination.

The four stage determination of the eight price indexes for the various categories of consumption goods may be explained as follows. First, there are fourteen equations for the unit industrial prices of the outputs of the various producing sectors. These equations, estimated from the 1963 U.K. input-output table, are completely cost-determined, relating the unit industrial price of the sector to the costs of intermediate inputs (from the other producing sectors), to the sector's unit labour costs, and to the unit costs of sectoral imports. (In the model, a lag of one quarter is assumed to operate in these relationships, so that the current quarter levels of the sectoral unit industrial price indexes are effectively predetermined.) The second set of relationships is eight category relationships in which the dependent variable is an index of unit costs for these eight categories of final consumption goods and services. The explanatory variables in this second set of relationships are a proper subset of the fourteen industrial price indexes (the number of included industrial price indexes varies from two to nine, depending on the consumption category) and in general on a price index of a category of goods imported directly for final consumption.⁵⁵ It may be noted that these eight category relationships operate without a lag, as do in general the third and fourth stage category relationships described in the next paragraph.

The third stage relationships explain the consumer price indexes net of indirect tax (and plus subsidy) for the eight categories of final consumption. The key explanatory variable in these relationships are, of course, the indexes of unit costs for the corresponding categories, which of course are determined in the second stage relationships; the marginal coefficients range from 0.5 to

⁵⁵In the case of the category, furnishings and floor coverings, there is no import price explanatory variable for this particular unit cost equation. In the case of the unit cost equation for other goods and services (obviously a catch-all category), there are four specific import price indexes included as explanatory variables.

1.6, and all eight of these coefficients are highly significant. It is in these third stage relationships that the direct influence of demand pressures variables can in principle make its appearance; in practice the only additional variables (aside from the seasonal dummies) are the ratio of real gross domestic product to its trend value in the radio and electrical goods category relationship and a time trend in the automobiles category relationship. The former supplementary variable has a positive but only marginally significant regression coefficient, which is consistent with the view that this explanatory variable is a proxy for the pressures of excess demand. The time trend variable has a negative and highly significant influence on the net-of-tax price index of the automobiles category. The exact interpretation of this result eludes me (possibly increasing foreign competition?), but I doubt very much that the time trend is a suitable proxy for excess demand.⁵⁶ The fourth stage relationships simply assume that all of the unit indirect taxes paid (less subsidies received) are passed along to the final consumer; the treatment also assumes (in the case of ad valorem excise taxes) that the retail margin is a constant proportion of the wholesale price of the good in question. Hence one ends up with eight fourth stage relationships in which the dependent variable is the final consumer price level (at retail) for eight of the ten categories of total consumer expenditure and the two explanatory variables are the corresponding net-of-tax price indexes (from the third stage relationships) and the indexes of unit indirect taxes paid (less subsidies received) for the category in question. The treatment of the price aspects of indirect taxation in this model contrasts to

⁵⁶The third stage relationships are all simultaneous relationships, except for the food category relationship, in which the current level of the net-of-tax consumer price index of food is related to the preceding quarter's level of the index of the unit cost of food.

a large extent with the treatment in the KBHV model; the treatment in Ball-Burns is, in my view, a definite improvement.

Again, there would appear to exist the potential for a conflict between the goals of high employment and stable prices in the Ball-Burns model. High demand, by reducing unemployment, should lead to an increase in labour productivity (via the short run production functions) and so to a faster rate of rise of money wages in the producing sectors of the economy than would otherwise have occurred. In turn, this will presumably raise unit labour costs in the producing sectors of the economy,⁵⁷ which in turn will raise the indexes of unit industrial prices. In turn this will percolate through the remaining three stages of the system (unit costs for eight of the categories of total consumption expenditures, net-of-tax price indexes for these eight categories, and final retail price indexes for these eight categories), resulting in a higher level of the price index for total consumption. In addition, the pressure of excess demand may give an additional fillip to the net-of-tax price index for the category of radio and electrical goods. The process continues in succeeding periods, as the higher level of the price index of total consumption leads to a further adjustment of money wages in the economy's producing sectors. In their discussion paper, Ball and Burns promise some future simulation studies

⁵⁷The word "presumably" is there in the text because I am not certain how Ball and Burns define unit labour costs. If unit labour costs are defined as the reciprocal of current labour productivity times the average wage and salary payment, then there will very likely be no pressure on unit labour costs and hence on prices from a short term rise in labour productivity. If a more sensible definition of unit labour costs is used (say one that corresponds to that utilized in input-output analysis), then for unit labour costs there is little or no offset to the rise in money wages and the discussion in the text applies.

of alternative economic policies. It will be interesting to note what light these simulation studies may shed on the issue of the trade-off between unemployment and inflation.

Two final observations may be offered. First, this model depicts import prices as having an important influence on domestic British prices. The level of import prices have their primary influence in the determination of the indexes of unit industrial prices (the first stage relationships, in the discussion of this section); while important in these relationships, they would appear (in twelve of the fourteen producing sectors) to be secondary in importance to unit labour costs. Import prices also have a small additional influence in the determination of the indexes of unit costs for the categories of consumption expenditure (the second stage relationships described above). The impression persists that, on occasion, the British economy can experience inflationary pressures of foreign origin. Secondly, the over-all structure of wage and price determination in the United Kingdom economy appears to display the well-known lags; inflationary forces would appear to require a number of quarters to work out their full (or nearly full) impacts.

