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This paper contains preliminary findings from research work still in progress and should not be quoted without prior approval of the author.

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ABSTRACT

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Recent years have witnessed a growing interest in the economics of capital utilization. The existing literature develops the theory of the optimal level of capital utilization almost exclusively in a partial equilibrium context. The present paper incorporates the degree of capital utilization as a choice variable in the two-sector model of general equilibrium. This incorporation leads to reversal of some of the standard results. For example, neither the Heckscher-Ohlin nor the factor-price-equalization theorem is valid in the presence of endogenous capital utilization. Moreover, the price-output response in a small open economy can be perverse. The paper contains a stability analysis that shows that these and other non-standard results do not necessarily violate the stability condition.

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1. Introduction

Recent years have witnessed a growing interest in the economics of capital utilization. In a study of British industry, Marris (1964) argued that as a part of their investment decision, firms consider the use or idleness of capital as a choice variable. Georgescu-Roegen (1970) explicitly incorporated the choice of daily duration of operations in a production model and made a clear distinction between the instantaneous rates of capital service flows and the stock of capital. Since these two pioneering contributions, numerous aspects of capital utilization at the firm level have been intensively studied by economists.[1]

The existing literature develops the theory of the optimal level of capital utilization almost exclusively in the context of partial equilibrium or one-sector growth models. Consequently, the general equilibrium effects of the optimal choice of capital utilization have so far remained completely unexplored. Specifically, to our knowledge, no one has yet analyzed questions such as what determines the level of capital utilization in general equilibrium and how its presence affects the relationships among commodity prices, factor prices, outputs and factor endowments.

Our purpose in the present paper is to fill this gap in the literature by explicitly incorporating the degree of capital utilization as a choice variable in the standard two-sector general equilibrium model. We also intend to analyze the implications of a variable degree of capital utilization for relationships among certain key variables in the economy.

In particular, we study the way in which the inclusion of capital utilization in the two-sector model affects the conclusions regarding the standard theorems of trade theory, namely, the Stolper-Samuelson, Rybczynski, factor-price-equalization, and Heckscher-Ohlin theorems. We also study the effects of an exogenous change in commodity prices on the optimal degree of capital utilization and on the outputs.

An important feature of our analysis is the explicit consideration of the stability of equilibrium. Since the presence of endogenous capital utilization as modelled in the present paper can lead to a reversal of the conventional results under certain circumstances, this stability analysis is of special interest.

In order to bring out the implications of capital utilization most clearly, we assume that the level of capital utilization is exogenously given in one sector while it is endogenously determined in the other sector The first sector could be called agriculture and the second industry; for in fact shift-work is not feasible in most of agriculture (Georgescu-Roegen 1969) while it is at least feasible in most of industry.

Shift-work requires at least some employees to work during abnormal hours, say, at night or on weekends. According to the available empirical evidence, the work performed during abnormal hours has to be rewarded by a higher wage. We incorporate this feature explicitly into the model via a shift premium function. An important consequence of the higher wage at abnormal hours is the introduction of the possibility of a divergence between relative factor intensities in the two industries in physical and value terms.

The results of this paper can be summarized as follows. First, the Stolper-Samuelson theorem remains valid or breaks down as the value and physical factor intensities in the two industries do or do not correspond. Second, the Rybczynski theorem continues to be valid in the presence of endogenous capital utilization. Third, in a two-country model, if tastes towards work during abnormal hours are identical in the two countries and there are no value factor-intensity reversals, commodity-price equalization via international trade also leads to factor-price equalization across the countries. Fourth, given the standard assumptions of the Heckscher-Ohlin model and the absence of value factor-intensity reversals, a restrictive version of the Heckscher-Ohlin theorem can be shown to be valid. In general, however, neither the factor-price-equalization nor the Heckscher-Ohlin theorem is valid in the presence of shift-work. Fifth, the stability of equilibrium in a small open economy does not require that the relative factor intensities in value and physical terms correspond with each other. Sixth, an increase in the price of the good produced by the shift-working industry can lower the level of capital utilization. Finally, the price-output response in a small open economy can be perverse without violating the stability condition. Moreover, these perversities cannot be ruled out on the basis of plausible values of the various parameters.

The paper is organized as follows. In Section 2, we lay out the model in detail. In Section 3, we consider the validity of the standard theorems of trade theory in the presence of capital utilization. Section 4 derives a stability condition in the context of a small open economy. In Section 5, we analyze the effects of a change in the commodity-price ratio on the level of capital utilization and the outputs. Finally, Section 6 summarizes our results.

2. The Model

Assume that our economy produces two goods, 1 and 2, using two primary factors of production, capital and labor. Denote by X_i, L_i and K_i, respectively, the flow of output, the flow of labor services and the stock of capital in industry i (i = 1,2) during a given period of calendar time, say, a week or a day. Define the units of capital services so that the capital stock K_i, when used for U_i shifts, generates U_iK_i amount of capital services per unit of calendar time. If the capital stock is operated during all the normal hours but not beyond, then $U_i = 1$ and the flow of capital services during the calendar time equals the stock of capital.[2] In line with the usual practice, let the production functions for the two goods be linear homogenous in labor and capital services.

As noted in the introduction, we assume that capital utilization is a choice variable in only one of the two industries. Let this industry be 1 so that $U_1 \equiv U \geq 1$ and $U_2 \equiv 1$. We also assume a putty-clay technology for both industries. Thus, while the capital-services-to-labor-services ratio can be varied between any two units of calendar time, it cannot be changed within any one unit of calendar time. If U (>1) shifts are operated--one during the normal hours and (U - 1) during the abnormal hours--the number of workers operating on the capital stock at a given instant must remain unchanged across the shifts. In other words we assume zero ex-post substitutability between capital and labor so that once the machines have been built, the same capital-services-to-labor-services ratio must be maintained during the normal and the abnormal hours of operation. The assumption of a putty-clay technology also implies the existence of positive ex-ante

substitutability between the two factors. Before factories are built, firms can choose different types of machines to substitute capital for labor or vice-versa, in response to a change in some parameter, say, the commodity price ratio. Given this description of the technology, we can write the flow of output of good 1 in any one period as[3]

$$(1) \quad X_1 = F_1(K_1, \frac{L_1}{U}) + F_1\,[(U - 1)K_1, \frac{U-1}{U} L_1]$$

$$\equiv F_1(UK_1, L_1),$$

where F_1 determines the form of the production function and is linear homogeneous in capital and labor services. Observe that $F_1(K_1, L_1/U)$ and $F_1[(U - 1)K_1, (U - 1)L_1/U]$, respectively, denote the outputs of good 1 produced during the normal and abnormal hours in a given calendar period. The industry employs L_1/U units of labor services during the normal hours and $(U - 1)L_1/U$ units during the abnormal hours. The ratio of capital services to labor services is the same at UK_1/L_1 at all times.

Next, let the output of good 2 be represented by

$$(2) \quad X_2 = F_2(K_2, L_2),$$

where F_2 is linear homogeneous and K_2 represents both the stock and services of capital in industry 2 employed in any one period. Denoting the economy's endowments of labor services and capital stock by L and K, respectively, and assuming full employment of resources, we have

$$(3)\quad K_1 + K_2 = K$$

$$(4)\quad L_1 + L_2 = L \quad .$$

The operation of more than one shift causes inconvenience to those employed during the abnormal hours. In order to compensate for this inconvenience, a premium has to be paid on top of the normal day wage for the work performed during abnormal hours, for example, at night or on weekends. We assume that this premium--referred to as the 'shift premium' in the present paper--is an increasing function of the level of utilization, U. There is substantial empirical evidence supporting the existence of a positive shift premium for the evening shift and of a higher one for the night or 'graveyard' shift, e.g., Betancourt and Clague (1981, Ch. 12, Section 2.3). An increasing premium can be modelled as a step function of utilization, in which case a corner solution will result, or as a continuous function of utilization, in which case an interior solution can be obtained. Indeed, the partial equilibrium literature contains instances of both approaches. For an example of the former approach the reader is referred to Betancourt and Clague (1981, Chs. 1,2); for an example of the latter approach the reader is referred to Winston and McCoy (1974). Here we find the continuous formulation to be more attractive because it allows U to vary continuously and thus enables us to analyze the general equilibrium aspects of varying levels of capital utilization in the comparative statics exercises involving small changes. If we were to choose a step function to represent the shift-premium function, U will be at a corner and it will not change in response to small changes in the exogenous variables.[4] Thus, the model of the firm developed below is similar to the one put forth by Winston and McCoy (1974).

