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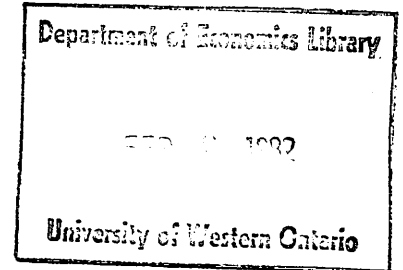
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DYNAMIC NON-SUBSTITUTION AND

LONG-RUN PRODUCTION POSSIBILITIES

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ABSTRACT

Capital is modelled as a produced input whose quantity is determined in an optimal fashion rather than by an arbitrary savings rule or by a time-phased intermediate-goods production structure. The long-run properties of such a model provide an interesting contrast to the alternative dynamic formulations as well as to static Heckscher-Ohlin theory. Despite a "non-substitution theorem" which shows that long-run production possibilities are linear, the model has several long-run properties which are remarkably similar to those characterizing the static Heckscher-Ohlin model, while simultaneously avoiding the technical morass of the traditional two-sector growth models.

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I. Introduction

The names of Heckscher and Ohlin have long been associated with a production model in which all factors are costlessly mobile between sectors yet in which total factor supplies are fixed. There has in recent years been some dissatisfaction with these two assumptions appearing in a model simultaneously. Neary (1978) has argued that some factors are fixed (sector specific) in the short run while factor endowments vary in the long run, leading him to suggest that the Heckscher-Ohlin production model "bears little or no relation to any economically relevant time horizon" (Neary, 1978, p. 509).

As an alternative to the assumption that all factors are mobile, the sector-specific-factors model has been advanced by Jones (1971). Alternatives to the assumption of fixed endowments have been developed by Oniki and Uzawa (1965) and more recently by Ethier (1979), Metcalfe and Steedman (1973), and Brecher and Parker (1977).

Several limitations exist in current treatments of dynamic trade with factor accumulation. Perhaps the most serious problem with the two-sector growth tradition of Oniki and Uzawa is the assumption of a fixed savings rule; that is, a fixed proportion of income is reinvested regardless of the position of the system. Such a specification is not in general consistent with utility maximization and, as we shall show below, many of the Oniki-Uzawa results depend crucially on this assumption.

A related but distinct limitation exists in the Ethier-Metcalfe-Steedman tradition of time-phased intermediate goods.[1] They assume that the rate of interest is free to vary within the production sector or alternatively that it is determined within the production sector alone.

For common utility functions like the one we use below, there is no reason to believe that the steady-state rate of interest derived in the production sector is consistent with a steady-state in the demand side of the model. These two main veins of literature are thus similar insofar as both tend to either ignore the demand sector or to specify it in a highly arbitrary way.

A somewhat less important limitation in both traditions is the restrictive assumption that one or more outputs can be used both as a final consumption good and as a factor of production (referred to either as "capital" or "time-phased intermediate goods"). This assumption was also used in early papers on intermediate goods (Melvin (1968, 1969), Warne (1971)) and is restrictive in the sense that capital and final outputs are joint products of a rather special sort. Alternatively, this approach assumes, for example, that a two-final-good economy has only two production activities which produce three outputs: capital (or time-phased intermediate goods) and the two final goods.

More recent papers on intermediate goods or "middle products" (Sanyal and Jones (1981)) take an alternate approach and assume that intermediate goods and final goods are distinct and produced with different technologies. Recent papers by Manning (1980, 1981) have introduced similar assumptions into dynamic trade models where one factor is produced and accumulated with a distinct technology. This approach includes of course the earlier work as the special case where two production functions are identical. Brecher and Parker also have distinct intermediate inputs, but their formulation unfortunately makes comparisons with static Heckscher-Ohlin models difficult.

The purpose of the present paper is to use the basic Manning model to explore an alternative formulation of the factor accumulation problem. The most important difference between this approach and the two earlier approaches cited above is that we explicitly introduce an intertemporal utility function and view the economy as maximizing its utility stream. Unlike the models of Ethier and others, this ties the rate of interest to demand-side parameters and unlike the model of Oniki and Uzawa allows capital to be accumulated in a systematic rather than in an ad hoc fashion. We are not claiming that our formulation is completely general (e.g., the utility function suffers from well-known limitations) but we would argue that it captures some important considerations not dealt with by earlier treatments.

The next section of the paper develops a two-final-good, two-factor model in which one factor (labour) is in inelastic supply while the stock of the second factor (capital) can be augmented by current production. The model thus has two factors and three non-joint production activities. Whereas Manning (1981) focussed on the dynamic characteristics of the model, this paper will focus on the shape of long-run production possibilities and characteristics of long-run trading equilibria, and then relate them to the properties of the usual static Heckscher-Ohlin production model. Short-run dynamics are treated only briefly in Section IV.

Several results are obtained for the two-factor (one primary factor) case of Sections II to V. A "non-substitution theorem" shows that the long-run production frontier is linear and that the price ratio necessary for diversified production is unique.[2] This result follows from the explicit introduction of demand into the model, and the consequent link between the return to capital and the rate of time preference. The slope

of the long-run production frontier is related to but differs in a systematic way from the static concept of a "Rybczynski line" for reasons developed in the text. Similarly, a positive rate of time preference implies that the supply price ratio is not equal to the slope of the long-run transformation frontier (a result also found in Brecher and Parker).

In spite of the non-substitution result, the model has several long-run properties which are consistent with the well-known results of the static Heckscher-Ohlin model. First, in a long-run trading equilibrium, countries will export the good using intensively the country's relatively abundant factor. Second, a free-trade equilibrium will be characterized by factor price equalization, even if one or both countries are specialized. Third, a somewhat modified version of the Rybczynski theorem holds and changes in endowments have Heckscher-Ohlin effects on the volume of trade. Fourth, tariffs will produce changes similar to those that occur in the Heckscher-Ohlin model if factor movements can occur (Mundell (1957)). Countries will accumulate or deaccumulate capital in response to tariffs until trade has been eliminated. Fifth, the Stolper-Samuelson theorem holds in the short-run though it is rendered irrelevant in the long run simply because the price ratio is unique.

