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HETEROSCEDASTICITY IN RETURNS: ARCH EFFECTS VERSUS THE MIXTURE OF DISTRIBUTIONS HYPOTHESIS

by

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Submitted in partial fulfillment
of the requirements for the degree of
Doctor of Philosophy

Faculty of Graduate Studies
The University of Western Ontario
London, Ontario
May, 1993

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ISBN 0-315-83933-3

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ABSTRACT

The main purpose of this thesis is to examine and compare the Mixture of Distributions Hypothesis versus Autoregressive Conditional Heteroscedasticity (ARCH) models as an explanation for the distribution of stock returns and the relationship of returns to measures of trading activity. The conjecture has been made by Lamoureux and Lastrapes, [1990a], that ARCH modelling of stock returns does not contribute any information if a variable representing the rate of flow of information is accounted for in the variance of the stock return. This thesis directly challenges this conjecture.

Three measures of trading activity, namely, the number of intraday changes in the bid-ask quotes, the number of daily transactions, and the volume of shares traded, are examined and the relationship of these variables to the variance of stock returns is studied. These variables are used as proxies for the rate of flow of information about a specific stock and are modelled using both Exponential ARCH, (EARCH), and Generalized ARCH, (GARCH), models. The data sample consists of daily returns and daily trading activity variable data for twenty securities traded on the Toronto Stock Exchange and twenty securities traded on the New York Stock Exchange.

The results of this thesis show that when stock returns are modelled using a GARCH model that the addition of a trading activity variable in the variance portion of the GARCH model renders the ARCH components insignificant. However when the more general EARCH model is utilized, then both the ARCH components and the trading activity variable become significant. These results contradict the results of Lamoureux and Lastrapes and show the limitations of using the GARCH model to model stock returns over an extended period of time. Model selection criteria always select an EARCH model over a GARCH model demonstrating the superiority of EARCH modelling.

Concerning the trading activity variables, the results of this thesis show that the number of changes in the intraday quotes for a stock is the best measure for modelling the rate of flow of information about a specific stock. This conclusion is particularly strong for the Canadian stocks in the sample. The results on the proper trading activity variable to use is more mixed for the American data although changes in quotes is still shown to be preferred.

The results of this thesis are important for both modelling volatility of stock returns and for determining the distribution of stock returns. An accurate knowledge of the distribution of stock returns is critical for testing hypotheses concerning stock market variables, especially in an event study setting.

DEDICATION

to my family

Lori, Sarah and Winston

for all your love, support, sacrifices, and understanding

ACKNOWLEDGEMENTS

In the preparation of this thesis I was helped and encouraged by a number of people. I would especially like to acknowledge my thesis supervisor, Robert W. White, for his support and interest in this thesis. I am also grateful to Mark D. Griffiths of the University of Wisconsin-Milwaukee for both his comments and encouragement throughout the process of producing this thesis and throughout my doctoral studies. I am indebted to Michael J. Robinson of the University of Calgary and Alasdair S. Turnbull of Memorial University for their assistance with the collection of the data from the computer databases.

I gratefully acknowledge the financial assistance that I received from the Social Sciences and Humanities Research Council of Canada.

Finally, I would like to thank my parents, my sister, and the many friends I have at the University of Western Ontario and in London, for their always present support and encouragement during the course of my doctoral studies.

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CHAPTER 1

INTRODUCTION

1.1 INTRODUCTION TO THESIS

The purpose of the thesis is to examine the Mixture of Distributions Hypothesis (MDH) as an explanation for the relationship between measures of trading activity and the distribution of stock returns and to compare the MDH against the Autoregressive Conditional Heteroscedasticity (ARCH) effects observed in stock returns.

Another goal and contribution of this thesis is to examine the role of the trading activity variables: (1) quotes, the number of intraday changes in the bid-ask quotes, (2) transactions, the number of daily transactions, and (3) volume, the percentage of shares outstanding which are traded during a day. The role and importance of these variables will be examined in the context of the MDH and ARCH models.

This thesis also examines the usefulness of the Exponential Autoregressive Conditional Heteroscedasticity (EARCH) model versus the simpler but more widely used Generalized Autoregressive Conditional Heteroscedasticity (GARCH) model.

The results of this thesis will lead to a better understanding of the distribution of stock returns and more specifically to a better understanding of volatility. Volatility has always been an important variable in the pricing of derivative securities, but since the events of October 1987, volatility has also gained in importance as a concern of equity investors. As Stoll and Whaley [1991] highlight, " volatility affects the public's confidence in security markets and ultimately the level of saving and investment in the economy." By separating out the GARCH effects from the MDH of equity returns, better inferences regarding volatility shocks will be possible.

One of the goals of financial researchers is to describe accurately the distribution of stock returns. Mandelbrot [1963] and Fama [1965] were among the first to demonstrate that stock returns are not normally distributed. Mandelbrot proposed that cotton prices are from the stable Paretian class of distributions. Fama [1963,1965] presents evidence that is consistent with stock prices also being from the stable Paretian class. Officer [1972], while providing evidence that stock returns are not normally distributed, uncovers evidence which is inconsistent with the stable Paretian hypothesis.

The MDH is a theoretical attempt to merge the empirical evidence on trading activity with the empirical evidence on return distributions showing non-normality. The MDH, first postulated by Press [1967] and later by Clark [1973], Westerfield [1977] and Harris [1987] among others, states that the return series comes from a set of normal distributions where the rate of information arrival determines the actual distribution or, more specifically, the mean and variance of the distribution for a given period is dependent upon the rate of arrival of information for that period.

A different approach is given by Engel [1982] who developed the autoregressive conditional heteroscedascity (ARCH) model to account for the behaviour of returns. ARCH models, as originally conceived by Engel, are purely statistical models of economic time series without a basis in economic theory. Despite this, ARCH type models have been used very successfully in modelling several types of financial time series, including equity return series. The ARCH model is very similar to the familiar AR or autoregressive econometric model, the only difference being that instead of the residual error term itself being lagged and modelled, the square of the error is lagged and modelled. In this sense, the ARCH framework allows modelling of the variance of the time series. Various extensions of Engel's model have been proposed to account for various stylized facts including the GARCH model by Bollerslev [1986], and the EARCH model developed by Nelson [1991].

However the question of the distribution of stock returns is still open. In addition, recent theoretical papers by Admati and Pfleiderer [1988,1989], Foster and Viswanathan [1990] and Blume Easley and O'Hara [1991], attempt to explain the distribution of returns and trading activity in terms of the patterns of arrival of information to the stock markets. In this manner, volume and the number of transactions also become important variables in the theoretical literature on the distribution of stock returns.

Attempting a partial solution to the problem, Lamoureux and Lastrapes [1990a] (LL) tested the GARCH model of Bollerslev [1986] in the context of the mixture of distributions model, using volume as the mixing variable. They conclude that no ARCH effects exist when contemporaneous volume is used as a mixing variable.

While it has been confirmed in several studies that stock returns follow an ARCH type process, it can also be easily confirmed that daily volume follows an ARCH type process as well. ARCH tests show significant ARCH effects for daily trading volume for both indices and individual securities. Therefore, it can be hypothesized that both returns and volume are being driven by the same mixing variable, a point which was first made by Press [1967]. Press hypothesized that the number of changes in the equilibrium price was the mixing variable for both the return series and for the volume series. This hypothesis, combined with other theoretical models of Tauchen and Pitts [1983] and Karpoff [1987], which show that volume and returns are related, along with empirical work on ARCH models by Pagan and Schwert [1991], raises serious questions about the specification and conclusions of the LL model and tests. If the earlier studies and hypotheses are correct, then the LL's model is seriously flawed in that returns and volume are contemporaneously related and a simultaneity bias of undetermined effect is present, a problem of which Lamoureux and Lastrapes are aware.

Since volume also follows an ARCH process, and volume is closely related to returns, then it is quite possible that the volume variable in the LL model is merely acting as a substitute for the previously well documented ARCH effects of returns. This hypothesis, if true, invalidates LL's conclusion that returns do not follow an ARCH process.

In a later paper, LL [1990b] demonstrate that the GARCH model is sensitive to deterministic structural changes that are not specified. When an extended time series of returns is being analyzed such structural changes are likely to occur, bringing into question the use of GARCH modelling for testing the MDH. The results of Lamoureux and Lastrapes [1990a] are consistent with the fact that the GARCH model cannot accurately model oscillatory shifts in volatility. The EARCH model however does not possess this limitation. In this sense then, the GARCH model of LL [1990a], with volume in the volatility equation, may be acting as a substitute EARCH process, where volume is acting as a proxy for the EARCH process. This can be tested by using the EARCH model directly.

A contribution of this thesis is to extend the work of LL by utilizing the EARCH model of Nelson [1991] which overcomes many of the limitations of the LL model. Several nested EARCH models will be developed and tested utilizing trading volume, the number of changes in mean of bid and ask quotes, and the number of transactions as variables. By utilizing several different trading activity variables or proxies for the rate of flow of information, the simultaneity problem inherent in the LL study can be reduced, or at least examined under different contexts. The EARCH framework will allow direct testing of the LL model against the theoretical model of Press [1967].

The results will shed light on the validity of the mixture of distributions hypothesis, and give insight into the best specification of the mixing variable. An improved understanding of the relationship of trading activity to stock returns and the operation of a securities market is an important by-product of this study. With the current high level of investor concern over market volatility, the results of this thesis are especially germane to stock exchange operators and regulators.

The question of the proper mixing variable is important to the distinguishing between volatility and trading activity being generated by information traders, or being generated by noise traders. Press [1967] suggests the mixing variable is the number of events that cause price changes, while Harris [1987] suggests using the number of transactions as mixing variable. In an efficient market framework, the equilibrium price should only change when new information is available. However, it is quite possible to conceive of transactions occurring even in the absence of new information. Thus it is suspected that the number of intraday equilibrium price changes would be a better proxy for the flow of information than the number of transactions. If so, the number of intraday equilibrium price changes, proxied by the number of intraday changes in the quotes, would be a better mixing variable than the number of daily transactions. The results of this thesis will help to resolve the issue of what the proper mixing variable should be.

Another by-product of this study will be a better understanding of the distribution of trading activity. A better understanding of the distribution of returns, and the role of trading activity variables, is critical for testing hypotheses concerning stock market variables, especially in an event study setting. Schwert and Seguin [1990] highlight problems of tests that do not take stock return heteroscedasticity into account. For example, Morgan and Morgan [1987] examine the small firm effect utilizing a GARCH framework. They find that with the GARCH model, test statistics are greatly improved and that the small firm results are strengthened. A major contribution of this study will be to improve further such inference tests and studies.

