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Fertility And Economic Growth

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by

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ABSTRACT

The thesis comprises three chapters which model fertility and economic growth simultaneously in overlapping generations frameworks.

Chapter 1 focuses on the relation between fertility and wage rates and examines the effects on fertility and growth of subsidies for education and for the cost of rearing children by assuming that agents care about the consumption and number of children. Without education, this model reconciles two conflicting results about the relationship between fertility and wage rates: while Malthus and others predict a positive relation, Barro and Becker find a negative relation. With education, the positive relation between fertility and wage rates can no longer exist. An education subsidy may reduce or may increase fertility if bequests are operative and has no net effect on fertility otherwise, and is most likely to speed up growth of per capita income. Depressing growth, a child rearing subsidy cannot raise fertility unless bequests are operative.

Chapter 2 compares fertility and economic growth between economies with or without markets and firms by assuming that agents are concerned about the consumption of their old parents, and/or the consumption and the number of their children. It is shown that transforming a traditional economy into a market economy brings about lower fertility (as in the literature) but faster growth of per capita output if altruism is one-sided towards parents. Even if altruism is two-sided, this transformation reduces fertility. In this case, when gifts are operative it also speeds up growth unless tastes for the number of children are much stronger than those for the consumption of children. When bequests are operative, the two economies appear to have similar rates of economic growth. The results may help to explain why countries which were the first to establish market economies are richer and have lower fertility than countries where some people still live in a traditional way.

The effects of social security on fertility and economic growth are examined in Chapter 3. It is shown that an unfunded social security program may speed up economic growth by reducing fertility and increasing the ratio of human capital investment per child to family income even if saving rates fall, and may bring about faster economic growth than a funded one. Even if fertility is exogenous and private intergenerational transfers are operative, the neutrality of unfunded social security fails to hold due to human capital investment in children, although the saving rate is unchanged.

To Zhaoxi, my wife

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TABLE OF CONTENTS

LIST OF TABLES

LIST OF APPENDICES

Chapter 1

Investment in Children, Fertility, and Economic Growth

1.1 Introduction

Recently economists have been increasingly interested in the interactions between fertility and economic growth. The purposes of this chapter are modest: analyzing the relationship between fertility and wage rates, and examining effects of subsidies for education and child rearing on fertility and per capita growth.

Economic growth and fertility appear to have gone through two distinct stages in the last several centuries in Britain and other western countries. Before the early nineteenth century, fertility and wage rates seem to have been positively related and wage rates seem to have had long swings around a constant level.[1] Thereafter the positive relation between fertility and wage rates has disappeared and wage rates have shown continuous growth.[2] In his seminal work, Malthus (1890) argued that fertility and wage rates are positively related and increases in fertility prevent wage rates from perpetual growth. This result has been more rigorously shown in Bernhard (1990) and Raut (1990). However, Barro and Becker (1989) find a negative relation between fertility and the wage rate. What causes the conflicting results in these models? In

[1] See, e.g., Wrigley, 1981 (also see Lee, 1986; Lindert, 1986).
[2] Also see Wrigley, 1981.

the present chapter, their results are reconciled in an overlapping generations general equilibrium model.

In principle, there may be two offsetting effects on fertility of changes in wage rates. Higher wage rates can cause both income and substitution effects that have opposite signs (see Barro and Becker, 1989). The income effect is obvious; the substitution effect is due to that higher wage rates lead to higher opportunity costs of spending time rearing children. The net effect would thus depend on the relative magnitude of the two effects. In this chapter it is shown that whether bequests are positive plays a critical role in signing the net effect.

Moreover, bequests may take the form of goods invested in children (I distinguish investment in children from bequests in this thesis). However, investment of goods in children (education spending or child-rearing costs) has been assumed to be exogenous or negligible in the literature dealing with fertility and economic growth.[3] In fact, the investment of goods in children is neither exogenous nor negligible. Expenditure per child ranged between 15 and 17 percent of family disposable income in the United States in the early 1970's (Espenshade, 1984). Thus, with the average family having about two children, nearly one-third of family disposable income was spent on children in the United States. Also in the same country, total expenditure on education has averaged about six percent of Gross National Product in the post-war period.[4] With investment of goods in the quality of children, the positive relation between fertility and wage rates can no longer exist in contrast to the case without investment in the quality of children, and wage rates can grow. Thus, in order for the present model to be able to explain the different relationships between fertility and wage rates before and after Malthus bequests would have to have been nonpositive in most of the families and education not widely available prior to Malthus. Also, incorporating the

[3]For example, Becker, Murphy, and Tamura (1990) assume a negligible amount of goods needed to rear a child, and Ehrlich and Lui (1991) abstract from investment of goods in children.

[4]See U.S. Bureau of the Census, 990.

investment of goods in children, the present model is suited to examining government subsidies for education and child rearing.

Accounting for most of the total spending on education in developed countries as well as in many developing countries, public subsidies for education have drawn a great deal of attention. What is their impact on fertility? If fertility does respond to education subsidies, both the magnitude of their effect on economic growth and the mechanism through which they affect the economy will be different from existing analyses with exogenous fertility. This model addresses the responses of fertility and growth to the education subsidy simultaneously. Also, economists have recently started paying more attention to the child-rearing subsidy.[5] However, it has not been examined in the context of a growing economy with endogenous fertility. We need to do so because the developed countries are facing a shrinking population and various tax-incentives (family allowances and personal tax exemptions for dependents, for example) have been introduced to raise fertility back to its replacement level.

The main results of this chapter are as follows:

1. When investment in children is exogenous, fertility and wage rates are positively related if bequests are nonoperative, and negatively related otherwise. The 'Malthusian equilibrium' is most likely stable. Stability conditions are derived under which a temporary rise (fall) in wage rates will be followed by higher (lower) fertility which in turn will force the wage rate back to its steady-state level. When investment in children is endogenous, fertility is constant and wage rates grow over time.

2. With operative bequests, an education subsidy speeds economic growth unless tastes for the number of children are extremely stronger than those for the consumption of children. In this case, it may reduce or may increase fertility, depending on tastes for the consumption relative to the number of children. It may increase fertility

[5] Whittington, Alm, and Peters (1990) show empirically that the personal tax exemption for dependents raises fertility in the United States.

even with fairly strong tastes for the consumption of children. Without operative bequests, this subsidy has no net effect on fertility, while it accelerates economic growth unless the tax rate is too high. Depressing growth, a child-rearing subsidy can raise fertility when bequests are positive and has no net effect on fertility otherwise.

This chapter is organized as follows. Section 1.2 introduces the model. Section 1.3 focuses on the steady-state equilibrium in an economy where investment in children is exogenous. Section 1.4 deals with fertility and perpetual growth of per capita income in an economy where investment in children is endogenous. Section 1.5 discusses policy issues. The last section concludes the chapter.

1.2 The Model

There is an infinite number of periods and overlapping generations of three-period-lived agents. Let subscript t denote a period in time and superscript t the generation born in period $t-1$. Let L_t denote the number of middle-aged agents in period t. I assume that every parent has children at the beginning of middle age. A single good can be produced, consumed, invested in children, accumulated as physical capital, but cannot be stored for future consumption.[6]

Everyone is identical, learns when young, lives in retirement in old age, and is endowed with one unit of time in middle age which can be provided to the labor market or spent in rearing children. Let v (positive, less than one) denote the units of time needed to rear a child.[7]

The utility of agents depends on the number (N_t) and consumption of children (C_{t+1}^{t+1}) as well as on own middle-aged and old-aged consumption (C_t^t, C_{t+1}^t):

$$U(C_t^t,\ C_{t+1}^t,\ C_{t+1}^{t+1},\ N_t).$$

[6]Alternatively, I can assume that the return to investing in physical capital or human capital exceeds that to storing goods for future consumption without changing the results.

[7]Fertility thus has an upper bound.

The justification for ignoring younger generations' fertility choices and their consumption except C_{t+1}^{t+1} will be discussed later in this section. I assume that $U(\cdot,\cdot,\cdot,\cdot)$ is increasing and (strictly) concave. Also, I assume that agents care mainly about their own consumption, especially middle-aged consumption.

The production function for goods has the form:

$$y_t = f(K_t,\ L_t l_t H_t),$$

where K_t and l_t denote inputs of physical capital and labor respectively, and H_t refers to a middle-aged agent's human capital. I assume that $f(\cdot,\cdot)$ is increasing, concave, and homogeneous of degree one.

The education technology converts investment in children (q_t per child) into human capital where the economy-wide average human capital investment per child ($\bar{q}_t$) exhibits an externality:[8]

$$H_{t+1} = h(q_t,\ \bar{q}_t).$$

I assume that $h(\cdot,\cdot)$ is increasing, concave, and homogeneous of degree one.

To obtain a balanced growth path, I assume that $f(\cdot,\cdot)$, $h(\cdot,\cdot)$, and $U(\cdot,\cdot,\cdot,\cdot)$ are Cobb-Douglas.[9] However, I also model the relationship between wage rates and fertility by assuming a constant elasticity utility function and discuss it briefly with general utility functions.

Each middle-aged agent in period t spends vN_t units of time rearing children, supplies $1 - vN_t$ units of time to the labor market and earns $(1 - vN_t)w_t$. This agent

[8] Externalities in human capital accumulation have been stressed in the literature. For example, Lucas (1990) illustrates how the externality in human capital accumulation could explain why interest rates in developed and developing countries are much closer than the Neoclassical growth model predicts.

[9] See, e.g., Lucas, 1988; Becker, et al., 1990. In these models, production functions are Cobb-Douglas, while utility functions are constant elasticity of substitution functions (CES) with separability assumed for utilities of different generations in Becker, et al. or periods in Lucas. In this thesis, I do not assume separability for the utility function.

receives bequests, $(1+r_{t-1})b_{t-1}S_{t-1}$, where b is the fraction of the return to saving which is bequeathed to each child (or the bequest ratio)[10] from the old-aged parent at the beginning of period t, and leaves bequests out of saving (S_t), $(1+r_t)b_tS_t$, to each child at the beginning of children's middle-age. The middle-aged agent spends the earning and the inheritance, $(1-vN_t)w_t+(1+r_{t-1})b_{t-1}S_{t-1}$, on own middle-aged consumption C_t^t, on saving for old-aged consumption (C_{t+1}^t), S_t, on bequests to children, $N_tb_tS_t$, and on investments in children, q_tN_t. For simplicity, the child-rearing cost, which has no direct contribution to the quality of children, is not to be discussed until in section 1.5. Thus the middle-aged's budget constraint implies:

$$C_t^t = w_t(1-vN_t)+(1+r_{t-1})b_{t-1}S_{t-1}-q_tN_t-S_t, \tag{1.2.1}$$

$$C_{t+1}^t = (1+r_t)(1-b_tN_t)S_t \tag{1.2.2}$$

where w and r denote the wage rate and the interest rate respectively. I assume that bequests are nonnegative since agents are unable to force other generations to submit goods to them.

To keep the model tractable, I adopt the following assumption:

Assumption 1.2.1: *Agents make decisions (b_t, q_t, N_t, and S_t) while knowing the decisions of the older generations and taking the decisions of the younger generations as given.*

Without this assumption, it would be extremely hard to work out the interaction among infinite generations as shown by Nishimura and Zhang (1992). Under this assumption, generations act like Nash game players except that they play in different periods. Also under this assumption, middle-aged agents living in period t cannot directly affect younger generations' fertility choices, N_{t+2}, $N_{t+3}\cdots$, and consumption

[10] Equivalently, I can define $B_{t+1}=(1+r_t)b_tS_t$ and let agents choose B_{t+1}.

of descendents other than the middle-aged consumption of their own children (C_{t+1}^{t+1}), C_{t+2}^{t+1}, C_{t+2}^{t+2} $\cdots$. Thus I do not include these future fertility and consumption in the utility function.

Firms are perfect competitors maximizing profits by choosing the amount of physical capital K_t and labor l_t, given H_t and L_t. Physical capital lasts for one period in production, hence firms' problems are static. The profit function is defined as:

$$f(K_t, L_t H_t l_t) - (1 + r_{t-1})K_t - L_t l_t w_t.$$

Since people are identical, the symmetric condition is: $q_t = \bar{q}_t$.

Labor market clearing requires:

$$l_t = 1 - vN_t. \tag{1.2.3}$$

Capital market clearing needs:

$$K_t = L_{t-1}S_{t-1}. \tag{1.2.4}$$

Constant returns to scale and perfect competition imply that profits should be zero in every period. By Walras' law, the goods market clears as well.

Definition: *An equilibrium is a collection of sequences* $\{w_t, r_t, b_t, q_t, N_t, S_t, K_t, l_t\}_{t=0}^{\infty}$ *such that:*

(1) Given $\{w_t, r_t\}_{t=0}^{\infty}$*,* $\{b_t, q_t, N_t, S_t\}_{t=0}^{\infty}$ *solves agents' problems and* $\{K_t, l_t\}_{t=0}^{\infty}$ *solves firms' problems;*

(2) Markets clear.

1.3 Growth with Exogenous Investment in Children

In this section, it is first shown that the Malthusian equilibrium prevails if bequests are nonoperative. Then it is shown that when bequests are operative fertility will be positively related to interest rates and negatively related to wage rates.

Without operative bequests, the utility function can be written as:

$$V(C_t^t, C_{t+1}^t, N_t),$$

where we suppress C_{t+1}^{t+1} since parents can no longer affect children's consumption.

We assume:

$$V(C_t^t, C_{t+1}^t, N_t) = {C_t^t}^{\alpha} {C_{t+1}^t}^{\beta} N_t^{1-\alpha-\beta} \ , 0 < \alpha, \beta < 1 \ .$$

Profit maximizing and perfect competition require:

$$w_t = f(k_t) - k_t f'(k_t) \ , \tag{1.3.1}$$

$$1 + r_{t-1} = f'(k_t), \tag{1.3.2}$$

where $k_t = \frac{K_t}{l_t L_t}$ (the capital/effective labor ratio).

The middle-aged solve:

$$\max_{C_t^t, C_{t+1}^t, N_t} V(C_t^t, C_{t+1}^t, N_t)$$

subject to (1.2.1) and (1.2.2) with $b = 0$. The optimization problem corresponds to the Lagrangian:

$$V(C_t^t,\ C_{t+1}^t,\ N_t) + \lambda\{(1 + r_t)[w_t(1 - vN_t) - N_t q - C_t^t] - C_{t+1}^t\},$$

where λ is the Lagrange multiplier. Note above that S_t has been substituted out by using (1.2.1) and (1.2.2).

The first-order conditions of the middle-aged are given by:

$$C_t^t : \quad V_1 = \frac{\alpha V}{C_t^t} = \lambda(1 + r_t) \ , \tag{1.3.3}$$

$$C^t_{t+1}: \quad V_2 = \frac{\beta V}{C^t_{t+1}} = \lambda , \tag{1.3.4}$$

$$N_t: \quad V_3 = \frac{(1-\alpha-\beta)V}{N_t} = \lambda(1+r_t)(w_t v + q) , \tag{1.3.5}$$

$$\lambda: \quad (1+r_t)w_t(1-vN_t) = (1+r_t)(N_t q + C^t_t) + C^t_{t+1} , \tag{1.3.6}$$

where V_i indicates the partial derivative of V with respect to the argument i in V, $i = 1,2,3$.

Equations (1.3.3) and (1.3.4) imply that the utility forgone from saving one more unit now should equal the utility gained by consuming $1 + r_t$ units more in the next period. Equations (1.3.4) and (1.3.5) mean that the utility forgone by consuming less via earning vw_t units fewer and spending q units more to have one more child equals the utility obtained by enjoying the child.

The equilibrium is characterized by (1.2.1)-(1.3.6) and the symmetric condition with $b = 0$. We concentrate on the steady-state equilibrium:

$$C^t_t = C^{t+j}_{t+j} = C_1,$$

$$C^t_{t+1} = C^{t+j}_{t+j+1} = C_2,$$

$$N_t = N_{t+j} = N,$$

$$K_{t+1}/K_t = K_{t+j+1}/K_{t+j} = 1 + g,$$

for every t and j, where g and N are the steady-state rate of capital accumulation and fertility respectively. The demand for children is found to be an increasing, concave function of the wage rate:

$$N = \frac{(1-\alpha-\beta)w}{wv+q}\,. \tag{1.3.7}$$

This result agrees with Malthus (1890), Bernhard (1990) and Raut (1991) in that fertility and wage rates are positively related. Consequently, fertility and the capital/effective labor ratio (k) are also positively related since $\frac{\partial w}{\partial k} = -kf''(k) > 0$. Equation (1.3.7) also implies that fertility decreases with the exogenous investment of goods in children (q) and the time needed to rear a child (v) but increases with the degree of the taste for the quantity of children $(1-\alpha-\beta)$.

The positive effect of the steady-state wage rate on fertility is due to that its income effect on fertility is greater than its substitution effect. This can be shown by differentiating the first-order conditions (1.3.3)-(1.3.6) with respect to w:

$$\begin{bmatrix} V_{11} & V_{12} & V_{13} & -(1+r) \\ V_{21} & V_{22} & V_{23} & -1 \\ V_{31} & V_{32} & V_{33} & -(1+r)(wv+q) \\ 1+r & 1 & (1+r)(wv+q) & 0 \end{bmatrix} \begin{bmatrix} \frac{\partial C^1}{\partial w} \\ \frac{\partial C^2}{\partial w} \\ \frac{\partial N}{\partial w} \\ \frac{\partial \lambda}{\partial w} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \lambda(1+r)v \\ (1+r)(1-vN) \end{bmatrix}$$

where V_{ij} indicates the partial derivative of V_i with respect to the argument j, and $i, j = 1, 2, 3$. Solving for $\frac{\partial N}{\partial w}$ via Cramer's rule, we have:

$$\frac{\partial N}{\partial w} = \frac{(1+r)v\lambda}{H}\begin{vmatrix} V_{11} & V_{12} & -(1+r) \\ V_{21} & V_{22} & -1 \\ 1+r & 1 & 0 \end{vmatrix}$$

$$+\frac{(1+r)(1-vN)}{H}\begin{vmatrix} V_{11} & V_{12} & 1+r \\ V_{21} & V_{22} & 1 \\ V_{31} & V_{32} & (1+r)(wv+q) \end{vmatrix}$$

$$= \frac{-(1-\alpha-\beta)vw}{(vw+q)^2} + \frac{1-\alpha-\beta}{vw+q} = \frac{(1-\alpha-\beta)q}{(wv+q)^2},$$

where H stands for the determinant of the bordered Hessian matrix which is positive. The first term on the right hand side of the equation is the negative substitution effect, while the second one is the positive income effect. Note that the substitution effect would be zero if v, the time cost per child, is zero. The overall effect ($\frac{\partial N}{\partial w}$) equals $\frac{(1-\alpha-\beta)q}{(wv+q)^2}$ which is positive. This result ($\frac{\partial N}{\partial w} > 0$) is similar to the downward sloping n_2 curve in Barro and Becker (1989, p.489).[11]

If bequests are operative, it is shown that fertility will be positively related to the interest rate instead of the wage rate. In this case, children's middle-aged consumption will be affected by parental decision. Thus it should remain in parental utility function: $U(C_t^t, C_{t+1}^t, C_{t+1}^{t+1}, N_t)$. Then we assume:

$$U(C_t^t, C_{t+1}^t, C_{t+1}^{t+1}, N_t) = \left(C_t^{t\ \phi} C_{t+1}^{t\ \ 1-\phi}\right)^{\sigma} \left(C_{t+1}^{t+1\ \varepsilon} N_t^{\ 1-\varepsilon}\right)^{1-\sigma} .$$

We substitute (1.2.1) and (1.2.2) into $U(\cdot,\cdot,\cdot,\cdot)$ for C_t^t, C_{t+1}^t, and C_{t+1}^{t+1} (by updating (1.2.1) one period). Then middle-aged agents seek to maximize their utility by choosing b_t, N_t, and S_t.

The first-order conditions with respect to b_t and S_t imply:

$$N = \frac{\varepsilon(1-\sigma)}{\sigma\phi}(1+r) = \frac{\varepsilon(1-\sigma)}{\sigma\phi} f'(k)\,, \qquad (1.3.8)$$

which means that fertility is positively related to the interest rate, or negatively related to the capital/effective labor ratio, k, if bequests are operative.[12] The intuition

[11] Since the n_2 curve does not use the first-order condition for bequests in Barro and Becker (1989), it is comparable to our analysis here.

[12] If we assume an exogenous economic growth rate, then:

$$N = \frac{\varepsilon(1-\sigma)(1+r)}{\sigma\phi(1+g_e)}\,,$$

in the case with operative bequests is that higher interest rates increase the return to leaving one unit of goods now to children in the next period, and therefore encourage agents to leave more bequests and to have more children.[13] The positive (negative) relationship between fertility and interest (wage) rates is consistent with data after Malthus.

The bequest ratio is given implicitly by:

$$b = \frac{\theta\epsilon(1-\sigma)(1-vN)[\sigma+(1-\sigma)(1-\epsilon)] - \sigma^2\phi(1-\phi)(1-\theta)}{\theta N(1-vN)\{\epsilon(1-\sigma)[\sigma+(1-\sigma)(1-\epsilon)] + \sigma^2\phi(1-\phi)\}}\ .$$

Clearly, bequests are operative only if:

$$\theta\epsilon(1-\sigma)[\sigma+(1-\sigma)(1-\epsilon)] > \sigma^2\phi(1-\phi)(1-\theta)\ .$$

This condition may not be satisfied if agents' tastes for own consumption are strong relative to tastes for children's consumption.

1.3.1 Stability of the Steady-State Equilibrium without Operative Bequests

This subsection derives the stability conditions for the equilibrium with Malthus' feature.

We assume:

where $1+g_c$ is the growth rate of consumption. This equation is similar to equation (23) of Barro and Becker (1989) where an exogenous technology progress is assumed to be the driving force of growth.

[13]To maintain the optimal condition with respect to bequests, $U_1 = \frac{(1+r_t)}{N_t}U_3$, agents could leave more bequests per child and have more children. Increasing fertility would partly offset the effect of a rise in interest rates on the equality of the optimal condition. (This optimal condition means that an additional unit of goods consumed by middle-aged agents now could be converted to $1+r_t$ units in the next period and be $\frac{1+r_t}{N_t}$ units for consumption of each child such that marginal utilities from both ways are the same.)

$$f(K_t, L_t H_t l_t) = D K_t^{\theta} (L_t l_t)^{1-\theta}, D > 0, 1 > \theta > 0 .$$[14]

Thus equations (1.3.1) and (1.3.2) become:

$$w_t = D(1-\theta)\left(\frac{K_t}{L_t l_t}\right)^{\theta}, \qquad (1.3.1)'$$

$$1 + r_{t-1} = D\theta\left(\frac{K_t}{L_t l_t}\right)^{\theta-1}. \qquad (1.3.2)'$$

Equations (1.3.3)-(1.3.6) and the last two equations imply that saving is proportional to the real wage rate:

$$S_t = \beta w_t. \qquad (1.3.9)$$

Combining (1.3.9) with the market clearing conditions in the last section and denoting $1 + g_k = \frac{K_{t+1}}{K_t}$ give:

$$(1 + g_k) K_t = L_t S = \beta L_t w, \qquad (1.3.10)$$

which means that the capital/labor ratio $K_t/L_t \equiv k^l$ is constant in the steady-state, which in turn implies: $N = 1 + g_k$. Therefore, in the steady-state, we have:

$$S = N k^l.$$

This equation is similar to the fundamental equation in the Neoclassical growth model which equates the supply of capital (S) with the demand for capital ($N\ k^l$) in order to maintain the steady-state capital/labor ratio (k^l). It can be shown that k^l satisfies:

[14] With the Cobb-Douglas production function, we can easily show that output per middle-aged, $\frac{f(K_t, L_t l_t)}{L_t} = (1 - vN_t) f(k_t)$, increases with k_t and hence with w_t in both cases with operative and nonoperative bequests. Thus the relationship between fertility and wage rates is essentially the one between fertility and per worker output. Of course, in the case with nonoperative bequests higher k_t means higher fertility and in turn implies less labor supply. Falls in labor supply may reduce per capita output. However, we would like to restrict the strength of the substitution effect of higher wage rates on fertility by assuming v is not too large such that higher k_t or w_t results in higher per worker output when more general forms of the production function are assumed.

$$k^{l^2}[v\beta D(1-\theta)]^{1/\theta} = [(1-\alpha-\beta)k^l - \beta q]^{1/\theta}[\beta q + (\alpha+\beta)k^l]\,. \tag{1.3.11}$$

Like the Neoclassical growth model and the Malthusian theory, there is no sustained growth of per capita income.

