1970

# The Production Possibility Curve and Factor Intensity Reversal

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RESEARCH REPORT 7020

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## AND FACTOR INTENSITY REVERSAL

While the phenomenon of factor intensity reversal is familiar to all
international trade economists in the partial equilibrium context where prop-
erly chosen isoquants for the two commodities intersect two or more times,
some of the general equilibrium implications of factor intensity reversal do
not seem to have been discussed in the literature. We might wonder, for ex-
ample, how factor intensity reversal will affect the relative shapes of the
production possibility curves. From H. G. Johnson's analysis[1] we know that
when the relative factor endowments of the two countries are separated by a
single reversal, either good may be exported by either country, but regardless
of the trade pattern both countries will export either their labour intensive
good or their capital intensive good, for the good that is capital intensive in
one country is labour intensive in the other. When the factor endowments are
separated by an even number of reversals, we can unambiguously state that one
good is capital intensive and the other labour intensive, but it is quite possi-
ble for the country which is relatively well endowed with capital to export the
labour intensive good. The purpose of this paper is to show how the production
possibility curves can be used to illustrate these results. Our approach will
be first of all to present a geometric method of deriving the production possi-
bility curve from the representative isoquants of the two goods, and then to
use this derivation to illustrate how the relative shapes of the production
possibility curves differ when factor intensity reversals prevail. We will then
use these curves to illustrate the Johnson results. Our analysis will also

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[1] H. G. Johnson, _International Trade and Economic Growth_. (London: Unwin
University Books, 1958), Ch. I.

indicate the importance for the Heckscher-Ohlin theorem of explicitly ex-
cluding the possibility of factor intensity reversal, and will make clear the
factors which determine the shape of the production possibility curve.
Throughout our analysis we will make the usual Heckscher-Ohlin assumptions
that two goods are produced in two countries using labour and capital, that
the goods vary in their factor intensities and the countries vary in their
factor endowments, that both countries possess the same linear homogeneous
production functions for the two goods, that perfect competition prevails,
and that tastes for the two countries are identical and can be represented by
utility functions which are homothetic.[2]

In Figure 1, point E on the ray OZ represents the endowment of capital
and labour for the country under consideration. $X_0$ and $Y_0$ are the two iso-
quants for commodities X and Y which pass through E. These levels of output
therefore represent the maximum possible output for the two commodities.
These maximum output quantities are translated to the axes by the line AB,
arbitrarily drawn except that it must go through E. Thus, measuring Y and X
along the K and L axes respectively, A and B represent the maximum producible
quantities of Y and X respectively. Now consider the line·FG tangent to the
two isoquants. If the wage-rental ratio corresponding to the slope of this
line is to prevail, then the parallelogram OHEJ illustrates the allocation of
capital and labour between the two industries, and OH and OJ represent the
levels of output for X and Y, respectively. OH can then be compared to output
OE by drawing HK parallel to FE. Then by drawing MK parallel to AB the output
OH can be represented on the Y axis as point M. Similarly output OJ of X can

