# Posture-Dependent Projection-Based Force Reflection Algorithms for Bilateral Teleoperators 

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# Posture-Dependent Projection-Based Force Reflection Algorithms for Bilateral Teleoperators 

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by

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Graduate Program
in
Electrical and Computer Engineering

A thesis submitted in partial fulfillment of the requirements for the degree of Master of Engineering Science

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## Abstract

It was previously established that the projection-based force reflection (PBFR) algorithms improve the overall stability of a force reflecting teleoperation system. The idea behind the PBFR algorithms is to identify the component of the reflected force which is compensated by interaction with the operator's hand, and subsequently attenuate the residual component of the reflected force. If there is no a priori information regarding the behaviour of the human operator, the PBFR gain is selected equal to a sufficiently small constant in order to guarantee stability for a wide range of human operator responses. Small PBRF gains, however, may deteriorate the transparency of a teleoperator system. In this thesis, a new method for selecting the PBFR gain is introduced which depends on the human operator posture. Using an online human posture estimation, the introduced posture-dependent PBFR algorithm has been applied to a teleoperation system with force feedback. It is experimentally demonstrated that the developed method for selection of the PBFR gain based on human postures improves the transparency of the teleoperator system while the stability is preserved. Finally, preliminary results that deal with an extension of the developed methods towards a more realistic model of the human arm with 4 degrees of freedom and three dimensional movements are presented.

Keywords: Teleoperation system, stability, transparency, passivity, small gain theorem, projection-based force algorithm, velocity and force transmission rates.

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To Yalda, Giti and
Memory of $M$.

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## List of Notations

## Notation

| $q, \dot{q}, \ddot{q}$ | Generalized coordinate, velocity and acceleration |
| :--- | :--- |
| $v_{h}$ | Human velocity |
| $v_{l}, v_{r}$ | Local-remote manipulator velocity |
| $v_{r d}$ | Delayed remote manipulator velocity |
| $f_{e}, f_{e n v}$ | Environment force |
| $f_{l}$ | Local manipulator force |
| $f_{l d}$ | Delayed local force |
| $\varphi_{e n v}$ | Interaction component of reflected force |
| $M\left(q_{l}\right), M\left(q_{r}\right)$ | Local-remote inertia(metric) tensor |
| $C(q, \dot{q})$ | Centrifugal/Coriolis matrix |
| $G(q)$ | Gravity vector filed |
| $\Gamma_{i j k}$ | Christoffel symbols |
| $L$ | Lagrangian |
| $K$ | Kinetic energy |
| $\tau$ | Torque/input control |
| $H$ | Hybrid matrix |
| $\langle\rangle$, | Inner product |
| $\langle,\rangle_{t}$ | Finite time inner product |
| $\\|\cdot\\|_{p}$ | $p$-norm |
| $\lambda_{m i n}, \lambda_{m a x}$ | Maximum and minimum eigenvalues |
| $Z(s)$ | Impedance transformation |
| $S(s)$ | Scattering transformation |
| $S^{*}$ | Adjoint of scattering matrix |
| $T, T_{i}(t), \tau(t)$ | Constant and variable time delay |
| $\gamma()$. | gain of a nonlinear system |

## Notation

| $\operatorname{Sat}_{[a, b]}(x)$ | Saturation function |
| :--- | :--- |
| $\sup$ | Supremum |
| $\lim \sup _{t \rightarrow \infty}$ | Limit supremum |
| $\theta$ | Joint angles of a manipulator |
| $J$ | Jacobian matrix |
| $J^{T}$ | Jacobian transpose |
| $\beta_{u}$ | Velocity transmission rate in direction $u$ |
| $m_{u}$ | Effective mass in direction $u$ |
| $K_{p}, K_{d}$ | PID controller coefficients |
| $\phi_{e n v}$ | Projected force component in PBFR |
| $\alpha$ | Gain of PBFR |

# List of Acronyms 

PBFR Projection based force reflection
ISS Input-to-state stability
IOS Input-to-output stability
WIOPS Weak input-to-output practical stability
RTT Round-Trip Time

## List of Figures

2.1 Teleoperator system; $v_{h}, v_{l}, v_{r d}, v_{r}$ represent human velocity, local ve-locity, delayed velocity received at remote manipulator, and remote ve-locity. $f_{e}, f_{r}, f_{l d}, f_{l}$ are environment force, remote force, delayed forcereceived at the local manipulator (master), and the local force. . . . . 7
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## Chapter 1

## Introduction

Telerobotics is a part of robotics which deals with remote control of robots. An important part of telerobotics is teleoperation, which means operating at a distance. A teleoperator system consists of two separate manipulators, called a local manipulator and a remote manipulator which communicate information, usually position, velocity and/or force, through a communication channel. The remote manipulator follows the local manipulator to execute a task on a remote environment. When the remote manipulator is in contact with its environment, it may be desirable to send the contact force information to the local manipulator, since this provides the human operator with better understanding of the contacted environment and hence can improve the performance of the system during task execution.

Although force feedback provides the human operator with a better understanding of contact tasks, at the same time it may create problems such as instability of the system due to unwanted motion caused by the reflected force from the contacted environment. In certain tasks where a high level of precision is needed, such as telesurgery, handling hazardous materials, etc., this instability may result in dangerous situation or even cause irreversible damage. Projection-Based Force Reflection (PBFR) algorithms were introduced to address this issue [58, 59, 60]. The idea behind PBFR algorithms is to decompose the force reflection signal into interaction and motion-generating components, and subsequently attenuate the latter while applying the former in full. The interaction component of the reflected force is defined as

$$
\varphi_{e n v}:=\operatorname{Sat}_{[0,1]}\left\{\frac{\bar{f}_{h}^{T} f_{e n v}}{\left|\bar{f}_{h}\right|^{2}+\epsilon}\right\} \bar{f}_{h}
$$

where $f_{\text {env }}$ is the force feedback from the remote side, $\bar{f}_{h}$ is the estimate of the human force applied to the haptic device, and $\epsilon>0$ is a small number introduced to avoid the ambiguity arising when $\bar{f}_{h}=0$. The algorithm finds the interaction component as the projection of force feedback on the direction of the human force with magnitude bounded by the magnitude human force. The PBFR algorithm suggests to generate the force reflection signal as a convex combination of the direct force feedback and the projection-based component:

$$
f_{r}=\alpha f_{e n v}+(1-\alpha) \varphi_{e n v}
$$

where $\alpha \in[0,1]$. It is shown [61] that the PBFR algorithm improves the stability of the force reflecting teleoperator system with haptic interface without considerable transparency deterioration. In addition, it has been shown 61] that the algorithm guarantees the convergence of the reflected force to the contact force in teleoperator systems when in contact with the environment. Because of the fact that there is no a priori information of the human operator forces, the PBFR gain $\alpha$ is chosen small enough to cope with a wide range of human operator responses. Although a small PBFR gain $\alpha>0$ implies stability of the system, it increases the transient time and hence decreases the performance of the system. Therefore, selecting the gain so that it keeps a reasonable level of stability and transparency at the same time will be a question of interest and is the core of this thesis. To be more precise, we look at an issue that has not been discussed in earlier research works on PBFR algorithm which is the effect of the human posturing on the stability of a teleoperator system when the PBFR algorithm is applied. The main idea is to adjust the PBFR gain $\alpha$ based on the human postures (or, more precisely, based on the force transmission ratio of the human hand in the direction of the reflected force, which is roughly speaking a measure of the human capability for compensating an external force). Our method suggests that there is no need to select a very small constant as the PBFR gain during
a task; instead, one can select the gain according to human postures, with small values only selected for the cases where the human hand cannot compensate for the reflected force. The method is applied to a single-local single-remote force feedback teleoperator system using a webcam to detect the human posture online and update the gain at each instant in time during execution of a task. Our suggested method is supported by simulations and experimental results which show how the performance of the system is improved in comparison with the usual PBFR algorithm.

### 1.1 Contributions

The main objectives/contributions of this thesis can be summarized as follows:

- Survey of the research studies on a teleoperator system from a control theoretic point of view which covers the stability and transparency issues associated with a teleoperator system with force feedback. Two approaches, i.e., passivitybased and small gain methods are discussed and the (selected) related issues are addressed as well as suggested solutions.
- Development of a new method for on-line selection of PBFR gain based on the human operator postures (instead of the non-posture dependent gain applied in the earlier related works).
- Improvement of the transparency of a force reflecting teleoperator system, using the new posture dependent projection-based gain and online posture estimation, while the stability of the system is preserved, compared to the projection-based force reflection algorithm with a non-posture dependent gain.


### 1.2 Thesis outline

Here is the outline of this thesis. Chapter 2 is a literature review of the stability and transparency of a teleoperator system with force feedback and also of the two main
approaches to study stability of the system, i.e., passivity based and small gain methods. In Chapter 3, we review elements of task manipulability and also investigate the effect of human posturing on the performance of a haptic interface. In Chapter 4, a new posture-dependent projection-based force reflection gain dependent on human postures is introduced and experimental results are provided to demonstrate the performance of the system with the new posture-dependent PBFR gain versus constant PBFR gain. Chapter 5 is devoted to applying the mentioned posture-dependent gain to teleoperation systems, using an online human posture estimation. Chapter 6 discusses a generalization of our method to a more realistic model of the human arm with 4 degrees of freedom and 3-dimensional movement, unlike the 2-DOF planar model of the arm adopted in the earlier chapters. Finally, in Chapter 7, the thesis contribution as well as possible future work are briefly summarized.

## Chapter 2

## A Survey of Teleoperation

Over the past few decades the progress in telerobotics has changed our vision as well as our expectation of robots. The idea that a robot can be controlled in a remote environment brings a number of capabilities with extensive amount of applications, from undersea to space exploration, and from robotic telesurgery and high precision assembly to handling heavy loads and hazardous materials.

An important part of telerobotics is teleoperation, that is operating at a distance. A teleoperator system usually consists of two separate manipulators, called local manipulator and remote manipulator which are connected through a communication channel. The local and remote manipulators exchange information (usually, force, position and/or velocity) through this communication channel. The remote manipulator, which might have some degrees of autonomy, will follow the local manipulator to execute a task on a remote environment (Figure 2.1). A teleoperation system with one local and one remote manipulator is called single-local single-remote teleoperation system and a teleoperation system with more than one local and more than one remote manipulators is called multi-local multi-remote teleoperation system. When the remote manipulator is in contact with an environment, in order to provide a better understanding of the contact environment for the human operator at the local side and hence achieve a higher level of performance, it is frequently desirable to send the interaction force information to the local manipulator. Such a force feedback, however, may cause a number of difficulties such as instability which will be explained later in this chapter. For example, the current commercial telesurgical minimally invasive surgery (MIS) systems do not provide force feedback due to the above
mentioned issues. However, there are experimental results demonstrating that the haptic feedback improves the task performance. Two main requirements associated with the force reflecting teleoperation systems are stability and transparency of the system. There are several definitions of stability such as Lyapunov stability, inputoutput stability, input-to-state (input-to-output) stability, etc.; some of them will be explained later in this chapter. A rigorous definition of transparency will be presented later in this chapter; informally, a system is called transparent if the received signals at local (respectively, remote) side are the same as sent signals from remote (respectively, local) side. Transparency also can be defined in terms of impedance matching, in the sense that the transmitted to human impedance is equal to the environmental impedance [42]. In this case, the difference between these two impedances can be considered as a measure of transparency. Cooperation between local and/or remote manipulators during a task execution is advantageous in many applications such as robotic telesurgery, high precision assembly, heavy loadings and handling hazardous materials. In these tasks, lack of stability and/or transparency may result in unsuccessful task execution and even irreversible damage. It is known that [72], in the presence of the delay in the communication channel, the teleoperation system may become unstable. In some cases, communication delay is a known constant but this does not hold in the case of teleoperation over the Internet which has recently been widely accepted as a communication medium. The time delay function in the case of communication over the Internet is typically time-varying and unknown. The stabilization techniques used in the case of constant communication delays will not guarantee stability of the system in the presence of time-varying delays. On the other hand, the transparency of the system, which is usually a conflicting goal with the stability, can be achieved by a different approach. In addition to the stability and transparency issues of the teleoperator systems, the possibility of data dropouts in the communication channel is another problem that might occur because of the Internet-based teleoperation. In the case where some packets are lost, it may be


Figure 2.1: Teleoperator system; $v_{h}, v_{l}, v_{r d}, v_{r}$ represent human velocity, local velocity, delayed velocity received at remote manipulator, and remote velocity. $f_{e}, f_{r}, f_{l d}, f_{l}$ are environment force, remote force, delayed force received at the local manipulator (master), and the local force.
useful to forget the old packet and send new packets which contain the recent information [75], but significant amount of data dropouts will cause discontinuity of the reference trajectories and forces transmitted between the local and the remote manipulators.

In the next section, we discuss some recent results that address the issues of the teleoperation systems in the presence of the communication delay, as well as two main approaches to the design of teleoperator systems, i.e., passivity-based and small-gain-based approaches, to tackle problems arising in this area.

### 2.1 Passivity based approach

A challenging problem in teleoperation systems design is to achieve stability and transparency of the system at the same time which are frequently conflicting goals; therefore, usually some trade-off between stability and transparency is required. To find an accurate relationship between these two determining factors of performance is a challenging problem.

An important step in the design of control algorithms in robotics is determination of the dynamics of the system. Dynamics (or equations of motion) of a robotic
system in our context is given by

$$
\begin{equation*}
\sum_{i} m_{k j}(q) \ddot{q}_{j}+\sum_{i, j} \Gamma_{i j k} \dot{q}_{i} \dot{q}_{j}+g_{k}(q)=\tau_{k}, \quad k=1,2, \cdots, n, \tag{2.1}
\end{equation*}
$$

where $q=\left(q_{1}, q_{2}, \cdots, q_{n}\right)^{T}$ describes trajectory of the motion on a $n$-dimensional smooth manifold. Velocity and acceleration of the motion are given by the time derivatives $\dot{q}, \ddot{q}$. We recall that an $n$-dimensional manifold is a (topological) space that locally looks like the Euclidean space $\mathbb{R}^{n}$. In the cases that are of interest for our work, the manifold is usually a compact smooth sub-manifold of a $N$-dimensional Euclidean space, for some positive integer $N$. For example, in the case of a two degrees-of-freedom manipulator shown in Figure 2.2, the configuration space is the compact 2-dimensional manifold $S^{1} \times S^{1}$ (Figure 2.3), where $S^{1}$ is the unit circle defined as $S^{1}=\{z \in \mathbb{C}| | z \mid=1\}$.


Figure 2.2: A 2-DOF manipulator.

The elements $m_{i j}$ are the components of the metric/inertia matrix (more precisely, the type 2-covariant metric tensor). The Christoffel symbols $\Gamma$ 's are given by the partial derivatives of the metric tensor, $m_{i j}$, as

$$
\begin{equation*}
\Gamma_{i j k}:=\frac{1}{2}\left(m_{k j, i}+m_{k i, j}-m_{i j, k}\right) . \tag{2.2}
\end{equation*}
$$

Here, as it is conventional in differential geometry and physics, subscripts will be used for the covariant components of a tensor, and $f_{i}=\frac{\partial f}{\partial q_{i}}$. The gravity term is given by $g_{k}$ and $\tau_{k}$ is the external force.


Figure 2.3: Two dimensional torus $S^{1} \times S^{1}$

In control literature, the dynamics are typically described by the Euler-Lagrange equations of the form

$$
\begin{equation*}
M(q) \ddot{q}+C(q, \dot{q}) \dot{q}+G(q)=\tau \tag{2.3}
\end{equation*}
$$

where $\tau=\left(\tau_{1}, \tau_{2}, \cdots, \tau_{n}\right)^{T} \in \mathbb{R}^{n}$ is the external force, $M(q)=\left(m_{i j}(q)\right)$ is the inertia matrix, $C(q, \dot{q})=\left(c_{i j}(q, \dot{q})\right)$ is called the Coriolis and centrifugal matrix given by

$$
c_{i j}=\sum_{l} \Gamma_{l i j} \dot{q}_{l},
$$

and $G(q)=\left(g_{1}(q), \cdots, g_{n}(q)\right)^{T}$ is the gravity vector field. These equations of motion can be obtained for example by applying the Euler-Lagrange equations to the Lagrangian $L=K-P$ of the system, where $K$ and $P$ are the kinetic and potential energy of the system, respectively. We assume that the kinetic energy is given by a quadratic form defined by a symmetric positive definite matrix $M=\left(m_{i j}(q)\right)$ as

$$
K=\frac{1}{2} \sum_{i j} m_{i j}(q) \dot{q}_{i} \dot{q}_{j},
$$

and the potential energy depends only on the coordinate, i.e. $P=P(q)$. The Euler-

Lagrange equations are given by

$$
\frac{d}{d t} \frac{\partial L}{\partial \dot{q}_{i}}-\frac{\partial L}{\partial q_{i}}=\tau_{i}, \quad i=1,2, \cdots, n
$$

The terms $\frac{\partial L}{\partial \dot{q}_{i}}$ and $\frac{\partial L}{\partial q_{i}}$ are called generalized momentum and generalized force, respectively. In telerobotics, the dynamics of the system is given by a pair of equations of motion, one for the local manipulator and one for the remote manipulator as follows,

$$
\left\{\begin{array}{l}
M_{l} \ddot{q}_{l}+C_{l} \dot{q}_{l}+G_{l}=\tau_{l}  \tag{2.4}\\
M_{r} \ddot{q}_{r}+C_{r} \dot{q}_{r}+G_{r}=\tau_{r} .
\end{array}\right.
$$

In this chapter, we use subscripts $l, r$ for local and remote manipulators and drop the argument of the matrices, whenever there is no place for confusion. For example, we use $M\left(q_{l}\right)$ for the local inertia matrix instead of $M_{l}\left(q_{l}\right)$ and also we use the brief notation of $M_{l}:=M\left(q_{l}\right)$.

### 2.1.1 General properties of the robot dynamics

The matrices in the above equations of motion enjoy a few important properties that we mention here.

Property 1. The matrix $\dot{M}-2 C$ is skew symmetric, that is, for any vector $v$, one has $(v,(\dot{M}-2 C) v)=0$. Here, $(u, v)=\sum_{i} u_{i} v_{i}$ is the usual inner product for vectors $u, v \in \mathbb{R}^{n}$. In fact the skew-symmetric property of $\dot{M}-2 C$ follows from the equation

$$
\dot{m}_{i j}-2 c_{i j}=\sum_{k}\left(\frac{\partial m_{j k}}{\partial q_{i}}-\frac{\partial m_{i k}}{\partial q_{j}}\right) \dot{q}_{k}
$$

We note that, for any size $n$ matrix $A$ and vector $v \in \mathbb{R}^{n}$, by expanding with respect to an orthonormal basis, one has $(v, A v)=\sum_{i, j} a_{i j} v_{i} v_{j}$, where $v_{i}$ 's and $a_{i j}$ are components of the vector $v$ and the matrix $A$. This sum simply vanishes if $A$ is
skew symmetric. The skew symmetric property of $\dot{M}-2 C$ is also equivalent to $\dot{M}=C+C^{T}$.

An immediate consequence of this property is the passivity of the system. Consider a nonlinear system given by

$$
\left\{\begin{array}{l}
\dot{x}(t)=f(x(t), u(t))  \tag{2.5}\\
y(t)=h(x(t))
\end{array}\right.
$$

where $x(t)=\left(x_{1}(t), \cdots, x_{n}(t)\right)$ is the state of the system which belongs to Euclidean space $\mathbb{R}^{n}$ at each instant time $t, u(\cdot):[0, \infty) \rightarrow \mathbb{R}^{p}$ is the input, control or disturbances depending on the context, and $y(t)=\left(y_{1}(t), \cdots, y_{p}(t)\right)$ is the output of the system for some positive integers $n, p$. The function $f: \mathbb{R}^{n+p} \rightarrow \mathbb{R}^{n}$ is locally Lipschitz and $h: \mathbb{R}^{n} \rightarrow \mathbb{R}^{m}$ is continuous. We omit the argument $t$ usually. The solution $x\left(t, x_{0}, u\right)$ is considered on a maximal interval $\left[0, t_{\max }\left(x_{0}, u\right)\right)$ for the initial state $x_{0}$ and the input $u$. For a system with no input,

$$
\begin{equation*}
\dot{x}(t)=f(x(t)) \tag{2.6}
\end{equation*}
$$

the solution is denoted by $x\left(t, x_{0}\right)$. The zero input system associated with $\dot{x}=f(x, u)$ is the system $\dot{x}=f(x, 0)$.