Furthermore, we derive the conditions under which the steady-state path is unique and stable. The system of equations characterizing the equilibrium without operative bequests boils down to the following two equations:

$$k^l_{t+1} = \frac{q\beta}{1-\alpha-\beta-vN_t}\,, \tag{1.3.12}$$

$$N_t = \frac{D(1-\alpha-\beta)(1-\theta)k_t^{l^\theta}}{vD(1-\theta)k_t^{l^\theta} + q(1-vN_t)^\theta}\,. \tag{1.3.13}$$

Equations (1.3.12) and (1.3.13) implicitly define k^l_{t+1} as a function of k^l_t: $k^l_{t+1} = g(k^l_t)$. The steady-state capital/labor ratio k^l is unique and stable if $g(\cdot)$ is globally increasing and strictly concave, and if $k^l_{t+1} > k^l_t$ when k^l_t approaches zero.

From (1.3.12) and (1.3.13), it can be shown that $g(\cdot)$ is an increasing function if $1 > (1+\theta)vN_t$, and is strictly concave if $\theta < 1/2$ and $1 > (1+\theta)(1-\alpha-\beta)$ (note that $1-\alpha-\beta > vN_t$ is needed for $k^l_{t+1} > 0$ in (1.3.12)). In reality, these conditions seem to be satisfied, because $\theta = .25$ is widely accepted, and $1-\alpha-\beta < 1/2$ (hence $1 > (1+\theta)(1-\alpha-\beta)$) under the assumption that parents care about their own consumption more than the quantity of children. With $\theta = .25$, $vN_t < \frac{1}{(1+\theta)}$ says that the time spent in rearing children is less than eighty percent of the labor endowment of the middle-aged, which is also conceivable. Under the last condition, $k^l_{t+1} > k^l_t$ when $k^l_t \to 0$ because $N_t \to 0$ as $k^l_t \to 0$ by (1.3.13), and then by (1.3.12), $k^l_{t+1} \to \frac{q\beta}{1-\alpha-\beta}$ which is finite and positive. Under these conditions, the steady-state equilibrium is stable and the implications of this model are similar to those of the Malthusian theory.

If $v = 0$ as in Bernhard (1990) and Raut (1990) (i.e., no time is needed to rear a child), it will be obvious from (1.3.12) and (1.3.13) that the system is stable and the steady-state will be realized in one period. However, their conclusions seem to be inconsistent with data in England where fertility and wage rates seem to have shown long swings from the 16th century to the beginning of the 19th century.[15]

1.3.2 Functional Forms

In the last subsection, we assumed Cobb-Douglas utility functions. (Note, however, no specific production functional form was assumed to have the positive or negative relation between fertility and wage rates.) To see whether the results about the relationship between fertility and wage rates are robust to different functional forms, a CES utility function is now adopted as in Barro and Becker (1989):

$$U(C_t^t, C_{t+1}^t, C_{t+1}^{t+1}, N_t) =$$

$$\left(\beta_1 C_t^{t(\rho-1)/\rho} + \beta_2 {C_{t+1}^t}^{(\rho-1)/\rho} + \beta_3 C_{t+1}^{t+1(\rho-1)/\rho} + \beta_4 {N_t}^{(\rho-1)/\rho}\right)^{\frac{\rho}{(\rho-1)}} ,$$

where $0 \leq \rho < \infty$. When $\rho = 1$, it is a Cobb-Douglas utility function used earlier.

When bequests are operative, fertility is given by:

$$N = \frac{\beta_3}{\beta_1} \left(\frac{1}{1+g}\right)^{\frac{1}{\rho}} (1+r) , \tag{1.3.14}$$

which is essentially the same as (1.3.8). Hence fertility and wage rates are negatively related as before. From the optimal condition with respect to bequests in footnote 13, one may expect a positive response of fertility to a rise in interest rates even with more general forms of the utility function.

However, when bequests are nonoperative, fertility is found to be:

[15] See, Wrigley, 1981.

$$N = \frac{\left(\frac{\beta_4}{\beta_1}\right)^{\rho}(1+r)^{1-\rho}w}{(wv+q)^{\rho}\left[\left(\frac{\beta_2}{\beta_1}\right)^{\rho}+(1+r)^{1-\rho}\right]+\left(\frac{\beta_4}{\beta_1}\right)^{\rho}(1+r)^{1-\rho}(wv+q)} \,. \tag{1.3.15}$$

In this case, fertility may or may not be positively related to wage rates, depending on the degree of the constant elasticity ρ. If w and r are treated separately (i.e., if we ignore the link between w and r via the production function), it can be shown that $\rho < 1$ corresponds to $\frac{\partial N}{\partial w} > 0$. Even if $\rho > 1$, $\frac{\partial N}{\partial w} > 0$ can still hold unless ρ is large enough. Roughly put, fertility and wage rates are still positively related as long as the elasticity of substitution is not too large.

However, when w and r are determined simultaneously via k, it would be difficult to determine the relation between fertility and wage rates (or interest rates). Nevertheless, "reasonable" parameter values may give a positive relation between fertility and wage rates. Expressing w and r in terms of k in (1.3.15) and differentiating N with respect to k give rise to:

$$\frac{\partial N}{\partial k} = -qkf''(k)\beta_4{}^{\rho}\left(\beta_4{}^{\rho}+\rho\beta_1{}^{\rho}(wv+q)^{\rho-1}+\rho\beta_2{}^{\rho}(1+r)^{\rho-1}(wv+q)^{\rho-1}\right)/d^2 -$$

$$f''(k)(1-\rho)\beta_2{}^{\rho}\beta_4{}^{\rho}(wv+q)^{\rho}(1+r)^{\rho-2}\left[\left(2+\left(\frac{\beta_1}{\beta_2}\right)^{\rho}(1+r)^{1-\rho}\right)kf'(k)-f(k)\right]/d^2 \,,$$

where $d = (wv+q)^{\rho}\left[\left(\frac{\beta_2}{\beta_1}\right)^{\rho}+(1+r)^{1-\rho}\right]+\left(\frac{\beta_4}{\beta_1}\right)^{\rho}(1+r)^{1-\rho}(wv+q)$.

Suppose that the share of capital income, kf', to output, $f(k)$, ranges between 1/4 and 1/3 and that $0 < \rho \leq 1$ (see Auerbach and Kotlikoff, 1987, pp.50-52), $\left(\frac{\beta_1}{\beta_2}\right)^{\rho}(1+r)^{1-\rho} \geq 2$ would guarantee that fertility is positively related to k (hence w). This condition can be satisfied if the taste for own middle-aged consumption (β_1) is at least twice as strong as that for own old-aged consumption (β_2), and if the annual interest rate is no less than 3 percent (i.e., $1 + r \geq 2$ if one period in this model equals 25 years). Alternatively, it can be shown that the middle-aged's first-order conditions imply that $\frac{C^t_t}{C^t_{t+1}} = \left(\frac{\beta_1}{\beta_2}\right)^{\rho}(1+r)^{-\rho}$. Thus the above condition

$\left(\frac{\beta_1}{\beta_2}\right)^{\rho}(1+r)^{1-\rho} \geq 2$ would hold if $\frac{(1+r)C_t^t}{C_{t+1}^t} \geq 2$, or equivalently if $\frac{C_t^t}{S_t} \geq 2$ (note that $C_{t+1}^t = (1+r_t)S_t$). It appears reasonable to assume that the ratio of working-period consumption to saving is over two.

When $v = 0$ (i.e., no time is needed to have a child), Bernhard (1990) shows that the positive relationship between fertility and wage rates holds with a general utility function. Thus, we can infer that if v is small enough (i.e., the substitution effect is weak), the positive relation between fertility and wage rates may not rely on any particular form of the utility function.

1.4 Growth with Endogenous Investment in Children

With endogenous investment in children, it is illustrated in this section that sustained steady-state growth of per capita income is possible, and that fertility is no longer related positively to wage rates even if bequests are nonoperative.

Assume that:

$$f(K_t, L_t l_t H_t) = DK_t^{\theta}(L_t H_t l_t)^{1-\theta}, D > 0, 1 > \theta > 0 \ ,$$

$$h(q_t, \bar{q}_t) = Aq_t^{\ \psi}\bar{q}_t^{\ 1-\psi}, A > 0, 1 \geq \psi > 0 \ .$$

Thus equations (1.3.1)′ and (1.3.2)′ become:

$$w_t = D(1-\theta)\left(\frac{K_t}{L_t l_t H_t}\right)^{\theta} H_t \ , \tag{1.3.1''}$$

$$1 + r_{t-1} = D\theta\left(\frac{K_t}{L_t l_t H_t}\right)^{\theta-1} \ . \tag{1.3.2''}$$

We substitute (1.2.1) and (1.2.2) into the utility function, $U(\cdot,\cdot,\cdot,\cdot)$, for C_t^t, C_{t+1}^t, C_t^{t-1} (by backdating (1.2.2) one period), C_{t+1}^{t+1} (by updating (1.2.1) one period). The first-order conditions of the middle-aged are given by:

$$b_t: \quad N_t(1+r_t)S_tU_2 = (1+r_t)S_tU_3 + \mu, \;\; \mu \geq 0, \;\; \mu b_t = 0; \tag{1.4.1}$$

$$q_t: \quad N_tU_1 = \frac{\psi(1-vN_{t+1})w_{t+1}}{q_t}U_3;^{16} \tag{1.4.2}$$

$$N_t: \quad [vw_t + q_t]U_1 = -(1+r_t)b_tS_tU_2 + U_4; \tag{1.4.3}$$

$$S_t: \quad U_1 = (1+r_t)(1-b_tN_t)U_2 + b_t(1+r_t)U_3. \tag{1.4.4}$$

Here, μ is the Lagrange multiplier associated with b_t.

Equation (1.4.1) means that utility forgone by leaving an additional unit of bequests to children should be no less than the utility obtained from improving children's consumption by the bequest; the equality holds if bequests are operative ($b_t > 0$). Equation (1.4.2) equates the loss in utility from investing one more unit in the human capital of children to the gain in utility from increasing consumption of children due to higher earnings of children by the investment. Equation (1.4.3) means that the utility forgone from consuming less to have one more child should equal the utility obtained from enjoying the child. Equation (1.4.4) requires that the utility forgone from saving one more unit now should equal the utility derived from consuming $1+r_t$ units more when old.

When bequests are operative, the equilibrium is characterized by (1.2.1)-(1.2.4), (1.3.1)′, (1.3.2)′, (1.4.1)-(1.4.4), and by the symmetric condition.

The bequest ratio (bN) is found to be:

[16] Note that when q is endogenous, w_{t+1} is a function of H_{t+1} and hence a function of q_t.

$$bN = \frac{\varepsilon(1-\sigma)[\theta+\psi(1-\theta)(1-\phi)] - \sigma\phi(1-\phi)(1-\theta)}{\sigma\phi\theta(1-\phi)+\varepsilon\theta\phi(1-\sigma)} . \tag{1.4.5}$$

The bequest ratio increases with ε, the taste for the consumption of children relative to the number of children.

Saving is proportional to the labor income:

$$S_t = \frac{\varepsilon\theta(1-\sigma)}{\sigma\phi(1-\theta)}(1-vN)w_t \equiv \gamma_s w_t, \tag{1.4.6}$$

where the saving rate, denoted by γ_s, increases with ε, and decreases with σ and ϕ. That is, agents save more when they care more about their own old-aged consumption and the consumption of children. Of course, γ_s increases with the share parameter of physical capital, θ.

Fertility is given by:

$$N = \frac{x}{v[\sigma\theta(1-\phi)+x]} , \tag{1.4.7}$$

where:

$$x = (1-\varepsilon)(1-\sigma)[(1-bN)\theta + (1-\theta)] - \sigma(1-\phi)\psi(1-\theta) - bN\sigma\theta(1-\phi).$$

Fertility decreases with ε. Namely, fertility is higher when tastes for the number of children are stronger. From (1.4.5) and (1.4.7), we can see that fertility is no longer related to wage rates.

The investment per child is:

$$q_t = \frac{\psi\varepsilon(1-\sigma)}{N\sigma\phi}(1-vN)w_t \equiv \gamma_q(1-vN)w_t. \tag{1.4.8}$$

The investment per child as a fraction of labor income, denoted by γ_q, is negatively related to fertility. Also, it increases (decreases) with the taste for the consumption of children (own consumption) and increases with the degree of individual returns to investment in the quality of children.

Without operative intergenerational transfers, the equilibrium is characterized by (1.2.1)-(1.2.4), (1.3.1)′, (1.3.2)′, (1.4.2)-(1.4.4), and the symmetric condition, with $b_t = 0$.

Fertility in this case is given by:

$$N = \frac{(1-\sigma)\{\sigma\phi(1-\varepsilon) - \psi\varepsilon[\sigma + (1-\varepsilon)(1-\sigma)]\}}{v[\sigma\phi - \psi\varepsilon(1-\sigma)][\sigma + (1-\varepsilon)(1-\sigma)]} \,. \tag{1.4.9}$$

Note that in both (1.4.7) and (1.4.9), fertility is a constant determined by preferences and technology. The investment per child as a fraction of labor income is found to be the same as that in (1.4.8).

Saving is also proportional to labor income:

$$S_t = \gamma_s(1 - vN)w_t, \tag{1.4.10}$$

where: $\gamma_s = \frac{(1-\phi)[\sigma\phi - \psi\varepsilon(1-\sigma)]}{\sigma\phi}$. The saving rate now decreases with tastes for the consumption of children in contrast to (1.4.6) where bequests are operative.

Combining these solutions with production functions for both goods and human capital, we have that evolution of both human and physical capital is determined jointly by the following two equations:

$$K_{t+1} = L_t S_t = \gamma_s(1 - vN_t)L_t D(1-\theta)K_t^{\theta}[L_t H_t(1 - vN_t)]^{-\theta} H_t \,, \tag{1.4.11}$$

$$H_{t+1} = Aq_t = A\gamma_q(1 - vN_t)D(1-\theta)K_t^{\theta}[L_t H_t(1 - vN_t)]^{-\theta} H_t \,. \tag{1.4.12}$$

Equation (1.4.7) or (1.4.9) implies that the evolution of population is simply determined by:

$$L_{t+1} = N L_t \,.$$

Denoting accumulation rates of physical capital and human capital as $1 + g_{kt} \equiv K_{t+1}/K_t$ and $1 + g_{ht} \equiv H_{t+1}/H_t$ respectively and imposing steady-state growth conditions ($g_{ht} = g_h$, $g_{kt} = g_k$, and $N_t = N$), we rewrite (1.4.11) and (1.4.12) to be:

$$\frac{K_t}{H_t L_t} = \left(\frac{D\phi_1(1-\theta)}{1+g_k} \right)^{\frac{1}{1-\theta}} (1 - vN)^{\frac{-\theta}{1-\theta}} \,, \tag{1.4.13}$$

$$\frac{K_t}{H_t L_t} = \left(\frac{1+g_h}{AD\phi_3(1-\theta)} \right)^{\frac{1}{\theta}} (1 - vN) \,. \tag{1.4.14}$$

Equations (1.4.13) and (1.4.14) imply that the steady-state physical capital/effective labor ratio is constant:

$$\frac{K_{t+1}}{H_{t+1} L_{t+1}} = \frac{K_t}{H_t L_t} \equiv k^* \,,$$

Which is equivalent to:

$$1 + g_k = (1 + g_h) N \,. \tag{1.4.15}$$

The equation above means that physical capital per capita grows at the same pace as an individual's human capital.

Equating the right hand side of (1.4.13) to that of (1.4.14), and substituting (1.4.15) into (1.4.13) for $1 + g_k$ yield the steady-state rate of human capital accumulation:

$$1 + g_h = A^{1-\theta} D[(1-\theta)\gamma_e]^{\theta} N^{-\theta}[(1-\theta)\gamma_q]^{1-\theta}(1 - vN)^{1-\theta} . \tag{1.4.16}$$

It turns out that the rate of human capital accumulation is also the growth rate of per capita output.[17] Clearly, the growth rate of per capita income is related positively to the saving rate and the investment per child as a fraction of labor income, and negatively to fertility. Wage rates now can grow over time.

1.5 Policy Issues

Empirical work (e.g., Barro, 1989) suggests that government policies may have profound effects on economic growth and population growth. This section is devoted to examining the impacts of the government's subsidies for education and child-rearing on fertility and economic growth.

1.5.1 An Education Subsidy

The government taxes labor earnings at a flat rate τ to subsidize investments in children at a flat rate χ, and balances its budget in every period:

$$\tau w_t(1 - vN_t) = \chi q_t N_t . \tag{1.5.1}$$

Then (1.2.1) becomes:

[17]Per capita output in period t is:

$$\frac{f\Big(K_t, L_t H_t(1 - vN_t)\Big)}{L_{t-1} + L_t + L_{t+1}} = D\left(\frac{K_t}{H_t L_t}\right)^{\theta}(1 - vN)^{1-\theta} H_t/(1/N + 1 + N) ,$$

where $f\Big(K_t, L_t(1 - vN_t)H_t\Big)$ is aggregate output (recall that $\frac{k_t}{l_t H_t} = \frac{K_t}{L_t(1-vN_t)H_t}$) and where fertility and the steady-state physical capital/effective labor ratio are constant. Thus per capita income grows at the same pace as that of H_t.

$$C_t^t = w_t(1-\tau)(1-vN_t) + (1+r_{t-1})b_{t-1}S_{t-1} - (1-\chi)q_tN_t - S_t\,. \qquad (1.2.1)'$$

Following the same steps in the last section, we can solve for the decisions as functions of the tax rate. In the steady-state growth equilibrium, the bequest ratio is given by:

$$bN = \frac{\varepsilon(1-\sigma)[\theta+\psi(1-\tau)(1-\theta)(1-\phi)] - \sigma\phi(1-\phi)(1-\tau)(1-\theta)}{\theta\phi[\varepsilon(1-\sigma)+\sigma(1-\phi)]}\,. \qquad (1.5.2)$$

It is increasing with τ under the assumption that agents' tastes for their own middle-aged consumption are stronger than those for children's. The ratio of bequests per child to the returns of saving may rise as well with the increase in τ unless fertility rises substantially.

In the steady-state growth regime, it can be shown that fertility decreases with τ if and only if:

$$\varepsilon(1-\varepsilon)(1-\sigma) + \sigma[\varepsilon - \phi(1-\varepsilon)] > 0\,,$$

which holds when the taste for the consumption of children is strong enough relative to the quantity of children, at least when $\varepsilon > \frac{\phi}{1+\phi}$. With $\varepsilon < \frac{1+\sigma\phi-\sqrt{(1+\sigma\phi)^2-4\sigma\phi(1-\sigma)}}{2(1-\sigma)}$, fertility increases with τ.

The saving rate is the same as that in (1.4.6) and is independent of the subsidy. The ratio of investment in all children to labor income is an increasing function of τ:

$$q_tN = \frac{[\psi\varepsilon(1-\sigma)(1-\tau)+\tau\sigma\phi]}{\sigma\phi}(1-vN)w_t\,, \qquad (1.5.3)$$

under $\sigma\phi > \varepsilon(1-\sigma)$. When fertility falls due to the rise in τ, investment per child as a fraction of labor income will rise.

Consequently, the subsidy speeds the growth of per capita income if $\varepsilon > \frac{\phi(1-\phi)}{1+\phi(1-\phi)}(\leq$

$\frac{1}{2}$) under $\sigma\phi > \varepsilon(1-\sigma)$. That is, it speeds growth unless tastes for the number of children are substantially stronger than those for the consumption of children.

In short, when bequests are operative, the education subsidy has no net effect on the saving rate, and raises investment in all children as a fraction of labor income if parents care mainly about their own consumption, hence it can speed up economic growth. It will accelerate economic growth if it reduces fertility since then investment per child as a fraction of labor income goes up, and since the saving rate is independent of the tax rate. It can stimulate growth even if it increases fertility.[18]

When bequests are nonoperative, fertility is independent of the subsidy:

$$N = \frac{\sigma\phi(1-\varepsilon)(1-\sigma) - \psi\varepsilon(1-\sigma)[\sigma + (1-\varepsilon)(1-\sigma)]}{v[\sigma\phi - \psi\varepsilon(1-\sigma)][\sigma + (1-\varepsilon)(1-\sigma)]} \,. \tag{1.5.4}$$

The saving rate now decreases with τ:

$$S_t = \frac{(1-\phi)(1-\tau)[\sigma\phi - \psi\varepsilon(1-\sigma)]}{\sigma\phi}(1-vN)w_t \,. \tag{1.5.5}$$

Under the condition $\sigma\phi > \varepsilon(1-\sigma)$, investment in all children as a fraction of labor income still increases with τ:

$$q_t N = \frac{[\tau\sigma\phi + \psi\varepsilon(1-\sigma)(1-\tau)]}{\sigma\phi}(1-vN)w_t \,. \tag{1.5.6}$$

Since fertility is independent of τ, investment per child as a fraction of labor income is raised by the subsidy. It can be shown that the education subsidy accelerates the growth of per capita income if and only if $\tau < \frac{1-2\theta}{1-\theta}$ under $\sigma\phi > \varepsilon(1-\sigma)$.

[18]Using the data set and framework of Barro (1991), I found that the ratio of public expenditures on education to Gross Domestic Product has positive impacts on fertility. Also, this ratio has (insignificant) positive effects on growth as reported by Barro (1989).

Overall, when bequests are nonoperative, the education subsidy has no net effect on fertility, reduces the saving rate, but raises investment per child as a fraction of labor income. In this case, the effect on economic growth depends on how low the tax rate is, or on whether the negative impact on saving dominates the positive impact on investment per child as a fraction of labor income. If $\theta < 1/2$, then human capital plays a more important role in the production of goods than physical capital does. Thus, with $\theta < 1/2$ the positive effect on human capital accumulation will outweigh the negative effect on the physical capital accumulation, bringing about faster economic growth. One can see that when $\theta = 1/2$, it is impossible for the subsidy to speed economic growth (recall the restriction on the tax rate above). If the tax rate is so high that its negative effect on saving dominates its positive effect on the investment in children, $\theta > 1/2$, then economic growth will be inhibited.

It is seen that middle-aged agents choose different combinations of instruments in response to the subsidy in different cases with or without operative bequests. Since the education subsidy transfers goods across generations, varying the bequest ratio seems more effective than changing the saving rate[19]. Moreover, as in (1.5.2), the bequest ratio, b, is inversely related to fertility, hence fertility varies in response to τ in general. When bequests are nonoperative, on the other hand, the subsidy induces a switch of resources from saving to investment in children, without changing fertility.

No matter whether bequests are operative or not, the education subsidy affects fertility in two opposite ways: it lowers the after-tax wage rate (the opportunity cost of spending time rearing children), and also the cost of the quality of children (q). The former tends to raise fertility while the latter may induce parents to trade the quantity of children for the quality of children since agents care about consumption of each child and due to the negative relation between the quantity and quality of

[19]Barro (1974) showed that when bequests are operative, public transfers like the pay-as-you-go pensions have no effect on the saving rate.

children (see 1.5.3 and 1.5.6). It happens that these two effects exactly offset each other if bequests are nonoperative. Otherwise, the net effect on fertility depends on tastes for the number relative to the consumption of children.

1.5.2 A Child-Rearing Subsidy

In this subsection, the model is extended to incorporate the child-rearing cost which exhibits no contribution to the quality of children except providing young-aged consumption for children. The effects of the subsidy on economic growth and fertility are investigated simultaneously.