Denoting the normal shift wage by \$W per unit of time and the abnormal shift wage by $\$W_a$ per unit of time, we can write

$$(5) \quad W_a = (1 + \beta(U))W$$

where β is the shift premium and $\partial\beta/\partial U \equiv \beta_U > 0$.[5] Remembering that industry 1 employs L_1/U workers during the normal hours and $(U - 1)L_1/U$ workers during the abnormal hours, we may write the average wage in industry 1 as

$$(6) \quad W_1 = \frac{1}{L_1}\left[\frac{L_1}{U} W + \frac{U-1}{U} L_1 W_a\right]$$

$$= \left[1 + \frac{U-1}{U}\beta(U)\right]W$$

$$= [1 + \alpha(U)]W$$

where for convenience we have replaced the average shift premium $(U - 1)\beta/U$ by $\alpha(U)$. For future reference, we note the following relationships among the first and second derivatives of α and β:

$$7(a) \quad \alpha_U = \frac{U-1}{U} \beta_U + \frac{1}{U^2} \beta > 0$$

$$7(b) \quad \alpha_{UU} = \frac{U-1}{U} \beta_{UU} + \frac{2}{U^2} \beta_U - \frac{2}{U^3} \beta \gtrless 0 \quad .$$

Let us denote the price of commodity i by p_i and the rental rate on capital by r. Given the normal shift wage W, in equation (6), the total cost of producing output X_1 will be given by $rK_1 + [1 + \alpha(U)]WL_1$. Therefore,

cost minimization by firms in sector 1 leads to the following Lagrangean expression

$$(8)\quad \text{Min } Z = rK_1 + [1 + \alpha(U)WL_1] + \lambda[\bar{X}_1 - F_1(UK_1, L_1)]$$

where λ is the Lagrange multiplier and the assumption of perfect competition by firms in industry 1 leads to $\lambda = p_1$. Thus, the first-order conditions implied by (8) can be written as

$$(9)\quad p_1 \frac{\partial F_1}{\partial L_1} = [1 + \alpha(U)]W$$

$$(10)\quad p_1 \frac{\partial F_1}{\partial K_1} \equiv p_1 \frac{\partial F_1}{\partial (UK_1)} U = r$$

$$(11)\quad p_1 \frac{\partial F_1}{\partial U} \equiv p_1 \frac{\partial F_1}{\partial (UK_1)} K_1 = \alpha_U WL_1$$

Parenthetically, in the next section we use the fact that by making use of equation (10) we can replace (11) with

$$(11')\quad \frac{rK_1}{U} = \alpha_U WL_1 \quad .$$

Equations (9) and (10) are the usual conditions for the optimal employment of labor and capital respectively. Equation (11) gives the condition for the optimal utilization of capital. If we were to view capital utilization as a third factor of production, according to (11), we must equate the value of marginal product of capital utilization to its marginal cost ($= WL_1 \partial\alpha/\partial U$). The latter is incurred in the form of the additional wage paid

to the workers exposed to working the abnormal hours. Finally, we note that in order to ensure an interior solution to the cost minimization problem, the following condition must be satisfied:[6]

$$(12) \quad (\theta_{1L} - \sigma_1) + \theta_{1L}(1 + R) > 0$$

where σ_1 is the elasticity of substitution between capital services and labor services in industry 1; θ_{1L} is the share of labor in the value of the product in sector 1, i.e., $\theta_{1L} = \frac{[1 + \alpha(U)]WL_1}{p_1X_1}$; and $R = \frac{U\alpha_{UU}}{\alpha_U}$. This new term R is simply the percentage change in the marginal costs of increased utilization (given W and L in (11)) that results from a percentage change in utilization. Remembering that $\sigma_1 \geq 0$, (12) implies that $R > -2$. As we shall see the second order condition plays an important role in the stability condition derived in Section IV.

Finally, cost minimization by firms together with perfect competition in industry 2 yields the standard first-order conditions

$$(13) \quad p_2 \frac{\partial F_2}{\partial L_2} = W$$

$$(14) \quad p_2 \frac{\partial F_2}{\partial K_2} = r \quad .$$

Our model is now completely specified. Equations (1) - (4), (9) - (11), (13), and (14) constitute a system of 9 equations in 9 endogenous variables, names, X_1, X_2, L_1, L_2, K_1, K_2, U, W and r. As usual, we can set $p_2 \equiv 1$ and $p \equiv p_1/p_2 = p_1$ and obtain the effects of the exogenous changes in

each of p, K and L on the endogenous variables by differentiating totally the nine equations and solving the resulting system of equations. At present, we note that given the homogeneity of the production functions and the cost minimizing conditions (9) - (11), (13) and (14), we can write

$$(15) \quad rK_1 + [1 + \alpha(U)]WL_1 = p_1X_1$$

$$(16) \quad rK_2 + WL_2 = p_2X_2 \quad .$$

It must be remembered that (15) and (16) have been derived from the cost minimizing conditions of the model and as such do not constitute additional independent equations of the model.

Before concluding this section, it is perhaps appropriate to note how our model applies to the analysis of both the rotating and non-rotating types of multiple-shift operation. Under the rotating shift system, all workers in the shift-working industry must be seen as partially working during the abnormal hours. In this case, W_1 in equation (6) will be the actual wage received by each worker. Under non-rotating multiple-shift operation on the other hand, only a part of the industrial labor force will be exposed to the abnormal hours of work. In this case, those who work during normal hours in the shift-working industry will receive W while the others will receive $[1 + \beta(U)]W$ as the wage.[7]

3. Capital Utilization and the Main Theorems of Trade Theory

In this section, we discuss the implications of capital utilization for the main theorems of trade theory, namely, the Stolper-Samuelson,

Rybczynski, factor-price equalization and Heckscher-Ohlin theorems. For this purpose, let us totally differentiate equations (1) - (4), (11'), (15) and (16). Denoting the proportionate change in a variable by a circumflex (^), we have

$$(17)\quad \hat{X}_1 = \theta_{1K}\hat{K}_1 + \theta_{1K}\hat{U} + \theta_{1L}\hat{L}_1$$

$$(18)\quad \hat{X}_2 = \theta_{2K}\hat{K}_2 + \theta_{2L}\hat{L}_2$$

$$(19)\quad \lambda_{1K}\hat{K}_1 + \lambda_{2K}\hat{K}_2 = \hat{K}$$

$$(20)\quad \lambda_{1L}\hat{L}_1 + \lambda_{2L}\hat{L}_2 = \hat{L}$$

$$(21)\quad \hat{U} = \frac{1}{(1+R)}\,[\hat{r} - \hat{W} + \hat{K}_1 - \hat{L}_1]$$

$$(22)\quad \theta_{1K}\hat{r} + \theta_{1L}\hat{W} = \hat{p}_1 + \hat{X}_1 - \theta_{1K}\hat{K}_1 - \theta_{1L}\hat{L}_1 - \frac{U\alpha_U}{1+\alpha}\,\theta_{1L}\hat{U}$$

$$(23)\quad \theta_{2K}\hat{r} + \theta_{2L}\hat{W} = \hat{p}_2 + \hat{X}_2 - \theta_{2K}\hat{K}_2 - \theta_{2L}\hat{L}_2$$

where recall that $1 + R \equiv 1 + U\alpha_{UU}/\alpha_U$. For notational convenience, we have also made use of the following definitions:

$$\theta \equiv \begin{pmatrix} \theta_{1K} & \theta_{1L} \\ \theta_{2K} & \theta_{2L} \end{pmatrix} \equiv \begin{pmatrix} \dfrac{rK_1}{p_1X_1} & \dfrac{(1+\alpha)WL_1}{p_1X_1} \\ \dfrac{rK_2}{p_2X_2} & \dfrac{WL_2}{p_2X_2} \end{pmatrix}$$

and

$$(\lambda) \equiv \begin{pmatrix} \lambda_{1K} & \lambda_{2K} \\ \lambda_{1L} & \lambda_{2L} \end{pmatrix} \equiv \begin{pmatrix} \frac{K_1}{K} & \frac{K_2}{K} \\ \frac{L_1}{L} & \frac{L_2}{L} \end{pmatrix}$$

Note that

$$|\theta| \equiv \det(\theta) = \theta_{1K} - \theta_{2K} = \theta_{2L} - \theta_{1L} = \frac{WrL_1L_2}{p_1p_2X_1X_2}\left[\frac{K_1}{L_1} - (1+\alpha)\frac{K_2}{L_2}\right]$$

$$|\lambda| \equiv \det(\lambda) = \lambda_{1K} - \lambda_{1L} = \lambda_{2L} - \lambda_{2K} = \frac{L_1L_2}{LK}\left[\frac{K_1}{L_1} - \frac{K_2}{L_2}\right]$$

It is evident that $|\lambda| \gtrless 0$ as industry 1 is capital or labor intensive in physical terms. As regards $|\theta|$, if commodity 1 is labor intensive in physical terms, it is negative; if commodity 1 is capital intensive in physical terms, its sign is ambiguous. Thus, if the shift-working industry is capital intensive, the conventional definitions of the relative factor intensities in physical and value terms need not correspond with each other. The reason for this divergence lies in the existence of the shift premium. Even though industry 1 happens to use more capital relative to labor than does industry 2, the presence of a high enough shift premium may make labor's share relative to that of capital higher in industry 1 than in industry 2. This possibility has important implications for the Stolper-Samuelson theorem when stated in terms of physical factor intensities.

