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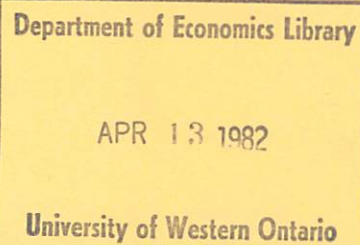# On the Estimation of Import and Export Demand Elasticities and Elasticity Pessimism

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ON THE ESTIMATION OF IMPORT AND EXPORT DEMAND
ELASTICITIES AND ELASTICITY PESSIMISM

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This paper contains preliminary findings from research work still in
progress and should not be quoted without prior approval of the author.

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On the Estimation of Import and Export Demand
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Prepared by Ahsan H. Mansur

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I. Introduction

Controversy around the estimation of price elasticities in interna-
tional trade models goes back to the empirical works of the inter-war
period which indicate that the price elasticities for imports and exports
are low. Since the pioneering article by Orcutt (1950), which indicates
some potential sources of bias in the data used and estimation methods
employed, various studies have attempted to explain this phenomenon and
sometimes suggested alternative methods which could yield better esti-
mates. However, in none of these works (some of which have been summa-
rized in Section II) the specified import and export demand functions
have been considered as a part of a more complete foreign trade sector
specification. Import and export demand functions, along with the trade
balance condition which closes the system, is a kind of specification
that has been recognized in the pure trade theory literature and also
in the empirically oriented general equilibrium models. Incorporation
of trade balance condition endogenizes the exchange rate and price elasti-
cities (for exportables and importables) take into account the simultaneous
variations of foreign prices along with exchange rate variations (i.e.,

effective foreign prices) relative to corresponding domestic prices. Moreover, if we are interested in the estimation of long run import and export demand elasticities trade balance condition should be a binding constraint and its omission (as has been shown in this paper) would make the estimates biased towards zero. 1/ This bias may be quite significant giving substantially lower estimates of the parameters and consequent concern over the elasticity pessimism. From an analytical point of view the evaluation of the effectiveness of exchange rate policies or other policy actions in the form of tariffs or subsidies, is made on the basis of long run outcomes and it is in the long run estimates (of the elasticities) that we should be more interested.

In the existing literature some works indicate potential sources of bias and discuss some ways to remove those. However, no attention has been paid to the effects of trade balance condition on long run estimators and in most of those studies import demand (or supply) and export demand functions are specified and estimated independently ignoring the fact that these two are linked to each other through the trade balance condition.

1/ For a comprehensive bibliography of the price elasticities in international trade, see Stern, R. et.al. (1976). Survey of the literature indicates that most of the specifications of export demand functions have terms containing logarithm of index of export prices to the index of world export prices, the associated coefficients measuring the price elasticity But in practice it is well recognized that variations of foreign prices relative to domestic prices can be offset or further reinforced by somewhat independent variation of exchange rate. What in fact matters is the variation of foreign prices relative to domestic prices of exportables net of exchange rate variation. This is also true for conventional import demand specifications.

The role of the trade balance condition in this framework is somewhat analogous to the role of budget constraint in the demand analysis, endowment constraint in an exchange model and income identity in a macro model. In the short run there may be differences between export receipts and import payments along with exogenously determined capital account adjustments, but that situation is temporary and cannot prevail in the long run. For any economy, imports are financed by export receipts and since surplus or deficit cannot exist indefinitely, in the long run trade balance condition should act as a binding constraint for the long run elasticity estimators. Exchange rate movements would exert sufficient pressure to insure that. 1/

This kind of specification is recognized in the pure trade theory literature and also in the applied general equilibrium models. In the pure theory of trade the foreign offer curve (or the import supply function) and the home country's offer curve (or the export demand function) intersect each other (under general conditions in a two good economy) and this insures trade balance. In the recent applied general equilibrium models a system of import demand (or supply) and export demand functions along with a trade balance condition are introduced to specify the external sector. Most of these models are constructed for single economies sometimes using Armington (1969) type procedure (which distinguishes

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1/ If an economy enjoys transfer of foreign exchange (due to some unspecified reasons) so that imports exceed exports the trade balance condition can be modified accordingly to take that into account without changing any of the conclusions in the following sections.

imports from domestic products of the same sectoral classification) simultaneously incorporating price taking behavior for imports, constant price elasticity for export demand and a zero trade balance condition. 1/

In this paper we would argue that:

(1) If we are interested in determining the long run effects, import and export demand elasticities should not be estimated independently because of the presence of trade balance condition. Least squares estimation of these elasticities ignoring the inherent simultaneity would give us biased and inconsistent estimators.

(2) The direction of bias can be determined under certain general conditions. If the sum of the true (absolute) values of export demand (a) and import demand or supply (b) elasticities is greater than 1, the estimated elasticities ($\hat{a}$ and $\hat{b}$) would both be asymptotically biased towards zero. That is, if $|a| + |b| > 1$, the independent estimation of the equations without regard to the rest of the system would make,

$$|\hat{a}| < |a| \quad \text{and} \quad |\hat{b}| < |b| \quad \text{asymptotically.}$$

Here a and b are the true values of export demand and import demand elasticities and $\hat{a}$ and $\hat{b}$ are their respective least squares estimators.