Three main hypotheses are tested in this thesis. The first hypothesis tests for misspecification in the LL model. The hypothesis examines whether or not coefficients on lagged squared residual error terms are significant in an EARCH model even when there is a trading activity variable also present in the variance equation. The second hypothesis tests for the significance of the trading activity variable and determines the proper trading activity variable to utilize. The third hypothesis tests for the superiority of the EARCH model over the more widely used GARCH model.

1.2 SUMMARY OF INTRODUCTION

In summary, the contributions of this thesis are:

(1) to compare directly the MDH versus ARCH models,

(2) to provide insight into the role of trading activity variables and how they fit into the correct specifications of the MDH or ARCH type models,

(3) to demonstrate the usefulness of using the advanced EARCH model over the simpler and more widely used GARCH models,

(4) to re-examine the results and conclusions of the LL study, and,

(5) to advance the debate on the distribution of stock returns and as a result, improve inferences in which heteroscedasticity may play a role.

The primary results of this thesis are:

(1) The lagged squared residual error terms do add information about the variance of equity terms even when a trading activity variable is present in the variance equation and accounted for. Thus both MDH and ARCH effects are present simultaneously. This result contradicts the conclusions of LL.

(2) The trading activity variables quotes and volume are significant with quotes appearing to be the preferred variable. The significance of the transactions variable is doubtful.

(3) For long time series, the EARCH model is superior to the GARCH models for modelling equity returns. For short time series the GARCH model may be superior.

Chapter two of this thesis provides a literature review of the relevant background material. Chapter three discusses the LL paper which forms the starting point for the subsequent models and hypotheses which are presented in chapter four. The data is discussed in chapter five. The results and conclusions are presented and discussed in chapter six. Chapter seven gives a summary of the thesis.

CHAPTER 2

REVIEW OF RELEVANT LITERATURE

II.1 DISTRIBUTION OF RETURNS

Early theories on the distribution of stock price changes postulated that returns follow a random walk. Bachelier [1900], and later Osborne [1959] developed the theoretical basis for such a hypothesis. In this hypothesis, it is assumed that price changes are independent, identically distributed random variables from transaction to transaction. If a finite variance for the series is assumed, and the number of transactions in any period is large, then the central limit theorem implies that the series of returns measured over a given period of time will follow a normal distribution.

Kendall [1953] and Moore [1962] provided empirical support for the hypothesis. However, the empirical studies of both Moore and Kendall find evidence that the distributions are leptokurtic relative to a normal distribution.

These discrepancies compelled Mandelbrot [1963] to propose the class of distributions known as the stable Paretian. The normal distribution is a special subset of the stable Paretian class. What distinguishes the normal distribution is the existence of a finite variance in the series in question. All other distributions in the stable Paretian class have infinite variance. Fama and Roll [1968,1971] provide a review of the properties of the stable Paretian distribution as well as the algorithms for estimation of its parameters.

Fama [1963,1965] models stock market data to see whether they fit better with Mandelbrot's stable Paretian distribution or with the normal distribution. After extensive testing, Fama concludes that the data fit the stable Paretian class best.

Officer [1972], conducts further examination of stock market returns and concludes that while the return distribution is best described by the symmetric stable class of distributions, there is also ample evidence against the hypothesis of stable distributions. Officer suggested that the search continue for an analytic distribution function that has a finite second moment for the distribution of returns. Thus the best that can be said is that stock returns are not normal, but the debate about a stable or non-stable class of distributions is not finished.

II.2 MIXTURE OF DISTRIBUTIONS HYPOTHESIS (MDH)

The early work on the distribution of returns led to the development of the MDH. The MDH is an attempt to merge the empirical evidence on trading activity with the empirical evidence on return distributions showing non-linearity. The MDH, first postulated by Press [1967] and later by Clark [1973], Westerfield [1977] and Harris [1987] among others, states that the return series comes from a set of normal distributions where the rate of arrival of information determines the actual distribution, i.e. the variance.

Kon [1984] provides a review of stock return models and various forms of the MDH.

Press's notation is shown in equation [2.1].

where: $Z(t)$ = the natural logarithm of price,

C = $Z(0)$ and is assumed to be known, [2.1] $$Z(t)=C+\sum_{k=1}^{\eta(t)} Y_k+X(t)$$

Y_t = a sequence of mutually independent random variables normally distributed as $N(\theta,\sigma^2_2)$.

$\eta(t)$ = a random variable which represents the number of random events that cause price changes in time period (t), and

$X(t)$ is a Markov error term which is independent of $\eta(t)$ and Y_L and is distributed $N(0,\sigma^2_1(t))$.

In this model, Z(t) will be normally distributed as;

[2.2] $$Z(t)\sim N(\theta\eta(t),\sigma_2^2\eta(t))$$

and $\eta(t)$ is called the mixing variable or sometimes the directing variable. The proxy for this variable and the assumptions about its distribution have been the focus of several studies. The MDH can explain the leptokurtic behaviour of returns, and is intuitively appealing. It makes sense that the market behaves differently during periods of peak activity from lull periods. Although Officer's [1972] study showed stability of the standard deviation of returns, thus contradicting Press's hypothesis, the MDH has remained a popular explanation for describing the distribution of stock returns.

Variations of the MDH include time deformation models in which returns are considered to be distributed normally in economic time, rather than in calendar time. An example of this model is given by Poon [1989] who frames his model in an ARCH framework. The intuition behind Poon's model (and all MDH models), is that returns evolve in economic time, which in turn is dependent on the flow of information. If an event occurs such that information flows rapidly to the market, then economic time moves faster. Thus comparing a time period in which there is little information flow, to a time period with much greater information flow, is analogous to comparing monthly returns to daily returns.

There remains the question of what to use as the mixing variable, η_t, the variable for the rate of flow of information. Press [1967] hypothesizes that η_t is the number of events causing price changes and is distributed as a Poisson process. More recently, Harris [1987] develops a MDH model in which i. is postulated that η_t is the number of transactions. Lamoureux and Lastrapes [1990a] (LL), implicitly assume that volume itself is the mixing variable as they usr volume as a proxy for the number of intraday price changes in a MDH type framework. The problem is, of course, that the rate of flow of information is unobservable and thus the question can only be answered empirically.

II.3 ARCH EFFECTS IN RETURNS

A more recent line of research has been to examine new econometric models to account for heteroscedasticity in stock returns. Engel [1982] proposed the autoregressive conditional heteroscedasticity (ARCH) model as a suitable framework for modelling the distribution of economic time series. The ARCH model, and the later extensions of it, is a time series technique. Since its introduction, the ARCH framework and extensions of it have been successfully and widely utilized in financial modelling. Bollerslev, Chou and Kroner [1992] provide a review of the use of ARCH applications in finance. The ARCH model, as first proposed by Engel, is shown in equation [2.3].

[2.3a] $$\varepsilon_t = y_t - x_t' b$$

[2.3b] $$\varepsilon_t | \psi_{t-1} \sim N(0, h_t)$$

[2.3c] $$h_t=\alpha_0+\sum_{i=1}^{q}\alpha_i\varepsilon_{t-i}^2$$

where:

y_t = the dependent variable,

x_t = the set of independent variables,

b = the set of regression parameters,

ψ_t = the information set at time t, and

α_i = parameters to be estimated.

In this framework, the variance is allowed to be time-varying. Time varying components of the variance can be incorporated into the α_0 term. For this reason, the ARCH framework is well suited for modelling time series with a changing variance.

The ARCH model and the MDH can, of course, be compatible. For example, and as already discussed, Poon [1989] modelled the MDH in an ARCH framework. By setting part of the α_0 term equal to the MDH mixing variable, one can set the MDH in an ARCH framework. If the MDH mixing variable is the only determinate of the variance, then all $\alpha_i = 0$, for $i > 0$. The MDH, as formulated by Press [1967] does not emphatically exclude other factors from also affecting the variance. The correct and exact specification is an empirical question which this thesis addresses.

Early studies utilizing the simple ARCH model frequently uncovered a long lag process in the variance equation, that is a large value for q. To allow for more parsimonious modelling, Bollerslev [1986] developed the Generalized ARCH model (GARCH). Bollerslev's model is shown in equation [2.4].

[2.4b] $$\varepsilon_t|\psi_{t-1}\sim N(0,h_t)$$

[2.4a] $$\varepsilon_t = y_t - x_t' b$$

[2.4c] $$h_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{i=1}^{p} \beta_i h_{t-i}$$

where:

y_t = the dependent variable,

x_t = the set of independent variables,

b = the set of regression parameters,

ψ_t = the information set at time t, and

α_i = parameters to be estimated.

In both the Engel and the Bollerslev models, there are restrictions on the parameter coefficients in the variance equation. More specifically, the restrictions are:

$p \geq 0$, $q > 0$, $\alpha_0 > 0$, $\alpha_i \geq 0$ for $i = 1,....q$

and $\beta_i \geq 0$ for $i = 1,....p$

Because of these restrictions, the ARCH and GARCH models cannot possibly model a series in which the variance shifts directions several times. For modelling over a short period of time, a one time shock to the variance poses no problem. However, in modelling a longer time series, this becomes a major limitation.

To overcome these limitations and to allow for the modelling of some well documented empirical facts, Nelson [1991] proposed an extension of the ARCH model called Exponential Autoregressive

Conditional Heteroscedascity (EARCH). The EARCH model as proposed by Nelson is shown in equation [2.5].

[2.5a] $$R_t = a + bR_{t-1} + c\sigma_t^2 + \varepsilon_t$$

[2.5b] $$\ln(\sigma_t^2) = \alpha_t + \frac{(1+\psi_1 L + \cdots + \psi_q L^q)}{(1-\Delta_1 L - \cdots - \Delta_p L^p)} g(Z_{t-1})$$

[2.5c] $$g(Z_t) = \theta Z_t + \gamma(|Z_t| - E|Z_t|)$$

[2.5d] $$\alpha_t = \alpha + \ln(1 + \delta n_t)$$

[2.5e] $$Z_t = \sigma_t^{-1}(R_t - a - bR_{t-1} - c\sigma_t^2)$$

where R_t = the daily index return,

n_t = the number of preceding non-trading days,

L = the lag operator,

E = the expectations operator,

Z_t = the normalized residuals, and

Ψ_i, Δ_i, θ, γ, α, δ, a, b, and c are parameters to be estimated.

