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Essays In International Finance And Macroeconomics

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Essays in International Finance and Macroeconomics

by

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ABSTRACT

This thesis consists of three essays in international finance and macroeconomics to study the link between money and economic activity.

The first essay, entitled "Liquidity, Exchange Rates, and Business Cycles," presents a two-country, two-good, two-currency model to study the role of liquidity effects in exchange rate determination and the international transmission of economic fluctuations. The monetary authority's injections of cash are funneled into the economy through financial markets. The asymmetry of economic agents' access to the newly injected cash induces liquidity effects. The model provides an exchange rate equation which is different from the simple purchasing-power-parity law of exchange rate determination. Both monetary injections and real disturbances can lead to exchange rate fluctuations and comovements of interest rates, prices and output of the two economies. Whether the covariances of variables in the two countries are positive, negative, or zero depends critically upon the substitutability of the two consumption goods in consumers' preferences.

The second essay, "Capital Controls, Foreign Exchange Controls, and Liquidity," continues the study of liquidity effects. By using a similar model to that constructed in the first essay, this essay analyses the liquidity effects generated by restrictions on international financial markets. Taxes on international financial transactions induce redistribution of liquidity in international financial markets which results in comovements of macroeconomic aggregates in the two economies, fluctuations in exchange rates and interest rates, and changes in welfare of economic agents of each country.

The third essay, "Imperfect Information, Money, and Economic Growth," presents an endogenous growth model with financial market imperfections to study the effects of money on economic growth, and to examine the role of informational imperfections in the determination of the equilibrium growth path. It is found that the economy will grow slower if there is imperfect information. Changes in money growth have qualitatively similar effects on the economies with and without private information. However, the economy with private information will be less responsive to monetary shocks. The results contradict the popular view that informational imperfections in credit markets or borrowing constraints tend to amplify the impacts of policy interventions.

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OVERVIEW

This thesis consists of three essays in international finance and macroeconomics to study the link between money and economic activity.

The first essay, entitled "Liquidity, Exchange Rates, and Business Cycles," presents a two-country, two-good, two-currency model to study the role of liquidity effects in an open economy. The monetary authority's injections of cash are funneled into the economy through financial markets. There is an asymmetry of monetary injections. An unanticipated monetary injection will redistribute purchasing power in favor of those economic agents who have access to the newly injected cash. This redistribution of liquidity will affect asset prices and real activity in the economy. This essay investigates how liquidity effects affect the determination of the exchange rate and the international transmission of economic fluctuations. The model provides an exchange rate equation which is different from the simple purchasing-power-parity law of exchange rate determination. Both monetary injections and real disturbances can lead to exchange rate fluctuations and comovements of interest rates, prices and output of the two economies. Whether the covariances of variables in the two countries are positive, negative, or zero depends critically upon the substitutability of the two consumption goods in consumers' preferences.

The second essay, "Capital Controls, Foreign Exchange Controls, and Liquidity," continues the study of liquidity effects. By using a similar model to that constructed in the first essay, this essay analyses the liquidity effects generated by restrictions on international financial markets. Taxes on international financial transactions induce redistribution of liquidity in international financial markets which results in comovements of macroeconomic aggregates in the two economies, fluctuations in exchange rates and interest rates, and changes in welfare of economic agents of each country. As the cash-in-advance constraints generate distortions in the world economy, there will be a welfare improvement if these restrictions relax the liquidity constraints.

The third essay, "Imperfect Information, Money, and Economic Growth," presents an endogenous growth model with financial market imperfections to study the effects of money on economic growth, and to examine the role of informational imperfections in the determination of the equilibrium growth path. This essay introduces money to the model as one of the portfolio choices of economic agents. Private information is modeled explicitly in an endogenous growth environment. It is found that the economy will grow slower if there is imperfect information. Changes in money growth have qualitatively similar effects on the economy in the full information case and in the imperfect information case. However, the economy with imperfect information will be less responsive to monetary shocks. The results contradict the popular view that informational imperfections in credit markets or borrowing constraints tend to amplify the impacts of policy interventions.

Chapter 1

Liquidity, Exchange Rates, and Business Cycles

1.1 Introduction

A number of recent studies have provided strong empirical support for the view that positive shocks to money lead to reductions in nominal interest rates and increases in output. For instance, in the U.S. data, Bernanke and Blinder (1990) find evidence consistent with the view that monetary policy works, using innovations to the interest rate on Federal Funds as a measure of changes in monetary policy. Christiano and Eichenbaum (1991) find that the Federal Funds rate is negatively correlated with different measures of monetary growth and with current and future growth rates of output. These observed negative correlations can be interpreted as the results of two opposing effects associated with unanticipated monetary shocks. The first is a liquidity effect, where nominal interest rates fall to induce economic agents to absorb the extra money, which in turn stimulates real activity. The second is an expected inflation effect. If monetary shocks are positively serially correlated, a surprise increase in money supply drives the nominal interest rate up and leads to reductions in employment and output. The evidence suggests that the liquidity effect dominates, at least in the short run.

The liquidity effect is absent in monetary versions of real business cycle models.

Money has been introduced into the standard real business cycle model by imposing cash-in-advance constraints on transactions [as in Greenwood and Huffman (1987), and Cooley and Hansen (1989)]. These models have only the expected inflation effect, where interest rates rise and output falls in response to a positive monetary shock.

Recently, Lucas (1990) has suggested a methodology to capture liquidity effects in modified versions of cash-in-advance models in which the convenience of the representative household fiction can be retained. In contrast to the transaction-based models of Grossman and Weiss (1983) and Rotemberg (1984), Lucas's methodology can entirely eliminate the problematic wealth redistribution effects induced by monetary shocks, and allows us to isolate the liquidity effects.

Fuerst (1992) extends Lucas' work by introducing production in the exchange economy of Lucas (1990), to study the link between money and real activity in a closed economy. Monetary injections are assumed to occur through financial intermediaries. Intermediaries channel the newly injected cash to borrowers, and intermediate between borrowers and savers. The crucial assumption is that there is an asymmetry of monetary injections. Savers are unable to alter their deposit decisions after each monetary injection. Only borrowers have access to the newly injected cash through intermediaries.[1] As a consequence, an unanticipated monetary injection will redistribute purchasing power in favor of borrowers. This redistribution of liquidity will affect real activity by altering the composition of current output, shifting it toward the goods and services borrowers consume. These compositional effects distinguish Fuerst's model from other cash-in-advance models. With the liquidity effects, a monetary shock will also generate fluctuations in asset prices for non-Fisherian reasons.

There are some further studies of liquidity effects as a channel of monetary trans-

[1] These assumptions are to capture the institutional fact that corporations have more access to the loans market, while consumers have difficulties in getting credit, and face an interest rate which is much higher than that faced by corporations.

mission. Christiano (1991) and Christiano and Eichenbaum (1991) modify Fuerst's model to generate a dominant, persistent liquidity effect so as to improve its ability to confront the data.

This paper is an attempt to study the role of liquidity effects in an open economy, to investigate how they affect the determination of the exchange rate and the international transmission of economic fluctuations, following Fuerst's approach to modeling the asymmetry of monetary injections. The literature on international economics contains many cash-in-advance approaches to exchange rate determination. The two-country general equilibrium models developed in Stockman (1980), Lucas (1982), Helpman and Razin (1985), and Svensson (1985) provide theoretical studies of the determination of prices, interest rates and currency exchange rates with liquidity constraints. However, they did not incorporate the liquidity effects from an asymmetry of monetary injections into their models. In addition, their attention was concentrated mainly on exchange rate determination, and the transmission mechanism for fluctuations in output was ignored.

In the literature on the generation and transmission of international business fluctuations, most work studies the transmission of real business cycles [e.g. Dellas (1986), Cantor and Mark (1987) and (1988)]. The performance of these open economy real business cycle models is not satisfactory. Backus, Kehoe and Kydland (1992) find that the most striking discrepancy between theory and data concerns the cross-country correlations of consumption and output. In the data, output fluctuations are more highly positively correlated across countries than consumption fluctuations, while the theory predicts the opposite. Some explanations for the low cross-country consumption correlations have been provided by Stockman and Tesar (1990), Tesar (1990), and Devereux, Gregory and Smith (1992), however, these open economy real business cycle models consider only the non-monetary features of the world economy.[2]

[2] The low cross-country consumption correlations can be explained by incorporating nontraded

The role of monetary disturbances, or more precisely, the role of liquidity effects has not been emphasized in open economy studies.

This paper presents a two-country, two-good, two-currency model to take both exchange rate determination and the transmission of economic fluctuations into consideration. The model is a two-country version of the model in Fuerst (1992). As in Fuerst's model, cash-in-advance constraints are imposed on all transactions. The two countries are linked together by trade in goods and trade in assets. The restrictions of using the sellers' currency for transactions in goods markets and using the buyers' currency for transactions in bonds markets assign intermediaries a role in international trade financing.[3]

An alternative explanation (with monetary features) for the low cross-country consumption correlations is presented in this paper. The particular financial structure imposed in this model seems to be capable of generating lower consumption correlations and higher output correlations across countries than the usual open economy real business cycle models do. This model focuses attention on the role of international financial markets in allocating liquidity. This particular financial structure prohibits economic agents from insuring themselves against the liquidity risk of dealing with financial intermediaries, and this can lead to a reduction in consumption correlations across countries. Meanwhile, the cash-in-advance constraints restrain labor from being used too intensively when its productivity is high, and this can increase the correlations of output across countries.

This model provides an exchange rate equation with new elements which is different from the usual purchasing-power-parity law of exchange rate determination. Liquidity effects are incorporated into the determination of the exchange rate. Both

goods [as in Tesar (1990)], or by introducing taste shocks [as in Stockman and Tesar (1990)], or by assuming preferences with non-separability between consumption and labor supply [as in Devereux, Gregory and Smith (1992)].

[3] Although Rotemberg (1985) presents a two-country version of the closed economy model in Rotemberg (1984) to examine the connection between money and the terms of trade, it is unable to generate the compositional effects as Fuerst (1992) and this paper do.

monetary injections and real disturbances can lead to exchange rate fluctuations and comovements of the interest rates on bonds denominated in either currency, prices and output (employment level) of the two economies. The international transmission mechanism of business fluctuations in this model is different from those of the traditional Mundell-Fleming framework or some recent work on open economy disequilibrium models with sticky goods prices [e.g. Svensson and van Wijnbergen (1989)]. In this model, all prices are flexible. In addition to the cash-in-advance constraints, the only rigidity is in the deposit decision of households. By affecting the allocation of liquidity in international financial markets, the effects of the home country's economic disturbances are transmitted to the foreign economy. How the foreign economy responds to these liquidity shocks depends critically upon the substitutability of the two consumption goods in consumers' preferences. This result is similar to that in Svensson and van Wijnbergen (1989), though we have different channels of monetary transmission.

The remainder of the paper is organized as follows. The model is presented in Section 2. A stationary rational expectations equilibrium is described in Section 3. The effects of monetary shocks and productivity disturbances on the world equilibrium are discussed in Section 4. Section 5 is the conclusion.

1.2 The Model

This is a two-country, two-good, two-currency model, formulated in discrete time with an infinite horizon. The home country and the foreign country have identical constant numbers of infinitely-lived households. All variables will then be expressed in per (own country) household terms. Foreign variables and parameters are indexed with an asterisk (*).

The infinitely-lived households of each country are identical. The world economy can be considered as an economy with two heterogeneous representative households.

The objective of the home country representative household is to maximize its expected lifetime utility,

$$E_0 \left[\sum_{t=0}^{\infty} \beta^t U(c_{1t}, c_{2t}, l_t) \right], \qquad 0 < \beta < 1, \tag{1}$$

$$U(c_{1t}, c_{2t}, l_t) = \ln \left[{c_{1t}}^{\alpha} + {c_{2t}}^{\alpha} \right]^{\frac{1}{\alpha}} + [1 - l_t], \qquad \alpha \leq 1, \tag{2}$$

where E_0 is the mathematical expectation conditional upon information available in period 0. The instantaneous utility function $U(c_{1t}, c_{2t}, l_t)$ is discounted by the subjective discount factor β. The consumption of good j in period t is c_{jt}, $j = 1, 2$.[4] Define $\sigma \equiv \frac{1}{1-\alpha}$, to be the constant elasticity of (intratemporal) substitution in consumption between the two goods. In every period, the household is endowed with one unit of time. The household supplies l_t units of work effort to the labor market in period t.[5] Labor is internationally immobile. For simplicity, the representative households of the two countries are assumed to have identical preferences.

There is a complete specialization in production, and trade allows agents in each country to consume both goods. Firms in the home country produce only good 1, while those of the foreign country produce good 2. The production functions are identical within a country, but different across countries. Each production process requires only labor input. Each of the home firms faces the production function,

$$\mathcal{F}(H_t) = \theta_t \, H_t \,, \tag{3}$$

[4] Taking the logarithm of the CES utility function of the two consumption goods results in a unit elasticity of intertemporal substitution in consumption, which simplifies the calculation.

[5] As in Fuerst (1992), the instantaneous utility function is linear in leisure. This assumption is required for simplifying the calculation and obtaining a closed-form solution of the optimization problem. In addition, this assumption implies a perfectly elastic labor supply curve. As a result, both of the intertemporal and intratemporal responses of labor supply to the economic disturbances are eliminated. Rather than being a drawback, this specification of the utility function allows the model to stress the fluctuations in employment level brought about by shifts in labor demand.

where θ_t is the country-wide productivity shock, and H_t denotes units of labor employed by the home firm. Similarly, the foreign firm's production function of good 2 is given by $\mathcal{F}^*(H_t^*) = \theta_t^* H_t^*$. With the constant returns technology, perfect competition implies that firms earn zero profits, so that trading in equity claims can be ignored.

1.2.1 The Timing of Information and Transactions

The world economy is a monetary economy with cash-in-advance constraints on all transactions, making use of the methodology suggested by Lucas (1990). This approach allows for the introduction of an asymmetry of monetary injections while retaining the convenience of the representative household fiction.[6] In this model, each household is composed of five members: a shopper, a worker, a firm, an importer, and a financial intermediary. Cash injections are asymmetric within a household, but symmetric across households of the same country. Injections are asymmetric across countries. Let the beginning-of-period per (own country) household home money stock and foreign money stock be denoted by M_t and M_t^*, respectively. From now on, I will use the term "per capita" instead of "per (own country) household" for convenience.

The timing of information and transactions is shown in Figure 1.1. The representative home country household enters a period with cash balances carried over from last period, which include M_{ht} units of home currency and M_{ft} units of foreign currency. It deposits N_t units of home currency in the home intermediary. Transaction costs are prohibitively high so that once the deposit is made, it cannot be withdrawn until the end of the current period. This assumption is crucial to generate the asymmetry of monetary injections within the household and will be illustrated in more

[6]The international problem is complicated by the fact that the two representative households cannot be treated as a single representative household. In order to eliminate the redistribution of wealth across the two heterogeneous households induced by liquidity shocks and to keep the model tractable, the assumptions of no physical capital and no intertemporal trade across countries are required.

Figure 1.1 :

Timing of Information and Transactions of the Home Representative Household

Period t

- Beginning-of-period cash holdings, M_{ht} and M_{ft}.
- Deposits N_t units of home currency.
- The household separates.
  - * Shopper $M_{ht} - N_t$
  - * Importer M_{ft}
  - * Financial Intermediary N_t
  - * Firm
  - * Worker
- Current state of the world, s_t, is revealed to everybody. Monetary injections occur. Financial intermediary's cash holdings, $N_t + X_t + Y_t$.
- Markets are opened.
  - * Loans markets
  - * Foreign exchange market
  - * Labor markets
  - * Goods markets
- Markets are closed.
- Foreign exchange market is reopened.
- Loan repayments are made.
- The household is reunited, all remaining cash is pooled, and goods purchased by shopper are consumed.
- Lump-sum rebates of interest payments from the home government, Γ_t.
- End-of-period cash holdings, M_{ht+1} and M_{ft+1}.

Period $t+1$

⋮

detail below. In addition, for simplicity, the home country household is not allowed to hold foreign currency-denominated deposits. This restriction will not make any qualitative difference to the results.

After the home country deposit is made, the household separates. The shopper takes the remaining $M_{ht} - N_t$ units of home currency to the local goods markets. The worker goes to the home labor market. The importer takes the foreign cash balance M_{ft}. The firm, the intermediary and the importer go to the financial market where they will meet their counterparts of the foreign country. In the financial market, both home currency-denominated bonds and foreign currency-denominated bonds can be traded. Assume that the number of households of each country is large enough that every individual acts as a price-taker.

Once the household is separated, the state of the world is revealed to everybody. The state of the world, s_t, is independently and identically distributed across time, and its distribution function $G(s_t)$ is public knowledge. The country-wide productivity shocks, and the monetary injections of each currency from its own country's monetary authority are determined by the state of the world.

Each country has a monetary authority which injects newly issued currency of its own country into the financial market through two channels. The first is by way of lump-sum transfers. The home monetary authority transfers X_t units of home currency to each of the home intermediaries. These monetary injections increase the money stock of the economy permanently. The second channel is open market purchases. Each home intermediary is free to choose the quantity of home currency borrowed from the home monetary authority, Y_t, taking the one-period nominal interest rate, i_{0t}, as given. Given the fixed volume of open market purchases, $\bar{Y}_t$, determined by the home monetary authority, the interest rate will adjust to clear this market. By assuming these are one-period loans and the monetary authority rebates all of its interest income to its households at the end of the current period, after all transactions are completed, this type of monetary injection is temporary and leaves the end-of-period (beginning-of-period) economy-wide money stock unchanged.

In the goods markets of the home country, the home shopper purchases good 1 and good 2 for consumption subject to the cash-in-advance constraint

$$M_{ht} - N_t \geq P_{1t} c_{1t} + P_{2t} c_{2t}, \tag{4}$$

where P_{jt}, $j = 1, 2$, is the price of good j, in units of home currency.

In the financial market, each intermediary is assumed to be able to make loans denominated in its own country's currency only, while it is allowed to lend to any firms and importers of both countries.[7] There are three sources of home currency available tc the home intermediary: the deposit by the home household, N_t, the lump-sum transfer, X_t, and the borrowings from the home monetary authority, Y_t. An implicit assumption is that financial intermediaries are not allowed to issue inside money as a medium of exchange. The representative home intermediary has the following cash-in-advance constraint

$$N_t + X_t + Y_t \geq B_{ht} + Z_{ht} + B^*_{ht} + Z^*_{ht}. \tag{5}$$

Taking the market interest rate on the home currency-denominated bonds, i_t, and the demand for funds of the foreign borrowers as given, the home intermediary allocates the loans B_{ht}, Z_{ht}, B^*_{ht} and Z^*_{ht} to the home firm, the home importer, the foreign firm and the foreign importer, respectively.

The home firm borrows B_{ht} units of home currency from the home intermediary and B_{ft} units of foreign currency from the foreign intermediary. The home importer borrows Z_{ht} units of home currency from the home intermediary and Z_{ft} units of foreign currency from the foreign intermediary. However, home firms have to pay the

[7] That is, it is assumed that each bond is denominated in the currency of the lender's country. This assumption is for simplicity. Even if we let intermediaries borrow from the intermediaries of another country, then lend to their borrowers, the equilibrium will not be affected. But many accounting procedures will be involved.

home workers in home currency, while home importers have to pay the foreign firms in foreign currency. Their foreign counterparts also face a similar problem. At this time, the foreign exchange market is opened so that the firms and the importers of both countries can trade for the desired currency. Assume that a flexible exchange rate regime is adopted by the two countries. Let $\hat{e}_t$ be the exchange rate at the beginning of the period, after the state of the world s_t is known. This exchange rate will adjust to clear the foreign exchange market.

In addition, firms and importers of the same country may trade bonds with each other. These bonds may be either home currency-denominated or foreign currency-denominated. They are not allowed to trade bonds with the firms or importers of another country. Only intermediaries are assumed to have the expertise to deal with the borrowers of other countries. However, in equilibrium, the net supply of the bonds traded among firms and importers of the same country will be zero in this representative agent economy. Thus, it does not matter whether these kinds of asset trading among firms and importers of the same country are allowed or not. But for deriving the asset pricing rules, two kinds of asset trading will be considered. The first is the trading of home currency-denominated bonds among the home firms. The second is the trading of foreign currency-denominated bonds among the home importers. The representative home firm purchases K_{ht} units of the one-period, one-unit home currency-denominated bonds, issued by other home firms, at the price of $q_t = \frac{1}{1+i_t}$ units of home currency. Similarly, the representative home importer purchases K_{ft} units of the one-period, one-unit foreign currency-denominated bonds, issued by other home importers, at the price of $q_t^* = \frac{1}{1+i_t^*}$ units of foreign currency.

The home firm then takes the home currency obtained after asset trading, $B_{ht} + B_{ft}\hat{e}_t - K_{ht}\, q_t$, to the home labor market, hires H_t units of labor at the market wage rate, W_t, to produce good 1 according to the production function given by equation (3). The cash-in-advance constraint of the home firm is given by

$$B_{ht} + B_{ft}\hat{e}_t - K_{ht}q_t \geq W_t H_t . \tag{6}$$

The output will be shipped to the home market of good 1 and sold to home shoppers and foreign importers at the price of P_{1t} units of home currency. The home worker receives his wage, $W_t l_t$, and goes home to enjoy his leisure, $1 - l_t$.

The home importer goes to the foreign goods markets with the cash balances obtained after the asset trading, $M_{ft} + Z_{ft} + \frac{Z_{ht}}{\hat{e}_t} - K_{ft}q_t^*$, purchases foreign goods at the price of P_{2t}^* units of foreign currency, and sells them to home shoppers at the price of P_{2t} units of home currency in the home market for good 2. Thus, the cash-in-advance constraint for the home importer is

$$M_{ft} + Z_{ft} + \frac{Z_{ht}}{\hat{e}_t} - K_{ft}q_t^* \geq P_{2t}^* c_{2t} . \tag{7}$$

At the end of the period, loan repayments are made.[8] However, the home borrowers do not have foreign currency to repay the loans from the foreign intermediaries, while the foreign borrowers do not have home currency to repay the loans from the home intermediaries. It is assumed that the transaction costs in the foreign exchange market are significantly less than in the loans market so that currency trading can be more frequent than other asset trading.[9] Consequently, only the foreign exchange market can be reopened at the end of the period. Borrowers can trade for the currency they need for repaying their foreign debts.[10] The end-of-period exchange rate, e_t, responds to clear this foreign exchange market. After all transactions are completed, the household is reunited, all remaining cash is pooled, and goods purchased by the shopper are consumed. The home monetary authority rebates its interest income to

[8]Whether the loan repayments are made at the end of the current period or in the next period before the borrowers obtain their new loans makes no difference to our analysis.

[9]This assumption is also used by Helpman and Razin (1982) and Svensson (1985). Svensson (1985) calls this "continuous currency trade".

[10]Since the cash-in-advance constraints are imposed, borrowers are always able to repay their debts. The market-clearing condition of the foreign exchange market implies that there is no intertemporal trade across countries in every period, which helps to focus our discussion on the role of liquidity effects.

the home representative household as a lump-sum transfer, Γ_t. The household then holds the remaining cash balances M_{ht+1} and M_{ft+1} for next period, where

$$\begin{aligned} M_{ht+1} &= M_{ht} + X_t - Y_t\, i_{0t} + P_{1t}\,\mathcal{F}(H_t) - P_{1t}\,c_{1t} - W_t\,H_t + W_t\,l_t + B_{ft}\,\hat{e}_t - Z_{ht} \\ &\quad + (B^*_{ht} + Z^*_{ht})\,i_t - (B_{ft} + Z_{ft})\,(1 + i^*_t)\,e_t + K_{ht}\,i_t\,q_t + \Gamma_t\,, \qquad (8) \\ M_{ft+1} &= M_{ft} + Z_{ft} + \frac{Z_{ht}}{\hat{e}_t} + K_{ft}\,i^*_t\,q^*_t - P^*_{2t}\,c_{2t}\,. \qquad (9) \end{aligned}$$

Due to the assumptions of a flexible exchange rate regime and no debts carried over periods, the current account of each country is always balanced.[11] Let $\Lambda_t = (\theta_t, X_t, \bar{Y}_t, \Gamma_t)$ and $\mathcal{P}_t = (P_{1t}, P_{2t}, i_t, i_{0t}, i^*_t, W_t, \hat{e}_t, e_t)$ denote the vector of the economic disturbances and the vector of the market prices faced by the home representative household, respectively.

The activities of the foreign representative household are analogous to those of the home representative household, and the foreign variables and parameters are defined in a similar way.

1.2.2 The Optimization Problem of the Home Representative Household

The home representative household faces the following dynamic optimization problem. Also, it is noted that there is an analogous optimization problem for the foreign representative household, which is presented in Appendix 1.

Given the distribution function G and the collection of sequences

$\{\ \Lambda_t, \mathcal{P}_t, M_t, M^*_t, M^*_{ht}, M^*_{ft}, B^*_{ht}, Z^*_{ht}\ \}_{t=0}^{\infty}$,

the home representative household chooses the collection of sequences

$\{N_t, c_{1t}, c_{2t}, Y_t, B_{ht}, Z_{ht}, B_{ft}, Z_{ft}, l_t, H_t, K_{ht}, K_{ft}\}_{t=0}^{\infty}$

to maximize the expected lifetime utility with preferences given by (1) and (2), subject

[11] In this model, the existence of the cash-in-advance constraints implies that the current account consists of trade in goods and trade in financial services. Each household not only purchases the goods of another country but also employs the financial services provided by the intermediaries of that country.

to the technology constraint (3), the cash-in-advance constraints (4)-(7), and the two evolution equations of the household's beginning-of-period cash balances (8) and (9).

In restricting the discussion to stationary rational expectations equilibrium, in which the prices and decision rules are fixed functions of the state of the world, s_t, and the ratio of the beginning-of-period per capita money stocks of the two countries, $\delta_t \equiv \frac{M_t^*}{M_t}$, we have to rescale the nominal variables. Every nominal variable is divided by the beginning-of-period per capita money stock of its own country. That is, normalize the beginning-of-period per capita money stock of each country at unity. Let lowercase letters represent the rescaled values. For instance, the vector of the economic disturbances and the vector of the market prices faced by the home representative household are now denoted by $\lambda_t = (\theta_t, x_t, \tilde{y}_t, \gamma_t)$ and $p_t = (p_{1t}, p_{2t}, i_t, i_{0t}, i_t^*, w_t, \hat{e}_t, e_t)$, respectively. Dynamic programming can be applied to solve the household's optimization problem. Define the value function corresponding to the household's problem $J(m, \delta, \tilde{s})$ by

$$J(m,\delta,\tilde{s}) = \max_{0 \le n \le m_h} \int \max_{c_1,c_2,y,b_h,z_h,b_f,z_f,l,H,k_h,k_f} \{U(c_1,c_2,l) + \beta J(m',\delta',s)\} dG(s)\,, \quad (1')$$

subject to

$$m_h - n \;\ge\; p_1 c_1 + p_2 c_2\,, \quad (4')$$

$$n + x + y \;\ge\; b_h + z_h + b_h^* + z_h^*\,, \quad (5')$$

$$b_h + b_f \hat{e}\,\delta - k_h q \;\ge\; wH\,, \quad (6')$$

$$m_f + z_f + \frac{z_h}{\hat{e}\,\delta} - k_f q^* \;\ge\; p_2^* c_2\,, \quad (7')$$

$$m_h' \;=\; \frac{1}{1+x}\,[\, m_h + x - y\, i_0 + p_1 \mathcal{F}(H) - p_1 c_1 - w\,H + w\,l + b_f\, \hat{e}\,\delta - z_h$$
$$+ (b_h^* + z_h^*)\, i - (b_f + z_f)(1+i^*) e\,\delta + k_h\, q\, i + \gamma \,]\,, \quad (8')$$

$$m_f' \;=\; \frac{1}{1+x^*}\,[\, m_f + z_f + \frac{z_h}{\hat{e}\,\delta} + k_f\, q^*\, i^* - p_2^* c_2 \,]\,, \quad (9')$$

where the time subscripts for current period have been omitted while those for the next period have been replaced by primes. The realization of last period's state of the world is $\tilde{s}$. The collection of the beginning-of-period money holdings of the home and foreign representative households is represented by $m = (m_h, m_f, m_h^*, m_f^*)$. The growth rates of the beginning-of-period per capita money stocks of the home and the foreign country are denoted by x and x^*, respectively. It is noted that the decision for n is made before the realization of the state of the world s. Thus, n is independent of s.