(3) This result has obvious bearings on the issue surrounding the controversy of 'elasticity pessimism'. On the aggregate level policy makers may

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1/ The exact specification of the external sector import and export demand functions in applied general equilibrium models vary from model to model but their broad characteristics are somewhat similar. The models presented by Adelman and Robinson (1977), Boadway and Treddnick (1978), Deardorff and Stern (1979), Dervis and Robinson (1978) etc. all belong to this category of external sector specifications.

need to determine what effects exchange rate realignments may have on the trade balance of different countries. Other issues like the effects of uniform changes in tariffs and in domestic indirect taxes on the composition of trade, level of income and employment, etc., depend upon the elasticity magnitudes. Our results indicate that most statistical estimates (obtained without the consideration of the inherent simultaneity) lower the absolute values of estimated elasticities, leading to considerable underestimation of the effectiveness of different policy actions.

(4) The basic conclusion holds under different specifications of the relevant functions, including one where the import demand funtion is obtained from choice theoretic foundations. Defining a commodity which is a CES aggregation of imports (M) and domestically produced and used commodities (D) and assuming cost minimization by the producers and consumers (in using the composite good), we can derive the import demand function. This specification has been used in different World Bank models but has never been estimated. This formulation (as described in Model II) hs two desirable advantages in the sense that it does not assume separability between imports and domestic goods and can explain the use of imports as intermediate good.

(5) Computational experience with Japanese data also supports our theoretical (asymptotic) assertion that the OLS estimators of a and b are substantially lower than those of asymptotically unbiased estimators like 2SLS, NL2SLS or FIML estimators of the corresponding parameters.

(6) In view of very frequent evidence of serial autocorrelation in the disturbance terms (in various studies), we extend our analysis to determine the asymptotic bias of OLS estimators of export demand elasticity with serially correlated error terms (w_t). It is straightforward to show that if $w_{1t} = p \cdot w_{1,t-1} + v_{1t}$, the asymptotic bias increases quadratically as the value of p, the autocorrelation coefficient increases. What is more important is that the bias gets much worse if $p > 0$ and the quasi first difference transformation of export demand function is made to correct the serial correlation problem. Thus if the underlying true model is simultaneous and due to misspecification we are applying OLS, so far as the bias is concerned we may be better off by not making the transformation to correct serial correlation (when $p > 0$ which is the most frequent case). This result is not necessarily true for only our specification but may very well be of relevance to other class of models as well.

The plan of the paper is as follows:
Section II summarizes some of the existing works on the estimation of trade elasticities and derivation of the directions of bias under various conditions. In Section III we introduce the basic models and demonstrate that single equation estimation by OLS would be asymptotically biased and inconsistent. Then we demonstrate and comment on the direction of bias. Estimates of various import and export demnd systems would be presented and compared with the corresponding OLS counterparts in Section IV. This would be followed by a discussion on the sensitivity of the bias

to the degree of autocorrelation and particularly on the possible adverse

effects of correcting the autocorrelation without considering the simul-

taneous equation bias. Main conclusiosn are summarized in Section VI.

Somewhat detailed mathematical deductions are given in the Appendix A-1.

II. Some Observations on the Existing Methods and Measurements of the Directions of Bias

Most of the empirical research on the estimation of import and export

demand functions is based on multivariable regression analysis with time-

series data. Investigators have estimated import demand equations by re-

gressing the logarithm of a measure of imports on the national income and

the ratio of imports to the price of domestic value added, all expressed

in logarithms. Export demand functions are also expressed in analogous

linear or log-linear forms. One striking feature of these estimates which

has come to the surface is that the estimates for both import and export

price elasticities appear to be surprisingly low. 1/ In an early attempt

to address the issue Orcutt (1950) investigated the various sources of

bias in the elasticity estimates in a non-simultaneous equation model

framework. He emphasized on the bias due to shifts in the demand surface,

the neglect of lagged prices and errors in the measurement of the price

variable. Using a linear import demand function with measurement error

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1/ It may also be noted that reported estimates of export price elasti-
cities do not appear to follow the ranking we expect from the relative
sizes of countries. Since it is believed that small economies are price
takers, they should face higher export price elasticities than the same
for large price making countries. This does not correspond to the values
of -1.41 for the U.S. versus -0.79 for Canada and -1.25 for Japan versus
-0.70 for New Zealand.

in the price variable, Orcutt shows that least squares estimates of
price elasticity would be biased towrds zero provided error in the quan-
tity variable is not highly and negatively correlated with the error in
the price variable.

In a follow-up paper, Kemp (1962) considered a modified version of
Orcutt's model where the dependent variable is constructed by dividing
the total money value with price index containing measurement errors.
This model differs from the usual errors-in-variable model in the sense
that the error in the dependent variable is highly and negatively corre-
lated with the error in the independent variable. Assuming that the
error in the price index is independent of the true price, Kemp demon-
strates that large sample bias is not towards zero but towards minus one.
As a continuation of the same issue, Kakwani (1972) proves that Kemp's
conclusions are vald even for small sample and for a model with a stocha-
stic import demand function. However, these results are concerned with
errors in the measurement and are discussed in the context of a single
equation model.

