

1980

OPEC versus the West: A Robust Duopoly Situation

Arthur J. Robson

Follow this and additional works at: https://ir.lib.uwo.ca/economicscsier_wp

 Part of the [Economics Commons](#)

Citation of this paper:

Robson, Arthur J.. "OPEC versus the West: A Robust Duopoly Situation." Centre for the Study of International Economic Relations Working Papers, 8001. London, ON: Department of Economics, University of Western Ontario (1980).

ISSN 0228-4235
ISBN 0-7714-0032-2

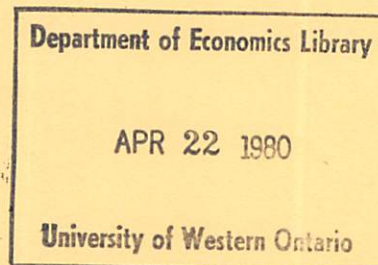
CENTRE FOR THE STUDY OF INTERNATIONAL ECONOMIC RELATIONS

WORKING PAPER NO. 8001

OPEC VERSUS THE WEST: A ROBUST DUOPOLY SITUATION

Arthur J. Robson

This paper contains preliminary findings from research still in progress and should not be quoted without prior approval of the author.



DEPARTMENT OF ECONOMICS
UNIVERSITY OF WESTERN ONTARIO
LONDON, CANADA
N6A 5C2

20028

OPEC VERSUS THE WEST:
A ROBUST DUOPOLY SOLUTION*

Arthur J. Robson
Department of Economics
University of Western Ontario
London, Ontario, Canada

Revised
January 1980

*I would like to thank John McMillan for discussion.

ABSTRACT

A simple two-period model analyzes the optimal exploitation of Western oil reserves in the face of OPEC. The Cournot-Nash equilibrium is equivalent to global competition if it is interior. A familiar elasticity condition then implies the OPEC share of the market rises over time. Whether the equilibrium is interior or not, it is equivalent to that obtained if merely Western oil companies are competitive. It is "robust" in that it is also the outcome if either the West or OPEC is the Stackelberg leader. The model thus provides no strategic justification for non-competitive Western domestic oil supply.

1. Introduction

How should the West react optimally to the existence of OPEC?

There still exist in the West substantial conventional and nonconventional oil reserves. An important aspect of Western strategy is thus the exploitation of these reserves over time. Clearly any Western decision in this regard should account for the actions of OPEC and vice versa.

In order to focus on the above aspect of the international oil market a simple model is constructed. Both OPEC and the West have reserves of oil which is produced at zero (or constant and identical) cost. Consumption, however, takes place solely in the West. (OPEC consumers are neglected as a first approximation.) OPEC receives the market price for oil. (A Western tariff on OPEC oil is thus not considered, although this might actually be an attractive option.) The inherent dynamic nature of the market for oil is modelled by considering two periods. OPEC and the West are explicitly taken to be "monolithic", with each agent facing a decision on how to allocate its fixed amount of oil between the two periods. (This is an "open-loop" strategy.)

Section 2 considers firstly the reaction function of OPEC, derived on the basis of the Cournot-Nash assumption. It is shown that OPEC may respond to an increase in first-period Western output either by increasing or by decreasing its own first period output. However, total first period output rises. Next the objective function of the West is considered. Profits of Western oil companies are not, of course, an adequate measure of Western welfare--a measure of the utility of consumers should also be considered. The Western maximand is thus taken to be the present value of oil rents plus consumer surplus. The Nash reaction function of the

West is then derived. This prescribes that the West lower its own first period output in response to an increase in first period OPEC output. Again, total first period output rises nonetheless.

The properties of the two Nash reaction functions immediately establish the existence and uniqueness of the Nash equilibrium. It is shown that an interior Nash equilibrium is equivalent to global competition, although a determinate allocation of output between OPEC and the West is obtained. OPEC output will actually rise over time if demand is concave. More generally, given a familiar condition on the elasticity of demand, the share of OPEC in the oil market will rise over time. Most corner solutions can then also be ruled out. The only corner solution of interest arises when the West exhausts its oil in the first period but OPEC supplies oil in both periods. In this case, the price of oil rises at a rate lower than the rate of interest. Whether the equilibrium is interior or not, it is equivalent to the outcome where Western oil companies are competitive but OPEC is a monopoly.

Section 3 considers the situation in which either the West or OPEC is sophisticated in the Stackelberg sense. The results are simply stated-- in either case, the Stackelberg outcome is the Nash equilibrium of Section 2. This is true whether the Nash equilibrium is interior or not. Thus a threat by either side in the sense of a precommitted "open-loop" strategy pays no dividend. (See Schelling [4], for a discussion of the general gain from precommitment.) In particular, it follows that the West cannot gain in a strategic sense from noncompetitive domestic supply of oil.

Section 4 discusses a simple case in which demand is linear. Explicit solutions can be obtained for the reaction functions and the Nash-Stackelberg equilibrium. In this case, the West's strategy is "dominant"--that is, best against any OPEC strategy.

Section 5 discusses extensions of the analysis. A continuous-time infinite horizon model has an analogous "open-loop" Nash-Stackelberg equilibrium. This, however, adds only a little and is not presented. "Closed-loop" and "feedback" strategies are also discussed. In the present two period model, the only new equilibrium arising is the Stackelberg "closed-loop" equilibrium. An example of this is given. Finally the problems introduced by more general assumptions regarding extraction costs are noted.

The conclusion stresses the basic points made by the paper.

It should be noted parenthetically that the inclusion of consumer surplus in the objective function of the West is responsible for the divergence between the results of the present paper and other work on imperfect competition in the oil market. (See Salant [3], or Lewis and Schmalensee [1].)

2. Cournot-Nash Behavior

(a) OPEC Reaction Function

Demand in the West is taken to be given by a downward-sloping demand function, taken to be twice differentiable,

$$(1) \quad P = P(S) \quad P'(S) < 0$$

where S is the amount of oil supplied in either of the two periods. (The assumption that demand is identical in the two periods is inessential.) The amounts of oil supplied in each period are then

$$(2) \quad S_A^1 + S_W^1 = S^1 \quad \text{and} \quad S_A^2 + S_W^2 = S^2$$

where

$$(3) \quad S_A^1 + S_A^2 = \bar{S}_A, \quad S_W^1 + S_W^2 = \bar{S}_W$$

where S_A^1 and S_A^2 are the amounts supplied by OPEC in period 1 and period 2, respectively, given a total endowment of \bar{S}_A ; similarly for the West. More generally, inequality might be permitted in (3). However, the conditions under which exhaustion will occur are straightforward and this seems realistic. (Exhaustion is readily seen to be inevitable in infinite horizon models with constant demand.)