Section VI concludes the analysis by showing briefly how the structure of the model can be summarized in matrix notation and thereby related to the Ethier-Metcalfe-Steedman formulation. This exercise also suggests the more general idea that our optimal capital accumulation formulation amounts to reducing the dimensionality of the production sector. Alternatively, the properties of the long-run production surface are determined by the number of primary factors relative to the number of goods, not by the total number of factors.

II. Long-Run Equilibrium

It is assumed that there are three production activities producing two final goods (X_1 and X_2) and capital (X_3). Each production function uses both labour (L) and capital (K), with each production function character-ized by constant returns, smooth factor substitution, and the other usual neoclassical properties. Total supplies of both factors are fixed at any point in time. Omitting time subscripts for the sake of clarity, these relationships are denoted as follows:

(1) $X_i = F_i(L_i, K_i)$ $i = 1, 2, 3;$

$L_1 + L_2 + L_3 = L;$ $K_1 + K_2 + K_3 = K.$

Throughout the paper, L is assumed fixed though it will hopefully be clear that all the results remain essentially valid if L grows at a constant rate. Change in the capital stock with respect to time is given as follows:

(2) $\dot{K} = F_3(L_3, K_3) - \delta K$ $\dot{K} = dK/dt$

where $\dot{K}$ denotes the time derivative and δ ($0 < \delta < 1$) denotes the depreciation rate.

Although we will be concerned primarily with the production sector of the model in this section, the model is completed with the specification of a utility function and a balance of payments constraint. Letting C_1 and C_2 denote consumption quantities of X_1 and X_2, respectively, and letting p denote the price of X_2 in terms of X_1, we have

(3) $U^* = \int_0^\infty e^{-\rho t} U(C_1, C_2)$

(4) $pX_1 + X_2 = pC_1 + C_2$ $p = p_1/p_2$

where ρ is the usual discount rate. The economy will seek to maximize U^*

subject to (1), (2), and (4) which requires the value of production to

equal the value of consumption at each point in time (exports equal imports).

The Hamiltonian function for (1)-(4) is given as follows.

(5) $\quad H = e^{-\rho t}[U(C_1, C_2) + \lambda_1(K-K_1-K_2-K_3) + \lambda_2(L-L_1-L_2-L_3) +$

$\quad\quad \mu_1(X_1-F_1(L_1,K_1)) + \mu_2(X_2-F_2(L_2,K_2)) +$

$\quad\quad \mu_3(\delta K-F_3(L_3,K_3)) + \sigma(pC_1+C_2-pX_1-X_2)]$

First-order necessary conditions for an interior optimum follow directly.

(6) $\quad \dfrac{\partial H}{\partial K_i} = - \lambda_1 - \mu_i F_{ik} = 0 \quad\quad\quad\quad\quad\quad i=1,2,3$

(7) $\quad \dfrac{\partial H}{\partial L_i} = - \lambda_2 - \mu_i F_{i\ell} = 0$

(8) $\quad \dfrac{\partial H}{\partial C_i} = U_i + \sigma p_i = 0 \quad\quad\quad\quad\quad\quad\quad i=1,2$

(9) $\quad \dfrac{\partial H}{\partial X_i} = \mu_i - \sigma p_i = 0 \quad\quad\quad\quad\quad\quad\quad p_1 = p, \; p_2 = 1$

(10) $\quad \dfrac{\partial H}{\partial K} = \dfrac{d(e^{-\rho t}\mu_3)}{dt}$

plus the first-order conditions relating to the constraints. The left- and

right-hand sides of (10) can be rewritten as

(11) $\quad e^{-\rho t}(\lambda_1+\mu_3\delta) = \dot{\mu}_3 e^{-\rho t} - \rho e^{-\rho t}\mu_3$.

Setting $\dot{\mu}_3 = d\mu_3/dt$ equal to zero in the steady state, we have

(12) $\quad e^{-\rho t}(\lambda_1+\mu_3(\delta+\rho)) = 0$.

Replacing λ_1 in (12) with $-\mu_3 F_{3k}$ from (6), equation (12) becomes

(13) $\quad F_{3k} = (\delta+\rho)$.

In the steady state, the marginal product of capital in the production of

capital is equal to the (constant) sum of the depreciation rate (δ) and the

discount rate (ρ). It is well known that constant returns implies that marginal products depend only on the ratio of inputs. Since (13) "pins down" F_{3k}, it also pins down $k_3 = K_3/L_3$ and therefore also determines $F_{3\ell}$ in the steady state.

The first-order conditions in (6) and (7) combined with competitive pricing imply

$$(14) \qquad \frac{F_{i\ell}}{F_{ik}} = \frac{\lambda_2}{\lambda_1} = \frac{w}{r} \qquad\qquad i=1,2,3$$

where w and r are respectively the wage and rental rates (in terms of X_2) in competitive equilibrium. The steady-state condition (13) thus determines a unique wage-rental ratio and under the usual assumptions unique capital labour ratios in all industries.

The first-order conditions in (6) through (9) also imply

$$(15) \qquad \frac{U_1}{U_2} = p = \frac{\mu_1}{\mu_2} = \frac{F_{2\ell}}{F_{1\ell}} = \frac{F_{2k}}{F_{1k}}$$

The fact that the F_{ij}'s are fixed by (13) and (14) implies that there will be a unique price ratio associated with a non-specialized equilibrium. In a trading context, this will, of course, be true for each country provided that they have identical technologies and discount rates (note from (13) that p will depend on ρ).[3]

III. Long-Run Production Possibilities

The results of equations (13)-(15) do not imply that the long-run production frontier is linear nor do they imply that the price ratio is equal to the long-run MRT. This section will investigate both these questions as well as the question of how long-run production possibilities relate to the static Heckscher-Ohlin production frontier.

