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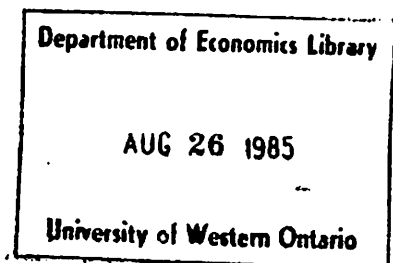
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with Costly Communication

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THE REVELATION PRINCIPLE WITH COSTLY COMMUNICATION[1]

by

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Abstract

The Revelation Principle in the principal-agent framework is extended to a case in which it is costly for the principal to communicate with any agent. It is shown that there is a _direct sequential_ mechanism which is optimal in the class of all mechanisms. This result is then applied to the problem of a buyer procuring a good from one of a set of possible suppliers with unobservable production costs. With costly communication, the optimal mechanism is a combination of reservation-price search and auction.

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One party to an exchange often knows something relevant to the transaction that another party does not know. Such asymmetries of information are pervasive in economic activity: for example, in the relationship between buyer and seller when the value of the item is uncertain; or between employer and employee when the employee's effort cannot perfectly be monitored; between the stockholders and the managers of a firm; between insurer and insured; between a regulated firm and the regulatory agency; between the supplier and the consumers of a public good; or between a taxpayer and the taxation authority.

The Revelation Principle[2] is a tool for analyzing exchange under asymmetric information. The Revelation Principle characterizes resource-allocation mechanisms which are optimal subject to the constraints imposed by the informational asymmetries. Myerson (1982) extended the Revelation Principle to a principal-agent setting: that is, to a situation where there is not only adverse selection (the agents have information which the principal cannot directly observe) but also moral hazard (the principal cannot control the agents' decisions).

In the usual formulation of the Revelation Principle, information, although asymmetric, can be transmitted without cost. This paper generalizes the Revelation Principle in a principal-agent framework to a case where communication is costly to the principal. It will be shown that what will be called a sequential direct mechanism is optimal in the class of all mechanisms. In a sequential direct mechanism, the principal asks the agents their types (and nothing else) in sequence; at any time the principal may stop asking agents their types, so that he need not communicate with all of the

1

agents. In addition, it can be assumed without loss of optimality that those agents who are asked reveal their types truthfully and execute the decision the principal recommends for them. Thus, one can construct an optimal mechanism by optimizing over the class of sequential direct mechanisms subject to the usual incentive-compatibility constraints.

The extended Revelation Principle is then applied (in Section 4) to analyze the problem facing a buyer who wishes to acquire a good from one of a set of possible suppliers whose production costs he cannot observe. In the absence of costs of communication, the buyer's optimal mechanism is well known to be a sealed-bid or oral auction. With costly communication, it will be shown that the optimal mechanism has the buyer approaching the potential suppliers in sequence; the optimal mechanism is a combination of reservation-price search and auction.

1.   Communication in the Principal-Agent Framework

There is a single principal, who is assumed to be able to commit himself to a mechanism. This is a nontrivial assumption, for the resulting equilibrium is not subgame perfect for the principal. In general, after the agents have revealed their private information, the principal would be able to use this information to his advantage by reneging on his pre-announced mechanism. However, the commitment is needed in order to induce the agents to reveal their private information. The same problem arises in any mechanism constructed in the principal-agent framework. In the present model, this difficulty is compounded by the principal's incurring communication costs. The need for commitment by the principal means that application of the revelation principle to be derived below is limited, as with the usual principal-agent problem, to situations where it can be presumed that such

commitment is possible.[3]

There are $n < \infty$ agents, indexed by $i \in \{1,2,\ldots,n\} \equiv N$, who behave noncooperatively. Denote by Ω the set of ordered subsets of agents: $\Omega = \{(i_1,\ldots,i_k) | k \leq n, i_j \in N, \text{ and } j < m \leq k \Rightarrow i_j \neq i_m\}$. On occasion, we shall refer to a member $\omega \in \Omega$, and we shall, in a loose but not misleading use of notation, refer to agent $i \in \omega$, meaning that $\omega = (i_1,\ldots,i_{j-1},i,i_{j+1},\ldots,i_k)$. Agent i has type $t_i \in T_i$, which only he can observe.[4] Let $T = X_{i \in N} T_i$. There is a probability distribution $P:T \to [0,1]$ which is common knowledge, and we assume that, given his type t_i, agent i uses the Bayesian posterior of P as the probability of the vector of types. Agent i can make decision $d_i \in D_i$; the principal cannot directly control the agent's decision. The principal makes a decision $d_o \in D_o$. Let $D = X_{i=o}^n D_i$. The agents have von Neumann–Morgenstern utility functions $U_i:D \times T \to R$.

Since communicating with an agent is costly for the principal, the principal's utility depends upon his communications. We shall assume that his utility depends only on $\omega \in \Omega$ and $(d,t) \in D \times T$. Interpret $\omega = (i_1,i_2,\ldots,i_k)$ to mean that the principal communicated first with agent i_1, second with i_2, and so forth. This requires that, once the principal has communicated with agent i, there are no additional costs incurred for further communication with agent i. (Corollary 2 will show that this restriction is without loss of generality.) Let $U_o:D \times T \times \Omega \to R$ be the principal's utility function. Thus, we are in essence assuming fixed costs of communication. Although communication costs vary with types and decisions, they are invariant to the actual message sent. It is in this sense that costs are fixed.

We restrict attention to what we shall call <u>principal-centered</u> mechanisms; by this is meant that the only type of communication that takes place is agent-to-principal or principal-to-agent. If, on the contrary,

agents were allowed to communicate among themselves, it would be in the principal's interest to incur the cost of communicating with only one agent, and to have that agent learn the types of the other agents. (It appears to be intractable to make agent-to-agent communication possible but costly.) Note that, in the absence of communication costs as in the usual formulation of the revelation principle, allowing only communication between principal and agent is not a restriction, because the principal could commit to passing signals from agent i to agent j without himself observing the signal (that is, the principal would commit himself not to condition any of his future signals or decisions on this communication). Thus there is a sense in which the restriction to principal-centered mechanisms is no less general than the usual Revelation Principle analysis with costless communication, in that the usual analysis can be interpreted as being principal-centered. Note also that there is a relationship between the assumption of no agent-to-agent communication and the assumption that the agents behave noncooperatively, in that communication is a prerequisite for cooperation in a static game: in this sense, the absence of agent-to-agent communication is a sufficient condition for noncooperative behavior.