Let μ_1, μ_2, μ_3 and μ_4 be the multipliers associated with the cash-in-advance constraints (4')-(7') respectively. Necessary conditions for the home representative household are:

$$E[\mu_1(s)] = E[\mu_2(s)], \tag{10}$$

$$\frac{U_1(s)}{p_1(s)} = \frac{U_2(s)}{p_2(s)} = \frac{1}{1-n}, \tag{11}$$

$$\frac{U_1(s)}{p_1(s)} = \mu_1(s) + \frac{1}{1+x(s)}\,\beta E\left[\frac{U_1(s')}{p_1(s')}\right], \tag{12}$$

$$-U_l(s) = \frac{w(s)}{1+x(s)}\,\beta E\left[\frac{U_1(s')}{p_1(s')}\right], \tag{13}$$

$$w(s)[1+i(s)] = p_1(s)\mathcal{F}'(H(s)), \tag{14}$$

$$\mu_2(s) = \mu_3(s) = \frac{i(s)}{1+x(s)}\,\beta E\left[\frac{U_1(s')}{p_1(s')}\right], \tag{15}$$

$$\mu_4(s) = \frac{i^*(s)e(s)\delta}{1+x(s)}\,\beta E\left[\frac{U_1(s')}{p_1(s')}\right], \tag{16}$$

$$\frac{1}{1+i(s)} = \frac{\hat{e}(s)}{e(s)[1+i^*(s)]} \tag{17}$$

$$m_h(s) - n \geq p_1(s)c_1(s) + p_2(s)c_2(s) \qquad \text{with equality if } \mu_1(s) > 0\,, \tag{4''}$$

$$n + x(s) + y(s) \geq b_h(s) + z_h(s) + b_h^*(s) + z_h^*(s) \qquad \text{with equality if } \mu_2(s) > 0\,, \tag{5''}$$

$$b_h(s) + b_f(s)\,\hat{e}(s)\,\delta \geq w(s)H(s) \qquad \text{with equality if } \mu_3(s) > 0\,, \tag{6''}$$

$$m_f(s) + z_f(s) + \frac{z_h(s)}{\hat{e}(s)\delta} \geq p_2^*(s)c_2(s) \qquad \text{with equality if } \mu_4(s) > 0\,. \tag{7''}$$

where the ratio $1/[1 + x(s)]$ is for rescaling the values relative to next period's beginning-of-period per capita money stock rather than the current one. Arbitrage implies $i(s) = i_0(s)$. Also, as mentioned above, $K_h(s) = K_f(s) = 0, \;\; \forall\; s,$ in equilibrium. These have been substituted into the necessary conditions.

Equation (10) is the first-order condition for deposits. The expected marginal utility from one unit of home currency allocated to the shopper, $E(\mu_1)$, must be equal to the expected marginal utility from one unit of home currency-denominated deposit put into the financial market, $E(\mu_2)$.

Equation (11) is the usual first-order condition for intratemporal allocation of consumption. Equations (12)-(14) are the intertemporal first-order conditions for consumption, labor supply and labor demand, respectively. The effective wage rate faced by the firm is $w(s)[1+i(s)]$. Equation (12) shows that the existence of a binding liquidity constraint drives a wedge between the marginal utility of consumption and the discounted expected marginal utility of consumption of next period.

Equation (15) is the first-order condition for borrowing from the home intermediary. The marginal utility of one unit of home currency in the financial market, μ_2, should equal the marginal utility from one unit of home currency allocated to the home firm, μ_3, which should also equal the discounted expected marginal utility derived from the interest payment, $i(s)$. Equation (16) is the first-order condition for borrowing from the foreign intermediary. The marginal utility from the use of one unit of foreign currency, μ_4, and the discounted expected marginal utility loss from the interest payment paid at the end of current period must be equalized. Equation (17) is the first-order condition for the home borrowers to borrow from both intermediaries. That is, they are indifferent between issuing bonds denominated in either currency of the two countries. It must be the case that, the price of a one-period, one-unit home currency-denominated bond, $\frac{1}{1+i(s)}$, is equal to the price of the one-period, foreign currency-denominated bond with face value equivalent to one unit of home currency, measured in terms of home currency, $\frac{\dot{e}(s)}{e(s)[1+i^*(s)]}$. This implies that the cost of borrowing home currency directly from the home intermediary is equal to the effective cost of borrowing foreign currency from the foreign intermediary, and then obtaining home currency through foreign exchange. Arbitrage ensures that this condition always holds. So it is also an arbitrage condition of the financial market.

Equations (4")-(7") are the standard complementary slackness conditions. If the cash-in-advance constraints are non-binding, their associated multipliers (shadow prices of the marginal units of currency in their corresponding markets.) will be zero.

Asset Pricing

From the necessary conditions (12), (15) and (16), the asset pricing rules can be derived.

$$1 + i(s) = \frac{\mu_2(s) - \mu_1(s) + \left[\frac{U_1(s)}{p_1(s)}\right]}{\frac{1}{1+x(s)}\,\beta E\left[\frac{U_1(s')}{p_1(s')}\right]}, \qquad 1 + i^*(s) = \frac{\frac{\mu_4(s)}{e(s)\delta} - \mu_1(s) + \left[\frac{U_1(s)}{p_1(s)}\right]}{\frac{1}{1+x(s)}\,\beta E\left[\frac{U_1(s')}{p_1(s')}\right]}.$$

If the terms $[\mu_2(s) - \mu_1(s)]$ and $[\frac{\mu_4(s)}{e(s)\delta} - \mu_1(s)]$ did not exist in these pricing equations, they will become the standard asset pricing equations.[12] However, the cash-in-advance constraints on all transactions make the prices of financial assets (interest rates) quite different from those predicted on the basis of Fisherian fundamentals. The term $[\mu_2(s) - \mu_1(s)]$ is the difference of the values (in terms of utility) of one unit of home currency in the financial market and in the home good market.[13] If $\mu_2(s)$ is larger than $\mu_1(s)$, that means that the home goods markets is relatively more liquid than the home currency-denominated bonds market. Then, the price of the home currency-denominated bonds will be relatively lower than that implied by Fisherian fundamentals. There are two effects determining $i(s)$, which are the liquidity effect, represented by $[\mu_2(s) - \mu_1(s)]$, and the expected price effect, represented by $[1 + x(s)]p_1(s')$.[14] These two effects act in opposite directions which generate the ambiguity of the movements of the interest rate.

Similarly, $\mu_4(s)$ reflects the level of liquidity in the foreign currency-denominated bond market. For comparing with $\mu_1(s)$, it has to be adjusted by the exchange rate $e(s)$ and the ratio δ. It is obvious that the prices of the home currency-denominated bonds and the foreign currency-denominated bonds are in general different. The only case where they are equalized is $\mu_2(s) = \frac{\mu_4(s)}{e(s)\delta}$. That is, the two bonds markets are equally liquid. However, there is no mechanism to equate them, so this condition does not necessarily hold. Arbitrage ensures the effective costs of financing in these two markets to be identical, that is, equation (17) holds, rather than equalizes the nominal interest rates, $i(s)$ and $i^*(s)$. The price of a home currency-denominated

[12]In the standard model, [e.g. Lucas (1978)], the representative agent is assumed to maximize time-additive expected utility $E_0\{\sum_{t=0}^{\infty} \beta^t U(c_t)\}$, where E_0 denotes the expectation conditional on information at time 0, β is the subjective discount factor, U is the instantaneous utility function, and c_t is consumption. This model implies that the price of an asset is the representative agent's rational expectation of future returns multiplied by the marginal rate of substitution between present and future consumption — the Fisherian fundamentals of interest rate determination.

[13]As the household has to choose n before the realization of the current state of the world is revealed, an optimal decision requires that the expected values, $E[\mu_1(s)]$ and $E[\mu_2(s)]$, must be equalized. Thus the term $[\mu_2(s) - \mu_1(s)] \neq 0$ represents a forecasting error.

[14]The increase in the money supply results in an expected one-time jump in next period's prices. In addition, there will be an expected inflation effect if money growth rates are positively serially correlated.

bond will be higher if its market is relatively more liquid than that of a foreign currency-denominated bond. In addition, the beginning-of-period exchange rate will be higher than the end-of-period exchange rate, $\hat{e}(s) > e(s)$, and the home currency will appreciate. These exchange rates will be equal only when $i(s)$ and $i^*(s)$ are the same. Thus, in this model, monetary injections cause not only fluctuations and differentials of the interest rates in the two bonds markets for non-Fisherian reasons but also exchange rate fluctuations.[15]

1.3 Stationary Rational Expectations Equilibrium

In the stationary rational expectations equilibrium of the world economy, the prices and decision rules are fixed functions of the ratio of the beginning-of-period per capita money stocks of the two countries, δ, and the state of the world, s. Note that n and n^* are independent of s. They depend on δ only. As mentioned above, $k_{jt} = k^*_{jt} = 0$, $j = h, f$, $\forall\, t$, in equilibrium. From now on, they will be omitted in the following discussion. An equilibrium is defined as follows:

Definition: An equilibrium is a set of initial conditions $\{m_{h0}, m_{f0}, m^*_{h0}, m^*_{f0}\}$, and a collection of sequences $\{\lambda_t, \lambda^*_t, p_t, p^*_t, m_t, n_t, y_t, b_{ht}, b_{ft}, z_{ht}, z_{ft}, c_{1t}, c_{2t}, l_t, H_t, n^*_t, y^*_t, b^*_{ht}, b^*_{ft}, z^*_{ht}, z^*_{ft}, c^*_{1t}, c^*_{2t}, l^*_t, H^*_t\}_{t=0}^{\infty}$, such that

(a) Given $\{\lambda_t, p_t, m^*_{ht}, m^*_{ft}, b^*_{ht}, z^*_{ht}\}_{t=0}^{\infty}$, $\{n_t, c_{1t}, c_{2t}, y_t, b_{ht}, b_{ft}, z_{ht}, z_{ft}, l_t, H_t\}_{t=0}^{\infty}$ solve the home representative household's optimization problem (1') subject to (4')-(9').

Given $\{\lambda^*_t, p^*_t, m_{ht}, m_{ft}, b_{ft}, z_{ft}\}_{t=0}^{\infty}$, $\{n^*_t, c^*_{1t}, c^*_{2t}, y^*_t, b^*_{ht}, b^*_{ft}, z^*_{ht}, z^*_{ft}, l^*_t, H^*_t\}_{t=0}^{\infty}$ solve the analogous optimization problem of the foreign representative household subject to the constraints.

[15] It is noted that the beginning-of-period exchange rate in period $t+1$, $\hat{e}_{t+1}$, has incorporated the information about s_{t+1} which is not available in period t. Thus $\hat{e}_{t+1}$ is in general different from the end-of-period exchange rate in period t, e_t. The exchange rate may fluctuate both within a period and across periods.

(b) The budget constraint of the each country's monetary authority must be satisfied. The monetary injections are financed by printing money. The lump sum rebates of interest income must satisfy

$$\gamma_t = y_t i_t \,, \tag{18}$$

$$\gamma_t^* = y_t^* i_t^* \,. \tag{19}$$

(c) All markets are cleared.

Goods markets

$$c_{1t} + c_{1t}^* = \theta_t H_t \,, \tag{20}$$

$$c_{2t} + c_{2t}^* = \theta_t^* H_t^* \,. \tag{21}$$

Labor markets

$$l_t = H_t \,, \tag{22}$$

$$l_t^* = H_t^* \,. \tag{23}$$

Financial markets

(i) open market purchases

$$y_t = \bar{y}_t \,, \tag{24}$$

$$y_t^* = \bar{y}_t^* \,, \tag{25}$$

(ii) money markets

$$m_{ht} + m_{ht}^* = 1 \,, \tag{26}$$

$$m_{ft} + m_{ft}^* = 1 \,, \tag{27}$$

(iii) loans markets

$$n_t + x_t + y_t = b_{ht} + z_{ht} + b_{ht}^* + z_{ht}^* \,, \tag{28}$$

$$n_t^* + x_t^* + y_t^* = b_{ft} + z_{ft} + b_{ft}^* + z_{ft}^* \,, \tag{29}$$

(iv) foreign exchange market

$$(b^*_{ht} + z_{ht}) \;=\; \hat{e}_t\delta_t\,(b_{ft} + z^*_{ft}), \tag{30}$$

$$(b^*_{ht} + z^*_{ht})(1 + i_t) \;=\; e_t\delta_t\,(b_{ft} + z_{ft})(1 + i^*_t). \tag{31}$$

(d) Arbitrage conditions hold.

$$\hat{e}_t\,(1 + i_t) = e_t\,(1 + i^*_t), \tag{17'}$$

$$\frac{p_{2t}}{p_{1t}} = (1 + i_t)(1 + i^*_t)\frac{p^*_{2t}}{p^*_{1t}}, \tag{32}$$

$$p^*_{1t} \;=\; \frac{p_{1t}(1 + i_t)}{e_t\delta_t}, \tag{33}$$

$$p_{2t} \;=\; p^*_{2t}(1 + i^*_t)e_t\delta_t. \tag{34}$$

This completes the definition of equilibrium.

As shown in equation (32), because of the cash-in-advance constraints for importers, the terms of trade faced by the two representative households are different. So given the same instantaneous utility function, for any value of $\sigma \neq 0$, the consumption ratios are different, $(c_{1t}/c_{2t}) > (c^*_{1t}/c^*_{2t})$, $\forall\, t$, if any nominal interest rate is positive. Each representative household consumes more domestic goods and less imported goods than its trading partner.

The Determination of the Exchange Rate

In this model, the exchange rate is determined simultaneously with the home price of good 1 and the foreign price of good 2. The exchange rate responds to clear the foreign exchange market by equating the supply and demand for foreign exchange of the two countries. Assume that nominal interest rates for bonds denominated in either currency of the two countries are positive in all states, so that the cash-in-advance constraints for the intermediaries, firms and importers of the two countries are always binding. By combining equations (17'), (30) and (31) with the following binding constraints for the firm and the importer of each country,

$$b_t \equiv b_{ht} + b_{ft}\,\hat{e}_t\,\delta_t = w_t\,l_t\,, \tag{35}$$

$$z_t \equiv \frac{z_{ht}}{\hat{e}_t\,\delta_t} + z_{ft} = p^*_{2t}\,c_{2t}\,, \tag{36}$$

$$b^*_t \equiv \frac{b^*_{ht}}{\hat{e}_t\,\delta_t} + b^*_{ft} = w^*_t\,l^*_t\,, \tag{37}$$

$$z^*_t \equiv z^*_{ht} + z^*_{ft}\,\hat{e}_t\,\delta_t = p_{1t}c^*_{1t}\,, \tag{38}$$

the exchange rate equations can be derived,

$$\hat{e}_t = \frac{P_{1t}\,c^*_{1t}}{P^*_{2t}\,c_{2t}} = \frac{1}{\delta_t}\left[\frac{(x_t + y_t + n_t)(1 + i_t) - (1 - n_t)}{(x^*_t + y^*_t + n^*_t)(1 + i^*_t) - (1 - n^*_t)}\right], \tag{39}$$

$$e_t = \frac{(1 + i_t)}{(1 + i^*_t)}\,\hat{e}_t = \frac{(1 + i_t)\,P_{1t}\,c^*_{1t}}{(1 + i^*_t)\,P^*_{2t}\,c_{2t}} = \frac{P_{2t}\,c_{2t}}{P^*_{1t}\,c^*_{1t}}\,, \tag{40}$$

where b_t, z_t, b^*_t and z^*_t are the ultimate demands for currency of the home firm, the home importer, the foreign firm, and the foreign importer, respectively.

Equation (39) states that $\hat{e}_t$ is determined by the relative quantities of the two currencies used in international trade financing, $(P_{1t}c^*_{1t})/(P^*_{2t}c_{2t})$, where

$$P_{1t}\,c^*_{1t} = M_t\,p_{1t}\,c^*_{1t} = M_t\,[(x_t + y_t + n_t)(1 + i_t) - (1 - n_t)]\,, \tag{41}$$

$$P^*_{2t}\,c_{2t} = M^*_t\,p^*_{2t}\,c_{2t} = M^*_t\,[(x^*_t + y^*_t + n^*_t)(1 + i^*_t) - (1 - n^*_t)]\,. \tag{42}$$

Define $\Omega_t \equiv (x_t + y_t + n_t)$ and $\Omega_t^* \equiv (x_t^* + y_t^* + n_t^*)$, which are the supplies of loanable funds in the markets for home currency-denominated bonds and the foreign currency-denominated bonds, respectively. As n_t and n_t^* are independent of the realization of the current state of the world, s_t, the exchange rate, $\hat{e}_t$, depends only upon how the two terms, $\Omega_t(1+i_t)$ and $\Omega_t^*(1+i_t^*)$, are affected by s_t.

$$\frac{1}{\Omega_t(1+i_t)}\,\frac{d[\Omega_t(1+i_t)]}{ds_t} = \frac{1}{\Omega_t}\,\frac{d\Omega_t}{ds_t} + \frac{1}{(1+i_t)}\,\frac{di_t}{ds_t}, \tag{43}$$

$$\frac{1}{\Omega_t^*(1+i_t^*)}\,\frac{d[\Omega_t^*(1+i_t^*)]}{ds_t} = \frac{1}{\Omega_t^*}\,\frac{d\Omega_t^*}{ds_t} + \frac{1}{(1+i_t^*)}\,\frac{di_t^*}{ds_t}. \tag{44}$$

If the home country's monetary authority's injections of cash, by way of changes in either x_t or y_t, are the only economic disturbances in the world economy, we have

$$\frac{\Omega_t}{\Omega_t(1+i_t)}\,\frac{d[\Omega_t(1+i_t)]}{d\Omega_t} = 1 + \frac{\Omega_t}{(1+i_t)}\,\frac{di_t}{d\Omega_t}, \tag{45}$$

$$\frac{\Omega_t}{\Omega_t^*(1+i_t^*)}\,\frac{d[\Omega_t^*(1+i_t^*)]}{d\Omega_t} = \frac{\Omega_t}{(1+i_t^*)}\,\frac{di_t^*}{d\Omega_t}, \tag{46}$$

where $\frac{\Omega_t}{\Omega_t(1+i_t)}\frac{d[\Omega_t(1+i_t)]}{d\Omega_t}$ and $\frac{\Omega_t}{\Omega_t^*(1+i_t^*)}\frac{d[\Omega_t^*(1+i_t^*)]}{d\Omega_t}$ are the elasticities of the changes in $\Omega_t(1+i_t)$ and $\Omega_t^*(1+i_t^*)$ with respect to the changes in Ω_t , respectively. The elasticities of the changes in the gross interest rates, $(1+i_t)$ and $(1+i_t^*)$, with respect to the changes in Ω_t are denoted by $\frac{\Omega_t}{(1+i_t)}\frac{di_t}{d\Omega_t}$ and $\frac{\Omega_t}{(1+i_t^*)}\frac{di_t^*}{d\Omega_t}$, respectively.

Similarly, if the home country's productivity shock is the only economic disturbance in the world economy, we have

$$\frac{\theta_t}{\Omega_t(1+i_t)}\,\frac{d[\Omega_t(1+i_t)]}{d\theta_t} = \frac{\theta_t}{(1+i_t)}\,\frac{di_t}{d\theta_t}, \tag{47}$$

$$\frac{\theta_t}{\Omega_t^*(1+i_t^*)}\,\frac{d[\Omega_t^*(1+i_t^*)]}{d\theta_t} = \frac{\theta_t}{(1+i_t^*)}\,\frac{di_t^*}{d\theta_t}. \tag{48}$$

The elasticities of the changes in $\Omega_t(1+i_t)$ and $\Omega_t^*(1+i_t^*)$ with respect to the changes in θ_t are denoted by $\frac{\theta_t}{\Omega_t(1+i_t)}\frac{d[\Omega_t(1+i_t)]}{d\theta_t}$ and $\frac{\theta_t}{\Omega_t^*(1+i_t^*)}\frac{d[\Omega_t^*(1+i_t^*)]}{d\theta_t}$, respectively. The elasticities of the changes in the gross interest rates, $(1+i_t)$ and $(1+i_t^*)$, with respect to the changes in θ_t are denoted by $\frac{\theta_t}{(1+i_t)}\frac{di_t}{d\theta_t}$ and $\frac{\theta_t}{(1+i_t^*)}\frac{di_t^*}{d\theta_t}$, respectively. The analogous elasticity equations for the changes in Ω_t^* and θ_t^* can be derived in the similar way.

Equations (43)-(48) show that the quantity of each currency used in international trade financing depends upon the liquidity in the market for bonds denominated in that currency, which is reflected by the interest rate on those bonds. Hence, $\hat{e}_t$ depends on the relative liquidity of the two markets for bonds denominated in the home currency and the foreign currency, respectively. How $P_{1t}c_{1t}^*$ and $P_{2t}^*c_{2t}$ are affected by the economic disturbances is determined by the elasticities of the changes in $(1+i_t)$ and $(1+i_t^*)$ with respect to the changes in the realizations of those exogenous random variables. The following discussion will show that

1. A monetary injection by way of an open market purchase, $\tilde{y}_t$, has only a "pure liquidity effect" such that $\frac{di_t}{dy_t} < 0, \quad \forall\ \sigma$.

2. A monetary injection in the form of a lump-sum transfer, x_t, has a liquidity effect and an expected price effect. These two effects operate in opposite directions. However, it can be shown that the liquidity effect dominates and $\frac{di_t}{dx_t} < 0, \quad \forall\ \sigma$.

3. The elasticity of the changes in $(1+i_t)$ with respect to the changes in the home country's productivity shock, θ_t, is determined by the elasticity of the changes in p_{1t} with respect to the changes in θ_t. $\frac{\theta_t}{(1+i_t)}\frac{di_t}{d\theta_t} = 1 + \frac{\theta_t}{p_{1t}}\frac{dp_{1t}}{d\theta_t}$ and sign $[1 + \frac{\theta_t}{p_{1t}}\frac{dp_{1t}}{d\theta_t}]$ = sign $(1-\sigma)$, where σ is the constant elasticity of substitution between the consumption of the two goods.

4. How the economic disturbances of the home economy affect i^* depends critically

upon the value of σ. sign $\frac{di_t^*}{dy_t}$ = sign $\frac{di_t^*}{dx_t}$ = sign $\frac{di_t^*}{d\theta_t}$ = sign $(\sigma - 1)$.

The illustration of these results is in Section 4.

As mentioned before, the beginning-of-period exchange rate, after the current state of the world is known, $\hat{e}_t$, is in general different from the end-of-period exchange rate, e_t. The only case where they are equal is if the two nominal interest rates are identical. Thus, the exchange rate may fluctuate both within a period and across periods. However, once e_t, i_t and i_t^* are determined, $\hat{e}_t$ will be pinned down by equation (17'). So the following discussion of the exchange rate will focus on the end-of-period exchange rate, e_t.

As shown in equation (40), the exchange rate (end-of-period), e_t, depends upon the relative size of the flows of the two countries' currencies to the foreign exchange market at the end of the period. This is determined by two factors. The first is the interest rate ratio $(1 + i_t)/(1 + i_t^*)$. The second is the relative quantities of the two countries' currencies used in international trade financing, $(P_{1t}c_{1t}^*)/(P_{2t}^*c_{2t})$. Both of these two factors reflect the relative liquidity of the home currency-denominated bond market and the foreign currency-denominated bond market. If the market for home currency denominated bonds is relatively more liquid, $(1 + i_t)/(1 + i_t^*)$ is less than one, and the quantity of home currency used in international trade financing, $P_{1t}c_{1t}^*$, is smaller than the quantity of foreign currency used in international trade financing, $P_{2t}^*c_{2t}$, then the home currency will appreciate.

The special feature of this model is the dependence of the exchange rates on the relative liquidity of the home currency-denominated bond market and the foreign currency-denominated bond market. This special feature is absent in other models, for example, Stockman(1980), Lucas (1982), and Svensson (1985). They simply have the purchasing-power-parity law of exchange rate determination which does not capture the liquidity effects.

Solving for an Equilibrium

Assume that there exists an equilibrium in which all of the cash-in-advance constraints for both countries are always binding, which implies $\mu_j(s) > 0$ and $\mu_j^*(s) > 0$, $\forall\, j = 1,2,3,4$. In this equilibrium, $m_h(s) = m_f^*(s) = 1$, $m_f(s) = m_h^*(s) = 0$, $\forall\, s$, n and n^* are fixed numbers which are between zero and one. The representative home country household's necessary conditions (11)-(14) and the analogous necessary conditions of the foreign representative household imply

$$E\left[\frac{1+i(s)}{1+x(s)}\right] = \frac{1}{\beta}, \tag{49}$$

$$E\left[\frac{1+i^*(s)}{1+x^*(s)}\right] = \frac{1}{\beta^*}, \tag{50}$$

$$c_j(p_1(s),p_2(s)) = \frac{(1-n)p_j(s)^{-\sigma}}{[p_1(s)^{1-\sigma}+p_2(s)^{1-\sigma}]}, \qquad j = 1,2, \tag{51}$$

$$c_j^*(p_1^*(s),p_2^*(s)) = \frac{(1-n^*)p_j^*(s)^{-\sigma}}{[p_1^*(s)^{1-\sigma}+p_2^*(s)^{1-\sigma}]}, \qquad j = 1,2, \tag{52}$$

$$w(s) = \frac{p_1(s)\theta(s)}{1+i(s)} = \frac{1}{\beta}[1+x(s)](1-n), \tag{53}$$

$$w^*(s) = \frac{p_2^*(s)\theta^*(s)}{1+i^*(s)} = \frac{1}{\beta^*}[1+x^*(s)](1-n^*), \tag{54}$$

$$q(s) = \frac{1}{1+i(s)} = \frac{[1+x(s)](1-n)}{\beta\theta(s)p_1(s)}, \tag{55}$$

$$q^*(s) = \frac{1}{1+i^*(s)} = \frac{[1+x^*(s)](1-n^*)}{\beta^*\theta^*(s)p_2^*(s)}. \tag{56}$$

Expressions (53)-(56) show that open market purchases, y and y^*, do not change the wage rates and they affect the bond prices only through their effects on p_1 and p_2^*. As pointed out in Lucas (1990), this kind of open market purchase only has a "pure liquidity effect" and the inflationary effect has been isolated.

Substitute the expressions (51)-(56), equations (20)-(23), (30) and (33)-(38) into equations (28), (29) and (40), we have a simultaneous equation system of three equations, (57)-(59), in three unknowns, p_1, p_2^* and e.

$$n + x + y = \frac{1+x}{\theta\beta v}\,[\, c_1(p_1,p_2) + c_1^*(\,p_1^*,p_2^*\,)\,] + p_1\, c_1^*(\,p_1^*,p_2^*\,)\,, \tag{57}$$

$$n^* + x^* + y^* = \frac{1+x^*}{\theta^*\beta^* v^*}\,[\, c_2(\,p_1,p_2\,) + c_2^*(\,p_1^*,p_2^*\,)] + p_2^*\, c_2(\,p_1,p_2\,)\,, \tag{58}$$

$$\left[\frac{\theta^*\beta^* v^*}{1+x^*}\right] e\,\delta\, p_2^{*2}\, c_2(\,p_1,p_2\,) = \left[\frac{\theta\beta v}{1+x}\right] p_1{}^2\, c_1^*(\,p_1^*,p_2^*\,)\,, \tag{59}$$

where $v \equiv \dfrac{1}{1-n}$, $v^* \equiv \dfrac{1}{1-n^*}$, $p_1^* = \dfrac{p_1{}^2\,\theta\,\beta\,v}{(1+x)\,e\,\delta}$ and $p_2 = \dfrac{p_2^{*2}\,e\,\delta\,\theta^*\,\beta^*\,v^*}{1+x^*}$.

The equilibrium prices, p_1 and p_2^*, and the exchange rate, e, can then be expressed as functions of s, n and n^* only. Given the distribution function $G(s)$, the equilibrium values of n and n^* can be solved by substituting the expressions (55) and (56), and the three functions, $p_1(s,n,n^*)$, $p_2^*(s,n,n^*)$ and $e(s,n,n^*)$, into equations (49) and (50).

There is a continuum of equilibria which have the common set of equilibrium values of $\{\, n,\ p(s),\ \gamma(s),\ y(s),\ c_1(s),\ c_2(s),\ l(s),\ b(s),\ z(s),\ n^*,\ p^*(s),\ \gamma^*(s),\ y^*(s),\ c_1^*(s),\ c_2^*(s),\ l^*(s),\ b^*(s),\ z^*(s)\,\}$. Each of these equilibria is characterized by a distinct vector of equilibrium values, $[\, b_h(s), b_f(s), z_h(s), z_f(s), b_h^*(s), b_f^*(s), z_h^*(s), z_f^*(s)\,]$, which satisfies equations (28)-(31) and (35)-(38). In spite of having eight equations in these eight unknowns, there are only seven independent equations. Consequently, the equilibrium values of these eight decision variables are indeterminate. This indeterminacy arises because what really matters are the ultimate allocations of currency across the borrowers: b, z, b^* and z^*, but not the ways to finance them. In equilibrium, the effective costs of financing by issuing bonds denominated in either currency are equalized.

All of these equilibria are actually equivalent as they have identical real allocation and pricing rules. For simplifying the discussion of the effects and the transmission mechanism for economic fluctuations in the following section, the particular equilibrium in which $b_h = b$, $z_f = z$, $b_f^* = b^*$, $z_h^* = z^*$ and $b_f = z_h = b_h^* = z_f^* = 0$ will be chosen to illustrate the results of the comparative statics exercises. In this particular equilibrium, there is no need for the foreign exchange market to be opened at the beginning of the period, after the current state of the world is revealed, because there are no market participants.

1.4 Comparative Static Analysis

Some comparative statics exercises will now be performed to analyze the impacts of the monetary shocks and real disturbances on the world equilibrium.[16] Given the distribution function G, n and n^* are independent of the realization of s. By totally differentiating the simultaneous equation system, equations (57)-(59), we can derive the effects of these disturbances on the equilibrium values of the exchange rate, e, the price of good 1 in units of home currency, p_1, and the price of good 2 in units of foreign currency, p_2^*. The effects on other macroeconomic aggregates can then be calculated. The details of these comparative statics exercises are presented in Appendix 2. Tables 1.1 — 1.3 summarize the results of these exercises, which show

[16]Though all of the economic disturbances are determined by the realization of the current state of the world, these comparative statics exercises are performed by allowing only one of these exogenous variables to vary while assuming the rest being constant.

that the effects depend critically on the value of the constant elasticity of substitution in consumption, σ, between the two goods.[17] The directions of the responses of e, p_1, and p_2^* are unambiguous except in the case of monetary injections through lump-sum transfers. In that case, the changes in e and p_1 are ambiguous. The reasons for the ambiguity will be illustrated below.