Definition 2.1.1. The system given by (2.5), is called passive if

$$
\begin{equation*}
\langle y, u\rangle \geq-\beta \quad \text { for some } \beta \geq 0 \tag{2.7}
\end{equation*}
$$

Here, the inner product of the two signals $v(t)=\left(v_{1}(t), \cdots, v_{p}(t)\right)$ and $w(t)=$ $\left(w_{1}(t), \cdots, w_{p}(t)\right)$ is given by

$$
\langle v, w\rangle=\int_{0}^{\infty} v^{T} w d t
$$

This integral shows the energy absorbed by the system, and $\beta$ at the right side of the definition can be thought as the initial energy stored at the system. To avoid the complexity arising from the Lebesgue integral, one can consider just piecewise continuous signals. As it can be seen, passivity is an input-output property of the system. In fact, it measures the exchanged power/energy between the interconnected subsystems. The passivity of a dynamical system can also be defined equivalently as follows [62]:

Definition 2.1.2. The dynamical system (2.5) is called passive if there exists a positive semidefinite $C^{1}$-function $V: \mathbb{R}^{n} \rightarrow \mathbb{R}$, such that

$$
\dot{V} \leq u^{T} y
$$

The function $V$ is called a storage function of the system. Establishing the passivity of a system allows one to consider sum of storage functions of the subsystems as a Lyapunov function candidate for the system to prove the stability, assuming the environment and the human operator are passive. We recall that the system (2.5) is Lyapunov stable at $x=0$ if for any $\epsilon>0$, there exists a $\delta>0$ such that $|x(0)|<\delta$ yields that $x(t)<\epsilon$ for all $t>0$. It is well known that the existence of a so called Lyapunov function, which is a positive definite function on a region containing origin with a negative definite time derivative, implies the Lyapunov stability. We will refer to 62] for more details on generalizations of this definition.

Note that here and in the following, we use the notation $|\cdot|$ for the standard Euclidean norm, i.e., $|v|=\sqrt{v_{1}^{2}+\cdots+v_{n}^{2}}$ for a vector $v \in \mathbb{R}^{n}$.

We will use the notation $\langle v, w\rangle_{t}$ for the integration on the finite time interval $[0, t]$, that is,

$$
\langle v, w\rangle_{t}=\int_{0}^{t} v^{T} w d t
$$

The norm induced by $\langle$,$\rangle is called the L_{2}$-norm and is denoted by $\|v\|_{2}=\langle v, v\rangle^{1 / 2}$. The $L_{p}-$ norm is defined by

$$
\|v\|_{p}=\left(\int_{0}^{\infty}|v|^{p}\right)^{\frac{1}{p}}
$$

A bounded signal is a signal whose norm, which is appropriately chosen depending on the context, is finite. When the Lebesgue integral is applied, the space of signals with bounded $L_{p}-$ norm is called the $L_{p}$ space, which is a Banach (and hence a normed) space. In the special case that $p=2$, this will also be a Hilbert space with a complete set of orthonormal basis. This is a crucial fact in proving Parsvall's identity, which gives us the ability to find the norm of a signal in the frequency domain instead of time domain.

As it is already mentioned, passivity of a system is equivalent to the existence of a storage function [36], which is closely related to the Lyapunov function. An important feature of passive systems is the fact that the negative feedback interconnection of passive systems is also passive and stable [36].

Now, let us go back to the dynamic equations of a teleoperation system.
Property 2. The system (2.5) is passive, i.e., the following holds for some $\beta \geq 0$ and for all $t_{1}>0$,

$$
\int_{0}^{t_{1}} \dot{q}(t)^{T} \tau(t) d t \geq-\beta
$$

The quantity $\dot{q}^{T} \tau$ is the power flow of the system. To see the passivity of the system, one can consider the total energy of the system $E=K+P$. Then the time derivative of the total energy is given by

$$
\dot{E}=\dot{q}^{T} M \ddot{q}+\frac{1}{2} \dot{q}^{T} \dot{M} \dot{q}+\dot{q}^{T} \partial_{q} P=\dot{q}^{T} \tau
$$

where $\partial_{q} P$ is the gradient of the potential energy. Therefore, one has

$$
\int_{0}^{t_{1}} \dot{q}(t)^{T} \tau(t) d t=E\left(t_{1}\right)-E(0) \geq-E(0)
$$

which yields the passivity of the system for $\beta=E(0)$. Note that, the passivity holds for both local and remote manipulators.

Property 3. Another important fact about the symmetric positive definite matrix $M$ is the following inequality which holds at each point $q$

$$
\lambda_{\min }(M(q))|v|^{2} \leq v^{T} M(q) v \leq \lambda_{\max }(M(q))|v|^{2} .
$$

Here $\lambda_{\min }(M(q)), \lambda_{\max }(M(q))>0$ are minimum and maximum eigenvalues of the positive definite matrix $M(q)$. This is a point-wise property, but if the configuration space is a compact space, which is the case when joints are revolute and/or prismatic with finite range of motion, we have

$$
\lambda_{1}|v|^{2} \leq v^{T} M(q) v \leq \lambda_{2}|v|^{2}, \quad \text { for some } \quad \lambda_{1}, \lambda_{2}>0
$$

Property 4. The next important property is about the matrix $C$ and its boundedness. In fact,

$$
|C(q, \dot{q}) \dot{q}|<K|\dot{q}|^{2}, \quad \text { for some } K>0
$$

Property 5. The last property that we mention here is, the equations of motion can be linearly parameterized as $M \ddot{q}+C \dot{q}+G=Y(q, \dot{q}, \ddot{q}) \Theta$. For details on proof of the mentioned properties, reader is referred to [41, 62].

It was around 1980s that it was realized that a local-remote teleoperator system can be modeled as a two port network (Figure 2.4), which was already introduced and studied in literatures [8, [26, 27, 28]. The two-port network properties can be analyzed using different models, such as impedance, hybrid and admittance matrices.


Figure 2.4: Two-port network

The impedance matrix for the 2-port network is relating velocities to forces as

$$
\binom{f_{1}}{f_{2}}=Z(s)\binom{v_{1}}{v_{2}}
$$

Each component of $Z_{i j}(s)$ has an expression in terms of local, remote impedance and controllers. Here, $f_{i}, v_{i}$ are the force and velocity signals in frequency domain.

The hybrid matrix of the two-port network system, $H$, is defined as

$$
\binom{f_{1}}{-v_{2}}=\left(\begin{array}{ll}
h_{11} & h_{12}  \tag{2.8}\\
h_{21} & h_{22}
\end{array}\right)\binom{v_{1}}{f_{2}}
$$

where $h_{i j}$ are the components of the matrix $H$; the above equation is written in the frequency domain. Each entry $h_{i j}(s)$ of the hybrid matrix has a natural meaning related to the system [32]. In fact, diagonal entries are input and inverse output impedance and off-diagonal entries are force and velocity scaling.

### 2.1.2 Stability vs. Transparency

The relation between stability (passivity) and transparency has been studied for several configurations of teleoperator systems [42], including position-position and position-force schemes. It was found that passivity and transparency are conflicting goals. This means that, selecting design parameters appropriately, a system might become more passive but at the cost of transparency deterioration, or one might
achieve a more transparent system at the expense of lower stability margin. The following is a well known example that demonstrates this trade-off. Considering the usual two-port network given by the system equations (2.8), the ideal hybrid matrix which is related to the perfect transparency is given as follows:

$$
H_{i d e a l}=\left(\begin{array}{cc}
0 & 1 \\
-1 & 0
\end{array}\right)
$$

On the other hand, it can be shown that the system given by this matrix is marginally stable, and even a small disturbance (such as small communication delays) might make it unstable. This can be seen by analyzing the system given by

$$
H=\left(\begin{array}{cc}
0 & e^{-s T}  \tag{2.9}\\
-e^{-s T} & 0
\end{array}\right)
$$

In the next section, while discussing scattering transformation, we will show, using the relation between the scattering matrix and the hybrid matrix, that the above system is not passive. We refer the reader to [26] for methods of how to obtain the ideal transparency matrix that is mentioned above. The hybrid method has also been used to address several issues of 2-port networks, such as four-channel setup [42].

### 2.1.3 The effect of time delay on stability and transparency

The communication delay for teleoperator systems over a long distance is almost unavoidable despite today's technological development. The existence of time delay affects performance of a teleoperator system, including its stability and transparency characteristics. The effect of delay on transparency can be explained as follows: while the human operator performs a task and is in contact with an environment, he/she cannot feel the feedback until after a round trip time delay, which excludes the possibility for the system to be ideally transparent (for example, in the sense of
impedance matching [42]).
On the other hand, the effect of communication delay on stability has been shown first in [25], where it was demonstrated that the system in the presence of time delay as small as 0.1 sec may become unstable.

### 2.1.4 Scattering transformation

A breakthrough step in investigating the stability of a teleoperator system in presence of time delay was the work [3], where the authors considered transmitting the socalled incident and reflected scattered information instead of the original velocity and force signals in the presence of a constant time delay (see also [4]). The scattering transformation $S$ is given by the formula

$$
f-v=S(f+v)
$$

where the force and the velocity $f, v$ are considered to be $L_{2}$ bounded signals. A relation between the scattering matrix and the hybrid matrix in the 2-port channel case is given by the formula

$$
S=\left(\begin{array}{cc}
1 & 0 \\
0 & -1
\end{array}\right)(H-I)(H+I)^{-1}
$$

Here, $I$ is the identity matrix. It is not hard to see that the passivity of the system which is given by $\langle v, f\rangle \geq 0$ is equivalent to $\|S\|_{\infty} \leq 1$. Note that $\|A\|_{\infty}=$ $\sqrt{\lambda_{\max }\left(A A^{*}\right)}$. With this in mind, we can see that the system given by 2.9 is not passive (and hence stability is not guaranteed), since its scattering transformation is given by
$S=\left(\begin{array}{cc}1 & 0 \\ 0 & -1\end{array}\right)\left(\begin{array}{cc}-1 & e^{-s T} \\ -e^{-s T} & -1\end{array}\right)\left(\begin{array}{cc}1 & e^{-s T} \\ -e^{-s T} & 1\end{array}\right)^{-1}=\left(\begin{array}{cc}-\tanh (s T) & \cosh ^{-1} s T \\ \cosh ^{-1} s T & \tanh (s T)\end{array}\right)$
and its norm is

$$
\|S\|_{\infty}=\sup _{\omega}(|\tan (\omega T)|+|\sec (\omega T)|)=\infty .
$$

In [3], the teleoperation system is described by equations

$$
\begin{aligned}
& M_{l} \dot{v}_{l}+B_{l} v_{l}=F_{h}-F_{r e f} \\
& M_{r} \dot{v}_{r}+B_{r} v_{r}=F_{r}-F_{e}
\end{aligned}
$$

where $F_{r}=K_{r} \int\left(v_{r d}-v_{r}\right) d t+D_{r}\left(v_{r d}-v_{r}\right)$ and $F_{r e f}=F_{r}, v_{r d}=v_{l}$. Note that here $M_{l}, M_{r}, B_{l}, B_{r}$ are constant matrices with appropriate size. It is shown that the natural control law $F_{r e f}(t)=F_{r}(t-T)$ and $v_{r d}(t)=v_{l}(t-T)$ results in an unbounded scattering matrix. Here, $T$ is a constant time delay. However, the following control law

$$
\begin{aligned}
& F_{r e f}(t)=F_{r}(t-T)-v_{r d}(t-T)+v_{l}(t) \\
& v_{r d}(t)=v_{l}(t-T)-F_{r}(t)+F_{r e f}(t-T)
\end{aligned}
$$

yields the scattering matrix

$$
S=\left(\begin{array}{cc}
0 & e^{-s T} \\
e^{-s T} & 0
\end{array}\right)
$$

which has norm equal to 1 and hence, the communication channel is passive. Indeed, one can see that

$$
S S^{*}=\left(\begin{array}{ll}
1 & 0 \\
0 & 1
\end{array}\right)
$$

and $\lambda_{\max }\left(S S^{*}\right)=1$.

### 2.1.5 Wave variables method

There is a similar concept to scattering which is called wave variables and was introduced in [46, 47]. It addresses the passivity issue of the communication channel with the time delay. In this method, one transfers the signals $U$ and $V$ instead of the original signals $f$ and $v$, Figure 2.5, which are given by


Figure 2.5: Wave variable method

$$
U_{1}=\frac{b v_{l}+f_{l}}{\sqrt{2 b}}, \quad V_{2}=\frac{b v_{r}-f_{r}}{\sqrt{2 b}},
$$

where $b>0$ is called the characteristic wave impedance. A simple computation shows that

$$
2\left\langle v_{l}, f_{l}\right\rangle=\left\langle U_{1}, U_{1}\right\rangle-\left\langle V_{1}, V_{1}\right\rangle, \quad 2\left\langle v_{r}, f_{r}\right\rangle=\left\langle U_{2}, U_{2}\right\rangle-\left\langle V_{2}, V_{2}\right\rangle
$$

Note that $U_{2}(t)=U_{1}(t-T)$ and $V_{2}(t)=V_{1}(t+T)$. Therefore, the absorbed energy over a finite time interval $\left[0, t_{1}\right]$ can be computed as (when $T_{1}=T_{2}=T$ )

$$
\begin{aligned}
2 E\left(t_{1}\right) & =2\left\langle v_{1}, f_{1}\right\rangle_{t_{1}}-2\left\langle v_{2}, f_{2}\right\rangle_{t_{1}} \\
& =\int_{0}^{t_{1}}\left(\left|U_{1}(t)\right|^{2}-\left|U_{2}(t)\right|^{2}-\left|V_{1}(t)\right|^{2}+\left|V_{2}(t)\right|^{2}\right) d t \\
& =\int_{0}^{t_{1}}\left(\left|U_{1}(t)\right|^{2}-\left|U_{1}(t-T)\right|^{2}-\left|V_{2}(t-T)\right|^{2}+\left|V_{2}(t)\right|^{2}\right) d t \\
& =\int_{t_{1}-T}^{t_{1}}\left(\left|U_{1}(t)\right|^{2}+\left|V_{2}(t)\right|^{2}\right) d t \geq 0,
\end{aligned}
$$

hence the communication channel is passive.

### 2.1.6 Geometry behind the scattering method

A geometrical approach to the passivity concept has been introduced in [33], which clarifies the geometry behind the notion. The main idea is as follows: let $D=V \times V^{*}$, where $V$ is a vector space with a dual $V^{*}$. For $\left(f_{i}, e^{i}\right) \in D$, one can define a nondegenerate 2 form, $\langle,\rangle_{+}$as

$$
\left\langle\left(f_{1}, e^{1}\right),\left(f_{2}, e^{2}\right)\right\rangle_{+}=e^{1}\left(f_{2}\right)+e^{2}\left(f_{1}\right)
$$

Here, $e^{i}\left(f_{j}\right)$ is the dual pairing. Fixing a basis $B=\left(v_{1}, \cdots, v_{n}\right)$ for the space $V$, one can define a basis for $D$ as $\bar{B}=\operatorname{diag}\left(B, B^{*}\right)$, where $B^{*}$ is the associated dual basis. Then the + paring has the following form in components

$$
T_{i j}=\bar{B}_{*}\left(\begin{array}{cc}
0 & I \\
I & 0
\end{array}\right) \bar{B}_{*}^{T}
$$

Using a metric on $V$, associated to the characteristic impedance $B_{*} Z B_{*}^{T}$, one can define a two-contravariant tensor on $D$ as

$$
Y^{l i}=\bar{B}\left(\begin{array}{cc}
Z^{-1} & 0 \\
0 & Z
\end{array}\right) \bar{B}^{T}
$$

Now, considering the (1,1)-tensor

$$
L_{j}^{l}:=Y^{l i} T_{i j}
$$

one can see that eigenvalues of $L$ are $\pm 1$, so one has the following decomposition of $D=D_{+} \oplus D_{-}$, where $D_{+}, D_{-}$are associated eigenspaces. This implies that there is a unique way to express $(f, e) \in D$ as sum of two elements $s_{ \pm} \in D_{ \pm}$.

Theorem 2.1.1. [33] Given any $(f, e) \in D$ and any positive definite, symmetric 2-covariant tensor $Z$, the following holds

$$
e(f)=1 / 2\left\|s_{+}\right\|^{2}-1 / 2\left\|s_{-}\right\|^{2}
$$

where $s_{ \pm} \in D_{ \pm},(f, e)=s_{+}+s_{-}$and $\|\cdot\|_{ \pm}$are the induced norms on $D_{ \pm}$.

As explained in [33], this orthogonal decomposition is fundamental since it shows that we can write the power flow algebraically as the sum of positive and negative power (power going in the opposite directions) only on the two scattering variables.

Although the scattering transformation and wave variables methods make the communication channel passive, at the same time there are some inefficiencies, such as asymptotically divergent behaviour of velocity, wave reflection, tracking and transparency issues. Here, we mention briefly a few results that address these issues. In [4], a control scheme was introduced that implies the ultimate zero convergence of the velocity. Another issue is the wave reflection phenomenon which appears when
the impedance of the terminal load differs from the characteristic impedance. This causes a wave reflection which decreases the performance of the bilateral teleoperation system. The idea of impedance matching as explained in [49] is introduced to address this issue. As it is discussed in [9], impedance matching at both sides of the communication channel impacts on position tracking, while considering the matching only at the remote side will cause a smaller position drift.

The problem of mismatch between the transmitted power from one side of the teleoperator to the other side, such that the human can handle the environment power is addressed in [11, 39] using a scaling scheme.

In general, in the scattering method and wave variables, no position information is transmitted (just velocity and force signals are communicated), that might result in a position mismatch between the remote and the local systems. This is mostly because of initial transient response or numerical roundoff errors and the situation might become worse in the presence of a time varying delay. We refer the reader to [48, 50], for suggested solutions of the position drift problem in the presence of the constant/time varying delay. In [48], a method is suggested for the constant time delay case which consists of transmitting a combination of the wave signal and its integral and separating them at the receiver side. In [50], an attempt has been made to address the time varying case which proposes to transmit the integral of $u$ and $u^{2}$, where the former contains the position information and the latter contains the energy information. Sending the position information in addition to the scattered signals, Chopra et al. [22, 23], introduced a control scheme that guarantees position tracking. On the other hand, in [23] a new control scheme is proposed that sends the position information explicitly, both from local side to remote side and vice versa, together with velocity and force, which results in boundedness of the position error under appropriate assumptions, as well as velocity convergence to zero. It is also proved that, in free motion, the tracking error converges to zero.

### 2.1.7 Time varying delay

In all of the above mentioned results, the communication delay is considered to be a known constant delay, but in the presence of time varying delays which occur when the Internet is used as a communication medium, these methods are not applicable. In 44], using a time dependent gain of interconnection and imposing appropriate conditions on that, it is proved that the energy will not be generated through the information exchange in the channel and hence the communication channel will be passive (Figure 2.6).


Figure 2.6: Time dependent gain for interconnection medium with time varying delay

In fact, considering the time varying delays $T_{i}(t)$ for $i=1,2$, corresponding to the forward and the backward delays, and $g_{i}(t)$ for the communication gains, the absorbed energy can be computed as

$$
\begin{aligned}
E(t) & =\int_{t_{1}-T_{1}(t)}^{t_{1}}\left|U_{1}(t)\right|^{2} d t+\int_{t_{1}-T_{2}(t)}^{t_{1}}\left|V_{2}(t)\right|^{2} d t+\gamma_{1} \int_{0}^{t_{1}-T_{1}(t)}\left|U_{1}(t)\right|^{2} d t \\
& +\gamma_{2} \int_{0}^{t_{1}-T_{2}(t)}\left|V_{2}(t)\right|^{2} d t
\end{aligned}
$$

where

$$
\gamma_{i}(t)=\frac{1-\dot{T}_{i}(t)-g_{i}(t)^{2}}{1-\dot{T}_{i}(t)}
$$

It is clear that $1-\dot{T}_{i} \geq g_{i}^{2}$, implies passivity and therefore stability. This requires information about the rate of change of the time delay. Along the same line of research, the boundedness of the position error and position tracking for the time dependent gain are also proved in [19, 51].

In the next section, we discuss another method for investigating performance of a teleoperation system, i.e., the small gain approach.

### 2.2 Small gain approach

As pointed out in the previous section, the presence of even small time delay in the communication channel of the force feedback teleoperation system will cause performance deterioration [72]. It is also known that the stability of a system with a constant time delay cannot guarantee the stability of the teleoperator system with time varying delay. Even if the stability is achieved, performance deterioration will occur [42]. It is known that to achieve stability, when force is reflected, a high level of damping is needed at the local side, but this decreases the transparency of the system, because human does not feel the actual contact force and feels the stabilizing force instead. The idea of time varying damping is suggested in [43, 30, 13], to address this issue. As another approach to the problem, we discuss the small gain approach as introduced in [54]. In this method, one adopts other definitions of stability called input-to-state stability (ISS) and input-to-output stability (IOS) which have been shown to be more flexible and more appropriate for this setup.

