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MODERN AND DECENTRALIZED
CONTROL FOR MULTIVARIABLE PROCESSES

by

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Submitted in partial fulfilment of
the requirement for the degree of
Doctor of Philosophy

Faculty of Graduate Studies
The University of Western Ontario
London, Ontario, Canada
February, 1993

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ISBN 0-315-83932-5

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ABSTRACT

Control methodologies to cope with multivariable process systems are studied in this thesis. Topics on both modern and conventional decentralized control strategies are covered with the major focus on the latter.

In the modern control aspect, the two most common approaches -- the LQG optimal control and the eigenvalue assignment design are discussed. Emphasis is placed on investigation of LQG applications to complex real-time processes, and eigenvalue assignment design for improved steady state and robust performance. By careful consideration of practical issues and innovative use of model identification techniques, a strongly interactive and non-linear multivariable pressure tank system is satisfactorily controlled by the LQG scheme. A PI state feedback controller is proposed, and an eigenvalue assignment design for robust performance is discussed.

With respect to decentralized control, various issues including interaction measurement, variable pairing, stability and stability robustness, robust performance, controller design and integrity, are systematically addressed. Major results include: (1). A new interaction measure capable of measuring the absolute interaction and the nature of interaction is developed. (2). A new comprehensive variable pairing criterion, based on the Niederlinski Index, is

presented. (3). The use of the RGA as a direct measure of integrity is expanded. (4). A series of stability conditions for decentralized control under independent design and variable pairing are established. (5). A new stability robustness measure to evaluate the effects of model error on process gains is developed. (6). New disturbance directionality indices and robust performance criteria with respect to model uncertainty associated with manipulated variables are developed, and robust design procedures for general multivariable controllers are provided.

ACKNOWLEDGEMENTS

I sincerely express my gratitude to Dr. A. Jutan, my supervisor, for his invaluable guidance, warm encouragement, and great support throughout the course of this thesis work.

I would like to thank my fellow graduate students in the process control group, Venu Krishnapura, Ian Edwards, Edward Katende, Emdad Hoque, Ethel Nakano, and Anh Vo Nguyen, for their friendship, help, and cooperation.

I also wish to thank Mrs. Margaret Mousseau, the departmental secretary, for her kind assistance and help in typing some of the thesis and managing the correspondence for the papers included in this thesis.

Financial assistance from the Department of Chemical and Biochemical Engineering, Faculty of Engineering Science, and University of Western Ontario, and financial support from the National Science and Engineering Research Council of Canada are gratefully appreciated.

I would like to dedicate this thesis to my motherland, the People's Republic of China, where I received excellent education -- an absolute asset for this work.

Finally, I wish to express my appreciation to my wife, Ming-Jing, and my daughter, Weixi, for their wholehearted support, without whom this work would not have been attempted.

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NOMENCLATURE

All the notation used in the text have been defined at appropriate places. Only some of the common and important notations are listed below.

$C(s)$	= Controller transfer function matrix
$C(0)$	= Steady state gain of C(s)
$G(s)$	= Transfer function matrix of plants
$G(0)$	= Steady state gain of G(s)
$G^{-1}(0)$	= The inverse of G(0)
$[G(0)]_{ij}$	= The (i, j)-th element of G(0)
$G^{(ii)}(0)$	= The reduced matrix of G(0) with i-th row and i-th column removed
$\bar{G}(0)$	= The subsystem of G(0) containing diagonal elements only
$H(s)$	= G(s)C(s)
I	= The unitary matrix
k_p	= Proportional gain of PID controllers
n	= The number of input and output variables

s = Laplace variable

T_i = Derivative time of PID controllers

T_i = Reset time of PID controller

Greek Symbols

γ = Condition number of a matrix

λ_{ij} = The (i,j)-th element of RGA

ω = Frequency

Δ = model error

Abbreviations

diag = Diagonal

DIC = Decentralized Integral Controllability

det = Determinant (of a matrix)

IC = Integral Controllability

IM = Interaction Measurement

NI = The Niederlinski's Index

RGA = The Relative Gain Array

VP = Variable Pairing

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CHAPTER 1

INTRODUCTION

1.1 Opening Remarks

Process control has come to play a more and more important role in the process industry. In fact, as global competition, coupled with tougher environmental regulations and economic conditions, becomes ever stronger, process control often represents a vital factor in achieving optimal operation of chemical processes. On the other hand, as the chemical and petroleum industries are driven to become increasingly large in scale and integrated in operation, the requirements for process control become stringent. The most important characteristic of modern chemical processes, in terms of control system design, is that almost all of them are multi-input multi-output (MIMO) systems.

Essentially, there are two types of design strategies to tackle MIMO systems: modern control, such as state space analysis and model based control, and conventional decentralized control. With their strong advances in the last decade, modern control techniques have found successful applications, especially in electrical, military and aerospace industries. In particular, Linear-Quadratic (LQ) optimal control and Pole Placement (also commonly referred to Eigenvalue Assignment) design are the most widely used techniques. However, despite the sophisticated development of a now vast body of such knowledge , at least in theory, and despite some spectacular applications of this theory to practical situations, modern control has not achieved widespread acceptance in process industries as was initially expected since its establishment in the 1960's.

Instead, classical control has remained a predominant scheme in process practice. Since it is a direct extension of classical Single-Input Single Output (SISO) control philosophy in nature, conventional decentralized structure is commonly used to tackle MIMO problems in practice. There are many practical advantages using decentralized control strategy. For instance, it is simple to implement and tune, easy to understand by engineers, and easy to make failure tolerant. However, in contrast to SISO problems, there exist distinct characteristics associated with MIMO systems, such as interactions among different control loops, directionality of inputs, and correlated model uncertainties. However, These characteristics of decentralized control systems have not been completely understood yet. As a result, the design of decentralized control remains, to a large extend, a trial and error procedure. Straightforward application of classical control methods to MIMO systems usually results in inefficient and even unsuccessful design. Clearly, any advance in decentralized control will have immediate and important implications for practical applications.

1.2 Objective and Scope of the Thesis

This thesis covers topics on both modern control and decentralized control for multivariable processes with the focus on the latter. In the modern control (with a full scale controller) aspect, attempts are made to investigate how LQ control can be successfully applied to complex multivariable process systems, and how eigenvalue assignment scheme can be improved for better steady state and robust performance. With regard to decentralized control, a systematic approach is taken to investigate various issues involved in the analysis and design of such a control strategy, such as interaction measurement, variable pairing, controller tuning, stability and integrity, robust stability and robust performance, and configuration and design of robust controllers.

1.3 The Organisation of the Thesis

The main body of this thesis (Chapter 3 to 11) is essentially based on papers or notes published or submitted for publication to various scientific journals or conferences, during the author's Ph.D program. The integrity of each paper or note on a specific topic is maintained, so each chapter is virtually self-contained and independent in terms of writing and use of notation. However, there does exist a main theme which follows through the entire thesis. The topics in all the chapters fall into the scope and the overall objective outlined in the last section with individual chapters focusing on specific issues. In addition, a great deal of effort is made to interrelate individual chapters. Each chapter includes its own introduction and brief but concise literature review of the relevant issues. The main thrust and its relevance to other chapters are interrelated in the literature review (Chapter 2).

The thesis consists of two parts and twelve chapters. The first part (Chapter 3, 4, and 5) covers topics on modern control, and the second part (Chapter 6 to 11) is devoted to issues involved in decentralized control. A brief summary of all the chapters are outlined below.

Chapter 2: Literature Review and Relation to Present Work

A general review of literature on modern control theories (LQG control and eigenvalue assignment design) as well as decentralized control techniques is presented. The relevance to present work in the thesis and inter-relationship between different chapters are provided.

Chapter 3: Model Identification and Optimal Stochastic Control of A Multivariable Pressure Tank System

The chapter provides a successful real-time application of LQG control to a strongly interactive and nonlinear pressure tank process by using model identification techniques and taking practical considerations into account in design and implementation. The content of this chapter is based on the paper with the same title accepted for publication by *Chem. Eng. Res. Des.*(1993).

Chapter 4: A PI-Type State Feedback Controller and Robust Eigenvalue Assignment Design

A PI state feedback controller is constructed to achieve perfect steady state (off-set free) control by explicitly introducing integral action in the formation of state feedback design problem. Eigenvalue assignment design is then used to design the controller, and the extra degree of freedom in addition to eigenvalue assignment in MIMO systems is utilized to achieve robust performance within an 'engineering' framework without resorting to complex robust control theories. This chapter is based on the paper submitted for publication to *Trans. Inst. Meas. Contr.* (1992).

Chapter 5: A Dynamic Simulator for Control Design in Distributed Systems

This chapter is devoted to the development of design tools to facilitate control system design for complex systems described by partial and ordinary differential equations. A general dynamic simulator, called MDSS (Micro Dynamic System

Simulator), is developed. This chapter is published in *Trans. Inst. Meas. Contr.*, 14 (2), 65-70, (1992).

Chapter 6: A New Variable Pairing Criterion Based on Niederlinski Index

A new variable pairing criterion for decentralized control systems is presented in this chapter, which is based on a paper accepted for publication by *Chem. Eng. Comm.* (1992).

Chapter 7: RGA as A Measure of Integrity for Decentralized Control Systems

RGA is shown to be capable of directly measuring the integrity problem for decentralized control systems.

Chapter 8: Dynamic Interaction Analysis In Decentralized Control Systems

Based on a paper presented at the *40th Can. Chem. Eng. Conf.* and submitted for publication, a new dynamic interaction measure, capable of measuring both the absolute and relative interaction, is presented here.

Chapter 9: Consistency Principles for Stability in Decentralized Control Systems

A number of stability conditions, called 'consistency principles for stability', in decentralized control systems are developed in this chapter. This chapter is based on the paper with the same title submitted for publication.

Chapter 10: Stability Robustness for Decentralized Control Systems

This chapter addresses the robust stability problem in face of model errors in independent and correlated individual gains. Immediate implications for variable pairing from robust stability point of view are also provided. The main context of this chapter is published in *Chem. Eng. Sci.*, 48 (13), 2337-2343, (1993).

Chapter 11: Robust Multivariable Control Against Model Uncertainty and Disturbance Directionality

Robustness indices against strong disturbance directionality and model uncertainty are developed, and systematic design approach to achieve a best compromise for robust performance against both disturbance directionality and model error is presented. This chapter is based on a paper submitted for publication.

Chapter 12: Conclusions and Future Work

The main contributions of this thesis are summarized, and some recommendations for future work are also discussed.

CHAPTER 2

LITERATURE REVIEW

2.1 GENERAL

As the chemical industry advances towards increasingly large scale and integrated operation, most control systems involved constitute MIMO systems. There are basically two sorts of strategies to cope with the design of MIMO systems -- conventional decentralized control and modern (model-based) control.

Decentralized control uses a diagonal controller and thus naturally divides a MIMO system into several interacting control loops. The design strategy of such a structure is usually to approximately design individual loops first as though they were interaction-free, followed by a tuning procedure, usually by trial and error, with interactions taken into account (Shinsky, 1989). Hence, the theoretical basis for decentralized control design rests on classical SISO control methods, and the key issue is interaction measurement.

In classical control theory, only the input, output, and error signals are considered important, and the analysis and design of control systems are carried by transfer functions. Classical design methods are a combination of analytical ones (e.g., Laplace transform, Routh test), graphical ones (e.g., root locus, Bode diagram, Nyquist plots). These methods are covered by a number of textbooks (Franklin, at al., 1991; Ogata, 1990; Luyben, 1990; Seborg, et al., 1989).

The primary aim of classical design methods is to stabilize a plant, whereas secondary aims may involve obtaining a certain transient response, disturbance rejection, steady state error, and robustness to uncertainties. Essentially, however, classical control methods are based on trial and error procedures (Ogata, 1990). On the other hand, an accurate mathematical description of the plant is not strictly required by trial and error design in practical applications. This is mainly why this design scheme has remained predominant in process industry due to the fact that a precise model is usually unavailable.

In contrast to SISO systems, multivariable systems have many distinct characteristics. Various issues associated with these characteristics in decentralized control, such as interaction measurement, variable pairing, and controller tuning, must be addressed to facilitate and to ensure successful design of decentralized control systems. A review of existing literature dealing with these issues is given, and the main thrust of this thesis provided in the next section.

Modern control theory emerged in the 1960's. The investigations of state space approach by Kalman (1960) and Gilbert (1963) and the frequency-domain work of Rosenbrock (1970) enabled systems theory to progress to the analysis of multivariable linear processes. As opposed to classical control, modern control is aimed to de-empiricize system design and to present solutions to a much wider class of control problems. Usually, we refer to modern control as the state space based method. One advantage of the state space method is that the increase of state variables and input/outputs in a system does not structually increase the complexity of the equations.

System design by modern control theory via state space methods enables engineers to design such systems having desired closed-loop poles (or desired characteristic equations) or optimal control systems with respect to given performance indices (such as the most widely used linear-quadratic index). In fact, pole placement (or eigenvalue assignment) design and linear-Quadratic (LQ) optimal control form two pillars in modern control theory (Ogata, 1990; Brogan, 1991; Franklin et al., 1991; Anderson and Moore, 1990; Kwakernaak and Sivan, 1972;). Notably, Both strategies end up with the same control law -- state feedback and allow for analytical solutions. Also, they both involve the same design steps: controller design (state feedback law), state estimator design (estimation of states from measurements and outputs), and combining of control law with estimator.

However, design by modern control theory requires accurate mathematical model of system dynamics. Despite its rapid development and its successful applications to electrical, military and aerospace systems, modern model-based control has not found widespread application in process industries. One major reason is the limited understanding of the details of internal mechanisms for processes, coupled with their often strong non-linearities and constantly changing conditions (Foss, 1973; Ray, 1983; Morari and Lee, 1991). Thus, direct application of the standard modern control theory without modification would lead to unsuccessful design and implementation in practical process environment (Morari and Doyle, 1986; Ray, 1983).

The various techniques in using pole placement and LQ design with Gaussian noise in the framework of stochastic control are reviewed, and our modifications to improve and ensure their successful application to process systems are presented below.

2.2 MODERN CONTROL METHODS

Design issues involved in pole placement design and LQG control are reviewed separately in this section. Our improvement over standard methods are stressed.

2.2.1 LQG Optimal Control

The concept of optimality means optimizing some mathematical performance index or cost function, by the selection of control design parameters and structure. The LQ and LQG control design philosophies utilize a quadratic performance index. they have provided the most complete multivariable design and synthesis theory yet available (Johnson and Grimble, 1987). This is supported by the wealth of research papers over the last decades on the subjects of optimal control and estimation theory (Brogan, 1991; Anderson and Moore, 1990; Johnson and Grimble, 1987; Grimble, 1992).

The basis of LQ and LQG control is Bellman's (1962) principle of optimality. Dynamic programming (Bellman and Dreyfus, 1962) and Pontryagin's (1962) minimum principle represent the most elegant approaches to optimal control problems. Newton et al. (1957) and Chang (1961) first presented design procedures for univariate processes in the frequency domain for continuous-time LQ output controller. Kalman (1960) introduced a more general LQ state space control theory. The noticeable development in frequency domain approaches, which use transfer function models, leads to a Wiener-Hopf equation solved by spectral factorization methods (Youla et al., 1976; Youla and Bongiorno, 1985; Grimble, 1979; Harris and MacGregor, 1987). The state space based method, nevertheless, has today become so well established (Astrom, 1970; Kwakernaak and Sivan, 1972; Anderson and Moore, 1991) that the mere mention of LQ control conjures up to the vision of the state space models and Riccati equations (solving for state feedback control law) of this time domain approach. We shall focus on the state space design methodology.

The first step in the LQ and LQG control design is to obtain the control law by solving Riccati equations for deterministic or stochastic systems respectively, as discussed above (Astrom, 1970; Anderson and Moore, 1991). The resulting control laws explicitly require state feedback while state variables usually have no physical meanings and are not directly available. Thus, state observer (LQ control) or state estimator (LQG), which provides reconstruction or estimations for states from measurements and outputs of the physical system, has to be designed. State observer (full and reduced order) for LQ control, is introduced and its design method is developed by Luenberger (1971). For stochastic processes, Kalman filter (Kalman and Bucy, 1961) provides an optimal estimation of states in the presence of white noises in the system. The second step in LQ and LQG control system design, the design of the state estimator, can be readily accomplished following the same procedure as for the controller design. This is due to the duality between the state feedback law and the state estimator, (Astrom, 1970; Anderson and Moore, 1991).

The third step in LQ and LGQ control system design is to combine both the controller and the estimator. The separation principle (Kwakernaak and Sivan, 1972; Anderson and Moore, 1991) allows both to be designed separately. Hence, it is clear that the core problem in LQ and LQG control is to solve the matrix Ricatti equation. Various software packages for this purpose are now readily available.

Recently topics on LQ and LQG control include robustness and integrity considerations in their design (Rosenbrock and McMorran, 1971; Doyle, 1978; Doyle and Stein, 1979; Grimble, 1992).

LQ and LQG are model-based controls. Hence, an accurate model is a prerequisite for their design and application. Techniques abound in the literature on model identification of dynamic systems (Ljung, 1987; Box and Jinkins, 1976). However, an adequate model remains rare for most process systems. This is why model based control schemes, including LQ/LQG method, have not found wide acceptance in chemical processes, despite their sophistication in theory and rather successful applications in military and aerospace systems (Grimble, 1987). Hence, modifications of standard identification and modern control approaches are often a key factor for their successful implementation in real time process environment (Ray, 1983). This is the focus of Chapter 3 in this thesis.

2.2.2 Eigenvalue Assignment Design

The primary requirement in any control system design is stability. System stability and major dynamics are solely determined by the poles of the characteristic equation of the closed-loop system. One of the fundamental design objective is thus the achievement of suitable pole locations in order to ensure stability and satisfactory transient response. This is exactly what the eigenvalue assignment design approach is aimed at. Eigenvalue assignment design method most elegantly and simply demonstrates the major design philosophy and various issues, such as controllability, observability, stability, state feedback, state estimator, and the separation principle, involved in the state space approach. Hence, it is usually a necessary and often the key content in most textbooks on state space method (Ogata, 1990; Franklin et al., 1991; Brogan, 1991).

Wonham (1967) proved that eigenvalues of the closed loop system resulting from state feedback control to liner time-invariant systems can be arbitrarily assigned by adjusting controller parameters, if and only if the open loop system is completely state controllable and observable. For SISO system, a state feedback law can be uniquely determined by pre-assigned eigenvalues of the closed-loop system (Ogata, 1990). However, for MI systems, there are infinite solutions for the controller for a given set of eigenvalues (Wonham, 1967; Davison, 1968; Bragon, 1974; Moore, 1976).

Various approaches have been proposed in utilizing the degree of freedom in addition to eigenvalue assignment to achieve other design objectives, such the eigenstructure assignment (Dooren, 1981; Fahmy and O'Reilly, 1982; White, 1991; Andry, et al., 1983), and optimal eigenvalue assignment (Amin, 1985; Lean et al., 1988).

Originally, eigenvalue assignment design is primarily aimed at eliminating disturbances due to initial states for regulatory problems. However, the resulting system will result in unacceptable performance in the presence of external disturbances (Johnston, 1968). The inherent feature of integral action in achieving perfect steady state performance (off-set free) in face of such external perturbations and system parameter variations calls for the introduction of integrator to standard state feedback control problems.

To date, various methods of incorporating integral action into eigenvalue assignment in the formation of design problem have been proposed, especially for continuous systems (Porter and Crossley, 1970; Davison and Smith, 1971; Park and Seberg, 1974; Aida and Kitamari, 1990). For discrete systems, the integral action is obtained usually by integrating the outputs to accommodate constant disturbances (Seraji, 1983; Wittenmark, 1985; Ogata, 1991). Unfortunately, this approach usually lead to rank deficiency problems, and the final design procedure is often very complex. MacGregor and Wong (1980) suggested artificially introducing integral action by using the velocity control algorithms in the final stage of design. However, this design method does not explicitly include the integrator as part of controller design, thus performance of the resulting closed loop system may not be the same as designed. Salama (1983) proposed a method to incorporate integral action into state feedback controllers and demonstrated various advantages of this approach over output feedback.

In Chapter 4, a new PI state feedback control structure with fewer parameters and no constraint on the choice of controller parameters, compared to Salama's (1983) structure, is presented. The eigenvalue assignment design for the controller, with the extra degree of freedom used to achieve robust performance from an 'engineering' point of view, is then proposed.

2.2.3 Control of Distributed Systems

The control of distributed systems governed by partial differential equations, as opposed to those described usually by ordinary differential equations, represents a difficult problem. There is no systematic approach available so far to deal with control system design for such systems (Ray, 1981). Perhaps the only realistic way to design control systems for these processes is to analyses and evaluate the control system with given controllers by numerical simulation, using a trial and error procedure. Chapter 5 presents a dynamic simulator for this purpose.

2.3 DECENTRALIZED CONTROL OF MULTIVARIABLE SYSTEMS

Multivariable control problems are commonly tackled by using a decentralized structure, particularly a multiloop control scheme with PID controller in each control channel, especially in process control practice. Skogestad and Morari (1989) pointed out many practical advantages of decentralized control systems. A more important advantage of such a strategy, in our view, is that a great deal of practical experience gained by engineers in adopting classical control theory over decades can be inherited.

Basically, decentralized design can be divided into two stages: variable pairing and controller tuning (Shinsky, 1988; Grosdidier, et al., 1985). In order to accomplish a systematic approach for decentralized control system design with respect to the best variable pairing and effective controller tuning, a tremendous effort in process control field has been devoted to addressing important issues, which follow.

2.3.1 Variable Pairing and Interaction Measurement

Ideally, a decentralized control system may be effectively reduced to several SISO problems if variables are carefully paired (Grosdidier et al., 1985). Consequently, the objective of variable pairing has traditionally been to pair variables such that the resulting system most closely resembles a set of independent SISO loops (Mijares et al., 1986). In other words, the best pairing is the one that shows the minimum interaction. Hence, the Variable Pairing (VP) problem is closely correlated with and, in fact, relies largely on the Interaction Measure (IM) (McAvoy, 1983).

So far numerous approaches addressing the IM/VP problem have been proposed, such as the RGA (Bristol, 1966), singular value analysis (Bruans and Smith, 1982; Lau, et al., 1985), the RDGA (Witcher and McAvoy, 1977; Tung and Edgar, 1977; 1981), the Average Dynamic Gain Array (Gagepain and Seborg, 1982), the μ interaction measure (Grosdier and Morari, 1986;

1987), the direct Nyquist array (Jensen et al., 1986), the Jacobi eigenvalue criterion (Majares et al., 1986), and the structural interaction array (Jonston, 1990). Nevertheless, only those using steady state information (Bristol, 1966; Bruans and Smith, 1982; Mijares, et al., 1986), particularly the RGA (Bristol, 1966), are of practical importance.

However, the role of the RGA may have been exaggerated in addressing IM/VP issue. There are, in fact, some fundamental limitations associated with RGA. For example, it provides no clear choice regarding the best pairing when several alternatives satisfying the RGA rules exist (McAvoy, 1983). This situation can arise frequently in practice. More importantly, the RGA also offers uncertain information regarding system stability under a specific pairing (Grosdidier et al., 1985). Other techniques such as the Niederlinski Index (Niederlinski, 1971) have to be jointly used to eliminate unstable pairings (McAvoy, 1983).

The NI provides a necessary condition for stability, and constitutes a parallel tool in screening pairing alternatives. Naturally, the NI corresponding to subsystems with loops removed provides a necessary condition for integrity (Chiu and Arkun, 1990). Surprisingly, only the sign of the NI has been considered and little attention has been paid to its size.

The size of the NI in relation to the strength of interaction is investigated in Chapter 6. It shows that the NI provides a measure of the overall interaction in a system and so its use can overcome the ambiguity associated with the

RGA. As a result, a more comprehensive IM/VP criterion, which encompasses the size of interaction, stability and integrity in one quantity, is accomplished.

2.3.2 Interaction Measure and Controller Tuning

So far considerably less attention has been paid to the controller tuning problem, compared to that of variable pairing. We still lack an effective method to guide controller tuning, despite some of the approaches that have been proposed (Yu and Luyben, 1985; Seborg et al., 1989). In particular, no direct connection has been established between the two basic issues in decentralized control -- interaction measure and controller tuning. Instead, interaction measure has been solely utilized for variable pairing purpose. In fact, interaction is the most distinct feature of decentralized control systems, in contrast to SISO systems. Hence, it has to contain important implications for controller tuning.

Most notably, we still lack a unified definition of interaction (Rosenbrock, 1974; Jensen et al., 1986; Grosdidier and Morari, 1987; Johnston, 1990), and lack a common sense on the purpose of interaction measurement (Jensen, 1985; McAvoy, 1985). So far, only relative interaction measures, along the line of Bristol's (1966) RGA, are targeted. No one has ever attempted to measure the *absolute* interaction based on a meaningful definition of interaction. Another long standing problem is to measure the nature of interaction (Witcher and McAvoy, 1977). The solutions to utilize interaction measures for controller tuning and to develop effective guidelines would most likely rely on the measure of absolute interaction and the nature of interaction. An interesting attempt along this line is made in Chapter 8.

2.3.3 Controller Tuning and System Stability

The most widely used method in controller tuning is to design a controller on the basis of interaction-free subsystem followed by a second fine tuning usually performed by trial and error (Shinskey, 1988; Seborg et al., 1989; Luyben, 1990). However, the stability of a decentralized control system can be very sensitive to controller settings (Grosdidier, et al., 1985). As a result, an independently stable system can be easily destabilized in the process of the 'ad hoc' tuning. On the other hand, stability is the prime concern in the design of any control system. General stability theory (Morari and Zafiriou, 1989) requires the analysis of the overall dynamics. This is discouraging due to the complexity for high order systems, and further constrained by the scarcity of adequate dynamic models since there are often not available in industry. Hence, it is of great importance to provide conditions on stability using steady state information only (Grosdidier et al., 1985).

Stability in a decentralized control system based on steady state gains has been studied by Grosdidier et al.

(1985). However, only 'integral stabilizability' was investigated, and this result has a limited practical value (Jonston, 1990). On the other hand, stability has also been closely associated with VP problem. The most widely accepted method has been the NI (Niederlinski, 1971; Grosdidier and Morari, 1986). Nevertheless, only process gains are utilized with the controller omitted, so the conclusion is restricted to stability under variable pairing rather than controller tuning (Grosdier et al., 1985).

In Chapter 9, a comprehensive treatment of control system stability under overall closed loop control, independent design and variable pairing is performed, using steady state information of both process and controller. As a result, various stability conditions are developed and direct guidance for controller tuning to ensure stability is available.

2.3.4 System Integrity

Integrity considerations in variable pairing has been extensively studied by Chiu and Arkun (1990), using the concept of Brock Relative Gain (BRG). Morari and Zafiriou (1989), and Grosdidier et al. (1985) also addressed system integrity properties based on the concepts of Integral Stabilizability(IS), Integral Controllability (IC), and Decentralized Integral Controllability (DIC). A general treatment of system integrity against any combination of loop failure is available in Chiu and Arkun's paper.

However, the most important integrity problem is the ability of decentralized control systems to tolerate single loop failure. A simplified procedure to check this integrity property, using the RGA directly on the basis of its definition, is presented in Chapter 7.

2.3.5 Stability Robustness

Because of the complexity of many industrial processes, good models are hard to come by. Controller designs which are robust to model mismatch are desirable (Marari and Zafiriou, 1989). Conventionally, variable pairing decisions are based solely on a nominal model of process gains by means of RGA and NI. In decentralized control systems, a control structure or pairing satisfying nominal stability and integrity requirements can be rather risky in the presence of inherent model mismatch (Grosdidier et al., 1985; Skogestad and Morari, 1987a; Yu and Luyben, 1987). Therefore, stability robustness must be taken into account when making decisions regarding the best pairing. Steady state criteria is certainly of practical importance.

Grosdidier et al. (1985) proposed an approximate bound for steady state model uncertainty in terms of integral controllability. Skogestad and Morari (1987a) presented conditions on the magnitude of model mismatch for each independent element for robust singularity of the process. Yu and Luyben (1987) reported a stability robustness measure based on the RGA.

Nevertheless, only uncertainties associated with uncorrelated individual gains has been accommodated. Chapter 10 presents a new stability measure based on a rigorous analysis of the behaviour of the NI stability index in the face of single gain error, and their approach can be extended to include multiple gain errors.

2.3.6 Robust Performance and Robust Controller

Robust control has provided engineers with very useful tools for explicitly accounting for process model uncertainty (often found in industry) in control system design (Morari and Zafiriou, 1989). However, the quantification of model uncertainty remains a difficult task (Goberdhansingh et al., 1992). Hence, robust control with the uncertainties considered but appropriately separated in the design has recently gained much attention (Skogestad and Morari, 1987a; 1987b; Brambilla and D'Elia, 1992).

In process control, a major source of model uncertainty arises from the operational error in manipulated variables (Skogestad and Morari, 1987a). Skogestad and Morari (1987a) presented an approximate method to evaluate robustness against this model mismatch by means of RGA. Brambilla and D'Elia (1992) used this method as a design criterion for robust control. Another important source of uncertainties in process systems is the directionality of disturbance and setpoint changes (Skogestad and Morari, 1987b; Grosdidier, 1990;

Brambilla and D'Elia, 1992). Skogestad and Morari (1987b) introduced a 'Disturbance Condition Number' to evaluate the effect of disturbance directions on closed-loop performance. Grosdidier (1990) demonstrates how disturbance directions affect control performance. Brambilla and D'Elia (1992) discussed design issues to achieve robust performance in the presence of strong disturbance directionality.

Chapter 11 presents a more general and effective design approach to achieve robust control against both disturbance uncertainty and disturbance directionality.

PART I MODERN MULTIVARIABLE CONTROL

CHAPTER 3

MODEL IDENTIFICATION AND LQG CONTROL OF A MULTIVARIABLE PRESSURE TANK SYSTEM

Synopsis *The dynamic and stochastic components of a state model are identified for a real time multivariable pressure tank process. Some simplying assumptions for the stochastic components of the model allow for easy determination of the Kalman filter and subsequent on-line implementation of a Linear Quadratic control law. The controller behaves well for both load and set point disturbances for this process which is strongly interactive and nonlinear. Some practical suggestions for real time implementation are given.*

3.1. INTRODUCTION

Modern control theory, based on a state space description of process systems, has found some successful applications in the process industry, especially for Single-Input Single-Output (SISO) systems. Nevertheless, modern control still has not achieved the widespread acceptance in the process control industries as was initially expected since its establishment in the 1960's. One reason for this is the relative scarcity of studies on real time systems with their encumbent non linearities and implementation details. Classical control is still dominant in practice (Shinskey, 1988; Luyben, 1990).

On the other hand, there is a strong need for advanced control in modern industry, especially for multivariable systems. Classical control often fails here due to highly integrated and constrained process characteristics and strong interactions between control loops. Moreover, the processes where classical control fails to achieve high control performance are frequently the key points in modern plants (Ray, 1983). However, the applications of modern control techniques to multivariable systems are not as frequent compared to SISO systems. One major cause is the limited understanding of the processes along with their inherently non-linear and uncertain characteristics (Foss, 1973). Therefore, the first and most important step in the application of modern control techniques lies in the development of a sufficiently accurate dynamic model of the process. Various model identification methods such as Time Series Analysis (Box and Jenkins, 1976) have been developed and widely applied (Ljung, 1987), especially for univariate systems. Multivariate methods are less well known, and have seen substantially fewer applications. Another important problem in applying modern control techniques is that special considerations associated with process practice must be accommodated. Direct application of the usual theories without modification would be prone to failure in a practical environment (Morari and Doyle, 1986; Ray, 1983). For example in most application studies the stochastic or process noise component of the modeling procedure is treated as an afterthought. Typically, the noise covariance matrices are treated as single tuning parameters which are adjusted by trial and error in the Kalaman Filter Design for the system. However, if a full process identification procedure is carried out using, say, Time Series Analysis, the state space process model as well as the stochastic noise terms can be properly identified. In fact in the identified model form that naturally appears in this procedure (Innovations form), the states are the optimal filtered estimates (MacGregor, 1973).

One difficulty that occurs in multivariable identification is the need to deal with a multivariable noise model. A simplification that is adopted in this chapter is to assume that the noise terms for each output are uncorrelated. This allows a considerable simplification in parameter estimation while still giving rise to a suitable dynamic stochastic state model which led to a good multivariable controller.

This chapter presents a successful modelling and optimal stochastic control of a multivariable pressure tank system. Conventional PID controllers were not able to satisfactorily control this process due to the strong non-linearities and strong interactions between control loops (Zhu and Jutan, 1989). In the article an analytical process model for the pressure tank was developed. This highly nonlinear model was linearised around nominal operating conditions and the relative gain array (Bristol, 1966) evaluated. The array showed strong interaction (most relative gains close to 0.5). These factors gave rise to very poor single loop control and made the pressure tank a good candidate for a multivariate control scheme.

Time Series Analysis (TSA) (Box and Jenkins, 1976) was used, in conjunction with other methods, to develop a dynamic-stochastic model for the process. In spite of some simplifications regarding noise structure, an adequate state model was developed and used to implement an optimal feedback control. The usual trial and error 'tuning' stage for the Kalman Filter design was avoided and the process was identified directly in a form which leads to integral action in the control law (MacGregor and Wong, 1980). In addition, some practical considerations for a successful implementation are discussed.

3.2. MODEL IDENTIFICATION

In practice, mathematical models based on fundamental principles are rarely used directly for control purposes for the following reasons: (1) the resulting models are usually too complex for controller design, leading to controllers of very high order which are often difficult to implement; (2) these models usually need to be largely simplified, resulting in largely reduced reliability; (3) there are frequently many parameters associated with the models that need to be provided or estimated with added inaccuracies; (4) most of these models are nonlinear and hence need to be linearized in order to use linear control theories, further introducing model errors. As a result, control models are usually established using various model identification approaches.

In the process industry, there are usually various unknown stochastic disturbances affecting a process. These are ignored by classical control design which is based on deterministic upsets, such as pulse and step functions. However, stochastic upsets can be significant and even dominant in a system.

In modern stochastic control, the actual disturbance experienced in a system is characterized by a suitable stochastic model along with an input-output dynamic model. This disturbance is taken into account in the controller design and leads to an optimal stochastic controller. Several approaches have been proposed to model these stochastic systems such as Wiener's (1949) spectral factorization method, Box and Jenkins' (1976) Time Series Analysis, and Kalman's (1964) state space approach. In this chapter, the Time Series Analysis method is used to identify the multivariate system.

3.2.1 Multivariate Time Series Analysis

Consider a system described by

$$Y(s) = G(s) \cdot U(s) + N(s) \tag{3.1}$$

where Y(s) is an (nx1) vector of outputs, U(s) is an (nx1) vector of inputs, G(s) is an (nxn) matrix of transfer functions (square system), and N(s) is an (nx1) vector of noises or disturbances.

A discrete time series model representation for (3.1) which uses a transfer function and an autoregressive integrated-moving-average (ARIMA) model to characterize process dynamics and stochastic disturbances respectively can be expressed as

$$Y_t = V(B)\ U_t + N_t \tag{3.2}$$

where Y_t, V(B), U_t and N_t have corresponding dimensions for equation (3.1). The transfer function between the i-th input and j-th output has the form

$$V_{ij}(B) = \frac{\omega_{ij} \cdot B^{b_{ij}}}{\delta_{ij}\ (B)} \qquad ,\forall \quad i,j \tag{3.3}$$

where $\omega_{ij}(B)$ and $\delta_{ij}(B)$ are polynomials of a certain order in the backward shift operator B (sometimes denoted q or z^{-1}), and b_{ij} is the time delay interval.

In general the noise can be modelled by a full multivariate ARIMA series. However, this is often not practical and in this chapter we assume that a relatively simple MA(1) model is adequate for describing the noise characteristics. This is an important simplifying assumption which is often true in practise (Jutan, et al 1977). In general we can represent the noise as

$$N_t = D(B)a_t \tag{3.4}$$

where $\{a_t\}$ is a multivariate noise sequence with covariance matrix Σ, and D(B) is an (nxn) matrix of ARIMA structure with the form

$$d_{ij}(B) = \frac{\theta_{ij}(B)}{\phi_{ij}(B) \cdot \Delta^{d_{ij}}} \qquad \forall\ i,j \tag{3.5}$$

where $\theta_{ij}(B)$ and $\phi_{ij}(B)$ are polynomials in B, Δ is a backward difference operator (1-B) to account for nonstationarity, and d_{ij} is the degree of nonstationarity.

In an nxn system, a_t is an nx1 vector of noise sequences which we assume are basically uncorrelated with each other. This again may not be true in general, but is an important simplifying assumption that led to a practical design. This implies that the covariance matrix Σ of a_t is approximately diagonal. In practice, this means that each noise sequence a_{kt}, produces one disturbance N_{kt} that affects its corresponding output independently. The adequateness of this assumption was confirmed with model diagnostic procedures (see later).

From (3.3) above and our assumption of independent noise, the i-th output can be expressed as

$$y_i(t) = \sum_{j=1}^{n} \frac{\omega_{ij}(B) \cdot B^{b_{ij}}}{\delta_{ij}} u_j(t) + n_i(t) \quad , \quad \forall\ i \qquad (3.6)$$

and from (3.4) and (3.5)

$$n_i(t) = \frac{\theta_i(B)}{\phi_i(B) \cdot \Delta^{d_{ij}}} a_i(t) \quad , \quad \forall i,j \qquad (3.7)$$

Transfer Function Model Identification

In equation (3.6) and (3.7) if $\{u_j(t)\}$ and $\{a_i(t)\}$ are uncorrelated with each other (unautocorrelated and uncross-correlated), it is a simple matter to identify the individual transfer function $V_{ij}(B)$ using correlation techniques proposed by Box and Jenkins (1976) or by investigating the impulse and/or step response from time series data collected. For a specific output series $y_i(t)$ we can obtain all the elements of i-th row of transfer function matrix V(B) by correlating $y_i(t)$ with each input sequence $\{u_j(t)\}$ separately. The identification of each element of the transfer function becomes a SISO problem which can be easily obtained. We can thus form the whole transfer function matrix V(B) (Box and Jenkins, 1976).

Noise Model Identification

With the specified model structure (independent, uncorrelated) and initial estimates of parameters in the transfer functions, an estimate of noise sequence $n_i(t)$ associated with an output $y_i(t)$ can be generated using equation

(3.6) to calculate the difference between the model and the data:

$$n_i(t) = y_i(t) - \overline{y_i}(t) \quad , \qquad \forall i \qquad (3.8)$$

where

$$\overline{y_i}(t) = \sum_{j=1}^{n} \frac{\omega_{ij}(B)\, B^{b_{ij}}}{\delta_{ij}} u_j(t) \quad , \qquad \forall\, i \qquad (3.9)$$

The noise sequences can then be identified using ARIMA models, such as equation (3.7), by examining their auto-correlation and cross-correlation properties. Both model structure and initial estimates of parameters in the model can be obtained. However, as stated earlier it is often adequate to model the multivariate noise using an MA(1) model.

The Role of Disturbances

Often in the design of optimal controllers, the disturbance model is not explicitly stated, but rather indirectly implied in the problem formulation, e.g., the LQ design implies that the disturbance is simply an impulse type function. Such unrealistic disturbance assumptions often leads to poor controllers, such as ones without integral action (Harris and MacGregor, 1987). In modern stochastic control, tha disturbances in the system are modelled and then used for controller design such that the controller would be optimal in the presence of these disturbances.

Notice that the noise or disturbance terms in equation (3.8) represent the difference between model output calculated by the linear transfer functions in equation (3.9) and the actual time series data for a specific output. These disturbance terms can include both randomly occurring deterministic disturbances and stochastic disturbances affecting the system (MacGregor, 1984). In fact, they contain any source causing model mismatch from the real system, including nonlinearity. Inclusion of a disturbance model improved the adequacy of the identified model, and allowed satisfactory optimal controller design for the pressure tank system in this chapter.

The noise is modelled by ARIMA models (equation (3.7)) and usually simple, low-order models are adequate in practice. However, the model structure for a given series is generally not unique. Nevertheless, the assumed structure of the disturbance model can affect the performance of the controller (Bergh and MacGregor, 1987). A suitable choice of disturbance model structure can facilitate the state space model realization and facilitate the controller design. We chose an MA(1) model.

Another important consideration is the nonstationarity of the noise, which has an important effect on controller design and system performance. Industrial processes often tend to drift away from their equilibrium values when the manipulated variable is held constant, due to nonstationary disturbances. If there are nonstationary disturbances, i.e., $d>1$ in equation

(3.7), then integral action is required in the controller to prevent drift and to eliminate offset in the process. Integral action is ensured by incorporating the backward difference Δ in equation (3.6) and (3.7) into the input series u_j. This modification is conveniently incorporated in the identification stage.

Parameter Estimation

Parameter estimation of the combined dynamic-stochastic model was achieved using the Prediction Error Method (Ljung, 1987). A proper model diagnosis using auto and cross correlation of the model residuals was used (Box and Jenkins, 1976) to check the adequacy of the model.

Many studies ommit this final stage and rely purely on visual adequacy checks. This may be why a more extensive trial and error tuning stage using the Kalman filter parameters is often necessary. One criticism of the approach used here, is that the model may not be robust in the sense that there is no guarentee that the model, particularly the stochastic component, is of fixed structure. Often the 'stochastic component' is simply a 'grab bag' of true measurement and process noise but also includes process nonlinearities, which may in fact dominate. If the control engineer believed that the model was changing, an adaptive control design would be preferable, since the inherent assumption here requires both constant model structure and parameter values.

3.2.2 The Pressure Tank System

The system considered here is a pressure tank through which air flows from a regulated source. Control valves are installed on both inlet and exit flow. The pressure in the vessel and the flow rate in exit flow are measured and transmitted to a computer. Data collection and system control are accomplished by use of a microcomputer with an input-output (I/O) interface board. The computer used is a Zenith-AT which is compatible with an IBM-AT. The I/O interface used is from TAURUS products. The programs which perform real time data acquisition and on-line control are written in Microsoft's Quick BASIC language. The schematic diagram and the configuration of the system is shown in Figure 3.1.

The system is a two-input two-output system with inlet and exit valve position as manipulated variables and pressure in the tank and flow rate in the exit flow as the controlled variables. The process design parameters were chosen especially to ensure a strongly interactive system. In fact, the Relative Gain Array (Bristol,1966) has coefficients close to 0.5 for this process.

At first, semi-theoretical and empirical models were developed and conventional PID control for the system was tried, but it was found that the process was difficult to control. Because of nonlinearities and strong interactions the selection of the control system configuration (variable pairing), controller tuning and even maintaining stability of the closed-loop system, proved very difficult (Zhu and Jutan, 1989).

3.2.3 Data Collection

In the data collection stage, the valves were operated according to an MA(1) model. The pressure and exit flow rate were sampled every 4 seconds. These stochastic input and output sequences were collected and stored for further processing using the identification procedures discussed previously.

The sample interval and the maximum number of observations are user selected. In order to identify system dynamics efficiently an appropriate input series structure should be carefully chosen (Box and Jenkins, 1976).

Input Signal Selection

A basic and crucial step in identifying process dynamics from real data is to collect a good set of process data with sufficient information about the dynamics of the process. One key factor is the proper selection of an input series for the process. The following conditions must apply:

(1) The separate input series should be uncross-correlated for multi-input time series, in order to identify the effects of individual inputs on the outputs. The cross-correlation functions can be readily examined for to confirm this in the data collection stage.

(2) The input sequences must have proper frequency content suitable for the specific process to be identified. Usually a simple structure such as MA(1) can be used to generate input sequences. For an MA(1) series

$$u_t = \frac{1}{1 - \phi B} a_t \tag{3.10}$$

where a_t is a Gaussian white noise, ϕ is a parameter which determines the frequency content of input u_t.

The proper frequency content of input series is determined by the dynamics of the process and the sample interval used. Following the recommendation of Box and Jenkins (1976) and from familiarity with the process, the parameter ϕ in equation (3.10) was selected for the two inputs as 0.65 and 0.5 respectively, with a sample time of 4 seconds.

(3) The magnitude of variation of the input series (from its steady state) should allow the contribution of each input to be uncovered, i.e., the effect each input has on the outputs should be well balanced such that individual transfer functions can be distinguished from each other -- especially for multivariate systems.

In the pressure tank system, the effect of inlet and exit valve position to (normalized) pressure and exit flow rate outputs are quite different. When both inputs were manipulated with the same magnitude the individual effect of the exit valve position on the output variables could not be identified. The corresponding transfer function was poorly estimated. To avoid this problem, different magnitudes for the two inputs should be used. An alternative approach is to run the system with one input constant, while the other one is manipulated. This requires separate runs, and was the method used for the pressure tank system.

After satisfying requirements for the structure, frequency content and the magnitude of inputs, the variances of the corresponding white noise terms in equation (3.10) can be determined (Box and Jenkins, 1976). The input sequences for both valves are then generated.

3.2.4 Model Identification

Several iterations of the identification procedure described above resulted in the final model form given by:

$$\begin{bmatrix} y1_t \\ y2_t \end{bmatrix} = \begin{bmatrix} \dfrac{0.0685+0.0127B}{1-0.89B} & \dfrac{-0.043-0.0087B}{1-0.92B} \\ \dfrac{0.0477}{1-0.91B} & \dfrac{0.495}{1-0.22B} \end{bmatrix} \begin{bmatrix} u1_{t-1} \\ u2_{t-1} \end{bmatrix}$$

$$+ \begin{bmatrix} \dfrac{1-0.188B}{1-0.96B} & 0 \\ 0 & \dfrac{1-0.3B}{1-0.86B} \end{bmatrix} \begin{bmatrix} a1_t \\ a2_t \end{bmatrix} \qquad (3.11)$$

and the covariance matrix of the residuals (assumed independent) is given by

$$\Sigma = \begin{bmatrix} 0.04205 & 0 \\ 0 & 0.085 \end{bmatrix} \tag{3.12}$$

The diagnostics for the model (Box and Jenkins, 1976) failed to reveal any serious inadequacies in this fitted model. The comparisons of the model and the real time series are shown in Figures 3.2 and 3.3 for pressure Y_1 and flow rate Y_2 respectively. It is clear that the model fits the data very well.

3.3. STATE SPACE REALIZATION

In previous sections, a transfer function - MA noise model was used during the model identification process. However, the design of an optimal controller is more easily solved using a state space model for the process.

The state space representation of the system is equivalent to that given by the transfer function - noise model in equation (3.2). Equation (3.11) can be expressed as

$$X_{t+1} = AX_t + GU_t + \Gamma a_t$$

$$Y_t = HX_t + a_t \tag{3.13}$$

where X_t, Y_t, U_t and a_t are state, output, input and white noise vectors, and A, G, Γ and H are parameter matrices. Although other alternative state space forms can be realized (MacGregor, 1973), the form (3.13) is used and leads to a simple state estimator design.

In the state space model, the state vector lacks any physical meaning in contrast to the input and output vectors. Hence the state variables are usually not measured, and instead a state estimator is required for state feedback in optimal control.

Given the state space model (3.13), the equivalent transfer function-ARIMA model can be written as

$$Y_t = [H(I-AB)^{-1} GB] U_t + [H(I-AB)^{-1} \Gamma] a_t \quad (3.14)$$

Nevertheless, the reverse problem of transforming a transfer function-noise model into state space form is less straightforward because the solution is not unique. Minimal realization algorithms are usually employed (Sinha, 1975). Here we used a canonical form of the state space equation to obtain a realization for the multivariate systems.

Minimal realization can be examined by checking the controllability and observability of the resulting state space equations. If the state space equation is not a minimal realization, extra states can be removed through various methods (Sinha, 1975). However, if individual subsystems are realized minimally and there are no common zeros and poles among these subsystems (most industrial processes fit this case, as does the pressure tank system here), the resulting realization is often minimal.