Some authors, Ball and Marwah (1962), Balassa and Kreinin (1967),
argue for substantially higher elasticities than those obtained from time
series estimation, which are to be taken as a king of lower limit.
Because of the potential reasons for downward bias associated with the
statistical methods applied, as upper bounds they suggest adding the
corresponding standard errors or multiples of standard errors to the least
squares estimates. This kind of upward adjustment has also been justified

on the basis of so-called 'tariff elasticities', proposed by Kreinin
(1961) and Krause (1962). Kreinin and Krause empirically demonstrate
that the elasticitiy of the demand for imports with respect to tariff
change is considerably higher than the elasticities calculated with
respect to price. A possible intuitive explanation may be that tariff
changes are regarded as more permanent while import price changes may be
considered as temporary and a kind of 'ratchet' effect may also be opera-
tive in the latter case. These are, however, in addition to the downward
bias in the least squares estimation of price elasticities, as has been
referred to earlier.

In a simultaneous approach Goldstein and Khan (1978) estimate supply
and demand functions for exports in a full-information framework.
According to them, while the assumption of infinite price elasticity
appears to be reasonable for a small country facing world supply, it is
not equally applicable to the supply of exports of an individual country.
They point out that an increase in the world demand for a country's
exports can normally not be satisfied without an increase in the price of
its exports and to ignore this simultaneous relationship between demand
and supply of exports would bias the estimated export demand elasticites.
Their empirical results indicate that export price elasticities are con-
siderably larger than those reported by other researchers for the same
group of countries. However, they do not investigate the direction of
potential bias from an analytical point of view and no comparison has

been reported vis-a-vis the least squares estimates from the same functional forms. Moreover, their specification has not been cast explicitly or implicitly in the form of an external sector closing system and hence is not so directly relevant to the present context.

III. External Sector Specifications

1. Model I

In this section we would deal with various forms of external sector specifications which can be used to estimate the (long run) elasticities and also have been used in various applied general equilibrium models. A simple system of external sector closure suggested by Boadway and Treddnick (1978) can be represented by the following 'simple' foreign export demand and import supply functions.

$$(II-1)..E_t = E_0 \; (PE_t/e_t)^a \; ; \quad -\alpha < a < 0 \; ; \quad t = 1, .., T.$$

$$(II-2)..M_t = M_0 \; (PM_t/e_t)^b \; ; \quad 0 < b < \alpha \; ; \quad t = 1, .., T$$

where E_t and M_t are exports and imports and E_0 and M_0 are 'base year' imports and exports respectively. PE_t and PM_t are the home country prices paid for exports and imports. a and b are the export demand and import supply price elasticities while e_t is the exchange rate between domestic and foreign currencies. Here (PE_t/e_t) is the price paid by the foreigners and (PM_t/e_t) is the price charged by the foreigners on home country imports and so enter the functions (II-1) and (II-2) respectively.

Balance of payments condition,

$$PM_t \cdot M_t = PE_t \cdot E_t \quad \quad \dots (II-3)$$

closes the system.

Given this specification it is convenient to treat E_t, M_t and e_t as endogenous variables; E_0 and M_0 as constants and PM_t, PE_t and any other terms (if appears) are exogenous variables. When it comes to estimation, the specified system is modified by the introduction of stochastic disturbances. This along with the logarithms of equations furnish the following system of equations:

$$\log E_t = \log E_0 + a \cdot \log PE_t - a \cdot \log e_t + v_{1t} \quad \dots (II-1')$$
$$\log M_t = \log M_0 + b \cdot \log PM_t - b \cdot \log e_t + v_{2t} \quad \dots (II-2')$$
$$\log M_t + \log PM_t = \log PE_t + \log E_t \quad \quad \dots (II-3')$$

The additive error terms are assumed to be independent of each other and follow the conventional classical assumptions, v_i having mean zero and constant variance ($\sigma_{v_i}^2$ (for all i=1,2). It can be noted from the reduced form for e_t' ($= \log e_t$) using all the equations of the system, that e_t' is correlated with v_{1t} and v_{2t}, the error terms of equations (II-1') and (II-2'). If ignoring this simultaneity we use OLS to estimate a and b from equations (II-1') and (II-2') independently, the estimators would be asymptotically biased and inconsistent.

This is a kind of result that we would generally expect but what is more important is the direction of bias. It has been shown in the Appendix A-1 that the asymptotic bias for OLS estimators $\hat{a}$ and $\hat{b}$ (of a and b) are respectively positive and negative under very general conditions. As has been shown in the Appendix:

$$\text{Plim } (\hat{a} - a) = \frac{-(1/(a-b)) \cdot \sigma_{v_1}^2}{q} \qquad \ldots (II-4)$$

where $q = \underset{T \to \infty}{\text{Plim }} (1/T) \overset{T}{\underset{t=1}{\Sigma}} (e_t'' - \bar{e}'') > 0$

$e_t'' = (\log PE_t - \log e_t)$, $\bar{e}''$ being the corresponding mean.

$\sigma_{v_1}^2$, being the variance due to the error term v_1 is also positive.

If $a < 0$ and $b > 0$, which we expect the coefficients to be according to theoretical justification, $\text{Plim } (\hat{a} - a) > 0$. Since algebraically a is negative, the limiting value of $\hat{a}$ in absolute terms would be smaller than the corresponding true (absolute) value of a, i.e., $\underset{T \to \infty}{\text{Plim }} |\hat{a}| < |a|$.