The function $g(Z_t)$ allows for asymmetric relationships with regard to past shocks to volatility. The overall variance relation, [2.5b], is a non-linear function of past values of $g(Z_t)$.

Nelson applies his framework to CRSP value weighted index returns over the period July 1962 to December 1987, and finds that the fit of the model "... seems remarkably good." There are several advantages to using the EARCH framework over the earlier ARCH type frameworks. These advantages are: (1) the "g" function allows for asymmetric relationships among the variables in the variance relation, (2) the constraints on the parameters of the model are reduced, (3) oscillatory behaviour of the variance is allowed, (4) it provides an easier method to evaluate the persistence of shocks to variance, and (5) assumptions about the underlying distribution are relaxed by utilizing the Generalized Error Distribution (GED).

Pagan and Schwert [1990], compare the effectiveness of several models of monthly stock returns, including ARCH, GARCH and EARCH, and conclude that among the parametric models, EARCH works best. They attribute the improved explanatory power to the ability of the EARCH model to reflect the asymmetric relationship between volatility and past returns that was first observed by Black [1976].

Hsieh [1989], models daily foreign exchange rates and concludes that EARCH models provide slightly better fit than GARCH models. A significant finding of Hsieh is that by using EARCH models, one can show for foreign exchange data that variances are not integrated and thus that unconditional variances are finite. This is generally not the case for GARCH models as for example in the LL study. The apparent ability of the EARCH model to avoid models with integrated variances is potentially important for studies which attempt to examine a structural break in variances. If the model employed consistently results in integrated variances, then such studies will be indeterminate and unfruitful. Intuitively, it is hard to ascertain exactly what an indeterminate or infinite unconditional variance really means.

In contrast to Hsieh [1989] and Pagan and Schwert [1990], Day and Lewis [1992], in studying weekly index returns, provide evidence that GARCH models may have slightly superior forecasts of volatility than EARCH models. It is not clear what the effect of the frequency of th data is on whether or not EARCH is superior to GARCH or vice-versa. However, it is suspected that the higher the frequency of the data, the more suitable the EARCH model would be since the EARCH model allows for directional shifts in the variance which are more likely to be found in high frequency data.

The most appropriate form of ARCH model to use is still an open question in the finance literature. This study contributes to the debate.

II.4 TRADING ACTIVITY VARIABLES

Trading activity variables examined in this study include volume, (defined as the percentage of outstanding shares traded in any given period), number of transactions in a day and the number of daily changes in the mean of the bid-ask spread, (which is given the variable name "quotes"). In both the MDH and the ARCH models, it appears that trading activity plays a central role in the distribution of returns. An important contribution of this thesis is a better understanding of the role of volume and other trading activity variables, and the information that can be gathered from the statistics of these variables.

Measures of trading activity such as volume or number of transactions are often mentioned but rarely studied. The lack of studies on volume in particular has been attributed to both a lack of data and the common assumption of homogenous investors (Karpoff [1987]). However interest in volume has recently increased in connection with studies testing technical trading rules, for example Blume, Easley and O'Hara [1991].

The set of empirical studies on trading activity can be divided into those that examine (1) intraday patterns, (2) intraweek patterns, (3) seasonal patterns, and (4) bursts of trading activity.

Intraday patterns were first studied by Cootner [1964] and more recently by Amihud and Mendelson [1987], Mulherin and Gerety [1988], and Jain and Joh [1988]. All of these studies find a significant "U" shaped pattern throughout the day with volume high both early and late in the trading day.

Both Mulherin and Gerety [1988] and Jain and Joh [1988] find a strong intraweek pattern with volume being low on Monday, increasing through to Wednesday, and then declining through to Friday.

As for a seasonal pattern, Cootner [1964] finds that volume is high at the turn of the year, declines through the summer and rises in the fall. This finding is supported by Lakonishok and Smidt [1984].

Anecdotal evidence suggests that trading tends to occur in bursts. Studies by Osborne [1964], Granger and Morgenstern [1970] and Morse [1980] confirm this finding. Clustering of peaks in the volume time series is significant since it suggests the use of an ARCH model whenever volume or transactions are modelled. ARCH testing of the volume and transactions series in this thesis confirms that the two series do indeed conform to an ARCH process. A daily series constructed by examining the number of changes in the bid-ask quotes for individual stocks also follows an ARCH process. The fact that all three of these trading activity variables exhibit ARCH characteristics implies that information flows to the market in bursts and/or trading generates trading.

II.5 THE RELATIONSHIP OF VOLUME TO RETURNS

The impetus for studying the relationship of volume to returns comes from two familiar Wall Street adages; (1) it takes volume to move prices, and (2) volume is higher in bull markets than in bear markets. The literature in this area is quite well developed and is reviewed in Karpoff [1987]. The empirical studies focus on the relationship of; (1) volume to the absolute value of returns, and (2) volume to returns per se.

The bulk of the evidence seems to support a positive simultaneous relationship between volume and the absolute value of returns. Ying [1966], Crouch [1970] , Epps and Epps [1976], Wood, McInish and Ord [1985] and Jain and Joh [1986] all support a positive simultaneous relationship, confirming the adage that it takes volume to move prices. The only known dissenting study is that of Godfrey, Granger and Morgenstern [1964] who do not find evidence of a relationship between the two variables.

The evidence for a relationship between volume and returns per se is much more ambiguous. Ying [1966], Epps [1977], Rogalski [1978] and Jain and Joh [1986], all find evidence of a positive relationship between volume and returns per se. Studies which do not find a relationship include Godfrey, Granger and Morgenstern [1964], James and Edmister [1983] and Wood McInish and Ord [1985].

To account for both the volume versus return relationship and the volume versus absolute value of return relationship, Karpoff [1987], proposes an asymmetric volume-price change relation. Karpoff bases his model on costly short selling constraints which limit the participation of traders on the "downside" vis à vis the "upside" movement of a stock's price. Karpoff thus hypothesizes that volume is related to both positive and negative returns, but the slope of the relationship is greater for positive returns. In this way, empirical studies will find both a relationship between volume and the absolute value of returns and between volume and returns per se.

Epps [1975] provides a different explanation for the same effects based upon behaviourial differences in the actions of "bulls" and "bears". In his model, Epps hypothesizes that interpretations of new information tends to reinforce existing opinions. However, "good news", tends to be more reinforcing than "bad news", and thus the slope of the relationship between volume and returns is steeper for positive returns than it is for negative returns.

II.6 VOLUME AND RETURN VOLATILITY

Granger and Morgenstern [1970] were among the first to examine the relationship between volume and return volatility. Using interday highs and lows in stock prices as their measure of return volatility, they find a positive relationship with volume. Epps and Epps [1976] demonstrate that volume has explanatory power in a regression of return volatility on contemporaneous volume. Finally, Rogalski [1978] examined the causality between returns and volume, showing that they are related but contemporaneous. He concludes however that volume can be used to predict the volatility of price changes.

Tauchen and Pitts [1983] construct a model of the market to examine the effects of a changing number of traders in a security. From their theory, they develop a joint likelihood function of price variability and volume. Their model, and an empirical examination of the T-bill futures market, shows that with the number of traders fixed, the daily price change is leptokurtic and the square of the daily price change is positively related to trading volume. From their model, a MDH model, identical in structure to Press [1967] evolves, in which both price and volume series are drawn from a mixture of normals with the mixing variable being the number of equilibrium price changes.

Gallant, Rossi, Tauchen and Pitts [1990], study the co-movement of returns and volume in an ARCH framework. They uncover four empirical regularities. They are; "(1) there is a positive correlation between conditional volatility and volume, (2) large price movements are followed by high volume, (3) conditioning on lagged volume substantially attenuates the 'leverage' effect, and (4) after conditioning on lagged volume, there is a positive risk / return relation."

II.7 SUMMARY OF LITERATURE

There appears to be agreement that the distribution of stock returns is not normal and that volume and returns are somehow related. Both returns and volume seem to exhibit ARCH type distribution effects. Two models, not necessarily competing, are proposed to explain the empirical findings; the mixture of distributions hypothesis and the ARCH model and its extensions. As will be discussed in the next chapter, LL propose that modelling returns in the MDH framework with volume as the mixing variable is superior to the straight ARCH modelling of returns.

Knowledge of the distribution of returns, and the role of trading activity, is important for many reasons but primarily for use in conducting event studies, especially those that attempt to quantify an amount of information. More specifically, current investor concern about excess market volatility justifies further studies into return variance characteristics.

Due to the richness of the data available for this study, it is possible to examine other measures of trading activity, namely the number of changes in the bid-ask quotes and the number of transactions in a day. Intuitively and theoretically, these are important variables to examine in the context of studying the distribution of stock returns. However these variables have received scant attention in the empirical literature. This thesis addresses this shortfall of the current literature.

CHAPTER 3

THE LAMOUREUX AND LASTRAPES [1990a] STUDY

III.1 THE LAMOUREUX AND LASTRAPES [1990a] (LL) STUDY AND FINDINGS

Synthesizing previous research results, LL tie together autoregressive conditional heteroscedasticity (ARCH) modelling of returns, the relationship of returns to volume, and the Mixture of Distributions Hypothesis (MDH). They test empirically the notion, first suggested by Diebold [1986], Gallant, Hsieh, and Tauchen [1988] and Stock [1987,1988] that the ARCH process is capturing the time series properties of the mixing variable of the MDH.

LL fit the following Generalized Autoregressive Heteroscedasticity (GARCH) model, an extension of the ARCH model developed by Bollerslev [1986]. LL's adaptation is shown in equation [3.1].

[3.1a] $$R_t = \mu_{t-1} + \varepsilon_t$$

[3.1b] $$\varepsilon_t | (\varepsilon_{t-1}, \varepsilon_{t-2}, \ldots) \sim N(0, h_t)$$

[3.1c] $$h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \alpha_2 h_{t-1} + \alpha_3 V_t$$

where R_t = the daily stock return,

μ_t = the average daily stock return, (LL set $\mu_t = 0$),

V_t = the daily volume, and

α_i are parameters to be estimated.

LL let δ_{it} denote the ith intraday equilibrium price increment in day t, which implies that

{3.1d} $$\varepsilon_t = \sum_{i=1}^{\eta_t} \delta_{it}$$

where ε_t is the residual term in equation (3.1a), and

η_t is the number of changes in intraday equilibrium price.

LL state that η_t, the unobservable number of changes in intraday equilibrium price, is the mixing variable which represents the stochastic rate at which information flows into the market. The model is thus consistent with the earlier models of the MDH. LL use daily volume as a proxy for the mixing variable η_t.