Consider now the "open-loop" Nash reaction function for OPEC. (Open-loop strategies specify quantity as a function of time alone.) If the rate of interest is r , the present value of OPEC profits is

$$(4) \quad \pi_A = S_A^1 P(S^1) + \frac{1}{1+r} S_A^2 P(S^2)$$

where (2) and (3) hold. For a maximum of OPEC profits, given the exploitation pattern of the West, the necessary condition is

$$(5) \quad \begin{aligned} &\leq 0 \quad S_A^1 = 0 \\ &\frac{\partial \pi_A}{\partial S_A^1} = 0 \quad 0 < S_A^1 < \bar{S}_A \\ &\geq 0 \quad S_A^1 = \bar{S}_A \end{aligned}$$

where

$$(6) \quad \frac{\partial \pi_A}{\partial S_A^1} = (P(S^1) + S_A^1 P'(S^1)) - \frac{1}{1+r} (P(S^2) + S_A^2 P'(S^2))$$

(The solution for S_A^1 exists by compactness of $[0, \bar{S}_A]$.) This condition has a unique solution if, also,

$$(7) \quad \frac{\partial^2 \pi_A}{(\partial S_A^1)^2} = (2 P'(S^1) + S_A^1 P''(S^1)) + \frac{1}{1+r} (2 P'(S^2) + S_A^2 P''(S^2)) < 0$$

which is ensured if

$$(8) \quad 2 P'(S) + S P''(S) < 0 \quad \forall S \geq 0$$

Consider now the slope of the OPEC reaction function where S_A^1 is interior.

From (6),

$$(9) \quad \frac{dS_A^1}{dS_W^1} = - \frac{(T_1 + T_2)}{(2 T_1 + T_2)} > - 1$$

where the following abbreviations are introduced

$$(10) \quad \begin{aligned} T_1 &= P'(S^1) + \frac{1}{1+r} P'(S^2) < 0 \\ T_2 &= S_A^1 P''(S^1) + \frac{S_A^2}{1+r} P''(S^2) \end{aligned}$$

so that, by (7),

$$(11) \quad 2T_1 + T_2 < 0$$

Hence the complete OPEC Nash reaction function has perhaps the form represented in Figure 1. Note that the reaction of OPEC to an increase in Western output in the first period may be either to increase or to decrease its own first period output. However by (9) total first period output inevitably rises given an interior solution. This property is all that is subsequently required.

(b) Western Reaction Function

Now consider the Nash reaction function of the West. The objective function of the West is taken to include both the profits from Western oil and Western consumer surplus. Thus

$$(12) \quad \pi_W = \left(\int_0^{S^1} P(S) dS - S_A^1 P(S^1) \right) + \frac{1}{1+r} \left(\int_0^{S^2} P(S) dS - S_A^2 P(S^2) \right)$$

The use of consumer's surplus is approximately valid if oil is a relatively small item in Western consumers' budgets. (Similarly the use of a common rate of discount for consumer surplus and profits is approximately valid.) Despite current hysteria over energy, this is not unrealistic. Note that this formulation of the Western objective function assumes explicitly that Western oil companies are run in the "national" interest. It turns out, however, as noted elsewhere, that domestic competition yields equivalent results. Consider now the Nash open-loop reaction function for the West. The necessary condition for π_W to be maximized, given a pattern of OPEC output, is

$$\begin{aligned} &\leq 0 \quad S_W^1 = 0 \\ (13) \quad &\frac{\partial \pi_W}{\partial S_W^1} = 0 \quad 0 < S_W^1 < \bar{S}_W \\ &\geq 0 \quad S_W^1 = \bar{S}_W \end{aligned}$$

where

$$(14) \quad \frac{\partial \pi_W}{\partial S_W^1} = (P(S^1) - S_A^1 P'(S^1)) - \frac{1}{1+r} (P(S^2) - S_A^2 P'(S^2))$$

(Again, existence of a solution for S_W^1 is guaranteed by the compactness of $[0, \bar{S}_W]$.) This condition has a unique solution if

$$(15) \quad \begin{aligned} \frac{\partial^2 \pi_W}{(\partial S_W^1)^2} &= (P''(S^1) - S_A^1 P'''(S^1)) + \frac{1}{1+r} (P''(S^2) - S_A^2 P'''(S^2)) \\ &= T_1 - T_2 < 0 \end{aligned}$$

This is guaranteed if

$$(16) \quad P''(S) \geq 0 \quad \forall S \geq 0$$

Hence sufficiency of both OPEC's and the West's Nash choices is guaranteed by (8) and (16). It seems plausible that the demand curve is convex, since this implies that the rate of reduction in quantity demanded decreases as price rises and the use of oil is restricted to more and more essential uses. (Convexity is not, however, necessary for sufficiency of the Western Nash choice.) Consider now the slope of the Western reaction function when the solution for S_W^1 is interior. From (14) it follows that

$$(17) \quad \frac{dS_W^1}{dS_A^1} = \frac{T_2}{T_1 - T_2}, \quad 0 \geq \frac{dS_W^1}{dS_A^1} > -1$$

since (15) implies $T_1 - T_2 < 0$. Hence the Western reaction function, R^W , say, is as represented in Figure 1. Thus the West always reacts to an increase in OPEC first period output by reducing its own first period output. Again, however, total first period output always rises.

(c) Cournot-Nash Equilibrium

The existence and uniqueness of a Nash equilibrium is immediate from the properties of the reaction functions established above. (Existence follows from continuity and compactness, uniqueness is a consequence of (9) and (17).) The equilibrium might, on the face of it, involve a corner solution for S_W^1 or S_A^1 or both.