A similar demonstration for the import supply function shows that the asymptotic bias for the OLS estimator $\hat{b}$ can be put as:

$$\underset{T \to \infty}{\text{Plim }} (\hat{b}-b) = \frac{(1/(a-b)) \cdot \sigma_{v_2}^2}{S} \qquad \ldots (II-5)$$

where $S = \text{Plim} \dfrac{1}{T} \cdot (p''_{mt} - \bar{p}''_m)^2 > 0$
$\quad\quad\quad\quad T \rightarrow$

$p''_{mt} = (\log PM_t - \log e_t)$ and $\bar{p}''_m$ is the corresponding mean and

σ^2_{v2} is the variance due to the error term in the second equation.

Once again since $a < 0$ and $b > 0$, Plim $(\hat{b}-b) < 0$, i.e., $\hat{b}$ underestimates

the true value of b asymptotically.

2. **Model II**

Another formulation would be to consider the economy under investigation as facing fixed world prices for imports while simultaneously facing a constant elasticity demand function for exports. As a buyer of imports the country is a price taker (a typical small country assumption) while in the case of exports the demand depends on the export price charged relative to the world price and the corresponding export demand elasticity. In this price taking formulation (for imports) the investigators generally specify some kind of ad hoc import demand function which facilitates estimation (see Houthakker and Magee (1969), Leamer and Stern (1970)). Logarithm of the measure of import (quantity) is expressed as a linear function of the logarithm of income and logarithm of the ratio of import prices to the domestic price index. The estimated coefficients are then expressed as the measure of income and price elasticities of imports. [1]/

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[1]/ There are some exceptions to this where import demand functions are obtained from microeconomic foundations, e.g., Gregory (1970), Burgess (1974a, 1974b).

This specification of import demand (along with its variants) is not well integrated with the behavioral relationships of the rest of the economy; neither do we have a satisfactory explanation for the import demand. One possible explanation may be that imports implicitly enter the utility function of the consumers as final goods. This, however, conflicts with the empirical evidence that most the import is used as intermediate goods. Also from the empirical point of view we need to drop the most unrealistic assumption of the conventional trade theory that foreign and domestic goods of the same sectoral classification are identical. This enables us to assert that domestic prices of tradables are not fully tied to the prices of imports and extreme sensitivity in the production and consumption structure due to slight variation in relative prices can be avoided. Following Armington (1969) we can define a 'composite' commodity that is a CES (we can also use a Cobb-Douglas form) aggregation of imports, M and the commodities produced and consumed domestically, D. The aggregation can be put as: 1/

$$Q_t = (d \cdot M_t^{-P} + (1-d) \cdot D_t^{-P})^{-(1/p)}$$

1/ In the context of imports this type of aggregation was originally suggested by Ahluwalia, Lysy and Pyatt (World Bank mimeo). It can also be shown that as the elasticity of substitution $b = 1/(1+p) \to \infty$, the equation tends to

$$Q_t = d \cdot M_t + (1-d) \cdot D_t$$

Assuming that the producers and consumers minimize the cost of obtaining the 'composite good', Q_t, solving the corresponding first order conditions the import demand function can be expressed as:

$$M_t = (d/(1-d))^b \cdot (PD_t/(e_t \cdot PM_t))^b \cdot D_t \; ; \quad b > 0$$

where $b = (1/(1+p))$ defines the elasticity of import substitution;

PD_t: domestic good price

PM_t: imported good price (fixed in foreign currency).

In a somewhat similar spirit we distinguish between the world price of the product and the price received by an individual exporting country. The export demand function depends on the normal level of exports (E_0) and the export price of the country relative to the world price. This can be put as:

$$E_t = E_0 \cdot \left[\frac{\overline{\Pi}_t \cdot e_t}{PE_t} \right]^a \; ; \qquad a > 0$$

where $PE_t = PD_t$, the domestic goods price (in the absence of export

taxes and subsidies);

$\overline{\Pi}_t$: price of exportables in the international market in

foreign currency.

Since the ratio in the parenthesis on the right hand side is the price of exportables in the international (or world) market relative to the domestic price obtainable domestically, a, the price elasticity of export demand would be positive. Taxes and subsidies on imports and

exports can also be captured. The system is closed with the trade balance condition:

$$PM_t \cdot M_t = (PE_t/e_t) \cdot E_t$$

Once again we can log-linearize the system and show that OLS estimate of a and b would be asymptotically biased and inconsistent. We can examine the direction of bias and its bearing on the estimated measure of elasticity.

$$\text{Plim } (\hat{a} - a) = \frac{\underset{T \to \infty}{\text{Plim }} (1/T) \cdot \sum_{t=1}^{T} (\Theta_t - \bar{\Theta}) \cdot (v_{1t} - \bar{v}_1)}{\text{Plim } (1/T) \cdot \sum_{t=1}^{T} (\Theta_t - \bar{\Theta})^2}$$

$$= \frac{-(1/(a + b - 1)) \cdot \sigma_{v_1}^2}{Q}$$

where $\Theta_t = (\log \bar{\Pi}_t + \log e_t - \log PE_t)$ and $\bar{\Theta}$ is the corresponding mean.

$Q = \underset{T \to \infty}{\text{Plim }} (1/T) \cdot \sum_{t=1}^{T} (\Theta_t - \bar{\Theta})^2$ is assumed to exist and be positive.