We also restrict attention to what we shall call principal-initiated mechanisms. This is to avoid the possibility that the principal might use the mechanism itself as a communication device and in so doing avoid incurring the costs of communication.[5] For example, suppose the principal wishes to sell a good and chooses a mechanism that dictates that he will accept the first bid of not less than x^* that he receives from an agent. Then any agent who values the good at more than x^*, knowing that this is the principal's decision rule, will submit a bid of x^*. The principal then accepts the first bid he receives and shuts off communication, incurring the cost of only one communication. In

this example, the mechanism itself communicates the value x* costlessly to all agents. In a principal-initiated mechanism, this cannot happen. A mechanism is principal-initiated if no communication occurs between the principal and a particular agent unless the principal first sends a message to that agent; agents never initiate communication. (For example, one may imagine that the principal must contact the agent, explain the mechanism to him, and demonstrate that he is bound to it.) We assume, then, that if the principal does not communicate with agent i, then agent i chooses a decision $\hat{d}_i(t_i)$, which is the same regardless of which mechanism the principal has chosen.

To the extent that communication takes time, the principal and the agents are assumed not to discount future returns (or, alternatively, the communication process is sufficiently fast that discounting can be ignored as an approximation).[6] Thus simultaneous communication can be ignored without loss of generality. The principal has the option of mimicking a simultaneous mechanism by receiving communications sequentially but committing himself to make no use of the information until all of the information is received; that is, until the "simultaneous" information is fully received, no action or signal of the principal is conditioned on the initial information. Hence, in what follows, attention will be restricted to sequential mechanisms. At some increased notational complexity, real time could be included in the communication model. This will, in general, lead to simultaneous signals, as in Morgan's (1985) search model. The extension to this case is straightforward if it is feasible for the principal to delay sending signals. Indeed, the proof of the revelation principle is consistent with this case.

2.   Description of a Mechanism

A principal-initiated, principal-centered mechanism allows an exchange of signals between the principal and some or all of the agents and, when this exchange of information has ended, culminates in a vector of decisions $(d_0, d_1, \ldots, d_n) \in D$.

Since communication is costly for the principal, there will be some agents with whom the principal will not want to communicate at any stage. It is notationally useful to introduce a nonsignal, ξ_ϕ; ξ_ϕ is interpreted as meaning no signal was sent.

The kth stage of a mechanism occurs in two parts. First, the principal sends signals $\xi_k = (\xi_k^i)_{i=1}^n$ to the n agents. (Many of these may be ξ_ϕ.) Denote the principal's earlier signals by $_{k-1}\xi = (\xi_j)_{j=1}^{k-1}$ and his earlier signals to agent i in particular by $_{k-1}\xi^i = (\xi_j^i)_{j=1}^{k-1}$. The principal, having sent ξ_k to the agents, then receives from them signals $\eta_k = (\eta_k^1, \ldots, \eta_k^n)$. Let $_{k-1}\eta = (\eta_j)_{j=1}^{k-1}$ and $_{k-1}\eta^i = (\eta_j^i)_{j=1}^{k-1}$. The signals ξ_k^i and η_k^i are members of some signal spaces given by the mechanism (note that ξ_ϕ is a member of both signal spaces).

When the principal chooses his k^{th}-round signals ξ_k, he has previously chosen $_{k-1}\xi$ and observed $_{k-1}\eta$, and so he can condition his choice of ξ_k on these. To describe the choice of signal by the principal, let $\mu_k(\xi_k, {}_{k-1}\xi, {}_{k-1}\eta)$ be a probability distribution describing the choice of ξ_k.

Any agent's response η_k^i to the signal he receives, ξ_k^i, can be conditioned on $_k\xi^i$ and $_{k-1}\eta^i$ as well as the particular agent's type t_i. Let $\sigma_k^i(\eta_k^i, {}_k\xi^i, {}_{k-1}\eta^i, t_i)$ be the probability distribution governing the generation of the agent's reply η_k^i.

The fact that communication is principal-initiated means that σ_k^i must satisfy two requirements. First, a signal ξ_ϕ must be answered by ξ_ϕ, as ξ_ϕ means that no signal was sent by the principal to this agent. This implies that σ_k^i must satisfy:

(1) $\quad \sigma^i_k(\eta^i_k,(_{k-1}\xi^i,\xi^i_\phi),_{k-1}\eta^i,t_i) = \begin{cases} 1 & \text{if } \eta^i_k = \xi^i_\phi \\ \\ 0 & \text{otherwise .} \end{cases}$

Second, let $_{k+1}\hat{\xi}^i = (\xi^i_1,\dots,\xi^i_j,\xi^i_\phi,\xi^i_{j+1},\dots,\xi^i_k)$ and

$_k\hat{\eta}^i_1 = (\eta^i_1,\dots,\eta^i_j,\xi^i_\phi,\eta^i_{j+1},\dots,\eta^i_{k-1})$ for some $j \le k$. Then

(2) $\quad \sigma^i_{k+1}(\eta^i_o,_{k+1}\hat{\xi}^i,_k\hat{\eta}^i,t_i) = \sigma^i_k(\eta^i_o,_k\xi^i,_{k-1}\eta^i,t_i)$ for any η^i_o .

This is because $_{k+1}\hat{\xi}^i$ and $_k\hat{\eta}^i$ differ from $_k\xi^i$ and $_{k-1}\eta^i$ only by the addition of a nonsignal which, not being received, cannot affect the outcome of any future stage. Conditions (1) and (2) embody the fact that ξ_ϕ is merely a record-keeping device, and not a true signal.

Without loss of generality, the outcome ξ_ϕ from the distribution μ_k is used to denote the end of the communication process, where ξ_ϕ is the vector of nonsignals. Since it is possible that this point is reached before all of the agents have been communicated with, there must be some way of informing those agents who were not contacted that the process has ended. Introduce another fictitious signal ϵ, which the principal can, without cost, send to those agents he did not communicate with to tell them that the communication is finished.[7]

Upon communication finishing at the kth stage, the principal chooses a decision d_o according to the probability distribution $\mu_o(d_o,_k\xi,_k\eta)$, while each agent chooses his decision d_i given by the probability distribution $\sigma^i_o(d_i,_k\xi^i,_k\eta^i,t_i)$. The distribution σ^i_o must satisfy two conditions. First, since (as already noted) it must not be possible for the principal to use the mechanism so as to avoid incurring communication costs, there must be some exogenous decision $\hat{d}_i(t_i)$ which agent i takes if he has never been

communicated with; that is,

$$(3) \quad \sigma_o^i(d_i,(\xi_\phi,\ldots,\xi_\phi,\epsilon),(\xi_\phi,\ldots,\xi_\phi)t_i) = \begin{cases} 1, & \text{if } d_i = \hat{d}_i(t_i) \\ \\ 0, & \text{otherwise .} \end{cases}$$

Second, the addition of a nonsignal to the signals received by i must not

change i's decision; that is, defining $_{k+1}\xi^i$ and $_k n^i$ as before,

$$(4) \quad \sigma_o^i(d_i, {}_{k+1}\hat{\xi}^i, {}_k \hat{n}^i, t_i) = \sigma_o^i(d_i, {}_k \xi^i, {}_{k-1} n^i, t_i), \text{ for any } d_i.$$

(3) and (4) are analogous to (1) and (2). (Conditions (1), (2), (3), and (4)

together can be taken to be a formal definition of principal-initiated

communication.)

