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Estimates of energy fluxes associated with sprites in the mesosphere

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Estimates of energy fluxes associated with sprites in the mesosphere

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Abstract

We present calibrated estimates of photon flux, lightning peak Poynting flux and Joule heating associated with the brightest region of sprites observed in the mesosphere over South Africa. The sprites' photon fluxes were estimated using 28 sprites events (observed during the 2019 sprites campaign) calibrated by stars in the sprite's image background. The lightning driven background electric field associated with the brightest region of sprites were found to vary from 0.1 to 6.7 E_k (local air breakdown field). The lightning return stroke at lower frequencies (4 Hz - 2 kHz) has more influence in carrot sprites than in column sprites formation processes. The lightning peak Poynting flux and Joule heating were estimated from calibrated electromagnetic field measurements made in parallel with eight sprites events (observed during the 2020 sprites campaign). The photon flux, peak Poynting flux, and peak Joule heating associated with the brightest region of sprites were found to be 1.1×10^{-7} W/m^2, 12.7 W/m^2, and 4.7×10^{-3} W/m^2 on average, respectively. The altitude/distance-normalised lightning peak Poynting flux decreases with increasing atmospheric altitude. The photon flux from column sprites decreased with increased altitude of the brightest region. Column sprites have a shorter time delay (<30 ms) from their parent lightning strokes than carrot sprites (up to 145 ms).

Keywords: Sprite's Joule heating, sprite's Poynting flux, sprite's photon flux, lightning return stroke.

1. Introduction

The energy from the quasi-static electric field generated mainly by large positive cloud to ground (CG) lightning discharges of Mesoscale Convective Systems (MCS) initiates collisions between electrons, neutral atoms and molecules, resulting in photon emissions forming the Transient Luminous Events (TLEs) called sprites in the middle atmosphere (Franz et al., 1990; Liu et al., 2015; Pasko, 2010; Surkov and Hayakawa, 2020). The sprites initiation process involves the downward and upward movement of streamers (Bór, 2013; Luque and Ebert, 2009). Collisions between electrons, neutral atoms and molecules lead to the emission of photons from transitions of excited states of neutral nitrogen (N_2) and ionised nitrogen (N_2^+) predominantly at an altitude above and below 50 km, respectively (Sentman et al., 1995; Heavner et al., 2010; Surkov and Hayakawa, 2012; Nnadih et al., 2021). The emissions of N_2 and N_2^+ occur as a result of excitation of the ^{1}P and 1NG states, respectively, in the ~300-1000 nm visible band (Mende et al., 1995; Hampton et al., 1996; Heavner et al., 2010). $N_2(^1P)$ has a band emission around 640 nm (~550-1070 nm wavelength range (Shibusawa and Funatsu, 2019)), with a lifetime of about 100 ms. N_2^+(1NG) has a band emission around 427.8 nm (~300-460 nm wavelength range (Heavner et al., 2010)), with a lifetime of about 5 ms (Armstrong et al., 1998; Suszcynsky et al., 1998; Heavner et al., 2010; Nnadih et al., 2021). Quenching accounts for the short lifetime of the N_2^+ emission, whereas Rayleigh scattering accounts for its low intensity (Heavner et al., 2010; Nnadih et al., 2021). The emissions of $N_2(^1P)$ and N_2^+(1NG) have lifetimes of about 6 μs and 70 ns, and quenching altitudes of about 53 km and 48 km, respectively (Heavner et al., 2010).

The altitude of the brightest region of sprites varies from 50 to 84.1 km (Füllekrug et al., 2019; Luque et al., 2016; Mashao et al., 2021; Malagón- Romero et al., 2020; Sentman et al.,1995; Stenbaek-Nielsen et al., 2010; Wescott et al., 1998; Wescott et al., 2001). The lightning electric field accelerates the electrons, and the conversion of electrons to negative ions intensifies the local electric field. The brightest region of the sprite is a region with relatively low mesospheric conductivity, enhanced electric field and electrons are converted to negative ions, which results in maximum photon production (Malagón- Romero et al., 2020; Mashao et al., 2021).

Sprites result from Joule heating of the mesosphere associated with lightning electric field (Füllekrug, 2006). Füllekrug et al. (2006) reported on stratospheric Joule heating determined from the lightning continuing current measurement. They found that the Joule heating varied from 1×10^{-4} J/m^3 to 1×10^{-8} J/m^3 for altitudes ranging from 25 to 45 km, respectively (Füllekrug et al., 2006). Gordillo-Vázquez et al. (2018) investigated, by spectroscopic means, the temperature of mesospheric regions between 65 km and 76 km and found no measurable heating associated with sprites. We are not aware of any reported estimates in the literature of lightning Joule heating and Poynting flux in the mesosphere in relation to sprites.

For non-delayed sprites to occur the lightning driven electric field must exceed the local air breakdown field (120 Townsend (Td)) (Pasko et al. 2013; Surkov and Hayakawa, 2020). However, the lightning driven electric field lower than the local air breakdown field can lead to delayed sprites due to associative detachment (Luque and Gordillo- Vázquez, 2012).

We evaluate lightning initial apparent peak Poynting flux and Joule heating projected to the altitude of the brightest regions of sprites in the mesosphere for the first time. We also compare photon flux at the altitude of the brightest region of the optical observations column and carrot sprites using a camera with a red (N_2) emission filter. The lightning-driven electric field associated with the altitude of the brightest regions of sprites were determined and compared with the published literature.

2. Optical and radio recordings

The reported sprites were recorded during the 2019 and 2020 sprites campaigns from the Square Kilometre Array (SKA) (30.97º S, 21.98º E) and the South African Astronomical Observatory (SAAO) (32.38º S, 20.81º E), Northern Cape, South Africa. The sprites were observed on the following nights: 01 February 2019 from SKA and 24 January 2020 from SKA and SAAO. Figure 1 shows the map of South Africa, the MCS associated with some of the observed sprites, and the SKA and SAAO sites marked with red and yellow dots, respectively. The +CG, -CG, and IC lightning discharges are marked with plus, negative, and dash-like symbols within the thundercloud, respectively. Lightning electric field observations were made in 2019 and 2020. Lightning magnetic field observations were made in 2020 only. Eight sprites events with no calibrated stars present were recorded during the 2020 sprites campaign and are used for estimating the lightning Poynting flux and Joule heating associated with the altitude of the brightest regions of sprites only. The 2020 sprites campaign data comprised of column, carrot, wishbone, jellyfish, tree, angel, and sprites with halo. From the 2019 sprites campaign, we selected column and carrot sprites that occurred individually or in a group with calibrated stars of known photon flux. We found 14 column and 14 carrot sprites red emission optical data with calibrated stars and used them for estimating the photon flux as well as Joule heating (Alekseeva et al., 1996; Bór, 2013).

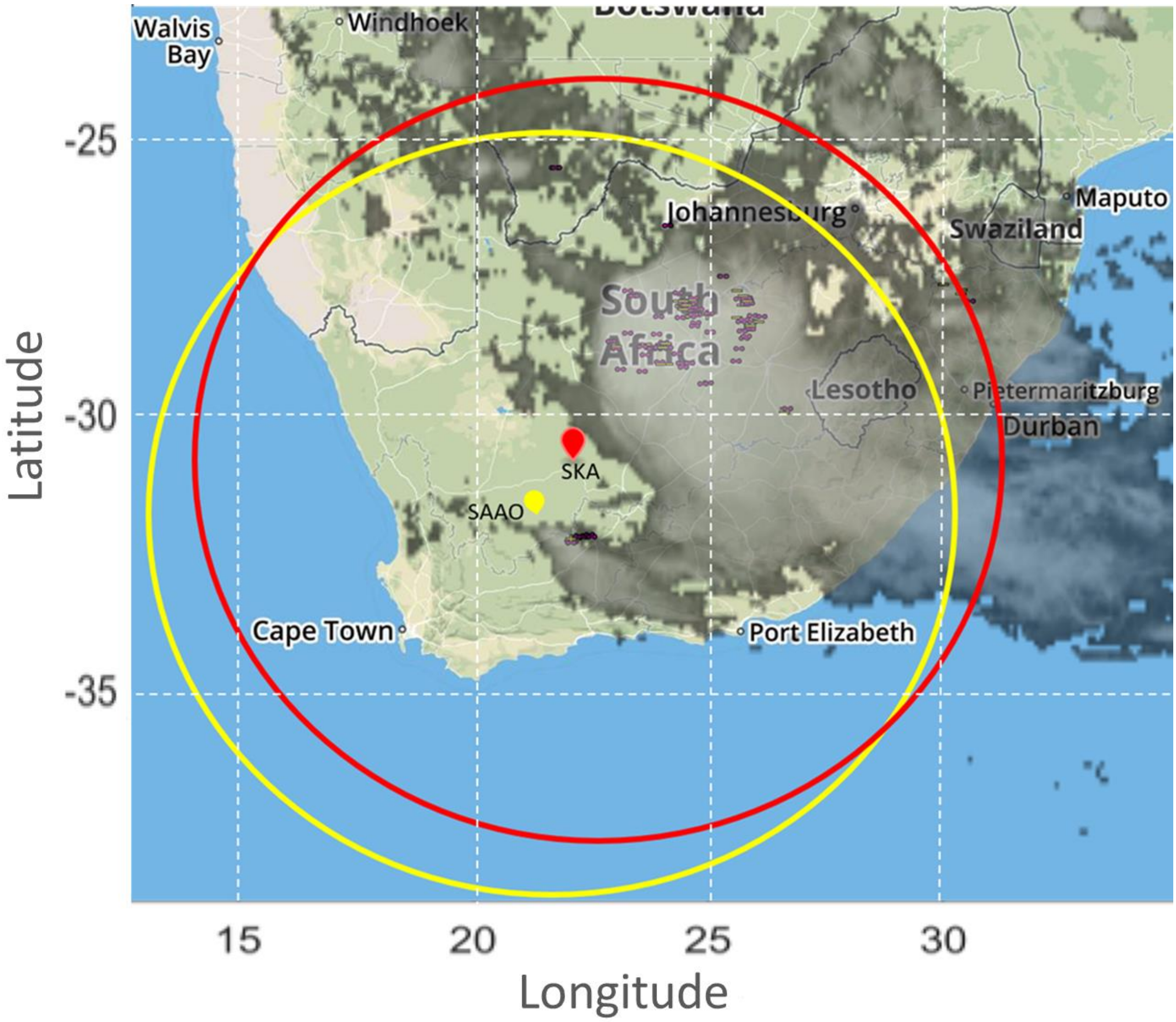

Figure 1. Map of South Africa captured on 1 February 2019 at 22:41:00 UTC. The SKA/ SAAO sites are marked with red/yellow dots, and the MCS associated with some observed sprites is shown in grey. The circles mark the 900 km radius, which is the maximum distance our cameras can observe from SKA and SAAO, respectively (Earthnetworks.com, 2020).

2.1 Optical recording

Watec 910Hx and Allied Vision Pike night vision cameras, mounted on the same tripod, were used to record the sprites from SKA/SAAO (Mashao et al., 2021; Nnadih et al., 2018, Nnadih et al., 2021). The stars in our field of view (FOV) were utilised to co-align the cameras. The cameras' viewing directions were North, East, and South of SKA and SAAO.

The Watec 910Hx cameras, with an 8.0 mm f/1.4 C-mount lens, operated with 0.45 gamma factor and 8-bit intensity resolution. The Watec 910Hx cameras recorded 25 video frames per second (fps) with a 40 ms frame period. A Global Positioning System (GPS) video timer provided time with millisecond timing to the camera system. The Watec 910Hx cameras had a 29°/46.2° FOV Vertical/Horizontal (V/H), 0.061°/0.072° V/H angular resolution per pixel, and 640×480 (H×V) pixels.

The Allied Vision Pike camera, coupled to a Ceramic xx1332 Mullard image intensifier with Xenon F0.95 50 mm C-mount TV lens, captured images at 25 fps and a 40 ms frame period, 640×480 video image size, 15°/21° FOV V/H, and 14-bit intensity resolution. The Ceramic xx1332 Mullard image intensifier has a wavelength range of about 360 nm to 850 nm (Allied Vision, 2022; Worthpoint, 2022). A red N_2 longpass cut off filter with a 640-650 nm cutoff was used to record the $N_2(^1P)$ emission of sprites. A Network Timing Protocol server provided timing accuracy of about 1 ms for the Pike camera system at SKA.

2.2 Electromagnetic recording

The lightning electric (2019 and 2020 campaigns) and magnetic (2020 campaign only) fields were recorded in parallel with sprites optical recordings at SKA. A wideband digital ELF/VLF/LF radio receiver that detected the lightning vertical electric field strength observed with about 4 Hz to 400 kHz frequency range, with a sampling frequency of 1 MHz, and timing accuracy of 20 ns (Füllekrug, 2010; Füllekrug et al., 2019). The lightning magnitude values were converted to Td by using the modelled N_2 densities associated with the targeted sprites altitude. We then obtain the ratio between the local lightning electric field (E_s) and local air breakdown field (E_k), see Tables 1 to 4. All N_2 densities are obtained from the NRLMSISE profile (https://ccmc.gsfc.nasa.gov/modelweb/models/nrlmsise00.php). Two orthogonal induction coils are used to record the horizontal magnetic field in the frequency range of ~4 Hz to ~60 kHz with a sampling frequency of 500 kHz and a timing accuracy of ~20 ns. Lightning magnetic field observations were conducted during the 2020 sprites campaign only from SKA.

The Earth Network Total Lightning Network (ENTLN) and South African Lightning Detection Network (SALDN) provided lightning data (time, position, peak current, type of discharge, and polarity) related to the observed sprites. ENTLN and SALDN have position accuracy of about 0.2 km and 0.5 km, respectively (Gijben, 2012; Bui et al., 2015; Zhu et al., 2017). All CG lightning events associated with the sprites reported here have positive polarities. The peak lightning current varied from 28 to 142 kA, see Tables 1 to 4.

3. Data analyses

3.1 Sprite altitude estimation

We estimate the lightning peak Joule heating and apparent Poynting flux, and average photon flux associated with the sprites brightest regions using optical data as well as lightning electric and magnetic field data. The right ascension and declination of stars on the sprites image background and the geographic position of the camera were used to determine the azimuth angle, elevation angle, and field of view of the cameras. The azimuth and elevation angles of stars were computed and used to fit onto the real stars in the background image of sprites. The altitude of the sprite's brightest region was obtained by employing spherical and planar trigonometry in the horizontal and vertical planes, respectively (Mashao et al., 2021; 2022a; 2022b), assuming that the sprites events occurred directly above their parent lightning locations. The latter assumption is commonly used in sprites optical research (Füllekrug et al., 2019; Li et al., 2008; Luque et al., 2016; Mashao et al., 2021; 2022a; 2022b; McHarg et al., 2007; Stenbaek-Nielsen and McHarg, 2008). The uncertainty in altitude of sprites computed from the angular resolution of the camera spanned from ± 0.33 to 0.47 km. The uncertainty in the altitude of sprites varies with slant distance from the location of camera to the sprites.

3.2 Poynting flux estimation

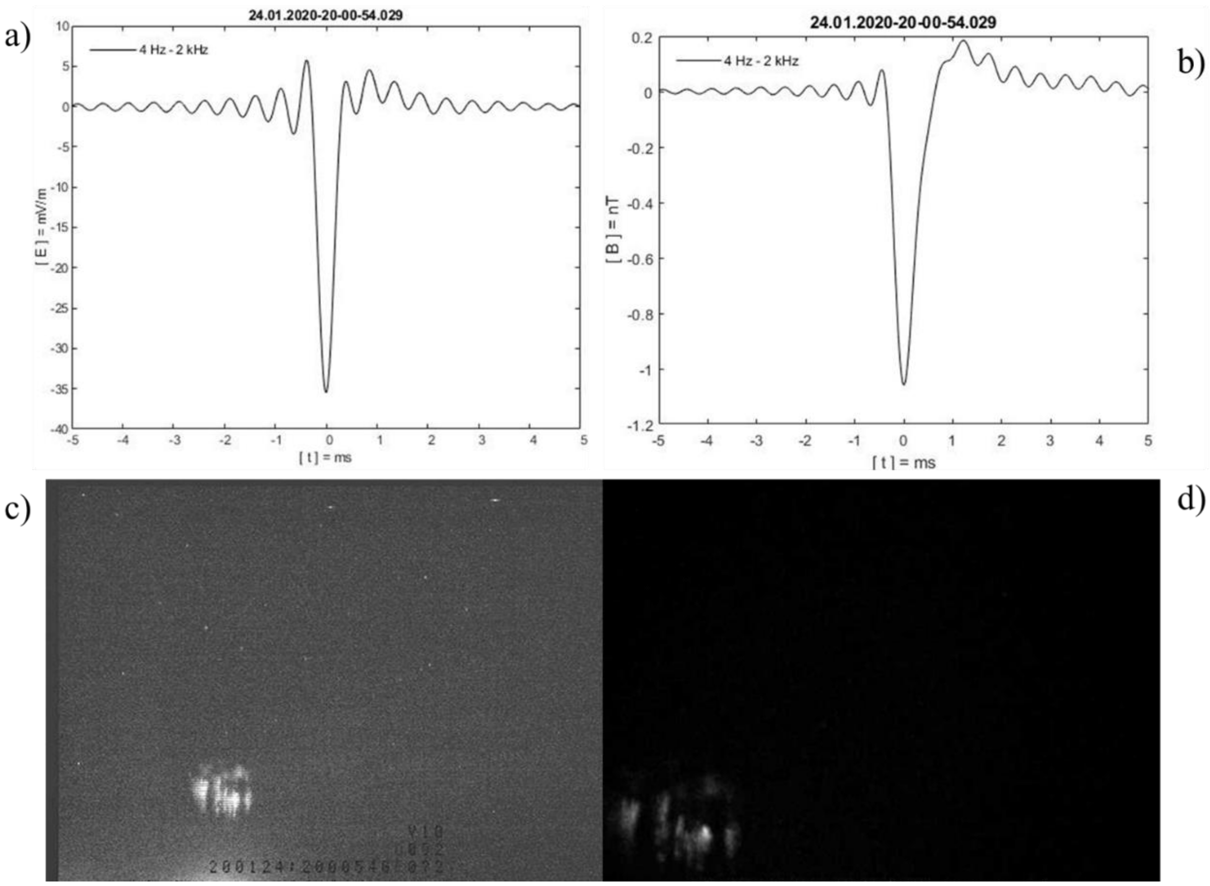

Figure 2. The lightning electric (a) and magnetic (b) waveforms associated with the sprite event observed on 24 January 2020 at 20:00:54.072 UTC from SKA recorded by monochrome camera without a filter (c) and with a red N_2 filter (d). The correct sprite event time stamp is displayed on the image with no filter.

Figure 2 shows the lightning electric (a) and magnetic (b) field waveforms associated with the sprites event recorded simultaneously by a monochrome camera without a filter (c) and with the red N_2 filter (d) from SKA. We used the simultaneous lightning peak electric and magnetic field strengths associated with the optical sprites to determine the maximum apparent Poynting flux associated with the eight sprites analysed in the frequency range of 4 Hz to 2 kHz. The Poynting flux (**S**) is given by (Farrell et al., 2006):

$$\mathbf{S} = \frac{(\mathbf{E} \times \mathbf{B})}{\mu} \quad (1)$$

where (**E**) is the lightning electric field, (**B**) is the lightning magnetic field, and (μ) is the permeability of the medium. The free space wave impedance is modified near a conductor, in this case the Earth, which is studied in the field of magnetotellurics in detail (Bór et al. 2022). Here we assume a vacuum. To compare the lightning electric and magnetic field strengths, we filter the data to obtain the lightning return stroke at lower frequencies information (4 Hz - 2 kHz), see Figure2 (a) and (b) (Constable, 2016; Cummer et al., 2013; Fraser-Smith and Bowen, 1992; Füllekrug, 2010). ELF/VLF/LF lightning radio receivers with lower minimum frequency (<4 Hz) are required to measure lightning continuing current. These data are not available to us. Lightning continuing current plays an essential role in initiating delayed sprites, sustaining the quasi-static electric field in the mesosphere and the brightness of sprites longer than the local relaxation time (Gomez Kui et al., 2021; Kitagawa et al., 1962; Ren et al., 2021; Tomicic et al., 2021).

Due to a technical problem, we only recorded the dominant horizontal component of the magnetic field aligned in the North-South geographic direction. Since the lightning occurred between 209.5 and 562.3 km away from the receiver, we used Equation (2) to estimate the total horizontal magnetic field ($\mathbf{B_T}$):

$$\mathbf{B_T} = \mathbf{B} / \sin(AZ) \quad (2)$$

where *AZ* is the azimuth angle between the lightning position and the North-South coil orientation.

The radiation field terms decrease as 1/d, $1/d^2$, and $1/d^3$, depending on the horizontal distance between the lightning event and the receiver, where d is the distance from the lightning location to the receiver location (Cooray and Lobato, 2020). At short distances, radiation field terms decrease as 1/d, and $1/d^2$.

From the recorded data, we estimate the lightning electric and magnetic fields associated with the altitude of the brightest regions of sprites. The lightning electric and magnetic fields associated with the altitude of the brightest regions of sprites were normalised as follows: We project horizontally the lightning electric and magnetic radiated fields as ${}^{1}/_{\mathrm{d}^2}$, to 1 km away from the lightning location (Cooray and Lobato, 2020; Taylor and Jean, 1959). This was done by multiplying the observed lightning electric and total magnetic field values by the distance squared in km. For our geometry, the parent lightning source of the fields is assumed to be a finite line current. It is known that the lightning quasi-static electric field strength, which may initiate sprites, lessens with increasing altitude as $\frac{1}{a^3}$, where *a* is atmospheric altitude (Pasko et al., 2013). We assumed that the CG lightning charges, which initiated the sprites events, were removed at 10 km altitude (Asano et al., 2008; Pasko et al., 2013), and we then determined lightning electric (E_s) and magnetic field strength associated with the sprite's brightest region altitude as $\left(\frac{10}{a}\right)^3$, where *a* is the calculated altitude (km) of the sprite's brightest region (see example in the Appendix). Finally, we obtain the distance/altitude-normalised peak apparent Poynting flux associated with the sprite's brightest region altitude using Equations 1 and 2. The lightning peak Poynting flux associated with the altitude of the brightest regions of sprites estimates are presented in Table 1 (see Appendix). An example calculation of the lightning peak Poynting flux associated with the altitude of the brightest regions of sprites is presented in the Appendix.