The government imposes a tax at a flat rate τ on wage income to subsidize the child-rearing cost at a flat rate χ and balances its budget:

$$w_t(1 - vN_t)\tau = \chi N_t e_t \ , \tag{1.5.7}$$

where e_t is the amount of goods spent in rearing a child. Here, the subsidy is proportional to e_t. However, its lump-sum version can be examined similarly.

Subsequently, in addition to (1.2.2), the middle-aged agent's budget constraint now implies:

$$C_t^t = w_t(1 - vN_t)(1 - \tau) + (1 + r_{t-1})b_{t-1}S_{t-1} - (1 - \chi)e_t N_t - q_t N_t - S_t \ , \quad (1.2.1)''$$

$$C_t^{t+1} = e_t \ , \quad (1.2.2)''$$

where C_t^{t+1} is the consumption of every child in young age.

Preferences are now defined as:

$$U(C_t^t, C_{t+1}^t, C_t^{t+1}, C_{t+1}^{t+1}, N_t) \ ,$$

and once again we assume that the utility function is Cobb-Douglas:

$$U(.) = (C_t^{t\ \phi} C_{t+1}^{t\ \ 1-\phi})^{\sigma} (C_t^{t+1\ \varepsilon} C_{t+1}^{t+1\ \delta} N_t^{\ 1-\varepsilon-\delta})^{1\ \ \sigma} .$$

Choosing b_t, e_t, N_t, q_t, and S_t, middle-aged agents maximize their own utility given future generations' decisions and w_t, r_t. The first-order condition with respect to e_t is given by:

$$(1-\chi) N_t U_1 = U_3 \ , \tag{1.5.8}$$

which equates the utility forgone by spending one more unit of goods in rearing children to the utility obtained from raising children's young-aged consumption by the spending.

Following the same steps as before, we can show that bequests are operative if and only if:

$$\delta(1-\sigma)[\sigma\theta + \theta\varepsilon(1-\sigma) + \sigma\psi(1-\theta)(1-\tau)(1-\phi)] - \sigma^2\phi(1-\tau)(1-\phi)(1-\theta) > 0 \ ,$$

and the bequest ratio rises with τ under $\sigma\phi > \delta(1-\sigma)$.

When bequests are operative, from the first-order condition with respect to b_t, we have:

$$N_t C_t^{t+1} = \frac{\varepsilon(1-\sigma)}{\sigma\phi(1-\chi)} C_t^t \ .$$

The equation above means that the subsidy increases the ratio of children's young-aged consumption to parental middle-aged consumption.

On the steady-state growth path, the subsidy also affects fertility in two opposite ways: taxing labor earning encourages agents to have more children, but subsidizing the cost of spending goods rearing children may induce parents to trade the quantity of children for the young-aged consumption of each child due to the similar reason I gave in the case with education subsidies. Thus, when bequests are operative, the net effect on fertility is ambiguous:

$$sign(\frac{\partial N}{\partial \tau}) = sign[-\delta(1-\varepsilon-\delta) - \delta\sigma(\varepsilon+\delta) + \sigma\phi(1-\varepsilon-\delta) - \sigma\phi\varepsilon] .$$

The sign is less likely to be positive when tastes for the consumption of children (δ, ε) relative to those for the number of children $(1-\delta-\varepsilon)$ are stronger.

The saving rate is the same as that in (1.4.6) and is independent of the subsidy. Investment in all children as a fraction of labor income decreases with τ:

$$q_t N = \frac{\psi\delta(1-\sigma)(1-\tau)}{\sigma\phi}(1-vN)w_t , \tag{1.5.9}$$

which implies that wl .n fertility does not fall in response to a rise in τ, investment per child as a fraction of labor income will go down.

Clearly, from (1.4.16) and the solutions above, if the child-rearing subsidy raises fertility, it will depress economic growth:

$$sign(\frac{\partial g_h}{\partial \tau}) = sign[-(1-vN) - (1-\tau)v\frac{\partial N}{\partial \tau}] < 0 .$$

When bequests are nonoperative, τ has no net effect on fertility:

$$N = \frac{\sigma\phi(1-\sigma)(1-2\varepsilon-\delta) - \delta\psi(1-\sigma)[\sigma+(1-\sigma)(1-\varepsilon-\delta)]}{v[\sigma+(1-\sigma)(1-\varepsilon-\delta)][\sigma\phi-\delta\psi(1-\sigma)]} . \tag{1.5.10}$$

The child-rearing subsidy lowers the saving rate:

$$S_t = \frac{[\sigma\phi - \psi\delta(1-\sigma)](1-\phi)(1-\tau)}{\phi\epsilon(1-\sigma)+\sigma\phi}(1-vN)w_t\ . \tag{1.5.11}$$

Also, investment in all children as a fraction of labor income falls with τ. Investment per child as a fraction of labor income will be reduced since fertility is unchanged.

Clearly, when bequests are nonoperative, this subsidy will inhibit economic growth by reducing the saving rate and investment per child as a fraction of labor income without changing fertility.

1.6 Conclusion

It has been shown that when investment in children is exogenous, fertility is positively related to wage rates if bequests are nonoperative, and positively related to interest rates otherwise. When human capital investment in children is endogenous, however, a positive relation between fertility and wage rates can no longer exist, and wage rates grow over time. When bequests are nonoperative, the results of this model seem to be consistent with what Malthus observed when education was not widely available, and is in line with his theory. Thus, bequests and education change the relationship between wage rates and fertility, and education is necessary for an economy to take off from a Malthusian equilibrium.

Also it has been seen that education subsidies are most likely to speed up economic growth, whereas child-rearing subsidies slow it down. Net effects of these subsidies on fertility depend on whether bequests are operative or not, and on people's preferences over both the consumption and the quantity of children. The child rearing subsidy aimed at raising fertility can achieve its goal at the cost of hindering economic growth. The education subsidy, however, seems to be suitable for countries seeking to raise growth rates of per capita income even though it may increase or may decrease fertility.

Chapter 2

Markets, Fertility, and Economic Growth

2.1 Introduction

Introducing markets and firms into the representative agent economy of the Neoclassical growth model with exogenous fertility has no effect on economic growth. Put another way, there are some different ways to assign property rights and to impose market structure which yield the same Pareto optimal allocation. However, it is evident that countries which were the first to transform their traditional economies into market economies are richer and have lower fertility than countries where many people still live in a traditional way. These observations suggest that introducing markets and firms may have a prolonged effect on economic growth and may be an important factor causing the "population transition". The Neoclassical growth model abstracts from fertility choices. Though it has been used to study how economic growth is affected if population grows at different paces, it ignores the response of population growth to changes in the economic environment.

Contributions have recently been made to modeling endogenous fertility and sustained economic growth by Becker, Murphy, and Tamura (1990), and Ehrlich and Lui (1991). Ignoring old-aged consumption, Becker, et al. (1990) find that countries starting with abundant human capital have low fertility and perhaps growing per capita output, whereas countries starting with scarce human capital have high fertility and constant per capita output.[1] Stressing old-age security, Ehrlich and Lui (1991) show that an exogenous fall in mortality rates reduces fertility and increases growth rates (or levels) of per capita output.[2]

Zhang and Nishimura (forthcoming) show that introducing capital markets may reduce fertility when children are strictly regarded as investment goods to secure old-aged consumption. However, the implication of adopting capital markets for economic growth is unclear since the production of both goods and human capital is ignored in their model. It is also necessary to relax one-sided altruism towards parents in their model to permit preferences for the consumption and number of children,[3] because fertility may not fall in response to the emergence of capital markets when children are taken care of by parents.

The purpose of this chapter is to compare fertility and economic growth between economies with or without markets and firms in an overlapping generations framework. The economy without markets and firms is referred to as the traditional economy, and the economy with markets and firms as the market economy. In the traditional economy, without capital markets and public transfers, the consumption of the old comes from gifts presented by children, and the production of goods takes place in families. In the market economy, profit-maximizing firms organize the production of goods, markets channel the trade of labor, capital and goods among firms and families, and agents can save for old-aged consumption on capital markets. In this model, human capital is the engine of sustained economic growth and the economy-wide average investment in children exhibits an externality in the production of human

[1] However, the postwar evidence does not confirm their underdevelopment equilibrium as pointed out by Barro (1989, pp. 26-27). Also see Summers and Heston, 1991.

[2] Ehrlich and Lui (1991) point out that mortality may be partly endogenous. It is unclear how their prediction will be modified if mortality is partly endogenous.

[3] Preferences for the consumption and number of children have been emphasized in the literature (see, e.g., Becker and Barro, 1988; Becker, et al., 1990).

capital.

It is found in this chapter that transforming a traditional economy into a market economy brings about lower fertility (as in the literature) but faster growth of per capita output if altruism is one-sided towards parents. Even if altruism is two-sided, this transformation reduces fertility. In this case, when gifts are operative it also speeds up growth unless tastes for the number of children are much stronger than those for the consumption of children. The two economies appear to have similar rates of economic growth when bequests are operative. Moreover, these two economies are found to have the same fertility and the same growth rate of per capita output if agents care equally about their own consumption and other generations'. Thus the conclusion of the Neoclassical model is a special case of this model.

Fertility falls because children become less essential for old-age security when parents can save for their own old-aged consumption. Also, human capital investment in all children as a fraction of output per middle-aged falls in the transition towards market economies. However, when gifts are operative in both economies investment per child as a fraction of output per middle-aged may be higher in the market economy. This can happen when tastes for the consumption relative to the number of children are strong. Furthermore, when gifts are operative in both economies the saving/output ratio is higher in the market economy. The fall in fertility and the rise in the saving/output ratio imply that the market economy has higher per capita physical capital. As a consequence, with operative gifts per capita output grows faster in the market economy unless tastes for the number relative to the consumption of children are so strong that investment per child as a fraction of output per middle-aged drops substantially.

When gifts are operative in both economies,[4] the results in this chapter can pro-

[4]While Becker, et al. (1990) stress bequests, some economists emphasise gifts (see, e.g., Ehrlich and Lui, 1991; Nishimura and Zhang, 1992).

vide a possible explanation to the empirical relation among the changes in economic structure, population growth, and economic growth in the last several centuries.[5] In the 18th century, living standards were low but fertility was high in both the West and the East (see, Day, 1933; Wrigley, 1981). In 1800, for example, the average per capita gross national product (in 1960 US dollars) has been estimated to have been $198 in current developed countries and $188 in current less developed countries (see Bairoch, 1981). Unprecedentedly rapid economic growth and drastic declines in fertility were experienced while the transition towards market economies was taking place in the West in the 19th century, dividing the world into two extremes: the developed West and the less developed East.[6] In the 20th century, many less developed countries have become market economies, and observed similar changes in fertility and growth rates of per capita output to those happened in the West in the last century. Since World War II, the average fertility (the average number of children per woman) in less developed countries has dropped from 6.19 during 1950-1955 to 3.94 during 1985-1990,[7] and the average growth rate of per capita output in these countries is significantly higher than it was in the 19th century.

In the transition towards a market economy, both traditional and market sectors may coexist for a period of time. The implication of this model for growth is that growth rates of per capita output increase in the transition process. Consequently, advanced market economies (or rich countries) in which traditional sectors have disappeared should have faster growth of per capita output than economies in the transition process (or poor countries). These predictions are consistent with data: both the West and East have faster per capita growth than they had in the 19th century; and initially rich countries tend to growth faster than poor ones (see Barro and

[5] If at least some countries have operative gifts after the transition the results can also reflect the positive effects of the transition on growth on average.

[6] In 1913, the average per capita gross national product (in 1960 US dollars) reached $662 in current developed countries and $192 in current less developed countries (also see, Bairoch, 1981).

[7] See, United Nations, 1991, p.25.

Sala-i-Martin, 1992, pp. 241-243).

Unfortunately, we do not have precise ways to measure the transition towards market economies. Thus the above lines are only suggestive. However, disparities in economic growth and fertility between the West and the East in the 19th century can hardly be explained by the difference in initial living standards or in mortality rates because these initial conditions had been similar in both regions.

This chapter is organized as follows. Section 2.2 introduces the model. Section 2.3 and Section 2.4 analyse the traditional economy and the market economy respectively. Section 2.5 compares fertility and economic growth in the traditional economy with those in the market economy. Section 2.6 concludes the chapter.

2.2 The Model

There is an infinite number of periods and overlapping generations of three-period-lived agents. Let subscript t denote a period in time and superscript t the generation born in period $t-1$. Let L_t be the number of middle-aged agents living in period t. We assume that every parent has children at the beginning of middle age. A single good can be produced, consumed, invested in children, accumulated as physical capital, but cannot be stored for future consumption.[8]

Everyone is identical, learns when young, lives in retirement in old age, and is endowed in middle age with one unit of time which can be used in the production of goods or spent in rearing children. Let v (positive, less than one) denote the units of time needed to rear a child.[9]

When one-sided altruism towards an old-aged parent is assumed, children are strictly investment goods for their parent's old-aged consumption. Utility then depends positively on the old-aged consumption of the retired parent (C_t^{t-1}) as well as

[8] Alternatively, we can assume that the return of investing in physical capital or human capital exceeds that of storing goods for future consumption without changing the results.

[9] As mentioned in chapter 1, fertility has an upper-bound.

on own middle-aged and old-aged consumption $(C_t^t,\ C_{t+1}^t)$:

$$U(C_t^t,\ C_{t+1}^t,\ C_t^{t-1}).$$

When two-sided altruism towards both an old-aged parent and children is assumed, utility also depends positively on the consumption (C_{t+1}^{t+1}) and the number (N_t) of children:

$$V(C_t^t,\ C_{t+1}^t,\ C_t^{t-1},\ C_{t+1}^{t+1},\ N_t),$$

where C_{t+1}^{t+1} is the middle-aged consumption of every child. The justification of ignoring younger generations' fertility choices and consumption except C_{t+1}^{t+1} will be discussed later in this section. We assume that $U(\cdot,\cdot,\cdot)$ and $V(\cdot,\cdot,\cdot,\cdot,\cdot)$ are increasing and concave.

The production function for goods has the form:

$$y_t = f(k_t,\ l_t H_t) \equiv f_t,$$

where k_t and l_t denote inputs of physical capital and labor respectively, and H_t refers to an individual's human capital. We assume that $f(\cdot,\cdot)$ is increasing, concave, and homogeneous of degree one.

The education technology converts investments in children (q_t per child) into human capital where the economy-wide average investment per child $(\bar{q}_t)$ exhibits an externality:

$$H_{t+1} = h(q_t,\ \bar{q}_t).$$

We assume that $h(\cdot,\cdot)$ is increasing, concave, and homogeneous of degree one.

As in chapter 1, we assume Cobb-Douglas functions (see, Lucas, 1988; Becker, et al., 1990):

$$V(C_t^t, C_{t+1}^t, C_{t-1}^t, C_{t+1}^{t+1}, N_t) = \left({C_t^t}^{\phi} {C_{t+1}^t}^{1-\phi}\right)^{\sigma} {C_t^{t-1}}^{\epsilon} \left({C_{t+1}^{t+1}}^{\delta} {N_t}^{1-\delta}\right)^{1-\sigma-\epsilon},$$

where $0 < \sigma,\ \phi,\ \varepsilon,\ \delta < 1$, and $1 - \sigma - \varepsilon \geq 0$;

$$f_t(k_t, l_t) = Dk_t^{\theta}(l_t H_t)^{1-\theta}\ ,\ h(q_t, \bar{q}_t) = A{q_t}^{\beta}{\bar{q}_t}^{\ 1-\beta},$$

where $D > 0$, $A > 0$, and $0 < \theta,\ \beta < 1$. Here, β measures the individual returns to scale of human capital investment and $1 - \beta$ the degree of the externality in the education technology. Altruism is one-sided towards parents if $1 - \sigma - \varepsilon = 0$, and two-sided when $1 - \sigma - \varepsilon > 0$. We assume $\sigma\phi > \delta(1 - \sigma - \varepsilon)$ and $\sigma^2\phi(1 - \phi) > \varepsilon\delta(1 - \sigma - \varepsilon)$. That is, tastes for own middle-aged consumption are stronger than those for consumption of each child, and tastes for own two-period consumption are stronger than those for the consumption of both the parent and each child.

In the traditional economy, the production of goods takes place in every family, and the consumption of the old comes from gifts presented by children.[10] In this economy, each middle-aged agent living in period t spends vN_t units of time rearing children, works for $1 - vN_t$ units of time, and produces $f\big(k_t,\ (1 - vN_t)H_t\big)$. Here k_t is the bequest in the form of physical capital[11] received from the old-aged parent at the

[10]Here, I implicitly assume that middle-aged agents have the output they produce and then, due to altruism, they present gifts to parents to equalise marginal utilities of all agents in a family. There may be other ways to determine the shares of output between agents across generations within a family in the traditional economy. For example, family members equally share the output produced in the family and then intergenerational transfers may occur, due to altruism, so as to equalise marginal utilities of the old and the middle-aged. This setup may not have analytical solutions because of nonlinearity in fertility it introduces. However, computational results can show that fertility goes down and growth can speed up when markets and firms are introduced (i.e., the market force determines shares of output). Nevertheless, it seems unreasonable to assume that shares of output in the traditional economy are determined by the same way as in the market economy.

[11]Bequests in the form of consumption goods are found to be nonoperative in the traditional economy under the assumption $\sigma^2\phi(1 - \phi) > \varepsilon\delta(1 - \sigma - \varepsilon)$. Thus this model does not incorporate such kind of bequests in the traditional economy.

beginning of period t and H_t is the middle-aged agent's human capital. We assume that physical capital lasts one period in the production of goods. The middle-aged agent allocates the produced goods to own middle-aged consumption, C_t^t, gifts to the old-aged parent, $g_t f_t$, investments in children, $q_t N_t$, and bequests to children at the beginning of children's middle-age, $N_t k_{t+1}$. This agent receives gifts from children, $N_t g_{t+1} f_{t+1}$, for old-aged consumption (C_{t+1}^t) in the next period. Thus the middle-aged agent's budget constraint implies:

$$C_t^t = f\Big(k_t,\ (1 - vN_t)H_t\Big)(1 - g_t) - q_t N_t - N_t k_{t+1}, \tag{2.2.1}$$

$$C_{t+1}^t = N_t g_{t+1} f\Big(k_{t+1},\ (1 - vN_t)h(q_t,\ \bar{q}_t)\Big), \tag{2.2.2}$$

where g_t and k_{t+1} are the gift ratio and bequests per child respectively. Note that in the traditional economy even if altruism is one-sided towards an old-aged parent, there are incentives to leave bequests, since gifts and bequests are the only source of old-aged consumption and physical capital respectively.

In the <u>market economy</u>, there are many firms and markets. Firms organize the production of goods; markets channel the trade of labor, physical capital, and goods. In period t, each middle-aged agent spends vN_t units of time rearing children, supplies $1 - vN_t$ units of time to the labor market, and earns $(1 - vN_t)w_t$. This agent receives bequests, $(1 + r_{t-1})b_{t-1}S_{t-1}$,[12] from the old-aged parent at the beginning of period t, and leaves bequests out of saving (S_t), $(1 + r_t)b_t S_t$, to each child at the beginning of children's middle-age. I refer to b as the bequest ratio. The middle-aged agent spends the earning and inheritance, $(1 - vN_t)w_t + (1 + r_{t-1})b_{t-1}S_{t-1}$, on own middle-aged consumption C_t^t, on saving for old-aged consumption (C_{t+1}^t), S_t, on bequests to children, $N_t b_t S_t$, on investments in children, $q_t N_t$, and on gifts for the old-aged

[12] Alternatively, we can assume that agents choose B_{t+1} where $B_{t+1} = (1 + r_t)b_t S_t$, instead of choosing b_t, without changing the results.

parent, $g_t(1-vN_t)w_t$. Each middle-aged agent in period t expects gifts from children, $N_t g_{t+1}(1-vN_{t+1})w_{t+1}$, in the next period. Thus the middle-aged agent's budget constraint implies:

$$C_t^t = w_t(1-vN_t)(1-g_t) + (1+r_{t-1})b_{t-1}S_{t-1} - q_tN_t - S_t, \qquad (2.2.3)$$

$$C_{t+1}^t = (1+r_t)(1-b_tN_t)S_t + N_t g_{t+1}(1-vN_{t+1})w_{t+1}, \qquad (2.2.4)$$

where w and r denote the wage rate and the interest rate respectively.

In the market economy, when one-sided altruism towards an old-aged parent is assumed there are no incentives to bequeath, because the middle-aged agents can save for their own old-aged consumption and do not care about the consumption of children. It is also unnecessary for middle-aged agents to leave bequests, since firms' physical capital can come from the saving of the preceding generation. Thus in this case, $b_t = 0$. In any case in both economies, bequests and gifts are nonnegative since agents are unable to force other generations to submit goods to them.

To keep the model tractable, we adopt the following assumptions:

Assumption 2.2.1: *Agents make decisions (g_t, k_{t+1}, q_t, and N_t in the traditional economy; b_t, g_t, q_t, N_t, and S_t in the market economy) while knowing the decisions of the older generations and taking the decisions of the younger generations as given.*

Assumption 2.2.2: *The middle-aged agents within each family make their gift decisions cooperatively.*

Without Assumption 2.2.1, it would be extremely hard to work out the interaction among infinite generations as shown by Nishimura and Zhang (1992). Under this assumption, generations act like Nash game players except that they play in different periods. Also under this assumption, middle-aged agents living in period t cannot directly affect younger generations' fertility choices, N_{t+2}, $N_{t+3}\cdots$, and consumption of descendants other than the middle-aged consumption of their own children (C_{t+1}^{t+1}), C_{t+2}^{t+1}, C_{t+2}^{t+2} $\cdots$. Thus we do not include these future fertility and consumption in the utility function. Under Assumption 2.2.2, each middle-aged agent makes the gift decision as if he/she were the head of his/her siblings.

Therefore, in the traditional economy each middle-aged agent in period t chooses q_t, N_t, k_{t+1}, and g_t to maximize own utility subject to (2.2.1), (2.2.2), and $g_t \geq 0$, taking future generations' decisions as given. On the other hand, in the market economy, each middle-aged agent chooses b_t, g_t, N_t, q_t, and S_t to maximize own utility subject to (2.2.3), (2.2.4), $b_t \geq 0$, and $g_t \geq 0$, taking wage rates, interest rates, and future generations' decisions as given.

In the market economy, firms are perfect competitors maximizing profits by choosing physical capital, k_t, and labor, l_t. We also assume that physical capital lasts for one period in the production of goods in the market economy. Thus firms' problems are static. The profit function is defined as:

$$f(k_t, l_t H_t) - (1 + r_{\ -1})k_t - l_t w_t.$$

The first-order conditions of firms maximizing profits are given by:

$$w_t = (1-\theta)D\left(\frac{k_t}{l_t H_t}\right)^{\theta} H_t, \tag{2.2.5}$$

$$1 + r_{t-1} = \theta D\left(\frac{l_t H_t}{k_t}\right)^{1-\theta}. \tag{2.2.6}$$

Clearly, firms' physical capital/effective labor ratios, $k_t/(l_t H_t)$, are equal, and are the same as the economy-wide physical capital/effective labor ratio, $K_t/(L_t^d H_t)$, where K_t and L_t^d are aggregate physical capital and aggregate labor demand respectively.

Labor market clearing requires:

$$L_t^d = L_t(1 - vN_t). \tag{2.2.7}$$

Capital market clearing needs:

$$K_t = L_{t-1}S_{t-1}. \tag{2.2.8}$$

Constant returns to scale and perfect competition imply that profits should be zero in every period. By Walras' law, the goods market clears as well in the market economy.

Since agents are identical, the symmetric condition in both economies is: $q_t = \bar{q}_t$.

Definition 1: *An equilibrium in the traditional economy is a collection of sequences* $\{g_t, q_t, N_t, k_{t+1}\}_{t=0}^{\infty}$ *such that individual agents of each generation optimize.*

Definition 2: *An equilibrium in the market economy is a collection of sequences* $\{w_t, r_t, b_t, g_t, q_t, N_t, S_t, l_t, k_t\}_{t=0}^{\infty}$ *such that:*

(1) Given $\{w_t, r_t\}_{t=0}^{\infty}$, $\{b_t, g_t, q_t, N_t, S_t\}_{t=0}^{\infty}$ *solves individual agents' problems and* $\{l_t, k_t\}_{t=0}^{\infty}$ *solves firms' problems.*

(2) Markets clear.