Let $\sigma^i = (\sigma_j^i)_{j=0}^\infty$; σ^i is a __strategy__ for agent i. A __message__ for agent i

is $m_i = ({}_k \xi^i, {}_k n^i)$; denote by M_i the set of messages. Formally, a __mechanism__

consists of the signal spaces and the probability distribution $\mu = (\mu_j)_{j=0}^\infty$.

Any mechanism, when combined with a vector of strategies $\sigma =$

$(\sigma^1,\ldots,\sigma^n)$, produces a distribution of outcomes $\zeta(d|t,\sigma)$. To solve for the

distribution of outcomes, one generates the distribution of messages at the

first stage, using μ_1 and σ_1. This is then used as an input into the second

stage, using μ_2 and σ_2; and so on. This gives rise to a distribution of

messages, which is then used in conjunction with μ_o and σ_o to produce a

distribution of decisions d ϵ D.

Agent i's expected utility is

$$(5) \quad V_i(\sigma) = \int_T \int_D U_i(t,d)\zeta(d|t,\sigma)P(t)dt \, dd .^{[8]}$$

Consider a strategy vector σ. Let $\tilde{\sigma}^{-i} = (\sigma^1,\ldots,\sigma^{i-1},\tilde{\sigma}^i,\sigma^{i+1},\ldots,\sigma^n)$.

Then σ is a Bayesian Nash equilibrium for the principal-initiated mechanism

μ if and only if, for all $\tilde{\sigma}^i$ satisfying (1), (2), (3), and (4),

(6) $V_i(\sigma) \geq V_i(\tilde{\sigma}_i)$ for all $i=1,\ldots,n$.

3. Direct Sequential Mechanisms

In a __direct__ mechanism,[9] each agent simply reports his type from T to the principal, and the principal responds by suggesting a decision from D_i for the agent.

The viability of an agent to communicate without prompting in a principal-initiated mechanism causes us to introduce yet another fictitious signal, ξ_o. The principal, by sending ξ_o to agent i, opens the communication and is the cue for the agent to respond with his type.

Formally, a mechanism is a __direct sequential__ mechanism if the set of messages M_i contains only elements of the type (7) or (8). (Here it is supposed that the principal chooses to cease communication after the kth stage; and in (7), the (ξ_o, t_i) element occurs at the jth component; that is, the jth stage of the sequential process):

$$
(7) \quad (_k\xi^i, _kn^i) =
\begin{bmatrix}
\xi_\phi & \xi_\phi \\
\vdots & \\
\xi_\phi & \xi_\phi \\
\xi_o & t_i \\
\xi_\phi & \xi_\phi \\
\vdots & \vdots \\
d_i & \xi_\phi
\end{bmatrix}
\equiv \gamma_j ;
$$

$$(8) \quad (_k\xi^i, _k\eta^i) = \begin{bmatrix} \xi_\phi & \xi_\phi \\ \cdot & \cdot \\ \cdot & \cdot \\ \xi_\phi & \xi_\phi \\ \epsilon & \xi_\phi \end{bmatrix}.$$

With the message (7), the principal communicates with agent i at the jth stage, asking and being told i's type. The principal then, at the kth and last stage, signals to i that the communication is over by suggesting the decision d_i for agent i. With the message (8), the principal never communicates with i except at the end to inform the agent, without incurring any cost of communication, that the process is ended.

An agent is <u>honest</u> is

$$(9) \quad \sigma_j^i(t_i, \xi_0, t_i) = 1 \quad \text{for all } t_i.$$

(9) implies, using (2), that agent i correctly reports his type when asked (that is, upon receiving the message ξ_0 as in (7)). An agent is <u>obedient</u>[10] if, with γ_j given by (7),

$$(10) \quad \sigma_0^i(d_i, \gamma_j, t_i) = 1 \quad \text{for all } t_i;$$

that is, the agent takes the decision recommended by the principal.

The direct sequential mechanism is <u>incentive compatible</u> when honest and obedient strategies form a Bayesian Nash equilibrium.

We now show how the Revelation Principle extends to the case of costly communication. A direct sequential mechanism is optimal in the class of all principal-initiated mechanisms.

THEOREM 1: <u>Corresponding to any equilibrium σ for any principal-initiated mechanism μ, there is a direct sequential incentive-compatible mechanism μ^* in which, for each vector of types t ϵ T, the honest and obedient strategy σ^* produces the same distribution of decisions and agents communicated with</u> (d, ω), d ϵ D, ω ϵ Ω.

PROOF: The proof begins by constructing μ^* from μ, σ. It is then demonstrated that μ^* is incentive compatible. It will be clear from the construction that, for any t, the distribution of (d,ω) is unchanged. The construction is by induction. Denote by ν_k the set of agents communicated with in the direct sequential mechanism μ^* by the end of the kth stage.

The base of the induction requires .

$$(11) \quad \mu_1^*(\xi_\phi,\ldots,\xi_\phi,\xi_0,\xi_\phi,\ldots,\xi_\phi) = \sum_{\substack{i \\ \xi_1^i \neq \xi_\phi}} \mu_1(\xi_\phi,\ldots,\xi_\phi,\xi_1^i,\xi_\phi,\ldots,\xi_\phi).$$

Let ν_0 be the empty tuple. This provides the base of the induction.

Now suppose $(_{k-1}\xi,_{k-1}\eta)$, an internal vector to μ^*, has been given. The following steps represent the internal workings of the mechanism.

1. Choose ξ_k from the distribution $\mu_k(\xi_k,_{k-1}\xi,_{k-1}\eta)$. If $\xi_k = \xi_\phi$, go to 4. If there is no $i_k \epsilon \omega$ with $_k\xi^{i_k} = (_{k-1}\xi_\phi,\xi_k^k)$ and $\xi_k^k \neq \xi_\phi$, go to 3. Otherwise, proceed to 2.

2. To be here, there must be an agent i_k receiving his first signal. Send i_k the signal ξ_0. Let $\nu_k = (\nu_{k-1},i_k)$. i_k responds with his type t_{i_k}. Go to 3.

3. Since t_i is known for all types who have been sent any message other than ξ_ϕ, we may operate σ_k^i on $(_k\xi^i,_{k-1}\eta^i)$ and generate a signal, internal to the mechanism, η_k^i, which is added to $_{k-1}\eta^i$ to produce $_k\eta^i$. For other $i \notin \nu_k$, $_k\xi^i = \xi_\phi$, so set $\eta_k^i = \xi_\phi$. Return to 1.

4. The last value of ξ_k was $_k\xi_\phi$. For all $i \epsilon \nu_k$, draw a decision d_i from the distribution $\sigma_0^i(d_i,_k\xi^i,_k\eta^i,t_i)$. Send this decision d_i to i, and send ϵ to all other agents. Draw a decision d_0 from the distribution $\mu_0(d_0,_k\xi,_k\eta)$.