Similarly for the import demand elasticity:

$$\text{Plim } (\hat{b} - b) = \frac{-(1/(a + b - 1))\sigma_{v_2}^2}{S}$$

where $S = \underset{T \to \infty}{\text{Plim}} \; (1/T). \overset{T}{\underset{t=1}{\Sigma}} \; (Z_t - \bar{Z})^2 > 0$

and $Z_t = \log PD_t - \log PM_t - \log e_t$ and Z is the corresponding mean value. Since a and b are both positive, if a + b > 1 (so that a + b - 1 > 0),

$$\underset{T \to \infty}{\text{Plim}} \; (\hat{a} - a) < 0$$

and $\quad \underset{T \to \infty}{\text{Plim}} \; (\hat{b} - b) < 0$

i.e., the bias is negative for both the estimators. This bias pulls down the estimated sum of the import and export demand elasticities if the true value is greater than 1.

These demonstrations in terms of a somewhat simplified model have some bearing on the controversy of the elasticity pessimism. Conventionally, the export and import demand functions are thought to be independent and are estimated separately by OLS. As we have indicated in terms of Models I and II, this gives a lower absolute value of the estimated elasticities sometimes causing unnecessary concern regarding the effectiveness of various commercial policies. The qualifying restriction (for Model II) that a + b > 1 is important in determining the critical point. Bias would tend to reduce the absolute values of both $\hat{a}$ and $\hat{b}$ when the true sum of the absolute values of a and b (i.e., $|a| + |b|$) is greater than 1, the condition (so called Marshall-Lerner condition) under which devaluation would be effective in improving the trade balance of the depreciating countries.

Before going on to further extensions we probably need to make some comments on the models specified above. Due to the imposition of trade balance condition our models may be referred to as a king of 'equilibrium model' and since no short run adjustment to equilibrium has been specified this essentially depicts a long run situation. Hence the elasticities a and b can be considered as long run export and import demand elasticities where full adjustment to price and exchange rate changes are taken care of. Analogously, the estimates obtained without the consideration of trade balance condition may be (somewhat crudely), called short run price elasticity estimates. Thus as one would expect, the long run price elasticities of imports and exports (when exchange rate adjustment and its consequent effects are allowed to take place) would be greater (in absolute value) than the corresponding short run ones.

In another variation of the model we can incorporate the income term explicitly in the export demand function to take into account the variations due to income changes. This would make our export demand function somewhat similar to that of Houthakker and Magee (1969) except for the allowance for exchange rate changes. Given the export demand function of the form:

$$\log E_t = C + a.\log \left[\frac{\bar{\Pi}_t \cdot e_t}{PD_t} \right] + c.\log YW_t + v_{1t}$$

where YW_t is the weighted index of the GNP of the countries which are importing the home country products.

Defining, $\Theta_t = \log \bar{\Pi}_t + \log e_t - \log PD_t$ and $\bar{\Theta}_t$ the corresponding mean, the specified equation can be expressed as:

$$ex_t = C + a. \Theta_t + c.y_t + v_{1t}$$

where $ex_t = \log E_t$ and $y_t = \log YW_t$. Then the OLS estimator for a is given by:

$$\hat{a} = \frac{m_{y,y} \cdot m_{\Theta,ex} - m_{\Theta,y} \cdot m_{y,ex}}{m_{\Theta,\Theta} \cdot m_{y,y} - m_{\Theta,y}^2}$$

where $m_{\Theta,ex} = \sum_{t=1}^{T} (\Theta_t - \bar{\Theta}).(ex_t - \overline{ex})$

$m_{y,y} = \sum_{t=1}^{T} (y_t - \bar{y}).(y_t - \bar{y})$ etc.

Taking probability limit, on simplification we once again end up with the same condition that:

$$Plim (\hat{a} - a) < 0 \quad if \ a + b - 1 > 0.$$

IV. Empirical Observations

The models outlined in Section III have been applied to the Japanese data to estimate the relevant parameters. Here we present the estimated parameters of the Model III which is a simple extension of Model II (due to the explicit incorporation of income term of the rest of the world). The estimators used are Two Stage Least Squares (2SLS), Nonlinear Two-Stage Lest Squares (NL2SLS), Full-Information Maximum Likelihood (FIML) and OLS.

Data is taken from International Financial Statistics, International Monetary Fund, for the period beginning 1972 to 1978. This is quarterly data adjusted for seasonal variations. The world price index Π_t has been calculated using the method employed by Houthakker and Magee. The weighted index incorporates export prices and trade shares of five other countries (e.g., Canada, France, Germany, U.K. and U.S.A.). Rest of the world real income is expressed as an index (following Houthakker-Magee), the weights coming from the relative shares of five other countries in their total GNP. Due to non-availability of GDP figures as a quarterly time series GNP values have been used as the proxy.

Tables 1 and 2 show the estimates of the parameters of the export and import demand respectively as obtained under different methods. For each of the estimated values of the parameters the corresponding T-statistics are presented in the parenthesis immediately below. Durbin-Watson statistics (D.W.) are presented for each of the methods in a separate column.