3.3 Joule heating estimation

To determine the lightning Joule heating associated with the sprite's brightest region altitude, we used the lightning electric field strength ($\mathbf{E}$) and the atmospheric conductivity ($\boldsymbol{\sigma}$) adopted from Liu et al. (2015), integrated over the observed vertical extent of the brightest region of the sprites. The vertical extent of the brightest region of sprites spanned from 1.2 to 2.4 km. Joule heating ($\mathbf{J_h}$) is given by (e.g. Kosch and Nielsen, 1995; Foster et al., 1998):

$$\mathbf{J_h} = \boldsymbol{\sigma}\mathbf{E}^2 \qquad (3)$$

The same altitude/distance normalised lightning electric field data used to estimate the peak Poynting flux was used to determine the peak Joule heating (first-order approximation). The atmospheric conductivity corresponding to the vertical extent of the brightest region altitude of sprites was used to calculate the lightning peak Joule heating associated with the altitude of the brightest regions of sprites using Equation (3). Sprite streamer formation involves ionisation of the neutral atmosphere, which will modify the local conductivity and electric field within the sprites. Hence, we can only estimate the initial peak Joule heating before streamer ionisation takes over. The atmospheric conductivity was obtained from rocket measurements, which have been used in other TLE studies (Holzworth et al.,1985; Liu, 2012; Liu et al., 2015). For the 2019 sprites campaign, the height-integrated atmospheric conductivity varied from 1.2×10^{-6} to 2.1×10^{-4} S, for altitudes ranging from 61.5 to 75.3 km. For the 2020 sprites campaign, the height-integrated atmospheric conductivity corresponding to the column and carrot sprites' brightest region altitudes varied from 1.1×10^{-7} to 1.1×10^{-5} S, for altitudes ranging from 53.1 km and 69 km. The values of conductivities used here are lower limits.

Dowden et al. (2001) used measurements of VLF scattering in millisecond time scale, through horizontal angles of up to 180° to demonstrate that the sprites plasma has high conductivity. They set an approximate lower bound for conductivity of sprites plasma to 3×10^{-7} S/cm. Liu et al. (2009b) sprite simulations were extrapolated by Luque and Ebert (2010) which showed conductivity of about 8×10^{-8} and 5×10^{-8} S/cm, respectively. The sprite conductivity increases in time with respect to the background and the re-enhancement of the electric field in the sprites' streamer wake results in further increase in conductivity (Gordillo-Vázquez and Luque, 2010).

The lightning initial peak Joule heating is due to the lightning discharge within the bandwidth of the measured electric field (4 Hz – 2 kHz), corresponding to the lightning return stroke, and atmospheric conductance, corresponding to the vertical extent of the brightest region altitude of sprites. The results of lightning peak Joule heating associated with the altitude of the brightest regions of sprites estimate are shown in Tables 2, 3, and 4 (see Appendix). An example calculation of the lightning peak Joule heating associated with the altitude of the brightest regions of sprites is presented in the Appendix.

3.4 Photon Flux

To determine the sprite's photon flux out of the brightest region, we acquired sprites optical data observed with red N_2 emission filter simultaneously with calibrated stars of known photon flux (in W/m^2), as described in detail by Nnadih et al. (2021). The flux values of selected stars can be found in the Pulkovo Spectrophotometry Bright Stars Catalogue (Alekseeva et al., 1996). Only the sprites recorded during the 2019 sprites campaign from SKA had images containing calibrated stars. Figure 3 shows an example of a recorded sprite with calibrated star.

Figure 3. Images recorded with no filter (left) and red N_2 (right) filter on 01 February 2019 at 19:08:24.202 UTC from SKA. The images contain the same sprites event. The red circle in the image on the right denotes the calibrated star. The correct sprite event time stamp is displayed on the image with no filter.

We can estimate photon flux since we recorded the sprites events and the stars simultaneously with fixed camera gain setting and similar atmospheric losses over the same night. We averaged the sprites video clip frames over one second in order to determine the average intensity pixel value of the calibrated star ($\mathrm{Str}_{\mathrm{avg}}$). The sky background ($\mathrm{Sky}_{\mathrm{bkg}}$) value was obtained by using a 7×7 median filter to remove the stars on the video frame prior to the frame which contained the sprites event. To determine the sprite's average brightness ($\mathrm{Sprs}_{\mathrm{avg}}$), we average the image pixel values over the sprite's brightest region altitude range. The average intensity pixel value ($\mathrm{Str}_{\mathrm{avg}}$) and known photon flux value of the calibrated star ($\mathrm{Str}_{\mathrm{flux}}$, integrated over the filter's longpass), sky background ($\mathrm{Sky}_{\mathrm{bkg}}$), and the average image intensity pixel value over the sprite's brightest region altitude range ($\mathrm{Sprs}_{\mathrm{avg}}$) were used to obtain the photon flux at the sprite's brightest region ($\mathrm{Sprs}_{\mathrm{flux}}$), by means of Equation (4) (Nnadih et al., 2021).

$$\mathrm{Sprs}_{\mathrm{flux}} = \frac{\mathrm{Str}_{\mathrm{flux}}(\mathrm{Sprs}_{\mathrm{avg}} - \mathrm{Sky}_{\mathrm{bkg}})}{\mathrm{Str}_{\mathrm{avg}} - \mathrm{Sky}_{\mathrm{bkg}}} \, (\mathrm{W/m^2}) \tag{4}$$

An example of the photon flux calculation at the sprite's brightest region is presented in the Appendix. The estimated column and carrot sprites' photon flux are shown in Tables 3 and 4 in the Appendix.

4. Results and discussion

The radiated electric fields of lightning discharges associated with 8 sprites events for the 2020 campaign (Table 1) at the ELF/VLF/LF receiver, projected to 1 km from the lightning position at ground level, and projected to the sprites' brightest region altitude varied from 2.6×10^{-3} to 5.7×10^{-1} V/m, 454.1 to 25059.6 V/m, and 1.1 to 68.9 V/m, respectively. The radiated magnetic field of lightning discharges associated with 8 sprites events for the 2020 campaign (Table 1) at the receiver, projected to 1 km from the lightning position at ground level, and projected to the sprites' brightest region altitudes varied from 2.95×10^{-10} to 1.09×10^{-9} T, 5.26×10^{-5} to 6.33×10^{-4} T, and 1.23×10^{-7} to 2×10^{-6} T, respectively. The radiated electromagnetic fields values are associated with the lightning return stroke at lower frequencies (4 Hz - 2 kHz). The radiated lightning electric and magnetic fields uncertainties were about ±0.05 V/m and $\pm2.5\times10^{-8}$ T, respectively, at the sprites' brightest region altitudes. Uncertainties associated with the use of first-order distance projection estimates are not included. The estimated altitude of the brightest region of 8 sprites events ranged from 61.5 to 75.3 km. The altitude of the brightest region of sprites is within that previously reported (i.e. 50 - 84.1 km) (Füllekrug et al., 2019; Luque et al., 2016; Mashao et al., 2021; Malagón- Romero et al., 2020; Sentman et al.,1995; Stenbaek-Nielsen et al., 2010; Wescott et al., 1998).

The N_2 density at the altitude of the brightest region of sprites (61.5—75.3 km) varied from 6.85×10^{14} to 4.39×10^{15} cm^{-3}, see Tables 1 and 2. All N_2 densities are obtained from the NRLMSISE profile (https://ccmc.gsfc.nasa.gov/modelweb/models/nrlmsise00.php). The altitude/distance normalised lightning driven peak electric field (E_s) at the altitude of the brightest region of sprites spanned from 0.1 to 0.5 E_k, associated with the lightning electric field in the frequency range 4 Hz to 400 kHz. The frequency range of 4 Hz to 400 kHz provides the lightning electric field information that the ELF/VLF/LF radio receiver can measure. These E_k values fall into the range reported in the literature (Gamerota et al., 2011: Hu et al., 2007; Kanmae et al., 2012; Li and Cummer, 2012). The E_s at the altitude of the brightest region of sprites associate with the lightning return stroke at lower frequencies (4 Hz - 2 kHz) varied from 0.01 to 0.4 E_k. The reduction of the lightning driven electric field is due to the filtering of the lightning electric fields data to obtain the lightning return stroke fields at lower frequencies. Gamerota et al. (2011) model demonstrated that a lightning electric field of 0.08 E_k is not sufficient to produce sprites. However, Gamerota et al. (2011)

model doesn't consider the intracloud lightning electric field which enhances the electric field energy at sprites altitudes (Füllekrug et al., 2019). The electric field energy might make it possible for lightning discharges with 0.08 E_k or less to initiate sprites. The lower E_k may be due to lower temporal resolution of the observations (Pasko et al., 2013). We note that some of our analysed E_s values are greater than the previous reports (Gamerota et al., 2011; Hu et al., 2007; Li and Cummer, 2012). However, the E_s values are within the same order (3-5 times the E_k) obtained by Kanmae et al. (2012) for sprites streamers peak electric field. Our peak electric field values are the background estimated electric field associated with the altitude of the brightest region of sprites. During sprites formation processes, conversion of electrons to negative ions intensifies the electric field (Malagón- Romero et al., 2020; Mashao et al., 2021). The electric field value of about 8 E_k has been associated with gigantic jet discharge (Kuo et al., 2009; Pasko et al., 2013). None of our events are gigantic jets, as confirmed by the cameras.

For the same 8 sprites, the lightning peak apparent Poynting flux (see Table 1) associated with the sprite's brightest region, using distance/altitude normalised data, was found to span from 0.6 to 30.1 W/m^2 (0.08 W/m^2), with an average of 12.7 W/m^2. The value in parentheses is the uncertainty. Figure 4 shows the peak Poynting flux associated with the brightest region of sprites increased with decreasing atmospheric altitude. The linear correlation coefficient between these variables is -0.7. This outcome is expected as both the electric and magnetic fields decrease as $\frac{1}{a^3}$ with increasing altitude.

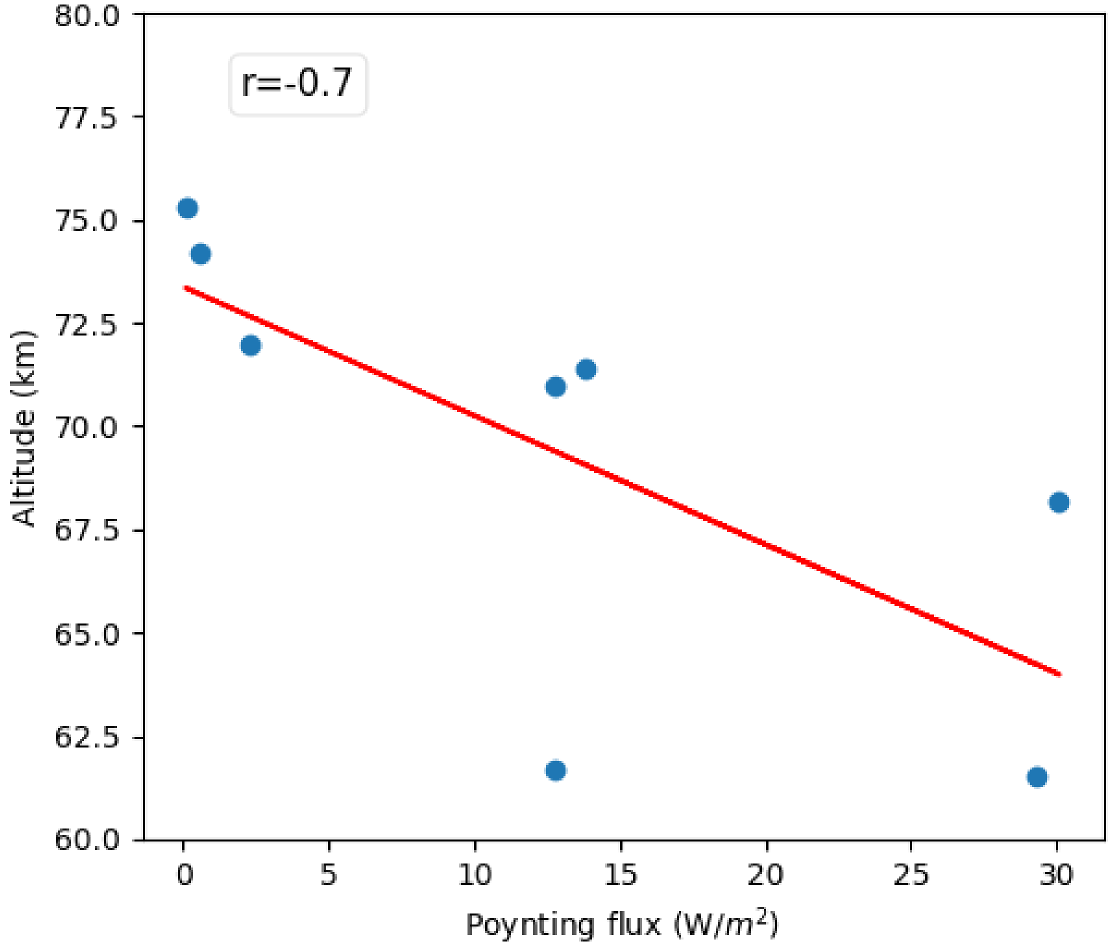

Figure 4. The relationship between the distance/altitude normalised lightning peak apparent Poynting flux associated with the brightest region of sprites and atmospheric altitude. r shows the linear correlation coefficient.

For the same 8 sprites, for the estimates of lightning initial peak Joule heating (see Table 2) associated with the sprites' brightest region, we used the atmospheric conductivity values from Liu et al. (2015), integrated over the vertical extent of the brightest region of the sprites. The vertical extent of the brightest region of sprites varied from 1.2 to 2.4 km. The altitude/distance normalised initial peak Joule heating ranged from 2.6×10^{-5} to 1.4×10^{-2} W/m^2 ($\pm2.4\times10^{-6}$ W/m^2) over the same altitude range (61.5 – 75.3 km). The value in parentheses is the uncertainty. The average peak Joule heating was found to be 4.7×10^{-3} W/m^2. The initial peak Joule heating associated with sprite's brightest region altitude decreases with an increase in atmospheric altitude, with a linear correlation coefficient of -0.4 (not shown). This outcome is expected as the electric field decreases as $\frac{1}{a^3}$ with increasing altitude. A linear correlation coefficient of -0.04 was found for the E_s/E_k and altitude of brightest region of sprites (not shown).

From the 2019 campaign (see Tables 3 and 4), the N_2 red photon flux from the sprites (14 column and 14 carrot sprites) brightest region spanned from 2.5×10^{-8} to 2.7×10^{-7} W/m^2 (±2.5×10^{-9} W/m^2) with an average value of about 1.1×10^{-7} W/m^2, for altitudes between 53 to 69 km. The value in parentheses is the uncertainty. There is no significant linear correlation between the peak photon flux of sprites and atmospheric altitude, with a correlation coefficient of -0.2 (not shown).

The apparent radiated lightning peak electric field at the column and carrot sprites' brightest region altitudes ranged from 59.9 to 479.6 V/m and 22.3 to 438 V/m, respectively, for altitudes between 53.1 and 69 km. The average apparent radiated lightning electric field within the column and carrot sprites were 193.3 and 190.4 V/m, respectively. The sprites associated with radiated lightning electric field lower than the breakdown electric field (120 Td) may be generated due to associative detachment (Luque and Gordillo- Vázquez, 2012). The radiated electric field values are associated with the lightning return stroke at lower frequencies (4 Hz - 2 kHz). For the lightning electric fields in the frequency range 4 Hz to 400 kHz, the apparent radiated lightning peak electric field at the column and carrot sprites' brightest region altitudes varied from 537.4 to 2727.9 V/m and 596.9 to 2477.9 V/m, respectively, for altitudes between 53.1 and 69 km. These values are consistent with the expected quasi-electrostatic field produced by lightning in the mesosphere (Liu et al., 2009a).

The N_2 number density at the altitude of the brightest region of column and carrot sprites (53.1—69 km) vary from 1.67×10^{15} to 1.27×10^{16} cm^{-3}. For column sprites, the E_s at the altitude of the brightest region ranged from 0.6 to 5.8 E_k (4 Hz to 400 kHz) and 0.1 to 0.8 E_k (4 Hz - 2 kHz), respectively. For carrot sprites, the E_s at the altitude of the brightest region varied from 0.6 to 6.6 E_k (4 Hz to 400 kHz) and 0.03 to 1 E_k (4 Hz - 2 kHz), respectively. This is in good agreement with Gamerota et al. (2011), Hu et al. (2007), Kanmae et al. (2012), Li and Cummer (2012), and Qin et al. (2013).

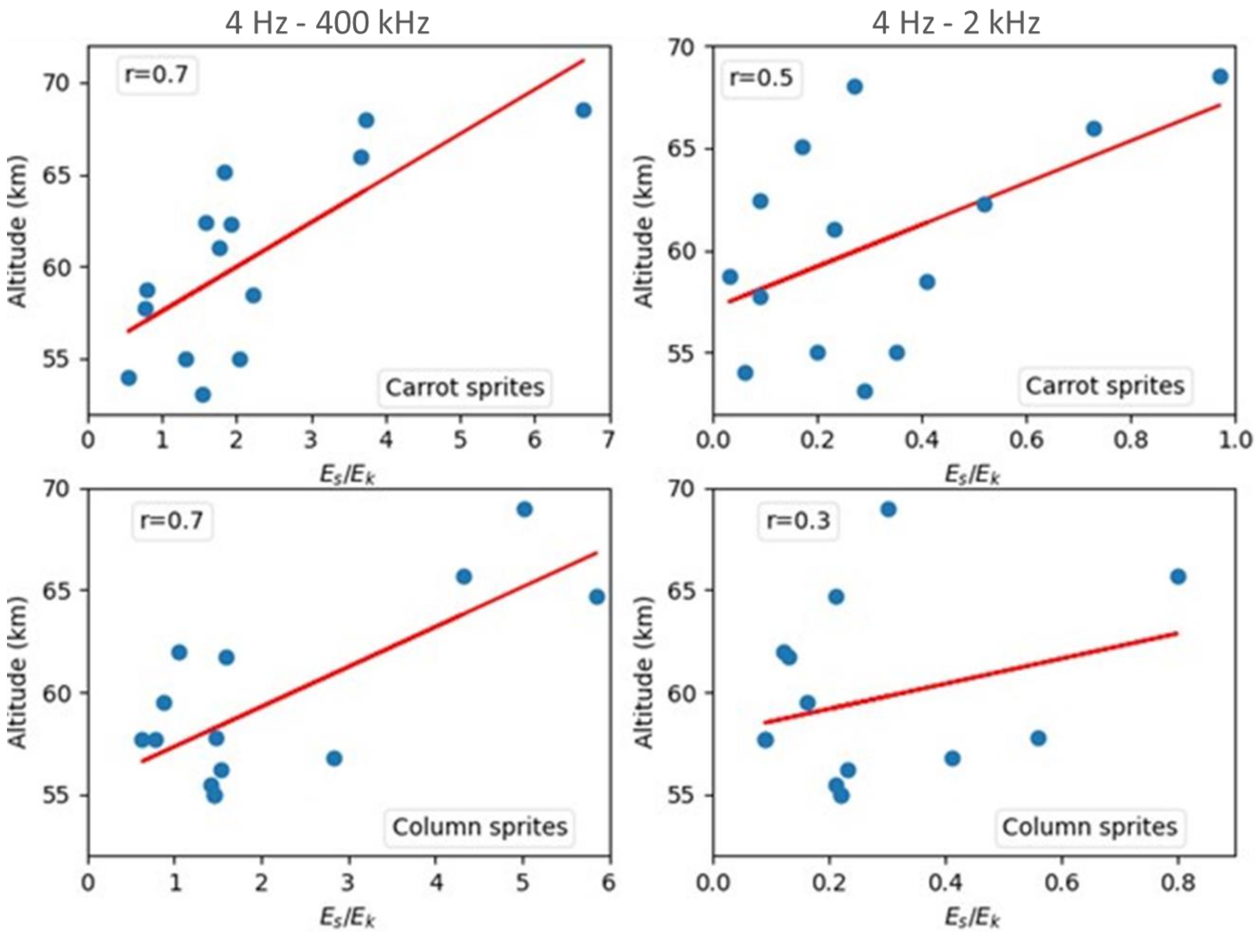

Figure 5. The relationship between the altitude/distance normalised lightning peak electric field over local air breakdown field (E_s/E_k) at the brightest region of column and carrot sprites versus atmospheric altitude. The lightning peak electric fields for the panels on the left panels were obtained in the frequency ranging from 4 Hz to 400 kHz, whereas the panels on the right were obtained in the frequency ranging from 4 Hz - 2 kHz.

Figure 5 shows the relationship between E_s/E_k at the brightest region of column and carrot sprites versus atmospheric altitude. A good linear correlation coefficient of 0.7 and 0.7 were found between E_s/E_k (4 Hz to 400 kHz) associated with column and carrot sprites' brightest region versus the brightest region altitude, respectively, see Figure 5 left panels. A good linear correlation coefficient of 0.5 was found between E_s/E_k (4 Hz - 2 kHz) associated with carrot sprites' brightest region versus the brightest region altitude, whereas a weak linear correlation coefficient of 0.3 was found for column sprites, see Figure 5 right panels. The weak correlation coefficient shows that the lightning return stroke at lower frequencies (4 Hz - 2 kHz) has less influence on the column sprites than on the carrot sprites formation processes.