Note that the resulting mechanism μ^* is direct and sequential. By construction, $\nu_k = \omega$. Since the same distributions are used, the same distribution of outcomes occurs for each $t \epsilon T$, provided the agents are

honest and obedient. Finally, to see that this mechanism is incentive compatible, suppose that at stage j agent i responds with type t_i' when his true type is t_i. Then, in the original mechanism, the σ_j^i arising from type t_i' must dominate the σ_j^i arising from the type t_i. Thus, σ^i was suboptimal, contrary to hypothesis. Similarly, if the agent does not choose d_i as suggested by the principal, there must be a preferred decision d_i', contrary to the hypothesis that σ_0^i was optimal.

<div align="right">

Q.E.D.
</div>

If it is desirable to include real time and discounting in the model, the definition of sequential mechanisms must be modified to allow asking several agents their type simultaneously. The only modification necessary to the proof is allowing the principal to let real time pass in the computation of ε_k, in step 3, so that μ^* involves the same timing as μ.

Note that this proof is essentially the same as the proof of the usual Revelation Principle (Myerson, 1982), except that the fact that communication is costly means that care must be taken over the timing of messages: in particular, the proof must keep track of exactly which agents have been communicated with, and which have not, at any stage. This is why the fictitious signals ξ_ϕ, ξ_0, and ε are needed.

The foregoing analysis assumed that the principal incurred a cost only upon the first communication with any particular agent. However, the proof showed that the principal need only communicate at most twice with each agent; either not at all, or once to ask his type and once to suggest his decision. Clearly, therefore, if the second and subsequent communications with any one agent are costly to the principal, the principal optimizes by using a direct sequential mechanism. The assumption that only the first communication with any agent is costly for the principal is therefore without loss of generality. This is stated in the following corollary.

COROLLARY 2: <u>Suppose the principal's utility depends upon the whole vector</u>

$\omega = (i_1, \ldots, i_k)$, $i_j \epsilon$ N, <u>of agents communicated with. Let $\hat{\omega}$ =</u>

$(i_1, \ldots, i_j, \hat{i}_j, i_{j+1}, \ldots, i_k)$ <u>and suppose that, for all</u> $(d, t) \epsilon$ D x T,

$U_0(d, t, \omega) \leq U_0(d, t, \hat{\omega})$. <u>Then for any principal-initiated mechanism μ and any</u>

<u>equilibrium σ, there is a direct sequential incentive-compatible mechanism</u>

<u>which produces the same equilibrium distribution of</u> d <u>given</u> t, <u>and which</u>

<u>leaves the principal no worse off</u>.

4.   <u>Searching for the Lowest Bid</u>

The level of abstraction of the foregoing analysis is such that it is

not apparent whether the incentive-compatibility constraints are tractable

even in straightforward problems. We now illustrate the application of this

costly communication version of the Revelation Principle.

Consider a monopsonist who wishes to buy one unit of an indivisible

good. The buyer is assumed to be able to commit himself to a purchasing

policy. There are n potential suppliers, who vary in that they may have

different production costs, which only they themselves know. Denote a

supplier's production cost by x, and suppose that costs are identically and

independently distributed as $F(x)$, with $F'(x) = f(x)$ and $F \epsilon C^1$, $F(0) = 0$.

The buyer is able to produce the good himself at a cost of c_o. Both the

potential sellers and the buyer are risk neutral.

The buyer incurs a cost c > 0 every time he contacts a potential

supplier. For example, it may be that there is competition over design as

well as price, so that the bids differ in several quality dimensions as well

as in price. The buyer must reduce these multidimensional characteristics to

a single-dimensional comparison in order to decide which is the best offer.

Doing this may be time-consuming and therefore costly for the buyer.[11]

In the absence of the buyer's cost of search or evaluation c, the buyer's optimal procurement policy is well known: the buyer holds an auction; for example, either a sealed-bid auction or an oral auction will minimize the buyer's expected procurement costs.[12] It will be shown that, when taking bids is costly for the buyer, the usual auction forms are no longer optimal. Instead, the optimal mechanism works like a marriage of auction and reservation-price search. In particular, we show that, with an infinite number of potential suppliers, reservation-price search is optimal, while with a finite number of potential suppliers, the optimal mechanism involves a period of reservation-price search followed by an auction. (It is worthy of note that, while it is conceivable that the presence of information costs could result in the optimal mechanism being very complicated, it is in fact, as will be seen, remarkably simple.)

The model to be presented can be interpreted as a model of the process of negotiating a price. Whereas, in the public sector, procurement is usually done by means of sealed-bid auctions, in the private sector, negotiations tend to be used for procurement (McAfee and McMillan (1985c, Ch. 1)). This analysis suggests an explanation for profit-maximizing firms' preference for negotiations over auctions.

Before developing the mechanism, it is useful to summarize the notation to be used. Define

$$(12) \qquad J(x) = x + \frac{F(x)}{f(x)} ,$$

and assume J is strictly increasing, on $\{x | 0 < F(x) < 1\}$. (This is the analog--for buying as opposed to selling--of the J function of Maskin and Riley (1983) and the c_i function of Myerson (1981): it is assumed to be strictly increasing for the same reasons as in those papers.) Define

$$x_0 = J^{-1}(c_0).$$

Since the function J will figure prominently in the analysis that follows, the following lemma is useful as an aid to understanding.

LEMMA 3: <u>Let s^i represent the expected value of the ith order statistic of the set of suppliers' costs. Then</u>

(13) $E[J(s^1)] = E[s^2].$

PROOF: The density of the second order statistic is

$$n(n-1)[1-F(x)]^{n-2}F(x)f(x) \ .$$

Thus the expected value of the second order statistic is

$$(14) \quad Es^2 = \int_0^\infty xn(n-1)[1-F(x)]^{n-2}F(x)f(x)dx$$

$$= -n \int_0^\infty [xF(x)] \frac{d}{dx} [1-F(x)]^{n-1}dx$$

$$= -nxF(x)[1-F(x)]^{n-1}\Big|_0^\infty + n \int_0^\infty [xf(x) + F(x)][1-F(x)]^{n-1}dx$$

$$= \int_0^\infty J(x)n[1-F(x)]^{n-1}f(x)dx \ .$$

Thus, since $n[1-F(x)]^{n-1}f(x)$ is the density of the first order statistic, (13) holds.

<div align="right"><u>Q.E.D.</u></div>

It follows from (13) that the expected difference between the lowest-cost supplier's cost, x, and the second-lowest-cost supplier's cost is $F(x)/f(x)$. Thus, by the usual auction-theory intuition (see McAfee and McMillan (1985b), for example), the winning bidder's profit is on average $F(x)/f(x)$ and his expected payment is $J(x)$.

From Theorem 1, we know that the optimal direct mechanism has the buyer sequentially asking the potential suppliers their types (production costs). Let the subscript i denote the ith potential supplier asked. Let $x_1,\ldots,x_n$ be the random variables that are their responses. Let $y_k = \min\{x_0,x_1,\ldots,x_k\}$ be the lowest of the first k responses, together with x_0.