2.3 The Traditional Economy

We substitute (2.2.1) and (2.2.2) into the utility function $V(\cdot,\cdot,\cdot,\cdot,\cdot)$ for C_t^t, C_{t+1}^t, C_t^{t-1} (by backdating (2.2.2) one period), C_{t+1}^{t+1} (by updating (2.2.1) one period). Under the assumptions in the last section, the first-order conditions of the middle-aged in the traditional economy are as follows:

$$g_t: \quad V_1 = N_{t-1}V_3 + \lambda, \lambda \geq 0, \lambda g_t = 0; \tag{2.3.1}$$

$$q_t: \quad N_t V_1 = \frac{\beta(1-\theta)N_t g_{t+1} f_{t+1} V_2}{q_t} + \frac{\beta(1-\theta)(1-g_{t+1}) f_{t+1} V_4}{q_t}; \tag{2.3.2}$$

$$k_{t+1}: \quad N_t V_1 = \frac{\theta N_t g_{t+1} f_{t+1} V_2}{k_{t+1}} + \frac{\theta(1-g_{t+1}) f_{t+1} V_4}{k_{t+1}}; \tag{2.3.3}$$

$$N_t: \quad \left(\frac{v(1-\theta)(1-g_t)f_t}{1-vN_t} + q_t + k_{t+1}\right) V_1 = g_{t+1} f_{t+1} V_2 -$$

$$\frac{v(1-\theta)N_{t-1} g_t f_t}{1-vN_t} V_3 + V_5. \tag{2.3.4}$$

Equation (2.3.1) means that the utility forgone from presenting one more unit of gifts to the old-aged parent should be no less than that obtained from increasing the parent's old-aged consumption by the gift; the equality holds if gifts are operative ($g_t > 0$). Equation (2.3.2) equates the loss in utility from investing an additional unit in the quality of children to the gain in utility from both receiving more gifts from children and improving the consumption of children due to the rise in the productivity of children by the investment. Equation (2.3.3) says that the utility forgone from leaving one more unit of bequests to children should equal the utility obtained from both receiving more gifts from children and increasing the consumption of children by the bequest. Equation (2.3.4) means that the utility forgone from consuming less and from presenting fewer gifts to the parent to have one more child should equal that obtained from both receiving more gifts from and enjoying the child.

The equilibrium in this case is characterized by (2.2.1), (2.2.2), (2.3.1)-(2.3.4), and the symmetric condition. On the steady-state growth path, the gift ratio, the ratio of bequests to output, fertility, and the growth rate of per capita output should be constant. Combining these steady-state conditions with the equations characterizing the equilibrium, we solve for optimal choices of middle-aged agents.

The gift ratio is found to be:

$$g = \frac{\sigma\phi\varepsilon - [\theta + (1-\theta)\beta]\delta\varepsilon(1-\sigma-\varepsilon)}{\sigma^2\phi^2 + \sigma\phi\varepsilon + [\theta + (1-\theta)\beta][\sigma^2\phi(1-\phi) - \delta\varepsilon(1-\sigma-\varepsilon)]}\,, \tag{2.3.5}$$

which is positive and less than unity under the assumptions $\sigma\phi > \delta(1-\sigma-\varepsilon)$ and $\sigma^2\phi(1-\phi) > \varepsilon\delta(1-\sigma-\varepsilon)$. The gift ratio decreases with tastes for the consumption relative to the number of children (measured by δ) and increases with tastes for the consumption of the old-aged parent (measured by ε).

Fertility is given by:

$$N = \frac{x}{v[\sigma\phi\varepsilon(1-\theta) + x]}\,, \tag{2.3.6}$$

where:

$$x = [\sigma^2\phi(1-\phi)(1-\theta)(1-\beta) + \sigma\phi(1-\delta)(1-\sigma-\varepsilon)]g - [\theta + (1-\theta)\beta]\delta\varepsilon(1-\sigma-\varepsilon)(1-g).$$

Fertility here is positively related to the gift ratio. From (2.3.5) and (2.3.6), fertility decreases with δ, the taste for the consumption relative to the number of children, under the assumption $\sigma\phi > \delta(1-\sigma-\varepsilon)$.

The investment rate per child, denoted by γ_q, is positively related to the gift ratio under the assumption $\sigma^2\phi(1-\phi) > \varepsilon\delta(1-\sigma-\varepsilon)$, and negatively related to fertility:

$$q_t = \frac{\beta(1-\theta)}{N\sigma\phi\varepsilon}[\sigma^2\phi(1-\phi)g + \delta\varepsilon(1-\sigma-\varepsilon)(1-g)]f_t \equiv \gamma_q f_t. \tag{2.3.7}$$

It can be shown that γ_q increases with δ. From (2.3.6) and (2.3.7), agents have fewer children and invest more in each child when tastes for consumption of children relative to the number of children are stronger.

The ratio of bequests to output, denoted by γ_s, is also positively related to the gift ratio:

$$Nk_{t+1} = \frac{\theta}{\sigma\phi\varepsilon}[\sigma^2\phi(1-\phi)g + \delta\varepsilon(1-\sigma-\varepsilon)(1-g)]f_t \equiv \gamma_s f_t. \quad (2.3.8)$$

It can be shown that γ_s decreases with ε but increases with δ. That is, the stronger is the taste for consumption of children relative to that of parents, the higher is the ratio of bequests to output. Note that the ratio of bequests to output is also the physical capital/output ratio in the traditional economy where physical capital comes solely from bequests.

The steady-state rate of physical capital accumulation $(1+g_k)$ arises from substituting the production function for goods into (2.3.8) for f_t:

$$1 + g_k \equiv k_{t+1}/k_t = \frac{\gamma_s D}{N}\left(\frac{H_t}{k_t}\right)^{1-\theta}(1-vN)^{1-\theta}. \quad (2.3.9)$$

This rate is positively related to the bequest ratio and the human/physical capital ratio, and negatively related to fertility.

Analogously, from (2.3.7) and the education technology, we have the steady-state rate of human capital accumulation $(1+g_h)$:

$$1 + g_h \equiv H_{t+1}/H_t = A\gamma_q D\left(\frac{k_t}{H_t}\right)^{\theta}(1-vN)^{1-\theta}. \quad (2.3.10)$$

The rate of human capital accumulation is positively related to the investment rate per child and the physical/human capital ratio, and negatively related to fertility.

Equations (2.3.9) and (2.3.10) determine the human/physical capital ratio:

$$H_t/k_t = \frac{A\gamma_q N}{\gamma_e} = \frac{A\beta(1-\theta)}{\theta}\,, \tag{2.3.11}$$

implying in turn that an individual's human capital and physical capital per middle-aged grow at the same pace.

Since the human/physical capital ratio and fertility are constant on the steady-state growth path, output per middle-aged agent (hence per capita) f_t is linear in H_t from the production function for goods. Therefore, the rates of human capital accumulation and physical capital accumulation are the same as the growth rate of per capita output, denoted by $1+g_y$, which is given by substituting (2.3.11) into (2.3.10):

$$1+g_y = A^{1-\theta} D\gamma_e^{\theta} N^{-\theta}\gamma_q^{1-\theta}(1-vN)^{1-\theta}\,. \tag{2.3.12}$$

Obviously, the growth rate of per capita output is positively related to the bequest ratio and the investment rate per child, but it is negatively related to fertility.

2.4 The Market Economy

We substitute (2.2.3) and (2.2.4) into the utility function, $V(\cdot,\cdot,\cdot,\cdot,\cdot)$, for C_t^t, C_{t+1}^t, C_t^{t-1} (by backdating (2.2.4) one period), C_{t+1}^{t+1} (by updating (2.2.3) one period). Under Assumptions 2.2.1 and 2.2.2 in Section 2.2, the first-order conditions of the middle-aged living in the market economy are given by:

$$b_t: \quad N_t(1+r_t)S_tV_2 = (1+r_t)S_tV_4 + \mu, \;\; \mu \geq 0, \;\; \mu b_t = 0; \tag{2.4.1}$$

$$g_t: \quad w_t(1-vN_t)V_1 = N_{t-1}w_t(1-vN_t)V_3 + \lambda, \lambda \geq 0, \lambda g_t = 0; \tag{2.4.2}$$

$$q_t: \quad N_tV_1 = \frac{\beta N_t g_{t+1}(1-vN_{t+1})w_{t+1}}{q_t}V_2 + \frac{\beta(1-vN_{t+1})(1-g_{t+1})w_{t+1}}{q_t}V_4; \quad (2.4.3)$$

$$N_t: \quad [vw_t(1-g_t)+q_t]V_1 = [g_{t+1}(1-vN_{t+1})-$$

$$(1+r_t)b_tS_t]V_2 - N_{t-1}g_tvw_tV_3 + V_5; \quad (2.4.4)$$

$$S_t: \quad V_1 = (1+r_t)(1-b_tN_t)V_2 + b_t(1+r_t)V_4. \quad (2.4.5)$$

Here, μ and λ are Lagrange multipliers associated with b_t and g_t respectively.

Equation (2.4.1) means that utility forgone by leaving an additional unit of bequests to children should be no less than that obtained from improving children's consumption by the bequest; the equality holds if bequests are operative ($b_t > 0$). Equation (2.4.2) says that the utility forgone by presenting one more unit of gifts should be no less than that obtained from enhancing the parent's old-aged consumption by the gift; the equality holds if gifts are operative ($g_t > 0$). Equation (2.4.3) equates the loss in utility from investing one more unit in human capital of children to the gain in utility from both receiving more gifts and increasing consumption of children due to higher earnings of children by the investment. Equation (2.4.4) means that the utility forgone from consuming less and presenting fewer gifts to the old-aged parent to have one more child should equal the utility obtained from both receiving more gifts from and enjoying the child. Equation (2.4.5) requires that the utility forgone from saving one more unit now should equal the utility achieved from consuming $1 + r_t$ units more when old.

It is shown that when gifts are operative ($g > 0$) bequests will be nonoperative ($b = 0$) under the assumption $\sigma^2\phi(1-\phi) > \varepsilon\delta(1-\sigma-\varepsilon)$. From (2.2.3)-(2.2.6), and (2.4.1), we have:

$$N_tC_{t+1}^{t+1} = \frac{\delta(1-\sigma-\varepsilon)}{\sigma(1-\phi)}C_{t+1}^t + \frac{\mu}{(1+r_t)S_tV_2}, \quad \mu \geq 0, \quad \mu b_t = 0. \tag{2.4.6}$$

Similarly, (2.2.3)-(2.2.6), and (2.4.2) give:

$$\frac{\sigma\phi}{\varepsilon}C_{t+1}^t = N_tC_{t+1}^{t+1} + \frac{\lambda}{(1-vN_t)w_tV_1}, \quad \lambda \geq 0, \quad \lambda g_t = 0. \tag{2.4.7}$$

Therefore, when gifts are operative (hence $\lambda = 0$) (2.4.6) and (2.4.7) lead to:

$$\mu = \frac{\sigma^2\phi(1-\phi) - \varepsilon\delta(1-\sigma-\varepsilon)}{\sigma\phi\varepsilon}C_{t+1}^t(1+r_t)S_tV_2,$$

which is positive under the assumption: $\sigma^2\phi(1-\phi) > \varepsilon\delta(1-\sigma-\varepsilon)$ in Section 2.2.

Thus, there are three possible cases: (i) $g > 0$ and $b = 0$; (ii) $g = 0$ and $b > 0$; (iii) $g = b = 0$. We focus on the one with operative gifts in the present paper. In case (i), without public transfers from the working generation to the retired in a market economy, children in the present model are, at least partly, investment goods for old-age security.[13] Cases (ii) and (iii) are analyzed in Appendix II.

When gifts are operative, the equilibrium in this case is characterized by (2.2.3)-(2.2.8), (2.4.2)-(2.4.5), and the symmetric condition, with $b = 0$. Similarly to what we did in the last section, we solve these equations for the steady-state growth path by imposing steady-state conditions.

The gift ratio is found to be:

$$g = \frac{\sigma\phi\varepsilon(1-\theta) - \beta(1-\theta)\varepsilon\delta(1-\sigma-\varepsilon) - \sigma^2\phi\theta}{(1-\theta)[\sigma^2\phi^2 + \sigma\phi\varepsilon + \beta\sigma^2\phi(1-\phi) - \beta\varepsilon\delta(1-\sigma-\varepsilon)]}, \tag{2.4.8}$$

[13]This assumption does not rule out operative bequests in a market economy with old-age social security instituted. It is shown in Chapter 3 that old-age social security can turn a market economy with operative gifts into one with operative bequests.

which increases with the taste for the retired parent's old-aged consumption, ε, as in the traditional economy.

Fertility is positively related to the gift ratio:

$$N = \frac{x}{v[\sigma\phi\varepsilon(1-\theta)+x]}\ , \tag{2.4.9}$$

where: $x = \sigma^2\phi(1-\phi)(1-\theta)(1-\beta)g + (1-\sigma-\varepsilon)\{\sigma\phi(1-\delta)[\theta+(1-\theta)g] - \beta(1-\theta)\varepsilon\delta(1-g)\}$. Fertility decreases with the taste for consumption of children, δ, as in the traditional economy.

The investment per child is positively related to the gift ratio and negatively related to fertility:

$$q_t = \frac{\beta}{\sigma\phi\varepsilon N}[\sigma^2\phi(1-\phi)g + \varepsilon\delta(1-\sigma-\varepsilon)(1-g)](1-vN)w_t \equiv \gamma_q(1-vN)w_t\ . \tag{2.4.10}$$

Since $(1-vN_t)w_t$ is $1-\theta$ per cent of output per middle-aged, investment per child as a fraction of output per middle-aged is $(1-\theta)\gamma_q$. Thus investment per child as a fraction of output per middle-aged in the two economies has the same expression.

The saving function is given by:

$$S_t = \frac{\sigma\theta(1-\phi)}{\varepsilon(1-\theta)}(1-vN)w_t \equiv \gamma_s(1-vN)w_t. \tag{2.4.11}$$

The saving rate, γ_s, increases with the taste for own old-aged consumption, $1-\phi$, and with the share parameter of physical capital in the production of goods, θ, but decreases with the taste for the retired parent's old-aged consumption, ε. Also, $(1-\theta)\gamma_s$ is the saving/output ratio.

Combining (2.2.5)-(2.2.8), (2.4.10), (2.4.11), and production functions for both goods and human capital, we have that evolution of both human and physical capital is determined jointly by the following two equations:

$$K_{t+1} = L_t S_t = \gamma_s(1 - vN_t)L_t D(1-\theta)K_t^{\theta}[L_t H_t(1 - vN_t)]^{-\theta} H_t \, , \tag{2.4.12}$$

$$H_{t+1} = Aq_t = A\gamma_q(1 - vN_t)D(1-\theta)K_t^{\theta}[L_t H_t(1 - vN_t)]^{-\theta} H_t \, . \tag{2.4.13}$$

Equation (2.4.9) implies that the evolution of population is simply determined by:

$$L_{t+1} = NL_t \, .$$

Denoting accumulation rates of physical and human capital as $1 + g_{kt} \equiv K_{t+1}/K_t$ and $1 + g_{ht} \equiv H_{t+1}/H_t$ respectively and imposing steady-state growth conditions ($g_{ht} = g_h$, $g_{kt} = g_k$, and $N_t = N$), we rewrite (2.4.12) and (2.4.13) to be:

$$\frac{K_t}{H_t L_t} = \left(\frac{D\phi_1(1-\theta)}{1+g_k}\right)^{\frac{1}{1-\theta}} (1 - vN)^{\frac{-\theta}{1-\theta}} \, , \tag{2.4.14}$$

$$\frac{K_t}{H_t L_t} = \left(\frac{1+g_h}{AD\phi_3(1-\theta)}\right)^{\frac{1}{\theta}} (1 - vN) \, . \tag{2.4.15}$$

These two equations imply that the steady-state physical capital/effective labor ratio is constant:

$$\frac{K_{t+1}}{H_{t+1}L_{t+1}} = \frac{K_t}{H_t L_t} \equiv k^* \, .$$

This is equivalent to:

$$1 + g_k = (1 + g_h)N \, , \tag{2.4.16}$$

which means that physical capital per capita grows at the same pace as an individual's human capital.

Equating the right hand side of (2.4.14) to that of (2.4.15) and substituting (2.4.16) into (2.4.14) for $1 + g_k$ yield the steady-state rate of human capital accumulation:

$$1 + g_h = A^{1-\theta} D[(1-\theta)\gamma_s]^{\theta} N^{-\theta}[(1-\theta)\gamma_q]^{1-\theta}(1 - vN)^{1-\theta} \, . \tag{2.4.17}$$

That is, the rate of human capital accumulation is related positively to the saving rate and the investment rate per child, and negatively to fertility. It turns out that the rate of human capital accumulation is also the growth rate of per capita output.[14] Note that (2.3.12) and (2.4.17) are the same function since investment per child as a fraction of output per middle-aged and the saving/output ratio are $(1-\theta)\gamma_q$ and $(1-\theta)\gamma_s$ respectively in the market economy, and γ_q and γ_s respectively in the traditional economy.

When $\sigma^2\phi(1-\phi) = \varepsilon\delta(1-\sigma-\varepsilon)$, the equilibrium is characterized by (2.2.3)-(2.2.8), (2.4.1)-(2.4.5), and by the symmetric condition, since in this case both gifts and bequests may be operative. However, we cannot solve for the steady-state gift rate and bequest rate separately.

Fertility in this case is found to be:

[14] Per capita output in period t is:

$$\frac{f\Big(K_t, L_t H_t(1 - vN_t)\Big)}{L_{t-1} + L_t + L_{t+1}} = D\left(\frac{K_t}{H_t L_t}\right)^{\theta}(1 - vN)^{1-\theta} H_t/(1/N + 1 + N) \, ,$$

where $f\Big(K_t, L_t(1 - vN_t)H_t\Big)$ is aggregate output (recall that $\frac{k_t}{l_t H_t} = \frac{K_t}{L_t(1-vN_t)H_t}$) and where fertility and the steady-state physical capital/effective labor ratio are constant.

$$N = \frac{x}{v[\varepsilon(1-\theta)+x]},$$

where:

$$x = \sigma(1-\phi)y + (1-\delta)(1-\sigma-\varepsilon)(\theta+y) - \beta\sigma(1-\phi)(1-\theta),$$

with

$$y \equiv (1-\theta)g - \theta N b = \frac{\varepsilon(1-\theta) - \sigma(1-\phi)[\theta+(1-\theta)\beta] - \sigma\phi\theta}{\sigma\phi+\varepsilon}.$$

Here y can be viewed as the net intergenerational transfers. It can be shown that fertility in (2.4.9) approaches fertility in the above equation as $\sigma^2\phi(1-\phi) \to \varepsilon\delta(1-\sigma-\varepsilon)$. Moreover, human capital investment per child and saving are the same functions as (2.4.10) and (2.4.11) respectively. Consequently, the growth rate of per capita output in this case is the limit of the growth rate defined by (2.4.17) and (2.4.8)-(2.4.11) as $\sigma^2\phi(1-\phi) \to \varepsilon\delta(1-\sigma-\varepsilon)$. Therefore equations (2.4.8)-(2.4.11) and (2.4.17) represent the solution in the market economy when gifts are operative and when $\sigma\phi(1-\phi) \geq \varepsilon\delta(1-\sigma-\varepsilon)$.

2.5 The Comparison between the Two Economies

This section compares fertility and growth rates of per capita output between the traditional economy and the market economy. In Subsection 2.5.1 gifts are operative in both economies. Subsection 2.5.2 makes the comparison of the two economies when gifts are not operative in the market economy. Subsection 2.5.3 discusses the difference in results between cases with and without operative gifts in the market economy. Superscripts T and C refer to the traditional economy and market economy respectively.

2.5.1 The Case with Operative Gifts in Both Economies

In a traditional economy, children finance the consumption of the old. When capital markets have been developed, agents can save for their own old-aged consumption and hence depend less on gifts from children. Thus the gift ratio is lower in the market economy than that in the traditional economy from (2.3.5) and (2.4.8) under the assumption $\sigma^2\phi(1-\phi) > \varepsilon\delta(1-\sigma-\varepsilon)$ where the sign of $g^T - g^C$ is determined by:

$$[\sigma^2\phi(1-\phi) - \delta\varepsilon(1-\sigma-\varepsilon)][\theta + (1-\theta)\beta] + \sigma\phi\varepsilon[1-(1-\phi)(1-\theta)(1-\beta)]+$$

$$\phi[\sigma^2\phi - \varepsilon\delta(1-\sigma-\varepsilon)(1-\theta)(1-\beta)],$$

which is positive.

Fertility falls as well in response to the transition towards market economies which can be seen from (2.3.6) and (2.4.9) that $N^T - N^C$ is positive:

$$(1-\beta)[\sigma^2\phi(1-\phi) - \varepsilon\delta(1-\sigma-\varepsilon)][1-(1-\phi)(1-\beta)(1-\theta)].$$

When tastes for own consumption are the same as those for other generations', i.e., $\sigma^2\phi(1-\phi) = \varepsilon\delta(1-\sigma-\varepsilon)$, however, fertility will be the same in both economies.

From (2.3.8) and (2.4.11), it is easy to see that the saving rate in the market economy is higher than the bequest rate in the traditional economy.[15] Also the physical capital/output ratio in the market economy, $(1-\theta)\gamma_s^C$, is higher than that, γ_s^T, in the traditional economy: $(1-\theta)\gamma_s^C - \gamma_s^T$ is signed by $[\sigma^2\phi(1-\phi) - \varepsilon\delta(1-\sigma-\varepsilon)](1-g^T)$ which is positive under the assumption $\sigma^2\phi(1-\phi) > \varepsilon\delta(1-\sigma-\varepsilon)$.

Which of the ratios of investment per child to output per middle-aged in the two economies is higher is ambiguous. It has been shown that $N^C < N^T$ and $(1-\theta)\gamma_s^C >$

[15]Recall that physical capital comes solely from bequests in the traditional economy.

γ_s^T. Thus, per capita output in the market economy will grow faster if investment per child as a fraction of output per middle-aged in the market economy, $(1-\theta)\gamma_q^C$, is not substantially lower than that in the traditional economy, γ_q^T.

Comparing growth rates of per capita output in these two economies with each other, we have that $sign(g_y^C - g_y^T) = sign\left([1]^{1-\theta}[2]^{\theta} - [3]^{1-\theta}[4]^{\theta}]\right)$ where:

$$[1] = \frac{[\sigma^2\phi(1-\phi)g^C + \varepsilon\delta(1-\sigma-\varepsilon)(1-g^C)](1-vN^C)}{N^C};$$

$$[2] = \frac{\sigma(1-\phi)}{N^C};$$

$$[3] = \frac{[\sigma^2\phi(1-\phi)g^T + \varepsilon\delta(1-\sigma-\varepsilon)(1-g^T)](1-vN^T)}{N^T};$$

$$[4] = \frac{\sigma^2\phi(1-\phi)g^T + \varepsilon\delta(1-\sigma-\varepsilon)(1-g^T)}{\sigma\phi N^T}.$$

Clearly, $g_y^C > g_y^T$ if $[1] > [3]$ and $[2] > [4]$.

It turns out that $[2]-[4] > 0$ when $\sigma^2\phi(1-\phi) > \varepsilon\delta(1-\sigma-\varepsilon)$ because the sign of $[2]-[4]$ is positive:

$$\sigma^2\phi(1-\phi)(N^T - N^C g^T) - N^C\varepsilon\delta(1-\sigma-\varepsilon)(1-g^T) \;>\; N^T - N^C \;>\; 0.$$

We have $[2] = [4]$ if and only if $\sigma^2\phi(1-\phi) = \varepsilon\delta(1-\sigma-\varepsilon)$ since $N^T = N^C$ under the same condition.