Consider firstly the properties of the equilibrium when both S_W^1 and S_A^1 are interior, which is the situation represented in Figure 1. Analytically, from (5), (6), (13) and (14), it follows that

$$(18) \quad P(S^1) = \frac{1}{1+r} P(S^2)$$

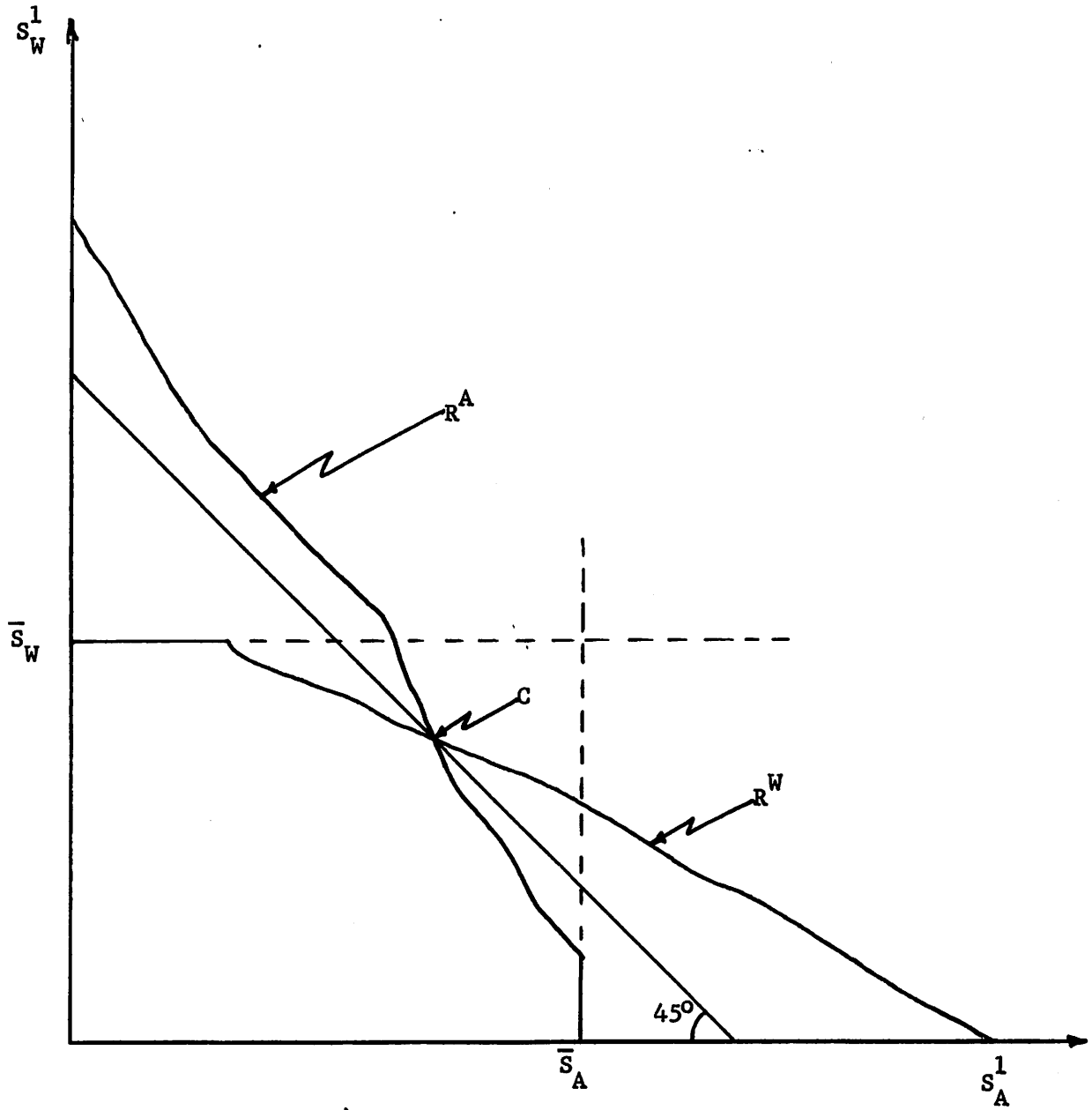


Figure 1

Cournot-Nash Reaction Functions
and Equilibrium

Thus the overall allocation of oil between the two periods is exactly that obtaining under global competition. Thus each side obtains the competitive payoff. Suppose that the values of \bar{S}_W and \bar{S}_A are such that (18) holds. Then it will also hold if only Western oil companies are competitive, despite the monopoly power of OPEC. However, global or domestic competition will in these circumstances lead to an indeterminate allocation of output between OPEC and the West. The Nash equilibrium entails a specific allocation. Indeed, from (5), (6), (13) and (14)

$$(19) \quad S_A^1 P'(S^1) = \frac{1}{1+r} S_A^2 P'(S^2)$$

This has the immediate consequence that if $P''(S) \leq 0$,

$$(20) \quad S_A^1 < S_A^2$$

so that OPEC output can actually rise over time. More generally, from (18) and (19),

$$(21) \quad \frac{S_A^1 / S^1}{S_A^2 / S^2} = \frac{P(S^2)}{S^1 P'(S^1)} \bigg/ \frac{P(S^2)}{S^2 P'(S^2)} \\ = \varepsilon_{S,P}(S^1) / \varepsilon_{S,P}(S^2)$$

where $\varepsilon_{S,P}(S)$ is the elasticity of demand. Thus, if

$$(22) \quad 0 > \varepsilon_{S,P}(S^1) > \varepsilon_{S,P}(S^2), \quad S^1 > S^2$$

then the share of OPEC in the world oil market rises over time. (If, on the other hand, the reverse relationship holds in (22), the share of OPEC declines.) Condition (22) implies that a single monopoly will "overconserve" in the sense of restricting first-period output below the optimal level. (See Stiglitz [6].) This seems to have gained acceptance as a stylized fact about demand. (See Salant [3], or Lewis and Schmalensee [1].) In

the present model, of course, Western oil output must then fall over time.

Consider now the possibility of corner solutions. Firstly, it is shown that (22) rules out any corner solution with $S_W^1 = 0$. For consider Figure 2 in which this corner solution obtains at C. Assuming that a competitive equilibrium exists there must be a "virtual" Nash equilibrium at C', with the competitive overall allocation between periods, but with $S_W^1 \leq 0$. The equilibrium at C' is characterized by (18) and (19). Thus, at C',

$$(23) \quad \frac{S_A^1}{S^1} \geq 1 \geq \frac{S_A^2}{S^2}$$

which contradicts (21) and (22). Hence no Nash equilibrium such as C can exist, given (22).

Now corner solutions with $S_A^1 = 0$ can also be ruled out. For suppose $S_A^1 = 0$. Then (5) and (6) imply

$$(24) \quad P(S^1) \leq \frac{1}{1+r}(P(S^2) + \bar{S}_A P'(S^2))$$

but since $0 < S_W^1 \leq \bar{S}_W$, (13) and (14) imply

$$(25) \quad P(S^1) \geq \frac{1}{1+r}(P(S^2) - \bar{S}_A P'(S^2))$$

contradicting (24) since $P'(S) < 0$.