Observing the model in equation (3.11), the denominators in the matrix multiplying $a_1(t)$ and $a_2(t)$ can be approximated by a backward difference operator Δ, and then using a canonical form, the state space representation for the pressure tank system can be obtained as,

$$X(t+1) = \begin{bmatrix} 2.77 & 1 & 0 & 0 & 0 & 0 \\ -2.56 & 0 & 1 & 0 & 0 & 0 \\ 0.786 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 1 & 0 \\ 0 & 0 & 0 & -1.18 & 0 & 1 \\ 0 & 0 & 0 & 0.177 & 0 & 0 \end{bmatrix} X(t)$$

$$+ \begin{bmatrix} 0.0645 & -0.044 \\ -0.0423 & 0.0262 \\ -0.0156 & 0.0116 \\ 0.0486 & 0.5 \\ -0.011 & -0.455 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta u1(t) \\ \Delta u2(t) \end{bmatrix} \tag{3.15}$$

$$+ \begin{bmatrix} 0.76 & 0 \\ -1.38 & 0 \\ 0.62 & 0 \\ 0 & 0.55 \\ 0 & 0.62 \\ 0 & 0.112 \end{bmatrix} \begin{bmatrix} a1(t) \\ a2(t) \end{bmatrix}$$

and

$$\begin{bmatrix} y1(t) \\ y2(t) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} X(t) + \begin{bmatrix} a1(t) \\ a2(t) \end{bmatrix} \quad (3.16)$$

3.4. OPTIMAL STOCHASTIC CONTROL

3.4.1 Methodology

Consider a general system described by

$$\begin{aligned} X_{t+1} &= AX_t + GU_t + \omega_t \\ Y_t &= HX_t + \upsilon_t \end{aligned} \quad (3.17)$$

where ω_t and ν_t are independent noise sequences. The optional N-stage control policy is sought which will minimize the linear quadratic performance criterion

$$J = E\ (\sum_{s=1}^{n} X_s' Q_1 X_s + U_s' Q_2 U_s) \quad (3.18)$$

where Q_1 and Q_2 are positive semi-definitive matrices which contain adjustaple weighting parameters.

The solution to this optimal control problem by dynamic programming is given by Astrom (1970)

$$U_t = -L_t \hat{X}_{t/t} \quad (3.19)$$

where L_t is the feedback control matrix and is given by the solution to the matrix Riccati equations

$$L_t = (Q_2 + G' S_t G)^{-1} G' S_t A \tag{3.20a}$$

$$S_t = A' S_{t+1} A + Q_1 - A' S_{t+1} G (Q_2 + G' S_{t+1} G)^{-1} G' S_{t+1} A \tag{3.20b}$$

with initial condition $S_N = Q_1$.

We are interested in the steady state solution (N --> ∞), and S_t in equation (3.19) usually converges quickly to a fixed value S_∞. The desired steady state control matrix L_∞ is then obtained by substituting S_∞ into equation (3.20a). Software packages to obtain L_∞ are readily available.

$\hat{X}_{t|\tau}$ is the conditional expectation of state vector, i.e. $E\{X_t/Y_\tau\}$, given information $y_\tau = (y_\tau, y_{\tau-1} \ldots, y_0)$ available at time τ. For Gaussian noise, the best estimates of the state vector X_t and X_{t+1} are obtained by the Kalman filter equations (Astrom, 1970).

In the pressure tank system, the noise terms with their specific structures and properties have been identified. This allows us to specify the properties of ω_t and υ_t in (3.17), without having to resort to a trial and error iteration. In particular, for the state space model in the form of equation (3.13) it can be shown that the state estimator X_{t+1} is in fact the state vector X_{t+1} itself which can be obtained directly from state equation (3.13) (MacGregor, 1973).

$$\begin{aligned} \hat{X}_{t+1/t} &= A\hat{X}_{t/t-1} + Gu_t + \Gamma a_t \\ &= A\hat{X}_{t/t-1} + Gu_t + \Gamma (y_t - H\hat{X}_{t/t-1}) \end{aligned} \tag{3.21}$$

Equation (3.21) uses information up to time (t-1) to estimate state X_t at time t. This is suitable for the pressure tank system since the system has one time delay (equation

(3.15)) (Jutan, 1977). The final optimal control can then be implemented as

$$\Delta U_t = -L_\infty \hat{X}_{t/t-1} \tag{3.22}$$

Also, in the pressure tank system we are interested in output control, i.e., our objective function here is

$$J = \sum_{s=1}^{n} E(Y_s' Q_3 Y_s + U_s' Q_2 U_s') \tag{3.23}$$

which can be easily expressed in the form of (3.18).

<u>Selection of the Weighting Matrix</u>

Usually, Q_1 is fixed in equation (3.18) and we adjust Q_2 to obtain an acceptable L_∞. Here $Q_3 = I$, so $Q_1 = H'H$.

Q_2 is now selected by trial and error and the corresponding L_∞ can be obtained by solving the Ricatti Equation. The expected performance of the closed-loop system under equation (3.22) can be evaluated by computing the theoretical variances and covariances of outputs and inputs for system (3.15) using the following formula (MacGregor, 1973):

$$V_x = (A-GL_\infty)\ V_x\ (A-GL_\infty)' + \Gamma\Sigma\Gamma'$$

$$V(\Delta U_t) = E\ (\Delta U_t\ (\Delta U_t)') = L_\infty V_x L'_\infty$$

$$V(y_t) = E\ (y_t y'_t) = HV_x H' + \Sigma$$

where Σ is the covariance of a_t in equation (3.12).

By iterating the above equations given an initial guess for V_x, variances for a specific L_∞ corresponding to a trial Q_2 can be evaluated.

The final Q_2 and corresponding L_∞ is chosen such that the variances of input and output are jointly acceptable. Since small inputs were used to identify the system in the data collection stage (equation 11), the identified model is valid within a small region around the operating point. Model mismatch usually arises when the system undergoes larger disturbances or setpoint changes, hence controller robustness should also be taken into account. Intuitively, by taking less severe control actions one would expect more robust control (Kozub et al, 1987). Thus it makes sense to choose Q_2 such that the variances of inputs are substantially decreased even though this results in an increase (usually small) of the variances of outputs. The closed-loop system would thus exhibit greater robustness to model error because less demand is being placed on the model to predict the effect of large changes in inputs (Harris and MacGregor, 1987).

Following this approach, a value of Q_2 was chosen as Q_2= diagonal (20, 20) and the corresponding feedback matrix was

$$L_\infty = \begin{bmatrix} 3.75 & 3.18 & 2.59 & 0.43 & 0.41 & 0.386 \\ -2.43 & -2.03 & -1.62 & 0.699 & 0.634 & 0.554 \end{bmatrix} \qquad (3.24)$$

The resulting variances of inputs and outputs were: $V(\Delta u1)=0.0041$, $V(\Delta u2)= 0.0025$, $V(y1)= 0.20$ and $V(y2)= 0.17$.

3.4.2 State Estimator Design

Equation (3.21) can be directly used to design a state estimator. Substituting the parameter matrices in equation (3.15) into equation (3.21), the state estimator equation becomes:

$$\hat{X}_{t/t-1} = \begin{bmatrix} 2.01 & 1.0 & 0 & 0 & 0 & 0 \\ -1.18 & 0 & 1.0 & 0 & 0 & 0 \\ 0.166 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1.45 & 1.0 & 0 \\ 0 & 0 & 0 & -0.56 & 0 & 1.0 \\ 0 & 0 & 0 & 0.065 & 0 & 0 \end{bmatrix} \hat{X}_{t-1/t-2}$$

$$+ \begin{bmatrix} 0.0645 & -0.044 \\ -0.0423 & 0.0262 \\ -0.0156 & 0.0116 \\ 0.0485 & 0.5 \\ -0.011 & -0.455 \\ 0 & 0 \end{bmatrix} \Delta U_{t-1} + \begin{bmatrix} 0.76 & 0 \\ -1.38 & 0 \\ 0.62 & 0 \\ 0 & 0.55 \\ 0 & -0.62 \\ 0 & 0.112 \end{bmatrix} Y_{t-1}$$

(3.25)

3.4.3 On-Line Implementation

With the optimal control law equation (3.22), the feedback matrix (3.24) and state estimator (3.25), an on-line algorithm of LQG control system can be implemented.

The real time control system was written in Microsoft Quick Basic on a Zenith-AT Computer (see Figure 3.1).

3.4.4 Results

<u>Regulatory Control</u>

The LQG controller was originally developed based on the dynamic stochastic model (3.15) and (3.16) aimed at overcoming disturbances modelled in the noise component of the stochastic model. The ability to keep the process outputs at the reference setpoints is the first requirement of the optimal controller.

The response curves shown in Figure 3.4 before 250 sample times represent the controlled pressure and exit flow rate responses under normal operating conditions. It can be seen that the controller works very well. The process is able to maintain its set point in the presence of the inherent noise in the system.

<u>Load Disturbance Rejection</u>

Although LQG controller is designed to overcome process disturbances identified in the modelling stage, the controller is also capable of rejecting unmodelled load changes (Wong and

MacGregor, 1980). This property of the LQG controller was tested as follows:

A load disturbance was introduced by plugging the exit pipe for certain time. This introduced a serious load disturbance into the system. The responses in Figure 3.4 after 250 sample times show the pressure and flow rate response to the load disturbance introduced at 260 sample times. It can be seen that the controller is easily able to overcome this serious disturbance.

Setpoint Tracking

In order to achieve setpoint tracking in a LQG control system, the state space equations are usually augmented with a given setpoint trajectory and a feed-forward compensation design is required (Kwakernaak and Sivan, 1972). In our system, however, the LQG controller was able to achieve satisfactory setpoint tracking due to the integral action in (3.22). Figure 3.5 shows the pressure and flow rate response under step setpoint change in pressure with 0.1 magnitude. The controller output for both valves are shown in Figure 3.6. It can be seen that the system is able to track the setpoint changes. The decreased scale in these plots emphasises the inherent transmitter noise.

3.5. FINAL REMARKS

A multivariable pressure tank process was successfully modelled using discrete Time Series methods to produce a dynamic-stochastic model of the process. This model was used to develop an LQG optimal stochastic controller and state estimator. Optimal Regulatory Control as well as Set Point Control was successfully implemented.

This process is quite nonlinear and highly interactive and a previous attempt at control using single loop classical methods was not successful. The suggested approach thus consists of the following main steps. The stochalistic component of the state model was directly identified using an MA(1) time series. A simplifying assumption of uncorrelated noise components was used to obtain the covariance structure for the filter estimates avoiding the need for a iterative approach. Iterative choice of the optimal weighting matrices for the overall objective function is easily guided using variance transmission formula such that both input and output variances are jointly acceptable. Finally, set point tracking is conveniently included as a property of the controller at the identification stage by assuming a non-stationary noise.

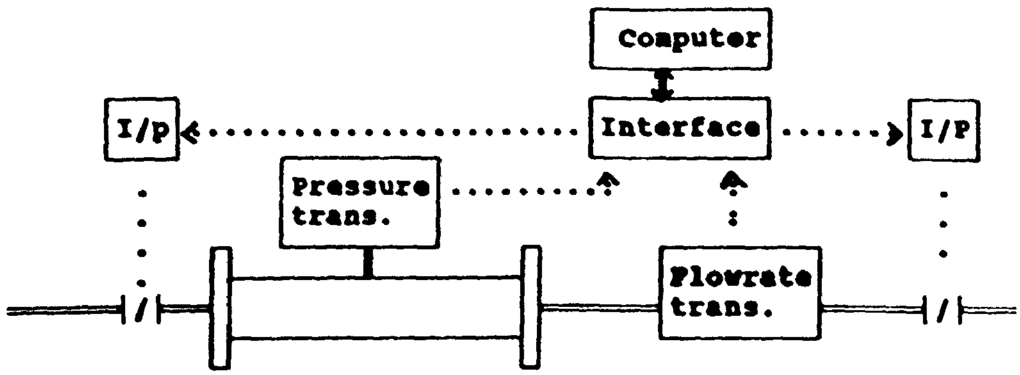


Figure 3.1 Schematic diagram of pressure tank system

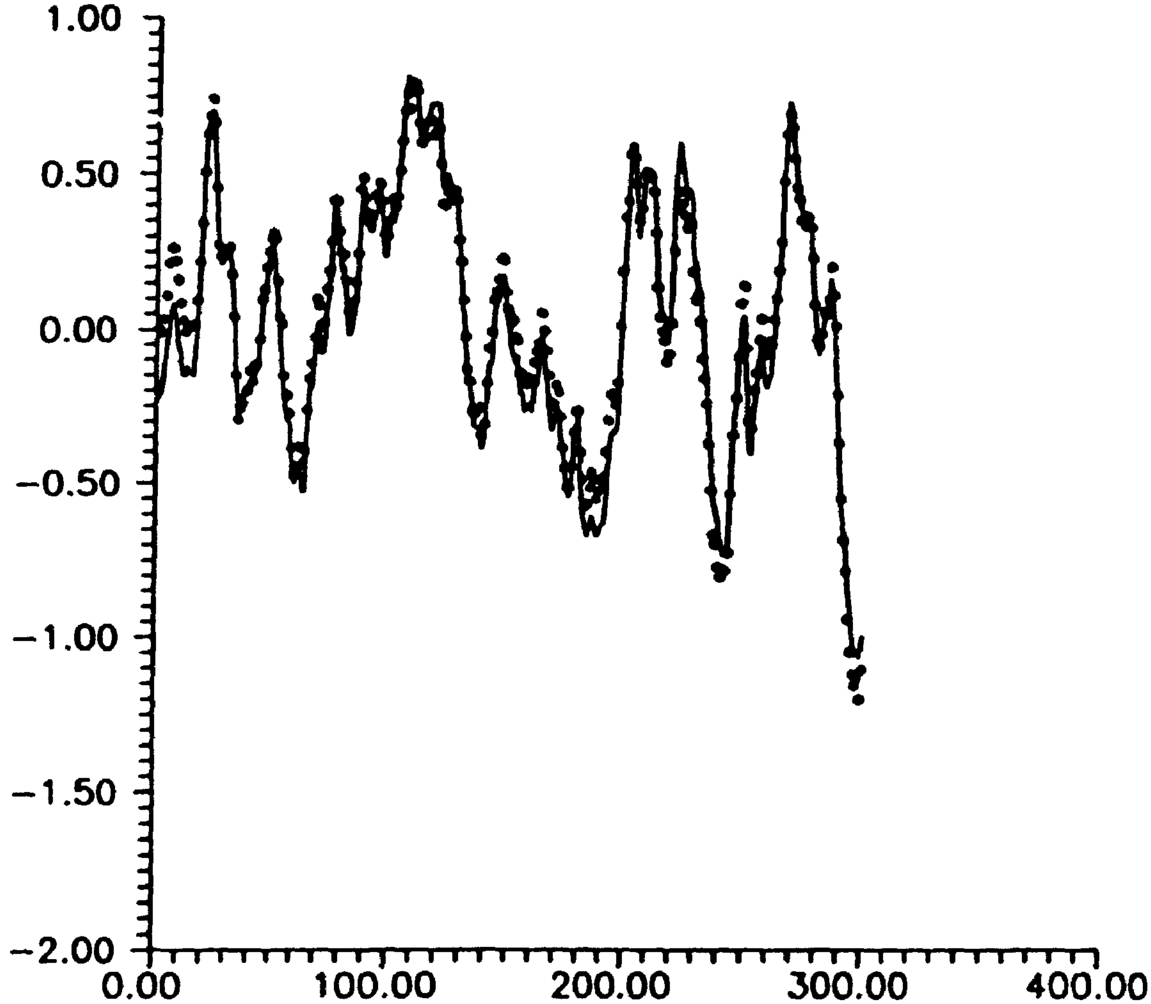


Figure 3.2 y1 vs data

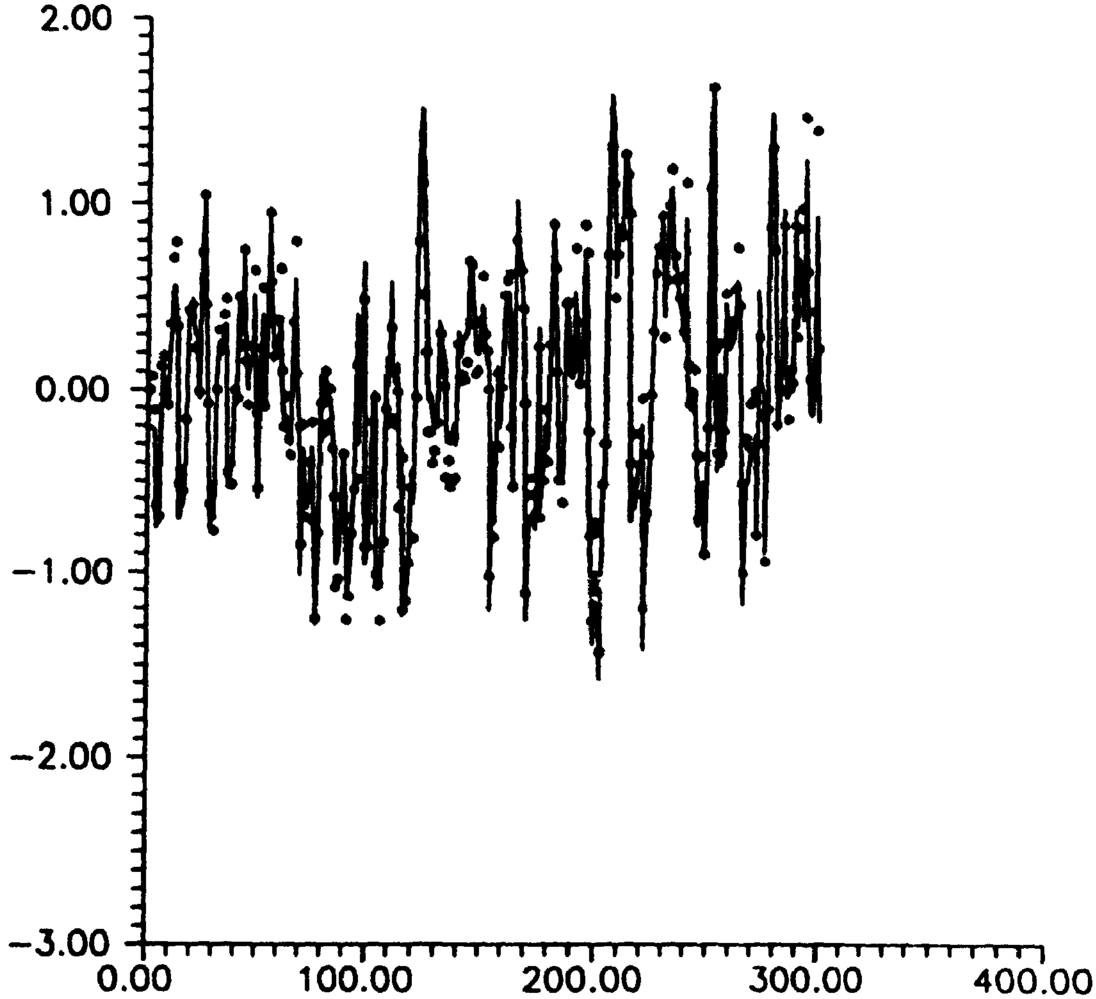


Figure 3.3 y2 vs data

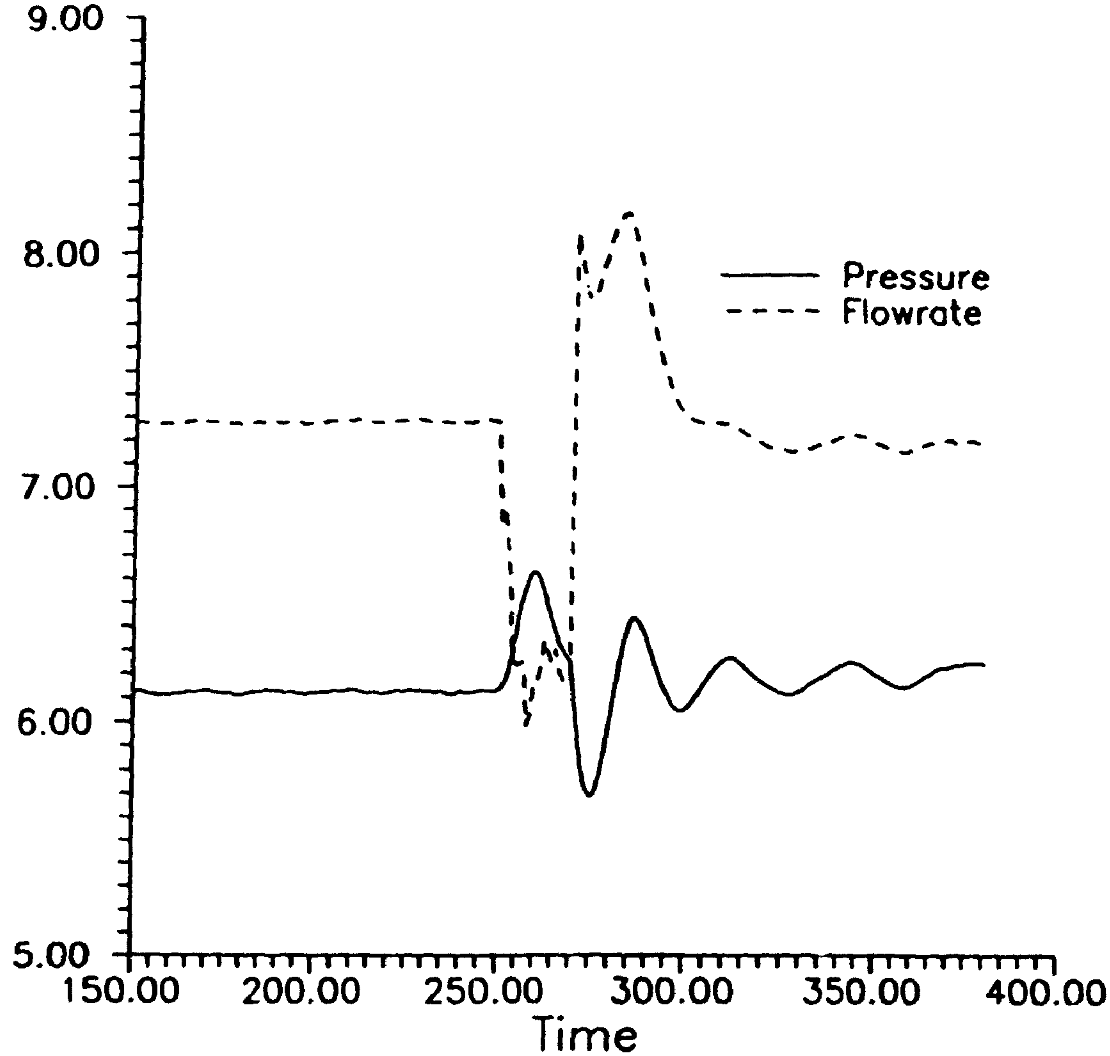


Figure 3.4 System response under load disturbance

for example, would make performance and stability characteristics more sensitive to the accuracy of the system model and lead to a decrease in the robustness of the closed-loop system (Harris and MacGregor,1987). Thus if satisfactory control is obtainable with less control action, we expect the system to be less sensitive to modelling errors. This is analogous to the minimum variance controller which often leads to excessive control action and reaction. Constrained minimum variance controllers while suffering from some small loss in performance were much more successful since their corresponding controller action was considerably reduced (Clarke and Gawthrope, 1975; 1979; 1981).

As a result, to make a system more robust the norm of the control action can be minimized, e.g., for the control system with the PI state feedback law discussed here we require

$$\min \| [L_p \; L_i] \| \qquad (4.14)$$

where $\|\cdot\|$ denote the matrix Euclidean norm.

For our controller structure the controller transfer function can be written as

$$G_c(z) = L_i T \, z/(z-1) - L_p \qquad (4.15)$$

One solution to equation (4.14) using (4.15) is to simply reduce both gain matrices L_i and L_p individually. However, we emphasize that this reduction is subject to the prescribed system performance as specified by the closed loop eigenvalues, that is, equation (4.11) must be satisfied simultaneously with equation (4.14). Under these conditions we will have achieved a reduction in controller norm without

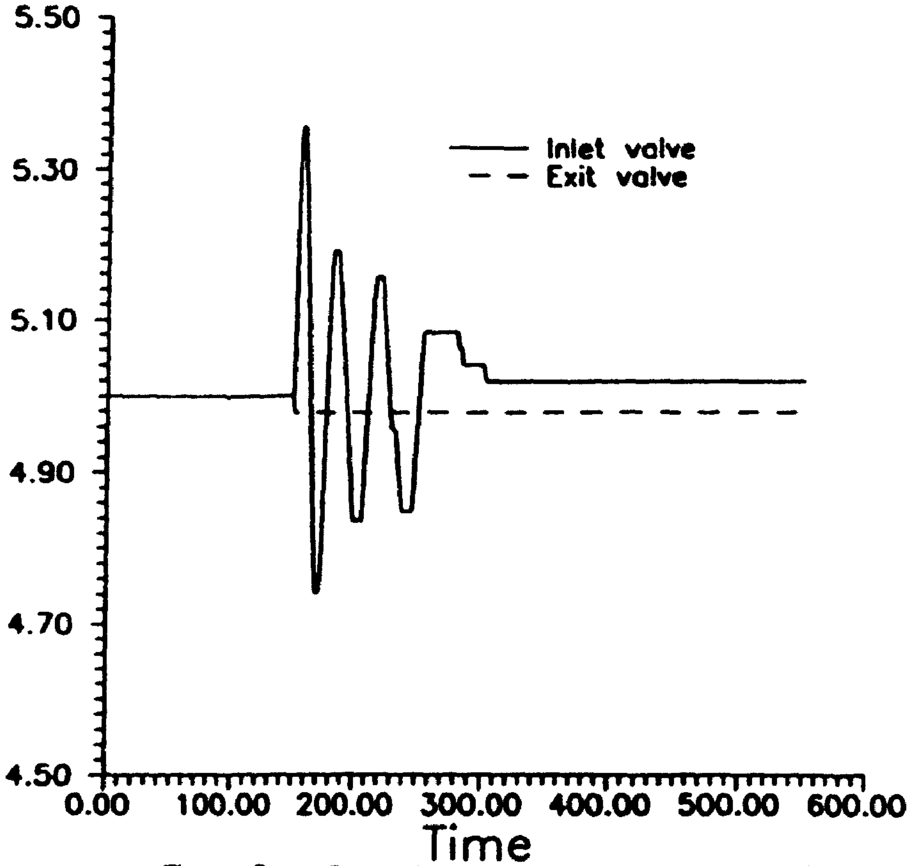


Figure 3.6 Controller response under setpoint change

CHAPTER 4

ROBUST EIGENVALUE ASSIGNMENT OF A PI STATE FEEDBACK CONTROLLER FOR DISCRETE MIMO SYSTEMS

Synopsis *A PI state feedback controller for multivariable discrete systems is proposed, and the robust design of the controller is addressed. The resulting design structure, in contrast to other schemes, is always controllable and observable and can thus allow for developing control algorithms using standard methods such as eigenvalue assignment. Robust performance is achieved through a minimum norm design using a matrix pseudo inverse. Simulation examples show the effectiveness of the resulting controller.*

4.1 INTRODUCTION

State feedback plays a distinct role in modern control system design, since it leads to arbitrary eigenvalue assignment and linear quadratic optimal control for linear time-invariant systems. Although it is an effective technique for regulator problems aimed primarily at eliminating disturbances due to initial state values, the use of such a proportional control law would usually result in unacceptable performance in the presence of external disturbances (Johnson,1968). The inherent feature of integral action in eliminating the off-set in face of such perturbations and system parameter variations calls for the introduction of integrators (Wittenmark, 1985).

So far various methods of incorporating integral action in state feedback using either pole placement or LQ optimal design technique have been proposed, especially for continuous systems (Johnson, 1968; Porter and Crossley, 1970; Davison and Smith, 1971; Park and Seborg, 1974; Yahagi, 1977; Fukata et al, 1980; Aida and Kitamor, 1990). In discrete systems, the introduction of integral action has been focused on integrating the output signal mainly to accommodate constant disturbances (Seraji, 1983; Wittenmark, 1985; Ogata, 1987). Salama (1985) proposed discrete state feedback controllers with integral action, and showed that the state feedback allows for greater freedom in design than output feedback, and that the design was much simpler than the corresponding continuous system. However, more design parameters are required and their choice is constrained in his PI structure. Also only eigen-structure assignment, which is the direct extension of the problem in continuous systems (Moore, 1976), was considered in that paper. All the PI design procedures, either for continuous or discrete systems, are far more complicated than those for the proportional only control problems, and this limits their applications in practice (Fakata, et al, 1980).

In the last decade, on the other hand, robust control in face of model uncertainty has drawn extensive attention and has dramatically changed design concepts in modern control area (Morari and Zafiriou, 1989). Robustness should be taken into account for any model based design technique since the performance of the closed-loop system rests heavily on an accurate model of the system. This, unfortunately, has not been considered in the literature for PI state feedback control.

In this chapter, a PI-type controller integrating state signal, for discrete multivariable systems, is introduced. Compared with other strategies, the structure of this controller leads to several design advantages: i). State integration is implemented with fewer parameters and there is no constraint on the choice of these parameters. ii). The controllability and observability of the resulting closed-loop system are conserved, and as a result, system design can be significantly simplified. iii). The controller takes a typical proportional plus integral control form in both state space and transfer function domains. Thus design techniques based on state space notation, such as eigenvalue assignment, and those based on transfer functions, such as robust control, can be directly implemented. A somewhat different view of robustness is considered here, and in contrast to the formal singular value based methods (Morari and Zafiriou, 1989), robustness is improved using a "minimum norm control" approach by means of a matrix pseudo inverse. This is achieved utilizing the extra degree of freedom provided by the integral terms.

4.2 THE PI STATE FEEDBACK CONTROLLER

Consider a linear discrete time-invariant system described by

$$x(k+1) = Ax(k)+Bu(k) \tag{4.1a}$$

$$y(k) = Cx(k) \tag{4.1b}$$

where x is the nx1 state vector, u is the mx1 control input vector, y is the rx1 output vector. The scalar parameter k specifies the sampling instant and assumes integer values, i.e., k=0, 1, 2, ..., with actual time t=kT (the sample interval T is omitted here for simplicity). A is an nxn constant matrix, B and C are nxm and rxn constant matrices, respectively.

The regular problem is to design a state feedback matrix K in the following control law

$$u(k) = -Kx(k)$$

such that the closed-loop system satisfies some design requirements, such as, the closed loop system takes on given eigenvalues or achieves LQ optimal control.

It is known that integrator will eliminate steady-state error for some variables and reduce the sensitivity of closed-loop system to model error in general (Wittenmark, 1985). There are several ways to incorporate integral action in a system:

i). Use velocity control algorithms in the final stage of design (MacGregor and Wong, 1980; Zhu and Jutan, 1991). But this approach does not explicitly include the integrator as part of the controller design.

ii). Differentiate the input signal in state feedback control. However, this method is proved to be inflexible (Wittenmark, 1985).

iii). Integrate the output signal (Seraji, 1983; Wittenmark, 1985). Unfortunately, this scheme is usually accompanied by inherent rank deficiency problems, and thus results in complicated design procedures (Seraji, 1983).

iv). Integrate the state signal. Salama (1985) showed that this strategy allows for greater freedom and easy implementation in design. However, in Salama's structure, an extra parameter matrix is needed and certain conditions must be imposed on parameters to ensure controllability of the system.

Proposed PI Controller

Our configuration for a PI state feedback controller is shown in figure 4.1.

The control law can be expressed as

$$u(k) = -L_p x(k) + v(k) \qquad (4.2a)$$

$$\nabla v(k) \equiv v(k) - v(k-1) = L_i x(k) \qquad (4.2b)$$

where L_p and L_i are the mxn proportional and integral action gains respectively, v is an mx1 vector with initial condition v(0)=0, and ∇ is the difference operator.

It can be seen from figure 4.1 and equations (4.2a) and (4.2b) that in this controller structure, only m integrators are used to integrate the linear combinations of all the n state variables. Note that only two design matrices are required. The first term on the right hand side of equation (4.2a) is the proportional control and equation (4.2b) provides the integral action.

Combining equations (4.2a) and (4.2b) gives

$$u(k) = (L_i - L_p)x(k) + v(k-1) \qquad (4.3)$$

Applying the control law given by equation (4.3) to system (4.1a), we have

$$u(k+1) = (L_i - L_p)(Ax(k) + Bu(k)) + v(k) \qquad (4.4)$$

Manipulating the above equation and using equation (4.2a), we obtain

$$u(k+1) = ((L_i - L_p)A + L_p)x(k) + (I_m + (L_i - L_p)B)u(k) \qquad (4.5)$$

Combining the system model described by equations (4.1a) and (4.1b) with the controller equation (4.5), an (n+m) order augmented system is obtained as

$$\begin{bmatrix} x(k+1) \\ u(k+1) \end{bmatrix} = \begin{bmatrix} A & B \\ (L_i - L_p)A + L_p & I_m + (L_i - L_p)B \end{bmatrix} \begin{bmatrix} x(k) \\ u(k) \end{bmatrix}$$

$$\equiv A^* \begin{bmatrix} x(k) \\ u(k) \end{bmatrix} \qquad (4.6a)$$

and

$$y(k) = [\ C \ \ 0\ ] \begin{bmatrix} x(k) \\ u(k) \end{bmatrix} \qquad (4.6b)$$

The above homogeneous equation represents the resulting closed-loop system with input vector treated as extra state variables due to the addition of integral action. System characteristics are solely determined by the (n+m)x(n+m) matrix A^* in equation (4.6a). We now need to choose proper gain matrices L_p and L_i such that the system satisfies suitable design specifications.

We need to determine real values for L_p and L_i which result from a suitable choice of closed loop eigenvalues λ_j. Through the characteristic equation the λ_j will be function of the design parameters, L^p and L_i, as shown in equation (4.7).

$$\lambda_j(A^*) = f_i(L_p, L_i), \qquad j = 1, 2, \ldots, n+m \qquad (4.7)$$

4.3 EIGENVALUE ASSIGNMENT DESIGN

Equation (4.6) can be modified to read

$$\begin{bmatrix} x(k+1) \\ u(k+1) \end{bmatrix} = \begin{bmatrix} A & B \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x(k) \\ u(k) \end{bmatrix} + \begin{bmatrix} 0 \\ I_m \end{bmatrix} w(k) \tag{4.8}$$

where

$$w(k) = [(L_i - L_p)A + L_p \; : \; I_m + (L_i - L_p)B] \begin{bmatrix} x(k) \\ u(k) \end{bmatrix} \tag{4.9}$$

and

$$y(k) = [C \;\; 0] \begin{bmatrix} x(k) \\ u(k) \end{bmatrix} \tag{4.10}$$

Defining

$$\xi(k) = \begin{bmatrix} x(k) \\ u(k) \end{bmatrix} \quad \text{-- (n+m)-vector}$$

$$\hat{A} = \begin{bmatrix} A & B \\ 0 & 0 \end{bmatrix} \quad \text{-- (n+m)x(n+m)-matrix}$$

$$\hat{G} = \begin{bmatrix} 0 \\ I_m \end{bmatrix} \quad \text{-- (n+m)xm-matrix}$$

$$\bar{L} = -[(L_i - L_p)A + L_p \; : \; I_m + (L_i - L_p)B] \quad \text{-- mx(n+m)-matrix} \tag{4.11}$$

equations (4.10a) and (4.10b) become

$$\xi(k+1) = \hat{A}\xi + \hat{G}w(k) \tag{4.12a}$$

and

$$w(k) = -\bar{L}\xi(k) \tag{4.12b}$$

Notice that system described by equations (4.12a) and (4.12b) is now in the form of proportional state feedback and hence is an standard regulator problem. Standard regulator design methods such as eigenvalue assignment apply for this equivalent system. It is also worthwhile to note that equation (4.11) separates eigenvalue assignment from the overall design of the PI controller so that other specifications can be easily accomplished. Robustness considerations will be discussed later in this chapter.

The controllability matrix for system governed by equation (4.12a) is

$$[\hat{G} : \hat{A}\hat{G} : \cdots : \hat{A}^{n+m-1}\hat{G}] = (n+m) \text{ x } m(n+m) \text{ matrix}$$

The above matrix can be written in terms of A and B as

$$[\hat{G} : \hat{A}\hat{G} : \cdots : \hat{A}^{n+m-1}\hat{G}]$$

$$= \left[\begin{array}{c:c:c:c:c:c:c} 0 & B & AB & \cdots & A^{n-1}B & \cdots & A^{n+m-2}B \\ \hdashline I_m & 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right] \tag{4.13}$$

If the open loop system (4.1a) is completely state controllable, we have

$$\text{rank } [B : AB : \cdots : A^{n-1}B] = n$$

From equation (4.13) we have

$$\text{rank } [\hat{G} : \hat{A}\hat{G} : \cdots : \hat{A}^{n+m-1}\hat{G}] = n+m$$

Therefore, we see that $(\hat{A},\hat{G})$ is completely state controllable if (A,B) is completely controllable. Hence, the rank deficiency, which occurs in corresponding continuous systems (Fukata et al., 1980) and output feedback systems (Seraji, 1983), does not exist in systems under this proposed PI controller. As a result, the controller design can be carried out in a standard way.

It is well known that the necessary and sufficient conditions for arbitrary eigenvalue assignment (eigenvalues are given in self-conjugate forms), for the standard regulator problem described by equation (4.12a) and (4.12b), is that the system is completely state controllable (Wonham, 1967; Ogata, 1987). Thus system (4.12a) and (4.12b) is eigenvalue assignable with real gain $\bar{L}$ if (A,B) is controllable. Note that the solution is not unique for single input systems and extra degrees of freedom exists for multi-input systems.

From equation (4.11), it is seen that we have mx(n+m) linear equations whereas there are 2xnxm variables. In practice, we usually have $n > m$ (i.e., the number of states is greater than the number of manipulated variables), so there are an infinite number of real solutions for L_p and L_i from equation (4.11) for any given real $\bar{L}$. This means that the PI control system offers additional degrees of freedom in pole placement design due to the introduction of integral action.

From the above observations, it can be seen that eigenvalue assignment design for the system described by equation (4.6) can be achieved through following two steps:

i). Design a proportional feedback gain $\bar{L}$ with a chosen set of eigenvalues for the equivalent standard regulator system governed by equations (4.12a) and (4.12b).

ii).Find the PI feedback gains L_p and L_i by solving equation (4.11) with $\bar{L}$ obtained in the first step.

As part of the design procedure, the additional degrees of freedom in solving for L_i and L_p can be removed by improving additional constraints which, for example, improves system robustness.

4.4 ROBUST DESIGN

4.4.1 Robustness Requirement

It is known that model-based design techniques, such as the eigenvalue assignment design discussed above, are prone to failure in the presence of model uncertainties since the performances of the resulting closed-loop system depend heavily on a precise model of the process which is often not available in practice.

General robust control theory using singular value analysis in the frequency domain has been widely advocated in recent years (Doyle and Stein, 1981; Doyle, 1982). However, considerable knowledge about the uncertainties in a system is required for its use and this is difficult to obtain in practice.

Intuitively, however, some constraints on the controller design can make the system less sensitive to model inaccuracies hence improve its 'robustness'. Tighter control, for example, would make performance and stability characteristics more sensitive to the accuracy of the system model and lead to a decrease in the robustness of the closed-loop system (Harris and MacGregor,1987). Thus if satisfactory control is obtainable with less control action, we expect the system to be less sensitive to modelling errors. This is analogous to the minimum variance controller which often leads to excessive control action and reaction. Constrained minimum variance controllers while suffering from some small loss in performance were much more successful since their corresponding controller action was considerably reduced (Clarke and Gawthrope, 1975; 1979; 1981).

As a result, to make a system more robust the norm of the control action can be minimized, e.g., for the control system with the PI state feedback law discussed here we require

$$\min || [L_p \; L_i] || \qquad (4.14)$$

where $||\cdot||$ denote the matrix Euclidean norm.

For our controller structure the controller transfer function can be written as

$$G_c(z) = L_i T \, z/(z-1) - L_p \qquad (4.15)$$

One solution to equation (4.14) using (4.15) is to simply reduce both gain matrices L_i and L_p individually. However, we emphasize that this reduction is subject to the prescribed system performance as specified by the closed loop eigenvalues, that is, equation (4.11) must be satisfied simultaneously with equation (4.14). Under these conditions we will have achieved a reduction in controller norm without changing system poles, and the resulting system will show both good dynamics and robustness.

From the above discussion, we know that robustness is closely related to the norm of control action and that robust control can be achieved by the "mininorm" control. As a result, the sensitivity of eigenvalues to model mismatches is minimized simply by seeking the mininorm control, although this is not a specific goal in the sense of Kautsky et. al (1985). However, this approach is quite easily implemented.

4.4.2 Minimum Norm Solution for L_p and L_i

After a gain $\bar{L}$ has been calculated based on a given set of poles, a minimum norm solution for the controller gains can be found as follows.

The design equation (11) can be rearranged as

$$\bar{L} = [(L_p-L_i)A-L_p \ : \ (L_p-L_i)B-I_m]$$

$$= [L_p \ L_i] \begin{bmatrix} A-I_n & B \\ -A & -B \end{bmatrix} - [0_{m,n} \ : \ I_m] \qquad (4.16)$$

Denoting

$$L = [L_p \ L_i] \text{ -- mx2n matrix} \qquad (4.17a)$$

$$\Phi = \begin{bmatrix} A-I_n & B \\ -A & -B \end{bmatrix} \text{ -- 2nx(n+m) matrix} \qquad (4.17b)$$

$$\Theta = \bar{L} + [0_{m,n} : I_m] \text{ -- } m\times(n+m) \text{ matrix} \quad (4.17c)$$

we have following relationship from equation (4.16),

$$L\Phi = \Theta \quad (4.18)$$

The minimum norm solution for $L=[L_p\ L_i]$ in the matrix-matrix equation (4.18) can be expressed as (Ogata, 1987),

$$L^o = \min ||L|| = \Theta\Phi^{LM} \quad (4.19)$$

where

$$\Phi^{LM} = (\Phi^T\Phi)^{-1}\Phi^T \quad (4.20)$$

is the left pseudo inverse of matrix Φ.

4.4.3. Disturbance Rejection

This section deals with the disturbance attenuation property of the proposed state feedback controller.

Consider a system with an external disturbance described by

$$x(k+1) = Ax(k) + Bu(k) + d(k) \quad (4.21a)$$

and

$$y(k) = Cx(k) \quad (4.21b)$$

where d(k) is the nx1 disturbance vector.

Applying the PI controller given by equations (4.2a) and (4.2b) to above system, we obtain the following augmented system

$$\begin{bmatrix} x(k+1) \\ u(k+1) \end{bmatrix} = A^* \begin{bmatrix} x(k) \\ u(k) \end{bmatrix} + \begin{bmatrix} I_n \\ L_i - L_p \end{bmatrix} d(k) \quad (4.22a)$$

and

$$y(k) = [C \ 0] \begin{bmatrix} x(k) \\ u(k) \end{bmatrix} \tag{4.22b}$$

where A^* is the same as in equation (4.6a), and is stable for $d(k)=0$.

We now have

$$\lim_{k\to\infty} \begin{bmatrix} x(k) \\ u(k) \end{bmatrix} = (I_{n+m} - A^*)^{-1} \begin{bmatrix} I_n \\ L_i - L_p \end{bmatrix} \lim_{k\to\infty} d(k) \tag{4.23}$$

For any external disturbance satisfying

$$\lim_{k\to\infty} d(k) = 0 \tag{4.24}$$

we have from equation (4.24)

$$\begin{bmatrix} x(\infty) \\ u(\infty) \end{bmatrix} = 0 \tag{4.25}$$

This means that the proposed PI controller can accommodate any pulse-type external disturbance for all the states , inputs and outputs.

However, from equation (4.23), the controller is unable to eliminate static offsets in the presence of step disturbances since equation (4.24) is not satisfied. Instead, due to the inherent property of integral action, we have

$$\lim_{k\to\infty} L_i \, x(k) = 0 \tag{4.26}$$

This leads to

$$y(\infty) = 0$$

for equal number of inputs and outputs if the controller is designed with the integral gain satisfying

$$L_I = C$$

4.5 STATE ESTIMATION

The design approach discussed above is based on state feedback control. The states are often not measurable, and an estimator is required. Since the observability is conserved in our design conserved, the observer can easily be constructed.

A state feedback control system with state observer is shown in figure 4.2.

The observer can be expressed as,

$$x(k+k) = A\hat{x}(k) + Bu(k) + K[y(k) - C\hat{x}(k)] \tag{4.27}$$

where K represents the gain matrix of the observer.

The PI state feedback controller becomes,

$$u(k) = - L_p\hat{x}(k) + v(k) \tag{4.28a}$$

$$ov(k) = L_I\hat{x}(k) \tag{4.28b}$$

Defining,

$$e(k) = x(k) -\hat{x} (k)$$

and incorporating the observer given by equation (4.27) and the controller in equations (4.28a) and (4.28b) into the system described by equations (4.1a) and (4.1b), we obtain the following augmented svstem:

$$\begin{bmatrix} e(k+1) \\ x(k+1) \\ u(k+1) \end{bmatrix} = \bar{A} \begin{bmatrix} e(k) \\ x(k) \\ u(k) \end{bmatrix} \tag{4.29a}$$

where

$$\bar{A} = \begin{bmatrix} A-KC & \vdots & 0 & 0 \\ \cdots\cdots & \cdots & \cdots & \cdots \\ 0 & \vdots & A & B \\ (L_p-L_i)(A-KC)-L_p & \vdots & (L_i-L_p)A+L_p & I_m+(L_i-L_p)B \end{bmatrix}$$

(4.29b)

The characteristic equation of the above closed-loop system can be expressed as follows,

$$| zI-\hat{A} | = | zI-A+KC | \; | zI-A^{*} | = 0 \qquad (4.30)$$

where A^{*} is the same as in equation (4.6a).

Since the eigenvalues of the observer can be chosen separately the complete design will be quite flexible.

4.6 EXAMPLES

The following examples demonstrate the use of the proposed design procedure in finding the 'mininorm' control subject to given eigenvalues and the robustness of the resulting closed loop system.

4.6.1 Example 1

Consider a system

$$x(k+1) = Ax(k) + Bu(k) + d(k)$$

$$y(k) = Cx(k)$$

where

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -0.12 & -0.01 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$C = [0.5 \ 1 \ 0]$$

with

$$x(0) = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$d(t) = \begin{bmatrix} 2te^{-t} \\ 0 \\ 0 \end{bmatrix}$$

Design a PI state feedback controller as shown in equation (4.2a) and (4.2b) such that the closed-loop system is robust and takes on the eigenvalues -0.2, -0.1, 0.1, 0.2.

It can be easily verified that the open loop system is controllable, and so the system is eigenvalue assignable and the method proposed in this chapter can be employed. First an equivalent regulator problem as described by equations (4.12a) and (4.12b) can be constructed with

$$\hat{A} = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ -0.12 & -0.01 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\hat{G} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

Using eigenvalue assignment the controller gain can be obtained as follows,

$$\tilde{L} = [-0.12 \quad -0.13 \quad 0.94 \quad 1.0]$$

Next, robust or the minimum norm control subject to the above given eigenvalues under the PI controller can be achieved by calculating the minimum norm solution for L_p and L_i from equation (4.11). Constructing the robust design equation (4.18) gives

$$\Phi = \begin{bmatrix} -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ -0.12 & -0.01 & 0 & 1 \\ 0 & -1.0 & 0 & 0 \\ 0 & 0 & -1.0 & 0 \\ 0.12 & 0.01 & -1.0 & -1 \end{bmatrix}$$

$$\Theta = [-0.12 \quad -0.13 \quad 0.94 \quad 2.0]$$

The <u>minimum norm solution</u> is given by

$$|| \; [L_p \; L_i] \; ||_{min}$$

$$= [-0.12 \quad -0.589 \quad 0.5262 \quad -0.359 \quad 0.974 \quad -1.474]$$

with a norm of 3.9.

The responses of all the states and output of the system under the designed PI controller are shown in figure 4.3. It can be seen from figure 4.3 that the system shows good control performance with the given eigenvalues and the minimum norm control. All states and output rapidly return to the steady state in less than 10 time steps in response to an initial state upset of one unit and an external disturbance of exponential decay form.

4.6.2 Example 2

To show the robustness of the closed-loop system under minimum norm control in example 1.

In order to show the robustness of the system in example 1, we investigate the system response in the presence of model uncertainty, and compare it with the corresponding response with another non-minimum norm control which has the eigenvalues as in example 1.

It can be easily shown that the control gain for the proposed PI controller

$$[L_p \; L_i] = [-0.12 \quad 2.94 \quad 3.0 \quad -2.95 \quad 1.0 \quad 1.0] \qquad (4.31)$$

is also a solution (but with a non minimum norm) to equation (4.11) or (4.18) leading to the same eigenvalues for the closed-loop system. This norm is 28.4.

Consider a multiplicative model error in system matrix A in example 1 (Morari and Zafiriou, 1989),

$$A_M = (I+\Delta)A \qquad (4.32)$$

with

$$\Delta = 0.17I \qquad (4.33)$$

i.e., 17% model error in system matrix A, where A_M represents the true system matrix.

The closed-loop system responses under the non-minimum norm control given by the gains in equation (4.31) which were designed from the nominal model, are shown in figure 4.4. The system now has sustained oscillations, which implies that equation (4.32) induces the maximum uncertainty into the system for stability.

However, system responses under the minimum norm control, with the same model error, still show acceptable dynamic performance and return to steady state within 10 time steps (see figure 4.5). Clearly, the minimum norm controller has led to a more robust controller design.