The sign of $[1]-[3]$ is determined by:

$$[\sigma^2\phi(1-\phi) - \varepsilon\delta(1-\sigma-\varepsilon)]$$

$$\{\sigma\phi(1-\phi)(1-\delta)(1-\beta)(1-\theta)+\delta(1-\delta)(1-\sigma-\varepsilon)[\beta+(1-\beta)\theta+\phi(1-\beta)(1-\theta)]$$

$$+\sigma\delta(1-\phi)[\beta+(1-\beta)\theta]+\sigma\phi[\delta-(1-\delta)]\}.$$

The sign is positive if $\sigma^2\phi(1-\phi) > \varepsilon\delta(1-\sigma-\varepsilon)$ and if the taste for the consumption of children measured by $\delta(1-\sigma-\varepsilon)$ is not much weaker than that for the number of children measured by $(1-\delta)(1-\sigma-\varepsilon)$ (or equivalently, δ is not much smaller than $1-\delta$). Clearly, if $\delta \geq 1-\delta$ and $\sigma^2\phi(1-\phi) > \varepsilon\delta(1-\sigma-\varepsilon)$, then [1] > [3], hence $g_y^C > g_y^T$. When $\sigma^2\phi(1-\phi) > \varepsilon\delta(1-\sigma-\varepsilon)$ holds, even if $\delta < 1-\delta$, there is a range of the values of δ such that $\delta < 1-\delta$ and [1] > [3], hence $g_y^C > g_y^T$. It is also obvious that [1] = [3] if $\sigma^2\phi(1-\phi) = \varepsilon\delta(1-\sigma-\varepsilon)$.

When children are strictly investment goods for old-age security (i.e., $1-\sigma-\varepsilon = 0$), the growth rate of per capita output in the market economy is higher than that in the traditional economy since then $sign(g_y^C - g_y^T) = sign[1-(1-\phi)(1-\beta)(1-\theta)] > 0$.

In summary, with operative gifts in both economies, we have: fertility is lower in the market economy; per capita output grows faster in the market economy unless tastes for the number of children are much stronger than those for the consumption of children. Fertility and growth rates are the same in both economies if agents care equally about their own consumption and other generations'.

2.5.2 Cases without Operative Gifts in the Market Economy

In the market economy, bequests may or may not be operative when gifts are nonoperative. (See Appendix II where the market economy is analyzed when bequests are operative or when both gifts and bequests are nonoperative.)

When bequests are operative in the market economy, it is found that fertility is lower in the market economy, but growth rates of per capita output are similar in both economies. Fertility is lower in the market economy because children are no longer investment goods for old-age security when gifts are nonoperative. Giving no gifts to parents, the middle-aged agents in the market economy choose higher γ_s and γ_q than in the traditional economy. However, it can be shown from (II.2) in appendix II and (2.3.8): $(1-\theta)\gamma_s^C < \gamma_s^T$. That is, the saving/output ratio is lower in the market economy than in the traditional economy. Also, it can be shown from (II.4) in appendix II and (2.3.7) that the ratio of investment per child to output per middle-aged is lower in the market economy than in the traditional economy: $(1-\theta)\gamma_q^C < \gamma_q^T$.

Reducing fertility, introducing firms and markets has a positive effect on growth of per capita output. Decreasing the saving/output ratio and the ratio of investment per child to output per middle-aged, on the other hand, the transition towards market economies has negative effects on growth of per capita output. The net effect on economic growth is found to be extremely small. If one period in this model equals 30 years the annual growth rate of per capita output in the market economy is systematically under 0.01% lower than that in the traditional economy when bequests are operative.[16] Practically, such a small difference in growth rates of per capita output is trivial.

When intergenerational transfers are nonoperative, it is found that fertility is lower in the market economy than in the traditional economy. Economic growth is faster in the market economy than in the traditional economy except that when the values

[16]Many combinations of parameter values have been chosen to see what the difference is in growth rates between the two economies when bequests are operative. The procedure is as follows. The values of parameters are chosen to have a base case where gifts are operative. Then, the value of the taste for the parent's old-aged consumption, ε, is reduced gradually to 0.001 in order to make gifts nonoperative, and bequests operative. We repeat the process by varying the values of β, σ, ϕ, and δ in the base case one after another. (Note that the comparison of growth rates between the two economies does not depend on A, D, and v.) Furthermore, θ is set at 0.25 as widely accepted.

Parameter values chosen in the base case are given in Table 2.1 and the various values of parameters in other cases in Table 2.2. The upper and lower bounds of parameters in Table 2.2 are selected such that beyond which fertility may not be positive or the assumed restrictions of parameter values may not hold.

In the base case, for example, $g_y^T = 3.635$ and $g_y^C = 3.628$ with $b > 0$ when ε is reduced to 0.02. Their annual rates are 4.396% and 4.389% respectively. The difference is 0.007%.

of parameters are close to those in the case with operative bequests growth rates in both economies are similar.

2.5.3 Discussion

It has been shown that transforming a traditional economy into a market economy results in a higher or a similar growth rate of per capita output, depending on whether gifts or bequests are operative in the market economy. To understand what causes the difference, it appears helpful to see what happens to the determinants of the growth rate of per capita output if we change the value of a key parameter. This parameter determines whether a market economy has operative gifts, operative bequests, or neither of them. It is found that ε, the taste for parents' consumption relative to the consumption and the number of children $(1-\sigma-\varepsilon)$, is such a parameter. When ε is large enough gifts are operative in a market economy. When ε drops gifts eventually fall to zero and we have the case without operative intergenerational transfers in the market economy. When ε goes down further, bequests become operative in the market economy.

The difference in the effect on growth seems to come mainly from the change in saving rates in response to the drop in the value of ε. With operative gifts (i.e., ε large), we have seen that the saving rate is so high in the market economy that the saving/output ratio is higher than in the traditional economy. (Note that saving/output ratios are higher in both economies when ε becomes smaller if intergenerational transfers are operative.) In the case where intergenerational transfers are nonoperative after the transition, however, it can be seen from (II.7) in appendix II that the saving rate, hence the saving/output ratio, is lower when ε is smaller in the market economy. The reason is: without operative intergenerational transfers agents in the market economy choose higher ratios of investment in all children to their middle-age income by (II.6) in appendix II when tastes for children relative to parents are stronger (i.e., smaller ε). Thus, agents in the market economy have to reduce saving rates in order to invest more in all children in the case without operative intergenerational transfers. This is different from the cases with operative gifts or operative bequests in the market economy. With operative gifts in the market economy, since decreases in ε also imply decreases in gifts, the saving rate rises when investment in all children as a fraction of labor income rises in response to the fall in ε. With operative bequests in the market economy, since decreases in ε mean rises in bequests, agents increase saving rates in order to leave more bequests. However, the saving rate (the saving/output ratio) in the traditional economy always increases with ε. The saving/output ratio in the market economy is eventually lower than that in the traditional economy when bequests are about to be operative where ε is small enough. When bequests are operative, although saving/output ratios in both economies increase with ε, the market economy has a lower saving/output ratio than the traditional economy does. The negative impact on growth of the lower saving/output ratio is offset almost completely by the positive impact of lower fertility on growth. Thus the net effect on growth is negligible.

2.6 Conclusion

This chapter has shown that transforming a traditional economy into a market economy reduces fertility and can increase the growth rate of per capita output. With operative gifts, it is found that such a transformation can result in higher growth rates of per capita output unless tastes for the number of children are much stronger than those for consumption of children. When bequests are operative, growth rates of per capita output are similar in both economies. The results in this chapter also provide a possible structural explanation of the "population transition" while allowing sustained economic growth in both market and traditional economies and permitting preferences for the quality and quantity of children.

It is unclear, however, whether bequests or gifts had been operative in market economies before the setup of old-age social security programs. If indeed gifts were operative after the transition towards market economies, at least in some countries, this model may help us to understand disparities in economic growth between the West and the East.

The evolution of economic system is a complicated process. It is usually related to the evolution of political system. Historically to say, some countries were more or less reluctant to change their traditional economies into market economies than others, which might have impacts on fertility and economic growth. (For example, compare Japan to China.) Thus, to some extent, we may view changes in economic system as exogenous to producers and consumers, and examine their economic and demographic consequences. The message this chapter delivers is that governments in less developed countries should encourage the transition towards market economies if they are aimed at reducing fertility and speeding up economic growth.

Table 2.1: Parameter Values in the Base Case

A	D	v	ε	θ	β	σ	ϕ	δ
8.0	8.0	0.02	0.25	0.25	0.65	0.55	0.85	0.5

A: the productivity parameter in the production function for goods.

D: the productivity parameter in the education technology.

v: units of time needed to rear a child.

ε: the taste for parents' old-aged consumption.

θ: the capital's share parameter in the production function for goods.

β: the share parameter of an individual's investment in the education technology.

σ: the taste for own consumption relative to tastes for other generations'.

ϕ: the taste for old-aged consumption relative to that for middle-aged consumption.

δ: the taste for children's consumption relative to that for the number of children.

Table 2.2: Various Parameter Values

β	σ	ϕ	δ
0.9	0.7	0.9	0.55
0.8	0.65	0.85	0.5
0.7	0.6	0.8	0.45
0.6	0.55	0.75	0.4
0.5	0.5	0.7	0.35

Chapter 3

Social Security, Fertility, and Economic Growth

3.1 Introduction

Social security programs are common in developed countries, and transfer a significant part of national income from the working generation to the retired.[1] Their effects on fertility and particularly on economic growth have been debated for decades. However, investment in human capital and its relation with fertility and economic growth have been neglected in the debate. If fertility, human capital accumulation, and economic growth are indeed closely related as stressed recently by Becker, Murphy, and Tamura (1990), and Ehrlich and Lui (1991), the analyses of the impacts of social security on fertility and growth in the literature may be seriously incomplete. Moreover, like the Neoclassical growth model, the existing literature on this issue takes growth rates as given, and thus is unable to look at how social security affects the growth rate of per capita income.

The purpose of this chapter is to examine the effects of social security on economic growth in an overlapping generations model in which investment in human capital of children is the engine of endogenous economic growth. The main results

[1] On average, the amount of the transfer comprised 8.8 per cent of the Gross Domestic Product in the seven major OECD countries in 1981 (see, Hansson and Stuart, 1989).

in this chapter are as follows. First, unfunded social security may speed up growth of per capita income by reducing fertility and increasing the ratio of human capital investment per child to family income even though saving rates fall, and may bring about faster economic growth than funded social security. Second, even when fertility is exogenous and private intergenerational transfers are operative as in Barro (1974), the neutrality of unfunded social security fails to hold although saving rates are unchanged.

The possible positive effect of the unfunded program on economic growth arises from the responses of fertility, private saving, and human capital investment in children to the program. With operative gifts, for example, unfunded social security affects fertility in two opposite ways: taxing labor income lowers the opportunity cost of spending time rearing children, and transferring the tax revenue to the old makes children less essential for their parents' old-aged consumption. Total human capital investment in children as a fraction of family income declines due to the increase in the public transfer. However, if tastes for the consumption of children are sufficiently strong relative to those for the number of children, fertility (total fertility rate, or children per woman)[2] falls and the ratio of human capital investment per child to family income may rise in response to unfunded social security.[3] Moreover, unfunded social security does not alter saving rates when private intergenerational transfers are

[2]Within the framework of Barro (1991), social insurance and welfare payments as a fraction of gross domestic products have negative impacts on both the estimated total fertility rate and the fertility rate net of infant mortality.

[3]This rise in the ratio of investment per child to family income is consistent with data for the United States. After the introduction of unfunded social security at the end of 1930's, total expenditure for education as a fraction of gross national product (GNP) rose from about 3 percent in 1940's to about 7 percent in 1970's (see U.S. Bureau of the Census, 1978, Table 6/6). In the meantime, fertility fell. As a result, expenditure per child for education as a fraction of per family GNP went up. After 1970, the ratio of total education expenditure to GNP dropped to 6.5 per cent in 1984 (see U.S. Department of Education, 1986, P. 15). However, education expenditure per student as a fraction of per family GNP rose from 6.5% in 1970 to 7.1% in 1984, or as a fraction of GNP per person aged 16-65 rose from 14.9% to 17% (for the numbers of families, see U.S. Bureau of the Census, 1988, P. 427). This is partly because of the fall in school enrollment caused by the fall in fertility.

operative, hence the physical capital/labor ratio rises if fertility falls. Consequently, unfunded social security may result in faster growth of per capita income.

Even if fertility is exogenous and private intergenerational transfers are operative, the response of investment in human capital of children to unfunded social security affects economic growth. Thus, the neutrality of unfunded social security does not hold in general once human capital investment is endogenous.

This chapter is organized as follows. Section 2 explains the relation of the present chapter to previous work. Section 3 introduces the model. Sections 4 and 5 focus on the unfunded and funded programs respectively. Section 6 provides examples to compare effects of an unfunded social security program on growth with those of a funded one. The last section concludes the chapter.

3.2 Relation to Previous Work

Conventional analyses on social security focus on its impacts on saving. Feldstein (1974) argued that unfunded social security depresses saving, so has negative impacts on economic growth.[4] As is well known, Barro (1974) responded by pointing out that in a dynastic family model incorporating operative intergenerational transfers unfunded social security would be neutral. The voluminous empirical work which has sought to resolve this debate has been inconclusive.

For many observers conclusions have appeared warranted, however, concerning the relative impact of unfunded vs. funded social security. Funded social security is generally believed to have positive impacts on economic growth if governments require extramarginal saving, and at worst to be neutral if forced saving is inframarginal.[5] As mentioned earlier, the present paper casts doubt on this supposed advantage of funded relative to unfunded social security, by incorporating endogenous fertility and

[4] In practice, unfunded social security is much more popular.
[5] See, e.g., Ferrara, 1980.

human capital investment.

Recently, some authors have begun to analyze the impact of social security in models where fertility is endogenous (Becker and Barro, 1988; Lapan and Enders, 1990; Wildasin, 1990; Zhang and Nishimura, forthcoming). Some of their results are echoed in the more general analysis presented here, which also makes human capital investment endogenous and explicitly models the impact of social security on the growth rate of per capita income.

Becker and Barro (1988) find that a permanent increase in unfunded social security benefits reduces fertility temporarily and raises "capital intensity". While this is an intriguing result, and one which is echoed in the model analyzed here, Becker and Barro point out that their model is not ideally suited to evaluating social security since they do not model old-age consumption explicitly. Lapan and Enders (1990) extend the work of Becker and Barro (1988) to analyze the effects of government debt on fertility and capital intensity. They find that increases in government debt reduce fertility and raise the capital/labor ratio. Wildasin (1990) reaches similar conclusions in a simpler model. However, like Becker and Barro (1988), those two models ignore human capital investment, old-age consumption, and the time cost of childbearing. My model does not suffer from those limitations.

Incorporating the time cost of childbearing,[6] I consider the positive substitution effect of social security on fertility since social security contributions reduce after-tax wage rates (hence the opportunity cost of spending time rearing children). It is not obvious whether fertility will fall and capital/labor ratios will rise in response to social security in those models mentioned above if they include the time cost.

Including old-age consumption and hence life-cycle saving, the present model is able to examine social security when gifts from children to retired parents are operative. Without old-age consumption, in contrast, one has to rely on operative bequests

[6]See Barro and Becker (1989) and Ehrlich and Lui (1991).

from parents to children to have positive physical capital, and thus ignores the case with operative gifts. It is quite possible that gifts from children to retired parents within families had been essential for old-age consumption prior to the setup of social security.

With human capital investment, this model has endogenous growth, and is able to analyze the effects of social security on the growth rate of per capita income. In addition, with human capital investment in the model, we cannot simply conclude that faster growth will be warranted by showing drops in fertility and increases in the (physical) capital/labor ratio because human capital investment may fall in response to social security when old-age consumption is more secured. It can be inferred in this paper that a drastic fall in the ratio of human capital investment per child to family income due to social security can result in slower growth of per capita income even if fertility falls and the (physical) capital/labor ratio rises.

Using cross-country data, Barro (1989) shows empirically that government transfers have negative effects on fertility (significant) and saving (insignificant), but a positive effect on economic growth (insignificant). He is puzzled by the coexistence of the negative effect on saving and the positive effect on growth. One interesting feature of the present paper is that it predicts that this coexistence occurs when fertility falls and human capital investment per child as a fraction of family income rises under unfunded social security.

Ignoring the production of both goods and human capital and assuming children as simply investment goods for their parent's old-age consumption, Zhang and Nishimura (1993) find that the old-age security hypothesis may not be valid under unfunded social security.[7] However, this hypothesis is consistent with the data in at least a gross sense: since the setup of social security in the first half of this century, fertility

[7]This hypothesis states that if children are strictly investment goods for their parents' old-age consumption fertility falls whenever old-age consumption is more secured.

in developed countries has fallen drastically and been negatively related to public transfers as found by Barro (1989). One of the virtues of the present paper is that it allows us to get to grips better with the important issue of the old-age security hypothesis. With essentially the same utility function as that in Zhang and Nishimura (1993), the present paper find that this hypothesis is valid under both funded and unfunded social security.

3.3 The Model

There is an infinite number of periods and overlapping generations of three-period-lived agents. Let subscript t denote a period in time and superscript t the generation born in period $t-1$. Let L_t be the number of middle-aged agents living in period t. We assume that every parent has children at the beginning of middle age. A single good can be produced, consumed, invested in children, or accumulated as physical capital, but cannot be stored for future consumption.[8]

Everyone is identical, learns when young, lives in retirement in old age, and is endowed in middle age with one unit of time which can be supplied to labor markets or spent in rearing children. Let v (positive, less than unity) denote the units of time needed to rear a child.

When one-sided altruism towards an old-aged parent is assumed, children are simply investment goods for their parent's old-aged consumption. Utility then depends positively on the old-aged consumption of the retired parent (C_t^{t-1}) as well as on own middle-aged and old-aged consumption (C_t^t, C_{t+1}^t):

$$V(C_t^t,\ C_{t+1}^t,\ C_t^{t-1}).$$

[8]Instead of this assumption, we could assume that the return of investment in human capital or physical capital exceeds that of storing goods for future consumption without changing the results.

When two-sided altruism towards an old-aged parent and children is assumed, utility also depends positively on the consumption and number (N) of children:

$$U(C_t^t,\ C_{t+1}^t,\ C_t^{t-1},\ C_{t+1}^{t+1},\ C_{t+2}^{t+1},\ N_t),^9$$

where C_{t+1}^{t+1} and C_{t+2}^{t+1} are the middle-aged and old-aged consumption of every child respectively.[10] The justification for ignoring younger generations' fertility choices and consumption except C_{t+1}^{t+1} and C_{t+2}^{t+1} will be discussed later on in this section. We assume that $V(\cdot,\cdot,\cdot)$ and $U(\cdot,\cdot,\cdot,\cdot,\cdot,\cdot)$ are increasing and concave.

An individual firm's production function for goods has the form:

$$y_t = f(k_t,\ l_t H_t),$$

where k_t and l_t denote inputs of physical capital and labor respectively, and H_t refers to a middle-aged agent's human capital. We assume that $f(\cdot,\cdot)$ is increasing, concave, and homogeneous of degree one.

The education technology converts investment in children (q_t per child) into human capital where the economy-wide average human capital investment per child ($\bar{q}_t$) exhibits an externality:[11]

$$H_{t+1} = h(q_t,\ \bar{q}_t).$$

[9] In contrast to chapter 2, here we have C_{t+2}^{t+1} in the utility function due to that in this chapter we will let agents choose saving rates instead of the amount of saving. This setup generates the neutrality under funded social security when forced saving is inframarginal without other substantial differences in results compared to the alternative setup.

[10] Assuming altruism towards children, the dynastic family models of Becker and Barro (1988) and Becker, et al. (1990) abstract from parents' consumption in the utility function. Assuming altruism towards parents, on the other hand, the models of Zhang and Nishimura (1992) and Ehrlich and Lui (1991) neglect children's consumption in the utility function.

[11] Externalities in human capital accumulation have been stressed in the literature. For example, Lucas (1990) illustrates that the externality in human capital accumulation can explain why real interest rates among developed and developing countries are much closer than the Neoclassical growth model predicts.

We assume that $h(\cdot,\cdot)$ is increasing, concave, and homogeneous of degree one.

To obtain a balanced growth path, we assume Cobb-Douglas functions (see, Lucas, 1988; Becker, et al., 1990):

$$U(C_t^t,\ C_{t+1}^t,\ C_t^{t-1},\ C_{t+1}^{t+1},\ C_{t+2}^{t+1},\ N_t) =$$
$$\left({C_t^t}^{\omega}{C_{t+1}^t}^{1-\omega}\right)^{\sigma}{C_t^{t-1}}^{\epsilon}\left(\left({C_{t+1}^{t+1}}^{\omega}{C_{t+2}^{t+1}}^{1-\omega}\right)^{\delta}{N_t}^{1-\delta}\right)^{1-\sigma-\epsilon},$$

with $0<\sigma,\ \omega,\ \varepsilon,\ \delta<1$ and $1-\sigma-\varepsilon\geq 0$;

$$f_t(k_t,l_tH_t)=Dk_t^{\theta}(l_tH_t)^{1-\theta}\ ,\ \ h(q_t,\bar{q}_t)=A{q_t}^{\beta}{\bar{q}_t}^{\ 1-\beta},$$

with $D>0$, $A>0$, and $0<\theta,\ \beta<1$. Here, β measures the individual returns to scale of human capital investment and $1-\beta$ the degree of externalities in the education technology. Altruism is two-sided if $\varepsilon<1-\sigma$, and one-sided if $\varepsilon=1-\sigma$. We further assume: $\sigma>\delta(1-\sigma-\varepsilon)$ and $\sigma^2\omega(1-\omega)>\varepsilon\delta\omega(1-\sigma-\varepsilon)$. That is, tastes for own consumption are stronger than those for children's, and tastes for own two-period consumption are stronger than those for the retired parent's old-aged consumption and children's middle-aged consumption.

There are many firms organizing the production of goods, and markets channeling the trade of labor, physical capital, and goods. In period t, each middle-aged agent spends vN_t units of time rearing children, supplies $1-vN_t$ units of time to labor markets, and earns $(1-vN_t)w_t$. This agent receives bequests, b_t, from his/her old-aged parent at the beginning of period t, and leaves bequests b_{t+1}, to each child at the beginning of the child's middle-age. The middle-aged agent spends the earning and inheritance, $(1-vN_t)w_t+b_t$, on own middle-aged consumption, C_t^t, on saving for old-aged consumption (C_{t+1}^t), $s_t[(1-vN_t)w_t+b_t]$, on bequests to children, N_tb_{t+1}, on investments in children, q_tN_t, and on gifts to the old-aged parent, $g_t[(1-vN_t)w_t+b_t]$.

The middle-aged agent also expects gifts from children, $N_t g_{t+1}[(1-vN_{t+1})w_{t+1}+b_{t+1}]$. Here, g and s are the gift rate and the ratio of saving to labor income plus inheritance respectively. Thus the middle-aged's budget constraint implies:

$$C_t^t = [(1 - vN_t)w_t + b_t](1 - g_t - s_t) - q_t N_t, \tag{3.3.1}$$

$$C_{t+1}^t = (1 + r_t)s_t[(1 - vN_t)w_t + b_t]+$$

$$N_t g_{t+1}[(1 - vN_{t+1})w_{t+1} + b_{t+1}] - N_t b_{t+1}, \tag{3.3.2}$$

where w and r denote the real wage rate and real .nterest rate respectively. When one-sided altruism towards an old-aged parent is assumed, there are no incentives for the middle-aged to bequeath since agents do not care about children's consumption. Thus in this case, $b = 0$. In any case, bequests and gifts are nonnegative since agents are unable to force other generations to submit goods to them.