For their sample, LL examined twenty actively traded stocks which also had options traded on the CBOE. Their source for returns was the CRSP data base and they used Standard and Poor's Daily Stock Price Records to gather the volume data.

When they used a simple GARCH model, (equation [3.1] but without the volume term included in equation [3.1c]), LL found that at least one of the GARCH parameters, (i.e. either α_1 or α_2) were statistically significant at the 5 percent level. They conclude that by itself, the GARCH model can adequately model stock returns. With the simple GARCH model they also find that the average sum of α_1 and α_2 terms is 0.728.

The results were quite different when they included the volume term in the model. The coefficient α_1 was only significant at the 5 percent level four times out of twenty and the coefficient α_2 was not significant for any of the twenty stocks. However the coefficient on the volume term, α_3, was significant in all twenty cases. In addition the average sum of α_1 and α_2, was now only 0.073.

Thus when the volume term is excluded from the variance equation, parameters α_1 and α_2 are both significant. LL also find that the sum of α_1 and α_2 is closer to one than it is when the volume term is included. When $\alpha_1 + \alpha_2 = 1$, then the variances are said to be integrated and the unconditional variance will be infinite. As will be discussed shortly, the finding of integrated variances is a common problem encountered with GARCH models.

When the volume term is included, both α_1 and α_2 are insignificant while α_3 is significant and positive. In addition, the sum of α_1 and α_2 is greatly reduced. LL conclude that ARCH effects in returns are a result of a dependence on the rate of arrival of information which can be proxied by volume. In effect, they provide evidence for the MDH with volume as the mixing variable and no other variable affecting the variance. LL suggest that, "lagged squared residuals contribute little if any additional information about the variance of the stock return process after accounting for the rate of information flow as measured by contemporaneous volume."

III.2 LIMITATIONS OF THE LL STUDY

There exist several limitations of the LL study. To begin with, LL utilize the GARCH framework of Bollerslev [1985]. As highlighted by Nelson [1991], and Pagan and Schwert [1991], the GARCH model will be inferior to the EARCH model due to the fact that the GARCH model cannot model oscillatory behaviour in the volatility due to parameter restrictions. In light of this, it is quite possible that the volume variable is simply a method to overcome the constraints of the GARCH model. In fact, it is likely that volume itself follows an EARCH process, thus supplanting ARCH effects of returns in the LL framework.

A second problem concerns the LL findings of an integrated variance with the plain GARCH model. When Hsieh [1989] modelled foreign exchange rates with a GARCH process, he found the variances were integrated whereas the variances were not integrated when modelled with a EARCH model. If a GARCH model, with volume included as an exogenous variable in the variance equation was in effect a substitute for a EARCH process, with the volume variable accounting for the EARCH effects, then the results of LL would be consistent with the results of Hsieh concerning integrated variances.

A third major potential problem with the LL model has to do with a possible simultaneity bias between contemporaneous volume and returns. As Press [1967] originally proposed, (see also Tauchen and Pitts [1983]), volume and price change are hypothesized to be a joint random function of information flow. LL state, "if this specification is correct, our estimation is subject to an unquantified specification bias." The review by Karpoff [1987], suggests that empirically such a relationship does exist. LL note that if volume is not exogenous, then any study that regresses volatility on volume will be subject to a simultaneity bias of indeterminate direction.

Other variables for use in testing the mixture of distributions hypothesis seem to be more appropriate than volume. The original model of Press [1967], suggests using the number of daily changes in the equilibrium price or, by extension, the number of daily changes in the mean of the bid and ask. (Blume and Stambaugh [1983] suggest the use of the mean of the bid and ask as an accurate estimate of the true equilibrium price. See also Roll [1984].) The model of Harris [1987] suggests the use of the number of transactions as the mixing variable. In MDH models, including the model of LL, η_t is the rate at which information flows to the market. Since this variable is unobservable, it is an empirical issue which variable is the best proxy. It is possible to have trading volume without information due to liquidity reasons. However if there is information, then the Efficient Markets Hypothesis implies that there will be a change in equilibrium prices. Thus, it can be argued that changes in equilibrium price means there is a flow of information.

III.3 SUMMARY OF LL PAPER

The LL paper ties together the MDH and the ARCH models in finance. LL utilize a GARCH framework and add an additional variance term for the rate of information flow, proxied by the volume. They find that ARCH effects disappear when the additional variance term for rate of information flow is included. They conclude that ARCH models do not improve model fit for equity returns once the rate of information flow is properly accounted for.

The LL findings are significant and interesting because they:

(1) provide support for the MDH,

(2) motivate the use of trading activity variables, and,

(3) shed more information on the use of ARCH modelling of equity returns.

The major drawbacks of the LL study are:

(1) the use of the GARCH model and thus all of its limitations, and,

(2) the questionable use of volume as a proxy for the rate of information flow.

Because of the historical and theoretical importance of the debate over the MDH, the current popularity of ARCH models, and the concern over volatility in financial markets, this thesis will extend the model of LL in a more rigorous and appropriate manner and test in more detail the specification of the MDH.

CHAPTER 4

ARCH MODELS AND TESTABLE HYPOTHESES

IV.1 THE GENERAL MODEL

Several models are examined in this study. The equations for all the models are presented together for reference purposes in Exhibit 1.

The general model to be tested is a direct extension of the LL model. However in this study, the basic model to be utilized is the EARCH model in place of the GARCH model used by LL. More specifically, the model is as follows:

[4.1a] $$R_t = a + bR_{t-1} + c\sigma_t^2 + \varepsilon_t$$

[4.1b] $$\ln(\sigma_t^2) = \alpha_t + \frac{(1+\Psi_1 L + \cdots + \Psi_q L^q)}{(1-\Delta_1 L - \cdots - \Delta_p L^p)} g(Z_{t-1})$$

[4.1c] $$\alpha_t = \alpha + \ln(1 + \delta u_t + f\eta_t)$$

[4.1d] $$g(Z_t) = \theta Z_t + \gamma(|Z_t| - E|Z_t|)$$

[4.1e] $$Z_t = \sigma_t^{-1}\varepsilon_t$$

where R_t = the daily return,

η_t = the mixing variable,

u_t = the number of days non-trading,

E = the expectations operator,

L = the lag operator, and

$a, b, c, \delta, f, \psi, \Delta, \alpha, \theta, \gamma$ are model parameters to be estimated.

The model is identical to the model of Nelson's [1991], with the exception of the additional term in equation [4.1c] to include the mixing variable, η_t.

To ease the exposition, henceforth this model will be referred to as Model 1. If there is no mixing variable in the model, (that is the $f\eta_t$ term is left out), then it will be refe~~ed to as Model 2.

The values of the lag parameters, p and q, could be determined by evaluating the equation for several values of p and q and then using the Akiake Information Criteria (AIC) or the Schwarz Information Criteria (SIC) to select the most appropriate model specification. For this study, the values of p and q were taken to be 2 and 1 respectively. These were the values determined to be most suitable in the study by Nelson [1991].

The models were fitted using maximum likelihood and the Berndt, Hall, Hall and Hausman [1974] (BHHH) algorithm. The likelihood function is given by Nelson [1991] and is reproduced in equation [4.2]. This representation of the likelihood function utilizes for the underlying distribution the more general Generalized Error Distribution (GED).

[4.2a]

$$L_t = \sum_{t=1}^{T} [\ln(\nu/\lambda) - \frac{1}{2}|(\varepsilon_t)/(\sigma_t\lambda)|^{\nu} - (1+\nu^{-1})\ln(2) - \ln[\Gamma(\nu^{-1})] - \frac{1}{2}\ln(\sigma_t^2)]$$

[4.2b]

$$\lambda = [2^{(-2/\nu)}\Gamma(1/\nu)/\Gamma(3/\nu)]^{1/2}$$

Model specification tests are given by Newey [1985] and presented in Nelson [1991]. The specification test equations are presented in Exhibit 2. The tests are based upon orthogonality conditions which a correct model specification would impose on the standardized error residuals, Z_t. The first two tests test for misspecification of the distribution. Conditions three through seven, test for misspecification of the conditional heteroscedasticity. Tests eight through twelve check for misspecification in the mean equation.

Standard t-tests can be used to determine the significance of variable coefficients if the usual assumption of the maximum likelihood estimator being consistent and asymptotically normal is invoked. Nelson [1991], however, highlights that invoking these usual assumptions may not be so innocent. Verifying that the conditions necessary for the maximum likelihood estimator to be consistent and asymptotically normal is exceedingly difficult. The complete asymptotic theory of EARCH models is yet to be developed. In spite of these difficulties, it is assumed for the purposes of this thesis, that the maximum likelihood estimator is asymptotically normal and thus standard t-tests can be used to determine the significance of coefficients. The reader is cautioned, however, that this assumption lacks a sound statistical basis.

IV.2 ADVANTAGES OF THE GENERAL MODEL

The variable u_t is included to account for the effect of non-trading as noted by French and Roll [1986] and verified in the original model of Nelson [1991]. Using the EARCH model avoids the non-negativity parameter constraints that are implicit in the GARCH model of LL. The "g" term in equation [6d] allows for differing volatilities in bull and bear markets which the models of Karpoff [1987] and Epps [1975] suggest and the empirical work of Black [1976] demonstrates.

In addition, the more encompassing GED is used to weaken the distributional constraints. The "v" in the likelihood function is a parameter which determines the form of the distribution. If v equals 2, then the distribution is normal. If v < 2, the distribution has thicker tails than normal, while if v > 2 the distribution has thinner tails than normal.

IV.3 ALTERNATIVE MODELS

Model 1 belongs to the class of models generally known as ARCH-in-Mean models, namely because the variance term appears in the mean regression equation. This places the onus on the modeller to correctly specify the entire system accurately and correctly. Namely, both the mean and variance equations must be correctly specified. In a non-ARCH-in-Mean model, consistent estimates of the parameters in the mean equation can be obtained even if the variance equation is misspecified. In addition, Nelson [1991] used index data to develop his model while this study uses individual security data. Using index data tends to introduce index induced serial correlation which is not present when individual security returns are examined. As a result it is questionable whether the ARCH-in-Mean and the autoregressive term in the mean equation of Model 1 are appropriate. For these reasons, and to allow better comparison to the results of LL, the following EARCH model was also examined.