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[2]A function is said to be homothetic if it is a monotonic transformation
of a linear homogeneous function. For our purposes, the important property of
such functions is that all indifference curves have the same slope along any
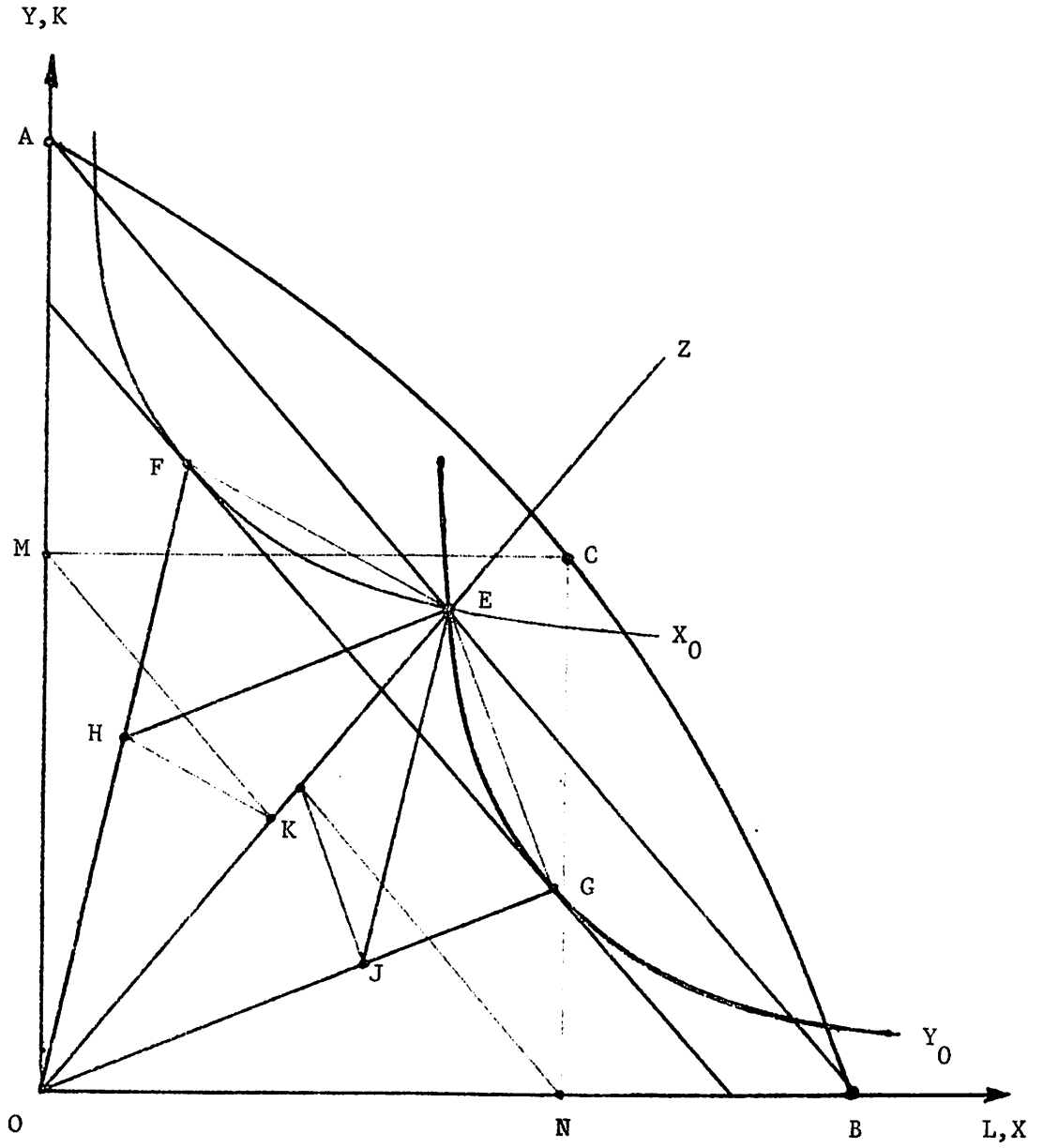ray from the origin.

Figure 1

be represented on the X axis by point N, and M and N give rise to point C in output space. By considering other factor price ratio lines and finding their tangents to $X_0$ and $Y_0$ and repeating the procedure, the production possibility curve ACB can be traced out.[3]

Before proceeding it is worthwhile observing from Figure 2 that there are two factors which will affect the curvature of the production possibility curve. The first is the degree·of difference in the factor intensities of the two goods, and the second is the elasticity of substitution in production for the two goods, i.e., the curvature of the isoquants. It is clear, for example, that if we move F and G further apart, then the point C will move further away from the line AB. And with given factor intensities, if we make the isoquants more curved, by moving E out further along OZ, for example, C will also move further away from AB. Johnson has recently examined the curvature of the pro-duction possibility curve under the assumption of Cobb-Douglas functions, and has concluded that the curves are much flatter than they are traditionally drawn.[4] This result depends in part on the fact that Cobb-Douglas functions have unitary elasticity of substitution, and had he used functions with lower elasticity coefficients he would have found much more curvature. Indeed it is easy to show that as the factor intensities approach opposite axes, and as the elasticities of substitution approaches zero, the production possibility set

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[3] Other approaches to the derivation of the production possibility curve are due to K. M. Savosnick, "The Box Diagram and the Production Possibility Curve," Ekonomisk Tidskrift, (November 1958), and W. P. Travis, The Theory of Trade and Protection (Cambridge, Mass.: Harvard University Press, 1964). Our method can be considered as a variant of the Travis construction, although all three methods are really variants of one another. The Savosnick method is not useful for our purpose for it does not allow two production possibility curves to be simultaneously constructed. And while two curves can be constructed by the Travis method they would have different scales and would therefore not be comparable.