From the 2020 data, the sprites events which occurred at 19:55:41.681 and 21:41:29.422 UTC had halos with diameters of about 65 and 61 km, respectively, see Tables 1 and 2. The E_s at the altitude of the brightest region of these events were 0.2 and 0.5 E_k (4 Hz to 400 kHz), and 0.1 and 0.4 E_k (4 Hz - 2 kHz), respectively. From the 2019 data, the sprites event which occurred at 19:41:09.728 UTC had a halo with diameter of about 57 km, see Table 3. The E_s at the altitude of the brightest region of this event were 1.5 E_k (4 Hz to 400 kHz) and 0.6 E_k (4 Hz - 2 kHz), respectively. An electric field value of greater than about 0.5 E_k has been associated with sprites halo discharge (Qin et al., 2013; Pasko et al., 2013). Note that halos were not seen by the camera with the red N_2 filter.

The photon flux and lightning initial peak Joule heating associated with the brightest region altitude of 14 column sprites were found to vary from 2.5×10^{-8} to 2.6×10^{-7} W/m^2 ($\pm2.4\times10^{-9}$ W/m^2) and 7.2×10^{-4} to 6.7×10^{-1} W/m^2 ($\pm6.2\times10^{-7}$ W/m^2), with an average value of approximately 8.4×10^{-8} W/m^2 and 7.5×10^{-2} W/m^2, respectively, for altitudes from 55 to 69 km. The photon flux and lightning peak Joule heating associated with 14 carrot sprites at the brightest region altitude ranged from 4.8×10^{-8} to 2.2×10^{-7} W/m^2 ($\pm2.6\times10^{-9}$ W/m^2) and 5.3×10^{-5} to 6.8×10^{-1} W/m^2 ($\pm4.3\times10^{-9}$ W/m^2) with an average value of approximately 1.3×10^{-7} W/m^2 and 9.4×10^{-2} W/m^2, respectively, for altitudes between 53 to 68.5 km. The value in parentheses is the uncertainty. The vertical extent of the brightest region for column and carrot sprites ranged from 0.6 to 2.2 km and 0.6 to 1.4 km, respectively.

To within the timing uncertainty of the video frame (±20 ms), we found that column and carrot sprites have a time delay that varied from about 18 to 27 ms and 30 to 145 ms from their parent lightning strokes, respectively. The time delay between the parent lightning strokes and column (< 30 ms) and carrot sprites (up to 145 ms) agrees with van der Velde et al. (2006). The sprite delay depends on the charge moment change in milliseconds (Cummer and Stanley, 1999).

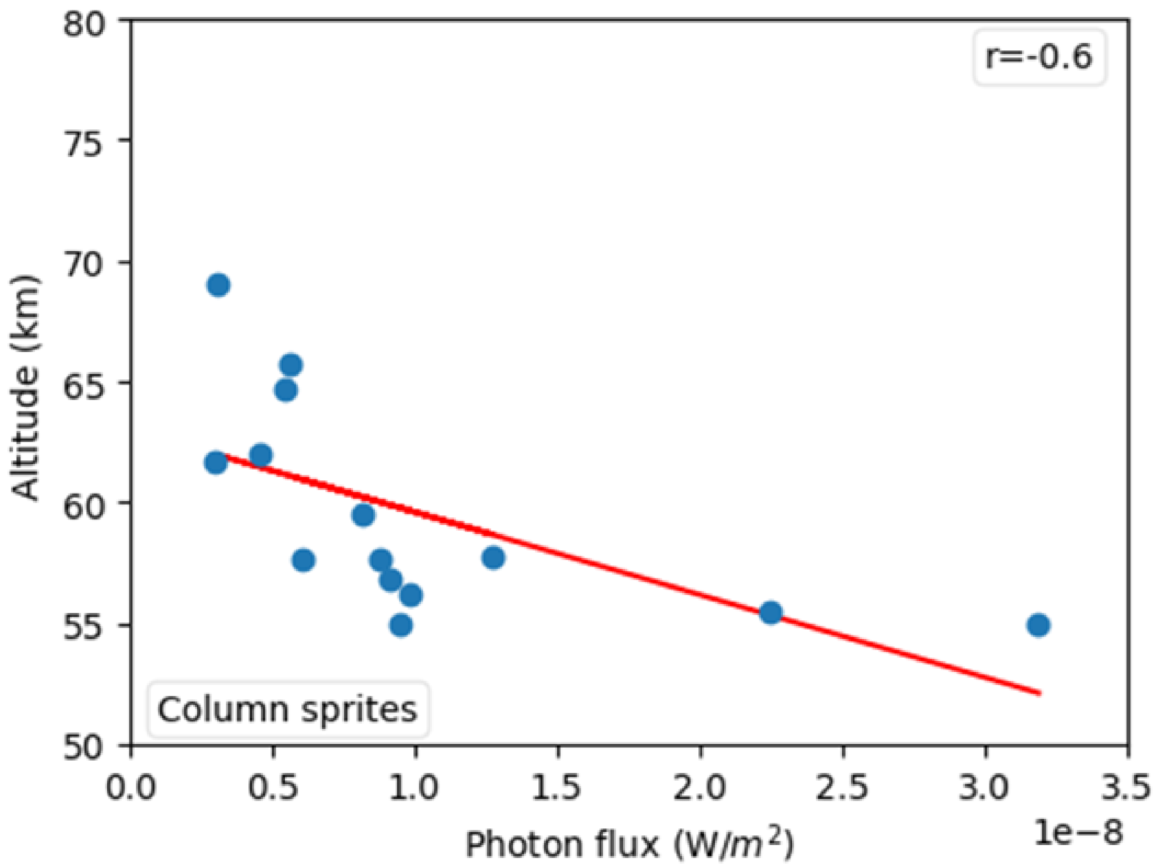

Figure 6. The relationship between the column sprite's photon flux versus sprite's brightest region altitude, respectively. r shows the linear correlation coefficient.

Figure 6 shows the column sprites' photon flux versus the sprites' brightest region altitude. r denotes the linear correlation coefficient. The photon flux out of column sprites' brightest region decreased with an increase in sprites' brightest region altitude, with a linear correlation coefficient of -0.6. The photon flux from carrot sprites' brightest region versus the brightest region altitude showed a weak positive correlation of 0.2 (not shown). A weak-moderate linear correlation coefficient of 0.4 and 0.3 was found between the altitude/distance normalised lightning initial peak Joule heating associated with column and carrot sprites' brightest region versus the brightest region altitude, respectively (not shown).

5. Conclusions

We have estimated the lightning peak apparent Poynting flux and peak initial Joule heating associated with the altitude of the brightest regions of sprites only. We investigated the lightning electric fields in frequencies ranging from 4 Hz to 400 kHz and lightning return stroke at lower frequencies (4 Hz - 2 kHz). The E_s/E_k values (0.1 to 6.6) are comparable with the sprites values from the published literature (Gamerota et al.,2011; Hu et al., 2007; Kanmae et al., 2012; Li and Cummer, 2012; Qin et al, 2013). The lightning discharges have an influence on carrot and column sprites formation processes. The lightning return stroke at lower frequencies (4 Hz - 2 kHz) has more influence on carrot sprites than on column sprites brightest region altitude.

From 2019, the averaged photon flux and lightning initial peak Joule heating associated with the brightest region of column sprites were found to be approximately 8.4×10^{-8} W/m^2 and 7.5 $\times10^{-2}$ W/m^2, respectively, for altitudes from 55 to 69 km. The averaged photon flux and lightning initial peak Joule heating associated with the brightest region of carrot sprites were found to be approximately 1.3×10^{-7} W/m^2 and 9.4×10^{-2} W/m^2, respectively, for altitudes between 53 to 68.5 km. The column sprites' brightest region altitude had a linear correlation coefficient of -0.6 against the photon flux of the brightest region.

From 2020, the investigation of the altitude/distance normalised lightning peak apparent Poynting flux and initial peak Joule heating associated with the brightest region of sprites yield averaged energy fluxes of 12.7 W/m^2 and 4.7×10^{-3} W/m^2, respectively. A linear correlation coefficient of -0.7 was found between the peak apparent Poynting flux associated with the brightest region of sprites and atmospheric altitude, whereas a correlation coefficient of -0.4 was obtained between the initial peak Joule heating associated with the sprite's brightest region and atmospheric altitude. Our study shows for the first time that the apparent Poynting flux emanating from the sprite dominates over the Joule heating and photon flux by several orders of magnitude.

Our photon flux values are less than Armstrong et al. (1998) by up to 2 orders of magnitude. However, Armstrong et al. (1998) photon fluxes are also less than our Poynting flux and Joule heating of sprites and so consistent with our conclusion. Factors such as local clouds, viewing direction, spatial resolution, atmospheric scattering, light pollution, camera type and setting, distance to sprite, technical approach of measurements and humidity affect camera sensitivity, therefore sprites visibility (Mashao et al., 2022a; Mlynarczyk et al., 2015; Pasko et al., 2013).

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Appendix

Table 1: Summary of estimations of lightning peak apparent Poynting flux at the brightest region altitude of sprites observed from SKA and SAAO sites on 24 January 2020. All CG lightning events are of positive polarity. E_s is the electric field at sprite's brightest region altitude.

Time (UT)	Lightning peak current (kA)	Distance from the receiver to the lightning stroke (km)	Sprite's brightest region altitude (km)	Azimuth angle from the magnetic coil to the lightning stroke (°)	Adjusted Magnetic field at receiver (T)	Magnetic field at 1 km from lightning location (T)	Electric field at 1 km from lightning location (V/m)	N_2 density at sprite's brightest region altitude (cm^{-3})	Electric field at sprite's brightest region altitude (E_s) (V/m)	Magnetic field at sprite's brightest region altitude (T)	Lightning peak apparent Poynting flux at sprite's brightest region altitude (W/m^2)	Es/E_k 4 Hz to 400 kHz	Es/E_k 4 Hz - 2 kHz
19:55:41.682 (SKA)	142	389.9	61.5	99.1	1.8E-09	2.8E-04	7.1E+03	4.4E+15	3.1E+01	1.2E-06	29.3	0.2	0.1
20:00:54.029 (SKA)	101	390.1	61.7	101.1	1.1E-09	1.6E-04	5.4E+03	4.4E+15	2.3E+01	7.0E-07	12.8	0.2	0.04
20:03:13.971 (SKA)	74	431.9	74.2	93.8	5.6E-10	1.0E-04	1.1E+03	8.1E+14	2.8E+00	2.6E-07	0.6	0.1	0.03
20:03:14.307 (SKA)	62	417.9	75.3	101.7	3.0E-10	5.3E-05	4.5E+02	6.9E+14	1.1E+00	1.2E-07	0.1	0.1	0.01
20:07:29.608 (SKA)	92	403.0	72.0	98.9	8.2E-10	1.3E-04	3.0E+03	1.1E+15	8.1E+00	3.6E-07	2.3	0.3	0.1
20:07:29.248 (SKA)	127	449.6	71.0	108.6	1.2E-09	2.3E-04	8.9E+03	1.3E+15	2.5E+01	6.5E-07	12.8	0.5	0.2
20:23:43.573 (SAAO)	66	562.3	68.2	30.9	2.0E-09	6.3E-04	6.0E+03	1.9E+15	1.9E+01	2.0E-06	30.1	0.3	0.1
21:41:29.422 (SKA)	133	209.5	71.4	159.1	2.1E-09	9.2E-05	2.5E+04	1.3E+15	6.9E+01	2.5E-07	13.8	0.5	0.4

Table 2: Summary of estimations of lightning peak initial Joule heating associated with the brightest region altitude of sprites observed from SKA and SAAO sites on 24 January 2020. All CG lightning events are of positive polarity. E_s is the electric field at sprite's brightest region altitude.

Time (UT)	Lightning peak current (kA)	Distance from the receiver to the lightning stroke (km)	Electric field at 1 km from lightning location (V/m)	Sprite's brightest region altitude (km)	Atmospheric conductivity (S/m)	Sprites brightest region vertical extent (km)	Electric field at sprite's brightest region altitude (E_s) (V/m)	Lightning peak initial Joule heating at the sprite's brightest region altitude (W/m^2)
19:55:41.682 (SKA)	142	389.9	7.1E+03	61.5	8.0E-09	1.3	3.1E+01	9.6E-03
20:00:54.029 (SKA)	101	390.1	5.4E+03	61.7	8.0E-09	1.3	2.3E+01	5.6E-03
20:03:13.971 (SKA)	74	431.9	1.1E+03	74.2	1.0E-08	1.5	2.8E+00	1.1E-04
20:03:14.307 (SKA)	62	417.9	4.5E+02	75.3	1.0E-08	2.3	1.1E+00	2.6E-05
20:07:29.608 (SKA)	92	403.0	3.0E+03	72.0	9.8E-08	2.2	8.1E+00	1.4E-02
20:07:29.248 (SKA)	127	449.6	8.9E+03	71.0	1.0E-09	2.4	2.5E+01	1.5E-03
20:23:43.573 (SAAO)	66	562.3	6.0E+03	68.2	3.0E-09	1.4	1.9E+01	1.5E-03
21:41:29.422 (SKA)	133	209.5	2.5E+04	71.4	1.0E-09	1.2	6.9E+01	5.8E-03

Table 3: Summary of estimations of photon flux and lightning peak initial Joule heating at the brightest region of column sprites observed from the SKA site on 01 February 2019. All CG lightning events are of positive polarity. E_s is the electric field at sprite's brightest altitude.

Time (UT)	Lightning peak current (kA)	Distance from the receiver to the lightning stroke (km)	Sprites photon fluxes (W/m^2)	Sprite's brightest region altitude (km)	Sprite's brightest region vertical extent (km)	N_2 density at sprite's brightest region altitude (cm^{-3})	Electric field at sprite's bright region altitude (ELF/VLF/LF radio receiver) (E_s) (V/m)	Lightning peak initial Joule heating at the sprite's brightest region altitude (W/m^2)	Es/E_k 4 Hz to 400 kHz	Es/E_k 4 Hz - 2 kHz
19:05:21.584	50	357.3	2.7E-07	55.0	1.1	1E+16	263.5	1.6E-02	1.4	0.2
19:05:21.584	50	357.3	1.9E-07	55.5	1.1	1E+16	256.4	1.5E-02	1.4	0.2
19:05:21.584	50	357.3	8.0E-08	55.0	1.1	1E+16	263.5	1.6E-02	1.4	0.2
19:05:21.584	50	357.3	8.3E-08	56.2	1.1	9E+15	246.9	1.4E-02	1.5	0.2
19:06:49.098	107	360.4	7.7E-08	56.8	0.6	8E+15	399.0	1.9E-02	2.8	0.4
19:35:00.707	80	334.4	4.7E-08	65.7	2.2	2.6E+15	248.1	6.7E-01	0.8	4.3
19:43:09.728	103	333.6	1.1E-07	57.8	1.1	7.1E+15	479.6	2.5E-02	1.5	0.6
19:47:59.520	64	332.1	6.9E-08	59.5	1.1	6.3E+15	124.6	1.5E-01	0.9	0.2
19:56:54.499	28	345	5.1E-08	57.7	1.1	7.1E+15	80.3	7.2E-04	0.8	0.1
19:56:54.499	28	345	7.4E-08	57.7	1.1	7.1E+15	80.3	7.2E-04	0.6	0.1
19:56:54.499	28	345	3.9E-08	62.0	1.1	4.4E+15	64.7	3.7E-02	1.0	0.1
19:56:54.499	28	345	2.5E-08	61.7	1.1	4.4E+15	65.7	3.8E-02	1.6	0.1
20:12:08.503	41	436.9	2.6E-08	69.0	1.4	1.7E+15	59.9	1E-02	5.0	0.3
20:12:08.503	41	436.9	4.6E-08	64.7	1.4	2.9E+15	72.9	3.7E-02	5.8	0.2

Table 4: Summary of estimations of photon flux and lightning peak initial Joule heating at the brightest region of carrot sprites observed from the SKA site on 01 February 2019. All CG lightning events are of positive polarity. E_s is the electric field at sprite's brightest region altitude.

Time (UT)	Lightning peak current (kA)	Distance from the receiver to the lightning stroke (km)	Sprites photon fluxes (W/m^2)	Sprite's brightest region altitude (km)	Sprites brightest region vertical extent (km)	N_2 density at sprite's brightest region altitude (cm^{-3})	Electric field at sprites bright region altitude (ELF/VLF/LF radioreceiver) (E_s) (V/m)	Lightning peak initial Joule heating at the sprite's brightest region altitude (W/m^2)	Es/E_k 4 Hz to 400 kHz	Es/E_k 4 Hz - 2 kHz
19:06:49.098	107	360.4	1.7E-07	68.5	0.6	2E+15	227.5	9.4E-02	6.7	1
19:08:24.111	92	309.1	1.5E-07	66.0	0.7	2.6E+15	225.2	1.7E-01	3.7	0.7
19:12:42.451	62	351.3	8.6E-08	62.3	1.1	4.4E+15	273.7	6.8E-01	1.9	0.5
19:26:48.376	42	333.1	2.0E-07	65.1	1.1	3E+15	60.7	2.0E-02	1.8	0.2
19:35:00.707	80	334.4	2.2E-07	58.5	1.1	7.1E+15	351.5	1.3E-02	2.2	0.4
19:36:23.568	42	409.4	1.2E-07	55.0	0.9	1E+16	238.1	9.9E-03	1.3	0.2
19:44:48.189	31	338.9	6.9E-08	54.0	0.7	1.1E+16	76.8	8.5E-04	0.6	0.1
19:49:30.929	49	345.9	1.5E-07	61.0	1.1	4.9E+15	136.5	1.7E-01	1.8	0.2
19:51:17.824	29	332	1.3E-07	58.7	1.1	6.3E+15	22.3	5.3E-05	0.8	0.03
19:53:59.597	48	413.9	7.3E-08	62.4	1.3	4.4E+15	46.4	2.3E-02	1.6	0.1
19:56:54.499	28	345	1.5E-07	57.7	1.1	7.1E+15	80.3	7.2E-04	0.8	0.1
19:57:34.604	92	352.4	2.0E-07	55.0	1.1	1E+16	425.7	4.1E-02	2	0.4
20:12:08.503	41	436.9	1.0E-07	68.0	1.4	1.9E+15	62.6	1.7E-02	3.7	0.3
20:38:25.718	68	431.6	4.8E-08	53.0	1.4	1.3E+16	438	8.0E-02	1.5	0.3

Poynting flux estimation calculation

The lightning peak apparent Poynting flux associated with the sprite's brightest region altitude calculation is demonstrated below using the sprites event in Figure 2. The lightning electric field (**E**) and magnetic field at the receivers' location were observed to be 3.6×10^{-2} V/m and 1.06×10^{-9} T, respectively (see Figure 2). The lightning discharge which initiates the sprites event was located 390.1 km away from the receivers' location. The sprite's brightest region altitude was 61.7 km. We only recorded the dominant horizontal component of the magnetic field due to technical issues with the second orthogonal induction coil. We first determine the azimuth angle from the magnetic coil to the lightning stroke in order to calculate the total horizontal magnetic field ($\mathbf{B_T}$) using Equation (A1) below:

$$\mathbf{B_T} = \mathbf{B}/\sin(AZ) \qquad (1A)$$

where *AZ* is the azimuth angle between the lightning position and the North-South coil orientation. *AZ* was found to be 101.1°. By using Equation (A1), we obtain $\mathbf{B_T}$ = 1.08×10^{-9} T.

To normalise **E** and $\mathbf{B_T}$ values to 1 km from the lightning location, we then multiply **E** and $\mathbf{B_T}$ values by the distance (390.1 km) squared in km. We found **E** and $\mathbf{B_T}$ at 1 km from the lightning location to be:

$\mathbf{E} = 5.4 \times 10^{3}$ V/m

$\mathbf{B_T}$=1.6×10^{-4} T

Assuming that the CG lightning charges, which initiated the sprites events, were removed at 10 km altitude, we then project **E** and $\mathbf{B_T}$ at 1 km from the lightning location to the sprite's brightest region altitude (61.7 km) by multiplying **E** and $\mathbf{B_T}$ at 1 km from the lightning location by $(10/r)^3$, where *r* is the altitude (km) of the sprite's brightest region (*r* = 61.7). We obtained **E** and $\mathbf{B_T}$ at the sprite's brightest region altitude to be:

$\mathbf{E} = 2.3 \times 10^{1}$ V/m

$\mathbf{B_T}$=7.0×10^{-7} T

Finally, we estimated the lightning peak apparent Poynting flux (**S**) associated with the sprite's brightest region altitude using Equation (A2) below:

$$\mathbf{S} = \frac{(\mathbf{E} \times \mathbf{B})}{\mu} \tag{A2}$$

where μ is the permeability of the medium (assumed vacuum). For the event in Figure 2, we found that **S** = 12.8 W/m^2. This calculation is summarised in Table 1 (row 3).

Joule heating estimation calculation

The lightning peak initial Joule heating associated with the sprite's brightest region altitude calculation is demonstrated below using the sprites event in Figure 2. The lightning electric field (**E**) at the receiver's location was obtained to be 3.6×10^{-2} V/m (see Figure 2a). The lightning discharge which initiates the sprites event was located 390.1 km away from the receiver's location. The sprite's brightest region altitude was 61.7 km.

To normalise the **E** value to 1 km from the lightning location, we then multiply the **E** value by the distance (390.1 km) squared in km. We found **E** at 1 km from the lightning location to be:

$\mathbf{E} = 5.4 \times 10^{3}$ V/m

Assuming that the CG lightning charges, which initiated the sprites events, were removed at 10 km altitude, we then project **E** at 1 km from the lightning location to the sprite's brightest region altitude (61.7 km) by multiplying **E** at 1 km from the lightning location by $(10/r)^3$, where *r* is the altitude (km) of the sprite's brightest region (*r* = 61.7). We obtained **E** at the sprite's brightest region altitude to be:

$\mathbf{E} = 2.3 \times 10^{1}$ V/m

We adopted the atmospheric conductivity ($\boldsymbol{\sigma}$) from Lui et al. (2015), integrated over the vertical extent of the brightest region altitude of the sprite. The vertical extent of the brightest region altitude of sprite was found to be 1.3 km, and the corresponding $\boldsymbol{\sigma}$ was found to be:

$\boldsymbol{\sigma} = 8 \times 10^{-9}$ S/m

To determine the lightning peak initial Joule heating ($\mathbf{J_h}$) associated with the sprite's brightest region altitude, we used Equation (A3) below:

$$\mathbf{J_h} = \boldsymbol{\sigma}\mathbf{E}^2 \tag{A3}$$

For the event in Figure 2, we found that $\mathbf{J_h} = 5.6 \times 10^{-3}$ W/m^2. This calculation is summarised in Table 2 (row 3).

