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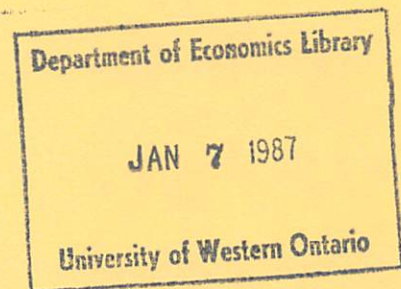
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and

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This paper contains preliminary findings from research work
still in progress and should not be quoted without prior
approval of the authors.

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GENERAL EQUILIBRIUM WORLD TRADE
UNDER BILATERAL QUOTAS†

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ABSTRACT

This paper presents computational general equilibrium techniques for analyzing models of world trade with bilateral quota restrictions imposed on sales by specified sellers to specified buyers. The analysis is motivated by voluntary restraint agreements, and other similar measures which now restrict a sizeable fraction of world trade. The bilateralism in such arrangements generates the added efficiency cost that buyers will typically not purchase from their least-cost source of supply. From a computational point of view, the presence of bilateral quotas increases the dimensionality of the equilibrium problem by a factor equal to the number of trading countries and commodities for which quota restrictions apply, and requires that the Gale-Nikaidô mapping be modified for the existence and computation of equilibrium. The analysis is first presented for a pure exchange case, followed by a discussion of the extensions needed to incorporate production. A numerical example is provided and areas of potential policy application discussed.

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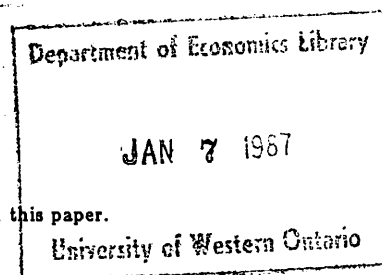
1. INTRODUCTION

This paper presents computational general equilibrium techniques for analyzing models of world trade with bilateral quota restrictions imposed on sales of commodities by specified sellers to specified buyers. The main rationale for the analysis is the form that quota restrictions now take place in several major product areas of international trade. The Multi Fiber Arrangement (MFA), recently renegotiated for a fourth time, covers trade in textiles and clothing and involve a series of bilateral export restraint agreements between importing countries in the developed world and exporting countries from the developing world (see, e.g., Pelzman [1984]). A wide range of other bilateral restraint and orderly marketing agreements cover global trade in automobiles, consumer electronics, steel, machine tools, shipbuilding, and other products. In addition, internal regulation of production and trade in agricultural and other products also takes the same form, such as with milk in the EEC where quota in various regional markets is allocated to specific producers.

Unlike quotas imposed on the buying side of markets (such as a general import quota) these bilateral arrangements generally discriminate among sellers. Under the MFA and other voluntary export restraint agreements, importing countries not only transfer quota rents to exporting countries, but also typically import from several countries at different supply prices. The bilateralism in such arrangements imposes the added cost, beyond those of traditional trade restrictions, that buyers will typically not purchase from their least-cost source of supply. The trade literature has thus far not recognized that export restraint agreements negotiated by buyers with sellers are inevitably bilateral, and that compared to general import quotas, the added efficiency costs of this bilateralism can be large.

We first illustrate our approach for a pure exchange case. To analyze the general equilibrium effects of bilateral quotas, we formulate an appropriate equilibrium mapping incorporating these restraints on trades, to which a fixed-point algorithm can be applied. The mapping has the property that if bilateral quotas are binding, prices paid on trades in quota-restricted goods can vary from country to country, and the bilateral transfers of quota rents between traders are fully incorporated in the calculation of excess demand functions. We then discuss the extensions needed to incorporate production. A final section presents an illustrative numerical example, and discusses some areas of potential policy application.

2. THE BASIC FRAMEWORK

We begin by considering the classical pure trade economy into which we introduce bilateral quotas on *net* trades between specified traders. We present the modifications needed for the well-known Gale-Nikaidô mapping in this model, and indicate how computation can proceed.