Open Market Purchases

Consider a larger realization of the monetary injection by way of an open market purchase, $\tilde{Y}$. As this monetary injection is through the home financial intermediaries, the first impact of this monetary shock is a decrease in the interest rate on the home currency-denominated bonds, i. Because this monetary injection is temporary, it does not affect the wage rate of the home country workers. Given the fixed wage rate and the decrease in the cost of financing, firms are able to reduce the home price of good 1. If other variables remain unchanged, the initial decrease in p_1 will raise the relative prices of good 2, p_2/p_1 and p_2^*/p_1^*. Therefore, for $\sigma > 1$, $\sigma = 1$ and $0 \leq \sigma < 1$, the expenditure share of good 2 in each shopper's budget becomes smaller, constant and larger, respectively. This, in turn, changes the demand for currency of the importers in the loans market, and also changes their demand for currency in the foreign exchange market at the end of the period. Equation (40) states that e is determined by the ratio of the expenditure shares of the imported goods in the two shoppers' budget constraints, $(P_2c_2)/(P_1^*c_1^*)$. Thus e is expected to fall, be unaffected and rise in the cases of $\sigma > 1$, $\sigma = 1$ and $0 \leq \sigma < 1$, respectively. The expected movement of the exchange rate induces further adjustment in the world economy. The home importer raises (reduces) his ultimate demand for foreign currency, z, if he expects e to fall (rise). With a fixed supply of foreign currency in the financial market, the interest rate i^* will adjust to clear the market. Liquidity in the foreign currency-

[17]The utility function is Cobb-Douglas if $\sigma = 1$. The two goods are gross substitutes if $\sigma > 1$. They are gross complements if $0 \leq \sigma < 1$.

denominated bond market is reallocated from the foreign firm (home importer) to the home importer (foreign firm). Hence, in addition to the impacts on the home economy, the effects of this monetary shock are transmitted to the foreign economy. The interest rate i^* and the foreign price of good 2, p_2^*, rise (fall), while the foreign employment level, l^*, and foreign output, Q^*, fall (rise). Only in the case of $\sigma = 1$, the foreign economy is insulated from the shocks of the home country.

The correlations of output fluctuations across countries depend highly on the substitutability of the two consumption goods in the consumers' preferences. The lower the substitutability of the two goods is, the higher the (positive) correlations of output across countries will be. However, the cross-country consumption correlations are determined by how close the comovements of the goods prices in the two economies are. Equations (33) and (34) state that the difference between the price of a good in the importing country and the price of the same good in the exporting country is determined by the cost of financing and the exchange rate. An economic disturbance will affect liquidity in the financial markets, reflected in the fluctuations in the nominal interest rates and the exchange rate. Because of the presence of the cash-in-advance constraints, both the relative prices and the nominal prices of the two goods faced by the two representative shoppers are affected by the economic disturbances differently. Hence, the consumption of the two households will adjust differently, and result in low correlations in consumption across countries.

As shown in Table 1.1, the effects on the two representative households' consumption of good 2, c_2 and c_2^*, depend on the value of σ. If $\sigma > 1$, c_2 rises while c_2^* falls. If $\sigma = 1$, c_2 and c_2^* are not affected by this monetary injection. If $0 \leq \sigma < 1$, c_2 falls while c_2^* rises. In addition, for any value of σ, the consumption of good 1 by the foreign representative household, c_1^*, increases. However, the changes in the home representative household's consumption of good 1, c_1, are ambiguous. Because of the ambiguity of the changes in c_1, the effects on the home country's employment level, l, and output, Q, are also ambiguous. There are three effects on c_1. They are a substitution effect caused by changes in p_2/p_1 and two income effects induced by changes in p_1 and p_2, respectively. If c_1 increases, l will also increase.[18]

If $\sigma > 1$, the effect of e dominates the effect of p_2^* on p_2, so that p_2 falls, which results in an ambiguous effect on p_2/p_1. If p_2/p_1 increases, the substitution effect caused by the increase in p_2/p_1 and the income effects respectively caused by the decreases in p_1 and p_2 will act in the same direction and raise c_1. This, in turn, implies an increase in the output (employment) of the home country. If p_2/p_1 decreases, the substitution effect will reduce c_1. However, it is expected that the substitution effect will be dominated by the income effects, so c_1 and the home country's output are expected to rise. Thus if the two consumption goods are gross substitutes ($\sigma > 1$), the national output movements of the two countries are negatively correlated. The home country's output (employment) rises, while the foreign country's output (employment) falls. The movements of the nominal interest rates are also negatively correlated. The interest rate on the home currency-denominated bonds falls, while the interest rate on the foreign currency-denominated bonds rises. The exchange rate decreases, and the home currency appreciates.

If $\sigma = 1$, p_2 is not affected by the monetary injection of the home economy. There are a substitution effect induced by the increase in p_2/p_1 and an income effect induced by the fall in p_1. These imply an increase in c_1. The home country's output, Q, and the employment level, l, rise.

If $0 \leq \sigma < 1$, the two goods are complements. The substitution effect induced by the increase in p_2/p_1, raises c_1. However, the income effects are ambiguous because p_1 decreases while p_2 increases. If the substitution effect dominates, c_1 and the home

[18] A sufficient (but not necessary) condition for $(dc_1/dy) > 0$ is $[\sigma + \pi_{12}^*(1 - \pi_{12}^*)] > 0$

country's output will increase. Thus, in this case, the correlations of the interest rates and the national output (employment) movements are both positive. Both countries increase their real output and the interest rates, i and i^*, both fall. The home currency depreciates.

When the value of σ is sufficiently small, the substitution effect is weak. Since the expenditure share of c_2 in the home representative shopper's budget is large, the income effect of p_2 on c_1 may dominate, c_1 will fall, and the effect on Q will be ambiguous. However, Q can increase if the increase in c_1^* is larger than the decrease in c_1. For the extreme case in which $\sigma = 0$, the movements of the two countries' output are perfectly positively correlated, while the correlations of consumption are negatively correlated across countries. Both countries increase their output. The consumption of each good by the home representative household falls, but consumption of each good by the foreign representative household rises. Though this is an extreme case, it indicates that, when the complementarity of the two consumption goods is high and the goods prices move quite differently in the two economies, this model is able to generate output fluctuations across countries which are more highly positively correlated across countries than consumption fluctuations.

The effects of a larger realization of the monetary injection by way of an open market purchase can be summarized as follows. It can be shown that

$$\frac{dp_1}{dy} < 0\,, \quad \frac{di}{dy} < 0\,, \quad \frac{d(p_2^*/p_1^*)}{dy} > 0\,, \quad \frac{dc_1^*}{dy} > 0\,, \quad \forall\ \sigma\,,$$

$$\text{sign}\,\frac{de}{dy} = \text{sign}\,\frac{dp_2}{dy} = \text{sign}\,\frac{dl^*}{dy} = \text{sign}\,\frac{dc_2^*}{dy} = \text{sign}\,\text{cov}(i, i^*) = \text{sign}\,(1-\sigma)\,,$$

$$\text{sign}\,\frac{dp_2^*}{dy} = \text{sign}\,\frac{di^*}{dy} = \text{sign}\,\frac{dc_2}{dy} = \text{sign}\,(\sigma - 1)\,,$$

$$\frac{d(p_2/p_1)}{dy} > 0\,, \qquad \frac{dp_1^*}{dy} < 0\,, \qquad \forall \;\; \sigma \leq 1\,,$$

$$\frac{dc_1}{dy}\begin{cases} > 0 & \text{if } \sigma = 1 \\ < 0 & \text{if } \sigma = 0 \\ = ? & \text{otherwise}\,, \end{cases} \qquad \frac{dl}{dy}\begin{cases} > 0 & \text{if } \sigma = 1 \\ > 0 & \text{if } \sigma = 0 \\ = ? & \text{otherwise}\,, \end{cases}$$

$$\text{if} \quad \frac{dc_1}{dy} > 0 \quad \text{for} \quad \sigma > 1 \quad \text{and} \quad 0 < \sigma < 1\,,$$

$$\text{then} \quad \frac{dl}{dy} > 0\,, \quad \forall \; \sigma \quad \text{and} \quad \text{sign cov}(l, l^*) = \text{sign }(1-\sigma)\,.$$

Lump Sum Transfers

The effects of a larger monetary injection to the representative home intermediary in the form of a lump-sum transfer, X, are not as clear as in the case of open market purchases. The comparative statics exercise shows that the direction of the movement of p_2^* and i^* are the same as with a larger volume of open market purchases. The movement of p_1 is ambiguous due to the two opposing forces, namely, an increase in the wage rate and a decrease in the opportunity cost of using cash to finance production in the home country. This permanent increase in the per capita stock of the home currency will increase the wage rate because workers request a higher wage rate for maintaining their purchasing power in next period. However, the monetary injection increases liquidity in the home currency-denominated bond market and lowers the market interest rate, i. This implies that the expected price effect is dominated by the liquidity effect, $\frac{di}{dx} < 0, \;\; \forall \;\; \sigma$. The change in the effective wage rate $w(1+i)$ is ambiguous and so is the change in p_1. Only in the case of $\sigma = 1$, can it be shown that p_1 decreases.

For the exchange rate, the movement is unambiguous in the cases of $0 \leq \sigma < 1$ (e will rise) and of $\sigma = 1$ (e will be unaffected). However, in the case of $\sigma > 1$,

e will decrease (as that of open market purchases) only if π^*_{11} is sufficiently small. This can be explained in the following way. Since the change in p_1 is ambiguous, the decrease in i may only reduce p^*_1 marginally. The demand for good 1 by the foreign shoppers will increase. We have to restrict π^*_{11} to be sufficiently small such that the demand for good 1 and the ultimate demand for home currency, z^*, will not increase too much. Otherwise, the interest rate will increase to offset part of the original reduction and thus put a upward pressure on p_1. Thus if π^*_{11} is sufficiently small, a monetary injection through a lump-sum transfer can have similar effects on the national output and interest rate movements as with monetary injections in the form of open market purchases. Basically, the transmission mechanism of the effects of home country's lump-sum transfers of newly issued currency on the foreign economy is the same as that of open market purchases. Loosely speaking, a monetary injection in the form of a lump-sum transfer to intermediaries is more inflationary and less expansionary than a monetary injection in the form of open market purchases. This is because the wage rate of the home workers increases in the former case while it remains constant in the latter case.

Productivity Shocks

As the home country has a larger realization of the productivity parameter θ, its workers become more productive, so that home firms are able to reduce the home price of good 1. Both countries' relative prices of good 2, in units of good 1, rise. Therefore, this positive productivity shock in the home country will affect the world economy through the same mechanism as that of the open market purchases case [see Table 1.2].

Unlike the impacts of monetary shocks, the output of the home economy increases unambiguously in this case. Moreover, the effects on the employment level l and the interest rate i are quite different from those of monetary shocks. The rise in productivity allows home firms to reduce l, which tends to reduce i, while the decrease in p_1 induces a larger demand for good 1, which increases l and has an upward force on i. Consequently, the effects on l are ambiguous, and the changes in i are determined by the elasticity of the changes in p_1 with respect to the changes in θ, $-\frac{\theta_t}{p_{1t}}\frac{dp_{1t}}{d\theta_t}$. $\frac{\theta_t}{(1+i_t)}\frac{di_t}{d\theta_t} = 1 + \frac{\theta_t}{p_{1t}}\frac{dp_{1t}}{d\theta_t}$ and $\text{sign}\,[1 + \frac{\theta_t}{p_{1t}}\frac{dp_{1t}}{d\theta_t}] = \text{sign}\,(1-\sigma)$. In the case of $\sigma > 1$, the elasticity is larger than one. The effect of the decrease in p_1 dominates the effect of the increase in θ on i. As a result, i decreases. In the case of $\sigma = 1$, the elasticity is equal to one. The two forces of θ and p_1 on i are entirely offset by each other, so that i is unaffected by the productivity shock. In the case of $0 < \sigma < 1$, the elasticity is smaller than one. Thus the effect of θ dominates, and i increases.

It can be shown that a productivity shock in the home economy has the following effects,

$$\frac{dp_1}{d\theta} < 0\,, \quad \frac{d(p_2/p_1)}{d\theta} > 0\,, \quad \frac{d(p_2^*/p_1^*)}{d\theta} > 0\,, \quad \frac{dc_1^*}{d\theta} > 0\,, \quad \forall\ \sigma\,,$$

$$\text{sign}\,\frac{de}{d\theta} = \text{sign}\,\frac{dp_2}{d\theta} = \text{sign}\,\frac{di}{d\theta} = \text{sign}\,\frac{dl^*}{d\theta} = \text{sign}\,\frac{dc_2^*}{d\theta} = \text{sign}\,(1-\sigma)\,,$$

$$\text{sign}\,\frac{dp_2^*}{d\theta} = \text{sign}\,\frac{di^*}{d\theta} = \text{sign}\,\frac{dc_2}{d\theta} = \text{sign}\,(\sigma-1)\,,$$

$$\text{sign}\,\text{cov}(i,i^*)\begin{cases} = 0 & \text{if } \sigma = 1 \\ < 0 & \text{otherwise}\,, \end{cases}$$

$$\frac{dp_1^*}{d\theta} < 0\,, \qquad \forall\ \sigma \le 1\,,$$

$$\frac{dQ}{d\theta} > 0\,, \qquad \forall\ \sigma \ge 1\,,$$

$$\frac{dc_1}{d\theta}\begin{cases} > 0 & \text{if } \sigma \ge 1 \\ < 0 & \text{if } \sigma = 0 \\ = ? & \text{otherwise}\,, \end{cases}$$

$$\text{if} \quad \frac{dc_1}{d\theta} > 0 \quad \text{for} \quad 0 < \sigma < 1 \,, \quad \text{then} \quad \text{sign}\,\text{cov}(Q, Q^*) = \text{sign}\,(1 - \sigma)\,,$$

where $Q \equiv c_1 + c_1^*$ and $Q^* \equiv c_2 + c_2^*$.

1.5 Conclusion

In summary, both monetary shocks and real disturbances can generate fluctuations in exchange rates and interest rates, and result in comovements of economic variables of the two economies. An exchange rate equation, which incorporates liquidity effects, is derived in this model. Liquidity effects also act as the transmission mechanism for economic fluctuations. The correlations among the two economies' macroeconomic aggregates depend on the substitutability of the two consumption goods in consumers' preferences.

Svensson and van Wijnbergen (1989) analyze the international transmission of monetary policy shocks in an open economy disequilibrium model with sticky goods prices and excess capacity. Though they also have similar findings, which show that the effect of a home country's monetary expansion on foreign output depends on whether the two goods are complements or substitutes, the transmission mechanism of the effect is quite different from that in this model.[19] In their model, output is demand determined. The home country's monetary expansion has real effects on the world economy as it depreciates the home currency and implies an expected inflation, which affects the demands for goods of the economic agents. In this model, the only rigidity is in the deposit decision, while all prices are flexible. This paper stresses the role of international financial markets in allocating liquidity across market participants in the two countries. Monetary injections induce a redistribution of liquidity in the financial

[19]Given the intertemporal elasticity of substitution in consumption is unity, our classifications of complements and substitutes are equivalent to the "Edgeworth-Pareto" complements and substitutes used in Svensson and van Wijnbergen (1989).

markets. For different values of the constant elasticity of substitution between the two goods in consumers' preferences, the allocation of liquidity will be affected differently. Hence, real activity in the world economy will be affected differently, and different patterns of fluctuations in exchange rates, interest rates and goods prices will be generated.

With the imposed financial structure here, this model is able to generate higher output correlations and lower consumption correlations across countries, which may be an explanation (with monetary features) for the discrepancies between data and the predictions given by the usual open economy real business cycle models.

This paper develops a theoretical analysis of the role of liquidity effects in an open economy. To evaluate the model quantitatively would require introducing physical investment and international debt carried over periods. However, that would be a topic for future research.

Table 1.1 : The Effects of Open Market Purchases

	$\frac{de}{dy}$	$\frac{dp_1}{dy}$	$\frac{dp_2^*}{dy}$	$\frac{dp_2}{dy}$	$\frac{dp_1^*}{dy}$	$\frac{di}{dy}$	$\frac{di^*}{dy}$	$\frac{d\rho}{dy}$	$\frac{d\rho^*}{dy}$	$\frac{dz^*}{dy}$	$\frac{dz}{dy}$	$\frac{dl}{dy}$	$\frac{dc_1}{dy}$	$\frac{dc_1^*}{dy}$	$\frac{dl^*}{dy}$	$\frac{dc_2}{dy}$	$\frac{dc_2^*}{dy}$
$\sigma > 1$	−	−	+	−	?	−	+	?	+	?	+	?	?	+	−	+	−
$\sigma = 1$	0	−	0	0	−	−	0	+	+	+	0	+	+	+	0	0	0
$0 < \sigma < 1$	+	−	−	+	−	−	−	+	+	+	−	?	?	+	+	−	+
$\sigma = 0$	+	−	−	+	−	−	−	+	+	+	−	+	−	+	+	−	+

where $\rho \equiv p_2/p_1$, $\rho^* \equiv p_2^*/p_1^*$, $z^* = p_1 c_1^*$, $z = p_2^* c_2$.

Table 1.2 : The Effects of Productivity Shocks

	$\frac{de}{d\theta}$	$\frac{dp_1}{d\theta}$	$\frac{dp_2^*}{d\theta}$	$\frac{dp_2}{d\theta}$	$\frac{dp_1^*}{d\theta}$	$\frac{di}{d\theta}$	$\frac{di^*}{d\theta}$	$\frac{d\rho}{d\theta}$	$\frac{d\rho^*}{d\theta}$	$\frac{dz^*}{d\theta}$	$\frac{dz}{d\theta}$	$\frac{dl}{d\theta}$	$\frac{dc_1}{d\theta}$	$\frac{dc_1^*}{d\theta}$	$\frac{dl^*}{d\theta}$	$\frac{dc_2}{d\theta}$	$\frac{dc_2^*}{d\theta}$
$\sigma > 1$	−	−	+	−	?	−	+	+	+	?	+	?	+	+	−	+	−
$\sigma = 1$	0	−	0	0	−	0	0	+	+	0	0	0	+	+	0	0	0
$0 < \sigma < 1$	+	−	−	+	−	+	−	+	+	?	−	?	?	+	+	−	+
$\sigma = 0$	+	−	−	+	−	+	−	+	+	?	−	?	−	+	+	−	+

where $\rho \equiv p_2/p_1$, $\rho^* \equiv p_2^*/p_1^*$, $z^* = p_1 c_1^*$, $z = p_2^* c_2$.

Table 1.3 : The Effects of Lump-sum Transfers

	$\frac{de}{dx}$	$\frac{dp_1}{dx}$	$\frac{dp_2^*}{dx}$	$\frac{di}{dx}$	$\frac{di^*}{dx}$
$\sigma > 1$	?	?	+	−	+
$\sigma = 1$	0	−	0	−	0
$0 \leq \sigma < 1$	+	?	−	−	−

Chapter 2

Capital Controls, Foreign Exchange Controls, and Liquidity

2.1 Introduction

This paper analyzes the liquidity effects generated by restrictions on international financial markets in a two-country, two-good, two-currency world economy. The impacts of these liquidity effects on the welfare and macroeconomic aggregates of the two countries are studied. Two types of controls on international financial transactions are examined. The first is capital controls, which are taxes on the purchases of bonds issued by foreign borrowers. The second is foreign exchange controls, which are taxes on the purchases of foreign currency.

Though there is a worldwide trend toward financial liberalization, the use of various forms of taxes and quantitative restrictions on international financial transactions continues to be widespread in the world economy, and is likely to remain so in the future. According to the International Monetary Fund (IMF) Annual Report on Exchange Arrangements and Exchange Restrictions (1992), out of 157 IMF member countries, 88 have restrictions on payments for current transactions, while 122 have restrictions on payments for capital transactions. It is important to have a clear understanding of how these controls affect the world economy, and what the likely consequences of financial liberalization are.

The effects of capital controls and foreign exchange controls of various forms have been studied in previous work. Adams and Greenwo. l (1985) investigate the welfare aspects of a dual exchange rate system. Greenwood and Kimbrough (1985, 1987) analyze how foreign exchange and capital controls affect macroeconomic aggregates such as output, employment, and consumption. These papers show that these controls are equivalent to some forms of commercial policies and should be evaluated by applying standard real traue theory. However, all of these studies are restricted to small open economy setups. Consequently, it is clear that all of these controls are undesirable for a distortion-free small open economy. In addition, the role of international financial markets has not been emphasized in areas of the above papers.

Stockman and Hernandez (1988) highlight one of the many crucial functions of international financial markets: assets markets allow individuals to hedge against unwanted risks. They examine the effects of restrictions on international financial markets in a general-equilibrium, rational-expectations model of a two-country world. They show that the effects of capital and foreign exchange controls depend critically upon the availability of international financial market for assets with state-contingent real returns. They get these results because contingent assets eliminate all diversifiable wealth-redistributed effects of changes in policies, and only the aggregate wealth effects and substitution effects remain.

Unlike Stockman and Hernandez (1988), this paper stresses another crucial function of international financial markets, which is allocating liquidity across market participants. Financial intermediaries are explicitly modeled in the form of credit institutions, which accept deposits from savers and make loans to finance production and international trade. The taxes on international financial transactions affect the distribution of liquidity in international financial markets. This paper investigates the effects of this redistribution of liquidity on the world economy.

A number of recent papers have studied liquidity effects induced by monetary injections in modified versions of closed economy cash-in-advance models [e.g. Grossman and Weiss (1983), Rotemberg (1984), Lucas (1990) and Fuerst (1992)]. In Grossman and Weiss (1983) and Rotemberg (1984), government open market operations not only induce liquidity effects, but also alter the distribution of wealth. This redistribution of wealth complicates the analysis. Lucas (1990) suggests a simple approach that abstracts from these wealth redistribution effects. This approach allows for the study of situations in which different agents face different trading opportunities while retaining the convenience of the representative household fiction. He analyzes a series of models in which money is required for asset transactions as well as for transactions on goods. In these modified versions of cash-in-advance models, government open market operations induce liquidity effects that lead to fluctuations in interest rates for non-Fisherian reasons in a closed economy. Fuerst (1992) introduces production to Lucas (1990)'s pure exchange economy and analyzes how monetary injections generate liquidity effects and influence the real activity of a closed economy.

This paper extends the line of research of Lucas (1990) and Fuerst (1992) to study liquidity effects in an open economy. A two-country version of the model in Fuerst (1992) is presented to study liquidity effects induced by the taxes on international financial transactions in an open economy. If cash is required for international financial transactions, then the quantity of each country's currency (liquidity) in financial markets will influence exchange rates and the interest rates on bonds denominated in currency of each country in the world economy. Taxes on international financial transactions generate liquidity shocks to the international financial markets. As different economic agents face different trading opportunities, they are affected by the liquidity shocks asymmetrically. These taxes not only withdraw some cash from the world economy, but also redistribute liquidity across economic agents. This redistribution of liquidity affects real activity and generates fluctuations in exchange rates and interest rates.

In this model, capital controls are shown to be sufficiently prohibitive that each of the borrowers will take loans from the intermediaries of his own country only. However, when capital controls are the only limitations on international financial transactions, they have no real effects on the world economy. They only restrict the world economy to be in an equilibrium in which each borrower takes loans only from the financial intermediaries of his own country, but the real allocation of the economy is identical to that in the equilibrium of the model with no controls on international financial transactions. It is the availability of unrestricted trading in the foreign exchange market which makes capital controls become ineffective. To have an effect on international capital flows, capital controls and foreign exchange controls have to be imposed simultaneously. These controls act to restrict economic agents' access to liquidity and to redistribute liquidity in international financial markets. Different economic agents will be affected by the controls differently.

How the liquidity effects operate depends critically upon the value of the constant elasticity of substitution in consumption between the two goods in consumers' preferences. As the taxes are imposed on the home representative household, the initial liquidity effect is going to occur in the economy of the home country. Liquidity in the market for bonds denominated in home currency is redistributed. This liquidity effect affects the relative prices of the consumption goods in both economies. The value of the constant elasticity of substitution in consumption between the two goods then determines how the liquidity in the market for bonds denominated in foreign currency has to be reallocated. This model can generate different comovements of the macroeconomic aggregates in the two economies. Nevertheless, the home currency appreciates as a result of the imposition of the foreign exchange controls, which is independent of the substitutability of the two consumption goods in consumers' preferences.

The impacts on the welfare of each household in the world economy depend upon how the liquidity constraints of this household are affected by the changes in these controls. As the cash-in-advance constraints are imposed on all transactions in this monetary economy, they produce distortions in the world economy. There will be a welfare improvement if the restrictions relax the liquidity constraints.

The remainder of the paper is organized as follows. The model is presented in Section 2. Section 3 describes a stationary rational expectations equilibrium. The impacts of changes in the taxes on the home country's residents' purchases of foreign currency, given a fixed policy rule, are investigated in Section 4. Section 5 provides a discussion of the welfare implications of changes in policy rules of restrictions on international financial transactions. The conclusion is in Section 6.

2.2 The Model

The model used is a two-country, two-good, two-currency cash-in-advance model with endogenous production. The home country and the foreign country have identical constant numbers of infinitely-lived households. All variables will then be expressed in per (own country) household terms. Foreign variables and parameters are indexed with an asterisk (*). The infinitely-lived households of each country are identical. The world economy can be considered as an economy with two heterogeneous representative households. Each representative household consists of five members: a shopper, a worker, a firm, an importer, and a financial intermediary. The objective of the home country representative household is to maximize its expected lifetime utility

$$E_0\left[\sum_{t=0}^{\infty}\beta^t U(c_{1t},c_{2t},l_t)\right], \qquad 0<\beta<1, \tag{1}$$

$$U(c_{1t},c_{2t},l_t)=\ln\left[c_{1t}{}^{\alpha}+c_{2t}{}^{\alpha}\right]^{\frac{1}{\alpha}}+\left[1-l_t\right], \qquad \alpha\leq 1, \tag{2}$$

where E_0 is the mathematical expectation conditional upon the information available in period 0. The instantaneous utility function $U(c_{1t}, c_{2t}, l_t)$ is discounted by the subjective discount factor β. The consumption of good j in period t is c_{jt}, $j = 1,2$.[1] Define $\sigma \equiv \frac{1}{1-\alpha}$, which is the constant elasticity of substitution in consumption between the two goods. In every period, the household is endowed with one unit of time. The household supplies l_t units of work effort to the labor market in period t.[2] Labor is internationally immobile. For simplicity, the instantaneous utility functions of the representative households of the two countries are assumed to be identical.

Firms in the home country produce only good 1, while those of the foreign country produce good 2. However, every household consumes both goods. Thus there is a complete specialization in production, and trade allows agents in each country to consume both goods. The production functions are identical within a country, but different across countries. Each production process requires only labor input. Each of the home firms faces the production function

$$\mathcal{F}(H_t) = \theta H_t , \tag{3}$$

where θ is the productivity parameter which is assumed to be constant over time, and H_t denotes units of labor employed by the home firm. Similarly, the foreign firm's production function of good 2 is given by $\mathcal{F}^*(H_t^*) = \theta^* H_t^*$. With constant returns technology, perfect competition implies that firms earn zero profits, so that trading in equity claims can be ignored.

In this model, cash-in-advance constraints are imposed on all transactions. The restrictions of using the sellers' currency for transactions in goods markers and using

[1]Taking the logarithm of the CES utility function of the two consumption goods results in a unit elasticity of intertemporal substitution in consumption, which simplifies the calculation.

[2]As in Fuerst (1992), the instantaneous utility function is linear in leisure. This assumption is required for simplifying the calculation and obtaining a closed-form solution of the optimization problem. In addition, this assumption implies a perfectly elastic labor supply curve. As a result, both of the intertemporal and intratemporal responses of labor supply to the economic disturbances are eliminated. Rather than being a drawback, this specification of the utility function allows the model to stress the fluctuations in employment level brought about by shifts in labor demand.

the buyers' currency for transactions in bond markets assign financial intermediaries a role in international trade financing.

This model is similar to the one constructed in Ho (1992), except for the following two modifications. First, since our focus is on the effects of foreign exchange and capital controls, the production parameters and the money supply of each country are assumed to be constant. There are no monetary injections. The only exogenous variables are the taxes on international financial transactions imposed by the home government on the home country's residents. The home government will rebate all of its tax revenue to the households of the home country as a lump-sum transfer at the end of each period. Thus, changes in money stock of the world economy caused by these taxes can be prevented from occurring. It is convenient to use a normalization so that the money stock of each country is unity. Second, for simplicity, the trading of bonds among firms and importers of either country will be ignored.[3] Only financial intermediaries are assumed to have the expertise to deal with the borrowers of other countries.

2.2.1 The Timing of Information and Transactions

The timing of information and transactions is shown in Figure 2.1. The representative home country household enters a period with cash balances carried over from last period, which include m_{ht} units of home currency and m_{ft} units of foreign currency. It deposits n_t units of home currency in the home intermediary. Transaction costs are prohibitively high so that once the deposit is made, it cannot be withdrawn until the end of the current period. In addition, for simplicity, the home country household is not allowed to hold foreign currency-denominated deposits. This restriction will not make any qualitative difference to the results.

After the home country deposit is made, the household separates. The shopper

[3] As in equilibrium, the net supply of bonds traded among firms and importers of the same country will be zero in this representative agent economy.

Figure 2.1 :

Timing of Information and Transactions of the Home Representative Household

Period t

- Beginning-of-period cash holdings, m_{ht} and m_{ft}.
- Deposits n_t units of home currency.
- The household separates.
  - Shopper $m_{ht} - n_t$
  - Importer m_{ft}
  - Financial Intermediary n_t
  - Firm
  - Worker
- Current state of the world, s_t, is revealed to everybody.
- Markets are opened.
  - Loans markets
  - Foreign exchange market
  - Labor markets
  - Goods markets
- Markets are closed.
- Foreign exchange market is reopened.
- Loan repayments are made.
- The household is reunited, all remaining cash is pooled, and goods purchased by shopper are consumed.
- Lump-sum rebates of tax revenue from the home government, γ_t.
- End-of-period cash holdings, m_{ht+1} and m_{ft+1}.