At the kth stage, the buyer chooses between producing the good himself at cost c_o; buying the good from person i, i=1,...,k; or continuing to ask further potential suppliers. Given $x_o, x_1, ..., x_{k-1}$, denote by γ_k^o, γ_k^i, i=1,...,k, and α_k the set of kth responses x_k in which he makes these respective decisions. Thus, if $x_k \epsilon \gamma_k^o(x_o, x_1, ..., x_{k-1})$, the buyer decides after the kth stage to produce the good himself; if $x_k \epsilon \gamma_k^i(x_o, x_1, ..., x_{k-1})$, he purchases it from supplier i; and if $x_k \epsilon \alpha_k(x_o, x_1, ..., x_{k-1})$, he takes his $(k+1)^{th}$ observation. Let

(15) $\quad \Gamma_k^i = \{(x_1, ..., x_k) | x_k \epsilon \gamma_k^i(x_o, x_1, ..., x_{k-1})\}$, i=o,...,k

and

(16) $\quad \beta_k^i(z) = \{(x_1, ..., x_{i-1}, x_{i+1}, ..., x_k) / (x_1, ..., x_{i-1}, z, x_{i+1}, ..., x_k) \epsilon \Gamma_k^i\}$,

$$i = 0, 1, ..., k$$

Thus Γ_k^i is the set of others' responses such that bidder i wins in round k if he reports x_i; $\beta_k^i(z)$ is the set of others' responses such that i wins in the kth round if he reports z. The arguments of α_k and γ_k^i will be suppressed for clarity.

It follows that, if a seller reports a cost of z, his probability of winning the contract is

(17) $\quad \mu(z) = \sum_{k=1}^{n} \sum_{i=1}^{k} \int_{\beta_k^i(z)} f_k(x_{-i}) dx_{-i}$

where $x_{-i} = (x_1, ..., x_{i-1}, x_{i+1}, ..., x_k)$ and $f_k(x_{-i}) = \prod_{\substack{j=1 \\ j \neq i}}^{k} f(x_j)$.

It follows from Theorem 1 that there is a function $A_k^i(x)$ which represents the amount i is paid, given that he is asked to supply the good in the kth round. Assume that i is paid if and only if he is asked to supply the good: this is without loss of generality because of risk neutrality. Denote by p(z) an agent's average payment given that he is asked to supply the good

and that his reported cost is z; p(z) satisfies

$$(18) \qquad \mu(z)p(z) = \sum_{k=1}^{n} \sum_{i=1}^{k} \int_{\beta_k^i(z)} A_k^i(x_{-i},z)f_k(x_{-i})dx_{-i} \ .$$

If a seller's true cost is x and he reports z, his expected profit is

$$(19) \qquad \pi(z) = \mu(z)[p(z) - x].$$

The incentive-compatibility constraint requires that $\pi(z)$ is maximized at z = x. This requires

$$(20) \qquad \frac{d}{dz}[p(z)\mu(z)]\bigg|_{z=x} = x\mu'(z)\bigg|_{z=x} ,$$

or

$$(21) \qquad p(x)\mu(x) = -\int_x^{x_m} z\mu'(z)dz$$

$$= x\mu(x) + \int_x^{x_m} \mu(z)dz \ ,$$

where x_m is defined by $p(x_m)\mu(x_m) = 0$. Since $p(x) \geq x$ for $\mu(x) > 0$ by the assumption of free exit, x_m satisfies $x_m = \inf\{x|\mu(x) = 0\}$. In addition, the second-order condition requires $\mu'(x) \leq 0$, which is assumed to hold, and will hold in the solution.

LEMMA 4: <u>The expected payment to the successful bidder is</u>:

$$(22) \qquad \tau = \sum_{k=1}^{n} \sum_{i=1}^{k} \int_{\Gamma_k^i} J(x_i)f(x)dx \ .$$

PROOF:

$$\tau = \sum_{k=1}^{n} \sum_{i=1}^{k} \int_{\Gamma_k^i} A_k^i(x)f(x)dx \qquad \text{(by definition of } A_k^i)$$

$$= \sum_{k=1}^{n} \sum_{i=1}^{k} \int_0^{\infty} [\int_{\beta_k^i(z)} A_k^i(x_{-i},z)f_k(x_{-i})dx_{-i}]f(z)dz$$

$$= \int_{o}^{\infty} \sum_{k=1}^{n} \sum_{i=1}^{k} \int_{\beta_k^i(z)} A_k^i(x_{-i}, z) f_k(x_{-i}) dx_{-i}] f(z) dz$$

$$= \int_{o}^{\infty} p(z) \mu(z) f(z) dz \qquad \text{(from (18))}$$

$$= \int_{o}^{x_m} p(z) \mu(z) f(z) dz$$

$$= \int_{o}^{x_m} z f(z) \mu(z) dz + F(x) \int_{z}^{x_m} \mu(x) dx \Big|_{o}^{x_m} + \int_{o}^{x_m} f(z) \mu(z) dz \qquad \text{(from (21))}$$

$$= \int_{o}^{\infty} J(z) f(z) \mu(z) dz \qquad \text{(from (12))}$$

$$= \int_{o}^{\infty} J(z) f(z) \sum_{k=1}^{n} \sum_{i=1}^{k} \int_{\beta_k^i(z)} f_k(x_{-i}) dx_{-i} \, dz \qquad \text{(from (17))}$$

$$= \sum_{k=1}^{n} \sum_{i=1}^{k} \int_{o}^{\infty} \int_{\beta_k^i(z)} J(z) f(z) f_k(x_{-i}) dx_{-i} \, dz$$

$$= \sum_{k=1}^{n} \sum_{i=1}^{k} \int_{\Gamma_k^i} J(x_i) f(x_i) f_k(x_{-i}) dx_{-i} \, dx_i$$

$$= \sum_{k=1}^{n} \sum_{i=1}^{k} \int_{\Gamma_k^i} J(x_i) f(x) dx.$$

<div align="right">Q.E.D.</div>

Define some more notation: let

$$(23) \qquad h_k = c + \sum_{i=o}^{k} \int_{\gamma_k(x_o, x_1, \ldots, x_{k-1})} J(x_i) f(x_k) dx_k ;$$

$$(24) \qquad Q_k = h_k(x_o, x_1, \ldots, x_{k-1}) + \int_{\alpha_k(x_o, x_1, \ldots, x_{k-1})} f(x_k) Q_{k+1}(x_o, x_1, \ldots, x_k) dx_k ;$$

$$(25) \qquad Q_n = h_n.$$

The interpretation of these variables is as follows. Suppose the principal

has observed $x_1, \ldots, x_{k-1}$. His expected cost if he takes exactly one more

observation is h_k since, when he accepts a type x_i, he pays him $J(x_i)$ as implied by Lemmas 3 and 4. His total expected cost given that he does not stop at the $(k-1)^{th}$ observation is Q_k because he pays h_k if he stops at the kth observation and he pays Q_{k+1} if he continues beyond the kth observation. Hence Q_k is the principal's total expected cost associated with taking the kth observation.