4.7. FINAL REMARKS

A PI controller using state feedback for linear, multivariable discrete systems is proposed and a simple method to achieve robust control subject to any prescribed set of eigenvalues is presented. It has been shown that the proposed controller has several advantages, comparing other existing methods. The resulting closed-loop system can be designed with pre-assigned eigenvalues, while the increased design freedom can be used to improve the robustness of the system. The design procedure to achieve robust control, subject to prescribed eigenvalues, involves two separate steps: an eigenvalue assignment design for a standard regulator problem, followed by a minimum norm solution for a matrix equation.

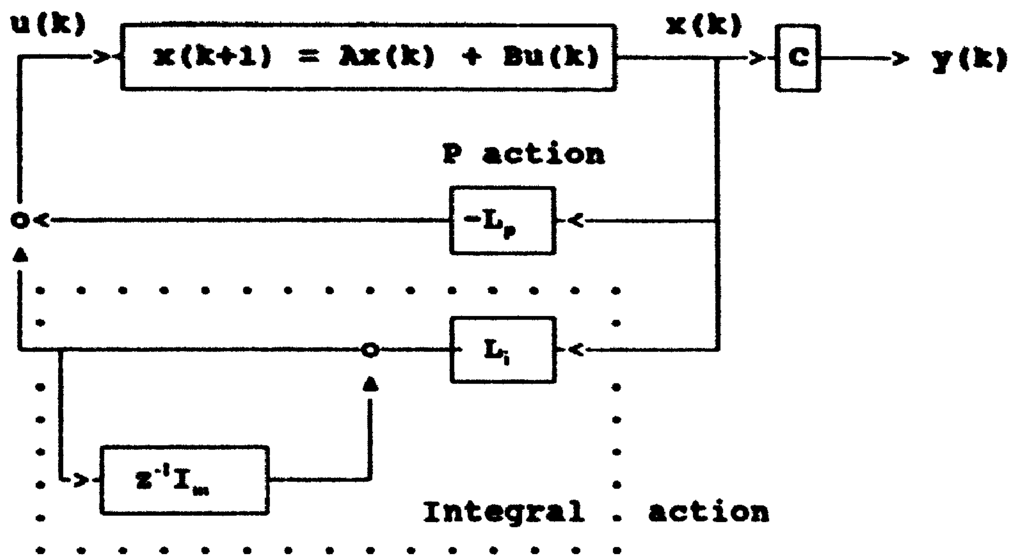


Figure 4.1. Block diagram of proposed PI-type controller

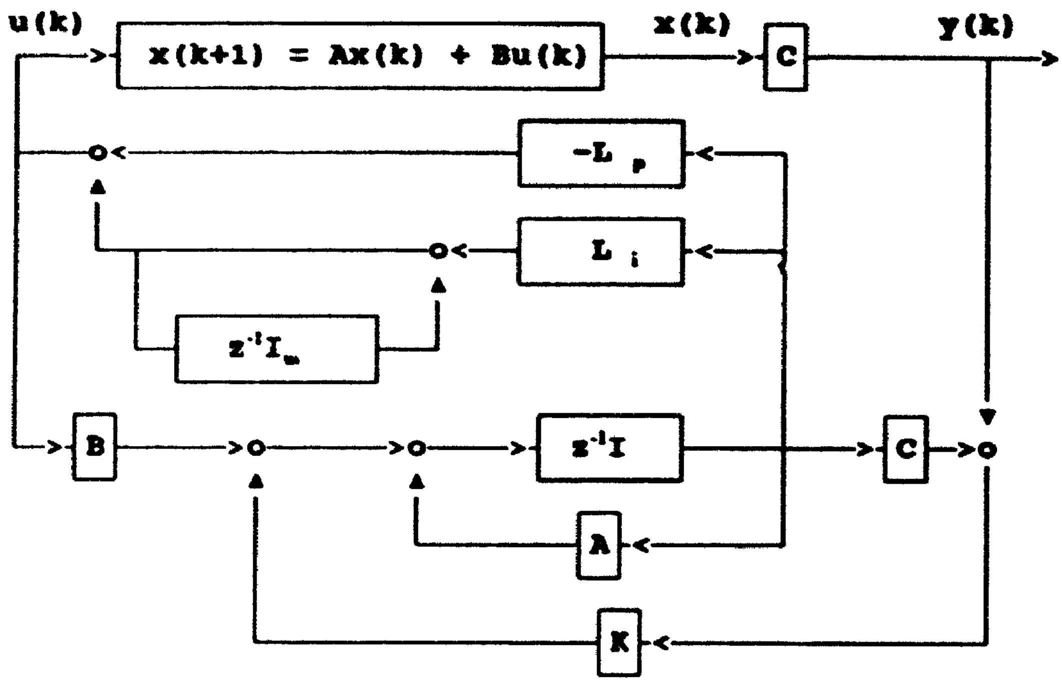


Figure 4.2. Observed-state PI control system configuration

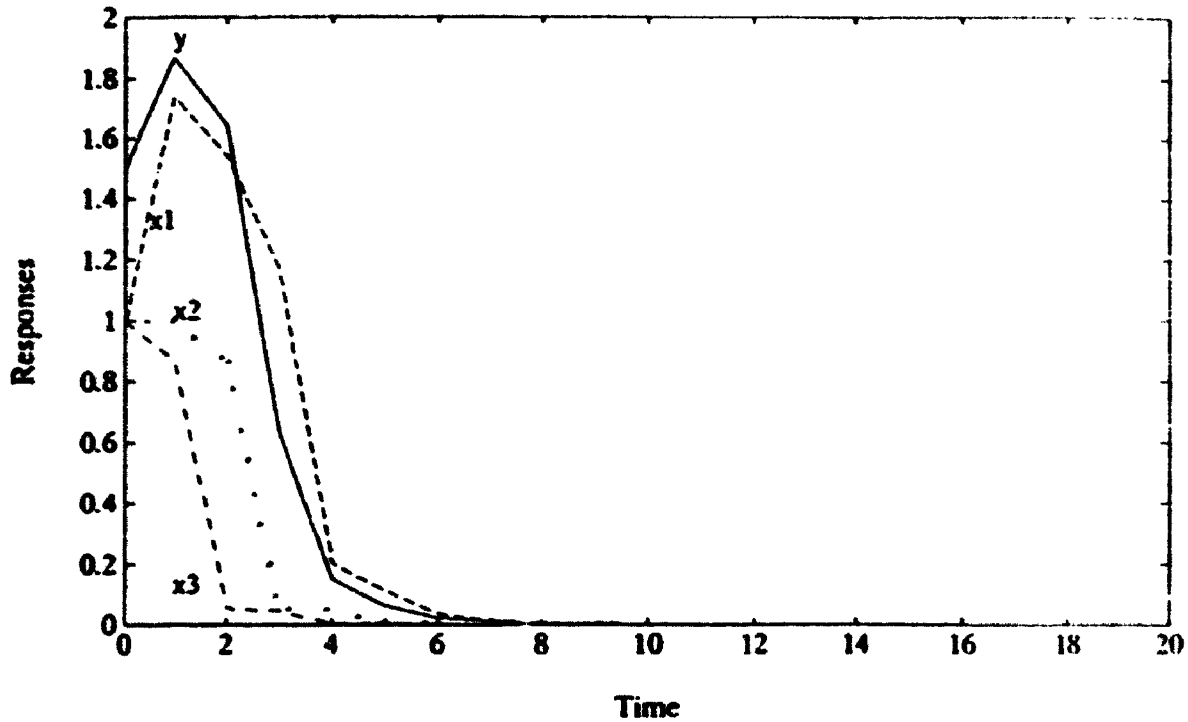


Figure 4.3 Responses under mini norm feedback

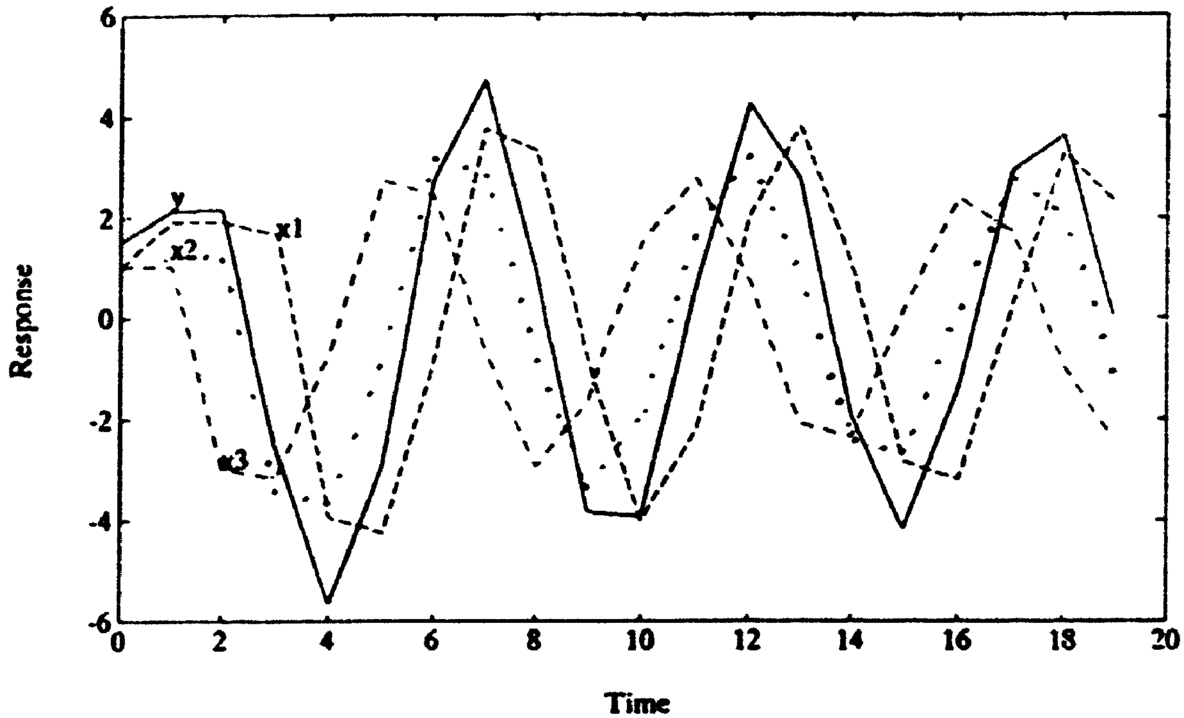


Figure 4.4 non-mini norm gains with model error

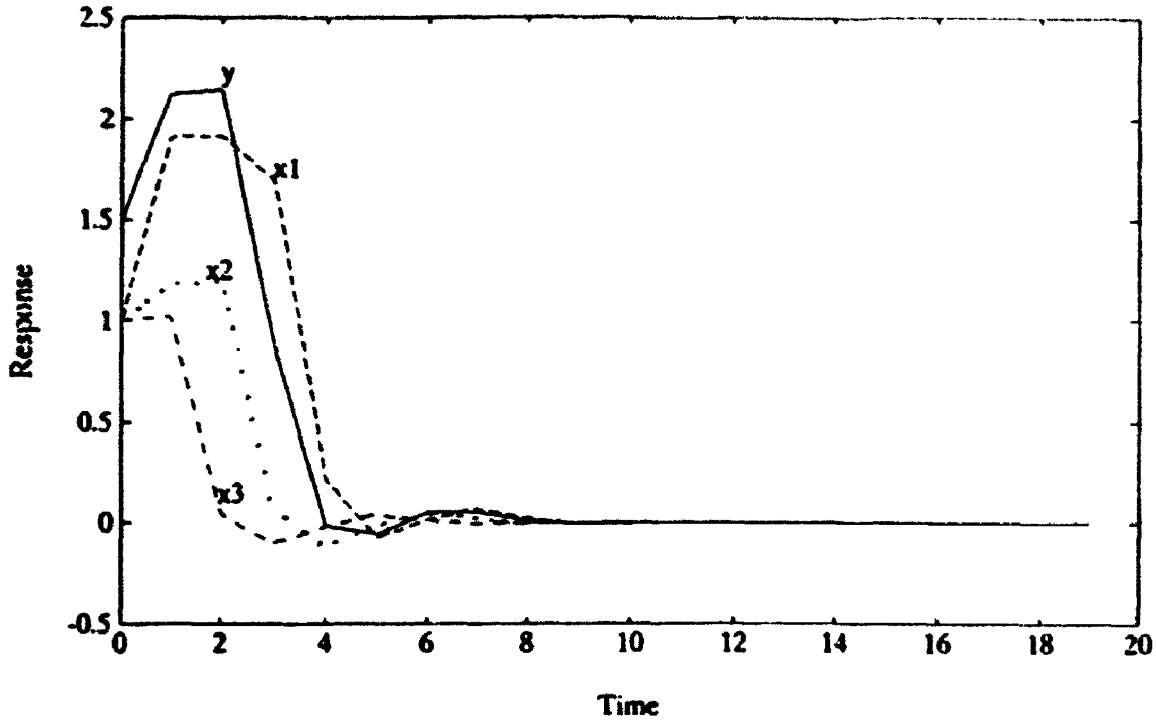


Figure 4.5 mini-norm control with model error

CHAPTER 5

A PC-BASED DIFFERENTIAL SYSTEM SIMULATOR FOR PROCESS DYNAMICS

Synopsis *Dynamic simulators were originally developed for batch use in computer 'main frame' environment. With the availability of powerful personal computers, there is a need for the development of simulators with strong 'user friendly' interactive capabilities. This involves the need for comprehensive, user interface management systems which are modular and flexible. A new management system MDSS has been developed and implemented using an existing integration 'engine'. The unique structure of MDSS allows for a powerful interactive tool for analysis of complex dynamic systems, described by both partial and ordinary differential equations.*

5.1 INTRODUCTION

Dynamic simulation is an important tool for scientists and engineers. Many dynamic simulators have been developed both for general dynamic simulation such as DSS2 (Schiesser, 1976), CSMP (Anon, 1971) and ACSL (Anon, 1976), and for process design such as DYFLO (Franks, 1972), DYSCO (Lopez, 1974) and DPS (Thambynayagam, 1981).

Generally speaking, the main components of a dynamic simulator is the capacity to solve equations (integration 'engine') and the management system to make the system more flexible and user-friendly. Another major requirement for a dynamic simulator is the ability to model different problems. Most of the above simulators can only be run on large mainframe computers in order to accommodate this level of flexibility.

In the process control area, for example, new control algorithms are constantly proposed and control system design for processes described by partial differential equations is largely a trial and error procedure. Here the need for a portable and efficient interactive system with the capabilities for easily introducing new control algorithms and adjusting controller parameters is valuable. MDSS was developed using DSS2 as the integration engine, partly to meet these process control needs. Henceforth, we call the integrated system MDSS. It has an open block (or modular) structure and allows the user to specify the process dynamic and control algorithms in a standard user subroutine. It has powerful management system and color graphic functions and allows the user to design control system efficiently. DSS2 was developed at Lehigh University (Pirkle and Schiesser, 1987). Its powerful integration 'engines' form part of the run time library of MDSS. The capabilities of DSS2 extend from the simple integration of non stiff Ordinary Differential Equations (ODE's) to integration of 3-dimensional Partial Differential Equations (PDE's) using the Method of Lines. More recent versions of DDS2 allow for other integration methods such as Collocation (Finlayson, 1980).

The program is mainly written in FORTRAN language whereas the management system is written partly in BASIC and MSDOS batch language. Colour Graphics Routines are written in ASSEMBLER language. Thus MDSS makes full use of the advantages of these languages. The complete package runs under MicroSoft Disk Operation System (MSDOS) 3.2 or higher, on an IBM(c) PC/AT or compatible. Use of a math co-processor is recommended. Microsoft (MS) Fortran version 4.01 was used for program development and compilation.

5.2 GENERAL DESCRIPTION OF MDSS

The modular structure of MDSS allows the user to specify the process model equations in Fortran subroutines. Some predefined structure is required which allows for information exchange between this user module and the MDSS manager. The management system compiles the user module, links it with the overall system, including the run-time libraries, and starts executable file automatically. The user can interactively create data files, plots at each stage, and perform a variety of on line design tasks. The main features of MDSS are:

(1) Solve up to 250 ODE's and/or PDE's using DSS2's extensive integration 'engine'.

(2) Modular structure for easy implementation of user modules.

(3) Automatic management

(4) Run time library for facilitating systems management and saving memory space.

(5) Incorporate Fortran, BASIC, Assembler languages and DOS command.

(6) Data Exchange using either (a) interactive mode or (b) batch files.

(7) Results in color graphical and/or tabular form.

(8) On line facility for modifying user defined process model and control parameters.

(9) Extensive Help File.

In order to provide the above features, MDSS is composed of five functional parts:

(1) Management System

(2) Executive System

(3) Equation solving engine

(4) User Modules

(5) Input-Output System

The management system co-ordinates all the part resources to make optimal use of them. The major functions are task management and task handling. These are described in more detail later.

The executive system is the most important system in MDSS. Its tasks are control of input and output information, simulation control, selection of numerical methods, generation of colour graphics and control system design, tuning, etc. The structure and function of this system is presented in the next section.

As previously noted, the equation solving 'engine' selected here is DSS2. It is an extremely versatile system with 18 integration methods (for Ordinal Differential equations) based on Runge-Kutta and Gear algorithms. For Partial Differential Equations there are 20 methods based on the Method of Lines with the Collocation Method available in more recent versions.

MDSS has an open module structure for the user to define any dynamic problem and test for example, new controller algorithms. This module has a flexible structure, however several procedures are required for information exchange between modules. MDSS requires a single user file which contains two subroutines (see Appendix):

(1) DERV defines differential equations

(2) INITAL defines initial conditions

For partial differential systems user can call appropriate subroutines available in the run time library. This supplies ODE's for integration and calculates spatial derivatives for the PDEs (see example below).

The Input-Output system's major function is to act as a user friendly interface between MDSS and the user. For efficient use of program organisation a run time library is created for MDSS which contains, individual integration routines in separate files, colour graphics routines and other service routines. This ensures that the link manager for MS fortran selects only those routines necessary for a specific simulation to create an executable file of minimum size. These files were typically less than 250 KB in length.

The overall system can thus be subdivided into the following component sections (see Figure 5.1): (1) Management System; (2) User File; (3) Main Prog; (4) Run time Library

5.3 THE STRUCTURE OF MDSS

The flexible nature of MDSS is due its unique structure for its management, executive and input-output systems. These are discussed in more detail below.

5.3.1 Management System Structure

Management system structure has two functions: task management and task handling. Its structure is shown in Figure 5.2.

From Figure 5.2 we see that there are four basic tasks:

(1) On-line help

(2) 'First Run' for a specific simulation

(3) Run executable file directly

(4) Exit MDSS environment

After the user executes MDSS an interactive menu is presented. This code is written using the powerful screen management capabilities of MSDOS BASIC. Advanced batch commands are used to handle program flow.

An online help file is available for new users. The user will have already developed the required user file and will be prompted to name them. The management system then compiles this file and links it with a general main program and the appropriate run time library routines to create a tailored executable code for the specific problem.

For subsequent runs the compile/link stages above are bypassed automatically.

5.3.2 Executive System Structure

The user supplies a userfile with his problem description. (Essentially the right hand side of the differential equations describing his problem.)

The executive system selects a user chosen integration method and passes the user subroutine to the integration 'engine' which solves the equations simultaneously. The executive system takes care of integration time and saving of intermediate results, for tabulated or graphical output.

After the final simulation time is reached the user can display system dynamic variables graphically, increase final time, change process controller parameters and then continue with another simulation run.

Different runs with different operating conditions can be stacked in a batch file for sequential runs.

The details of the executive system structure is shown in Figure 5.3.

5.3.3 Input-Output System

There are two Input-Output (IO) options: Batch and Interactive. In batch mode the user supplies a prepared data file, whereas in interactive mode, the system prompts the user. The new user would typically use the interactive mode. His answers are then saved to a batch file for subsequent runs.

There are two options for data output: A full report option, which provides values of all user specified variables at each integration time step in a data file and Plot report, which can be used for plotting selected variables using offline plotting routines.

Up to 20 variables can be displayed on the graphics monitor.

All information exchange between the various modules in Figure 3 are handled via a common data base.

A general print module is supplied to the user, but he may opt to use his own and interface it to the executive.

5.4 A SIMULATION EXAMPLE

A three-pass shell and tube heat exchanger with a PI control loop shown as Figure 5.4 is presented here to demonstrate the use of MDSS.

The Partial Differential equations for this 3-pass heat exchanger are given by:

$$\frac{\delta T_1}{\delta t} + v_t \frac{\delta T_1}{\delta x} = C_t (T_s - T_1)$$

$$\frac{\delta T_2}{\delta t} - v_t \frac{\delta T_2}{\delta x} = C_t (T_s - T_2)$$

$$\frac{\delta T_3}{\delta t} + v_t \frac{\delta T_3}{\delta x} = C_t (T_s - T_3)$$

$$\frac{\delta T_s}{\delta t} - v_s \frac{\delta T_s}{\delta x} = C_s [(T_1 - T_s) + (T_2 - T_s) + (T_3 - T_s)]$$

where T_1, T_2, T_3 - temperature of fluid in 3 tubes, resp.

T_s - temperature of fluid in shell

x - distance along the exchanger

t - time

v_t, v_s - average velocity of tube and shell side fluids, resp.

C_t, C_s - constants defining heat transfer rate to tube and shell side fluids, resp.

The following operating conditions apply

$T_1 = T_2 = T_3 = T_s = 25$ at $t = 0$ initial conditions

$T_{1o} = 100, T_{2o} = T_{1o}, T_{3o} = T_{2o}, T_{so} = 220$ boundary conditions

$T_3set = 170$ set point

$T_{s_i} = 220$ inlet condition

This problem requires an interactive trial and error method for selecting the tuning constants K_c and T_i for the Proportional Integral Controller in order to stabilize the exit temperature from the third pass, T_{3o}. Controller tuning and design for systems described by partial differential equations are difficult and represent an ideal application for an interactive simulator. This example was set up on MDSS using the user file shown in the Appendix. The exit temperature dynamics was observed and the tuning constants were changed interactively by trial and error until a satisfactory response was obtained.

<u>Results</u>

The results are shown in Figures 5.5, 5.6, 5.7 and 5.8.

In Figure 5.5 we have only proportional control (Kc = 0.1) and T_{3o} is stable but has large offset.

Figure 5.6 (Kc = 4.0, Ti=10) shows a faster response for T_{3o} but at the expense of heavy oscillation in T2.

Figure 5.7 shows dynamics from initial conditions with Kc = 4.0, Ti = 30.0. T_{3o} is sluggish and other responses are too oscillatory.

Figure 5.8 (Kc = 0.1, Ti = 30) is the best tuning and gives responsive stable dynamics for T_{3o} and T_{2l}

5.5 FINAL REMARKS

Certain problems, especially non linear and partial differential equations, are currently best solved with powerful PC based interactive simulators.

Many dynamic simulators were developed for mainframe environment and are not as useful. By judicious construction of flexible and user friendly interfaces, a PC based interactive environment can be created which bridges this gap. MDSS is such an interface which, with an open block structure and management system, allows the user to focus on his specific problem description, without concern for the usual program development 'baggage' that often accompanies the complete problem solution.

MDSS is modular in nature, uses DSS2 as the integration 'engine', and allows the user great flexibility.

An example is presented which shows trial and error tuning of a PI control for a heat exchanger described by partial differential equations.

Computation time on AT (8 Mz, Zero wait state, 80287 co-processor) was 30 min for the complete problem solution and tuning.

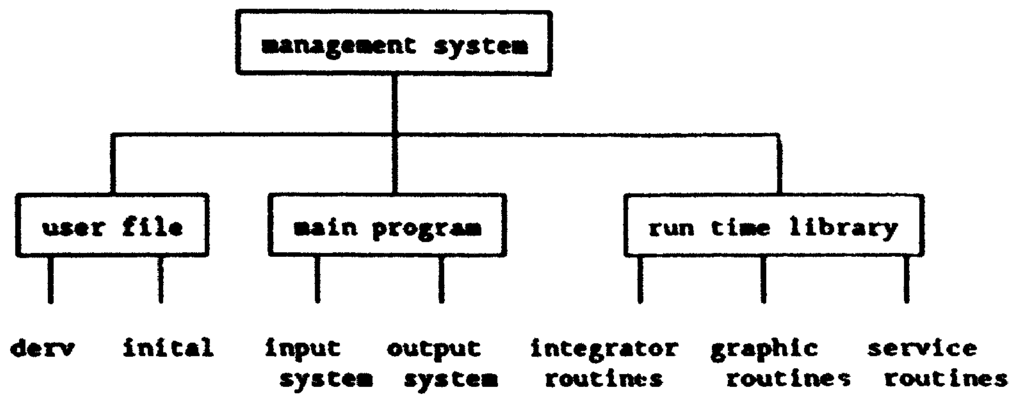


Figure 5.1 Overall Structure of MDSS

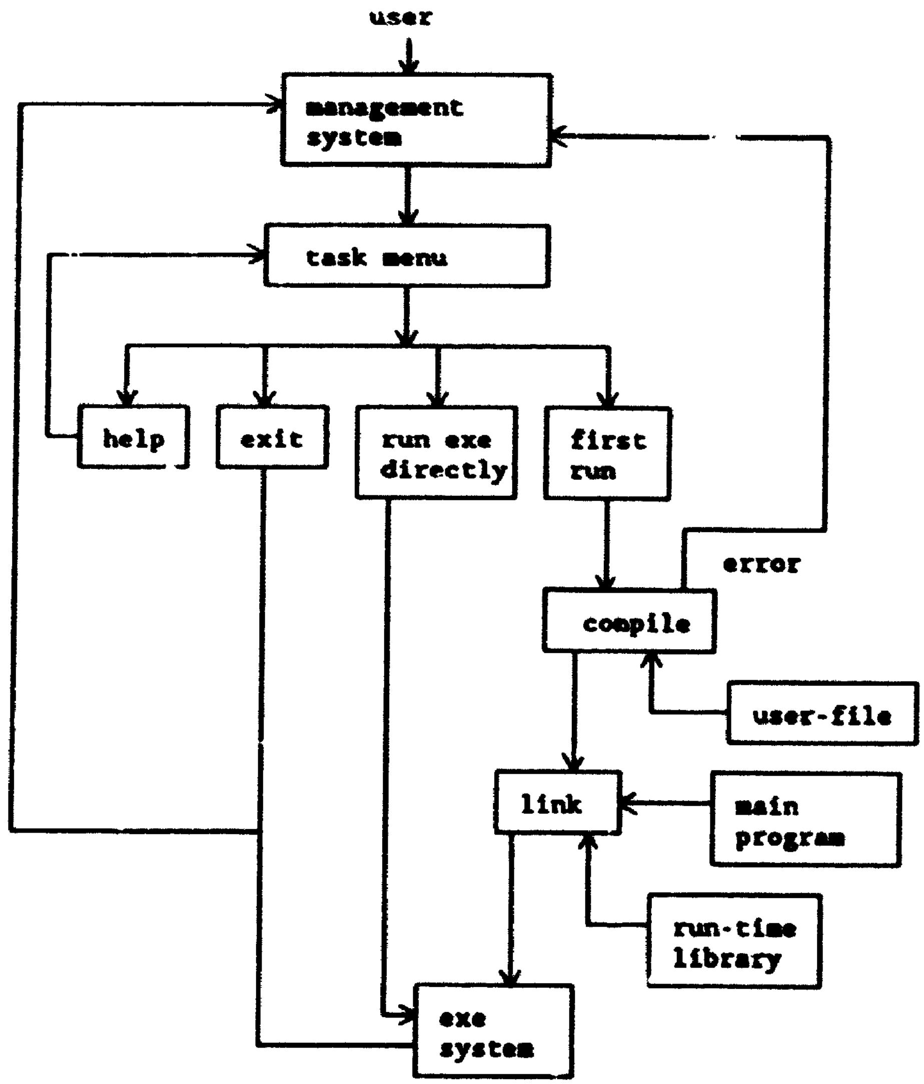


Figure 5.2 Management System Structure

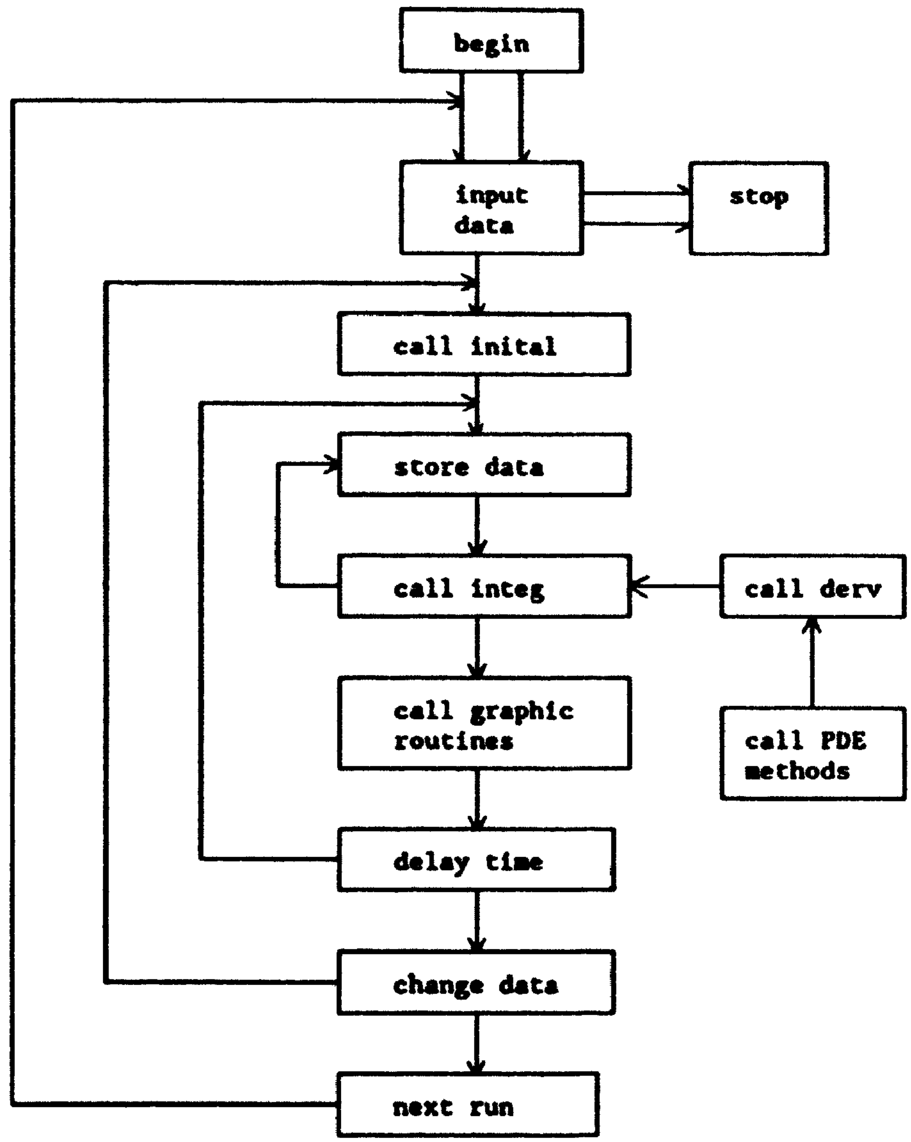


Figure 5.3 Executive Structure

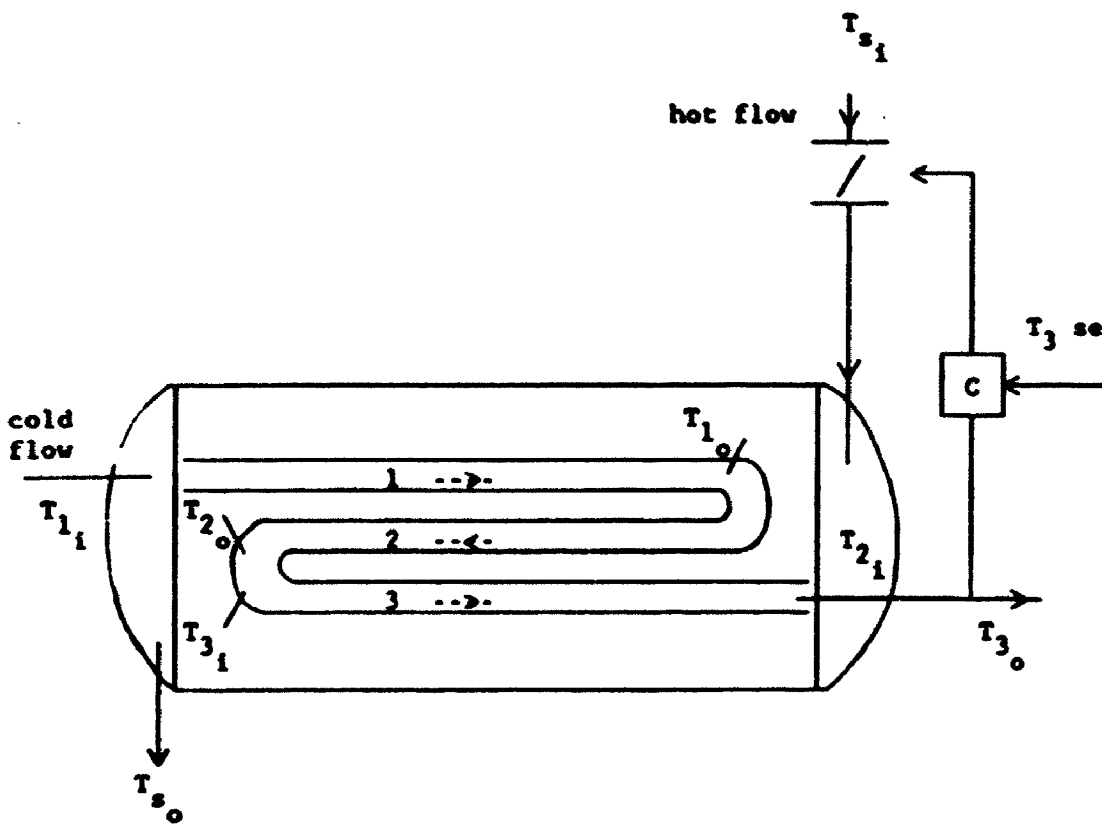


Figure 5.4 Heat Exchange and Control System

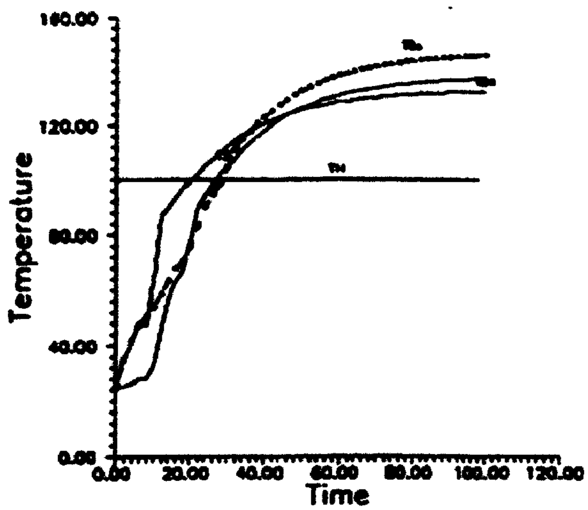


Fig. 5.5 Temperature Response under Proportional Action with K_c = 0.1.

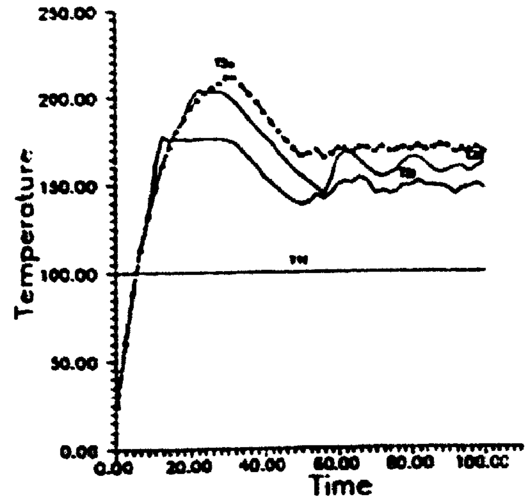


Fig. 5.6 Temperature Response under PI Control with K_c = 4.0, T_1 = 10.

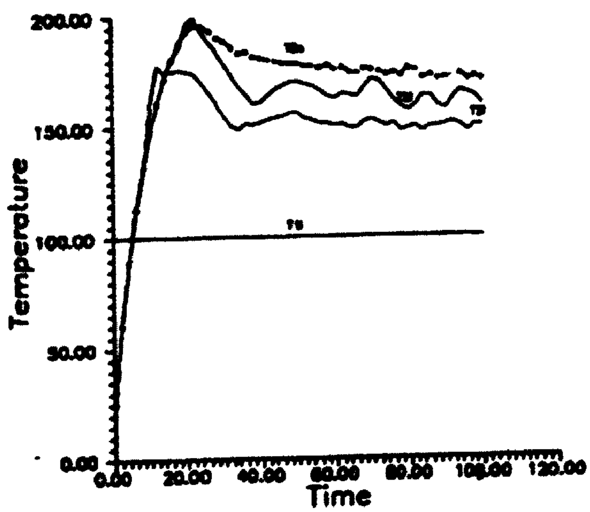


Fig. 5.7 Temperature Response under PI Control with K_c = 4.0, T_2 = 30.

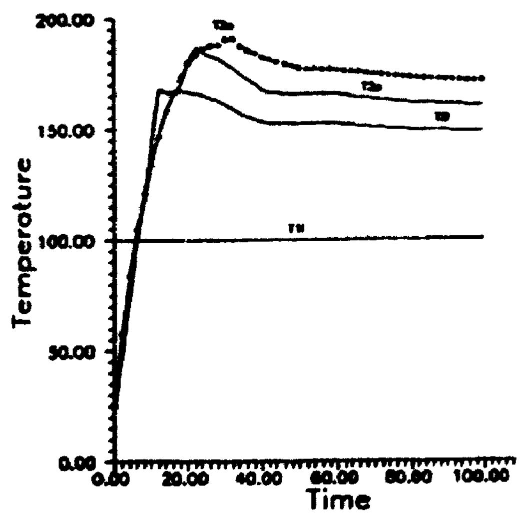


Fig. 5.8 Temperature Response under PI Control with K_c = 0.1, T_a = 30.

PART II DECENTRALIZED CONTROL

CHAPTER 6

A NEW VARIABLE PAIRING CRITERION BASED ON THE NIEDERLINSKI INDEX

Synopsis *The Niederlinski Index (NI) has been used only as a stability condition to date. This chapter expands upon the use of the NI and shows that, in addition to the traditional function of the sign of the NI, the size of the NI is, in fact, an overall interaction measure itself, and that it provides important insights into Interaction Measurement (IM) and Variable Pairing (VP) issues. The use of the NI is thus extended to propose a new more comprehensive steady state VP criterion. This criterion is shown to avoid ambiguities present in other VP rules and IMs. Theoretical justifications for the new pairing rule are provided with supporting examples.*

6.1 INTRODUCTION

Decentralized structure is a strong alternative in multivariable control system design, particularly for industrial processes, because of its simplicity in design, implementation and tuning. Two steps are usually involved in the design of such a scheme: variable pairing and controller tuning, with the key often being the first one. Ultimately, various variable pairings should be evaluated in terms of control performance of the resulting configurations, for instance, the ability of the system to reject disturbances (Stanley et al., 1985). However, pairing choice has traditionally been aimed at achieving the minimum interaction among control loops (Mijares, et al., 1986), since the subsequent control tuning can be largely facilitated due to the fact that a multivariable control problem may be effectively reduced to several Single Input Single Output (SISO) problems (Grosdidier, et al., 1985). Consequently, Variable Pairing (VP) problem is intimately correlated with and in fact relies, to a great extent, on the Interaction Measure (IM).

So far numerous approaches addressing the IM/VP problem have been proposed (Bristol, 1966; Bruns and Smith, 1982; Witcher and McAvoy, 1977; Tung and Edgar, 1981; Grosdidier and Morari, 1987; Jensen et al., 1986; Mijares, et al., 1986; Johnston, 1990). Nonetheless, only those using steady state information (Bristol, 1966; Bruns and Smith, 1982; McAvoy, 1983; Mijares, et al., 1986) are of practical importance. In particular, the RGA has been by far the most widely accepted method.

The role of the RGA in IM/VP, nevertheless, may have been exaggerated. There are in fact some fundamental limitations associated with the RGA. For instance, it provides no definite choice regarding best pairing when several alternatives satisfying the RGA rules exist. As a result, other techniques have to be used to help eliminate unstable RGA pairing (McAvoy, 1983) or make the final choice among alternatives (Yu and Luyben, 1987).

Fortunately, the Niederlinski's Index (NI) (Niederlinski, 1971) provides a necessary condition for system stability, and constitutes a parallel tool in scanning pairing alternatives. Consequently, the joint use of the RGA and NI has been widely advocated (McAvoy, 1983). However, the incorporation of the NI cannot overcome the inherent limitations of the RGA other than addressing the stability problem. Surprisingly, only the sign of the NI is considered important and little attention has been focused on its size.

In this chapter, a new criterion to measure the overall interaction and achieve the best pairing in decentralized control systems, using both the sign and, more importantly, the size of the NI, is proposed. Theoretical justification is provided. It is shown that the NI is more than a stability index. The size of the NI, in fact, provides a direct measure of the strength of interaction as a whole, whereas the sign gives the necessary stability condition. As an IM, the NI offers more insights into the interaction and variable pairing problem. Finally, simplification procedures in calculating the NI's and implementing the new pairing criterion are discussed. Nevertheless, We would like to point out that the new pairing rule is only a steady state criterion and thus, like other steady state rules, has its limitations when considering dynamic behavior. Dynamic analysis is required when the NI or RGA is a strong function of frequency, and in this case the behavior around the crossover frequency should be investigated (Hovd and Skogestad, 1992). In addition, many researchers point out that, for some systems, there can be a trade off between minimum interaction and disturbance rejection.

6.2 THE RGA AND ITS LIMITATIONS

We assume throughout the chapter that a diagonal controller, C(s), containing integral action is to be used to control a nxn plant, G(s), and that G(s) is stable and G(s)C(s) is rational and proper. For a given G(s), the RGA is defined using the steady state plant gains only as (Ray, 1981),

$$RGA[G(0)] = [\lambda_{ij}] \qquad (6.1)$$

and

$$\lambda_{ij} = [G(0)]_{ij} \cdot [G^{-1}(0)]_{ji} \qquad (6.2)$$

where G(0) denotes the steady state gain matrix of G(s),

$[G(0)]_{ij}$ denotes the (i,j)-th elements of G(0), and $[G^{-1}(0)]_{ij}$ represents the (i,j) elements of $G^{-1}(0)$.

The RGA rule for variable pairing is to pair on the positive λ_{ij} that are closest to unity (Bristol, 1966). We can see from the pairing rule that two types of information in the RGA are required:

1. The signs of the RGA elements. Positive λ_{ij} are required since it is claimed that negative elements would lead to unstable pairings (Bristol, 1966; Grosdidier, et al., 1985, Morari and Zafiriou, 1989). Based on the relationship between the RGA and the NI, Chiu and Arkun (1990) showed that the sign of the RGA is explicitly related to the system integrity (stability under loop failures). Hence, the final pairing suggested by the RGA is subject to other techniques, such as the Niederlinski Index for stability constraints.

2. The size of the RGA elements. The closeness of the RGA elements to unity as a measure of interaction is well understood. However, from the definition in equation (2) we see that only <u>individual</u> elements of the RGA are defined and hence a RGA element only provides a measure of interaction in a <u>single</u> loop. No wonder why the RGA rule frequently results in several satisfactory pairing options but fails to distinguish between them. The final choice usually relies on experience. One quantitative method of selecting the final RGA pairing is proposed in Remark 1.

<u>Remark 1</u> A final pairing among alternatives satisfying the RGA rule may be decided by means of

the 'minimum overall difference' of the paired RGA elements and unity, i.e.,

$$\min \sum_{i=1}^{n} |\lambda^{k}_{ij} - 1| , \qquad \forall k \qquad (6.3)$$

where λ^{k}_{ij} represents the paired RGA elements corresponding to the k-th alternative, and n is the number of input/output variables.

However, this approach provides only a working guide and may result in incorrect pairing since the intrinsic character of the overall system is not accounted for. A more fundamental approach is called for.

3. According to the original definition (Bristol, 1966), the magnitude of the RGA provides information about how much a process gain can change on a relative basis when other loops are closed, indicating the ease of subsequent controller design. Consequently, pairings containing large and small RGA elements should be avoided if possible.

6.3 THE NI PAIRING CRITERION

When a system is diagonally paired and a diagonal controller with integral action is designed such that each individual loop, obtained from the multivariable system by opening any n-1 feedback loops, is stable, the NI is defined, using steady state gains as,

$$NI\ [G(0)] = \frac{\det\ [G(0)]}{\det\ [\bar{G}(0)]} \qquad (6.4)$$

where det(A) denotes the determinant of matrix A, and $\bar{G}(0)$ denotes the subsystem of G(0) with all the off-diagonal elements ignored, i.e.,

$$\bar{G}(0) = diag\,[G(0)] \tag{6.5}$$

The sign of the NI is used as a stability check for a given variable pairing scheme. Nevertheless since any good variable pairing rule should be closely coupled with a suitable measure of interaction, one could surmise that the NI contains some information on interaction as well.

The NI is defined as the ratio of the determinant of the gain matrix G(0), to the determinant of the gain matrix for the corresponding diagonal system $\bar{G}(0)$. The value of the NI is thus sensitive to the relationship between these two elements. For an interaction free (diagonal) system NI would be unity. The further away from unity we move the less G(0) resembles its diagonal $\bar{G}(0)$ and the more interactive its behaviour. Clearly if we were able to select a pairing arrangement such that NI=1 we would have a system that behaved as though it were several independent SISO loops operating on a purely diagonal system. Consequently, interaction can be characterized by the size of the NI.

Given that the NI also provides a necessary condition for stability we propose the following Pairing Rule:

Pairing Criterion : The input/output variables in a decentralized control system should be paired such that the corresponding NI is positive and closest to unity. In addition, in order to guarantee integrity, the corresponding RGA elements should also be positive.

The new criterion on variable pairing is referred to as the 'NI Pairing Criterion'. Before proceeding to a theoretical justification by comparing the NI with other techniques, several distinct features of the NI as an IM can be observed:

i) Both the sign and size of the NI are utilized. The former provides a necessary stability condition, and the latter provides an interaction measure. In addition, the NI itself also contains direct information about system integrity upon being applied to reduced subsystems with single loops removed (see later). Hence the NI plays a triple role, and is thus a comprehensive tool for the IM/VP problem.

ii) In contrast to the RGA which highlights individual loop interaction, the NI provides an overall measure of system interaction since it compares the full interactive system to a corresponding diagonal non interactive system. The best pairing with truly minimum interaction can be identified, and thus ambiguities in the RGA avoided.

iii) From an information viewpoint the NI appears to follow the pairing options more realistically. For a nxn system there are n! pairing combinations and also n! unique indexes calculated from the NI. However there are only nxn RGA elements which themselves contain dependencies since RGA rows and columns must sum to unity. As n increases the RGA falls further behind in terms of providing separate information on all possible n! pairings.

iv) The additional statement regarding integrity is discussed later below Remark 6.

v) The new pairing rule is aimed at the pairing which shows the minimum overall interaction, with stability and integrity requirements satisfied. However, a practitioner may take other considerations, such as the easiness of control (the size of RGA helps in this regard, as previously noted), in conjunction with it, into account to make the final decision (see example later).

6.4 THE CONSISTENCY BETWEEN NI AND RGA

6.4.1 Extension of IM to SISO Systems

We know that only multivariable (at least 2x2) systems are meaningful as far as interaction is concerned, and that SISO systems as special cases are obviously interaction-free. An effective IM should reveal the interaction-free nature for SISO systems if applied to them. The following remark extends the NI and RGA to SISO systems and can be easily verified from equations (6.2) and (6.4):

<u>Remark 2</u> For SISO systems

$$NI = RGA = 1$$

Therefore, the NI and RGA share the same starting point and are equivalent indicators for SISO systems.

6.4.2 2X2 Systems

This simplest multivariable system has been extensively studied (Morari and Zafiriou, 1989; Chiu and Arkun, 1991). The following remark allows for a thorough analysis and clarifies some previous results on IM/VP for 2x2 systems.

<u>Remark 3</u> For 2x2 systems, following relationships hold:

$$\lambda_{11} = \lambda_{22} = \frac{1}{NI} \qquad (6.6a)$$

$$\lambda_{12} = \lambda_{21} = \frac{1}{NI'} \qquad (6.6b)$$

$$\frac{1}{NI'} = 1 - \frac{1}{NI} \qquad (6.7)$$

where NI' denotes the NI corresponding to the off-diagonal pairing.