We consider a world economy with ℓ countries indexed by $k \in K \equiv \{1,...,\ell\}$ and n goods indexed by $j \in J \equiv \{1,...,n\}$. Each country $k \in K$ has a fixed initial endowment $\overline{W}_{kj} > 0$ of each good $j \in J$. The countries are assumed to negotiate a series of bilateral quota restraint agreements

under which $\overline{Q}_{kjh}$ establishes the maximum amount of good j that country k can sell to country h. Quotas do not apply to sales of domestic goods on the home markets. Hence, if Y_{kjh} denotes the amount of good j sold by country k to country h, then $Y_{kjh} \leq \overline{Q}_{kjh}$ and.

$$\overline{Q}_{:jk} \equiv \sum_{h \in K} \overline{Q}_{hjk} \tag{1a}$$

establishes the total quota on good j that all countries $h \in K$ can sell to country k, while

$$\overline{Q}_{kj:} \equiv \sum_{h \in K} \overline{Q}_{kjh} \tag{1b}$$

establishes the total quota on good j that country k can sell to all countries $h \in K$.

In quota-constrained trades, sellers will typically be able to sell at higher prices than they can realize by selling in their own domestic markets. As a result, unlike the traditional pure trade model, $n\ell$ prices need to be considered rather than only n prices. We consider the $n\ell$-dimensional price vector $\mathbf{p} \equiv [p_{kj}]$, where p_{kj} denotes the market price of good j in country k. If neither of the bilateral quotas $\overline{Q}_{kjh}$, $\overline{Q}_{hjk}$ is binding, then $p_{kj} = p_{hj}$.

Depending upon preferences, endowments, and the size of the quotas, some or all of these bilateral constraints may be non-binding. If they are binding, their effect is to segment markets in which trades occur. No single market price for goods prevails globally; the internal price in each country reflects the effect of quota restrictions in that country, and typically differs from prices elsewhere. Buyers in any segmented market pay a uniform price for all purchases they make, but sellers receive different prices depending into which market they are selling.

2.1. Quota Rents

On the demand side, the relevant price for buyers of good j in country k is p_{kj}. However, on the supply side, sellers of good j face a range of prices $\{p_{hj} \mid h \in K\}$ which vary from country to country. In quota-constrained trades, it is possible for sellers to exploit these price differentials and extract *quota rents*. To maximize their quota rents, sellers rank countries in terms of their internal market prices for good j and then use up their quotas according to a rank order (i.e., selling to countries with the highest prices first, and subsequently to countries with lower prices) until either their quotas or initial endowments are exhausted.

For each good $j \in J$ world prices $\{p_{1j},...,p_{\ell j}\}$ can be arranged in a *nonincreasing* sequence[1] $p_{1j} \geq \cdots \geq p_{\ell j}$. For each country $k \in K$ we can then partition the world into three mutually exclusive regions:

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[1] Strictly speaking, different notation should be used for the original, *unordered* set of world prices $\{p_{1j},...,p_{\ell j}\}$, and the new, *ordered* set of world prices $\{\pi_{1j},...,\pi_{\ell j}\}$. One can then establish a one-to-one correspondence $\phi : K \rightarrow K$ defined by $\pi_{\phi(k),j} = p_{kj}$ for $k \in K$. Each country k can then be equivalently referred to either by the index k in the old sequence or by the index $\phi(k)$ in the new sequence. To minimize notational complexity, we assume, without loss of generality, that the ranking of countries has already been done, and use the *same* price notation $p_{1j},...,p_{\ell j}$ throughout the analysis.

$$H_{kj} \equiv \{h \in K \mid p_{hj} > p_{kj}\}, \tag{2a}$$

$$L_{kj} \equiv \{h \in K \mid p_{hj} < p_{kj}\}, \tag{2b}$$

$$D_{kj} \equiv \{h \in K \mid p_{hj} = p_{kj}\} \supseteq \{k\}. \tag{2c}$$

H_{kj}, L_{kj}, D_{kj} denote countries in which the price of good j is higher, lower, or the same when compared to the domestic price p_{kj}. If, for any two countries h and k, $p_{hj} = p_{kj}$ then $D_{hj} = D_{kj}$, $H_{hj} = H_{kj}$, and $L_{hj} = L_{kj}$.