Differentiating the production functions for X_1 and X_2 we have

(16) $dX_2 = F_{2\ell}(- dL_1 - dL_3) + F_{2k}(dK - dK_1 - dK_3)$

   $dX_1 = F_{1\ell}dL_1 + F_{1k}dK_1$

since $dL_2 = -dL_1 - dL_3$ and $dK_2 = - dK_1 - dK_3 + dK$. Dividing the two equations in (16) gives us the MRT:

(17) $- \dfrac{dX_2}{dX_1} = \dfrac{F_{2\ell}dL_1 + F_{2k}dK_1}{F_{1\ell}dL_1 + F_{1k}dK_1} + \dfrac{F_{2\ell}dL_3 + F_{2k}(dK_3 - dK)}{F_{1\ell}dL_1 + F_{1k}dK_1}$

Noting from (15) that $F_{1i} = F_{2i}/p$, the first of the two quotients on the right-hand side of (17) can be rewritten as follows:

(18) $\dfrac{F_{2\ell}dL_1 + F_{2k}dK_1}{F_{1\ell}dL_1 + F_{1k}dK_1} = p \dfrac{F_{2\ell} + F_{2k}(dK_1/dL_1)}{F_{2\ell} + F_{2k}(dK_1/dL_1)} = p \;.$

Equation (17) can then be rewritten as follows by substituting in (18) and factoring (dL_3/dL_1) out of the second right-hand term in (17).

(19) $- \dfrac{dX_2}{dX_1} = p + p[\dfrac{F_{2\ell} + F_{2k}(dK_3 - dK)/dL_3}{F_{2\ell} + F_{2k}(dK_1/dL_1)} \dfrac{dL_3}{dL_1}]$

The MRT thus equals the price ratio plus a rather formidable looking term. In long-run equilibrium with p taking on the unique value for diversified production discussed above, it is however easy to show that this term is both constant and positive (negative) if X_2 is capital intensive ·(labour intensive) relative to X_1.

The first step is to recall that in long-run equilibrium the F_{ij}'s are constant in (19) as are the $dK_i/dL_i = k_i$. To show that (19) is a constant thus requires us only to show that dK/dL_3 and dL_3/dL_1 are constant. The steady state requires that $\delta K = F_3(L_3, K_3)$ and therefore that

$$(20) \quad \delta \frac{dK}{dL_3} = F_{3\ell} + F_{3k} \frac{dK_3}{dL_3} = F_{3\ell} + F_{3k}k_3$$

which from earlier results implies that dK/dL_3 is therefore a constant.

The resource constraints in (1) can be expressed as

$$(21) \quad L_1 + L_2 + L_3 = L$$

$$k_1 L_1 + k_2 L_2 + K_3 = K \qquad k_i = K_i/L_i$$

Differentiating, we have

$$(22) \quad \begin{bmatrix} 1 & 1 \\ k_1 & k_2 \end{bmatrix} \begin{bmatrix} dL_1 \\ dL_2 \end{bmatrix} = \begin{bmatrix} -dL_3 \\ dK - dK_3 \end{bmatrix} .$$

Since the k_i's are constant in the steady state, (22) will give us

$$(23) \quad \frac{dL_3}{dL_1} = \frac{(k_2 - k_1)}{- k_2 - (dK - dK_3)/dL_3} .$$

Since all the terms on the right-hand side of (23) are constant in the steady state, (23) is a constant and thus so is the long-run MRT in (19). As shown below $(dK - dK_3)/dL_3$ must be positive: changes in the long-run capital stock must exceed the change in capital used to produce it $(dK > dK_3)$. (23) is thus negative if X_2 is the capital intensive good, an assumption we will make for the remainder of the paper $(k_2 > k_1)$.

The last piece of the puzzle is simply to show that $(-dX_2/dX_1) > p$

given our assumption that X_2 is capital intensive. Since we have just

shown that $dL_3/dL_1 < 0$, (19) only requires us to show that

$$(24) \qquad F_{2\ell} + F_{2k}\left(\frac{dK_3-dK}{dL_3}\right) < 0, \quad \text{or} \quad \frac{F_{2\ell}}{F_{2k}} = \frac{F_{3\ell}}{F_{3k}} < \frac{dK-dK_3}{dL_3}$$

From (20), we have

$$(25) \qquad \frac{dK}{dL_3} - \frac{dK_3}{dL_3} = \frac{F_{3\ell}}{\delta} + \frac{F_{3k}k_3}{\delta} - k_3 = \frac{F_{3\ell}}{\delta} + \frac{\rho}{\delta}k_3$$

since $F_{3k} = (\delta+\rho)$ from the steady-state condition in (13). The right-hand

inequality in (24) can then be rewritten as

$$(26) \qquad \frac{F_{3\ell}}{F_{3k}} = \frac{F_{3\ell}}{(\delta+\rho)} < \frac{F_{3\ell}}{\delta} + \frac{\rho}{\delta}k_3$$

which obviously holds.[4] Thus given the results from (23) and (24), the bracketed

term in (19) is positive for $k_2 > k_1$. In a steady state, we have

$$(27) \qquad -\frac{dX_2}{dX_1} = \text{constant, with} \quad -\frac{dX_2}{dX_1} > p \text{ iff } k_2 > k_1$$

Figure 1 shows a long-run equilibrium for a closed economy. LL' is the

long-run production frontier, and p the long-run price ratio. The equilibrium

(point E) occurs at a point where p equals the marginal rate of substitution

along an indifference curve, denoted $\bar{U}$ in Figure 1. The assumption that demand can

be represented by a homothetic community utility function should be sufficient

for E to be unique.