Finally, it can be shown that $S_A^1 = \bar{S}_A$ only if also $S_W^1 = \bar{S}_W$. For suppose $0 < S_W^1 < \bar{S}_W$ but $S_A^1 = \bar{S}_A$. Then (5) and (6) imply

$$(26) \quad P(S^1) + \bar{S}_A P'(S^1) \geq \frac{1}{1+r} P(S^2)$$

but (13) and (14) imply that

$$(27) \quad P(S^1) - \bar{S}_A P'(S^1) = \frac{1}{1+r} P(S^2)$$

which is, again, a contradiction.

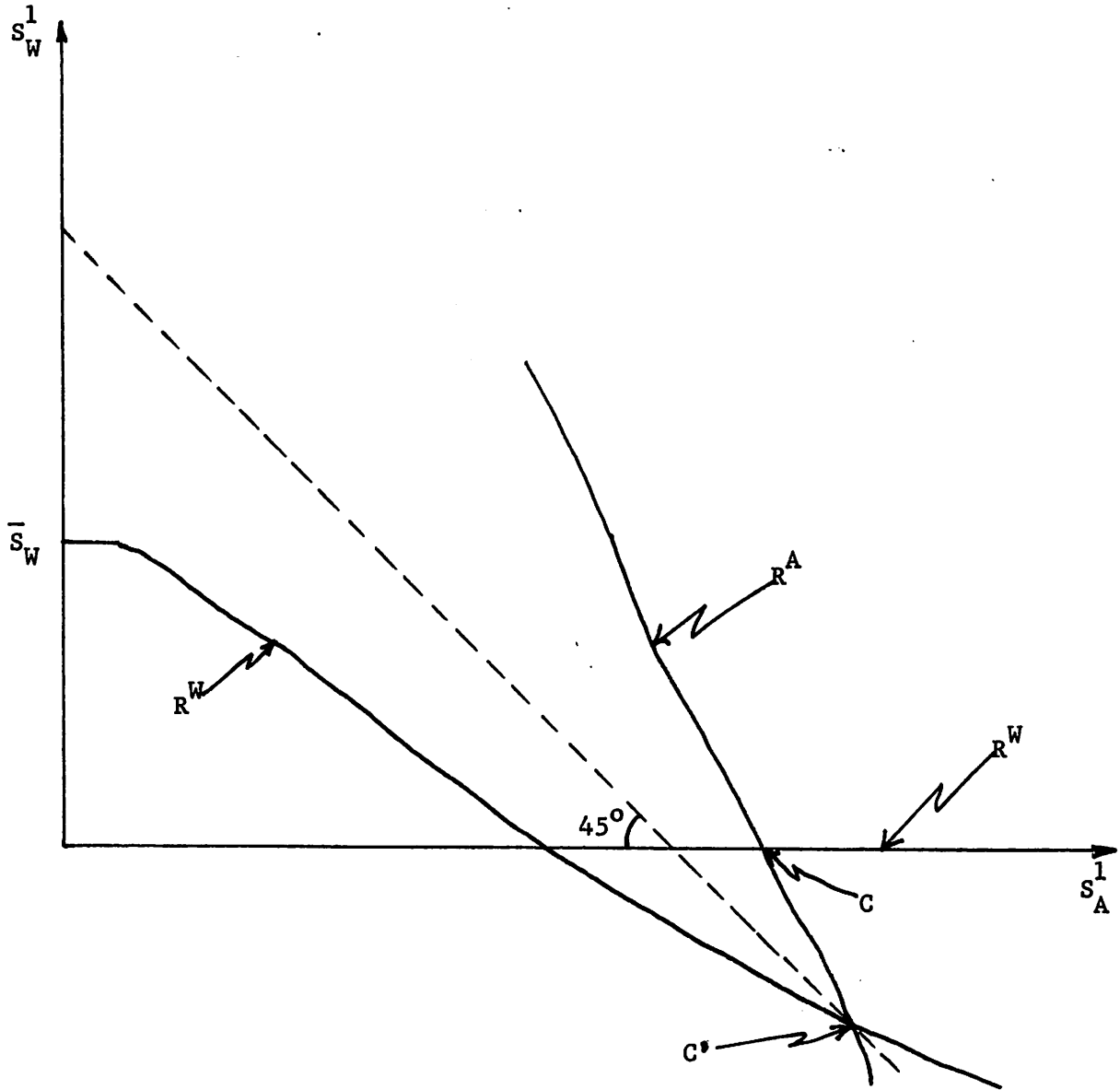


Figure 2

Cournot-Nash Equilibrium

with $S_W^1 = 0$

Hence the only possible corner solutions are $S_W^1 = \bar{S}_W$ and $0 < S_A^1 < \bar{S}_A$ or $S_W^1 = \bar{S}_W$ and $S_A^1 = \bar{S}_A$. The second possibility is readily seen to arise only when the rate of interest is large. It is trivially easy to consider this possibility but it does not seem to hold any interest. The remaining possible corner solution is represented in Figure 3. Now (5), (6), (13) and (14) imply that

$$(28) \quad P(S^1) \geq \frac{P(S^2)}{1+r}$$

so that the price of oil rises at a rate lower than the rate of interest. This is qualitatively the result holding in the monopoly case, given (22).

It is easily shown that this outcome would obtain if Western oil companies were competitive and OPEC were a monopolist. In this case, however, Western oil companies hold too little oil to bring about global competition.

3. Stackelberg Behavior

(a) OPEC Leader

Consider now the outcome if OPEC maximizes its profits subject to the reaction function of the West. This is equivalent to a threat by OPEC to precommit its pattern of exploitation over time. (Thus OPEC plays first.) OPEC's objective criterion is, as before,

$$(29) \quad \pi_A = S_A^1 P(S^1) + \frac{1}{1+r} S_A^2 P(S^2)$$

and the Western reaction function, R^W , is described by

$$(30) \quad \begin{aligned} & \leq 0 & S_W^1 &= 0 \\ & (P(S^1) - S_A^1 P'(S^1)) - \frac{1}{1+r} (P(S^2) - S_A^2 P'(S^2)) = 0 & 0 < S_W^1 < \bar{S}_W \\ & \geq 0 & S_W^1 &= \bar{S}_W \end{aligned}$$