We denote countries with non-binding quotas by $k \in N_j$, and those with binding quotas by $k \in B_j$, where

$$N_j \equiv \bigcup_{k \in K} [D_{kj} - \{k\}], \tag{3a}$$

$$B_j \equiv K - N_j. \tag{3b}$$

In the special case where $N_j = K$, $B_j = \varnothing$ for all $j \in J$, none of the bilateral quota constraints are binding. In this case, a single world market price applies across all countries for all goods, and the model degenerates to the classical pure trade model.

Since quotas are used by sellers according to the rank order of prices by country, the amount of good j that sellers of country k sell to country h is constrained by the size of the quota as well as the available supply, i.e.,

$$Y_{kjh} \equiv \min (\overline{Q}_{kjh}, S_{kjh}) \tag{4a}$$

where

$$S_{kjh} \equiv \max (\overline{W}_{kj} - \sum_{t \in H_{hj}} \overline{Q}_{kjt}, 0). \tag{4b}$$

That is, S_{kjh} is the remainder of the endowment $\overline{W}_{kj}$ after country k has used quotas issued by all countries $t \in H_{hj}$ having higher prices than country h. By construction, $S_{kjh} = Y_{kjh} = 0$ for $h \in D_{kj} - \{k\}$ since quotas do not apply to domestic markets, and sellers do not sell their goods to countries with prices lower than domestic prices.

Quota rents T_{kj} realized by sellers of good j located in country k are defined as the added revenues from sales abroad due to the price differentials between foreign and domestic markets, i.e.

$$T_{kj} \equiv \sum_{h \in K} \max (p_{hj} - p_{kj}, 0) \, Y_{kjh} = \sum_{h \in H_{kj}} (p_{hj} - p_{kj}) \, Y_{kjh} \geq 0. \tag{5a}$$

Summing over all goods $j \in J$, we obtain the quota rents which accrue to all sellers located in country k, i.e.

$$T_k \equiv \sum_{j \in J} T_{kj} = T_k(\mathbf{p}).$$ (5b)

2.2. Demands

For simplicity we characterize the demand side of each country by either a single consumer or many consumers with identical homothetic preferences. This can be easily generalized to a multi-consumer case for each country, but is notationally more complex. Such a model extension would, however, allow the effects of bilateral quota arrangements across household groups to be analyzed.

Country k is assumed to have initial endowments $\overline{W}_{kj} > 0$ for all $j \in J$. The consumer demand functions in country k, $X_{kj}(\mathbf{p})$, are assumed to reflect utility maximization subject to the budget constraint

$$\sum_{j \in J} p_{kj} X_{kj} = I_k$$ (6a)

where consumer income in country k, I_k, is given by

$$I_k \equiv \sum_{j \in J} p_{kj} \overline{W}_{kj} + T_k.$$ (6b)

Thus when quota-constrained trades occur, consumer incomes have two components: the usual endowment income evaluated at *domestic* prices, and quota rents reflecting the added revenue from sales abroad at higher prices. These quota rents are treated here as being redistributed to consumers.

The excess demand for good j in country k is defined by

$$Z_{kj} \equiv X_{kj}(\mathbf{p}) - \overline{W}_{kj} + E_{kj} - M_{kj}$$ (7a)

where E_{kj} and M_{kj} are exports and imports, respectively, and

$$E_{kj} \equiv \sum_{h \in K} Y_{kjh} = \sum_{h \in H_{kj}} Y_{kjh},$$ (7b)

$$M_{kj} \equiv \sum_{h \in K} Y_{hjk} = \sum_{h \in L_{kj}} Y_{hjk}.$$ (7c)

Using the definitions of quota rents (5ab) and excess demands (7abc) above, the budget constraint (6a) implies that

$$\sum_{j \in J} p_{kj} Z_{kj} = BT_k$$ (8a)

where

$$BT_k \equiv \sum_{j \in J} \sum_{h \in H_{kj}} p_{hj} Y_{kjh} - \sum_{j \in J} \sum_{h \in L_{kj}} p_{kj} Y_{hjk}. \qquad (8b)$$

BT_k is the difference between earnings from selling goods in all other countries (hence, valued at *foreign* prices) and purchases of goods sold by all other countries in the domestic market (hence, valued at *domestic* prices) for each country $k \in K$. In other words, BT_k is the *balance of trade* for country k, modified for the presence of different prices across countries due to bilateral quotas. In the special case where quotas are non-binding in all countries for all goods, BT_k degenerates to the simple form

$$BT_k = \sum_{j \in J} p_{kj} (E_{kj} - M_{kj}), \qquad (8c)$$

i.e., the usual definition of trade balance in standard trade theory.