[4.3a] $$R_t = \varepsilon_t$$

[4.3b] $$\ln(\sigma_t^2) = \alpha_0 + \alpha_1 \ln(f\eta_t) + \frac{(1+\Psi L)}{(1-\Delta_1 L - \Delta_2 L^2)} g(Z_{t-1})$$

[4.3c] $$g(Z_t) = \theta Z_t + \gamma(|Z_t| - E|Z_t|)$$

where R_t = the daily return,

[4.3d] $$Z_t = \sigma_t^{-1}\varepsilon_t$$

η_t = the mixing variable,

u_t = the number of days non-trading,

E = the expectations operator,

L = the lag operator, and

a,b,c,δ,f,ψ,Δ,α,θ,γ are model parameters to be estimated.

This model will henceforth be referred to as Model 3. A model identical to Model 3, but without a mixing variable included, (that is without the $\alpha_t \ln(f\eta_t)$ term), will be named Model 4.

Model 3 is more closely aligned with the model of LL in that the mean equation is identical. If the EARCH parameters, namely, α,Ψ,Δ,θ,γ, are all insignificant, then the model collapses to be identical in form to the LL model. In addition Model 3, unlike Model 1, is not an ARCH-in-Mean type model. Therefore a slight misspecification in the variance equation should not have drastic consequences for the system as a whole.

Finally, the LL model will be referred to as Model 5, and a model identical to the LL model but without a mixing variable will be referred to as Model 6. The LL model is shown in equation [4.4].

[4.4] $$R_t = \mu_{t-1} + \varepsilon_t$$

[4.4b] $$\varepsilon_t | (\varepsilon_{t-1}, \varepsilon_{t-2}, \ldots) \sim N(0, h_t)$$

[4.4c] $$h_t = \alpha_0 + \alpha_1\varepsilon_{t-1}^2 + \alpha_2 h_{t-1} + \alpha_3\eta_t$$

where R_t = the daily stock return,

μ_t = the average daily stock return, (LL set μ_t = 0),

η_t = the daily volume, and

α_i are parameters to be estimated.

The models are listed for reference purposes together in Exhibit 1.

IV.4 TESTABLE HYPOTHESES

There are three main testable hypotheses. The first concerns misspecification in the LL model and the nonlinearities noted by Nelson [1991] and Pagan and Schwert [1991]. The LL GARCH model cannot account for non-linearities in the variance equation, and remembering the asymmetric price-volume relationship of Karpoff [1987], it may be that volume is proxying for the nonlinearities, since the α's in the LL model are constrained to be non-negative. More directly, it can be shown that volume series themselves tend to follow an ARCH process. Thus, as hypothesized earlier, the volume ARCH process may be masking the return ARCH process in the LL model. Therefore, the first hypothesis, which tests for misspecification in the LL model is :

Hypothesis 1:

H0 : $f > 0$; ψ, Δ_i, θ, γ all = 0

H1 : at least one of ψ, Δ_i, θ, $\gamma \neq 0$

If the conditions of the null hypothesis are satisfied, then the model is similar in form to the model of LL. If the null hypothesis is rejected, then it implies misspecification in the LL model. This misspecification will be most likely due to the constraints on the GARCH model which LL use.

The second hypothesis tests the significance of the possible simultaneity bias in the LL study. In this test, the mixing variable is set equal to the number of daily changes in the mean of the bid and ask quotes. If Press's [1967] model is correct, then the simultaneity problem will be reduced with this formulation. Therefore, with η_t defined to be the number of daily changes in the mean of the bid and ask quotes, (henceforth referred to as the variable quotes), the hypothesis to be tested is:

<u>Hypothesis 2</u>:

H0 : $f = 0$; and at least one of ψ, Δ_t, θ, $\gamma \neq 0$

H1 : $f > 0$; and at least one of ψ, Δ_t, θ, $\gamma \neq 0$

If the null hypothesis is rejected, then it implies that a mixing variable adds information to the variance of the distribution and Press's model is supported. Harris's [1987] model suggests the use of transactions as the most appropriate mixing variable to use. This conjecture will be examined as well.

Hypothesis 1 and hypothesis 2 will be examined using both model 1 and model 3.

To clarify the effect of using EARCH versus GARCH, the simpler EARCH models, namely Model 3 and Model 4, can be constructed and compared against the GARCH models, Model 5 and Model 6. The comparisons can be made using either the Akaike Information Criteria (AIC) or Schwarz's Information Criteria (SIC).

The AIC chooses the model which minimizes Akaike's information which is given in equation [4.5].

[4.5] $$AI = -2\max L(\theta_Q) + 2Q$$

where max $L(\theta_Q)$ is the maximum likelihood value for a set of parameters and Q is the number of parameters in the equation. Likewise the SIC chooses the model which minimizes Schwarz's information which is given in equation [4.6].

[4.6] $$SI = -2\max L(\theta_Q) + Q\log(T)$$

where T is the number of observations used to calculate the likelihood value. When the data set is large, the SIC tends to favour the model with fewer parameters. In this study, the SIC will exhibit a tendency toward selecting the simpler GARCH type models and rejecting the EARCH type models which have more parameters. If the GARCH model is truly limiting, then the SIC will select the EARCH models, and thus the SIC is chosen as the model selection criteria. This leads to the third hypothesis:

<u>Hypothesis 3</u>:

H0 : The SIC selects the GARCH models

H1 : The SIC selects the EARCH models

The GARCH likelihood functions were maximized using a scanning process, (also known as a grid search), over 360 initial starting values for the maximization algorithm. The grid consisted of starting the maximization procedure with all possible pairs of α_1 and α_2 from 0.1 to 0.9 in steps of 0.1. For example the maximization was started with the pairs (0.1, 0.1), (0.2, 0.1), (0.3, 0.1), ... (0.8, 0.1), (0.2, 0.1), etc. The α_3 variable was also searched over an order of magnitude in ten equal steps. This gives strong confidence that the likelihood functions for the GARCH models are maximized.

The EARCH maximization were all started with only one set of starting parameters. It was not practical to perform a grid search for the EARCH likelihood functions due to the excessive computer time that each EARCH maximization took. The differences in maximization routines biases the selection procedure toward the GARCH models. Thus if the null of hypothesis 3 is rejected, then it is strong evidence that the EARCH models have superior modelling power over GARCH models when modelling daily equity returns.

The MDH model of Press [1967], (where the mixing variable is the number of daily price changes), can be tested against the model of Harris [1987], (where the mixing variable is the number of transactions), by reformulating the model in hypothesis (2) by defining η_t to be the number of daily transactions. By examining the likelihood values for the resulting model, it can be determined empirically which formulation fits the data best. The determination of the proper mixing variable is critical in new theories which model the information implicit in both the size and rate of stock transactions, (for example Blume, Easley and O'Hara [1991]).

The determination of the mixing variable can also assist in the determination of the causes of volatility. Two commonly cited causes of return volatility are information based trading, and the generation of trading being self-generated. While there is no clear cut distinction, one would expect that changes in the quotes would reflect the results of information trading, in part because of the implications of the Efficient Markets Hypothesis. Conversely, and in contradiction of the Efficient Markets Hypothesis, the number of transactions would tend to reflect trading being self-generating, i.e. traders preferring to trade only when other traders are trading which in turn leads to even more trading etc.

The choice of transactions would however be consistent with the models of Admati and Pfleiderer, [1988], [1989], in which they conjecture that both uninformed and informed traders will prefer to trade when the market is active with traders versus a market in which few traders are participating. Admati and Pfleiderer's model suggests that there will be higher return variability when the market is more active with traders. Thus if Admati and Pfleiderer's model is accurate, one would expect transactions or volume to be the most appropriate trading activity variable.

IV.5 SUMMARY

The models to be examined in this study consist of four EARCH models, (two of which are ARCH-in-Mean models), and two GARCH models which are duplicates of the models which LL examined. The models are presented together in Exhibit 1.

There are three main testable hypotheses. The first two hypotheses are concerned with the estimated parameters of Model 1. The first hypothesis tests the significance of the EARCH specific parameters when there is also a trading activity variable included in the variance equation. The hypothesis is:

Hypothesis 1:

H0 : $f > 0$; and ψ_i, Δ_i, θ, γ all $= 0$

H1 : at least one of ψ_i, Δ_i, θ, $\gamma \neq 0$

Rejection of the null of Hypothesis 1 implies misspecification of the LL model. In addition it throws doubt on their assertion that "lagged squared residuals contribute little if any additional information about the variance of the stock return process after accounting for the rate of information flow as measured by contemporaneous volume."

The second hypothesis examines the use of the variable "quotes" as the trading activity variable relative to the variable "volume" which LL used. The second hypothesis is:

Hypothesis 2:

H0 : $f = 0$; and at least one of ψ_i, Δ_i, θ, $\gamma \neq 0$

H1 : $f > 0$; and at least one of ψ_i, Δ_i, θ, $\gamma \neq 0$

Rejection of the null, with the trading activity variable being either quotes or volume, implies that a trading activity variable adds additional information along with lagged squared values of the residual error term. The variable "transactions" is also included in the model of Hypothesis 2 to check on the validity of Harris's [1987] MDH model and the trading activity theories of Admati and Pfleiderer [1988, 1989].

The third hypothesis tests whether or not EARCH models are superior to GARCH models in the modelling of equity returns. The third hypothesis is:

Hypothesis 3:

H0 : The SIC selects the GARCH models

H1 : The SIC selects the EARCH models

Rejection of the null demonstrates that EARCH models are superior.

CHAPTER 5
DATA

V.1 DESCRIPTION OF DATA

The data for this study consists of two samples of twenty equity securities each. The first sample consists of twenty Canadian securities traded on the Toronto Stock Exchange (TSE). The second sample consists of twenty American companies which are traded on the New York Stock Exchange (NYSE).

Lamoureux and Lastrapes [1990a], (LL), use the data for twenty stocks in their study for a period of time of approximately 18 months. The number of securities to be tested in this study is double that number and for a significantly greater period of time. The number of observations per security in this study is approximately 1900 days of data. Lamoureux and Lastrapes [1990b] demonstrate that using Generalized Autoregressive Conditional Heteroscedasticity, (GARCH), models for long time series may be inappropriate since the GARCH framework does not allow for deterministic structural shifts in the variance. Using an Exponential Autoregressive Conditional Heteroscedasticity, (EARCH), framework, this limitation is greatly reduced since there are limited constraints on the EARCH parameters and the EARCH model allows for oscillatory behavior in the variance. Thus the use of a longer time series is appropriate in order to improve the quality of parameter estimates.