LEMMA 5: <u>The total expected cost incurred by the principal is</u>, for $1 \leq k \leq$ n-1:

(26) $\Phi = h_1 + \int_{\alpha_1} f(x_1)[h_2(x_1) + \int_{\alpha_2} f(x_2)[\ldots[\int_{\alpha_k} f(x_k)Q_{k+1}(x_0,x_1,\ldots,x_k)dx_k]\ldots]dx_1$

PROOF: From (22),

(27) $\Phi = \sum_{k=1}^{n} [(c + kc)\int_{\Gamma_k^0} f(x)dx + \sum_{i=1}^{k} \int_{\Gamma_k} [J(x_i) + kc]f(x)dx]$

$= \sum_{k=1}^{n} [\sum_{i=o}^{k} \int_{\Gamma_k} [J(x_i) + kc]f(x)dx]$

$= c + \sum_{k=1}^{n} \int_{\alpha_1}\ldots\int_{\alpha_{k-1}} \prod_{j=1}^{k-1} f(x_j)\{c\int_{\alpha_k} f(x_k)dx_k + \sum_{i=o}^{k} \int_{\gamma_k} J(x_i)f(x_k)dx_k\}$

$dx_{k-1}\ldots dx_n$ (by (15))

$= h_1 + \int_{\alpha_1} f(x_1)[h_2(x_1) + \int_{\alpha_2} f(x_2)[\ldots[\int_{\alpha_{n-1}} f(x_{n-1})h_n(x_{-n})dx_{n-1}]\ldots]dx_1$

     (by (23));

$= h_1 + \int_{\alpha_1} f(x_1)[h_2(x_1) + [\ldots[+ \int_{\alpha_{n-2}} f(x_{n-2})Q_{n-1}(x_0,\ldots,x_{n-2})dx_{n-2}]\ldots]dx_1$.

(26) is obtained by backward induction.

<u>Q.E.D.</u>

Clearly, the cost to the principal of searching further, Q_k, depends on the reported costs $x_0,x_1,\ldots,x_{k-1}$. However, we now show it depends only on the minimum of these, $y_{k-1} = \min\{x_0,x_1,\ldots,x_{k-1}\}$.

LEMMA 6: <u>Minimizing the total expected cost incurred by the principal, Φ,</u>

<u>implies that the cost of continuing, Q_k, depends only on the lowest previously</u>

<u>observed cost, y_{k-1}.</u>

PROOF: The proof is by induction. For the base, note that, from the last

line of the proof of Lemma 5, minimizing Φ requires minimizing $Q_n = h_n$. By

(23), this occurs by putting x_k in γ_n^i when $J(x_i)$ is smallest, which occurs at

$x_i = y_{n-1}$ if $x_n \geq y_{n-1}$, or $x_i = x_n$ if $x_n < y_{n-1}$. Thus

$$(28) \qquad Q_n = h_n = c + \int_0^{y_{n-1}} J(x_n)f(x_n)dx_n + \int_{y_{n-1}}^\infty J(y_{n-1})f(x_n)dx_n$$

$$= c + \int_0^{y_{n-1}} J(x_n)f(x_n)dx_n + [1-F(y_{n-1})]J(y_{n-1}).$$

This proves the base of the induction. From (26), we must minimize Q_k over

α_k, γ_k^i. Suppose Q_{k+1} depends only on y_k. Let $y_k = \min\{y_{k-1}, x_k\}$.

$$(29) \qquad Q_k = c + \sum_{i=0}^{k} \int_{\gamma_k^i} J(x_i)f(x_i)dx_k + \int_{\alpha_k} f(x_k)Q_{k+1}(y_k)dx_k \qquad \text{(by (23), (24))}$$

$$= c + \int_0^{y_{k-1}} \min\{J(x_k), Q_{k+1}(x_k)\}f(x_k)dx_k$$

$$+ \int_{y_{k-1}}^\infty \min\{J(x_k), Q_{k+1}(y_{k-1})\}f(x_k)dx_k$$

$$= c + \int_0^{y_{k-1}} \min\{J(x), Q_{k+1}(x)\}f(x)dx + [1-F(y_{k-1})]\min\{J(y_{k-1}),$$

$$Q_{k+1}(y_{k-1})\} .$$

Since this depends only on y_{k-1}, the proof is complete.

<div align="right"><u>Q.E.D.</u></div>

COROLLARY 7: <u>The set of reports for which, at the kth stage, the buyer</u>
<u>decides to continue searching, is</u>

$$(30) \qquad \alpha_k(y_{k-1}) = \{x \mid z = \min\{y_{k-1}, x\} \Rightarrow Q_{k+1}(z) \leq J(z)\} \ .$$

Define

$$(31) \qquad \psi_k(y) = Q_k(y) - J(y)$$

Thus $\psi_k(y)$ is the difference between the expected cost to the principal of searching further and the cost of purchasing at the current best observation.

LEMMA 8: $\psi_k(y)$ <u>is strictly decreasing in</u> y.

PROOF: As the base of the induction, note that $\psi_n' = -J' < 0$ by assumption.

From (31) and the last line of the proof of Lemma 6,

$$(32) \qquad \psi_k(y) = c + \int_0^y \min\{0, \psi_{k+1}(x)\}f(x)dx + \int_0^y J(x)f(x)dx + F(y)J(y)$$
$$+ [1-F(y)] \min\{0, \psi_{k+1}(y)\}$$

$$= c + \int_0^y [J(x)-J(y)]f(x)dx + \int_0^y \min\{0, \psi_{k+1}(x)\}f(x)dx$$
$$+ [1-F(y)] \min\{0, \psi_{k+1}(y)\}$$

$$= c - \frac{F^2(y)}{f(y)} + \int_0^y \min\{0, \psi_{k+1}(x)\}f(x)dx + [1-F(y)]\min\{0, \psi_{k+1}(y)\},$$

the last line following because

$$(33) \qquad \int_0^y J(x)f(x)dx = xF(x)\Big|_0^y - \int_0^y F(x)dx + \int_0^y F(x)dx \qquad \text{(by (12))}$$
$$= yF(y),$$

so that

$$(34) \qquad \int_0^y [J(x)-J(y)]f(x)dx = yF(y) - [y + \frac{F(y)}{f(y)}]F(y)$$
$$= \frac{-F^2(y)}{f(y)} \ .$$

It follows from the last line of (32) that

$$(35) \qquad \psi_k' = -J' + [1-F(y)] \frac{d}{dy} \min\{0, \psi_{k+1}(y)\} < 0.$$

<u>Q.E.D.</u>

Define x_k^* by $\psi_{k+1}(x_k^*) = 0$. Since $\psi_{k+1}(y)$ is strictly decreasing for $0 < F(y) < 1$, there is at most one interior solution to $\psi_{k+1}(x_k^*) = 0$. From Corollary 7, x_k^* acts like a reservation price (or, more accurately, a reservation type), since the principal continues to search if and only if $y_k > x_k^*$.