Summing the budget constraints (8a) across all countries, and noting that $Y_{kjh} = 0$ for $h \notin H_{kj}$ (i.e., country k will not export to countries having lower prices than its domestic prices) and $Y_{hjk} = 0$ for $h \notin L_{kj}$ (i.e., country k will not import from countries having higher prices than its domestic prices), we obtain the following global version of Walras Law

$$\sum_{k \in K} \sum_{j \in J} p_{kj} Z_{kj} = \sum_{k \in K} BT_k = 0. \qquad (9)$$

That is, in aggregate, the terms BT_k sum to zero. This is due to the fact that *net* quota rents cancel out across countries as quota rents accrued to exporting countries are also rents transferred from importing countries.

2.3. General Equilibrium

Given initial endowments and bilateral quota arrangements, a global general equilibrium in this model can be defined as $n\ell$-dimensional price vector $\mathbf{p} \equiv [p_{kj}]$ such that markets clear in all countries for all goods. That is, for each good $j \in J$,

- markets clear in all countries with binding quotas, i.e.

$$Z_{kj} \leq 0 \; (\; = 0 \text{ if } p_{kj} > 0) \quad \forall \, k \in B_j, \qquad (10a)$$

- markets clear *in total* for all countries with non-binding quotas, i.e.

$$Z_{N_j} \equiv \sum_{k \in N_j} Z_{kj} \leq 0 \; (\; = 0 \text{ if } p_{kj} > 0). \qquad (10b)$$

This equilibrium concept generalizes that used in the classical model of pure trade. Quota-constrained trades segment markets. *Ex ante*, which quotas will be binding and which will be non-binding is not known. In the special case where quotas are non-binding in all countries for all goods (i.e., $N_j = K$, $B_j = \varnothing$ for $j \in J$) conditions (10a) do not apply while conditions (10b) reduce to the usual market-clearing conditions for *aggregate* excess demand functions.

If there are binding quotas in all countries for all goods (i.e., $N_j = \emptyset$, $B_j = K$ for all $j \in J$) equilibrium conditions (10a) together with the budget constraints (6ab) imply that

$$T_k + \sum_{j \in J} p_{kj} \left(E_{kj} - M_{kj} \right) = 0 \qquad \forall k. \tag{11}$$

That is, the value of net trades evaluated at *domestic* prices equals the quota rents realized under bilateral quota arrangements. In other words, in equilibrium each country can show a current account *surplus* $(T_k > 0)$ when trade flows are valued at its domestic prices. This is, however, not inconsistent with conventional notions of trade balance, since in this model bilateral quotas give rise to different commodity prices across countries.

For computational purposes a modification of the well-known Gale-Nikaidô mapping can be used to determine an equilibrium solution for such a model, using Scarf's [1973] algorithm, or some more recent extension such as van der Laan [1980], Talman [1980]. The same modification also yields an existence proof via a more conventional application of Brouwer's fixed-point theorem. Let S be an $(n\ell - 1)$-dimensional unit price simplex, and define G: S → S such that for each $p \in S$,

$$G_{kj}(\mathbf{p}) \equiv \begin{cases} \dfrac{p_{kj} + \max\left(Z_{kj}, 0 \right)}{1 + D} & \forall\ k \in B_j \\[3mm] \dfrac{p_{kj} + \max\left(\dfrac{1}{n_j} Z_{N_j}, 0 \right)}{1 + D} & \forall\ k \in N_j \end{cases} \tag{12}$$

where n_j is the number of elements of the index set N_j (i.e., the number of countries which can be thought of as a consolidated whole) and

$$D \equiv \sum_{j \in J} \left[\sum_{k \in B_j} \max \left(Z_{kj}, 0 \right) + \sum_{k \in N_j} \max \left(\frac{1}{n_j} Z_{N_j}, 0 \right) \right],$$

$$= \sum_{j \in J} \sum_{k \in B_j} \max \left(Z_{kj}, 0 \right) + \sum_{j \in J} \max \left(Z_{N_j}, 0 \right).$$