[4] Harry G. Johnson, "Factor Market Distortions and the Shape of the Trans-formation Curve," Econometrica, 34 (July, 1966).
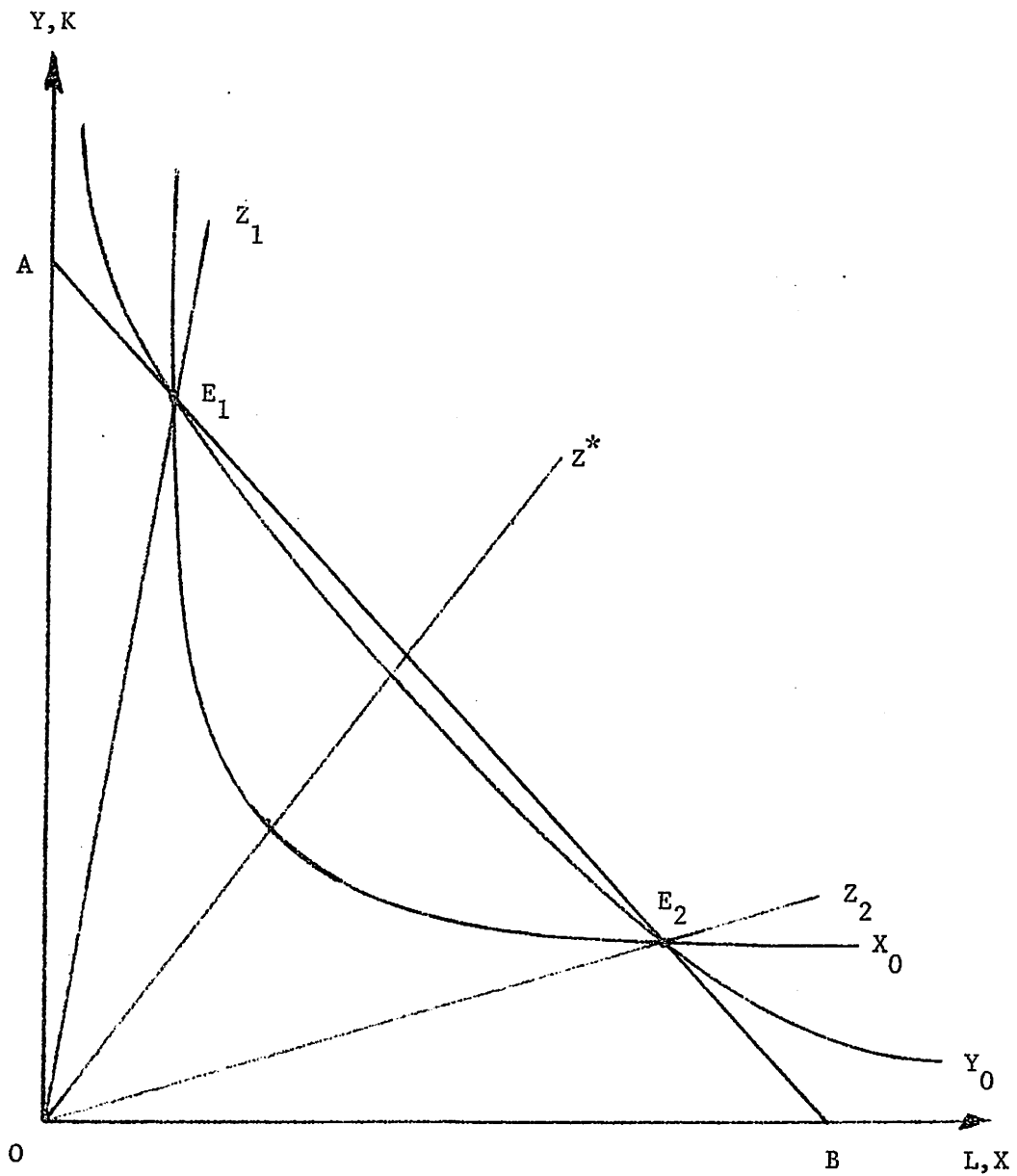
Figure 2

approaches a rectangle.