The database for this study includes all date and time stamped bid-ask quotes, transaction prices and volume for every security traded on the TSE from January 1984 to the end of July 1991, for over 1900 trading days of data. Similar data for 311 NYSE and AMEX securities were obtained from a direct data feed employed by the TSE. The 311 securities on the U.S. data feed are: 68 Canadian-AMEX interlisted securities, 53 U.S. based TSE interlisted securities, 33 Canadian based NYSE interlisted securities, 27 non-interlisted DOW 30 securities and 130 other NYSE securities.

Returns and shares outstanding data were obtained from the TSE-Western database and the Center for Research in Security Prices (CRSP) database.

Utilizing the intraday data, for each security and for each day, the number of transactions was first counted. The daily volume was found by adding together the total volume for each transaction. The volume variable was scaled by the listed number of shares outstanding for that security. Thus the volume variable is the fraction of shares outstanding being traded on any specific day.

For the quotes variable, each time that either the bid and/or the ask price was changed, the quotes variable was increased by one. The initial quote of the day was counted only if it differed from the ending quote on the previous day. This counting method did not directly take into account changes in the mean of the bid and ask, (for instance the quotes could have changed by symmetrically widening or narrowing without changing the mean of the bid and ask.) When the specific number of changes in the mean of the bid and ask was counted, the resulting difference in the values of the quote variable was very trivial.

The days between trading was determined by examining the number of days for which each stock did not have an opportunity to trade due to the exchange being closed or trading in the stock being halted for the day. For those days with missing data, the days between trading was determined by the number of days that the exchange had been closed to trading. This is consistent with the meaning of non-trading days as used in French and Roll [1986].

The securities in this study were selected arbitrarily from a pool of stocks on the data tapes which had the most complete and continuous trading history available over the entire period of the study. There was no other selection criteria. A sample size of twenty Canadian securities and twenty American securities was thought to be adequate based on the trade-off between sample size and time of completion of study. The results achieved are so consistent that a larger sample size is felt to be unwarranted.

The selection criteria and the database characteristics induces a strong survivorship characteristic among the securities selected. In addition the securities selected, particularly in the American sample, tend to be some of the larger and more actively traded stocks. This characteristic of the data does not have any obvious implications for the purposes and outcomes of this study. The fact that the stocks selected tend to be among the more actively traded securities makes the sample similar to the sample of LL, since they purposely chose only actively traded securities with listed options.The effect of this selection bias is not obvious. The advantage of having a large complete data set was considered to be greater than any drawbacks from using a more random sample.

The securities and the number of observations for each security used in this study are listed in Table 1 for the Canadian sample and Table 2 for the American sample.

CHAPTER 6

RESULTS AND ANALYSIS

VI.1 EXAMINATION OF TRADING ACTIVITY VARIABLES

Averages for the three trading activity variables examined are presented in Table 3 and 4. The number of changes in quotes for the Canadian and American sample are roughly equivalent. However, the number of transactions was almost three times greater and the volume series was also almost three times greater for the American sample. Taking the quote variable to be indicative of the rate of flow of information to the market, and the transaction and volume variables to be indicative of the amount of trading activity, the conclusion to be drawn is that the rate of information is flowing to the two markets is at roughly equal rates but that the amount of trading for non-information reasons is significantly greater for the American sample.

A test for Autoregressive Conditional Heteroscedasticity, (ARCH), based upon the methodology suggested by Engel [1982], and using the base or mean equation used in LL, was conducted. The average results for the two data samples are presented in Table 5. The results presented are the Chi-Squared statistics against the null hypothesis of no ARCH effects for up to six lags. The returns series and the series for the three trading activity variables all rejected the null of no ARCH in their respective time series.

The fact that the returns series exhibited ARCH effects is no surprise. The use of ARCH models in modelling equity returns has already been well established. The finding that the trading activity variables also exhibit ARCH effects is new. The quotes and transactions series for all of the stocks rejected the null of no ARCH every time. Not all of the volume series however exhibited ARCH effects. The null of no ARCH could not be rejected at any reasonable significance level for ten volume series from the Canadian data and six volume series from the American data.

Press's [1967] model of the Mixture of Distributions Hypothesis, (MDH), also stated that the distribution of the volume variable was a mixture of normal distributions with the mixing variable being the rate of flow of information. If one accepts that MDH variables can be modelled in an ARCH framework, (for example what Lamoureux and Lastrapes [1990a], (LL), did in their study), then it is not surprising to see that at least the volume series tested positive for ARCH.

The presence of ARCH effects in the trading activity variables is significant in that it supports the suggestion that the LL results are a consequence of the trading activity variable, which follows an ARCH process, masking the return ARCH process. The fact that volume did not always reject the null hypothesis of no ARCH effects however weakens this conjecture, since volume was the specific trading activity variable used by LL. The ARCH test used does not directly test for Exponential Autoregressive Conditional Heteroscedasticity, (EARCH), effects, and as such one can not automatically conclude that the variables do, or do not follow an EARCH process. However, the evidence and intuition suggests a high probability that they do.

A similar ARCH test was conducted, but with an autoregressive term for the variable in question in the mean equation. The results were similar and thus are not presented.

VI.2 COMPARISON OF EARCH MODELS

The results for EARCH Models 1 and 2 are shown in Table 6 for the Canadian data and in Table 7 for the American data. The results for EARCH Models 3 and 4 are likewise shown in Table 8 for the Canadian data and Table 9 for the American data.

Models 1 and 2 differ from Models 3 and 4 in that Models 1 and 2 have an expanded mean equation, namely the presence of a lagged return term and an ARCH-in-Mean term. When focusing on index returns, the lagged return term is appropriate, but with individual security return data, the use of the term is questionable. In addition, while it is intuitively plausible to have a component in the mean return equation proxying for the risk or variability of the returns, it is not clear whether an ARCH-in-Mean term is the best way to model a variability effect. The coefficient "b", on the lagged return term was significant at the five percent level in the Canadian data only seven times out of a possible twenty when the trading activity variable quotes was present and five times and four times when transactions and volume were the trading activity variables respectively and five times when the mixing variable was not present, (i.e. in Model 2). The equivalent results for coefficient "b" with the American data is six times out of twenty with quotes, ten times with transactions, eleven times with volume and six times when there was no trading activity variable. These results indicate that the use of a lagged return in the mean equation is questionable for these models.

The ARCH-in-Mean coefficient, "c", was significant at the five percent level in the Canadian sample nine times out of twenty with quotes as the trading activity variable, eight times with either transactions or volume and only once when the model did not include a trading activity variable. The significance of the coefficient "c" was much greater in the American sample with the coefficient being significant at the five percent level fifteen times out of twenty when either quotes, transactions or volume was used as the trading activity variable. The coefficient was significant six times out of twenty in Model 2. The difference in the results between the Canadian and American data may be a consequence that the

American securities tended to have a much larger capitalization and thus tended to trade more like an index than the stocks in the Canadian sample.

The third difference between Models 1 / 2 and Models 3 / 4, is the variable in the variance equation to account for the number of days non-trading effect, u_t. This variable was never significant at the five percent level in the Canadian data when either quotes or transactions was used as the trading activity variable and the variable was significant only once when volume was used and seven times when there was not a trading activity variable present. The results were similarly weak in the American sample with the variable being significant five times when quotes was the trading activity variable, once for transactions and never for volume. The variable was significant seven times in Model 2. An obvious explanation for lack of significance when there is a trading activity variable present in the equation is that the French and Roll effect [1986], which was accounted for by the number of days non-trading, is better represented by a trading activity variable. This is not a surprising result since French and Roll hypothesized that the effect was caused mainly by the arrival of information to the markets. The relative lack of significance when there is not a trading activity variable present contradicts the observations of French and Roll.

The lack of significance of the variables representing a lagged return, the ARCH-in-Mean, and the number of days non-trading, suggests the preference of Models 3 and 4 over the use of Models 1 and 2. The results of both sets of Models will be used in the rest of the analysis but the focus will be on Models 3 and 4.

The average value for the G.E.D. parameter, v, was 1.35 for the Canadian data and 1.44 for the American data in Models 1 and 2. The corresponding values for Model 3 and 4 are 1.12 and 1.37 for the Canadian and American data respectively. This implies that the distributions had thicker tails than normal and that t-statistics will be biased upwards. Use of conventional t-statistic tables will not be perfectly valid since the t-value needed for a given critical area will have to be larger in magnitude, given the fatter tails of the distribution.

It should be noted that the maximization algorithm used restricted the value of the parameter v to lie between the values of 0.5 and 3.0. For Models 3 and 4, in thirty-one cases in the Canadian data the lower bound of 0.5 was landed on while in the American data the lower bound was reached in only eight cases. The upper bound of 3.0 was reached once in the Canadian and twice in the American sample. Excluding these values where a bound was reached for calculating the average value of the parameter v, the averages were 1.49 for the Canadian data and 1.47 for the American data. The fact that the lower bound was reached so often in the Canadian sample implies that the distribution of returns in Canadian is extremely thick-tailed relative to the normal distribution. The reaching a boundary constraint of the maximization procedure also implies that the EARCH likelihood functions are not maximized and the model itself with the estimated parameter values may be misspecified.

VI.3 RESULTS CONCERNING HYPOTHESIS 1

Hypothesis 1 challenges LL's assertion that "lagged squared residuals contribute little if any additional information about the variance of the stock return process after accounting for the rate of information flow...". If LL's assertion is correct then the addition of a trading activity variable should change the number of significant lag coefficients in the EARCH framework. Namely, the number of the group of parameters consisting of, ψ, Δ_i, θ, and γ, which are significant should be smaller for Models 1 and 3 than for Models 2 and 4 which do not have a mixing variable in the model.

Tables 10 and 11 present a summary of the number of the EARCH parameters, ψ, Δ_i, θ, and γ, which are significant at the five percent level in Models 1 / 2 and Models 3 / 4 respectively. Examination of the tables shows that the presence of a trading activity variable does not affect the significance of the parameters ψ, Δ, θ, and γ. Over the two Models with trading activity variables present, Models 1 and 3, there was only one case for the Canadian sample and one case for the American sample were none of the EARCH parameters were significant. The sole exception with Model 1 was with volume as the trading activity variable and the two exceptions with Model 3 were with transactions as the trading activity variable. In no cases with transactions, was there no EARCH parameters significant when quotes was the trading activity variable. In a large majority of the cases there were at least three of the variables significant.