First we need to define a couple of concepts. A function $\gamma: \mathbb{R}^{\geq 0} \rightarrow \mathbb{R}^{\geq 0}$ is called a class $\mathcal{K}_{\infty}$ function if it is continuous, strictly increasing and unbounded with $\gamma(0)=0$. The set of all such functions is also denoted by $\mathcal{K}_{\infty}$. A function $\beta(\cdot, \cdot): \mathbb{R}^{\geq 0} \times \mathbb{R}^{\geq 0} \rightarrow \mathbb{R}^{\geq 0}$ is called a class $\mathcal{K} \mathcal{L}$ function if $\beta(\cdot, t) \in \mathcal{K}_{\infty}$ for each $t$, and $\beta(s, t) \searrow 0$, that is, $\beta(s, t)$ decreasingly converges to zero, as $t \rightarrow \infty$.

Definition 2.2.1. The system (2.5) is called ISS if

$$
|x(t)| \leq \beta(|x(0)|, t)+\gamma\left(\|u\|_{\infty}\right),
$$

holds for all the solutions, i.e., all admissible inputs, all initial conditions and all $t \geq 0$.

Here, $\|u\|_{\infty}=\sup _{0 \leq t^{\prime} \leq t}\left|u\left(t^{\prime}\right)\right|$ is the norm supremum of (or maybe depending on the context, essential sup norm of) the input signal. The ISS condition, simply says that the system is Lyapunov stable in zero input case and is state bounded by the input magnitude, such that small inputs result in small states. IOS will be defined accordingly, i.e., the system (2.5) is called IOS if

$$
|y(t)| \leq \beta(|x(0)|, t)+\gamma\left(\|u\|_{\infty}\right),
$$

holds for all admissible inputs, all initial conditions and all $t \geq 0$. For details about this definition as well as generalization, the reader is referred to 65, 66]. Another important property of ISS/IOS is its independence of coordinates, see 65], unlike the exponential stability case.

A sufficient condition for ISS is the existence of a so called ISS-Lyapunov function. Suppose that $D$ is a (simply connected) domain in $\mathbb{R}^{n}$ containing 0 , the $C^{1}$ function $E: D \rightarrow \mathbb{R}$ is said to be an ISS-Lyapunov function if for class $\mathcal{K}$ functions $\alpha_{1}, \alpha_{2}, \alpha_{3}$ and $\chi$, one has

$$
\begin{aligned}
& \alpha_{1}(|x(t)|) \leq E(x(t)) \leq \alpha_{2}(|x(t)|) \quad x \in D, t>0 \\
& \nabla_{x} E \cdot f(x, u) \leq-\alpha_{3}|x(t)| \quad x \in D, u \in D_{u}:|x| \geq \chi(|u|)
\end{aligned}
$$

Here, $\nabla_{x} E=\frac{\partial E}{\partial x}$. The existence of the ISS-Lyapunov function guarantees the ISS of the system. In fact, one can prove the necessity as well, 65]. An important fact that makes the input-to-state stability a useful technique for the teleoperation systems is
the small-gain theorem, which says that interconnection of two ISS systems is ISS in an appropriate sense. For details as well as generalization, we refer the reader to [54], where it is shown that in the presence of the time delay, the small gain theorem holds but under mild conditions on the time delay function. First, the forward and backward time delays $T_{1}(t), T_{2}(t)$ have an upper bound $\tau(t)$ such that

$$
\begin{equation*}
\tau\left(t_{2}\right)-\tau\left(t_{1}\right) \leq t_{2}-t_{1}, \quad \text { for } \quad t_{1}, t_{2} \geq 0 \tag{2.10}
\end{equation*}
$$

Secondly,

$$
\begin{equation*}
t-\max \left\{T_{1}(t), T_{2}(t)\right\} \rightarrow \infty \quad \text { as } \quad t \rightarrow \infty \tag{2.11}
\end{equation*}
$$

The small-gain approach has the advantages that stability even in the presence of a time varying delay is guaranteed and also position tracking can be achieved for sufficiently smooth delay.

### 2.3 Projection-based force reflection algorithms

Although the force feedback information, especially when the remote robot is in contact with an environment will improve that task performance when the reflected force cannot be compensated by the human operator completely, an unwanted motion will be caused which makes the system unstable. The effect of this motion, which is called the induced motion, can be lowered by down-scaling the reflected force, but this is at the cost of transparency deterioration of the system [40]. The projection-based force reflection (PBFR) algorithm, is introduced to address this issue [58, 59, 60]. Here, we briefly discuss the idea of the PBFR algorithm and we will get back to this concept in the next chapter with more details. The idea of the PBFR algorithm is to project the reflected force on the human force direction which can be compensated by human and attenuate the remaining part which causes the unwanted motion. In
fact, one can decompose the environment force as $f_{e n v}=\varphi_{e n v}+\left(f_{e n v}-\varphi_{e n v}\right)$, where

$$
\begin{equation*}
\varphi_{e n v}=\operatorname{Sat}_{[0,1]}\left(\frac{f_{e n v}^{T} f_{h}}{\left|f_{h}\right|^{2}}\right) f_{h}, \quad \text { if } \quad f_{h} \neq 0 \tag{2.12}
\end{equation*}
$$

and $\varphi_{\text {env }}=0$ if $f_{h}=0$. Here, $f_{\text {env }}$ and $f_{h}$ are environment and human forces. The saturation function is defined as

$$
\operatorname{Sat}_{[a, b]}(x)= \begin{cases}a & x<a  \tag{2.13}\\ x & x \in[a, b] \\ b & x>b\end{cases}
$$

The algorithm can be written as

$$
\begin{equation*}
\varphi_{e n v}=\operatorname{Sat}_{[0,1]}\left(\frac{f_{e n v}^{T} f_{h}}{\max \left\{\epsilon,\left|f_{h}\right|^{2}\right\}}\right) f_{h} \tag{2.14}
\end{equation*}
$$

where the sufficiently small $\epsilon>0$ is to remove the singularity caused by $f_{h}=0$. The above algorithm is a rule to identify the interaction component of the external force as will be explained here. When $\left|f_{h}\right|^{2} \geq \epsilon$ and $0 \leq \frac{f_{e n v}^{T} f_{h}}{\left|f_{h}\right|^{2}} \leq 1$, one can see that $\varphi_{e n v}$ is the projection of $f_{e}$ on $f_{h}$. The lower saturation at 0 guarantees that $-\varphi_{e n v}$ and $f_{h}$ are directed opposite to each other and the upper saturation limit at 1 ensures that $\left|\varphi_{e n v}\right|$ does not exceed $\left|f_{h}\right|$. So it is clear that the algorithm computes the interaction component of $f_{e n v}$ which is directed against the human force $f_{h}$ and its magnitude is bounded by the magnitude of human force.

The PBFR algorithm suggests to generate the force reflection signal as a convex combination of the direct force feedback and the projection based component, as follows

$$
f_{r}=\alpha f_{e n v}+(1-\alpha) \varphi_{e n v}
$$

for $\alpha \in[0,1]$. This relation also can be written as

$$
f_{r}=\varphi_{e n v}+\alpha\left(f_{e n v}-\varphi_{e n v}\right)
$$

which states that PBFR algorithm reflects the projection-based component $\varphi_{e n v}$ and attenuates the residual momentum generating component $f_{e n v}-\varphi_{e n v}$. The component $\varphi_{e n v}$ which can be compensated by the human hand is transmitted in full, but the remaining part $f_{e n v}-\varphi_{e n v}$ which produces the induced motion is attenuated by the gain factor $\alpha$. An appropriate $\alpha \in[0,1]$ will guarantee the overall stability in a suitable sense. For example, in [60], a general stability result for bilateral teleoperator systems with PBFR algorithm has been proved. In fact, it has been shown that the overall stability of the teleoperator system can be obtained under some assumptions on subsystems, communication channel and dynamics of the human operator. More results related to the PBFR algorithm will be provided in the next section.

In applying the PBFR algorithm, one needs to estimate the human force to know whether the operator is capable of compensating the reflected force. This implies a selection of a small gain $\alpha$ which is suitable for the worst case scenario. The small (and constant) value of the PBFR gain $\alpha$ guarantees stability; however it increases the transient time of the convergence of the reflected force to the actual contact force, and hence transparency deteriorates. An objective of this thesis, that will be discussed through subsequent chapters, is to introduce a new method of selecting the PBFR algorithm gain $\alpha$ depending on human postures.

### 2.4 ISS \& IOS for functional differential equation

The ISS definition has been developed for more general dynamical systems, such as systems described with functional differential equations, especially, delayed differential equations [67]. This idea also is developed for cooperative teleoperator systems with time varying delays and interaction between local manipulators. For the math-
ematical background, the reader is referred to [24]. A delay differential equation with bounded delay is described by

$$
\dot{x}=f\left(t, x\left(\delta_{1}(t)\right), x\left(\delta_{2}(t)\right), \cdots, x\left(\delta_{n}(t)\right)\right)
$$

where

$$
t-r \leq \delta_{i}(t) \leq t \quad \text { for some } r \geq 0, t \geq t_{0}, i=1,2, \cdots, n
$$

The initial condition is of the form of

$$
x(t)=\theta(t), \quad \text { for } t_{0}-r \leq t \leq t_{0}
$$

It is assumed that $f$ is defined on $\left[t_{0}, \beta\right) \times D$ for some $\beta>t_{0}$ and $D \subset \mathbb{R}^{n}$. The conditions on $f$ such that the system has a (unique) solution will be discussed. First, let us mention that, for brevity, we will use a simpler notation $\dot{x}=F\left(t, x_{t}\right)$. One needs to give meaning to $F$ and $x_{t}$. First, for the trajectory $x$ and a given $t$, define $x_{t}:[-r, 0) \rightarrow \mathbb{R}^{n}$ by

$$
x_{t}(\sigma)=x(\sigma+t) \quad \text { for } \quad-r \leq \sigma \leq 0
$$

Obviously, continuity of $x(\cdot)$ on $[t-r, t)$ implies continuity of $x_{t}$. Now, let $\mathcal{C}_{A}$ to be the set of all continuous functions from $[-r, 0)$ to $A$. Hence, if $x(\cdot)$ is continuous on $[t-r, t) \rightarrow A$, then $x_{t} \in \mathcal{C}_{A}$. We consider $F$ as a function $F: I \times \mathcal{C}_{A} \rightarrow \mathbb{R}^{n}$. For $\psi \in \mathcal{C}_{A}$, define $\|\psi\|_{r}=\sup _{-r \leq t \leq 0}\|\psi(t)\|$. Note that, not all $\mathcal{C}_{A}$ are linear spaces, but $\mathcal{C}$ is and $\|.\|_{r}$ is a norm on $\mathcal{C}$. Then, $F$ is called Lipschitz on $\mathcal{E} \subset I \times \mathcal{C}$ with constant $K>0$ if

$$
\left|F(t, \psi)-F\left(t, \psi^{\prime}\right)\right| \leq K| | \psi-\psi^{\prime} \|_{r},
$$

for $(t, \psi),\left(t, \psi^{\prime}\right) \in \mathcal{E}$. The continuity of $F$ on $t$, Lipschitzness on $\psi$ and boundedness on delay imply the existence and uniqueness of the corresponding functional solution of a
differential equation. Taking this to account, Teel proved a Razumikhin-type theorem using a nonlinear small gain theorem for the system described by the functional differential equation 67]

$$
\dot{x}=f\left(t, x_{d}(t), w_{d}(t)\right), \quad x_{d}\left(t_{0}\right)=\xi
$$

where for a given function $x:\left[-t_{d}, \infty\right) \rightarrow \mathbb{R}^{m}$, one defines $x_{d}(t)(\cdot)$ as $x_{d}(t)(\tau)=$ $x(t-\tau)$ on the interval $\left[0, t_{d}\right]$ for some $t_{d} \geq 0$. This is generalized in 55] for multiinput multi-output systems coupled with disturbances and a small gain condition has been obtained using minimal cycles of the gain matrix. A system described by the functional differential equation

$$
\dot{x}=f\left(x_{t}, u_{t}\right)
$$

is said to be input-to-state stable at $t=0$ with $t_{d} \geq 0$ and gain $\gamma \in \mathcal{K}$, if there exist positive constants $\Delta_{x}, \Delta_{u}$ such that $\sup _{t \in\left[-t_{d}, 0\right]}|x(t)|<\Delta_{x}$ and $\sup _{t \geq-t_{d}}|u(t)|<\Delta_{u}$ imply that solutions of the system are well defined for all $t \geq 0$, and for some $\beta \in \mathcal{K}_{\infty}$ one has

$$
\sup _{t \geq 0}|x(t)| \leq \max \left(\beta\left(\sup _{s \in\left[-t_{d}, 0\right]}|x(s)|\right), \gamma\left(\sup _{s \geq-t_{d}}|u(t)|\right)\right)
$$

and

$$
\lim \sup _{t \rightarrow \infty}|x(t)| \leq \gamma\left(\lim \sup _{t \rightarrow \infty}|u(t)|\right)
$$

This definition can be extended to multi-input systems and also to multi-input multioutput systems and also to input to output stability. Using the small-gain theorem for the interconnection of two IOS systems (described by functional differential equations) [59, and also projection-based the force reflection algorithm, Polushin and co-authors showed that the overall stability of the system can be achieved without increasing the damping at the local side, and in addition, "almost perfect" tracking can be achieved. In [58], the problem of significant data dropouts which may result
in discontinuity of the reference trajectory transmitted through the communication channel is discussed, In fact, the author proposed a control scheme including a filter that provides smooth approximation of a possibly discontinuous reference trajectory and the overall stability is also guaranteed by IOS small gain theorem. Along this line of research, in [68] Takhmar, et al., suggested to separate the high and low frequency signals that play different roles in stability and attenuate high frequencies which cause instability. In [55], using a new WIOPS (weakly input-to-output practical stability) small gain theorem, the authors designed a (multi-master multi-slave) force-reflecting teleoperator system which is demonstrated to be stable in the presence of multiple network-induced communication constraints. In fact, the stability analysis of a teleoperator system is analyzed in the presence of irregular communication delays and communication errors.

### 2.5 Conclusions

In this chapter, we reviewed teleoperator systems from a control theoretic point of view, which means that we mostly focus on stability and transparency issues arising in the study of teleoperator systems. The mathematical theory for teleoperator systems and related definitions, such as stability, passivity and transparency are explained in details as needed to follow the survey. It is known that force feedback in a teleoperator system provides a better understanding of contact for human operators and therefore, it can improve the performance of the task, but at the same time it causes several issues such as instability of the teleoperator system which might lead to irreversible damage in cases like telesurgery, handling hazardous materials, etc. The problems might become even worse when there exist communication channel delays. We addressed the two main approaches in the study of the performance of a teleoperator system in the presence of the time delays in the communications. These two are passivity-based and small-gain-based methods. These methods and their short-
comings and suggested solutions are addressed. It should be mentioned that there are well known surveys such as [72, 32, 52] which cover the passivity-based approach in control of teleoperator systems.

## Chapter 3

## The Effect of Human Postures on the Stability of Teleoperator Systems

As mentioned in the previous chapter, force feedback in a teleoperation system (particularly when the remote manipulator is in contact with an environment) may be very useful, since it enables the operator to have a better control of the interaction with the environment, but at the same time it might cause harm because of the momentum generated by the reflected force. The component of the reflected force that is compensated by the human hand creates the contact feeling, while the residual part of the reflected force generates the momentum and, potentially, instability. The projection based force reflection (PBFR ) algorithm [59, 61], as will be explained below, suggests to identify these two components of the reflected force, apply the component that can be compensated by the human completely and attenuate the residual part to improve the stability. It is shown that this improves drastically the overall stability of the system [61]. The interaction component of the force is defined as

$$
\varphi_{e n v}:=\operatorname{Sat}_{[0,1]}\left\{\frac{\bar{f}_{h}^{T} f_{e n v}}{\left|\bar{f}_{h}\right|^{2}+\epsilon}\right\} \bar{f}_{h},
$$

where, $f_{\text {env }}$ is the force feedback from the remote side, $\bar{f}_{h}$ is the estimate of the human force applied to the haptic device and $\epsilon>0$ is a small number to avoid the ambiguity arising when $\bar{f}_{h}=0$. The algorithm finds the interaction component as the projection of force feedback on human force with magnitude bounded by human force. The PBFR algorithm suggests that it should be possible to generate the force
reflection signal as a convex combination of direct force feedback and the projection based component:

$$
\begin{equation*}
f_{r}=\alpha f_{e n v}+(1-\alpha) \varphi_{e n v} \tag{3.1}
\end{equation*}
$$

for $\alpha \in[0,1]$. This relation can also be written as

$$
f_{r}=\varphi_{e n v}+\alpha\left(f_{e n v}-\varphi_{e n v}\right)
$$

which states that the PBFR algorithm reflects the projection-based component $\varphi_{\text {env }}$ and attenuates the residual momentum generating component $f_{\text {env }}-\varphi_{\text {env }}$.

It is shown that 61] the PBFR algorithm improves the stability of the force reflecting teleoperator system with haptic interface for interaction with environment. It also has been shown that [61] the algorithm guarantees the force convergence to the contact force in the teleoperator systems when in contact with the environment. In this method, since there is no prior information of the reflected force, the PBFR gain will usually be selected as a small constant to cope with a wide range of human behaviors. Although a small PBFR gain $\alpha$ implies the stability of the system, but on the other hand the performance of the system deteriorates, since it increases the transient time.

An issue that has not been discussed in earlier research work on the PBFR algorithm and is the main idea of this chapter, is to discuss the effect of the human postures on the PBFR algorithm. In fact, there are positions at which human hand is more capable of compensation the reflected force. For instance, consider when the human hand is in a horizontal position (perpendicular to the body), it can handle a horizontal force easier comparing to a vertical force. Our main objective in this chapter is first of all, to quantify the human capability of a task manipulation and, second to demonstrate simulations and experimental results that show that the induced motion by the reflected force can be decreased by the projection-based force reflection algorithm introduced for all human postures.

The structure of this chapter is as follows. First, we review the theoretical background related to task manipulability. Through this part, we define the velocity and force transmission rates as well as velocity and force ellipsoids related to the human hand. In the second part, we present the simulation and experimental results. We consider a 2-DOF manipulator model representing (a simple model of ) the human arm when an external disturbance force is applied. The objective is to show the dependence of the human postures to the unwanted induced motion in response to an external disturbance force. All our simulation has been done in MATLAB. It is worth mentioning that, this is a first step in introducing a new PBFR gain as a function of human posture which will be done in the upcoming chapter.

### 3.1 Theoretical background on task manipulability

In this section, we will investigate the effects of human postures in the stability of the teleoperation system. First we need to quantify the human capability for a task execution. This will be done by using previously introduced concepts of force and velocity transmission rates corresponding to the force and velocity ellipsoids related to the end-effectors. In the following, we recall the definition of different ellipsoids related to the end-effector of the manipulator, such as velocity, force ellipsoid and effective mass ellipsoid as in [10, 73, 74, 37].

### 3.1.1 Velocity and force ellipsoids

We recall the definition of the velocity ellipsoid and force ellipsoid related to the end-effector of an arm/manipulator as discussed in [10].

Velocity ellipsoid: Given a smooth coordinate transformation as

$$
x_{i}=x_{i}\left(\theta_{1}, \cdots, \theta_{n}\right), \quad i=1,2, \cdots, m
$$

with the Jacobian $J=\frac{\partial\left(x_{1}, \cdots, x_{m}\right)}{\partial\left(\theta_{1}, \cdots, \theta_{n}\right)}$, the $n$-sphere

$$
|\dot{\theta}|^{2} \leq 1
$$

will be mapped to the $m$-ellipsoid given by

$$
\begin{equation*}
\dot{x}^{T}\left(J J^{T}\right)^{-1} \dot{x} \leq 1 . \tag{3.2}
\end{equation*}
$$

In fact, in the case $m=n$ and $J$ is invertible, one concludes that this is an easy result of $\dot{x}=J \dot{\theta}$. Since $\dot{\theta}=J^{-1} \dot{x}$,

$$
|\dot{\theta}|^{2}=\dot{\theta}^{T} \dot{\theta}=\left(J^{-1} \dot{x}\right)^{T} J^{-1} \dot{x}=\dot{x}^{T}\left(J^{-1}\right)^{T} J^{-1} \dot{x}=\dot{x}^{T}\left(J J^{T}\right)^{-1} \dot{x}
$$

However, in the case $m<n$, and $J$ is not invertible, this mapping of $|\dot{\theta}|^{2} \leq 1$ to


Figure 3.1: Principle axes of a ellipsoid are given by the eigenvectors of the symmetric matrix associated with the quadratic form of the ellipsoid. The ellipsoid diameters are given by the $1 / \sqrt{\lambda_{i}}$ 's, where $\lambda_{i}$ 's are eigenvalue of the ellipsoid defining symmetric quadratic form.
$\dot{x}^{T}\left(J J^{T}\right)^{-1} \dot{x} \leq 1$ is still true, but one needs to use the generalized (pseudo-) inverse matrix. In the example that we investigate in this chapter there is no need to consider
redundancy, since the $J$ would be invertible. However, in Chapter 6, where we outline a more general case of a 4-DOF human arm model, the role of redundancy is crucial. The ellipsoid given by (3.2) is called the velocity ellipsoid.