THEOREM 9: $x_1^* = x_2^* = \ldots = x_{n-1}^*$.

PROOF: Suppose, by way of induction, $x_k^* = x_{k+1}^* = \ldots = x_{n-1}^*$, for some $k \le n-1$. This is true for $k = n-1$. From (32)

$$(36) \quad \psi_k(x_k^*) = c - \frac{F^2(x_k^*)}{f(x_k^*)} + [1-F(x_k^*)]\min\{0,\psi_{k+1}(x_k^*)\}$$

$$+ \int_0^{x_k^*} \min\{0,\psi_{k+1}(x)\}f(x)dx$$

$$= [c - \frac{F^2(x_{n-1}^*)}{f(x_{n-1}^*)}] + \int_0^{x_k^*} \min\{0,\psi_{k+1}(x)\}f(x)dx$$

$$= 0,$$

since the first term in brackets is $\psi_n(x_{n-1}^*) = 0$ and $x < x_k^* \Rightarrow \psi_{k+1}(x) \ge \psi_{k+1}(x_k^*) = 0$. Thus $x_{k-1}^* = x_k^*$.

Q.E.D.

Define x* by

$$(37) \quad c = \frac{F^2(x^*)}{f(x^*)}.$$

From the proof of Theorem 9, x* is the constant cutoff reported cost which determines whether or not the principal continues searching. For some intuitive understanding of why this cutoff is determined by (37), recall from Lemma 3 that a supplier with cost x* makes a profit equal to the difference on

average between his own cost and the cost of the second-lowest bidder, or $F(x^*)/f(x^*)$. Suppose the lowest-cost bidder the principal has so far observed has a cost of x^*. If the principal stops searching now and buys from this bidder, the price he pays is $x^* + F(x^*)/f(x^*)$. If instead he takes one more observation and finds a lower-cost supplier, he must pay the new bidder a price equal to the cost of the second-lowest cost bidder, which is now x^*. Thus, if he searches once more and finds a lower-cost supplier, he saves $F(x^*)/f(x^*)$ on average. The probability of finding a lower-cost supplier with the next observation is $F(x^*)$. Hence the marginal expected benefit to one more observation is $F^2(x^*)/f(x^*)$. The marginal cost is c. Hence (37) simply equates marginal benefit to marginal cost.

The optimal mechanism can now be summarized.

THEOREM 10: <u>The optimal strategy for the buyer is</u>:

(a) <u>If</u> $x_0 = J^{-1}(c_0) < x^*$, <u>the buyer consults no firms</u>.

(b) <u>If</u> $x_0 > x^*$ and $F(x^*) = 1$, <u>the buyer takes one observation and pays</u> $x_{max} = \inf\{x|F(x) = 1\}$.

(c) <u>If</u> $x_0 > x^*$ and $F(x^*) < 1$, <u>the buyer sequentially samples the firms until the first with a cost of no greater than</u> x^* <u>is found; otherwise he samples all of the firms and either buys from the lowest-cost firm or produces the good himself if the lowest-cost firm's cost exceeds</u> x_0. <u>The payment to a firm with cost x is then</u>

$$(38) \quad p(y) = \begin{cases} y + [1-F(y)]^{-(n-1)} \int_y^{x_0} [1-F(x)]^{n-1} \, dx, & y > x^* \\ x^* + \int_{x^*}^{x_0} [1-F(x)]^{n-1} \, dx & y \leq x^*. \end{cases}$$

<u>On average, the total cost to the buyer is</u>

$$(39) \quad \Phi = J(x_o)[1-F(x_o)]^n + [x^* + \frac{c}{F(x^*)}][1-(1-F(x^*))^n]$$

$$+ \int_{x^*}^{x_o} J(x)n[1-F(x)]^{n-1}f(x)dx.$$

PROOF: (38) and (39) follow from the fact that, given that the cutoff type is defined by (37), the probability of a firm with cost x winning the contract is

$$(40) \quad \mu(x) = \begin{cases} 1 & x \leq x^* \\ n[1-F(x)]^{n-1} & x^* < x \leq x_o \\ 0 & x > x_o \end{cases}$$

Q.E.D.

Just as the optimal direct incentive-compatible auction in the usual case of costless communication can be implemented as a sealed-bid or oral auction, so the optimal direct sequential incentive-compatible mechanism has its bidding counterpart. The principal in sequence invites potential suppliers to submit price quotations. Bidder i, with production cost x_i, rationally bids $p(x_i)$. The buyer either awards the contract to the first bidder who bids less than $p(x^*)$, or produces the item himself if no bid is less than $J(x_o)$. This may be regarded as a model of a negotiating process. Note that, analogously with the usual equivalence between sealed-bid and oral auctions in the case of risk-neutral bidders, in the sequential auction the seller is indifferent between receiving bids openly or in secret; in particular, the seller gains nothing on average by informing a bidder about the levels of previous bids. The fact that the buyer rejects all bids and produces the good himself if the lowest bidder's production cost exceeds x_o means that, as in the usual auction model, the buyer sets a reserve cost x_o which is strictly less than his own production cost c_o (since $c_o = J(x_o) > x_o$), so that there is some probability of an inefficient outcome, with the

buyer producing the item himself even though he has found a firm with a lower production cost.[13]

Implementing the optimal mechanism by a sequence of price quotations makes complete the analogy with search theory. The cutoff production cost x^* defined by (37) implies a reservation-price rule. Let G represent the cumulative distribution of offered prices in the usual search formulation. Then, with c as the unit search cost, the reservation price r is defined in the usual search model by[14]

$$(41) \qquad c = \int_0^r (r-p)G'(p)dp = \int_0^r G(p)dp \ .$$

(This simply equates the marginal cost of taking one more observation with the marginal expected gain.) In the present model, the distributions of offered prices is $G(p) = F(J^{-1}(p))$, from Lemma 3.

THEOREM 11: <u>The reservation price r satisfies</u>

$$(42) \qquad r = J(x^*).$$

PROOF: Note that both x^* and r are functions of the search cost c. Suppose $c > 0$. Differentiating (37) and using (12) yields

$$(43) \qquad \frac{dc}{dx^*} = J'(x^*)F(x^*).$$

Thus

$$(44) \qquad \frac{dJ(x^*)}{dc} = \frac{1}{F(x^*)}$$

Also, from (41),

$$(45) \qquad \frac{dr}{dc} = \frac{1}{G(r)} = \frac{1}{F(J^{-1}(r))}$$

If the search cost c were zero, then $J(x^*(0)) = J(x_{min}) = x_{min} = r(0)$, where $x_{min} = \inf\{x|F(x) > 0\}$. Thus, since r and J have the same derivatives and are equal at one point, they are the same function.