To simplify the analysis, we adopt the following assumptions:

Assumption 3.3.1: *Agents make decisions (b_{t+1}, g_t, N_t, q_t, and s_t) while knowing the decisions of the older generations and taking the decisions of the younger generations as given.*

Assumption 3.3.2: *The middle-aged agents within each family make their gift (to their old-aged parent) decisions cooperatively.*

Without Assumption 3.3.1, it would be extremely hard to work out the interaction among infinite generations, as shown by Nishimura and Zhang (1992). Under this assumption, generations act like Nash game players except that they play in different periods. Also under Assumption 3.3.1, the middle-aged agents living in period t cannot directly affect younger generations' fertility choices, N_{t+1}, N_{t+2} $\cdots$, and consumption of descendents other than their own children's, C_{t+2}^{t+2}, C_{t+3}^{t+2} $\cdots$. Thus we do not include these future fertility and consumption in the utility function. Under Assumption 3.3.2, each middle-aged agent makes the gift decision as if he/she were the head of his/her siblings.

Therefore, each middle-aged agent chooses b_{t+1} (if altruism is two-sided), g_t,[12] N_t, q_t, and s_t to maximize own utility subject to (3.3.1), (3.3.2), $b_{t+1} \geq 0$, and $g_t \geq 0$, taking wage rates, interest rates, and future generations' decisions as given.

Firms are perfect competitors maximizing profits by choosing physical capital, k_t, and labor, l_t. We assume that physical capital lasts for one period in the production of goods, hence firms' problems are static. The profit function is defined as:

$$f(k_t, l_t H_t) - (1 + r_{t-1})k_t - l_t w_t.$$

The first-order conditions of firms maximizing profits are given by:

$$w_t = (1-\theta)D\left(\frac{k_t}{l_t H_t}\right)^{\theta} H_t, \tag{3.3.3}$$

$$1 + r_{t-1} = \theta D\left(\frac{l_t H_t}{k_t}\right)^{1-\theta}. \tag{3.3.4}$$

Clearly, firms' physical capital/effective labor ratios, $k_t/(l_t H_t)$, are equal, and are the same as the economy-wide physical capital/effective labor ratio, $K_t/(L_t^d H_t)$, where K_t and L_t^d are aggregate physical capital and labor demand respectively. Equations (3.3.3) and (3.3.4) imply that a middle-aged agent's real wage rate is positively related to his/her own human capital.

Labor market clearing requires:

$$L_t^d = L_t(1 - vN_t). \tag{3.3.5}$$

[12] If g_t is an amount of gifts instead of the gift rate, parents would not take into account the effects of investment in children's quality on gifts under Assumption 3.3.1. Thus in this case, parents would not invest in children's quality if altruism is one-sided towards old-aged parents. However, the change in the results is trivial when altruism is two-sided.

Capital market clearing needs:

$$K_t = L_{t-1}s_{t-1}[(1 - vN_{t-1})w_{t-1} + b_{t-1}]. \tag{3.3.6}$$

Constant returns to scale and perfect competition imply that profits should be zero in every period. By Walras' law, the goods market clears as well.

Since agents are identical, the symmetric condition is: $q_t = \bar{q}_t$.

Definition: *An equilibrium is a collection of sequences* $\{r_t, w_t, b_t, g_t, s_t, q_t, N_t, k_t, l_t\}_{t=0}^{\infty}$ *such that:*

(1) Given $\{w_t, r_t\}_{t=0}^{\infty}$, $\{b_t, g_t, q_t, N_t, s_t\}_{t=0}^{\infty}$ *solves every individual agent's problem and* $\{l_t, k_t\}_{t=0}^{\infty}$ *solves firms' problems.*

(2) Given $\{b_t, g_t, q_t, N_t, s_t, l_t, k_t\}_{t=0}^{\infty}$, $\{w_t, r_t\}_{t=0}^{\infty}$ *clears markets.*

3.4 Unfunded Social Security

This section examines the responses of fertility and economic growth to unfunded social security.

We assume in this section that there is a long-lived government taxing labor income of middle-aged agents at a flat rate τ and transferring the tax revenue to old-aged agents in each period:

$$T_t = \bar{N}_{t-1}\tau(1 - v\bar{N}_t)\bar{w}_t, \tag{3.4.1}$$

where T_t is the benefit received by every old-aged agent living in period t. Note that here $\bar{N}$'s and $\bar{w}_t$ are the economy-wide average fertility and wage rate respectively, and in equilibrium we have $\bar{N}_t = N_t$ and $\bar{w}_t = w_t$. Thus agents take the benefit as given.

Then the unfunded program changes (3.3.1) and (3.3.2) to:

$$C_t^t = [(1 - vN_t)w_t + b_t](1 - g_t - s_t) - \tau(1 - vN_t)w_t - q_tN_t, \tag{3.4.2}$$

$$C_{t+1}^t = (1 + r_t)s_t[(1 - vN_t)w_t + b_t] +$$

$$N_tg_{t+1}[(1 - vN_{t+1})w_{t+1} + b_{t+1}] + T_{t+1} - N_tb_{t+1}. \tag{3.4.3}$$

We substitute (3.4.2) and (3.4.3) into the utility function for C_t^t, C_{t+1}^t, C_t^{t-1} (by backdating (3.4.3) one period), C_{t+1}^{t+1} (by updating (3.4.2) one period), and C_{t+2}^{t+1} (by updating (3.4.3) one period). Under the assumptions in the last section, the first-order conditions of the middle-aged maximizing own utility are as follows:[13]

$$N_t(1 - g_{t+1})U_2 = (1 - g_{t+1} - s_{t+1})U_4 + (1 + r_{t+1})s_{t+1}U_5 + \mu, \quad \mu \geq 0,$$

$$\mu b_{t+1} = 0; \tag{3.4.4}$$

$$[(1 - vN_t)w_t + b_t]U_1 = N_{t-1}[(1 - vN_t)w_t + b_t]U_3 + \lambda, \lambda \geq 0, \lambda g_t = 0; \tag{3.4.5}$$

$$N_tU_1 = \frac{\beta N_tg_{t+1}(1 - vN_{t+1})w_{t+1}}{q_t}U_2 + \frac{\beta(1 - vN_{t+1})(1 - g_{t+1} - s_{t+1} - \tau)w_{t+1}}{q_t}U_4 +$$

$$\frac{\beta(1 + r_{t+1})s_{t+1}(1 - vN_{t+1})w_{t+1}}{q_t}U_5;\text{[14]} \tag{3.4.6}$$

$$[vw_t(1 - g_t - s_t - \tau) + q_t]U_1 = \{g_{t+1}[(1 - vN_{t+1})w_{t+1} + b_{t+1}] -$$

$$b_{t+1} - (1 + r_t)s_tvw_t\}U_2 - N_{t-1}g_tvw_tU_3 + U_6; \tag{3.4.7}$$

[13]Note that w_{t+1} is a function of q_t from (3.3.3), (3.3.4), and the education technology.

[14]Updating (3.3.3) and (3.3.4) for one period and substituting (3.3.4) into (3.3.3) for $\frac{k_{t+1}}{l_{t+1}H_{t+1}}$, we have: $w_{t+1} = (1-\theta)D\left(\frac{\theta D}{1+r_t}\right)^{\frac{\theta}{1-\theta}}H_{t+1}$. Thus we get $\frac{\partial w_{t+1}}{\partial q_t} = \frac{\beta w_{t+1}}{q_t}$ by substituting $h(\cdot,\cdot)$ into the last equation for H_{t+1}.

$$U_1 = (1 + r_t)U_2. \tag{3.4.8}$$

Here, μ and λ are Lagrange multipliers associated with b_{t+1} and g_t respectively.

Equation (3.4.4) means that the utility forgone by leaving one more unit of bequests to children should be no less than that obtained from improving children's consumption by the bequest; the equality holds if bequests are operative ($b > 0$). Equation (3.4.5) says that the utility forgone by submitting one more unit of gifts to an old-aged parent should be no less than that obtained from enhancing the parent's old-aged consumption by the gift; the equality holds if gifts are operative ($g > 0$). Equation (3.4.6) requires that the utility forgone by investing one more unit in children's human capital should equal the utility gained from both enjoying more gifts due to children's higher earnings and improving children's consumption by the investment. Equation (3.4.7) means that the loss of utility by consuming less and presenting fewer gifts to an old-aged parent to have one more child should equal the gain in utility by both receiving more gifts when old from and enjoying the child. Equation (3.4.8) equates the utility forgone by saving one more unit now to that obtained from consuming more when old by the saving.

Equations (3.3.3), (3.3.4), (3.4.1)-(3.4.5) imply that when gifts are operative bequests are nonoperative (zero) under the assumption $\sigma^2\omega(1-\omega) > \varepsilon\delta\omega(1-\sigma-\varepsilon)$ in Section 3.3 If gifts are operative, then $\lambda = 0$. Therefore, we have:

$$\mu = \frac{\sigma^2\omega(1-\omega) - \varepsilon\delta\omega(1-\sigma-\varepsilon)}{\sigma^2\omega(1-\omega)}(1-g_{t+1})U_2,$$

which is positive with $0 < g_{t+1} < 1$ if and only if $\sigma^2\omega(1-\omega) > \varepsilon\omega\delta(1-\sigma-\varepsilon)$.

Thus there are three possible cases: (i) $b = 0$ and $g > 0$, (ii) $b > 0$ and $g = 0$, and (iii) $b = g = 0$. We analyze these cases in turn.

3.4.1 The Case with Operative Gifts

The equilibrium in this case is characterized by (3.3.3)-(3.4.3), (3.4.5)-(3.4.8), and the symmetric condition, with $b = 0$. Steady-state growth requires fertility, the gift rate (or the bequest rate later on), the saving rate, accumulation rates of both physical and human capital, and the growth rate of per capita income to be constant. Solving these equations characterizing the equilibrium under the steady-state growth conditions gives the reduced form solutions.[15]

With operative gifts, the ratio of saving to labor income, denoted by γ_s, is independent of τ:

$$\gamma_s = \frac{\sigma\theta(1-\omega)}{\varepsilon(1-\theta)}. \tag{3.4.9}$$

Clearly, we have $s = \gamma_s$ without operative bequests. Moreover, in this simple model economy, a middle-aged's labor income is also his/her family income. We define γ_s as the saving rate.[16]

The gift rate is found to be:

$$g = \frac{\sigma\omega\theta\varepsilon(1-\theta)(1-s-\tau) - \beta\omega\theta\varepsilon\delta(1-\theta)(1-\sigma-\varepsilon)(1-\tau)}{(1-\theta)\{\sigma^2\omega^2\theta + \sigma\omega\varepsilon\theta + \beta\omega\varepsilon[\sigma(1-\theta)s - \theta\delta(1-\sigma-\varepsilon)]\}} - \frac{\sigma^2\omega^2\theta[\theta + (1-\theta)\tau]}{(1-\theta)\{\sigma^2\omega^2\theta + \sigma\omega\varepsilon\theta + \beta\omega\varepsilon[\sigma(1-\theta)s - \theta\delta(1-\sigma-\varepsilon)]\}}, \tag{3.4.10}$$

which decreases with τ under the assumption $\sigma > \delta(1-\sigma-\varepsilon)$ in Section 3.3.

Fertility is given by:

[15] From (3.3.3)-(3.3.6) and $\frac{k_t}{l_t H_t} = \frac{K_t}{L_t^d H_t}$, we get $\frac{w_t}{1+r_{t-1}} = \frac{(1-\theta)s_{t-1}[(1-vN_{t-1})w_{t-1} + b_{t-1}]}{\theta N_{t-1}(1-vN_t)}$. Substituting this equation into the middle-aged's first-order conditions and budget equations of both the government and agents, we get a closed form solution.

[16] Since labor income is $1-\theta$ per cent of total output with a Cobb-Douglas production function, γ_s has a constant relation with the saving/output rate. Thus if τ does not affect the ratio of saving to labor income it will not affect the saving/output ratio.

$$N = \frac{x}{v[\sigma\theta\varepsilon(1-\theta)(1-\tau)+x]}\,, \tag{3.4.11}$$

where: $x = \sigma\varepsilon(1-\theta)^2(1-\beta)gs + \theta(1-\sigma-\varepsilon)\{\sigma(1-\delta)[\theta+(1-\theta)(g+\tau)] - \beta\varepsilon\delta(1-\theta)(1-g-\tau)\}$. Whether fertility falls when τ rises is ambiguous in general: a rise in τ has direct positive effects on fertility but indirect negative effects on fertility by reducing the gift rate. The ambiguity stems from the fact that increasing τ makes children less essential for their parents' old-age security and reduces the opportunity cost of spending time rearing children (the after-tax real wage rate). Whether fertility falls in response to a rise in τ depends on how strong the taste for the number of children is relative to that for the consumption of children. If tastes for children's consumption are strong enough relative to those for the number of children, fertility will fall as τ rises.[17]

If altruism is one-sided towards an old-aged parent, fertility decreases with τ. This can be shown by setting $1-\sigma-\varepsilon = 0$ in (3.4.11), and the sign of $\frac{\partial N}{\partial \tau}$ is then negative. Thus the old-age security hypothesis is valid in the case of unfunded social security. In addition, the labor supply $(1-vN)$ increases due to the rise in τ. However, fertility may not fall without incorporating the production of goods and human capital as predicted by Zhang and Nishimura (1992).[18]

[17] The condition for the fall in fertility due to the rise in τ is given by:

$$\delta \quad \frac{\sigma(1-\sigma-\varepsilon)[\beta(1-\theta)s+\theta(1-s)] - (1-\theta)s[\sigma^2\omega(1-\beta)}{\sigma(1-\sigma-\varepsilon)[\beta(1-\theta)s+\theta(1-s)] + \beta(1-\sigma-\varepsilon)[\tau\varepsilon(1-\beta)(1-\theta)^2(1-\tau)s+\sigma\omega\theta]} +$$

$$\frac{\sigma\varepsilon(1-\theta)(1-\beta)s+\beta\varepsilon\theta(1-\sigma-\varepsilon)]}{\sigma(1-\sigma-\varepsilon)[\beta(1-\theta)s+\theta(1-s)] + \beta(1-\sigma-\varepsilon)[\tau\varepsilon(1-\beta)(1-\theta)^2(1-\tau)s+\sigma\omega\theta]}\,.$$

The right hand side of this inequality is clearly less than unity.

[18] With exogenous interest rates and virtually the same utility function, they find that unfunded social security reduces fertility only if tastes for own old-aged consumption are strong enough relative to those for the parent's old-aged consumption.

The investment per child is proportional to labor income:

$$q_t = \frac{\beta}{\sigma\theta N}[\sigma(1-\theta)gs + \theta\delta(1-\sigma-\epsilon)(1-g-\tau)](1-vN)w_t$$

$$\equiv \gamma_q(1 - vN)w_t \ , \tag{3.4.12}$$

where γ_q is the investment rate per child (recall that $(1 - vN_t)w_t$ is the labor earning). This rate is positively related to the gift rate but negatively related to fertility. Whether the investment rate per child decreases with τ is ambiguous as well. The rate increases with τ only if fertility falls when τ rises.[19] In other words, a rise in the ratio of investment per child to family income in response to a rise in τ implies that fertility decreases with τ.

Equation (3.4.12) also implies that total investment in children as a fraction of labor income $\frac{q_t N_t}{(1-vN_t)w_t}$ falls when τ rises. It is important to distinguish the total investment rate from the investment rate per child since we deal with per capita growth.

Combining (3.3.3)-(3.3.6), (3.4.9), (3.4.12), and production functions for goods and human capital, we have that evolution of both human and physical capital is jointly determined by the following two equations:

$$K_{t+1} = L_t s_t(1 - vN_t)w_t = \gamma_s(1 - vN_t)L_t D(1-\theta)K_t^{\theta}[L_t H_t(1 - vN_t)]^{-\theta}H_t \ , \tag{3.4.13}$$

$$H_{t+1} = Aq_t = A\gamma_q(1 - vN_t)D(1-\theta)K_t^{\theta}[L_t H_t(1 - vN_t)]^{-\theta}H_t \ . \tag{3.4.14}$$

From (3.4.11), the evolution of population is simply determined by:

$$L_{t+1} = NL_t \ .$$

[19]Note that $\frac{\partial\gamma_q}{\partial\tau} < 0$ if $\frac{\partial N}{\partial\tau} > 0$.

Denoting accumulation rates of both physical and human capital as $1 + g_{kt} \equiv K_{t+1}/K_t$ and $1 + g_{ht} \equiv H_{t+1}/H_t$ respectively, and imposing steady-state growth conditions ($g_{ht} = g_h$, $g_{kt} = g_k$, and $N_t = N$), we rewrite (3.4.13) and (3.4.14) as:

$$\frac{K_t}{H_t L_t} = \left(\frac{D\gamma_s(1-\theta)}{1+g_k}\right)^{\frac{1}{1-\theta}} (1 - vN) \,, \tag{3.4.15}$$

$$\frac{K_t}{H_t L_t} = \left(\frac{1+g_h}{AD\gamma_q(1-\theta)}\right)^{\frac{1}{\theta}} (1 - vN)^{\frac{\theta-1}{\theta}} \,. \tag{3.4.16}$$

Equations (3.4.15) and (3.4.16) imply that the steady-state physical/human capital ratio is constant:

$$\frac{K_{t+1}}{H_{t+1}L_{t+1}} = \frac{K_t}{H_t L_t} \equiv k^* \,.$$

This is equivalent to:

$$1 + g_k = (1 + g_h)N \,, \tag{3.4.17}$$

which means that physical capital per capita grows at the same pace as that of an individual's human capital.

Equating the right hand side of (3.4.15) to that of (3.4.16) and substituting (3.4.17) into (3.4.15) for $1 + g_k$ yield the steady-state rate of human capital accumulation:

$$1 + g_h = N^{-\theta} D(1-\theta)(\gamma_q A)^{1-\theta} \gamma_s^{\theta} (1 - vN)^{1-\theta} \,. \tag{3.4.18}$$

That is, the rate of human capital accumulation is related positively to the ratio of saving to labor income and the investment rate per child, and negatively to fertility.

It turns out that the rate of human capital accumulation is also the growth rate of per capita income.[20]

Under unfunded social security programs with operative gifts, the growth rate of per capita income may or may not increase with τ. Since $\frac{\partial \gamma_k}{\partial \tau} = 0$ and since $\frac{\partial \gamma_z}{\partial \tau} < 0$ if $\frac{\partial N}{\partial \tau} \geq 0$, $\frac{\partial N}{\partial \tau} \geq 0$ implies $\frac{\partial g_z}{\partial \tau} < 0$. Thus $\frac{\partial N}{\partial \tau} < 0$ is necessary for $\frac{\partial g_z}{\partial \tau} > 0$. Moreover, since $\frac{\partial \gamma_z}{\partial \tau} > 0$ implies $\frac{\partial N}{\partial \tau} < 0$ by (3.4.12), $\frac{\partial \gamma_z}{\partial \tau} > 0$ is sufficient for $\frac{\partial g_z}{\partial \tau} > 0$ by (3.4.18) with operative gifts. When tastes for children's consumption are strong enough, fertility falls as τ rises, and the ratio of investment per child to labor income may increase, hence it is likely that a rise in τ results in faster economic growth.

Summing up this subsection, we have the following propositions:

Proposition 3.4.1: *With operative gifts, fertility falls in response to a rise in the tax rate under an unfunded social security program when tastes for the consumption of children are strong enough relative to those for the number of children.*

Proposition 3.4.2: *With operative gifts, if the ratio of investment per child to family income rises in response to a rise in the tax rate under an unfunded social security program fertility will fall and the growth rate of per capita income will increase.*

Proposition 3.4.3: *When children are strictly investment goods for their parent's old-aged consumption, fertility falls as the tax rate rises under an unfunded social security program.*[21]

[20] Per capita income in period t is proportional to H_t:

$$\frac{f\Big(K_t, L_t H_t(1 - vN_t)\Big)}{L_{t-1} + L_t + L_{t+1}} = D\left(\frac{K_t}{H_t L_t}\right)^{\theta} \frac{(1 - vN)^{1-\theta} H_t}{1/N + 1 + N},$$

where $f\Big(K_t, L_t H_t(1 - vN_t)\Big)$ is aggregate output (recall that $\frac{k_t}{l_t H_t} = \frac{K_t}{L_t(1-vN_t)H_t}$), and both fertility and the steady-state physical/human capital ratio are constant.

[21] Note that when gift are nonoperative and children are strictly investment goods for their parent's old-aged consumpt on parents will not have any children. Thus, in the present model, the old-age security hypothesis is relevant only when gifts are operative.

3.4.2 The Case with Operative Bequests

When bequests are operative, the equilibrium is characterized by (3.3.3)-(3.3.6), (3.4.1)-(3.4.4), (3.4.6)-(3.4.8), with $g = 0$, and by the symmetric condition.

Saving per middle-aged agent is found to be:

$$s_t[(1 - vN_t)w_t + b_t] = \frac{\theta\delta(1 - \sigma - \varepsilon)}{\sigma(1 - \theta)}(1 - vN_t)w_t \equiv \gamma_s(1 - vN_t)w_t \; , \tag{3.4.19}$$

where γ_s is the ratio of saving to labor income. The saving rate is clearly independent of τ.

The ratio of bequests ($N_t b_{t+1}$) to the return of total saving ($(1+r_t)s[(1-vN)w_t + b_t]$), denoted by γ_b (the bequest rate), increases with τ under the assumption $\sigma > \delta(1 - \sigma - \varepsilon)$ in Section 3.3:

$$\gamma_b = \frac{\delta(1 - \sigma - \varepsilon)[\theta + \omega(1 - \theta)\tau + \beta(1 - \omega)(1 - \theta)(1 - \tau)]}{\theta[\sigma(1 - \omega) + \omega\delta(1 - \sigma - \varepsilon)]} - \frac{\sigma(1 - \omega)(1 - \theta)(1 - \tau)}{\theta[\sigma(1 - \omega) + \omega\delta(1 - \sigma - \varepsilon)]} \, . \tag{3.4.20}$$

The rise in the bequest rate and the fall in the gift rate due to a rise in τ indicate that private intergenerational transfers move in an opposite direction to that of the unfunded government transfers.

When bequests are operative, s_t is the ratio of saving to labor income plus inheritance, which is given by:

$$s = \frac{(1 - \theta)\gamma_s}{1 - \theta(1 - \gamma_b)} \, . \tag{3.4.21}$$

With operative bequests, s is clearly decreasing with τ. Note that s_t is no longer the saving/labor income ratio when $b > 0$.

Fertility is negatively related to γ_b:

$$N = \frac{x}{v[\sigma^2(1-\theta)(1-\omega)(1-\tau)+x]}, \tag{3.4.22}$$

where:

$$x = \delta(1-\delta)(1-\sigma-\varepsilon)^2[\theta+(1-\theta)\tau] - \theta\delta(1-\sigma-\varepsilon)[\sigma(1-\omega)+(1-\delta)$$

$$(1-\sigma-\varepsilon)]\gamma_b - \beta\sigma\delta(1-\omega)(1-\theta)(1-\tau)(1-\sigma-\varepsilon).$$

Fertility may drop when τ rises since the sign of $\frac{\partial N}{\partial \tau}$ is negative at least when $\delta \geq \frac{1}{1+\theta+\omega(1-\theta)}$. That is, fertility falls due to unfunded social security with operative bequests when tastes for the consumption of children are strong enough relative to those for the number of children.

The investment per child is:

$$q_t = \frac{\beta\delta(1-\sigma-\varepsilon)(1-\tau)}{N\sigma}(1-vN)w_t \equiv \gamma_q(1-vN)w_t, \tag{3.4.23}$$

where the investment rate may or may not increase with τ. It increases with τ only if fertility decreases with τ. Again, the total investment in children as a fraction of labor income decreases with τ.