Period $t+1$

⋮

takes the remaining $m_{ht} - n_{ht}$ units of home currency to the local goods markets. The worker goes to the home labor market. The importer takes the foreign cash balance m_{ft}. The firm, the intermediary and the importer go to the financial market where they will meet their counterparts of the foreign country. In the financial market, both home currency-denominated bonds and foreign currency-denominated bonds can be traded. Assume that the number of households of each country is large enough that every individual acts as a price-taker.

Once the household is separated, the current state of the world is revealed to everybody. Let the state of the world be denoted by a vector $s_t = (\tau_t, \eta_t)$, where τ_t is the tax rate on the purchases of foreign currency imposed by the home government on the home country's residents (foreign exchange controls), and η_t is the tax rate on the purchases of bonds issued by foreigners, imposed by the home government on the home financial intermediaries (capital controls).[4] The state of the world, s_t, is independently and identically distributed across time, and the joint distribution function $G(\tau_t, \eta_t)$ is public knowledge.[5]

In the goods markets of the home country, the home shopper purchases good 1 and good 2 for consumption subject to the cash-in-advance constraint

$$m_{ht} - n_t \geq p_{1t} c_{1t} + p_{2t} c_{2t}, \tag{4}$$

where p_{jt}, $j = 1, 2$, is the price of good j, in units of home currency.

In the financial market, each intermediary is assumed to be able to make loans denominated in its own country's currency only, while it is allowed to lend to any firms and importers of both countries.[6] The source of home currency available to the

[4] Alternatively, these controls can be in the forms of quantitative restrictions. If the home government auctions off the quotas on these international financial transactions in every period, there will exist some taxes that exactly duplicate these quotas. In this case, the state of the world is denoted by the quotas set by the home government.

[5] The treatment of government policy as uncertain and exogenous can be considered as a fixed policy rule while having some elements of randomness attached to its inputs, so that the resulting policies will be stochastic.

[6] Even if we let intermediaries borrow from the intermediaries of another country, then lend to

home intermediary is the the deposit by the home household, n_t. Taking the market interest rate on the home currency-denominated bonds issued by the home country's borrowers, i_t, the tax rate η_t, and the demand for funds of the foreign borrowers as given, the home intermediary allocates the loans b_{ht}, z_{ht}, b^*_{ht} and z^*_{ht} to the home firm, the home importer, the foreign firm and the foreign importer, respectively. The representative home intermediary has the following cash-in-advance constraint

$$n_t \geq b_{ht} + z_{ht} + (b^*_{ht} + z^*_{ht})(1 + \eta_t). \tag{5}$$

It is noted that with the assumption of perfect competition, the home financial intermediaries will lend to the foreigners at the interest rate $[(1 + i_t)(1 + \eta_t) - 1]$.

The home firm borrows b_{ht} units of home currency from the home intermediary and b_{ft} units of foreign currency from the foreign intermediary. The home importer borrows z_{ht} units of home currency from the home intermediary and z_{ft} units of foreign currency from the foreign intermediary. However, home firms have to pay the home workers in home currency, while home importers have to pay the foreign firms in foreign currency. Their foreign counterparts also face a similar problem. At this time, the foreign exchange market is opened so that the firms and the importers of both countries can trade for the desired currency. The home government levies taxes on its residents' purchases of foreign currency at the tax rate τ_t. These taxes are paid in units of home currency.[7] Assume that a flexible exchange rate regime is adopted by the two countries. Let $\hat{e}_t$ be the exchange rate at the beginning of the period, after the state of the world s_t is known. This exchange rate will adjust to clear the foreign exchange market.

their borrowers, the equilibrium will not be affected. But many accounting procedures will be involved.

[7]In this model, it will matter whether the taxes on purchases of foreign currency are paid in units of home currency or in units of foreign currency. However, only the case in which the taxes are paid in units of home currency will be considered here. For the taxes on the home financial intermediaries' purchases of bonds issued by foreigners, it does not matter whether they are paid in units of the currency of either country. This is because in equilibrium, the foreign borrowers will not take any loans from the home intermediaries.

The home firm then takes the home currency obtained after asset trading, $b_{ht} + b_{ft}\hat{e}_t$, to the home labor market, hires H_t units of labor at the market wage rate, w_t, to produce good 1 according to the production function given by equation (3). The cash-in-advance constraint of the home firm is given by

$$b_{ht} + b_{ft}\,\hat{e}_t \geq w_t H_t \,. \tag{6}$$

The output will be shipped to the home market of good 1 and sold to home shoppers and foreign importers at the price of p_{1t} units of home currency. The home worker receives his wage, $w_t l_t$, and goes home to enjoy his leisure, $1 - l_t$.

The home importer goes to the foreign goods markets with the cash balances obtained after the asset trading, $m_{ft} + z_{ft} + z_{ht}(1 - \tau_t)/\hat{e}_t$ purchases foreign goods at the price of p^*_{2t} units of foreign currency, and sells them to home shoppers at the price of p_{2t} units of home currency in the home market for good 2. Thus, the cash-in-advance constraint for the home importer is

$$m_{ft} + z_{ft} + \frac{z_{ht}(1-\tau_t)}{\hat{e}_t} \geq p^*_{2t}\, c_{2t} \,. \tag{7}$$

At the end of the period, loan repayments are made. However, the home borrowers do not have foreign currency to repay the loans from the foreign intermediaries, while the foreign borrowers do not have home currency to repay the loans from the home intermediaries. It is assumed that the transaction costs in the foreign exchange market are significantly less than in the loans market so that currency trading can be more frequent than other asset trading.[8] Consequently, only the foreign exchange market can be reopened at the end of the period. Borrowers can trade for the currency they need for repaying their foreign debts. The end-of-period exchange rate, e_t, responds to clear this foreign exchange market. After all transactions are completed, the household is reunited, all remaining cash is pooled, and goods purchased by the

[8]This assumption is also used by Helpman and Razin (1982) and Svensson (1985). Svensson (1985) calls this "continuous currency trade".

shopper are consumed. The home government rebates its tax revenue to the home representative household as a lump-sum transfer.

$$\gamma_t = \left[z_{ht} + \frac{(b_{ft} + z_{ft})(1 + i_t^*)\, e_t}{1 - \tau_t} \right] \tau_t + (b_{ht}^* + z_{ht}^*)\, \eta_t \,. \tag{8}$$

The household then holds the remaining cash balances m_{ht+1} and m_{ft+1} for next period, where

$$\begin{aligned} m_{ht+1} &= m_{ht} + p_{1t}\,\mathcal{F}(H_t) - p_{1t}\,c_{1t} - w_t\,H_t + w_t\,l_t + b_{ft}\,\hat{e}_t - z_{ht} \\ &\quad + (b_{ht}^* + z_{ht}^*)\,(1 + \eta_t)\, i_t - \frac{(b_{ft} + z_{ft})(1 + i_t^*)\, e_t}{1 - \tau_t} + \gamma_t \,, \end{aligned} \tag{9}$$

$$m_{ft+1} = m_{ft} + z_{ft} + \frac{z_{ht}\,(1 - \tau_t)}{\hat{e}_t} - p_{2t}^*\, c_{2t} \,. \tag{10}$$

Due to the assumptions of a flexible exchange rate regime and no debts carried over periods, the current account of each country is always balanced. The activities of the foreign representative household are analogous to those of the home representative household, and the foreign variables and parameters are defined in a similar way.

2.2.2 The Optimization Problem of the Home Representative Household

The home representative household maximizes its expected lifetime utility with preferences given by (1) and (2), subject to the technology constraint (3), the cash-in-advance constraints (4)-(7), and the two evolution equations of the household's beginning-of-period cash balances (9) and (10). There is an analogous optimization problem for the foreign representative household.

Dynamic programming can be applied to solve the household's optimization problem. Define the value function corresponding to the household's problem $J(m, \tilde{s})$ by

$$J(m, \tilde{s}) = \max_{0 \le n \le m_h} \int \max_{c_1, c_2, b_h, z_h, b_f, z_f, l, H} \{ U(c_1, c_2, l) + \beta J(m', s) \}\, dG(s) \,, \tag{11}$$

subject to (3)-(7), (9) and (10). The time subscripts for current period have been omitted while those for the next period have been replaced by primes. The realization of last period's state of the world is $\tilde{s}$. The collection of the beginning-of-period money holdings of the home and foreign representative households is represented by $m = (m_h, m_f, m_h^*, m_f^*)$. It is noted that the decision for n is made before the realization of the state of the world s. Thus, n is independent of s.

The first-order conditions for the home representative household's optimization problem and those for the foreign representative household's optimization problem imply the following arbitrage condition for borrowing from both countries' financial intermediaries,

$$(1+i)\hat{e} = (1+i^*)e, \tag{12}$$

which is the same as in the case with no restrictions on international financial transactions. This implies that the foreign borrowers will not borrow from the home financial intermediary. Also, the home firm will not borrow from the foreign financial intermediary. That is, $b_f = b_h^* = z_h^* = 0$. Consequently, there will be no suppliers of foreign currency in the foreign exchange market at the end of the period. The foreign exchange market will not be opened at the end of those periods in which capital controls are imposed. Thus, the home importer will not borrow from the foreign financial intermediary as he is not able to obtain foreign currency through trading in the foreign exchange market for repaying the loans, and $z_f = 0$. It is noted that whenever capital controls are imposed, the foreign borrowers will stop taking loans from the home financial intermediaries. Thus, any taxes on the home financial intermediaries' purchases of bonds issued by foreigners will be prohibitive. These results are valid no matter whether there are foreign exchange controls or not, because equation (12) always holds.

In a model in which capital controls are the only limitation on international financial transactions, capital controls have no real effects on the world economy. They only restrict the world economy to be in an equilibrium in which each borrower only takes loans form the financial intermediaries of his own country, but the real allocation of the economy is identical to that in the equilibrium of the model with no controls on international financial transactions. In Ho (1992) it is shown that there is a continuum of equilibria with identical real allocations, but with different loan quantities. For the controls on international capital flows to be effective, capital controls need to be imposed together with foreign exchange controls.[9]

2.3 Stationary Rational Expectations Equilibrium

In the stationary rational expectations equilibrium of the world economy, the prices and decision rules are fixed functions of the current state of the world, $s_t = (\tau_t, \eta_t)$. It is noted that n_t and n_t^* are independent of s_t.

Assume that there exists an equilibrium in which all of the cash-in-advance constraints for both countries are always binding. In this equilibrium, n and n^* are fixed numbers which are between zero and one, $m_{ht} = m_{ft}^* = 1$, $m_{ft} = m_{ht}^* = 0$, $b_{ft} = z_{ft} = b_{ht}^* = z_{ht}^* = 0$, $\forall\, t$, all markets are cleared,

$$\text{money markets} \qquad m_{ht} + m_{ht}^* = 1\,, \qquad m_{ft} + m_{ft}^* = 1\,, \tag{13}$$

$$\text{loans markets} \qquad n = b_{ht} + z_{ht}\,, \qquad n^* = b_{ft}^* + z_{ft}^*\,, \tag{14}$$

$$\text{foreign exchange market} \qquad z_{ht}(1 - \tau_t) = \hat{e}_t z_{ft}^*\,, \tag{15}$$

[9]If foreign exchange controls are the only limitation on international financial transactions, there will be a continuum of equilibria. Each of these equilibria is characterized by a distinct vector of $[z_h, z_f, z_h^*, z_f^*]$ with non-negative values. These equilibria are quantitatively different from each other as the magnitudes of the liquidity effects induced by changes in the tax rate τ depend upon the values of $[z_h, z_f, z_h^*, z_f^*]$. However, the qualitative properties of all of these equilibria are the same. For simplifying the analysis, capital controls are assumed to be imposed together with foreign exchange controls so that the particular equilibrium in which $z_f = z_h^* = 0$ will be chosen to illustrate the results.

goods markets $c_{1t} + c^*_{1t} = \theta H_t\,,$ $c_{2t} + c^*_{2t} = \theta^* H^*_t\,,$ (16)

labor markets $l_t = H_t\,,$ $l^*_t = H^*_t\,,$ (17)

and the arbitrage conditions hold,

$$\hat{e}_t\,(1 + i_t) \;=\; e_t\,(1 + i^*_t)\,, \tag{18}$$

$$p^*_{1t} \;=\; \frac{p_{1t}(1 + i^*_t)}{\hat{e}_t}\,, \tag{19}$$

$$p_{2t} \;=\; \frac{p^*_{2t}(1 + i_t)\hat{e}_t}{1 - \tau_t}\,. \tag{20}$$

The Determination of Exchange Rates

Equation (15) and the binding cash-in-advance constraints for the importer of each country imply the nominal exchange rate,

$$\hat{e}_t \;=\; \frac{z_{ht}\,(1 - \tau_t)}{z^*_{ft}} \;=\; \frac{p_{1t}c^*_{1t}}{p^*_{2t}c_{2t}}\,, \tag{21}$$

while the real exchange rate (terms of trade) is given by

$$e^R_t \;=\; \hat{e}_t\frac{p^*_{2t}}{p_{1t}} \;=\; \frac{c^*_{1t}}{c_{2t}}\,. \tag{22}$$

By combining equations (14) and (21), and the binding cash-in-advance constraints for the shopper and the firm of each country, the loans market clearing conditions become,

$$n \;=\; w_t l_t \;+\; \frac{p_{1t}c^*_{1t}}{1 - \tau_t} \;=\; \frac{1 - n}{1 + i_t} \;+\; \frac{p_{1t}c^*_{1t}}{1 + i_t}\,, \tag{23}$$

$$n^* \;=\; w^*_t l^*_t \;+\; p^*_{2t}c_{2t} \;=\; \frac{1 - n^*}{1 + i^*_t} \;+\; \frac{p^*_{2t}c_{2t}}{1 + i^*_t}\,. \tag{24}$$

The quantity of each currency used in international trade financing is given by

$$p_{1t}c_{1t}^* = n(1+i_t) - (1-n), \tag{25}$$

$$p_{2t}^*c_{2t} = n^*(1+i_t^*) - (1-n^*). \tag{26}$$

The exchange rate equation can then be derived,

$$\hat{e}_t = \frac{p_{1t}c_{1t}^*}{p_{2t}^*c_{2t}} = \frac{n(1+i_t) - (1-n)}{n^*(1+i_t^*) - (1-n^*)}. \tag{27}$$

Equation (27) states that $\hat{e}_t$ is determined by the relative quantities of the two currencies used in international trade financing. This is also the relative size of the flows of the two currencies to the foreign exchange market at the beginning of the period, after the current state of the world is known. Given the distribution function G, the supplies of loanable funds in the two bond markets, n and n^*, are independent of the realization of the current state of the world, so they are constant over time. Equations (25) and (26) state that the quantity of each currency used in international trade financing depends only upon the interest rate on the bonds denominated in that currency, which reflects liquidity of the market for those bonds. When the liquidity in the market is high, the interest rate is low, and the proportion of loanable funds allocated to international trade financing is small. How $\hat{e}_t$ is affected by the taxes on the international financial transactions is determined by how the interest rates, i_t and i_t^*, respond to the realizations of the current values of these tax rates. For example, if τ_t is the only exogenous random variable, and η_t is assumed to be positive and constant over time, we have

$$\frac{1}{\hat{e}_t}\frac{d\hat{e}_t}{d\tau_t} = \frac{n}{p_{1t}c_{1t}^*}\frac{di_t}{d\tau_t} - \frac{n^*}{p_{2t}^*c_{2t}}\frac{di_t^*}{d\tau_t}.$$

Hence, the changes in $\hat{e}_t$ reflect the changes in the relative liquidity of the home currency-denominated bond market and the foreign currency-denominated bond market. When the market for home currency-denominated bonds becomes relatively more liquid, the home currency appreciates, and $\hat{e}_t$ decreases.

The Allocations of Loanable Funds

Though equation (14) describes an allocation of loanable funds across economic agents in the two bond markets, b_{ht}, z_{ht}, b^*_{ft}, and z^*_{ft}, what really matters is for whom each of the economic activities facilitated by the funds serves. Another allocation of loanable funds can be derived according to this criterion. Equations (23) and (24) provide this allocation. The loanable funds in the home currency-denominated bond market, n, is allocated across the following three uses. The first one, $\frac{p_{1t}c_{1t}}{1+i_t}$, is to finance the production of good 1 for the consumption of the home representative household. The second one, $\frac{p_{2t}c_{2t}}{1+i_t}$, is to finance the import of good 2 for the consumption of the home representative household. The third one, $\frac{p_{1t}c^*_{1t}}{1+i_t}$, is to finance the production of good 1 for the consumption of the foreign representative household. Similarly, n^* is divided into $\frac{p^*_{2t}c^*_{2t}}{1+i^*_t}$, $\frac{p^*_{1t}c^*_{1t}}{1+i^*_t}$ and $\frac{p^*_{2t}c_{2t}}{1+i^*_t}$, which are for financing the production of good 2 for the consumption of the foreign representative household, the import of good 1 for the consumption of the foreign representative household, and the production of good 2 for the consumption of the home representative household, respectively. Thus, the home representative household has access to $\frac{1-n}{1+i_t}$ units of home currency and $\frac{p^*_{2t}c_{2t}}{1+i^*_t}$ units of foreign currency. The foreign representative household has access to $\frac{p_{1t}c^*_{1t}}{1+i_t}$ units of home currency and $\frac{1-n^*}{1+i^*_t}$ units of foreign currency. Equations (23) and (24) state that a change in the interest rate of a bond market represents a redistribution of liquidity across the home representative household and the foreign representative household in that market. An increase in i (i^*) represents a redistribution of liquidity in the home (foreign) currency-denominated bond market from the home (foreign) representative household to the foreign (home) representative household.

Solving for an Equilibrium

The first-order conditions of the representative household's optimization problem of each country imply that their consumption, c_{jt} and c^*_{jt}, $j = 1, 2$, the wage rates, w_t and w^*_t, the market interest rates, i_t and i^*_t, and the real exchange rate, e^R_t, are given by

$$c_j(p_{1t}, p_{2t}) = \frac{(1-n)p_{jt}{}^{-\sigma}}{[p_{1t}{}^{1-\sigma} + p_{2t}{}^{1-\sigma}]}, \qquad c^*_j(p^*_{1t}, p^*_{2t}) = \frac{(1-n^*)p^*_{jt}{}^{-\sigma}}{[p^*_{1t}{}^{1-\sigma} + p^*_{2t}{}^{1-\sigma}]}, \tag{28}$$

$$w_t = \frac{1-n}{\beta}, \qquad w^*_t = \frac{1-n^*}{\beta^*}, \tag{29}$$

$$1 + i_t = \frac{\beta\theta p_{1t}}{(1-n)}, \qquad 1 + i^*_t = \frac{\beta^*\theta^* p^*_{2t}}{(1-n^*)}, \tag{30}$$

$$E\,[\,1 + i_t\,] = \frac{1}{\beta}, \qquad E\,[\,1 + i^*_t\,] = \frac{1}{\beta^*}, \tag{31}$$

$$e^R_t = \frac{c^*_{1t}}{c_{2t}} = \left[\left(\frac{n}{1-n} - \frac{1}{1+i_t}\right)\beta\,\theta\right]\left[\left(\frac{n^*}{1-n^*} - \frac{1}{1+i^*_t}\right)\beta^*\,\theta^*\right]^{-1}. \tag{32}$$

Substitute the expressions (28)-(30), equations (19) and (20), into equations (23), (24) and (27), we have a simultaneous equation system of three equations, (33)-(35), in three unknowns, p_{1t}, p^*_{2t} and $\hat{e}_t$.

$$\hat{e}\,p^*_2\,c_2\,(\,p_1\,,\,p_2\,) = p_1\;c^*_1\,(\,p^*_1,\,p^*_2\,)\,. \tag{33}$$

$$n = \frac{1-n}{\theta\beta}\,[\,c_1\,(\,p_1,\,p_2\,)\;+\;c^*_1\,(\,p^*_1,\,p^*_2\,)\,] + \frac{1}{1-\tau}\,p_1\,c^*_1\,(\,p^*_1,\,p^*_2\,)\,, \tag{34}$$

$$n^* = \frac{1-n^*}{\theta^*\,\beta^*}\,[\,c_2\,(\,p_1,p_2\,)\;+\;c^*_2\,(\,p^*_1,p^*_2\,)\,]\;+\;p^*_2\,c_2\,(\,p_1,p_2\,)\,. \tag{35}$$

where $p_2 = \dfrac{\hat{e}\, p_1\, p_2^*\, \theta\, \beta}{(1-\tau)(1-n)}$ and $p_1^* = \dfrac{p_1\, p_2^*\, \theta^*\, \beta^*}{\hat{e}\,(1-n^*)}$.

The equilibrium prices, p_{1t} and p_{2t}^*, and the exchange rate, $\hat{e}_t$, can be expressed as functions of τ_t, n and n^* only. By substituting these three functions and the expressions in (30) into (31), the equilibrium values of n and n^* can be solved.

2.4 Comparative Static Analysis

How alternative realizations of the taxes on international financial transactions affect equilibrium allocations and prices can be analyzed by performing a comparative statics exercise. As mentioned in Section 2, the taxes on the home financial intermediaries' purchases of bonds issued by foreigners are prohibitive. What really matters is the presence of the taxes but not the level of the tax rate, η. The imposition of capital controls by the home government restricts the world economy to be in the equilibrium in which each of the borrowers only borrows from the financial intermediaries of his own country. From now on, η is assumed to be positive and constant over time. The only exogenous random variable is the tax rate on the home country residents' purchases of foreign currency imposed by the home government, τ. Given the distribution function G, n and n^* are constant over time. By totally differentiating the simultaneous equation system, equations (33)-(35), we can derive the effects of the changes in τ, on the equilibrium values of the exchange rate, $\hat{e}$, the home price of good 1, p_1, and the foreign price of good 2, p_2^*. The effects on other macroeconomic aggregates can then be calculated. Table 2.1 summarizes the results of this comparative statics exercise, while the details are in Appendix 1. It is shown that the effects depend critically upon the value of the constant elasticity of substitution in consumption between the two goods, σ.[10] These results will be illustrated in the

[10]The utility function is Cobb-Douglas if $\sigma = 1$. The two goods are gross substitutes if $\sigma > 1$. They are gross complements if $0 \leq \sigma < 1$.

following discussion.

Given a fixed policy rule, represented by the distribution function G, this comparative statics exercise is performed across states of the world. Consider a larger realization of the tax rate, τ. As the taxes are paid in units of home currency, this increase in τ reduces the supply of home currency in the foreign exchange market at the beginning of the period. The home government withdraws τz_h units of home currency from the world economy. From equation (21), it is clear that if the other variables do not adjust in response to the increase in τ, $\hat{e}$ and c_1^* will fall as the same proportion as $(1 - \tau)$ does. These initial changes in $\hat{e}$ and c_1^* will generate liquidity effects in the bond markets of both countries.

As the foreign importer obtains less home currency through the transactions in the foreign exchange market, the demand for home good by the foreign importer, c_1^*, falls. This, in turn, reduces the demand for home currency of the home firm for financing its production. There is an excess supply in the home currency-denominated bond market. The interest rate i falls to clear the market. The loanable funds in the home currency-denominated bond market is reallocated from the home firm to the home importer. The home economy's employment level, l, and output, Q, decrease, while p_1 rises.

Though the home government withdraws home currency through the foreign exchange controls, liquidity of the home currency-denominated bond market increases. This is because the taxes reduce the foreign representative household's access to home currency. Equation (23) states that the decrease in i represents a redistribution of liquidity in the home currency-denominated bond market from the foreign representative household to the home representative household. The imposition of the foreign exchange controls on the home representative household effectively places a restriction on the foreign representative household's access to the liquidity in the market for home currency-denominated bond.

How the liquidity effects in the home economy transmit the impacts of a larger realization of the tax rate, τ, to the foreign economy depends critically upon the value of the constant elasticity of substitution in consumption between the two goods, σ. If the nominal good prices, p_1 and p_2^*, and the interest rates, i and i^*, remain unchanged, the decrease in $\hat{e}$ will reduce the relative good price in the foreign economy, p_2^*/p_1^*, while it will not affect the relative good price in the home economy, p_2/p_1. The liquidity shocks induced by the increase in τ will be transmitted to the foreign economy by way of the decrease in p_2^*/p_1^*. There is an expenditure switching effect. The expenditure share of good 1 in the foreign shopper's budget constraint, $p_1^* c_1^*$, falls, is unaffected and rises in the cases of $\sigma > 1$, $\sigma = 1$ and $0 \leq \sigma < 1$, respectively. Therefore, for $\sigma > 1$, $\sigma = 1$ and $0 \leq \sigma < 1$, the demand for loanable funds by the foreign importer for financing the import of good 1, z_f^*, decreases, remains unchanged and increases, respectively. There is an excess supply (demand) in the foreign currency-denominated bond market in the case of $\sigma > 1$ ($\sigma < 1$). The interest rate i^* falls (rises) to clear this bond market. The loanable funds in the foreign currency-denominated bond market is reallocated from the foreign importer (foreign firm) to the foreign firm (foreign importer). The foreign employment level, l^*, and output, Q^*, increase (decrease), while p_2^* falls (rises). Only in the case of $\sigma = 1$, the foreign economy is insulated from the liquidity shocks induced by the changes in the policy of the home country's government.

The reallocation of the loanable funds in the two bond markets affects the supply of each currency in the foreign exchange market, and results in further changes in the exchange rate, $\hat{e}$. However, it can be shown that for any value of σ, the nominal exchange rate, $\hat{e}$, falls. The real exchange rate, e^R, is shown to fall too. There is an improvement in the home country's terms of trade.

The results of this comparative statics exercise can be summarized as follows:

$$\frac{d\hat{e}}{d\tau} < 0\,, \quad \frac{de^R}{d\tau} < 0\,, \quad \frac{dp_1}{d\tau} < 0\,, \quad \frac{di}{d\tau} < 0\,, \quad \frac{dz_h}{d\tau} > 0\,, \quad \frac{dl}{d\tau} < 0\,, \quad \forall\ \sigma\,,$$

$$\text{sign}\,\frac{dp_2^*}{d\tau} = \text{sign}\,\frac{di^*}{d\tau} = \text{sign}\,\frac{dz_f^*}{d\tau} = -\,\text{sign}\,\frac{dl^*}{d\tau} = \text{sign}\,(1-\sigma)\,.$$

For different values of σ, different comovements of the variables in the two economies will be observed, sign $\text{cov}(l, l^*) = \text{sign cov}(Q, Q^*) = -\text{sign cov}(i, i^*) = \text{sign}(1-\sigma)$.

Welfare Analysis

Stockman and Hernandez (1988) find that given the probability distributions of the home government policies, the realization of a higher tax rate on acquisitions of foreign currency generally reduces the utility of the home country's representative household, while raising the utility of the foreign representative household. These welfare changes are due to aggregate wealth effects and substitution effects generated by the increase in the tax rate. This paper emphasizes the role of liquidity effects. The following discussion shows that in this model, the welfare implications of changes in the tax rate are different from those in Stockman and Hernandez (1988).

We now investigate how a larger realization of the tax rate τ, given the distribution function G, affects the welfare of the two countries' representative households. From the comparative statics exercise, we have

$$\frac{dc_1}{d\tau} > 0\,, \quad \frac{dc_1^*}{d\tau} < 0\,, \quad \frac{dl}{d\tau} < 0\,, \qquad \forall\ \sigma\,,$$

$$-\,\text{sign}\,\frac{dc_2}{d\tau} = \text{sign}\,\frac{dc_2^*}{d\tau} = \text{sign}\,\frac{dl^*}{d\tau} = \text{sign}(\sigma-1)\,.$$

It is obvious that the welfare level of the home country's household is improved by the increase in τ if $0 \leq \sigma \leq 1$. Consumption increases and work effort decreases. While for the case of $\sigma > 1$, the change in welfare of the home country's household is ambiguous because c_1 rises, l falls, but c_2 falls.[11] The changes in the welfare of the foreign representative household are ambiguous for the cases of $\sigma > 1$ and $0 \leq \sigma < 1$.

[11]It can be shown that the proportional change in c_1 is larger than the proportional change in c_2. If initially, c_1 is either larger or not substantially less than c_2, the welfare of the home country's household will be improved.

Only for the case of $\sigma = 1$ can it be shown that the welfare of the foreign representative household is worse off. The changes in welfare of the two countries' households are induced by the redistribution of liquidity in the two bond markets. There are two sources of welfare changes. The first one is from the changes in the employment levels. The second one is from the changes in consumption.

Let consider first how liquidity in the home currency-denominated bond market is redistributed as a result of a larger realization of τ. On the one hand, the allocation of n across economic agents, given by equation (14), indicates that the allocation of liquidity to the home firm, b_h, falls. Thus the employment level (work effort) of the home economy, l, falls. This improves the welfare of the home representative household. On the other hand, the allocation of n across the two countries' representative households, given by equation (23), indicates that an imposition of taxes on the home country residents' purchases of foreign currency effectively places a restriction on the foreign representative household's access to liquidity in the market for the home currency-denominated bonds. A larger value of τ makes this restriction become tighter, and this benefits the home representative household by increasing its access to the liquidity in this market. Recall that equation (23) implies the allocation of liquidity across the two representative households for them to facilitate the acquisitions of their consumption goods. Hence, redistributing liquidity from the foreign representative household to the home representative household improves the home country household's welfare, while reducing the welfare of the foreign household.