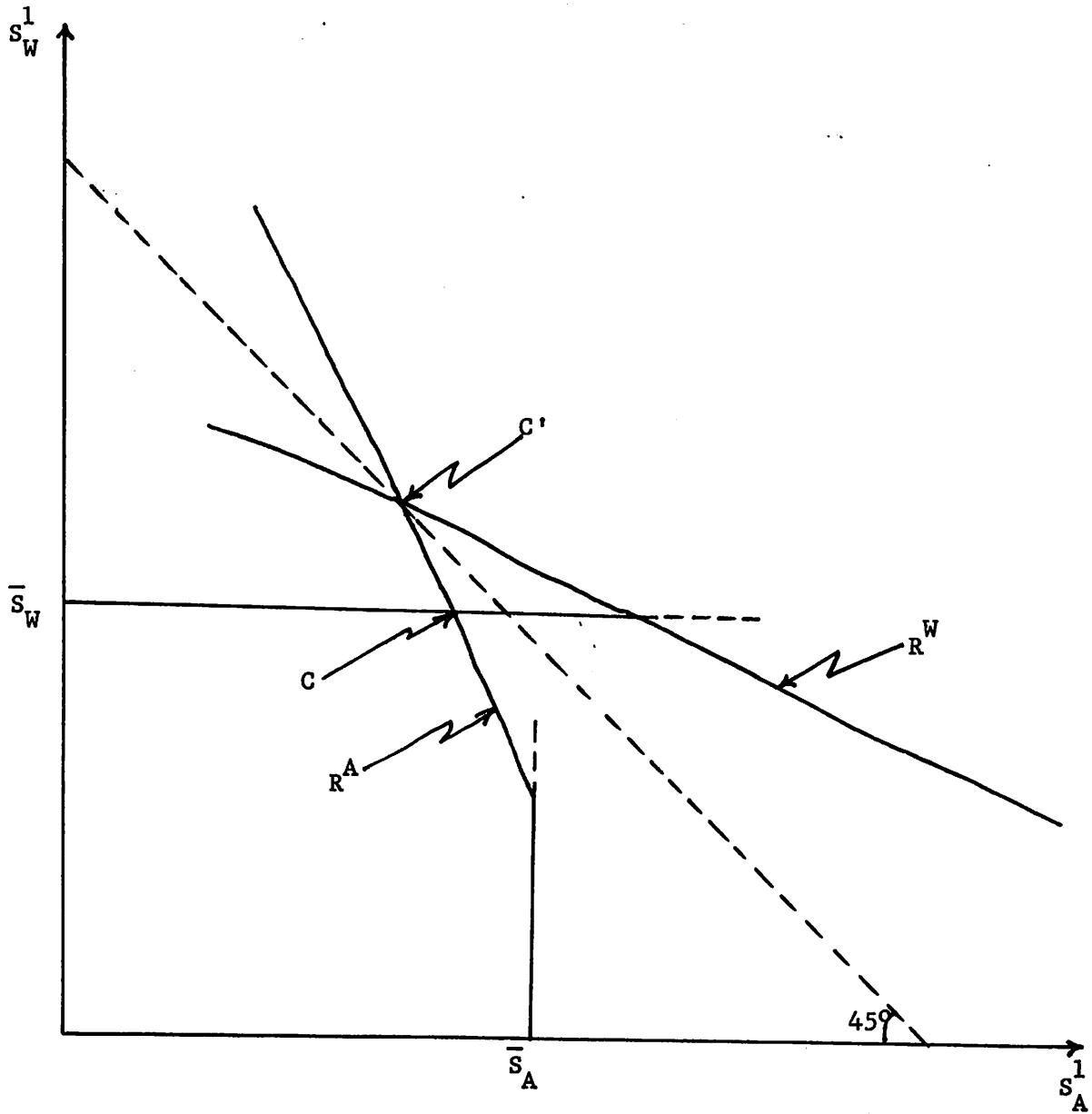


Figure 3

Cournot-Nash Equilibrium

with $S_W^1 = \bar{S}_W$

Suppose firstly that the Nash equilibrium of the previous section occurred at an interior value of S_W^1 . Consider now the derivative of π_A with respect to movement along R^W as parameterized by S_A^1 . Then from (29) and (30),

$$(31) \quad \frac{d\pi_A}{dS_A^1} = [(P(S^1) + S_A^1 P'(S^1)) - \frac{1}{1+r} (P(S^2) + S_A^2 P'(S^2))] \\ + [S_A^1 P'(S^1) - \frac{1}{1+r} S_A^2 P'(S^2)] \frac{dS_W^1}{dS_A^1}$$

Thus, from (18) and (19), at C,

$$(32) \quad \frac{d\pi_A}{dS_A^1} = 0$$

This follows since the Nash equilibrium is a stationary point of π_A . The present method of direct substitution facilitates discussion of sufficiency. (Takayama [7], for example, presents only second-order conditions for the general case using the Lagrange multiplier technique.) For now, using (17),

$$(33) \quad \text{sgn} \frac{d\pi_A}{dS_A^1} = \text{sgn}\{(P(S^1) - \frac{1}{1+r} P(S^2))(T_2 - T_1) - T_1 (S_A^1 P'(S^1) - \frac{1}{1+r} S_A^2 P'(S^2))\}$$

Then, referring to Figure 1, if $(S_A^1) > (S_A^1)^C$, then $S_A^1 + S_W^1 > (S^1)^C$. Hence $P(S^1) < \frac{1}{1+r} P(S^2)$ and from (30) $S_A^1 P'(S^1) < \frac{1}{1+r} S_A^2 P'(S^2) < 0$. Hence

$$(34) \quad \text{sgn} \frac{d\pi_A}{dS_A^1} = \{(-)(+) - (-)(-)\} = (-)$$

Similarly, if $S_A^1 < (S_A^1)^C$,

$$(35) \quad \text{sgn} \frac{d\pi_A}{dS_A^1} = \{(+)(+) - (-)(+)\} = (+)$$

Also given the position of the Nash equilibrium and the OPEC reaction function, R^A , it follows that

$$(36) \quad \begin{aligned} \operatorname{sgn} \frac{d\pi_A}{dS_A^1} &= (+), & S_W^1 &= \bar{S}_W \\ & & & \\ &= (-), & S_W^1 &= 0 \end{aligned}$$

Hence the Nash equilibrium is the global maximum for the Stackelberg problem when OPEC is the leader, given that the Nash equilibrium is interior.

This result is clearly only possible because the Cournot equilibrium is a stationary point for π_A . It is, in fact, a saddle-point. For

$$(37) \quad \begin{aligned} &\frac{\partial^2 \pi_A}{(\partial S_A^1)^2} \cdot \frac{\partial^2 \pi_A}{(\partial S_W^1)^2} - \left(\frac{\partial^2 \pi_A}{\partial S_A^1 \partial S_W^1} \right)^2 \\ &= (2 T_1 + T_2) T_2 - (T_1 + T_2)^2 = - T_1^2 < 0 \end{aligned}$$

This implies, incidentally, that the value of S_W^1 maximizing π_A must be a corner solution.

