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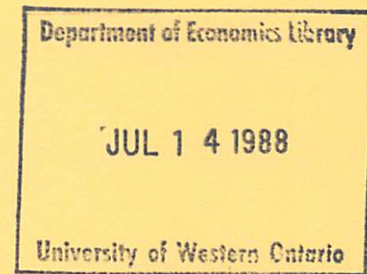
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NATIONAL PRODUCT FUNCTIONS IN COMPARATIVE
STEADY-STATE ANALYSIS

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This paper contains preliminary findings from research work still in progress and should not be quoted without prior approval of the author.

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NATIONAL PRODUCT FUNCTIONS IN COMPARATIVE
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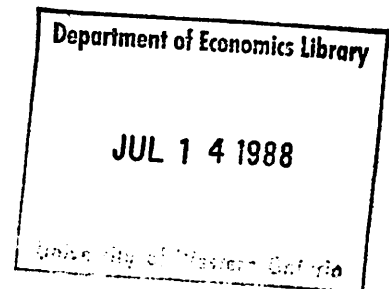
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Abstract

The structural properties of steady states are well understood for multi-sector models, but the comparative dynamics of longrun growth are much less developed. By exploiting duality theory, this paper obtains conclusions about these effects for a small, internationally-trading, economy. Arbitrary numbers of consumption and capital goods, and a very general technology, are admitted. Definite conclusions about the shortrun and longrun impact on outputs and factor prices are not always available. Some correlations can be established between vectors of parameter changes and endogeneous variable changes, however. To confirm that some correlations hold it is necessary to impose strong assumptions.



1. Introduction

The comparative dynamic properties of a multisectoral model of economic growth and international trade are investigated. The economy is assumed to face world prices (or changes in world prices) that are given, and to be in an equilibrium steady state (or to change between such states). Results are obtained comparing the shortrun and longrun effects on output and factor rewards of changes in world prices, primary resource endowments, time preference, and taxes. Even with the strong simplifying assumptions about prices and steady-state growth it is, in general, impossible to get results which describe the impact of a particular parameter change on a particular endogeneous variable. What can be established is the correlation between vectors of parameter changes and endogeneous variable changes. Some correlations hold quite generally, but others can be established only on strong assumptions about the technology and the relationships between factors.

Many papers have been written on the topic of economic growth and international trade. These papers have, for the most part, been designed to make specific points, and they develop models accordingly. Simple models which illustrate the consequences of capital accumulation for an open economy have been analyzed by Bruce (1977), Inada (1968), Johnson (1971), Oniki and Uzawa (1965) and Stiglitz (1970): In these models a single capital good is used, with labor, to produce several tradeable commodities, one of which is new capital. Educational policy, which concerns the accumulation of human capital, was considered by Manning (1982) in a structurally similar model. Still simpler models have been used to discern the effects of international capital movements on economic growth. Long (1973), Manning (1975), Negishi (1965) and Ruffin (1979, 1985) developed models in which capital and labor

produce a single all-purpose commodity. No commodity trade occurs. The only international transactions are capital flows and interest payments. More complex models have been presented by Acheson (1970), Ethier (1979), Kemp and Khang (1974), Petith (1976) and Samuelson (1975). These models were designed to illustrate particular features of the timing of investment and the structure of production: Although they recognize more than one capital good they are not completely general.

From the existing literature many results are known about economic growth and international trade. Few general results are available, however. It is not known which conclusions rest on details of the economic structure and which hold generally. With the multi-sectoral model developed in this paper it is possible to distinguish between these kinds of result, and to identify the type of conditions needed for particular conclusions.

Any number of consumer goods, any number of capital goods, any number of primary resources, and a very general technology, are admitted in the economy that is considered. The economy is assumed to be in a steady state in which identical consumers are maximizing (the present value of) their utility. This is in contrast to many of the papers cited above: It is commonly assumed that savings obey a simple behavioural rule (savings are a constant fraction of national income, for example).

For closed economies, the analysis of multi-sectoral optimal (or equilibrium) growth models is well advanced. It is known that there may not be a steady-state equilibrium for these economies. McKenzie (1976) showed that only if time preference is sufficiently low will a steady state be achieved. In an important paper, Benhabib and Nishimura (1979) demonstrated that the economy approaches an endless cycle if the rate of time preference exceeds a critical value.¹ Although prices are endogeneously determined for

a closed economy, the possibility of cyclic growth in a small, open, multi-sector economy cannot be ruled out. (Indeed, Benhabib and Nishimura's investigation of the stability properties of the economy assumed that prices were constant in a neighbourhood of the steady state.) Implicit, therefore, in the present formulation of the trade and growth problem is an assumption that parameter values (time preference in particular) admit a steady state as the longrun equilibrium.

Well-known properties of the national product function are exploited here to analyse the multisectoral model. Dixit and Norman (1980) popularized the "dual" approach to international trade theory, but the present analysis is closest to that of Woodland (1982, Chapters 14 and 15) who treated the problem of economic growth and international trade with the techniques of duality theory. As previously noted, precise conclusions about the effects of parameter changes are not found here; correlations are discovered instead. These are of the same sort as presented in different contexts by Neary (1985), Markusen and Svensson (1985), and Neary and Ruane (1985).

Issues more general than those commonly considered in international trade theory are dealt with here. To some extent, the analysis also extends earlier work on the longrun effects of taxes on capital. Feldstein (1974a, 1974b) worked out the incidence of a tax on capital, using a model with a single capital good. Most subsequent analyses have retained this structure. When capital taxes are allowed in the multisectoral model, more general conclusions about their effects become available than have previously been found.

2. The Economy

To begin, no taxes are allowed. These will be introduced later, after

developing and analysing the economy in their absence. The small, open, economy has available n primary resources, which are used with m capital goods to produce l final (consumer) goods and m investment goods. These inputs and outputs are related by a transformation equation:

$$T(y, x, k, v) = 0 \tag{1}$$

where y is the vector of final goods outputs, x is the vector of investment goods outputs, k is the vector of capital stocks, v is the vector of primary resources,² and T is increasing in all its arguments. Various restrictions may usefully be imposed on T :

(T1) T is strictly quasi-convex in y and x ;

(T2) T is strictly quasi-convex in k ;

(T3) T is strictly quasi-convex in v ;

(T4) $T(\lambda y, \lambda x, \lambda k, \lambda v) = 0$, for all $\lambda > 0$.