In addition, the liquidity effects in the home economy will be transmitted to the foreign economy by way of the changes in the relative good price in the foreign economy, p_2^*/p_1^*. Liquidity in the foreign currency-denominated bond market has to be reallocated for the case of $\sigma > 1$ and $0 \leq \sigma < 1$. For the case $\sigma = 1$, liquidity and the interest rate, i^*, are unaffected. Thus the welfare of the two representative households are affected by the liquidity effects in the home economy only. The home representative household has a welfare improvement while the foreign representative household's welfare decreases.

For the case $\sigma > 1$, less foreign currency is allocated to the foreign importer, z_f^* falls, and i^* drops. The employment level (work effort) of the foreign representative household, l^*, increases, and this reduces welfare. Equation (24) shows that liquidity is redistributed from the home representative household to the foreign representative household. This redistribution of liquidity in the foreign currency-denominated bond market reduces the welfare of the home country's household and increases the welfare of the foreign household. As these redistributions of liquidity in the two bond markets affects the welfare of each household in opposite directions, the welfare change for each household is ambiguous.

For the case $0 \leq \sigma < 1$, liquidity in the market for foreign currency-denominated bonds falls, z_f^* increases, and i^* rises. The employment level, l^*, falls, which results in a welfare improvement for the foreign household. The increase in i^* represents a redistribution of liquidity from the foreign representative household to the home representative household. The redistribution of liquidity in the market for foreign currency-denominated bonds reinforces the welfare effects caused by the redistribution of liquidity in the market for home currency-denominated bonds. The home country's household is better off, while the foreign household's welfare change is ambiguous. This is because the reduction in the supply of work effort, l^*, affects its welfare in a direction opposite to those effects induced by the redistribution of liquidity across the two representative households in the two bond markets.

To summarize, the existence of the cash-in-advance constraints imposed on all transactions produces distortions in the world economy. The effects of the foreign exchange controls on the welfare of each of the households in the world economy depends upon how the liquidity constraints of each household are affected. There will be a welfare improvement if the restrictions effectively loosen the liquidity constraints.

2.5 Welfare Implications of Changes in Policy Rules

The previous discussion investigates alternative realizations of the tax rate on the home representative household's purchases of foreign currency, τ_t, given a fixed distribution function G. In this section, the effects of changes in G are studied. The distribution function, G, is interpreted as a fixed policy rule with randomness. Thus changes in G can be interpreted as changes in the home government's policy rule. To keep the calculation simple and manageable, we adopt the following two simplifications.

First, only the case in which $\sigma = 1$ will be considered. Here, liquidity effects occur in the home economy only. There is no redistribution of liquidity in the market for foreign currency-denominated bonds. With the assumption of $\sigma = 1$ (Cobb-Douglas utility function), we have $p_{1t}c_{1t} = p_{2t}c_{2t} = \frac{1}{2}(1-n)$, and $p^*_{1t}c^*_{1t} = p^*_{2t}c^*_{2t} = \frac{1}{2}(1-n^*)$. Equations (23) and (24) can be rewritten as

$$n = \frac{1-n}{1+i_t} + \frac{(1-n)(1-\tau_t)}{2(1+i_t)^2} = (1-n)\,q_t + \frac{1}{2}(1-n)(1-\tau_t)\,q_t^2, \quad (36)$$

$$n^* = \frac{1-n^*}{1+i^*_t} + \frac{(1-n^*)}{2(1+i^*_t)^2} = (1-n^*)\,q^*_t + \frac{1}{2}(1-n^*)\,q_t^{*2}, \quad (37)$$

where $q_t \equiv \dfrac{1}{1+i_t}$ and $q^*_t \equiv \dfrac{1}{1+i^*_t}$.

It is obvious that i^*_t and q^*_t are constant and independent of the realization of τ_t.

Second, assume that there are only two possible states of the world, (τ_A, η) and (τ_B, η), where η is positive and constant over time. The distribution function G is a function of τ_t only. The random variable τ has density given by

$$g(\tau) = \begin{cases} \frac{1}{2} & \text{for} \quad \tau = \tau_A \quad \text{or} \quad \tau = \tau_B\,, \\ 0 & \text{otherwise}\,. \end{cases}$$

Let $\bar{\tau}$ and Var(τ) denote the mean and the variance of τ, respectively,

$$\bar{\tau} = \frac{1}{2}(\tau_A + \tau_B) \ , \qquad \mathrm{Var}(\tau) = \frac{1}{4}(\tau_A - \tau_B)^2 .$$

From the results in Section 4, if $\tau_A > \tau_B$, then we have

$$\hat{e}_A < \hat{e}_B , \quad \hat{e}^R_A < \hat{e}^R_B , \quad i_A < i_B , \quad i^*_A = i^*_B , \quad q_A > q_B , \quad q^*_A = q^*_B ,$$

$$c_{1A} > c_{1B} , \quad c_{2A} = c_{2B} , \quad l_A < l_B , \quad c^*_{1A} < c^*_{1B} , \quad c^*_{2A} = c^*_{2B} , \quad l^*_A = l^*_B .$$

where the subscripts $_A$ and $_B$ denote the equilibrium values of the variables in the two possible states, (τ_A, η) and (τ_B, η), respectively. Equations (31), (36) and (37) imply that $q^*_A = q^*_B = \beta^*$, n^* can be solved by using equation (37), and it can be shown that $n^* \in (0,1)$. In addition, we have a simultaneous equation system in three unknowns, n, q_A and q_B, so that the equilibrium can be solved.

$$1 \ = \ (\frac{1}{n} - 1)\, q_A \ + \ \frac{1}{2}\,(\frac{1}{n} - 1)(1 - \tau_A)\, {q_A}^2 , \tag{38}$$

$$1 \ = \ (\frac{1}{n} - 1)\, q_B \ + \ \frac{1}{2}\,(\frac{1}{n} - 1)(1 - \tau_B)\, {q_B}^2 , \tag{39}$$

$$\frac{1}{2q_A} + \frac{1}{2q_B} = \frac{1}{\beta} , \tag{40}$$

where $(\frac{1}{n} - 1)q_j$ and $\frac{1}{2}(\frac{1}{n} - 1)(1 - \tau_j){q_j}^2$, $j = A, B$, are the shares of liquidity in the home currency-denominated bond market allocated to the home representative household and the foreign household, respectively.

Two types of changes in G will be examined. The first is changes in the variance of τ, while keeping its mean constant. The second is changes in the mean of τ, while its variance is constant. The household of each country adjusts its deposit in response to the change in the distribution function G. However, in the case of $\sigma = 1$, the foreign economy is insulated from the economic disturbances of the home economy. The values of n^*, i^*, q^*, and p_2^* are constant over time. In the home currency-denominated bond market, the supply of home currency, n, and the interest rate, i, are adjusted. There is a redistribution of liquidity in this market. The impacts of the changes in G on the world equilibrium are illustrated as follows. The details of this comparative statics exercise are in Appendix 2.

Changes in the Variance of τ

Assume $\tau_A = \tau_o + \varepsilon$ and $\tau_B = \tau_o - \varepsilon$, with mean $\bar{\tau} = \tau_o$ and variance $\mathrm{Var}(\tau) = \varepsilon^2$. An increase in ε represents an increase in $\mathrm{Var}(\tau)$, while $\bar{\tau}$ is constant, that is, the randomness of the foreign exchange controls increases. By totally differentiating equations (38)–(40), it can be shown that the increase in the randomness of τ results in a higher variability of the equilibrium value of each of the following variables, $\hat{e}$, q, c_1, c_1^*, p_1 and p_1^*. However, the expected utility of the home representative household rises as the proportional increase in c_{1A} is larger than the proportional decrease in c_{1B}, and the expected work effort, $\bar{l} \equiv \frac{1}{2}(l_A + l_B)$, falls. The expected utility of the foreign representative household falls as the decrease in c_{1A}^* is in a larger proportion than the increase in c_{1B}^*.

The reasons for the welfare changes are as follows. Let consider an increase in ε. If the supply of the loanable funds, n, is not adjusted in response to the larger ε, there is an expected excess supply of home currency in the bond market. From equation (36), it is noted that

$$\frac{1}{q}\frac{\partial q}{\partial \varepsilon} = \frac{1}{2[1+(1-\tau)q]}\frac{\partial \tau}{\partial \varepsilon} \quad \Rightarrow \quad -\frac{1}{(1+i_A)}\frac{\partial i_A}{\partial \varepsilon} > \frac{1}{(1+i_B)}\frac{\partial i_B}{\partial \varepsilon}.$$

Equation (40) is not satisfied. The expected gross interest rate, $E[1+i]$, is lower than the rate of time preference, $1/\beta$. Hence, at the beginning of the period, the home representative household reduces the deposit of home currency in the home financial intermediary. This will have an upward force on i, and n will be reduced until

$E[1+i]$ equals to $1/\beta$. In equilibrium, i_A falls and i_B rises. When $\tau = \tau_A$, the redistribution of the liquidity in the market for home currency-denominated bonds is from the foreign representative household to the home representative household, which improves the welfare of the home representative household while it makes the foreign representative household worse off. When $\tau = \tau_B$, the redistribution of the liquidity is from the home representative household to the foreign representative household, which reduces the welfare of the home representative household while improving the foreign representative household's welfare. It can be shown that the welfare changes in the state $\tau = \tau_A$ outweigh those in the state $\tau = \tau_B$. The expected share of liquidity allocated to the home representative household, $E[(\frac{1}{n} - 1)q]$, rises. Thus the expected utility of the home representative household is increased while that of the foreign representative household is reduced. In addition, the expected quantity of home currency allocated to the home firm falls so that the expected work effect, $\bar{l}$, falls too. This improves the welfare of the home country's households further.

Changes in the Mean of τ

Assume $\tau_A = \tau_a + \varepsilon$ and $\tau_B = \tau_b + \varepsilon$, where $\tau_a > \tau_b$. Thus the mean is $\bar{\tau} = \frac{1}{2}(\tau_a + \tau_b) + \varepsilon$ and the variance is $\text{Var}(\tau) = \frac{1}{4}(\tau_a - \tau_b)^2$. A change in ε represents a change in $\bar{\tau}$, while $\text{Var}(\tau)$ is constant. A decrease in ε can be interpreted as a financial liberalization. Performing a comparative statics exercise by totally differentiating the equations (38)-(40), we find that some of the results are similar to those of the changes in $\text{Var}(\tau)$ [see Appendix 2].

The home representative household has a welfare improvement while the foreign representative household's welfare is lower. The reasons for the changes in welfare are similar to those of the previous case. In this case, as both τ_A and τ_B rise, there is a downward force on i_A and i_B. The expected gross interest rate, $E[1+i]$, falls. The home representative household reduces the size of its deposit which generates an upward pressure on $E[1+i]$. The decrease in n will continue until $E[1+i]$ equals to the rate of time preference, $1/\beta$. It is expected that the reduction in n will be larger than in the case of an increase in $\mathrm{Var}(\tau)$, because an increase in the randomness of τ affects the interest rates of the two states in opposite directions. In equilibrium, i_A falls and i_B rises. It is noted that for equation (40) to be satisfied, the movements of i_A and i_B have to be in opposite directions. As the proportional change in i_A is larger than the proportional change in i_B, the welfare changes caused by the redistribution of liquidity in the state $\tau = \tau_A$ outweigh those in the state $\tau = \tau_B$. Hence, the expected utility levels of the representative households of the home country and the foreign country rises and falls, respectively

2.6 Conclusion

This paper presents a two-country, two-good, two-currency cash-in-advance model to study the effects of taxes on international financial transactions. The role of financial intermediaries in allocating the liquidity across the participants in the international financial markets is emphasized. The liquidity effects induced by these controls on international financial transactions are studied. In this model, when capital controls are the only limitation on international financial transactions, capital controls have no real effects on the world economy. The availability of unrestricted trading in the foreign exchange market makes these controls become ineffective. They only restrict the world economy to be in an equilibrium in which each borrower takes loans only from the financial intermediaries of his own country, but the real allocation of the economy is identical to that in the equilibrium of the model with no controls on international financial transactions. Ho (1992) has shown that, in that model, there is a continuum of equilibria with identical real allocations, while each has a distinct allocation of the loans. However, when capital controls are imposed simultaneously with the foreign exchange controls, international capital flows can be controlled effectively.

These controls have real effects on the world equilibrium.

The impacts on welfare and macroeconomic aggregates depend critically upon the value of the constant elasticity of substitution in consumption between the two goods. Though the taxes are explicitly imposed on the home representative household, they effectively restrict the foreign representative household's access to the liquidity in the market for home currency-denominated bonds. These restrictions induce redistribution of liquidity in each of the bond markets. Hence, there are fluctuations in the exchange rate and the interest rates on bonds denominated in the currency of either country. As real activity of the world economy is affected, these taxes also have impacts on the welfare of each representative household. If a representative household's liquidity constraints are effectively loosened by these taxes, its welfare will be improved.

In addition, the discussion of the changes in the home government's policy rule may be interpreted as financial liberalization or deliberalization, depending upon the directions of the changes. It suggests that the common argument for financial liberalization, which claims that it can improve the country's welfare, is not necessarily true. The controls on international capital flows can restrict foreigners' access to liquidity in the domestic financial market, which is in favor of the domestic residents. When the economic agents are subject to liquidity constraints, liquidity effects induced by these controls should be taken into account. Previous work on capital and foreign exchange controls has overlooked the role of these liquidity effects in the determination of the effectiveness of these controls.

Table ? 1 : The Effects of Foreign Exchange Controls

The effects of a larger realization of the rate of taxes on the home country's residents' purchases of foreign currency, τ, given a fixed policy rule.

	$\frac{d\hat{e}}{d\tau}$	$\frac{de^R}{d\tau}$	$\frac{dp_1}{d\tau}$	$\frac{dp_2^*}{d\tau}$	$\frac{dp_2}{d\tau}$	$\frac{dp_1^*}{d\tau}$	$\frac{di}{d\tau}$	$\frac{di^*}{d\tau}$	$\frac{d\rho}{d\tau}$	$\frac{d\rho^*}{d\tau}$	$\frac{dz_h}{d\tau}$	$\frac{d(Tz_h)}{d\tau}$	$\frac{dz_f^*}{d\tau}$	$\frac{dl}{d\tau}$	$\frac{dc_1}{d\tau}$	$\frac{dc_1^*}{d\tau}$	$\frac{dl^*}{d\tau}$	$\frac{dc_2}{d\tau}$	$\frac{dc_2^*}{d\tau}$
$\sigma > 1$	−	−	−	−	?	?	−	−	+	−	+	?	−	−	+	−	+	−	+
$\sigma = 1$	−	−	−	0	0	+	−	0	+	−	+	−	0	−	+	−	0	0	0
$0 \leq \sigma < 1$	−	−	−	+	?	+	−	+	?	−	+	−	+	−	+	−	−	+	−

where $e^R \equiv \frac{p_2^*}{p_1} e$, $\rho \equiv \frac{p_2}{p_1}$, $\rho^* \equiv \frac{p_2^*}{p_1^*}$, $T \equiv (1-\tau)$, $Tz_h = p_1 c_1^*$, $z_f^* = p_2^* c_2$.

Chapter 3

Imperfect Information, Money, and Economic Growth

3.1 Introduction

This paper presents an endogenous growth model with financial market imperfections to study the effects of money growth on economic growth, and to examine the role of informational imperfections in the determination of the equilibrium growth path. The connection between money and economic activity and the economic significance of financial market imperfections are both investigated here within the context of an endogenous growth model.

Money has been incorporated into the neoclassical growth model in a large literature on money and growth. Tobin (1965) models money as a net asset of the private sector. The existence of uncertainties with respect to the timing of payments generally causes individuals to hold a portfolio which consists of both physical capital and real money balances. Higher money growth reduces the real value of existing cash balances. This results in portfolio substitution which in turn leads an increase in the economy's steady state capital stock. Levhari and Patinkin (1968) assign two different roles to money – either as a consumer's or a producer's good. That means, the role of money determines whether money enters the utility function or the production function. Their results suggest that money growth can either raise or lower the steady state capital stock. Some studies of money and growth using the cash-in-advance approach, [see for example Stockman (1981)], show that when a cash-in-advance constraint is imposed on both consumption and investment, the correlation of money and capital stock of the economy is negative.[1]

Recently, endogenous growth models have become the new direction in the literature on economic growth. The neoclassical growth model is modified so that it can display endogenous steady state growth. However, in most of these endogenous growth models, there are no roles for money and financial structure in the economy. With the development of endogenous growth theories, it seems to be beneficial to reexamine the link between money and economic growth within the context of an endogenous growth model. Howitt (1990) modifies the money-and-growth model of Levhari and Patinkin (1968) to incorporate the transaction-impeding aspect of inflation, endogenous growth of technology, and externalities. He finds that the model can magnify the nonsuperneutralities of money. The negative effects of long-term monetary expansion on economic growth and the real rate of interest can also be derived in his model. Gomme (1991) studies the welfare costs of money growth in an endogenous growth model with cash-in-advance constraints on consumption. With the existence of the cash-in-advance constraints, money growth reduces labor supply by lowering the effective return on working. Consequently, the real rate of growth of the economy is lowered by the money growth. In contrast, this paper introduces money to the model as one of the portfolio choices of individuals.[2] This approach to modeling the role of money results in a positive correlation of money growth and economic growth.

For the financial aspects of the economy, the recent developments in modeling

[1]It is noted that in all of these neoclassical growth models, economic growth is exogenously determined by technological change. Money can affect the level of output by altering the level of capital stock of the economy, while it has no effect on the economic growth rate.

[2]Unlike Tobin (1965), there is no uncertainty in the timing of payments in this model. The diminishing returns to capital in the production technologies induce individuals to include money in their portfolios.

financial intermediation have been applied within the context of business cycle models, for example, Williamson (1987a) and (1987b), and Bernanke and Gertler (1988). Their results suggest that the explicit modeling of private information can provide new insight into understanding macroeconomic fluctuations. This paper follows their line of research to model private information explicitly, but applying this in an endogenous growth environment. The impacts of the asymmetric distribution of information across lenders and borrowers on the determination of the equilibrium growth path are examined. Effects of changes in money growth on the economic growth rates in economies with and without private information are also compared.

There are some recent studies on the relationship between financial intermediation and economic growth. For example, Greenwood and Jovanovic (1990) study the link between economic growth and the distribution of income, and the connection between the development of financial structure and economic development. Bencivenga and Smith (1991) analyze the conditions which imply that the development of financial intermediation will increase the real growth rate. Bencivenga and Smith (1990) investigate the optimal degree of financial repression in a developing country faced with a sustained deficit that must be monetized. Unlike these papers, our focus is on the role of private information in investment opportunities in the determination of the equilibrium financial structure and economic growth, which has not been addressed in these other analyses.

In this paper, a set of risky investment projects is introduced to a Romer (1986) type growth model. The financial structure of the economy depends on the distribution of information across lenders and borrowers in the credit market. If the riskiness of the investment projects is private information, internal finance constraints will arise endogenously in equilibrium. The loan market is shut down by a severe adverse selection problem. Agents with high quality projects (projects with high success probabilities) face binding internal finance constraints, so that their projects suffer from underinvestment problems. Moreover, there is a misallocation of investment in the economy with private information. Hence, the economy will grow slower in the imperfect information case than in the full information case.

It is shown that changes in the money growth rate have qualitatively similar effects on the endogenous growth rate of the economy in the full information case and in the imperfect information case. However, these effects are quantitatively different. An increase in money growth will enlarge the the difference between the economic growth rates in these two cases. Because of the existence of binding finance constraints, those agents subject to binding finance constraints are restrained from adjusting their portfolio in response to the changes in the market rate of return. Therefore, the economy with private information is less responsive to monetary shocks than the economy with full information.

This result contradicts the popular view that informational imperfections in credit markets or borrowing constraints tend to amplify the impacts of policy interventions. Models of imperfect information in credit markets like Stiglitz and Weiss (1981), and Greenwald and Stiglitz (1988a) typically imply that competitive credit markets may be characterized by credit rationing if there are informational asymmetries in credit markets. Greenwald and Stiglitz (1988b) analyze the macroeconomic consequence of informational imperfections in equity market by simply imposing exogenous finance constraints on the economic agents. These models suggest that, by changing the financial positions of the agents who have been credit-constrained, economic shocks will have important and significant effects on the behavior of these agents. Consequently, these models conclude that, with the presence of imperfect information, the economy will be affected more dramatically by the changes in the economic environment and policies than if there is full information. However, our model predicts the opposite.

The remainder of the paper is organized as follows. The model is presented in Section 2. The equilibrium growth paths of the economies with and without private information are compared. In Section 3, the economic implications of informational asymmetries to the effects of changes in money growth are discussed. The conclusion is in Section 4.

3.2 The Model

An endogenous growth model with risky investment projects is now presented. Romer (1986) makes sustained growth feasible through a combination of aggregate increasing returns and diminishing private returns to capital. This paper follows Romer's approach to modeling the externalities in production. This approach allows the long-run growth rate to be an endogenous outcome of time invariant technologies. Economic growth is driven by the endogenous augmentation of the capital stock in the economy. Preferences and production possibilities are restricted so that the economy can display steady state growth. In order to study the role of informational imperfections in financial markets for the determination of the equilibrium growth path, a set of risky investment projects is introduced to the capital good sector of the model. The demand for and supply of external financing arise endogenously from the heterogeneity of the projects owners' investment decisions. In the presence of credit market imperfections, their investment decisions will be affected asymmetrically by changes in the economic environment or in policies.

3.2.1 Economic Environment

This economy consists of an infinite sequence of two-period-lived overlapping generations. Time is discrete, and indexed by $t = 0, 1, \cdots$. There are two goods in this economy, a non-storable consumption good and a capital good.

Preferences and Endowments

At $t = 1$, there is an initial old generation endowed with an initial per capita capital stock, k_o. Assume that there is no population growth in the economy. In each period, a continuum of young agents, distributed over the unit interval, is born. Each agent lives for two periods. All agents are assumed to have identical preferences. The utility function of agent i who is born in period t is given by

$$U_i^t\,(C_{it}^t, C_{it+1}^t) = C_{it+1}^t, \tag{1}$$

where C_{ij}^t is the consumption of agent i from generation t in period j, $j = t, t+1$. Each young agent does not value the first-period consumption, his welfare depends only on his second-period consumption. In his first period of life, each agent is endowed with a single unit of labor, which is supplied inelastically, and an investment project characterized by a distinct success probability, which can convert the current-period consumption good into the next-period capital good in the event of success.

Production

The non-storable consumption good is produced from capital and labor by perfectly competitive firms according to the following production function,

$$Q_t = Q(z_t, l_t, Z_t) = A\, z_t^{\sigma}\, l_t^{1-\sigma}\, Z_t^{1-\sigma}, \qquad \sigma \in (0, 1), \tag{2}$$

where A is a positive parameter. The output level of the consumption good produced by a firm, Q_t, depends on the firm-specific inputs of labor, l_t, and capital, z_t, and on the aggregate level of knowledge in the economy, which is represented by the economy-wide capital stock in period t, Z_t. The aggregate level of knowledge has a positive external effect in the production of the consumption good. Hence the production function Q exhibits social increasing returns to scale in production. Given the specification of this production function, $Z_t\, l_t$ can be interpreted as the "efficiency units of labor".

Because of the assumed homogeneity of Q with respect to the factors of production, z_t and l_t, perfectly competitive firms earn zero profits. The scale and number of firms will be indeterminate. However, the specification of the number of firms in the consumption good market is irrelevant for our analysis.

The capital good is produced by an investment technology, which can convert the current-period consumption good into the next-period capital good in the event of success. No labor input is required for the production of the capital good. In each period, there is a continuum of risky investment projects distributed over the unit interval $[0, 1]$. The investment projects differ in risk (the success probabilities). The success probability of a project is known by the project owner when he is born. Investment projects are not transferable. Each investment project can be operated by its owner only. There is a time lag of one period between the investment and the realization of the output. Hence each agent who decides to operate his project will invest when he is young. In addition, the capital good cannot be consumed but can be used in the production of the consumption good.

In period t, the owner of project i (the project with probability of success equal to ρ_{it}), chooses to invest $x_{it} + y_{it}$ units of the consumption good in his own project, where x_{it} is the quantity of the consumption good provided by the project owner (internal financing), while y_{it} is the quantity of the consumption good borrowed from other young agents (external financing). The details of the financing decisions will be discussed in the following sections. The outcome of investment project i, k_{it}, is a random variable

$$k_{it} = \begin{cases} \hat{k}_{it} & \text{with probability} \quad \rho_{it}\,, \\ 0 & \text{with probability} \quad 1 - \rho_{it}\,. \end{cases}$$

If the investment project is successful, $\hat{k}_{it}$ units of the capital good will be produced, and available to the production of the consumption good in period $t + 1$. If the investment project fails, nothing can be produced. The distribution of ρ_{it} across all of the investment projects in period t is characterized by a time invariant distribution function $F(\rho_{it}) = \rho_{it}$, with density function $f(\rho_{it}) = 1$, $\rho_{it} \in (0, 1)$, $\forall\, t$. The realization of k_{it} is revealed to everybody at the beginning of period $t+1$. In the event of success, the output level of project i, $\hat{k}_{it}$, is a concave function of the input of the consumption good, $(x_{it} + y_{it})$,

$$\hat{k}_{it} = \hat{k}_i(x_{it} + y_{it}, Y_{t-1}) = a(x_{it} + y_{it})^{\alpha}\, Y_{t-1}{}^{1-\alpha}, \qquad \alpha \in (0, 1), \tag{3}$$

where a is a positive parameter. The total investment of the economy in period t is

$$Y_t = \int_0^1 (x_{it} + y_{it})\, d\rho_{it}\,. \tag{4}$$

The term Y_{t-1} in equation (3) represents the positive external effect in the production of the capital good. The productivity of investment in period t depends on the level of technology (knowledge) of the economy which is captured by the total investment in period $t-1$, Y_{t-1}.

It is noted that Y_{t-1} is positively correlated with the economy-wide capital stock in period t, Z_t. Thus the role of Y_{t-1} in equation (3) is similar to that of Z_t in equation (2). In order to have sustained, endogenous economic growth, it is necessary to have externalities in both of the production technologies for the consumption good and for the capital good. These positive external effects allow all of the factors of production (capital and efficiency units of labor) to be reproducible. As production functions Q and $\hat{k}_i$ are constant returns to scale with respect to capital and efficiency units of labor, it will be feasible for the economy to sustain constant growth rates.

Though the outcomes of individual projects are uncertain, by the law of large numbers, there is no aggregate uncertainty in this economy. Assume that the capital good depreciates completely after the production of the consumption good. Then, the capital stock of the economy in period $t+1$ equals the total output of all investment projects undertaken in period t, which is given by

$$Z_{t+1} = \int_0^1 \rho_{it}\, \hat{k}_{it}\, d\rho_{it}\,. \tag{5}$$

In period $t+1$, the old generation obtains Z_{t+1} units of the capital good. The firms which produce the consumption good will rent the capital at the market rental rate. q_{t+1} units of the consumption good, for their production.

Government

The government issues fiat money and distributes it to the old agents as lump-sum transfers. At the beginning of period 1, the government gives m_o units of money to each of the initial old agents. At the beginning of period t, each old agent of generation $t-1$ receives a lump-sum transfer of T_{t-1} units of the newly issued money from the government. Let the growth rate of the money supply of the economy be denoted by μ, which is assumed to be constant over time and public knowledge to everybody. The evolution equation of the money supply in this economy is given by

$$M_t = (1 + \mu) M_{t-1},$$

where M_t is the economy-wide money stock at the end of period t. The government is assumed to engage in no economic activity except for the monetary transfers, so the budget constraint of the government is

$$T_{t-1} = \mu M_{t-1}.$$

Then, the old agents will sell their money holdings, which include the lump-sum transfers from the government and the money carried from their first period of life, to the young agents of generation t. Let p_t denote the price of money in terms of the consumption good in period t.

Financial Contracts

The assumption of risk neutrality of the economic agents allows us to ignore the role of financial intermediaries in this economy. Since the outcome of each investment project has only two possible realizations and a failure implies zero output, it is obvious that the optimal financial contract is a standard debt contract. This means, the financial contract specifies the fixed repayment in nonbankruptcy, and the maximum repayment in bankruptcy. As a borrower gets nothing from his project in the event of failure, he will declare bankruptcy. With limited liability, the maximum repayment of the debt in bankruptcy will be zero.

3.2.2 Portfolio Decisions

Agents only value their second-period consumption, while they are able to earn labor income only in their first period of life. They must save to finance their second-period consumption. In every period, each agent can save by way of holding money, investing in his own project, and / or lending to finance other agents' projects.

Young agent i of generation t supplies his labor effort to the firms which produce the consumption good at the market wage rate, w_t. Hence his savings, s_{it}, is equal to w_t. Given the preferences specified in (1), the objective of agent i is to maximize the expected value of his second-period consumption, $\mathrm{E}\,[\,C^t_{it+1}\,]$. The agent chooses to hold the portfolio which maximizes expected return. Let p^e_{t+1} be the expected price of money in period $t+1$. The expected gross rates of return on money and on loans traded among the young agents in period t are denoted by p^e_{t+1}/p_t and R_t respectively. Both p^e_{t+1}/p_t and R_t are denominated in terms of the consumption good. It is obvious that for the existence of a monetary equilibrium, p^e_{t+1}/p_t and R_t should be equalized.