The EARCH parameters are significant when the trading activity variable is quotes, transactions or even volume, the variable that LL used in their study. In addition the EARCH parameters are significant if the trading activity variable was significant or not significant. The distribution of the number of significant EARCH parameters is virtually identical for the models with the trading activity variable present, (Models 1 and 3) as it is for the models where the trading activity variable is not present (Models 2 and 4).

These results lead to a very strong rejection of the null of Hypothesis 1. Lagged values of the squared residuals do add information about the stock return even if the rate of flow of information is accounted for.

Rejection of the null of Hypothesis 1 implies that LL's GARCH model was misspecified. A possible source of the misspecification could lie with the limitations of the GARCH model framework. More specifically, the GARCH model only allows for a linear structure for the relationship between the current variance and the past variance. It is obvious however that the variance of returns has a tendency to oscillate up and down. In addition, Press [1967] originally hypothesized that volume, as well as returns, follows a mixture of normals distribution. With the fact that the trading activity variables used, including volume, test positive for ARCH effects, then it is quite possible that the LL results are a consequence of the volume variable proxying for the ARCH effects of the returns. With the more general EARCH model, the limitations on the structure of current variance to past variance is reduced, and thus the trading activity variables are not mathematically forced to proxy for the ARCH process of the stock returns.

VI.4 GARCH MODEL RESULTS

The log-likelihoods, t-statistics on the trading activity variables, and the sum of the ARCH parameters for the GARCH models are summarized in Table 12 for the Canadian data and in Table 13 for the American data. Model 5 is a (1,1) GARCH model which incorporates a trading activity variable. Model 6 is identical to Model 5 except that it does not include a trading activity variable. The GARCH models are identical to the GARCH models utilized in the LL study.

One of the main observations from the LL study was that the sum of the coefficients α_1 and α_2 had a value closer to one in the models without the trading activity variable but the sum of the coefficients was greatly reduced in the model with the trading activity variable. This result is replicated in this study. Across all the securities and for the various different trading activity variables, the average sum of the alohas was 0.91 before the addition of a trading activity variable and only 0.42 after the addition of a trading activity variable. In the Canadian sample, the sum of the alphas was reduced from 0.90 when there was not a trading activity variable in the model to 0.36 when quotes was included, or 0.29 with transactions and 0.35 with volume. The American sample results were a reduction in the sum from 0.92 to 0.42 with quotes, 0.55 with transactions and 0.53 with volume. The fact that the LL results were verified shows that the effect is not sample specific and gives validity to the EARCH results in comparing the outcomes of the various models.

For comparison purposes, the results of the EARCH models are presented in a similar format in Table 14 for the Canadian data and in Table 15 for the American data.

VI.5 TEST OF SIGNIFICANCE OF TRADING ACTIVITY VARIABLES

Hypothesis 2 is concerned with the significance of the trading activity variables and in particular the quotes variable. Table 16 provides a summary of the results concerning the significance of the various trading activity variables.

For the Canadian sample, the quotes variable is significant at the five percent level seventeen times out of twenty in Model 1 and eighteen times out of twenty in Model 2. Thus for the Canadian sample the null of Hypothesis 2 is strongly rejected. The average t-statistic of the coefficient of the variable was 2.73 in Model 1 and 4.38 in Model 2. The average t-statistic when the variable was significant in Model 1 was 3.06 and 0.87 when insignificant. The corresponding results in Model 3 are 4.71 and 1.40 respectively.

The results concerning the significance of the quotes variable are slightly different for the American data. Examining the results for Model 3, shows strong support that the quotes variable is superior. The average t-statistic for the quotes coefficient over the twenty American stocks in Model 3 is 6.42 with the variable being significant in sixteen cases out of twenty. For the sixteen significant cases the average t-statistic was 7.89 and 0.56 when the variable was insignificant. For Model 1, the results concerning the significance of the quotes parameter are not as strong. The variable was significant only eight times out of twenty with an overall t-statistic of 1.62.

Overall there is strong evidence to reject the null of Hypothesis 2. If one accepts the conjecture that the mean of the bid-ask quotes is the equilibrium price, and thus everytime the mean of the quotes changes there is a change in the equilibrium price, then the fact that the quotes variable is significant supports the MDH as first postulated by Press [1967]. Press claimed that the number of changes in the equilibrium price is the directing variable that determines the variance of the stock return distribution. This is consistent with the quotes variable being significant in the EARCH models.

The trading activity variable volume was significant in the Canadian sample but not in the American sample. The volume variable in the Canadian sample was significant at the five percent level sixteen times out of twenty in Model 1 and fifteen times out of twenty in Model 3 with an average t-statistic of 2.85 and 3.22 respectively. The results for the volume variable in the American sample were significance four and six times out of twenty for Model 1 and Model 3 respectively with average t-statistics of 1.73 and 1.22.

The dramatic difference for the results between the Canadian and American samples for the volume variable could be due to the relative differences in liquidity in the two markets. The average relative trading volume for the stocks in the American sample was three times greater than the average relative volume in the Canadian sample. Coupled with a similar difference in the average number of transactions in the American and Canadian sample, it implies that there is more liquidity trading occurring in the American stocks. Liquidity trading will not affect the equilibrium price. Only trade based on information will affect the equilibrium price. The probability of a trade being an information trade is greater for the Canadian sample than the American. For this reason volume of trades in Canada may convey more information to the markets than a similar volume in the American stocks. If this is the case, then the volume variable should be more significant in the Canadian sample than it is in the American sample.

A similar argument holds for the transaction variable. Although the transaction variable is not often significant, the results show that it is more important in the Canadian sample than in the American sample. Indeed the transactions variable is significant at the five percent level seven times out of twenty in Model 1 of the Canadian sample and five times in Model 3. The transaction variable is only significant for one stock in Model 1 and is never significant in Model 3 of the American sample.

The results showing insignificance of the transaction variable in the EARCH models casts doubt on Harris' [1987] model of the MDH when he claims that the number of transactions may be a suitable variable to use as the mixing variable. In addition it contradicts the theories of Admati and Pfleiderer [1988, 1989], which show that return variability should be higher when the market has the highest number of traders transacting.

The fact that transactions seem to contribute little information about the variance of equity returns is interesting in light of concerns that excessive trading leads to excessive stock return volatility. The results found seem to contradict such a conclusion. In fact it could be stated that the number of transactions has little or no effect on the volatility of returns when the past volatility of returns is taken into account.

Overall, the evidence points convincingly towards quotes being the preferred trading activity variable, (or mixing variable in MDH terminology), to utilize.

The result that quotes appear to be more significant a trading activity variable than transactions is consistent with the implications of the Efficient Markets Hypothesis. Changes in quotes signals the arrival of information in the markets. It is thus not surprising that the quotes variable is significant in determining the distributions of returns. If transactions was a significant variable, then it would be evidence that trading in some sense was self generating, i.e. traders trading simply because many other traders are trading which in turn leads to even more traders trading etc. This concept of how trading develops contradicts the Efficient Markets Hypothesis. In part this is what the models of Admati and Pfleiderer [1988,1989], are trying to capture. The evidence of this thesis however supports the Efficient Markets Hypothesis.

It is interesting to note that all three of the trading activity variables are strongly significant in almost all of the sample securities when they are modelled with the GARCH model. This result is consistent with the conjecture made earlier that the mixing variable in the LL model is acting as a substitute ARCH process to overcome the limitations of the GARCH framework.

VI.6 SIC RESULTS AND HYPOTHESIS 3

The log-likelihood results for the GARCH and EARCH models presented in Tables 12 through 15 are summarized in Table 17, along with the results of model selection by the Schwarz Information Criteria (SIC).

Hypothesis 3 is to test whether the EARCH models or the GARCH models provide a better overall fit for modelling stock returns. The SIC was chosen as the selection criteria because it is biased against the EARCH models which have more parameters.

The results presented in Table 17 resoundingly show that EARCH models are superior to GARCH models. The null of Hypothesis 3 that GARCH models are superior is rejected in every single case. The results are remarkable in that firstly the selection criteria is biased against the EARCH models. Secondly, the GARCH models were maximized using an extensive grid of starting points for the maximization. Thus the chances of the maximization routine stopping at a local maximum for the

GARCH models is slight. The EARCH maximizations were only started from a single beginning point due to computational time constraints. As such the probability of being at the global maximum is much greater for the GARCH models than for the EARCH models. One would expect this methodology to favour the GARCH models. The favouritism of the methodology however turns out to be irrelevant.

For the Canadian sample there is a strong tendency for the SIC to select Model 3 over the more complicated Model 1 with the SIC choosing Model 3 fifteen times, Model 4 once, and Model 1 only four times. The results for the American sample are more balanced. Model 1 is chosen nine times and Model 3 is chosen eleven times. In part the difference in results between the Canadian and American sample may be that the companies in the American sample are much larger and resemble a portfolio or index much more than the stocks in the Canadian sample. This being the case, the American stocks may fit into Model 1 which was originally developed by Nelson [1990] for an index rather than Model 3, which this study developed for use with individual securities.

It is interesting to note that the SIC, with one exception, always selects an EARCH model which includes a trading activity variable in the formulation. The EARCH Models 2 and 4, which do not include trading activity variables, are inferior to their counterparts which include the trading activity variable. This demonstrates again the importance of the trading activity variables in describing stock return series.

The selection of trading activity variable using the SIC criteria within models is difficult because the difference of SIC value within models is generally of the order of one percent. It is impossible to ascertain if the maximum log-likelihoods calculated are near the global maximum log-likelihoods by this amount. Consequently, it is unreliable to use the SIC criteria to chose the proper mixing variable to utilize. With that as a caveat, there is a strong preference for the SIC to chose the quotes variable in the Canadian sample for Model 1 and Model 3. The quotes variable is selected sixteen and seventeen times respectively in Model 1 and Model 3 while transactions is selected four and three times respectively. The American results are more evenly distributed but with a slight preference in Model 1 for the quotes variable which is selected thirteen times versus four times for transactions and three times for volume.

VI.7 SPECIFICATION TEST RESULTS

Results of specification tests for Model 3 and for Model 5 are shown in Table 18 and Table 19 respectively. The actual tests themselves are presented in Exhibit 2.

The first two specification tests are for misspecification of the distribution. Tests three through seven test for misspecification of the conditional heteroscedasticity. Tests eight through twelve check for misspecification in the mean equation.

The specification test results for Canadian security MTT were very high, (value greater than 1000), for Model 3 when transactions was the trading activity variable. Thus the Canadian results are reported without the MTT results included in the averages for the transactions set. Likewise, the American security IRC gave high t-statistic results and thus the American results are reported with IRC results included and with IRC results excluded.