These require, respectively, that the production-possibility set is strictly convex, that there are diminishing returns to capital, that there are diminishing returns to primary resources, and that there are constant returns to scale.

Although there is no need to explain the transformation equation in more basic terms, some examples may help to relate it to the underlying production conditions. Suppose, for instance, that there are two commodities produced according to homogeneous of degree one, quasi-strictly concave, production functions which have as inputs capital and labor. If factor intensities differ between sectors, then T1, T2, T3 and T4 are satisfied. Alternatively, suppose that each commodity is produced according to a strictly concave production function (so there are both decreasing returns to scale and a diminishing rate of factor substitution): Then T1, T2 and T3 are satisfied, but T4 is not.

Let p be the vector of final goods prices, and q the vector of capital goods prices. The value of output is $p'y + q'x$ (where $'$ denotes transpose). Competitive behaviour ensures that this is maximized.

DEFINITION 1: The gross national product function is given by

$$G(p, q, k, v) = \frac{\text{Maximum}}{y, x} [p'y + q'x : T(y, x, k, v) = 0]$$

It is well-known that G is homogeneous of degree one, and convex, in p and q . In addition, under T2 G is strictly concave in k , while under T3 G is strictly concave in v . Under T2 and T3 G is concave in k and v jointly.

Furthermore, under T1,

$$y = G_p \quad \text{and} \quad x = G_q \tag{2}$$

and also

$$r = G_k \quad \text{and} \quad w = G_v \tag{3}$$

where G_p is the vector of derivatives of gross national product with respect to final goods prices, G_q , G_k and G_v are similarly defined, r is the vector of rentals of capital, and w is the vector of primary resource prices. The results stated in (2) and (3) are intuitive. For example, a one unit increase in a particular capital stock will permit an increase in gross national product. This increase is the marginal value of that capital good, which, in a competitive market, is its rental price.

Primary resources are assumed to be in fixed supply. The shortrun is defined to be a period in which the capital stocks are fixed independently of economic considerations: Comparative static effects are developed directly from the gross national product function on this assumption. In the longrun, the size of capital stocks depends on their contribution to the well-being of their owners.

Let c be the rate of consumption of final goods by consumers in the economy, and let s be their rate of gross investment in capital goods.³

These are limited by the budget constraint

$$p'c + q's = p'y + q'x \quad (4)$$

Installed capital is assumed to be internationally immobile, and no lending occurs, so the budget constraint requires that the value of current expenditure equals the value of current production. Capital stocks evolve according to

$$\dot{k} = s - D(\delta)k \quad (5)$$

where k is the vector of rates of change of capital stocks, and $D(\delta)$ is a diagonal matrix with its i th diagonal element the exponential rate of depreciation of the i th capital stock.

The following is assumed about decision-making in the economy:

(U) Capital is accumulated as if there is a single consumer who maximizes

$$\int_0^{\infty} e^{-\rho t} u(c) dt \text{ subject to (1), (4) and (5), where } \rho > 0 \text{ is the rate of}$$

time preference and u is a strictly concave instantaneous utility function.

This implies a path through time for consumption, gross investment, and capital stocks. In the limit as time tends to infinity, the longrun values of these variables are obtained. The work of Benhabib and Nishimura shows that longrun values need not be constants: But if they are constants, the economy has achieved a steady state. The following is assumed:

(S) The longrun equilibrium for the economy is a steady state.

PROPOSITION 1: Under T1, T2, U and S in the longrun

$$G_k = D(\rho + \delta)q \quad (6)$$

If G is twice differentiable and G_{kk} is non-singular,
then the longrun capital stocks are determined by a
differentiable function $K(p, q, v, \rho)$ for which

$$K_p = -G_{kk}^{-1} \cdot G_{kp} \quad (7)$$

$$K_q = -G_{kk}^{-1} \cdot (G_{kq} - D(\rho + \delta)) \quad (8)$$

$$K_v = -G_{kk}^{-1} \cdot G_{kv} \quad (9)$$

$$K_\rho = G_{kk}^{-1} \cdot q \quad (10)$$

Proof: Let $e^{-\rho t} \mu_1$ and $e^{-\rho t} \mu_2$ be the multipliers associated with (1) and (4) and let $e^{-\rho t} \lambda$ be the co-state variable for (5). Then necessary (and sufficient, in view of the assumptions of concavity) conditions for the problem U include:

$$u_c - \mu_2 p = 0 \quad (11)$$

$$\lambda - \mu_2 q = 0 \quad (12)$$

$$\mu_2 p - \mu_1 T_y = 0 \quad (13)$$

$$\mu_2 q - \mu_1 T_x = 0 \quad (14)$$

$$\dot{\lambda} = D(\rho + \delta)\lambda + \mu_1 T_k \quad (15)$$

Now (13) and (14) imply that gross national product is maximized. From Definition 1,

$$G_k = -p_i T_{ik} / T_{yi}, \quad i=1, \dots, l$$

Using this in (13) implies that

$$-\mu_2 G_k / \mu_1 = T_k \quad (16)$$

In a steady state $\dot{\lambda} = 0$. Then (6) follows from (12), (15) and (16).

Since G_{kk} is non-singular, then G_{kk}^{-1} exists.⁴ The implicit function theorem can then be applied to (6) to ensure the existence of a differentiable function $K(p, q, v, \rho)$, with derivative (7), (8), (9) and (10).

□

The longrun behaviour of capital stocks is described in Proposition 1. By substituting into the gross national product function, the longrun (steady-state) gross national product function $G(p, q, K(p, q, v, \rho), v)$ is obtained. From this can be calculated the longrun responses of the endogeneous output and price variables to parameter changes.

3. Shortrun and Longrun Responses to Price Changes.

By differentiating (2) the shortrun output responses to price changes are found.