Force ellipsoid: It is known that the $\dot{x}=J \dot{\theta}$ implies the following relation between the joint torques and the force at end-effector:

$$
\tau=J^{T} f
$$

Analogous to the velocity ellipsoid, one can define the force ellipsoid. First, note that the sphere

$$
|\tau|^{2} \leq 1
$$

using $\tau=J^{T} f$ will be mapped to

$$
\begin{equation*}
f^{T} J J^{T} f \leq 1 \tag{3.3}
\end{equation*}
$$

This equation will define the force ellipsoid, similarly to the velocity ellipsoid, it can be utilized for force transmission characteristics of the manipulator at a given posture. This is the same ellipsoid that have been employed by Asada and Youcef-Toumi [6, 7] in analysis of the power to force conversion. As it will be explained in the following, the principal axes of these two ellipsoids, velocity and force ellipsoids, are identical, but in a reciprocal fashion.

### 3.1.1.1 Duality between velocity and force ellipsoid

A square matrix $A$ and its inverse $A^{-1}$ share the same eigenvectors with inverse eigenvalues, since, if $A v=\lambda v$ for some $v \neq 0$, then $A^{-1} v=\lambda^{-1} v$. This implies that $J J^{T}$ and $\left(J J^{T}\right)^{-1}$ will share the same eigenvectors with reciprocal eigenvalues, i.e., the velocity and force ellipsoids share the principle axes but with the reciprocal magnitudes. This means that the velocity can be controlled more accurately in the
direction that the manipulator can resist larger disturbance forces, and force can be more accurately controlled in the direction that the manipulator can adopt its motion as quick as possible.

### 3.1.1.2 Transmission velocity and force rates

As it is discussed in [10], although effective capability of the manipulator can be increased by adopting the postures that align the optimal directions with task directions, but this is not sufficient for the overall optimality of the posture. For instance, in a situation such as writing, we would like to control the force vertically and velocity horizontally. Although, a standard form of the ellipsoid (horizontal ellipsoid) is aligned with task directions, but this is not the optimal direction. This shows that a measure of task compatibility is not only the alignment of the optimal directions with human/manipulator's posture, but is a combination of transmission velocity ratio and/or force ratio along the task directions. Given a unit vector $u$, the constant $\gamma$ which makes the vector $\gamma u$ lie on the force ellipsoid is called the force transmission rate (see Figure 3.2.) This implies that


Figure 3.2: The transmission force rate in direction of the unit vector $u$ is the length of the segment line from the centre of ellipse (ellipsoid) to the boundary of the ellipse (ellipsoid) in the direction $u$.

$$
(\gamma u)^{T}\left(J J^{T}\right)(\gamma u)=1
$$

which means

$$
\begin{equation*}
\gamma=\left(u^{T}\left(J J^{T}\right) u\right)^{-\frac{1}{2}} \tag{3.4}
\end{equation*}
$$

The transmission velocity rate will be defined in the same manner by

$$
\begin{equation*}
\beta=\left(u^{T}\left(J J^{T}\right)^{-1} u\right)^{-\frac{1}{2}} \tag{3.5}
\end{equation*}
$$

To achieve the optimal manipulator performance during a task, one should optimize the transmission rate in the task directions [10]. More precisely, suppose that we would like to control the force in $l$ different directions given by unit vectors $u_{1}, u_{2}, \cdots, u_{l}$ and control the velocity in $n-l$ directions given by unit vectors $u_{l+1}, \cdots, u_{n}$. Also assume that the force and velocity transmission ratio in direction $u_{i}$ are denoted by $\gamma_{i}$ and $\beta_{i}$, respectively. Then, the index of task compatibility is defined as

$$
c=\sum_{i=1}^{l} w_{i} \gamma_{i}^{ \pm 2}+\sum_{i=l+1}^{n} w_{i} \beta_{i}^{ \pm 2}
$$

The + sign will be used when the magnitude is of interest and the - sign when accuracy. The weighting factors $w_{i}$ 's indicate the relative magnitude and accuracy requirements in the respective task directions. An important problem is searching a posture that maximize this index which is a weighted sum of squares of transmission ratios and squares of reciprocals of transmission ratios.

### 3.1.2 Remarks

1. Redundant case: A real model of a human arm comes with redundancies. We refer the reader to [70, 71, 69, 72] for the extensive amount of research that has been done on the redundant single- and dual-arm manipulators. Note that in the case of redundancy, the higher dimensional joint velocity sphere will be mapped to lower
dimensional task velocity ellipsoid. As it is known, the inverse problem has infinitely many solutions. The general solution can be given by

$$
\dot{\theta}=J^{+} \dot{x}+\left(I-J^{+} J\right) \kappa,
$$

where $J^{+}$is the pseudo inverse of $J$ given by $J^{+}=J^{T}\left(J J^{T}\right)^{-1}$ and $\kappa$ is an arbitrary vector in $\mathbb{R}^{n}$. Note that $\left(I-J^{+} J\right) \kappa \in \operatorname{null}(J)$, that is, $J\left(I-J^{+} J\right) \kappa=0$. This means that $\left(I-J J^{+}\right) \kappa$ does not produce a motion of the end-effector. The fact that $\kappa$ can be selected arbitrarily, suggests that there might be a solution for $\kappa$ that maximize the performance. For more details on such a solution of $\kappa$, the reader is referred to [73].

## 2. Velocity ellipsoid, optimal directions and task compatibility directions:

 The velocity ellipsoid given by (3.2) is what Yoshikawa [73] called the "manipulability ellipsoid" ; it was proposed that the kinematic redundancy should be used to maximize the volume of this ellipsoid. For avoiding singularity this is of absolute importance, since the volume of this ellipsoid will be zero at singularities. For the non-redundant case, which is our main interest in this chapter, the volume of this ellipsoid can also be used [74]. It is not hard to see that the eigenvectors of $\left(J J^{T}\right)^{-1}$ determine the diameters of the ellipsoid and the square root of the inverse of associated positive eigenvalues determine the magnitude of diameters. The velocity transmission characteristics of a manipulator at a certain posture can be understood by this velocity ellipsoid. Note that the optimal direction of control velocity is along with the diameter associated to the minimum eigenvalue (longer diameter) and on the other hand the optimal direction for effecting velocity is in the direction associated to the maximum eigenvalue. It is worth mentioning, that as explained in [10], the optimal force control is not always aligned with the task direction, but it is posture dependent. For examples of this phenomena and more details, we refer the reader to [10].3. Effective mass: Here, we introduce another type of ellipsoid associated with the end-effector as discussed in [37. We recall that the quantifying approach to the task manipulability that we adapted in this chapter (and which will be used in upcoming chapters) is based on the velocity and force transmission rates associated with velocity and force ellipsoids of the end effector. However, this can also be done using through the effective mass and the belted ellipsoid introduced by Khatib [37] and explained below.

As mentioned already, a critical part of control of robots is considering the dynamics of the system. Recall that the dynamics of a $n$-DOF manipulator are given by the equation (2.3). Because of the fact that the impact force at some point at the end-effector is of interest, it is useful to have a formulation of dynamics of the system with respect to the operational point [37]. In fact, it is convenient to take position and orientation of the end effector with respect to the base as new coordinate, $x$, with dimension $m$. When the dimension of the configuration space described in joint space and task space are the same, i.e., $m=n$, the Jacobian of the change of coordinates is a square matrix. The kinetic energy in the operational space can be described as a new quadratic form $\frac{1}{2} \dot{x}^{T} \Lambda(x) \dot{x}$. Hence, the relation between the two matrices $\Lambda$ and $M$ is given by

$$
\Lambda=J^{-T} M J^{-1}
$$

where $J$ is the Jacobian of $x=x(q)$, i.e., $J=\frac{\partial x}{\partial q}$. The dynamics with respect to the operational point will be given by the following equation of motion

$$
\Lambda(x) \ddot{x}+\mu(x, \dot{x}) \dot{x}+p(x)=f
$$

A manipulator is said to be redundant if $n>m$, that is the number of degrees of freedom of the joint configuration space is more than that of task space. Obviously, the coordinate of the end effector in the redundant case does not uniquely determine the configuration. Hence, the dynamic of the entire system cannot be explained just
with the dynamics of the end-effector. In this case, the pseudo kinetic energy matrix $\Lambda$ is given by

$$
\Lambda^{-1}=J M^{-1} J^{T}
$$

In the non-redundant case, this equation describes the dynamics of the whole system, but in the redundant case, it only describes the dynamics of the end-effector, that is dynamics projected to the lower dimensional task space. In fact, the remainder dynamics will affect the $n-m$ dimensional null space caused by redundancy. The relation between the joint torques and the force applied to the end effector is given by

$$
\tau=J^{T} f+\left(I-J^{T} J^{T^{\#}}\right) \tau_{0}
$$

where $\tau_{0}$ is an arbitrary joint torques which will be projected in the null space of the pseudo inverse $J^{T^{\#}}$. It is known that the pseudo inverse is not unique, but there is only one that is consistent with the dynamics [37], and is given by

$$
\bar{J}=M^{-1} J^{T} \Lambda .
$$

Here to be consistent with the dynamics means

$$
J M^{-1}\left(I-J^{T} J^{T^{\#}}\right) \tau_{0}=0
$$

In fact, $\bar{J}$ is the generalized inverse of the Jacobian corresponding to the $\dot{x}=J \dot{q}$, which minimizes the manipulator kinetic energy [37]. Finally, the equation of motion in the end-effector set of coordinates will be obtained by pre-multiplying the joint space equation of motion by $\bar{J}$.

To analyze the inertial properties of the manipulator, usually one can consider the end effector translational and rotational tasks. For positioning at the end-effector, one defines

$$
\Lambda_{v}^{-1}=J_{v} M^{-1} J_{v}^{T}
$$

where $J_{v}$ is the linear velocity associated Jacobian. The matrix $\Lambda^{-1}$, encodes the end effectors translational response to the force.

### 3.1.2.1 Mass ellipsoid and effective mass

It is possible to analyze inertial properties of the end-effector in an arbitrary direction. If $u$ is a unit vector in the required direction, then the effective mass, $m_{u}$ at the operational point in the direction of $u$ is given by 37]

$$
m_{u}^{-1}=u^{T} \Lambda_{v}^{-1} u
$$

The effective inertia will be defined accordingly.
A possible representation of the mass properties of the end effector is the mass ellipsoid given by

$$
v^{T} \Lambda_{v}^{-1} v=1
$$

The inertial ellipsoid is defined in the same manner. In [37], Khatib defines another ellipsoid-like illustration of the mass properties, which is called belted ellipsoid and given by

$$
\frac{v^{T} \Lambda_{v}^{-1} v}{|v|}=1
$$

This is obtained, in fact, by re-scaling vectors $v$ on the mass ellipsoid by the $v|v|$.

### 3.1.2.2 Effective mass from mathematical point of view

In this part, we give a more detailed mathematical explanation of the notion of effective mass. Given a symmetric transformation $T$ on $\mathbb{R}^{2}$, what does $m_{u}=(T u, u) /|u|^{2}$ represent?

In fact, when $T$ is symmetric, we can write $T=U^{T} D U$ for some orthogonal matrix $U$ and a diagonal matrix $D$. Then

$$
(T u, u)=\left(U^{T} D U u, u\right)=(D U u, U u)=\left(D u_{1}, u_{1}\right)=\sum \lambda_{i} x_{i}^{2}
$$

where $u_{1}=\left(x_{1}, \cdots, x_{n}\right)$. One knows that $|U u|=|u|$, so maximum of $m_{u}$ will be $\lambda_{\max }$.

### 3.2 Simulation results of the human arm response to a disturbance force for different postures

As discussed earlier in this chapter, our goal is to investigate the effect of human arm postures on the performance of a teleoperator system. In order to do so, we first investigate the capability of human hand to compensate a reflected force from the remote environment. We consider a simple model of human arm as a two degrees of freedom (2-DOF) model shown in Figure 2.2. We explain the dynamics of the system and the associated velocity ellipsoids of the end-effector for different postures. Then, using the simulation that has been performed in MATLAB, we present evaluation results of the effect of the human postures on the stability of a teleoperator system under a disturbance force.

It is known that for the 2-DOF manipulator modelled as Figure 2.2, the change of coordinate from joint space to task space is given by:

$$
\left\{\begin{array}{l}
x=l_{1} \cos \theta_{1}+l_{2} \cos \theta_{12} \\
y=l_{1} \sin \theta_{1}+l_{2} \sin \theta_{12}
\end{array}\right.
$$

This yields the following Jacobian (of the change of coordinate)

$$
J=\frac{\partial(x, y)}{\partial\left(\theta_{1}, \theta_{2}\right)}=\left(\begin{array}{cc}
-l_{1} \sin \theta_{1}-l_{2} \sin \theta_{12} & -l_{2} \sin \theta_{12} \\
l_{1} \cos \theta_{1}+l_{2} \cos \theta_{12} & l_{2} \cos \theta_{12}
\end{array}\right)
$$

where $\theta_{12}=\theta_{1}+\theta_{2}$. The dynamics of the above 2-DOF system is given by

$$
M(\theta) \ddot{\theta}+C(\theta, \dot{\theta}) \dot{\theta}+G(\theta)=\tau
$$

with the inertia matrix $M(\theta)$ described as

$$
\begin{aligned}
& m_{11}=m_{1} l_{1 c}^{2}+m_{2}\left(l_{1}^{2}+l_{2 c}^{2}+2 l_{1} l_{2 c} \cos \theta_{2}\right)+I_{1}+I_{2} \\
& m_{12}=m_{21}=m_{2}\left(l_{2 c}^{2}+l_{1} l_{2} \cos \theta_{2}\right)+I_{2} \\
& m_{22}=m_{2} l_{2 c}^{2}+I_{2}
\end{aligned}
$$

the Coriolis/centrifugal matrix $C(\theta, \dot{\theta})$ as

$$
\begin{aligned}
& c_{11}=h \dot{\theta}_{2}, \\
& c_{12}=h\left(\dot{\theta}_{2}+\dot{\theta}_{1}\right), \\
& c_{21}=-h \dot{\theta}_{1}, \\
& c_{22}=0,
\end{aligned}
$$

and the gravity vector field $G(\theta)$ :

$$
\begin{aligned}
& g_{1}=\left(m_{1} l_{1 c}+m_{2} l_{1}\right) g \cos \theta_{1}+m_{2} g l_{2 c} \cos \theta_{12}, \\
& g_{2}=g m_{2} l_{2 c} \cos \theta_{12}
\end{aligned}
$$

Here $h=-m_{2} l_{1} l_{2 c} \sin \theta_{2}, g=9.81 \mathrm{~m} / s^{2}$ and $l_{i c}=l_{i} / 2, i=1,2$. In the following, the arm masses are considered $m_{1}=m_{2}=0.5 \mathrm{Kg}$, and arm lengths are $l_{1}=l_{2}=0.5 \mathrm{~m}$.

As we discuss in details in the following, for different human postures, a disturbance force in two different directions $x$ and $y$ is applied to the human arm; the human reaction, displacement, as well as velocity ellipsoid at each case are shown. Here, for the stability of the system, we consider a PD-controller as

$$
\tau=K_{p}\left(\theta-\theta_{d}\right)+K_{d}\left(\dot{\theta}-\dot{\theta}_{d}\right)
$$

where $\theta_{d}$ and $\dot{\theta}_{d}$ are desired position and velocity, $K_{p}$ and $K_{d}$ are positive definite matrices which in our experiments are given by $K_{p}=50 I, K_{d}=25 I$, where $I$ is the
identity matrix of rank 2 . The equation of an ellipse is given as follows

$$
A\left(x-x_{0}\right)^{2}+B\left(y-y_{0}\right)^{2}+C\left(x-x_{0}\right)\left(y-y_{0}\right)=1
$$

where $A, B, C$ are parameters of the ellipse. When the ellipse is given by a quadratic form, as in our case which is given by the symmetric matrix $\left(J J^{T}\right)^{-1}$, parameters $A, B, C$ can be obtained from the eigenvalues and eigenvectors of the quadratic form as following. Suppose $v_{1}, v_{2}$ are eigenvectors of the matrix $\left(J J^{T}\right)^{-1}$ with corresponding eigenvalues $\lambda_{1}, \lambda_{2}$, then parameters of the ellipse with the centre $\left(x_{0}, y_{0}\right)$ is given by

$$
\begin{aligned}
A & =\lambda_{1} \cos ^{2} \phi+\lambda_{2} \sin ^{2} \phi \\
B & =\lambda_{2} \cos ^{2} \phi+\lambda_{1} \sin ^{2} \phi \\
C & =\left(\lambda_{1}-\lambda_{2}\right) \sin 2 \phi
\end{aligned}
$$

Here

$$
\tan \phi=\frac{v_{1} \cdot e_{1}}{v_{1} \cdot e_{2}}
$$

where, $e_{1}=(1,0)$ and $e_{2}=(0,1)$ are the standard basis in $\mathbb{R}^{2}$.
Below, we demonstrate the simulation results. Our simulations include three sets of experiments which will be explained in detail below. For each case, we move the human arm along a trajectory over a period of 10 seconds and a disturbance force will be applied between $t=5 \mathrm{~s}$ and $t=7 \mathrm{~s}$. The associated ellipsoids as well as transmission rates will be investigated. It should be mentioned that we perform three sets of experiments to cover different directions of motions and postures of human arm during a task execution when a disturbance force is applied.

Simulation results: experiment one. We consider a 2-DOF model shown in 2.2 as a simple model of the human arm. As shown in Figure 3.3, we will consider eleven different arm positions in a horizontal line. The disturbance force, $f_{x}$, as shown in Figure 3.4, is applied to the end effector at each of the given positions. The force
is a rectangular pulse with the magnitude 10 N and the duration 2 s , and is applied starting from $t=5 \mathrm{~s}$. The transmission rates $\beta_{x}$ for the given positions are shown in Figure 3.5, it can be seen that $\beta_{x}$ has its highest value in the second position and gradually decreases to its lowest value, which happens at position number 11. This can be understood in the way that in position 1 the human hand is less capable of resisting a horizontal force in comparison with the final position, where the hand is more capable of resisting an external force. The end-effector displacements in the $x$-direction, which is shown in Figure 3.5, agrees with the velocity ellipsoids and transmission rates, in the sense that, a smaller transmission rate $\beta_{x}$ corresponds to a smaller displacement for the same applied force. Under the vertical disturbance force, $f_{y}$, which is shown in Figure 3.6, the transmission rates $\beta_{y}$ and the corresponding $y$ displacement are shown in Figure 3.7. The corresponding velocity ellipsoids and transmission rates $\beta_{x}, \beta_{y}$, are also shown in Figure 3.8 .


Figure 3.3: Experiment 1: The manipulator movement in a horizontal line; eleven positions of the end effector on the horizontal line $y=-0.5$ for $x=-0.4+0.1 i, i=$ $0, \cdots 10$.


Figure 3.4: Experiment 1: The disturbance force in $x$-direction


Figure 3.5: Experiment 1: Transmission rate $\beta_{x}$ (left) and displacement in $x$-direction in the horizontal movement (right)


Figure 3.7: Experiment 1: The transmission rate $\beta_{y}$ (left) and displacement in $y$ direction in the horizontal movement (right)


Figure 3.6: Experiment 1: The disturbance force in $y$-direction

Experiment two. In the second part of our simulation, as shown in Figure 3.9, we will consider eleven different arm positions along a vertical line. The transmission rates $\beta_{x}$ for the positions are shown in Figure 3.10. We apply the same disturbance rectangular force pulse as in the experiment one to the end effector at each of the given positions. It can be seen that at the position that $\beta_{x}$ has its lowest value (Position 3), the minimum displacement will occur, and at the position number 10 , where $\beta_{x}$ is the highest rate among these postures, the maximum displacement happens. The endeffector displacements in the $x$-direction which are shown in Figure 3.10 show that where the transmission rate $\beta_{x}$ is higher, the displacement also has a greater value


Figure 3.8: Experiment 1: Velocity ellipsoids during horizontal movement along the line $y=-0.5$ for $x=-0.4+0.1 i, i=0, \cdots 10$, the red dashed lines show the horizontal and vertical transmission rates $\beta_{x}, \beta_{y}$
during the time period that force $f_{x}$ is applied. Under the vertical disturbance force $f_{y}$ which is shown in Figure 3.6, the transmission rates $\beta_{y}$ and the corresponding $y$ displacement is shown in Figure 3.11. The corresponding velocity ellipsoids are shown in Figure 3.12 .