It will prove useful to define the "Heckscher-Ohlin production frontier"

through E as the locus of final outputs which can be produced from the same

bundle of resources that is allocated to final goods production at E (i.e.,

a locus of (X_2,X_1) such that $L_1 + L_2$ and $K_1 + K_2$ take on the same values as

FIGURE 1
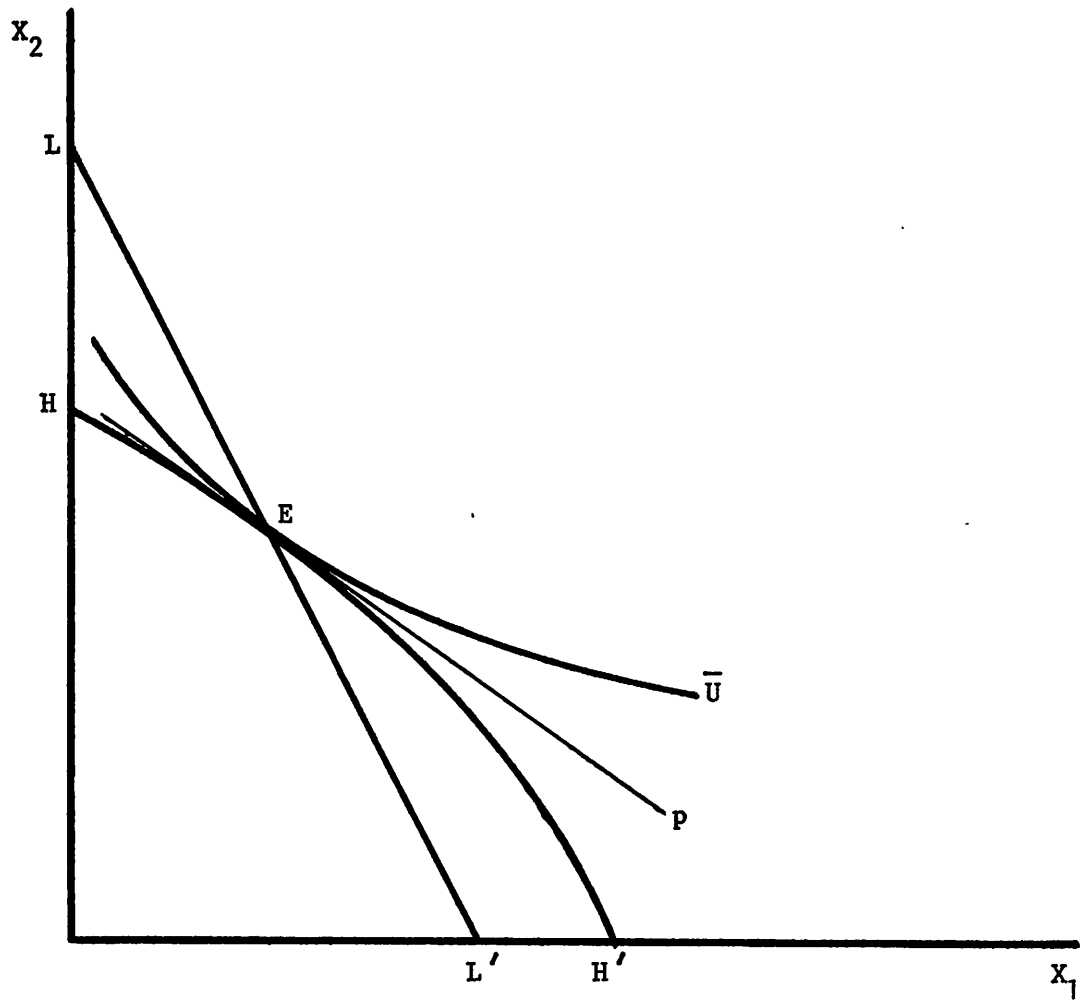
they do at E). This Heckscher-Ohlin production frontier is shown by HH'
in Figure 1 and must be tangent to p at E. With $dL_2 = -dL_1$ and $dK_2 = -dK_1$,
this tangency result follows from equations (16)-(18) which imply that
$-dX_2/dX_1 = p$ as shown in Figure 1. This Heckscher-Ohlin frontier and
shifts in it will of course be characterized the well-known Stolper-
Samuelson and Rybczynski properties.

We should be quick to point out that a disturbance such as a demand
shift will not move the economy along HH' even in the very short run when
K is fixed. The reason is that such a disturbance will reallocate some
of the (instantaneously) fixed endowment into or out of X_3 and thus change
the total allocation available for final goods production. Nevertheless,
HH' will prove useful in analyzing short-run dynamics and it is to this
problem that we now turn.

IV. A Note on Short-Run Adjustments

Manning (1981) was principally concerned with dynamic adjustment in
this type of model whereas the present paper is more concerned with long-
run properties and their relation to static Heckscher-Ohlin theory. Since
Manning used a phase-diagram approach, a detour here into short-run dynamics
using the output diagrams of the previous section may be of some use. We
can in particular show how the static Heckscher-Ohlin production frontier of
the previous section and the static concept of a Rybczynski line put bounds
on the short-run outputs of final goods that might be observed following
a disturbance from an initial long-run equilibrium.

Let E in Figure 2 denote a long-run equilibrium as discussed in the
previous section with LL' and HH' defining the corresponding long-run and
Heckscher-Ohlin production frontiers. Define a "Rybczynski line" through
E with respect to capital stock changes as the locus of final outputs for
different levels of K holding (A) the total labour allocation to final
goods production constant and (B) the MRT (or price ratio) along the
Heckscher-Ohlin frontier constant and equal to that at E. This Rybczynski line
is shown by RR' and is formed by the points of equal MRT along all the
Heckscher-Ohlin production frontiers, each formed by the same value for
$L_1 + L_2$ but a different value for $K_1 + K_2$.