We can rewrite equation (11') as

$$(11'') \quad \frac{U\alpha_U}{1+\alpha} = \frac{rK_1}{(1+\alpha)WL_1} = \frac{\theta_{1K}}{\theta_{1L}}$$

Substituting for $\hat{X}_1$ and $\hat{X}_2$ from (17) and (18), respectively, and for $U\alpha_U/[1+\alpha]$ from (11") equations (22) and (23) yield

$$(22') \quad \theta_{1K}\hat{r} + \theta_{1L}\hat{W} = \hat{p}_1$$

$$(23') \quad \theta_{2K}\hat{r} + \theta_{2L}\hat{W} = \hat{p}_2 \quad .$$

As in the standard two-sector model, factor prices in the present model depend on commodity prices alone. Specifically, they do not depend on output levels or factor endowments. If one thinks of the level of utilization as a third factor of production, this result may seem surprising. The explanation, however, lies in the fact that the level of utilization is determined endogenously in our model. In the market place, we have only two factor markets to clear. Therefore, when the factor endowments change at fixed commodity prices, we can clear the two factor markets by simply adjusting the two outputs at the constant values of W, r and U.

We now demonstrate

Proposition 1 (Stolper-Samuelson): If the relative factor intensities are defined in terms of the distributive shares, a rise in the price of a commodity necessarily raises the real and relative return to the factor used more intensively in that industry and lowers the return to the other factor. If the relative factor intensities are defined in the conventional physical

terms, however, this result (i.e., the Stolper-Samuelson theorem) remains valid or is reversed as the physical factor intensities do or do not correspond with the value factor intensities.

The proof of Proposition 1 follows straightforwardly from equations (22') and (23'). These equations can be solved to yield

$$(24)\quad \hat{r} - \hat{p}_i = \frac{\theta_{iL}}{|\theta|}\hat{p} \qquad i = 1,2$$

$$(25)\quad \hat{W} - \hat{p}_i = -\frac{\theta_{iK}}{|\theta|}\hat{p}$$

$$(26)\quad \hat{r} - \hat{W} = \frac{1}{|\theta|}\hat{p}$$

where $\hat{p} = \hat{p}_1 - \hat{p}_2$. If industry 1 is capital intensive (labor intensive) in terms of the distributive shares, $|\theta|>0$($|\theta|<0$), it follows from equations (24) and (25) than an increase in the relative price of good 1 raises (lowers) the real return to capital and lowers (raises) that to labor in terms of each good. From (26), the rise in the price also raises (lowers) the relative return to capital. Thus the Stolper-Samuelson theorem is shown to be valid. On the other hand, if factor intensities are defined in physical terms, it is possible for the Stolper-Samuelson theorem to be reversed. That is, if industry 1 is capital intensive in physical terms ($|\lambda|>0$) but not in value terms ($|\theta|<0$), an increase in the price of good 1 will lower the real and relative returns to capital and raise those to labor.

Observe that the necessary and sufficient condition for the breakdown of the Stolper-Samuelson theorem is given by $|\lambda||\theta|<0$. In the presence of factor-market distortions, Neary (1978) has shown that in a small open

economy, $|\theta||\lambda|<0$ implies instability. If this were the case in the presence of capital utilization too, the breakdown of the theorem would be uninteresting, for this result would never be observed. We demonstrate later in the paper, however, that in our model of a small open economy, instability neither implies nor is implied by the condition $|\theta||\lambda|<0$. Thus, the paradoxical effect of a change in commodity prices on factor returns cannot be ruled out on stability grounds in the presence of endogenous capital utilization.

Next, we demonstrate

Proposition 2: (Rybczynski) The Rybczynski theorem continues to be valid in the presence of multiple-shift operation. Thus, in a small open economy, an exogenous growth in a factor leads to the expansion of the industry using it more intensively in physical terms and to a contraction of the other industry.

To prove this proposition, first note that the elasticities of substitution between labor and capital services in the two sectors can be written as[8]

$$(27)\quad \sigma_1 = \frac{\hat{U} + \hat{K}_1 - \hat{L}_1}{\hat{U} + \frac{U\alpha_U}{1+\alpha}\hat{U} + \hat{W} - \hat{r}}$$

$$(28)\quad \sigma_2 = \frac{\hat{K}_2 - \hat{L}_2}{\hat{W} - \hat{r}} \quad .$$

Given equations (22') and (23'), constant commodity prices imply constant factor prices. With $\hat{W} = \hat{r} = 0$, equations (21), (27) and (28) yield $\hat{U} = 0$, $\hat{K}_1 = \hat{L}_1$ and $\hat{K}_2 = \hat{L}_2$.[9] Given these results, we obtain from equations (17) - (20)

$$(29)\quad \hat{X}_1 = \frac{1}{|\lambda|}\left(\lambda_{2L}\hat{K} - \lambda_{2K}\hat{L}\right)$$

$$(30)\quad \hat{X}_2 = \frac{-1}{|\lambda|}\left(\lambda_{1L}\hat{K} - \lambda_{1K}\hat{L}\right)$$

The validity of the Rybczynski theorem follows straightforwardly from these equations.

The intuitive reason for continued validity of the Rybczynski theorem lies in the fact that even with endogenous capital utilization, factor prices continue to depend on commodity prices alone. Therefore, in a small open economy, growth does not alter factor prices. The constancy of factor prices implies that the profitability of shift-work remains unchanged, thus yielding $\hat{U} = 0$. Given constant U, W, W_1 and r, the physical capital-labor ratios also remained unchanged. It then follows that in order to maintain full employment, growth in a factor must lead to the Rybczynski type of expansion in a small open economy.

It may be of interest to note here that as shown in the appendix growth via Hicks-neutral technical change in an industry in a small open economy has ambiguous effects on outputs if capital utilization is endogenous. For example, in the present model, if Hicks-neutral technical progress takes place in industry 2 at constant commodity prices, any one of the following three may happen: (i) X_1 contracts and X_2 expands; (ii) X_1 expands and X_2 contracts, or (iii) both X_1 and X_2 expand. Thus, Johnson's (1955) result that in the standard two-sector two-factor model of a small open economy, Hicks-neutral technical progress in an industry leads to its expansion and the other industry's contraction cannot be generalized to situations involving endogenous capital utilization.

Let us now consider the question of factor-price equalization. For this purpose, assume that there are two countries, A and B, and denote variables for each country by a country subscript. We can demonstrate

Proposition 3 (Factor-Price Equalization): Assuming that there are no value factor-intensity reversals over the relevant range of factor endowments, that tastes with respect to work during abnormal hours are identical across the countries, and that other standard assumptions of the Heckscher-Ohlin model are valid, commodity price equalization via international trade will also lead to factor-price equalization.

The proof of this proposition is simple. Given identical tastes towards work during abnormal hours, the shift premium functions will be the same in the two countries, that is, $\beta_A = \beta_B$. It follows from the cost minimizing conditions (9) - (11), (13), (14), the definition of the average shift premium $\alpha(U)$, and the assumption of a lack of value factor-intensity reversal that the two countries face the same one-to-one relationship between the factor returns and commodity prices. Hence, equalization of commodity prices necessarily implies factor-price equalization. Observe that under the assumptions of Proposition 3, the levels of capital utilization will also be equalized across the countries.

Finally, consider the Heckscher-Ohlin theorem.