Experiment three. In this part of our simulation, we will consider eleven different arm positions along a slant line given by $y=-0.5+x$, as shown in Figure 3.13. The corresponding velocity ellipsoids and transmission rates $\beta_{x}, \beta_{y}$, are illustrated in Figure 3.16. The transmission rate $\beta_{x}$ for the positions is shown in Figure 3.14. As in the previous two experiments, the rectangular force $f_{x}$, is applied at each of the given positions. In position that $\beta_{x}$ has its lowest value (Position 3), the minimum displacement will occur, while in position number 10 , where $\beta_{x}$ is the highest rate among these postures, the maximum displacement happens. The end-effector displacement in the $x$-direction which is shown in Figure 3.14 shows that where the transmission rate $\beta_{x}$ is higher, the displacement also has a greater value during the time period that force $f_{x}$ is applied. Under the vertical disturbance force $f_{y}$ which is shown in Figure 3.6, the transmission rates $\beta_{y}$ and the corresponding $y$ displacement is shown in Figure 3.15 .


Figure 3.9: Experiment 2: The manipulator movement in a vertical line; eleven positions of the end effector on the vertical line $x=0.5$ for $y=-0.4+0.1 i, i=$ $0, \cdots 10$.


Figure 3.11: Experiment 2: The transmission rate $\beta_{y}$ (left) and displacement in $y$ direction in the vertical movement (right)


Figure 3.10: Experiment 2: The transmission rate $\beta_{x}$ (left) and displacement in $x$ direction in the vertical movement (right)


Figure 3.12: Experiment 2: Velocity ellipsoids during the vertical movement along the line $x=0.5$ for $y=-0.4+0.1 i, i=0, \cdots 10$, the red marble lines show the horizontal and vertical transmission rates $\beta_{x}, \beta_{y}$.


Figure 3.13: Experiment 3: The manipulator positions along a slanted line


Figure 3.14: Experiment 3: The transmission rate $\beta_{x}$ (left) and displacement in $x$ direction in the slanted movement (right).


Figure 3.15: Experiment 3: The transmission rate $\beta_{y}$ (left) and displacement in $y$ direction in the slanted movement (right).


Figure 3.16: Experiment 3: Velocity ellipsoid during slant movement along the line $y=-0.5+x$ for $x=-0.4+0.1 i, i=0, \cdots 10$, the red dashed lines show the horizontal and vertical transmission rates $\beta_{x}, \beta_{y}$.

### 3.3 Experimental results: human postures and stability

In this section, we present some results of our experimental evaluation of the effect of human posturing on the stability of the teleoperation system with force reflection. The experiment, which is explained below in detail, is to consider the human arm in different postures. The human holds the manipulator device in each of the three positions, and an external force is applied to the human hand. The motions induced by the reflected force are illustrated. The objective of this set of experiments is to show that the human behavior in response to an external force depends on the human postures. During the experiment, a simple 2-DOF model shown in Figure 2.2 has been used as the human arm model. We should also mention that the human hand holds the device in each case with the same amount of effort.

Experimental setup: The setup, which is illustrated in Figure 3.17, consists of a haptic device, where the Phantom Omni ${ }^{\text {TM }}$ haptic device manufactured by SensAble Technologies Inc. is used, see Figure 3.18.


Figure 3.17: Exprimental setup

The device has 6-DOF position sensing and 3-DOF force feedback, and is programmed using the OpenHaptics toolkit. The device is controlled from a PC, which
is connected over a local area network. All the experiments were run at a sampling frequency of 1000 Hz .


Figure 3.18: PhantomOmni ${ }^{\text {TM }}$ haptic device

Experiment one: In this experiment, we consider three human postures as shown in Figures 3.19, 3.20 and 3.21. The human holds the manipulator device and the vertical force $f_{y}$ is applied to the human hand as shown in Figure 3.22 . The induced motion is shown in Figure 3.23 right, and the human force in Figure 3.23 left. It can be seen that, for the vertical force, the induced motion for Posture 1 has the highest value and for Posture 3 has the smallest value among the three. On the other hand, the human force for Posture 3, at the time $t=2 s$, is the highest, and for Posture 1 is the lowest. Comparing this with the velocity ellipsoid as shown on Figure 3.19, it can be seen that for the Posture 1, the minimal principal diagonal of the ellipse is in the horizontal direction and for Posture 3 is in vertical direction. This is in agreement with the observed induced motion, i.e., for Posture 3, where the human is capable of compensating the force we have less induced motion and more stability, but for Posture 1, where human is not capable of handling the force, the induced motion has the highest value. Note that, to avoid the ambiguity coming from infinity in computation of transmission ratios, we consider Postures 1 and 3 almost horizontal and vertical, instead of exact horizontal and vertical. In fact, Posture 1 is given by $\left(\theta_{1}, \theta_{2}\right)=\left(-5^{\circ}, 10^{\circ}\right)$ and Posture 3 is given by $\left(\theta_{1}, \theta_{2}\right)=\left(-95^{\circ}, 10^{\circ}\right)$. In fact, we make this estimation because since the velocity ellipsoid is obtained from the equation (3.2) and $J^{-1}$ does not exist for extreme positions $\left(\theta_{1}, \theta_{2}\right)=\left(0^{o}, 0^{o}\right)$
(Posture 1) and $\left(\theta_{1}, \theta_{2}\right)=\left(-90^{\circ}, 0^{\circ}\right)$ (for Posture 3). In these extreme cases, velocity ellipsoid turns to a straight line.

Experiment two: In this experiment, we consider the same three human postures as shown in Figures 3.19, 3.20 and 3.21 under the the horizontal force $f_{x}$ applied to the human hand, see Figure 3.24 . The induced motion on the remote side is shown in Figure 3.25 right and the human force in Figure 3.25 left. It can be seen that for the horizontal force, the induced motion for Posture 1 has the lowest value and for Posture 3 has the highest among the three. For the human force, on the other hand, it can be seen that for Posture 1, at the time $t=2 s$, is the highest and for Posture 3 is the lowest amount, although soon after that they all will be the same value. This is in agreement with the velocity ellipse shown in Figure 3.19, as explained above.



Figure 3.19: The velocity ellipse and the human posture 1


Figure 3.20: The velocity ellipse and the human posture 2


Figure 3.21: The velocity ellipse and the human posture 3


Figure 3.22: The disturbance force in the vertical direction


Figure 3.23: The human force and induced motion according to the vertical disturbance force for postures 1,2 and 3


Figure 3.24: The disturbance force in the horizontal direction


Figure 3.25: The human force and induced motion according to the horizontal disturbance force for postures 1, 2 and 3

### 3.4 Conclusions

In this section, we first reviewed the theoretical background on task manipulability and introduced the concepts of velocity and force transmission rates associated with the velocity and force ellipsoids of the end-effector. We also investigated the effect of human postures when the human arm is under an external force. We showed that the induced motion caused by the same disturbance force depends on the human posture. In fact, for those postures for which the human hand can compensate the force, the unwanted induced motion will be lower in comparison with the cases where the human hand is not capable of compensating the force completely. In the next section, we will show how applying the PBFR algorithm will decrease the unwanted induced motion for different human postures, and will introduce a new method for selecting the PBFR gain depending on human postures.

## Chapter 4

## A Posture-Dependent Algorithm for Selecting the PBFR Gain

As mentioned in the previous chapters, the PBFR algorithm will improve stability of the teleoperator system; however, in the original PBFR algorithm, the gain $\alpha$ is considered to be a constant small value. The reason is when there is no a priori information of the response of the human operator, the PBFR gain is considered as a small value to work for a wide range of human responses. The small value of PBFR gain $\alpha$ guarantees stability but, at the same time increases the transient time and hence decreases transparency. In this chapter, we first perform several experiments to show that how applying the PBFR algorithm decreases the unwanted induced motion caused by the reflected force for different human arm postures; subsequently, we present a new method of selecting the PBFR algorithm gain $\alpha$ which is, unlike the original PBFR gain, a posture dependent gain. We also show how this method improves the performance of the teleoperation system with force feedback. In fact, the new gain $\alpha$ depends on the human operator postures or, more precisely, is a function of the force transmission rate. Our method suggests that there is no need to select a very small constant $\alpha$ as the PBFR gain during task performance, but instead one can chose $\alpha$ according to the human postures, selecting small values just for the cases where the human hand cannot compensate for the reflected force. We also perform a set of experiments for different human postures and present the induced motion for the three cases of the direct force reflection, PBFR algorithm with constant gain,
and PBFR algorithm with new adaptive gain to show the overall improvement of the performance of the system.

### 4.1 Human posture and the stability of the teleoperator system: experimental results

In the previous chapter, we observed how the human hand reacts to a disturbance force in different postures, and also, we have seen the associated induced motion due to the force applied. In this section, we will show the the projection-based force reflection algorithm for different human postures will decrease the induced motion caused by the reflected force and, hence, improve stability. In the next section, we will discuss how to select the PBFR gain appropriately (as a function of transmission rates) to improve also the transparency while preserving stability.

### 4.1.1 Experimental results

In this section, we present some experimental results to show how the PBFR algorithm decreases the unwanted induced motion for different human postures and hence improves the performance of the system. The experimental setup (which is similar to the one used in the previous chapter) is illustrated in Figure 3.17, and consists of a haptic device (the Phantom Omni ${ }^{\mathrm{TM}}$ device manufactured by SensAble Technologies Inc., see Figure 3.18). The device has 6-DOF position sensing and 3-DOF force feedback, and they were programmed using the OpenHaptics toolkit. The device is controlled from a PC. All the experiments were run at the sampling frequency of 1000 Hz . The estimates of the human force which are used in the PBFR algorithms are obtained using the high-gain force observer designed in [57]. We recall that in the last chapter, we studied the posture dependent behavior of a human arm while holding a haptic device, and when an external disturbance force is applied. In fact,
we presented the induced motion associated to the three human hands postures as in Figures 3.19, 3.20 and 3.21 and showed that for the postures where the human hand can compensate for the force, the induced motion caused by the reflected force has lower values comparing to the postures in which human hand cannot compensate for the force. In this chapter, we first perform an experiment similar to the one described in the previous chapter, but this time with PBFR algorithm. We show that the performance of the system for each posture with PBFR algorithm applied will be improved in the sense that the induced motion will be smaller comparing to the case where the contact force is directly reflected. The experiment will be as follows: the human hand holds the haptic device in each of the three different postures 1,2 and 3 shown in 3.19 , 3.20 and 3.21 , where a horizontal force $f_{x}=3 N$ is applied to the device from $t=2 s$ to $t=4 s$ and the PBFR gain is considered as $\alpha=0.3$. Figure 4.1-right shows the induced motion of the end-effector when human is positioned at those three postures. As can be seen in Figure 4.1-right, in all postures the induced motion with PBFR algorithm applied is less than the induced motion when the force is directly reflected [61. However, it is clear that the induced motion is substantially dependent on the human postures, even if PBFR algorithm is applied. When PBFR algorithm is applied, the reflected force is computed based on human force measurement, i.e. according to the formula (3.1). It is known that the human capability in compensating for the reflected force is an important fact in PBFR algorithms, and it can be shown that the human postures is one of the factors in this capability. In Posture 1, where the human is strongly capable of compensating the force, the induced motion even without the PBFR algorithm is less than that for the other two postures; on the other hand, in Posture 3, where the human has the minimum capability in compensating for the force, even with PBFR algorithm applied, still the induced motion is larger than for the other two postures.

In Figure 4.1 (left), the human force and the reflected force for all three postures are shown with and without PBFR algorithm applied. It can be seen that in Posture

3, even with small reflected force and hence small human force, human is not able to compensate the force completely while in Posture 1 is. On the other hand, in Posture 3 where human hand is less capable of handling the force, there exists a longer transient time.

### 4.2 A posture dependent selection of the PBFR gain

In this section, we introduce a new method for on-line selection of the PBFR gain depending on the human hand posture. Specifically, given a unit vector $u \in \mathbb{R}^{2}$, we introduce the PBFR gain $\alpha$ as a function of the configuration of human arm quantified with the force transmission rate $\gamma_{u}$ given by

$$
\begin{equation*}
\gamma_{u}=\left(u^{T}\left(J J^{T}\right) u\right)^{-\frac{1}{2}} \tag{4.1}
\end{equation*}
$$

We select $\alpha$ as a linear function of the force transmission rate in the direction of the reflected force as,

$$
\begin{equation*}
\alpha\left(\gamma_{u}\right)=\operatorname{Sat}_{\left[\alpha_{\min }, \alpha_{\max }\right]}\left\{\alpha_{\max }+\frac{\left(\alpha_{\max }-\alpha_{\min }\right)}{\left(\gamma_{\max }-\gamma_{\min }\right)}\left(\gamma_{u}-\gamma_{\max }\right)\right\} . \tag{4.2}
\end{equation*}
$$

Here, the unit vector $u=\hat{u}_{f_{e}}$ is the unit vector in the direction of the reflected force and $\alpha_{\min }=\alpha\left(\gamma_{\min }\right), \alpha_{\max }=\alpha\left(\gamma_{\max }\right)$.

To select appropriate values for our linear function, we have a look at Figures 3.19, 3.20 and 3.21 , where corresponding velocity ellipsoids and also maximum and minimum of velocity transmission rate for each posture are shown. As can be seen in Figure 3.19, the minimum and maximum force transmission rate are $\gamma_{\text {min }}=1.12$, and $\gamma_{\max }=30$ respectively. Here, we consider the $\alpha\left(\gamma_{\min }\right)=0.1$ and $\alpha\left(\gamma_{\max }\right)=1$. Using the equation (4.2), we suggest a new PBFR gain $\alpha$ as a function of the force


Figure 4.1: The human force and reflected force for postures 1-3(top to bottom) with constant PBFR gain in response to a horizontal disturbance force
transmission rate $\gamma$ given by

$$
\begin{equation*}
\alpha(\gamma)=\underset{[0.1,1]}{\operatorname{Sat}}\left\{1+\frac{.9}{28.88}(\gamma-30)\right\} \tag{4.3}
\end{equation*}
$$

The $\alpha(\gamma)$ is taken to be a simple transformation relating $\alpha(1.12)=.1$ to $\alpha(30)=$ 1. The saturation function at 0.1 and 1 makes sure that, for the cases where the force transmission rate is high (more precisely higher than or equal 30) and human hand is capable of compensating for the force, the PBFR gain $\alpha=1$ is applied. On the other hand, for the cases where force transmission rate is low (here lower than 1.12) and the human hand is not capable of compensating for the force completely, a rather small value of $\alpha=0.1$ is applied. These values, i.e., 1.12 and 30 , correspond to the force transmission rate when the human arm is in the position $\left(\theta-5^{\circ}, 10^{\circ}\right)$, where $\theta=u_{2} / u_{1}$. For the cases $u=(1,0)$ and $u=(0,1)$, these values are illustrated in Figure 3.19. In addition, the $\beta_{u}^{-1}$ is 'almost' the force transmission rate in the direction $u \in \mathbb{R}^{2}$ which here is the direction of the force applied; for this reason, we also use the notation $\alpha_{u}=\alpha\left(\beta_{u}\right)$. One should note that in general, the velocity transmission rate and force transmission rate are not inverse of each other. In fact, if the vector $u$ is a unit eigenvector of $J J^{T}$, then

$$
\beta^{2} \gamma^{2}=u^{T}\left(J J^{T}\right)^{-1} u u^{T} J J^{T} u=\lambda^{-1}|u|^{2} \lambda\left|u^{2}\right|=1
$$

and hence one can use $\gamma=1 / \beta$ when it is known that the unit reflected force is (almost) in direction of principle axes.

### 4.3 The posture dependent PBFR gain and stability: experimental results

In this section, we present some experimental results to show the effect of applying the new posture dependent PBFR gain introduced in the last section and given by (4.3), on the stability and transparency of the haptic system. The experimental setup is similar to the previous section, see Figure 3.17. As the previous section, we apply a horizontal force while human hand holds the haptic device in three different Postures 1, 2 and 3 shown in Figures 3.19- 3.21. We perform the experiment for three different cases; direct force reflection, PBFR with constant gain $\alpha=0.3$, and the variable PBFR gain introduced by formula 4.3). In our experiment, the horizontal force applied to the human hand is $f_{x}=3 N$.

As can be seen in Figure 4.1-right, in Postures 2 and 3, the induced motion with varying PBFR gain applied is smaller than the induced motion when the force is directly reflected (in Posture 2, between $t=2$ to $t=3.5$ seconds). The induced motion is dependent on the human postures, even if the PBFR algorithm is applied. Recall that in the PBFR case, the reflected force is computed based on human force measurement, i.e., $f_{r}=\alpha f_{e n v}+(1-\alpha) \varphi_{e n v}$.

As can be seen in the Figure 4.2, in Posture 3 where the human hand has lower capability to compensate for the force compared to Postures 2 and 1, a smaller PBFR gain $\alpha=0.1$ is implemented which causes a lower induced motion and hence higher level of stability. Moreover, it can be seen that, the transient time decreases slightly, which corresponds to a higher level of transparency and also in a position like Posture 1, where the human hand is capable of handling the force, a high PBFR gain (such as $\alpha=1$ ) provides the same level of transparency and stability (Figure 4.2-top). Therefore, there is no need to decrease the PBFR gain for all postures, but for each posture, the gain can be assigned proportional to its capability of force compensation.


Figure 4.2: The human force vs. the reflected force in postures 1-3 with direct force reflection, constant PBFR gain and variable PBFR gain (left), the induced motion for direct force reflection, constant PBFR gain and variable PBFR gain in postures 1-3 (right)

### 4.4 Conclusion

It is known that the PBFR algorithm improves stability of force reflecting teleoperator systems and haptic interfaces; however, it is also known that choosing small PBFR algorithm gain will cause slower convergence of the reflected force to the actual contact force which causes deterioration of transparency. In this chapter, we suggested a method that improves the transparency pf PBFR algorithms. In fact, by applying a varying PBFR gain depending on the human posture, we showed that there is no need to select a small PBFR gain for those cases where human hand can handle the reflected force. This improves the transparency of the system while the same level of stability is preserved.

In the next chapter, by means of online estimation of the human postures, we compute the velocity transmission rates at each instant of time. The obtained estimate of the velocity transmission rates will be used to update the PBFR gain in real time.

## Chapter 5

## A Posture Dependent PBFR Gain for Teleoperation System with Online Posture

## Estimation

In the previous chapter, we developed a method to select the PBFR gain as a posture dependent function. Applying our method in the haptic interface case, we provided the experimental results that show how the transparency of the system will be improved. In this chapter, our goal is to generalize the method of the last chapter to a teleoperation system with force feedback. Our teleoperation system consists of a local and a single remote manipulator which communicate through a communication channel with time varying delay. In order to update our posture dependent PBFR gain, we use a webcam which detects transmission rate of human postures during the task execution. In the following, we first describe our teleoperation system in details and then explain the experimental results that compare the performance of the teleoperator system for the cases of direct force reflection, constant PBFR gain and posture dependent PBFR gain.

### 5.1 Online posture estimation

In this section, we explain how by using the library OpenCV and a web-cam we estimate the human posture online to find the transmission rates of the arm at each instant. The OpenCV (Open Source Computer Vision) is a library of programming functions under the open source BSD license. Its main focus is on real-time image
processing. Having transmission rates $\beta_{x}, \beta_{y}$ at each time instant, we can apply the posture dependent PBFR gain by the formula 4.3. To have the model of the human arm at each instant of time, we consider three green markers on human arm as shown in Figure 5.1, one on the shoulder, one on the elbow and one on the wrist. Based on the size and the color, markers will be detected during the task execution. Connecting the centres of the areas of these markers, we will be able to detect configuration of the human arm. With this information, we will find the corresponding angles and hence, establish the velocity ellipsoids and find the associated transmission rates.


Figure 5.1: The process of online posture estimations by detecting the wrist, elbow and shoulder positions using a camera. The center of the area of the green markers will be detected and connected to make a 2 -DOF model of human arm.

In the next section, we describe the experimental setup, and present some results of our experimental evaluation of the effect of human posturing on the stability and transparency of the teleoperation system with force reflection.

### 5.2 Experimental setup

The structure of the teleoperator system with PBFRalgorithm and online posture estimation is shown in Figure 5.2. The experimental setup, which is illustrated in Figure 5.3, consists of a single-local single-remote force reflecting telerobotic system.


Figure 5.2: Structure of teleoperator system with projection-based force reflection and on-line posture estimation.

During this experiment, the Phantom Omni ${ }^{\text {TM }}$ haptic devices manufactured by SensAble Technologies Inc., were used as the local and remote devices. The devices have 6-DOF position sensing and 3-DOF force feedback, and they were programmed using the OpenHaptics toolkit. The devices are controlled from two different PC's, which are connected over a local area network. There is an artificially created time varying delay between local and remote devices using internal buffers, and an algorithm is implemented that generates random delay and packet dropouts with prescribed characteristics. All the experiments were run at the sampling frequency of 1000 Hz .