The Rybczynski theorem tells us that RR' will be steeper than HH'
in Figure 2 given that X_2 is relatively capital intensive. We can also
show that RR' is steeper than LL'. The intuition behind this result is
that a movement from E up RR' increases the capital available to final
goods production while, by definition, leaving the labour allocation to

FIGURE 2
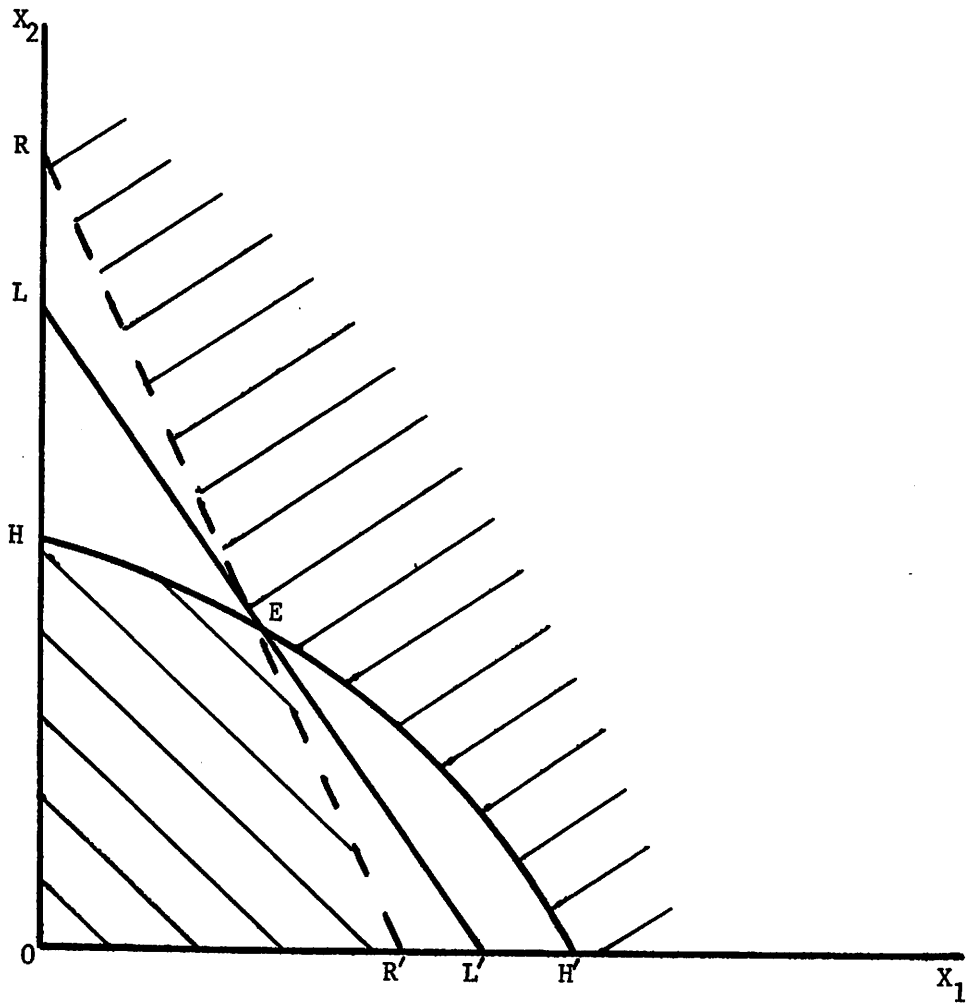
final goods constant. Movements from E up LL′ on the other hand require higher levels of (L_3, K_3) to sustain the higher capital stock, and thus labour is withdrawn from final goods production ($L_1 + L_2$ decreases) as we move from E toward L in Figure 2. With X_2 capital intensive, this withdrawl of labour shifts production more towards X_2 than is found along the Rybczynski line RR′.

Technically, this result can be seen by breaking equation (19) up into three terms.

$$(28) \qquad -\frac{dX_2}{dX_1} = p - p \frac{F_{2k}(dK/dL_1)}{F_{2\ell} + F_{2k}(dK_1/dL_1)} + p \frac{F_{2\ell}dL_3 + F_{2k}dK_3}{F_{2\ell}dL_1 + F_{2k}dK_1}.$$

At any point in time, the first two additive terms on the right-hand side of (28) give the MRT along the Rybczynski line ($dK_3 = dL_3 = 0$) while all three terms equal the slope of LL′. Assumptions of the model imply that

$$(29) \qquad p \frac{F_{2k}(dK/dL_1)}{F_{2\ell} + F_{2k}(dK_1/dL_1)} < 0 \quad \text{and} \quad p \frac{F_{2\ell}dL_3 + F_{2k}dK_3}{F_{2\ell}dL_1 + F_{2k}dK_1} < 0,$$

since dL_1 and dK_1 are negatively related to dK and to dL_3 and dK_3 as shown earlier. Thus the slopes of RR′ and LL′ are related by

$$(30) \qquad -[\frac{dX_2}{dX_1}]_R = p - p \frac{F_{2k}(dK/dL_1)}{F_{2\ell} + F_{2k}(dK_1/dL_1)} > -[\frac{dX_2}{dX_1}]_L$$

where $-[dX_2/dX_1]_L$ is the long-run MRT given in (19) and (28).

Beginning at E in Figure 2, suppose that p decreases due to some demand shift. With this increase in the relative price of the capital intensive good, the price and rental rate of capital will rise and resources will be diverted into X_3 in order to produce more K (Manning (1981)). Immediately following the price changes instantaneous outputs of X_1 and X_2 must be in the hatched region OHER′ in Figure 2. The fact that some resources will be

withdrawn from final goods production implies that final outputs are bounded from above by HH' in Figure 2. The region of possible outputs can be narrowed down further by supposing that X_3 uses only capital, so that only capital is withdrawn from final goods production. In this case, the point on the new Heckscher-Ohlin production frontier with the same slope as that at E will lie along the Rybczynski segment ER'. But since this whole process was started by an increase in the relative price of X_2, the new short-run outputs must lie to the left of ER' where the MRT exceeds that along ER'. Any labour withdrawn from final goods production will strengthen this initial shift to the left of RR' via the corresponding type of Rybczynski effect. It can similarly be argued that an increase in p will generate instantaneous outputs in the hatched region to the northeast of REH' in Figure 1.

While this analysis does place some limitations on the possible short-run responses to price changes, it does unfortunately still leave a great deal of indeterminacy. We can, however, conclude that in the short run the model shares some properties of the Heckscher-Ohlin model but not others. First, an increase in the price of the capital intensive good will increase the real rental rate and decrease the real wage rate so that the Stolper-Samuelson theorem remains valid. (We have not proved this here since it follows directly from Manning (1981).) On the other hand, outputs in the very short run may not respond in the usual Heckscher-Ohlin manner. With resources being withdrawn from final goods production, the output of the capital intensive good may fall and even fall relative to the labour intensive good if the production function for capital is very capital intensive. Such a problem does not arise in the long run, which forms the subject of the next section.