In the optimal portfolio of agent i, he holds m_{it} units of money, invests $x_{it} + y_{it}$ units of the consumption good in his investment project, and lends $s_{it} - m_{it}\,p_t - x_{it}$ units of the consumption good to other young agents. To accomplish this optimal portfolio decision, agent i borrows y_{it} units of the consumption good from other young agents. He has to pay the gross interest rate of r_{it} units of the capital good in the event of nonbankruptcy. If the project fails, he will receive nothing from the project, and will declare bankruptcy. Thus, the expected return to agent i of generation t on his project, R_{it}, is

$$R_{it} = \rho_{it} \left[a \left(x_{it} + y_{it} \right)^{\alpha} Y_{t-1}^{\ 1-\alpha} - r_{it} \, y_{it} \right] q_{t+1} \, .$$

The expected marginal gain from internal financing, which is defined as the expected marginal return from internal financing net of its opportunity cost R_t, is given by

$$\hat{R}_{it} = \rho_{it} \, \alpha \, a \left(x_{it} + y_{it} \right)^{\alpha-1} Y_{t-1}^{\ 1-\alpha} q_{t+1} - R_t \, ,$$

while the expected marginal return from financing the project externally is

$$\tilde{R}_{it} = \rho_{it} \left[\alpha \, a \left(x_{it} + y_{it} \right)^{\alpha-1} Y_{t-1}^{\ 1-\alpha} - r_{it} \right] q_{t+1} \, .$$

How investment project i will be financed is determined by the relative size of the expected marginal returns $\hat{R}_{it}$ and $\tilde{R}_{it}$. Project i will be jointly financed by internal and external funds only if $\hat{R}_{it} = \tilde{R}_{it}$.

As each investment project is characterized by a project-specific success probability, the gross interest rate of external funds in the state of nonbankruptcy, r_{it}, may also be project-specific. The following discussions will show that the determination of r_{it} and the young agents' optimal portfolio decisions depend crucially on the distribution of information across lenders and borrower in the economy. Two cases will be analyzed. The first is the full information case in which the probability of success of each investment project is costlessly observable to everybody. The second is the imperfect information case in which the probability of success of each project can be observed by its owner only.

3.2.3 A Model with Full Information

As the success probability of each project can also be observed by agents other than the project owner, lenders are able to distinguish projects with different expected returns. They will charge a higher interest rate, r_{it}, on loans to a project with a lower value of ρ_{it}. Assume that the loan market is perfectly competitive. This implies that the expected gross rates of return from lending to different borrowers will be equalized,

$$R_t = q_{t+1}\,\mathrm{E}[\,r_{it}\,], \qquad \forall\, i,$$

so r_{it} is determined by

$$r_{it} = \frac{R_t}{q_{t+1}\,\rho_{it}}.$$

Furthermore, the zero-profit condition for lenders implies that the expected rate of return on loans should be equal to that on money, that is, $R_t = p^e_{t+1}/p_t$. As there is no aggregate uncertainty in the economy, the expected values of the rates of return are respectively equal to their actual values in equilibrium.

The optimal level of investment, $x_{it} + y_{it}$, is implied by the following first-order condition

$$\rho_{it}\,\alpha\,a\,(\,x_{it} + y_{it}\,)^{\alpha-1}\,Y_{t-1}{}^{1-\alpha}\,q_{t+1} - R_t = 0. \tag{6}$$

In the equilibrium the expected gross market rate of return on loans (the opportunity cost of internal funds), R_t, is always equal to the expected cost of external funds, $q_{t+1}\,\mathrm{E}[\,r_{it}\,]$. Consequently, the quantities of internal and external financing, x_{it} and y_{it}, are indeterminate. It is because the expected marginal returns $\hat{R}_{it} = \tilde{R}_{it}$, agent i is indifferent between internal and external financing. This indeterminacy in the financial structure is just an example of the Modigliani-Miller theorem. With complete markets and complete information, the financing decision of an investment project does not matter. There is also an indeterminacy in the portfolio decision, $(\,x_{it},\,m_{it}\,)$, because the expected rates of return to holding money, lending and the expected cost of external funds are equalized in the equilibrium.

The total investment of the economy in period t can be derived by substituting equation (6) into equation (4). Then we have

$$Y_t = Y_{t-1} \left[\frac{\alpha\, a\, q_{t+1}}{R_t} \right]^{\frac{1}{1-\alpha}} \frac{1-\alpha}{2-\alpha}. \tag{7}$$

The allocation of the aggregate savings of generation t, S_t, between money and investment should satisfy

$$S_t = s_{it} = w_t = M_t\, p_t + Y_t. \tag{8}$$

By using equations (3), (5) and (6), the capital stock of the economy in period $t+1$, Z_{t+1}, will be

$$Z_{t+1} = a\, Y_{t-1} \left[\frac{\alpha\, a\, q_{t+1}}{R_t} \right]^{\frac{\alpha}{1-\alpha}} \frac{1-\alpha}{2-\alpha}. \tag{9}$$

Equilibrium

In the competitive equilibrium of the economy, each young agent maximizes the expected return of his portfolio, taking the set of pricing rules and the government's monetary transfers as given, and all markets are cleared each period. The market-clearing conditions for the economy are as follows,

labor market: $$l_t = 1, \tag{10}$$

the capital good market: $$z_t = Z_t, \tag{11}$$

the consumption good market: $$\int_0^1 C_{it}^{t-1}\, d\rho_{it-1} + \int_0^1 (\, x_{it} + y_{it}\,)\, d\rho_{it} = Q_t, \tag{12}$$

money market: $$M_t = \int_0^1 m_{it}\, d\rho_{it}, \tag{13}$$

loan market: $$\int_0^1 y_{it}\, d\rho_{it} = \int_0^1 (\, s_{it} - m_{it} - x_{it}\,)\, d\rho_{it}. \tag{14}$$

The equilibrium rental rate of capital, q_t, and wage rate, w_t, in period t are,

$$q_t = \sigma A \tag{15}$$

$$w_t = (1-\alpha)\, Q_t = (1-\alpha)\, A\, Z_t. \tag{16}$$

In addition, the gross rates of return on loans and money are equalized in equilibrium,

$$R_t = \frac{p_{t+1}}{p_t}. \tag{17}$$

Balanced Growth

The long-run properties of this economy will now be examined. With the specification on preferences and technologies in this model, the economy exhibits steady state growth. The balanced growth path of this economy is endogenous and deterministic. Denote the growth rate of a variable n as g_n (i.e., $1 + g_n = n_{t+1}/n_t$). Along a balanced growth path, output levels of the consumption good and the capital good, consumption, investment, wage rate of labor and the price of money are all growing at the same rate, g. Any feasible balanced growth path requires

$$g_Q = g_Z = g_{C_i} = g_Y = g_{x_t} = g_{y_t} = g_w = g_p = g,$$

and the growth rate of money supply, μ, the gross interest rates, R_t and r_{it}, the gross rate of return on money, p_{t+1}/p_t, and the rental rate of capital, q_t, are all constant over time. These implies that the share of money in the aggregate savings of each young generation, $(M_t p_t/S_t)$, is also constant over time. Then the relationship between the endogenous growth rate of the economy, g, and the exogenous growth rate of money supply, μ, is

$$(1+g) = R(1+\mu). \tag{18}$$

From equations (7), (15) and (18), the endogenous constant economic growth rate and the gross interest rate can be determined

$$1 + g = \left[\alpha a A \sigma \, (1+\mu) \left[\frac{1-\alpha}{2-\alpha}\right]^{1-\alpha}\right]^{\frac{1}{2-\alpha}}, \tag{19}$$

$$R = \left[\alpha a A \sigma \, (1+\mu)^{-(1-\alpha)} \left[\frac{1-\alpha}{2-\alpha}\right]^{1-\alpha}\right]^{\frac{1}{2-\alpha}}. \tag{20}$$

From equations (7)-(9) and (18)-(20), the fraction of aggregate savings of each generation invested in the investment projects, (Y_t/S_t), can be derived

$$\frac{Y_t}{S_t} = 1 - \frac{M_t p_t}{S_t} = \frac{\sigma \alpha}{1-\sigma}(1+\mu). \tag{21}$$

Equations (19)-(21) shows that the exogenous money growth rate, μ, plays an important role in the economy. It can be shown that

$$\frac{dR}{d\mu} < 0, \qquad \frac{d(Y_t/S_t)}{d\mu} > 0, \qquad \frac{dg}{d\mu} > 0.$$

An increase in μ implies a faster growth in the money supply. The price of money in units of the consumption good, p_t, decreases over time. Young agents shift from holding money to lending or investment. The shifts in their portfolio decisions reduce the gross market interest rate, R, and increase the share of total investment in aggregate savings, (Y_t/S_t). As a result, the endogenous growth rate, g, becomes higher.

3.2.4 A Model with Imperfect Information

To understand the role of informational imperfections in the economy, we now assume that the success probabilities are private information. If the success probability of each project can be costlessly observed by its owner only, lenders are unable to distinguish the borrowers, and to charge different interest rates depending on the riskiness of individual projects. As all of the economic agents are risk neutral, it is obvious that a separating equilibrium does not exist. Hence we only consider pooling equilibria.

Though lenders have no information on the success probabilities of individual projects, they know the distribution of success probabilities of the population of borrowers, $F(\rho_{it})$. In a pooling equilibrium, all agents face the same gross interest rate in nonbankruptcy, $r_{it} = r_t, \ \forall\, i$. The expected gross rate of return to lenders is,

$$R_t = r_t \bar{\rho}_t q_{t+1}, \tag{22}$$

where $\bar{\rho}_t = \mathrm{E}[\rho_{jt}]$ is the expected (average) success probability among all agents j who seek external financing in period t.

To determine how project i should be financed, young agent i compares the expected marginal returns $\hat{R}_{it}$ and $\tilde{R}_{it}$,

$$\hat{R}_{it} = [\rho_{it} \alpha a (x_{it} + y_{it})^{\alpha-1} Y_{t-1}{}^{1-\alpha} - r_t \bar{\rho}_t] q_{t+1},$$

$$\tilde{R}_{it} = [\rho_{it} \alpha a (x_{it} + y_{it})^{\alpha-1} Y_{t-1}{}^{1-\alpha} - r_t \rho_{it}] q_{t+1}.$$

If an agent has a success probability lower than the average success probability among agents seeking external funds, (i.e., $\rho_{it} < \bar{\rho}_t$), this implies that $\hat{R}_{it} < \tilde{R}_{it}$. This is because the expected cost of external funds faced by this agent, $r_t \rho_{it} q_{t+1}$, is lower than the opportunity cost of internal funds, R_t. Then, it is optimal for this agent to use only external finance for his project, that is, $x_{it} = 0$. All his labor income should be saved in the forms of money and loans only. The optimal quantity of external financing is determined by letting $\tilde{R}_{it} = 0$. This implies,

$$\alpha a y_{it}{}^{\alpha-1} Y_{t-1}{}^{1-\alpha} - r_t = 0. \tag{23}$$

As equation (23) is independent of ρ_{it}, the demand for external funds by each agent i seeking external financing will be the same, $y_{it} = y_t$. In addition, for external financing to be feasible, it has to be the case that $y_t < w_t$.

If $\rho_{it} > \bar{\rho}_t$, the expected cost of external financing of agent i is higher than his opportunity cost of internal financing. That means, $\hat{R}_{it} > \tilde{R}_{it}$. This agent will choose to use only internal finance for his project, thus $y_{it} = 0$. Agents with $\rho_{it} = \bar{\rho}_t$ are indifferent between internal and external financing as $\hat{R}_{it} = \tilde{R}_{it}$. It is assumed that they will use internal financing only. The higher the success probability of a project is, the more the investment the project owner would like to make. However, the quantity of internal financing is constrained by the labor income of the project owner. Let ρ_t^* be the success probability of the marginal project which satisfies

$$\rho_t^* \, \alpha \, a \, s_t^{\alpha-1} \, {Y_{t-1}}^{1-\alpha} \, q_{t+1} \; - \; R_t \; = \; 0. \tag{24}$$

Therefore, agents with $\rho_{it} \geq \rho_t^*$ essentially face binding internal finance constraints. They will choose the maximum level of investment which is feasible for them, that is, $x_{it} = s_{it}$.

For agents with $\bar{\rho}_t \leq \rho_{it} < \rho_t^*$, the optimal quantity of internal financing, x_{it}, is determined by letting $\hat{R}_{it} = 0$, that is,

$$\rho_{it} \, \alpha \, a \, {x_{it}}^{\alpha-1} \, {Y_{t-1}}^{1-\alpha} \, q_{t+1} \; - \; R_t \; = \; 0. \tag{25}$$

Agent i will save some of his labor income, $s_{it} - x_{it}$ units of the consumption good, by way of holding money and lending. From equations (22), (23) and (25), it is clear that the quantity of investment of each agent with $\rho_{it} \geq \bar{\rho}_t$, x_{it}, is larger than the quantity of investment of each agent with $\rho_{it} < \bar{\rho}_t$, y_t.

Equilibrium

There is a severe adverse selection problem in the loan market. Agents who use external funds are those with high risk, $\rho_{it} < \bar{\rho}_t$. However, in equilibrium it cannot be the case that all agents have probabilities of success below the average success probability of all borrowers. As a result, the loan market is shut down by private information.[3] Since loan market does not exist, the opportunity cost of internal funds is equal to the gross rate of return on money, p_{t+1}/p_t, only. However, we will keep using R_t to represent p_{t+1}/p_t in the following discussions.

In equilibrium, all projects will be financed internally. Agents with $\rho_{it} \geq \rho_t^*$ will choose $x_{it} = s_{it}$. Agents with $\rho_{it} < \rho_t^*$ will determine x_{it} from equation (25), and the demand for money of agent i will be

$$m_{it} \; = \; s_{it} \; - \; x_{it} \, .$$

[3] This is an extreme example of Akerlof's (1970) "lemons principle." He shows that it is quite possible to have the bad driving out the good so that there is no trade on the market.

From equations (3), (4), (24) and (25), the total investment of the economy in period t and the economy-wide capital stock in period $t+1$ are

$$Y_t = \left[1 - \frac{\rho_t^*}{2-\alpha}\right] S_t\,, \tag{26}$$

$$Z_{t+1} = \frac{1}{2}\, a\, S_t^{\alpha}\, Y_{t-1}{}^{1-\alpha} \left[1 - \frac{\alpha\, \rho_t^{*2}}{2-\alpha}\right]. \tag{27}$$

In the competitive equilibrium of the economy with imperfect information, no loans are traded among agents. Each agent maximizes the expected return of his portfolio, subject to the fact that external financing is not available.[4] In addition, as in the full information case, the market clearing conditions (10)-(13) hold, and the equilibrium rental rate of capital and wage rate of labor are respectively given by equations (15) and (16).

Balanced Growth

The balanced growth path of the economy with imperfect information can now be determined. Along the balanced growth path, it is required that

$$g_Q = g_Z = g_{C_t} = g_Y = g_{x_i} = g_w = g_p = g,$$

and the growth rate of money supply, μ, the gross rate of return on money, p_{t+1}/p_t, the rental rate of capital, q_t, the success probability of the marginal project, ρ_t^*, and the share of money in the aggregate savings of each young generation, $(M_t\, p_t / S_t)$, are all constant over time. From equations (16), (18), (26) and (27), the equilibrium balanced growth path can be determined,

$$1 + g = \left[\alpha\, a\, A\, \sigma\, (1+\mu)\, \rho^* \left[1 - \frac{\rho^*}{2-\alpha}\right]^{1-\alpha}\right]^{\frac{1}{2-\alpha}}, \tag{28}$$

[4]Bernanke and Gertler (1989) have studied the importance of internal finance constraints in a business cycle model. They suggest that the presence of informational imperfections in the credit markets amplifies the magnitudes of business fluctuations. However, as we will show, this model will predict the opposite.

$$R = \left[\alpha\, a\, A\, \sigma\,(1+\mu)^{-(1-\alpha)}\, \rho^{*} \left[1 - \frac{\rho^{*}}{2-\alpha}\right]^{1-\alpha}\right]^{\frac{1}{2-\alpha}}, \tag{29}$$

$$1+\mu = \frac{1-\sigma}{2\,\sigma\,\alpha\,\rho^{*}} \left[1 - \frac{\alpha\,\rho^{*2}}{2-\alpha}\right]. \tag{30}$$

By combining equations (28) and (30), the endogenous constant growth rate of this economy, g, can be expressed in terms of the exogenous money growth rate and the parameters of preferences and technologies only. Furthermore, equations (26) and (28)-(30) imply that

$$\frac{d\rho^{*}}{d\mu} < 0, \quad \frac{dR}{d\mu} < 0, \quad \frac{d(Y_t/s_t)}{d\mu} > 0, \quad \frac{dg}{d\mu} > 0.$$

When the money growth rate increases, the price of money in terms of the consumption good decreases over time. The gross rate of return on money falls. Agents shift from holding money to investing in their own projects. Thus the measure of agents facing binding finance constraints, $(1-\rho^{*})$, increases. As a greater fraction of the aggregate savings of the economy is devoted to the investment projects, the economy will experience a higher growth rate. These effects of an increase in μ on the economy are qualitatively similar to those obtained in the full information case. However, these effects are quantitatively different in these two cases. A detailed discussion will be provided in Section 3.

3.2.5 Money and Growth

In the literature on money and growth, the roles of money have been modeled quite differently. In cash-in-advance models, [e.g. Gomme (1991)], money is introduced through a cash-in-advance constraint on consumption, and higher money growth discourages labor supply by reducing the effective return to working. This leads agents to substitute leisure for labor. As a result, output drops, and the economy's growth rate falls. In transaction-based models, [e.g. Howitt (1990)], money enters the models as a medium of exchange which helps to economize on the transaction costs of exchange. Higher money growth increases the opportunity cost of holding money. Agents reduce their money balances so as to minimize the loss of purchasing power. This implies that more real resources are devoted to facilitate transactions, consequently reducing economic growth. In contrast, this model follows the portfolio approach. Money competes with capital for a place in the portfolios of agents. An increase in money growth induces portfolio substitution. Economic agents shift away from money and toward physical investment, which in turn raises the economic growth rate. Obviously, the transmission mechanisms of the effects of higher money growth and the resulting impacts on economic growth depend on which role we assign to money in the economy. However, all these roles of money are important in practice.

Gomme (1991) examines the welfare costs resulting from higher money growth in an endogenous growth model. In our model, the study of the welfare effects of increases in the money growth are complicated by the externalities in production and the heterogeneity among economic agents' investment opportunities in this overlapping generation setup.[5] As higher money growth will affect each agent differently, a meaningful welfare comparison seems to be very difficult. Loosely speaking, the welfare for agents of earlier generations will decrease as the output devoted to their consumption decreases. Higher economic growth means that more output of the consumption good is devoted to physical investment and less can be allocated to consumption. With time, this negative effect of economic growth on the levels of consumption will be dominated by the positive effect of the higher balanced growth rate on consumption. Thus, the levels of consumption for generations far enough in the future will be increased by the higher money growth.

[5] As we know, the presence of externalities and the overlapping generation structure imply that the competitive equilibrium of the economy in either the full information case or the imperfect information case is not Pareto optimal.

3.3 Information and Economic Growth

The preceding analysis has shown that the economy can sustain constant growth rates in both the full information case and the imperfect information case. In general the endogenous growth rates can be greater or less than one. Hence the economy may experience either positive or negative real growth, depending on the values of the parameters of preferences, technologies and money growth rate. The role of informational imperfections in the determination of the equilibrium growth path can be studied by comparing the equilibrium growth paths of the economies with and without private information.

It can be shown that, for any set of parameter values, the economic growth rate in the full information case, g^F, given by equation (19), is greater than or equal to the growth rate in the imperfect information case, g^I, given by equation (28). From now on, we use the superscripts ^{F} and ^{I} to denote the variables of the full information case and the imperfect information case, respectively.

Define Δ as the ratio of the economic growth rates in the two cases. From equations (19), (20), (28) and (29), it is found that

$$\Delta \equiv \frac{g^I}{g^F} = \frac{R^I}{R^F} = \left[\rho^{*\frac{1}{1-\alpha}}\left[1 - \frac{\rho^*}{2-\alpha}\right]\left[\frac{2-\alpha}{1-\alpha}\right]\right]^{\frac{1-\alpha}{2-\alpha}}, \qquad \frac{d\Delta}{d\rho^*} > 0, \tag{31}$$

and $0 < \Delta \leq 1$, which is equal to one only when $\rho^* = 1$. It is obvious that the endogenous growth rates, g^F and g^I, will be the same if the equilibrium success probability of the marginal project in the imperfect information case, ρ^*, is equal to one. That is, the measure of agents who face binding internal finance constraints is zero. Equation (30) implies that

$$\rho^* = 1 \quad \Leftrightarrow \quad 1 + \mu \leq \frac{(1-\sigma)(1-\alpha)}{\sigma\alpha(2-\alpha)}. \tag{32}$$

This condition can be interpreted as follows. Given the parameter values from preferences and technology, when the growth rate of money supply is sufficiently small, agents do not have to worry about the loss of purchasing power of their money balances due to the growth in money supply. Each agent devotes a large fraction of his savings to money. As the opportunity cost of investment is so high that each agent chooses to invest less in his project. In fact, the quantity of investment of each agent is less than his labor income. In the full information case, there is no demand for external funds, thus no loans will be traded. In the imperfect information case, though external financing is not available, the desired quantity of investment of each agent can be sufficiently financed by his labor income. No agents face binding finance constraints. The equilibrium gross market rates of return, R^F and R^I, are equal. This implies that the equilibrium portfolio decisions in the full information case are the same as those in the imperfect information case. Thus, the fractions of aggregate savings invested in the projects are also the same, $(Y_t/S_t)^F = (Y_t/S_t)^I$. In both cases, the expected marginal benefits from investment are equalized across all projects in the economy. Hence the informational asymmetries between lenders and borrowers in the economy do not matter when condition (32) is satisfied.

When the growth rate of the money supply is sufficiently high, condition (32) fails to hold. The presence of private information on the success probabilities of individual projects results in an equilibrium growth rate which is relatively lower than the growth rate in the full information case. Since the money growth rate is high, the gross rate of return on money becomes lower. In the full information case, those agents with high success probabilities would like to invest more than their labor income, which they do through external funding. They do not hold any money. For those agents with low success probabilities, the share of money in each agent's portfolio is small. These agents not only invest in their own projects but also supply external financing to those agents with high success probabilities. In equilibrium, the expected marginal benefits from investment of each project will equal to the gross market rate of return, R^F.

In the imperfect information case, agents also prefer to invest a large fraction of their savings in projects as in the full information case. However, the loan market is shut down by the severe adverse selection problem. Agents who desire to invest more than their labor income are facing binding internal finance constraints. Their investment is constrained to be equal to their labor income. Agents with low success probabilities can choose only between money and investment. This implies that the share of money in each of these agents' savings will be higher than if there is full information.

There are two reasons for the endogenous growth rate of the economy to be lower in the imperfect information case than in the full information case, $g^F > g^I$. The first is the underinvestment problem in the economy with private information. As agents with high success probabilities are subject to binding finance constraints, they invest relatively less than if there is full information. From equations (21), (26) and (30), it can be shown that in the full information case, the economy will invest a greater fraction of its aggregate savings in projects than in the imperfect information case, $(Y_t/S_t)^F > (Y_t/S_t)^I$,

$$\lambda \equiv \frac{(Y_t/S_t)^F}{(Y_t/S_t)^I} = \frac{2\rho^*(2-\alpha-\rho^*)}{2-\alpha-\alpha\rho^{*2}} \quad \text{and} \quad \frac{d\lambda}{d\rho^*} > 0, \tag{33}$$

where $0 < \lambda \leq 1$, and $\lambda = 1$ when $\rho^* = 1$. The larger the measure of agents facing binding internal finance constraints is, the greater the difference between $(Y_t/S_t)^F$ and $(Y_t/S_t)^I$ will be.

The second reason for $g^F > g^I$ is the misallocation problem in the economy with private information. The expected marginal benefits from investment are equalized across those projects with $\rho \leq \rho^*$ only. For projects with $\rho > \rho^*$, their expected marginal benefits from investment are higher than the gross market rate of return,

R^I. That means, comparing with the allocation of investment in the full information case, the economy with private information invests too little in the projects with high success probabilities, and too much in the projects with low success probabilities. Hence, for any given level of total investment, Y_t, the total output of the capital good, Z_{t+1}, will be lower in the imperfection information case than in the full information case.

The Impacts of an Increase in the Money Growth Rate

Consider the equilibrium impacts of an increase in μ. In section 2, it has been shown that the impacts of any given change in μ on the economy in the full information case and the imperfect information case are qualitatively similar to each other. The quantitative differences of the impacts in these two cases are now examined.

The increase in the growth rate of the money supply implies a decrease in the rate of return on money. As the opportunity cost of investment becomes lower, agents will adjust their portfolio decisions by reducing their money holdings. In the full information case, each agent increases the fraction of savings invested in his own project. Thus the fraction of aggregate savings invested in projects, $(Y_t/S_t)^F$, increases, which results in a higher real growth rate of the economy.

However, in the imperfect information case, agents who have already been subject to binding internal finance constraints cannot increase their investment. They will not respond to the decrease in the gross market rate of return. Only agents with lower success probabilities can respond to the monetary shock by increasing their investment. Consequently, more agents' finance constraints will become binding. As the fraction of the economy's aggregate saving invested in projects, $(Y_t/S_t)^I$, increases, the economy will experience higher economic growth. However in the imperfect information case, the increase in μ will raise the growth rate of the economy less than in the full information case.

Actually, the increase in μ causes ρ^* to fall, which worsens both the the under-investment and the misallocation problems in the economy with private information. Equations (30) and (33) imply that the ratio λ decreases when μ increases.

$$\frac{d\lambda}{d\rho^*} > 0 \quad \text{and} \quad \frac{d\rho^*}{d\mu} < 0 \quad \Rightarrow \quad \frac{d\lambda}{d\mu} < 0.$$

More agents' finance constraints become binding. These agents cannot increase their investment, so that the difference between $(Y_t/S_t)^F$ and $(Y_t/S_t)^I$ is enlarged. When more projects' expected marginal benefits from investment are higher than the gross market rate of return, R^I, the misallocation of investment of the economy becomes more severe.

Hence the difference between g^F and g^I becomes larger. The ratio of these two endogenous growth rates, Δ, falls. From equations (19), (28) and (32), we have

$$\frac{1}{1+g^I}\frac{dg^I}{d\mu} = \frac{1}{1+g^F}\frac{dg^F}{d\mu} + \frac{1}{(2-\alpha)\delta}\frac{d\delta}{d\mu}$$

$$\Rightarrow \quad \frac{1}{1+g^I}\frac{dg^I}{d\mu} < \frac{1}{1+g^F}\frac{dg^F}{d\mu} \quad \text{and} \quad \frac{d\Delta}{d\mu} < 0\,.$$

where $\delta \equiv \rho^*\left[1 - \dfrac{\rho^*}{2-\alpha}\right]^{1-\alpha}, \quad \dfrac{d\delta}{d\rho^*} > 0 \quad \text{and} \quad \dfrac{d\rho^*}{d\mu} < 0 \quad \Rightarrow \quad \dfrac{d\delta}{d\mu} < 0.$

It is shown that changes in μ will have stronger impacts on the economy in the full information case than in the imperfect information case. With informational asymmetries between lenders and borrowers, agents with high success probabilities face binding internal finance constraints which restrain them from adjusting their investment decisions in response to the changes in μ. Consequently, the economy will not be affected as dramatically as in the full information case. The endogenous growth rate is less responsive to changes in money growth.

Discussion

The results of this model are contrasted with the macroeconomic implications in the literature. In the literature on credit market imperfections, it is argued that in an economy with informational imperfections in the credit markets, policy interventions will lead to larger changes in the quantities of lending than if there is full information. Models of imperfect credit markets, [see, for example, Stiglitz and Weiss (1981) and Greenwald and Stiglitz (1988a)], show that competitive credit markets may be characterized by credit rationing. Policy interventions which raise the supply of credit will have stronger impacts on the economy as the rationing in the credit markets is relaxed. Those firms which are previously credit-constrained can obtain credit to finance their production. The economy's output will be affected more dramatically than if there is full information. Therefore, these models conclude that the presence of imperfect information in the credit markets tends to amplify the effects of policy interventions on the economy.

In addition, in the consumption literature, empirical studies on consumption and borr wing constraints support the hypothesis that the existence of borrowing constraints makes consumption more responsive to policy changes. For instance, Zeldes (1989), Campbell and Mankiw (1990) and (1991) find that the permanent income / life cycle model is empirically rejected due to a significant portion of the population's inability to borrow against their future income. Their work indicates that the magnitudes of the changes in consumption are substantially greater than the predictions of permanent income model, in which borrowing constraints do not exist.

In this model, there is an explicit treatment of the informational asymmetries in the loan market. Unlike those models with fixed exogenous finance constraints, [for example Greenwald and Stiglitz (1988b)], in this model, finance constraints on investors arise endogenously in equilibrium. Since agents with high success probabilities are subject to binding finance constraints, this model is able to generate an under-investment result similar to what is obtained in credit rationing models. However, these finance constraints make the economy less responsive to policy interventions rather than amplifying their effects. Those agents who face binding constraints have already invested all of their resources in their projects. They are prevented from responding to the changes in the gross market rate of return. Since only a portion of the population is able to adjust their portfolio decisions in response to the policy changes, the impacts on the economy will be weaker than if there is full information. This prediction contradicts the popular view in the literature.

3.4 Conclusion

This paper presents an endogenous growth model with financial market imperfections to study the role of informational imperfections in the determination of economic growth. It is shown that the loan market of an economy with private information is shut down by a severe adverse selection problem. Agents with high quality projects (projects with high success probabilities) face binding internal finance constraints so that their projects suffer from underinvestment problems. Moreover, there is a misallocation of investment in the economy with private information. Any given level of total investment of the economy will yield a lower level of output of the capital good compared with the economy with full information. Consequently, the economy will grow slower if there is imperfect information.