Figure 5.3: Experimental setup

The estimates of the human force which are used in the PBFR algorithm are obtained using the high-gain force observer designed in [57]. There were two virtual walls located at $(-5 \mathrm{~mm}, 75 \mathrm{~mm})$, one orthogonal to $x$-axis and one to $y$-axis. The
remote manipulator interacts with these virtual walls. To display a hard contact with the environment, a method developed in [53] has been used. The contact force $f_{e}=\left(f_{e x}, f_{e y}\right)$ is generated by the formula

$$
\begin{aligned}
& f_{e x}=-K_{1}\left(x-x_{0}\right) \\
& f_{e y}=-K_{2}\left(y-y_{0}\right),
\end{aligned}
$$

where $x$ and $y$ are the depth of penetration of the end-effector proxy into the $x$ and $y$ wall's surfaces, respectively and $x_{0}=-5 \mathrm{~mm}, y_{0}=75 \mathrm{~mm}$ are the location of the virtual wall. The constants $K_{i} \geq 0, i=1,2$ are the walls stiffness and are chosen as $K_{1}=K_{2}=0.5 \mathrm{~N} / \mathrm{m}$ for our experiment. The time-varying communication delay has the minimum round-trip time (RTT) equal to 0.4 seconds. The controller used for


Figure 5.4: Experimental setup for on line estimation of the human postures to adjust the variable PBFR gain
the remote device is a PID plus gravity compensation controller given by

$$
u=-K_{p} \tilde{q}-k_{d} \dot{\tilde{q}}-K_{I} \int \tilde{q} d t+G(q)
$$

where $\tilde{q}=\hat{q}_{l}-q_{r}$ is the difference between the delayed remote and local position in joint space, $G(q)$ is the gravity vector, and $K_{p}, K_{d}, K_{I}$ are gains selected as $K_{p}=$ $2, K_{d}=0.005, K_{I}=5$.


Figure 5.5: On line estimation of the human postures to adjust the variable PBFR gain; in the web-cam picture in the monitor, the online attachment velocity based ellipsoid based on the human posture is shown.

In the experimental results presented below, the human operator performs a simple task using the local manipulator, and the remote manipulator will follow the local device to execute the task. The human operator moves the manipulator to the virtual wall for each of the three postures shown in Figures 5.6, 5.7, and the stability and transparency of the teleoperator system will be compared between the three cases: direct force reflection, PBFR algorithm with small PBFR gain, and PBFR algorithm with a posture dependent PBFR gain. It will be shown that, in all three cases the teleoperator system is stable, however, in the case that an adaptive PBFR gain is applied, the transparency of the system is improved while the same level of stability is preserved. In the following, $\beta_{u}, \gamma_{u}$ denote the velocity and force transmission rates in direction of $u$ and $\alpha_{u}:=\alpha\left(\gamma_{u}\right)$.

Figure 5.8 shows the contact force versus reflected force and the local device trajectory for the direct force reflection case, for the three Postures illustrated in Figures 5.6-5.7. The reflected force is caused by the contact with the virtual wall in $x$-direction. This experiment shows the effect of human postures on the stability of the teleoperator system in the case of direct force reflection. The associated velocity transmission rate has been shown in Figure 5.9. As it can be seen in Figure 5.8 . right, that in Posture 1, in which the human hand is capable of compensating for the reflected force, is more stable comparing to Posture 2 and 3 (Figure 5.8-right-
middle/bottom), where the human hand cannot compensate for the external force completely.


Figure 5.6: Posture 1


Figure 5.7: Posture 2 (left) and Posture 3 (right)

Figure 5.10 shows the contact force versus reflected force and the local device trajectory in the direct force reflection case, for the same three postures as above. The reflected force this time is caused by being in contact with the virtual wall in $y$ direction. The associated velocity transmission rate has been shown in Figure 5.9. As it can be seen in Figure 5.10-right, the system at Posture 3, in which human the hand is capable of compensating the reflected force, is more stable comparing to Posture 1 and 3, Figure 5.10 -right-top/middle, where the human hand cannot compensate the force completely. Accordingly, the local device trajectory for the Posture 3 is


Figure 5.8: Contact versus reflected force (left), local manipulator trajectory (right) for direct force reflection case for Postures 1-3 (top-bottom). The virtual wall perpendicular to $x$-direction.


Figure 5.9: Velocity transmission rate in postures 1, 2 and 3 (top to bottom) for direct force reflection case. The virtual wall orthogonal to $x$-direction.
a smooth trajectory and the system is stable, unlike the Posture 1 where it has a oscillatory graph due to instability.

In Figure 5.12, the contact force vs reflected force and the haptic device trajectory are shown when PBFR algorithm is applied. The human arm is in Posture 1, and the force is caused by the contact with the virtual wall in $x$-direction. In fact, we consider three different situations regarding the PBFR gain as it is shown that in Figure 5.13 left, i.e., constant high gain $\alpha_{x}=1$, constant small gain $\alpha_{x}=0.1$ and variable gain $\alpha\left(\gamma_{x}\right)$. In Figure 5.13 right, the associated velocity transmission rates are shown. As it can be seen in Figure 5.12-left-top corresponding to the contact vs reflected force for the $\alpha_{x}=1$, the situation is exactly as the direct force reflection, where there is no transient time and that is because for this selection of gain the whole force is reflected back. In Figure 5.12 left-middle, which corresponds to the case $\alpha_{x}=0.1$, because of the fact that a small value of $\alpha$ has been selected, the transient time (i.e., the time that the reflected force converges the actual force), increases which deteriorates the transparency. However, as it can be seen in Figure 5.12-leftbottom, applying a variable PBFR gain depending on the velocity transmission rate (or human posture), will improve the transparency, in the sense that it considerably reduces the transient time. In fact, in this case, since the human hand is capable of handling the contact force, there is no need to select a very small value of $\alpha_{x}$. Therefore, $\alpha\left(\gamma_{x}\right)$ varies around 0.4 versus 0.1 in the previous case where $\alpha_{x}$ was constant, while the stability of the teleoperator system is preserved. In fact, since the human arm is in Posture 1, it is fully capable of compensating for the force and, hence, the local device follows a smooth trajectory.

The above experiment is repeated for Posture 3, where the human/local device is in contact with the virtual wall in the $x$-direction. This is the case where the human hand is not capable of compensating for the force and, hence, the overall stability would need a small PBFR gain. The contact force vs. reflected force and the local device trajectory are shown in Figure 5.14, and the corresponding PBFR


Figure 5.10: Contact versus reflected force (left), local manipulator trajectory (right) for direct force reflection case for Postures 1-3 (top-bottom). The virtual wall perpendicular to $y$-direction.


Figure 5.11: Velocity transmission rate in postures 1, 2 and 3 (top to bottom) in direct force reflection case; $y$-direction.
gain in Figure 5.15. As it can be seen in Figure 5.15-left-bottom, the varying PBFR gain in this case is small, around 0.1 , the transparency improvement comparing to the constant PBFR gain $\alpha_{x}=0.1$, is very slight, as the transient time is a little less that 2 seconds, while for the constant PBFR gain it is a little more than 2 seconds. This is while the trajectory illustrated in Figure 5.14 right-bottom is slightly smoother than 5.14 right-middle, see the sharpness at $t=2$ and $t=10$ seconds.

In Figure 5.16, the contact force versus reflected force are shown when human is positioned in Posture 1 and is in contact with a virtual wall in $y$-direction. This is a case that human is not capable of compensating the force and stability would need a small PBFR gain. The corresponding PBFR gain illustrated in Figure 5.17. As it can be seen in Figure 5.17-left-bottom, the varying PBFR gain for this case that human hand is not capable of compensating the force is small, around 0.1 , and the transparency improvement comparing to the constant PBFR gain $\alpha_{y}=0.1$, is very slight, as represented in the transient (convergence) time. The trajectory illustrated in 5.14-right, for both cases, constant $\operatorname{PBFR} \alpha=0.1$ and varying PBFR gain are smoother than 5.16-right-top, the case of direct force reflection.

In Figure 5.18 the contact vs reflected force and the local device trajectory are shown when PBFR algorithm is applied. The human arm is in Posture 3, and the contact force $f_{y}$ is produced because of being in contact with a virtual wall in $y$-direction. Again, three different situation regarding the PBFR gain is considered represented in Figure 5.19-left which are constant high gain $\alpha_{y}=1$, constant small gain $\alpha_{y}=0.1$ and variable gain $\alpha\left(\gamma_{y}\right)$. In Figure 5.19-right, the associated velocity transmission rates are shown. As it can be seen in 5.18-left-top corresponding to the contact vs reflected force for the $\alpha_{y}=1$, there is no transient time, i.e., it is like the direct force reflection. In Figure 5.18-left-middle, corresponding to the case $\alpha_{y}=0.1$, because of the fact that a small value is selected for $\alpha_{y}$, the transient time will increase, which causes deterioration in transparency. However, in Figure 5.18 left-bottom, it can be seen that applying a variable PBFR gain depends on velocity


Figure 5.12: Contact vs. reflected force (left), local trajectory (right) with constant PBFR gain $\alpha=1$ (top), constant PBFR gain $\alpha=0.1$ (middle) and time varying $\alpha$ (bottom). Human is in Posture 1 and virtual wall orthogonal to $x$-direction.


Figure 5.13: PBFR gain (left) and velocity transmission rate (right) with constant PBFR gain $\alpha=1$ (top), constant PBFR gain $\alpha=0.1$ (middle) and time varying $\alpha$ (bottom). Human is in Posture 1 and virtual wall orthogonal to $x$-direction.


Figure 5.14: Contact vs. reflected force (left), local trajectory (right) with constant PBFR gain $\alpha=1$ (top), constant PBFR gain $\alpha=0.1$ (middle) and time varying $\alpha$ (bottom). Human is in Posture 3 and virtual wall orthogonal to $x$-direction.


Figure 5.15: PBFR gain (left) and velocity transmission rate (right) with constant PBFR gain $\alpha=1$ (top), constant PBFR gain $\alpha=0.1$ (middle) and time varying $\alpha$ (bottom). Human is in Posture 3 and virtual wall orthogonal to $x$-direction.
transmission rate (or human posture), will improve the transparency, in the sense that it reduces the transient time. In fact, in this case, since the human hand is capable of handling the contact force, there is no need to select a very small amount of $\alpha_{y}$ and the variable $\alpha_{y}$ varies around $0.3-0.4$ versus 0.1 in the previous case that provides a longer transient time. This improves the transparency while the stability of the system is persevered, that is, the human arm is positioned at Posture 1 and the human is capable of compensating the force, the local device follows a smooth trajectory.

The dependence of PBFR gain on the velocity transmission rate are shown in Figures 5.20 and 5.21. In Figure 5.20 the varying PBFR gain $\alpha_{x}$ is illustrated versus the velocity transmission rate $\beta_{x}$. It can be seen that whenever, $\beta_{x}$ increases, which is corresponding to the less capability of compensating the force in Posture 2, the PBFR gain decreases to a smaller value (such as the duration between $t=5 \mathrm{~s}$ to $t=7 \mathrm{~s}$ that $\alpha_{x}$ decreases to 0.1 ), and whenever, $\beta_{x}$ decreases, which is corresponding to the higher capability of compensating the force in Posture 2, the PBFR gain increases to larger value (such as the duration between $t=13 \mathrm{~s}$ to $t=19 \mathrm{~s}$ that $\alpha_{x}$ increases to 0.4), which improves the transparency of the system as already discussed. The same result for PBFR gain in $y$-direction and Posture 1 is presented in Figure 5.21.


Figure 5.16: Contact vs. reflected force (left), local trajectory (right) with constant PBFR gain $\alpha=1$ (top), constant PBFR gain $\alpha=0.1$ (middle) and time varying $\alpha$ (bottom). Human is in Posture 1 and virtual wall orthogonal to $y$-direction.


Figure 5.17: PBFR gain (left) and velocity transmission rate (right) with constant PBFR gain $\alpha=1$ (top), constant PBFR gain $\alpha=0.1$ (middle) and time varying $\alpha$ (bottom). Human is in Posture 1, and virtual wall orthogonal is to $y$-direction.


Figure 5.18: Contact vs. reflected force (left), local trajectory(right) with constant PBFR gain $\alpha=1$ (top), constant PBFR gain $\alpha=0.1$ (middle) and time varying $\alpha$ (bottom). Human is in Posture 3, and virtual wall orthogonal is to $y$-direction.


Figure 5.19: PBFR gain (left) and velocity transmission rate (right) with constant PBFR gain $\alpha=1$ (top), constant PBFR gain $\alpha=0.1$ (middle) and time varying $\alpha$ (bottom). Human is in Posture 3 and virtual wall orthogonal to $y$-direction.


Figure 5.20: Posture dependent $\operatorname{PBFR}$ gain $\alpha\left(\beta_{x}\right)$ and the velocity transmission rate $\beta_{x}$, Posture 3.


Figure 5.21: Posture dependent $\operatorname{PBFR}$ gain $\alpha\left(\beta_{y}\right)$ and the velocity transmission rate $\beta_{y}$, Posture 1.

### 5.2.1 Discussion on the stability

Although in practice the teleoperator system with local-remote manipulators and the above described adaptive PBFR algorithm demonstrates stable behavior, however one needs to prove this claim rigorously. In fact, one can see that the proof will not be a simple mimic of the original proof of IOS stability for a teleoperator system with PBFR as described in 60. One reason for that is our selection of function $\alpha$ does not turn to a class $\mathcal{K}$ function of the magnitude of the environment force $\left|f_{e}\right|$.

### 5.3 Conclusion

In this chapter, the method of adaptive PBFR gain developed in the previous chapter is applied to the force reflecting teleoperation system with time varying delay. We present the experimental results that compare the stability and transparency of the system in three different cases: direct force reflection, PBFR algorithm with constant gain and the adaptive PBFR gain. We also showed that application of the new posture-dependent PBFR gain based on human postures introduced in Chapter 4 improves the transparency of the teleoperator system, while stability is preserved.

## Chapter 6

## Towards a 4-DOF model of human arm

In this chapter, we discuss a more realistic model of the human arm with 4 degrees of freedom and explain how the methods of the previous chapters can be applied to this case. We recall that our goal is to investigate the effect of human postures on the overall stability of a teleoperator system with force feedback. Through Chapters 3 to 5. we investigated the effect of human postures on the stability of the teleoperator system, when the human arm is modelled as a 2-DOF (planar) model shown in Figure 2.2 and we also developed a new method of selecting PBFR gain based on human postures. It is also shown that our suggested method improves the performance of the force reflecting teleoperator system with time varying delay. In this chapter, we aim to explore the relation between human postures and the stability and performance of the system for a more realistic model of the human arm with 4-DOF. The model of the human arm that we would like to consider here has 4 degrees of freedom; three degrees of freedom at the shoulder (a spherical joint) and one degree of freedom at elbow (a revolute joint). This model allows us to capture a three dimensional movements of the human arm. We investigate this model thoroughly, which means that we solve the kinematics, inverse kinematics, and also find the dynamics of the model. The kinematics of the model which will be obtained from the Denavit-Hartenberg method together with the inverse kinematics will allow us to find the velocity/force transmission ratios and also velocity/force ellipsoids during the arm movement. Later in this chapter, we compute the dynamics of the model and demonstrate simulations similar to those performed in Chapter 3 to demonstrate the behaviour of the human
arm in response to an external disturbance for different postures. The teleoperation experiments as well as online posture estimation is postponed to future work.

### 6.1 A 4-DOF human arm model; Denavit-Hartenberg representation

In this following, we first use the Denavit-Hartenberg (DH-) representation to obtain the forward kinematics of the human arm modeled as Figure 6.1. The model that is considered here has 4-DOF with a spherical joint at shoulder and a revolute joint at elbow and the end-effector fixed at hand. The inverse kinematics problem associated with the model also will be solved afterwards.

### 6.1.1 The forward kinematics of the 4-DOF human arm model

As it is mentioned above, the model that we consider is a 4-DOF model of human arm with a spherical joint at shoulder, a revolute joint at elbow, and the end-effector at hand connected via upper arm and forearm with lengths $l_{1}, l_{2}$. The joint angles are denoted by $\theta_{1}, \theta_{2}, \theta_{3}, \theta_{4}$.

To find the linear transformation from the joint coordinates $\theta_{1}, \theta_{2}, \theta_{3}, \theta_{4}$ to the frame attached to the end-effector, we use the well known Denavit-Hartenberg method. First, we need to fix a frame at each joint. The origin of frames are denoted by points $P_{0}, P_{1}, P_{2}, P_{3}$ and $P_{4}$. Note that there are five frames, the first three are attached to the spherical joint at shoulder, the forth one to the revolute joint at elbow and the last one at the end-effector, see Figure 6.1. The coordinate of each $P_{i}, i=0, \cdots, 4$ is denoted by $P_{i}=\left(x_{i}, y_{i}, z_{i}\right)$. The length of upper arm and forearm here are denoted by $l_{1}, l_{2}$. For our simulation and experimental results in the future the length of upper arm and forearm are considered as $l_{1}=0.35 \mathrm{~m}$ and $l_{2}=0.3 \mathrm{~m}$.

After fixing a frame at each joint, we can find the four parameters $a_{i}, \alpha_{i}, d_{i}$ and $\theta_{i}$ of D-H method. These four parameters are usually called link length, link twist, link offset and joint angle. The $\alpha_{i}$ is the rotation about the $x_{i}$ axis to fit $z_{i-1}$ on $z_{i}, \theta_{i}$ is the rotation angle about the $z_{i-1}$ axis for $x_{i-1}$ to fit on $x_{i}$. The distance between $P_{i-1}$ and $P_{i}$ in the $x_{i}$ direction is $a_{i}$ and along the $z_{i-1}$ direction is $d_{i}$. The Denavit-Hartenberg parameters of our model are given in the following table.

| DH-parameters |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| $i$ | $\alpha_{i}$ | $a_{i}$ | $d_{i}$ | $\theta_{i}$ |
| 1 | $-\pi / 2$ | 0 | 0 | $\theta_{1}$ |
| 2 | $-\pi / 2$ | 0 | 0 | $\theta_{2}-\pi / 2$ |
| 3 | $\pi / 2$ | 0 | $l_{1}$ | $\theta_{3}+\pi / 2$ |
| 4 | 0 | $l_{2}$ | 0 | $\theta_{4}$ |

The general form of the transformation from the coordinate system attached to the $(i-1)^{t h}$ joint to the coordinate system attached to the $i^{t h}$ joint is denoted by $A_{i}^{i-1}$ and is given by

$$
A_{i}^{i-1}=\left(\begin{array}{cccc}
\cos \theta_{i} & -\sin \theta_{i} \cos \alpha_{i} & \sin \theta_{i} \sin \alpha_{i} & a_{i} \cos \theta_{i} \\
\sin \theta_{i} & \cos \theta_{i} \cos \alpha_{i} & -\cos \theta_{i} \sin \alpha_{i} & a_{i} \sin \theta_{i} \\
0 & \sin \alpha_{i} & \cos \alpha_{i} & d_{i} \\
0 & 0 & 0 & 1
\end{array}\right)
$$

Now, to find the transformation relating the first frame to the frame that is attached to the end-effector, one should compose the the successive transformations $A_{1}^{0}, A_{2}^{1}, A_{3}^{2}, A_{4}^{3}$. From the last column of $T:=A_{4}^{0}=A_{1}^{0} A_{2}^{1} A_{3}^{2} A_{4}^{3}$, one can find the coordinate of the end point. (Note that the first three columns of the matrix $T$ represent the rotation of the the last frame with respect to the first frame.) Therefore, the coordinate of the frame $P_{4}=\left(x_{4}, y_{4}, z_{4}\right)$, fixed at the end-effector can be computed


Figure 6.1: Coordinate system of the arm

$$
\begin{align*}
x_{4} & =l_{1} \cos \theta_{1} \cos \theta_{2}-l_{2} \cos \theta_{4}\left(\cos \theta_{3} \sin \theta_{1}-\cos \theta_{1} \sin \theta_{2} \sin \theta_{3}\right) \\
& +l_{2} \cos \theta_{1} \cos \theta_{2} \sin \theta_{4} \\
y_{4} & =l_{1} \cos \theta_{2} \sin \theta_{1}-l_{1} \cos \theta_{4}\left(\cos \theta_{1} \cos \theta_{3}+\sin \theta_{1} \sin \theta_{2} \sin \theta_{3}\right) \\
& +l_{2} \cos \theta_{2} \sin \theta_{1} \sin \theta_{4} \\
z_{4} & =-l_{1} \sin \theta_{2}-l_{2} \sin \theta_{2} \sin \theta_{4}-l_{2} \cos \theta_{2} \cos \theta_{4} \sin \theta_{3} . \tag{6.1}
\end{align*}
$$