Consider now the possibility that $S_W^1 = \bar{S}_W$ as in Figure 3. Of course, at C,

$$(38) \quad \frac{\partial \pi_A}{\partial S_A^1} = 0$$

and from (8), when $S_W^1 = \bar{S}_W$

$$(39) \quad \begin{aligned} \operatorname{sgn} \frac{\partial \pi_A}{\partial S_A^1} &= (+) & S_A^1 &< (S_A^1)^C \\ &= (-) & S_A^1 &> (S_A^1)^C \end{aligned}$$

There must exist, in this case also, a "virtual" equilibrium at C', say.

Now along any interior portion of R^W the argument for the interior case can be applied relative to C'. Thus, along such a segment,

$$(40) \quad \text{sgn} \frac{d\pi_A}{dS_A^1} = (-)$$

Finally the position of C^1 and R^A imply that

$$(41) \quad \text{sgn} \frac{\partial \pi_A}{\partial S_A^1} = (-), \quad S_W^1 = 0$$

Hence the Nash equilibrium is the global maximum for the Stackelberg problem when OPEC is sophisticated, even if a corner solution with $S_W^1 = \bar{S}_W$ is involved.

(b) West Leader

Consider now the outcome if the West maximizes its objective criterion subject to the OPEC reaction function. The West's objective is, as before,

$$(42) \quad \pi_W = \left(\int_0^{S^1} P(S) dS - S_A^1 P(S^1) \right) + \frac{1}{1+r} \left(\int_0^{S^2} P(S) dS - S_A^2 P(S^2) \right)$$

and the OPEC reaction function, R^A , is

$$\leq 0 \quad S_A^1 = 0$$

$$(43) \quad (P(S^1) + S_A^1 P'(S^1)) - \frac{1}{1+r} (P(S^2) + S_A^2 P'(S^2)) = 0 \quad 0 < S_A^1 < \bar{S}_A$$

$$\geq 0 \quad S_A^1 = \bar{S}_A$$

Suppose firstly that the Nash equilibrium is interior as in Figure 1. Consider the derivative of π_W with respect to S_W^1 , parameterizing movement along R^A .

If S_A^1 is interior,

$$(44) \quad \frac{d\pi_W}{dS_W^1} = \left[- S_A^1 P'(S^1) + \frac{1}{1+r} S_A^2 P'(S^2) \right] \frac{dS_A^1}{dS_W^1} + \left[(P(S^1) - S_A^1 P'(S^1)) - \frac{1}{1+r} (P(S^2) - S_A^2 P'(S^2)) \right]$$

Again by (18) and (19), the Nash equilibrium is a stationary point for π_W so that C satisfies the first-order condition. Furthermore, by (9),

$$(45) \quad \text{sgn} \frac{d\pi_W}{dS_W^1} = \text{sgn} \left\{ T_1 (S_A^1 P'(S^1) - \frac{S_A^2}{1+r} P'(S^2)) - (2T_1 + T_2) (P(S^1) - \frac{P(S^2)}{1+r}) \right\}$$

Hence, if $S_W^1 > (S_W^1)^C$, then $S^1 > (S^1)^C$ and thus $P(S^1) < \frac{P(S^2)}{1+r}$ and

$$0 > S_A^1 P'(S^1) > \frac{S_A^2 P'(S^2)}{1+r}. \quad \text{Hence}$$

$$(46) \quad \text{sgn} \frac{d\pi_W}{dS_W^1} = \text{sgn} \{ (-)(+) - (-)(-) \} = (-)$$

Similarly, if $S_W^1 < (S_W^1)^C$, then

$$(47) \quad \text{sgn} \frac{d\pi_W}{dS_W^1} = \text{sgn} \{ (-)(-) - (-)(+) \} = (+)$$

Again the position of C and R^W imply that the appropriate signs for the derivative hold on corner values of S_A^1 .

Hence the Nash equilibrium is the global maximum for the Stackelberg problem when the West is the leader, assuming the Nash equilibrium is interior. Again, it can be shown that π_A has a saddle point at the Cournot equilibrium. Thus the value of S_A^1 maximizing π_W must be a corner solution.

Suppose now that $S_W^1 = \bar{S}_W$ at C, as in Figure 3. The previous argument applied relative to C' yields that

$$(48) \quad \text{sgn} \frac{d\pi_W}{dS_W^1} = (+)$$

along any interior portion of R^A . Again the position of C and R^W imply that the derivative has the appropriate sign along any portions of R^A where S_A^1 is a corner solution.

Hence the Nash equilibrium provides the global maximum for the Stackelberg problem when the West is the leader, even if $S_W^1 = \bar{S}_W$.

4. Linear Demand

Suppose now demand in the West is given by

$$(49) \quad P(S) = \alpha - \beta S \quad \alpha, \beta > 0$$

This demand satisfies (8) and (16). The OPEC reaction function as in (5) and (6) is

$$(50) \quad S_A^1 = \frac{\bar{S}_A}{2+r} + \frac{\bar{S}_W}{2(2+r)} - \frac{S_W^1}{2} + \frac{r\alpha}{2(2+r)\beta}$$

assuming S_A^1 is interior. Thus OPEC reduces output in the first period by one half of any Western increase in first period output. The reaction function, R^A , is drawn in Figure 4.

The reaction function of the West, R^W , as in (13) and (14) is

$$(51) \quad S_W^1 = \frac{\bar{S}_W}{2+r} + \frac{r\alpha}{2(2+r)\beta} > 0$$

assuming this is interior. Note that this choice of S_W^1 is a "dominant" strategy--that is, is independent of OPEC's strategy. (See Luce and Raiffa [2].) Solution of (50) and (51) yields, for an interior Nash equilibrium,

$$(52) \quad S_W^1 = \frac{\bar{S}_W}{2+r} + \frac{r\alpha}{(2+r)\beta} > S_W^2 = \frac{1+r}{2+r} \bar{S}_W - \frac{r\alpha}{(2+r)\beta}$$