Table 1. Structural Equation Estimates of Export Demand
Functon: Quarterly Data: 1972-1978

$$\log E_t = a. \log (\overline{\Pi}_t \cdot e_t/PE_t) + c. \log YW_t + v_{1t}$$

Parameters → Methods ↓	a	c	D.W.
OLS	7.78 (1.85)	1.68 (20.37)	2.22
2SLS	12.96 (2.25)	1.68 (19.59)	2.02
NL2SLS	13.05 (2.31)	1.68 (19.55)	2.01
NLFIML	20.49 (4.83)	1.66 (16.72)	—

Table 2. Structural Estimation of the Parameters of the
Import Demand Function: Quarterly Data: 1972-1978

$$\log (m_t/D_t = C + b. \log (PD_t/e_t. PM_t) + v_{2t}$$

Parameters → Methods ↓	C	b	D.W.
OLS	-4.01 (-88.38)	0.52 (3.11)	0.26
Adjusted for Autocorrelation	-4.11 (-26.12)	0.36 (1.30)	1.70
2SLS	-4.05 (-86.58)	0.56 (3.24)	0.26
Adjusted for Autocorelation	-4.09 (-19.39)	0.43 (1.93)	
NL2SLS	d = 0.01 (insignificant)	0.59 (3.36)	0.26
NLFIML	d = 0.05 (2.555)	1.30 (3.21)	

V. Serial Correlation and the Bias

In Section III we implicitly assumed that the errors were temporally independent which would not always be valid in our applications with time series data. Presence of autocorrelation (in the context of import and export demand estimations) have been tested in many of the applications that appear in the literature (see Stern, et. al. (1976), Houthakker and Magee (1969), etc.) and also in our applications presented in Section IV. Violation of the classicial assumption of no autocorrelation does not, however, change the qualitative conclusions of Section III; but worsens the magnitude of the bias as the value of p, the autocorrelation coefficient deviates from zero in either direction.

To illustrate this let us start with Model II of Section III, the export demand function being written as:

$$\log E_t = C + a. \log \Pi_t + a. \log e_t - a. \log PE_t + w_t \quad ..(V-1)$$

where $w_t = p. w_{t-1} + v_{1t}$ and the rest of the system being unchanged (i.e., equations III-5 and III-6 come in).

If we do not make any correction for the serial correlation and apply OLS to equation V-1, the asymptotic bias would be as before:

$$\Plim_{T \to \infty} (\hat{a} - a) = \frac{-(1/(a + b - 1)). \sigma_w^2}{Q}$$

where $Q = \Plim_{T \to \infty} (1/T). \Sigma(\Theta_t - \bar{\Theta})^2$ and once again,

$$\Theta_t = \log \bar{\Pi}_t + \log e_t - \log PE_t$$

However, due to the first order autoregressive scheme that we have assumed for the error term,

$$\sigma_w^2 = \frac{\sigma_{v_1}^2}{(1-p^2)}$$

On substitution,

$$\text{Plim}_{T \to \infty} (a - a) = \frac{-\frac{1}{(a + b - 1)} \cdot \frac{\sigma_{v_1}^2}{(1 - p^2)}}{Q} \quad \ldots.(IV-3)$$

If p = 0, the bias exactly equals that of Section III and as p gets away from zero in either the positive or negative directions the bias would get worse very rapidly. The bias tends to infinity asymptotically as p → 1 (see the dotted curve in Figure I).

Now let us consider the situation where we take a quasi-first difference transformation of equation (V-1), so that we end up with:

$$ex_t - p \cdot ex_{t-1} = a \cdot (\Theta_t - p \cdot \Theta_{t-1}) + v_{1t}$$

or, $ex_t^* = a \cdot \Theta_t^* + v_{1t}$; where v_{1t} is not autocorrelated and $ex_t^* = ex_t - p \cdot ex_{t-1}$ and $\Theta_t^* = (\Theta_t - p \cdot \Theta_{t-1})$.

Then once again we can apply OLS to this transformed system and derive the asymptotic bias of the OLS estimator a*.

$$\text{Plim}_{T \to \infty} (\hat{a}^* - a) = \frac{\text{Plim}_{T \to \infty} (1/T). \; \Sigma (\Theta_t^* - \overline{\Theta}).(v_{1t} - \overline{v}_1)}{\text{Plim}_{T \to \infty} (1/T). \; \Sigma (\Theta_t^* - \overline{\Theta})^{*2}}$$

Substituting the values of Θ_t^* and $\overline{\Theta}^*$ in the numerator and the underlying reduced form expression for log e_t we get:

$$\text{Plim}_{T \to \infty} (\hat{a}^* - a) = \frac{-(1/(a + b - 1)) \cdot \sigma_{v_1}^2}{D}$$

where $D = \text{Plim}_{T \to \infty} (1/T) \; \Sigma (\Theta_t^* - \overline{\Theta}_t^*)^2$

Now, $D = \text{Plim}_{T \to \infty} (1/T). \; \Sigma [(\Theta_t - p \cdot \Theta_{t-1}) - (\overline{\Theta}_t - p \cdot \overline{\Theta}_{t-1})]^2$

$= \text{Plim}_{T \to \infty} (1/T). \; \Sigma [(\Theta_t - \overline{\Theta}_t) - p \cdot (\Theta_{t-1} - \overline{\Theta}_{t-1})]^2$

On simplification this reduces to:

$$[(1+p^2) - 2b \cdot \hat{d}_{\Theta_t, \Theta_{t-1}}] \cdot \text{Plim}_{T \to \infty} (1/T) \cdot \Sigma (\Theta_t - \overline{\Theta}_t)$$

where $\hat{d}_{\Theta_t, \Theta_{t-1}} = \dfrac{\Sigma (\Theta_t - \overline{\Theta}_t)(\Theta_{t-1} - \overline{\Theta}_{t-1})}{\Sigma (\Theta_t - \overline{\Theta}_t)^2}$

which is the least squares estimates of the slope coeffcients between Θ_t and Θ_{t-1}. Since Θ_{t-1} is simply the one period lagged value of Θ_t, $\hat{d}_{\Theta_t, \Theta_{t-1}}$ is normally not statistically significantly different from 1.