In Figure 2 we have shown a situation where factor intensity reversal occurs, and where OZ* is the factor intensity reversal ray. The diagram has been constructed so that $E_1$ and $E_2$, the points of intersection of the two isoquants, also represent the endowments for the two countries. With this construction it is clear that the maximum producible quantities of the two goods are the same for both countries, namely the quantities corresponding to $X_0$ and $X_0$. We have drawn our reference line through both $E_1$ and $E_2$ so that A and B represent the maximum producible quantities for Y and X, respectively, for both countries. We could now proceed as before to derive intermediate points and trace out both production possibility curves. Such an exercise is really unnecessary, however, for as we have already observed, the curvature of the production possibility curve will depend on the elasticity of substitution in the relevant range and on the difference in the factor intensities. Unless these are identical for both countries, which must be regarded as a pathological case,[5] the two curves will, in general, have different curvature so that we can assume that one of the curves lies outside the other at all points except the axes.[6]

We will now endow the country whose production possibility curve is innermost with slightly more labour and capital in the same proportion as the initial endowment. This will shift the curve out slightly and we get Figure 3.

Now imagine that I' is a representative member of the set of community

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[5]We shall have more to say about this case presently. Notice that this case is pathological because it requires not only the suggested symmetry but also the very special factor endowments we have assumed.

[6]I am assuming that the curves do not cross but I have not been able to rigorously prove this. And even if they do cross it makes no difference to our analysis nor to the conclusions drawn.
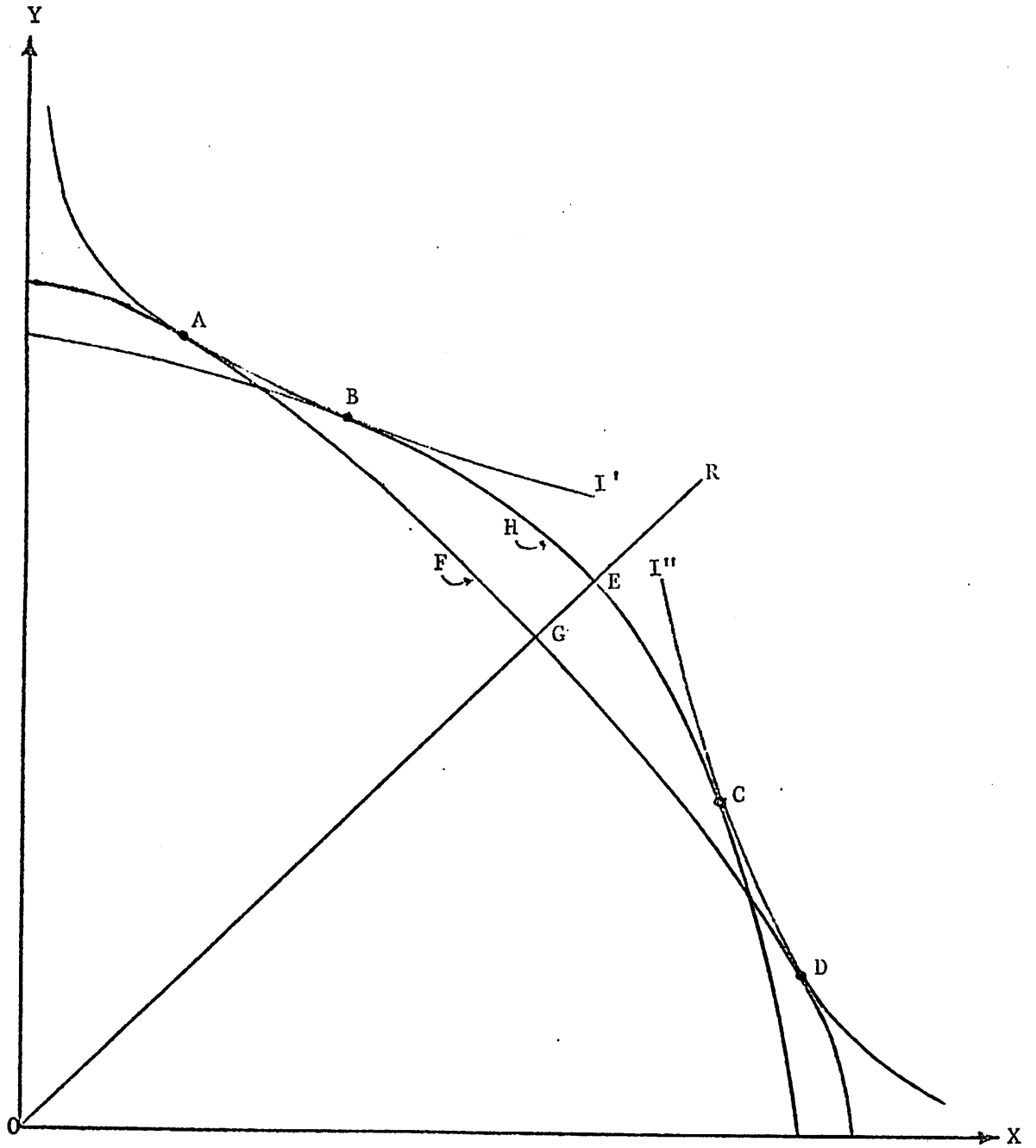
Figure 3