PROPOSITION 2: If T1 applies, then there is a non-negative correlation between the price changes and the vector of output changes in the shortrun: That is

$$dp'dy \geq 0 \text{ and } dq'dx \geq 0$$

Proof: From (2)

$$\begin{bmatrix} dy \\ dx \end{bmatrix} = \begin{bmatrix} G_{pp} & G_{pq} \\ G_{qp} & G_{qq} \end{bmatrix} \begin{bmatrix} dp \\ dq \end{bmatrix} \quad (17)$$

Therefore

$$[dp' \ dq'] \begin{bmatrix} dy \\ dx \end{bmatrix} = [dp' \ dq'] \begin{bmatrix} G_{pp} & G_{pq} \\ G_{qp} & G_{qq} \end{bmatrix} \begin{bmatrix} dp \\ dq \end{bmatrix} \geq 0 \quad (18)$$

since G is convex in p, q , which implies that $[G_{ij}]$ is positive semi-definite. □

Nothing new is involved in this Proposition. It is given as a basis of comparison for the longrun effects of price changes. Denote by Y and X the longrun outputs of final and investment goods. From the longrun gross national product function

$$\begin{bmatrix} dY \\ dX \end{bmatrix} = \begin{bmatrix} G_{pp} + G_{pk} K_p & G_{pq} + G_{pk} K_q \\ G_{qp} + G_{qk} K_p & G_{qq} + G_{qk} K_q \end{bmatrix} \begin{bmatrix} dp \\ dq \end{bmatrix} \quad (19)$$

This breaks into two parts the longrun response to a price change: There is first an output change at given capital stocks, and there is in addition an output change due to the price-induced change in capital stocks. The difference between the shortrun and the longrun response is entirely due to the changed capital stocks.

PROPOSITION 3: If T1, T2, U and S apply, then the correlation between a vector of final goods price changes and their output changes is greater in the longrun than it is in the shortrun:
That is

$$dp'dY > dp'dy \geq 0$$

Proof: Put $dq = 0$. Subtracting (17) from (19), and using (7) gives

$$dp'(dY-dy) = -dp'G_{pk} G_{kk}^{-1} G_{kp} dp > 0 \quad (20)$$

by the negative definiteness of G_{kk} . □

When a single price only goes up, according to this Proposition, the output of that final good expands in the longrun by more than it does in the shortrun. In the shortrun the output expands as more factors are drawn into the industry. There is an additional expansion in the longrun as the capital stock is adjusted to its appropriate configuration for the new prices.

Matters are more complex when a capital good increases in price. Such a price change increases the revenue from producing the capital good, and so raises its output in the shortrun. But the cost of capital is also increased, which dampens the incentive for capital formation. The balance of these opposing forces determines whether or not the longrun effect reinforces the shortrun effect on output. Before examining this, it is convenient to give some definitions and a preliminary result.

DEFINITION 2: Capital good i is a gross complement to (substitute for) capital good j if and only if an increase in the stock of capital good i raises (lowers) the rental price of capital good j : That is, $\partial r_j / \partial k_i > 0 (< 0)$, $i \neq j$.

Note that this definition of complementarity does not express an intrinsic property of capital goods. Rather, it rests on interconnections throughout the economy - it is a general equilibrium property, not a primitive concept about the system.

LEMMA: Suppose that all capital goods are gross complements. If T2 holds, then G_{kk}^{-1} is non-positive.

Proof: T2 implies that G_{kk} is negative definite. If all capital goods are gross complements then the off-diagonal elements of G_{kk} are all positive. A well known theorem on matrices then implies the result (see Takayama (1985, Theorem 4.D.3, p. 393)). □

DEFINITION 3: Newly-produced capital good i is capital-intensive if and only if $\partial x_j / \partial k_j > 0$, $j=1, \dots, m$. It is non-capital-intensive if and only if the opposite inequalities hold.

This definition also expresses a general equilibrium relationship. A similar definition applies to final goods. Neither for newly-produced capital goods, nor for final goods, are these categories exhaustive: There may be goods which are neither capital-intensive nor non-capital-intensive.

PROPOSITION 4: If T1, T2, U and S apply, then there is a presumption that the correlation between a vector of changes in the prices of newly-produced capital goods and the changes in their outputs is greater in the longrun than it is in the shortrun: More precisely, $dq' \cdot (dX-dx)$ consists of a positive term and a term that may be of either sign.

If only one price increases for a newly-produced capital good, then its output expands more in the longrun than it does in the shortrun if all capital goods are gross complements and if either (i) the capital good is non-capital-intensive or (ii) the capital good is "sufficiently" capital intensive.

Proof: Put $dp = 0$. Subtracting (17) from (19), and using (8) gives

$$dq'(dX-dx) = -dq'G_{qk} G_{kk}^{-1} G_{kq} dq + dq'G_{qk} G_{kk}^{-1} D(\rho+\delta)dq \quad (21)$$

The first term on the right-hand side is positive, by the negative-definiteness of G_{kk} . The second term may be of any sign.

If $dq_1 = 1, dq_2 = \dots = dq_m = 0$, then (21) gives

$$dX_1 - dx_1 = -G_{q_1k} G_{kk}^{-1} (G_{kq_1} - \Delta_1) \quad (22)$$

where G_{q_1k} is the first row of G_{qk} , G_{kq_1} is the first column of G_{kq} and

$\Delta_1 = (\rho + \delta_1, 0, \dots, 0)$. The Lemma implies that G_{kk}^{-1} is non-positive if all capital goods are gross complements. If the first capital good is non-capital-intensive, then G_{q_1k} ($= G_{kq_1}$) is negative. The sign of (22) is

then clearly positive. This proves (i). On the other hand, if the first capital good is capital intensive, then G_{q_1k} ($= G_{kq_1}$) is positive. Provided that

$$G_{q_1 k_1} = \frac{\partial x_1}{\partial k_1} > \rho + \delta_1 \quad (23)$$

the sign in (22) is again positive. Thus (ii) is established. \square

The condition (23) may be written as

$$\frac{\partial x_1}{\partial q_1} \cdot \frac{q_1}{r_1} > 1$$

on using (3) and (6): That is, the elasticity of the rental price of the capital good with respect to its output price exceeds unity. In this event, the increased price for the capital good will lead to a greater than proportional rise in its rental price. To achieve a steady state the capital good must be accumulated so that its marginal product is reduced. This accumulation raises the rental prices of all of the complementary capital goods, and they must also be accumulated to reach equilibrium. These increased capital stocks permit even greater increases in output than occur in the short run.