The Jacobian of the change of coordinate from the joint space $\left(\theta_{1}, \theta_{2}, \theta_{3}, \theta_{4}\right)$ to the task space $\left(x_{4}, y_{4}, z_{4}\right)$ can be found as $J=\left(J_{i j}\right)=\frac{\partial\left(x_{4}, y_{4}, z_{4}\right)}{\partial\left(\theta_{1}, \theta_{2}, \theta_{3}, \theta_{4}\right)}$. A straightforward
computation implies that

$$
\begin{align*}
J_{11} & =-l_{1} \cos \theta_{2} \sin \theta_{1}+l_{2} \cos \theta_{4}\left(\cos \theta_{1} \cos \theta_{3}+\sin \theta_{1} \sin \theta_{2} \sin \theta_{3}\right) \\
& -l_{2} \cos \theta_{2} \sin \theta_{1} \sin \theta_{4} \\
J_{12} & =-l_{1} \cos \theta_{1} \sin \theta_{2}-l_{2} \cos \theta_{1} \sin \theta_{2} \sin \theta_{4} \\
& -l_{2} \cos \theta_{1} \cos \theta_{2} \cos \theta_{4} \sin \theta_{3} \\
J_{13} & =-l_{2} \cos \theta_{4}\left(\sin \theta_{1} \sin \theta_{3}+\cos \theta_{1} \cos \theta_{3} \sin \theta_{2}\right) \\
J_{14} & =-l_{2} \sin \theta_{4}\left(\cos \theta_{3} \sin \theta_{1}-\cos \theta_{1} \sin \theta_{2} \sin \theta_{3}\right)+l_{2} \cos \theta_{1} \cos \theta_{2} \cos \theta_{4} \\
J_{21} & =l_{1} \cos \theta_{1} \cos \theta_{2}+l_{2} \cos \theta_{4}\left(\cos \theta_{3} \sin \theta_{1}-\cos \theta_{1} \sin \theta_{2} \sin \theta_{3}\right) \\
& +l_{2} \cos \theta_{1} \cos \theta_{2} \sin \theta_{4} \\
J_{22} & =-l_{1} \sin \theta_{1} \sin \theta_{2}-l_{2} \sin \theta_{1} \sin \theta_{2} \sin \theta_{4} \\
& -l_{2} \cos \theta_{2} \cos \theta_{4} \sin \theta_{1} \sin \theta_{3} \\
J_{23} & =l_{2} \cos \theta_{4}\left(\cos \theta_{1} \sin \theta_{3}-\cos \theta_{3} \sin \theta_{1} \sin \theta_{2}\right) \\
J_{24} & \left.\left.=l_{2} \cos \theta_{2} \cos \theta_{4} \sin \theta_{1}\right)+l_{2} \sin \theta_{4}\left(\cos \theta_{1} \cos \theta_{3}+\sin \theta_{1} \sin \theta_{2} \sin \theta_{3}\right)\right) \\
J_{31} & =0 \\
J_{32} & =-l_{1} \cos \theta_{2}-l_{2} \cos \theta_{2} \sin \theta_{4}+l_{2} \cos \theta_{4} \sin \theta_{2} \sin \theta_{3} \\
J_{33} & =-l_{2} \cos \theta_{2} \cos \theta_{3} \cos \theta_{4} \\
J_{34} & =-l_{2} \cos \theta_{4} \sin \theta_{2}+l_{2} \cos \theta_{2} \sin \theta_{3} \sin \theta_{4} . \tag{6.2}
\end{align*}
$$

Now, we can find the velocity and force transmission ratios from $\left(J J^{T}\right)^{-1}$ and $J J^{T}$. In fact, as mentioned in the previous chapters, the velocity transmission rate $\beta$ in the direction of the unit vector $u$ is given by $\beta=\left(u\left(J J^{T}\right)^{-1} u\right)^{-1 / 2}$ and the associated velocity ellipsoid at the end-effector can be obtained through eigenvalues and eigenvectors of $\left(J J^{T}\right)^{-1}$, see Figure 6.2. In the following we solve the inverse kinematics of the above mentioned 4-DOF human arm model.


Figure 6.2: The velocity ellipsoid of the 4 -DOF model of human arm; the principal axes are denoted by dashed red lines.

### 6.1.2 Inverse Kinematics of the 4-DOF human arm model

In the following, we solve the inverse kinematics problem of the 4-DOF human arm model introduced in the previous section. We should mention that, when we estimate human postures on-line, we need the inverse kinematics of the model to move from task space to joint space. This is needed because, the transmission ratios depend on Jacobian which is written in joint space coordinates. Recall that we denoted the frame bases which are fixed at joints by points $P_{0}, \cdots, P_{4}$ and the angle joints by $\theta_{1}, \cdots, \theta_{4}$. The points $P_{0}, P_{1}, P_{2}$ are at shoulder, $P_{3}$ is fixed at elbow, and $P_{4}$ at hand. We recall that our goal is to estimate postures of the arm on-line during a task performance. This is a necessary step to develop experiments which have been done in Chapter 5 from the point of view that it can capture a three dimensional movement of the arm using two cameras (although, we will not perform the on-line posture estimation in this thesis.) Let us formulate inverse problem that we are interested to solve.

Formulation of the inverse kinematics problem: Given positions of the elbow and hand, we will find joint angles. More precisely, we would like to find joint angles $\theta_{1}, \theta_{2}, \theta_{3}, \theta_{4}$ if $P_{3}$ and $P_{4}$ are known.

Note that, since $P_{0}$ is considered fixed as the origin of the base frame during the task, so we only need to detect $P_{3}$ and $P_{4}$. These angles can be found as following: first note that the point $P_{3}$ is given by the last column of the matrix $A_{3}^{0}=A_{1}^{0} A_{2}^{1} A_{3}^{2}$ as

$$
\begin{align*}
& x_{3}=l_{1} \cos \theta_{1} \cos \theta_{2} \\
& y_{3}=l_{1} \cos \theta_{2} \sin \theta_{1} \\
& z_{3}=-l_{1} \sin \theta_{2}, \tag{6.3}
\end{align*}
$$

and $P_{4}$ is given by (6.1). From (6.3), one can easily see that $\tan \theta_{1}=y_{3} / x_{3}$ and hence

$$
\theta_{1}=\operatorname{atan} 2\left(y_{3}, x_{3}\right),
$$

(we use atan 2 as the inverse tangent function in MATLAB). From 6.1) and by squaring the components one has

$$
\tan ^{2} \theta_{2}=\frac{z_{3}^{2}}{x_{3}^{2}+y_{3}^{2}}
$$

therefore,

$$
\theta_{2}=\operatorname{atan} 2\left( \pm z_{3}, \sqrt{x_{3}^{2}+y_{3}^{2}}\right)
$$

To find $\theta_{4}$ note that

$$
x_{4}^{2}+y_{4}^{2}+z_{4}^{2}-l_{1}^{2}-l_{2}^{2}=2 l_{1} l_{2} \sin \theta_{4},
$$

hence, one can find $\theta_{4}$ as

$$
\theta_{4}=\operatorname{asin}\left(\frac{x_{4}^{2}+y_{4}^{2}+z_{4}^{2}-l_{1}^{2}-l_{2}^{2}}{2 l_{1} l_{2}}\right)
$$

( asin is the inverse sine function in MATLAB). Although, from (6.3) and 6.1), $\theta_{3}$
can be found as

$$
\begin{gathered}
z_{4}-z_{3}=l_{1}\left(\sin \theta_{2} \sin \theta_{4}+\cos \theta_{2} \cos \theta_{4} \sin \theta_{3}\right) \\
\theta_{3}=\operatorname{asin}\left(\frac{\frac{1}{l_{1}}\left(z_{4}-z_{3}\right)-\sin \theta_{2} \sin \theta_{4}}{\cos \theta_{2} \cos \theta_{4}}\right)
\end{gathered}
$$

but we prefer to express $\theta_{3}$ as an inverse tangent function. First, let us consider the following notations:

$$
\begin{align*}
& \alpha:=l_{2} \cos \theta_{1} \cos \theta_{2} \sin \theta_{4} \\
& \beta:=l_{2} \cos \theta_{2} \sin \theta_{1} \sin \theta_{4} \\
& \gamma:=\left(\left(X_{34}-\alpha\right)^{2}+\left(Y_{34}-\beta\right)^{2}\right) /\left(l_{2}^{2} \cos ^{2} \theta_{4}\right)=\cos ^{2} \theta_{3}+\sin ^{2} \theta_{2} \sin ^{2} \theta_{3}, \tag{6.4}
\end{align*}
$$

where

$$
\left(X_{34}, Y_{34}, Z_{34}\right):=P_{4}-P_{3}=\left(x_{4}-x_{3}, y_{4}-y_{3}, z_{4}-z_{3}\right)
$$

Therefore, one has

$$
\begin{align*}
& \kappa:=\left(\gamma-\sin ^{2} \theta_{2}\right) /\left(\cos ^{2} \theta_{2}\right)=\cos ^{2} \theta_{3} \\
& \delta:=\left(Z_{34}+l_{2} \sin \theta_{2} \sin \theta_{4}\right)^{2} /\left(l_{2} \cos \theta_{2} \cos \theta_{4}\right)^{2}=\sin ^{2} \theta_{3} \tag{6.5}
\end{align*}
$$

which gives us

$$
\tan ^{2} \theta_{3}=\delta / \kappa
$$

and hence

$$
\begin{equation*}
\theta_{3}=\operatorname{atan} 2( \pm \sqrt{\delta}, \sqrt{\kappa}) . \tag{6.7}
\end{equation*}
$$

This concludes the solution of inverse kinematics problem of our model.

### 6.1.3 Simulation of the human arm model and associated velocity ellipsoids

In the following, we bring the simulation of the 4-DOF human arm considered above to illustrate the transmission ratios and also velocity ellipsoids attached to the endeffector during a three dimensional movement of the arm. The simulation is performed in MATLAB, see Appendix B for the required MATLAB code. Here, using the inverse kinematics solution of the 4 -DOF human arm model, we will find the joint angles $\theta_{i}$ 's, $i=1, \cdots, 4$, during a movement of the arm. Then, $\left(J J^{T}\right)^{-1}$ will provide us the information needed to find the transmission ratios and velocity ellipsoids. We recall that our model is a 4 -DOF model with a spherical joint at shoulder and a revolute joint at elbow as it is shown in Figure 6.1 and also explained in the previous section. The length of the upper arm and former arm are $l_{1}=0.35 \mathrm{~m}, l_{2}=0.3 \mathrm{~m}$, respectively. Figure 6.3 shows the velocity ellipsoids associated to the arm during a movement. The movement of the arm is along a a circular path given by $\left(l_{1} / \sqrt{2}+l_{2} \sin t, l_{2} \cos t, l_{1} / \sqrt{2}\right)$ from $t=-1$ to $t=0.2$ with the step time $t=0.2$.

In the next part, we aim to simulate the arm behavior in response to an external disturbance force for different postures. In order to do so, we need to investigate dynamics of the model discussed above.

### 6.2 Dynamics of the 4-DOF human arm model

In this section, we would like to repeat the simulation that has been performed in Chapter 3 to explore the behaviour of the human arm in response to an external force for different postures. For this aim, i.e., to show the dependence of the human arm performance to the configured posture, one needs the dynamics of the system. Through this section, we bring the steps of finding the dynamics of our model. One approach to dynamics equation of a model is applying the Euler-Lagrange formula on the Lagrangian of the system. The Lagrangian of the system is the difference between


Figure 6.3: The 4-DOF model of human arm: Velocity ellipsoid during a circular motion on the path given by $\left(l_{1} / \sqrt{2}+l_{2} \sin t, l_{2} \cos t, l_{1} / \sqrt{2}\right)$ for $t=-1: 0.2: 0.6$. The principle axes of the ellipsoid is shown by red dashed lines.
kinetic and potential energies, that is

$$
L(\theta, \dot{\theta})=T(\theta, \dot{\theta})-U(\theta)
$$

where in our case

$$
T=\frac{1}{2} \sum_{i} m_{i} \dot{X}_{i, c}^{2}
$$

Here, $X_{1, c}=0.5 P_{3}, X_{2, c}=0.5\left(P_{3}+P_{4}\right)$ are positions of the centre of the upper arm and forearm, respectively. The coordinate of $P_{3}, P_{4}$ are given by formulas (6.3) and (6.1). A straightforward computation gives us

$$
\begin{align*}
T(\theta, \dot{\theta}) & =\left(\dot{\theta}_{2}^{2} l_{1}^{2} m_{1}\right) / 8+\left(\dot{\theta}_{2}^{2} l_{1}^{2} m_{2}\right) / 2+\left(\dot{\theta}_{2}^{2} l_{2}^{2} m_{2}\right) / 8+\left(\dot{\theta}_{4}^{2} l_{2}^{2} m_{2}\right) / 8 \\
& +\left(\dot{\theta}_{1}^{2} l_{1}^{2} m_{1} \cos ^{2} \theta_{2}\right) / 8+\left(\dot{\theta}_{1}^{2} l_{1}^{2} m_{2} \cos ^{2} \theta_{2}\right) / 2+\left(\dot{\theta}_{1}^{2} l_{2}^{2} m_{2} \cos ^{2} \theta_{2}\right) / 8 \\
& +\left(\dot{\theta}_{1}^{2} l_{2}^{2} m_{2} \cos ^{2} \theta_{4}\right) / 8+\left(\dot{\theta}_{3}^{2} l_{2}^{2} m_{2} \cos ^{2} \theta_{4}\right) / 8-\left(\dot{\theta}_{2} \dot{\theta}_{4} l_{2}^{2} m_{2} \sin \theta_{3}\right) / 4 \\
& +\left(\dot{\theta}_{2}^{2} l_{1} l_{2} m_{2} \sin \theta_{4}\right) / 2-\left(\dot{\theta}_{1}^{2} l_{2}^{2} m_{2} \cos ^{2} \theta_{2} \cos ^{2} \theta_{4}\right) / 4 \\
& -\left(\dot{\theta}_{2}^{2} l_{2}^{2} m_{2} \cos ^{2} \theta_{3} \cos ^{2} \theta_{4}\right) / 8+\left(\dot{\theta}_{1}^{2} l_{2}^{2} m_{2} \cos ^{2} \theta_{2} \cos ^{2} \theta_{3} \cos ^{2} \theta_{4}\right) / 8 \\
& +\left(\dot{\theta}_{1} \dot{\theta}_{4} l_{2}^{2} m_{2} \cos \theta_{2} \cos \theta_{3}\right) / 4-\left(\dot{\theta}_{1} \dot{\theta}_{3} l_{2}^{2} m_{2} \cos ^{2} \theta_{4} \sin \theta_{2}\right) / 4 \\
& +\left(\dot{\theta}_{1}^{2} l_{1} l_{2} m_{2} \cos ^{2} \theta_{2} \sin \theta_{4}\right) / 2+\left(\dot{\theta}_{2} \dot{\theta}_{3} l_{1} l_{2} m_{2} \cos \theta_{3} \cos \theta_{4}\right) / 2 \\
& -\left(\dot{\theta}_{2} \dot{\theta}_{4} l_{1} l_{2} m_{2} \sin \theta_{3} \sin \theta_{4}\right) / 2+\left(\dot{\theta}_{2} \dot{\theta}_{3} l_{2}^{2} m_{2} \cos \theta_{3} \cos \theta_{4} \sin \theta_{4}\right) / 4 \\
& -\left(\dot{\theta}_{1} \dot{\theta}_{2} l_{1} l_{2} m_{2} \cos \theta_{3} \cos \theta_{4} \sin \theta_{2}\right) / 2+\left(\dot{\theta}_{1} \dot{\theta}_{3} l_{1} l_{2} m_{2} \cos \theta_{2} \cos \theta_{4} \sin \theta_{3}\right) / 2 \\
& +\left(\dot{\theta}_{1} \dot{\theta}_{4} l_{1} l_{2} m_{2} \cos \theta_{2} \cos \theta_{3} \sin \theta_{4}\right) / 2 \\
& -\left(\dot{\theta}_{1} \dot{\theta}_{2} l_{2}^{2} m_{2} \cos \theta_{2} \cos \theta_{3} \cos { }^{2} \theta_{4} \sin \theta_{3}\right) / 4 \\
& -\left(\dot{\theta}_{1}^{2} l_{2}^{2} m_{2} \cos \theta_{2} \cos \theta_{4} \sin \theta_{2} \sin \theta_{3} \sin \theta_{4}\right) / 4 \\
& -\left(\dot{\theta}_{1} \dot{\theta}_{2} l_{2}^{2} m_{2} \cos \theta_{3} \cos \theta_{4} \sin \theta_{2} \sin \theta_{4}\right) / 4 \\
& +\left(\dot{\theta}_{1} \dot{\theta}_{3} l_{2}^{2} m_{2} \cos \theta_{2} \cos \theta_{4} \sin \theta_{3} \sin \theta_{4}\right) / 4 \\
& -\left(\dot{\theta}_{1}^{2} l_{1} l 2 m_{2} \cos \theta_{2} \cos \theta_{4} \sin \theta_{2} \sin \theta_{3}\right) / 2 . \tag{6.8}
\end{align*}
$$

The potential energy, $U$, can be found from

$$
U=m_{1} g z_{1, c}+m_{2} g z_{2, c}
$$

where $z_{1, c}, z_{2, c}$ are the $z$-components of $X_{1, c}, X_{2, c}$. A simple computation shows that
$U=-g m_{2}\left(l_{1} \sin \theta_{2}+\left(l_{2} \sin \theta_{2} \sin \theta_{4}\right) / 2+\left(l_{2} \cos \theta_{2} \cos \theta_{4} \sin \theta_{3}\right) / 2\right)-\left(g l_{1} m_{1} \sin \theta_{2}\right) / 2$.

Now, the $G$-term of the dynamics equation (2.3), will be obtained as

$$
\begin{equation*}
G=\left(G_{1}, G_{2}, G_{3}, G_{4}\right)=\left(\frac{\partial U}{\partial \theta_{1}}, \frac{\partial U}{\partial \theta_{2}}, \frac{\partial U}{\partial \theta_{3}}, \frac{\partial U}{\partial \theta_{4}}\right) \tag{6.10}
\end{equation*}
$$

where

$$
\begin{align*}
& G_{1}=0 \\
& G_{2}=-g m_{2}\left(l_{1} \cos \theta_{2}+l_{2} \cos \theta_{2} \sin \theta_{4} / 2-l_{2} \cos \theta_{4} \sin \theta_{2} \sin \theta_{3} / 2\right)-g l_{1} m_{1} \cos \theta_{2} / 2 \\
& G_{3}=-\left(g l_{2} m_{2} \cos \theta_{2} \cos \theta_{3} \cos \theta_{4}\right) / 2 \\
& G_{4}=-g m_{2}\left(l_{2} \cos \theta_{4} \sin \theta_{2} / 2-l_{2} \cos \theta_{2} \sin \theta_{3} \sin \theta_{4} / 2\right) \tag{6.11}
\end{align*}
$$

It is an easy computation to show that the inertia terms $M=\left(M_{i j}\right)$ can be found from the following formula

$$
\begin{equation*}
M_{i j}=\frac{\partial^{2} T}{\partial \dot{\theta}_{i} \partial \dot{\theta}_{j}} \tag{6.12}
\end{equation*}
$$

and having $M_{i j}$, the Coriolis/centrifugal terms can be obtained from

$$
\begin{equation*}
C_{i j}=\sum_{k} c_{i j k} \dot{\theta}_{k} \tag{6.13}
\end{equation*}
$$

where the Christoffel's symbols will be computed as (2.2). We will omit bringing the lengthy terms of inertia and Coriolis/centrifugal here, but rather, for the sake of completeness, we will bring the required MATLAB code to compute the terms of the
associated matrices in Appendix B.

### 6.2.1 Simulation of effect of human postures on the stability of the system; a regulation problem

In this section, we present a set of simulation results to show the posture dependent behavior of human arm in response to an external force. In our simulations, we consider the previously mentioned 4-DOF model of the human arm with a spherical joint at shoulder and a revolute joint at elbow joined by two arms with length $l_{1}=$ 0.35 m and $l_{2}=0.3 \mathrm{~m}$. Dynamics of the model is given by (6.11), 6.12) and 6.13). In our simulation, the arm is set in different positions in the three dimensional space as it is shown in Figure 6.4.


Figure 6.4: The human arm movement on the path given by the (6.14).

Then disturbance forces, $F_{y}, F_{z}, F_{w}$ are applied to the end-effector at each of the given positions, where $F_{y}, F_{z}, F_{w}$ are rectangular pulses respectively, in directions $y, z$, and $w=P_{4}-P_{3}$ (direction of the forearm). The magnitude of all three forces are $\left|F_{z}\right|=\left|F_{y}\right|=\left|F_{w}\right|=3 N$ and the time duration of applying is 2 seconds starting
from $t=5 \mathrm{~s}$. To regulate the system at the desired position, we use a PD-controller with gravity compensation as

$$
\tau=G-K_{p}\left(\theta-\theta_{d}\right)-K_{d}\left(\dot{\theta}-\dot{\theta}_{d}\right)
$$

where $K_{p}, K_{d}$ are positive definite matrices given by $K_{p}=10 I, K_{d}=2 I$. Postures are due to the movement of the arm on a path determined by the positions of elbow and hand, i.e., $P_{3}, P_{4}$ as following

$$
\begin{align*}
& P_{3}=\left(0,-l_{1} \cos (1.2),-l_{1} \sin (1.2)\right) \\
& P_{4}=\left(0,-l_{1} \cos (1.2)-l_{2} \cos (-.1 t),-l_{1} \sin (1.2)-l_{2} \sin (-.1 t)\right),-3 \leq t \leq 3 \tag{6.14}
\end{align*}
$$

Here $t$ is sampled every 0.5 s . In the following we refer to the above positions as Posture 1 to Posture 6, that is, Posture i corresponds to $P_{3}, P_{4}$, where $t=-3.5+.5 i$ for $i=1,2, \cdots, 6$. The corresponding velocity transmission rate $\beta_{u}$ for each posture is shown in Figure 6.5.