The growth rate of per capita income is also defined by (3.4.18), where the saving/labor income rate, fertility, and the investment rate per child are defined in (3.4.19), (3.4.22), and (3.4.23) respectively. When bequests are operative, a fall in fertility in response to a rise in τ is necessary for unfunded social security to speed up economic growth since γ_q decreases with τ if fertility rises and since γ_s is constant. Also, per capita income will grow faster if the investment rate per child increases with τ. We have:

Proposition 3.4.4: *With operative bequests, fertility falls in response to a rise in the tax rate under an unfunded social security program when tastes for the consumption of children are strong enough relative to those for the number of children.*

Proposition 3.4.5: *With operative bequests, if the ratio of investment per child to family income rises in response to a rise in the tax rate under an unfunded social security program fertility will fall and the growth rate of per capita income will increase.*

Define $\underline{\tau}$ as the tax rate such that $g(\underline{\tau}) = 0$ in (3.4.10) and $\bar{\tau}$ as the tax rate such that $b(\bar{\tau}) = 0$ in (3.4.20) respectively. It can be shown that $\underline{\tau}$ is strictly lower than $\bar{\tau}$.[22] Since the gift rate decreases with and the bequest rate increases with τ, neither gifts nor bequests are operative when $\underline{\tau} < \tau < \bar{\tau}$. This case is studied in the next subsection.

3.4.3 The Case without Private Intergenerational Transfers

Without operative private intergenerational transfers, the equilibrium is characterized by (3.3.3)-(3.3.6), (3.4.1)-(3.4.3), (3.4.6)-(3.4.8), and the symmetric condition, with $g = 0$ and $b = 0$.

The ratio of saving to labor income now decreases with τ in contrast to the cases with operative gifts or bequests:

$$s = \gamma_s = \frac{\theta(1-\omega)(1-\tau)[\sigma - \beta\delta(1-\sigma-\varepsilon)]}{\sigma\omega[\theta + (1-\theta)\tau] + \theta\sigma(1-\omega)} \,. \tag{3.4.24}$$

[22]The sign of $\bar{\tau} - \underline{\tau}$ is determined by $\sigma - \beta\delta(1-\tau-\varepsilon)$ which is positive under the assumption $\sigma > \delta(1-\sigma-\varepsilon)$ in Section 3.3.

Fertility in this case is given by:

$$N = \frac{x}{v[\sigma\theta(1-\omega)(1-\tau)+x]}\,, \tag{3.4.25}$$

where: $x = (1-\delta)(1-\sigma-\varepsilon)[\theta+(1-\theta)\tau]s - \beta\theta\delta(1-\omega)(1-\sigma-\varepsilon)(1-\tau)$. It can be shown that fertility increases with τ.[23]

The investment rate per child falls in response to the rise in τ:

$$q_t = \frac{\beta\delta(1-\sigma-\varepsilon)(1-\tau)}{N\sigma}(1-vN)w_t \equiv \gamma_q(1-vN)w_t. \tag{3.4.26}$$

This equation is the same as (3.4.23). However, from (3.4.25) and (3.4.26), increasing τ lowers the after-tax wage rate (the opportunity cost of spending time rearing children) and encourages parents to trade the quality of children for the quantity of children when $\underline{\tau} < \tau < \bar{\tau}$.

Without operative private intergenerational transfers, to smooth consumption between life cycle and among generations after a rise in τ, agents save less for old-aged consumption and tend to substitute the quality of children for the quantity of children unless τ is raised above $\bar{\tau}$. Consequently, the growth rate of per capita income falls when τ rises by (3.4.18) and (3.4.24)-(3.4.26). When τ is higher than $\bar{\tau}$, agents leave bequests to children to offset partly the drop in children's consumption as shown in the last subsection.

From the analysis in this subsection, we have the following propositions:

Proposition 3.4.6: *Without operative private intergenerational transfers, fertility rises and the ratio of investment per child to family income falls in response to a rise in the tax rate under an unfunded social security program.*

[23] The sign of $\frac{\partial N}{\partial \tau}$ is determined by $\sigma - \beta\delta(1-\sigma-\varepsilon)$ which is positive under the assumption $\sigma > \delta(1-\sigma-\varepsilon)$ in Section 3.3.

Proposition 3.4.7: *Without operative private intergenerational transfers, a rise in the tax rate under an unfunded social security program decreases the growth rate of per capita income.*

3.4.4 Discussion

We have seen in this section that agents respond to the rise in the tax rate differently in different cases with operative gifts, with operative bequests, or without operative private intergenerational transfers. In general, a rise in τ reduces the opportunity cost of spending time rearing a child and the ratio of middle-aged consumption to old-aged consumption (see (3.4.1)-(3.4.3)). Thus it has offsetting effects on agents' decisions. The interpretation of the results could be very complicated due to interactions between q_t and N, and between g (or b) and N.

With operative gifts ($\tau < \underline{\tau}$), the rise in τ makes gifts from children less essential. Thus the gift rate drops and children become less essential as investment goods for their parents' old-age security. Agents then invest less in children (Nq_t). On the other hand, the rise in τ exerts a positive effect on fertility by reducing the opportunity cost of time in rearing a child. Fertility therefore may go up. The net effect on fertility is determined by how strong tastes are for the number of children relative to those for the consumption of children. When tastes for children's consumption are strong enough relative to those for the number of children, fertility falls while investment per child as a fraction of labor income (γ_q) may rise. As agents reduce the gift rate, they do not change their saving rates. As a result, per capita income may grow faster.

Without operative private intergenerational transfers ($\underline{\tau} < \tau < \bar{\tau}$), the rise in τ does not make children less essential for old-age security since children present no gifts to their parents. Also τ has not been so high such that parents leave bequests to offset partly the drop in the middle-aged/old-aged consumption ratio. However, the positive effect on fertility exists. Fertility thus rises and the investment rate per child falls. Also, with more secure old-aged consumption and without operative private intergenerational transfers, agents reduce the saving rate. Therefore the rise in τ hinders economic growth. This case is essentially similar to the model of Feldstein (1974) where fertility and private intergenerational transfers are ignored and the income effect of public transfers (from middle-aged to the old) has to be offset by reducing the saving rate.

With operative bequests ($\tau > \bar{\tau}$), agents react to a further rise in τ by leaving more bequests to children and keeping the saving/labor income rate and hence the saving/output rate unchanged.[24] That is, parents give back some resources to children when the government transfers too much. If fertility goes down bequests per child as a fraction of the return to total saving will go up, which can partly offset the income effect on life cycle consumption. Thus it is favourable in this respect to reduce fertility (recall that N and b are negatively related). On the other hand, there still exists the positive effect on fertility due to the lower opportunity cost of time in rearing children. The net effect on fertility also depends on how strong the tastes are for consumption relative to the number of children. As in the case with operative gifts, fertility falls and investment per child as a fraction of family income may rise (although investment in all children drops) when parents have sufficiently strong tastes for the consumption relative to the number of children.[25] Consequently, the rise in τ may accelerate economic growth.

Even if gifts or bequests are operative and fertility is exogenous, predictions of this model are different from those in Barro (1974). From (3.4.12), the rise in τ depresses the investment rate per child when fertility is exogenous and when gifts are operative. This is because it reduces the gift rate and makes the quality of children less essential

[24]However, the saving rate (γ_s) with operative bequests is lower than that with operative gifts, which can be seen from (3.4.9) and (3.4.19).

[25]Becker and Barro (1988) ignore the cost of time in rearing children and model social security contributions and benefits in a lump-sum manner. Thus they do not have positive effects of social security on fertility.

as a source of the parent's old-aged consumption. Similarly from (3.4.23), the rise in τ lowers the investment rate per child as well when fertility is exogenous and when bequests are operative. Therefore even when fertility is exogenous and the saving rate is unchanged, the negative effect of the unfunded program on the investment rate per child hinders economic growth, which can be inferred from (3.4.18). Thus the neutrality of unfunded social security as shown by Barro (1974) arises not only from the presence of operative private intergenerational transfers and the absence of fertility choices, but also from the neglect of investment in children's human capital.

3.5 Funded Social Security

To compare the effects of funded and unfunded social security on fertility and economic growth, we investigate equilibria under a funded program in this section.

The long-lived government in this case forces middle-aged agents to save at least τ per cent of their middle-aged earnings, and lets each of them have the returns of the forced saving back in old age. Thus the middle-aged agent's budget constraint is changed to:

$$C_t^t = [(1 - vN_t)w_t + b_t](1 - g_t - s_t) - (1 - vN_t)w_t\tau - q_tN_t, \tag{3.5.1}$$

$$C_{t+1}^t = (1 + r_t)\{s_t[(1 - vN_t)w_t + b_t] + (1 - vN_t)w_t\tau\}+$$

$$N_tg_{t+1}[(1 - vN_{t+1})w_{t+1} + b_{t+1}] - N_tb_{t+1}, \tag{3.5.2}$$

where s_t is the ratio of voluntary saving to labor earnings plus inheritance in period t. Here (3.5.1) is the same as (3.4 2) in the case of unfunded social security except that τ has different meaning. Equation (3.5.2) includes the return of the forced saving, $(1 + r_t)(1 - vN_t)w_t\tau$, and does not have the public transfer, T_{t+1}, in contrast to (3.4.3). Since there is no voluntary lending among identical agents, there is no voluntary borrowing among them as well. Thus we restrict s_t to be nonnegative.[26]

With two-sided altruism towards an old-aged parent and children, each middle-aged agent living in period t maximizes own utility by choosing q_t, N_t, g_t, b_{t+1}, and s_t, and taking w, r, and other generations' decisions as given, subject to $b_{t+1} \geq 0$, $g_t \geq 0$ and $s_t \geq 0$.

Recall that under the assumption $\sigma^2\omega(1-\omega) > \epsilon\delta\omega(1-\sigma-\epsilon)$ in Section 3.3, gifts and bequests cannot be operative simultaneously under an unfunded social security program. It can be shown that this conclusion holds under the same condition as well in the case with funded social security. In the remainder of this section we focus on the equilibrium with operative gifts. The analyses of the equilibria with operative bequests or without operative private intergenerational transfers are given in subsections 2 and 3 of Appendix III respectively.

In the case with operative gifts, the first-order condition with respect to g_t is the same as (3.4.5) with $b = 0$; the first-order condition with respect to s_t is similar to (3.4.8) except that it now has a non-negativity restriction.

The first-order condition with respect to q_t is:

$$q_tN_tU_1 = \beta N_tg_{t+1}(1 - vN_{t+1})w_{t+1}U_2 + \beta(1 - vN_{t+1})w_{t+1}(1 - g_{t+1} - s_{t+1} - \tau)U_4 +$$

$$\beta(1 + r_{t+1})(1 - vN_{t+1})w_{t+1}(s_{t+1} + \tau)U_5. \qquad (3.5.3)$$

The utility obtained here includes the positive effect of investing one more unit in children's quality to increase children's earnings on children's old-aged consumption via forced saving in contrast to (3.4.6).

[26]This restriction applies also to the case with unfunded social security. However, it is unlikely that the unfunded program will drive the saving rate to zero. Thus we ignore the non-negative restriction there.

The first-order condition with respect to N_t is given by:

$$[w_t v(1 - g_t - s_t - \tau) + q_t]U_1 = [g_{t+1}(1 - vN_{t+1})w_{t+1} -$$

$$(1 + r_t)w_t v(s_t + \tau)]U_2 - N_{t-1}g_t v w_t U_3 + U_6. \tag{3.5.4}$$

In contrast to (3.4.7) under an unfunded social security program, the utility forgone from the fall in old-aged consumption in (3.5.4) includes the fall in forced saving due to spending time rearing the one more child.

Since forced saving is absorbed by firms, the capital market clears when:

$$K_t = L_{t-1}(s_{t-1} + \tau)(1 - vN_{t-1})w_{t-1}. \tag{3.5.5}$$

The voluntary saving may be positive or zero, depending on how high the forced saving rate is. It is shown in Appendix III that the funded program is neutral when voluntary saving is positive and when gifts are operative.

From (III.1.1) in Appendix III that when $\tau \geq \frac{\sigma\theta(1-\omega)}{\varepsilon(1-\theta)}$, voluntary saving will be driven to zero. In this case the equilibrium with operative gifts is characterized by (3.3.3)-(3.3.5), (3.4.5), (3.5.1)-(3.5.5), and the symmetric condition, with $b = 0$ and $s = 0$.

Now, under the assumption that agents are mainly concerned about own consumption, the gift rate is decreasing with τ:

$$g = \frac{\sigma\varepsilon(1-\theta)(1-\tau) - \delta\beta(1-\sigma-\varepsilon)[\varepsilon(1-\theta)(1-\tau) + \sigma\theta(1-\omega)]}{(1-\theta)[\sigma\varepsilon + \beta\sigma^2(1-\omega) - \beta\delta\varepsilon(1-\sigma-\varepsilon) + \sigma^2\omega]} -$$

$$\frac{\sigma^2\omega\theta}{(1-\theta)[\sigma\varepsilon + \beta\sigma^2(1-\omega) - \beta\delta\varepsilon(1-\sigma-\varepsilon) + \sigma^2\omega]}\,. \tag{3.5.6}$$

Fertility is given by:

$$N = \frac{x}{v[\sigma\varepsilon(1-\theta)(1-\tau)+\sigma^2(1-\omega)\theta + x]}\,, \tag{3.5.7}$$

where: $x = \sigma^2(1-\omega)(1-\theta)(1-\beta)g + (1-\sigma-\varepsilon)\{\sigma(1-\delta)[\theta+(1-\theta)g] - \beta\varepsilon\delta(1-\theta)(1-g-\tau) - \beta\sigma\theta\delta(1-\omega)\}$. Like the case with operative private intergenerational transfers under unfunded programs, whether fertility falls as τ rises is ambiguous: the sign of $\frac{\partial N}{\partial \tau}$ is determined by $\sigma(1-\delta)(1-\sigma-\varepsilon)[\varepsilon+\sigma(1-\omega)] - \sigma^3\omega(1-\omega)(1-\beta) - \beta\delta(1-\sigma-\varepsilon)[\sigma^2(1-\omega)^2 + \sigma(1-\omega)(1-\delta)(1-\sigma-\varepsilon) + \varepsilon(1-\delta)(1-\sigma-\varepsilon) + \sigma^2\omega(1-\omega) + \sigma\varepsilon]$. The fall in the gift rate exerts a negative effect on fertility: agents demand fewer children for old-age security when they are forced to save more than they desire. But, agents then face a lower opportunity cost of spending time rearing children.[27] The fall in the opportunity cost has a positive effect on fertility. Fertility will decrease with τ if at least $\delta > \frac{\sigma(1-\sigma-\varepsilon)[\varepsilon+\sigma(1-\omega)] - \sigma^3\omega(1-\omega)(1-\beta)}{(1-\sigma-\varepsilon)\{\sigma[\varepsilon+\sigma(1-\omega)] + \beta[\sigma^2(1-\omega)^2 + \sigma\varepsilon + \sigma^2\omega(1-\omega)]\}}$.

The investment per child is:

$$q_t = \frac{\beta\{\sigma^2(1-\omega)(1-\theta)g + \delta(1-\sigma-\varepsilon)[\varepsilon(1-\theta)(1-g-\tau) + \sigma\theta(1-\omega)]\}}{N\sigma\varepsilon(1-\theta)}$$

$$(1-vN)w_t \equiv \gamma_q(1-vN)w_t. \tag{3.5.8}$$

[27] The utility forgone to have one more child in this case is: $vw_t(1-s_t-\tau)U_1 + (1+s_t+r_t)\tau w_t v U_2$, or $[vw_t(1-s_t-\tau) + (1+s_t+r_t)\tau w_t v \frac{U_2}{U_1}]U_1$ in terms of the marginal utility forgone with respect to middle-aged consumption. When $s_t > 0$, $U_1 = (1+r_t)U_2$ and hence the consumption forgone from having one more child is vw_t which is independent of τ given w_t. Thus there are no changes in the opportunity cost of spending time rearing children. But when $s_t = 0$, $U_1 > (1+r_t)U_2$ and $\frac{U_2}{U_1} = \frac{U_3}{N_{t-1}U_0}$ by (3.4.5). Thus by (3.3.3), (3.3.4), (3.5.5), and by the utility function, we have that the utility lost to have one more child equals $[vw_t(1-\tau) + \frac{\sigma\theta(1-\omega)}{\varepsilon(1-\theta)}vw_t]U_1$ where the consumption forgone to have one more child, $vw_t(1-\tau) + \frac{\sigma\theta(1-\omega)}{\varepsilon(1-\theta)}vw_t$, is decreasing with τ given w_t.

The investmen rate per child may or may not rise as τ rises as in the case of unfunded social security. A rise in the investment rate per child due to a rise in τ also implies a fall in fertility.

The growth rate of per capita income, now defined by (3.4.18), (3.5.6)-(3.5.8), and $\gamma_s = \tau$, may be reduced by funded social security if the investment rate per child falls and/or fertility rises. The fall in the growth rate as τ rises happens if tastes for children's consumption are extremely weak relative to those for the number of children. However, since a rise in τ is equivalent to a rise in the saving/income ratio and since $\frac{\partial \gamma_k}{\partial \tau} > 0$ implies $\frac{\partial N}{\partial \tau} < 0$, $\frac{\partial \gamma_k}{\partial \tau} > 0$ is sufficient for $\frac{\partial g_k}{\partial \tau} > 0$ when gifts are operative. Moreover, $\frac{\partial N}{\partial \tau} < 0$ is no longer necessary for $\frac{\partial g_k}{\partial \tau} > 0$ when forced saving rates are extramarginal in contrast to the case under unfunded social security.

It can be shown by setting $1 - \sigma - \varepsilon = 0$ in (3.5.6) and (3.5.7) that fertility falls as τ rises as under an unfunded social security program. That is, the old-age security hypothesis is valid under a funded social security program.

From the results in this section, we get:

Proposition 3.5.1: *With operative gifts, fertility will fall in response to an extramarginal rise in the forced saving rate under funded social security if tastes for the consumption relative to the number of children are strong enough.*

Proposition 3.5.2: *With operative gifts, if the ratio of investment per child to family income ri•es in response to an extramarginal rise in the forced saving rate under a funded social security program fertility will fall and the growth rate of per capita income will increase.*

Proposition 3.5.3: *When children are strictly investment goods for their parent's old-aged consumption, an extramarginal rise in the forced saving rate under a funded social security program decreases fertility.*[28]

[28]As under an unfunded social security program, the old-age security hypothesis is relevant only when gifts are operative.

3.6 Examples

Practically, it is important to show that an unfunded social security program may bring about faster economic growth than a funded one since the conventional view is that phasing out unfunded programs and replacing them with funded ones can achieve faster economic growth. It is shown in this section by two examples that starting from the same equilibrium, the unfunded programs can result in lower fertility and faster economic growth than the funded ones even when the tax rate and forced saving rate are raised from zero to 30 per cent of labor income. The calculations are, of course, merely meant to be illustrative.

Table 3.1 contains the parameter values used in these two experiments. Table 3.2 summarizes the responses of the gift rate (g), the ratio of bequests to total returns of saving (γ_b), the saving rate (γ_s), the investment rate per child (γ_q), fertility (N), and the growth rate of per capita income (g_y) to the rise in the tax rate (τ) of the unfunded program. Table 3.3 lists the effects of raising the forced saving rate (τ) of the funded program on the gift rate (g), the ratio of bequests to total returns of saving (γ_b), the saving rate (γ_s), the investment rate per child (γ_q), fertility (N), and on the growth rate of per capita income (g_y).

In Table 3.1, the value of θ is widely accepted as around 0.25. The values of other parameters are set to generate an equilibrium in which fertility and growth rates of per capita income are close to those in developed countries.[29] In particular, the value of δ is chosen to give a case where tastes for children's consumption relative to the number of children are strong enough to get faster per capita growth under unfunded programs than under funded ones. Note that since we abstract from the

[29]For example, fertility was about 2.5 during 1940-1944 (see U.S. Bureau of the Census, 1981, p. 58), and the average annual growth rate of per capita GNP during 1873 (1889)-1940 was about 1.77 (1.53) per cent in the United States (see U.S. Bureau of the Census, 1976, p. 244). However, this simple model differs from real economies in many ways, particularly in the composition of family income. Thus saving rates and the ratio of investment per child to family income in this model are noncomparable with those in real economies.

sex difference, the replacement level of fertility in this model (equals to unity) is half of that in reality, and that one period here equals several decades. To make the results comparable with data, we multiply N by 2 and convert g_y into its annual rate by assuming that one period in this model equals 30 years. In Tables 3.2 and 3.3, fertility and growth rates are the adjusted numbers.

In Table 3.2, the rise in τ drives the gift rate down to zero, makes bequests operative, and increases the ratio of bequests to total returns of saving. When gifts or bequests are operative ($b > 0$, or $g > 0$), the rise in τ leaves the saving rate unchanged, raises the investment rate per child, reduces fertility, and speeds up economic growth. Comparing the first two rows with the other in Table 3.2, we can see that even when the saving rate is reduced but economic growth is speeded up. If we have in mind that social security accounts for a very significant part of government transfers, this situation seems consistent with and may help to explain the empirical findings of Barro (1989) which reveal that government transfers reduce fertility significantly, and have a positive effect on economic growth (insignificant) but a negative effect on the saving rate (insignificant).

In Table 3.2, the changes in fertility and growth rates of per capita income are also consistent with U.S. data before and after the setup of unfunded social security at the end of 1930's. The payroll tax rate of old-age social security in the Unites States was about 10 per cent in 1980.[30] Fertility dropped below 1.9 after the mid-1970's from about 2.5 during 1940-1944, and the average annual growth rate of per capita GNP rose from 1.77 (%) during 1873-1940 to 2.15 (%) during 1940-1989.[31] Comparing the first row with the third in Table 3.2, the changes in fertility and the growth rate are close to those in U.S. data. Moreover, investment per child as a fraction of family income moves in the same direction as in U.S. data as aforementioned.

[30]See U.S. Bureau of the Census,1990, p. 847.

[31]Also see U.S. Bureau of the Census, 1976, p. 224; 1990, p.434.

In Table 3.3, when voluntary saving is positive, funded social security is neutral. When voluntary saving has been driven to zero, the rise in the forced saving rate increases the investment rate per child, reduces fertility and the gift rate, makes bequests operative, increases the ratio of bequests to total returns of saving, and raises the growth rate of per capita income.

Comparing Table 3.2 to Table 3.3, we can see that starting from the same equilibrium, the unfunded program brings about faster economic growth and slower population growth than the funded one. This result suggests that phasing out unfunded social security programs and replacing them with funded ones may not obtain faster economic growth.

These results are insensitive to the values of A, D, and v. However, they are sensitive to the values of other parameters. It is more likely for unfunded programs to have stronger positive effects on economic growth than funded ones if β is greater. Higher values of β mean weaker externalities in the education sector, or higher individual returns to investment in children's quality. Thus, facing higher values of β, parents demand fewer children but spend more on each child's education in response to a rise in τ. With larger θ, agents save more, choose lower gift rates, demand fewer children, invest more in each child, and have higher growth rates of per capita income under unfunded programs than under funded ones. When ε is small enough such that bequests are operative, it more likely that growth rates rise with τ under unfunded programs and are higher than those under funded programs. When ε is very large such that gifts are operative and the gift rate is high, unfunded programs are more likely to have faster growth than funded programs unless τ is too high. When ε is in between, equilibria without operative gifts and bequests come in under unfunded programs and growth rates are lower as τ rises.

With larger σ, agents have weaker altruism towards children, $1 - \sigma - \varepsilon$, and relatively weaker altruism towards parents. Agents then save more for their own old-aged consumption, choose lower gift rates and bequest rates. As a result, the equilibrium without operative private intergenerational transfers is more likely to prevail and the growth rates of per capita income are lower under unfunded programs than under funded ones. However, when σ is large enough (e.g., $\sigma > 0.57$), this model, with other parameter values unchanged, predicts drops in the investment rate per child and rises in fertility even with τ greater than 12 per cent. These predictions are inconsistent with the fact that expenditure per child for education as a fraction of per family income has gone up and fertility gone down in the last several decades as mentioned in Section 3.1. When $0.54 < \sigma < 0.57$, with other parameter values as given in Table 3.1, unfunded social security results in faster economic growth than do funded programs as τ is low or high where gifts or bequests are operative. But it brings about slower economic growth as τ is moderate where equilibria without operative private intergenerational transfers prevail.