Propositions 3 and 4 make an interesting contrast. The former provides a strong conclusion, which rests only on the most general features of the economy's structure. The latter, however, shows that to get any definite results about the effect on outputs of changes in capital goods prices it is necessary to make assumptions about the fine detail of inter-relationships in the economy. These assumptions may directly impose restrictions at the level of general equilibrium responses (as in Proposition 4), or they may impose restrictions at the more primitive level of the technology, in which case these restrictions must imply specific kinds of general equilibrium responses. It is, therefore, in the nature of the problem that no strong conclusions are available concerning the consequences of changes in capital goods prices. When the impact of output price changes on factor rewards are considered the outcome is similar.

From (3),

$$\begin{bmatrix} dr \\ dw \end{bmatrix} = \begin{bmatrix} G_{kp} & G_{kq} \\ G_{vp} & G_{vq} \end{bmatrix} \begin{bmatrix} dp \\ dq \end{bmatrix} \quad (24)$$

gives the shortrun responses of factor rewards to output price changes.

Using the longrun gross national product function, and (7) and (8), the longrun responses of factor rewards to output price changes is

$$\begin{bmatrix} dR \\ dW \end{bmatrix} = \begin{bmatrix} 0 & D(\rho+\delta) \\ G_{vp} - G_{vk} \cdot G_{kk}^{-1} G_{kp} & G_{vq} - G_{vk} \cdot G_{kk}^{-1} (G_{kq} - D(\rho+d)) \end{bmatrix} \begin{bmatrix} dp \\ dq \end{bmatrix} \quad (25)$$

where R and W respectively denote the steady state rental of capital goods and primary resource prices. Subtraction of (24) from (25) gives the difference between the longrun and shortrun responses:

$$\begin{bmatrix} dR - dr \\ dW - dw \end{bmatrix} = \begin{bmatrix} -G_{kp} & D(\rho+\delta) - G_{kq} \\ -G_{vk} \cdot G_{kk}^{-1} G_{kp} & -G_{vk} \cdot G_{kk}^{-1} (G_{kq} - D(\rho+\delta)) \end{bmatrix} \begin{bmatrix} dp \\ dq \end{bmatrix} \quad (26)$$

The only conclusions which do not depend on the details of the economy are (see (25)) that in the longrun the rates of return on capital are independent of the prices of final goods and are proportional to the prices of capital goods: These are, of course, obvious intuitively.

DEFINITION 4: Capital good i and primary resource j are complements (substitutes) if and only if an increase in the endowment of the primary resource raises (lowers) the rental price of capital: That is, $\partial r_i / \partial v_j > 0$ (< 0).

An example is now given of the restricted kind of result that can be obtained.

PROPOSITION 5: If T1, T2, U and S apply, then an increase in the price of a final good will in the longrun raise by more (or lower by less) than it does in the shortrun the price of any primary resource if capital goods are complementary to primary resources, capital goods are gross complements and the final good is capital intensive.

Proof: Without loss of generality, let $dp_1 = 1$, $dp_2 = \dots = dp_l = 0$, and consider the first primary resource. From (26)

$$dW_1 - dw_1 = -G_{v_1k} G_{kk}^{-1} G_{kp_1} \quad (27)$$

Since all capital goods are gross complements, by the Lemma G_{kk}^{-1} is non-positive. By definition, G_{v_1k} is positive if capital goods are complementary to primary resources. Finally, G_{kp_1} is positive if the final good is capital intensive. Thus $dW_1 - dw_1 > 0$. □

This result is intuitive, of course. If the final good is capital intensive, an increase in its price will raise capital stocks. This effect is accentuated by the complementarity of all capital goods. However, primary resources are also complementary to capital goods, so the price-induced increases in capital stocks raise the returns to the primary resources.

Other results of the same kind are available from (27). For example, primary resource returns rise by less in the longrun than in the shortrun in response to an increased price for a final good if capital goods are substitutes for primary resources, and otherwise the conditions of Proposition 5 apply. All of these results, however, rely on a detailed specification of the structure of the economy.

4. Shortrun and Longrun Responses to Changes in Primary Resource Endowments

By differentiating (3) the shortrun responses of factor prices to changes in factor endowments can be calculated.

PROPOSITION 6: If T2 and T3 apply, then there is a non-positive correlation between the vectors of factor price changes and factor endowment changes in the shortrun:

That is,

$$[dk' dv'] [dr' dw']' \leq 0$$

Proof: From (3)

$$\begin{bmatrix} dr \\ dw \end{bmatrix} = \begin{bmatrix} G_{kk} & G_{kv} \\ G_{vk} & G_{vv} \end{bmatrix} \begin{bmatrix} dk \\ dv \end{bmatrix} \quad (28)$$

Therefore

$$[dk' dv'] \begin{bmatrix} dr \\ dw \end{bmatrix} = [dk' dv'] \begin{bmatrix} G_{kk} & G_{kv} \\ G_{vk} & G_{vv} \end{bmatrix} \begin{bmatrix} dk \\ dv \end{bmatrix} \leq 0 \quad (29)$$

since T2 and T3 imply that G is concave in k, v, which implies that $[G_{ij}]$ is negative semi-definite. □

This provides a benchmark against which longrun effects can be measured. Of course, in the longrun the capital stocks are endogenously determined, and the return on capital is independent of the endowments of primary resources (see (6)). There is, accordingly, no reason to consider any longrun effect except that of primary resource endowments on primary resource prices.