$$S_A^1 = \frac{\bar{S}_A}{2+r} < S_A^2 = \frac{1+r}{2+r} \bar{S}_A$$

where the first inequality follows if

$$(53) \quad \frac{\bar{S}_W}{2} < \frac{\alpha}{\beta}$$

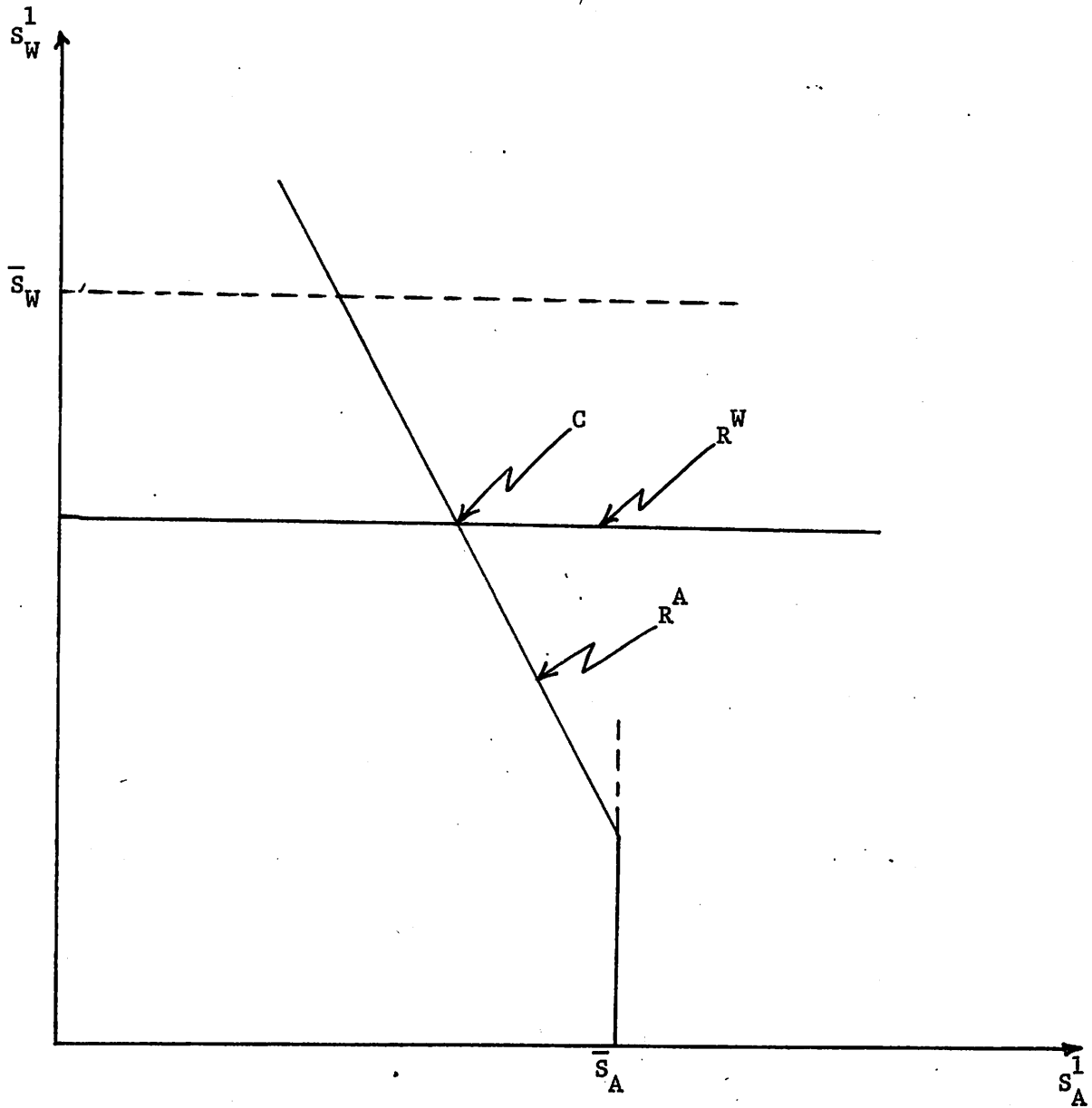


Figure 4

Linear Demand

which is necessary if prices are positive in both periods. The second inequality exemplifies the result in (20) that OPEC output can rise over time. It is readily checked that the price rises at the rate of interest as required in general for an interior solution.

Suppose now $S_W^1 = \bar{S}_W$ so that

$$(54) \quad r\alpha \geq (1+r)\beta\bar{S}_W$$

then the solution for S_A^1 is, if interior

$$(55) \quad S_A^1 = \frac{\bar{S}_A}{2+r} - \frac{1+r}{2(2+r)} \bar{S}_W + \frac{r\alpha}{2(2+r)\beta}$$

Thus $S_A^1 > 0$ by (54). However, $S_A^1 = \bar{S}_A$ if

$$(56) \quad r\alpha \geq (1+r)\beta(\bar{S}_W + 2\bar{S}_A)$$

It can be directly verified that both Stackelberg equilibria reduce to the Nash equilibrium.

5. Extensions

An obvious generalization of the model of the present paper would involve the study of open-loop Nash and Stackelberg equilibria in a continuous time, infinite horizon setting. The author has carried out this analysis but it does not add much additional insight. However, it does make the following apparent--the most general open-loop Nash equilibrium involves three stages. Firstly, there is a stage during which OPEC and the West exploit the resource, and its price rises at the rate of interest. Secondly there may be a stage during which Western oil has been exhausted and OPEC is a monopoly. Thirdly, there may be a stage during which all oil has been exhausted. Once again, it can be shown that the open-loop Nash equilibrium coincides with both the open-loop Stackelberg equilibria.

The open-loop strategies discussed so far do have simplicity militating in their favor. Strategies which make output conditional on previous output choices are "closed-loop" or "feedback" strategies. A closed-loop strategy is optimal from a fixed initial viewpoint. Feedback strategies are required to be consistent, and so can be obtained by a dynamic programming approach. It can be shown that, in general, Nash closed loop and Nash feedback strategies are equivalent. However, Stackelberg closed-loop and feedback equilibria generally differ. (See Simaan and Cruz [5] for greater detail.)

With two periods and fixed resource stocks, consistency simply means each side will exhaust its stock. Thus feedback strategies are identical to the open-loop strategies discussed explicitly. (These are also the closed-loop Nash strategies.) The only possible distinct outcome is then the closed-loop Stackelberg equilibrium. Consider the following example. Suppose demand is linear and the rate of interest is zero. Consider the case where OPEC is the Stackelberg leader. The OPEC objective function is, in the same notation as in Section 4,

$$(57) \quad \pi_A = (\alpha - \beta S^1_A) S^1_A + (\alpha - \beta S^2_A) S^2_A$$