The comparison in Tables 3.2 and 3.3 depends also on large values of ω. When $\omega = 0.89$, with other parameter values given in Table 3.1, economic growth is faster under unfunded programs than under funded programs only as τ is low (under 7 per cent of labor income) or high (above 28 per cent of labor income). However, in this case, investment rates per child with τ smaller than 15 per cent of labor income are lower than or about the same as the rate with $\tau = 0$. This is inconsistent with data in the United States where τ is about 10 per cent of labor income. When ω is even lower, not only are investment rates per child unlikely to rise, but also fertility tends to be unchanged or to rise, even with τ above 12 per cent of labor income under unfunded programs. With $\omega < 0.84$, fertility with $12 < \tau < 30$ per cent of labor income is higher than that with $\tau = 0$ under unfunded programs. This is because the lower ω is, the more likely are equilibria without operative private intergenerational transfers. With such small ω, positive effects of an unfunded program on economic growth may exceed those of a funded program only if gifts are operative.

For unfunded programs to bring about faster economic growth than funded ones, this model needs large δ. As $\delta = 0.54$, economic growth under funded programs is faster than that under unfunded programs unless $\tau < 6$ per cent of labor income. However, investment rates per child with $0 < \tau < 18$ per cent of labor income stay about the same level as that with $\tau = 0$. With even lower δ, unfunded programs slow down economic growth. On the other hand, funded programs still speed up economic growth unless δ is close to zero. Under unfunded programs, fertility falls little or may rise when $\delta < \frac{1}{2}$. Furthermore, small δ implies that a small fraction of income is invested in children. However, about one third of family disposable income in an average family in the United States is found to be invested in children (see Espenshade, 1984, p. 65-67). This evidence seems in favor of large $\delta(1 - \sigma - \varepsilon)$.

Other combinations of parameter values may also enable unfunded programs to have stronger positive effects on economic growth than funded ones with a wide range of values of τ. With smaller ε or larger β, smaller δ can preserve our basic properties in Tables 3.2 and 3.3. When ε is small such that bequests are operative throughout,[32] it is more likely that unfunded programs have faster economic growth than do funded programs since then the negative effects of the equilibrium without private transfers across generations under unfunded programs cannot come about.

3.7 Conclusion

The results of this paper challenge conventional views about social security. It is shown that an unfunded social security program may speed up growth of per capita income by reducing fertility and increasing investment per child as a fraction of family income even if saving rates fall, and may bring about faster economic growth than a funded one does. It is also shown that the neutrality of unfunded social security does

[32]Some economists claim that bequests are an important part of saving in the United States (see, Kotlikoff and Summers, 1981).

not hold once investment in children's quality has been incorporated.

The positive effect of social security on human capital investment per child as a fraction of family income and its implications for economic growth have been ignored in the literature. In fact, education expenditure per child has increased faster than per family income in the last several decades although total education expenditure as a fraction of gross national product has fallen since 1970 in the United States. If indeed the fall in fertility and the rise in the ratio of investment per child to family income are caused by unfunded social security, this model predicts that unfunded social security has exerted positive impacts on economic growth.

These results provide possible explanations to why government transfers have a negative effect on saving rates but a positive effect on economic growth as found by Barro (1989). The results are also consistent with the long term trend of fertility and per capita income in developed countries: fertility has fallen and economic growth has speeded up in the last two centuries, particularly since the setup of pay-as-you-go pensions in the 1940's and 1950's. Thus we should be cautious about claims that unfunded social security has hindered economic growth, and that replacing unfunded social security programs with funded ones will result in faster economic growth by forcing a higher saving rate.

Table 3.1: Parameter Values

A	D	v	ε	θ	β	σ	ω	δ
6.0	6.0	0.015	0.25	0.25	0.64	0.54	0.90	0.55

A: the productivity parameter in the production function for goods.

D: the productivity parameter in the education technology.

v: units of time needed to rear a child.

ε: the taste for parents' old-aged consumption.

θ: the capital's share parameter in the production function for goods.

β: the share parameter of an individual's investment in the education technology.

σ: the taste for own consumption relative to tastes for other generations'.

ω: the taste for old-aged consumption relative to that for middle-aged consumption.

δ: the taste for children's consumption relative to that for the number of children.

Table 3.2: Results of the Unfunded Program

τ(%)	g(%)	γ_b(%)	γ_s(%)	γ_q(%)	N	g_y(%)
0.0	4.9	0.0	7.2	11.7	2.34	1.77
5.0	0.1	0.0	7.2	12.2	2.12	1.98
10.0	0.0	13.2	7.1	12.9	1.82	2.18
15.0	0.0	27.5	7.1	14.1	1.66	2.55
20.0	0.0	41.8	7.1	16.2	1.36	3.10
25.0	0.0	56.1	7.1	20.4	1.00	3.94
30.0	0.0	70.4	7.1	31.3	0.62	5.50

τ: the (contribution) tax rate under an unfunded social security program.

g: the gift rate.

γ_b: the ratio of bequests to total returns of saving.

γ_s: the ratio of saving to labor income.

γ_q: the ratio of investment per child to labor income.

N: fertility.

g_y: the growth rate of per capita income.

Table 3.3: Results of the Funded Program

$\tau(\%)$	$g(\%)$	$\gamma_b(\%)$	$\gamma_s(\%)$	$\gamma_q(\%)$	N	$g_y(\%)$
0.0	4.9	0.0	7.2	11.7	2.34	1.77
5.0	4.9	0.0	7.2	11.7	2.34	1.77
10.0	4.0	0.0	10.0	11.7	2.28	2.10
15.0	2.6	0.0	15.0	11.8	2.14	2.50
20.0	1.1	0.0	20.0	12.0	1.98	2.86
25.0	0.0	0.4	25.0	12.2	1.84	3.16
30.0	0.0	4.8	30.0	12.8	1.65	3.53

τ: the forced saving rate.

g: the gift rate.

γ_b: the ratio of bequests to total returns of saving.

γ_s: the ratio of total saving to labor income.

γ_q: the ratio of investment per child to labor income.

N: fertility.

g_y: the growth rate of per capita income.

Appendix I

Appendix to Chapter 1

The appendix derives the effects of wage rates on fertility with operative bequests. In part A, interest rates and the capital/labor ratio are held constant to find out the direct effect of wage rates on fertility. In part B, interest rates and the capital/labor ratio are first converted into wage rates using the first-order conditions of firms to determine the (both direct and indirect) effect of wage rates on fertility.

For simplicity without changing the results, we rewrite (1.2.1) and (1.2.2) as:

$$C_t^t = w_t(1 - vN_t) + B_t - qN_t - S_t \ , \tag{I.1}$$

$$C_{t+1}^t = (1 + r_t)S_t - N_t B_{t+1} \ , \tag{I.2}$$

where the middle aged chooses bequests per child B_{t+1} instead of the bequest ratio b_t.

The Lagrangian of the middle-aged's problem is:

$$U\left(C_t^t, C_{t+1}^t, w_{t+1}(1 - vN_{t+1}) + B_{t+1} - N_{t+1}q - S_{t+1}, N_t\right) +$$

$$\lambda\{(1 + r_t)[w_t(1 - vN_t) + B_t - N_t q - C_t^t] - B_{t+1}N_t - C_{t+1}^t\}.$$

The first-order conditions are given by:

$$C_t^t : \quad U_1 = \frac{\sigma\phi U}{C_t^t} = \lambda(1 + r_t), \tag{I.3}$$

$$C_{t+1}^t : \quad U_2 = \frac{\sigma(1 - \phi)U}{C_{t+1}^t} = \lambda, \tag{I.4}$$

$$B_{t+1}: \quad U_3 = \frac{(1-\sigma)\varepsilon U}{C_{t+1}^{t+1}} = \lambda N_t, \tag{I.5}$$

$$N_t: \quad U_4 = \frac{(1-\sigma)(1-\varepsilon)U}{N_t} = \lambda[(1+r_t)(w_t v + q) + B_{t+1}], \tag{I.6}$$

$$\lambda: \quad (1+r_t)[w_t(1-vN_t) + B_t] = (1+r_t)(N_t q + C_t^t) + B_{t+1}N_t + C_{t+1}^t. \tag{I.7}$$

The solutions are the same as those in Section 1.3.

Part A: For given r and k, differentiating ($I.3$)-($I.7$) with respect to w, we have (at the steady-state):

$$\begin{bmatrix} U_{11}+U_{13} & U_{12} & 0 & U_{14} & -f' \\ U_{21}+U_{23} & U_{22} & 0 & U_{24} & -1 \\ U_{31}+U_{33} & U_{32} & 0 & U_{34}-\lambda & -N \\ U_{41}+U_{43} & U_{42} & -\lambda & U_{44} & -[f'(wv+q)+B] \\ f' & 1 & N-f' & B+f'(wv+q) & 0 \end{bmatrix} \begin{bmatrix} \frac{\partial C_1}{\partial w} \\ \frac{\partial C_2}{\partial w} \\ \frac{\partial B}{\partial w} \\ \frac{\partial N}{\partial w} \\ \frac{\partial \lambda}{\partial w} \end{bmatrix} =$$

$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ \lambda v f' \\ f'(1-vN) \end{bmatrix}$$

Solving for $\frac{\partial N}{\partial w}$ via Cramer's rule and denoting the bordered Hessian matrix as H, we have:

$$\frac{\partial N}{\partial w} = \frac{\lambda v f'}{H} \begin{vmatrix} U_{11}+U_{13} & U_{12} & 0 & -f' \\ U_{21}+U_{23} & U_{22} & 0 & -1 \\ U_{31}+U_{33} & U_{32} & 0 & -N \\ f' & 1 & N-f' & 0 \end{vmatrix}$$

$$+\frac{f'(1-vN)}{H}\begin{vmatrix} U_{11}+U_{13} & U_{12} & 0 & f' \\ U_{21}+U_{23} & U_{22} & 0 & 1 \\ U_{31}+U_{33} & U_{32} & 0 & N \\ U_{41}+U_{43} & U_{42} & \lambda & f'(wv+q)+B \end{vmatrix}=0$$

where both the first and second terms turn out to be zero in the right hand side of the above equation. Note that this outcome (i.e. $\frac{\partial N}{\partial w}=0$) using all the first-order conditions is the same as that from using (1.3.8) alone in Section 1.3.

Part B: Expressing r and k in terms of w and differentiating (I.3)-(I.7) with respect to w, we have (at the steady-state):

$$\begin{bmatrix} U_{11}+U_{13} & U_{12} & 0 & U_{14} & -f' \\ U_{21}+U_{23} & U_{22} & 0 & U_{24} & -1 \\ U_{31}+U_{33} & U_{32} & 0 & U_{34}-\lambda & -N \\ U_{41}+U_{43} & U_{42} & -\lambda & U_{44} & -[f'(wv+q)+B] \\ f' & 1 & N-f' & B+f'(wv+q) & 0 \end{bmatrix}\begin{bmatrix} \frac{\partial C_1}{\partial w} \\ \frac{\partial C_2}{\partial w} \\ \frac{\partial B}{\partial w} \\ \frac{\partial N}{\partial w} \\ \frac{\partial \lambda}{\partial w} \end{bmatrix}=$$

$$\frac{1}{k}\begin{bmatrix} -\lambda \\ 0 \\ 0 \\ -\lambda(wv+q)+\lambda f'kv \\ -[w+B-(wv+q)N-C_1]+kf'(1-vN) \end{bmatrix}$$

where k should be understood as a function of w, which is implicitly defined by (1.3.1).

Solving for $\frac{\partial N}{\partial w}$ via Cramer's rule and denoting the bordered Hessian matrix as H, we have:

$$\frac{\partial N}{\partial w}=\frac{\lambda/k}{H}\begin{vmatrix} U_{21}+U_{23} & U_{22} & 0 & -1 \\ U_{31}+U_{33} & U_{32} & 0 & -N \\ U_{41}+U_{43} & U_{42} & -\lambda & -[f'(wv+q)+B] \\ f' & 1 & N-f' & 0 \end{vmatrix}+$$

$$\frac{\lambda[wv+q-f'kv]/k}{H}\begin{vmatrix} U_{11}+U_{13} & U_{12} & 0 & f' \\ U_{21}+U_{23} & U_{22} & 0 & 1 \\ U_{31}+U_{33} & U_{32} & 0 & N \\ f' & 1 & N-f' & 0 \end{vmatrix}-$$

$$\frac{[w+B-(wv+q)N-C_1-f'k(1-vN)]/k}{H}\begin{vmatrix} U_{11}+U_{13} & U_{12} & 0 & f' \\ U_{21}+U_{23} & U_{22} & 0 & 1 \\ U_{31}+U_{33} & U_{32} & 0 & N \\ U_{41}+U_{43} & U_{42} & \lambda & f'(wv+q)+B \end{vmatrix}$$

$$=\frac{-\varepsilon(1-\sigma)}{\sigma\phi k}<0$$

where the second and third terms turn out to be zero in the right hand side of the above equation. Note that this outcome (i.e. $\frac{\partial N}{\partial w}=\frac{-\varepsilon(1-\sigma)}{\sigma\phi k}<0$) using all the first-order conditions is the same as that from using (1.3.8) and $\frac{\partial w}{\partial k}=-kf''(k)$ alone in Section 1.3.

Appendix II

Appendix to Chapter 2

This appendix analyzes equilibria in the capitalist economy with operative bequests or without operative intergenerational transfers in part A and part B respectively.

A. Equilibrium in the Capitalist Economy with Operative Bequests

When bequests are operative, the equilibrium is characterized by (2.2.3)-(2.3.8), (2.4.1), (2.4.3)-(2.4.5), with $g_t = 0$, and by the symmetric condition.

The steady-state bequest ratio (bN) is found to be:

$$bN = \frac{\delta(1-\sigma-\varepsilon)[\theta+\beta(1-\theta)(1-\phi)] - \sigma\phi(1-\phi)(1-\theta)}{\sigma\phi\theta(1-\phi)+\delta\theta\phi(1-\sigma-\varepsilon)} . \qquad (II.1)$$

The bequest ratio decreases with ε but increases with δ.

The saving rate is:

$$S_t = \frac{\delta\theta(1-\sigma-\varepsilon)}{\sigma\phi(1-\theta)}(1-vN)w_t \equiv \gamma_s w_t. \qquad (II.2)$$

The saving rate decreases with ε and ϕ as in the case with operative gifts. However, it now decreases with σ and increases with δ in contrast to the case with operative gifts.

Fertility is given by:

$$N = \frac{x}{v[\sigma\theta(1-\phi)+x]} , \qquad (II.3)$$

where:

$$x = (1-\delta)(1-\sigma-\varepsilon)[(1-bN)\theta+(1-\theta)] - \sigma(1-\phi)\beta(1-\theta) - bN\sigma\theta(1-\phi).$$

Fertility still decreases with δ.

The investment per child is:

$$q_t = \frac{\beta\delta(1-\sigma-\varepsilon)}{N\sigma\phi}(1-vN)w_t \equiv \gamma_q(1-vN)w_t. \qquad (II.4)$$

The growth rate of per capita income is again defined by (2.4.17), where the saving rate, fertility, and the investment rate per child are defined in ($II.2$), ($II.3$), and ($II.4$) respectively.

B. Equilibrium in the Capitalist Economy without Operative Intergenerational Transfers

Without operative intergenerational transfers, the equilibrium is characterized by (2.2.3)-(2.2.8), (2.4.3)-(2.4.5), and the symmetric condition, with $g_t = 0$ and $b_t = 0$.

Fertility in this case is given by:

$$N = \frac{(1-\sigma-\varepsilon)\{\sigma\phi(1-\delta) - \beta\delta[\sigma + (1-\delta)(1-\sigma-\varepsilon)]\}}{v[\sigma\phi - \beta\delta(1-\sigma-\varepsilon)][\sigma + (1-\delta)(1-\sigma-\varepsilon)]}. \qquad (II.5)$$

The investment rate per child is found to be:

$$q_t = \frac{\delta\beta(1-\sigma-\varepsilon)}{N\sigma\phi}(1-vN)w_t \equiv \gamma_q(1-vN)w_t, \qquad (II.6)$$

where γ_q is the same in ($II.4$) and ($II.6$).

The saving rate is:

$$S_t = \gamma_s(1-vN)w_t, \qquad (II.7)$$

where: $\gamma_s = \frac{(1-\phi)[\sigma\phi-\beta\delta(1-\sigma-\varepsilon)]}{\sigma\phi}$. Note that, differing from the case with operative gifts or bequests, here the saving rate increases with ε.

The growth rate of per capita income is defined by (2.4.17), and fertility, the investment rate per child, and the saving rate are defined by (II.5), (II.6), and (II.7) respectively.

Appendix III

Appendix to Chapter 3

III.1 The Neutrality of Funded Social Security with Positive Saving and Operative Gifts

In this case, the equilibrium is characterized by (3.3.3)-(3.3.5), (3.4.5), (3.4.8), (3.5.1)-(3.5.5), and the symmetric condition, with $b = 0$.

The voluntary saving rate is offset by τ:

$$s = \frac{\sigma\theta(1-\omega)}{\varepsilon(1-\theta)} - \tau. \qquad (III.1.1)$$

Thus, $s + \tau$ is constant.

The gift ratio is given by:

$$g = \frac{\sigma\theta\varepsilon(1-\theta)(1-s-\tau) - \beta\delta\varepsilon\theta(1-\theta)(1-\sigma-\varepsilon) - \sigma^2\omega\theta^2}{(1-\theta)\{\sigma^2\omega\theta + \sigma\theta\varepsilon + \beta\varepsilon[\sigma(1-\theta)(s+\tau) - \theta\delta(1-\sigma-\varepsilon)]\}}, \qquad (III.1.2)$$

which is unaffected by τ since $s + \tau$ is constant.

Fertility is found to be:

$$N = \frac{x}{v[\sigma\varepsilon\theta(1-\theta) + x]}, \qquad (III.1.3)$$

where: $x = \sigma\varepsilon(1-\theta)^2(1-\beta)g(s+\tau) + \theta(1-\sigma-\varepsilon)\{\sigma(1-\delta)[\theta + (1-\theta)g] - \beta\delta\varepsilon(1-\theta)$ $(1-g)\}$. Fertility is unchanged by τ as well.

The investment per child is:

$$q_t = \frac{\beta[\sigma(1-\theta)g(s+\tau) + \theta\delta(1-\sigma-\varepsilon)(1-g)]}{N\sigma\theta}$$

$$(1 - vN)w_t \equiv \gamma_3(1 - vN)w_t, \qquad (III.1.4)$$

where the investment rate per child will not change as τ rises.

Clearly, the funded program is neutral when voluntary saving is positive.

III.2 Equilibria under Funded Social Security with Operative Bequests

The capital market clears when:

$$K_{t+1} = L_t\{s_t[(1 - vN_t)w_t + b_t] + \tau(1 - vN_t)w_t\}. \qquad (III.2.1)$$

III.2.A. Equilibrium with Positive Voluntary Saving

Total saving (forced and voluntary saving) is found to be a fixed portion of labor income:

$$(s_t + \tau)[(1 - vN_t)w_t + s_t b_t] = \frac{\theta\delta(1 - \sigma - \varepsilon)}{\sigma(1 - \theta)}(1 - vN_t)w_t$$

$$\equiv \gamma_s(1 - vN_t)w_t. \qquad (III.2.2)$$

The ratio of bequests to total returns of saving is independent of τ:

$$\gamma_b = \frac{(1 - \theta)[\sigma\omega\gamma_s - \sigma(1 - \omega)(1 - \gamma_s) + \beta\delta(1 - \omega)(1 - \sigma - \varepsilon)]}{\sigma\omega\gamma_s(1 - \theta) + \sigma(1 - \omega)\theta}. \qquad (III.2.3)$$

The ratio of voluntary saving to labor earnings plus inheritance, s_t, is:

$$s = \frac{(1-\theta)(\gamma_s - \tau)}{1-\theta(1-\gamma_b)}. \tag{III.2.4}$$

Fertility is unaffected by τ:

$$N = \frac{x}{v[\theta\sigma(1-\omega)+x]} \tag{III.2.5}$$

where: $x = (1-\delta)\gamma_s(1-\sigma-\varepsilon)\theta(1-\gamma_b) - \sigma(1-\omega)\theta\gamma_b\gamma_s - \theta\beta\delta(1-\omega)(1-\sigma-\varepsilon)$. The investment per child has the form:

$$q_t = \frac{\beta\delta(1-\sigma-\varepsilon)}{\sigma N}(1-vN)w_t = \gamma_q(1-vN)w_t, \tag{III.2.6}$$

where γ_q is independent of τ.

Clearly, the funded program is neutral with operative bequests when voluntary saving is positive.

III.2.B. Equilibrium with Zero Voluntary Saving

The ratio of bequests to total returns of saving is:

$$\gamma_b = \frac{\delta(1-\sigma-\varepsilon)\{\sigma\omega\theta + \beta(1-\omega)[\sigma(1-\theta)(1-\tau)}{\sigma^2(1-\omega)\theta + \sigma\omega\delta\theta(1-\sigma-\varepsilon)} +$$

$$\frac{\theta\delta(1-\sigma-\varepsilon)]\} - \sigma^2(1-\omega)(1-\theta)(1-\tau)}{\sigma^2(1-\omega)\theta + \sigma\omega\delta\theta(1-\sigma-\varepsilon)} \tag{III.2.7}$$

Fertility is given by:

$$N = \frac{x}{v[\sigma^2(1-\omega)(1-\theta)(1-\tau) + \sigma\theta\delta(1-\omega)(1-\sigma-\varepsilon) + x]}, \tag{III.2.8}$$

where: $x = \delta(1-\delta)(1-\sigma-\varepsilon)^2\theta(1-\gamma_b) - \sigma\delta(1-\omega)\theta(1-\sigma-\varepsilon)\gamma_b - \beta\delta(1-\sigma-\varepsilon)[\sigma(1-\omega)(1-\theta)(1-\tau) + \theta\delta(1-\omega)(1-\sigma-\varepsilon)]$.

Investment per child is:

$$q_t = \frac{\beta\delta(1-\sigma-\varepsilon)[\sigma(1-\theta)(1-\tau)+\theta\delta(1-\sigma-\varepsilon)]}{\sigma^2(1-\theta)N}(1-vN)w_t$$

$$\equiv \gamma_q(1-vN)w_t. \qquad (III.2.9)$$

The growth rate of per capita income is defined by (3.4.18), and equations in this part.

III.3 Equilibria under Funded Social Security without Operative Gifts and without Operative Bequests

III.3.A. Equilibrium with Positive Voluntary Saving

The (voluntary) saving rate is found to be:

$$s = \frac{\sigma(1-\omega) - \beta\delta(1-\omega)(1-\sigma-\varepsilon)}{\sigma} - \tau. \qquad (III.3.1)$$

Here, $s+\tau$ is constant.

Fertility is given by:

$$N = \frac{x}{v[\sigma(1-\omega)+x]}, \qquad (III.3.2)$$

where: $x = (1-\delta)(1-\sigma-\varepsilon)(s+\tau) - \beta\delta(1-\omega)(1-\sigma-\varepsilon)$. Investment per child is:

$$q_t = \frac{\beta\delta(1-\sigma-\varepsilon)}{\sigma}(1-vN)w_t \equiv \gamma_q(1-vN)w_t. \qquad (III.3.3)$$

As in the cases with operative gifts (or bequests) and positive voluntary saving, funded social security is neutral.

III.3.B. Equilibrium with Zero Voluntary Saving

Different from other cases with zero voluntary saving, here fertility is independent of τ:

$$N = \frac{x}{v[\sigma^2 + x]}\,, \qquad (III.3.4)$$

where: $x = (1-\delta)(1-\sigma-\epsilon)[\sigma - \beta\delta(1-\sigma-\epsilon)] - \sigma\beta\delta(1-\sigma-\epsilon)$.

Investment per child is found to be:

$$q_t = \frac{\beta\delta(1-\sigma-\epsilon)(1-\tau)}{N[\sigma\omega + \beta\delta(1-\omega)(1-\sigma-\epsilon)]}(1-vN)w_t \equiv \gamma_q(1-vN)w_t. \qquad (III.3.5)$$

Here, the investment rate per child is decreasing with τ.

The growth rate of per capita income is defined by (3.4.18), and (*III*.3.3)-(*III*.3.5).

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