PROPOSITION 7: If T2, T3, U and S apply, then the correlation between a vector of changes in primary resource endowments and their price changes is smaller in the longrun than it is in the shortrun: That is

$$dv'dw < dv'dW$$

Proof: From the longrun gross national product function

$$dW = G_{vv} dv + G_{vk} \frac{K}{v} dv \quad (30)$$

Putting $dk = 0$ in (28), and subtracting this from (30), gives

$$dW-dw = -G_{vk} \frac{G_{kk}^{-1} G_{kv}}{kk} dv \quad (31)$$

on using (9). Hence

$$dv' [dW-dw] = -dv' G_{vk} \frac{G_{kk}^{-1} G_{kv}}{kk} dv > 0$$

since G_{kk} is negative definite. □

Whereas an increased endowment of primary resources tends to lower their prices in the shortrun, this effect is mitigated in the longrun by the induced accumulation of capital. Propositions 3 and 7 are alike: They each give general and unequivocal results about the longrun and shortrun relationship of quantities and prices of particular kinds of commodities. Other results rest on stronger assumptions, however,

DEFINITION 5: Final good i is intensive in the use of primary resource j if and only if $\partial w_j / \partial p_i > 0$. If the opposite inequality holds the good is non-intensive in the resource.

A similar definition holds for newly-produced capital goods. Note that Definitions 2-5 exhaust the combinations of goods and factors that can be related.

PROPOSITION 8: Suppose that T1 holds. An increase in the endowment of a primary resource will, in the shortrun, result in either an increase on average of the output of those goods using the factor intensively or a decrease on average of the output of those goods using the factor non-intensively. More generally,

$$dv' \begin{bmatrix} G_{vp} & G_{vq} \end{bmatrix} \begin{bmatrix} dy \\ dx \end{bmatrix} \geq 0 \quad (32)$$

Proof: From (2)

$$\begin{bmatrix} dy \\ dx \end{bmatrix} = \begin{bmatrix} G_{pv} \\ G_{qv} \end{bmatrix} dv \quad (33)$$

Therefore

$$dv' \begin{bmatrix} G_{vp} & G_{vq} \end{bmatrix} \begin{bmatrix} dy \\ dx \end{bmatrix} = dv' \begin{bmatrix} G_{vp} & G_{vq} \end{bmatrix} \begin{bmatrix} G_{pv} \\ G_{qv} \end{bmatrix} dv \geq 0 \quad (34)$$

Suppose that $dv_1 = 1$, $dv_2 = \dots = dv_n = 0$. Recall that

$$G_{1p_i} = \frac{\partial w_1}{\partial p_i}, \quad G_{1q_i} = \frac{\partial w_1}{\partial q_i}$$

Then (34) implies that

$$\begin{bmatrix} \frac{\partial w_1}{\partial p_1} & \dots & \frac{\partial w_1}{\partial p_l} & \frac{\partial w_1}{\partial q_1} & \dots & \frac{\partial w_1}{\partial q_m} \end{bmatrix} \begin{bmatrix} dy \\ dx \end{bmatrix} \geq 0 \quad (35)$$

At least one output must expand because of the increased availability of the first primary resource. For the inequality to hold, either the expanded output must be in an industry which uses intensively the primary resource, or the output must fall on average in those industries which use the resource non-intensively. □

This proposition is a generalized Rybczynski theorem. It does not rely on constant-returns-to scale, however.

A change in primary factor endowments will alter in the shortrun the

rates of return on capital. Whether capital goods are substitutes for or complements to primary resources will determine the direction of this alteration: See (28). The longrun adjustment in capital stocks depends on the relationship between factors, therefore.

The longrun gross national product function yields

$$\begin{bmatrix} dY \\ dX \end{bmatrix} = \begin{bmatrix} G_{pv} + G_{pk} \frac{K}{v} \\ G_{qv} + G_{qk} \frac{K}{v} \end{bmatrix} dv \quad (36)$$

as the longrun output effect of a change in primary resource endowments.

Subtracting (33) from (36) and using (9) gives

$$dv' \begin{bmatrix} G_{vp} & G_{vq} \end{bmatrix} \begin{bmatrix} dY-dy \\ dX-dx \end{bmatrix} = -dv' \begin{bmatrix} G_{vp} & G_{vq} \end{bmatrix} \begin{bmatrix} G_{pk} & G_{kk}^{-1} & G_{kv} \\ G_{qk} & G_{kk}^{-1} & G_{kv} \end{bmatrix} dv \quad (37)$$

The sign of (37) is, in general, ambiguous.

PROPOSITION 9: Suppose that T1, T2, T3, U and S apply. An increase in the endowment of a primary resource will have less effect on output on average in the longrun than in the shortrun if all capital goods are gross complements, capital goods are complementary to primary resources, and if every good is either intensive in the use of capital and non-intensive in the use of the primary resource, or is non-intensive in the use of capital and intensive in the use of the primary resource.

Proof: Without loss of generality, let $dv_1=1, dv_2=\dots=dv_n=0$. Then the right-hand side of (37) becomes

$$-\begin{bmatrix} G_{v_1p} & G_{v_1q} \end{bmatrix} \begin{bmatrix} G_{pk} \\ G_{qk} \end{bmatrix} \begin{bmatrix} G_{kk}^{-1} & G_{kv_1} \end{bmatrix} < 0 \quad (38)$$

The inequality follows from the assumptions that capital goods are gross complements (the Lemma shows that G_{kk}^{-1} is non-positive), that capital and primary resources are complementary (so that G_{kv_1} is positive), and the intensity assumptions (so that G_{v_1p} and G_{v_1q} are of opposite sign to G_{pk} and G_{qk} , respectively). □

Consider the second alternative. In the shortrun an addition to the stock of a primary resource will raise the return on the complementary factors, the capital stocks, and expand the output of those goods which are intensive in the use of the primary resource. Now, the higher return on capital signals an expansion in the stock of capital. However, those goods which are intensive in the primary resource are non-intensive in capital, so their outputs decline, which offsets the shortrun output expansion. A similar explanation can be developed for the first alternative.

5. Time Preference and the Steady State.

The rate of time preference has no influence in the shortrun. Its longrun effects operate through the steady-state capital stocks.

PROPOSITION 10: An increase in time preference reduces at least one capital stock in the steady state, and reduces the total value of capital stocks at any set of prices.