The first relationship has been demonstrated by Chiu and Arkun (1991), and the rest can easily be verified. Based on these results we can observe that:

i). From equations (6.6a) and (6.6b), requiring λ_{11} to be unity implies the same for the NI. Hence the closeness of the NI to unity also measures interaction in this 2x2 case.

ii). Since the NI plays a dual role (stability and interaction measure) as noted before, so does the RGA by virtue of equations (6.6a) and (6.6b) in this case. This means that the sign of λ_{11} also provides a necessary condition for stability. In this sense, the RGA is conclusive (Morari and Zafiriou, 1989), and can be exclusively used to address the IM/VP problem for 2x2 systems. However, this is not extendable to higher order systems. One should be careful to properly interpret the statement by Grosdidier et al. (1985) that the RGA can be generally used as a measure of stability. More accurately, the RGA provides a measure of integrity (see later).

iii). Equation (6.6a) or (6.6b) supports the fact that NI measures the overall interaction for each pairing, either diagonal or otherwise. In fact, only a single NI corresponds to each pairing whereas there are two RGA elements (although equal) associated with it. Moreover, each pairing actually undergoes the same interaction as indicated by two equal RGA elements and thus no ambiguity in pairing would arise for 2x2 systems by the RGA rule.

iv). From equation (6.7) it can also be noted that if NI is negative NI' is positive. This means that for 2x2 systems there always exists a pairing so that the system is stabilizable (Grosdidier, et al., 1985). Equation (6.7) can be used to provide a theoretical basis in this regard.

6.4.3 nxn Systems

Prior to appreciating the general case, two specific systems can be covered with the following remark:

Remark 4 For diagonal and triangular systems we have,

$$NI = \lambda_{ii} \equiv 1 \quad , \qquad \forall i \qquad (6.8)$$

The above remark can be readily verified. It can be seen that both the RGA and NI result in a unique solution (diagonal pairing) for variable pairing in both systems. This is true despite the fact that neither one is capable of detecting the one way interaction present in triangular systems. Thus for this specific case the two IM's coincide.

For general nxn systems, the following remark on the relationship between RGA and NI is fundamental for the analysis of the IM/VP problem:

Remark 5 For a nxn system the following relationship holds,

$$\lambda_{ii} \cdot NI = NI^{(i)} \quad , \qquad \forall i \qquad (6.9)$$

where $NI^{(i)}$ denotes the NI of the reduced subsystem of G(0) with i-th row and i-th column removed.

Equation (6.9) was first deduced from a general relationship based on the Block Relative Gain (BRG) concept by Chiu and Arkun (1990). We will present an independent proof of equation (6.9) based directly on the definitions of RGA and NI in the next chapter. Not surprisingly, applying equation (6.9) to 2x2 systems leads to equation (6.6a) with the aid of remark 2.

Note that the sign of $NI^{(i)}$ constitutes a necessary condition for stability against i-th loop failure. As a result, NI itself also provides a measure of integrity. In practice, however, instead of actually calculating it, the $NI^{(i)}$ can be evaluated in terms of NI and RGA using equation (6.9), and taking advantage of the calculational simplicity of RGA. In this sense, the joint calculation of NI and RGA can be used to address the integrity problem. Furthermore, the sign of RGA in turn provides a measure of integrity since stability (positive NI) is mandatory for any pairing. Consequently, we have:

<u>Remark 6</u> For closed loop stable systems with diagonal pairing, the system possesses integrity against i-th loop failure only if

$$\lambda_{ii} > 0 \quad , \qquad \forall i \qquad (6.10)$$

The proof of the above remark will be given in the next chapter. Remark 6 emphasizes the function of the sign of the RGA as a measure of system integrity against single loop failure, since this is often not directly addressed. Chiu and Arkun (1990) also discussed system integrity against any combination of loop failures. However, system integrity against single loop failure is often of greater practical concern. Hence positive RGA is more appropriately applied to system integrity. In particular, it is easy to see that 2x2 systems always possess integrity since removing one loop leaves a single loop which is assumed stable. This is also directly indicated by remark 3.

6.4.4 Integrity Considerations

We recall that the VP statement was extended to that both the NI and RGA are positive for the resulting system to possess integrity against single loop failure. Should the decision based on the NI closest to unity correspond to any negative RGA element, the pairing with the NI next closest to unity should be then selected etc., until both RGA and NI are positive. Note that in this case the minimum interaction requirement has to be sacrificed for integrity considerations.

6.4.5 Overall vs Individual Interactions

Again focusing on the size of the NI additional insights are possible. We are able to ensure that the NI conforms to the intuitively obvious remark that:

> **Remark 7** In a decentralized control system, the control loop with the smallest interaction will have the smallest effect on the overall system.

The above remark can be interpreted as follows. If the i-th loop shows very small interaction, λ_{ii} will be close to unity. It follows from equation (9) that the NI of the overall system will be close to that of the reduced subsystem with this loop removed. As a result, the characteristics of the reduced system, will also resemble those of the overall system. This implies that the single loop has little effect on the system. The smaller the interaction, the more closely the reduced system will resemble the overall system.

Furthermore, if every loop shows small interaction in a system, i.e., λ_{ii}, $\forall i$, are all close to unity, then the system will behave approximately as several independent SISO loops. In this case upon repeatedly using equation (9), any combination of loop removals will cause little change in the value of any NI corresponding to a subsystem, and eventually all NI's will be close to that of SISO system i.e. unity. As a result, the NI of the overall system will be close to unity, indicating that the overall interaction is also small. This further supports the fact that the NI provides a measure of the overall interaction in a system.

Notice, on the other hand, that the RGA indicates that other loops will impose small impact on a single loop that shows small interaction. As a result, the loop with the minimum interaction is the most isolated one in a multivariable control system.

6.5 THE NI AND THE JACOBI EIGENVALUE CRITERION (JEC)

Mijares et al. (1986) proposed a pairing rule called the 'Jacobi Eigenvalue Criterion' (JEC). This was based on purely algebraic principles in terms of the difficulty caused by the off-diagonal elements in finding the inverse of plant gain matrix G(0). A Jacobi iteration matrix is constructed as

$$A = I - \overline{G}^{-1}(0)\cdot G(0) \qquad (6.11)$$

where G(0) and $\bar{G}(0)$ are the same as before. The pairing criterion is to pair on that element whose corresponding Jacobi iteration matrix A has the smallest spectral radius among all possible pairings (Mijares et al., 1986), i.e.,

$$\min \rho(A) \tag{6.12}$$

where $\rho(A)$ denotes the spectral radius of A.

Note that the JEC also requires diagonal pairing, so input/output variables should be rearranged as in the NI. The relationship between the new NI pairing criterion proposed in this chapter and the JEC can be established as follows.

Manipulating equation (6.11) and taking determinants results in,

$$\det\,(I-A) = \det\,[\bar{G}^{-1}(0)\cdot G(0)] \tag{6.13}$$

The right hand side of equation (6.13) is referred to as the general NI (Grosdidier and Morari, 1986). To be consistent with the previous definition of NI, equation (6.13) can be in turn written as,

$$\det\,(I-A) = \frac{\det\,[G(0)]}{\det\,[\bar{G}(0)]} \equiv NI \tag{6.14}$$

Furthermore, the characteristic polynomial $P(\theta)$ of A can be expressed in terms of the eigenvalues θ_i of A as (Mijares et al., 1986),

$$P(\theta) = \det(\theta I-A) = \prod_{i=1}^{n} (\theta-\theta_i) \tag{6.15}$$

Setting $\theta=1$ and combining with equation (6.14), we obtain from equation (6.15),

$$NI = \prod_{i=1}^{n} (1-\theta_i) \tag{6.16}$$

From equation (6.16) it can be seen that if the spectral radius of A equals zero, all the eigenvalues of A have to be zero and hence NI has to be one. This means that zero spectral radius and unity NI are equivalent and represent a interaction-free pairing, i.e.,

$$\begin{aligned} \rho(A) = 0 \; &\leftrightarrow \; NI = 1 \\ &\leftrightarrow \; Interaction\text{-}free \end{aligned} \tag{6.17}$$

Moreover, from equation (6.16), the smaller the spectral radius of A, the closer NI is to unity. Thus the size of $\rho(A)$ and NI provide an equivalent measure of interaction most of the time, and the best pairing implies a $\rho(A)$ close to zero and an NI close to one. Hence we have,

<u>**Remark 8**</u> The NI pairing criterion and the JEC are equivalent as IM's, i.e.,

$$\min \rho(A) \; \leftrightarrow \; \min |NI-1| \tag{6.18}$$

where $|\bullet|$ denotes absolute value.

However, the spectral radius of the Jacobi iteration matrix and the NI provides different information regarding system stability. Whereas NI>0 is a necessary condition for stability, $\rho(A)$ contains no specific information about stability. Mijares et al. (1986) have shown that if the spectral radius of A is less than one, NI will be positive, i.e., $\rho(A)<1$ is only sufficient for NI>0. As a result, nothing can be concluded about stability when $\rho(A)>1$. Furthermore, from equation (6.16) we can see that the value of the NI represents the overall effects of all eigenvalues of A. Hence, the JEC may fail to measure interaction in the system and thus lead to incorrect final pairing, since the distribution of individual eigenvalues is not considered in JEC. In addition, in contrast to the NI and RGA, $\rho(A)$ contains no information about system integrity. This will be further illustrated by examples below. Thus the NI appears to be a more comprehensive measure of interaction than the JEC.

Finally, it is rather tedious to calculate the spectral radius of the Jacobi iteration matrix for all n! possible pairings (they have to be reorganized as diagonal pairings prior to the calculation), especially for large systems. It is also difficult to simplify the procedure, whereas the calculation for the NI can be simplified as discussed in the next section.

6.6 COMPUTATIONAL CONVENIENCE FOR THE NI

Although the NI is a powerful tool for achieving the best pairing in decentralized control systems, the calculational complexity (there are n! NI's for an nxn system and each pairing needs to be rearranged to produce a diagonal pairing) is discouraging. However this difficulty can be largely overcome by appropriate simplification of the calculation procedure.

It is known that any non-diagonal pairing needs to be rearranged as a diagonal first in order to calculate the NI, and that interchanging rows or columns of a matrix only affects the signs of the determinants. This property can be utilized to simplify the calculation of each NI, leading to the following remark:

<u>**Remark 9**</u> The NI's corresponding to all possible n! pairings can be determined by arranging the calculations in factorial order as follows,

$$NI^k = \frac{\pm \det [G(0)]}{\prod_{i=1}^{n} g^k_{ij}} , \qquad \forall\, k \tag{6.19}$$

where NI^k denotes the NI corresponding to the k-th pairing (k= 1 ~ n!), and g^k_{ij} represents the ij elements of G(0) corresponding to the k-th pairing, which would normally be rearranged to lie on the diagonal.

Equation (6.19) shows the calculation approach of finding NI^k for a particular k-th ordering of the inputs and outputs arranged factorially. Reordering in inputs/outputs to be along the diagonal simply changes the sign of det [G(0)]. The bottom product is equal to det [$\bar{G}$(0)] i.e. the diagonal matrix. However, rather than perform the actual rearrangement the appropriate g_{ij} elements that would be on the diagonal are selected and their product calculated as shown in equation (19).

As a result, the calculation of the NI can be facilitated. Nevertheless, it still remains computationally intensive.

In practice, a faster method is to use the RGA as an initial screening device, taking advantage of its calculational simplicity. This would quickly eliminate most of the pairing options. The NI stability requirement would further differentiate on the basis of stability. The final step would be the application of NI as measure of interaction (i.e., its size) to decide on the final pairing. For high order systems with many pairing alternatives one may still have to apply equation (19) in order to be sure of selecting the globally optimal pairing.

6.7 EXAMPLES

Examples are selected to demonstrate the effectiveness of the new 'NI pairing criterion' in comparison with the improved RGA rule (McAvoy, 1983) and the Jacobi Eigenvalue Criterion (JEC) (Mijares, et al., 1986).

6.7.1 Example 1 (Mijares, et al., 1986)

Example 1 illustrates the fact that NI suggests the right pairing even when the RGA fails to clearly identify it, and that the NI is more conclusive than the JEC.

The plant gain matrix and the RGA are shown below,

$$RGA = \begin{bmatrix} 0.53 & 0.59 & -0.12 \\ 0.43 & 1.59 & -1.02 \\ 0.04 & -1.18 & 2.14 \end{bmatrix}$$

The NI's and the eigenvalues of Jacobi iteration matrix

$$G(0) = \begin{bmatrix} 1.0 & 1.0 & -0.1 \\ 1.0 & -3.0 & 1.0 \\ 0.1 & 2.0 & -1.0 \end{bmatrix}$$

for each pairing are listed in table 1.

According to the improved RGA rule, pairing 1 and 3 are the two acceptable alternatives. Among these two pairings, 3 would be preferred since the differences between the relative gains and unity are minimized. This can be easily seen by applying the quantitative method suggested by remark 1 as follows.

The overall difference in pairing 1

$$= 0.47 + 0.59 + 1.14 = 2.20$$

The overall difference in pairing 3

$$= 0.41 + 0.57 + 1.14 = 2.12$$

Here pairing 3 shows the minimum difference. On the other hand, pairing 3 contains a relative gain of 0.43 which is less than 0.5, and this, according to Mijares (Mijares, et al., 1986) should be avoided. Hence some ambiguity arises here since the RGA is unable to clearly choose between the two options. However, from the NI pairing criterion, pairing 1 should be chosen as the best one since this corresponds to the NI closest to one. The JEC also recommends this pairing as shown by the original authors, and thus coincides with the NI pairing criterion.

6.7.2 Example 2

This example illustrates that the NI suggests a unique pairing with the minimum interaction, whereas direct use of the RGA provides no clear choice and the JEC fails to specify a suitable pairing.

The plant gain matrix and the RGA are as follows:

$$G(0) = \begin{bmatrix} 1.5 & -1.2 & -1.5 \\ 1.5 & -3.0 & 1.4 \\ -1.8 & 2.2 & -1.1 \end{bmatrix}$$

$$RGA = \begin{bmatrix} 0.073 & 0.23 & 0.70 \\ -1.53 & 2.88 & -0.35 \\ 2.46 & -2.12 & 0.66 \end{bmatrix}$$

The NI's and the eigenvalues of the Jacobi matrix for each pairing are shown in table 2.

By the RGA rule, both pairing 1 and 6 are acceptable, since all RGA elements and NIs for both pairings are positive. However, the original RGA rule again fails to differentiate between these two choices.

According to the NI pairing rule, pairing 1 should be selected in terms of minimum interaction requirement. Under pairing 1 both stability and integrity requirements are satisfied, in addition to satisfying minimum overall interaction, since NI and $NI^{(i)}$ or NI and RGA elements are all positive. However, using the JEC, pairing 4 would be chosen as the best pairing. Here, JEC fails to identify the pairing with minimum interaction. Moreover, as previously noted in the Chapter, the JEC offers no definite information about either stability or integrity. In this example, pairing 4 contains a negative RGA element, indicating that the system under this pairing will have no integrity. This further discredits the choice by JEC.

6.7.3 Discussion

It should be pointed out that the NI pairing criterion presented in this chapter is aimed purely at picking up the pairing showing minimum overall interaction. In practice, one may need to take other considerations into account. In particular, the final pairing should result in all RGA elements closest to unity. A pairing containing significantly large or small RGA elements should be avoided, since they may cause control problems. Specifically, a pairing containing large RGA elements will be very sensitive to model uncertainties (see Chapter 10).

On the other hand, a pairing containing very small RGA elements may result in a control system difficult to design and sensitive to loop failure. By traditional interpretation of the RGA, the size of the RGA indicates how much a process gain in one loop can change due to interaction (when other loops are closed), as pointed out by one of the examiners of this thesis. Hence, a pairing containing very small RGA elements implies that the process gain in a multivariable control environment is substantially different from that of the original independent SISO loop. As a result, controllers by independent design has to be largely tuned or even redesigned upon closing control loops in order to achieve satisfactory performance, making controller tuning very difficult. In addition, the closed loop process gain will largely depend on other control loops, indicating that the resulting system will be sensitive to the failure of other loops.

Therefore, using other considerations, one would lean towards pairing 6 in example 2, since pair 1 contains a very small RGA element, as pointed out by one of the examiners of this thesis. Another similar example is given by the examiner with the process gain matrix as below,

$$\begin{bmatrix} 1 & 0.4275 & 0 \\ 2 & 0.95 & -2 \\ 0 & 0.45 & 1.0526 \end{bmatrix}$$

The RGA is given by

$$\begin{bmatrix} 1.9 & -0.9 & 0 \\ -0.9 & 1.0 & 0.9 \\ 0 & 0.9 & 0.1 \end{bmatrix}$$

Two pairings -- (1-1, 2-2, 3-3) and (1-1, 2-3, 3-2) give positive NIs (0.99 and 1.11 respectively). By the NI pairing rule the first pairing should be chosen since it shows the least overall interaction. However, both NIs are very close to each other here, i.e., the amount of interaction resulting from both parings are essentially the same, but the first pairing involves a small 3-3 RGA element of 0.1 while the second one gives 2-3 and 3-2 RGA elements close to 1.0 (0.9).

Hence, the second pairing is more favourable.

6.8 CONCLUSIONS

The RGA has played a dominant role in solving steady state interaction and variable pairing problems for multiloop control systems. It's well understood that, in addition to its steady state limitations, other limitations have led to the use of Neiderlinski index (NI) as a supplement.

However, little attention has been focused on the NI as a measure of steady state interaction in its own right. We have shown that the NI can be used as a useful tool for both variable pairing decisions as well as for its stability and integrity measures. A new pairing criterion was proposed and shown to be consistent for simple problems with other criteria, but also able to distinguish among ambiguous pairing recommendations that sometimes result from using other criteria such as the RGA and JEC. In particular, the size of the RGA elements measures interactions in single loops, while their sign deals with system integrity as opposed to overall system stability. The Jacob Eigenvalue Criterion offers no definite information on either stability or integrity. By contrast the NI encompasses stability, integrity and interaction and provides a broader pairing criterion. Some theoretical relationships between these criteria are developed. Nevertheless, the proposed VP criterion is only a steady state one and thus, like other steady state criteria, has its limitations when considering dynamic behavior. In addition, the presented criterion is aimed only at achieving the minimum interaction. In practice, other considerations need to be considered in conjunction with the minimum interaction requirement, as shown in example 2.

Table 1. Example 1: NI's and Jacobi eigenvalues

No.	Pairings	NI's	$\{\theta_A\}$
1	(1,1)-(2,2)-(3,3)	0.62	-0.1, -0.53, 0.63
2	(1,1)-(2,3)-(3,2)	-0.93	-0.37, -0.98, 1.34
3	(1,2)-(2,1)-(3,3)	1.87	0.62, -0.31 ± 1.78i
4	(1,2)-(2,3)-(3,1)	18.7	-4.13, 2.02 ± 1.58i
5	(1,3)-(2,1)-(3,2)	-9.35	1.83, -0.91 ± 2.77i
6	(1,3)-(2,2)-(3,1)	-62.3	0.34, 9.65, -10.0

Table 2. Example 2: NI's and Jacobi eigenvalues

No.	Pairings	NI's	$\{\theta_A\}$
1	(1,1)-(2,2)-(3,3)	0.91	-0.3±0.75i, 0.59
2	(1,1)-(2,3)-(3,2)	-0.98	-0.65±0.78i, 1.30
3	(1,2)-(2,1)-(3,3)	-2.28	-0.8±0.66i, 1.62
4	(1,2)-(2,3)-(3,1)	1.50	±0.7i, 0
5	(1,3)-(2,1)-(3,2)	-0.91	-0.63±0.94i, 1.26
6	(1,3)-(2,2)-(3,1)	0.56	-0.4±0.92i, 0.80

CHAPTER 7

RGA AS A MEASURE OF INTEGRITY

FOR DECENTRALIZED CONTROL SYSTEMS

Synopsis *The relationship between the RGA (Relative Gain Array) and the NI (Niederlinski's Index) is established directly based on their definitions. The usefulness of the sign of RGA in addressing the integrity problem for decentralized systems is emphasized.*

7.1 INTRODUCTION

Decentralized control is common in process industry because full scale multivariable control often requires more cost and complexity of design. The first step in the design of such control systems is the variable pairing problem. The Relative Gain Array (RGA) (Bristol, 1966) plays a particularly important role in the selection of control structure. Introduced empirically as a measurement of interactions between control loops, the RGA has also proved to have a sound theoretical justification and to be much more than a simple measure of interaction (Grosdidier et. al., 1985). In particular, the linkage between the RGA and system stability is established by Grosdidier et al. (1985) and later Morari and Zafiriou (1989) in a general framework. However, this does not explicitly offer implications for variable pairing.

On the other hand, the Niederlinski Index (NI) (Niederlinski, 1971) provides a necessary condition for system stability (Grosdidier, et. al., 1985), and it constitutes a parallel but powerful tool in scanning alternatives for variable pairing. Consequently, the joint use of both RGA and NI as a pairing criteria has been widely advocated and found common acceptance (McAvoy, 1983; Yu and Luyben, 1986). Recently, Chiu and Arkun (1990) established a relationship between the RGA and the NI, and they found that the RGA and NI together exclusively define both stability and integrety.

This chapter presents an independent proof of the relationship between the RGA and the NI which is the special case for multiloop control system in Chiu and Arkun's (1990) paper. In contrast to Chiu and Arkun (1990), the derivation is directly based on the definitions of the RGA and the NI, and the intimate correlation between the RGA and the system integrity is emphasized. Some of our own interpretations on the variable pairing problem is also presented.

7.2 RGA, NI AND THEIR ROLES

Let G(s) denote the transfer function matrix of n inputs and n outputs plants (square systems), and $\bar{G}(s)$ denote the subsystem of G(s) containing the diagonal elements only

$$\bar{G}(s) = \mathrm{diag}[G(s)]$$

G(s) and $\bar{G}(s)$ are assumed stable. A diagonal controller C(s) containing integral action is to be used to control the plant (decentralized control), and G(s)C(s) is assumed rational and proper.

For a given G(s), the RGA is defined using only steady state plant gains as

$$RGA[G(0)] = [\lambda_{ij}] \tag{7.1}$$

and

$$\lambda_{ij} = [G(0)]_{ij} \cdot [G^{-1}(0)]_{ji} \tag{7.2}$$

where $[G(0)]_{ij}$ denotes the (i, j)th element of the gain matrix G(0).

When a diagonal controller C(s) with integral action is applied to the plant with diagonal pairing, and the controller is designed such that all one-variable control loops, obtained from the multivariable system by opening any n-1 feedback loops, are stable, i.e., the closed loop system

$$H(s)=\bar{G}(s)C(s)[I+\bar{G}(s)C(s)]^{-1}$$

is stable, and thus each individually independent loop

$$h_i=\frac{g_{ii}(s)c_i(s)}{1+g_{ii}(s)c_i(s)}$$

is stable, the NI is defined, also using steady state gains only, as

$$NI[G(0)]=\frac{\det[G(0)]}{\det[\bar{G}(0)]} \tag{7.3}$$

Both RGA and NI offer important insight into the issue of control structure selection. However, as individually inconclusive criteria, the RGA and NI play different roles in control structure scanning. RGA is mainly a interaction measure while the NI is essentially a stability index.

Another important problem in variable pairing is the system ability to maintain stability under loop failure, i.e., the system integrity problem. The establishment of the relationship between RGA and NI by Chiu and Arkun (1990) reveals that the signs of RGA elements are directly related to the integrity problem. The derivation of their relationship is based on the properties of the Block Relative Gain array (BRG). The following presents an independent proof of the relationship.

7.3 RELATIONSHIP BETWEEN RGA AND NI

Theorem 1

$$\lambda_{ii}[G(0)]\cdot NI[G(0)] = NI[G^{(i)}(0)] \qquad \forall i=1-n \qquad (7.4)$$

where $G^{(i)}(0)$ denotes the reduced plant matrix of G(0) with the i-th row and i-th column removed.

Proof.

The diagonal elements of RGA can be calculated from equation (7.2),

$$\lambda_{ii} = [G(0)]_{ii} \cdot [G^{-1}(0)]_{ii}, \qquad \forall\ i=1-n \qquad (7.5)$$

The inverse of G(0) can be expressed as

$$G^{-1}(0) = \frac{G^{*}(0)}{\det[G(0)]} \qquad (7.6)$$

Where

$$G^{*}(0) = [\alpha_{ij}] \qquad (7.7)$$

is the adjoint matrix of G(0), and

$$\alpha_{ij} = (-1)^{i+j}\det[G^{(ji)}(0)] \qquad (7.8)$$

is the i,j-cofactor of G(0), and $G^{(ji)}(0)$ represents the reduced matrix of G(0) with j-th row and i-th column removed.

Considering the i-th diagonal element of $G^{-1}(0)$, we have from equations (7.6), (7.7) and (7.8),

$$[G^{-1}(0)]_{ii} = \frac{\det[G^{(i)}(0)]}{\det[G(0)]} \qquad (7.9)$$

Thus from equation (7.5) we obtain

$$\lambda_{ii} = [G(0)]_{ii} \cdot \frac{\det[G^{(i)}(0)]}{\det[G(0)]} \qquad (7.10)$$

Manipulating the above equation and noticing the following relationships,

$$NI[G^{(i)}(0)] = \frac{\det[G^{(i)}(0)]}{\det[\bar{G}^{(i)}(0)]} \tag{7.11}$$

and

$$\det[\bar{G}(0)] = [G(0)]_{ii} \cdot \det[\bar{G}^{(i)}(0)] \tag{7.12}$$

where $\bar{G}^{(i)}(0)$ denotes the reduced matrix of $\bar{G}(0)$ with i-th row and column removed, we have

$$\lambda_{ii} = \frac{\det[\bar{G}^{(i)}(0)] \cdot [G(0)]_{ii}}{\det[G(0)]} \cdot \frac{\det[G^{(i)}(0)]}{\det[\bar{G}^{(i)}(0)} \tag{7.13}$$
$$= NI^{-1}[G(0)] \cdot NI[G^{(i)}(0)]$$

Rearranging equation (7.13) gives the relationship in theorem 1,

$$NI[G(0)] \cdot \lambda_{ii} = NI[G^{(i)}(0)] \tag{7.14}$$

QED.

In light of the fact that the use of NI requires diagonal pairing, only diagonal elements of RGA are considered. However, any off-diagonal pairing can be rewritten as diagonal pairing by interchanging the positions of input and output variables.

Theorem 1 is the special case for the multiloop system in Chiu and Arkun's (1990) paper. However, the above proof provides a more intuitive approach using the definitions of the RGA and NI directly and thus enhances the understanding of the relationship between the NI and RGA.

7.4 RGA AS A MEASURE OF SYSTEM INTEGRITY

Notice that in theorem 1 $NI[G^{(i)}(0)]$ measures system integrity against i-th loop failure from the definition of NI, and that only the sign of NI is meaningful. Thus the sign of the RGA element itself is also a direct measure of system integrity against single loop failure. We would like to emphasize the importance of the sign of RGA here because this is often not directly addressed.

Corollary 1. *For closed loop stable systems with diagonal pairing, the system possesses integrity against i-th loop failure only if*

$$\lambda_{ii}[G(0)] > 0, \qquad \forall\, i=1\sim n \tag{7.15}$$

Proof.

We know that a closed loop system with diagonal pairing is stable only if

$$NI[G(0)] > 0 \tag{7.16}$$

and that the system possesses integrity against i-th loop failure only if

$$NI[G^{(i)}(0)] > 0, \qquad \forall\, i=1\sim n \tag{7.17}$$

From theorem 1, the necessary condition for integrity against the i-th loop failure can be given by the sign of RGA as

$$\lambda_{ii}[G(0)] > 0, \qquad \forall i=1\sim n \tag{7.18}$$

Since NI provides only a necessary condition for stability, the sign of the RGA element also presents only a necessary condition for system integrity.

A direct consequence of corollary 1 is the requirement of both positive RGA and NI in variable pairing, for systems to possess both stability and integrity against single loop failure.

By combining the RGA and NI, the following observations can be directly inferred from theorem 1:

(1). If $\lambda_{ii} > 0$, and NI > 0, thus $NI[G^{ii}(0)] > 0$, then the closed loop system is integral stabilizable (Grosdidier et al, 1985) and system possesses integrity against single loop failure. This is inferred from corollary 1.

(2). If $\lambda_{ii} > 0$, but NI < 0, hence $NI[G^{ii}(0)] < 0$, then neither the closed loop system nor the reduced system (without the loop associated with the negative RGA element) is stable. The system thus lacks both stability and integrity against single loop failure. This shows that an RGA analysis is not sufficient in pairing variables. NI > 0 is the primary requirement in configuring control structures since system stability is the most important.

(3). If $\lambda_{ii} < 0$, but NI > 0, hence $NI[G^{ii}(0)] < 0$, then the closed loop system is integral stabilizable, but the system lacks integrity. This implies that we might pair variables with negative RGA element if we have to, but system integrity has to be sacrificed in this situation.

(4). If $\lambda_{ii} < 0$, and NI < 0, hence $NI[G^{ii}(0)] > 0$, then the system is closed loop unstable but will be stable if the i-th loop is opened, i.e., system possesses integrity against i-th loop failure. However, system integrity can not override stability since stability is always the most important requirement and NI must be positive.

7.5 FINAL REMARKS

The relationship between two important decentralized indexes, the RGA and the NI is established directly based on their definitions. The roles of the RGA and NI in control structure selection for decentralized control systems are elucidated. It has been shown that the sign of the RGA is a measure of system integrity while the size of it provides a measure of interaction, and the sign of NI gives a necessary condition for system stability.

CHAPTER 8

DYNAMIC INTERACTION ANALYSIS IN DECENTRALIZED CONTROL SYSTEMS

Synopsis *Based on a rigorous formulation and analysis, a decentralized control system is structurally decomposed into separate SISO control loops with interaction preserved and explicitly taken into account. Subsequently, both the absolute and relative interactions are defined and measured with structural significance on the basis of individual loops. As a result, the effects of interaction on a closed loop system can be evaluated in a transparent way. In particular, one is able to tell how much the performance of one output will deviate from its corresponding independent SISO counterpart with which the controller is usually designed. This in turn reveals whether interaction is likely to cause control problems and how other outputs will respond to a setpoint change in one specific loop. This analysis leads to important implications for controller tuning and variable pairing.*

8.1 INTRODUCTION

Multivariable control systems are generally much more difficult to control than single-input/single-output systems due to inherent interactions among control loops. Full scale control techniques, such as state space based modern control strategies, and more recently, model predictive control and robust control schemes, have found increasingly successful applications in the process industry (Morari and Zafiriou, 1989; Garcia and Prett, 1989; Eaton and Rawlings, 1992). Nevertheless, decentralized control still remains a strong, and often predominant, alternative due to its many practical advantages. Apparently, interaction measurement remains a the key issue in the design of decentralized control systems.

Over the last decade, interaction measurement has drawn substantial attention. This is evidenced by the tremendous number of papers published in the literature on this topic for both steady state (Bristol, 1966; Bruns and Smith, 1982; Mijares et al., 1986; Johnson, 1990) and dynamic systems (Rijnsdorp, 1965; Witcher and McAvoy, 1977; Bristol, 1978; Tung and Edgar, 1978 and 1981; Gagnepain and Seborg, 1982; Bequette and Edgar, 1988; Jensen et al., 1986; Balchen and Mumme, 1988). Nevertheless, with the lack of a unified definition of interaction itself (MacFarlane, 1972; Jensen, et al., 1986), the above interaction measures have had minimal impact on the analysis and design of multivariable control systems.

Jensen et al. (1986) presented a detailed analysis of interaction based on the closed loop responses of multivariable control systems in the frequency domain. However, their interaction measure does not provide explicit interpretation about how much control performance will deteriorate from the otherwise interaction-free SISO subsystems. It also fails to offer clear guidance for controller tuning and variable pairing in decentralized control systems. Tung and Edgar (1978; 1981), and Bequette and Edgar (1988) proposed dynamic interaction measures in both the frequency and time domains. Nevertheless, their methods only provide an approximate measure of interaction at low frequencies. Witcher and McAvoy (1977), Bristol (1978), and Gagnepain and Seborg (1982) have also developed dynamic interaction measures, but in a rather qualitative way. O'Reilly and Leithead (1991) proposed a design method by expressing a multivariable control system as equivalent individual channels. However, only 2x2 systems are considered, and implications for interaction measurement and independent design are not available.

In this paper, based on a structurally meaningful definition of interaction in a general dynamic framework, an interaction measure, capable of measuring both the absolute and relative interactions in multivariable control systems, is presented. The dynamic interaction measure allows for a detailed visual analysis of interactions and their effects on closed loop systems in the frequency domain, upon decomposing a multivariable control system into equivalent SISO loops with interaction preserved. More importantly, the impact of controller tuning and variable pairing on interaction and control system performance can be evaluated by means of the interaction measure. In addition, based on the interaction measure, a design approach is promising and a unified dynamic and steady state interaction measure may be developed. This will be topics of separate articles.

8.2 DYNAMIC INTERACTION ANALYSIS

8.2.1 General Considerations

An nxn multivariable process can be generally described by a network (similar to neural networks but with complex transfer functions as weighting coefficients) as shown in Figure 8.1. Upon closing control loops with a given controller structure, the resulting control system may become a rather complex network compounded by interactions among control loops. The relationships among any variables of interest can be established by investigating various transmittances by means of frequency domain, state space, or signal flow graph approaches, and, subsequently, interaction can be defined and extracted, but in different ways (Jensen, et al., 1986; Tung and Edgar, 1977; 1981; Bequette and Edgar, 1988; Balchen and Mumme, 1988). Nevertheless, a unified and generally acceptable definition is still lacking. Important issues, such as the effects of interaction on closed loop performance, the measure of the severity of interaction, and the impact of variable pairing and controller tuning on interaction and subsequently on closed loop performance, are far from being resolved.

The following aspects are important in the development of a measure of interaction:

1. Interaction must be defined on the basis of the dynamic framework, since a rigorous definition of interaction must be valid at more than just steady state. This is clear by noticing that there is no interaction at steady state if integral action is used and the system is stable.

2. Interaction must be defined in terms of individual control loops in relation to corresponding interaction-free subsystems, so that direct implications for controller tuning and variable pairing can be better explored.

3. It is desirable that an interaction measure have structural significance, in addition to theoretical justifications. This will ensure acceptance in industry.

4. With reasonable assumptions, the resulting dynamic interaction measure should be readily extendible to steady state case. This will provide a natural linkage between dynamic and steady state interaction measures.

8.2.2 Dynamics in Control Loops

Let us examine the transmittances in a control loop with the manipulated variable $u_j(s) \in \{u_1(s),\ldots,u_n(s)\}$ paired with the controlled variable $y_i(s) \in \{y_1(s),\ldots,y_n(s)\}$. When the controller, $c_j(s) \in R^n$, has to take action in response to setpoint changes and/or disturbances in the control loop, it affects the overall system in two ways: (1) it attempts to bring the output ($y_i(s)$) to its target (setpoint) with desirable dynamic performance, and (2) it causes perturbations in all the other loops in the system, forcing them to respond as well, which then further influence the original loop. This situation can be described by Figure 8.2 (Balchen and Mumme, 1988).

From Figure 8.1 we find that system responses are governed by the following equations,

$$y_m(s) = \sum_{l=1}^{n} g_{ml}(s)\, u_l(s) \;,\; \forall m$$
$$u_l(s) = c_l(s)\,[r_l(s) - y_l(s)] \tag{8.1}$$

Note that in the above equation u_l is assumed to be paired with y_l, however without loss of generality. Focusing on the u_j-y_i loop and assuming that only this loop undergoes setpoint changes, we thus have,

$$y_i(s) = g_{ij}(s)\, u_j(s) - \sum_{l=1,\, l\neq j}^{n} g_{il}(s)\, c_l(s)\, y_l(s) \tag{8.2}$$

Successively using equation 8.2 to express all the outputs in terms of $u_j(s)$, we find the transmittance between the control action $u_j(s)$ and its own output to be,

$$y_i(s) = g_{ij}(s)\, u_j(s) + a_{ij}(s)\, u_j(s) \tag{8.3}$$

and the perturbations caused by u_j to other loops as

$$y_k(s) = d_{kj}(s)\, u_j(s) \;,\; \forall k \tag{8.4}$$

where

$$a_{ij}(s) = a(c_l(s), g_{ml}(s)), \forall l \neq j, m \neq i \tag{8.5}$$

and

$$d_{ij}(s) = d(c_l(s), g_{ml}(s)), \forall l \neq j, m \neq i \tag{8.6}$$

The detailed expression for $a_{ij}(s)$ and $d_{ij}(s)$ can be readily obtained using the signal flow graph technique (Ogata, 1988).

8.2.3 Definition and Implications for Interaction

Note that $a_{ij}(s)$ in equation 8.3 represents the additional dynamics in the u_j-y_i loop resulting from other control loops, and that equation 8.3 separates it from the interaction-free transfer function of the loop. Therefore, $a_{ij}(s)$ can be defined as the absolute interaction.

A multivariable control system composed of individual loops with hidden structure, as shown in Figure 8.2, can be further decomposed into separate SISO loops with interactions explicitly taken into account. Figure 8.3 shows the physical structure of a particular loop. Further analysis can be performed based on these individual loops, making the analysis and design of multivariable control systems easier.

It must be emphasized that the individual SISO loops are structurally equivalent to the original multivariable system with no assumption made. Interactions among control loops are retained in individual loops and no structural information is lost. Further more, interaction in one control loop is expressed as an additive transmission in addition to the independent SISO process. Hence, interaction analysis can be performed in line with the independent design method (Seborg, et al., 1989).

The structural property in Figure 8.3 reveals the intrinsic connection of signal flow in a multivariable control systems. For instance, the perturbations caused by the control action become the sources of the interaction in the loop. The level of inter-loop connections is reduced by one. In particular, a 2x2 system can be treated as a SISO system with one level inter-connection.

The absolute interaction allows one to obtain an explicit mathematical expression of the interaction term for each loop and to determine to what degree interaction will affect control performance.

Moreover, a <u>relative interaction</u> on the basis of the interaction-free term, which is often more useful, can be directly defined. For instance, a relative interaction in the $u_j - y_i$ loop can be defined as the ratio of the magnitude of the interaction-free transmittance and that of the absolute interaction as,

$$\frac{|a_{ij}(s)|}{|g_{ij}(s)|} \qquad (8.7)$$

As a result, the severity of interaction in a control loop can be explored. Subsequently, one is able to gauge how much control performance of the loop will deviate in the face of interaction, in comparison to its corresponding independent SISO subsystem, which is frequently the design basis by independent design (Shinskey, 1988; Seborg, et al., 1989).

Furthermore, the nature of interaction can be evaluated. For instance, one is able to determine whether the interaction will likely cause control problems, and, more importantly, to predict if the resulting control system will be unstable by independent design. This can be analyzed by comparing both the magnitude and phase responses of the interaction-free term and the interaction. Specifically, a relative interaction greater than unity indicates that interaction plays a dominant role in the loop, and hence difficulties may arise in control. Furthermore, in case of dominant interaction, the control loop and subsequently the overall system will become unstable when the interaction acts in a different direction from the interaction-free term. Consequently, implications for variable pairing and controller tuning can be developed.

Notice that the perturbation terms represent an integrated part of the overall system and hence play an important role in dynamic analysis. However, they will disappear at steady state. The perturbation terms indicate how much control action in one loop would affect other loops.

With the above measure of absolute/relative interaction, a unified dynamic and steady state interaction measure can be developed. One can then develop a theoretical justification for steady state interaction measures such as the RGA (Grosdidier et al., 1985; Yu and Luyben, 1987; Chiu and Arkun, 1990) and also evaluate some implications and limitations of steady state measures. This final point will be further elaborated in a separate paper.

Jensen et al. (1986) analyzed the interaction problem in terms of the external input/output response of the closed-loop system. However, their definition and measure does not allow for a natural separation of the interaction from the corresponding independent SISO loop. Thus a well accepted measure of relative interaction is not available. Based on dynamic analysis in frequency and time domains, Tung and Edgar (1978; 1981), and Bequette and Edgar (1988) proposed interaction measures which reduce to the RGA at steady state. They also demonstrated some useful implications of their measures for variable pairing choices. Nevertheless, their analysis is less general, since they assumed a step change in the control action in response to a step change in the setpoint. In practice, control action has slow dynamics. As a result, their approach does not provide a measure of dynamic interaction at high frequencies. Witcher and McAvoy (1977), Bristol (1978), and Gagnepain and Seborg (1982) also proposed dynamic interaction measure however in a less rigorous fashion.

8.2.4 Interaction Analysis for 2X2 Systems

The largest subset of multivariable control problems is essentially 2-input 2-output (O'Reilly and Leithead, 1991). Apparently, a quantitative analysis of 2x2 systems using the principles previously developed is of great practical importance. For 2x2 systems, the mathematical expressions of the absolute interaction and the perturbations defined in equations (8.5) and (8.6) can be easily obtained. Consider a general diagonally paired 2x2 system as shown in Figure 8.4. The overall control action and the perturbation transmittances in loop u_1 - y_1 can be shown to be,

$$\frac{y_1(s)}{u_1(s)} = g_{11}(s) - \frac{c_2(s)\, g_{12}(s)\, g_{21}(s)}{1 + c_2(s)\, g_{22}(s)} \tag{8.8}$$

and

$$\frac{y_2(s)}{u_1(s)} \equiv d_{21}(s) = \frac{g_{21}(s)}{1 + c_2(s)\, g_{22}(s)} \tag{8.9}$$

respectively.

The absolute interaction in loop 1 can be found from equation (8.8) as,

$$a_{11}(s) = -\frac{c_2(s)\, g_{12}(s)\, g_{21}(s)}{1 + c_2(s)\, g_{22}(s)} \tag{8.10}$$

Outputs in response to changes in u_1 is shown in Figure 8.5 with interaction, interaction-free, and induced perturbation structurally separated out. From Figure 8.5 it can be seen that the perturbation from the first loop to the second loop is the only source of interaction in the original loop (first loop). Specifically, the perturbation from loop 1 to loop 2 is amplified by the cross transfer function of the process and controller 2 and then fed back to loop 1 as interaction, i.e.,

$$a_{11}(s) = -c_2(s)\, g_{21}(s)\, d_{21}(s) \qquad (8.11)$$

Also, the 2x2 system is decomposed into two separate SISO subsystems with interaction shown explicitly. The interaction in loop 1 depends only on controller 2. This allows one to analyze the effects of tuning controller 2 on loop 1. As a result, interaction in say loop 1, can be adjusted by properly tuning the other controller.

Moreover, the severity of interaction in one loop can be analyzed by means of relative interaction according to equation (8.7). This would offer immediate implications for variable pairing, noticing that interaction in loop 1 is independent of g_{11}.

A similar analysis shows that for loop 2

$$a_{22}(s) = -\frac{c_1(s)\, g_{12}(s)\, g_{21}(s)}{1 + c_1(s)\, g_{11}(s)} \qquad (8.12)$$

and the perturbation caused by controller 2 to loop 1 is

$$d_{12}(s) = \frac{g_{12}(s)}{1 + c_1(s)\, g_{11}(s)} \qquad (8.13)$$

Interaction and its effects on the closed loop system can be analyzed by investigating the frequency responses of the interaction-free, absolute interaction , and the perturbation terms. In particular, the relative magnitude of interaction and perturbation to the interaction-free term within the frequency range important for process control is of primary concern. Furthermore, the impact of controller tuning and variable pairing on the amount of interaction and control performance can be examined as well by means of the above frequency plots.

8.3 EXAMPLES

8.3.1 Example 1: Interaction Analysis

Consider a 2x2 system with the following transfer function matrix,

$$G(s) = \begin{bmatrix} \frac{1.0}{10s+} & \frac{0.6}{10s+1} \\ \frac{-0.6}{s+1} & \frac{1.0}{10s+1} \end{bmatrix} \tag{8.14}$$

The independent SISO subsystems for both control loops have the same response to a step setpoint change with the same controller. Figure 8.6 shows the response with a PI controller (k_c=4.0, T_i=2).

The magnitude of the interaction-free transfer function, absolute interaction, and the perturbation as a function of frequency for loop 1 and loop 2 is plotted in Figure 8.7 and Figure 8.8 respectively. From Figures 8.7 and 8.8, we can see that both loops exhibit the same amount of interaction and that the interaction does not play a major role in both loops since its magnitude is mostly less than that of the independent SISO transmittance. Hence, we can infer that the response of y_2 to setpoint change in y_1 is exactly the same as that of y_1 to setpoint change in y_2, and that upon closing both loops the response of one output (y_1 or y_2) in the face of setpoint changes in this loop will not deviate significantly from its independent SISO subsystem. However, the perturbation caused by one control action on the other loop is quite different for this system. Figures 8.7 indicates that control action in loop 1 in response to setpoint changes in the loop will cause a larger perturbation in y_2 than its influence on y_1, at high frequencies. On the other hand, Figure 8.8 shows that control action in loop 2 will cause a small perturbation in y_1. Indeed, responses shown in Figures 8.9 and 8.10 clearly demonstrate these observations.

8.3.2 Example 2: Effects of Controller Tuning

Example 2 is the continuation of example 1 and is intended to demonstrate that the frequency plots of the interaction-free term, the absolute interaction, and the perturbation can be used to evaluate the effects of controller tuning on the closed loop performance. Assume that both outputs are equally important in example 1. We would like to find controller settings resulting in appropriately balanced performances between y_1 and y_2 in the face of setpoint (or disturbance) changes in one loop. For loop 1, the perturbation caused in loop 2 needs to be reduced while the interaction can still remain quite low. This can be accomplished by tuning controller 2. Figure 8.11 shows the frequency response of the above interaction terms in loop 1 with controller 2 tightened (k_{c2}=12.0, T_{I2}=2). From Figure 8.11 we can see that interaction in loop 1 is slightly larger while the perturbation to loop 2 is largely reduced, particularly within the frequency range important for control (0.01 to 1 1/s). The responses of y_1 and y_2 with setpoint change in y_1 are given in Figure 8.12 which coincides with previous predictions.

8.3.3 Example 3: Effects of variable pairing

This example shows that interaction measure can indicate difficulties and even instability in the control of a multivariable system and that severe or unstable interaction can be avoided by means of proper variable pairing. Consider the same system given in example 1 however with off-diagonal pairing, i.e.,

$$G(s) = \begin{bmatrix} \frac{0.6}{10s+} & \frac{1.0}{10s+1} \\ \frac{1.0}{10s+1} & \frac{-0.6}{s+1} \end{bmatrix} \quad (8.15)$$

It can be shown that controllers with k_{c1}=6.7, T_{i1}=2, k_{c2}=-0.6, and T_{i2}=0.2 will give good performance for independent SISO subsystems for both loops. The frequency responses of $g_{11}(s)$, $a_{11}(s)$, and d_{21} (loop 1) are shown i Figure 8.13. From Figure 8.13 we can see that the resulting system exhibits a dominant dynamic interaction in loop 1 over a wide range of frequencies. This implies that the system will be very difficult to control.

Furthermore, the phase responses of $g_{11}(s)$ and $a_{11}(s)$ as functions of frequency, shown in Figure 8.14, indicate that $a_{11}(s)$ acts in a different direction from $g_{11}(s)$ (the phase difference is more than 45^0) over a wide frequency range.

Hence, we can predict that the system will become unstable upon closing both independently designed SISO loops. Notice that the steady state stability index -- the Niederlinski index (Niederlinski, 1971) is satisfied here. Consequently, controllers have to be detuned before closing individual loops. For example, it can be easily verified that the proportional gain of controller 2 in this example has to be detuned by a factor of twenty in order to bring the system under stable control (i.e., to decrease the interaction in loop 1 so that it will not dominate the control action).

Note that system stability is solely affected by interaction in one loop for open loop stable systems if independent SISO subsystems are stable (see equations (8.8) and (8.9)). Nevertheless, as shown in example 1, we have no difficulty in control if the system is diagonally paired. In fact, with diagonal pairing controllers can be either tuned up or detuned without inducing instability.

8.4 FINAL REMARK

A structurally meaningful definition of dynamic interaction in decentralized control systems has been proposed. A multivariable control system is decomposed into separate SISO loops with interactions explicitly taken into account. The effects of interaction on closed loop system, and the influence of controller tuning and variable pairing on the interaction in the system can be evaluated by investigating the frequency responses of the interaction free system, the absolute interaction and the perturbation terms.

The interaction measure explicitly exposes the structural properties of decentralized control systems and compares interaction with the independent SISO subsystems. it can be also extrapolated to a steady state measure. This will be discussed in a separate paper.

It should be pointed out that the expression of the interaction term in a control loop depends on how the other loops in the system are paired. As a result, it would be time consuming to use it as a tool for variable pairing for high order systems. Nevertheless, the conceptual framework developed here and used for low order systems provides an appealing visual tool for determining the effects of interaction on a multivariable dynamic system.