F. Conclusions and an Evaluation

Before attempting to say how well (or how poorly) the current generation of econometric models model the process of price formation, I should like to tie some strands together. There are several substantive conclusions that one can draw from our survey of wage and price formation in econometric models of Canada and her three principal trading partners. The first conclusion that one can draw is that, with the exception of Andersen and Carlson's monetarist model of the U.S. economy, the models surveyed describe price formation as principally a matter of cost factors, with the direct influence of demand pressures entering

secondarily or not at all. Wage costs are generally very prominent in the determination of the prices of final goods and services; for Canada and the United Kingdom, the costs of imported goods and services also have a prominent role. Secondly, our survey of wage determination suggests that demand forces play an important--and sometimes primary--role in the explanation of particular wage rates or their rates of change. However, even in the labour market, forces that have no role in the traditional theory of purely competitive markets appear to be important if at times secondary influences. Examples of such forces are illustrated by the use of the rate of change in the consumer price level, the level of profits or profit rates, and the lagged change in wages as explanatory variables. Thirdly, these two equations (or two types of equations) together form a subsystem which generally displays a considerable potential for a conflict between the policy objectives of full employment and stable prices. As demand presses on the availability of labour and capital resources, wage rate increases begin to accelerate, generally exceeding the "non-inflationary" rate of "normal" productivity growth. In addition, secondary demand pressures on price formation may appear as well, depending on the model under consideration.⁵⁸

It is interesting to rank these four economies with regard to their degree of responsiveness to price developments in world markets. On the basis of the econometric models surveyed in this paper, it would appear that Canada is the most sensitive to outside influences, while the United States appears to be the most insulated. The United Kingdom and Japan would appear to be intermediate cases, with the economy of the United Kingdom probably slightly

⁵⁸In Andersen and Carlson's model, these "secondary" demand pressures on price formation become the primary explanatory variable, if one judges by the t ratios.

more open to world price developments than the Japanese economy. This conclusion is suggested by a ranking of the coefficients on the rate of change of import prices as an independent variable in the explicit (but oversimplified) trade-off relationships derived in this paper.⁵⁹ It is also suggested by a number of qualitative factors. Thus, for example, from the Bank of Canada model of the Canadian economy and the KBHV model of the U.K. economy, there is some evidence that export price levels in these two economies are sensitive to the world prices of competing exports, while there is virtually no evidence that this is the case for either Japan or the U.S.

Finally, it is interesting to observe that, until very recently, neo-Keynesians had the field of econometric model building to themselves. With the publication of Andersen and Carlson's monetarist model of the U.S. economy, this monopoly has been broken, and in the decade to come, we can look forward to workable competition, if not to the pure competition of the textbooks, in this important field. As far as I am aware, there exists no econometric model of a developed economy in the spirit of the radical critics of capitalism, despite the fact that these critics have had something to say about the determination of the aggregate price level and the process of inflation.

We may ask ourselves how good in general are the neo-Keynesian econometric models in their description of price formation in these economies. (My evaluation of Andersen and Carlson's model, which is quite different in spirit as well as in detail from the other models surveyed here, appears in the section of this paper describing that model.) In particular, should we

⁵⁹These coefficients are: 0.55 for the Canadian economy [equation (5) of the text]; 0.27 for the U.K. economy [equation (28)]; and 0.16 for the Japanese economy [equation (20)]. For the U.S. economy, no calculated trade-off relationship includes the rate of change of import prices as an explanatory variable, so that the presumed universe value of this coefficient must be small, if not zero.

share the pessimism of Cooper [9] and some of the econometric model builders themselves, who regard their price equations as the weakest parts of the model? I am somewhat more sanguine. For one thing, the criterion of forecasting one quarter ahead loads the dice particularly against a structural explanation of prices and wages, due to the well known inertia which characterizes price formation in the economies surveyed. As our discussion of the Wharton model indicated, one gets a quite different picture if one's criterion is the ability to forecast price levels four quarters hence; here the standard "naive" models appear to be greatly inferior to a structural explanation of price formation. More importantly, one may have a different prospective if one looks at econometric model-building as a developmental process in which there is some technical progress. In the area of wage and price formation, the debt of the current generation of econometric model builders to work done by Lawrence R. Klein and A. W. Phillips in the early postwar period is enormous, and it is difficult to overstate this contribution. Nevertheless, the progress in modelling the wage and price sectors of the economy gained during the decade just ended has been quite large, as the introduction to this paper pointed out. It seems quite likely that there will be continued progress in the coming decade in modelling econometrically price formation for a developed, largely capitalistic economy. My own guess is that such progress will largely take the form of evolution, not revolution, and that the improved models of price formation of a decade or so in the future will be developments of our current, fairly crude models of this aspect of a developed, mixed economy. While econometricians have not been extremely successful in modelling this aspect of the economy, the control of inflation (by other than direct means) has similarly proved to be a rather intractable problem. This correspondence may be more than a coincidence.

My personal view is that the builders of the econometric models reviewed in this paper have in general done a reasonable job on a very difficult area in the developed, mixed economies; in any case, better models, with these models as antecedents, are very likely on the way.

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