Photon flux estimation calculation

To determine the photon flux out of the sprite's brightest region, we acquired sprites optical data observed with a red N_2 emission filter (https://www.schott.com/shop/advanced-optics/en/Matt-Filter-Plates/RG645/c/glass-RG645) simultaneously with calibrated stars of known photon flux, as described in Nnadih et al. (2021). The flux values of selected stars can be found in the Pulkovo Spectrophotometry Bright Stars Catalogue (Alekseeva et al., 1996). We demonstrate how we determine the photon flux out of the sprite's brightest region using the sprites event in Figure 3. We averaged the sprites video clip frames over one second in order to determine the average intensity pixel value of the calibrated star ($\mathrm{Str_{avg}}$). The sky background ($\mathrm{Sky_{bkg}}$) value was obtained by using a 7×7 median filter to remove the stars on the video frame prior to the frame which contains the sprites event. The sky background ($\mathrm{Sky_{bkg}}$) value was 3. The calibrated star in Figure 3 was recorded using cameras with red N_2 emission filter that has an absolute photon flux ($\mathrm{Str_{flux}}$) of 0.32058 W/m^2. The photon flux value of the calibrated star was integrated over the filter's longpass wavelength range, see Figure A1. The calibrated star ($\mathrm{Str_{avg}}$) observed with red N_2 emission filter had an average intensity pixel value of 17.1. We average the image pixel values at the sprite's brightest region altitude in order to determine the sprite's average brightness ($\mathrm{Sprs_{avg}}$). The $\mathrm{Sprs_{avg}}$ in Figure 3 had a pixel intensity value of 65.

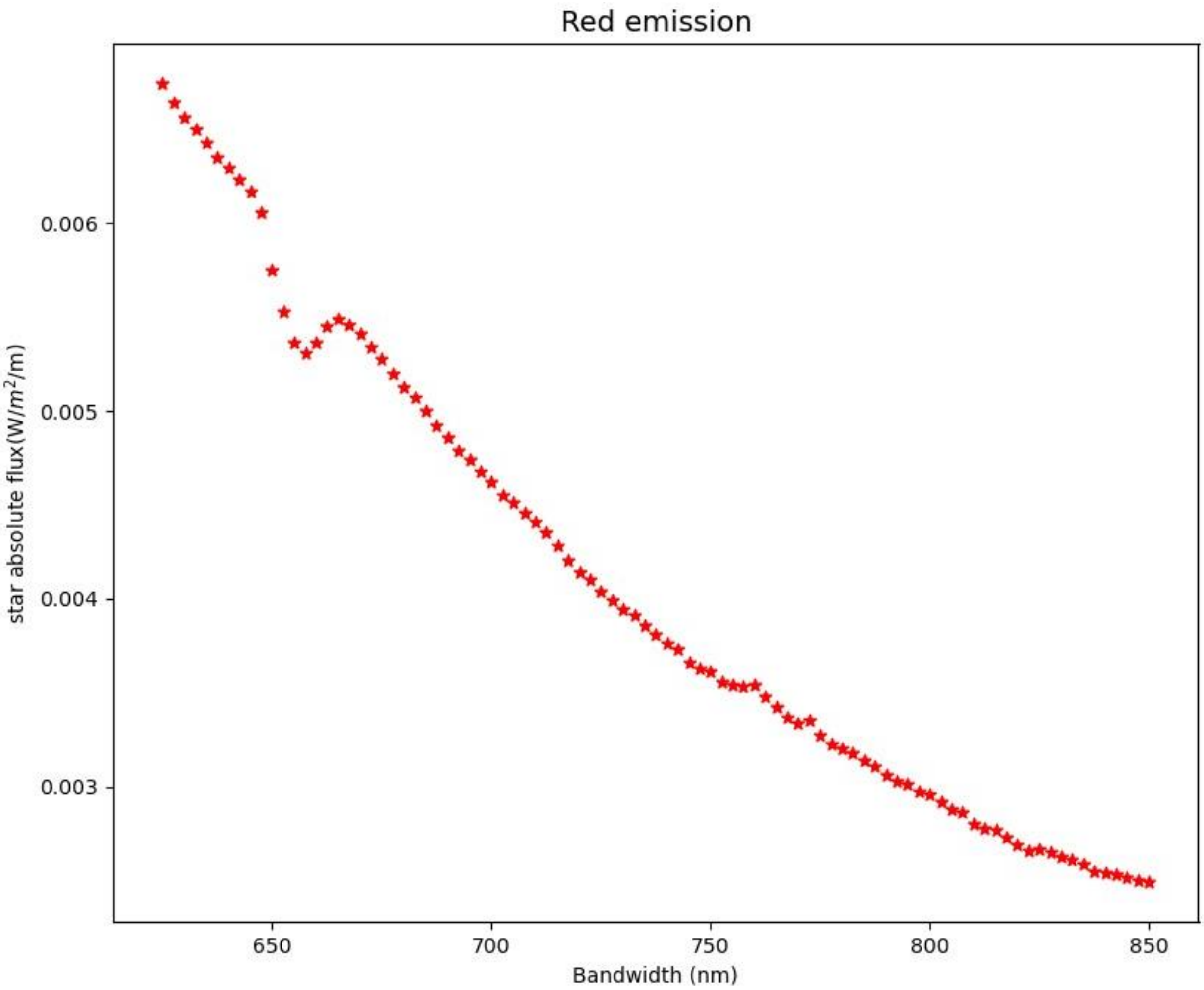

Figure A1. The calibrated star's absolute photon flux over the N_2[1]P emission range (Alekseeva et al., 1996).

We used data from the camera with a red filter to obtain the photon flux at the sprite's brightest region ($Sprs_{flux} = 1.7 \times 10^{-7}$ W/m^2), by means of Equation (4) (Nnadih et al., 2021).

$$Sprs_{flux} = \frac{Str_{flux}(Sprs_{avg} - Sky_{bkg})}{Str_{avg} - Sky_{bkg}} \text{ (W/m}^2\text{)} \quad \text{(A4)}$$

This calculation is summarised in Table 4 (row 3).

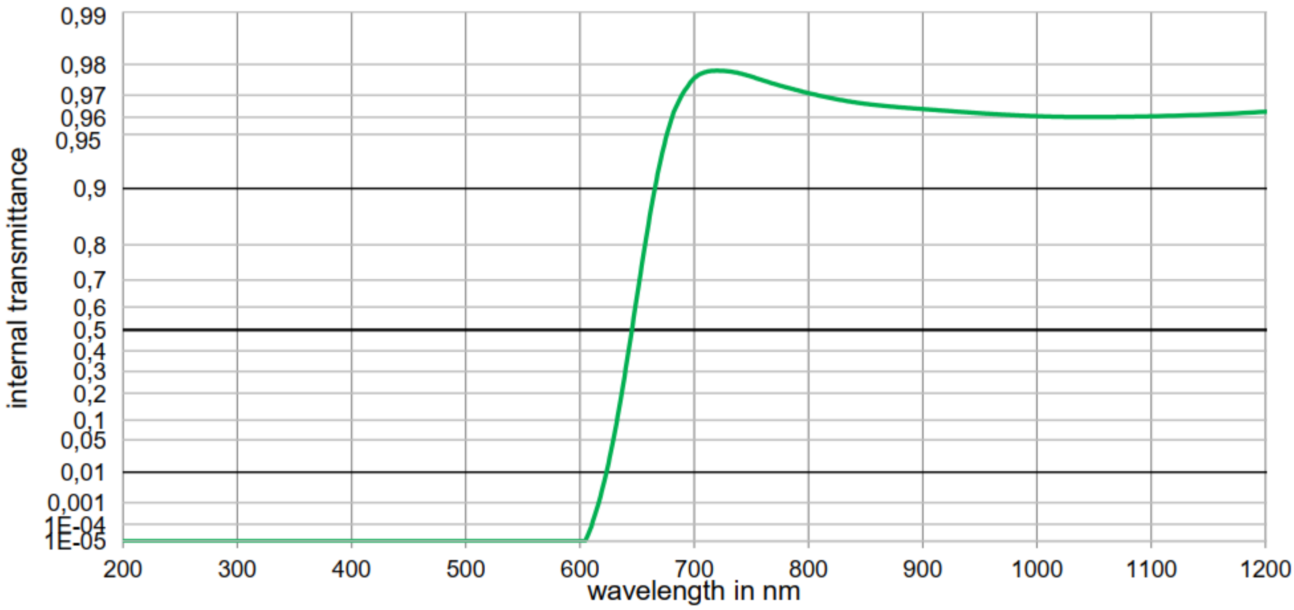

Figure A2. Internal transmittance of red N_2 filter with a 640-650 nm longpass (https://www.schott.com/shop/advanced-optics/en/Matt-Filter-Plates/RG645/c/glass-RG645).

To the Reviewers, Thank you so much for the fruitful response. We have revised the manuscript accordingly. Please see our comments and responses below in green. The major voluntary changes (new paragraphs and whole new sentences) are written in green text in the manuscript.

Comments to the Authors:

Review of **the revised** MS entitled "*Estimates of energy fluxes associated with sprites in the mesosphere"* by D. Mashao *et al.*, re-submitted for possible publication in Journal of Atmospheric and Solar-Terrestrial Physics.

Authors have only provide clarification to a number of the points I commented. There are still some points that regarding the mechanisms producing delayed sprites (the subject of this work) that are neither adequately answered nor properly cited. The worst is that authors write in their reply to my L 301-302 (of original paper): *"Thank you, we corrected the sentence and discussed as suggested in the revised manuscript"*, but they **did not.**

In addition, there is **an entire new paragraph** from **L 71 to L 87** **of the revised manuscript** that is quite unprecised and makes no sense to me in the context of this work that ***explores delayed sprites***. The authors write:

"The lightning driven electric field must exceed the local air breakdown field in order for sprites to occur (Pasko et al. 2013; Surkov and Hayakawa, 2020). The lightning radiated electromagnetic fields and numerical models have been used to estimate lightning driven electric field at sprites altitudes (Gamerota et al., 2011; Hu et al., 2007). The local air breakdown field Ek /N = 120 Townsend (Td), where N is the air number density (1 Td = 1 × 10^{-17} V cm^2) (Luque and Gordillo- Vázquez, 2012). Thus Ek is determined by the dissociative attachment and ionisation in air, which corresponds to the air density (Liu et al., 2009a). The lightning driven electric field needed to produce sprites reported by Hu et al. (2007) were about 0.2 Ek, 0.3—0.5 Ek, and greater than 0.5 Ek for dim, typical, and bright sprites, respectively, where Ek is the local air breakdown field. Gamerota et al. (2011) obtained the lightning driven electric field of about 0.1 to 5 Ek. Li and Cummer (2012) reported that positive sprites streamers induced by positive CG lightning propagates downwards to regions corresponding to 0.05 Ek. They found a normalised electric field ranging from 0.04 to 0.46 Ek for altitudes ranging from 42 to 74 km (Li and Cummer, 2012). Qin et al. (2018) reported carrot sprites occurred in the mesospheric region with electric field of about 0.8 Ek. Kanmae

et al. (2012) estimated the peak electric field of sprites streamers and found that the peak electric field varied between 90 and 150 kV/cm, which is 3 to 5 times the Ek (30 kV/cm)."

Clearly, **for non-delayed** sprites the *"the lightning driven electric field must exceed the local airbreakdown field"***, but this is not the case** in this work. However, it is written in a way that seems to be the case. The sprites investigated in this paper **are delayed sprites** for which **undervoltage electric fields** (those **below** conventional breakdown field E_k = 120 Td) can typically allow the occurence of delayed sprites because of the action due to associative detachment (AD) as explained in Luque and Gordillo-Vázquez, Nature Geoscience 2012. However, the authors have chosen to wrongly put in context this 2012 paper and, consequently, they will confuse readers by writting **in L 71 -72** of the revised manuscript that *"the lightning driven electric field must exceed the local air breakdown field".* The citation to Luque and Gordillo- Vázquez, 2012 in the above L71-L87 paragraph **is misleading**. Luque and Gordillo-Vázquez, 2012 does **not need** to be cited for such a known result, that is, that *"The local air breakdown field Ek /N = 120 Townsend (Td), where N is the air number density (1 Td = 1 × 10^{-17} V cm^2".* Luque and Gordillo- Vázquez, 2012 main **contribution deals with the** result that undervoltage electric fields (**made possible by associative detachment** (see below)) **can lead to delayed sprites**, that is, that delayed sprites are possible in an scenario of undervoltage electric fields (below 120 Td), so that, in such scenario, the driving electric field is not due to dissocative attachment and electron impact ionization of air but with a situation in which **there is no apparent electron attachment** in air (see below). Authors should read Luque and Gordillo- Vázquez, 2012 and cite it appropiately.

We rewrote the paragraph, see lines 71-74 in the manuscript and the text below.
"For non-delayed sprites to occur the lightning driven electric field must exceed the local air breakdown field (120 Td) (Pasko et al. 2013; Surkov and Hayakawa, 2020). However, the lightning driven electric field lower than the local air breakdown field can lead to delayed sprites due to associative detachment (Luque and Gordillo- Vázquez, 2012)."

The above is completely connected to my comment to **L 301-303 (of the original manuscript)** where I wrote:

L301-303 (of orignal paper): "Authors claim that "*the average apparent radiated lightning electric field within the column and carrot sprites were 193.3 and 190.4 V/m, respectively. These values are consistent with the expected breakdown electric field for air in the mesosphere*". I think this **is not so**. If you consider your 193 V/m = 1.93 V/cm and an average air density (between 53.1 km and 69 km) of, say, 4×10^{15} cm^{-3}, we get 0.5×10^{-15} V cm^2 = 50 Td, that is, the measured average electric fields are clearly lower than the conventional breakdown field of 120 Td. Obtaining 50 Td *is fine* **as long as the author would have clearly stated that this undervoltage values are a consequence of the effect due**

to associative detachment (AD) of O^- + N_2 -> N_2O + e (explained in Luque and Gordillo-Vazquez Nat.Geo 2012). This "AD breakdown" (as named in Luque and Gordillo-Vazquez Nat. Geo 2012) **is directly responsable** of the fact that delayed sprites can occur **within an undervoltage** environment, that is, when the **conventional breakdown field** (E_k = 120 Td) is not reached. These details must be clearly stated and properly included (and adequately cited) in the discussion section of the paper **so that readers can have a fair view** of why you can have sprites in undervoltage scenarios." to which **they replied by saying:**

"Thank you, we corrected the sentence and discussed as suggested in the revised manuscript."

But they **did not correct anything**, and they did not indicate what I mentioned above.

As written in Luque and Gordillo-Vazquez Nat. Geo (2012), the conventional breakdown assumption that electrons are inert after they form O- is valid only at high pressures, close to 1 atm, where three-body stabilization of O^- dominates. The relevance of associative detachment was known in the low-pressure plasma community already in the 1970s, when studies of low-pressure electric discharges showed that **there is no apparent electron attachment** in air below ~ 0.1 atm (corresponding to altitudes above ~ 15 km in the atmosphere).

We rewrote the sentence and included the suggested information, see lines 378-381 in the manuscript and the text below.

"The average apparent radiated lightning electric field within the column and carrot sprites were 193.3 and 190.4 V/m, respectively. The sprites associated with radiated lightning electric field lower than the breakdown electric field (120 Td) may be generated due to associative detachment (Luque and Gordillo- Vázquez, 2012)."

Recommendation:

The paper requires **major revision**. Authors need to **significatly improve the discussion including the mentioned points in this review**. In my view, the present manuscript could only be published in JASTP if authors correct the paper along the directions given here including appropriate citation of previous results.

Thank you.

Highlights:

- The lightning return stroke is important for the maximum brightness of sprites.
- The apparent Poynting flux emanating from the sprite dominates over the Joule heating and photon flux.
- The lightning peak Poynting flux decreases with increasing atmospheric altitude.
- Column sprites have a shorter time delay than carrot sprites from their parent lightning strokes.
- The brightest region of carrot sprites is associated with greater averaged photon flux and lightning initial peak Joule heating.

Figure 1

Click here to access/download;Figure;Figure 1.png

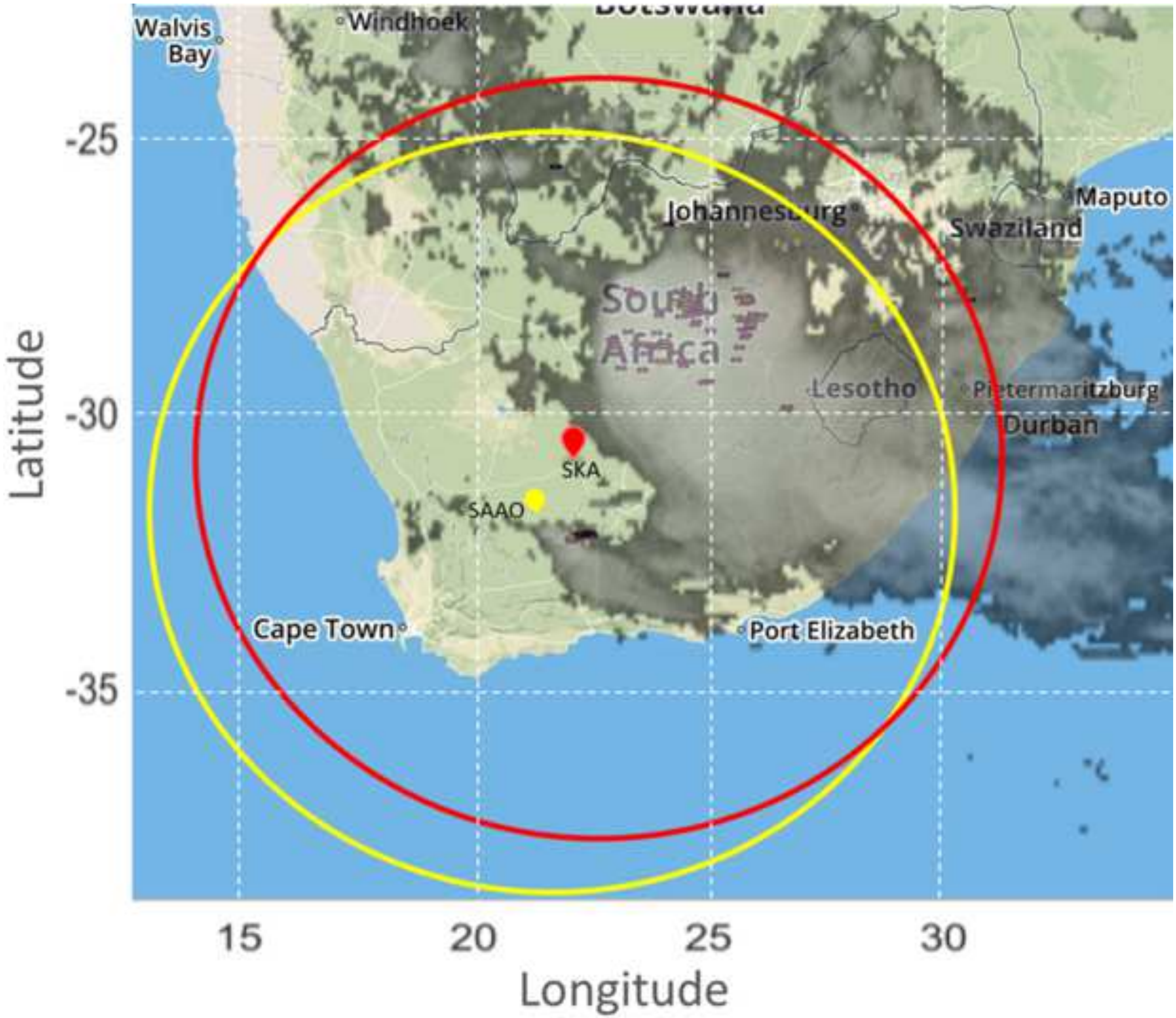

Figure 2

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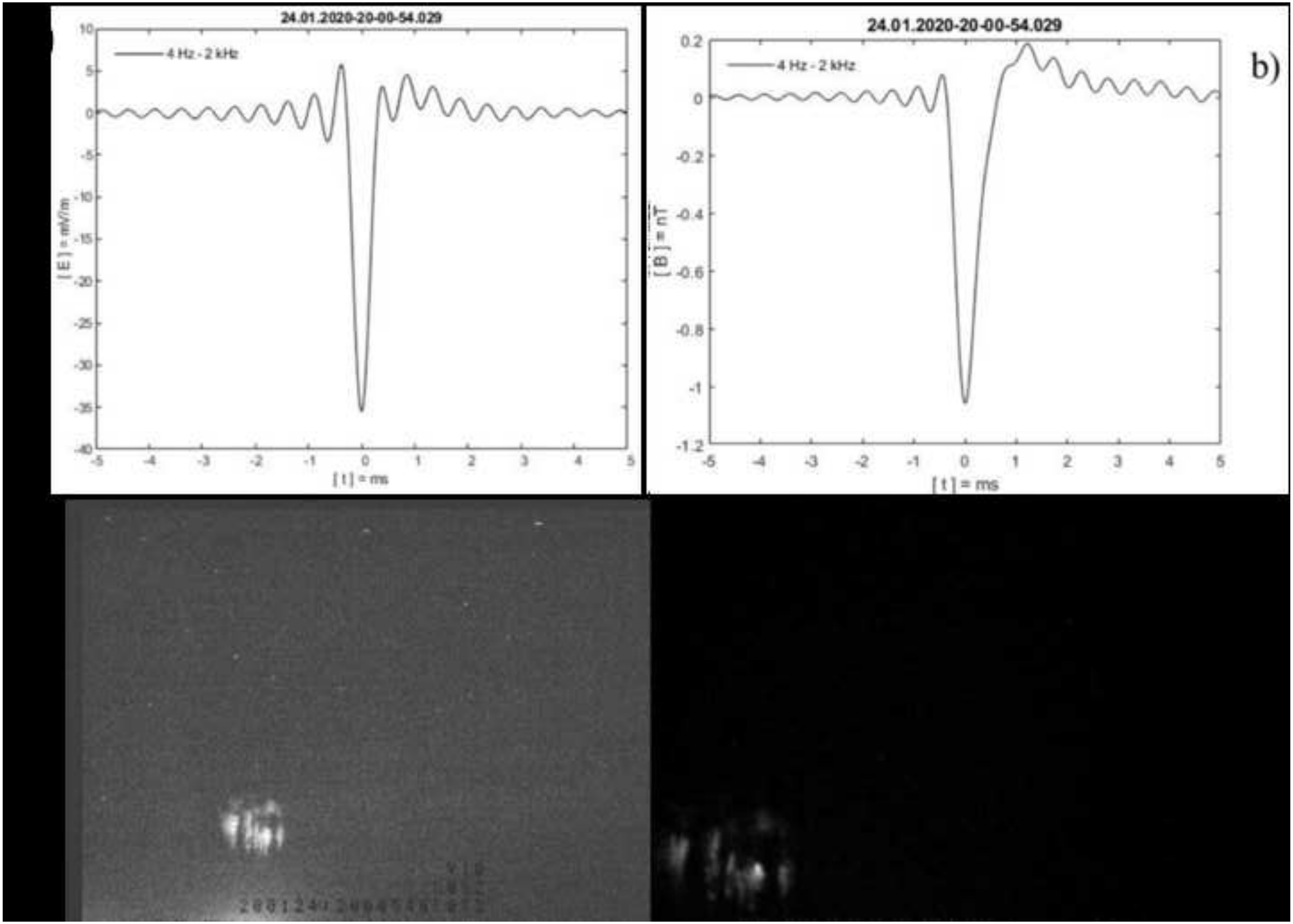