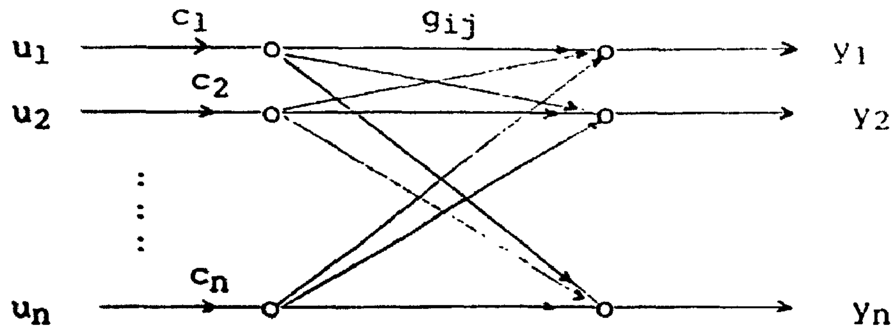


Figure 8.1 A decentralized control system as a network

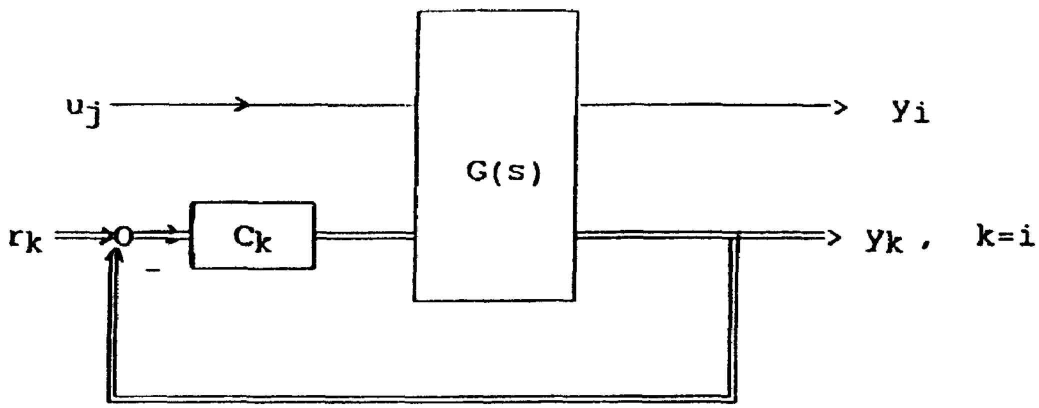


Figure 8.2 Impact of u_j on a closed loop system

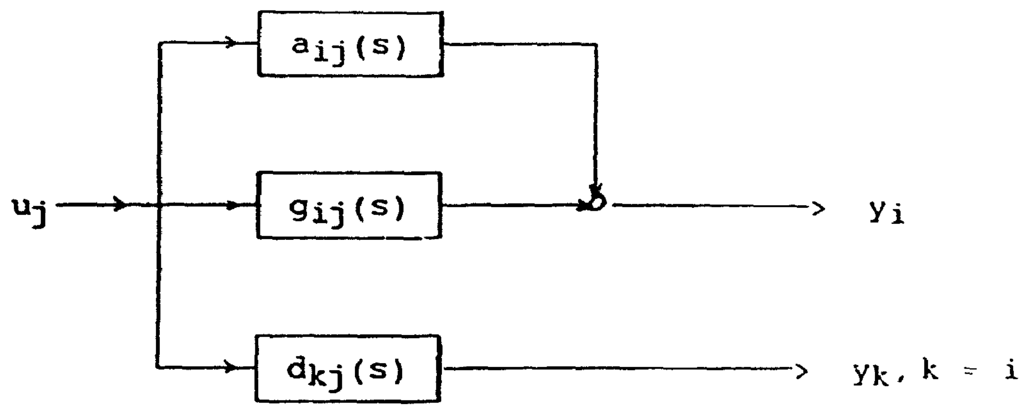


Figure 8.3 pseudo SISO system for u_j - y_i loop

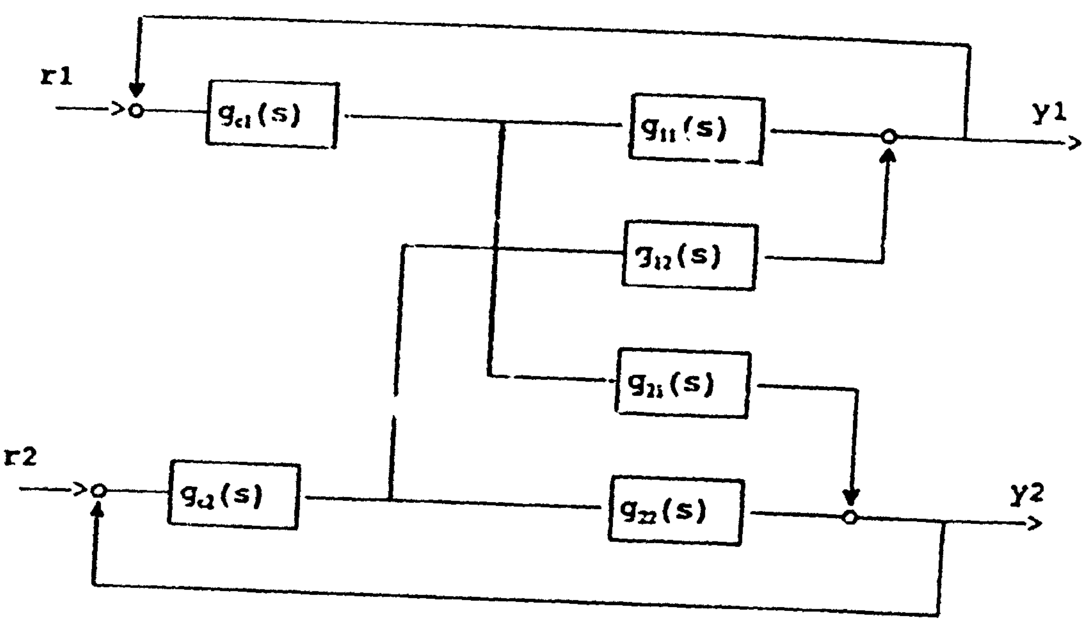


Figure 8.4 2x2 Control System

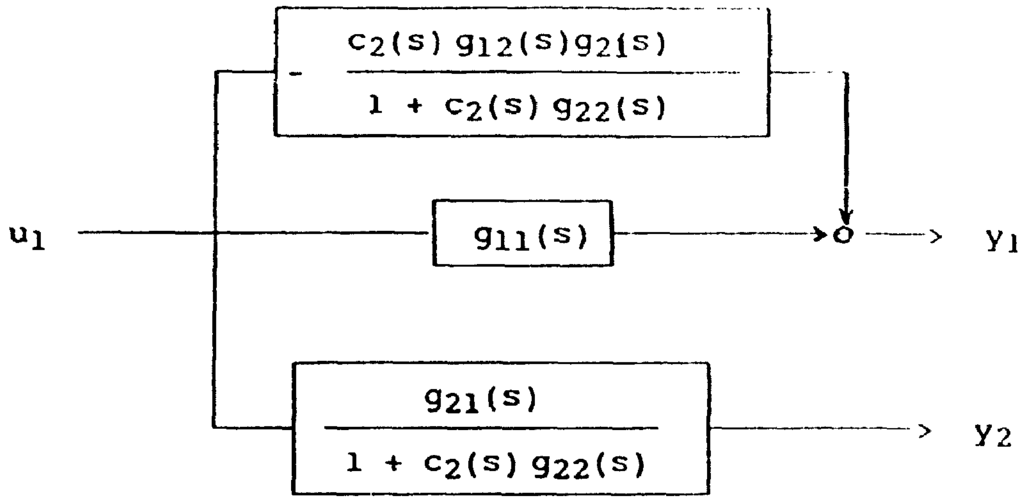


Figure 8.5 Structural decomposition of $u_1 - y_1$ loop

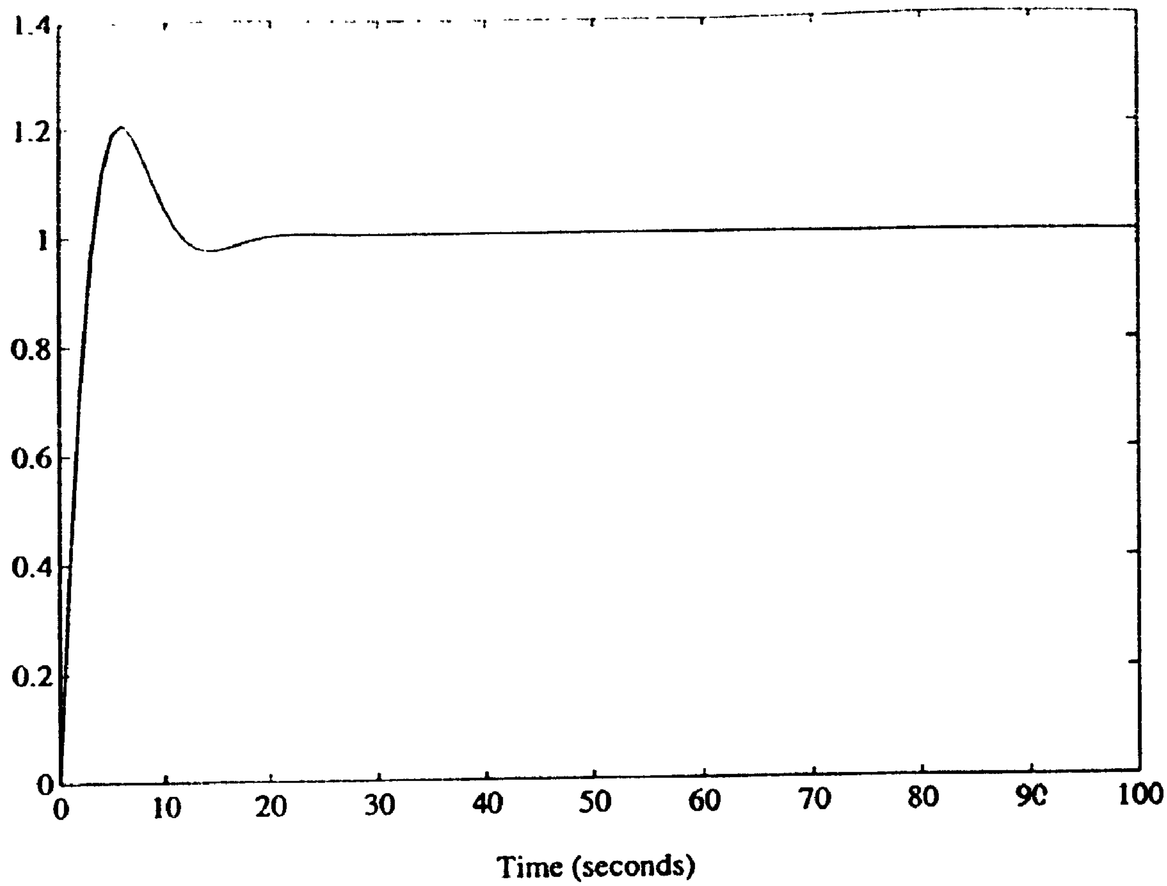


Figure 8.6 Example 1: Independent SISO output with r1

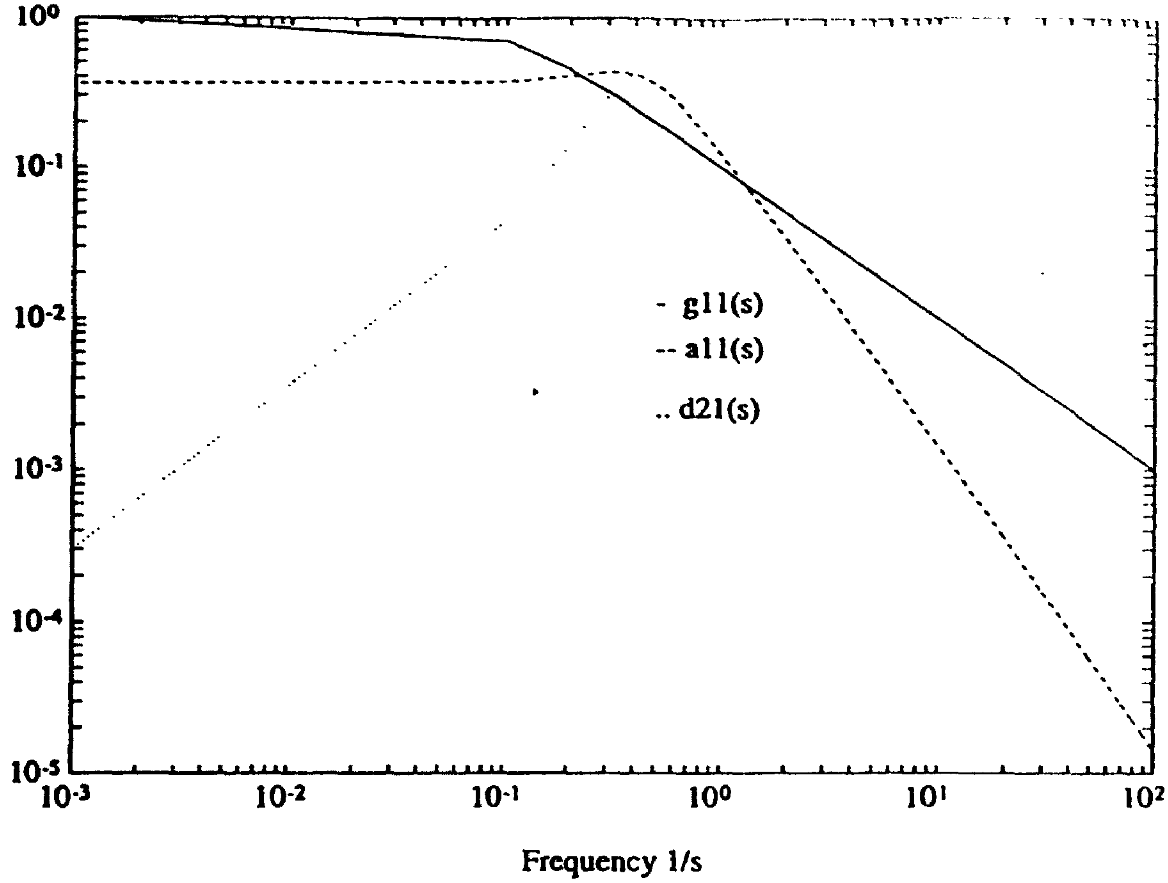


Figure 8.7 Example 1: Interaction terms in loop 1 vs frequency

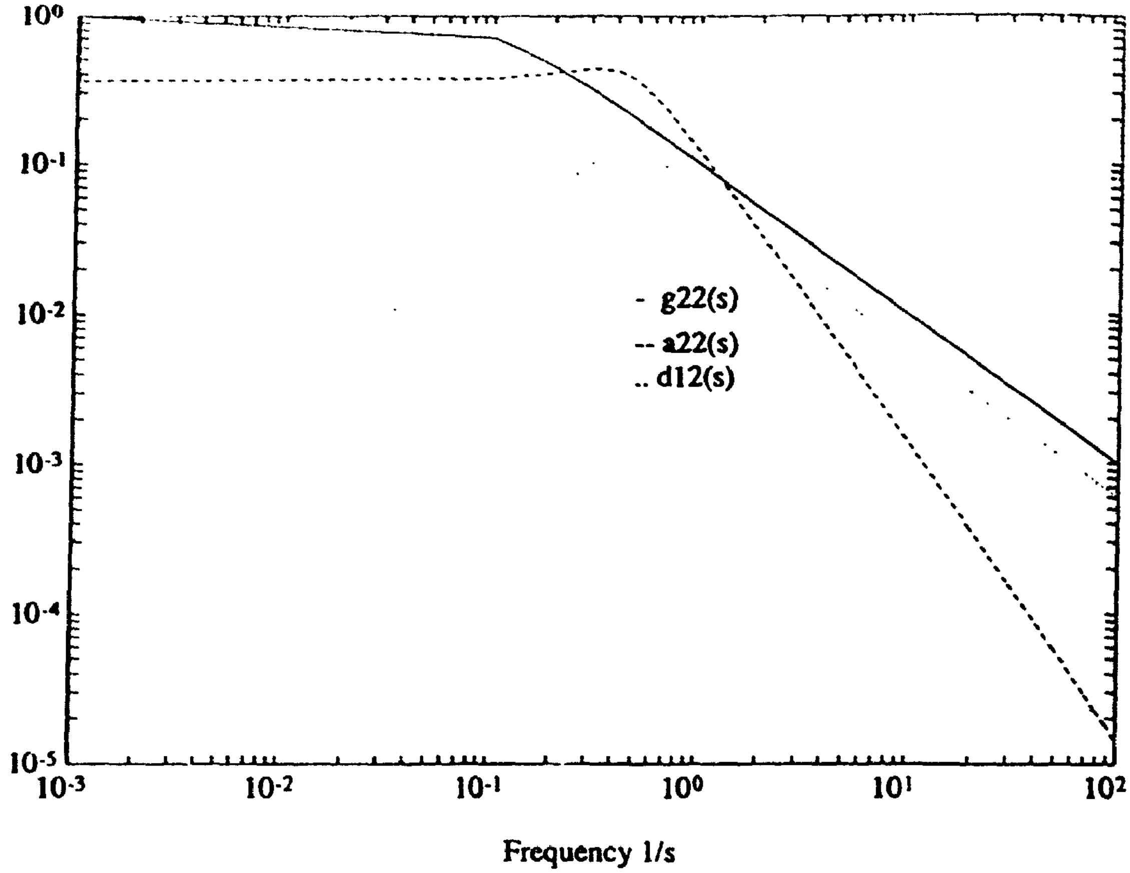


Figure 8.8 Example 1: interaction terms in loop 2 vs frequency

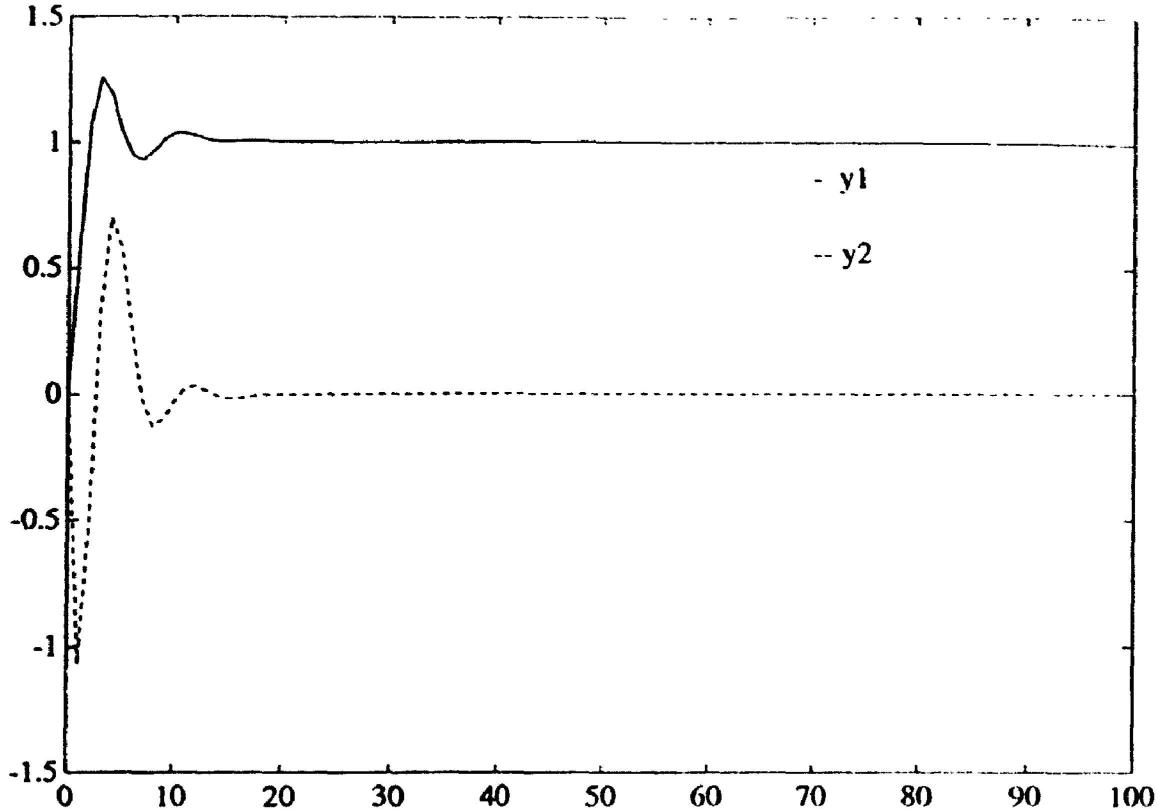


Figure 8.9 Example 1: setpoint change in y1

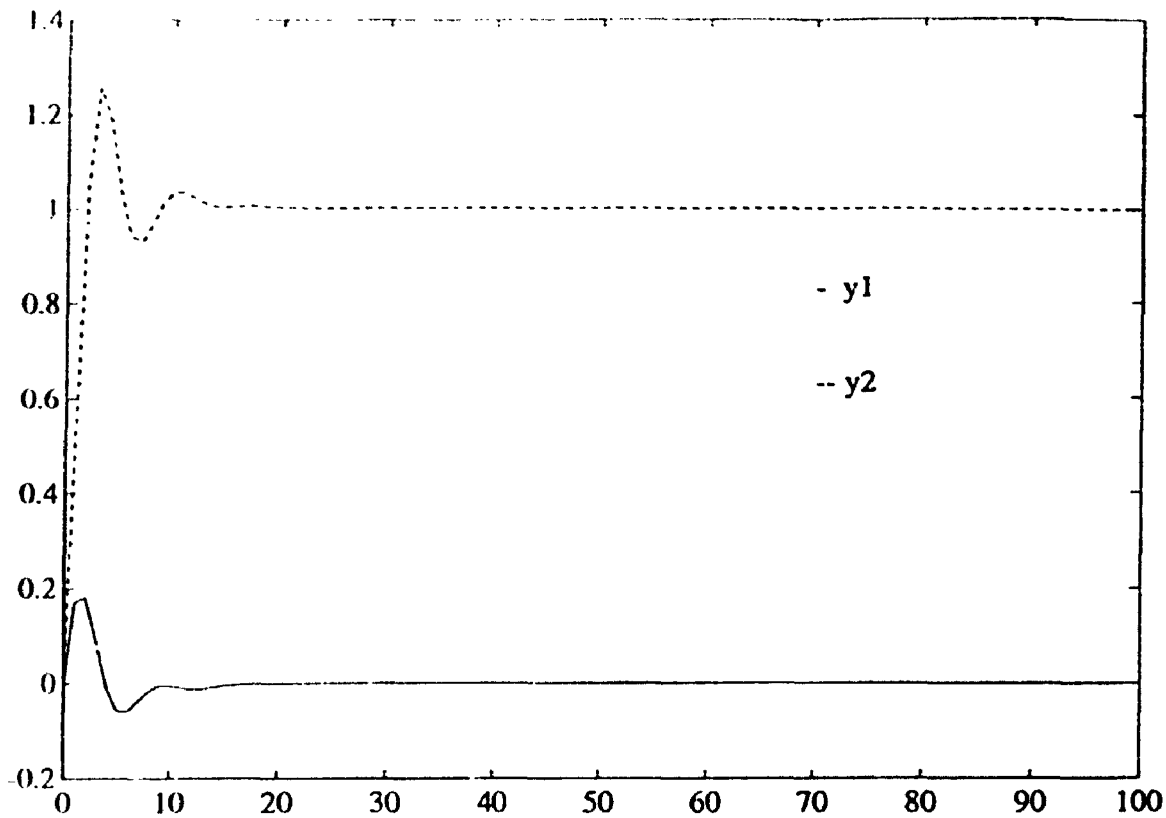


Figure 8.10 Example 1: setpoint change in y2

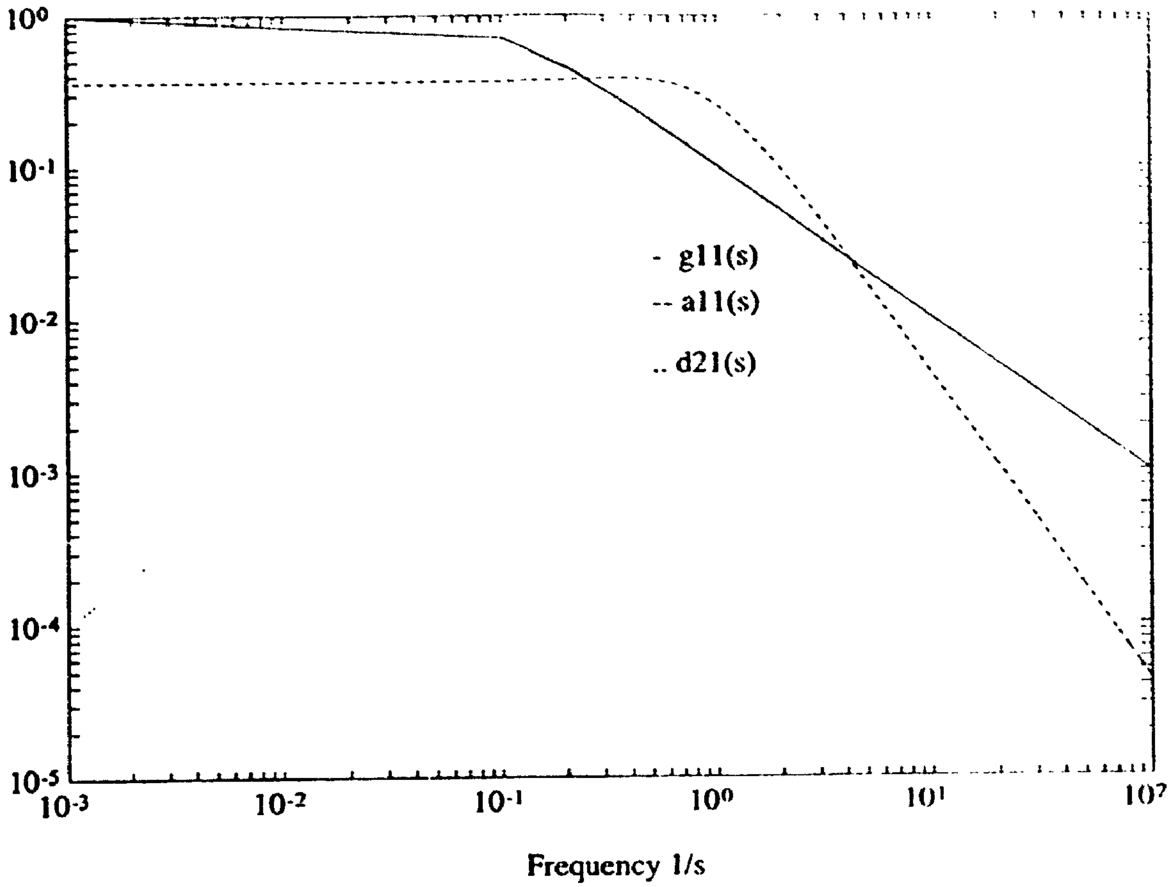


Figure 8.11 Example 2: Interaction terms vers frequency

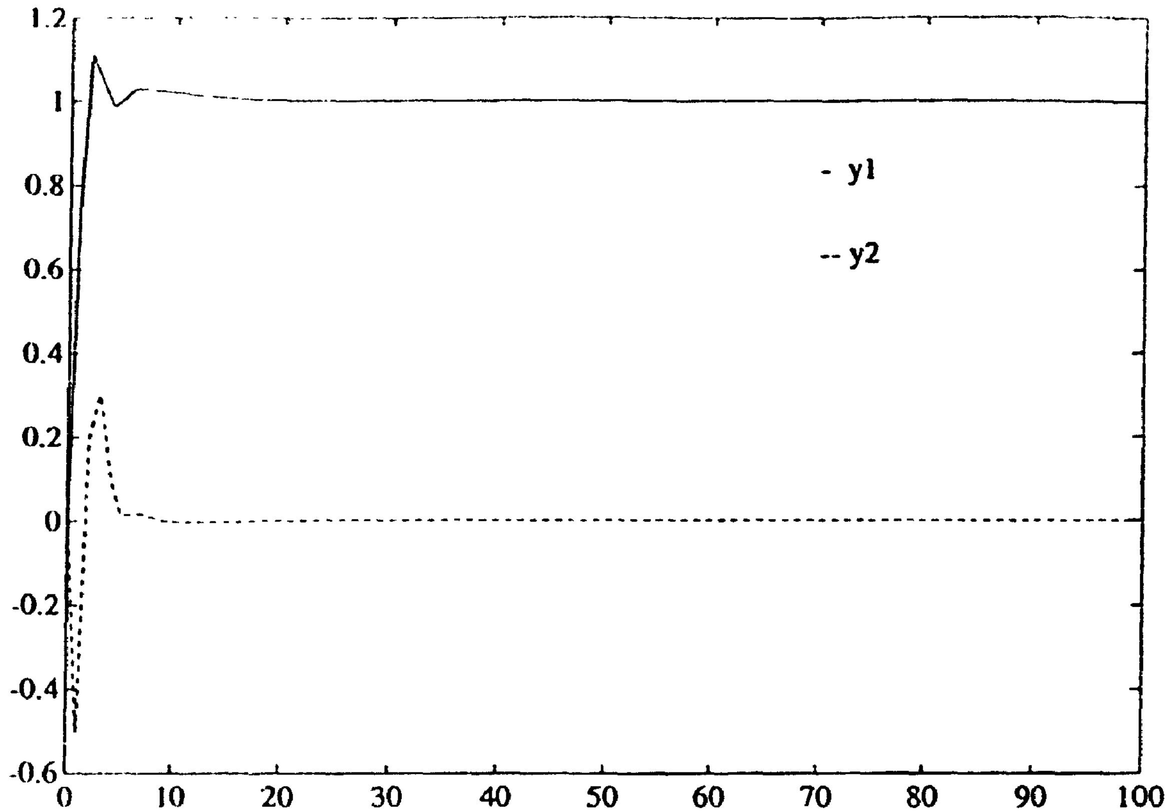


Figure 8.12 Example 2: setpoint change in y1

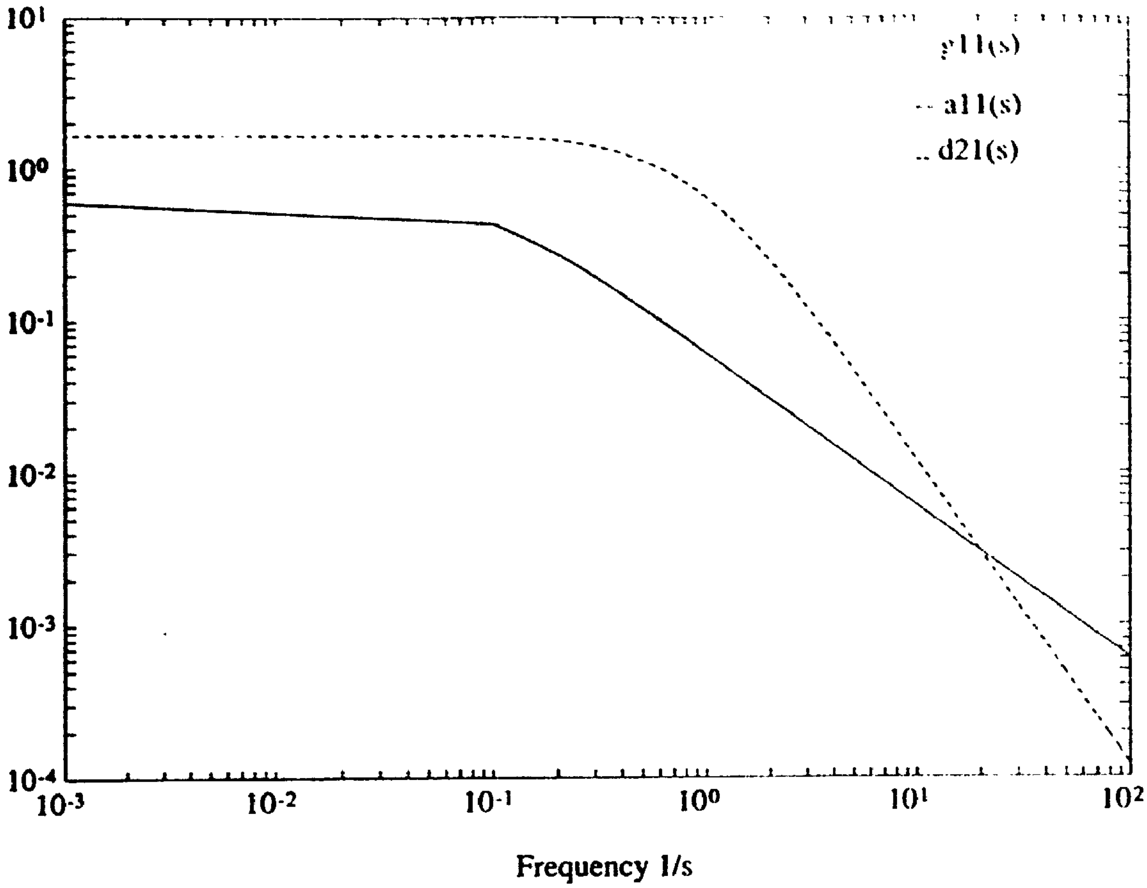


Figure 8.13 Example 3: Interaction terms in loop 1

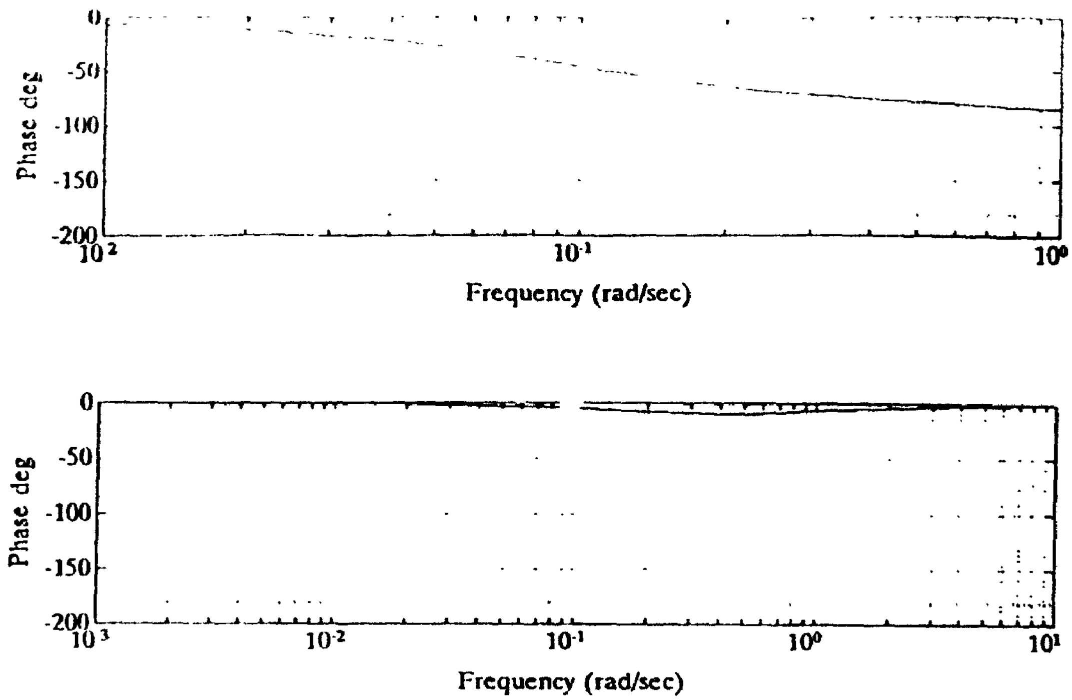


Figure 8.14 Example 3: Phase response vers frequency (a) $a_{11}(s)$
(b) $g_{11}(s)$

CHAPTER 9

CONSISTENCY PRINCIPLES FOR STABILITY IN DECENTRALIZED CONTROL SYSTEMS

Synnopsis *This chapter generalizes previous results on the stability of decentralized control systems from steady state information only. A series of results expressing stability conditions, called 'consistency principles for stability', are developed. Independent design and variable pairing rules are established, and previous results regarding system stability conditions are clarified.*

9.1 INTRODUCTION

Modern control based on state space techniques has a less than desired impact on industrial processes in spite of its tremendous development in theory over the last decades. Classical PID control remains the predominant approach in practice. As a result, many multivariable control problems are solved by using a decentralized control structure, particularly a multiloop control scheme with PID controller in each control channel. In addition to its many practical advantages (Skogestad and Morari, 1989), a more important feature of such a scheme is that a great deal of practical experience gained by engineers in adopting classical control theory over decades can be inherited. In fact, the design of multiloop control systems has essentially never gone beyond the scope of classical control.

Basically, this design can be divided into two stages: variable pairing and controller tuning. The objective in the first stage has traditionally been to minimize interactions in a system by properly choosing a control configuration. To date, substantial effort has been devoted to variable pairing problem by means of interaction measurement (Bristol, 1966; McAvoy, 1983; Bruans and Smith, 1981; Mijares et al., 1986; Zhu and Jutan, 1991). Ideally, a multivariable control problem may be effectively reduced to several SISO problems after variable pairing. Thus classical control theories can subsequently be applied to the pseudo SISO loops with minimal fine-tuning to counteract the residual interactions (Shinskey, 1988). Nevertheless, interactions in a system may not be sufficiently small for this purpose. They may be inherently severe no matter how variables are paired. Hence, control design as a whole may heavily rely on the second stage -- controller tuning.

Unfortunately, considerably less attention has been paid to the controller tuning problem, compared to that of variable pairing. We still lack an effective method in this regard, despite some of the approaches that have been proposed (Yu and Luyben, 1985; Seborg et al., 1989; Basualdo and Marchetti, 1991). So far, the most widely used approach is to design a controller on the basis of the interaction-free subsystem, followed by a second tuning procedure, usually performed by trial and error (Seborg et al., 1989). This is referred to as 'independent design'. However, this 'ad hoc' scheme may fail to provide satisfactory performance, especially in case of inherently strong interaction.

In particular, the stability of a multivariable control system can be very sensitive to controller settings. As a result, an independently stable system can be easily destabilized in the process of tuning (Grosdidier et al.,1985). On the other hand, stability is undoubtedly the most important and prime concern in the design of any control system. Hence, it is of great importance to impose conditions on a destabilized control system for stability. General stability theories require the analysis of the overall dynamics. This is discouraging due to the complexity for high order systems, and further constrained by the scarcity of adequate dynamic models. Consequently, it is clearly of practical significance to address stability problems using steady state information only, and more importantly, to explicitly relate system stability to controller tuning parameters.

Stability in a decentralized control system based on steady state gains has been studied by Grosdidier et al. (1985) and Morari and Zafiriou (1989). On the other hand, the stability problem has also been closely associated with variable pairing (Bristol, 1966; Niederlinski, 1971; Mijare et al., 1986). The most widely accepted method in this regard has been the Niederlinski Index (NI) (Niederlinski, 1971; Grosdidier et al., 1985). Nevertheless, so far only process gains have been utilized with the controller essentially omitted. Hence, the conclusions may be limited.

In this chapter, a general analysis of the stability problem in a decentralized control system, using steady state information of both process and controller, is performed. A comprehensive treatment of control system stability under overall closed loop control, independent design and variable pairing is presented. A series of results on stability conditions under the above cases, are established. These conditions express the constraints on the relative signs of process and controller quantities in the system, and are called 'the consistency principles for stability'. A generalized Niederlinski index, which allows for consideration of both the controller design and the variable pairing, is presented. System stability is related directly to the tuning parameters in multiloop control systems. Some important insights into the stability problem due to controller design and variable pairing are also provided, and some previous misunderstandings on this issue are clarified.

Throughout this chapter, it is assumed that a decentralized controller (diagonal), $G_c(s)$, containing integral action in each control channel, is to be used to control a square process, G(s). Here integral action is usually mandatory in practice since it leads to no offset at steady state. To emphasize the use of this integrator and to simplify mathematical treatment of the control system, the controller is separated into a integrator and the remaining compensator, C(s), i.e.,

$$G_c(s) = \frac{I}{s}C(s) \tag{9.1}$$

The following assumptions are used throughout the chapter unless otherwise specified:

Assumption 1

1). G(s) is stable;

2). H(s)=G(s)C(s) is rational and proper.

9.2 STABILITY CONSISTENCY UNDER DECENTRALIZED CONTROL

9.2.1 Stability Condition

Grosdidier et al. (1985) constructed a control system shown in figure 9.1 and then intruduced the concept of integral stabilizability (IS). The system is defined as integral stabilizable if there exists a k>0 such that the closed loop system is stable. A necessary condition was provided in that paper. However, IS only addresses existence and little guidance is provided for controller design.

To overcome the limitations of IS, let us consider the general control system shown in figure 9.2. This structure encompasses just the controller, which contains integral action and is expressed in equation (9.1), and the plant transfer function matrices. A necessary condition for stability is:

Theorem 1 Let assumption 1 hold. The closed loop control system shown in figure 9.2 is stable only if

$$\det[H(0)] > 0 \tag{9.2}$$

where H(0)=G(0)C(0).

Proof. see Appendix A.

The condition in equation (9.2) is the same as that for IS (Grosdidier et al., 1985). This is not surprising since the necessary condition for IS is certainly the same as for stability. However, as shown later, the sufficient condition for IS does not apply to stability. Moreover, the removal of the parameter k in figure 9.2 allows us to focus on system design.

Notice that integral action is explicitly required and that only steady state information is needed in equation (9.2). It can be seen that the use of integral action allows the stability problem to be addressed using only steady state information. This is another desirable feature of integral control, in addition to its ability to eliminate steady state offset.

Equation (9.2) can be further expressed as

$$\det[G(0)]\ \det[C(0)] > 0 \tag{9.3}$$

The above inequality indicates that the sign of the determinant of the plant gain must be consistent with that of the determinant of the compensator gain for the closed loop system to be stable. Therefore, this condition is called 'the consistency principle for stability' for the closed loop system. The signs of det[G(0)] and det[C(0)] can be generally considered as the 'sign' of the process and the compensator respectively in a decentralized control system. This is a direct extension of SISO systems where these signs are actually the signs of the respective gains. Moreover, since these signs determine the stability of the system, they can be viewed as the 'fundamental direction' of the process and the controller respectively.

9.2.2 Multiloop PID Control

For decentralized control, the determinant of the compensator can be expressed as

$$\det[C(s)] = \prod_{i=1}^{n} C_i(s) \tag{9.4}$$

In particular, for multiloop control with PID controller in each control loop, the controller transfer function matrix is given by

$$G_c(s) = diag(g_c^1(s), \ldots, g_c^n(s)) \tag{9.5}$$

and the transfer function of each individual PID controller is given by

$$\begin{aligned} g_c^i(s) &= k_p^i\left(1+\frac{1}{T_i^i s}+T_d^i s\right) \\ &= \frac{1}{s}\left(K_D^i s^2+k_p^i s+K_I^i\right), \qquad \forall i \end{aligned} \tag{9.6}$$

where

$$K_I^i = \frac{k_p^i}{T_i^i} \ , \qquad \forall i \tag{9.7}$$

$$K_D^i = k_p^i T_d \ , \qquad \forall i \tag{9.8}$$

are referred to as integral and derivative gain respectively, T_i and T_d are integral and derivative time, and k_p is the proportional gain. Superscript i denotes the i-th loop.

Separating the integrator, we obtain,

$$C_i(s) = K_D^i s^2 + k_p^i s + K_I^i \ , \qquad \forall i \tag{9.9}$$

Setting s=0 yields

$$C_i(0) = K_I^i \quad , \qquad \forall i \tag{9.10}$$

The above equation implies that the gain of each compensator depends only on the integral gain of the controller.

Finally, we have

$$\det[C(0)] = \prod_{i=1}^{n} K_I^i \tag{9.11}$$

It can be seen from above equation that the value of det[C(0)] can be changed either by tuning the proportional gain or integral time of any individual controller while the sign of it can be changed by alternating the direction of any one controller. Thus the value of det[C(0)] can be referred to as the gain of the controller and the sign of it as the direction of the controller in multiloop control system. Note that changing the direction of any odd number of individual controllers would alter the direction of the controller. We can change the control sign by changing the direction of just one controller.

Therefore, theorem 1 imposes a condition on the sign of the controller for a given process. In particular, for pure integral control theorem 1 leads to the condition for stability given by Lunze (1988).

9.2.3 SISO Systems

For SISO systems the stability condition can be expressed directly in terms of the signs of the process and controller as stated below.

Theorem 2 Let assumption 1 hold. A SISO system is stable only if

$$g(0)\,c(0) > 0 \qquad (9.12)$$

Theorem 2 says that for stability the signs of the process and the controller must be consistent. So the inequality (9.12) is called 'the consistency principle for stability' for SISO systems. Note that theorem 2 simply states the condition of negative feedback control. Similarly, theorem 1 is the generalization of the negative feedback condition for multivariable systems.

It must be emphasized that the condition in equation (9.12) still remains a necessary condition for stability. Grosdidier et al. (1985) showed that this is a necessary and sufficient condition for IS. However, the sufficient condition here is true only in the sense that there exists a controller such that the system can be made stable, rather than in the sense of stability, i.e., that the system is stable for all controllers. Here distinction must be made between stablizability and stability. The former does not explicitly address the controller design problem. The two terms are frequently confused. For instance, Chiu and Arkun (1991) used condition (9.12) as a necessary and sufficient condition for stability for SISO systems. In fact, negative feedback control does not necessarily guarantee stability unless properly tuned.

Example 1

Consider controlling the SISO process

$$g(s) = \frac{1}{(2s+1)(s+1)} \tag{9.13}$$

using a PI controller

$$g_c(s) = \frac{1}{s}c(s) = \frac{k_p}{T_i s}(T_i s+1) \tag{9.14}$$

with $T_i=1$.

It can be easily seen that g(s)c(s) is rational and proper, and $g(0)c(0)=k_p$. Clearly, a forward action in the controller must be used for the system to be stable. However, this is not sufficient. In fact, the characteristic equation of the closed loop system is given by

$$6s^3+5s^2+(1+k_p)s+k_p = 0 \tag{9.15}$$

According to the Routh test, the system is stable for $k_p<5$.

9.3 STABILITY CONSISTENCY UNDER INDEPENDENT DESIGN

9.3.1 Stability Condition

A common practice in tuning a MIMO controller under decentralized control is to use 'independent design' (Seborg et al., 1989; Skogestad and Morari, 1987). This tuning scheme can be best described by figure 9.3. Initial settings for all controllers are preliminarily obtained independently on the basis of a diagonal process with the switch δ off (δ=0). Final tuning (usually detuning) is then accomplished, usually by trial and error, taking into account the interactions arising from off-diagonal elements with the switch on (δ=1).

Nevertheless, the overall system may go unstable even though independent loops are tuned to have satisfactory performance. Thus the tuning process can be rather risky. It is necessary to determine conditions under which interactions can destabilize an independently stable system. Traditionally, the major goal has been to achieve a controller such that both individual loops and the overall system are stable, and as a result the stability condition is obtained only in terms of variable pairing (Niederlinski, 1971; Grosdidier et al., 1985; Grosdidier and Morari, 1986). However, in practice it is usually necessary to retune controllers when all loops are closed. A more important issue is how to tune individually designed controllers for a given process such that the overall system is stable. Furthermore, there is a need to systematically consider both controller design and variable pairing. The following theorem provides a general solution in this regard.

Theorem 3 Let assumption 1 hold. Assume that all independent loops are independently designed to be stable. The overall closed loop system is stable only if

$$\frac{\det[G(0)]}{\det[\bar{G}(0)]} \cdot \frac{\det[C(0)]}{\det[\bar{C}(0)]} > 0 \qquad (9.16)$$

where $\bar{C}(0)$ denotes the steady state gain of C(0) corresponding to independent design on the basis of $\bar{G}$ which is the diagonal subsystem of G(0).

Proof. see Appendix B.

Theorem 3 requires the sign of the overall process compared to the sign of the diagonal process (as calculated by the ratio of the determinants) to be consistent with the corresponding controller. Thus inequality (9.16) is referred to as 'the consistency principle for stability' under independent design.

Notice that the first ratio on the left hand side of equation (9.16) is the Niederlinski's index (Niederlinski, 1971) which compares the process gain of the overall system to that of the independent subsystem. Similarly, we can define a Niederlinski index for the <u>controller</u> as

$$NI[C(0)] \equiv \frac{\det[C(0)]}{\det[\bar{C}(0)]} \qquad (9.17)$$

Equation (9.16) can then be expressed as

$$NI[G(0)] \cdot NI[C(0)] > 0 \qquad (9.18)$$

Inequality (9.18) indicates that the NIs of the process and the controller must be consistent for stability. Compared with the usual NI defined for the process, equation (9.18) provides a necessary condition for stability using both NIs of the process and the controller, and thus can be viewed as a generalized Niederlinski index.