Thus for large samples:

$$D \approx (1-p)^2 \cdot \Plim_{T \to \infty} \left(\frac{1}{T}\right) \cdot \Sigma(\Theta_t - \overline{\Theta}_t)^2$$

$$\approx (1-p)^2 \cdot Q$$

Using this large sample approximation,

$$\Plim_{T \to \infty} (\hat{a}^* - a) = \frac{- \dfrac{1}{(a+b-1)} \cdot \dfrac{\sigma_v^2}{(1-p)^2}}{Q} \qquad \dots(IV-4)$$

Comparing (IV-3) and (IV-4) we can make some interesting conclusions:

(i) If $-1 < p < 0$, then $(1 - p^2) < (1 - p)^2$ and the bias in the uncorrected autocorrelation case is greater than for the case where quasi first difference transformation has been made to correct the autocorrelation. Note that in both cases we are applying OLS.

(ii) If $1 > p > 0$, then $(1 - p^2) > (1 - p)^2$ and the bias in the uncorrected case is less than that for the corrected one. Of course, once again we are applying OLS to both cases and so simultaneous equation bias is unattended.

Figure I shows how the asymptotic bias varies as a function of p, where corrections for serial correlation have or have not been made. This is represented for some arbitrary value of:

$$k = \frac{-(1/(a\ b-1)) \cdot \sigma_v^2}{Q} = 0.25 \text{ (say)}.$$

The dramatic increase of bias for the range $1 > p > 0$ indicates how serious could be the potential bias. Very often in econometric studies with time series data we face the existence of potential positive auto-correlation. Also in many of those (like the studies on international trade elasticities) we recognize the existence of potential simultaneity but without characterizing its exact nature and to make things simple apply OLS to get the parameter estimators. If we find evidence of auto-correlation (say $p > 0$) of first order normally we make necessary trans-formations to correct the situation. It is evident from Figure I that this could have potentially disastrous effects on the overall asymptotic bias.

Figure I: The Direction of Bias in the Presence of Autocorrelation for Uncorrected (dotted line) and corrected cases
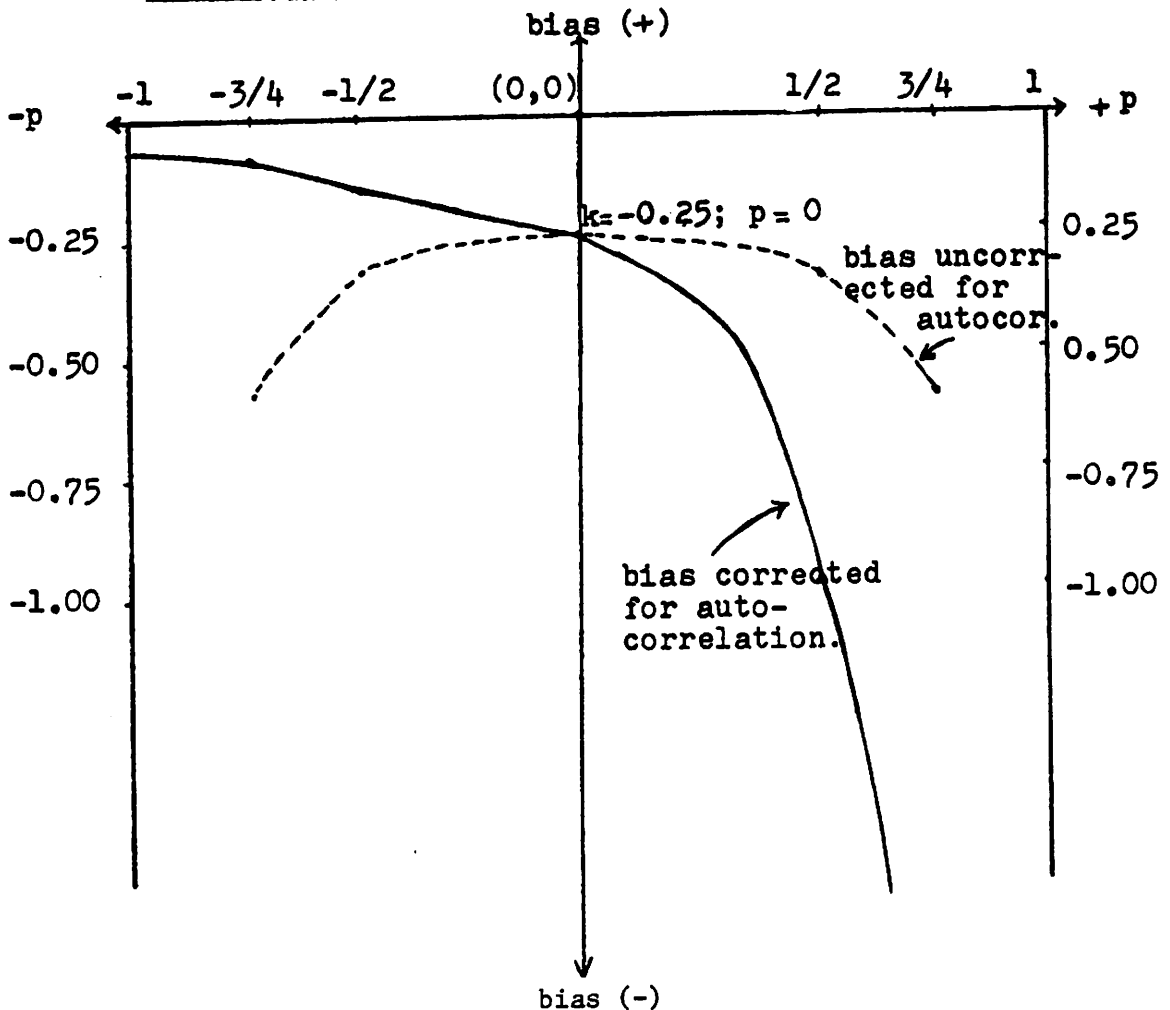
VI. Conclusions