V.   Long-Run Trading Equilibria

It is probably obvious that if the above model applies to both of two
countries and if they have in addition identical, homothetic demand structures,
then there will be no incentive to trade provided that they are initially in
long-run equilibrium. The "non-substitution" results of the previous section
imply that in such a situation, commodity prices are equalized between
countries without trade. In order to get a fully determinant pattern of
trade we would need another primary factor, but then we would have left the
Heckscher-Ohlin world and comparisons such as those of the previous section
would have little meaning (Ethier (1979)).

Manning (1981) notes that a determinant trade pattern can exist with
demand side differences and in particular differences in rates of time
preference. With the capital-market condition requiring $F_{3k} = (\delta + \rho)$, it should
be apparent that the slope of the long-run production frontier will depend
on ρ. With X_2 capital intensive, the country with the lower rate of time
preference will have the steeper production frontier and must specialize
in the capital-intensive good (Manning (1981)). Since the low-time preference
country will also have the higher capital/labour ratio, this implies that each
country will export the good using intensively the country's abundant factor.

We would like to make the added comment here that positive long-run
trade can exist if we dispense with the contrived view that countries are
initially in long-run autarky equilibrium and then all of a sudden trade is
permitted. If instead trade was initiated when one or both countries were
not in equilibrium or if factors such as technical change or endowment shifts
disturb an equilibrium, then both short-run and long-run trade can take place.

Suppose that two countries (A and B) have identical labour supplies,
identical homothetic utility functions, and identical rates of time preference.
Their long-run production frontiers will be given by LL' in Figure 3 with
their identical long-run autarky equilibria denoted by E in that diagram
(p gives the equilibrium price ratio). If the countries have different capital
stocks for various historical reasons, then an equilibrium such as that
shown in Figure 3 can exist. Figure 3 depicts a situation in which country A
has a larger capital stock than country B, with AA' and BB' denoting their
Heckscher-Ohlin production frontiers respectively. Factor accumulation/deaccum-
ulation does not occur by virtue of the fact that the equilibrium terms of
trade equals p. The long-run production bundles for countries A and B are
denoted A_p and B_p respectively with their corresponding consumption bundles
denoted A_c and B_c.

The equilibrium shown in Figure 3 is, of course, somewhat arbitrary,
but then so is any other equilibrium including the one where both countries
are producing (and not trading) at E. As in the case of the trading equilibrium
shown in Figure 3, any shock will generally move countries away from the
no-trade equilibrium at E to a new long-run equilibrium with positive trade.
We shall have more to say on this under point four below.

Since this model is obviously different from the static Heckscher-Ohlin
model, it is remarkable that long-run equilibria here share many of the
Heckscher-Ohlin properties. First, if the long-run equilibrium involves trade
(as in Figure 3), it must be the case that countries are observed to export
the good using intensively the country's abundant factor. Given identical
demand, countries in free trade will always consume in the same ratio with
their production ratios differing according to their relative factor endowments.

FIGURE 3
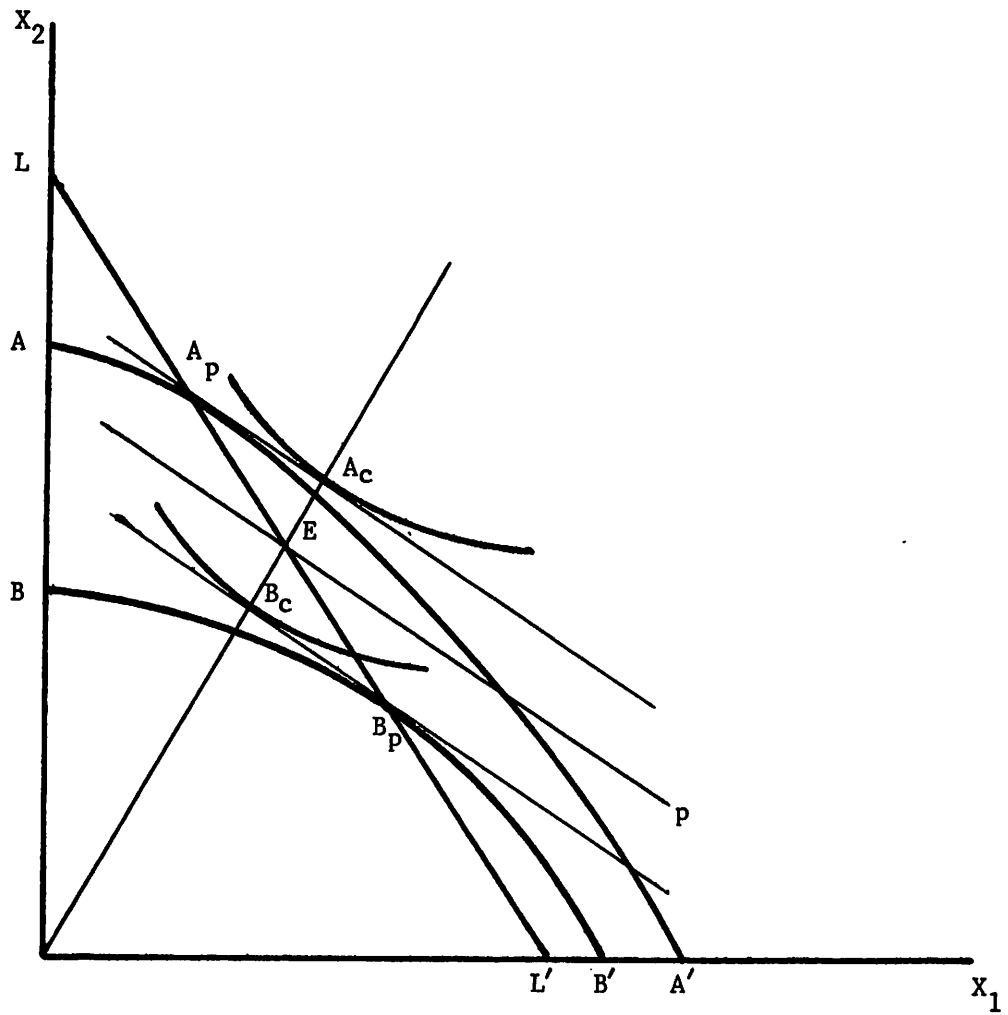
The relatively capital abundant country will produce relatively more of and thus export the capital intensive good as in Figure 3.