Figure 3

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Figure 4

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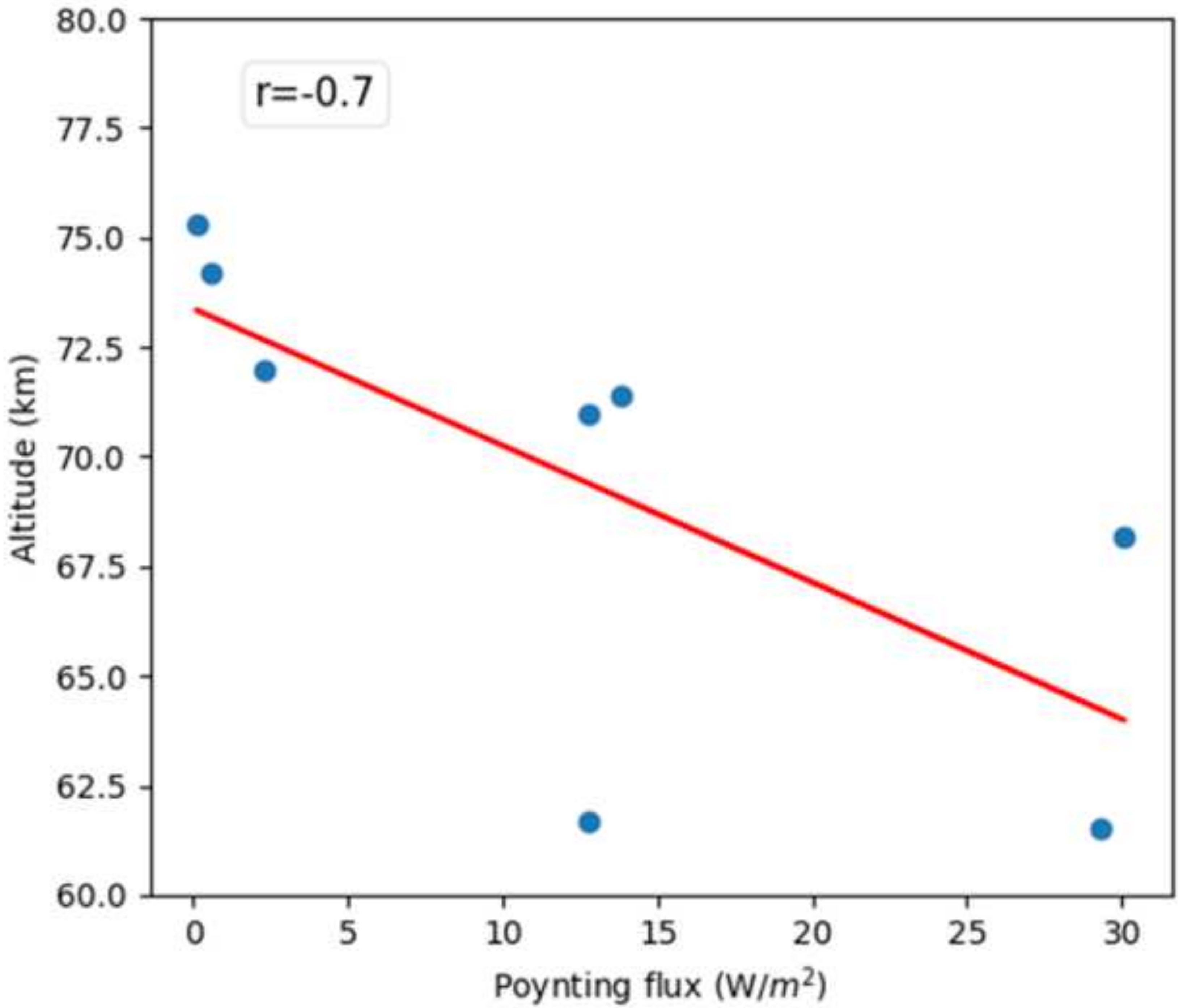

Figure 6

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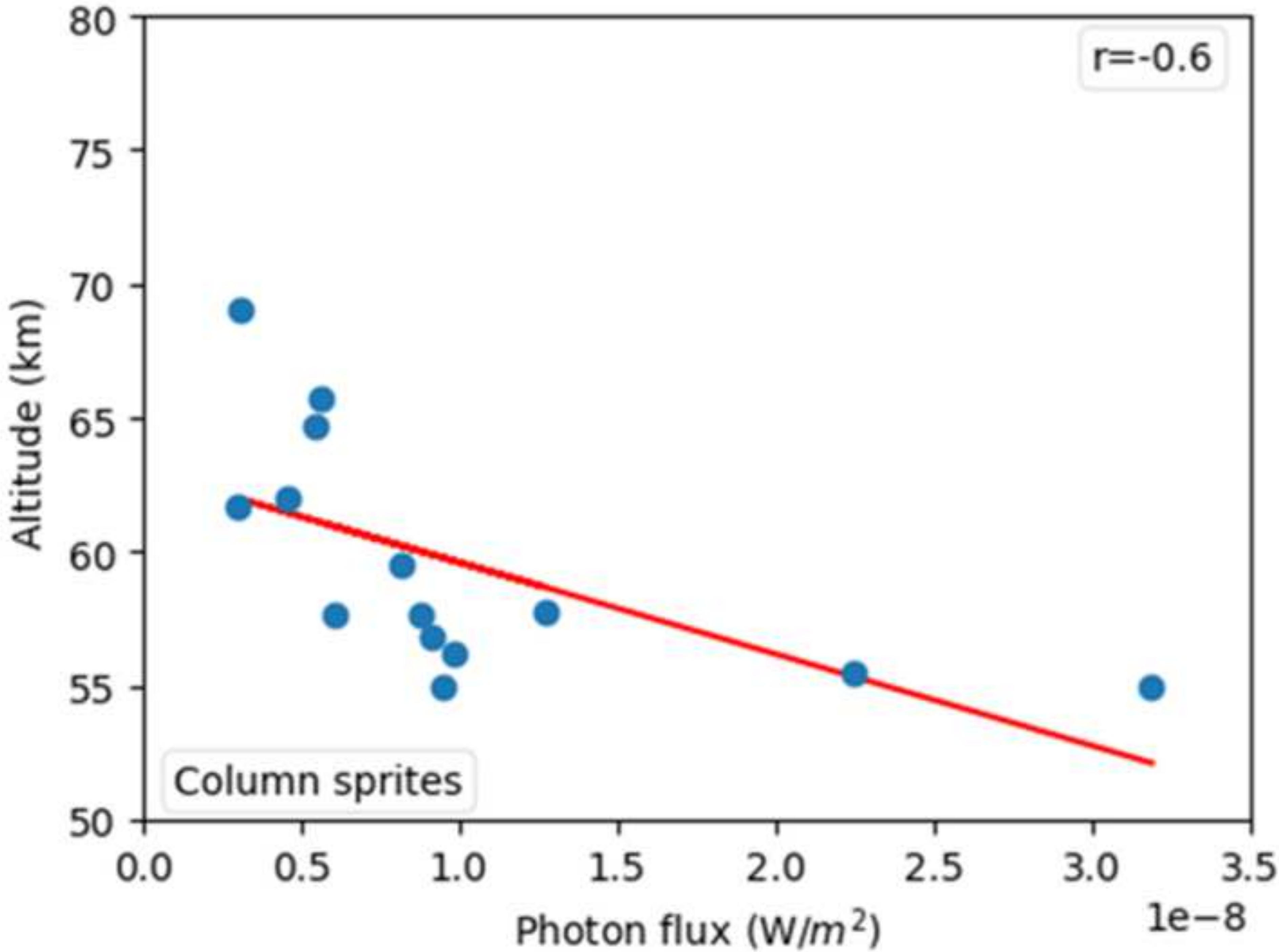

Figure 5

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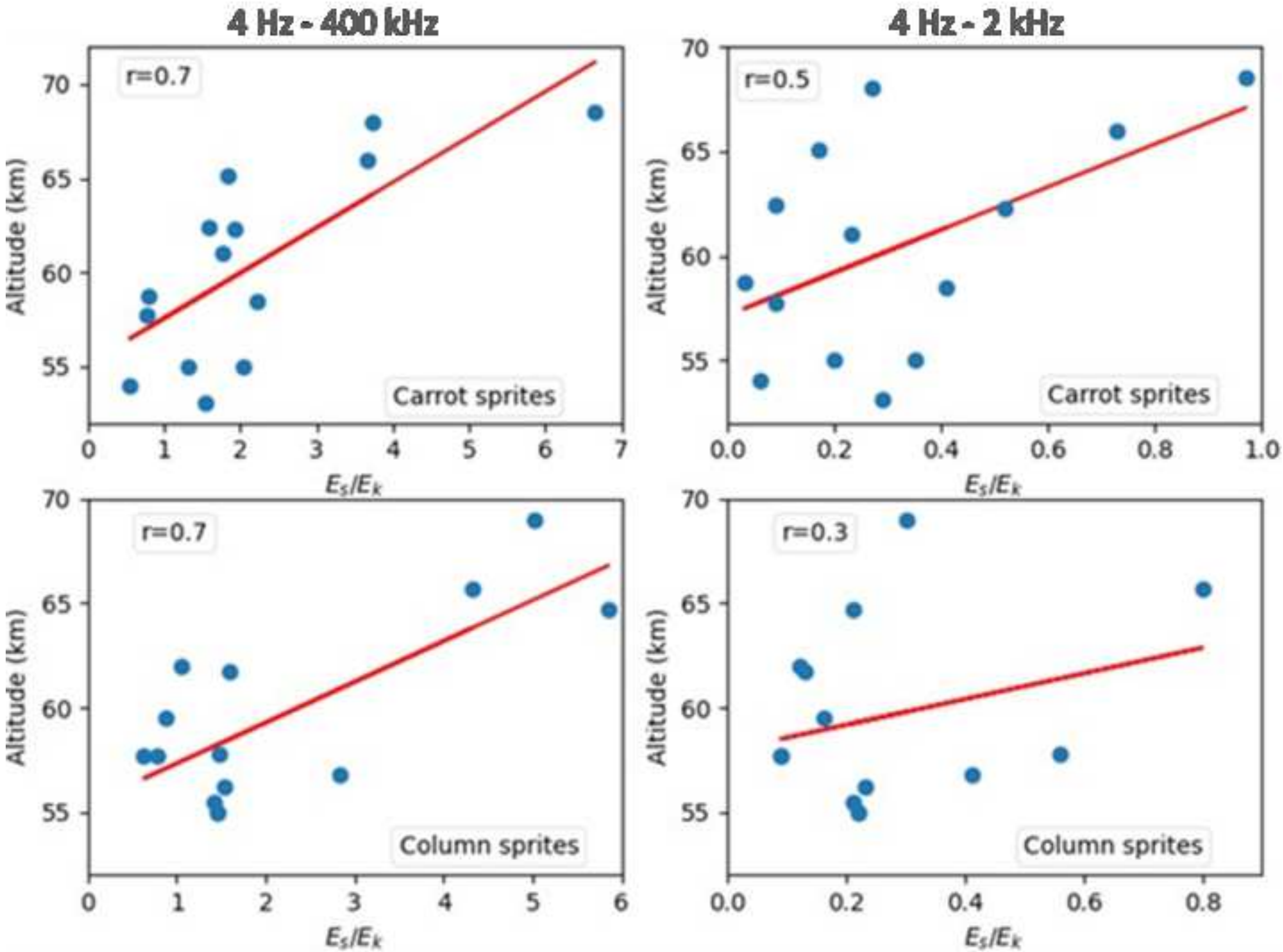

Declaration of interests

☒The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

☐The authors declare the following financial interests/personal relationships which may be considered as potential competing interests:

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Estimates of energy fluxes associated with sprites in the mesosphere

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Abstract

We present calibrated estimates of photon flux, lightning peak Poynting flux and Joule heating associated with the brightest region of sprites observed in the mesosphere over South Africa. The sprites' photon fluxes were estimated using 28 sprites events (observed during the 2019 sprites campaign) calibrated by stars in the sprite's image background. The lightning driven background electric field associated with the brightest region of sprites were found to vary from 0.1 to 6.7 E_k (local air breakdown field). The lightning return stroke at lower frequencies (4 Hz - 2 kHz) has more influence in carrot sprites than in column sprites formation processes. The lightning peak Poynting flux and Joule heating were estimated from calibrated electromagnetic field measurements made in parallel with eight sprites events (observed during the 2020 sprites campaign). The photon flux, peak Poynting flux, and peak Joule heating associated with the brightest region of sprites were found to be 1.1×10^{-7} W/m^2, 12.7 W/m^2, and 4.7×10^{-3} W/m^2 on average, respectively. The altitude/distance-normalised lightning peak Poynting flux decreases with increasing atmospheric altitude. The photon flux from column sprites decreased with increased altitude of the brightest region. Column sprites have a shorter time delay (<30 ms) from their parent lightning strokes than carrot sprites (up to 145 ms).

Keywords: Sprite's Joule heating, sprite's Poynting flux, sprite's photon flux, lightning return stroke.

1. Introduction

The energy from the quasi-static electric field generated mainly by large positive cloud to ground (CG) lightning discharges of Mesoscale Convective Systems (MCS) initiates collisions between electrons, neutral atoms and molecules, resulting in photon emissions forming the Transient Luminous Events (TLEs) called sprites in the middle atmosphere (Franz et al., 1990; Liu et al., 2015; Pasko, 2010; Surkov and Hayakawa, 2020). The sprites initiation process involves the downward and upward movement of streamers (Bór, 2013; Luque and Ebert, 2009). Collisions between electrons, neutral atoms and molecules lead to the emission of photons from transitions of excited states of neutral nitrogen (N_2) and ionised nitrogen (N_2^+) predominantly at an altitude above and below 50 km, respectively (Sentman et al., 1995; Heavner et al., 2010; Surkov and Hayakawa, 2012; Nnadih et al., 2021). The emissions of N_2 and N_2^+ occur as a result of excitation of the ^1P and 1NG states, respectively, in the ~300-1000 nm visible band (Mende et al., 1995; Hampton et al., 1996; Heavner et al., 2010). $N_2(^1P)$ has a band emission around 640 nm (~550-1070 nm wavelength range (Shibusawa and Funatsu, 2019)), with a lifetime of about 100 ms. N_2^+(1NG) has a band emission around 427.8 nm (~300-460 nm wavelength range (Heavner et al., 2010)), with a lifetime of about 5 ms (Armstrong et al., 1998; Suszcynsky et al., 1998; Heavner et al., 2010; Nnadih et al., 2021). Quenching accounts for the short lifetime of the N_2^+ emission, whereas Rayleigh scattering accounts for its low intensity (Heavner et al., 2010; Nnadih et al., 2021). The emissions of $N_2(^1P)$ and N_2^+(1NG) have lifetimes of about 6 μs and 70 ns, and quenching altitudes of about 53 km and 48 km, respectively (Heavner et al., 2010).

The altitude of the brightest region of sprites varies from 50 to 84.1 km (Füllekrug et al., 2019; Luque et al., 2016; Mashao et al., 2021; Malagón- Romero et al., 2020; Sentman et al.,1995; Stenbaek-Nielsen et al., 2010; Wescott et al., 1998; Wescott et al., 2001). The lightning electric field accelerates the electrons, and the conversion of electrons to negative ions intensifies the local electric field. The brightest region of the sprite is a region with relatively low mesospheric conductivity, enhanced electric field and electrons are converted to negative ions, which results in maximum photon production (Malagón- Romero et al., 2020; Mashao et al., 2021).

Sprites result from Joule heating of the mesosphere associated with lightning electric field (Füllekrug, 2006). Füllekrug et al. (2006) reported on stratospheric Joule heating determined from the lightning continuing current measurement. They found that the Joule heating varied from 1×10^{-4} J/m^3 to 1×10^{-8} J/m^3 for altitudes ranging from 25 to 45 km, respectively (Füllekrug et al., 2006). Gordillo-Vázquez et al. (2018) investigated, by spectroscopic means, the temperature of mesospheric regions between 65 km and 76 km and found no measurable heating associated with sprites. We are not aware of any reported estimates in the literature of lightning Joule heating and Poynting flux in the mesosphere in relation to sprites.

For non-delayed sprites to occur the lightning driven electric field must exceed the local air breakdown field (120 Townsend (Td)) (Pasko et al. 2013; Surkov and Hayakawa, 2020). However, the lightning driven electric field lower than the local air breakdown field can lead to delayed sprites due to associative detachment (Luque and Gordillo- Vázquez, 2012).

We evaluate lightning initial apparent peak Poynting flux and Joule heating projected to the altitude of the brightest regions of sprites in the mesosphere for the first time. We also compare photon flux at the altitude of the brightest region of the optical observations column and carrot sprites using a camera with a red (N_2) emission filter. The lightning-driven electric field associated with the altitude of the brightest regions of sprites were determined and compared with the published literature.

2. Optical and radio recordings

The reported sprites were recorded during the 2019 and 2020 sprites campaigns from the Square Kilometre Array (SKA) (30.97º S, 21.98º E) and the South African Astronomical Observatory (SAAO) (32.38º S, 20.81º E), Northern Cape, South Africa. The sprites were observed on the following nights: 01 February 2019 from SKA and 24 January 2020 from SKA and SAAO. Figure 1 shows the map of South Africa, the MCS associated with some of the observed sprites, and the SKA and SAAO sites marked with red and yellow dots, respectively. The +CG, -CG, and IC lightning discharges are marked with plus, negative, and dash-like symbols within the thundercloud, respectively. Lightning electric field observations were made in 2019 and 2020. Lightning magnetic field observations were made in 2020 only. Eight sprites events with no calibrated stars present were recorded during the 2020 sprites campaign and are used for estimating the lightning Poynting flux and Joule heating associated with the altitude of the brightest regions of sprites only. The 2020 sprites campaign data comprised of column, carrot, wishbone, jellyfish, tree, angel, and sprites with halo. From the 2019 sprites campaign, we selected column and carrot sprites that occurred individually or in a group with calibrated stars of known photon flux. We found 14 column and 14 carrot sprites red emission optical data with calibrated stars and used them for estimating the photon flux as well as Joule heating (Alekseeva et al., 1996; Bór, 2013).

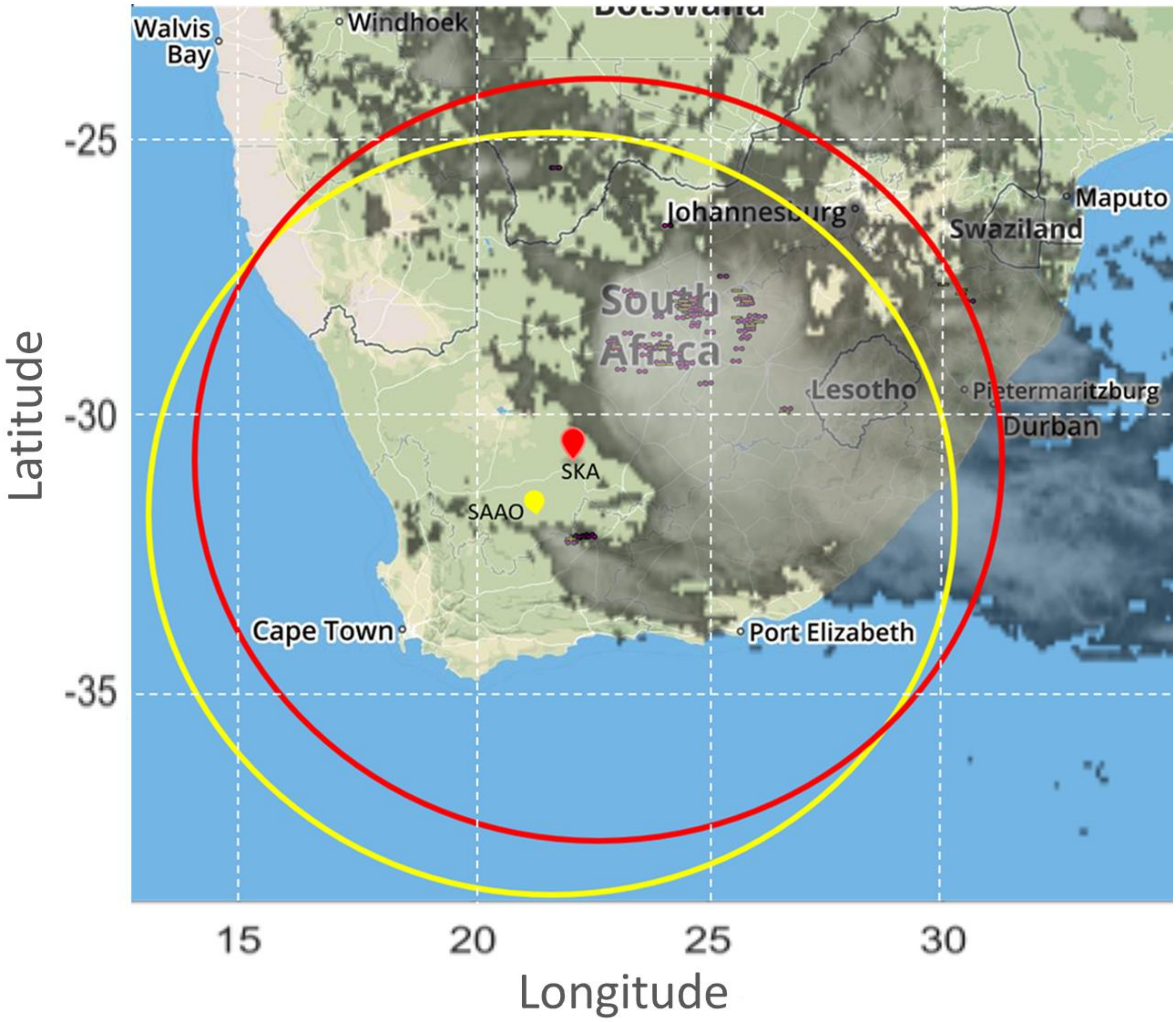

Figure 1. Map of South Africa captured on 1 February 2019 at 22:41:00 UTC. The SKA/ SAAO sites are marked with red/yellow dots, and the MCS associated with some observed sprites is shown in grey. The circles mark the 900 km radius, which is the maximum distance our cameras can observe from SKA and SAAO, respectively (Earthnetworks.com, 2020).