Proposition 4 (Heckscher-Ohlin): If the assumptions of the standard Heckscher-Ohlin theorem and those stated in Proposition 3 hold, each country will export the good which is more intensive in value terms in the use of the factor that is relatively cheap prior to trade.

Given our earlier results, the proof of this proposition is rather straightforward. Under the assumed conditions, both countries face the same

relationship between commodity and factor prices. Given equation (26), the commodity which uses the cheaper factor more intensively is relatively cheaper in each country before trade opens. Remembering that each country will export the good which is cheaper under autarky, Proposition 4 follows.

It is worth noting that Proposition 4 states the Heckscher-Ohlin theorem in a somewhat weak form by defining factor intensities in value terms and factor abundance in terms of factor prices. If either factor intensities or factor abundance are defined in physical terms, the Heckscher-Ohlin theorem need not hold. For example, if industry 1 is capital intensive in physical terms but labor intensive in value terms, it is possible for the country with relatively cheaper labor to export this physically capital-intensive good.

A final point that must be made with respect to both Propositions 3 and 4 is that they assume identical β functions in the two countries. According to the available empirical evidence, both the levels of utilization and the tastes with respect to work during abnormal hours differ considerably across countries. If we allow for these facts, the factor-price equalization and Heckscher-Ohlin theorems break down even under the restrictive assumptions made in stating Propositions 3 and 4. This and other related issues are discussed in detail in Betancourt, Clague and Panagariya (1982).

4. Stability of Equilibrium in a Small Open Economy

The presence of some paradoxical results in the last section and those to be presented in the next one suggest the possibility of instability of equilibrium. We therefore derive a condition for stability of equilibrium. Since most of our analysis is presented in the context of a small open economy,

the stability analysis assumes the same context. In can be safely assumed that the corresponding stability condition for a large open economy will be weaker than that for a small open economy.

Let us assume that all the assumptions of our model in Section 2 continue to be valid except one: while the labor market clears instantaneously to ensure the validity of conditions (9) and (13) at all times, capital goods are sector specific in the short run and move towards the sector with higher return in the medium run.[10]

Denote by r_i ($i = 1,2$) the rental on capital in sector i in the short run. Starting with an initial long-run equilibrium, let us consider an exogenous shock to the economy such as a change in world prices, a shift in the β function (due to a change in tastes towards work during the abnormal hours), or growth via factor augmentation or technical change. In the short run, this exogenous change will either raise or lower r_1 relative to r_2.[11] That is, denoting the source of shock by γ, in general $(\hat{r}_1 - \hat{r}_2)/\hat{\gamma} \gtrless 0$ in the short run. Suppose that $(\hat{r}_1 - \hat{r}_2)/\hat{\gamma} > 0$ for the moment. Remembering that the economy was initially in long-run equilibrium, this change will induce a movement of capital from sector 2 to sector 1 in the medium run. The system will be stable if, holding γ fixed at its new level, the movement of capital into sector 1 lowers the discrepancy between r_1 and r_2; that is, given $\hat{\gamma} = 0$, $(\hat{r}_1 - \hat{r}_2)/\hat{K}_1 < 0$.

Next, suppose that the exogenous shock lowers r_1, relative to r_2 in the short run; that is, holding K_1 and K_2 fixed, we have $(\hat{r}_1 - \hat{r}_2)/\hat{\gamma} < 0$. By an argument similar to that presented in the case of $(\hat{r}_1 - \hat{r}_2)/\hat{\gamma} > 0$, it can be seen that local stability requires $(\hat{r}_1 - \hat{r}_2)/\hat{K}_1 < 0$ with γ held fixed at its new level. Thus, the system is locally stable if a movement of capital

in the medium run leads to $(\hat{r}_1 - \hat{r}_2)/\hat{K}_1 < 0$. In the remainder of this section, we derive the condition for $(\hat{r}_1 - \hat{r}_2)/\hat{K}_1 < 0$.

Let us replace r by r_1 in equations (21), (22), and (11"), and by r_2 in equation (23). Making use of these substitutions and of (17) and (18), and remembering that the small country assumption implies $\hat{p}_1 = \hat{p}_2 = 0$, equations (22) and (23) can be rewritten as

$$(22'') \quad \theta_{1L}\hat{r}_1 + \theta_{1L}\hat{W} = 0$$

$$(23'') \quad \theta_{2K}\hat{r}_2 + \theta_{2L}\hat{W} = 0,$$

where $\theta_{iK} \equiv r_i K_i/p_i X_i$ $(i = 1,2)$. Let us also replace r by r_1 in (27) and by r_2 in (28) so that we have

$$(27') \quad \sigma_1 = \frac{\hat{U} + \hat{K}_1 - \hat{L}_1}{\frac{1}{\theta_{1L}}\hat{U} + \hat{W} - \hat{r}_1}$$

$$(28') \quad \sigma_2 = \frac{\hat{K}_2 - \hat{L}_2}{\hat{W} - \hat{r}_2}$$

where we have substituted $U\alpha_U/(1 + \alpha) = \theta_{1K}/\theta_{1L}$ from (11") in writing (27'). Equations (22"), (23"), (27'), (28'), (19) and (20) are six equations in 7 variables, $\hat{W}$, $\hat{r}_1$, $\hat{r}_2$, $\hat{K}_1$, $\hat{K}_2$, $\hat{L}_1$ and $\hat{L}_2$. We can therefore solve them to derive the effects of an exogenous change in K_1 in the medium run on the remaining six variables. Specifically, as shown in the appendix, we can obtain

$$(31) \quad \hat{r}_1 - \hat{r}_2 = -\frac{T_2}{\lambda_{2K}\sigma_2\lambda_{2L}(\theta_{2K}T_1 + \theta_{1K}T_2)}\,|\theta|\,|\lambda|\,\hat{K}_1$$

where use has been made of the following definitions:

$$T_1 \equiv \lambda_{1L}\{(\theta_{1L} - \sigma_1) + \sigma_1\theta_{1L}(1 + R)\}$$

$$T_2 \equiv \sigma_2\lambda_{2L}\{(\theta_{1L} - \sigma_1) + \theta_{1L}(1 + R)\} \quad .$$

We can thus write the stability condition as

$$(31') \quad \frac{(\theta_{1L} - \sigma_1) + \theta_{1L}(1 + R)}{\lambda_{2K}(\theta_{2K}T_1 + \theta_{1K}T_2)} |\lambda||\theta| > 0 \quad .$$

Observe that $|\theta||\lambda| > 0$ is neither necessary nor sufficient for stability. The system is stable if and only if $|\theta||\lambda| \gtrless 0$ as T_2 and $\theta_{2K}T_1 + \theta_{1K}T_2$ have the same or different signs. T_2 will always be positive, by the second order conditions for cost minimization which imply inequality (12). The sign of T_1, however, is ambiguous. It is easily verified that inequality (12) implies

$$(12') \quad \frac{T_1}{\lambda_{1L}} > (\sigma_1 - 1)(\sigma_1 - \theta_{1L}) \quad .$$

From (12') it follows that if $\sigma_1 > 1$, we necessarily have $\sigma_1 > \theta_{1L}$ and hence $T_1 > 0$. In this case, the stability of the equilibrium requires $|\theta||\lambda| > 0$ so that all the major theorems of trade theory discussed in the last section remain valid even when stated in the strong forms. On the other hand, if $\theta_{1L} < \sigma_1 < 1$, T_2 and $\theta_{2K}T_1 + \theta_{1K}T_2$ may or may not have the same sign. One set of sufficient conditions which yields the opposite signs for these terms and hence $|\theta||\lambda| < 0$ as the stability condition is presented below.[12]

5. The Effects of a Change in the Commodity Price Ratio on the Level of Capital Utilization and Outputs

In this section, we consider the effects of an exogenous change in the commodity-price ratio on the level of utilization and on the outputs. For this purpose, first note that given equation (11"), equations (17) and (27) yield

$$(32)\quad \hat{L}_1 = \hat{X}_1 - \sigma_1 \frac{\theta_{1K}}{\theta_{1L}} \hat{U} + \sigma_1 \theta_{1K}(\hat{r} - \hat{W})$$

$$(33)\quad \hat{K}_1 = \hat{X}_1 - (1 - \sigma_1)\hat{U} - \sigma_1 \theta_{1L}(\hat{r} - \hat{W})$$

Substituting for $\hat{L}_1$ and $\hat{K}_1$ from these equations and for $\hat{r} - \hat{W}$ from (26) into (21), we have

$$(34)\quad \hat{U} = \frac{\theta_{1L}\sigma_2\lambda_{2L}(1 - \sigma_1)}{T_2|\theta|} \hat{p} \quad .$$

It is evident from this equation that a rise in the price of the good produced by the shift-working industry will not necessarily raise the level of capital utilization. If $\sigma_1 < 1$ and the shift-working industry is relatively capital (labor) intensive in value terms, we have $\hat{U}/\hat{p} > 0$ (<0). If $\sigma_1 > 1$, the opposite result holds.