In this paper we have discussed the direction of bias in the conventional estimates of the import and export demand functions. This possibly could have caused underestimation of the values of the trade elasticity parameters which have also been supported by Japanese data. Although most of the arguments as specified in Section I, have been demonstrted in terms of specific models, most of these would possibly be valid under various other formulations. If we have multi-sectoral models, once again the trade balance condition would act as a binding constraint and the arguments should apply equally (although the algebra gets somewhat intractible). The observation regarding the effects of autocorrelation is valid for other models of this type. Its implications go beyond the context of estimating the international trade elasticity parameters and is of relevance to a wide class of potential applications to other studies.

Model I:

This model can be written as:

$$\log ex'_t = \log (E_t/E_0) = C'_1 + a \log (PE_t/e_t) + v_{1t}$$

$$\log m'_t = \log (M_t/M_0) = C'_2 + b.\log (PM_t/e_t) + v_{2t}$$

$$\log PM_t + \log m'_t + \log M_0 = \log PE_t + \log ex'_t + \log E_0$$

Through appropriate substitutions the reduced form expression for $\log e_t$ (= e'_t) can be expressed as:

$$e'_t = \frac{(a.C'_1+ex_0) - (m_0+C'_2)}{a-b} + \frac{(1+a).pe_t}{a-b} - \frac{(1+b).pm_t}{a-b} + \frac{(v_{1t}-v_{2t})}{a-b}$$

where $ex_0 = \log E_0$, $m_0 = \log M_0$, $C'_1 = \log C_1$, $C'_2 = \log C_2$

$pm_t = \log PM_t$, $pe_t = \log PE_t$, and $e'_t = \log e_t$.

The OLS estimator of export demand elasticity:

$$\hat{a} = \frac{\Sigma (e''_t - \bar{e}'') \cdot (ex'_t - \bar{ex}')}{\Sigma (e''_t - \bar{e}'')^2}$$

where $e''_t = \log (PE_t/e_t)$ and $\bar{e}''$ is the corresponding mean.

On substitution:

$$\text{Plim} (\hat{a} - a) = \frac{\underset{T \to \infty}{\text{Plim}} (\frac{1}{T}) \Sigma (v_{1t} - \bar{v}_1) \cdot (e_t'' - \bar{e}'')}{\underset{T \to \infty}{\text{Plim}} (\frac{1}{T}) \Sigma (e_t'' - \bar{e}'')^2}$$

$$= \frac{\underset{T \to \infty}{\text{Plim}} (\frac{1}{T}) \Sigma (v_{1t} - \bar{v}_1) \cdot (pe_t - \bar{P}_e)}{\underset{T \to \infty}{\text{Plim}} (\frac{1}{T}) \Sigma (e_t'' - \bar{e}'')^2} - \frac{\underset{T \to \infty}{\text{Plim}} (\frac{1}{T}) \Sigma (v_{1t} - \bar{v}_1) \cdot (e_t' - \bar{e}')}{\underset{T \to \infty}{\text{Plim}} (\frac{1}{T}) \Sigma (e_t'' - \bar{e}'')^2}$$

On probability limit the first term on the right hand side would vanish as $T \to \infty$; substituting the expression for the reduced form relation in the non-vanishing second term we get (II-4) in Section II. Analogous process with the import supply elasticity estimator (b) would yield the relation (II-5).

Model II:

The reduced form expression for $\log e_t$ corresponding to Model II is:

$$\log e_t = \frac{1}{(a + b - 1)} [(C_2' + \log D_t - ex_0) + (1 - b) \cdot pm_t + b \cdot \log PD_t$$

$$- a \cdot \log\Pi_t + (v_{2t} - v_{1t})]$$

where $C_2' = b \cdot \log (d/(1 - d))$ is a constant and other terms are as defined before in the text and in the appendex earlier.

The expressions for the asymptotic bias can then be obtained through appropriate substitutions and taking the probability limits.

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