2.1 Optical recording

Watec 910Hx and Allied Vision Pike night vision cameras, mounted on the same tripod, were used to record the sprites from SKA/SAAO (Mashao et al., 2021; Nnadih et al., 2018, Nnadih et al., 2021). The stars in our field of view (FOV) were utilised to co-align the cameras. The cameras' viewing directions were North, East, and South of SKA and SAAO.

The Watec 910Hx cameras, with an 8.0 mm f/1.4 C-mount lens, operated with 0.45 gamma factor and 8-bit intensity resolution. The Watec 910Hx cameras recorded 25 video frames per second (fps) with a 40 ms frame period. A Global Positioning System (GPS) video timer provided time with millisecond timing to the camera system. The Watec 910Hx cameras had a 29°/46.2° FOV Vertical/Horizontal (V/H), 0.061°/0.072° V/H angular resolution per pixel, and 640×480 (H×V) pixels.

The Allied Vision Pike camera, coupled to a Ceramic xx1332 Mullard image intensifier with Xenon F0.95 50 mm C-mount TV lens, captured images at 25 fps and a 40 ms frame period, 640×480 video image size, 15°/21° FOV V/H, and 14-bit intensity resolution. The Ceramic xx1332 Mullard image intensifier has a wavelength range of about 360 nm to 850 nm (Allied Vision, 2022; Worthpoint, 2022). A red N_2 longpass cut off filter with a 640-650 nm cutoff was used to record the $N_2(^1P)$ emission of sprites. A Network Timing Protocol server provided timing accuracy of about 1 ms for the Pike camera system at SKA.

2.2 Electromagnetic recording

The lightning electric (2019 and 2020 campaigns) and magnetic (2020 campaign only) fields were recorded in parallel with sprites optical recordings at SKA. A wideband digital ELF/VLF/LF radio receiver that detected the lightning vertical electric field strength observed with about 4 Hz to 400 kHz frequency range, with a sampling frequency of 1 MHz, and timing accuracy of 20 ns (Füllekrug, 2010; Füllekrug et al., 2019). The lightning magnitude values were converted to Td by using the modelled N_2 densities associated with the targeted sprites altitude. We then obtain the ratio between the local lightning electric field (E_s) and local air breakdown field (E_k), see Tables 1 to 4. All N_2 densities are obtained from the NRLMSISE profile (https://ccmc.gsfc.nasa.gov/modelweb/models/nrlmsise00.php). Two orthogonal induction coils are used to record the horizontal magnetic field in the frequency range of ~4 Hz to ~60 kHz with a sampling frequency of 500 kHz and a timing accuracy of ~20 ns. Lightning magnetic field observations were conducted during the 2020 sprites campaign only from SKA.

The Earth Network Total Lightning Network (ENTLN) and South African Lightning Detection Network (SALDN) provided lightning data (time, position, peak current, type of discharge, and polarity) related to the observed sprites. ENTLN and SALDN have position accuracy of about 0.2 km and 0.5 km, respectively (Gijben, 2012; Bui et al., 2015; Zhu et al., 2017). All CG lightning events associated with the sprites reported here have positive polarities. The peak lightning current varied from 28 to 142 kA, see Tables 1 to 4.

3. Data analyses

3.1 Sprite altitude estimation

We estimate the lightning peak Joule heating and apparent Poynting flux, and average photon flux associated with the sprites brightest regions using optical data as well as lightning electric and magnetic field data. The right ascension and declination of stars on the sprites image background and the geographic position of the camera were used to determine the azimuth angle, elevation angle, and field of view of the cameras. The azimuth and elevation angles of stars were computed and used to fit onto the real stars in the background image of sprites. The altitude of the sprite's brightest region was obtained by employing spherical and planar trigonometry in the horizontal and vertical planes, respectively (Mashao et al., 2021; 2022a; 2022b), assuming that the sprites events occurred directly above their parent lightning locations. The latter assumption is commonly used in sprites optical research (Füllekrug et al., 2019; Li et al., 2008; Luque et al., 2016; Mashao et al., 2021; 2022a; 2022b; McHarg et al., 2007; Stenbaek-Nielsen and McHarg, 2008). The uncertainty in altitude of sprites computed from the angular resolution of the camera spanned from ± 0.33 to 0.47 km. The uncertainty in the altitude of sprites varies with slant distance from the location of camera to the sprites.

3.2 Poynting flux estimation

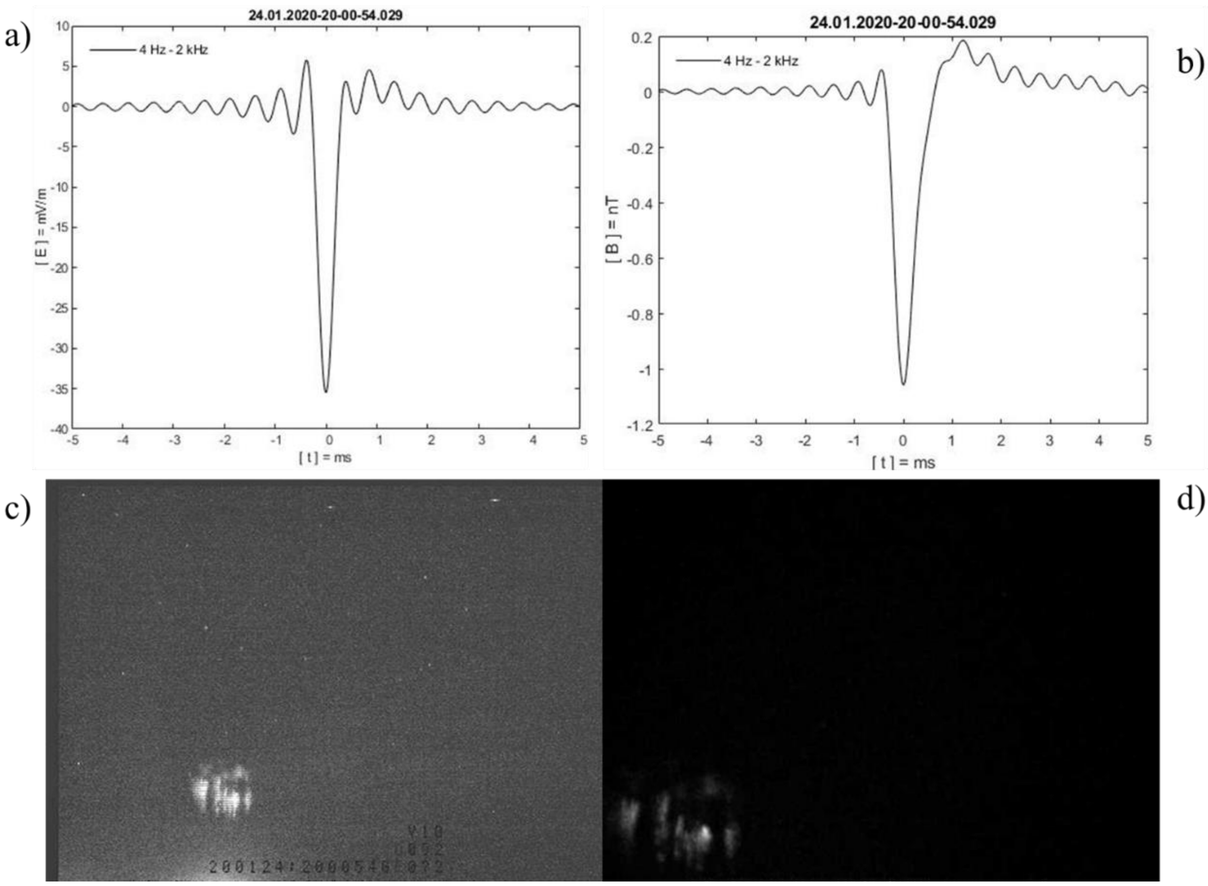

Figure 2. The lightning electric (a) and magnetic (b) waveforms associated with the sprite event observed on 24 January 2020 at 20:00:54.072 UTC from SKA recorded by monochrome camera without a filter (c) and with a red N_2 filter (d). The correct sprite event time stamp is displayed on the image with no filter.

Figure 2 shows the lightning electric (a) and magnetic (b) field waveforms associated with the sprites event recorded simultaneously by a monochrome camera without a filter (c) and with the red N_2 filter (d) from SKA. We used the simultaneous lightning peak electric and magnetic field strengths associated with the optical sprites to determine the maximum apparent Poynting flux associated with the eight sprites analysed in the frequency range of 4 Hz to 2 kHz. The Poynting flux (**S**) is given by (Farrell et al., 2006):

$$\mathbf{S} = \frac{(\mathbf{E} \times \mathbf{B})}{\mu} \quad (1)$$

where (**E**) is the lightning electric field, (**B**) is the lightning magnetic field, and (μ) is the permeability of the medium. The free space wave impedance is modified near a conductor, in this case the Earth, which is studied in the field of magnetotellurics in detail (Bór et al. 2022). Here we assume a vacuum. To compare the lightning electric and magnetic field strengths, we filter the data to obtain the lightning return stroke at lower frequencies information (4 Hz - 2 kHz), see Figure2 (a) and (b) (Constable, 2016; Cummer et al., 2013; Fraser-Smith and Bowen, 1992; Füllekrug, 2010). ELF/VLF/LF lightning radio receivers with lower minimum frequency (<4 Hz) are required to measure lightning continuing current. These data are not available to us. Lightning continuing current plays an essential role in initiating delayed sprites, sustaining the quasi-static electric field in the mesosphere and the brightness of sprites longer than the local relaxation time (Gomez Kui et al., 2021; Kitagawa et al., 1962; Ren et al., 2021; Tomicic et al., 2021).

Due to a technical problem, we only recorded the dominant horizontal component of the magnetic field aligned in the North-South geographic direction. Since the lightning occurred between 209.5 and 562.3 km away from the receiver, we used Equation (2) to estimate the total horizontal magnetic field ($\mathbf{B_T}$):

$$\mathbf{B_T} = \mathbf{B}/\sin(AZ) \quad (2)$$

where *AZ* is the azimuth angle between the lightning position and the North-South coil orientation.

The radiation field terms decrease as 1/d, $1/d^2$, and $1/d^3$, depending on the horizontal distance between the lightning event and the receiver, where d is the distance from the lightning location to the receiver location (Cooray and Lobato, 2020). At short distances, radiation field terms decrease as 1/d, and $1/d^2$.

From the recorded data, we estimate the lightning electric and magnetic fields associated with the altitude of the brightest regions of sprites. The lightning electric and magnetic fields associated with the altitude of the brightest regions of sprites were normalised as follows: We project horizontally the lightning electric and magnetic radiated fields as $1/_{d^2}$, to 1 km away from the lightning location (Cooray and Lobato, 2020; Taylor and Jean, 1959). This was done by multiplying the observed lightning electric and total magnetic field values by the distance squared in km. For our geometry, the parent lightning source of the fields is assumed to be a finite line current. It is known that the lightning quasi-static electric field strength, which may initiate sprites, lessens with increasing altitude as $\frac{1}{a^3}$, where *a* is atmospheric altitude (Pasko et al., 2013). We assumed that the CG lightning charges, which initiated the sprites events, were removed at 10 km altitude (Asano et al., 2008; Pasko et al., 2013), and we then determined lightning electric (E_s) and magnetic field strength associated with the sprite's brightest region altitude as $\left(\frac{10}{a}\right)^3$, where *a* is the calculated altitude (km) of the sprite's brightest region (see example in the Appendix). Finally, we obtain the distance/altitude-normalised peak apparent Poynting flux associated with the sprite's brightest region altitude using Equations 1 and 2. The lightning peak Poynting flux associated with the altitude of the brightest regions of sprites estimates are presented in Table 1 (see Appendix). An example calculation of the lightning peak Poynting flux associated with the altitude of the brightest regions of sprites is presented in the Appendix.

3.3 Joule heating estimation

To determine the lightning Joule heating associated with the sprite's brightest region altitude, we used the lightning electric field strength ($\mathbf{E}$) and the atmospheric conductivity ($\boldsymbol{\sigma}$) adopted from Liu et al. (2015), integrated over the observed vertical extent of the brightest region of the sprites. The vertical extent of the brightest region of sprites spanned from 1.2 to 2.4 km. Joule heating ($\mathbf{J_h}$) is given by (e.g. Kosch and Nielsen, 1995; Foster et al., 1998):

$$\mathbf{J_h} = \boldsymbol{\sigma}\mathbf{E}^2 \qquad (3)$$

The same altitude/distance normalised lightning electric field data used to estimate the peak Poynting flux was used to determine the peak Joule heating (first-order approximation). The atmospheric conductivity corresponding to the vertical extent of the brightest region altitude of sprites was used to calculate the lightning peak Joule heating associated with the altitude of the brightest regions of sprites using Equation (3). Sprite streamer formation involves ionisation of the neutral atmosphere, which will modify the local conductivity and electric field within the sprites. Hence, we can only estimate the initial peak Joule heating before streamer ionisation takes over. The atmospheric conductivity was obtained from rocket measurements, which have been used in other TLE studies (Holzworth et al.,1985; Liu, 2012; Liu et al., 2015). For the 2019 sprites campaign, the height-integrated atmospheric conductivity varied from 1.2×10^{-6} to 2.1×10^{-4} S, for altitudes ranging from 61.5 to 75.3 km. For the 2020 sprites campaign, the height-integrated atmospheric conductivity corresponding to the column and carrot sprites' brightest region altitudes varied from 1.1×10^{-7} to 1.1×10^{-5} S, for altitudes ranging from 53.1 km and 69 km. The values of conductivities used here are lower limits.

Dowden et al. (2001) used measurements of VLF scattering in millisecond time scale, through horizontal angles of up to 180° to demonstrate that the sprites plasma has high conductivity. They set an approximate lower bound for conductivity of sprites plasma to 3×10^{-7} S/cm. Liu et al. (2009b) sprite simulations were extrapolated by Luque and Ebert (2010) which showed conductivity of about 8×10^{-8} and 5×10^{-8} S/cm, respectively. The sprite conductivity increases in time with respect to the background and the re-enhancement of the electric field in the sprites' streamer wake results in further increase in conductivity (Gordillo-Vázquez and Luque, 2010).

The lightning initial peak Joule heating is due to the lightning discharge within the bandwidth of the measured electric field (4 Hz – 2 kHz), corresponding to the lightning return stroke, and atmospheric conductance, corresponding to the vertical extent of the brightest region altitude of sprites. The results of lightning peak Joule heating associated with the altitude of the brightest regions of sprites estimate are shown in Tables 2, 3, and 4 (see Appendix). An example calculation of the lightning peak Joule heating associated with the altitude of the brightest regions of sprites is presented in the Appendix.

3.4 Photon Flux

To determine the sprite's photon flux out of the brightest region, we acquired sprites optical data observed with red N_2 emission filter simultaneously with calibrated stars of known photon flux (in W/m^2), as described in detail by Nnadih et al. (2021). The flux values of selected stars can be found in the Pulkovo Spectrophotometry Bright Stars Catalogue (Alekseeva et al., 1996). Only the sprites recorded during the 2019 sprites campaign from SKA had images containing calibrated stars. Figure 3 shows an example of a recorded sprite with calibrated star.

Figure 3. Images recorded with no filter (left) and red N_2 (right) filter on 01 February 2019 at 19:08:24.202 UTC from SKA. The images contain the same sprites event. The red circle in the image on the right denotes the calibrated star. The correct sprite event time stamp is displayed on the image with no filter.

We can estimate photon flux since we recorded the sprites events and the stars simultaneously with fixed camera gain setting and similar atmospheric losses over the same night. We averaged the sprites video clip frames over one second in order to determine the average intensity pixel value of the calibrated star ($\mathrm{Str_{avg}}$). The sky background ($\mathrm{Sky_{bkg}}$) value was obtained by using a 7×7 median filter to remove the stars on the video frame prior to the frame which contained the sprites event. To determine the sprite's average brightness ($\mathrm{Sprs_{avg}}$), we average the image pixel values over the sprite's brightest region altitude range. The average intensity pixel value ($\mathrm{Str_{avg}}$) and known photon flux value of the calibrated star ($\mathrm{Str_{flux}}$, integrated over the filter's longpass), sky background ($\mathrm{Sky_{bkg}}$), and the average image intensity pixel value over the sprite's brightest region altitude range ($\mathrm{Sprs_{avg}}$) were used to obtain the photon flux at the sprite's brightest region ($\mathrm{Sprs_{flux}}$), by means of Equation (4) (Nnadih et al., 2021).

$$\mathrm{Sprs_{flux}} = \frac{\mathrm{Str_{flux}}(\mathrm{Sprs_{avg}} - \mathrm{Sky_{bkg}})}{\mathrm{Str_{avg}} - \mathrm{Sky_{bkg}}}\ (\mathrm{W/m^2}) \qquad (4)$$

An example of the photon flux calculation at the sprite's brightest region is presented in the Appendix. The estimated column and carrot sprites' photon flux are shown in Tables 3 and 4 in the Appendix.

4. Results and discussion

The radiated electric fields of lightning discharges associated with 8 sprites events for the 2020 campaign (Table 1) at the ELF/VLF/LF receiver, projected to 1 km from the lightning position at ground level, and projected to the sprites' brightest region altitude varied from 2.6×10^{-3} to 5.7×10^{-1} V/m, 454.1 to 25059.6 V/m, and 1.1 to 68.9 V/m, respectively. The radiated magnetic field of lightning discharges associated with 8 sprites events for the 2020 campaign (Table 1) at the receiver, projected to 1 km from the lightning position at ground level, and projected to the sprites' brightest region altitudes varied from 2.95×10^{-10} to 1.09×10^{-9} T, 5.26×10^{-5} to 6.33×10^{-4} T, and 1.23×10^{-7} to 2×10^{-6} T, respectively. The radiated electromagnetic fields values are associated with the lightning return stroke at lower frequencies (4 Hz - 2 kHz). The radiated lightning electric and magnetic fields uncertainties were about ±0.05 V/m and $\pm2.5\times10^{-8}$ T, respectively, at the sprites' brightest region altitudes. Uncertainties associated with the use of first-order distance projection estimates are not included. The estimated altitude of the brightest region of 8 sprites events ranged from 61.5 to 75.3 km. The altitude of the brightest region of sprites is within that previously reported (i.e. 50 - 84.1 km) (Füllekrug et al., 2019; Luque et al., 2016; Mashao et al., 2021; Malagón- Romero et al., 2020; Sentman et al.,1995; Stenbaek-Nielsen et al., 2010; Wescott et al., 1998).

The N_2 density at the altitude of the brightest region of sprites (61.5—75.3 km) varied from 6.85×10^{14} to 4.39×10^{15} cm^{-3}, see Tables 1 and 2. All N_2 densities are obtained from the NRLMSISE profile (https://ccmc.gsfc.nasa.gov/modelweb/models/nrlmsise00.php). The altitude/distance normalised lightning driven peak electric field (E_s) at the altitude of the brightest region of sprites spanned from 0.1 to 0.5 E_k, associated with the lightning electric field in the frequency range 4 Hz to 400 kHz. The frequency range of 4 Hz to 400 kHz provides the lightning electric field information that the ELF/VLF/LF radio receiver can measure. These E_k values fall into the range reported in the literature (Gamerota et al., 2011: Hu et al., 2007; Kanmae et al., 2012; Li and Cummer, 2012). The E_s at the altitude of the brightest region of sprites associate with the lightning return stroke at lower frequencies (4 Hz - 2 kHz) varied from 0.01 to 0.4 E_k. The reduction of the lightning driven electric field is due to the filtering of the lightning electric fields data to obtain the lightning return stroke fields at lower frequencies. Gamerota et al. (2011) model demonstrated that a lightning electric field of 0.08 E_k is not sufficient to produce sprites. However, Gamerota et al. (2011)

model doesn't consider the intracloud lightning electric field which enhances the electric field energy at sprites altitudes (Füllekrug et al., 2019). The electric field energy might make it possible for lightning discharges with 0.08 E_k or less to initiate sprites. The lower E_k may be due to lower temporal resolution of the observations (Pasko et al., 2013). We note that some of our analysed E_s values are greater than the previous reports (Gamerota et al., 2011; Hu et al., 2007; Li and Cummer, 2012). However, the E_s values are within the same order (3-5 times the E_k) obtained by Kanmae et al. (2012) for sprites streamers peak electric field. Our peak electric field values are the background estimated electric field associated with the altitude of the brightest region of sprites. During sprites formation processes, conversion of electrons to negative ions intensifies the electric field (Malagón- Romero et al., 2020; Mashao et al., 2021). The electric field value of about 8 E_k has been associated with gigantic jet discharge (Kuo et al., 2009; Pasko et al., 2013). None of our events are gigantic jets, as confirmed by the cameras.