To explain the relationship between U and p, first note that a change in p affects utilization by affecting factor prices (see equations (21) and (26)). From equation (21), it is evident that changes in factor prices affect utilization by affecting its profitability both directly, and indirectly through changes in K_1/L_1. The indirect effect can be derived by combining equations (21), (32) and (33) to obtain

$$(35)\quad \frac{\hat{K}_1 - \hat{L}_1}{\hat{r} - \hat{W}} = -\sigma_2 \frac{\lambda_{2L}}{\lambda_{1L}} \frac{T_1}{T_2} = -\frac{(\theta_{1L} - \sigma_1) + \sigma_1\theta_{1L}(1 + R)}{(\theta_{1L} - \sigma_1) + \theta_{1L}(1 + R)} \quad .$$

Now if $\sigma_1 > 1$, we necessarily have $1 + R > 0$ to ensure the positivity of T_2. It follows from (21) then that the direct effect of the change in r and W on U is positive. The indirect effect noted in (35), on the other hand, is negative because T_1 and T_2 are both positive. Given $\sigma_1 > 1$, we have $(\sigma_2\lambda_{2L}T_1)/(\lambda_{1L}T_2) > 1$, however, so that the (negative) indirect effect dominates the (positive) direct effect. Consequently, a rise in r/W lowers U.

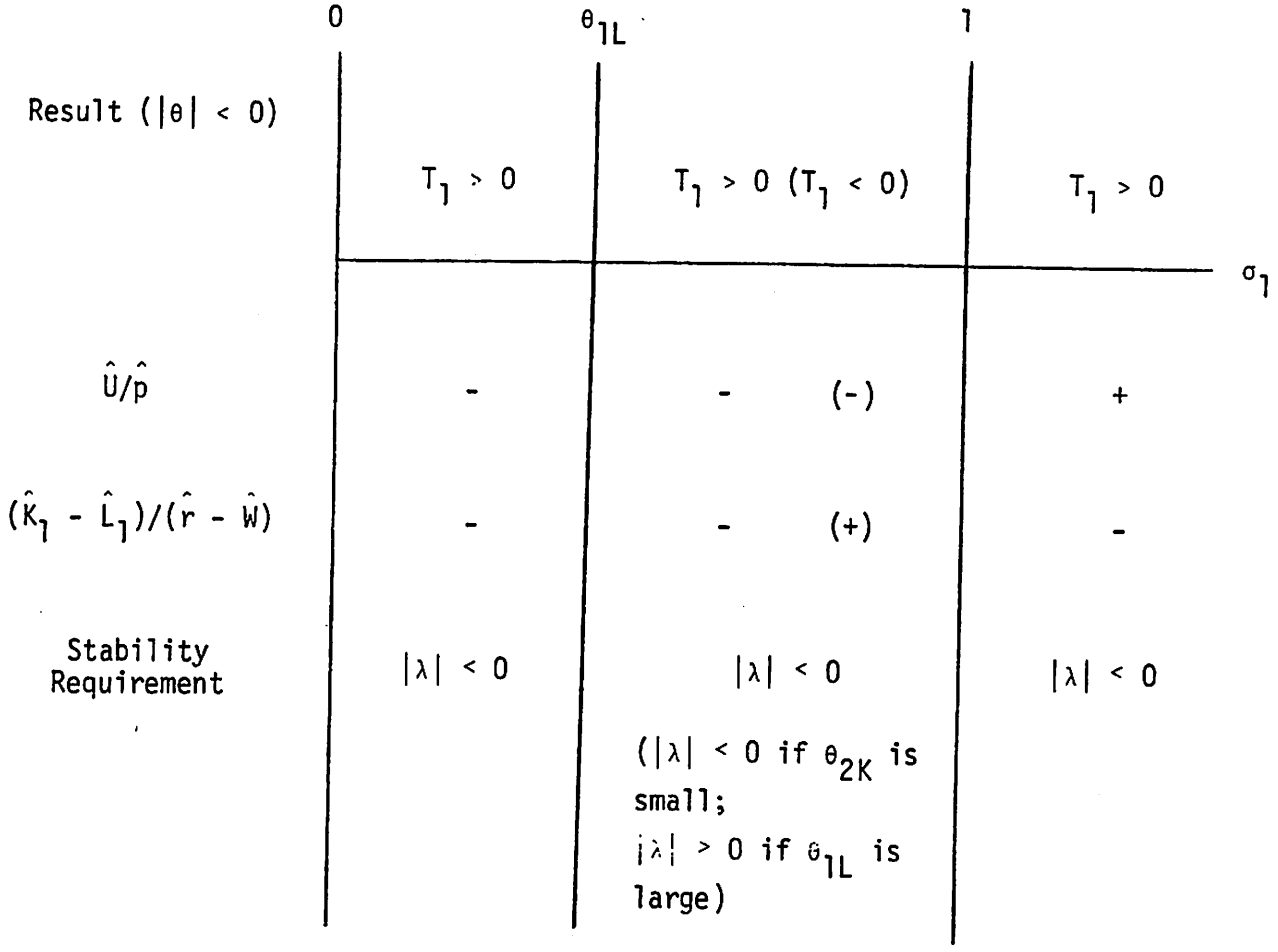
If $\sigma_1 < 1$, we can distinguish two cases on the basis of whether T_1 is negative or positive. If T_1 is negative, given $T_2 > 0$, $1 + R$ must be positive. As a result, both the direct and indirect effects are positive in this case thus causing U to rise whenever r/W rises.[13] If T_1 is positive, however, $1 + R$ may be positive or negative. When $1 + R$ is positive, the direct effect is positive and the indirect effect is negative but since $(\sigma_2\lambda_{2L}T_1)/(\lambda_{1L}T_2) < 1$, the former dominates. In the case that $1 + R$ is negative, which requires $\sigma_1 < \theta_{1L}$ so that T_2 can remain positive, the direct effect is negative and the indirect effect is positive but the latter dominates because $(\sigma_2\lambda_{2L}T_1)/(\lambda_{1L}T_2) > 1$. Once again a rise in r/W leads to a rise in U.

It may be useful to note that in the special case where $\sigma_1 = 1$, the direct and indirect effects of a change in r/W are of exactly equal magnitude but of opposite signs. In terms of equation (35), given $\sigma_1 = 1$, we obtain $\sigma_2\lambda_{2L}T_1 = \lambda_{1L}T_2$ so that $\hat{K}_1 - \hat{L}_1 = -(\hat{r} - \hat{W})$. Accordingly, a change in factor prices and hence in commodity prices has no effect on the level of utilization when $\sigma_1 = 1$.

The elasticity of substitution in the shift-working sector and its interaction with the value share of labor in this sector play a very important

role when capital utilization is endogenously determined. The nature of this role is best brought out by the results summarized in Figure 1, where we measure the value of the elasticity of substitution along the horizontal axis and critical values of this parameter by vertical lines. For simplicity of exposition we only present the results for the case where the shift-working sector is labor intensive, $|\theta| < 0$.