More generally, an NI for the closed loop system can be defined as

$$NI[H(0)] = \frac{\det[H(0)]}{\det[\bar{H}(0)]} \tag{9.19}$$

where $H(0)=G(0)C(0)$, $\bar{H}(0)=\bar{G}(0)\bar{C}(0)$.

Thus equation (9.16) can be written as

$$NI[H(0)] > 0 \tag{9.20}$$

The above inequality can be considered as the extension of theorem 1 to independent design.

From theorem 3 we see that it is essential that variable pairing and controller design be jointly considered. The significance of theorem 3 is that it allows two important design problems to be separately but systematically addressed: controller design with a given process configuration and variable pairing with constraints on the controller design. The latter will be discussed in the next section.

9.3.2 Controller Design

The sign of the NI corresponding to a specific process pairing plays an essential role in the controller design, particularly the setting of the controller sign or direction (reverse or forward action), according to theorem 3. Two cases can be separately discussed.

Controller Design With NI > 0

In this case, the sign of the independent subsystem (det[$\tilde{G}$(0)]) remains the same as that of the overall system (det[G(0)]), implying that the nature of the two systems does not change. Thus the sign of the controller does not need to be changed when all the loops are placed in automatic mode. In other words, the interaction in the system in this case is not strong enough to cause the change in the fundamental direction of the process and thus simply adjusting the magnitude of the controller action is sufficient to maintain stability. This is desirable from practical point of view. However, theorem 3 gives no guidance for controller tuning. This will be discussed later.

Controller Design With NI < 0

This case involves a change in the fundamental direction of the process for the overall system as compared to the diagonal subsystem and so requires the direction of the controller to be reversed when individual loops are closed, which were independently designed to be stable.

However, pairing on negative NI has long been regarded as 'inherently unstable pairing', i.e., the system would be unstable regardless of controller design (Grosdidier et al., 1985). Nevertheless, there might be situations where some pairings which correspond to negative NI are preferred for other reasons: for example, for operational convenience or for control effectiveness as dictated by process considerations.

From theorem 3, we can see that in this case (NI<0) the overall system can still be made stable if the controller action is reversed. Here the overall system is the prime concern while the independent subsystem serves only as a basis of comparison. As a result, the overall system 'globally' forms a negative feedback and constitutes the necessary condition for stability, even though some independent loops may contain 'locally' positive feedback. This is also true with state feedback control where individual control gains are not required to be negative.

Example 2

Consider a 2x2 system with process transfer matrix

$$G(s) = \begin{bmatrix} \frac{4}{s+0.3} & \frac{2}{s+0.1} \\ \frac{3}{s+0.15} & \frac{1}{s+0.2} \end{bmatrix} \qquad (9.21)$$

and PI controllers

$$G_c(s) = \begin{bmatrix} \frac{k_{p1}(s+0.1)}{s} & 0 \\ 0 & \frac{k_{p2}(s+0.1)}{s} \end{bmatrix} \qquad (9.22)$$

Let us design the controller for the diagonally paired system using independent design such that the closed loop system is stable.

The NI of the process can be calculated as -5.0 < 0. It can be easily verified that the two individual loops are stable if the two controllers are designed as $k_{p1}=1$ and $k_{p2}=0.1$. Indeed, the two loops have the closed-loop poles [-4.21, -0.095] and [-0.26, -0.038] respectively.

For the closed loop system to be stable either controller action needs to be reversed according to theorem 3. Suppose that the direction of the second controller is to be changed. Consider setting $k_{p2} = -0.1$, i.e., controller 2 is directionally reversed without any tuning. The characteristic equation of the resulting closed loop system can be shown, from the general equation (9.24) to be

$$s^5+4.55s^4+2.08s^3+0.48s^2+0.062s+0.003 = 0 \tag{9.23}$$

The corresponding closed loop poles can be calculated as

[-4.07, -0.1 ± 0.17i, -0.19, -0.1]

Hence the closed loop system is stable eventhough the pairing was chosen with a negative NI.

Note that some properties, particularly system integrity (Chiu and Arkun, 1990), of the closed loop system have to be sacrificed when pairing on a negative NI. For instance, the loop with control action changed will obviously become unstable when all the other n-1 loops fail. Hence, pairings corresponding to negative NIs should be avoided if integrity becomes important.

9.3.3 Controller Tuning under Independent Design

If we consider only stability, theorem 3 allows for arbitrary tuning of the controller since only the directions of the controller and the process are constrained. However, the sign of the NI can also have profound influence on the tuning of controllers.

If we take 2x2 systems with diagonal pairings, for example, (with possible extension to higher order systems), we can develop some guidance for controller tuning. The general structure of 2x2 systems is shown in figure 9.4.

The characteristic equation of the closed loop system is given by,

$$1+g_{c1}g_{11}+g_{c2}g_{22}+g_{c1}g_{c2}(g_{11}g_{22}-g_{12}g_{21}) = 0 \quad (9.24)$$

(9.25)

The Laplace variable s is omitted in each transfer function for simplicity.

Two cases corresponding to different signs of the NI can be distinguished.

<u>Controller Tuning With NI > 0</u>

Assume that PI controller is used in each control channel and each individual loop is independently designed to be stable. Then all zeros of the following characteristic equations for the two loops will lie in the left hand side of the complex plane. Also the following equations hold from theorem 2

$$1 + g_{c1} g_{11} = 0 \quad (9.26)$$

$$1 + g_{c2} g_{22} = 0 \quad (9.27)$$

$$g_{11}(0) c_1(0) > 0 \quad (9.28)$$

$$g_{22}(0) c_2(0) > 0 \quad (9.29)$$

Moreover, detuning each controller will result in larger stability margin for each independent loop.

For NI >0, the direction of each controller should remain the same as it would be when designed for independent control when both loops are closed. Suppose, without loss of generality, that we focus on the tuning of the proportional gain of the first controller with the second controller unchanged from the independently designed values or detuned. The characteristic equation (9.24) can be rearranged as

$$1 + \frac{g_{c1}(g_{11} + g_{c2} g_{11} g_{22} - g_{c2} g_{12} g_{21})}{1 + g_{c2} g_{22}} = 0 \quad (9.30)$$

The second term of the left hand side of the above equation can be considered as the open loop transfer function of the overall system in terms of the root locus movement. We see that none of the poles of this open loop transfer function lie in the right hand side of the s domain since the process is assumed stable and loop 2 is independently stable, except that some poles are located at the origin due to integrators in the controllers. The root locus of the closed loop system might begin to move to the right hand side of the s domain only due to branches starting from the stable open loop poles when k_{p1} in g_{c1} is increased, provided that decentralized PI control is feasible (Grosdidier and Morari, 1986). However, the poles of the closed loop system will stay in the left hand side of the s domain if k_{p1} is tuned small enough, since the root locus of the closed loop system starts from the open loop poles. Consequently, both controllers can be tuned starting from very small proportional gains, and detuning promotes stability for the overall system. This provides a justification for the common practice of detuning.

Controller Tuning With NI < 0

In this case, either one of the two controller directions has to be reversed upon closing the overall system according to theorem 3. Suppose, without loss of generality, that the direction of g_{c2} is reversed, and that we focus on the tuning of the proportional gain of the first controller, k_{p1}. Note that equation (9.26) in this case will have at least one unstable zero no matter how g_{c2} is tuned according to theorem 2. As a result, the system will be stable only when k_{p1} is set above a minimum value, since at least one branch of the root locus of the closed loop system as described by equation (9.29) starts from the unstable open loop pole. Hence, controller 1 can no longer be arbitrarily detuned as opposed to the case of NI>0. Instead, g_{c1} might have to be tuned up under some circumstances. As a result, the tuning of the controllers is difficult and becomes more risky by pairing on negative NI. This gives another reason for the preference of pairing with positive NIs.

9.4 STABILITY CONSISTENCY FOR VARIABLE PAIRING

9.4.1 Stability Condition

We shall show in this section that, upon imposing constraints on the design of the controller, stability conditions can be expressed in terms of variable pairing, using only process gains. In particular, if we require that both the overall and the individual subsystems be stable with the <u>same</u> controller, the generalized Niederlinski index of theorem 3 can thus be reduced to the usual one (Niederliski, 1971; Grosdidier et al., 1985), i.e., a necessary stability condition for variable pairing becomes

$$NI[G(0)] = \frac{\det[G(0)]}{\det[\tilde{G}(0)]} > 0 \qquad (9.31)$$

Equation (9.30) requires that variables be paired such that the signs of the overall system and the independent subsystem are consistent. Thus inequality (9.30) is referred to as 'the consistency principle for stability' under variable pairing.

It must be emphasized that the same controller for independent and the overall systems is assumed in the Niederlinski stability condition and that the NI only provides a necessary condition for stability. However, <u>retuning</u> of the controller is often necessary in practice upon closing individual loops. In fact a more general result can be inferred from theorem 3 as follows.

> ***Theorem 4*** Let assumption 1 hold. Assume that all independent SISO loops are designed to be stable. The closed loop system can be tuned stable, without reversing controller direction, only if inequality (9.30) holds.

Proof. See Appendix c.

Note that detuning is usually required as previously discussed.

Unstable pairings can be largly eliminated by equation (9.30), stable pairings can only be found within the remaining pairing alternatives. One can always design controllers for the remaining pairings such that the closed loop system is stable provided that decentralized control is feasible for the process. In other words, stability condition (9.30) is sufficient in terms of the existence of a controller which would make the system stable. However, for IS, positive NI only provides a necessary condition for stability except for 2x2 systems (Grosdidier et al., 1985). It is important to point out that for 2x2 systems NI>0 still remains only a <u>necessary</u> condition for <u>stability</u>. Nevertheless, positive NI has long been considered as a necessary and sufficient condition for stability (Chiu and Arkun, 1990) and even concluded in the textbook (Seborg et al., 1989), due to the commonly confused interpretation of stabilizability (Grosdidier et al., 1985) as stability. The following example will show that NI>0 does not guarantee stability for 2x2 systems.

Example 3

Consider a 2x2 system

$$G(s) = \begin{bmatrix} \frac{2}{(s+2)(s+1)} & \frac{3}{s+3} \\ \frac{1}{s+1} & \frac{4}{(s+2)(s+1)} \end{bmatrix} \tag{9.32}$$

with PI controllers

$$G_c(s) = \begin{bmatrix} \frac{k_{p1}(s+1)}{s} & 0 \\ 0 & \frac{2(s+1)}{s} \end{bmatrix} \tag{9.33}$$

using k_{p1} as a tuning parameter.

The NI is calculated as 0.5 >0. It can be easily verified that the two independent SISO loops are stable when k_{p1} =1.2. However, it can be shown that the overall system is unstable with the same controller. Indeed, by evaluating the characteristic equation given in equation (9.24), the poles of the resulting closed loop can be calculated as

[-3.36, -1.88±1.57i, 0.064±1.19i, -1.0±0.001i, -1.0±0.002i, -1.0]

This means that pairing on NI>0 does not guarantee stability unless the controller is appropriately tuned.

9.4.2 The Existence of Stable Pairing (NI >0)

It has previously been shown that pairing on NI>0 is desirable for system integrity and controller tuning. However, can one always find a pairing with NI>0 for any system? In other words, is there any 'inherent unstable pairing' (all NIs negative for all possible pairings)? The following theorem helps provide a solution.

Theorem 5 The NIs for all n! possible pairings can be calculated, upon arranging them in a factorial order, as

$$NI^k[G(0)] = \frac{(-1)^{k-1}\det[G(0)]}{\prod_{i=1}^{n} g_{ij}^k(0)}, \qquad \forall\ k=1 \sim n \tag{9.34}$$

where k is the factorial order of the NI, $g_{ij}^k(0)$ represents the pairing element of the process gain matrix corresponding to the k-th alternative.

Proof. See Appendix D.

Note that in theorem 5 the original diagonal pairing should be ordered as the first one. Expression (9.33) was first introduced by Zhu and Jutan (1991) without detailed proof as a simplified approach for calculating NIs. It also provides a basis for the following conjecture:

> ***Conjecture 1*** For any given plant gain matrix, there always exists at least one pairing with NI >0.

Justifications are provided in Appendix E.

Conjecture 1 implies that pairing on positive NI is not only desirable but also always possible. On the other hand, and NI should always be used to scan pairings since negative NIs are also accompanied for most systems. For 2x2 systems, it has been shown that there always exists a pairing for which the NI is positive (Grosdidier et al., 1985; Zhu and Jutan, 1991). Conjecture 1 extends this to the general case.

9.5 FINAL REMARKS

Closed loop stability under decentralized control, based on the steady state information, has been systematically addressed. A generalized Niederlinski index, which allows for the joint consideration of the controller design and the variable pairing problem, has been presented. The main conclusions can be summarised as follows.

(1). Systems with variable pairing corresponding to NI<0 can still be designed stable if the controller direction is properly chosen.

(2). Pairing with NI>0 is desirable for system integrity and controller tuning.

(3). NI>0 provides only a necessary stability condition for 2x2 systems.

(4). There exists at least one pairing with NI>0 for any decentralized control system.

(5). $h(0) > 0$ is only a necessary condition for stability of SISO systems.

9.6 PROOF OF THEOREMS

Proof of Theorem 1

Mathematically, the proof follows directly from Grosdidier at al. (1985) with the parameter k removed (set to 1). Refer to Grosdidier et al. (1985) for detailed proof.

Proof of Theorem 3

Since independent SISO loops are assumed stable, we have from theorem 2,

$$\bar{c}_i(0)\, g_{ii}(0) > 0 \quad , \forall\ i \tag{B1}$$

where $\bar{c}_i(0)$ represents the gain of the independently designed controller for the i-th loop.

Combining all individual loops yields,

$$\prod_{i=1}^{n} \bar{c}_i(0)\, g_{ii}(0) > 0 \tag{B2}$$

Thus,

$$\det[\bar{C}(0)] \cdot \det[\bar{G}(0)] > 0 \tag{B3}$$

where

$$\bar{C}(0) = diag[\bar{c}_1(0), \cdots, \bar{c}_n(0)] \tag{B4}$$

is a decentralized controller and independently designed.

For the overall system to be stable, on the other hand, from theorem 1 we require,

$$\det[G(0)] \cdot \det[C(0)] > 0 \qquad \text{(B5)}$$

Using the independently designed subsystem in equation (B3) as a basis, we see that the overall system is stable only if

$$\frac{\det[G(0)]}{\det[\overline{G}(0)]} \cdot \frac{\det[C(0)]}{\det[\overline{C}(0)]} > 0 \qquad \text{(B6)}$$

Proof of Theorem 4

Introducing a diagonal constant matrix K as a tuning device, the controller gain (with integrator split off) for the overall system can be expressed as,

$$C(0) = K\overline{C}(0) \qquad \text{(C1)}$$

where $\bar{C}(0)$ denotes the independently designed controller based on the diagonal process system.

From equation (C1), it can be seen that controller tuning without changing its direction requires

$$\det(K) > 0 \qquad \text{(C2)}$$

Since individual loops are designed stable, from theorem 1 we obtain

$$\det[\overline{G}(0)]\det[\overline{C}(0)] > 0 \qquad \text{(C3)}$$

The NI for a specific pairing can be expressed as,

$$NI[G(0)] \equiv \frac{\det[G(0)]}{\det[\overline{G}(0)]} = k_1 \qquad \text{(C4)}$$

Thus,

$$\det[G(0)] = k_1 \det[\overline{G}(0)] \tag{C5}$$

For the overall system to be stable, we require from theorem 1,

$$\det[G(0)]\det[C(0)] > 0 \tag{C6}$$

Expressing the above equation in terms of k_1 and the independent system and making use of equations (C1) and (C6) yields

$$k_1 \det(K)\det[\overline{G}(0)\,\overline{C}(0)] > 0 \tag{C7}$$

Finally, for the overall system to be stable we require

$$k_1 = NI[G(0)] > 0 \tag{C8}$$

due to equations (C4) and (C2).

Proof of Theorem 5

It is known that the NI is defined as,

$$NI[G(0)] = \frac{\det[G(0)]}{\det[\overline{G}(0)]} \tag{D1}$$

where $\bar{G}(0)$ represents the diagonal subsystem of the process gain matrix G(0), and the denominator of the right hand side of equation (D1) can be expressed as,

$$\det[\overline{G}(0)] = \prod_{j=1}^{n} g_{jj}(0) \qquad \text{(D2)}$$

Note that in the calculation of NI, any pairing is required to be arranged into a dianonal pairing by interchanging the positions of input/output variables of the plant. There are n! possible pairing options and so they can be ordered in certain factorial order for calculational convenience. It is reasonable to order the one with the original diagonal pairing whose NI is given by equation (D1) as the first.

As a result, the NI corresponding to the k-th pairing can be expressed as,

$$NI^{k} = \frac{\det[G^{k}(0)]}{\det[\overline{G}^{k}(0)]} \qquad \text{(D3)}$$

where $G^k(0)$ and $\bar{G}^k(0)$ represent the process gain matrix and corresponding diagonal one after rearrangement of variables.

We know that interchanging rows or columns only affects the sign of the determinant of a matrix. Hence, the numerator in equation (D3) can be expressed as,

$$\det[G^{k}(0)] = (-1)^{k-1}\det[G(0)] \quad , \qquad \forall\, k \qquad \text{(D4)}$$

And the determinant of a diagonal matrix can be written as the product of its elements as in (D1) for the original diagonal pairing. Thus the denominator in the right hand side of equation (D3) can be calculated as

where $g^k_{ij}(0)$ represent the pairing elements of G(0)

$$\det[\overline{G}^k(0)] = \prod_{i=1}^{n} g_{ij}^k(0) \quad , \qquad \forall k \tag{D5}$$

corresponding to the k-th option. Instead of performing the diagonal alignment, these elements can be directly picked up and multiplied.

Consequently, the NI corresponding the k-th pairing can be calculated by

$$NI^k = \frac{(-1)^{k-1}\det[G(0)]}{\prod_{i=1}^{n} g_{ij}^k(0)} \quad , \qquad \forall \; k \tag{D6}$$

Justification for Conjecture 1

From theorem 5 all the NIs can be calculated by

$$NI^k = \frac{(-1)^{k-1}\det[G(0)]}{\prod_{i=1}^{n} g_{ij}^k(0)} \quad , \qquad \forall \; k \tag{E1}$$

where k is the factorial order number of all n! pairings for a process. Notice the numerator in the right hand side of equation (E1) changes sign alternatingly. For any plant gain matrix with all the elements positive or negative, there are equal number of positive and negative NIs since the denominator has the same sign for all the pairing alternatives. In a general case, there exists no such a system for which the denominator in equation (E1) would change its sign alternatingly in exactly the same way to the numerator for all the n! options. Hence, one can always achieve a pairing with NI >0.

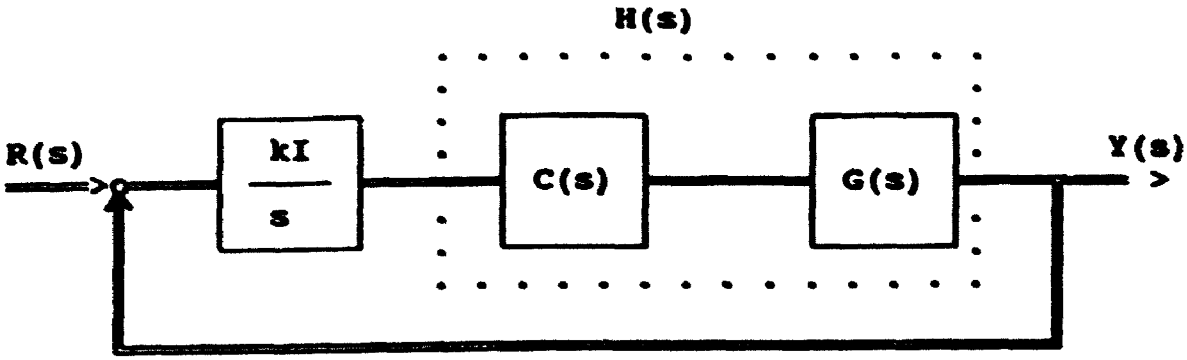


Figure 9.1 Control Structure for Stabilizability

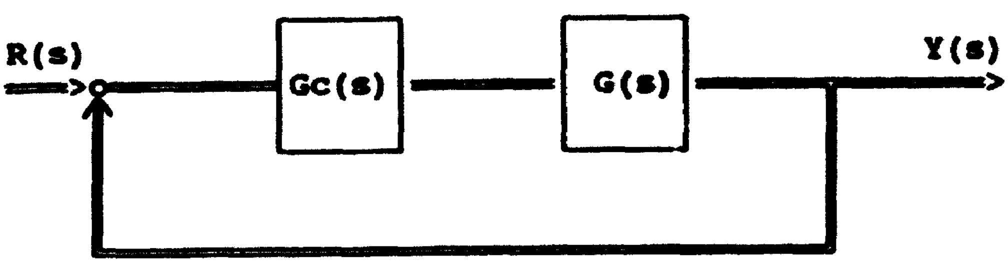


Figure 9.2 General Control Structure

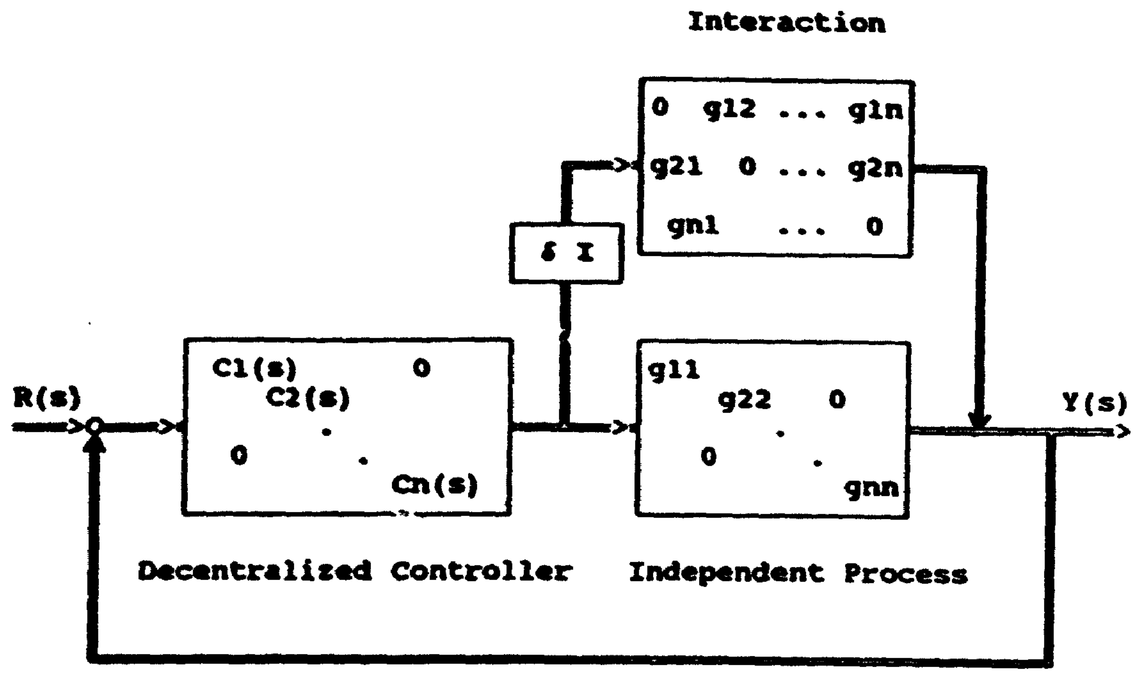


Figure 9.3 Control Structure for Independent Design

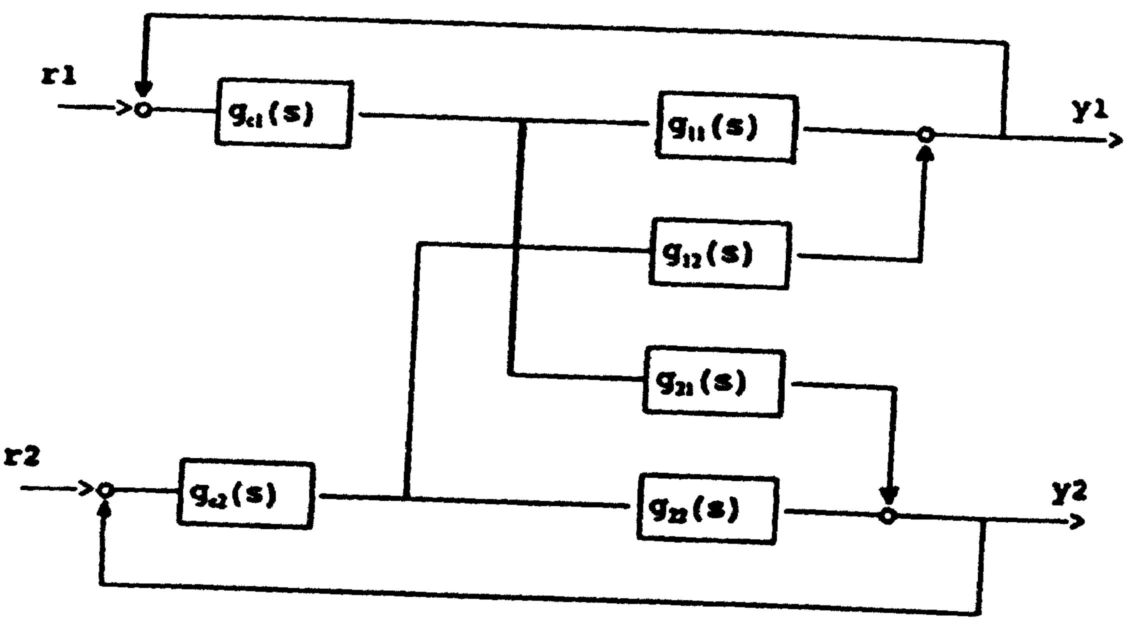


Figure 9.4 2x2 Control System

CHAPTER 10

STABILITY ROBUSTNESS FOR DECENTRALIZED CONTROL SYSTEMS

Synopsis *In decentralized control systems, a control structure or variable pairing satisfying the nominal stability and integrity requirements, by means of the RGA and NI, can be rather risky in the presence of inherent model mismatch. In this chapter, a stability robustness measure, based on mathematical and geometrical analysis of the behaviour of the NI and RGA in the face of model uncertainties, is developed. In contrast to existing methodology, rigorous justification is provided and more important insights into the interpretation and application of the measure are offered. The approach used also applies to the general case of multiple gain errors and a general stability condition is presented for 2x2 systems.*

10.1 INTRODUCTION

In decentralized process control design, the first step lies in the selection of the best control structure and/or variable pairing, mainly by means of interaction measures [Bristol, 1966; McAvoy, 1983; Mijare et al., 1985; Zhu and Jutan,1991]. In particular, the Relative Gain Array (RGA) (Bristol, 1966) and Niederlinski Index (NI) (Niederlinski, 1971) play a dominant role since they are closely related to closed loop properties such as system stability and integrity. Conventionally, variable pairing decisions are solely based on a nominal model of process gains. However, inherent modelling mismatch may cause a problem in system stability, which is of prime importance in any control system. Therefore, *stability robustness* against model mismatch must be taken into account when making decisions regarding the best control structure and pairing.

Grosdidier et al. (1985) proposed an *approximate* bound for steady state model uncertainty in terms of integral controllability (IC). Nevertheless, their approach contains no direct implication regarding the control structure selection and variable pairing choice. Skogestad and Morari (1987) provided conditions on the magnitude of model mismatch for each independent element for *robust singularity of the process*. However, these conditions do not clearly address robust stability of the closed loop system. Yu and Luyben (1987) presented an allowable model error for each single process gain for a system to maintain IC. Unfortunately, their derivation of the robustness measure does not have a wide theoretical base, and as a result some important issues are obscured.

In this chapter, a *stability robustness* measure is rigorously derived based on a mathematical and geometrical analysis of the NI stability index as a function of model mismatch in individual plant gains. The measure is conveniently expressed in terms of the RGA by virtue of the relationship between the NI and RGA. Although the final result is similar to the one by Yu and Luyben (1987), fundamental differences between them in the derivation and interpretation of the measure exist. More significantly, our method can be extend to the general case of multiple gain uncertainties.

10.2 NOMINAL STABILITY AND INTEGRITY

The Niederlinski Index (Niederlinski, 1971), defined as

$$NI(G) = \frac{\det(G)}{\det(\bar{G})} \tag{10.1}$$

where $\bar{G}$ = diag (G), provides a necessary condition (by its sign) for stability in decentralized control systems (Grosdidier, et al., 1985). Upon application to subsystems with loops removed, the NI can be used to address system integrity which is defined as the ability of a system to maintain stability in case of loop failure (Chiu and Arkun, 1990). For instance, $NI^i > 0$ (NI^i denotes the NI of the subsystem of G with i-th row and i-th column removed) is a necessary condition for integrity against i-th loop failure.

An alternative but more convenient tool to address integrity is the Relative Gain Array (RGA) (Bristol, 1966) by virtue of the following relationship (Chiu and Arkun, 1990; Zhu and Jutan, 1992):

$$\lambda_{ii} \cdot NI = NI^{i} \quad , \qquad \forall\, i \tag{10.2}$$

Clearly, the sign of the RGA plays the same role as NI^i, provided NI > 0. The RGA is defined by

$$\lambda_{ij} = g_{ij}\ \hat{g}_{ji} \tag{10.3}$$

where $\hat{g}_{ji}$ is the ji-th element of G^{-1}.

The investigation of the sign change in NI in the face of model mismatch leads to a measure of stability robustness, while the sign change in RGA indicates, as pointed out by Grosdidier et al. (1985), an integrity problem (provided that NI remains positive).

10.3 ROBUST STABILITY AND THE NI

Virtually all models used in control design are only approximate due to the intrinsic complexity and the inherent nonlinearity of many processes. Hence, robust stability against model uncertainties is an important issue. Here only model error arising from independent individual gains are considered, the more general case will be discussed later.

10.3.1 The Critical Value for the NI

It can be anticipated that the value of a paired element in the plant gain matrix (g_{ii}), corresponding to an NI equal to zero, plays a vital role in addressing the change in the sign of the NI. This value is thus defined as the *critical value* (denoted by c_{ii}) of the plant gain g_{ii} associated with the NI. This value can be calculated directly from the gain matrix as,

$$c_{ii} = \frac{1}{g_{ii} - \frac{\det G}{\det G^i}} \quad , \quad \forall\, i \tag{10.4}$$

It is clear that c_{ii} reflects the intrinsic property of the nominal gain matrix in terms of g_{ii}.

Also c_{ii} can be calculated from the NI as,

$$c_{ii} = \left(1 - \frac{NI}{NI^i}\right) g_{ii} \quad , \quad \forall i \tag{10.5}$$

or alternatively from the RGA by eq 10.2,

$$c_{ii} = \left(1 - \frac{1}{\lambda_{ii}}\right) g_{ii} \quad , \quad \forall i \tag{10.6}$$

10.3.2 NI as a Function of Model Errors

Further expressing the NI as a function of g_{ii} from eq 10.5, we have

$$NI = \left(1 - \frac{c_{ii}}{g_{ii}}\right) NI^i \tag{10.7}$$

Note that c_{ii} in the above equation is evaluated from the nominal process gain matrix, and NI^i is independent of g_{ii}. Let g^0_{ii} denote the nominal value of g_{ii}, NI^0 and λ_{ii}^0 represent the NI and RGA value calculated from the nominal gain matrix. Assume, without loss of generality, that $NI^i > 0$, i.e., NI^0 and λ_{ii}^0 have the same sign. The behavior of the NI under model mismatch in g^0_{ii} can be examined by analyzing eq 10.7 mathematically and geometrically for the following cases:

Case 1: $NI^0>0,\ g^0_{ii}>0$

It can be easily verified from eq 10.7 that this case corresponds to $g^0_{ii} > c_{ii} > 0$. Here, the nominal system is stable under the diagonal pairing. The NI as a function of g_{ii} (perturbed gain) is geometrically depicted in figure 10.1. The following important features can be observed from figure 10.1:

$$g_{ii} \rightarrow c_{ii} \Rightarrow NI \rightarrow 0\ , \quad \forall\ i$$

$$g_{ii} \rightarrow c_{ii}^{+} \Rightarrow NI < 0\ , \quad \forall\ i$$

$$g_{ii} \rightarrow 0 \Rightarrow NI \rightarrow -\infty\ , \quad \forall\ i$$

$$g_{ii} \rightarrow \infty \Rightarrow NI \rightarrow NI^i\ , \quad \forall\ i$$

where $\rightarrow$ means 'moves towards', $\Rightarrow$ implies 'results in', and c_{ii}^{+} represents 'beyond c_{ii}'. Most notably, when g_{ii} moves past the critical value, the NI will change its sign. Therefore, the distance between g_{ii}^0 and its corresponding critical value gives the allowable model mismatch in g_{ii}^0 for the system to maintain stability, i.e., this distance is a measure of stability robustness. Also, it is interesting to notice that model error may induce instability only when it occurs in the direction of the critical value for any paired gain. This phenomena is referred to as 'one way' stability robustness.

Case 2: $NI^0>0,\ g_{ii}^0<0$

This case also corresponds to a nominally stable system with $g_{ii}^0 < c_{ii} < 0$ and is described in figure 10.2. The following features are shown in figure 10.2:

$$g_{ii} \rightarrow c_{ii}^- \quad \Rightarrow \quad NI \rightarrow 0\ , \quad \forall\ i$$

$$g_{ii} \rightarrow c_{ii}^+ \quad \Rightarrow \quad NI < 0\ , \quad \forall\ i$$

$$g_{ii} \rightarrow 0 \quad \Rightarrow \quad NI \rightarrow -\infty\ , \quad \forall\ i$$

$$g_{ii} \rightarrow -\infty \quad \Rightarrow \quad NI \rightarrow NI^i\ , \quad \forall\ i$$

It can be seen from figure 10.2 that when g_{ii} moves towards and eventually past c_{ii}, the NI will move towards zero and ultimately change its sign, and the system will become unstable under the original pairing. The 'one way' robustness also exists.

Case 3: $NI^0<0,\ g_{ii}^0>0$

This case corresponds to a $c_{ii}>g_{ii}^0$ and an unstable pairing. The NI as a function of model mismatch in g_{ii}^0 is geometrically shown in figure 10.3. The following properties of the function can be seen from figure 10.3:

$$g_{ii} \rightarrow c_{ii}^- \quad \Rightarrow \quad NI \rightarrow 0\ , \quad \forall\ i$$

$$g_{ii} \rightarrow c_{ii}^+ \quad \Rightarrow \quad NI > 0\ , \quad \forall\ i$$

$$g_{ii} \rightarrow 0 \quad \Rightarrow \quad NI \rightarrow -\infty\ , \quad \forall\ i$$

$$g_{ii} \rightarrow \infty \quad \Rightarrow \quad NI \rightarrow NI^i\ , \quad \forall\ i$$

Again, in this case when g_{ii} moves past c_{ii}, the NI will change its sign. However, the system will become stable in this case if paired accordingly. Nevertheless, pairings with negative NI's should be dismissed for nominal stability.

Case 4: $NI^0<0,\ g_{ii}^0<0$

This case corresponds to an unstable pairing with $c_{ii}<g_{ii}^0$. The behavior of the NI in terms of the change in g_{ii}^0 is shown in figure 10.4. However, as in case 3, this pairing corresponds to a negative NI and is thus unacceptable for nominal stability.

10.3.3 A Stability Robustness Measure

Primarily, the pairing decision based on a nominal model should result in a positive NI for nominal stability. From the above discussions, the distance (denoted by Δg_{ii}) of a nominally paired element, g_{ii}^0, from its critical value, c_{ii}, provides a measure of stability robustness,

$$\Delta g_{ii} \equiv c_{ii} - g_{ii}^0 = -\frac{NI^0}{NI^i}\, g_{ii}^0 \quad , \qquad \forall i \tag{10.8}$$

In terms of the relative allowable model error, the above equation can then be expressed as,

$$\frac{\Delta g_{ii}}{g_{ii}^0} = -\frac{NI^0}{NI^i} \qquad \forall i \tag{10.9}$$

or equivalently in terms of the RGA,

$$\frac{\Delta g_{ii}}{g_{ii}^{0}} = -\frac{1}{\lambda_{ii}^{0}} \quad , \quad \forall i \tag{10.10}$$

Examining eq 10.9, one can see that the smaller the NI^0 is, the smaller the allowed model perturbation in g_{ii}, and hence the less robust the system. In contrast, from eq 10.10, the larger the RGA element, the smaller the allowable model mismatch. The effect of the RGA on system stability will be discussed in detail later.

From figure 10.1 and figure 10.2, one can observe that the size of the NI changes smoothly as it approaches zero. This is shown by differentiating eq 10.7 as,

$$\frac{dNI}{NI} = \left(\frac{NI}{NI^{i}} - 1\right) \frac{dg_{ii}}{g_{ii}} \tag{10.11}$$

It is important to point out that the robust stability measure given by eq 10.9 or 10.10 only indicates the effects of a single *paired* element on stability and is only applicable to decentralized control systems.

10.4 ROBUST STABILITY AND THE RGA

From eq 10.2, both the RGA and the NI ought to change sign simultaneously in the face of gain perturbations in individual paired elements. Hence, contrary to Grosdidier et al. (1985), the switch in the sign of the RGA also implies a stability problem rather than an integrity problem. Instead, integrity remains unchanged for this case of independent gain mismatch. Unlike the NI, however, the RGA is independent of pairing. This important property allows for the generalization of the stability robustness to include non-paired gains, leading to a more promising screening tool in variable pairing for robust stability requirements. Following the same strategy as above, we can *independently* analyze the behaviour of the RGA in the face of model error in a more general way.

10.4.1 The Critical Value for the RGA

As in the case of NI, the gain value at which the changeover of the sign of the RGA element occurs, is referred to the critical value (denoted by c_{ij}) of the plant gain g_{ij} associated with the RGA element λ_{ij}. Here c_{ij} corresponds to an infinite value of λ_{ij} (see later), which can be calculated from RGA element as,

$$c_{ij} = \left(1 - \frac{1}{\lambda_{ij}}\right) g_{ij} \quad , \quad \forall i, j \qquad (10.12)$$

(see Grosdidier et al., 1985). Note that c_{ij} is evaluated at the nominal value of g_{ij} and λ_{ij}.

For diagonal elements eq 10.12 reduces to eq 10.6, i.e., both the NI and the RGA share the same critical value and change sign at the same time. Hence, eq 10.12 can be used as a unified definition of critical value (for both the RGA and NI) corresponding to any gain g_{ij}. We call the matrix consisting of all the elements defined by eq 10.12 *'Critical Value Array'* (CVA).

10.4.2 λ_{ij} *as a Function of Model Errors*

Although it shares the same critical value with NI, λ_{ij}, however, as a function of perturbations in individual gains behaves quite differently. We consider the following cases (assume the system is nominally stable):

<u>Case 1: $\lambda_{ij}^0 > 0$, $g_{ij}^0 > 0$</u>

This case corresponds to $g_{ij}^0 > c_{ij}$ and is geometrically depicted in figure 10.5. The following features can be extracted from figure 10.5:

$$g_{ij} \to c_{ij}^- \Rightarrow \lambda_{ij} \to \infty , \quad \forall\ i, j$$

$$g_{ij} \to c_{ij}^+ \Rightarrow \lambda_{ij} \to -\infty , \quad \forall\ i, j$$

$$g_{ij} \to 0 \Rightarrow \lambda_{ij} \to 0 , \quad \forall\ i, j$$

$$g_{ij} \to \infty \Rightarrow \lambda_{ij} \to 1 , \quad \forall\ i, j$$

Most notably, λ_{ij} is a discontinuous function of g_{ij}, and it changes its value from positive infinity to negative infinity when g_{ij} changes from its nominal value, g_{ij}^0, towards and eventually past its critical value. Clearly, 'One way' robustness is also present here.

<u>Case 2: $\lambda_{ij}^0 > 0$, $g_{ij}^0 < 0$</u>

This case corresponds to $g_{ij}^0<c_{ij}<0$ and is portrayed in figure 10.6. The following characteristics can be observed:

$$g_{ij} \to c_{ij}^- \Rightarrow \lambda_{ij} \to \infty , \quad \forall\ i, j$$

$$g_{ij} \to c_{ij}^+ \Rightarrow \lambda_{ij} \to -\infty , \quad \forall\ i, j$$

$$g_{ij} \to 0 \Rightarrow \lambda_{ij} \to 0 , \quad \forall\ i, j$$

$$g_{ij} \to -\infty \Rightarrow \lambda_{ij} \to 1 , \quad \forall\ i, j$$

Also, λ_{ij} is a discontinuous function of g_{ij} and it drops from a positive infinite value to a negative infinite value. Again, when g_{ij} moves beyond its critical value the RGA will change sign.

Case 3: $\lambda_{ij}^{0} < 0,\ g_{ij}^{0} > 0$

This case, shown in figure 10.7 with $c_{ij} > g_{ij} > 0$, has no practical significance since nominal integrity is not satisfied.

Case 4: $\lambda_{ij}^{0} < 0,\ g_{ij}^{0} < 0$

This case is depicted by figure 10.8 with $c_{ij} < g_{ij}$. As in case of 3, nominal integrity is also not satisfied here.

10.4.3 Stability Robustness via RGA

In all cases discussed above, the critical value, c_{ij}, shows the direction, while the distance Δg_{ij} below defines the maximum magnitude of the model error for RGA to change sign. From eq 10.12, Δg_{ij} can be calculated as follows,

$$\Delta g_{ij} \equiv c_{ij} - g_{ij}^{0} = -\frac{g_{ij}^{0}}{\lambda_{ij}^{0}} \quad , \quad \forall i,\ j \qquad (10.13)$$

or in the form of relative model error,

$$\frac{\Delta g_{ij}}{g_{ij}^{0}} = -\frac{1}{\lambda_{ij}^{0}} \quad , \quad \forall\ i,\ j \qquad (10.14)$$

Applying eq 10.14 to paired elements leads to eq 10.10.

Hence, the RGA also provides a measure of stability robustness, subject to nominal stability. Note that sign change in NI indicates a stability problem in face of any gain uncertainties, while the sign of the RGA only applies to model mismatch in a single gain. As a result, eq 10.14 can be used as stability robustness measure only in case of paired elements. However, it is a powerful tool to screen control structures and variable pairings for robustness due to the calculational simplicity of the RGA. We call the matrix consisting of all the elements defined by eq 10.14 *'Stability Robustness Array'* (SRA).

From eq 10.14 and figures 10.5 to 10.8, it can be clearly seen that the larger the RGA element, the more sensitive it is to model errors. In particular, when g_{ij} is near its critical value, both λ_{ij} and its sensitivity approach infinity. This provides a geometrical justification for the following equation by Grosdidier et. al. (1985) and Yu and Luyben (1987):

$$\frac{d\lambda_{ij}}{(1 - \lambda_{ij})\,\lambda_{ij}} = \frac{dg_{ij}}{g_{ij}} \qquad (10.15)$$

10.4.4 Comparison with Previous Results

Although the stability robustness measure (eq 10.14) coincides with the Integral Robustness Array given by Yu and Luyben (1987), there are some important differences between them.

Yu and Luyben conjectured an infinite value of RGA as a limit for IC without a strong theoretical argument. In fact, the magnitude of the RGA is not directly related to the IC. Grosdidier et al. (1990) proposed an approximate bound for model error for robust IC in terms of the RGA, but the condition is only sufficient (but not necessary) for stability. Skogestad and Morari (1987) also presented conditions on model error which are directly related only to robust *singularity* of the process. This chapter has shown that the NI and RGA change sign when RGA approaches infinite magnitude. This provides a theoretical justification for the use of an infinite RGA as a stability index. However, the measure only applies to single gain mismatch.

The directionality of stability robustness and the applicability to only paired elements were obscured in Yu and Luyben's derivation, which limits the implications for control purposes. Also contrary to Yu and Luyben, both the RGA and the NI are indexes only for decentralized systems, just as the robustness measure is.

Another significant advantage of our approach is that the principle is easily extended to multiple model uncertainties as shown below.

10.5 EXTENSION TO MULTIPLE UNCERTAINTY

The robustness measure in eq 10.14 applies only to independent individual gain mismatch. In practice a system may not become unstable in the presence of individual gain mismatch due to, for example, multiple correlated gain errors.

Take 2x2 systems for example, with the possibility to extend to general cases, the NI can be calculated as,

$$NI = 1 - \frac{g_{12}g_{21}}{g_{11}g_{22}} \tag{10.16}$$

Upon expressing each perturbed gain in terms of the relative error δ_{ij} as $g_{ij}(1+\delta_{ij})$, the NI can then be related to multiple gain error by

$$NI = 1 - (1 - \frac{1}{\lambda_{11}^{o}}) \frac{(1+\delta_{12})(1+\delta_{21})}{(1+\delta_{11})(1+\delta_{22})} \tag{10.17}$$

Assuming that the system possesses nominal stability and integrity, we can see from eq 10.17 that if and only if

$$(1 - \frac{1}{\lambda_{11}^{o}}) \frac{(1+\delta_{12})(1+\delta_{21})}{(1+\delta_{11})(1+\delta_{22})} > 1 \tag{10.18}$$

the system will become unstable. The above equation gives a condition for robust stability in face of any model mismatch in some or all gains. Trivially, condition 10.18 reduces to eq 10.14 in case of single gain mismatch.

For systems with small RGA, particularly when $\lambda_{11} \leq 1$, eq 10.18 will never be satisfied for $|\delta_{ij}| < 1$ which is usually true. The closer to 1 the RGA, the more easily the system is able to tolerate a combination of gain errors.

On the other hand, when RGA is large (>>1) condition 10.18 becomes

$$\frac{(1+\delta_{12})(1+\delta_{21})}{(1+\delta_{11})(1+\delta_{22})} > 1 \qquad (10.19)$$

the condition may be easily satisfied, for instance, when uncertainties in off-diagonal gains exceed that in diagonal gains (assume all the errors are positive).

Often, a system may not be particularly prone to instability for some highly correlated gain uncertainties. This is clarified by checking condition 10.19, for example, when δ_{12} and δ_{11} or δ_{21} and δ_{22} undergoes the same change, or δ_{12} and δ_{21} take on the same value in the opposite direction, their effects can be cancelled out. However, the NI can still change sign in the presence of other correlated model errors.

The usefulness of the measure in eq 10.14 as a screening rule in pairing and structure selection remains, since pairings containing large RGA elements are more prone to instability. Notice that only the significance of paired gains is directly addressed by the measure in eq 10.4. However, model mismatch in gains other than those paired might also induce instability due to the fact that each row and column of RGA has to sum up to unity. In particular, for 2x2 systems each gain has an equal impact on stability. Therefore, particular attention should be paid to all the sensitive gains, i.e, those corresponding to large RGA values.

Note that variable pairing for stability is independent of controller design, and the only way to stabilize a system when instability is induced, is to reverse controller direction (this will cause other problems, see Zhu and Jutan, 1992). The primary focus of this chapter is on robust stability which would often be followed by a performance analysis. This second stage is not addressed here.

10.6 EXAMPLES

10.6.1 Example 1 (Skogestad et al., 1990a)

This example shows how to select control structure for the distillation column (column A) studied by Skogestad et al. (1990a) from the consideration of stability robustness. Since this is a 2x2 system, the study of the (1,1) element of the RGA, CVA, and SRA is sufficient. The RGA and SRA values for various structures are shown in table 10.1. Note that in table 1, L, V, D, L/D and B represent the reflux, boilup, distillate product flow, the ratio of L and D, and bottom product flow respectively.

From table 10.1, we can see that LV structure should be avoided since this structure can only tolerates 2.85% model error for the system to preserve stability. The DV scheme should be selected for stability robustness since it permits a maximum model error of 222%. Skogestad and Morari (1987), and Yu and Luyben (1987) also suggested the same structure.

Stability robustness can be demonstrated through dynamic simulations for LV structure using the following model (Skogestad and Morari, 1987; Skogestad et el., 1990a),

$$G(s) = \begin{bmatrix} \frac{0.878}{3.23s+1} & \frac{-0.878}{3.23s+1} + \frac{0.014}{0.25s+1} \\ \frac{1.082}{3.23s+1} & \frac{-1.082}{3.23s+1} - \frac{0.014}{0.25s+1} \end{bmatrix}$$

Note that in above model, the gains have been scaled, all time constants have a unit of hours. The decentralized PI controller designed for this system with diagonal pairing is

$$G_c(s) = \begin{bmatrix} \frac{50(s+90)}{s} & 0 \\ 0 & \frac{-60(s+40)}{s} \end{bmatrix}$$

The dynamic responses (y1, y2) under unit step change in the first setpoint (r1) for the nominal process are shown in figure 10.9. We can see from figure 10.9 that the system performs well. The responses of the system under the same step change for the process with 10% model error in g_{11} (which exceeds the allowable error of 2.85% in table 10.3) are shown in figure 10.10. It can be seen that now the system becomes unstable.