For the same 8 sprites, the lightning peak apparent Poynting flux (see Table 1) associated with the sprite's brightest region, using distance/altitude normalised data, was found to span from 0.6 to 30.1 W/m^2 (0.08 W/m^2), with an average of 12.7 W/m^2. The value in parentheses is the uncertainty. Figure 4 shows the peak Poynting flux associated with the brightest region of sprites increased with decreasing atmospheric altitude. The linear correlation coefficient between these variables is -0.7. This outcome is expected as both the electric and magnetic fields decrease as $\frac{1}{a^3}$ with increasing altitude.

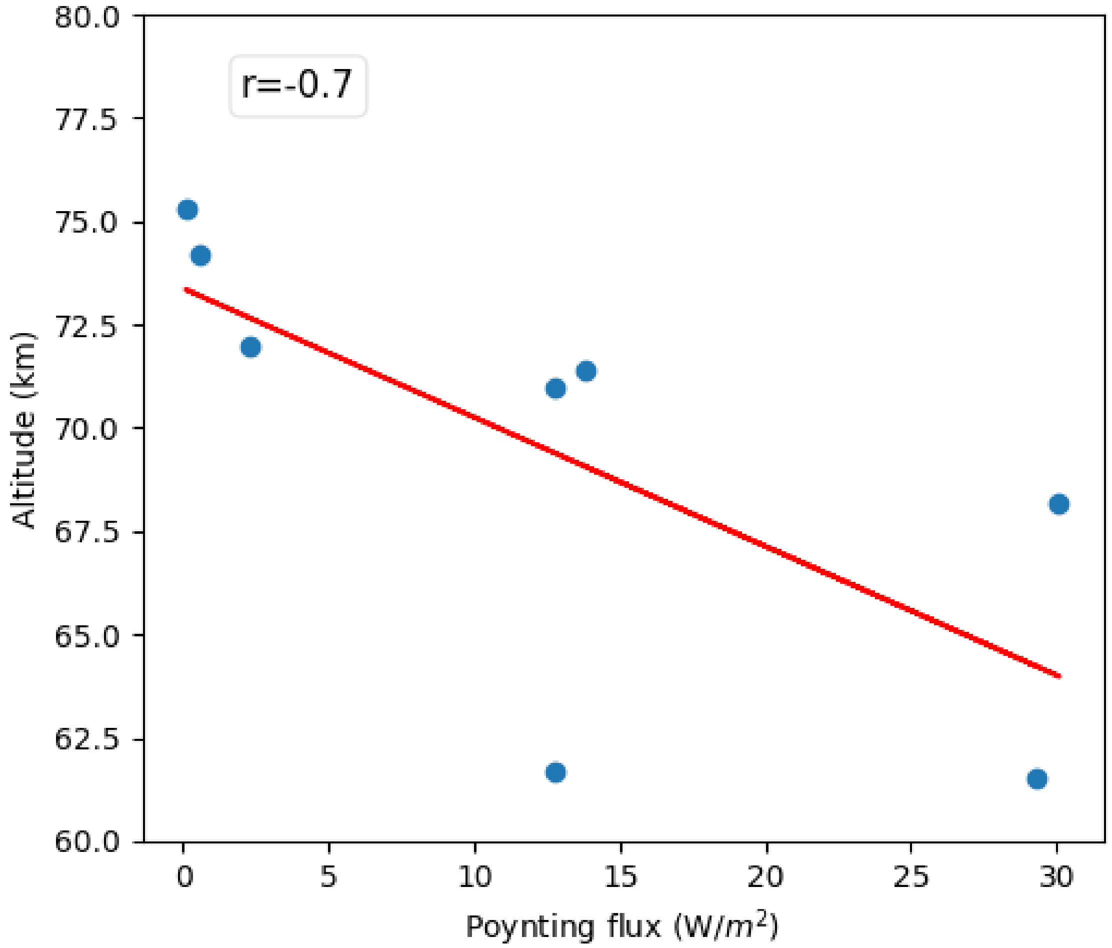

Figure 4. The relationship between the distance/altitude normalised lightning peak apparent Poynting flux associated with the brightest region of sprites and atmospheric altitude. r shows the linear correlation coefficient.

For the same 8 sprites, for the estimates of lightning initial peak Joule heating (see Table 2) associated with the sprites' brightest region, we used the atmospheric conductivity values from Liu et al. (2015), integrated over the vertical extent of the brightest region of the sprites. The vertical extent of the brightest region of sprites varied from 1.2 to 2.4 km. The altitude/distance normalised initial peak Joule heating ranged from 2.6×10^{-5} to 1.4×10^{-2} W/m^2 ($\pm2.4\times10^{-6}$ W/m^2) over the same altitude range (61.5 – 75.3 km). The value in parentheses is the uncertainty. The average peak Joule heating was found to be 4.7×10^{-3} W/m^2. The initial peak Joule heating associated with sprite's brightest region altitude decreases with an increase in atmospheric altitude, with a linear correlation coefficient of -0.4 (not shown). This outcome is expected as the electric field decreases as $\frac{1}{a^3}$ with increasing altitude. A linear correlation coefficient of -0.04 was found for the E_s/E_k and altitude of brightest region of sprites (not shown).

From the 2019 campaign (see Tables 3 and 4), the N_2 red photon flux from the sprites (14 column and 14 carrot sprites) brightest region spanned from 2.5×10^{-8} to 2.7×10^{-7} W/m^2 ($\pm2.5\times10^{-9}$ W/m^2) with an average value of about 1.1×10^{-7} W/m^2, for altitudes between 53 to 69 km. The value in parentheses is the uncertainty. There is no significant linear correlation between the peak photon flux of sprites and atmospheric altitude, with a correlation coefficient of -0.2 (not shown).

The apparent radiated lightning peak electric field at the column and carrot sprites' brightest region altitudes ranged from 59.9 to 479.6 V/m and 22.3 to 438 V/m, respectively, for altitudes between 53.1 and 69 km. The average apparent radiated lightning electric field within the column and carrot sprites were 193.3 and 190.4 V/m, respectively. The sprites associated with radiated lightning electric field lower than the breakdown electric field (120 Td) may be generated due to associative detachment (Luque and Gordillo- Vázquez, 2012). The radiated electric field values are associated with the lightning return stroke at lower frequencies (4 Hz - 2 kHz). For the lightning electric fields in the frequency range 4 Hz to 400 kHz, the apparent radiated lightning peak electric field at the column and carrot sprites' brightest region altitudes varied from 537.4 to 2727.9 V/m and 596.9 to 2477.9 V/m, respectively, for altitudes between 53.1 and 69 km. These values are consistent with the expected quasi-electrostatic field produced by lightning in the mesosphere (Liu et al., 2009a).

The N_2 number density at the altitude of the brightest region of column and carrot sprites (53.1—69 km) vary from 1.67×10^{15} to 1.27×10^{16} cm^{-3}. For column sprites, the E_s at the altitude of the brightest region ranged from 0.6 to 5.8 E_k (4 Hz to 400 kHz) and 0.1 to 0.8 E_k (4 Hz - 2 kHz), respectively. For carrot sprites, the E_s at the altitude of the brightest region varied from 0.6 to 6.6 E_k (4 Hz to 400 kHz) and 0.03 to 1 E_k (4 Hz - 2 kHz), respectively. This is in good agreement with Gamerota et al. (2011), Hu et al. (2007), Kanmae et al. (2012), Li and Cummer (2012), and Qin et al. (2013).

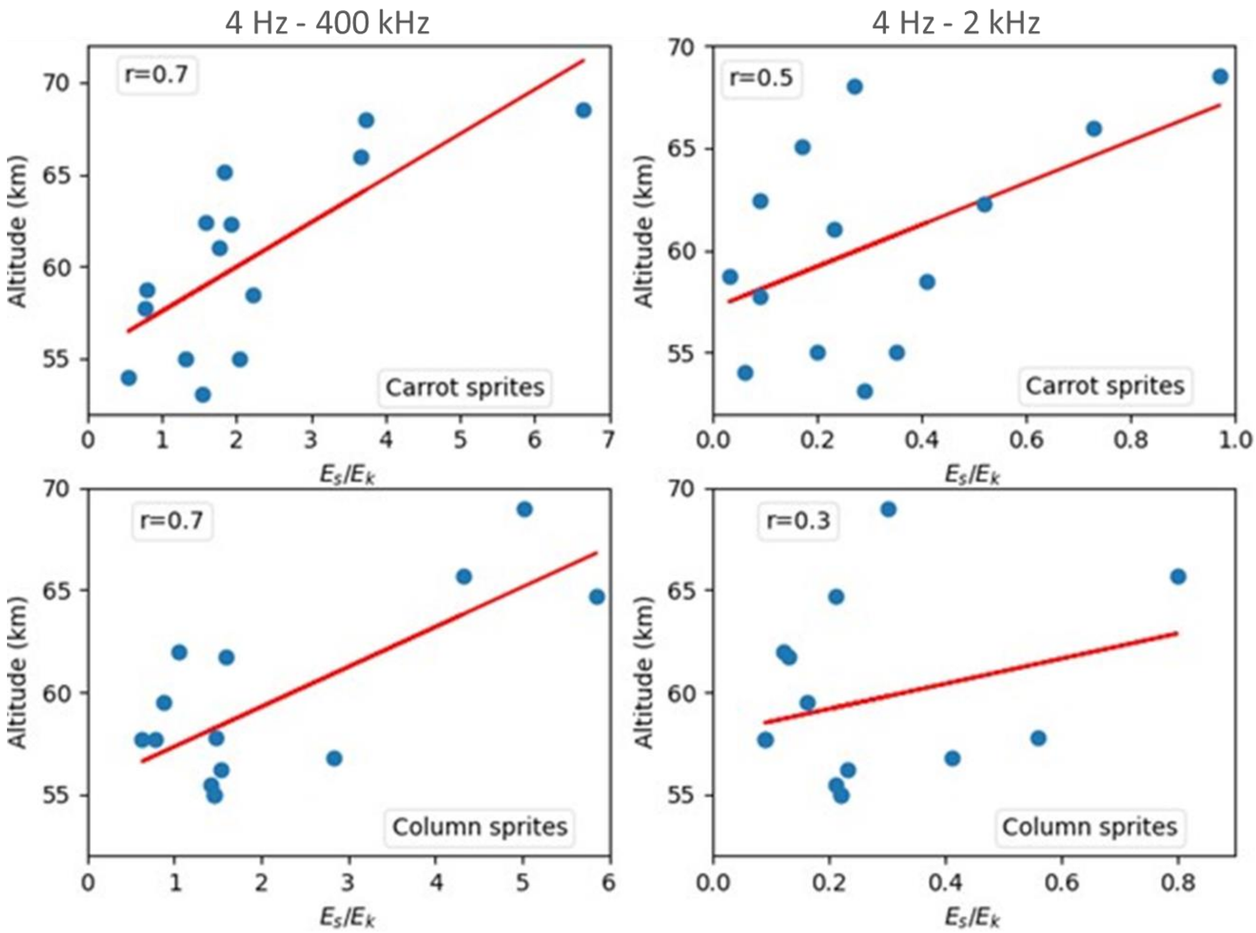

Figure 5. The relationship between the altitude/distance normalised lightning peak electric field over local air breakdown field (E_s/E_k) at the brightest region of column and carrot sprites versus atmospheric altitude. The lightning peak electric fields for the panels on the left panels were obtained in the frequency ranging from 4 Hz to 400 kHz, whereas the panels on the right were obtained in the frequency ranging from 4 Hz - 2 kHz.

Figure 5 shows the relationship between E_s/E_k at the brightest region of column and carrot sprites versus atmospheric altitude. A good linear correlation coefficient of 0.7 and 0.7 were found between E_s/E_k (4 Hz to 400 kHz) associated with column and carrot sprites' brightest region versus the brightest region altitude, respectively, see Figure 5 left panels. A good linear correlation coefficient of 0.5 was found between E_s/E_k (4 Hz - 2 kHz) associated with carrot sprites' brightest region versus the brightest region altitude, whereas a weak linear correlation coefficient of 0.3 was found for column sprites, see Figure 5 right panels. The weak correlation coefficient shows that the lightning return stroke at lower frequencies (4 Hz - 2 kHz) has less influence on the column sprites than on the carrot sprites formation processes.

From the 2020 data, the sprites events which occurred at 19:55:41.681 and 21:41:29.422 UTC had halos with diameters of about 65 and 61 km, respectively, see Tables 1 and 2. The E_s at the altitude of the brightest region of these events were 0.2 and 0.5 E_k (4 Hz to 400 kHz), and 0.1 and 0.4 E_k (4 Hz - 2 kHz), respectively. From the 2019 data, the sprites event which occurred at 19:41:09.728 UTC had a halo with diameter of about 57 km, see Table 3. The E_s at the altitude of the brightest region of this event were 1.5 E_k (4 Hz to 400 kHz) and 0.6 E_k (4 Hz - 2 kHz), respectively. An electric field value of greater than about 0.5 E_k has been associated with sprites halo discharge (Qin et al., 2013; Pasko et al., 2013). Note that halos were not seen by the camera with the red N_2 filter.

The photon flux and lightning initial peak Joule heating associated with the brightest region altitude of 14 column sprites were found to vary from 2.5×10^{-8} to 2.6×10^{-7} W/m^2 ($\pm2.4\times10^{-9}$ W/m^2) and 7.2×10^{-4} to 6.7×10^{-1} W/m^2 ($\pm6.2\times10^{-7}$ W/m^2), with an average value of approximately 8.4×10^{-8} W/m^2 and 7.5×10^{-2} W/m^2, respectively, for altitudes from 55 to 69 km. The photon flux and lightning peak Joule heating associated with 14 carrot sprites at the brightest region altitude ranged from 4.8×10^{-8} to 2.2×10^{-7} W/m^2 ($\pm2.6\times10^{-9}$ W/m^2) and 5.3×10^{-5} to 6.8×10^{-1} W/m^2 ($\pm4.3\times10^{-9}$ W/m^2) with an average value of approximately 1.3×10^{-7} W/m^2 and 9.4×10^{-2} W/m^2, respectively, for altitudes between 53 to 68.5 km. The value in parentheses is the uncertainty. The vertical extent of the brightest region for column and carrot sprites ranged from 0.6 to 2.2 km and 0.6 to 1.4 km, respectively.

To within the timing uncertainty of the video frame (±20 ms), we found that column and carrot sprites have a time delay that varied from about 18 to 27 ms and 30 to 145 ms from their parent lightning strokes, respectively. The time delay between the parent lightning strokes and column (< 30 ms) and carrot sprites (up to 145 ms) agrees with van der Velde et al. (2006). The sprite delay depends on the charge moment change in milliseconds (Cummer and Stanley, 1999).

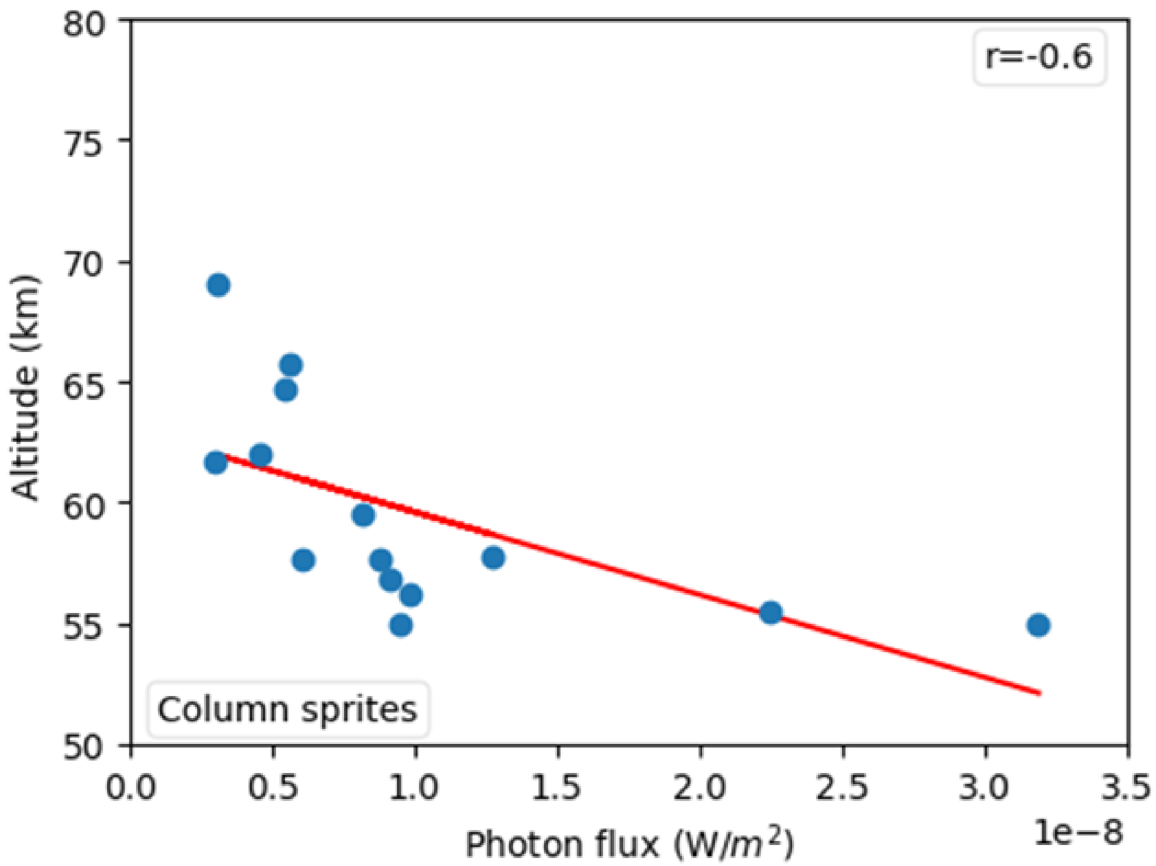

Figure 6. The relationship between the column sprite's photon flux versus sprite's brightest region altitude, respectively. r shows the linear correlation coefficient.

Figure 6 shows the column sprites' photon flux versus the sprites' brightest region altitude. r denotes the linear correlation coefficient. The photon flux out of column sprites' brightest region decreased with an increase in sprites' brightest region altitude, with a linear correlation coefficient of -0.6. The photon flux from carrot sprites' brightest region versus the brightest region altitude showed a weak positive correlation of 0.2 (not shown). A weak-moderate linear correlation coefficient of 0.4 and 0.3 was found between the altitude/distance normalised lightning initial peak Joule heating associated with column and carrot sprites' brightest region versus the brightest region altitude, respectively (not shown).

5. Conclusions

We have estimated the lightning peak apparent Poynting flux and peak initial Joule heating associated with the altitude of the brightest regions of sprites only. We investigated the lightning electric fields in frequencies ranging from 4 Hz to 400 kHz and lightning return stroke at lower frequencies (4 Hz - 2 kHz). The E_s/E_k values (0.1 to 6.6) are comparable with the sprites values from the published literature (Gamerota et al.,2011; Hu et al., 2007; Kanmae et al., 2012; Li and Cummer, 2012; Qin et al, 2013). The lightning discharges have an influence on carrot and column sprites formation processes. The lightning return stroke at lower frequencies (4 Hz - 2 kHz) has more influence on carrot sprites than on column sprites brightest region altitude.

From 2019, the averaged photon flux and lightning initial peak Joule heating associated with the brightest region of column sprites were found to be approximately 8.4×10^{-8} W/m^2 and 7.5×10^{-2} W/m^2, respectively, for altitudes from 55 to 69 km. The averaged photon flux and lightning initial peak Joule heating associated with the brightest region of carrot sprites were found to be approximately 1.3×10^{-7} W/m^2 and 9.4×10^{-2} W/m^2, respectively, for altitudes between 53 to 68.5 km. The column sprites' brightest region altitude had a linear correlation coefficient of -0.6 against the photon flux of the brightest region.

From 2020, the investigation of the altitude/distance normalised lightning peak apparent Poynting flux and initial peak Joule heating associated with the brightest region of sprites yield averaged energy fluxes of 12.7 W/m^2 and 4.7×10^{-3} W/m^2, respectively. A linear correlation coefficient of -0.7 was found between the peak apparent Poynting flux associated with the brightest region of sprites and atmospheric altitude, whereas a correlation coefficient of -0.4 was obtained between the initial peak Joule heating associated with the sprite's brightest region and atmospheric altitude. Our study shows for the first time that the apparent Poynting flux emanating from the sprite dominates over the Joule heating and photon flux by several orders of magnitude.

Our photon flux values are less than Armstrong et al. (1998) by up to 2 orders of magnitude. However, Armstrong et al. (1998) photon fluxes are also less than our Poynting flux and Joule heating of sprites and so consistent with our conclusion. Factors such as local clouds, viewing direction, spatial resolution, atmospheric scattering, light pollution, camera type and setting, distance to sprite, technical approach of measurements and humidity affect camera sensitivity, therefore sprites visibility (Mashao et al., 2022a; Mlynarczyk et al., 2015; Pasko et al., 2013).

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Appendix

Table 1: Summary of estimations of lightning peak apparent Poynting flux at the brightest region altitude of sprites observed from SKA and SAAO sites on 24 January 2020. All CG lightning events are of positive polarity. E_s is the electric field at sprite's brightest region altitude.