FIGURE 1


0
θ_{1L}
1
Result ($|\theta| < 0$)
$T_1 > 0$
$T_1 > 0$ ($T_1 < 0$)
$T_1 > 0$
σ_1
$\hat{U}/\hat{p}$
-
- (-)
+
$(\hat{K}_1 - \hat{L}_1)/(\hat{r} - \hat{W})$
-
- (+)
-
Stability Requirement
$|\lambda| < 0$
$|\lambda| < 0$
($|\lambda| < 0$ if θ_{2K} is small;
$|\lambda| > 0$ if θ_{1L} is large)
$|\lambda| < 0$

Let us now briefly discuss the relationship between outputs and commodity prices in our model. For this purpose, solve (18) and (28) to obtain

$$(36)\quad \hat{L}_2 = \hat{X}_2 + \sigma_2\theta_{2K}(\hat{r} - \hat{W})$$

$$(37)\quad \hat{K}_2 = \hat{X}_2 - \sigma_2\theta_{2L}(\hat{r} - \hat{W})$$

Substituting for $\hat{L}_1$, $\hat{K}_1$, $\hat{L}_2$ and $\hat{K}_2$ from (32), (33), (36) and (37), for $(\hat{r} - \hat{W})$ from (26) and for $\hat{U}$ from (34), we can solve equations (19) and (20) to obtain the effects of price changes on outputs at fixed factor endowments ($\hat{K} = \hat{L} = 0$).

$$(38)\quad \hat{X}_1 - \hat{X}_2 = \frac{\hat{p}}{|\theta||\lambda|}\left\{\frac{\sigma_2\lambda_{2L}(1-\sigma_1)}{T_2}\left[\lambda_{1K}\theta_{1L} - \sigma_1(\lambda_{1K}\theta_{1L} + \lambda_{1L}\theta_{1K})\right] + (\Delta_K + \Delta_L)\right\},$$

where $\Delta_K \equiv \sigma_1\lambda_{1K}\theta_{1L} + \sigma_2\lambda_{2K}\theta_{2L}$ and $\Delta_L \equiv \sigma_1\lambda_{1L}\theta_{1K} + \sigma_2\lambda_{2L}\theta_{2K}$. In this equation, the first term within the curly brackets on the R.H.S. gives the effect on the relative output of X_1 due to a change in the level of capital utilization while the second term yields the effect due to reallocation of capital and labor at constant U. As noted earlier, given $\sigma_1 = 1$, the level of utilization does not change and we obtain the conventional expression for the price-output relationship.

In light of the recent controversy regarding whether or not a perverse price-output response is consistent with the stability of equilibrium, it is of interest to consider this question in our model.[14] First observe that if we have $\sigma_1 < \theta_{1L} < 1$ and industry 1 is relatively capital intensive in

physical terms, T_1 and T_2 as well as the term in the square brackets in the R.H.S. of (38) are positive. Given our stability condition, we also have $|\theta||\lambda| > 0$. Thus, the price-output response is normal in this case. Similarly, if $\sigma_1 > 1$, the first term within the curly brackets will always be positive and the stability condition again implies $|\theta||\lambda| > 0$. Thus, we again have a normal price-output response. If $\theta_{1L} < \sigma_1 < 1$, however, the range of possibilities is considerably enhanced. Below we provide an example that shows the stability condition being satisfied while the price-output response is negative.[15]

6. Concluding Observations

This paper has introduced endogenous capital utilization into the two-sector general equilibrium model of international trade. The stability analysis of the model revealed that if $\sigma_1 < \theta_{1L}$ or $1 < \sigma_1$, stability requires $|\theta||\lambda| > 0$, which is to say that the physical and value factor intensities must correspond to each other. In this case, the Stolper-Samuelson and Rybczynski theorems continue to hold no matter how factor intensities are defined. In addition, if there are identical tastes toward work during abnormal hours in the two countries, the factor-price and Heckscher-Ohlin theorems continue to hold (in the absence of factor intensity reversals). Note, however, that even in these cases where $\sigma_1 < \theta_{1L}$ or $1 < \sigma_1$, if tastes toward work during abnormal hours are different in the two countries, factor price equalization cannot obtain (except by accident) and the Heckscher-Ohlin pattern of trade need not occur.

If $1 > \sigma_1 > \theta_{1L}$, the situation is more complicated. Suffice it to say that cases can arise in which all the parameters take on plausible values and the stability condition is met and yet $|\theta||\lambda| < 0$, which is to say that

industry 1 is more capital intensive than industry 2 in physical terms but not in value terms. In this situation, the Stolper-Samuelson theorem must be stated in terms of value factor intensities, and similarly the propositions regarding factor price equalization and the Heckscher-Ohlin pattern of trade need to be formulated with factor intensities defined in value terms and factor endowments defined in terms of factor prices. The Rybczynski theorem continues to hold when factor intensities are defined in physical terms. Finally, it is possible for the price-output response to be perverse even though the stability condition is met and the factor intensities correspond when defined in physical and value terms.

The paper also examined the effect of a commodity price change on the rate of capital utilization. A change in commodity prices affects utilization through its effect on factor prices, and the sign of the net effect of a change in factor prices on utilization will be entirely determined by whether the elasticity of substitution is greater or less than unity. Thus, the qualitative effect of a change in factor prices on utilization is the same in a general equilibrium context as in a partial equilibrium one; for the partial equilibrium result see, for example, Winston and McCoy (1974, Thm 1).

A distinctive feature of our model is that it assigns an important role to the elasticity of substitution and its interaction with value factor intensities, particularly labor's share in the shift-working sector. In this model, in contrast with the standard model, the elasticity of substitution, via the stability condition, plays a critical role in determining the qualitative effects of commodity prices on factor prices. Similarly, the qualitative and not just quantitative results regarding the price-output relationship depend directly on the elasticity of substitution and its interaction with physical factor intensities and distributive shares. That both the elasticity

of substitution and factor intensities are important in our model is interesting in light of the recent assertion by Jones and Easton (1981) who argue that such interactions are usually missing from the 'even' models with equal number of factors and commodities produced. Admittedly, these interactions in our model are less pronounced than in the 'uneven' models; they, nevertheless, play an important role in determining the qualitative results.

A related distinguishing feature of our model is that, in the presence of two different wage rates, $|\theta||\lambda| > 0$ is no longer a necessary condition for the stability of the model. Thus, in contrast to Neary's assertion (1978) with respect to the factor market distortions literature, in a small open economy equilibria where the value and factor intensity rankings of the two sectors differ are <u>not</u> necessarily unstable.

Footnotes

*We have benefited from comments on earlier versions of this paper by Ronald Jones, Murray Kemp, Edward Tower and Gordon Winston.

1. Among the other important contributions to the theory of capital utilization, mention must be made of Winston (1974a), Winston and McCoy (1974), Betancourt and Clague (1975) and Baily (1976). Winston (1974b) and Oi (1981) provide useful overviews of the literature on the subject. Recently, Betancourt and Clague (1981) have presented a comprehensive treatment of the theoretical and empirical issues involved in the analysis of this topic. They also provide a survey of the existing empirical studies in Chapter 5.

2. While it is easier to think in terms of a day as the unit of time and to view an eight-hour day shift, for example, as normal hours of operation, the model can be applied equally well to the week by viewing, for example, five eight-hour day shifts as normal hours of operation and all other shifts, including those that take place on the weekends, as abnormal hours.

3. Remembering that F_1 is linear homogeneous in capital and labor services, we have

$$F_1(K_1, \frac{L_1}{U}) + F_1[(U-1)K_1, \frac{U-1}{U} L_1] = \frac{L_1}{U} F_1(\frac{UK_1}{L_1}, 1) + \frac{U-1}{U} L_1 F_1(\frac{UK_1}{L_1}, 1)$$

$$= [\frac{L_1}{U} + \frac{U-1}{U} L_1]F_1(\frac{UK_1}{L_1}, 1) = L_1 F_1(U \frac{K_1}{L_1}, 1) = F_1(UK_1, L_1) \quad .$$

4. Interestingly enough this formulation also describes very aptly the costs of shift-work to the firm under a rotating shift system. In a rotation system the number of abnormal hours to which a worker is exposed will usually vary as the pattern of rotation is changed, and the costs to the firm of patterns of rotation that increase the number of abnormal hours per worker will be a continuously increasing function of utilization. Rotating shift systems are a common occurrence, especially in Europe (Betancourt and Clague 1981, pp. 224-25).

5. Observe that by writing β as a function of U alone and not of L_1 we are implicitly assuming that the workers are identical with respect to their distaste for work during abnormal hours. Given the level of capital utilization, U, all workers are indifferent between working during the normal and the abnormal hours so long as the two wage rates are related as described by equation (5). If the workers differed with respect to their distaste for work during the abnormal hours, at a <u>given</u> level of U, a higher premium will have to be paid whenever more workers are to be attracted to work during those hours; that is, β will depend on both U and L_1. To keep the analysis tractable, we choose to avoid this complication.

6. The derivation of (12) is straightforward but very lengthy. Thus, it is presented in an appendix available upon request.

7. As is well known from the literature on public finance and trade theory, the possibility of different wages in the two sectors also exists in the presence of partial factor taxation or a unionized wage. The presence of a shift premium is conceptually different, however, from the presence of factor taxes or the unionized wage. For while the latter constitute distortions in the economy from a social welfare point of view, the former does not.