<div align="right">Q.E.D.</div>

The number of potential suppliers, n, was assumed to be finite. However, taking limits, the buyer's expected total cost Φ approaches the reservation price r.[15] To see this, note from (39) that as $n \to \infty$,

$$(46) \qquad \Phi \to x^\star + \frac{c}{F(x^\star)} = x^\star + \frac{F(x^\star)}{f(x^\star)} = J(x^\star) = r.$$

Also, from (38), the expected payment received by the successful bidder, $p(y)$ approaches $x^\star$ as $n \to \infty$.

To summarize: whereas the buyer's optimal mechanism in the absence of communication costs is an oral or sealed-bid auction, with communication costs the optimal mechanism consists of sequential search together with some auction-like features.

5. Conclusion

This paper extends the revelation principle to a case involving costly communication. Although the model of the principal-agent framework is quite general, the costs of communication are fixed costs in the sense that they are invariant to the message that is sent. Thus, although the costs may depend on the actual types or decisions that emerge, they are invariant to the actual signals, and in particular to the size of the signals, that are sent. Thus, one might immediately inquire about a broader framework where the message space, or the message that is sent, matters.

We conjecture that the revelation principle fails to obtain in a broader framework in any meaningful way. An agent's type may be an exceedingly long vector of attributes and information, most of which is irrelevant to the principal. Indeed, the principal may only be concerned with a minor attribute, and if the size of the signals matter, will only ask the agent concerning this minor attribute. Thus, full revelation must certainly be suboptimal if the size of the signal matters.

The situation may be somewhat worse still. If an agent could be either type A, B, or C, and the size of the message space matter, it could be optimal for the principal to restrict the message space to {A,B}. Thus, an agent of type C may be forced to lie, simply to avoid the complexity of allowing the agent to be truthful. That is, honest revelation may be strictly suboptimal.

One might hope for a generalization that requires the agent to be truthful when he can be. There are two problems associated with this. First, it is unclear when the Revelation Principle ceases to be a revelation solution, in the sense that, if the mechanism is forcing the agent to lie, it is difficult to call this revelation. The second problem is more substantial. Rubenstein (1985) considered a repeated prisoner's dilemma played by finite machines called automata. The problem of choosing an automata is akin to choosing a mechanism, and in his model, the number of states, which is equivalent to the size of the signal space, mattered. Rubenstein observed that imposing costs on the number of states is difficult to do without an ad hoc approach. This problem is also discussed in McAfee (1984), where it is noted that there exist achievable outcomes to a problem that have no cheapest mechanism generating them. Thus, there must be examples where it is not possible to optimize over the cost of computation.

The second problem with this approach was raised by Rapullo. It may be the case that an indirect mechanism yields a unique Nash equilibrium, while any direct mechanism resulting in the same distribution of outcomes has multiple equilibria. Thus, it should be pointed out that the revelation principle only reveals feasible outcomes. There may be other advantages to indirect mechanisms, in particular by avoiding multiple equilibria, that make them desirable.

The use of sequential mechanisms will often be undesirable when time matters, and communication takes time. Thus, in this case, one might wish to send signals to several agents simultaneously, in order to reach a decision in less time. Our model generalizes to this case in a straightforward manner, with the caveat that a sequential mechanism may involve communications with several agents simultaneously. The proof of Theorem 1 is consistent with this generalization. It may be possible to show that the search strategy of Morgan (1983), involving samples of several observations, is optimal in the class of all mechanisms, using a method analogous to our construction.

Finally, the bargaining model of Rubenstein (1982) involves a fixed mechanism and costs of communication. In this model, agreement is reached after a single communication, although reaching this agreement requires an infinite sequence of strategies for the case when agreement fails to be reached, in the perfect equilibrium. This paper suggests an intuition for why agreement is reached immediately, and why we might anticipate that any optimal bargaining mechanism might have this feature. However, we are still a long way from having a reasonable model of optimizing over mechanisms in a bargaining context. Indeed, it is unclear how such a mechanism is chosen, except by bargaining using an existing mechanism, which subsumes the problem.

Footnotes

[1]We thank Ariel Rubenstein for helpful comments.

[2]For useful expositions and some of the history of the Revelation Principle, see Harris and Townsend (1985) and Myerson (1983a). This principle was developed by Dasgupta, Hammond, and Maskin (1979), Harris and Townsend (1981), Holmström (1978), 1984), and Myerson (1979, 1982, 1983b). Early applications of the principle include (the following list is not comprehensive) Gibbard (1973), Harris and Raviv (1981a, 1981b), Myerson (1981), and Rosenthal (1978).

[3]There are many ways such commitment can be achieved. For instance, in the case of government contracting, the government official responsible for the decision is required to follow procedures which are explicitly and precisely set out in a publicly available book of rules. See McAfee and McMillan (1985c, Chs. 1, 7, 8).

[4]Much of the terminology and notation in what follows is borrowed from Myerson (1982).

[5]Some different communication possibilities of the mechanism were explored by Myerson (1983b). To avoid questions that arise when the principal has private information, as analyzed by Myerson (1983b) we shall assume, as in Myerson (1982), that the principal has no private information.

[6]Thus later observations are assumed not to be discounted relative to earlier observations. On the implications of such discounting for sequential search within the usual search model, see Morgan (1983) and Morgan and Manning (1985).

[7]An alternative way of imagining how the agents not contacted know when communication is finished is to suppose that, while communication occurs arbitrarily quickly, there is a fixed time-point in real time at which decisions are made.

[8] This can be considered to be Lebesque integration, and thus full generality over mixed distributions is allowed. The abuse of notation dd is used, to remain consistent with Myerson (1982).

[9] The term "direct mechanism" is due to Dasgupta, Hammond, and Maskin (1979).

[10] The terms "honest" and "obedient" are due to Myerson (1982).

[11] These costs can be large in practise. For example, Fox (1974, p. 269) cited a U.S. Department of Defense contract in which government personnel used 182,000 man-hours in evaluating proposals from four prospective contractors.

[12] This was shown by Harris and Raviv (1981a), Holt (1980), Myerson (1981), Riley and Samuelson (1981), and Vickrey (1961). For more on optimal auction design, see Crémer and McLean (1985), Harris and Raviv (1981b), Maskin and Riley (1983, 1984), Matthews (1983), McAfee and McMillan (1985a, 1985b), and Milgrom and Weber (1982).

[13] This reserve cost (or type) as in Milgrom and Weber (1982), Myerson (1981), and Riley and Samuelson (1981). Note that "reserve cost" (from auction theory) is a different concept than "reservation price" (from search theory), which is about to be defined.

[14] On reservation-price search, see Carlson and McAfee (1983, 1984), Lippman and McCall (1976), McCall (1965), McMillan and Morgan (1984), and Rothschild (1974).

[15] In the context of the standard search model, a result analogous to this result (that, with perfect recall, the reservation price for search over a finite set of prices is the same as for search over an infinite set) was obtained by Landsberger and Peled (1977) and Lippman and McCall (1976).

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