The mapping (12) generalizes the usual Gale-Nikaidô mapping to include both countries with binding quotas and countries with non-binding quotas (i.e., those countries which can be thought of as a consolidated whole). In the special case where quotas are binding in all countries $(N_j = \emptyset, B_j = K$ for all $j \in J)$, the mapping (12) degenerates to the usual Gale-Nikaidô mapping.

In using a fixed-point algorithm, the only deviation from the traditional labelling rule for the classical pure exchange economy occurs at vertices for countries having non-binding quotas with uniform prices. In this case, these countries can be thought of as a consolidated whole and hence, assigned the *same* label as that for the consolidated whole. Alternatively, the excess demand of the consolidated market can be consolidated over the number of countries, i.e.,

$$Z_{kj} = \frac{1}{n_j} Z_{N_j},\tag{13}$$

and the same label assigned to each member country participating in the consolidated market.

3. PRODUCTION AND TRADE UNDER BILATERAL QUOTAS

Production can also be incorporated into this framework. To simplify our exposition, we restrict our discussion to smooth production functions only. We also abstract from intermediate goods and inter-industry flows.

We consider m internationally immobile primary factors of production indexed by $i \in I \equiv \{1,...,m\}$ The production side of each country k is characterized by a *constant returns to scale* production function

$$Q_{kj} = f_{kj}(\mathbf{R}_{kj})\tag{14a}$$

where Q_{kj} denotes the output of good j and $\mathbf{R}_{kj} \equiv [R_{kji}]_i$ the m-dimensional vector of factor inputs. Given a factor price vector $\mathbf{w} \equiv [w_{ki}]$, cost minimization yields *per-unit-output* factor demands

$$\frac{R_{kji}}{Q_{kj}} = r_{kji}(\mathbf{w}).\tag{14b}$$

In equilibrium, zero profit conditions yield cost-covering prices:

$$p_{kj} = \sum_{i \in I} w_{ki} r_{kji} = p_{kj}(\mathbf{w}).\tag{14c}$$

Relative to the pure trade model above which incorporates bilateral quotas, we need to modify equations (4ab) defining net sales of good j by country k to country h and equations (5ab) defining quota rents as follows:

$$S_{kjh} \equiv \max\left(Q_{kj} - \sum_{t \in H_{kj}} \overline{Q}_{kjt}, 0\right) = S_{kjh}(Q_{kj}),\tag{15a}$$

$$Y_{kjh} \equiv \min\left(\overline{Q}_{kjh}, S_{kjh}\right) = Y_{kjh}(Q_{kj}),\tag{15b}$$

$$T_{kj} \equiv \sum_{h \in H_{kj}} (p_{hj} - p_{kj}) Y_{kjh} = T_{kj}(\mathbf{p}, Q_{kj}),\tag{15c}$$

$$T_k \equiv \sum_{j \in J} T_{kj} = T_k(\mathbf{p}, \mathbf{Q}_k),\tag{15d}$$

where $\mathbf{Q}_k \equiv [Q_{kj}]$ is the output supply vector of country k. With production and trade under bilateral quotas, quota rents realized by sellers in any country thus depend not only on the *world* output price vector $\mathbf{p} \equiv [p_{kj}]$ but also on the output supply vector of the country, $\mathbf{Q}_k$.