Since money is introduced to our model as an asset in agents' portfolio, higher money growth will result in higher economic growth. Changes in the money growth rate have qualitatively similar effects on the endogenous economic growth rate in the full information case and the imperfect information case. However, these effects are quantitatively different. An increase in the money growth will enlarge the difference between the economic growth rates in these two cases. It is because the finance constraints make the economy with private information less responsive to higher money growth by worsening the underinvestment problem and the misallocation of investment of the economy. This prediction is quite different from the popular view in the literature that informational imperfections in credit markets or borrowing constraints tend to amplify the impacts of policy interventions.

APPENDIX I

Appendix 1 to Chapter 1:

The Optimization Problem of the Foreign Representation Household

The foreign representative household faces the following dynamic optimization problem. Given the distribution function G and the collection of sequences

$\{\Lambda_t^*, \mathcal{P}_t^*, M_t, M_t^*, M_{ht}, M_{ft}, B_{ft}, Z_{ft}\}_{t=0}^{\infty}$,

the household chooses the collection of sequences

$\{N_t^*, c_{1t}^*, c_{2t}^*, Y_t^*, B_{ht}^*, Z_{ht}^*, B_{ft}^*, Z_{ft}^*, l_t^*, H_t^*, K_{ht}^*, K_{ft}^*\}_{t=0}^{\infty}$

to maximize the expected lifetime utility

$$E_0\left[\sum_{t=0}^{\infty}\beta^{*t}U(c_{1t}^*, c_{2t}^*, l_t^*)\right], \qquad 0<\beta^*<1,$$

$$U(c_{1t}^*, c_{2t}^*, l_t^*) = \ln\,[c_{1t}^{*\,\alpha} + c_{2t}^{*\,\alpha}]^{\frac{1}{\alpha}} + [1-l_t^*], \qquad \alpha \le 1,$$

subject to

$$\begin{aligned}
M_{ft}^* - N_t^* &\ge P_{1t}^*\, c_{1t}^* + P_{2t}^*\, c_{2t}^*, \\
N_t^* + X_t^* + Y_t^* &\ge B_{ft}^* + Z_{ft}^* + B_{ft} + Z_{ft}, \\
B_{ft}^* + \frac{B_{ht}^*}{\hat{e}_t} - K_{ft}^*\, q_t^* &\ge W_t^*\, H_t^*, \\
M_{ht}^* + Z_{ht}^* + Z_{ft}^*\,\hat{e}_t - K_{ht}^*\, q_t &\ge P_{1t}\, c_{1t}^*,
\end{aligned}$$

$$\mathcal{F}^*(H_t^*) = \theta_t^*\, H_t^*,$$

$$\begin{aligned}
M_{ft+1}^* &= M_{ft}^* + X_t^* - Y_t^* i_{0t}^* + P_{2t}^*\,\mathcal{F}^*(H_t^*) - P_{2t}^*\, c_{2t}^* - W_t^*\, H_t^* + W_t^*\, l_t^* + \frac{B_{ht}^*}{\hat{e}_t} - Z_{ft}^* \\
&\quad + (B_{ft} + Z_{ft})\, i_t^* - \frac{(B_{ht}^* + Z_{ht}^*)\,(1+i_t)}{e_t} + K_{ft}^*\, i_t^*\, q_t^* + \Gamma_t^*,
\end{aligned}$$

$$M_{ht+1}^* = M_{ht}^* + Z_{ht}^* + Z_{ft}^*\hat{e}_t + K_{ht}^*\, i_t\, q_t - P_{1t}\, c_{1t}^*.$$

Define the value function corresponding to the foreign household's rescaled optimization problem $J^*(m, \delta, \tilde{s})$ by

$$J^*(m,\delta,\tilde{s}) = \max_{0 \le n^* \le m_f^*} \int \max_{c_1^*, c_2^*, y^*, b_h^*, z_h^*, b_f^*, z_f^*, l^*, H^*, k_h^*, k_f^*} \{U(c_1^*, c_2^*, l^*) + \beta^* J^*(m', \delta', s)\} dG(s),$$

subject to

$$\begin{aligned} m_f^* - n^* &\geq p_1^* c_1^* + p_2^* c_2^*, \\ n^* + x^* + y^* &\geq b_f^* + z_f^* + b_f + z_f, \\ b_f^* + \frac{b_h^*}{\hat{e}\,\delta} - k_f^* q^* &\geq w^* H^*, \\ m_h^* + z_h^* + z_f^* \hat{e}\,\delta - k_h^* q &\geq p_1 c_1^*, \end{aligned}$$

$$\begin{aligned} m_f^{\prime *} &= \frac{1}{1+x^*} [\, m_f^* + x^* - y^* i_0^* + p_2^* \mathcal{F}^*(H^*) - p_2^* c_2^* - w^* H^* + w^* l^* + \frac{b_h^*}{\hat{e}\,\delta} - z_f^* \\ &\quad + (b_f + z_f)\, i^* - \frac{(b_h^* + z_h^*)(1+i)}{e\,\delta} + k_f^* i^* q^* + \gamma^* \,], \end{aligned}$$

$$m_h^{\prime *} = \frac{1}{1+x} [\, m_h^* + z_h^* + z_f^* e\,\delta + k_h^* i\, q - p_1 c_1^* \,].$$

APPENDIX II

Appendix 2 to Chapter 1 : Results of the Comparative Static Analysis

By totally differentiating equations (57)-(59), we have

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} & a_{16} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} & a_{26} \\ a_{31} & a_{32} & a_{33} & a_{34} & a_{35} & a_{36} \end{bmatrix} \begin{bmatrix} de \\ dp_1 \\ dp_2^* \\ dx \\ dy \\ d\theta \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix},$$

where

$$a_{11} = -\frac{1}{e}[\pi_{22} + \pi_{11}^* - 1],$$

$$a_{21} = \frac{1}{e}\left[\frac{w}{\theta}(c_1\pi_{12} + c_1^*\pi_{11}^*) + z^*\pi_{11}^*\right],$$

$$a_{31} = -\frac{1}{e}\left[\frac{w^*}{\theta^*}(c_2\pi_{22} + c_2^*\pi_{21}^*) + z\pi_{22}\right],$$

$$a_{12} = \frac{1}{p_1}[\pi_{21} + 2(\pi_{11}^* - 1)],$$

$$a_{22} = -\frac{1}{p_1}\left[\frac{w}{\theta}(c_1\pi_{11} + 2c_1^*\pi_{11}^*) + z^*(2\pi_{11}^* - 1)\right],$$

$$a_{32} = \frac{1}{p_1}\left[\frac{w^*}{\theta^*}(c_2\pi_{21} + 2c_2^*\pi_{21}^*) + z\pi_{21}\right],$$

$$a_{13} = -\frac{1}{p_2^*}[2(\pi_{22} - 1) + \pi_{12}^*],$$

$$a_{23} = \frac{1}{p_2^*}\left[\frac{w}{\theta}(2c_1\pi_{12} + c_1^*\pi_{12}^*) + z^*\pi_{12}^*\right],$$

$$a_{33} = -\frac{1}{p_2^*}\left[\frac{w^*}{\theta^*}(2c_2\pi_{22} + c_2^*\pi_{22}^*) + z(2\pi_{22} - 1)\right],$$

$$a_{14} = -\frac{1}{1+x}[\pi_{11}^* - 1],$$

$$a_{24} = -\frac{1}{1+x}\left[1 + x - \frac{w}{\theta}[c_1 + c_1^*(\pi_{11}^* + 1)] - z^*\pi_{11}^*\right],$$

$$a_{34} = -\frac{1}{1+x}\left[\frac{w^*}{\theta^*}c_2^*\pi_{21}^*\right],$$

$$a_{15} = 0,$$

$$a_{25} = -1,$$

$$a_{35} = 0,$$

$$a_{16} = \frac{1}{\theta}[\pi_{11}^* - 1],$$

$$a_{26} = -\frac{1}{\theta}\left[\frac{w}{\theta}[c_1 + c_1^*(\pi_{11}^* + 1)] + z^*\pi_{11}^*\right],$$

$$a_{36} = \frac{1}{\theta}\left[\frac{w^*}{\theta^*}c_2^*\pi_{21}^*\right],$$

$$\pi_{ii} \equiv -\frac{\partial c_i}{\partial p_i}, \qquad \pi_{ij} \equiv \frac{\partial c_i}{\partial p_j}\frac{p_j}{c_i}, \qquad i,j = 1,2, \quad i \neq j.$$

Define

$$\frac{1}{(ep_1p_2^*)}\frac{1}{A} \equiv \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}.$$

The above derivation is for a general case. Now, we consider the CES utility function with the form

$$U(c_1, c_2) = \ln\left[c_1{}^{\alpha} + c_2{}^{\alpha}\right]^{\frac{1}{\alpha}}, \qquad \sigma \equiv \frac{1}{1-\alpha}, \qquad \alpha \leq 1,$$

$$\pi_{ii} = \left[\sigma + \frac{(1-\sigma)p_i{}^{1-\sigma}}{p_1{}^{1-\sigma} + p_2{}^{1-\sigma}}\right] > 0,$$

$$\pi_{ij} = -\left[\frac{(1-\sigma)p_j{}^{1-\sigma}}{p_1{}^{1-\sigma} + p_2{}^{1-\sigma}}\right], \qquad \text{sign}(\pi_{ij}) = \text{sign}(\sigma - 1),$$

$$\begin{array}{llll} \sigma > 1 & \Rightarrow & 1 < \pi_{ii} < \sigma, & 0 < \pi_{ij} < \sigma - 1, \\ \sigma = 1 & \Rightarrow & \pi_{ii} = 1, & \pi_{ij} = 0, \\ 0 \leq \sigma < 1 & \Rightarrow & \sigma < \pi_{ii} < 1, & \sigma - 1 < \pi_{ij} < 0, \end{array}$$

$$\begin{array}{ll} \pi_{ii} + \pi_{jj} = \sigma + 1, & \pi_{ii} = 1 + \pi_{ij}, \\ \pi_{ij} + \pi_{ji} = \sigma - 1, & \pi_{ii} + \pi_{ji} = \sigma, \end{array}$$

$$\pi_{22} + \pi_{11}^* - 1 = \pi_{21} + \pi_{11}^* = \pi_{22} + \pi_{12}^* > \sigma.$$

By using these properties, it can be shown that

$$\begin{aligned} \frac{1}{A} = & -\{\frac{w^*}{\theta^*}c_2[(\frac{w}{\theta}c_1 + z^*)\pi_{22} + \frac{w}{\theta}c_1^*(1 + \pi_{22})](1 + \pi_{11}^*) \\ & + (\frac{w^*}{\theta^*}c_2^* + z)[(\frac{w}{\theta}c_1 + z^*)(\pi_{22} + \pi_{12}^*) + \frac{w}{\theta}c_1^*\pi_{11}^*(1 + \pi_{22})]\}, \end{aligned}$$

$$A < 0, \qquad \forall \sigma \in [0, \infty).$$

Open Market Purchases (y)

$$\frac{y}{e}\frac{de}{dy} = A\,y\left[\frac{w^*}{\theta^*}c_2[2\pi_{21} + \pi^*_{12}(3\pi_{22}+1)] + (\frac{w^*}{\theta^*}c^*_2 + z)(\pi_{21} + 2\pi^*_{12})\right]$$

$$\frac{y}{p_1}\frac{dp_1}{dy} = \frac{y}{(1+i)}\frac{di}{dy} = A\,y\left[\frac{w^*}{\theta^*}c_2\pi_{22}(\pi^*_{11}+1) + (\frac{w^*}{\theta^*}c^*_2 + z)(\pi_{22} + \pi^*_{12})\right]$$

$$\frac{y}{p^*_2}\frac{dp^*_2}{dy} = \frac{y}{(1+i^*)}\frac{di^*}{dy} = -A\,y\frac{w^*}{\theta^*}c_2(\pi_{22}+1)\pi^*_{12}$$

$$\begin{aligned}\frac{y}{p_2}\frac{dp_2}{dy} &= \frac{y}{p^*_2}\frac{dp^*_2}{dy} + \frac{y}{(1+i^*)}\frac{di^*}{dy} + \frac{y}{e}\frac{de}{dy}\\ &= A\,y\left[\frac{w^*}{\theta^*}c_2\pi_{21}(\pi^*_{11}+1) + (\frac{w^*}{\theta^*}c^*_2 + z)(2\pi^*_{12} + \pi_{21})\right]\end{aligned}$$

$$\begin{aligned}\frac{y}{p^*_1}\frac{dp^*_1}{dy} &= \frac{y}{p_1}\frac{dp_1}{dy} + \frac{y}{(1+i)}\frac{di}{dy} - \frac{y}{e}\frac{de}{dy}\\ &= A\,y\left[\frac{w^*}{\theta^*}c_2(3-\pi^*_{11}) + \frac{w^*}{\theta^*}c^*_2 + z\right](\pi_{22}+1)\end{aligned}$$

$$\frac{y}{(\frac{p_2}{p_1})}\frac{d(\frac{p_2}{p_1})}{dy} = \frac{y}{p_2}\frac{dp_2}{dy} - \frac{y}{p_1}\frac{dp_1}{dy} = -A\,y\left[\frac{w^*}{\theta^*}c_2(\pi^*_{11}+1) + (\frac{w^*}{\theta^*}c^*_2 + z)(2-\pi^*_{11})\right]$$

$$\frac{y}{(\frac{p^*_2}{p^*_1})}\frac{d(\frac{p^*_2}{p^*_1})}{dy} = \frac{y}{p^*_2}\frac{dp^*_2}{dy} - \frac{y}{p^*_1}\frac{dp^*_1}{dy} = -A\,y\left[2\frac{w^*}{\theta^*}c_2 + \frac{w^*}{\theta^*}c^*_2 + z\right](\pi_{22}+1)$$

$$\frac{y}{p_1c_1}\frac{d(p_1c_1)}{dy} = \pi_{12}\left[\frac{y}{p_2}\frac{dp_2}{dy} - \frac{y}{p_1}\frac{dp_1}{dy}\right]$$

$$\frac{y}{p_2c_2}\frac{d(p_2c_2)}{dy} = -\pi_{21}\left[\frac{y}{p_2}\frac{dp_2}{dy} - \frac{y}{p_1}\frac{dp_1}{dy}\right]$$

$$\frac{y}{p^*_1c^*_1}\frac{d(p^*_1c^*_1)}{dy} = \pi^*_{12}\left[\frac{y}{p^*_2}\frac{dp^*_2}{dy} - \frac{y}{p^*_1}\frac{dp^*_1}{dy}\right]$$

$$\frac{y}{p^*_2c^*_2}\frac{d(p^*_2c^*_2)}{dy} = -\pi^*_{21}\left[\frac{y}{p^*_2}\frac{dp^*_2}{dy} - \frac{y}{p^*_1}\frac{dp^*_1}{dy}\right]$$

$$\begin{aligned}\frac{y}{p_1c^*_1}\frac{d(p_1c^*_1)}{dy} &= \frac{y}{p^*_1c^*_1}\frac{d(p^*_1c^*_1)}{dy} - \frac{y}{(1+i)}\frac{di}{dy} + \frac{y}{e}\frac{de}{dy}\\ &= -A\,y\left[\frac{w^*}{\theta^*}c_2(\pi^*_{11}+1) + (\frac{w^*}{\theta^*}c^*_2 + z)(1 + \pi^*_{12}\pi_{22})\right]\end{aligned}$$

$$\begin{aligned}\frac{y}{p^*_2c_2}\frac{d(p^*_2c_2)}{dy} &= \frac{y}{p_2c_2}\frac{d(p_2c_2)}{dy} - \frac{y}{(1+i^*)}\frac{di^*}{dy} - \frac{y}{e}\frac{de}{dy}\\ &= -A\,y\left[\frac{w^*}{\theta^*}c_2 + \frac{w^*}{\theta^*}c^*_2 + z\right](\pi_{22}+1)\,\pi^*_{12}\end{aligned}$$

$$
\begin{aligned}
\frac{dl^*}{dy} &= -\frac{1}{w^*}\frac{d(p_2^* c_2)}{dy} \\
\frac{y}{c_2}\frac{dc_2}{dy} &= \frac{y}{p_2 c_2}\frac{d(p_2 c_2)}{dy} - \frac{y}{p_2}\frac{dp_2}{dy} = -A\, y \left[\frac{w^*}{\theta^*} c_2^* + z\right](\pi_{22}+1)\pi_{12}^* \\
\frac{y}{c_2^*}\frac{dc_2^*}{dy} &= \frac{y}{p_2^* c_2^*}\frac{d(p_2^* c_2^*)}{dy} - \frac{y}{p_2^*}\frac{dp_2^*}{dy} \\
&= A\, y \left[\frac{w^*}{\theta^*} c_2(2\pi_{21}^* + \pi_{12}^*) + (\frac{w^*}{\theta^*} c_2^* + z)\pi_{21}^*\right](\pi_{22}+1) \\
\frac{y}{c_1}\frac{dc_1}{dy} &= \frac{y}{p_1 c_1}\frac{d(p_1 c_1)}{dy} - \frac{y}{p_1}\frac{dp_1}{dy} \\
&= -A\, y \left[\frac{w^*}{\theta^*} c_2(\pi_{11}^* + 1)\sigma + (\frac{w^*}{\theta^*} c_2^* + z)(\sigma + \pi_{12}^*(1-\pi_{12}^*))\right] \\
\frac{y}{c_1^*}\frac{dc_1^*}{dy} &= \frac{y}{p_1^* c_1^*}\frac{d(p_1^* c_1^*)}{dy} - \frac{y}{p_1^*}\frac{dp_1^*}{dy} \\
&= -A\, y \left[\frac{w^*}{\theta^*} c_2(\pi_{11}^* + 1) + (\frac{w^*}{\theta^*} c_2^* + z)\pi_{11}\right](\pi_{22}+1) \\
\frac{dl}{dy} &= \frac{1}{\theta}\left[\frac{dc_1}{dy} + \frac{dc_1^*}{dy}\right]
\end{aligned}
$$

Productivity Shocks (θ)

$$
\begin{aligned}
\frac{\theta}{e}\frac{de}{d\theta} &= A\,\{\frac{w^*}{\theta^*} c_2[(\frac{w}{\theta} c_1 + z^*)(\pi_{12}^*(\pi_{22}+1) + 2\pi_{21}) \\
&\qquad + 4\frac{w}{\theta} c_1^*(\pi_{21} + \pi_{12}^* + \pi_{21}\pi_{12}^*)] \\
&\quad + (\frac{w^*}{\theta^*} c_2^* + z)[(\frac{w}{\theta} c_1 + z^*)(\pi_{21} + \pi_{12}^*) + \frac{w}{\theta} c_1^*(\pi_{12}^*(\pi_{22}+1) + 2\pi_{21})]\} \\
\frac{\theta}{p_1}\frac{dp_1}{d\theta} &= A\,\{\frac{w^*}{\theta^*} c_2[\frac{w}{\theta} c_1 + 2\frac{w}{\theta} c_1^* + z^*]\pi_{22}(\pi_{11}^* + 1) \\
&\quad + (\frac{w^*}{\theta^*} c_2^* + z)[(\frac{w}{\theta} c_1 + z^*)(\pi_{22} + \pi_{12}^*) + \frac{w}{\theta} c_1^*(\pi_{11}^*(\pi_{22}+1) + \pi_{21})]\} \\
\frac{\theta}{p_2^*}\frac{dp_2^*}{d\theta} &= \frac{\theta}{(1+i^*)}\frac{di^*}{d\theta} = -A\,\frac{w^*}{\theta^*} c_2\left[\frac{w}{\theta} c_1 + \frac{w}{\theta} c_1^*(\pi_{22}+1) + z^*\right]\pi_{12}^* \\
\frac{\theta}{(1+i)}\frac{di}{d\theta} &= 1 + \frac{\theta}{p_1}\frac{dp_1}{d\theta} = A\,\frac{w}{\theta} c_1^*\left[\frac{w^*}{\theta^*} c_2(\pi_{11}^* + 1) + \frac{w^*}{\theta^*} c_2^* + z\right]\pi_{21}
\end{aligned}
$$

$$\frac{\theta}{p_2}\frac{dp_2}{d\theta} = \frac{\theta}{p_2^*}\frac{dp_2^*}{d\theta} + \frac{\theta}{(1+i^*)}\frac{di^*}{d\theta} + \frac{\theta}{e}\frac{de}{d\theta}$$

$$= A\,\{\frac{w^*}{\theta^*}c_2[\frac{w}{\theta}c_1 + 2\frac{w}{\theta}c_1^* + z^*]\pi_{21}(\pi_{11}^* + 1)$$

$$+ (\frac{w^*}{\theta^*}c_2^* + z)[(\frac{w}{\theta}c_1 + z^*)(\pi_{21} + \pi_{12}^*) + \frac{w}{\theta}c_1^*(\pi_{12}^*(\pi_{22} + 1) + 2\pi_{21})]\}$$

$$\frac{\theta}{p_1^*}\frac{dp_1^*}{d\theta} = \frac{\theta}{p_1}\frac{dp_1}{d\theta} + \frac{\theta}{(1+i)}\frac{di}{d\theta} - \frac{\theta}{e}\frac{de}{d\theta}$$

$$= A\,\{\frac{w^*}{\theta^*}c_2[\,2(\frac{w}{\theta}c_1 + z^*) + \frac{w}{\theta}c_1^*(\pi_{22} + 1)(3 - \pi_{11}^*)]$$

$$+ (\frac{w^*}{\theta^*}c_2^* + z)[\frac{w}{\theta}c_1 + \frac{w}{\theta}c_1^*(\pi_{22} + 1) + z^*]\}$$

$$\frac{\theta}{(\frac{p_2}{p_1})}\frac{d(\frac{p_2}{p_1})}{d\theta} = \frac{\theta}{p_2}\frac{dp_2}{d\theta} - \frac{\theta}{p_1}\frac{dp_1}{d\theta}$$

$$= -A\,\left[\frac{w^*}{\theta^*}c_2(\pi_{11}^* + 1) + \frac{w^*}{\theta^*}c_2^* + z\right]\left[\frac{w}{\theta}c_1 + 2\frac{w}{\theta}c_1^* + z^*\right]$$

$$\frac{\theta}{(\frac{p_2^*}{p_1^*})}\frac{d(\frac{p_2^*}{p_1^*})}{d\theta} = \frac{\theta}{p_2^*}\frac{dp_2^*}{d\theta} - \frac{\theta}{p_1^*}\frac{dp_1^*}{d\theta}$$

$$= -A\,\{\frac{w^*}{\theta^*}c_2[\frac{w}{\theta}c_1 + 2\frac{w}{\theta}c_1^* + z^*](\pi_{11}^* + 1)$$

$$+ (\frac{w^*}{\theta^*}c_2^* + z)[\frac{w}{\theta}c_1 + \frac{w}{\theta}c_1^*(\pi_{22} + 1) + z^*]\}$$

$$\frac{\theta}{p_1c_1}\frac{d(p_1c_1)}{d\theta} = \pi_{12}\,\left[\frac{\theta}{p_2}\frac{dp_2}{d\theta} - \frac{\theta}{p_1}\frac{dp_1}{d\theta}\right]$$

$$\frac{\theta}{p_2c_2}\frac{d(p_2c_2)}{d\theta} = -\pi_{21}\,\left[\frac{\theta}{p_2}\frac{ap_2}{d\theta} - \frac{\theta}{p_1}\frac{dp_1}{d\theta}\right]$$

$$\frac{\theta}{p_1^*c_1^*}\frac{d(p_1^*c_1^*)}{d\theta} = \pi_{12}^*\,\left[\frac{\theta}{p_2^*}\frac{dp_2^*}{d\theta} - \frac{\theta}{p_1^*}\frac{dp_1^*}{d\theta}\right]$$

$$\frac{\theta}{p_2^*c_2^*}\frac{d(p_2^*c_2^*)}{d\theta} = -\pi_{21}^*\,\left[\frac{\theta}{p_2^*}\frac{dp_2^*}{d\theta} - \frac{\theta}{p_1^*}\frac{dp_1^*}{d\theta}\right]$$

$$\frac{\theta}{p_1c_1^*}\frac{d(p_1c_1^*)}{d\theta} = \frac{\theta}{p_1^*c_1^*}\frac{d(p_1^*c_1^*)}{d\theta} - \frac{\theta}{(1+i)}\frac{di}{d\theta} + \frac{\theta}{e}\frac{de}{d\theta}$$

$$= A\,\{\frac{w^*}{\theta^*}c_2[(\frac{w}{\theta}c_1 + z^*)(\pi_{12}^*(\pi_{22} - \pi_{11}^*) + 2\pi_{21})$$

$$+ \frac{w}{\theta}c_1^*(2\pi_{12}^*(\pi_{22} - \pi_{11}^*) + \pi_{21}(\pi_{11}^* + 1))]$$

$$+ (\frac{w^*}{\theta^*}c_2^* + z)(\frac{w}{\theta}c_1 + \frac{w}{\theta}c_1^* + z^*)\pi_{21}\}$$

$$\frac{\theta}{p_2^* c_2}\frac{d(p_2^* c_2)}{d\theta} = \frac{\theta}{p_2 c_2}\frac{d(p_2 c_2)}{d\theta} - \frac{\theta}{(1+i^*)}\frac{di^*}{d\theta} - \frac{\theta}{e}\frac{de}{d\theta}$$

$$= -A\left[\frac{w^*}{\theta^*}c_2 + \frac{w^*}{\theta^*}c_2^* + z\right]\left[\frac{w}{\theta}c_1 + \frac{w}{\theta}c_1^*(\pi_{22}+1) + z^*\right]\pi_{12}^*$$

$$\frac{dl^*}{d\theta} = -\frac{1}{w^*}\frac{d(p_2^* c_2)}{d\theta}$$

$$\frac{dl}{d\theta} = -\frac{1}{w}\frac{d(p_1 c_1^*)}{d\theta}$$

$$\frac{\theta}{c_2}\frac{dc_2}{d\theta} = \frac{\theta}{p_2 c_2}\frac{d(p_2 c_2)}{d\theta} - \frac{\theta}{p_2}\frac{dp_2}{d\theta}$$

$$= -A\left[\frac{w^*}{\theta^*}c_2^* + z\right]\left[\frac{w}{\theta}c_1 + \frac{w}{\theta}c_1^*(\pi_{22}+1) + z^*\right]\pi_{12}^*$$

$$\frac{\theta}{c_2^*}\frac{dc_2^*}{d\theta} = \frac{\theta}{p_2^* c_2^*}\frac{d(p_2^* c_2^*)}{d\theta} - \frac{y}{p_2^*}\frac{dp_2^*}{d\theta}$$

$$= A\,\{\frac{w^*}{\theta^*}c_2[(\frac{w}{\theta}c_1 + z^*)(\pi_{21}^*(\pi_{11}^*+1) + \pi_{12}^*)$$

$$+ \frac{w}{\theta}c_1^*(2\pi_{21}^* + (\pi_{22}+1)\pi_{12}^*)]$$

$$+ (\frac{w^*}{\theta^*}c_2^* + z)[\frac{w}{\theta}c_1 + \frac{w}{\theta}c_1^*(\pi_{22}+1) + z^*]\pi_{21}^*\}$$

$$\frac{\theta}{c_1}\frac{dc_1}{d\theta} = \frac{\theta}{p_1 c_1}\frac{d(p_1 c_1)}{d\theta} - \frac{\theta}{p_1}\frac{dp_1}{d\theta}$$

$$= A\,\{\frac{w^*}{\theta^*}c_2[\frac{w}{\theta}c_1 + 2\frac{w}{\theta}c_1^* + z^*](\pi_{11}^*+1)\sigma$$

$$+(\frac{w^*}{\theta^*}c_2^* + z)[(\frac{w}{\theta}c_1 + z^*)\sigma + \frac{w}{\theta}c_1^*((\pi_{22}+1)(\pi_{12}^*+\pi_{22}) + (1-\pi_{12})\pi_{21})]\}$$

$$\frac{\theta}{c_1^*}\frac{dc_1^*}{d\theta} = \frac{\theta}{p_1^* c_1^*}\frac{d(p_1^* c_1^*)}{d\theta} - \frac{y}{p_1^*}\frac{dp_1^*}{d\theta}$$

$$= -A\,\{\frac{w^*}{\theta^*}c_2[(\frac{w}{\theta}c_1 + z^*)(\pi_{11}^{*\,2}+1) + \frac{w}{\theta}c_1^*(\pi_{22}+1)(\pi_{11}^*+1)]$$

$$+ (\frac{w^*}{\theta^*}c_2^* + z)[\frac{w}{\theta}c_1 + \frac{w}{\theta}c_1^*(\pi_{22}+1) + z^*]\pi_{11}^*\}$$