Time (UT)	Lightning peak current (kA)	Distance from the receiver to the lightning stroke (km)	Sprite's brightest region altitude (km)	Azimuth angle from the magnetic coil to the lightning stroke (°)	Adjusted Magnetic field at receiver (T)	Magnetic field at 1 km from lightning location (T)	Electric field at 1 km from lightning location (V/m)	N_2 density at sprite's brightest region altitude (cm^{-3})	Electric field at sprite's brightest region altitude (E_s) (V/m)	Magnetic field at sprite's brightest region altitude (T)	Lightning peak apparent Poynting flux at sprite's brightest region altitude (W/m^2)	Es/E_k 4 Hz to 400 kHz	Es/E_k 4 Hz - 2 kHz
19:55:41.682 (SKA)	142	389.9	61.5	99.1	1.8E-09	2.8E-04	7.1E+03	4.4E+15	3.1E+01	1.2E-06	29.3	0.2	0.1
20:00:54.029 (SKA)	101	390.1	61.7	101.1	1.1E-09	1.6E-04	5.4E+03	4.4E+15	2.3E+01	7.0E-07	12.8	0.2	0.04
20:03:13.971 (SKA)	74	431.9	74.2	93.8	5.6E-10	1.0E-04	1.1E+03	8.1E+14	2.8E+00	2.6E-07	0.6	0.1	0.03
20:03:14.307 (SKA)	62	417.9	75.3	101.7	3.0E-10	5.3E-05	4.5E+02	6.9E+14	1.1E+00	1.2E-07	0.1	0.1	0.01
20:07:29.608 (SKA)	92	403.0	72.0	98.9	8.2E-10	1.3E-04	3.0E+03	1.1E+15	8.1E+00	3.6E-07	2.3	0.3	0.1
20:07:29.248 (SKA)	127	449.6	71.0	108.6	1.2E-09	2.3E-04	8.9E+03	1.3E+15	2.5E+01	6.5E-07	12.8	0.5	0.2
20:23:43.573 (SAAO)	66	562.3	68.2	30.9	2.0E-09	6.3E-04	6.0E+03	1.9E+15	1.9E+01	2.0E-06	30.1	0.3	0.1
21:41:29.422 (SKA)	133	209.5	71.4	159.1	2.1E-09	9.2E-05	2.5E+04	1.3E+15	6.9E+01	2.5E-07	13.8	0.5	0.4

Table 2: Summary of estimations of lightning peak initial Joule heating associated with the brightest region altitude of sprites observed from SKA and SAAO sites on 24 January 2020. All CG lightning events are of positive polarity. E_s is the electric field at sprite's brightest region altitude.

Time (UT)	Lightning peak current (kA)	Distance from the receiver to the lightning stroke (km)	Electric field at 1 km from lightning location (V/m)	Sprite's brightest region altitude (km)	Atmospheric conductivity (S/m)	Sprites brightest region vertical extent (km)	Electric field at sprite's brightest region altitude (E_s) (V/m)	Lightning peak initial Joule heating at the sprite's brightest region altitude (W/m^2)
19:55:41.682 (SKA)	142	389.9	7.1E+03	61.5	8.0E-09	1.3	3.1E+01	9.6E-03
20:00:54.029 (SKA)	101	390.1	5.4E+03	61.7	8.0E-09	1.3	2.3E+01	5.6E-03
20:03:13.971 (SKA)	74	431.9	1.1E+03	74.2	1.0E-08	1.5	2.8E+00	1.1E-04
20:03:14.307 (SKA)	62	417.9	4.5E+02	75.3	1.0E-08	2.3	1.1E+00	2.6E-05
20:07:29.608 (SKA)	92	403.0	3.0E+03	72.0	9.8E-08	2.2	8.1E+00	1.4E-02
20:07:29.248 (SKA)	127	449.6	8.9E+03	71.0	1.0E-09	2.4	2.5E+01	1.5E-03
20:23:43.573 (SAAO)	66	562.3	6.0E+03	68.2	3.0E-09	1.4	1.9E+01	1.5E-03
21:41:29.422 (SKA)	133	209.5	2.5E+04	71.4	1.0E-09	1.2	6.9E+01	5.8E-03

Table 3: Summary of estimations of photon flux and lightning peak initial Joule heating at the brightest region of column sprites observed from the SKA site on 01 February 2019. All CG lightning events are of positive polarity. E_s is the electric field at sprite's brightest altitude.

Time (UT)	Lightning peak current (kA)	Distance from the receiver to the lightning stroke (km)	Sprites photon fluxes (W/m^2)	Sprite's brightest region altitude (km)	Sprite's brightest region vertical extent (km)	N_2 density at sprite's brightest region altitude (cm^{-3})	Electric field at sprite's bright region altitude (ELF/VLF/LF radio receiver) (E_s) (V/m)	Lightning peak initial Joule heating at the sprite's brightest region altitude (W/m^2)	Es/E_k 4 Hz to 400 kHz	Es/E_k 4 Hz - 2 kHz
19:05:21.584	50	357.3	2.7E-07	55.0	1.1	1E+16	263.5	1.6E-02	1.4	0.2
19:05:21.584	50	357.3	1.9E-07	55.5	1.1	1E+16	256.4	1.5E-02	1.4	0.2
19:05:21.584	50	357.3	8.0E-08	55.0	1.1	1E+16	263.5	1.6E-02	1.4	0.2
19:05:21.584	50	357.3	8.3E-08	56.2	1.1	9E+15	246.9	1.4E-02	1.5	0.2
19:06:49.098	107	360.4	7.7E-08	56.8	0.6	8E+15	399.0	1.9E-02	2.8	0.4
19:35:00.707	80	334.4	4.7E-08	65.7	2.2	2.6E+15	248.1	6.7E-01	0.8	4.3
19:43:09.728	103	333.6	1.1E-07	57.8	1.1	7.1E+15	479.6	2.5E-02	1.5	0.6
19:47:59.520	64	332.1	6.9E-08	59.5	1.1	6.3E+15	124.6	1.5E-01	0.9	0.2
19:56:54.499	28	345	5.1E-08	57.7	1.1	7.1E+15	80.3	7.2E-04	0.8	0.1
19:56:54.499	28	345	7.4E-08	57.7	1.1	7.1E+15	80.3	7.2E-04	0.6	0.1
19:56:54.499	28	345	3.9E-08	62.0	1.1	4.4E+15	64.7	3.7E-02	1.0	0.1
19:56:54.499	28	345	2.5E-08	61.7	1.1	4.4E+15	65.7	3.8E-02	1.6	0.1
20:12:08.503	41	436.9	2.6E-08	69.0	1.4	1.7E+15	59.9	1E-02	5.0	0.3
20:12:08.503	41	436.9	4.6E-08	64.7	1.4	2.9E+15	72.9	3.7E-02	5.8	0.2

Table 4: Summary of estimations of photon flux and lightning peak initial Joule heating at the brightest region of carrot sprites observed from the SKA site on 01 February 2019. All CG lightning events are of positive polarity. E_s is the electric field at sprite's brightest region altitude.

Time (UT)	Lightning peak current (kA)	Distance from the receiver to the lightning stroke (km)	Sprites photon fluxes (W/m^2)	Sprite's brightest region altitude (km)	Sprites brightest region vertical extent (km)	N_2 density at sprite's brightest region altitude (cm^{-3})	Electric field at sprites bright region altitude (ELF/VLF/LF radioreceiver) (E_s) (V/m)	Lightning peak initial Joule heating at the sprite's brightest region altitude (W/m^2)	Es/E_k 4 Hz to 400 kHz	Es/E_k 4 Hz - 2 kHz
19:06:49.098	107	360.4	1.7E-07	68.5	0.6	2E+15	227.5	9.4E-02	6.7	1
19:08:24.111	92	309.1	1.5E-07	66.0	0.7	2.6E+15	225.2	1.7E-01	3.7	0.7
19:12:42.451	62	351.3	8.6E-08	62.3	1.1	4.4E+15	273.7	6.8E-01	1.9	0.5
19:26:48.376	42	333.1	2.0E-07	65.1	1.1	3E+15	60.7	2.0E-02	1.8	0.2
19:35:00.707	80	334.4	2.2E-07	58.5	1.1	7.1E+15	351.5	1.3E-02	2.2	0.4
19:36:23.568	42	409.4	1.2E-07	55.0	0.9	1E+16	238.1	9.9E-03	1.3	0.2
19:44:48.189	31	338.9	6.9E-08	54.0	0.7	1.1E+16	76.8	8.5E-04	0.6	0.1
19:49:30.929	49	345.9	1.5E-07	61.0	1.1	4.9E+15	136.5	1.7E-01	1.8	0.2
19:51:17.824	29	332	1.3E-07	58.7	1.1	6.3E+15	22.3	5.3E-05	0.8	0.03
19:53:59.597	48	413.9	7.3E-08	62.4	1.3	4.4E+15	46.4	2.3E-02	1.6	0.1
19:56:54.499	28	345	1.5E-07	57.7	1.1	7.1E+15	80.3	7.2E-04	0.8	0.1
19:57:34.604	92	352.4	2.0E-07	55.0	1.1	1E+16	425.7	4.1E-02	2	0.4
20:12:08.503	41	436.9	1.0E-07	68.0	1.4	1.9E+15	62.6	1.7E-02	3.7	0.3
20:38:25.718	68	431.6	4.8E-08	53.0	1.4	1.3E+16	438	8.0E-02	1.5	0.3

Poynting flux estimation calculation

The lightning peak apparent Poynting flux associated with the sprite's brightest region altitude calculation is demonstrated below using the sprites event in Figure 2. The lightning electric field (**E**) and magnetic field at the receivers' location were observed to be 3.6×10^{-2} V/m and 1.06×10^{-9} T, respectively (see Figure 2). The lightning discharge which initiates the sprites event was located 390.1 km away from the receivers' location. The sprite's brightest region altitude was 61.7 km. We only recorded the dominant horizontal component of the magnetic field due to technical issues with the second orthogonal induction coil. We first determine the azimuth angle from the magnetic coil to the lightning stroke in order to calculate the total horizontal magnetic field ($\mathbf{B_T}$) using Equation (A1) below:

$$\mathbf{B_T} = \mathbf{B}/\sin(AZ) \qquad (1A)$$

where *AZ* is the azimuth angle between the lightning position and the North-South coil orientation. *AZ* was found to be 101.1°. By using Equation (A1), we obtain $\mathbf{B_T}$ = 1.08×10^{-9} T.

To normalise **E** and $\mathbf{B_T}$ values to 1 km from the lightning location, we then multiply **E** and $\mathbf{B_T}$ values by the distance (390.1 km) squared in km. We found **E** and $\mathbf{B_T}$ at 1 km from the lightning location to be:

$\mathbf{E} = 5.4 \times 10^{3}$ V/m

$\mathbf{B_T}$=1.6×10^{-4} T

Assuming that the CG lightning charges, which initiated the sprites events, were removed at 10 km altitude, we then project **E** and $\mathbf{B_T}$ at 1 km from the lightning location to the sprite's brightest region altitude (61.7 km) by multiplying **E** and $\mathbf{B_T}$ at 1 km from the lightning location by $(10/r)^3$, where *r* is the altitude (km) of the sprite's brightest region (*r* = 61.7). We obtained **E** and $\mathbf{B_T}$ at the sprite's brightest region altitude to be:

$\mathbf{E} = 2.3 \times 10^{1}$ V/m

$\mathbf{B_T}$=7.0×10^{-7} T

Finally, we estimated the lightning peak apparent Poynting flux (**S**) associated with the sprite's brightest region altitude using Equation (A2) below:

$$\mathbf{S} = \frac{(\mathbf{E} \times \mathbf{B})}{\mu} \quad \text{(A2)}$$

where μ is the permeability of the medium (assumed vacuum). For the event in Figure 2, we found that **S** = 12.8 W/m^2. This calculation is summarised in Table 1 (row 3).

Joule heating estimation calculation

The lightning peak initial Joule heating associated with the sprite's brightest region altitude calculation is demonstrated below using the sprites event in Figure 2. The lightning electric field (**E**) at the receiver's location was obtained to be 3.6×10^{-2} V/m (see Figure 2a). The lightning discharge which initiates the sprites event was located 390.1 km away from the receiver's location. The sprite's brightest region altitude was 61.7 km.

To normalise the **E** value to 1 km from the lightning location, we then multiply the **E** value by the distance (390.1 km) squared in km. We found **E** at 1 km from the lightning location to be:

$\mathbf{E} = 5.4 \times 10^{3}$ V/m

Assuming that the CG lightning charges, which initiated the sprites events, were removed at 10 km altitude, we then project **E** at 1 km from the lightning location to the sprite's brightest region altitude (61.7 km) by multiplying **E** at 1 km from the lightning location by $({}^{10}/_{r})^3$, where *r* is the altitude (km) of the sprite's brightest region (*r* = 61.7). We obtained **E** at the sprite's brightest region altitude to be:

$\mathbf{E} = 2.3 \times 10^{1}$ V/m

We adopted the atmospheric conductivity (**σ**) from Lui et al. (2015), integrated over the vertical extent of the brightest region altitude of the sprite. The vertical extent of the brightest region altitude of sprite was found to be 1.3 km, and the corresponding **σ** was found to be:

$\boldsymbol{\sigma} = 8 \times 10^{-9}$ S/m

To determine the lightning peak initial Joule heating ($\mathbf{J_h}$) associated with the sprite's brightest region altitude, we used Equation (A3) below:

$$\mathbf{J_h} = \boldsymbol{\sigma}\mathbf{E}^2 \quad \text{(A3)}$$

For the event in Figure 2, we found that $\mathbf{J_h} = 5.6 \times 10^{-3}$ W/m^2. This calculation is summarised in Table 2 (row 3).

Photon flux estimation calculation

To determine the photon flux out of the sprite's brightest region, we acquired sprites optical data observed with a red N_2 emission filter (https://www.schott.com/shop/advanced-optics/en/Matt-Filter-Plates/RG645/c/glass-RG645) simultaneously with calibrated stars of known photon flux, as described in Nnadih et al. (2021). The flux values of selected stars can be found in the Pulkovo Spectrophotometry Bright Stars Catalogue (Alekseeva et al., 1996). We demonstrate how we determine the photon flux out of the sprite's brightest region using the sprites event in Figure 3. We averaged the sprites video clip frames over one second in order to determine the average intensity pixel value of the calibrated star ($\mathrm{Str_{avg}}$). The sky background ($\mathrm{Sky_{bkg}}$) value was obtained by using a 7×7 median filter to remove the stars on the video frame prior to the frame which contains the sprites event. The sky background ($\mathrm{Sky_{bkg}}$) value was 3. The calibrated star in Figure 3 was recorded using cameras with red N_2 emission filter that has an absolute photon flux ($\mathrm{Str_{flux}}$) of 0.32058 W/m^2. The photon flux value of the calibrated star was integrated over the filter's longpass wavelength range, see Figure A1. The calibrated star ($\mathrm{Str_{avg}}$) observed with red N_2 emission filter had an average intensity pixel value of 17.1. We average the image pixel values at the sprite's brightest region altitude in order to determine the sprite's average brightness ($\mathrm{Sprs_{avg}}$). The $\mathrm{Sprs_{avg}}$ in Figure 3 had a pixel intensity value of 65.

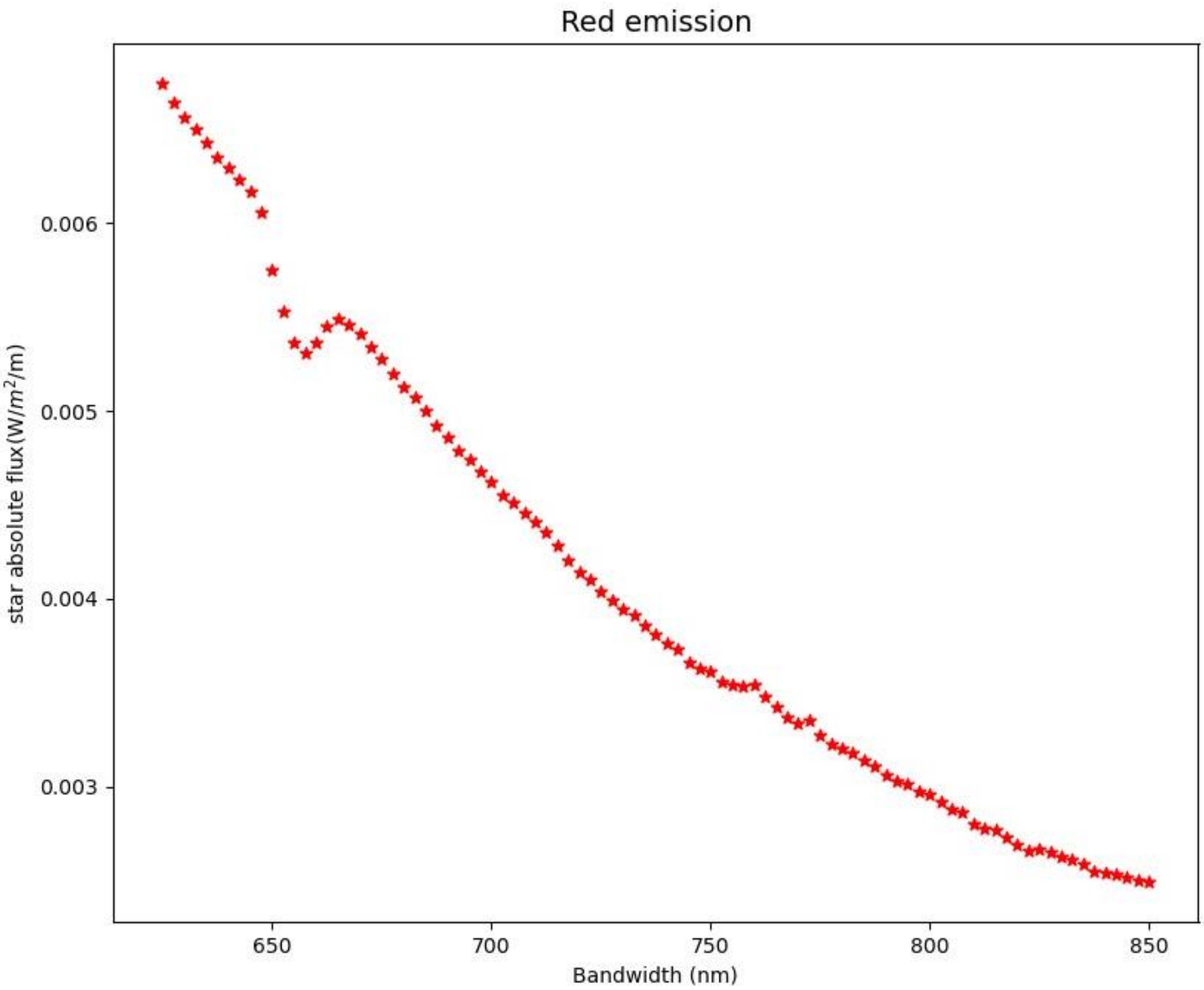

Figure A1. The calibrated star's absolute photon flux over the $N_2{}^1P$ emission range (Alekseeva et al., 1996).

We used data from the camera with a red filter to obtain the photon flux at the sprite's brightest region ($Sprs_{flux} = 1.7 \times 10^{-7}$ W/m^2), by means of Equation (4) (Nnadih et al., 2021).

$$Sprs_{flux} = \frac{Str_{flux}(Sprs_{avg} - Sky_{bkg})}{Str_{avg} - Sky_{bkg}} \text{ (W/m}^2\text{)} \quad \text{(A4)}$$

This calculation is summarised in Table 4 (row 3).

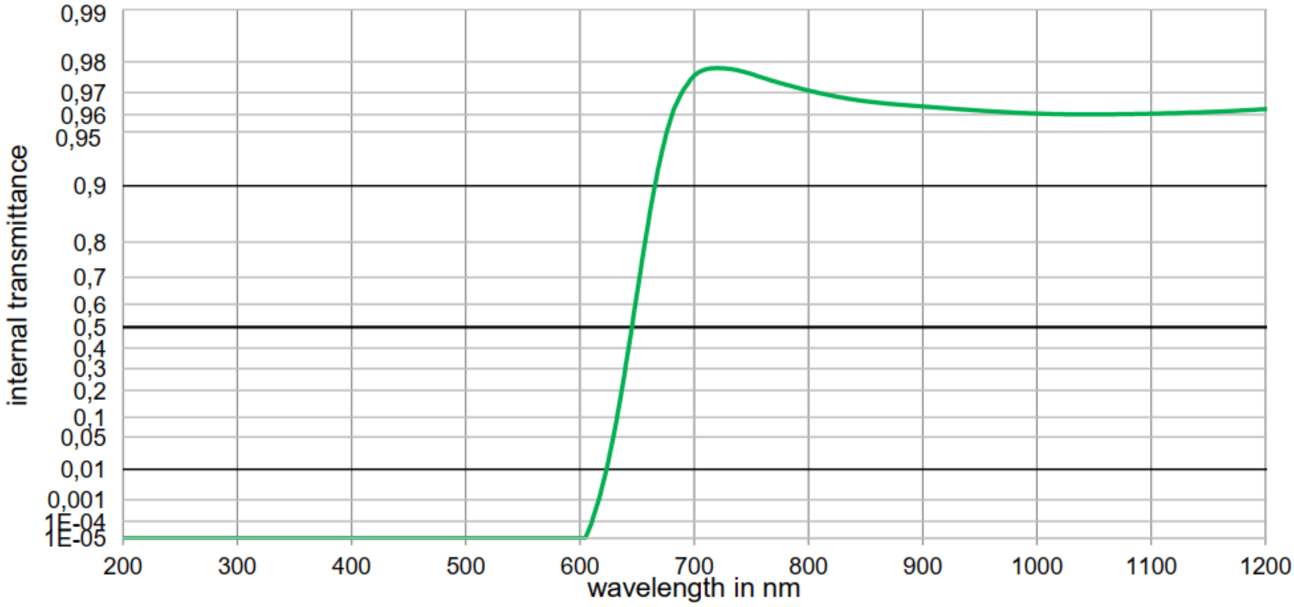

Figure A2. Internal transmittance of red N_2 filter with a 640-650 nm longpass (https://www.schott.com/shop/advanced-optics/en/Matt-Filter-Plates/RG645/c/glass-RG645).