10.6.2 Example 2 (Gagnepain and Seborg, 1982)

This example demonstrates the variable pairing decision using stability robustness considerations. Consider the 3x3 system studied in Gagnepain and Seborg (1982)
The process gain matrix, RGA, and SRA are shown in table 10.2. From the RGA and NI (not shown), two pairing alternatives possess nominal stability and integrity and deserve further

$$G(s) = \begin{bmatrix} -\frac{2}{10s+1} & \frac{1.5}{5s+1} & \frac{1}{s+1} \\ \frac{1.5}{5s+1} & -\frac{1}{s+1} & \frac{2}{10s+1} \\ \frac{1}{s+1} & \frac{2}{10s+1} & \frac{1.5}{5s+1} \end{bmatrix}$$

considerations: (1,1)-(2,3)-(3,2) and (1,3)-(2,2)-(3,1). From table 10.2, it can be seen that the system under the first pairing can tolerate 133% (SRA=-1.33) gain error in each paired gain element while the system under the second pairing allows 364% model error in each paired gain. Both system are quite robust against plant gain mismatch. However, the second pairing option is preferred from robust stability considerations. Gagnepain and Seborg (1982) also suggested the second pairing based on the average relative gain analysis. Arkun (1988) also favored the second pairing on the basis of relative sensitivity analysis.

10.7 FINAL REMARKS

Conventionally, control structure selection and variable pairing choice are determined solely on the basis of a nominal plant gain matrix. However, the resulting system may easily lose stability in the face of inherent model errors. From theoretical analysis of the characteristics of the NI and RGA as a function of individual process gain errors, a stability robustness measure consisting of a Stability Robustness Array (SRA) and a Critical Value Array (CVA) has been developed. The SRA defines the maximum model error, whereas the CVA defines the directionality of the error. Theoretical justification is provided based on stability implications of the NI. This provides important new insights into robust stability and allows a simple extension to multiple gain uncertainties.

Table 10.1 Robust stability of a distillation column (Skogestad et al., 1990)

	LV	DV	(L/D)V	DB
RGA, λ_{11}	35.1	0.45	5.85	∞
RSA, r_{11}	-2.85%	-222%	-17.1%	*

Table 10.2 Gagnepain and Seborg (1982) system

G(0)	-2.0	1.5	1.0
	1.5	-1.0	2.0
	1.0	2.0	1.5
RGA	0.7521	-0.0256	0.2735
	-0.0256	0.2735	0.7521
	0.2735	0.7521	-0.0256
SRA	-1.33	39.06	-3.64
	39.06	-3.64	-1.33
	-3.64	-1.33	39.06

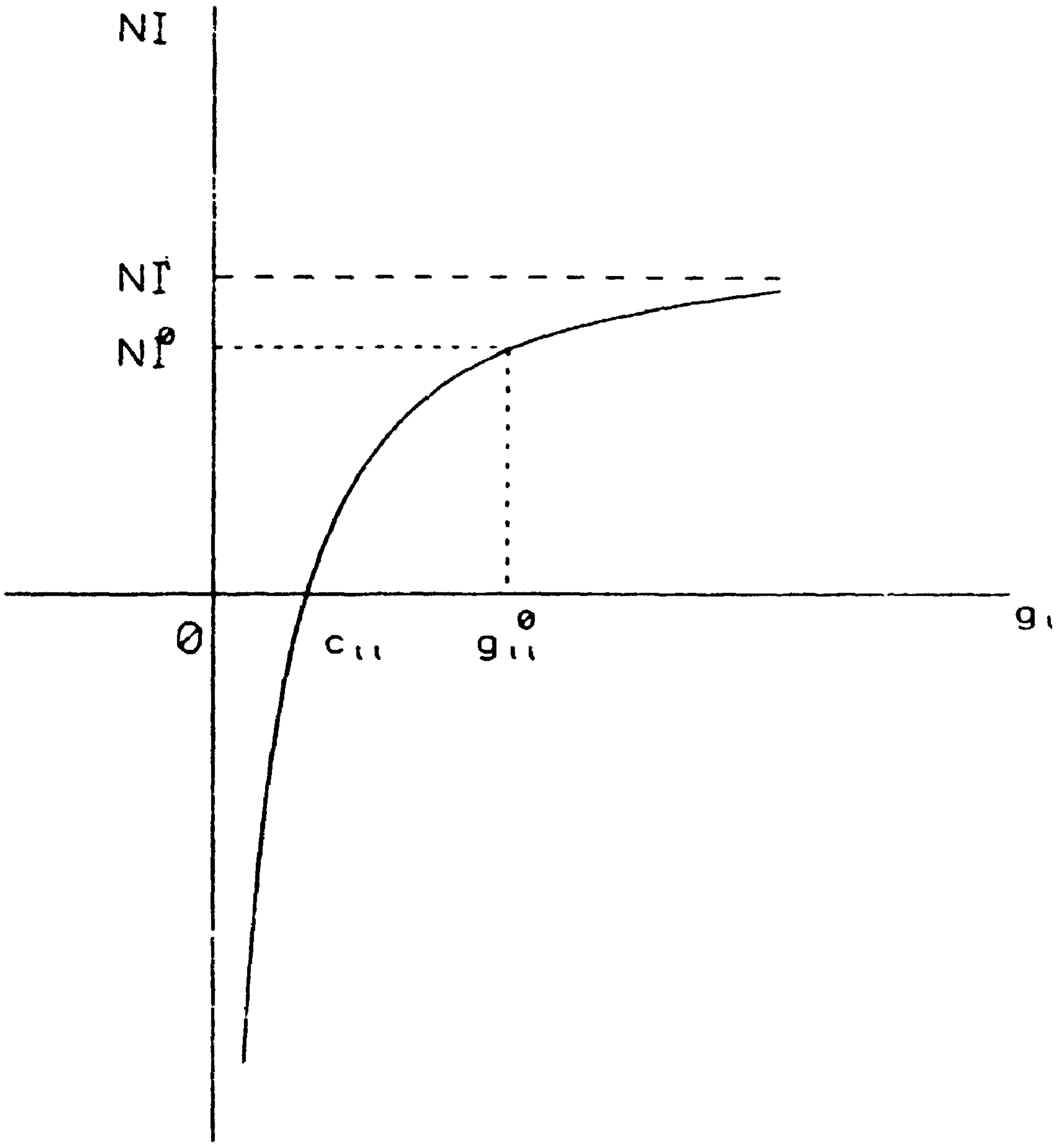


Figure 10.1 NI vs model error: case 1

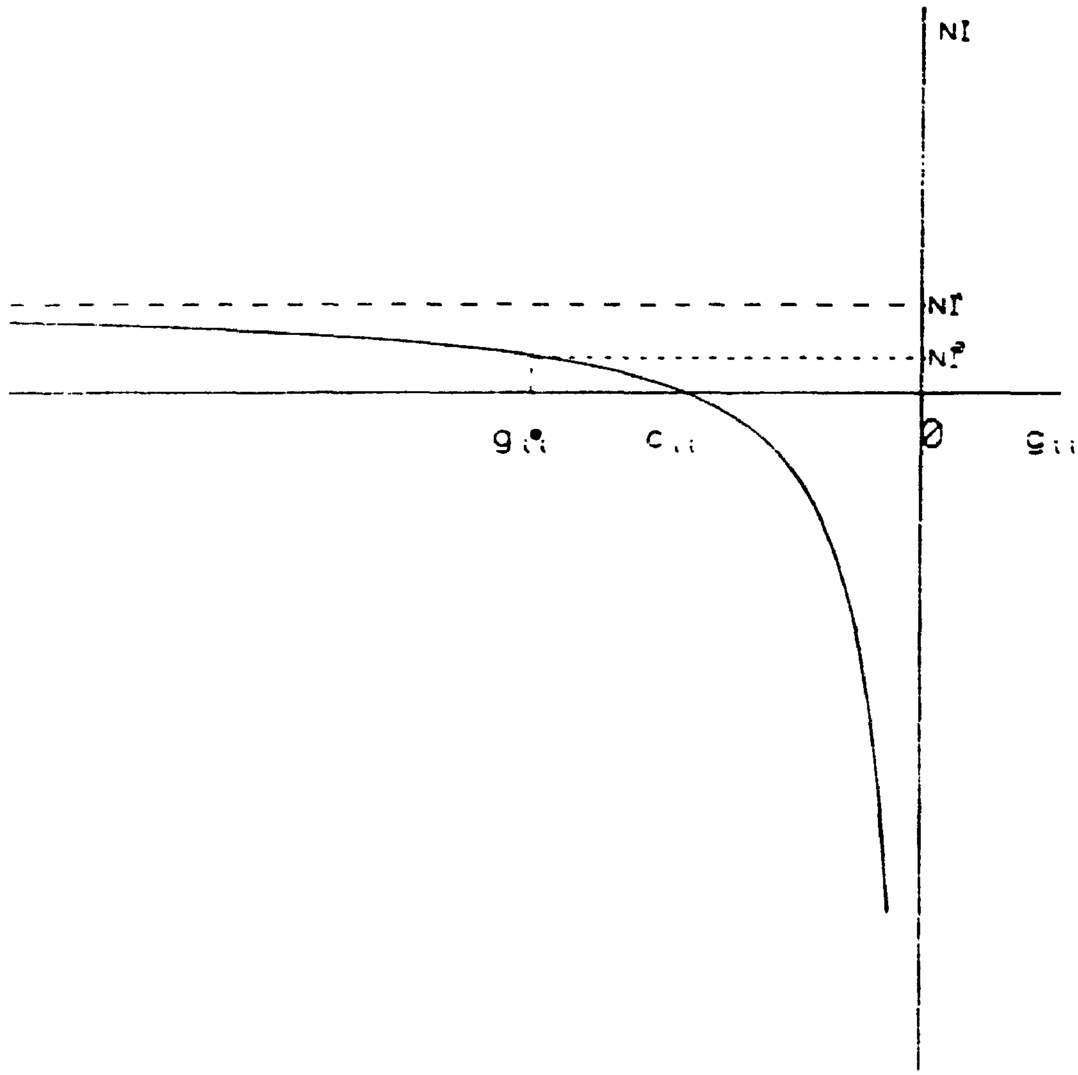


Figure 10.2 NI vs model error: case 2

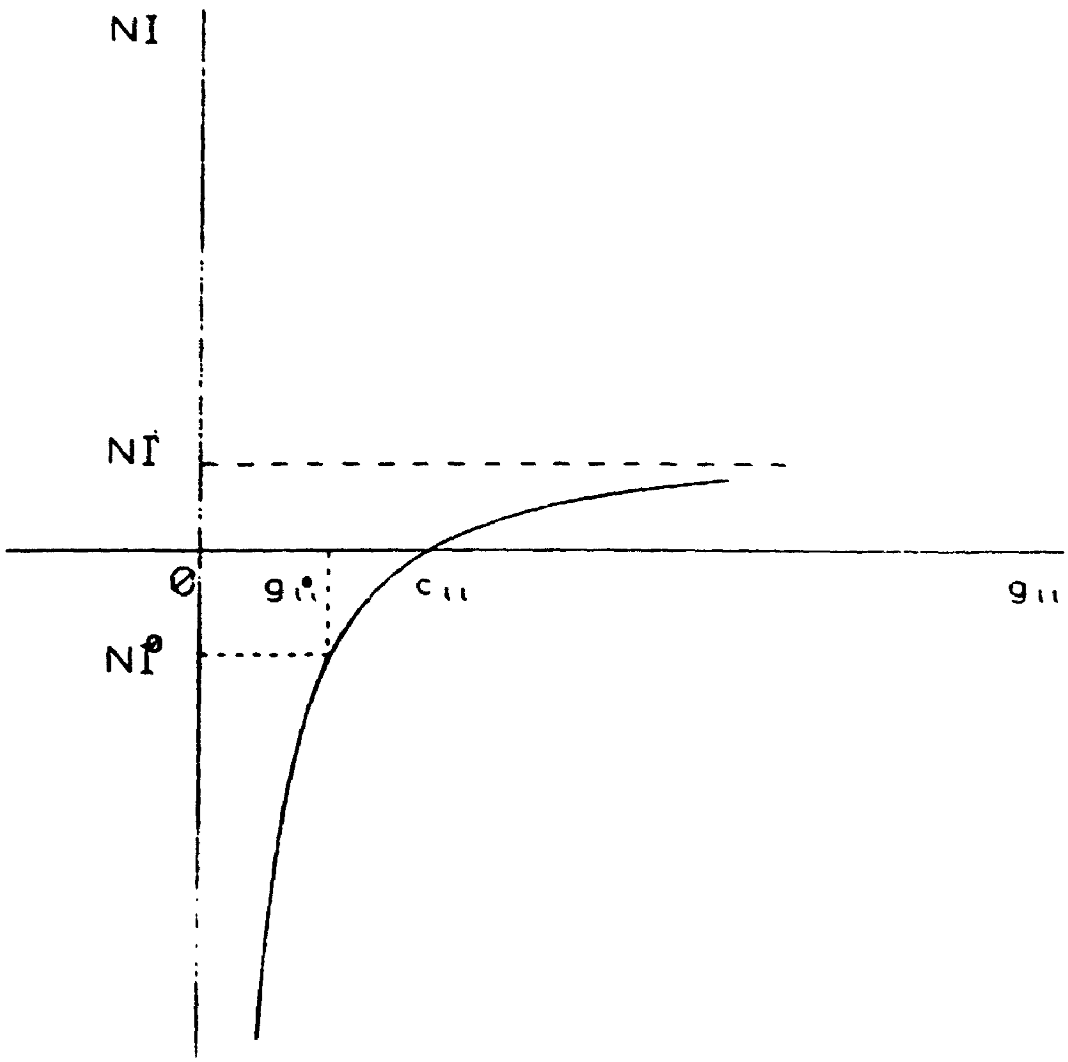


Figure 10.3 NI vs model error: case 3

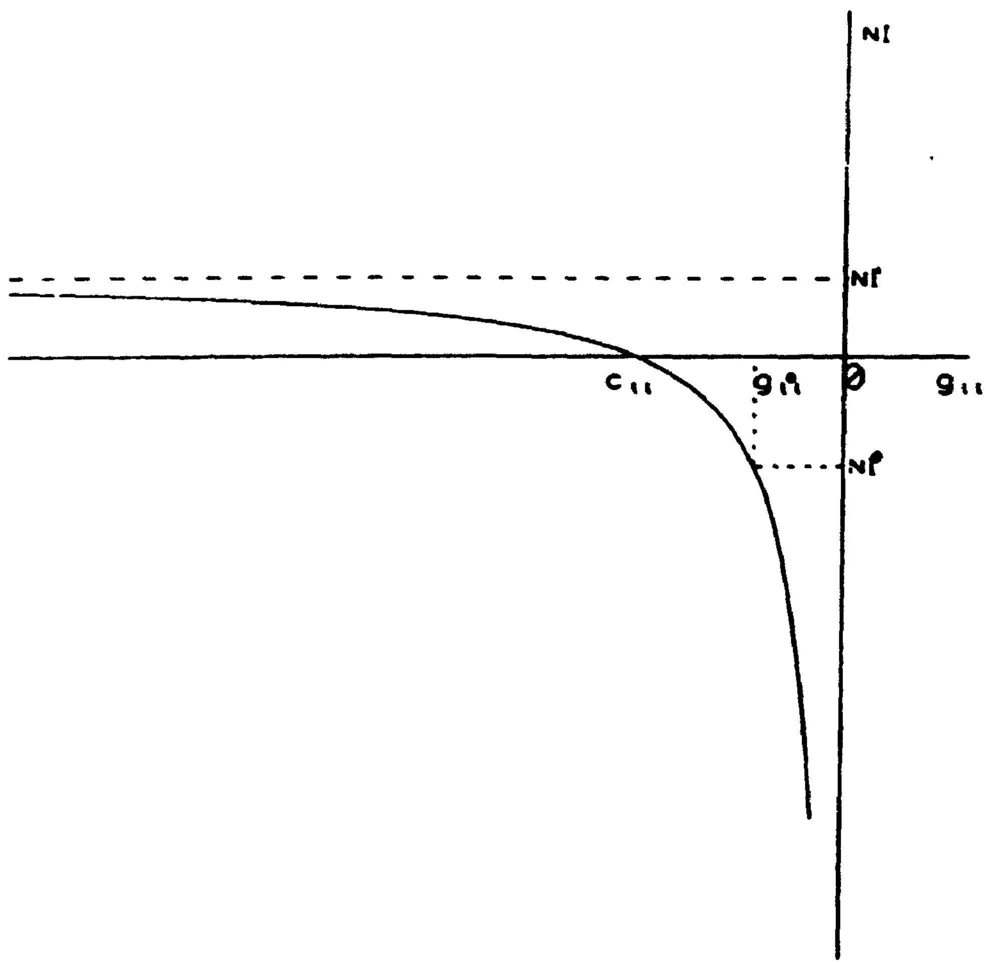


Figure 10.4 NI vs model error: case 4

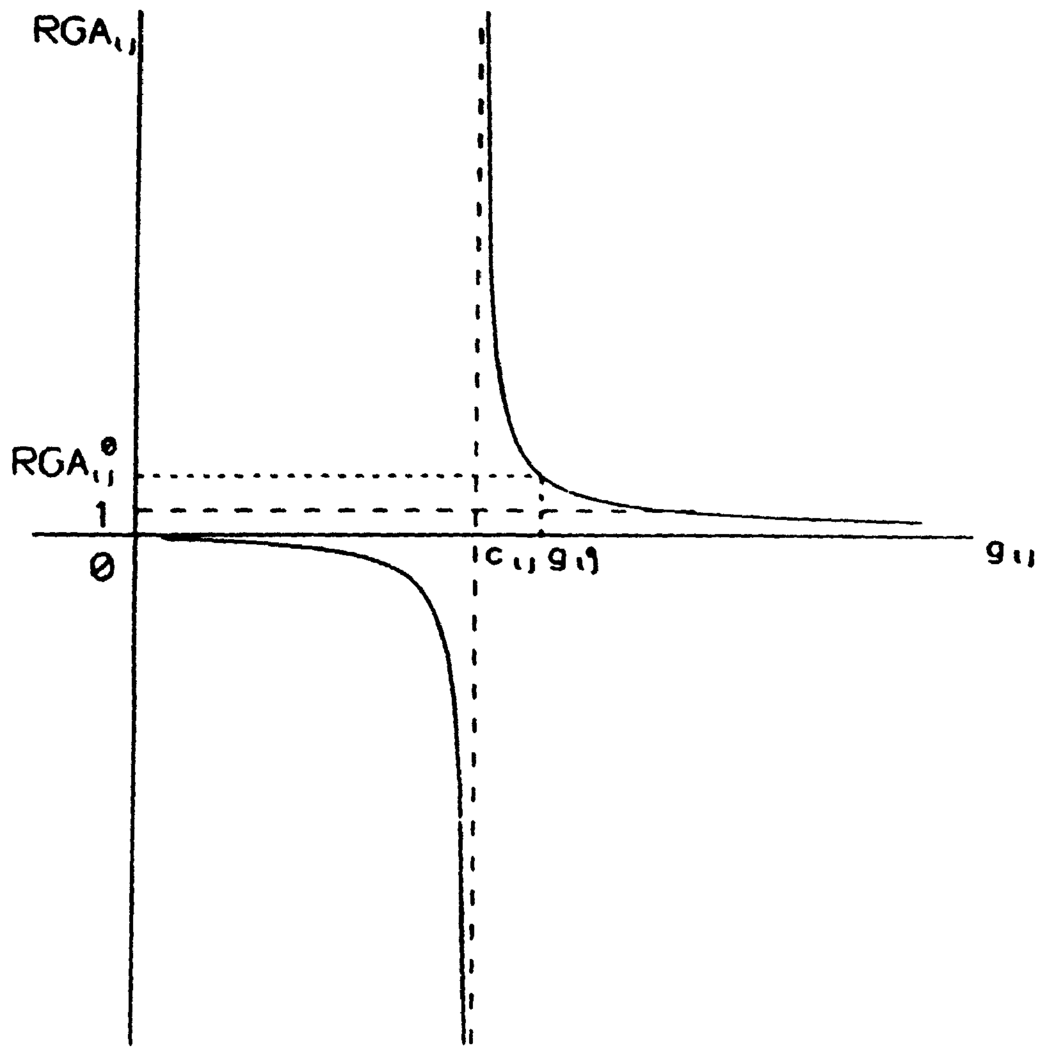


Figure 10.5 RGA vs model error: case 1

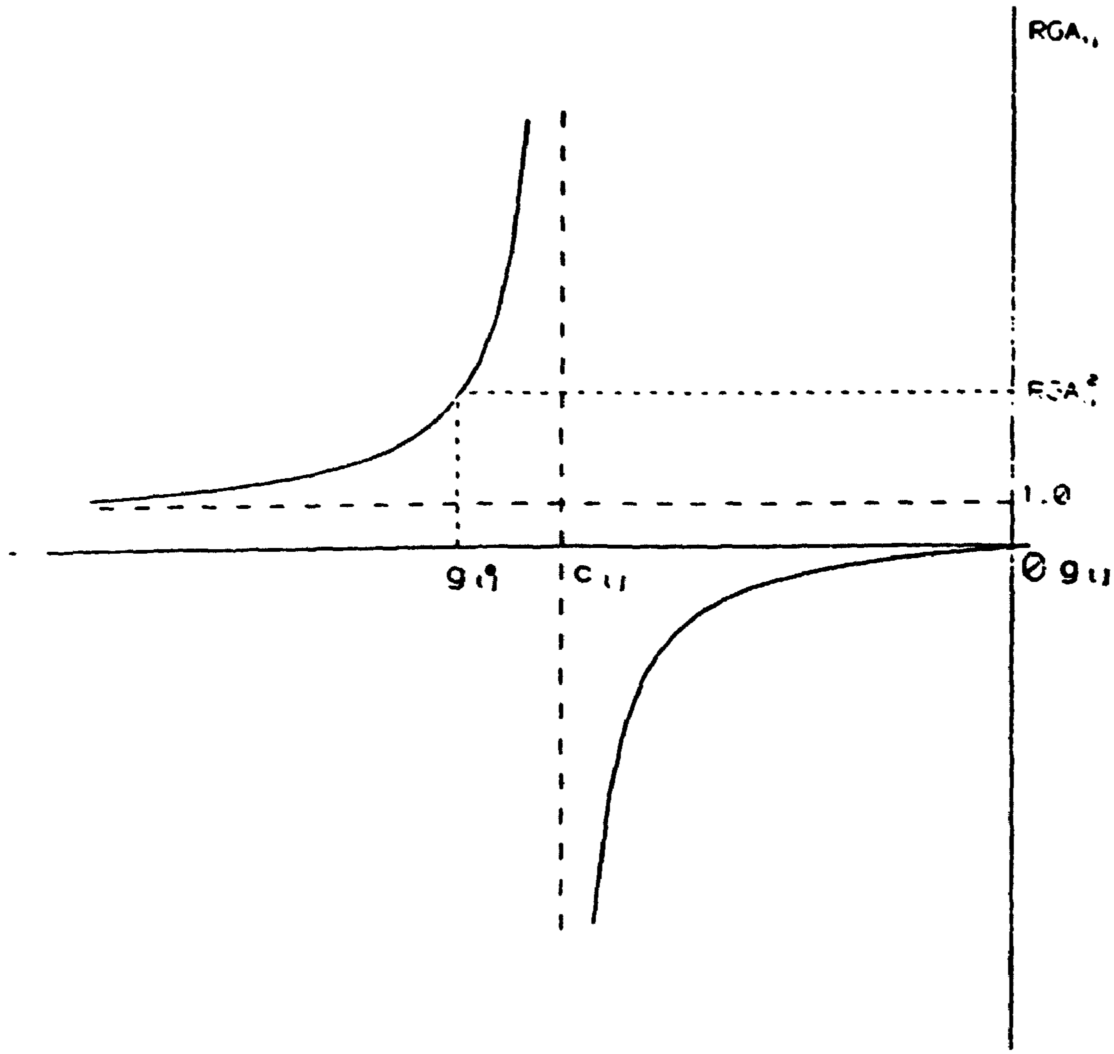


Figure 10.6 RGA vs model error: case 2

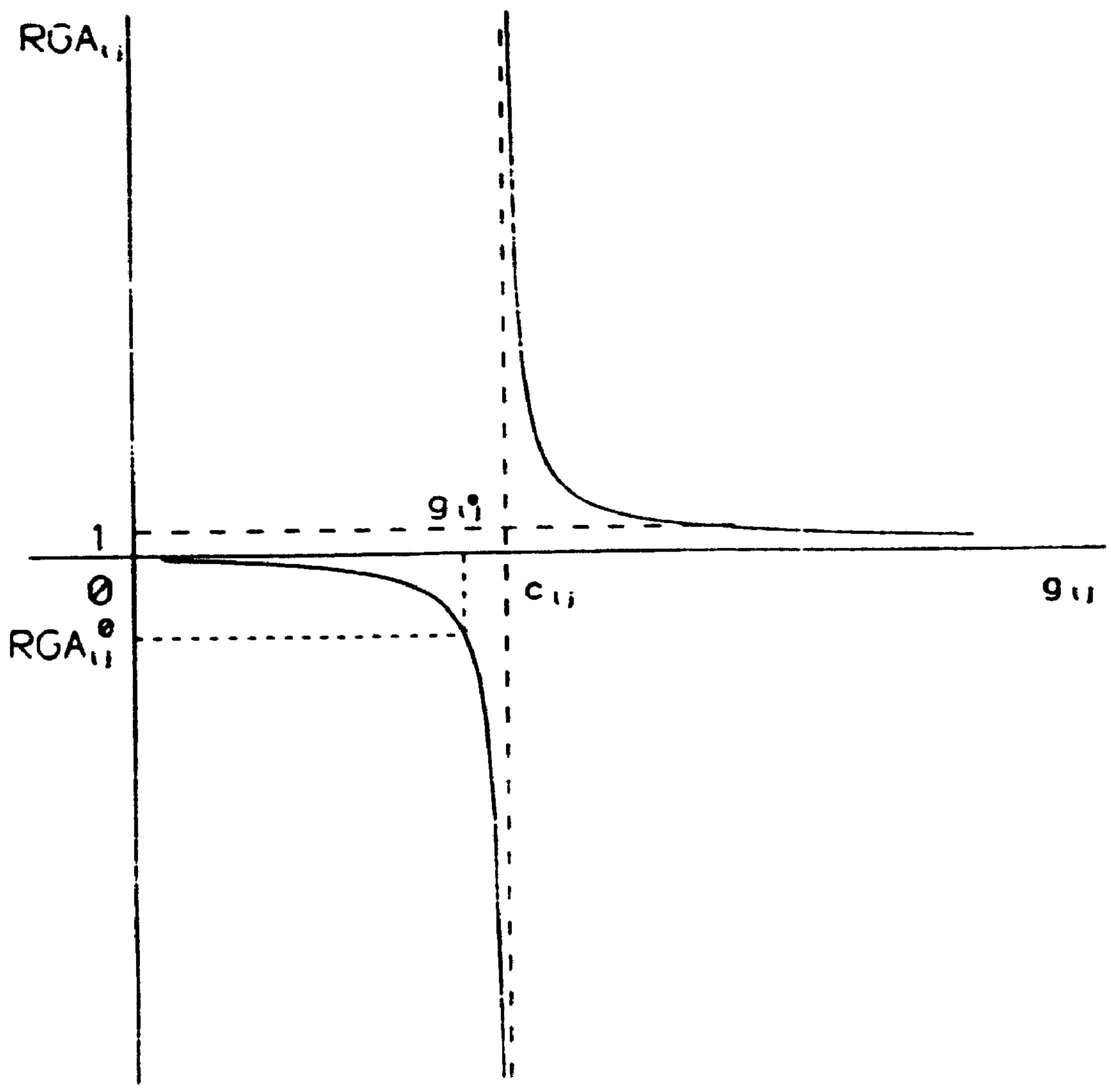


Figure 10.7 RGA vs model error: case 3

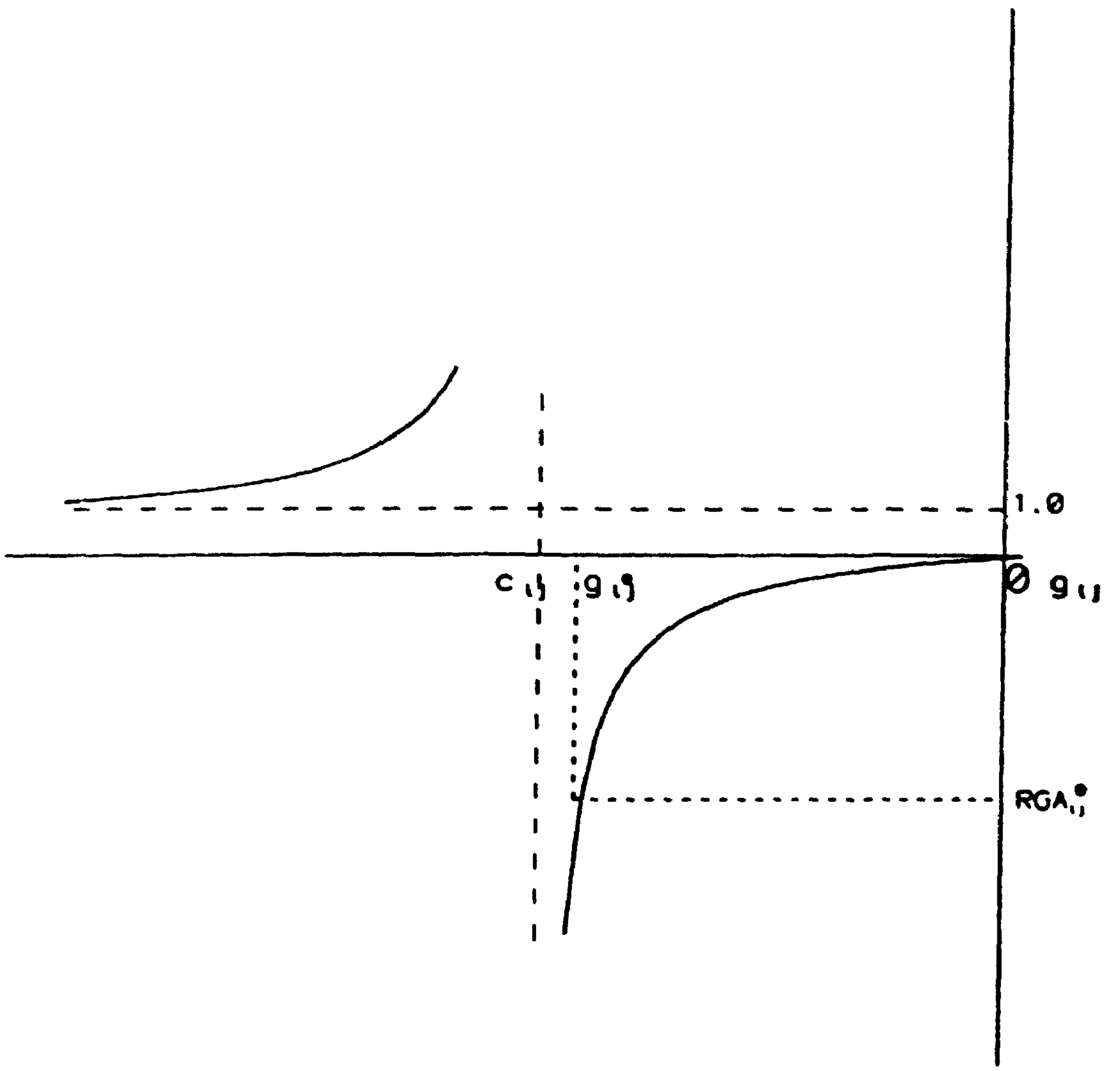


Figure 10.8 RGA vs model error: case 4

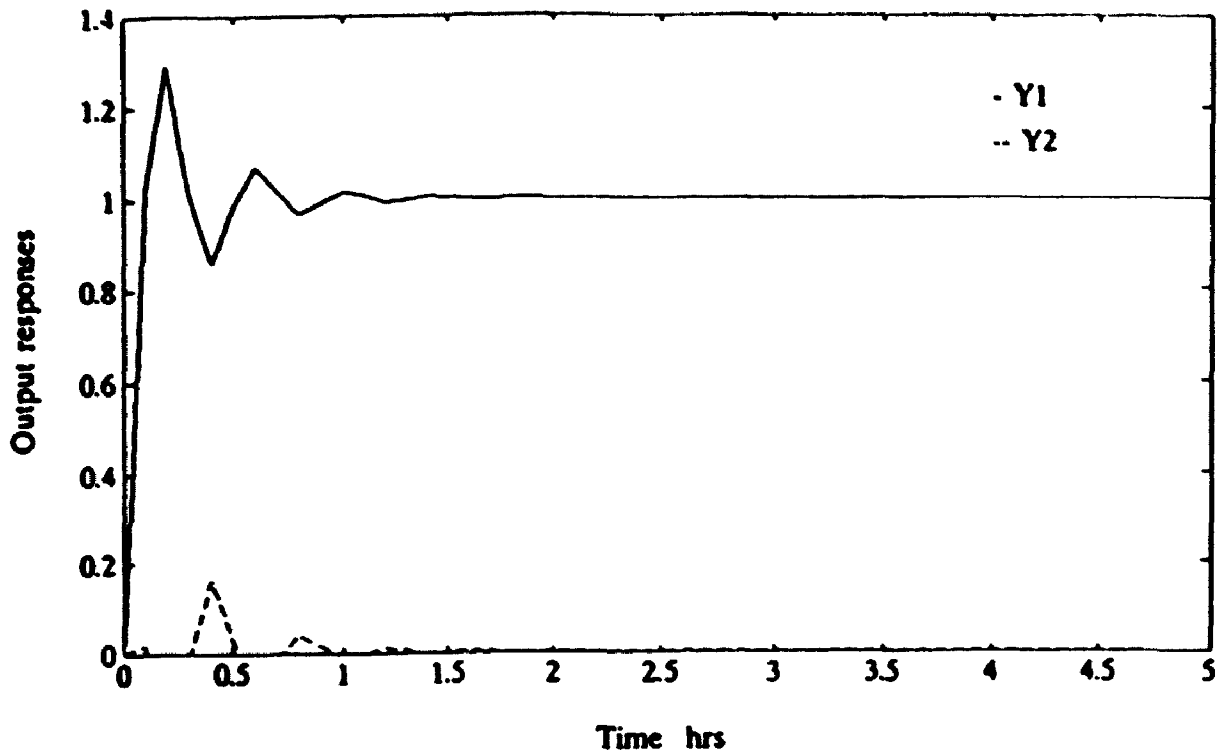


Figure 10.9 Nominal response to step change in r1

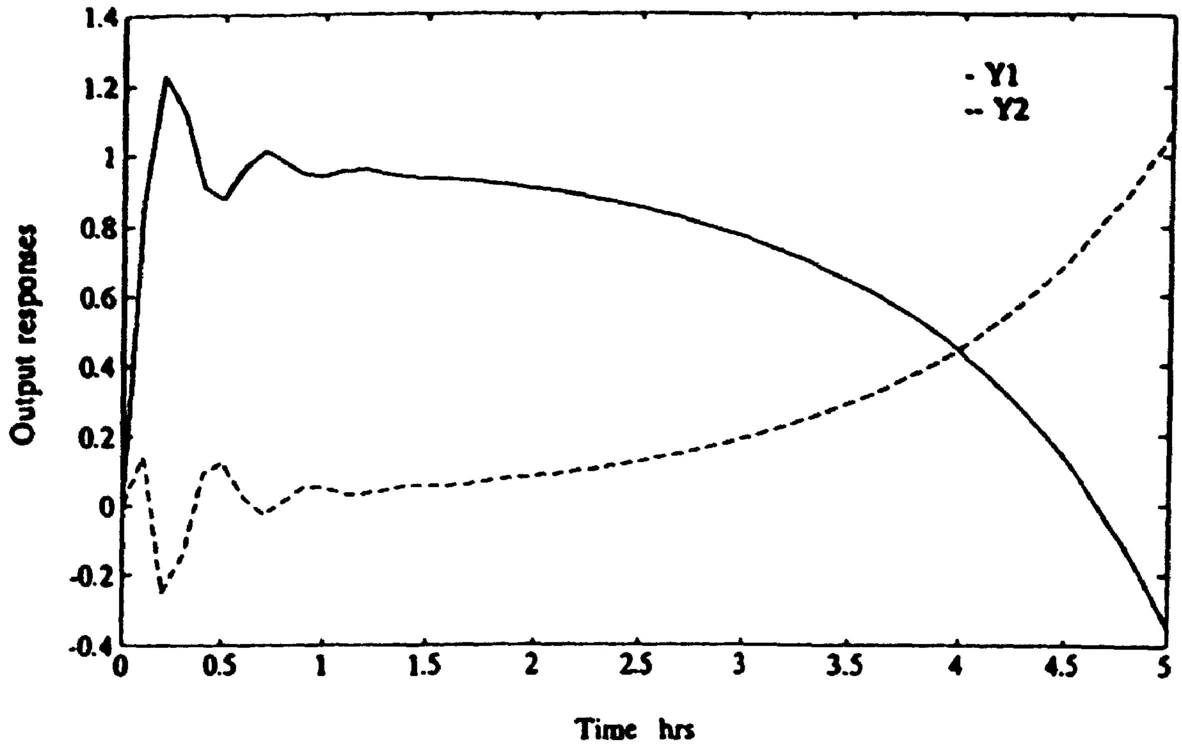


Figure 10.10 Response with 10% error in g_{11}

CHAPTER 11

ROBUST MULTIVARIABLE CONTROL AGAINST MODEL UNCERTAINTY AND DISTURBANCE DIRECTIONALITY

Synopsis Robust performance is addressed for multivariable control systems in face of strong disturbance directionality and model uncertainty associated with manipulated variables. New robustness indices applicable to any multivariable system are developed. These indices are then used to propose a systematic design strategy which uses the best compromise for robust control against both disturbance directionality and model error. The proposed design scheme is evaluated with examples and shown to be more effective than existing methods.

11.1 INTRODUCTION

Conventional decentralized control and model-based (inverse-based) control have gained wide acceptance in process control. Robustness is usually taken into account as an important issue in these designs. Nevertheless, in decentralized control research work has been mainly focused on *robust stability* rather than robust performance (Grosdidier et al., 1985; Yu and Luyben, 1987; Zhu and Jutan, 1992), and in most model-based control techniques, robust performance is considered only as an additional specification after the design rather than in the formation of design methodology. General robust control theory has provided engineers with very useful tools for explicitly accounting for process model uncertainty in control system design (Morari and Zafiriou, 1989). However, the quantification of model uncertainty remains a difficult task (Goberdhansingh et al., 1992), and the complexity involved in the design is discouraging to control engineers. Clearly, combining general robust control theory with decentralized control and model-based control, within an 'engineering framework', is of great practical importance.

In process control a major model uncertainty arises from the operational error in manipulated variables (Skogestad and Morari, 1987a). Skogestad and Morari (1987a) presented an *approximate* method to evaluate the robustness of different controller structures against this uncertainty by means of the Relative Gain Array [(RGA), Bristol(1966)]. Following this idea, Brambilla and D'Elia (1992) used a RGA-based sensitivity index to model errors for robust design of a multivariable controller they proposed. Unfortunately, this RGA-based criterion is applicable only to 2x2 systems since it is virtually impossible to derive an analytical relationship for higher order systems. More importantly, the design index may lead to misleading conclusions (see later in this chapter).

Another important uncertainty in multivariable process control stems from closed-loop input (disturbances or setpoints) directionality. Skogestad and Morari (1987b) introduced a 'disturbance condition number' to characterize closed-loop robustness against disturbance directionality. Nevertheless, only the steady state case has been addressed. In addition, no direct guidance is available for controller design to improve system robustness in face of strong disturbance directionality. Brambilla and D'Elia (1992) instead, used the RGA as a criterion for the design of their proposed controller. However, this design is restricted to 2x2 systems and, more importantly, it may result in unreliable design since large RGA is only a necessary (not sufficient) condition for a large condition number (Nett and Manousiouthakis, 1987).

In this chapter, robust performance against model uncertainty associated with manipulated variables under general multivariable, inverse-based, and decentralized controllers is systematically analyzed. A robustness index to evaluate the sensitivity of the closed-loop system to model mismatch is presented. A more reliable disturbance directionality index independent of controller, further to the analysis by Grosdidier (1990), is also provided. It is shown that both indices can be directly applied to the robust design of multivariable controllers, such as the one by Brambilla and D'Elia (1992), so that the best compromise between the robustness against model error and robustness against disturbance directionality can be readily achieved. Design procedures to achieve steady state and dynamic robust performance in general nxn multivariable systems are proposed. Examples demonstrate that the new design approach is not only more general but also more reliable than previous methods.

11.2 ROBUSTNESS AGAINST DISTURBANCE DIRECTIONALITY

In multivariable systems shown in Figure 11.1, the tracking error (e) is related to setpoints (r) and disturbance acting on plant output (d) through the sensitivity function S as

$$e = S(s)\,(r-d) \tag{11.1}$$

where

$$S(s) = [I+G(s)C(s)]^{-1} \tag{11.2}$$

where G(s) and C(s) denote the process and controller transfer function matrix respectively. Clearly the sensitivity operator can be used as an indication of closed-loop performance. Since disturbances and setpoints affect the error in a similar manner (see equation (11.1)) and are outside the control loop (see figure 11.1), they can be regarded as 'closed-loop inputs'. Without loss of generality, our subsequent analysis will focus only on the effects of disturbances.

Usually control systems are designed and tuned under certain assumed pattern of disturbances or setpoint changes (mostly step changes associated with one or several outputs). However, system performance may deteriorate drastically in practice due to the difference in the type, or more generally the directionality, of closed-loop inputs (Grosdidier, 1990; Brambilla and D'Elia, 1992).

11.2.1 Disturbance Directionality

The directionality in a multivariable control system implies that closed-loop gain changes with the type of inputs, and it can be analyzed using the SVD technique (Skogestad and Morari, 1987b; Brambilla and D'Elia, 1992). From Figure 11.1 the output y in response to disturbance d is given by

$$y = S(s)\,d \qquad (11.3)$$

The amplification factor of the closed-loop system to d is described by

$$\alpha = \frac{\|y\|}{\|d\|} = \frac{\|S(j\omega)\,d\|}{\|d\|}, \qquad \forall\,\omega \qquad (11.4)$$

where $\|v\|$ denotes the 2-norm of vector v and ω denotes frequency. The above equation normalizes output magnitude for the disturbance, so the amplification factor depends only on the direction of disturbance vector. It is desirable to reduce the effect of disturbance directionality, i.e., to achieve robust performance in face of disturbance uncertainty. Thus, the ratio (γ) between the largest and smallest amplification factor, which can be derived as

$$\gamma[S(j\omega)] = \frac{\bar{\sigma}[S(j\omega)]}{\underline{\sigma}[S(j\omega)]}, \qquad \forall\ \omega \tag{11.5}$$

where $\bar{\sigma}$ and $\underline{\sigma}$ denote the maximum and minimum singular value, should be minimized. It can also be verified that

$$\gamma[S(j\omega)] = \gamma[S^{-1}(j\omega)] = \frac{\bar{\sigma}[S^{-1}(j\omega)]}{\underline{\sigma}[S^{-1}(j\omega)]}, \qquad \forall\ \omega \tag{11.6}$$

$\gamma(S^{-1}(j\omega))$ is the condition number of the closed loop system and serves as a closed-loop directionality index.

11.2.2 Steady State Disturbance Directionality

At low frequencies, particularly at steady state, the controller gain is high due to the integral action usually required, and equation (11.6) can be simplified as,

$$\gamma(S^{-1}) \approx \gamma(G_0 C_0) = \frac{\bar{\sigma}(G_0 C_0)}{\underline{\sigma}(G_0 C_0)} \tag{11.7}$$

where G_0 and C_0 represent the value of G(s) and C(s) evaluated at steady state. The steady state closed-loop condition number is basically the same as the 'disturbance condition number' proposed by Skogestad and Morari (1987b) and serves then as a steady state disturbance directionality index.

$\gamma(G_0C_0)$ can be used as a preliminary criterion for robust performance against closed-loop input uncertainty before the more complex dynamic index (equation (11.6)) is used. In fact, steady state requirement is frequently sufficient in this regard, since most systems show the highest condition number at low frequencies. Figure 11.2 demonstrates this by displaying condition number as a function of frequency for the LPG splitter (Brambilla and D'Elia, 1992) described by

$$G(s) = \begin{bmatrix} \dfrac{-23.8e^{-11.4s}}{60s+1} & \dfrac{32.8e^{-8.8s}}{73s+1} \\ \dfrac{4.8e^{-5s}}{15.4s+1} & \dfrac{-8.3e^{-6s}}{14s+1} \end{bmatrix} \tag{11.8}$$

and the DV distillation process (derived from the LV process according to Skogestad et al., 1990) governed by

$$G(s) = \begin{bmatrix} \dfrac{-87.8}{194s+1} & \dfrac{1.4}{15s+1} \\ \dfrac{-108.2}{194s+1} & \dfrac{-1.4}{15s+1} \end{bmatrix} \tag{11.9}$$

Different controller structures differ in robustness against disturbance direction. For inverse-based controller ($C(s)=G^{-1}(s)$), we have $RGA[GC(s)]=I$ and $\gamma[GC(s)]=1$, i.e., this controller structure is capable of completely removing disturbance directionality. Here the plant model is assumed to be perfect. Model mismatch is to be considered separately later in this chapter. On the other hand, with a decentralized controller (denoted by $\bar{C}(s)$), the RGA of the loop gain is the same as that of the process, i.e., $RGA[G\bar{C}(s)]=RGA[G(s)]$ (RGA is independent of scaling). The RGA is closely related to the condition number by (Skogestad and Morari, 1987b; Nett and Manousiouthakis, 1987)

$$\gamma(G) \geq \|RGA\|_{m} - 1 \qquad (11.10)$$

and

$$\|RGA\|_{m} = 2\max\left(\|RGA\|_{i1}, \|RGA\|_{i\infty}\right) \qquad (11.11)$$

where subscript i and ∞ represent the 1- and infinity-norm. It is clear from equations (11.9) and (11.10) that processes containing large RGA elements correspond to a large closed-loop condition number. Hence, decentralized controller is sensitive to closed-loop input uncertainty in this case.

One major problem in using the steady state robustness index given in equation (11.7) is that $\gamma[GC(s)]$ is difficult to calculate due to the integral action in the controller. Skogestad and Morari (1987b) suggest evaluating the condition number with the integrator excluded. However, the resulting criterion is useful only for analysis rather than for design purpose, since it still depends on controller parameters. Brambilla and D'Elia (1992), instead, simply used RGA_{11} of the equivalent decentralized system (with a compensator) as a robustness criterion for 2x2 systems, taking the advantage of the independence of the RGA of a decentralized controller. It is very difficult, however, to extend this RGA-based method to 3x3 and higher systems. Moreover, a small RGA_{11} does not guarantee a small $\gamma[GC(s)]$ (see equations (11.10) and (11.11)), and thus does not guarantee good robustness. This further limits the usefulness of their approach. For example, the DV scheme of the distillation column given by equation (11.9) has a small RGA_{11} of 0.45 while the condition number of the process is 142.

11.2.3 An Alternative Disturbance Directionality Index

For decentralized control systems, the sensitivity function in equation (11.2) can be expressed in terms of that of the corresponding independent loops by (Grosdidier, 1990),

$$S(s) = \bar{S}(s)\,[I + (G(s) - \bar{G}(s))\,\bar{G}^{-1}(s)\,\bar{H}(s)\,]^{-1} \qquad (11.12)$$

where $\bar{G}(s)=\mathrm{diag}[G(s)]$, $\bar{S}(s)=[I+\bar{G}(s)\bar{C}(s)]^{-1}$, and $\bar{H}(s)=\bar{G}(s)\bar{C}(s)\bar{S}(s)+I$. At low frequencies, $\bar{H}(s)\approx I$ and equation (11.12) becomes:

$$S(j\omega) \approx \bar{S}(j\omega)\,R \qquad (11.13)$$

where

$$R = [I + (G - \bar{G})\,\bar{G}^{-1}]^{-1} = \bar{G}G^{-1} \qquad (11.14)$$

From equation (11.14) we can see that matrix R is independent of the controller and thus can be directly evaluated at steady state. The matrix R can be interpreted as the penalty imposed on the performance operator $\bar{S}(s)$ by interaction in the control system. The greater the magnitude of R, the greater the magnitude of $\bar{S}(s)$ will be with respect to that of $\bar{S}(s)$.

Grosdidier (1990) demonstrated that the singular values of R are indications of disturbance amplification factors to disturbances aligned with them. However, he did not give any indication about the way to improve control performance in the presence of strong disturbance directionality.