Lump Sum Transfers (x)

$$
\begin{aligned}
1+x &= (2+i)\left[\frac{w}{\theta}c_1 + \frac{w}{\theta}c_1^*\right] + (1+i)z^* - y \\
\frac{(1+x)}{e}\frac{de}{dx} &= A\,\{\pi_{12}^*[2(1+x) - \frac{w}{\theta}c_1(1-\pi_{12}) - 2\frac{w}{\theta}c_1^* - z^*][2\frac{w^*}{\theta^*}c_2 + \frac{w^*}{\theta^*}c_2^* + z] \\
&\quad + \pi_{21}[3(1+x) - \frac{w}{\theta}c_1 - 2\frac{w}{\theta}c_1^* - z^*][\frac{w^*}{\theta^*}c_2\pi_{12}^*] \\
&\quad + \pi_{21}[1+x - \frac{w}{\theta}c_1 - \frac{w}{\theta}c_1^*(\pi_{11}^* + 1) - z^*\pi_{11}^*][2\frac{w^*}{\theta^*}c_2 + \frac{w^*}{\theta^*}c_2^* + z]\} \\
\frac{(1+x)}{p_1}\frac{dp_1}{dx} &= A\,\{\frac{w^*}{\theta^*}c_2[1+x - \frac{w}{\theta}c_1 - 2\frac{w}{\theta}c_1^* - z^*]\pi_{22}(\pi_{11}^* + 1) \\
&\quad + (\frac{w^*}{\theta^*}c_2^* + z)[(1+x - \frac{w}{\theta}c_1 - 2\frac{w}{\theta}c_1^* - z^*)(\pi_{22} + \pi_{12}^*) - \frac{w}{\theta}c_1^*\pi_{21}\pi_{12}^*]\} \\
\frac{(1+x)}{p_2^*}\frac{dp_2^*}{dx} &= \frac{(1+x)}{(1+i^*)}\frac{di^*}{dx} \\
&= -A\,\frac{w^*}{\theta^*}c_2\left[(1+x - \frac{w}{\theta}c_1^*)(1+\pi_{22}) - \frac{w}{\theta}c_1(1-\pi_{12}) - z^*\pi_{22}\right]\pi_{12}^* \\
\frac{(1+x)}{(1+i)}\frac{di}{dx} &= \frac{(1+x)}{p_1}\frac{dp_1}{dx} - 1 \\
&= A\,\{\frac{w^*}{\theta^*}c_2[(1+x)\pi_{22} - \frac{w}{\theta}c_1^*\pi_{21}](1+\pi_{11}^*) \\
&\quad + (\frac{w^*}{\theta^*}c_2^* + z)[(1+x)(\pi_{21} + \pi_{11}^*) - \frac{w}{\theta}c_1^*\pi_{21}]\}
\end{aligned}
$$

APPENDIX III

Appendix 1 to Chapter 2 :

The Results of the Comparative Statics Exercise in Section 4

By total differentiating the simultaneous equation system, equations (33)-(35), we have the following results,

$$\begin{aligned}\Delta \equiv & -(\frac{w}{\theta}c_1 + \frac{p_1 c_1^*}{1-\tau})[\frac{w^*}{\theta^*}c_2\pi_{22}(\pi_{11}^* + 1) + (\frac{w^*}{\theta^*}c_2^* + p_2^* c_2)(\pi_{22} + \pi_{11}^* - 1)] \\ & - (\pi_{22} + 1)\frac{w}{\theta}c_1^*[\frac{w^*}{\theta^*}c_2(\pi_{11}^* + 1) + (\frac{w^*}{\theta^*}c_2^* + p_2^* c_2)\pi_{11}^*]\end{aligned}$$

$$\Delta < 0 \quad \forall \ \sigma$$

$$\begin{aligned}\frac{(1-\tau)}{\hat{e}}\frac{d\hat{e}}{d\tau} = & \frac{1}{\Delta}\pi_{22}[(\frac{w}{\theta}c_1 + \frac{p_1 c_1^*}{1-\tau})(2\frac{w^*}{\theta^*}c_2 + \frac{w^*}{\theta^*}c_2^* + p_2^* c_2) \\ & + \frac{w}{\theta}c_1^*(\frac{w^*}{\theta^*}c_2(\pi_{11}^* + 1) + (\frac{w^*}{\theta^*}c_2^* + p_2^* c_2)\pi_{11}^*)]\end{aligned}$$

$$\begin{aligned}\frac{(1-\tau)}{e^R}\frac{de^R}{d\tau} = & \frac{1}{\Delta}[(\frac{w}{\theta}c_1 + \frac{p_1 c_1^*}{1-\tau})(\frac{w^*}{\theta^*}c_2^* + p_2^* c_2) \\ & + \frac{w^*}{\theta^*}c_2[\frac{w}{\theta}c_1(2 + \pi_{12}^*(\pi_{22} + \pi_{12})) + \frac{p_1 c_1^*}{1-\tau}(\pi_{11}^* + 1)]\end{aligned}$$

$$\frac{(1-\tau)}{p_1}\frac{dp_1}{d\tau} = \frac{(1-\tau)}{(1+i)}\frac{di}{d\tau} = \frac{1}{\Delta}\frac{w}{\theta}c_1^*\pi_{22}\left[\frac{w^*}{\theta^*}c_2(\pi_{11}^* + 1) + (\frac{w^*}{\theta^*}c_2^* + p_2^* c_2)\pi_{11}^*\right]$$

$$\frac{(1-\tau)}{p_2^*}\frac{dp_2^*}{d\tau} = \frac{(1-\tau)}{(1+i^*)}\frac{di^*}{d\tau} = \frac{1}{\Delta}[\frac{w}{\theta}c_1(\pi_{22} + \pi_{12}) + \frac{p_1 c_1^*}{1-\tau}]\frac{w^*}{\theta^*}c_2\pi_{12}^*$$

$$\begin{aligned}\frac{(1-\tau)}{p_2}\frac{dp_2}{d\tau} = & \frac{1}{\Delta}[-(\frac{w}{\theta}c_1 + \frac{p_1 c_1^*}{1-\tau})(\frac{w^*}{\theta^*}c_2^* + p_2^* c_2)\pi_{12}^* \\ & + \frac{w}{\theta}c_1^*\pi_{21}[\frac{w^*}{\theta^*}c_2(\pi_{11}^* + 1) + (\frac{w^*}{\theta^*}c_2^* + p_2^* c_2)\pi_{11}^*]]\end{aligned}$$

$$\begin{aligned}\frac{(1-\tau)}{p_1^*}\frac{dp_1^*}{d\tau} = & -\frac{1}{\Delta}[(\frac{w}{\theta}c_1 + \frac{p_1 c_1^*}{1-\tau})(\frac{w^*}{\theta^*}c_2^* + p_2^* c_2) \\ & + \frac{w^*}{\theta^*}c_2[\frac{w}{\theta}c_1(2 - \pi_{12}^*(\pi_{22} + \pi_{12})) + \frac{p_1 c_1^*}{1-\tau}(1 - \pi_{11}^*)]]\end{aligned}$$

$$\begin{aligned}\frac{(1-\tau)}{\frac{p_2}{p_1}}\frac{d(\frac{p_2}{p_1})}{d\tau} = & -\frac{1}{\Delta}[(\frac{w}{\theta}c_1 + \frac{p_1 c_1^*}{1-\tau})(\frac{w^*}{\theta^*}c_2^* + p_2^* c_2)\pi_{12}^* \\ & + \frac{w}{\theta}c_1^*[\frac{w^*}{\theta^*}c_2(\pi_{11}^* + 1) + (\frac{w^*}{\theta^*}c_2^* + p_2^* c_2)\pi_{11}^*]]\end{aligned}$$

$$\frac{(1-\tau)}{\frac{p_2^*}{p_1^*}}\frac{d(\frac{p_2^*}{p_1^*})}{d\tau} = \frac{1}{\triangle}(\frac{w}{\theta}c_1 + \frac{p_1c_1^*}{1-\tau})[2\frac{w^*}{\theta^*}c_2 + \frac{w^*}{\theta^*}c_2^* + p_2^*c_2]$$

$$\begin{aligned}\frac{(1-\tau)}{z_h}\frac{dz_h}{d\tau} &= -\frac{1}{\triangle}[[\frac{w}{\theta}c_1(2\pi_{21}+\pi_{12}) + \frac{p_1c_1^*}{1-\tau}\pi_{21}]\frac{w^*}{\theta^*}c_2^*\pi_{12}^* \\ &\quad + \frac{w}{\theta}c_1^*[\frac{w^*}{\theta^*}c_2(\pi_{11}^*+1) + (\frac{w^*}{\theta^*}c_2^* + p_2^*c_2)\pi_{11}^*]]\end{aligned}$$

$$\begin{aligned}\frac{(1-\tau)}{z_h(1-\tau)}\frac{d(z_h(1-\tau))}{d\tau} &= \frac{1}{\triangle}[(\frac{w}{\theta}c_1 + \frac{p_1c_1^*}{1-\tau})(\frac{w^*}{\theta^*}c_2^* + p_2^*c_2)(\pi_{21}+\pi_{11}^*) \\ &\quad + \frac{w^*}{\theta^*}c_2[\frac{w}{\theta}c_1(3-\pi_{11}^*)\sigma + \frac{p_1c_1^*}{1-\tau}(2\pi_{22}+\pi_{11}^*)] \\ &\quad + \frac{w}{\theta}c_1^*\pi_{22}[\frac{w^*}{\theta^*}c_2(\pi_{11}^*+1) + (\frac{w^*}{\theta^*}c_2^* + p_2^*c_2)\pi_{11}^*]]\end{aligned}$$

$$\frac{(1-\tau)}{z_f^*}\frac{dz_f^*}{d\tau} = \frac{1}{\triangle}\pi_{12}^*\pi_{22}(\frac{w}{\theta}c_1 + \frac{p_1c_1^*}{1-\tau})[\frac{w^*}{\theta^*}c_2 + \frac{w^*}{\theta^*}c_2^* + p_2^*c_2]$$

$$\frac{dl}{d\tau} = -\frac{1}{w}\frac{dz_h}{d\tau}$$

$$\begin{aligned}\frac{(1-\tau)}{c_1}\frac{dc_1}{d\tau} &= -\frac{1}{\triangle}[\pi_{12}^*\pi_{22}(\frac{w}{\theta}c_1 + \frac{p_1c_1^*}{1-\tau})[\frac{w^*}{\theta^*}c_2^* + p_2^*c_2] \\ &\quad + (\pi_{12}+\pi_{22})\frac{w}{\theta}c_1^*[\frac{w^*}{\theta^*}c_2(\pi_{11}^*+1) + (\frac{w^*}{\theta^*}c_2^* + p_2^*c_2)\pi_{11}^*]]\end{aligned}$$

$$\begin{aligned}\frac{(1-\tau)}{c_1^*}\frac{dc_1^*}{d\tau} &= \frac{1}{\triangle}[(\frac{w}{\theta}c_1 + \frac{p_1c_1^*}{1-\tau})(\frac{w^*}{\theta^*}c_2^* + p_2^*c_2)(\pi_{21}+\pi_{11}^*) \\ &\quad + \frac{w^*}{\theta^*}c_2[\frac{w}{\theta}c_1(3-\pi_{11}^*)\sigma + \frac{p_1c_1^*}{1-\tau}(2\pi_{22}+\pi_{11}^*)]]\end{aligned}$$

$$\frac{dl^*}{d\tau} = -\frac{1}{w^*}\frac{dz_f^*}{d\tau}$$

$$\frac{(1-\tau)}{c_2}\frac{dc_2}{d\tau} = \frac{1}{\triangle}\pi_{12}^*\pi_{22}(\frac{w}{\theta}c_1 + \frac{p_1c_1^*}{1-\tau})(\frac{w^*}{\theta^*}c_2^* + p_2^*c_2)$$

$$\begin{aligned}\frac{(1-\tau)}{c_2^*}\frac{dc_2^*}{d\tau} &= -\frac{1}{\triangle}[\pi_{12}^*(\frac{w}{\theta}c_1 + \frac{p_1c_1^*}{1-\tau})(\frac{w^*}{\theta^*}c_2^* + p_2^*c_2) \\ &\quad + [\frac{w}{\theta}c_1(\pi_{12}^*(\pi_{12}+\pi_{22}) + 2\pi_{21}^*) + \frac{p_1c_1^*}{1-\tau}(2\pi_{21}^* + \pi_{12}^*)]\frac{w^*}{\theta^*}c_2]\end{aligned}$$

APPENDIX IV

Appendix 2 to Chapter 2:

The Results of the Comparative Statics Exercise in Section 5

Changes in the Variance of τ

$$\tau_A = \tau_o + \varepsilon \ , \ \tau_B = \tau_o - \varepsilon \ \Rightarrow \ \bar{\tau} = \tau_o \ , \ \mathrm{Var}(\tau) = \varepsilon^2$$

$$\Delta_1 = -\frac{1}{2q_A^2 q_B^2}\left[\, q_A^2\,(\,1 + (1-\tau_o-\varepsilon)\,q_A\,) + q_B^2\,(\,1 + (1-\tau_o+\varepsilon)\,q_B\,)\,\right] < 0$$

$$\frac{1}{1-n}\frac{dn}{d\varepsilon} = -\frac{1}{p_2}\frac{dp_2}{d\varepsilon} = \frac{n}{2\Delta_1 q_A q_B}\left[\, q_A - q_B \,\right] < 0$$

$$\frac{1}{q_A}\frac{dq_A}{d\varepsilon} = \frac{1}{c_{1A}}\frac{dc_{1A}}{d\varepsilon} = -\frac{1}{4\Delta_1 q_A q_B^2}\left[\, q_A^2 + q_B^2 \,\right] > 0$$

$$\frac{1}{q_B}\frac{dq_B}{d\varepsilon} = \frac{1}{c_{1B}}\frac{dc_{1B}}{d\varepsilon} = \frac{1}{4\Delta_1 q_A^2 q_B}\left[\, q_A^2 + q_B^2 \,\right] < 0$$

$$\frac{1}{p_{1A}}\frac{dp_{1A}}{d\varepsilon} = \frac{1}{4\Delta_1 q_A q_B^2}\left[\, q_A^2 + q_B^2(1+2n) - 2nq_A q_B \,\right] < 0$$

$$\frac{1}{p_{1B}}\frac{dp_{1B}}{d\varepsilon} = -\frac{1}{4\Delta_1 q_A^2 q_B}\left[\, q_A^2(1+2n) + q_B^2 - 2nq_A q_B \,\right] > 0$$

$$\frac{1}{\hat{e}_A}\frac{d\hat{e}_A}{d\varepsilon} = -\frac{1}{(1-\tau_o-\varepsilon)} - \frac{1}{4\Delta_1 q_A q_B^2}\left[\, q_A^2 + q_B^2(1-2n) + 2nq_A q_B \,\right] < 0$$

$$\frac{1}{\hat{e}_B}\frac{d\hat{e}_B}{d\varepsilon} = \frac{1}{(1-\tau_o+\varepsilon)} + \frac{1}{4\Delta_1 q_A^2 q_B}\left[\, q_A^2(1-2n) + q_B^2 + 2nq_A q_B \,\right] > 0$$

$$\frac{1}{p_{1A}^*}\frac{dp_{1A}^*}{d\varepsilon} = -\frac{1}{c_{1A}^*}\frac{dc_{1A}^*}{d\varepsilon} = \frac{1}{(1-\tau_o-\varepsilon)} + \frac{1}{2\Delta_1 q_A q_B^2}\left[\, q_A^2 + q_B^2 \,\right] > 0$$

$$\frac{1}{p_{1B}^*}\frac{dp_{1B}^*}{d\varepsilon} = -\frac{1}{c_{1B}^*}\frac{dc_{1B}^*}{d\varepsilon} = -\frac{1}{(1-\tau_o+\varepsilon)} - \frac{1}{2\Delta_1 q_A^2 q_B}\left[\, q_A^2 + q_B^2 \,\right] < 0$$

$$\frac{1}{z_{hA}}\frac{dz_{hA}}{d\varepsilon} = -\frac{1}{4\Delta_1 q_A q_B^2}\left[\, q_A^2 + q_B^2(1-2n) + 2nq_A q_B \,\right] > 0$$

$$\frac{1}{z_{hB}}\frac{dz_{hB}}{d\varepsilon} = \frac{1}{4\Delta_1 q_A^2 q_B}\left[\, q_A^2(1-2n) + q_B^2 + 2nq_A q_B \,\right] < 0$$

$$\frac{dl_A}{d\varepsilon} = \frac{1}{w}[-\frac{dz_{hA}}{d\varepsilon} + \frac{l_A}{\beta}\frac{dn}{d\varepsilon}] < 0$$

$$\frac{dl_B}{d\varepsilon} = \frac{1}{w}\left[-\frac{dz_{hB}}{d\varepsilon} + \frac{l_B}{\beta}\frac{dn}{d\varepsilon}\right] = ?, \qquad \frac{d\bar{l}}{d\varepsilon} < 0$$

$$\Rightarrow \qquad \frac{1}{q_A}\frac{dq_A}{d\varepsilon} > -\frac{1}{q_B}\frac{dq_B}{d\varepsilon}, \qquad \frac{d(p_{1B}-p_{1A})}{d\varepsilon} > 0,$$

$$\frac{d(\hat{e}_B-\hat{e}_A)}{d\varepsilon} > 0, \qquad \frac{d(p^*_{1B}-p^*_{1A})}{d\varepsilon} > 0$$

Changes in the Mean of τ

$$\tau_A = \tau_a + \varepsilon \;, \; \tau_B = \tau_b + \varepsilon \;, \; \tau_a > \tau_b$$

$$\Rightarrow \; \bar{\tau} = \frac{1}{2}(\tau_a + \tau_b) + \varepsilon \;, \; \mathrm{Var}(\tau) = \frac{1}{4}(\tau_a - \tau_b)^2$$

$$\Delta_2 = -\frac{1}{2q_A^2q_B^2}[\,q_A^2\,(\,1 + (1-\tau_a-\varepsilon)q_A\,) + q_B^2\,(\,1 + (1-\tau_b-\varepsilon)\,q_B\,)\,] < 0$$

$$\frac{1}{1-n}\frac{dn}{d\varepsilon} = -\frac{1}{p_2}\frac{dp_2}{d\varepsilon} = \frac{1}{\Delta_2}[\frac{1-n}{2} - \frac{n}{\beta}] < 0$$

$$\frac{1}{q_A}\frac{dq_A}{d\varepsilon} = \frac{1}{c_{1A}}\frac{dc_{1A}}{d\varepsilon} = -\frac{1}{4\Delta_2 q_A q_B^2}[\,q_A^2 - q_B^2\,] > 0$$

$$\frac{1}{q_B}\frac{dq_B}{d\varepsilon} = \frac{1}{c_{1B}}\frac{dc_{1B}}{d\varepsilon} = \frac{1}{4\Delta_2 q_A^2 q_B}[\,q_A^2 - q_B^2\,] < 0$$

$$\frac{1}{p_{1A}}\frac{dp_{1A}}{d\varepsilon} = \frac{1}{2\Delta_2\,\beta}[\,1 \cdot 2n - (1-n)\beta - \frac{q_A}{q_B}\,] = ?$$

$$\frac{1}{p_{1B}}\frac{dp_{1B}}{d\varepsilon} = \frac{1}{2\Delta_2\,\beta}[\,1 + 2n - (1-n)\beta - \frac{q_B}{q_A}\,] > 0$$

$$\frac{1}{\hat{e}_A}\frac{d\hat{e}_A}{d\varepsilon} = -\frac{1}{(1-\tau_a-\varepsilon)} + \frac{1}{q_A}\frac{dq_A}{d\varepsilon} - \frac{1}{1-n}\frac{dn}{d\varepsilon} < 0$$

$$\frac{1}{\hat{e}_B}\frac{d\hat{e}_B}{d\varepsilon} = -\frac{1}{(1-\tau_b-\varepsilon)} + \frac{1}{q_B}\frac{dq_B}{d\varepsilon} - \frac{1}{1-n}\frac{dn}{d\varepsilon} \quad < 0$$

$$\frac{1}{p^*_{1A}}\frac{dp^*_{1A}}{d\varepsilon} = -\frac{1}{c^*_{1A}}\frac{dc^*_{1A}}{d\varepsilon} = \frac{1}{(1-\tau_a-\varepsilon)} + \frac{1}{2\Delta_2 q_A q_B^2}\left[\, q_A^2 - q_B^2 \,\right] \quad > 0$$

$$\frac{1}{p^*_{1B}}\frac{dp^*_{1B}}{d\varepsilon} = -\frac{1}{c^*_{1B}}\frac{dc^*_{1B}}{d\varepsilon} = \frac{1}{(1-\tau_b-\varepsilon)} - \frac{1}{2\Delta_2 q_A^2 q_B}\left[\, q_A^2 - q_B^2 \,\right] \quad > 0$$

$$\frac{1}{z_{hA}}\frac{dz_{hA}}{d\varepsilon} = \frac{1}{q_A}\frac{dq_A}{d\varepsilon} - \frac{1}{1-n}\frac{dn}{d\varepsilon} \quad > 0$$

$$\frac{1}{z_{hB}}\frac{dz_{hB}}{d\varepsilon} = \frac{1}{q_B}\frac{dq_B}{d\varepsilon} - \frac{1}{1-n}\frac{dn}{d\varepsilon} \quad = ?$$

$$\frac{dl_A}{d\varepsilon} = \frac{1}{w}\left[-\frac{dz_{hA}}{d\varepsilon} + \frac{l_A}{\beta}\frac{dn}{d\varepsilon}\right] \quad < 0$$

$$\frac{dl_B}{d\varepsilon} = \frac{1}{w}\left[-\frac{dz_{hB}}{d\varepsilon} + \frac{l_B}{\beta}\frac{dn}{d\varepsilon}\right] = \frac{dc_{1B}}{d\varepsilon} + \frac{dc^*_{1B}}{d\varepsilon} \quad < 0, \qquad \frac{d\bar{l}}{d\varepsilon} < 0$$

$$\Rightarrow \quad \frac{1}{q_A}\frac{dq_A}{d\varepsilon} > -\frac{1}{q_B}\frac{dq_B}{d\varepsilon}$$

Bibliography

[1] Adams, C. and J. Greenwood (1985): "Dual Exchange Rate Systems and Capital Controls: An Investigation," *Journal of International Economics*, Vol.18, pp.43-63.

[2] Akelof, G.A. (1970): "The Market for Lemons: Qualitative Uncertainty and the Market Mechanism," *Quarterly Journal of Economics*, Vol.LXXXIV, pp.488-500.

[3] Backus, D.K., P.J. Kehoe, and F. Kydland (1992): "International Real Business Cycles," *Journal of Political Economy*, Vol.100, pp.745-775.

[4] Bencivenga, V.R. and B.D. Smith (1990): "Deficits, Inflation, and the Banking System in Developing Countries: The Optimal Degree of Financial Repression," Working Paper No.214, Rochester Center for Economic Research .

[5] Bencivenga, V.R. and B.D. Smith (1991): "Financial Intermediation and Endogenous Growth," *Review of Economic Studies*, Vol.58, pp.195-209.

[6] Bernanke, B. and A. Blinder (1990): "The Federal Funds Rate and the Channels of Monetary Transmission," Working Paper No.3487, National Bureau of Economic Research.

[7] Bernanke B. and M. Gertler (1989): "Agency Costs, Net Worth, and Business Fluctuations," *American Economic Review*, Vol.79, pp.14-31.

[8] Campbell J.Y. and N.G. Mankiw (1990): "Permanent Income, Current Income,

and Consumption," *Journal of Business and Economic Statistics*, Vol.8, pp.269-279.

[9] Campbell J.Y. and N.G. Mankiw (1991): "The Response of Consumption to Income: A Cross-Country Investigation," *European Economic Review*, Vol.35, pp.723-767.

[10] Cantor, R. and N.C. Mark (1987): "International Debt and World Business Fluctuations," *Journal of International Money and Finance*, Vol.6, pp.153-165.

[11] Cantor, R. and N.C. Mark (1988): "The International Transmission of Real Business Cycles," *International Economic Review*, Vol.29, pp.493-507.

[12] Christiano, L.J. (1991): "Modeling the Liquidity Effect of A Money Shock," *Federal Reserve Bank of Minneapolis Quarterly Review*, Winter, pp.1-34.

[13] Christiano, L.J. and M. Eichenbaum (1991): "Liquidity Effects, Monetary Policy and the Business Cycle," Manuscript, Department of Economics, Northwestern University.

[14] Cooley, T.F. and G.D. Hansen (1989) : "The Inflation Tax in a Real Business Cycle Model," *American Economic Review*, Vol.79, pp.733-748.

[15] Dellas, H. (1986): "A Real Model of the World Business Cycle," *Journal of International Money and Finance*, Vol.5, pp.381-394.

[16] Devereux, M.B., A.W. Gregory and G.W. Smith (1990): "Realistic Cross-Country Consumption Correlations in a Two-Country, Equilibrium, Business Cycle Model," *Journal of International Money and Finance*, Vol.11, pp.3-16.

[17] Fuerst, T.S. (1992): "Liquidity, Loanable Funds, and Real Activity," *Journal of Monetary Economics*, Vol.29, pp.3-24.

[18] Gomme, P. (1991): "Money and Growth Revisited," Discussion Paper No.55, Institute for Empirical Macroeconomics, Federal Reserve Bank of Minneapolis.

[19] Greenwald B. and J.E. Stiglitz (1988a): "Money, Imperfect Information, and Economic Fluctuations," in M. Kohn and S. Tsiang (eds.), *Finance Constraints, Expectations, and Macroeconomics*, pp.141-165, Oxford University Press.

[20] Greenwald B. and J.E. Stiglitz (1988b): "Imperfect information, Finance Constraints, and Business Fluctuations," in M. Kohn and S. Tsiang (eds.), *Finance Constraints, Expectations, and Macroeconomics*, pp.103-140, Oxford University Press.

[21] Greenwood, J and G.W. Huffman (1987): "A Dynamic Equilibrium Model of Inflation and Unemployment," *Journal of Monetary Economics*, Vol.19, pp.203-228.

[22] Greenwood, J. and K. Kimbrough (1985): "Capital Controls and Fiscal Policy in the World Economy," *Canadian Journal of Economics*, Vol.18, pp.743-765.

[23] Greenwood, J. and K. Kimbrough (1987): "An Investigation in the Theory of Foreign Exchange Controls," *Canadian Journal of Economics*, Vol.20, pp.271-288.

[24] Greenwood J. and B. Jovanovic (1990): "Financial Development, Growth, and the Distribution of Income," *Journal of Political Economy*, Vol. 98, pp.1077-1107.

[25] Grossman, S. and L. Weiss (1983): "A Transactions-Based Model of the Monetary Transmission Mechanism," *American Economic Review*, Vol.73, pp.871-880.

[26] Helpman, E. and A. Razin (1982): "A Comparison of Exchange Rate Regimes in the Presence of Imperfect Capital Markets," *International Economic Review*, Vol.23, pp.365-388.

[27] Helpman, E. and A. Razin (1985): "Floating Exchange Rates with Liquidity Constraints in Financial Markets," *Journal of International Economics*, Vol.19, pp.99-117.

[28] Ho, W.M. (1992): "Liquidity, Exchange Rates, and Business Cycles," Research Report No.9204, Department of Economics, University of Western Ontario.

[29] Howitt, P. (1990): "Money and Growth Revisited," Research Report No.9014, Department of Economics, University of Western Ontario.

[30] International Monetary Fund (IMF) Annual Report on Exchange Arrangements and Exchange Restrictions (1992): pp.569-575.

[31] Levhari, D. and D. Patinkin (1968): "The Role of Money in a Simple Growth Model," *American Economic Review*, Vol.58, pp.713-753.

[32] Lucas, R.E., Jr. (1978): "Asset Prices in an Exchange Economy," *Econometrica*, Vol.46, pp.1429-1445.

[33] Lucas, R.E., Jr. (1982): "Interest Rates and Currency Prices in a Two-country World," *Journal of Monetary Economics*, Vol.10, pp.335-359.

[34] Lucas, R.E., Jr. (1990): "Liquidity and Interest Rates," *Journal of Economic Theory*, Vol.50, pp.237-264.

[35] Romer, P.M. (1986): "Increasing Returns and Long-Run Growth," *Journal of Political Economy*, Vol. 94, pp.1002-1037.

[36] Rotemberg, J.J. (1984): "A Monetary Equilibrium Model with Transactions Costs," *Journal of Political Economy*, Vol.92, pp.41-58.

[37] Rotemberg, J.J. (1985): "Money and the Terms of Trade," *Journal of International Economics*, Vol.19, pp.141-160.

[38] Stiglitz, J.E. and A. Weiss (1981): "Credit Rationing and Markets with Imperfect Information," *American Economic Review*, Vol.71, pp.393-411.

[39] Stockman, A.C. (1980): "A Theory of Exchange Rate Determination," *Journal of Political Economy*, Vol.88, pp.673-698.

[40] Stockman, A.C. (1981): "Anticipated Inflation and the Capital Stock in a Cash-in-advance Economy," *Journal of Monetary Economics*, Vol.9, pp.387-393

[41] Stockman, A.C., and A.D. Hernandez (1988): "Exchange Controls, Capital Controls, and International Financial Markets," *American Economic Review*, Vol.78, pp.362-374.

[42] Stockman, A.C. and L.L. Tesar (1990): "Tastes and Technology in a Two-Country Model of the Business Cycle: Explaining International Comovements," Working Paper No.255, Rochester Center for Economic Research.

[43] Tesar, L.L. (1990): "International Risk-Sharing and Nontraded Goods," Working Paper No.15-90, Department of Economics, University of California at Santa Barbara.

[44] Svensson, L.E.O. (1985): "Currency Prices, Terms of Trade, and Interest Rates," *Journal of International Economics*, Vol.18, pp.17-41.

[45] Svensson, L.E.O. and S. van Wijnbergen (1989) : "Excess Capacity, Monopolistic Competition, and International Transmission of Monetary Disturbances," *Economic Journal*, Vol.99, pp.785-805.

[46] Tobin, J. (1965): "Money and Economic Growth," *Econometrica*, Vol.33, pp.671-684.

[47] Williamson, S.D. (1987a): "Financial Intermediation, Business Failure, and Real Business Cycles," *Journal of Political Economy*, Vol. 95, pp.1196-1216.

[48] Williamson, S.D. (1987b): "Recent Developments in Modeling Financial Intermediation," *Federal Reserve Bank of Minneapolis Quarterly Review*, Summer, pp.19-29.

[49] Zeldes, S.P. (1989): "Consumption and Liquidity Constraints: An Empirical Investigation," *Journal of Political Economy*, Vol. 97, pp.305-346.