indifference curves for both countries.[7] In autarky country H will produce and consume at B and country F will produce and consume at A. If trade is allowed, H will produce more X and less Y and will export X, and F will produce more Y and less A and will export Y. But now suppose that the representative indifference curve is I" so that the two autarky equilibrium points are C and D. Here trade patterns are just the opposite for H will produce more Y, less X and will export Y while F will produce more X, less Y and will export X. These, of course, are the Johnson results. The pattern of trade clearly depends on demand conditions. Observe that from the diagram it is impossible to know which is the labour-abundant and which the capital-abundant country.

Observe that for the situation of Figure 3 there exists a set of indifference curves for which there will be no trade even though factor endowments differ. If the ray OR represents the ray along which the slopes of the two transformation curves are the same, then if an indifference curve were tangent at E (and one at G)[8] autarky prices would be the same in both countries and there would be no trade. An even more interesting special case can be constructed from Figure 2, for if the diagram is completely symmetrical about the ray OZ*, then the two transformation curves would be identical so that regardless of demand conditions there would be no trade. While these cases are clearly pathological, they do illustrate that counter-examples to the Heckscher-Ohlin theorem can be found unless factor intensity reversal is explicitly excluded.

Before proceeding further it is important to observe two things. First,

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[7] In order to simplify the diagram we are considering the extremely special case where the same indifference curve is tangent to both production possibility curves. Because the utility functions are assumed to be homothetic, this very special assumption cannot affect our results.

[8] Recall that because the utility function is homothetic the set of indifference curves has constant slope along any ray from the origin.

the comments which we have just made do not depend on the countries being of relatively the same size as they are in Figure 3. Suppose, for example, that country H were only half as large as shown so that the production possibility curve would be half-way between the origin and its present position. In such a case our conclusion would be exactly the same; the special case of Figure 3 where the curves intersect is shown only to make the diagrammatic results easier to see.

Second, the situation just described is not a _necessary_ consequence of factor intensity reversal, for it might be impossible for the production possibility curves to cross more than once regardless of the scale of the two countries. It is quite possible that there does not exist any output price ratio which could be tangent to both curves. In such a case it is clear that trade will result in specialization in production by at least one country. That such a situation is possible is most easily seen by considering the Johnson diagram where the optimal capital-labour ratio and relative commodity prices are plotted as functions of the wage-rental ratio.[9] If the ranges of possible output price ratios for the two countries do not overlap then at least one country must specialize.

We will now illustrate the case where two factor intensity reversals are possible. Figure 4 shows one way in which the two production possibility curves could be related. Here the two curves intersect in three places, and as a consequence there is no longer any ambiguity with respect to which country is relatively well endowed with capital and which is relatively well endowed with labour. If Y is the capital intensive commodity then H is capital-abundant and F is labour-abundant. Now suppose I is the representative indifference curve