We also modify the demand side specification for each country by replacing initial *commodity* endowments $\overline{W}_{kj} > 0$, by initial *factor* endowments $\overline{R}_{ki} > 0$. Consumer income I_k in (6b) is therefore changed to

$$I_k \equiv \sum_{i \in \mathcal{J}} w_{ki} \overline{R}_{ki} + T_k(\mathbf{p}, \mathbf{Q}_k) = I_k(\mathbf{w}, \mathbf{p}, \mathbf{Q}_k), \tag{16a}$$

and utility maximization yields the consumer demand functions

$$X_{kj} = X_{kj}(\mathbf{p}, I_k) = X_{kj}(\mathbf{w}, \mathbf{p}, \mathbf{Q}_k), \tag{16b}$$

and hence, excess demand functions

$$Z_{kj} \equiv X_{kj} - Q_{kj} + E_{kj} - M_{kj} = Z_{kj}(\mathbf{w}, \mathbf{p}, \mathbf{Q}_k). \tag{16c}$$

In contrast to the standard general equilibrium trade model with production, this model features the added complexity that the output supply vector for each country also appears in the consumer demand functions, resulting a highly interdependent system of production and trade.

A general equilibrium of production and trade under bilateral quotas is defined by the triplet $(\mathbf{w}, \mathbf{p}, \mathbf{Q}) \equiv ([w_{ki}], [p_{kj}], [Q_{kj}])$ satisfying the $n\ell$ goods market equilibrium equations (9ab) as well as the additional conditions that

● all factor markets are in equilibrium:

$$\sum_{j \in J} Q_{kj} r_{kji} = \overline{R}_{ki} \qquad \forall \, ki, \tag{17a}$$

● zero profit conditions hold for all production processes:

$$p_{kj} = \sum_{i \in \mathcal{J}} w_{ki} r_{kji} \qquad \forall \, kj. \tag{17b}$$

This equilibrium structure is a *nontrivial* extension of the pure trade model above because it gives rise to a simultaneous system of $(2n+m)\ell$ equations in $(2n+m)\ell$ unknown factor and goods prices, and output levels. Hence, under bilateral quotas, information on prices alone is not sufficient to uniquely determine outputs in each country, as in the traditional goods and factors model. The system (10ab,17ab) has to be solved simultaneously for both outputs and prices.

For output levels to be uniquely determined from goods prices, it is necessary to impose further restrictions on the model. In the special case where $n = m$ (i.e., there are the *same* number of goods and factors as in a Samuelson-type [1953] trade model) and the zero profit conditions (17b) satisfy the Gale and Nikaidô [1965] conditions for global univalence, information on goods prices alone can be used to determine output levels *independently* of the demand side.

In this case, the zero profit conditions (17b) yield a system of $n\ell$ nonlinear equations which can be solved for factor prices from goods price, i.e.

$$w_{ki} = w_{ki}(\mathrm{p}) \quad \forall\, ki. \tag{18a}$$

Equations (14b) yield per-unit-output factor demands which together with factor market-clearing equations (17a) yield a system of $n\ell$ linear equations which can be solved for output levels from goods prices alone

$$Q_{kj} = Q_{kj}(\mathrm{w}(\mathrm{p})) = Q_{kj}(\mathrm{p}) \quad \forall\, kj. \tag{18b}$$

Once the production side of the model is determined from goods prices, the demand side can be determined in the same way as for the pure trade model above. In this case, the same solution strategy as in the traditional goods and factors trade model can be used, except that the model is solved using *goods* prices rather than *factor* prices.

4. A NUMERICAL EXAMPLE

To illustrate our approach we consider a simple numerical example of a global trading economy in which there are three countries and two goods. In this example, demands are represented by CES utility functions in each country, and production is represented by a *Constant Elasticity of Transformation* (CET) production possibility frontier. We consider a highly simplified example of a benchmark equilibrium (micro-consistent) data set, (see, e.g., Mansur and Whalley [1984]) reporting trade and production in these three countries. We have calibrated a model based on this specification of production and consumption to this data set, and into the model we build bilateral quotas between country A and countries B and C, with quota rents transferred to the exporting countries. We then evaluate a counter-factual equilibrium situation where the bilateral quotas are removed in both countries.

In Table 1, we present the numerical example of the benchmark equilibrium data set we use. The transfer of quota rents is reflected in the additional income component, besides the value of the production at domestic prices, for countries B and C. In Table 2 we report the calibrated preference and production parameters, and the counter-factual equilibrium solution where bilateral quotas are removed against both countries B and C.