Clearly, the condition number of R, $\gamma(\bar{G}_0 G_0^{-1})$, defined by

$$\gamma(\bar{G}_0 G_0^{-1}) = \frac{\bar{\sigma}(\bar{G}_0 G_0^{-1})}{\underline{\sigma}(\bar{G}_0 G_0^{-1})} \tag{11.15}$$

can be used to characterize the steady state disturbance directionality in decentralized control. Equation (11.15) provides direct implications for improving robust performance against disturbance directionality by means of variable pairing. It also represents a useful tool in robust design of any multivariable controller upon decomposition into a decentralized controller and a compensator (O'Reilly and Leithead, 1991), such as the one proposed by Brambilla and D'Elia (1992) (see later).

Like the index given by equation (11.7), the directionality index in equation (11.15) also reveals that decentralized control is sensitive to disturbance direction for processes with large RGA elements. This is clear by noticing that $RGA(\bar{G}G^{-1})=RGA(G^{-1})=RGA^T(G)$ and that large RGA will lead to large condition number as indicated by equation (11.10). On the other hand, the RGA can be viewed as an indicator of performance loss due to disturbance directionality. In fact, we have, $R_{ii}=RGA_{ii}$, $\forall i$. However, as previously noted, uncertainty exists regarding system robustness when RGA is small. Also, the new index indicates, trivially, that inverse-based control (decomposed into an inverse compensator and a unitary decentralized controller) is capable of removing disturbance directionality.

Most notably, the alternative disturbance directionality index in equation (11.15) overcomes the problem associated with the RGA-based index proposed by Brambilla and D'Elia (1992), and thus represents a more reliable tool. For instance, for the LPG splitter process given in equation (11.8), it can be easily shown that $RGA_{11}(G)=4.9$, which is not too large. By the RGA rule the system would be quite robust under decentralized control (without compensator). However, the system corresponds to a $\gamma(\bar{G}G^{-1})$ of 86.9, which indicates strong directionality. Indeed, decentralized control is very sensitive to disturbance direction for this process as demonstrated by Brambilla and D'Elia (1992).

11.3 ROBUSTNESS AGAINST MODEL MISMATCH

Model mismatch in a multivariable process can be described in different ways. In process control, as we never know the exact value of the process inputs acting on the plant, uncertainty on the manipulated variables is always present and thus must be considered (Skogestad and Morari, 1987a). Alternatively, this uncertainty can be equivalently described as model mismatch by

$$G_p(s) = G(s)(I+\Delta) , \qquad \Delta = diag(\delta_i) \qquad (11.16)$$

where $G_p(s)$ is the real plant, $G(s)$ is the model of the plant, and Δ represents the uncertainty associated with the manipulated variables. Note that individual model uncertainties in equation (11.16) represent the changes of manipulated variables relative to their nominal values (Skogestad and Morari, 1987a). From equations (11.1) and (11.2) it is clear that the loop transfer matrix, $G_p(s)C(s)$, is closely related to performance and can be written in terms of the process model and model error:

$$G_pC = GC(I+C^{-1}\Delta C) \qquad (11.17)$$

and

$$G_pC = (I+G\Delta G^{-1})GC \qquad (11.18)$$

In this section we are dealing with the dynamic case unless otherwise stated, so the Laplace variable s is omitted for simplicity throughout this section.

11.3.1 Robustness for 2x2 Systems

Skogestad and Morari (1987a) derived an analytical relationship between the error term in equation (11.17), $C^{-1}\Delta C$, and the RGA of the controller for 2x2 systems as follows,

$$C^{-1}\Delta C = \begin{bmatrix} RGA_{11}(C)\,\delta_1 + RGA_{21}(C)\,\delta_2 & RGA_{11}(C)\,\frac{C_{12}}{C_{11}}(\delta_1-\delta_2) \\ -RGA_{11}(C)\,\frac{C_{21}}{C_{22}}(\delta_1-\delta_2) & RGA_{12}(C)\,\delta_1 + RGA_{22}(C)\,\delta_2 \end{bmatrix} \qquad (11.19)$$

As the magnitude of the error matrix is directly related to the RGA elements, the RGA may be used as an indicator of robustness against model error. From equation (11.19) Skogestad and Morari (1987a) claimed that controllers with large RGA elements will lead to large elements in $C^{-1}\Delta C$ (and

G_pC) and thus poor performance in face of model error. As a result, they further concluded that a decentralized controller is robust against model mismatch since $RGA_{11}(\bar{C})=1$, while a inverse-based controller is sensitive to model error since

$$RGA(\bar{C}) = RGA(G^{-1}) = RGA^T(G) = RGA(G)$$

They also carried out similar analysis for the error term ($G^{-1}\Delta G$) associated with the process.

Based on this, Brambilla and D'Elia (1992) used the RGA of the controller as a robustness index against model mismatch. They further proposed a design procedure by minimizing RGA_{11} of the controller.

Nevertheless, the claim by Skogestad and Morari (1987a) may not be generally correct, e.g., a large RGA may lead to a small error, and on a relative basis, a smaller RGA may result in a large error, due to the impact of c_{12}/c_{11}, c_{21}/c_{22} and δ_i in equation (11.19). Hence the design by Brambilla and D'Elia (1992) may be unreliable. For example, Figure 11.3 shows the maximum magnitude of the error related to the process (the same applies to the controller), $\bar{\sigma}(G^{-1}\Delta G)$, as a function of frequency for the DV distillation given in equation (11.9) and the LV distillation described by (Skogestad and Morari, 1988)

$$G(s) = \begin{bmatrix} \frac{87.8}{194s+1} & \frac{-87.8}{194s+1} + \frac{1.4}{15s+1} \\ \frac{108.2}{194s+1} & \frac{-108.2}{194s+1} - \frac{1.4}{15s+1} \end{bmatrix} \qquad (11.20)$$

Note that the same error structure, $\delta_1 - \delta_2 = 1/RGA_{11}(s)$, is used for the both processes. Specifically, $\delta_1=1.5$ for the DV process and δ_1=0.014(194s+1)/(15s+1) for the LV process are used. From Figure 11.3 one can see that the maximum error in the DV process is always larger than that in the LV process, even though the RGA in the former is always smaller than that in the latter as shown in Figure 11.4.

Moreover, the error term in equation (11.19) may be independent of the RGA. For instance, $C^{-1}\Delta C=\delta I$ when $\delta_1=\delta_2=\delta$, which may be common in practice. This would be true when, say, the two control valves corresponding to the two manipulated variables have the same dynamic characteristics. The error term also depends only on the model error and scaling of output variables when $\delta_1-\delta_2$ = $[RGA_{11}(C)]^{-1}$ as shown in the above example. Hence, the RGA-based robust design may be misleading under some circumstances.

11.3.2 Robustness Index for nxn Systems

In multivariable systems, the error in loop gain caused by model mismatch can be characterized in different ways. Originally, model error can be independently described in an absolute or a relative form as shown in Figure 11.5(a) and 11.5(b). Mathematically, we have $G_pC=G(I+\Delta)C$. In relation to the nominal case, loop gain error is geometrically illustrated in Figure 11.6(a) and can be expressed as,

$$G_pC = GC+E_A\ , \quad E_A=G\Delta C \qquad (11.21)$$

where E_A denotes the absolute error of the loop gain. Considering the worst case, i.e., the maximum magnitude of the error E_A, we have,

$$\bar{\sigma}(E_A) \le \bar{\sigma}(G)\bar{\sigma}(\Delta) \le \bar{\sigma}(G)\bar{\sigma}(C)\bar{\sigma}(\Delta) \qquad (11.22)$$

From equation (11.22), we can see that the magnitude of the loop gain error is determined by the magnitude of all the three elements: process, controller and model error, and that loop gain error is guaranteed small enough, i.e., system is robust enough, if the magnitude of each element is made sufficiently small. This is also obvious from Figure 11.6(b). In contrast to Skogestad and Morari (1987a), however, loop gain error can be independent of the process and controller under some circumstances. For instance, when model errors associated with all the manipulated variables are equal, i.e., $\delta_i=\delta$, $\forall i$, equation (11.21) becomes

$$G_p C = (1+\delta)\, GC \qquad (11.23)$$

Clearly, equation (11.23) shows that loop gain error solely depends on the magnitude of the model error. Figure 11.6(b) also demonstrates this fact.

Model error can be associated with either the controller or the process in a multivariable control system. To reveal the effects of different controllers on performance robustness in face of input uncertainty, we first associate model error with the controller as described by Figure 11.7(a), the perturbed loop gain relative to the nominal case, geometrically depicted by Figure 11.7(b), can be expressed as,

$$G_p C = GC(I+E_c) \ , \quad E_c = C^{-1}\Delta C \tag{11.24}$$

where E_c denotes the relative error associated with the controller. It is clear that the difference between the nominal loop gain and the perturbed loop gain by model error depends only on E_c. The large the E_c is, the more significant the difference is, and the less robust the performance is (Skogestad and Morari, 1987a).

The worst case of the error can be easily shown to be,

$$\bar{\sigma}(E_c) \le \bar{\sigma}(C^{-1})\,\bar{\sigma}(\Delta_I)\,\bar{\sigma}(C) = \gamma(C) \max_i [\,|\delta_i|\,] \tag{11.25}$$

Note that the errors represent the relative changes of inputs and they are usually less than unity in magnitude. Hence, it is clear that under a given model error a well-conditioned controller lead to a small error in the loop gain, thus resulting in robust control against model uncertainties. Thus for robust design we require the condition number of the controller to be minimized, i.e.,

$$\min \gamma[C(j\omega)] \ , \ \forall \omega \tag{11.26}$$

For a decentralized controller, assuming that PI controllers are to be used in each control channel, the condition number of the controller is

$$\gamma(\bar{C}) = \frac{\max_i [\,|k_i|(1+\frac{1}{T_i s})\,]}{\min_j [\,|k_j|(1+\frac{1}{T_j s})\,]} \ , \ \forall i,\ j \tag{11.27}$$

where k_i and T_i denote the proportional gain and integral time of the i-th controller.

At low frequencies, equation (11.27) becomes,

$$\gamma(\overline{C}) \approx \frac{\max_i\left(\frac{|k_i|}{T_i}\right)}{\min_j\left(\frac{|k_j|}{T_j}\right)}, \quad \forall\, i \tag{11.28}$$

And at high frequencies, equation (11.27) can be simplified to,

$$\gamma(\overline{C}) \approx \left|\frac{k_{max}}{k_{min}}\right| \tag{11.29}$$

Therefore, the condition of decentralized controllers can be improved by tuning controller parameters so that the resulting decentralized control system is insensitive to model uncertainties. In contrast to Skogestad and Morari (1987a) who focused on the RGA properties of the controller, the above conclusions provide a more general basis.

On the other hand, for inverse-based controllers, the resulting control system may be very sensitive to model error, especially when the RGA of the process is large, as previously pointed out. However, no conclusion can be reached regarding the sensitivity of inverse-based control when RGA(G) is small. The impact of the process, separate from the controller, on the robustness of closed-loop control systems is evaluated as follows.

When model error is associated with the process as shown in Figure 11.8(a), the relative error in loop gain can be expressed by Figure 11.8(b). Mathematically,

$$G_pC = (I+E_G)GC\ ,\quad E_G = G\Delta G^{-1} \tag{11.30}$$

where E_G represents the error term associated with the process. Similar to E_t, the magnitude of E_G reflects the robustness of the closed-loop system in the presence of model uncertainty. Considering the worst case, we have,

$$\bar{\sigma}(E_G) \leq \bar{\sigma}(G)\,\bar{\sigma}(\Delta_I)\,\bar{\sigma}(G^{-1}) = \gamma(G)\max_i[|\delta_i|] \tag{11.31}$$

From equation (11.30), we can conclude that a well-conditioned process will lead to good robustness in control performance in face of input model error while ill-conditioned plant is difficulty to control since the closed-loop system is inherently sensitive to model error. Thus for robust design we require,

$$\min\gamma[G(j\omega)]\ ,\quad \forall\,\omega \tag{11.32}$$

The above requirement has immediate implications for variable pairing for robust performance. However, in terms of control system design, condition given by equation (11.26) has to be used.

11.4 ROBUST MULTIVARIABLE CONTROL

It is known that any multivariable controller can be expressed as a combination of a decentralized controller and a compensator, D, as shown in Figure 11.9 (O'Reilly and Leithead, 1991), i.e.,

$$C = D\overline{C} \tag{11.33}$$

The decomposition in equation (11.33) allows both nominal and robust performance to be accommodated separately. For instance, the design of the decentralized part may be focused mainly on achieving nominal performance while the design of the compensator takes care of robustness.

There can be various ways to construct the compensator. Brambilla and D'Elia (1992) proposed a multivariable compensator based on the Singular Value Decomposition (SVD) of the plant gain matrix as follows,

$$D = V[\alpha\Sigma^{-1} + (1-\alpha)U^T] \tag{11.34}$$

where

$$U\Sigma V^T = G_0 \tag{11.35}$$

The compensator is structurally attractive for robustness consideration, since its structure can be adjusted to range from pseudo decentralized to inverse, by tuning a single parameter α within the range 0-1 (Brambilla and D'Elia, 1992). As a result, robustness properties of both decentralized and inverse-based controller can be utilized. We use this controller structure but equip it with our new design tools previously presented.

The compensator D can also be viewed as a 'pre-conditioner' of the process (this is clear from Figure 9). Taking the compensator into account, the dynamic directionality index given in equation (11.6) can be modified as,

$$\min_{D} \gamma[I+G(j\omega)DC(j\omega)] \quad ,\forall\omega \tag{11.36}$$

and the new steady state disturbance directionality index given in equation (11.15) becomes,

$$\min_{D} \gamma[diag(G_0D)\,(G_0D)^{-1}] \tag{11.37}$$

The robustness index with respect to input model uncertainty, as shown in equation (11.26), now can be written as,

$$\min_{D} \gamma[D\overline{C}(j\omega)] \quad ,\forall\omega \tag{11.38}$$

At steady state, if we tune the decentralized controller in such a way that its condition number, as calculated by equation (11.28), is close to unity, then the scaling effect of the decentralized controller on D in equation (11.38) will be minimal. Consequently, the following steady state robustness index can be obtained:

$$\min_{D} \gamma[D] \tag{11.39}$$

The above criterion can be directly utilized as a design tool since it is independent of the controller and model error.

It must be pointed out that process outputs should be properly scaled in order to successfully use above indices. By combining directionality index with robustness index, both of which may represent conflicting requirements, proper compromise for robust performance against both strong directionality and model uncertainty can be obtained. A systematic design approach is presented as follows:

Step 1 <u>Preliminary steady state design of the compensator</u>: The following criterion, based on equations (11.37) and (11.39), is used to obtain a preliminary design for the compensator D by adjusting the tuning parameter, α, defined in equation (11.43):

$$\min_{D} J = \beta\gamma[diag(G_0D)\,D^{-1}G_0^{-1}] + (1-\beta)\,\gamma[D] \qquad (11.40)$$

In equation (11.40), β is a weighting parameter within the range 0-1 in order to adjust the relative importance of the robustness requirements with respect to disturbance directionality and input model error. To treat both requirements equally, one simply sets β=0.5.

Step 2 <u>Nominal tuning of the controller</u>: With the compensator designed in step 1, controller direction can be determined by satisfying the following stability condition (Grosdidier et al., 1985; Zhu and Jutan, 1993):

$$\det[G_0DC'(0)] > 0 \qquad (11.41)$$

where $C'(s)=1/\omega\ \tilde{C}(s)$, i.e., remaining part of $\tilde{C}(s)$ with the integrator separated. In addition, condition (11.28) should be followed in the tuning process ($\gamma[\tilde{C}]\approx 1$) to maintain robust performance against model uncertainty as shown in step 1. Trial and error method may have to be used to achieve the final tuning of controller parameters.

Step 3 Dynamic considerations: Conditions (11.36) and (11.38) should be satisfied, for the controller and compensator designed in step 1 and step 2, at their crossover frequencies (Hovd and Skogestad, 1992). If the dynamic design does not confirm the steady state design in step 1, further adjustment of D (using α) should be made. However, in practice this step may be bypassed by directly investigating the dynamic response of the closed-loop system under the designed controller and compensator.

11.5 ILLUSTRATIVE EXAMPLE

The LPG splitter process studied by Brambilla and D'Elia (1992), as given by equation (11.8), is used to demonstrate the effectiveness of the design strategy presented in this chapter. Note the pure time delays in the process will be ignored in the simulation for simplicity -- this does not affect steady state design.

According to the RGA criteria used by Brambilla and D'Elia (1992), this process would be of no particular concern regarding robust performance with a decentralized controller (D=I), since $RGA_{11}(G_0D)=4.9$, and $RGA_{11}(D)=1$. In fact, the closed-loop system is very sensitive to disturbance direction, as shown by (Brambilla and D'Elia, 1992). This is clearly indicated by our disturbance directionality index, calculated as since $\gamma[\bar{G}G^{-1}]=87$. Since the SVD-based compensator can never reach truly decentralized structure in nature (see equation (11.34)), the design with D=I accepted by their design criteria was obscured.

Brambilla and D'Elia designed a robust controller, and found the best compensator to be:

$$D = \begin{bmatrix} -0.70 & -0.67 \\ 0.58 & -0.73 \end{bmatrix} \qquad (11.42)$$

with α=0.1. By their criteria, the resulting closed-loop system should be quite robust against disturbance direction and model error, since $RGA_{11}(G_0D)$=2.8, and $RGA_{11}(D)$=0.57. Note that the RGA based rules fail again here, since $\gamma[G_0D]$ is large (39) even though RGA is small. Using our robustness criteria, we found that the resulting control system as such designed will still be sensitive to disturbance directions, since $\gamma[\mathrm{diag}(G_0D)(G_0D)^{-1}]$=28, while the system will be rather robust against model error since $\gamma[D]$=1.1. Hence, further improvement for a proper balance of these two robustness requirements is possible.

Figure 11.10 shows our design indices, given by equations (11.37), (11.39) and (11.40), as a function of α for the SVD based controller (equation (11.34)) with β=0.5. From Figure 11.10 we can easily see that α=0.81 will lead to the best compromise between the two robustness requirements. From equation (11.34), we have,

$$D = \begin{bmatrix} -0.30 & -0.76 \\ 0.031 & -0.60 \end{bmatrix} \qquad (11.43)$$

The resulting closed-loop control system will have much better robustness against directionality and robust performance against model error will also be very satisfactory, since now $\gamma[\text{diag}(G_0D)(G_0D)^{-1}]=\gamma[D]=4.7$.

Figure 11.11 shows the dynamic responses of both outputs under Brambilla and D'Elia's SVD controller (referred to as B-D's SVD controller) and our controller from equation (11.43) (referred to as improved SVD controller) to the 'good' setpoint change (r1=0.21, r2=0.98]. These responses can be regarded as nominal responses. Both controllers (decentralized part) are tuned to give approximately the save (nominal) dynamic characteristics (overshoot, response time). The controller parameters for the B-D's SVD controller are $k_1=3.1$, $T_1=0.1$, $k_2=12$, $T_2=0.7$, and the parameters for the improved SVD controller are $k_1=18$, $T_1=0.7$, $k_2=13$, $T_2=0.7$.

Figure 11.12 shows the dynamic responses of outputs under both controllers to the 'bad' setpoint change (r1=1.2, r2=-0.2). Clearly, from Figure 11.12 it can be observed that the improved SVD controller is more robust against setpoint direction change than the B-D's SVD controller.

Figure 11.13 shows the responses of the system under both controllers to the 'bad' setpoint change when model error is introduced ($\delta_1=0.1$, $\delta_2=-0.1$). Clearly, the improved SVD controller demonstrates better robust performance against combined uncertainties (both setpoint change and model error).

Notice that Figures 11.12 and 11.13 only show the improvement of robust performance with controller from our design strategy over the B-D's design. Compared with purely decentralized control and inverse-based control, the improvement will be much substantial (see Brambilla and D'Elia, 1992).

11.6 FINAL REMARKS

Robust performance of multivariable control systems in the presence of strong disturbance directionality and input associated model uncertainty has been systematically analyzed. New directionality indices and robustness criteria against model error were developed. These indices were then utilized to obtain a robust design strategy. This design strategy was shown to lead to satisfactory robust performance against both strong disturbance (or setpoint) direction changes as well as model errors. The design results in a best compromise between these two conflicting robust requirements. The proposed design scheme is applicable to arbitrarily sized multivariable systems and both steady state and dynamic considerations are accommodated. Simulations were used to demonstrate the superiority of our approach to other existing methods.

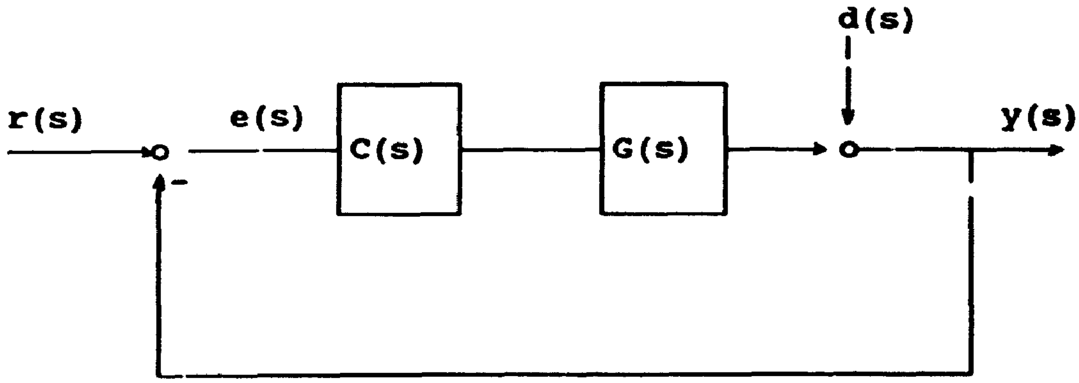


Figure 11.1 Multivariable system structure

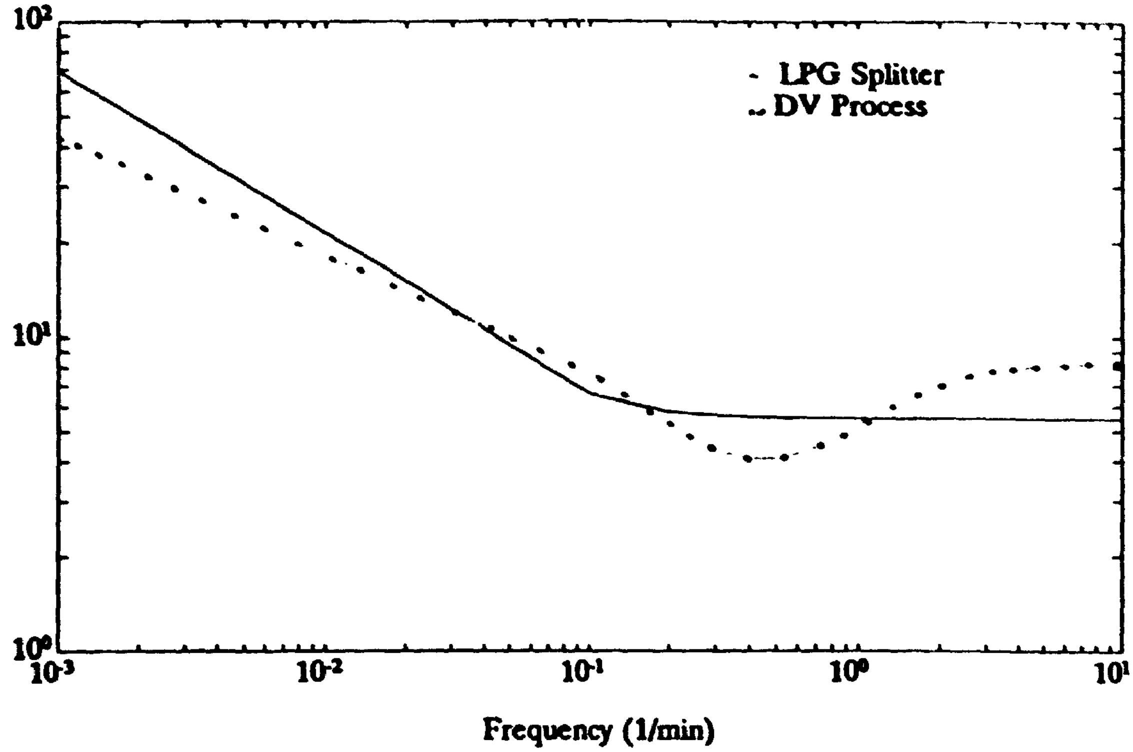


Figure 11.2 Condition number as a function of frequency

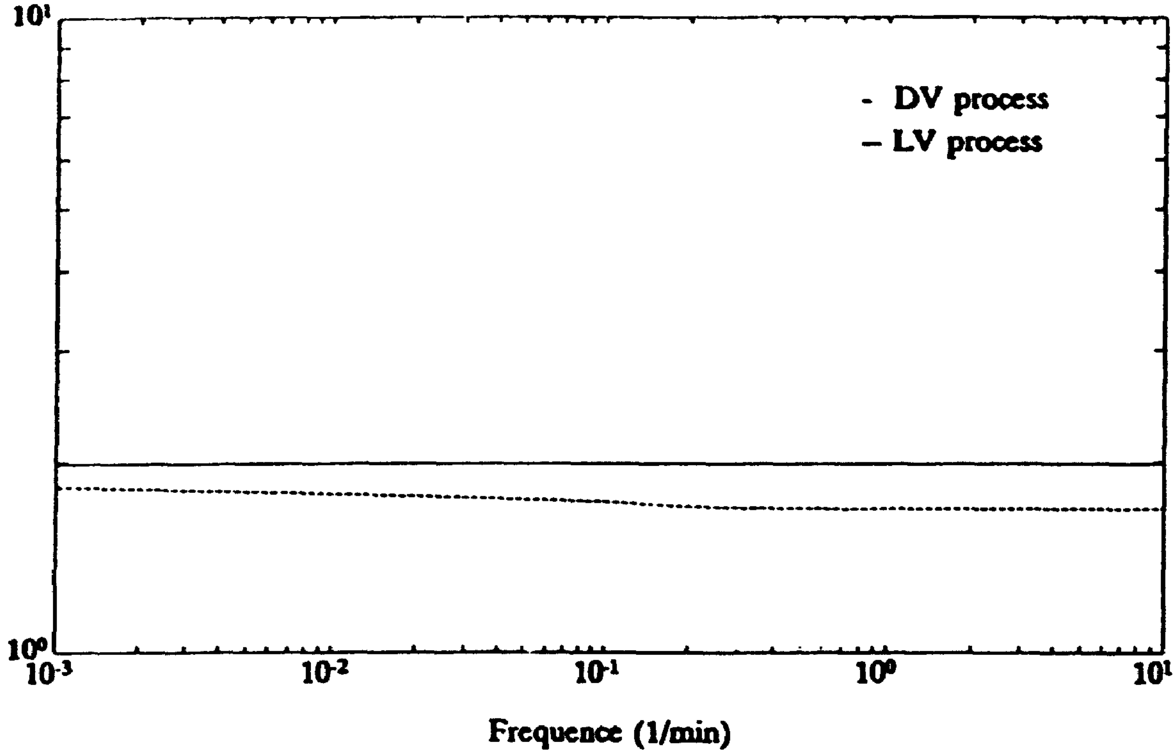


Figure 11.3 Maximum error $\bar{\sigma}[G^{-1}\Delta G]$ as a function of frequency

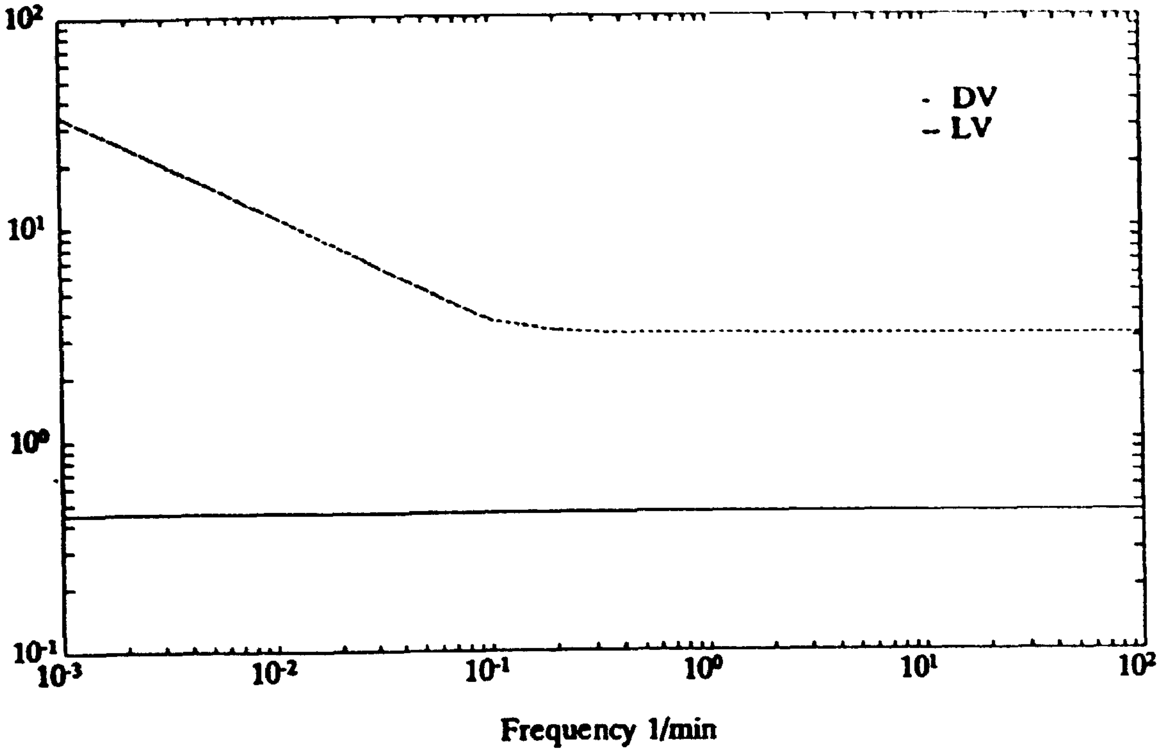
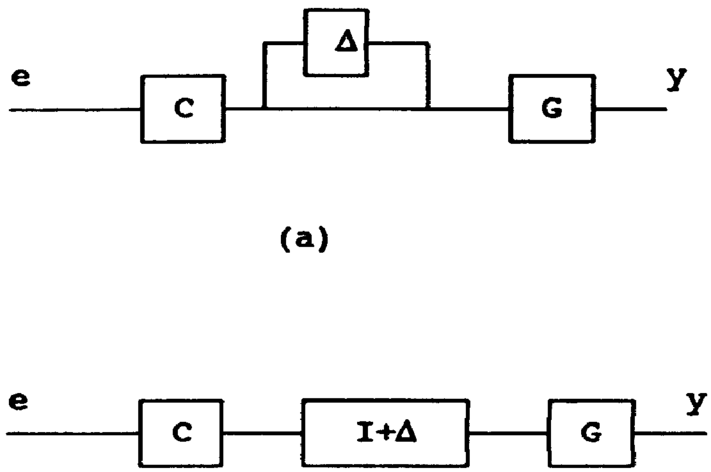


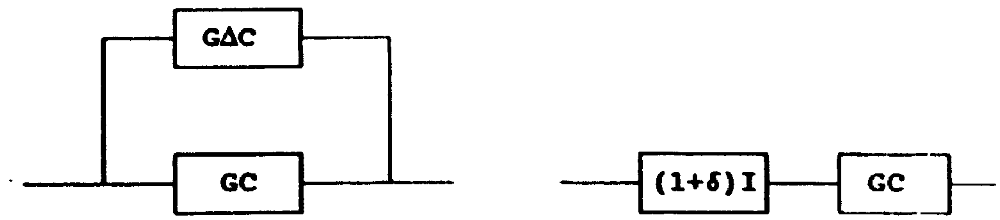
Figure 11.4. RGA as a function of frequency



(a)

(b)

Figure 11.5 Input model error structure
(a) multiplicate
(b) addicative



(a) (b)

Figure 11.6 Loop gain error
(a) absolute form
(b) equal model error

(a) (b)

Figure 7 Error associated with controller
(a) model error
(b) loop gain error

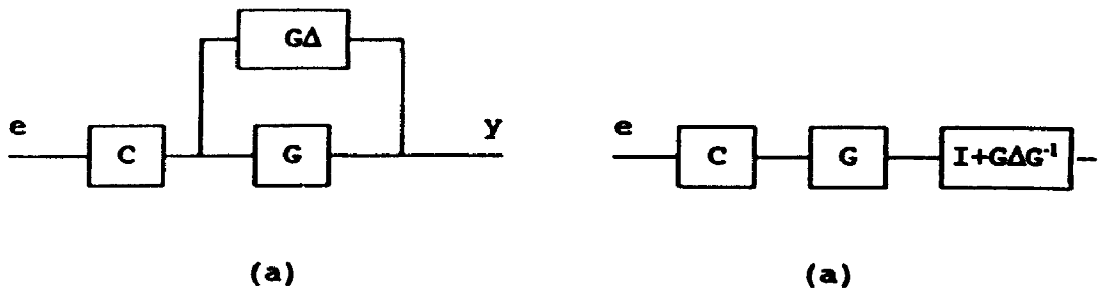


Figure 8 Error associated with process
(a) model error
(b) loop gain error

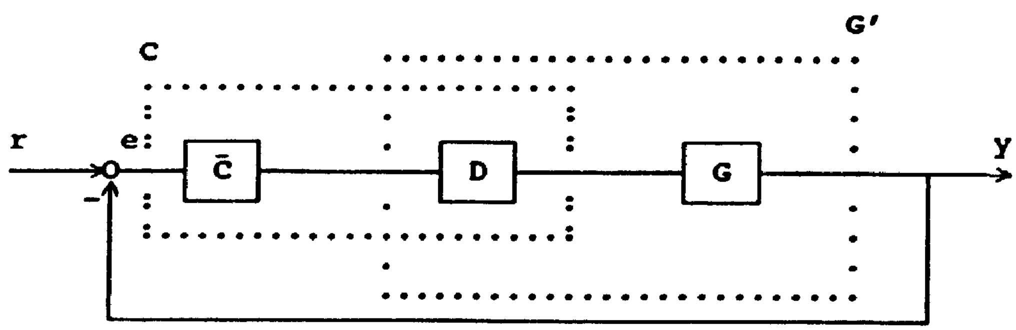


Figure 11.9 Control system with proposed robust controller

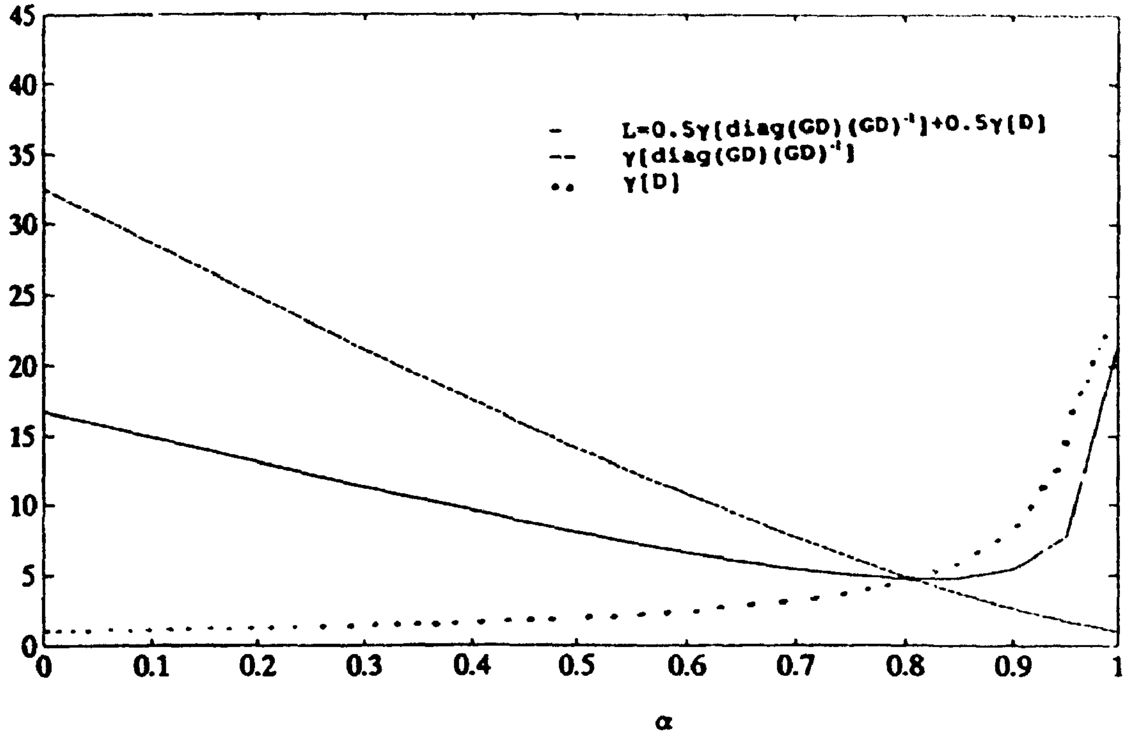


Figure 11.10 Robust Indices as a function of tuning parameter

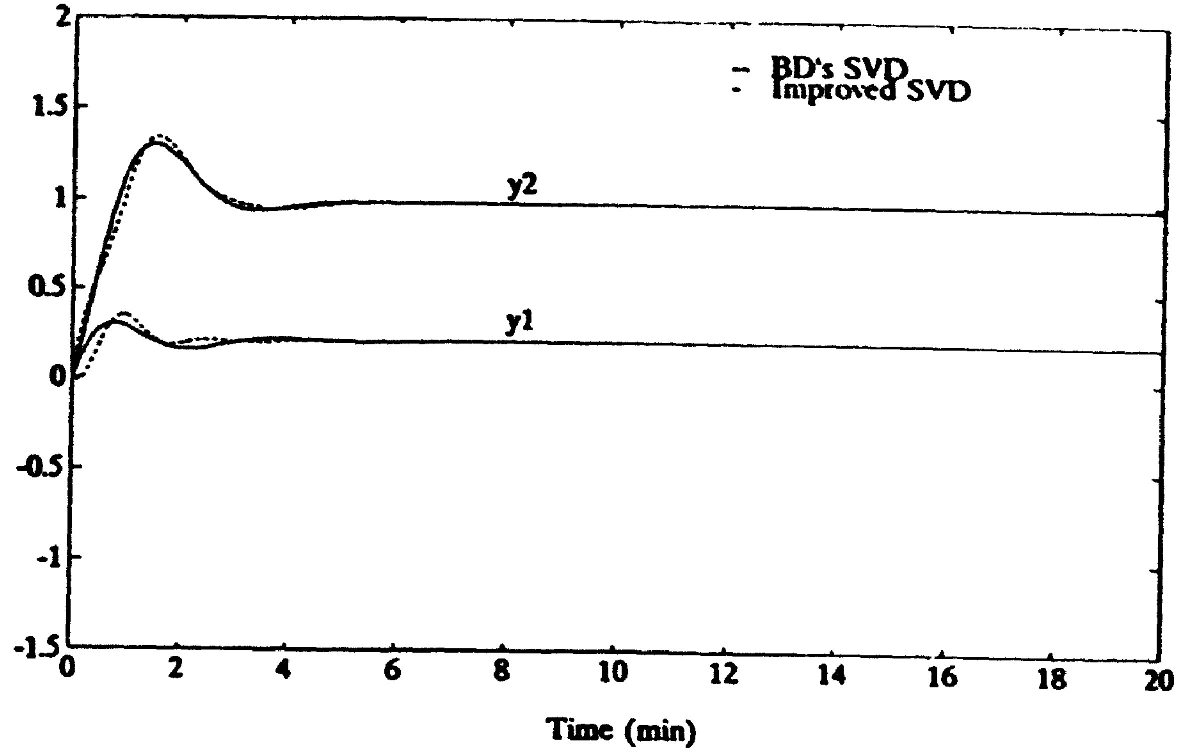


Figure 11.11 Nominal responses ("good" setpoint change)

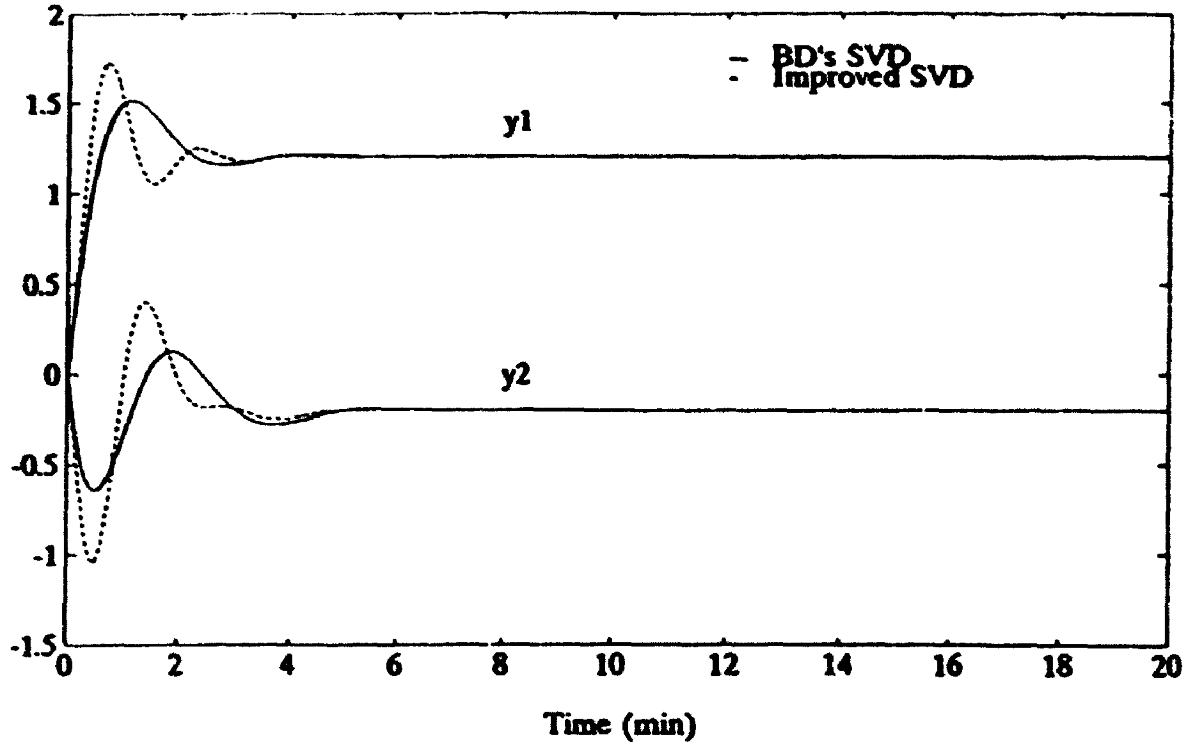


Figure 11.12 Responses to "bad" setpoint change

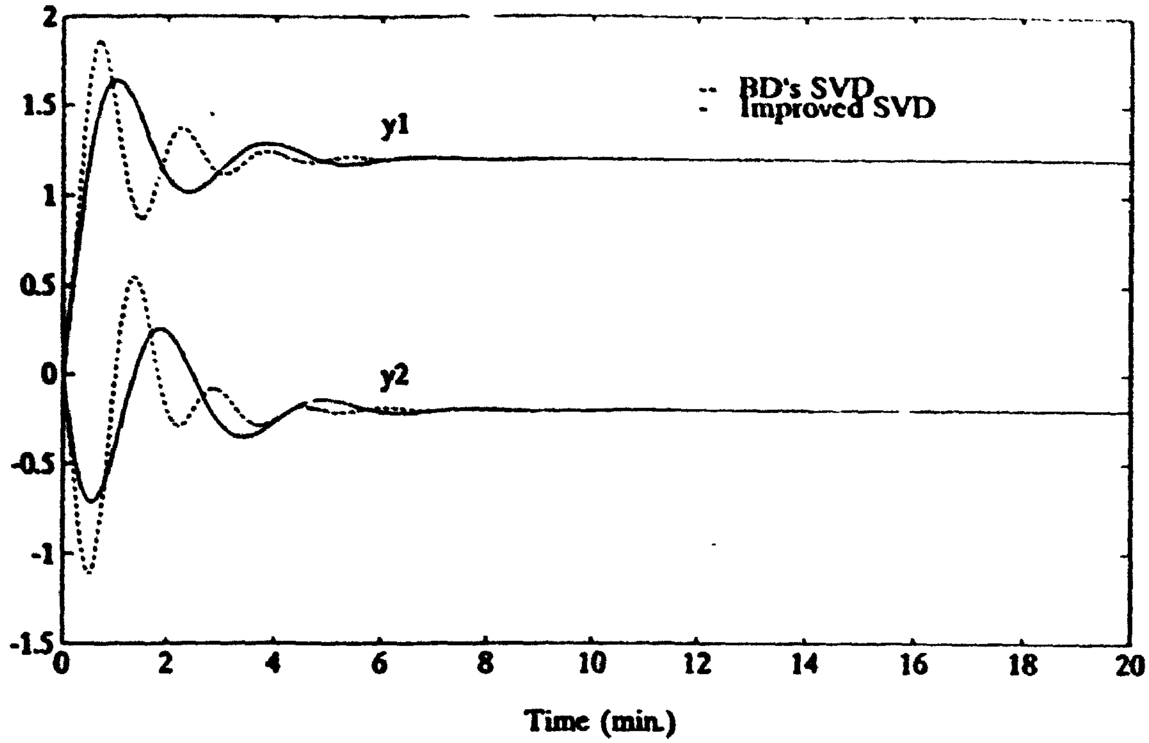


Figure 11.13 Responses to "bad" setpoint change with model error

CHAPTER 12

CONCLUSIONS

12.1 GENERAL

Topics on both modern control and decentralized control for multivariable processes have been studied with the latter emphasized. A number of important results and contributions, particularly on decentralized control, have been achieved, as outlined later in this chapter.

In the aspect of modern control, two most important approaches -- the LQG control and the eigenvalue assignment design, are discussed. Emphasis is placed on the investigation of the application of the standard LQG control method, in conjunction with proper model identification techniques, to complex real-time processes. Also the standard eigenvalue assignment design approach is improved for better steady state and robust performance.

With respect to decentralized control, various issues, such as interaction measurement, variable pairing, stability and integrity, stability robustness, robust performance, and control tuning, are systematically addressed.

12.2 CONTRIBUTIONS OF THE THESIS

The major contributions of this thesis, to both modern control and decentralized control, are summarized below.

1. LQG strategy is successfully implemented to control a strongly interactive and nonlinear multivariable pressure tank system, with innovative use of model identification procedures and consideration of practical problems associated with real-time applications.

2. A PI state feedback controller, with fewer controller parameters and no constraint on its design, is proposed, and a robust eigenvalue design method of the controller is presented.

3. A simulation package, called MDSS, for the simulation of distributed processes and control system design, is developed.

4. A new variable pairing criterion, using both the size and the sign of the NI, is presented.

5. A new dynamic interaction measure, capable of measuring both the absolute and relative interaction and addressing the nature of interaction, is developed.

6. Stability conditions under general decentralized control,

independent design, variable pairing are established in a systematic way.

7. A robustness measure to evaluate the effects on system stability of model errors, independent and correlated, in steady state gains is developed. Rigorous theoretical justifications are provided.

8. New reliable disturbance directionality indices and robustness criteria against model uncertainties associated with manipulated variables are developed. Effective design procedures for multivariable controllers to achieve robust performance in face of both disturbance directionality and model mismatch are subsequently proposed.

Among the above points, 4, 5, 6, 7, and 8 are significant contributions that have been communicated to the open literature.

12.3 RECOMMENDATIONS FOR FUTURE WORK

Future research to extend and strengthen present work should focus on decentralized control, including the following:

1. To further develop guidelines for effective tuning of

individual controllers in decentralized control systems, based on the new dynamic interaction measure proposed in Chapter 8.

2. To apply the new interaction measure, presented in Chapter 8, to steady state cases, so that a unified interaction measure for both dynamic and steady state cases can be accomplished.

3. To obtain sufficient stability conditions to supplement necessary conditions presented in Chapter 9, so that a feasible region regarding controller settings to stabilize decentralized control systems can be calculated.

4. To propose variable pairing rules for better disturbance rejection and robust performance, in contrast to the usual minimum interaction requirements, based on the disturbance directionality indices and robustness criteria developed in Chapter 11.

APPENDIX

PROGRAM LISTINGS

Some programs, in MATLAB M-files (examples), QUICKBASIC (real-time control implementation), or FORTRAN (MDSS), used in this study, are included in the floppy disk accompanying this thesis.

To facilitate the reader, programs related to specific issues covered by individual chapters are arranged in different subdirectories.

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