The West will exhaust its oil since OPEC cannot react to the West's second-period output. Consider then the global maximum of π_A over choice of S^1_W and S^1_A . It is easily shown that this obtains where

$$(58) \quad S^1_A = \frac{\bar{S}_A}{2} - \frac{\bar{S}_W}{4}, \quad S^2_A = \frac{\bar{S}_A}{2} + \frac{\bar{S}_W}{4}$$

$$S^1_W = \bar{S}_W, \quad S^2_W = 0$$

(It is assumed that $\bar{S}_A > \frac{1}{2} \bar{S}_W$.) Now consider a closed-loop Stackelberg strategy yielding this outcome. This is

$$\begin{aligned}
 s_A^1 &= \frac{\bar{s}_A}{2} - \frac{\bar{s}_W}{4} \\
 (59) \quad s_A^2 &= \frac{\bar{s}_A}{2} + \frac{\bar{s}_W}{4} \quad \text{if } s_W^1 = \bar{s}_W \\
 &= 0 \quad \text{if } s_W^1 < \bar{s}_W
 \end{aligned}$$

For consider the Western reaction to this precommitted strategy. If

$$(i) \quad s_W^1 = \bar{s}_W$$

then

$$(60) \quad \pi_W^{(i)} = \alpha \bar{s}_W - \frac{1}{2}\beta \bar{s}_W^2 + \frac{1}{2}\beta \left(\frac{\bar{s}_A}{2} - \frac{\bar{s}_W}{4} \right)^2 + \frac{1}{2}\beta \left(\frac{\bar{s}_A}{2} + \frac{\bar{s}_W}{4} \right)^2$$

If, on the other hand,

$$(ii) \quad s_W^1 < \bar{s}_W$$

then

$$(61) \quad \pi_W = \alpha s_W^1 - \frac{1}{2}\beta (s_W^1)^2 + \frac{1}{2}\beta \left(\frac{s_A}{2} - \frac{s_W^1}{4} \right)^2 + \alpha s_W^2 - \frac{1}{2}\beta (s_W^2)^2$$

Hence

$$(62) \quad s_W^1 = s_W^2 = \frac{\bar{s}_W}{2}$$

and

$$(63) \quad \pi_W^{(ii)} = \alpha \bar{s}_W - \frac{1}{4}\beta \bar{s}_W^2 + \frac{1}{2}\beta \left(\frac{\bar{s}_A}{2} - \frac{\bar{s}_W}{4} \right)^2$$

Thus

$$(64) \quad \pi_W^{(i)} - \pi_W^{(ii)} > 0 \quad \text{iff } s_A > \left(\sqrt{2} - \frac{1}{2} \right) \bar{s}_W > \frac{1}{2} \bar{s}_W$$

Thus, assuming this condition, the West will succumb to the OPEC threat. Note that the OPEC threat is not credible, in that it is not in OPEC's interest to actually carry it out.

It can be shown that the West does not have an analogous closed-loop threat in the Stackelberg sense. It could obtain an effective threat if quotas were added to the strategic set.

A final extension would be to consider positive costs of extraction. As long as these are constant and identical between OPEC and the West, the analysis is essentially unchanged. (Price merely must be reinterpreted as demand price net of cost.) If costs are not constant, or differ between OPEC and the West, the simple results of the present paper are jeopardized. This difficulty is, of course, quite typical of oligopoly models. (See Lewis and Schmalensee [1] for application of the standard Cournot model to the market for a depletable resource. Differential costs are discussed, in particular.)

6. Conclusions

The basic result of this paper is that both Stackelberg equilibria and the Nash equilibrium coincide in this model of the world market for a depletable resource. This is only possible because the criterion of each side has a saddlepoint at the Nash equilibrium. Along the reaction function of the other side, the criterion reaches a constrained maximum at this point. This saddlepoint property is reminiscent of two-person zero-sum games, in which the Stackelberg and Nash equilibria also coincide. (See Luce and Raiffa [2], p. 90.) The game of the present paper, while not constant-sum, retains this robustness of equilibrium.

In policy terms, a conclusion of note is that strategic behavior by the West (in either the Nash or Stackelberg sense) is equivalent, in equilibrium, to Western oil companies being competitive. This is true whether

Western oil companies hold enough oil that global competition obtains, or they do not, and deplete their oil in the first period. The model then provides no justification either for government intervention in the domestic oil market or for suspension of domestic antitrust laws.

REFERENCES

- [1] Lewis, T. R. and Schmalensee, R. L. "On Oligopolistic Markets for Nonrenewable Natural Resources," Quarterly Journal of Economics, forthcoming.
- [2] Luce, R. D. and Raiffa, H. Games and Decisions, New York: Wiley, 1957.
- [3] Salant, S. W. "Exhaustible Resources and Industrial Structure: A Nash-Cournot Approach to the World Oil Market," Journal of Political Economy, 84 (1976), 1079-1094.
- [4] Schelling, T. C. The Strategy of Conflict, Cambridge: Harvard University Press, 1960.
- [5] Simaan, M. and Cruz, J. B. "Additional Aspects of the Stackelberg Strategy in Non-Zero Sum Games," Journal of Optimization Theory and Applications, 11 (1973), 613-626.
- [6] Stiglitz, J. E. "Monopoly and the Rate of Extraction of Exhaustible Resources," American Economic Review, 66 (1976), 655-661.
- [7] Takayama, A. Mathematical Economics, Hinsdale, Illinois: Dryden Press, 1974.
- [8] Willig, R. D. "Consumer's Surplus Without Apology," American Economic Review, 66 (1976), 589-597.

CENTRE FOR THE STUDY OF INTERNATIONAL ECONOMIC RELATIONS

University of Western Ontario

Working Papers

- 8001. Robson, Arthur J. OPEC VERSUS THE WEST: A ROBUST DUOPOLY SITUATION
- 8002. McMillan, John and Ewen McCann. WELFARE EFFECTS IN CUSTOMS UNIONS
- 8003. Leith, J. Clark. MONEY, THE BALANCE OF PAYMENTS, AND GOVERNMENT DEBT IN A SMALL OPEN LDC: HAITI
- 8004. Mansur, Ashan and John Whalley. A DECOMPOSITION ALGORITHM FOR GENERAL EQUILIBRIUM COMPUTATION WITH APPLICATION TO INTERNATIONAL TRADE MODELS
- 8005. Schmid, Michael. OIL, EMPLOYMENT AND THE PRICE LEVEL: A MONETARY APPROACH TO THE MACROECONOMICS OF IMPORTED INTERMEDIATE GOODS UNDER FIXED AND FLEXIBLE RATES
- 8006. Markusen, James R. THE DISTRIBUTION OF GAINS FROM BILATERAL TARIFF REDUCTION
- 8007. Markusen, James R. TRADE AND THE GAINS FROM TRADE WITH IMPERFECT COMPETITION
- 8008. Markusen, James R. and James R. Melvin. TRADE, FACTOR PRICES, AND THE GAINS FROM TRADE WITH INCREASING RETURNS TO SCALE