In this example, under the change the volume of exports of country A falls but its imports rise because of its improved international terms of trade. The global price of good 1 falls but the price of good 2 rises reflecting the elimination of the bilateral quota restriction on good 1. For country B, its export volumes increase but its import volumes fall. Thus while there are increased export sales abroad, a deterioration of its terms of trade is reflected in the absence of the transfer of quota rents under the bilateral export restrictions. The result is a loss for country B. A similar situation also occurs for country C.

Thus in this numerical example, the country imposing the bilateral quota restrictions loses as a result of the restrictions, and countries exporting to it benefit because of the transfer of quota rents. This effect outweighs the costs of restricted market areas for exporting countries.

These techniques clearly offer the potential for application to a wide range of more realistic trade policy situations. Current trade restrictions in textiles which fit the description above have just been renegotiated under the Multi Fiber Arrangement, and extended to a wider range of product category. Similar restrictions are spreading in steel and machine tools. Domestic restrictions of agricultural markets in such areas as dairy products also have a similar form. More explicitly empirically based modelling may thus offer insights into the effects trade restricting instruments in these areas have.

5. CONCLUSION

In this paper, we present a general equilibrium analysis of a global trading system characterized by bilateral quota restrictions between specified buyers and sellers. The resulting computational scheme is motivated by the observation that these types of trade restrictions are now common in the global trading system, and yet numerical analysis of their impacts is not currently available in existing general equilibrium literature. The bilateralism in these arrangements has the important added efficiency cost that buyers typically do not purchase from their least-costs source of supply.

From a computational point of view, these bilateral quotas increase the dimensionality of the equilibrium problem by a factor equal to the number of trading countries and commodities in which quota restrictions apply. The traditional Gale-Nikaidô mapping needs to be modified accordingly. The approach we suggest is presented for the pure exchange case, followed by a discussion of the extensions needed to incorporate production. We also present a numerical example, with a brief discussion of areas of potential policy application.

TABLE 1

A SIMPLE NUMERICAL EXAMPLE OF A BENCHMARK EQUILIBRIUM
IN THE PRESENCE OF BILATERAL QUOTAS

	Country A	Country B	Country C
Prices			
Good 1	1.0	0.75	0.875
Good 2	1.0	1.0	1.0
Production			
Good 1	1000	1800	1000
Good 2	3000	1000	1000
Demands			
Good 1	2000	1200	600
Good 2	2000	1600	1400
Net Trades			
Good 1	-1000	600	400
Good 2	1000	-600	-400
Incomes			
Value of Production			
at Domestic Prices	4000	2350	1875
Quota Rents	0	150	25
Total	4000	2500	1900

TABLE 2

CALIBRATED PARAMETERS AND A COUNTERFACTUAL SOLUTION
FOR THE NUMERICAL EXAMPLE IN TABLE 1

A. Calibrated Preference and Production Parameters

	Country A	Country B	Country C
Consumption			
Share Parameter on Good 1	0.5	0.338	0.22
Share Parameter on Good 2	0.5	0.662	0.78
Elasticity of Substitution	1.0	0.75	0.75
Production			
Share Parameter on Good 1	0.293	0.546	0.467
Share Parameter on Good 2	0.707	0.454	0.533
Constant Term	2226.68	1389.82	1000.00
Elasticity of Transformation	1.25	1.25	1.25

B. Counterfactual Equilibrium Solution
When Bilateral Quotas In Country A Are Removed Against B and C

	Country A	Country B	Country C
Prices			
Good 1	0.903	0.903	0.903
Good 2	1.097	1.097	1.097
Production			
Good 1	1199.0	1713.0	1042.0
Good 2	2819.0	1068.0	965.0
Demands			
Good 1	225.7	1100.0	597.0
Good 2	1949.0	1572.0	1330.0
Net Trades			
Good 1	-1057.0	613.0	-445.0
Good 2	870.0	-504.0	-366.0
Welfare Gain or Loss			
Hicksian EV in Comparison			
With Benchmark Equilibrium	191.17	-104.09	-7265.00
(as % of Initial Income)	4.77%	-4.16%	-5.82%

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1984

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1985

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