Proof: Using (10), since G_{kk}^{-1} is negative definite,

$$q'K_{\rho} = q'G_{kk}^{-1}q < 0 \tag{39}$$

That is, the total value of capital stocks falls as time preference rises.

Since all prices are positive, at least one derivative in K_{ρ} is negative. □

The second part of this proposition is explicitly stated by Woodland (1982, p. 481), and the first part is almost given there too.

The value of consumption in any steady state equals net national product, which is given by the function

$$N(p, q, k, v) = G(p, q, k, v) - q'D(\delta)k \tag{40}$$

PROPOSITION 11: Net national product is maximized in the steady state when the rental price of each capital good equals the depreciation on its price: That is

$$r = D(\delta)q. \quad (41)$$

An increase in time preference reduces the value of steady-state consumption.

Proof: Recall that $G_k = r$. The first-order condition for the maximum of (40) with respect to k is (41). Now

$$N_p = G_k K_p - q'D(\delta)K_p$$

Substitution of (6) and (10) gives

$$N_p = q'D(\rho)G_{kk}^{-1}q < 0$$

□

The first of these results is a general version of the familiar golden-rule of economic growth. Its derivation is straightforward, despite the possibility of joint products and arbitrary numbers of consumption and capital goods. That increases in time preference reduce net national product is also well known, but it is established here in a particularly simple fashion.

6. Taxes.

The model presented in Section 2, and analyzed in Sections 3, 4 and 5, can be interpreted as concerning either optimal or equilibrium growth. By inserting taxes into the model attention is focused on the second interpretation.

In the presence of taxes a distinction must be drawn between the prices received by sellers and the prices paid by buyers. Thus,

$$p_c = p_s + \theta_p; \quad q_c = q_s + \theta_q; \quad r_s = r_c + \theta_r \quad (42)$$

Here, p_c is the price paid for final goods by consumers, which equals the price that producers receive, p_s , plus the specific taxes θ_p . All factors

are owned by consumers, so what they pay for capital goods, q_c , must equal what producers receive, q_c , plus taxes θ_q . Finally, the rental price that firms pay for capital, r_s , is made up of the rental to the factor owners, r_c , and the taxes θ_r . Firms and consumers are motivated by the (different) prices that confront them. Definition 1 must be modified accordingly.

DEFINITION 6: The gross national product function (at producer prices) is given by

$$G(p_s, q_s, k, v) = \underset{y, x}{\text{Maximum}} \{p'_s y + q'_s x : T(y, x, k, v) = 0\}$$

This enjoys most of those properties previously noted for the gross national product function, except that with taxes

$$y = G_{p_s} \quad \text{and} \quad x = G_{q_s} \tag{43}$$

and

$$r_s = G_k \quad \text{and} \quad w = G_v \tag{44}$$

take the place of (2) and (3).

The consumers' problem is to maximize utility subject to (1), (5) and the budget constraint, which becomes instead of (4)

$$p'_c c + q'_c s = p'_y y + q'_x x \tag{45}$$

This implicitly assumes that all tax revenues are returned to consumers. It can be shown that in the steady state

$$G_k(p_c - \theta_p, q_c - \theta_q, k, v) - \theta_r = D(\rho + \delta)q_c \tag{46}$$

which compares with (6). This implicitly defines the steady-state capital stocks as a function $K(p_c - \theta_p, q_c - \theta_q, \theta_r, v, \rho)$. Clearly,

$$-K_{p_c} = K_{\theta_p} = G_{kk}^{-1} G_{kp_s} \tag{47}$$

$$K_{\theta_q} = G_{kk}^{-1} G_{kq_s} \tag{48}$$

$$K_{\theta_r} = G_{kk}^{-1} \tag{49}$$

For given prices to consumers, (42) implies that

$$dp_s = -d\theta_p \text{ and } dq_s = -d\theta_q \quad (50)$$

That is, an increase in a tax on final or newly-produced capital goods sales is equivalent to a reduction in the price received by their producer. As a corollary of Proposition 2 the following result is immediate.

PROPOSITION 12: If T1 applies, there is a non-positive correlation between the vector of tax changes on final or newly-produced capital goods and the vector of their output changes in the shortrun: That is

$$d\theta'_p dy < 0 \text{ and } d\theta'_q dx < 0$$

The shortrun effects of the tax change are derived directly from the shortrun effects of a price change. However, in the longrun the effect of a tax increase is not simply the opposite of the effect of a price increase. Indeed, it turns out that a definite conclusion can be reached about the steady-state effect of an increased tax on newly-produced capital goods, in contrast to the ambiguity associated with an increase in their price.

PROPOSITION 13: If T1, T2, U and S apply, then the correlation between a vector of taxes on final or newly-produced capital goods and their output changes is more negative (that is, greater in absolute value) in the longrun than it is in the shortrun: That is

$$\begin{bmatrix} d\theta'_p & d\theta'_q \\ P & Q \end{bmatrix} \begin{bmatrix} dY \\ dX \end{bmatrix} < \begin{bmatrix} d\theta'_p & d\theta'_q \\ P & Q \end{bmatrix} \begin{bmatrix} dy \\ dx \end{bmatrix} \leq 0$$

Proof: From the longrun gross national product function with taxes

$$\begin{bmatrix} dY \\ dX \end{bmatrix} = - \begin{bmatrix} G_{p_s p_s} + G_{p_s k} K_{\theta_p} & G_{p_s q_s} + G_{p_s k} K_{\theta_q} \\ G_{q_s p_s} + G_{q_s k} K_{\theta_p} & G_{q_s q_s} + G_{q_s k} K_{\theta_q} \end{bmatrix} \begin{bmatrix} d\theta_p \\ d\theta_q \end{bmatrix} \quad (52)$$

On subtracting (51) from (52), and using (47) and (48),

$$\begin{bmatrix} d\theta'_p & d\theta'_q \end{bmatrix} \begin{bmatrix} dY-dy \\ dX-dx \end{bmatrix} = \begin{bmatrix} d\theta'_p & d\theta'_q \end{bmatrix} \begin{bmatrix} G_{p_s k} G_{kk}^{-1} G_{kp_s} & G_{p_s k} G_{kk}^{-1} G_{kq_s} \\ G_{p_s k} G_{kk}^{-1} G_{kp_s} & G_{p_s k} G_{kk}^{-1} G_{kq_s} \end{bmatrix} \begin{bmatrix} d\theta_p \\ d\theta_q \end{bmatrix} < 0$$

by the negative definiteness of G_{kk}^{-1} . □

Of course, the reason why this definite conclusion is reached about the longrun impact of capital taxes is that prices to consumers are held constant. This eliminates any impact by the tax on consumers' desired holdings of capital.