Second, a long-run free-trade equilibrium will involve factor-price equalization as required by the steady state conditions. Indeed, this property is more robust here than in the usual 2 x 2 Heckscher-Ohlin model. Here the result follows from the capital market condition alone and thus continues to hold if one or both countries are specialized or if there are factor intensity reversals.

Third, tariffs will lead to a Mundell-type result that factor endowments will adjust until trade has been eliminated (Mundell (1957) with reference to factor mobility). In Figure 3, a tariff will raise $p = p_1/p_2$ in A and lower it in B. Country A will deaccumulate capital, moving from A_p toward E and country B will accumulate capital moving from B_p toward E. Domestic prices cannot be equalized (and therefore take the equilibrium value in both countries) until after trade has been eliminated.

Fourth, certain shocks such as changes in factor endowments will have Heckscher-Ohlin effects. Suppose in Figure 3 that some additional capital is dumped on country A (the microeconomic equivalent of helicopter money). Relative differences in factor endowments have now increased and Heckscher-Ohlin theory would predict an increase in the volume of trade. In the present case, country A's (and therefore the world's) production of X_2 will increase and production of X_1 will decrease at the existing terms of trade by the usual Rybczynski analysis. This will cause p to fall and lead both countries to decrease capital. Long-run equilibrium will be reestablished when the world capital-labour ratio has fallen to its old value and prices are reestablished. But the fact that in the transition

country B has deaccumulated capital means that B will arrive at a new long-run equilibrium below B_p in Figure 3. Country A will in turn not reduce its capital stock by the full amount of the shock and thus will arrive at a new long-run equilibrium above A_p on LL′ in Figure 3. Since the consumption bundles of both countries must still lie along the ray OE, trade will have increased in the new long-run equilibrium. Similar arguments apply to labour changes: an increase (decrease) in the labour endowment of the initially labour abundant country will lead to an increase (decrease) in the long run volume of trade.

Note finally that this argument implies that the no-trade equilibrium with both countries at E in Figure 3 is no less arbitrary than any other equilibrium in the sense that any disturbance at E is likely to kick the countries to a new long-run equilibrium in which there exists a positive volume of trade. There is in simple terms a continuum of equilibria with all of them unstable in a certain sense. The important point for our purposes is just that if trade is observed, then the system has characteristics and properties which are remarkably similar to those of the static 2 x 2 Heckscher-Ohlin model.

VI. Time-Phased Intermediate Goods

The above model has quite a different algebraic structure to recent models on time-phased production (see especially Ethier (1979)). Yet in long-run equilibrium our model can in fact be quite easily expressed in a form consistent with that literature. Since we have shown that the 2-good 1-primary-factor case is degenerate (and we have noted why Ethier's is not) let us show this in a 2-good 2-primary-factor case.

In order to simplify the notation, assume that there are three inputs (R_1, R_2, R_3) with rental rates (r_1, r_2, r_3) used to produce three goods (X_1, X_2, X_3) with prices (p_1, p_2, p_3). Consistent with our earlier formulation, R_3 is assumed to be the same as X_3 (i.e., R_3 is the stock, X_3 is flow production). a_{ij} will denote the amount of resource i needed to produce one unit of good j. Cost/price equations are then given as follows

$$(31) \qquad a_{11}r_1 + a_{21}r_2 + a_{31}r_3 = p_1$$

$$a_{12}r_1 + a_{22}r_2 + a_{32}r_3 = p_2$$

$$a_{13}r_1 + a_{23}r_2 + a_{33}r_3 = p_3$$

p_3 and r_3 are, of course, closely related, with p_3 equaling the present value of the stream of returns from a unit of R_3. With ρ denoting the discount rate as before and δ the depreciation rate, this steady-state present value is equal to

$$(32) \qquad p_3 = \int_0^\infty e^{-(\rho+\delta)t} r_3 dt = [1/(\rho+\delta)]r_3.$$

Replacing p_3 in (31) with (32), the last equation in (31) becomes

$$(33) \qquad a_{13}r_1 + a_{23}r_2 + (a_{33} - (\rho+\delta)^{-1})r_3 = a_{13}r_1 + a_{23}r_2 + \alpha r_3 = 0; \quad \alpha < 0,$$

where $\alpha = (a_{33} - (\rho + \delta)^{-1})$ as noted. (33) allows us to eliminate the last equation from (31) and write the system as

(34) $\quad (a_{11} - a_{31}a_{13}/\alpha)r_1 + (a_{21} - a_{31}a_{23}/\alpha)r_2 = p_1$

$\quad\quad\quad (a_{12} - a_{32}a_{13}/\alpha)r_1 + (a_{22} - a_{32}a_{23}/\alpha)r_2 = p_2$

Equation (34) thus allows us to write the cost/price equations as the sum of direct primary factor costs plus the indirect costs of time-dated primary inputs embodied in R_3.

(35) $\quad \begin{bmatrix} a_{11} & a_{21} \\ a_{12} & a_{22} \end{bmatrix} \begin{bmatrix} r_1 \\ r_2 \end{bmatrix} + \begin{bmatrix} -a_{31}a_{13}/\alpha & -a_{31}a_{23}/\alpha \\ -a_{32}a_{13}/\alpha & -a_{32}a_{23}/\alpha \end{bmatrix} \begin{bmatrix} r_1 \\ r_2 \end{bmatrix} = \begin{bmatrix} p_1 \\ p_2 \end{bmatrix}$

$$= [A]r + [B]r = p$$

where all elements of [B] are positive since $\alpha < 0$. This formulation is reminiscent of the literature on intermediate goods (time dated or not) in which [B] is the matrix of indirect primary factor requirements embodied in the intermediate goods.