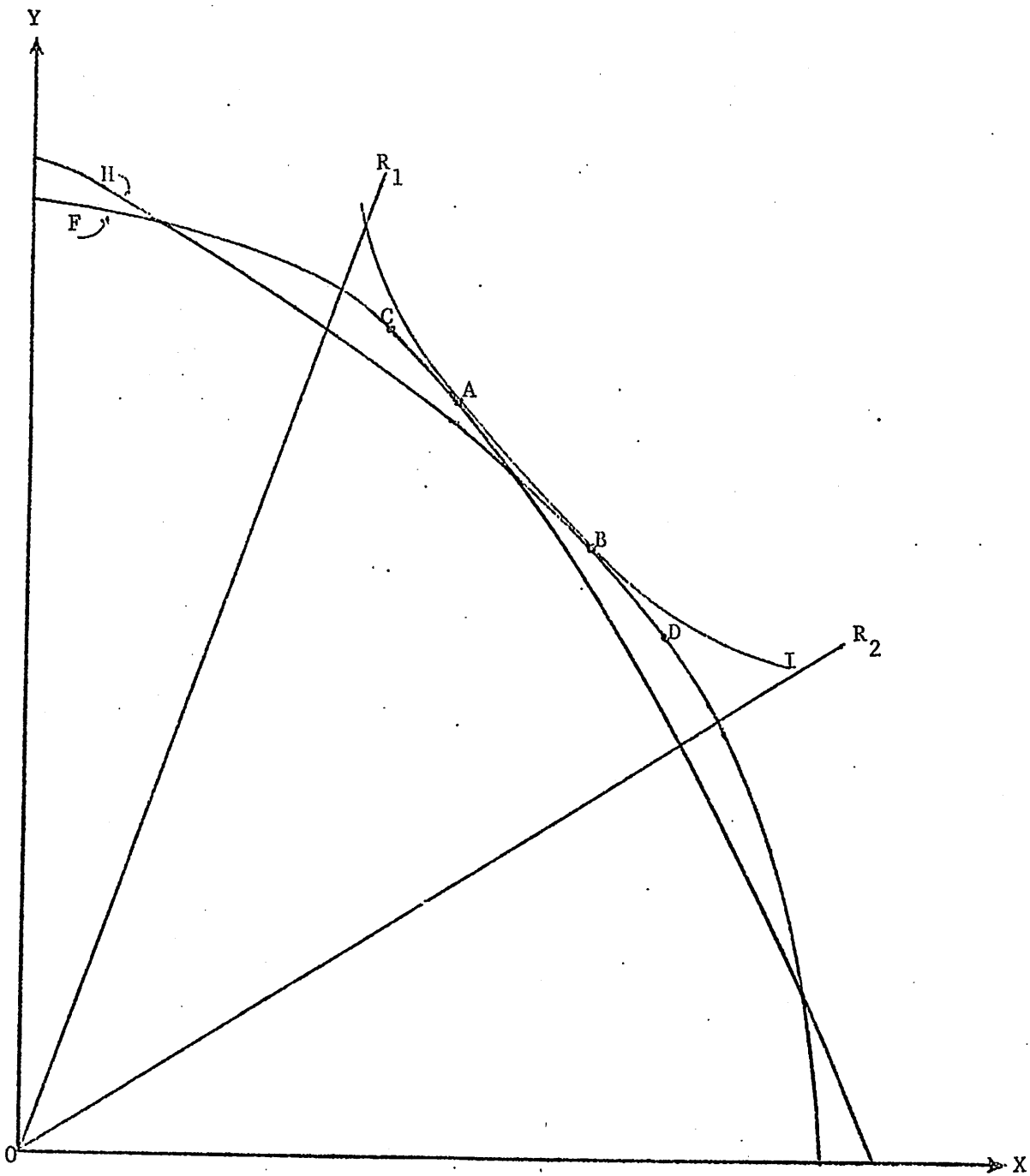[9]Johnson, _International Trade and Economic Growth_, 19.

Figure 4

for both countries and that it happens to be tangent to the two production possibility curves at A and B.[10] In autarky X is more expensive in F and Y more expensive in H so that when trade is allowed F will produce at a point such as C and H will produce at a point such as D, F will export Y and H will export X, and we have a situation where both countries are exporting the goods which use their scarce factor more intensively--the opposite result to that predicted by the Heckscher-Ohlin theorem. On the other hand, if the indifference curves were such that the initial tangencies were between the X axis and $OR_1$, or between the Y axis and $OR_2$, where $OR_1$ and $OR_2$ represent the rays along which the production possibility curves have the same slope, then the normal Heckscher-Ohlin results would prevail. Tangencies of the indifference curves and the production possibility curves along either $OR_1$ or $OR_2$ would result in no trade.

Again the reader is warned that the situation of Figure 4 is a special case and that it is not a necessary result of the endowment rays of the two countries being separated by two factor intensity reversals. Just as in the case where there is only one factor intensity reversal, it is possible that there does not exist an output price ratio which could be tangent to both curves with the result that trade would imply specialization by at least one of the two countries. Of course the above analysis is easily extended to cases where there are three or more factor intensity reversals.

In conclusion we can observe that the results we have described depend on the fact that at certain commodity price ratios we have a reversal in the

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[10] Again, this very special demand assumption is made only to simplify the diagram. It in no way affects the analysis. Also, as in Figure 3, the two countries are assumed to be about the same size only to make the results easier to see.

relative size of the ratios in which commodities are produced.[11] Thus we can say that when, in factor space, factor intensity reversal is possible, associated with each such reversal we have a commodity intensity reversal in output space.

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[11]The price ratio line tangent to the production possibility curve at point E in Figure 3 is one example.