No impact is felt in the shortrun from a change in the tax on capital earnings, θ_r . In the longrun, (46) implies that the return to consumers on capital is constant when the prices they face for capital goods are fixed. An increase in the taxes on capitals' earnings therefore affects the longrun equilibrium through its effects on the profitability of production.

PROPOSITION 14: If T1, T2, U and S apply, then there is a negative correlation between a vector of changes in taxes on the earnings of capital and the vector of capital stocks:

That is

$$d\theta'_r dk < 0$$

In particular, an increased tax on the earnings of a particular capital good will lower its steady-state stock.

If, in addition, all capital goods are gross complements, an increase in taxes on the earnings of capital will lead to a decrease (increase) in the output of capital-intensive (non-capital-intensive) goods.

Proof: From (49)

$$d\theta'_r dk = d\theta'_r G_{kk}^{-1} d\theta_r < 0 \quad (53)$$

since G_{kk}^{-1} is negative definite. If $d\theta_{r_1} = 1$, $d\theta_{r_2} = \dots = d\theta_{r_m} = 0$, then (53) implies that $dk_1 < 0$. Of course

$$\begin{bmatrix} dY \\ dX \end{bmatrix} = \begin{bmatrix} G_{psk} \\ G_{qsk} \end{bmatrix} G_{kk}^{-1} d\theta_r$$

If all capital goods are gross complements, the Lemma shows that G_{kk}^{-1} is non-positive. The i th row of G_{psk} is positive if and only if the i th final good is capital intensive. But then dY_i is negative when $d\theta_r > 0$. The other cases are similar. □

In the absence of taxes the present value of the flow of utility is maximized. Taxes reduce this present value, and alter the flow of utility. The extent of this alteration can be derived from the expenditure function $E(p_c, u)$. This gives the minimum expenditure necessary at consumer prices p_c to attain the utility level u . Under the small-country assumption consumer prices equal world market prices if there are no trade taxes. In the steady state the value of gross investment equals the value of the depreciation of capital stocks, so that

$$E(p_c, u) = p'_c y + q'_c x - q'_c D(\delta)k$$

Using (42) and Definition 6, this can be written as

$$E(p_c, u) = G(p_s, q_s, k, v) + \theta'_p y + \theta'_q D(\delta)k \quad (54)$$

where k is defined by (46). To discuss the effect of taxes on the steady-state flow of utility the following definitions are useful.

DEFINITION 7: Taxes on final goods are proportional if and only if

$$\phi_p = \frac{\theta_p}{p c_i} i, \quad i=1, \dots, l$$

Taxes on capital goods are proportional if and only if

$$\phi_q = \frac{\theta_q}{q c_i} i, \quad i=1, \dots, m$$

Commodity taxes are proportional if and only if

$$\phi_p = \phi_q$$

Taxes on the earnings of capital are proportional if and only if

$$\phi_r = \frac{\theta_r}{r_i} i, \quad i=1, \dots, m$$

PROPOSITION 15: If commodity taxes are proportional, then a tax increase has no effect on the shortrun equilibrium. If, however, depreciation rates are similar, then utility is reduced in the steady state. This also occurs if time preference is zero, or close to it.

Proof: Differentiation of (54) implies that

$$E_u \cdot du = q'_c D(\rho) G_{kk}^{-1} \begin{bmatrix} G_{kp_s} & G_{kq_s} \end{bmatrix} \begin{bmatrix} d\theta_p \\ d\theta_q \end{bmatrix}$$

$$- \begin{bmatrix} \theta'_p & \theta'_q \end{bmatrix} \begin{bmatrix} y_{p_s} & y_{q_s} \\ x_{p_s} & x_{q_s} \end{bmatrix} \begin{bmatrix} d\theta_p \\ d\theta_q \end{bmatrix}$$

$$+ \begin{bmatrix} \theta'_p & \theta'_q \end{bmatrix} \begin{bmatrix} y_k \\ x_k \end{bmatrix} G_{kk}^{-1} \begin{bmatrix} G_{kp_s} & G_{kq_s} \end{bmatrix} \begin{bmatrix} d\theta_p \\ d\theta_q \end{bmatrix} \quad (55)$$

where (43), (44) and (46) have been used.

The second term in (55) is zero for the tax increase considered here. This is a consequence of the fact that outputs are homogeneous of degree zero in producer prices. Now (42) and the definition of proportional commodity taxes imply that

$$d\theta_p = p_s d\phi (1+\phi) \text{ and } d\theta_q = q_s d\phi (1+\phi) \quad (56)$$

where $\phi = \frac{d\theta_p}{p_s} = \frac{d\theta_q}{q_s}$ and $d\phi$ is the common rate at which commodity taxes are increased. Substituting (56), and using Euler's theorem, yields the conclusion that the second term of (55) is zero. Since the other terms in (55) involve changes in the capital stock this also establishes the shortrun neutrality of the tax changes.

Recall that $y_k = G_{p_s k}$ and $x_k = G_{q_s k}$. The third term of (55) is then

$$\begin{bmatrix} \theta'_p & \theta'_q \end{bmatrix} \begin{bmatrix} G_{p_s k} \\ G_{q_s k} \end{bmatrix} G_{kk}^{-1} \begin{bmatrix} G_{kp_s} & G_{kq_s} \end{bmatrix} \begin{bmatrix} d\theta_p \\ d\theta_q \end{bmatrix} \quad (57)$$

But the definition of proportional taxes implies that

$$d\theta_p = \theta_p d\phi/\phi, \quad d\theta_q = \theta_q d\phi/\phi \quad (58)$$

Substitution of (58) into (57) reveals that term to be negative, in view of the negative definiteness of G_{kk}^{-1} .