For each of the above positions, the torque $\tau_{i}, i=1,2,3,4$ applied to each joint, trajectories and desired trajectories $\theta$ and $\theta_{d}$ when the external force is in direction of $z, w$ are illustrated in Figures 6.6 and 6.8. The transmission rate $\beta_{y}$, as it can be seen in Figure 6.5, attains its minimum value at Posture 4, i.e., the posture that human arm is more capable of compensating a force in $y$-direction. This is also confirmed in Figure 6.10, where it can be seen that the torque applied to the joint 4, to keep the hand in the desired position, is less than other postures and also the joint displacement is less than other postures. On the other hand the transmission rate $\beta_{z}$ has its maximum value in Postures 3 and 4 , so, hand should be less capable of compensating the force applied in $z$-direction, comparing to the other postures. This is also confirmed in Figure 6.10, where it can be seen that torques applied to the joint 4 at Postures 3 and 4 are higher than other postures and also joint displacement is higher than other postures, see Figure 6.6. The transmission rate $\beta_{w}$ has smaller value


Figure 6.5: The velocity transmission rates $\beta_{y}$ (up-left), $\beta_{z}$ (up-right) and $\beta_{w}$ (downleft), where $w$ is the forearm direction and human arm postures are given by 6.14 .
comparing to $\beta_{z}$, since hand in the direction of forearm is more capable of handing a disturbance force applied in the same direction. The small difference between $\beta_{w}$ and $\beta_{y}$, see Figure 6.5. is because of the fact that in our experiment, directions of forearm and $y$-axis are almost the same. Figure 6.10 shows that control input $\tau_{4}$ applied to the fourth joint for Postures 1-6, when the human arm is under forces $F_{y}, F_{z}, F_{w}$ (shown in black, blue and red, respectively). As it can be seen in this figure, for all the postures, the control applied to the joint 4 , when the arm is under force $F_{z}$, is higher than the cases that the arm is under force $F_{y}$ and $F_{w}$, as transmission ratios also confirm that human hand is less capable of compensating the force $F_{z}$. The reason that control $\tau_{4}$ is of more interest here is because of the fact that elbow experiences more force comparing to shoulder and is subject to higher displacement. (More precisely, joint 1 and 2 are fixed, but joint 3 which is along the upper arm feels more force.)


Figure 6.6: Left: trajectories $\theta$ vs desired trajectories $\theta_{d}$ for force $F_{z}=3$ N. Right: torques $\tau$ applied to the joints for Posture 1 (top) to Posture 3 (bottom). Postures are give by (6.14).


Figure 6.7: Left: trajectories $\theta$ vs desired trajectories $\theta_{d}$ for force $F_{z}=3 \mathrm{~N}$. Right: torques $\tau$ applied to the joints for Posture 4 (top) to Posture 6 (bottom). Postures are give by (6.14).


Figure 6.8: Left: trajectories $\theta$ vs desired trajectories $\theta_{d}$ for force $F_{w}=3 \mathrm{~N}$. Right: torques $\tau$ applied to the joints for Posture 1 (top) to Posture 3 (bottom). Postures are give by 6.14).


Figure 6.9: Left: trajectories $\theta$ vs desired trajectories $\theta_{d}$ for force $F_{w}=3 \mathrm{~N}$. Right: torques $\tau$ applied to the joints for Posture 4 (top) to Posture 6 (bottom). Postures are give by (6.14).


Figure 6.10: Control input $\tau_{4}$ applied to the fourth joint for Posture 1 (top-left) to Posture 6 (bottom-right). Blue: force applied in $z$-direction, black: force applied in $y$-direction and red: force applied in direction $w$. Postures are give by (6.14).

The simulation performed in MATLAB with a variety of other postures and
forces also confirms the above mentioned claim, that is, posture dependent behaviour of human arm in response to an external disturbance force. Since, we provide the required MATLAB code to perform similar simulations in other postures and force directions in Appendix B, we omit plotting similar results here.

### 6.3 An adaptive PBFR gain

In this short section, we suggest a new PBFR gain depending on human postures that is suitable when human arm is modelled with 4 degrees of freedom as discussed in the chapter. Given a unit vector $u \in \mathbb{R}^{3}$, the PBFR gain $\alpha$ is introduced as a linear function of the force transmission rate in the direction of the reflected force given by equation (4.2). As it is explained in Chapter 4, in the case of our interest, the unit vector $u=\hat{u}_{f_{e}}$ in formula (4.2) is the unit vector in the direction of the reflected force and also, $\alpha_{\min }=\alpha\left(\gamma_{\min }\right)$, and $\alpha_{\max }=\alpha\left(\gamma_{\max }\right)$.

Here, we consider the $\alpha\left(\gamma_{\min }\right)=0.1$ and $\alpha\left(\gamma_{\max }\right)=1$. Using (4.2), we suggest a new PBFR gain $\alpha$ given by

$$
\begin{equation*}
\alpha(\gamma)=\underset{[0.1,1]}{\operatorname{Sat}}\left\{1+\frac{.9}{2}(\gamma-4)\right\} \tag{6.15}
\end{equation*}
$$

which is a simple transformation relating $\alpha(1 / .25)=1$ to $\alpha(1 / .48)=0.1$. The saturation function at 0.1 and 1 , makes sure that for the cases that force transmission rate is higher than or equal 4 and human hand is capable of compensating the force, the PBFR gain is selected by $\alpha=1$. Also for cases that force transmission rate is lower than 2, and human hand is not capable of compensating the force completely, the small value of $\alpha=0.1$ is selected. These values 2 and 4 , are corresponding to the force transmission rate when the human arm is in the position $P_{3}=(0,-0.02,-0.34)$ and $P_{4}=(0,-0.32,-0.34)$. Although, we will not do it in this thesis, but this adaptive PBFR gain can be applied when the PBFR algorithm is employed to a force reflecting teleoperator system. In fact, as a future research, we suggest that one can
detect the human posture using at least two cameras to update postures of the human arm on-line and then based on this adaptive PBFR gain investigate the performance of the force reflecting teleoperator system with a time varying delay.

### 6.4 Concluding remarks

In this chapter, generalization of the methods of Chapters 4 and 5 to a more realistic model of the human arm with 4 degrees of freedom and three dimensional movement have been described. In fact, unlike the 2-DOF model considered in the last two chapters, this model is more close to reality when human arm is executing a task in three dimensional space. As mentioned in the previous chapters, our goal is to show the posture dependent behaviour of the human arm responding to an external force during a task execution and quantify it based on velocity/force transmission rates. Toward such a development, kinematics, inverse kinematics and dynamics of the model have been obtained. Also, simulation results have been performed to demonstrate the behavior of the human arm in different postures under a disturbance force. Moreover, an adaptive PBFR gain for this model has been suggested, however, experiments of on-line posture estimation when human arm performing a three dimensional task is postponed to a future work. It is worth mentioning that all requirements for such development now have been provided in this chapter, and the only part which is not discussed is how to setup two cameras to detect the human arm posture during a task.

## Chapter 7

## Conclusion and future work

In this final chapter, we briefly describe the major contribution of this thesis and also suggest possible future research work.

The focus of the thesis is on the performance of a teleoperator system with force reflection. A teleoperator system usually consists of a local and a remote manipulator which communicate through a communication channel. When the remote manipulator is in contact with an environment, to provide a better feel of contact for the human operator and to obtain a higher level of performance, it is desirable to send the interacting force information to the local manipulator. The contact force reflected back to the local side may cause irreversible damage. To address this issue, the projection based force reflection algorithm has been suggested. It has been already shown in the literature that the PBFR algorithm improves the stability of the force reflecting teleoperator system and haptic interface without significant transparency deterioration. In this method, because there is no a priori information of the contact force estimation, the PBFR gain is considered a constant small value for stabilizing the system for a wide range of forces. However, it is known that choosing a small PBFR gain will cause slower convergence of the reflected force to the actual force and hence transparency deterioration. Therefore, selecting an 'optimal' PBFR gain which is not necessarily a small constant would be of interest. In this thesis, studying the relation between the human operator postures and the stability of a local-remote force reflecting teleoperator system, we suggested a method of applying a new PBFR gain which depends on the human postures instead of a small constant gain (Chapter 4). Applying our varying and posture dependent PBFR gain $\alpha(\gamma)$, on an experiment
with a haptic device, we showed that there is no need to decrease the PBFR gain for cases where the human hand can compensate the reflected force. This improves the transparency of the system while the stability is preserved.

In Chapter 5, the above method was generalized to a teleoperator system with force feedback. Our teleoperation system consists of a local and a remote manipulator which are communicating through a communication channel with time varying delay. By means of a camera, we detected the human arm position at each instant time and updated the PBFR gain online during a task execution. Our experimental results compared the stability and transparency of the system in three different cases: direct force reflection, PBFR algorithm with constant gain and our suggested new adaptive PBFR gain. We showed that applying the adaptive PBFR gain based on human postures improved the transparency of the teleoperator system, while stability is preserved.

Chapter 6, was devoted to generalizing the method of Chapters 4 and 5 to a more realistic model of human arm with 4 degrees of freedom and three dimensional movement (unlike the 2-DOF model considered in Chapters 4 and 5). Towards such a development, kinematics, inverse kinematics and dynamics of the model were obtained. Also, simulation results were performed to demonstrate the behaviour of the human arm in different postures under a disturbance force. At the end, an adaptive PBFR gain for this model was also suggested; however, online posture detection during task performance was postponed to a future work.

### 7.1 Future work

Possible directions for future work can be listed as follows.

- An extension of methods developed in Chapter 5 to the case where the human arm is modelled with 4 (or even more) degrees of freedom (as explained in

Chapter 6). To develop such methods which allow for detection of three dimensional movements of the human operator, at least two cameras have to be setup. Here, the experimental setup will be the same as for the teleoperation system described in Chapter 5, with the difference that this time there will be a more realistic model of the human arm with 4-DOF, compared to the 2-DOF model in Chapter 5. Cameras are needed in order to detect the three dimensional movement of the human arm, by detecting the positions of the elbow and the hand (wrist).

- Development of a method for detection of muscles tensions while grabbing the manipulator.
- Rigorous stability analysis of the teleoperation system described in Chapter 4 with the new posture dependent PBFR gain.


## Appendix

## Appendix A: MATLAB codes

In this section, we present the MATLAB codes have been used in Chapter 6 to model a 4-DOF human arm, its inverse kinematics and dynamics. The first code (i.e., Human arm model with 4 -DOF) is to solve the inverse kinematics of the model and also to find transmission ratios and velocity ellipsoids at each human posture. The second code (i.e., Dynamics of the 4-DOF human arm) is to compute the gravity, inertia and Coriolis/centrifugal terms of dynamics of the system.

```
% Human arm model with 4-DOF
% This code solves the inverse kinematics problem and also plot the
% associated velocity ellipsoids and transmission ratios
% A. Moatadelro
%
clc
clear all
% Uper and fore arm length; for reference lu=l1 and lf=l2
lu=.35;
lf=.3;
%Path initialization
initialTime=-1;
Delta=.2;
finalTime=1;
t=initialTime:Delta:finalTime;
%P3 and P4 are frame origins fixed at elbow and end-effector
P3=[-.35*sqrt(2)*(t-t+1)/2;-.0*(t);-.35*sqrt(2)*(t-t+1)/2];
P4=[-.35*sqrt(2)* (t-t+1)/2-.3*\operatorname{sin}(t);-.3*\operatorname{cos(t);-.35*sqrt(2)*(t-t+1)/2];}
for i=1:length(t)
X3=P3(1,i);
Y3=P3(2,i);
Z3=P3(3,i);
X4=P4(1,i);
Y4=P4(2,i);
Z4=P4(3,i);
X34=P4(1,i)-P3(1,i);
Y34=P4(2,i)-P3(2,i);
Z34=P4(3,i)-P3(3,i);
tg=(X4.^2+Y4.^2+Z4.^2-lf.^2-lu.^2)/(2*lf*lu);
theta1=atan2(Y3,X3);
theta2=atan2(Z3,sqrt((X3.^2+Y3.^2)));
theta4=asin(tg);
if theta1<.1^(4)
    thetal=0;
end
if theta2<.1^(4)
            theta2=0;
end
if theta4 <.1^(4)
            theta4=0;
end
```

```
        Alpha=lf*cos(theta1)*cos(theta2)*sin(theta4);
        Beta=lf*cos(theta2)*sin(theta1)*sin(theta4);
        Gamma=simplify(((X34-Alpha).^2+(Y34-
Beta).^2)/(lf.^2*cos(theta4).^2));
    Kapa=simplify((Gamma-sin(theta2).^2)/(cos(theta2).^2));
Delta=((Z34+lf*sin(theta2)*sin(theta4)).^2)/((lf* cos(theta2)* cos(theta4)).^2)
;
    theta3=atan2(sqrt(Delta),sqrt(Kapa));
    theta=[theta1;theta2; theta3;theta4];
    J=jacHumanArm(theta);
    A=inv(J*J');
    [V,D]=eig(inv(J*J'));
        lambda1=D (1,1);
        lambda2=D (2,2);
        lambda3=D (3,3);
        r1=1/sqrt(lambda1);
        r2=1/sqrt(lambda2);
        r3=1/sqrt(lambda3);
        V1=[V(1,1); V(2,1); V (3,1)];
        v2=[V(1,2); V(2,2); V (3,2)];
        v3=[V(1,3); V(2,3); V (3,3)];
% The velocity ellipsoid in standard coordinate
[x, y, z] = ellipsoid(X4,Y4,Z4,.5*r1,.5*r2,.5*r3);
%Finding Euler angles of ellipsoid rotation (rotation matrix is V)
if V(3,1) ~= 1 && V (3,1) ~=-1
        th = - asin(V(3,1));
        psi = atan2(V(3,2)/cos(th),V(3,3)/cos(th));
        phi = atan2(V(2,1)/cos(th),V(1,1)/cos(th));
else
    phi= 0;
        if V(3,1)==-1
        th = pi/2;
        psi = phi + atan2(V(1,2), V(1,3));
        else
        th = -pi/2;
        psi = -phi + atan2(V(1,2), V(1,3));
        end
end
% Rotating the ellipsoid; Euler angles in degree
xangle=psi*180/pi; yangle=th*180/pi; zangle=phi*180/pi;
figure
```

```
h=surf(x, y, z);
rotate(h,[1 0 0], xangle, [X4,Y4,Z4]);
rotate(h,[0 1 0], yangle, [X4,Y4,Z4]);
rotate(h,[0 0 1], zangle, [X4,Y4,Z4]);
axis equal
hold on
axis([-.6 . . - -. 6 . . 6 -. 6 . 6])
grid on
line([0 X3 X4],[0 Y3 Y4],[0 Z3 Z4],'LineWidth',5,'Color',[0 0 0])
line([X4-r1*V1(1) X4+(r1)*v1(1)],[Y4-(r1)*v1(2) Y4+(r1)*V1(2)],...
    [Z4-(r1)*v1(3) Z4+(r1)*v1(3)],'LineWidth',2,'LineStyle','--','Color',[1 0
0])
line([X4-(r2)*v2(1) X4+(r2)*v2(1)],[Y4-(r2)*v2(2) Y4+(r2)*v2(2)],...
    [Z4-(r2)*v2(3) Z4+(r2)*v2(3)],'LineWidth',2,'LineStyle','--','Color',[1 0
0])
line([X4-(5*r3)*v3(1) X4+(5*r3)*v3(1)],[Y4-(5*r3)*v3(2) Y4 + (5*r3)*v3 (2)],...
    [Z4-(5*r3)*v3(3) Z4+(5*r3)*v3(3)],'LineWidth',2,'LineStyle','--
','Color',[1 0 0])
hold on
xlabel('x(m)','FontWeight','bold');ylabel('y(m)','FontWeight','bold');zlabel(
'z(m)','FontWeight','bold')
title('The velocity ellipsoid during human arm movement','FontWeight','bold')
%% Velocity Transmission Ratio
ux=[1;0;0];
betax=1/sqrt(ux'*inv(J*J')*ux);
uy=[0;1;0];
betay=1/sqrt(uy'*inv(J*J')*uy);
uz=[0;0;1];
betaz=1/sqrt(uz'*inv(J*J')*uz);
beta(:,i)=[betax;betay;betaz];
beta_inv(:,i)=[1/betax;1/betay;1/betaz];
end
temp=[11 2 3 3 4 5 6 7 8 9 10 11];
figure
plot (temp,beta(1,:))
title( 'Velocity transmission rate \beta in x-direction')
xlabel('Number of the location');ylabel('\beta_x')
grid on
figure
plot(temp,beta(2,:))
title( 'Velocity transmission rate \beta in y-direction')
xlabel('Number of the location');ylabel('\beta_y')
grid on
figure
plot(temp,beta(3,:))
```

title( 'Velocity transmission rate \beta in y-direction')
xlabel('Number of the location');ylabel('\beta_y')
grid on

```
% Dynamics of the 4-DOF human arm
% A.Moatadelro
clear all
clc
syms theta1 theta2 theta3 theta4 dtheta1 dtheta2 dtheta3 dtheta4 pi;
syms ddtheta1 ddtheta2 ddtheta3 ddtheta4 m1 m2 g l2 l1
A1=simplify(DH_convention(0,-pi/2, 0,theta1));
A2=simplify(DH_convention(0,-pi/2,0,theta2-pi/2));
A3=simplify(DH_convention(0, pi/2,l1,theta3+pi/2));
A4=simplify(DH_convention(l2, 0, 0,theta4));
T2=A1*A2;
T3=T2*A3;
T4=T3*A4;
%% Elbow and end effector coordinate
xel=T3(1,4);
yel=T3 (2,4);
zel=T3(3,4);
xend=T4(1,4);
yend=T4 (2,4);
zend=T4(3,4);
xc1=(1/2)*xel;
yc1=(1/2) *yel;
zc1=(1/2)*zel;
vxcl=diff(xc1,theta1)*dtheta1+diff(xc1,theta2)*dtheta2+...
    diff(xc1,theta3)*dtheta3+diff(xc1,theta4)*dtheta4;
vyc1=diff(yc1,theta1)*dtheta1+diff(yc1,theta2)*dtheta2+...
    diff(yc1,theta3)*dtheta3+diff(yc1,theta4)*dtheta4;
vzc1=diff(zc1,theta1)*dtheta1+diff(zc1,theta2)*dtheta2+...
    diff(zc1,theta3)*dtheta3+diff(zc1,theta4)*dtheta4;
```

```
xc2=.5* (xel+xend);
```

xc2=.5* (xel+xend);
yc2=.5* (yel+yend);
yc2=.5* (yel+yend);
zc2=.5*(zel+zend);
zc2=.5*(zel+zend);
vxc2=diff(xc2,theta1)*dtheta1+diff(xc2,theta2)*dtheta2+...
diff(xc2,theta3)*dtheta3+diff(xc2,theta4)*dtheta4;
vyc2=diff(yc2,theta1)*dtheta1+diff (yc2,theta2)*dtheta2+...
diff(yc2,theta3)*dtheta3+diff(yc2,theta4)*dtheta4;
vzc2=diff(zc2,theta1)*dtheta1+diff(zc2,theta2)*dtheta2+...
diff(zc2,theta3)*dtheta3+diff(zc2,theta4)*dtheta4;
T=.5*m1*(vxc1^2+vyc1^2+vzc1^2)+.5*m2*(vxc2^2+vyc2^2+vzc2^2);
U=m1*g*zc1+m2*g* zc2;
L=T-U;

```
```

q=[theta1; theta2; theta3; theta4];
dotq=[dtheta1; dtheta2; dtheta3; dtheta4];
syms M C G
G=[diff(U,theta1);diff(U,theta2);diff(U,theta3);diff(U,theta4)];
for i=1:4
for j=1:4
M(i,j)=0;
end
end
for i=1:4
for j=1:4
M(i,j)=diff(diff(T,dotq(i)),dotq(j));
end
end
for i=1:4
for j=1:4
C(i,j)=0;
end
end
for i=1:4
for j=1:4
for k=1:4
C(i,j)=C(i,j)+.5*(diff(M(i,j),q(k))+diff(M(i,k),q(j))-
diff(M(k,j),q(i)))*dotq(k);
end
end
end

```

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- A Riemann-Roch theorem for the noncommutative two torus, Canadian Operator Symposium Queen's University \& Noncommutative Geometry Conference, Ohio State University, 2012
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