8. Defining $K_1^* \equiv UK_1$ and denoting by r^* $(=r/U)$ the return on K_1^*, we have $\sigma_1 = (\hat{K}_1^* - \hat{L}_1)/\hat{W}_1 - \hat{r}^*)$. Remembering that W_1 is given by equation (6), equation (27) follows.

9. Equations (21) and (27) can be viewed as a subsystem of two linear equations in two variables (U and $\hat{K}_1 - \hat{L}_1$). If $\hat{W} = \hat{r} = 0$, then this subsystem of equations is homogeneous and the only solution values are $\hat{U} = 0$ and $\hat{K}_1 - \hat{L}_1 = 0$ as long as the two equations are independent.

10. This same assumption is made by Neary (1978) for stability analysis in the context of factor-market distortions. Eaton and Panagariya (1979) and Ethier (1979) postulate a very similar adjustment mechanism in a one-factor two-sector model with variable returns to scale where this factor, labor, is assumed to be sector specific in the short run.

11. In the special case where the effect is to leave the factor returns equal, the economy would move instantaneously to the new long-run equilibrium and no further adjustments would be required.

12. Let the β function be $\beta = .3 + .1U$ and let $U = 2$. Then $\alpha_U = .175$; $\alpha_{UU} = -.075$; from (11"), $\theta_{1L} = .7407$, and $R = -.857$, or $(1 + R) = .143$. As long as $.8285 < \sigma_1 < .8467$, $T_1 < 0$ and $T_2 > 0$. For instance, let us set $\sigma_1 = .83$. In this case if $\lambda_{1L} = .7$, and $\theta_{2K} = .3$, $\theta_{2K}T_1 + \theta_{1K}T_2 < 0$ for all values of $\sigma_2 < .21$. Moreover, the system will be stable for all values of $\lambda_{1K} > .7$, because these satisfy $|\theta||\lambda| < 0$. Clearly, a wide variety of other cases can be generated by selecting different β functions, or values for U, λ_{1L} and θ_{2K}.

13. Remembering that equation (35) allows for a change in the level of utilization, this result should not be surprising.

14. See Neary (1978, 1980) and Herberg and Kemp (1980a, 1980b).

15. Let the β function be $\beta = .3 + .1U$ and let $U = 2$, as in footnote 12. Then $\theta_{1L} = .7407$ and $(1 + R) = .143$. We let $\sigma_1 = .82$, which implies $T_1 > 0$, $T_2 > 0$; in fact, $T_2/\sigma_2\lambda_{2L} = .0267$. Stability requires $|\theta||\lambda| > 0$. Thus, let $\lambda_{1L} = .8$, $\lambda_{1K} = .3$, and $\theta_{2K} = .6$; then $|\lambda| = -.5$ and $|\theta| = -.3407$. If $\sigma_2 \leq 1.31$, the price-output response would be perverse, as can be easily checked using (38).

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Appendix

In this appendix, we derive the stability condition (31') and the effects of a Hicks-neutral technical change on outputs in a small open economy.

To derive the stability condition, we use equation (22"), (23"), (27'), (28'), (19), (20) and (21). From (27') and (28'), we have

$$(A1)\quad \hat{L}_1 = \hat{K}_1 + (1 - \frac{\sigma_1}{\theta_{1L}})\hat{U} - \sigma_1(\hat{W} - \hat{r}_1)$$

$$(A2)\quad \hat{L}_2 = \hat{K}_2 - \sigma_2(\hat{W} - \hat{r}_2) \quad .$$

Substituting for $\hat{L}_1$, $\hat{L}_2$ and $\hat{K}_2$ from (A1), (A2), and (19), we have

$$(A3)\quad \lambda_{1L}(\theta_{1L} - \sigma_1)\hat{U} - \theta_{1L}(\sigma_1\lambda_{1L} + \sigma_2\lambda_{2L})\hat{W} = \frac{\theta_{1L}}{\lambda_{2K}}|\lambda|\hat{K}_1 - \theta_{1L}(\sigma_1\lambda_{1L}\hat{r}_1 + \sigma_2\lambda_{2L}\hat{r}_2)$$

Similarly, substituting for $\hat{L}_1$ from (A1) into (21), we obtain

$$(A4)\quad [(\theta_{1L} - \sigma_1) + \theta_{1L}(1 + R)]\hat{U} + \theta_{1L}(1 - \sigma_1)\hat{W} = \theta_{1L}(1 - \sigma_1)\hat{r}_1 \quad .$$

(A3) and (A4) can be solved for $\hat{U}$ and $\hat{W}$. In matrix notation,

$$(A5)\quad \begin{pmatrix} \lambda_{1L}(\theta_{1L} - \sigma_1) & -\theta_{1L}(\sigma_1\lambda_{1L} + \sigma_2\lambda_{2L}) \\ [(\theta_{1L} - \sigma_1) + \theta_{1L}(1 + R)] & \theta_{1L}(1 - \sigma_1) \end{pmatrix} \begin{pmatrix} \hat{U} \\ \hat{W} \end{pmatrix} = \begin{pmatrix} \frac{\theta_{1L}}{\lambda_{2K}} |\lambda|\hat{K}_1 - \theta_{1L}(\sigma_1\lambda_{1L}\hat{r}_1 + \sigma_2\lambda_{2L}\hat{r}_2) \\ \theta_{1L}(1 - \sigma_1)\hat{r}_1 \end{pmatrix}$$

Denoting the 2 X 2 matrix on the L.H.S. by B, it can be shown that $|B| = \theta_{1L}(T_1 + T_2)$ where T_1 and T_2 are as defined in Section 4 of the text. The solution for $\hat{W}$ is then given by

$$(A6)\quad \hat{W} = \frac{1}{T_1 + T_2}\left\{\frac{-T_2|\lambda|\hat{K}_1}{\lambda_{2K}\sigma_2\lambda_{2L}} + T_1\hat{r}_1 + T_2\hat{r}_2\right\} \quad .$$

Now substitute for $\hat{W}$ from (A6) into (22") and (23"). After some rearrangement, these substitutions yield

$$(A7)\quad [\theta_{1K} + \frac{\theta_{1L}T_1}{T_1+T_2}]\hat{r}_1 + \frac{\theta_{1L}T_2}{T_1+T_2}\hat{r}_2 = \frac{\theta_{1L}T_2|\lambda|}{\sigma_2\lambda_{2L}\lambda_{2K}(T_1+T_2)}\hat{K}_1$$

$$(A8)\quad \frac{\theta_{2L}T_1}{T_1+T_2}\hat{r}_1 + [\theta_{2K} + \frac{\theta_{2L}T_2}{T_1+T_2}]\hat{r}_2 = \frac{\theta_{2L}T_2|\lambda|}{\sigma_2\lambda_{2L}\lambda_{2K}(T_1+T_2)}\hat{K}_1 \quad .$$

(A7) and (A8) can be rewritten in matrix form as

$$(A9)\quad \begin{pmatrix} \theta_{1K}(T_1 + T_2) + \theta_{1L}T_1 & \theta_{1L}T_2 \\ \theta_{2L}T_1 & \theta_{2K}(T_1 + T_2) + \theta_{2L}T_2 \end{pmatrix} \begin{pmatrix} \hat{r}_1 \\ \hat{r}_2 \end{pmatrix} = \begin{pmatrix} \theta_{1L} \\ \theta_{2L} \end{pmatrix} \frac{|\lambda|T_2}{\sigma_2\lambda_{2L}\lambda_{2K}}\hat{K}_1$$

Denoting the 2 X 2 matrix on the L.H.S. of (A9) by A, it can be shown that

$$|A| \equiv [\theta_{1L}\theta_{2K}T_1 + \theta_{1K}\theta_{2L}T_2 + \theta_{1K}\theta_{2K}(T_1 + T_2)](T_1 + T_2)$$

$$= (\theta_{2K}T_1 + \theta_{1K}T_2)(T_1 + T_2) \quad .$$

Solving (A9) for $\hat{r}_1$ and $\hat{r}_2$, we have

$$(A10)\begin{pmatrix}\hat{r}_1\\ \\ \hat{r}_2\end{pmatrix} = \frac{T_2|\lambda|(T_1 + T_2)\hat{K}_1}{\sigma_2\lambda_{2L}\lambda_{2K}|A|}\begin{pmatrix}\theta_{2K}\ \theta_{1L}\\ \\ \theta_{1K}\ \theta_{2L}\end{pmatrix}$$

From (A10) it follows that

$$(A11)\quad \frac{(\hat{r}_1 - \hat{r}_2)}{\hat{K}_1} = -\frac{T_2(T_1 + T_2)|\lambda||\theta|}{\sigma_2\lambda_{2L}\lambda_{2K}|A|}$$

Substituting $|A| \equiv (\theta_{2K}T_1 + \theta_{1K}T_2)(T_1 + T_2)$ into (A11) and remembering that stability requires $(\hat{r}_1 - \hat{r}_2)/\hat{K}_1 < 0$, condition (31') in the text follows.