It is interesting to note here that [B] is singular while [A] is non-singular provided only that primary factor intensities differ between industries. At this point, we have no particular intuition as to why [B] is singular nor is it apparent that this result buys us anything of interest. Stolper-Samuelson and Rybczynski properties of this problem depend on the determinant of [A + B] which need not bear any particular relationship to the individual determinants.

In any case, the "reduced form" in (35) is consistent with the non-substitution results of the previous section and suggests the possibility of a

more general result: with a produced factor of the type considered here, the economy might continue to display Heckscher-Ohlin properties provided that there are two primary factors and two goods. More generally, it is the number of primary factors rather than the total number of factors that matter. With more goods than primary factors, it can probably be demonstrated that the economy will have "Melvin-flats" (Melvin (1968)) of which our 2-good, 1-primary-factor model is simply a special case.

VII.   Summary and Conclusions

The model developed in this paper is a two-final-good, two-factor model in which one factor (capital) is produced using a distinct technology. This model differs from the two-sector "optimal" growth tradition of Oniki and Uzawa (1965) by assuming (A) that capital and one final good are not identical and more importantly (B) that capital stocks are adjusted according to an optimal program rather than by some ad hoc savings function. The model differs from the more recent literature on time-phased production (Ethier (1979)) by explicitly introducing the demand side and consequently introducing restrictions on the steady-state rate of interest.[5]

The short-run properties of the model were considered only briefly since certain aspects have been developed by Manning (1979, 1980, 1981). We did note however that the Stolper-Samuelson property holds in the short-run as does a somewhat modified version of the Rybczynski theorem. Only crude bounds can however be put on the short-run response of final outputs to changes in prices. The reason is simply that price changes induce changes in capital accumulation and thus changes in final outputs depend upon the amounts and relative proportions of resources withdrawn or added to final production. This is perhaps an intuitively plausible property that is not found in the other approaches.

The most striking property of long-run equilibrium here is the "non-substitution" result which shows that the long-run production frontier is linear and the supply price necessary for diversified production unique. In a general sense, the non-substitution result suggests that the optimal accumulation of one factor reduces the dimension of the problem by one, or that the properties depend on the number of goods relative to the number of primary factors, not the total number of factors. This result is not found

in the approach of Ethier and others due to their assumption that the rate
of interest is determined only by the production sector. We are not in
any sense claiming that our model is more general since it depends on the
restrictive form of the utility function in (3), but are simply documenting
the fact that this alternative formulation yields some interesting similarities
and contrasts. Finally, as noted in the introduction, Brecher and Parker
(1977) obtain a linear frontier from their model, but their production
specification makes comparisons with our paper and the others just mentioned
very difficult. As of writing this first draft, we have not disentangled things
to the point where we can offer a simple comparison.

The other interesting thing about long-run trading equilibria in our
model is that they display several characteristics of static Heckscher-Ohlin
theory.[6] First, countries will export the good using intensively the country's
abundant factor. Although there are a continuum of equilibria, any
equilibria in which there is positive trade will be characterized by this
property. Second, a free trade equilibrium will be characterized by factor
price equalization.

Third, tariffs will produce a long-run effect similar to that derived
by Mundell for the case of factor mobility: tariffs will induce countries
to accumulate or deaccumulate capital in a manner that leads to the
elimination of trade.[7] Fourth, a type of Rybczynski effect will hold here
(although strictly speaking the Rybczynski theorem is not valid): at
the constant long-run price ratio an increase in a country's endowment
of factor i will lead to an expansion in its production of the good using
that factor intensively and a decrease in the production of the other good.
The exact relationship between these changes and the static concept of a

Rybczynski line was shown to depend on the technology of capital production.

Trading equilibria are thus characterized by properties similar to three of the four main propositions of the Heckscher-Ohlin model: the Heckscher-Ohlin theorem, the factor-price equalization theorem, and the Rybczynski theorem. The only theorem which does not hold in the long run here is the Stolper-Samuelson theorem for the trivial reason that the long-run price ratio is unique. The Stolper-Samuelson theorem does however hold for short-run price changes as noted above. It is striking that Ethier found a similar consistency with Heckscher-Ohlin in his rather different model.

Footnotes

[1] There are actually quite a few papers in this literature. Rather than attempt to discuss them all, the reader is referred to the literature reviews in Ethier (1979) and Dixit (1981). Dixit refers to these models as "neo-Ricardian". See also Steedman (1979a,b).

[2] Such a result does not seem to occur in the Ethier-Metcalfe-Steedman models for two reasons. First, the rate of interest is free to vary independently of the demand side of the model. If it were pinned down, a non-substitution property would occur. Second, capital and final goods are joint products so that Samuelson's non-substitution theorem may not apply.

[3] As noted earlier, such a non-substitution result will also occur in Ethier's model if a demand sector of our type were added to the model. Also, we can easily show that our result continues to hold in the special case where capital and one final good are identical, which is, of course, the Oniki-Uzawa production formulation.

[4] Note that if $\rho = 0$, the inequality in (26) must be replaced by an equality, which will imply that p and MRT coincide.

[5] As noted earlier, Ethier and Metcalfe and Steedman also assume that capital and final goods are joint products, which normally violates the assumptions of the non-substitution theorem. But the important difference does seem to the rate of time preference rather than joint production, because if the rate of interest is tied to the rate of time preference by the demand sector, a non-substitution result does seem to occur in their models despite the joint production.

[6] The simple, straightforward results here provide a contrast to the complicated properties of the Oniki-Uzawa model. This is perhaps very speculative at this stage, but we regard their fixed savings function as a type of imposed distortion. Looked at in that light, complicated results are not surprising.

[7] Indeed, anything that prevents commodity price equalization will generate similar results as will factors that break the link between commodity prices and factor prices. See, for example, Markusen (1981) on imperfect competition or Markusen and Melvin (1981) on returns to scale.

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