Finally, consider the first term of (55). Recall that $G_{kp_s} = [\partial r_i / \partial p_{s_j}]$ and $G_{kq_s} = [\partial r_i / \partial q_{s_j}]$. If all producer prices rise in the same proportion then so do the value marginal products of all capital goods rise in the same proportion. The rows of $\begin{bmatrix} G_{kp_s} & G_{kq_s} \end{bmatrix}$ are, therefore, homogeneous of degree one in prices. Together with (56), this fact permits the first term of (55) to be written as

$$\begin{aligned} & q'_c D(\rho) G_{kk}^{-1} r_s d\phi / (1+\phi) \\ & = q'_c D(\rho) G_{kk}^{-1} D(\rho+\delta) q_c d\phi / (1+\phi) \end{aligned} \quad (59)$$

where the steady-state equilibrium condition (46) has been used to eliminate

r_s . This is negative if $\delta_1 = \dots = \delta_m$. By continuity, it is also negative if the depreciation rates are sufficiently close together.

If depreciation rates are close, (55) is negative, since the first and third terms are negative and the second is zero.

If the rate of time preference is zero then (59) is zero. For small enough ρ , by continuity, (59) is close to zero. If positive it will be outweighed by the unambiguously negative third term. \square

There is a presumption that taxes will lower the flow of utility in the steady state. This certainly occurs in the absence of time preference. Yet it is possible that taxes raise the steady state flow of utility. For this to happen the first term in (55) must be positive enough to offset the negative third term.

Proposition 15 gives a result about proportional taxes on all goods. A result is also available concerning the effect of a proportional tax on a subset of goods.

PROPOSITION 16: If time preference is zero, or close to it, then the imposition of a proportional tax on final goods (or on capital goods) reduces the flow of utility in a steady state.

Proof: Consider a proportional tax on final goods. Then $\theta_q = d\theta_q = 0$.

The second term of (55) is just

$$\theta'_p Y_p \frac{d\theta_p}{p} \leq 0$$

on using (56) and the negative semi-definiteness of the matrix.

The third term of (55) is negative, as in the Proof of Proposition 14.

The first term of (55) is zero if $\rho=0$, and is close to zero for small ρ .

It can, therefore, be outweighed by the other two negative terms. \square

Finally, the effect of a tax on capital can be discovered.

PROPOSITION 17: If depreciation rates are similar, or if the rate of time preference is zero (or close to it), then a proportional tax on capital earnings will reduce the flow of utility in the steady state when commodity taxes are proportional (or zero).

Proof: Differentiation of (54) implies that

$$\begin{aligned} E \frac{du}{u} = & q'_c D(\rho) G_{kk}^{-1} d\theta_r + \theta'_r G_{rk}^{-1} d\theta_r \\ & + \begin{bmatrix} \theta'_p & \theta'_q \end{bmatrix} \begin{bmatrix} y_k \\ x_k \end{bmatrix} G_{kk}^{-1} d\theta_r \end{aligned} \quad (60)$$

where (46) and (49) have been used.

Using the definition of proportional taxes on capital earnings, the second term of (60) is

$$\theta'_r G_{rk}^{-1} d\theta_r \frac{d\phi_r}{\phi_r} < 0$$

The first term of (60) is

$$q'_c D(\rho) G_{kk}^{-1} D(\rho + \delta) q_c \frac{d\phi_r}{\phi_r}$$

where (42), (46) and the definition of proportional taxes have been used. If $\delta_1 = \dots = \delta_m$ then this is negative. So is it for depreciation rates that are similar. It is zero if $\rho = 0$, and is small for small ρ .

Consider now the third term of (60). It is zero if commodity taxes are zero (and hence proportional). Otherwise, note that

$$\begin{bmatrix} \theta'_p & \theta'_q \end{bmatrix} \begin{bmatrix} y_k \\ x_k \end{bmatrix} = \frac{1}{1-\phi} \begin{bmatrix} p'_s & q'_s \end{bmatrix} \begin{bmatrix} y_k \\ x_k \end{bmatrix} = G'_k / (1-\phi) \quad (61)$$

using again the definition of proportional taxes with $\phi = \phi_p = \phi_q$, and recalling the definition of the gross national product function. But $G_k =$

r_s . The third term of (60) is therefore

$$r'_s dk / (1-\phi) = d\theta'_r dk (1+\phi_r) / \phi d\phi_r < 0$$

where Proposition 14 gives the inequality. □

In the absence of time preference the flow of utility is maximized in the steady state. Any tax or subsidy on capital earnings will reduce this flow. If, however, time preference is positive, then a proportional subsidy on capital earnings may raise the flow of utility: But this is guaranteed only if depreciation rates are similar.

7. Concluding Remarks

The advantages of the dual approach to static analysis are well known. This paper has shown that there are similar advantages when the technique is used in dynamic, steady-state, analysis. Where general results are available, they are derived quite simply from the longrun gross national product function. Where general results are not available, it is made clear what kinds of structural assumptions are needed to obtain particular results. In both situations average relationships are established, not precise links from parameters to endogenous variables. However, average relationships are the more useful in empirical tests of theory.

Footnotes

1. It seems intuitive that it is optimal to arbitrage away longrun cycles. This intuition is incorrect, however, as an example from a different context makes clear. Gallagher and Manning (1982) considered water storage, and showed that it is not worthwhile to eliminate cycles, even if it is technically possible to do so. Suppose that the price of water follows a seasonal pattern. As long as there is a positive rate of interest (or time preference) it is not profitable to arbitrage away all price differences, even if there are no costs of water storage. In fact, while water is held in storage its price must go up at the rate of interest. The price then falls when the storage facility is empty, and a new cycle begins. (This provides an extension of Hotelling's rule to cyclically available resources.)

2. Column vectors are considered throughout.

3. Both c and s , as well as y and x , are functions of time. But it is convenient (and appropriate in steady-state analysis) to ignore this argument.

4. Furthermore, G_{kk} is negative definite, as is G_{kk}^{-1} , under the assumption of non-singularity, since G is strictly concave in capital stocks. This assumption is maintained throughout.

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