Next, let us consider the effects of Hicks-neutral technical progress on outputs, holding p fixed. For this purpose, replace equation (2) by

$$(A12)\quad X_2 = \rho F_2(K_2, L_2)$$

where ρ is a technical change parameter. We must now replace (18), (22') and (23'), respectively, by

$$(A13)\quad \hat{X}_2 = \theta_{2K}\hat{K}_2 + \theta_{2L}\hat{L}_2 + \hat{\rho}$$

$$(A14)\quad \theta_{1K}\hat{r} + \theta_{1L}\hat{W} = 0$$

$$(A15)\quad \theta_{2K}\hat{r} + \theta_{2L}\hat{W} = \hat{\rho}$$

In writing (A14) and (A15), we have set $\hat{p}_1 \equiv \hat{p}_2 \equiv 0$. Setting $\hat{K} \equiv \hat{L} \equiv 0$, equations (17), (19) - (21), (27), (28) and (A13) - (A15) are 9 equations in 10 variables, $\hat{X}_1$, $\hat{X}_2$, $\hat{W}$, $\hat{r}$, $\hat{L}_1$, $\hat{L}_2$, $\hat{K}_1$, $\hat{K}_2$, $\hat{U}$ and $\hat{\rho}$. Given $\hat{\rho}$ exogenously, they can be solved for the remaining 9 endogenous variables. Solve (17) and (27) for $\hat{L}_1$ and $\hat{K}_1$.

$$(A16)\quad \hat{L}_1 = \hat{X}_1 - \frac{\sigma_1\theta_{1K}}{\theta_{1L}}\hat{U} - \sigma_1\theta_{1K}(\hat{W} - \hat{r})$$

$$(A17)\quad \hat{K}_1 = \hat{X}_1 + (\sigma_1 - 1)\hat{U} + \sigma_1\theta_{1L}(\hat{W} - \hat{r})$$

where use has been made of (11"). Next, solve (A13) and (28) for $\hat{L}_2$ and $\hat{K}_2$.

$$(A18)\quad \hat{L}_2 = \hat{X}_2 - \hat{\rho} - \sigma_2\theta_{2K}(\hat{W} - \hat{r})$$

$$(A19)\quad \hat{K}_2 = \hat{X}_2 - \hat{\rho} + \sigma_2\theta_{2L}(\hat{W} - \hat{r}) \quad .$$

Substituting from (A16) - (A19) into (19) and (20), we have

$$(A20)\quad \lambda_{1K}\hat{X}_1 + \lambda_{2K}\hat{X}_2 = \lambda_{1K}(1 - \sigma_1)\hat{U} + \lambda_{2K}\hat{\rho} - (\hat{W} - \hat{r})\Delta_K$$

$$(A21)\quad \lambda_{1L}\hat{X}_1 + \lambda_{2L}\hat{X}_2 = \frac{\lambda_{1L}\theta_{1K}\sigma_1}{\theta_{1L}}\hat{U} + \lambda_{2L}\hat{\rho} + (\hat{W} - \hat{r})\Delta_L$$

where Δ_K and Δ_L are defined in Section 5 (following equation (38)) in the text. Subtracting (A14) from (A13), we obtain

$$(A22)\quad \hat{r} - \hat{W} = -\frac{\hat{\rho}}{|\theta|} \quad .$$

From (A16) and (A17), we have

$$(A23)\quad \hat{K}_1 - \hat{L}_1 = -(1 - \frac{\sigma_1}{\theta_{1L}})\hat{U} + \sigma_1(\hat{W} - \hat{r}) \quad .$$

Substituting from (A22) and (A23) into (21), we obtain

$$(A24)\quad [(\theta_{1L} - \sigma_1) + \theta_{1L}(1 + R)]\hat{U} = \frac{\theta_{1L}}{|\theta|}(1 - \sigma_1)\hat{\rho} \quad .$$

Now substitute for $\hat{U}$ and $(\hat{W} - \hat{r})$ from (A24) and (A22), respectively, into (A20) and (A21).

$$(A25)\quad \lambda_{1K}\hat{X}_1 + \lambda_{2K}\hat{X}_2 = [\frac{\sigma_2\lambda_{2L}\lambda_{1K}\theta_{1L}}{T_2}(1 - \sigma_1)^2 - \Delta_K + \lambda_{2K}|\theta|]\frac{\hat{\rho}}{|\theta|}$$

$$(A26)\quad \lambda_{1L}\hat{X}_1 + \lambda_{2L}\hat{X}_2 = [\frac{\sigma_1\sigma_2\lambda_{1L}\lambda_{2L}\theta_{1L}\theta_{1K}}{T_2}(1 - \sigma_1) + \Delta_L + \lambda_{2L}|\theta|]\frac{\hat{\rho}}{|\theta|} \quad .$$

Or, in matrix form,

$$(A27)\begin{pmatrix} \lambda_{1K} & \lambda_{2K} \\ \lambda_{1L} & \lambda_{2L} \end{pmatrix}\begin{pmatrix} \hat{X}_1 \\ \hat{X}_2 \end{pmatrix} = \frac{\hat{\rho}}{|\theta|}\begin{pmatrix} \frac{\sigma_2\lambda_{2L}\lambda_{1K}\theta_{1L}}{T_2}(1-\sigma_1)^2 - \Delta_K + \lambda_{2K}|\theta| \\ \frac{\sigma_1\sigma_2\lambda_{1L}\lambda_{2L}\theta_{1L}\theta_{1K}}{T_2}(1-\sigma_1) + \Delta_L + \lambda_{2L}|\theta| \end{pmatrix}$$

The general solution for X_1 and X_2 is very complicated. Therefore, we consider two special cases which are sufficient for our purpose.

Case I: $\sigma_1 = 1$. In this case, we have

$$(A28)\begin{pmatrix} \hat{X}_1 \\ \hat{X}_2 \end{pmatrix} = \frac{\hat{\rho}}{|\theta||\lambda|}\begin{pmatrix} -(\lambda_{2L}\Delta_K + \lambda_{2K}\Delta_L) \\ (\lambda_{1L}\Delta_K + \lambda_{1K}\Delta_L) + |\lambda||\theta| \end{pmatrix}$$

In this case, stability requires $|\lambda||\theta| > 0$. Hence, we have $\hat{X}_1/\hat{\rho} < 0$ and $\hat{X}_2/\hat{\rho} > 0$ as in Johnson (1955).

Case II: Let σ_1 and σ_2 be small. In the limit, they can be set equal to zero. We have

$$(A29)\begin{pmatrix} \hat{X}_1 \\ \hat{X}_2 \end{pmatrix} = \frac{\hat{\rho}}{|\theta||\lambda|}\begin{pmatrix} \frac{\lambda_{2L}\lambda_{1K}}{2+R} \\ |\theta||\lambda| - \frac{\lambda_{1L}\lambda_{1K}}{2+R} \end{pmatrix}$$

Once again, stability requires $|\theta||\lambda| > 0$. Hence $\hat{X}_1/\hat{\rho} > 0$. The sign of $\hat{X}_2/\hat{\rho}$ is ambiguous in general. In the neighborhood of $X_1 = 0$, the first term will dominate because as X_1 approaches zero, $\lambda_{1L}\lambda_{1K}$ will approach zero faster than

$|\lambda|(\equiv \lambda_{1L} - \lambda_{1K} = \lambda_{2K} - \lambda_{2L})$. Hence, we have $\hat{X}_2/\hat{\rho} > 0$ near $X_1 = 0$. On the other hand, in the neighborhood of $X_2 = 0$, the negative term will dominate because as X_2 approaches zero, $|\lambda|$ approaches zero while $\lambda_{1L}\lambda_{1K}$ approaches 1. In this case, we obtain $\hat{X}_2/\hat{\rho} < 0$.

From cases I and II, it is evident that all possibilities noted in Section 3 of the text exist.

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