Examining the results for Model 3 first, it can be seen that the average mean of the normalized residual is not statistically different from zero for both the Canadian and American samples. However the square of the normalized residual is on average statistically different from the expected value of one for the Canadian stocks. The high average t-statistics for tests three through seven for the Canadian data indicate that the conditional variance is misspecified in Model 3. This conclusion is consistent for all the Model 3's and independent of what the trading activity variable used was. The results of tests three through seven are better for the American data, (when the results for IRC are excluded), but there is still evidence that the variance equation in the model is misspecified.

The average t-statistics for tests eight through twelve are all insignificant across the Canadian and American models with the exception of test number eight which fails in every case. Thus the mean equation appears to be well specified with the exception of first order autocorrelation in the mean, which is the conclusion from the results of test eight. This result is surprising since it implies that after correctly accounting for the variance in returns, there is mathematical predictability in returns using daily data.

The specification tests for the GARCH models, shown in Table 19 are much better, although there is slight evidence of misspecification of the variance in the Canadian sample. However test number eight still fails on average for all the various models.

It is difficult to judge the seriousness of the results of the specification test failures since there is no known study which uses these tests on models constructed for individual securities using daily data. Nelson [1989] finds similar failures when modelling a return index using the Standard 90 index, but gets much better specification test results using the CRSP value weighted index return, Nelson [1991]. It is ironic to note that Nelson finds the test for first order autocorrelation fails in both of his studies using index data.

It is likely that a more suitable lag structure may be found, (i.e. use different values for the length of lags p and q in Model 3 or Model 5). This study took as its values for p and q the precedents set in the previous literature. Namely this study used the lag structure (1,1) for the GARCH models as used by

LL and (2,1) for the EARCH models as used by Nelson [1991]. Experimenting and examining the effect of other lag structures is a topic for further study. Nelson also suggests that the use of a more general and encompassing distribution than the GED distribution may improve the results of the specification tests.

If computational time constraints were absent, a better model fit for some of the EARCH models may be found by using a grid search over a range of possible starting values for the maximization algorithm. However with fourteen variables to get starting values for the maximization algorithm for, this is not a practical technique.

VI.8 SHORTER DATA SAMPLES

To improve model fit, a long time series of approximately 1900 observations or seven and a half years of daily data was utilized. To examine the effect of using a shorter data series, a subset of the American stock sample was remodelled using only 750 days of data for Models 1,5 and 6. The log-likelihood values and SIC values are shown in Table 20.

Although the SIC values were close in magnitude, the surprising finding is that the SIC chose the GARCH model over the EARCH model in each of the ten cases examined. Thus when using shorter sets of data, it appears that GARCH models may be slightly preferred over EARCH models. A possible explanation for this surprising result is that the linear constraints may not matter over a short period of time. In other words, the variance may oscillate slowly over time such that if one uses a short time series the oscillations may not matter.

The debate started by Pagan and Schwert [1990] and continued by Day and Lewis [1992] over whether or not EARCH models are superior to GARCH models may depend on the length of the time series being used. The indications from this study is that the EARCH models are definitely superior for longer time series while the GARCH models are superior for shorter time series.

V.9 SUMMARY OF RESULTS

The empirical results are as follows:

(1) The EARCH parameters, Ψ, Δ, θ, and γ, are found to be significant and thus the null of Hypothesis 1 is rejected. This implies misspecification in the LL GARCH model. Furthermore it is evidence that lagged values of the error term squared do add information to the regression system even when the rate of information flow is accounted for. This finding contradicts the conclusions of LL. Both MDH and ARCH effects are present simultaneously.

(2) For the Canadian data in Model 1, and for both the Canadian data and the American data in Model 3, the trading activity variable coefficient is significant for the variables quotes and volume. Thus the null of Hypothesis 2 can be rejected, implying that a mixing variable adds information about the variance of a distribution and giving support to Press's MDH model. The trading activity variable transactions is generally not significant which is contradictory to the MDH model of Harris [1987].

(3) Overall the model selection criteria selected an EARCH model over a GARCH model in every instance when the full data series was utilized. Thus the null of Hypothesis 3 can be rejected. EARCH models appear to be superior to GARCH models when a long time series of returns is used. When a short time series, (750 data points), was used, the selection criteria selected the GARCH models over

the EARCH models. In part this could be because a GARCH model cannot model several shifts in the direction of volatility, something which is more likely to happen over an extended period of time versus a short period of time.

(4) The model selection criteria almost always chose an EARCH model which also had a trading activity variable as a component. This result demonstrates the importance of examining variables which measure trading activity or the rate of information flow, which up until now have all but been ignored in the empirical literature.

CHAPTER 7

SUMMARY

VII.1 CONCLUSIONS

One of the primary conclusions of this study is that the concepts of the Mixture of Distributions Hypothesis and ARCH models can and should co-exist when modelling equity returns. The model results show that a trading activity variable, acting as a proxy for the rate of flow of information to the markets adds information about the variance of a security's equity return as do the lagged squared values of the residual error term. Both MDH and ARCH effects are present and significant.

In this study three trading activity variables, proxying for the rate of flow of information were examined. The variables were the number of daily changes in the quotes of a security, the number of daily transactions, and the amount of daily volume. Both the quotes and volume variables proved to be significant while the evidence was much weaker for the significance of the transaction variable. The quotes variable being significant supports the MDH of Press [1967], (assuming that the number of changes in the quotes is an accurate measure of the number of information events causing equilibrium price changes in the market), while the absence of significance of the transactions variable contradicts the MDH of Harris [1987]. The results demonstrate that trading activity variables have a definite role in helping to model the variance of a distribution for equity returns.

Examination of the various models tested in this thesis show that for long time series of equity returns EARCH models are superior to GARCH models. However the results are reversed when a shortened time series of only 750 data points are used. The GARCH models are superior when shortened data sets are used. Since there is a high price to pay in complexity for using the EARCH models over the

GARCH models, it is important for the modeller to determine the appropriate model for the situation. Part of the explanation for the superior performance of GARCH models for shortened data sets but not for longer data sets is that the GARCH model is restricted in its capability to model several shifts in the direction of the variance.

The results of LL are verified but their conclusions cannot be supported by the results of this thesis. As stated above, both the trading activity variables and lagged squared values of the residual error term are significant when a full EARCH model is used. Thus even after accounting for the rate of flow of information, ARCH effects are present in equity returns. The results of LL and this thesis suggest that the GARCH model of LL with a trading activity variable in the variance equation is a substitute EARCH process. ARCH tests demonstrate that the trading activity variables themselves are ARCH processes. In addition the significance of the trading activity variables is much greater in the GARCH models than in the EARCH models. These facts suggest that the trading activity variables themselves follow an EARCH process along with the return series and when the trading activity variables are included in a GARCH model they attempt to overcome for the limitations of the GARCH model and thus supplant the GARCH variables. It should be noted that GARCH models with trading activity variables in the variance equation produce models with significantly larger log-likelihood values than GARCH models without trading activity variables included.

It seems plausible that the LL results are a consequence of the volume variable acting as a substitute EARCH process in the model. The volume variable itself was not constrained, and it is likely that the volume variable itself follows an EARCH type process. Thus in the LL model, the return series variance was better explained by the variance of the volume term than by the restricted GARCH parameters. In effect, it may be that LL formulated a substitute EARCH process by including the volume variable in the GARCH variance equation.

VII.2 IMPLICATIONS FOR FUTURE RESEARCH

The results of this thesis have advanced the debate on the distribution and modelling of stock returns. It has been shown that EARCH models with a trading activity variable included in the variance term, (particularly the trading activity variable quotes), provide a superior model for equity return series than GARCH models. As Schwert and Seguin [1990] demonstrate, it is important to correctly account for heteroscedasticity when testing hypotheses concerning stock market time series. This thesis shows that an EARCH model with a trading activity variable included in the variance equation is at least a good start to accounting for heteroscedasticity in equity returns.

Empirical testing of theories concerning the rate of information flow to the stock markets should focus on the variables of changes in quotes and volume. These were the variables, (especially quotes), that proved to be the most significant in the models examined. Researchers until now have focused relatively little attention on volume and virtually no attention on the rate of quote changes. The results of this thesis show that the neglect of these variables should cease.

Further studies should in part focus on the effect of changing the lag structure in the EARCH models. A lag structure of (1,1) is well accepted for GARCH models but little has been done in respect to the appropriate lag structure of EARCH models for individual security returns.

Additionally, intuition, and the results of the specification tests indicate that the specific lag structure and model is likely to be different for different securities. In this study, four EARCH models and two GARCH models were used for all securities. For further research, it seems appropriate to examine specific lag structures and models for specific securities.

EXHIBIT 1

ARCH MODELS EXAMINED

<u>Model 1</u> EARCH model including trading activity variable;

$$R_t = a + bR_{t-1} + c\sigma_t^2 + \varepsilon_t$$

$$\ln(\sigma_t^2) = \alpha_t + \frac{(1+\Psi L)}{(1-\Delta_1 L-\Delta_2 L^2)} g(Z_{t-1})$$

$$\alpha_t = \alpha + \ln(1+\delta u_t + f\eta_t)$$

$$g(Z_t) = \theta Z_t + \gamma(|Z_t| - E|Z_t|)$$

$$Z_t = \sigma_t^{-1}\varepsilon_t$$

where R_t = the daily return,
η_t = the mixing variable,
u_t = the number of days non-trading,
E = the expectations operator,
L = the lag operator,
and, $a, b, c, \delta, f, \psi_i, \Delta_i, \alpha, \theta, \gamma$ are model parameters to be estimated.

<u>Model 2</u> EARCH model without trading activity variable;

$$R_t = a + bR_{t-1} + c\sigma_t^2 + \varepsilon_t$$

$$\ln(\sigma_t^2) = \alpha_t + \frac{(1+\Psi L)}{(1-\Delta_1 L-\Delta_2 L^2)} g(Z_{t-1})$$

$$\alpha_t = \alpha + \ln(1+\delta u_t)$$

$$g(Z_t) = \theta Z_t + \gamma(|Z_t| - E|Z_t|)$$

$$Z_t = \sigma_t^{-1}\varepsilon_t$$

EXHIBIT 1 (cont'd)
ARCH MODELS EXAMINED

Model 3 EARCH model with trading activity variable but no ARCH-in-mean component:

$$R_t = \varepsilon_t$$

$$\ln(\sigma_t^2) = \alpha_t + \frac{(1+\Psi L)}{(1-\Delta_1 L - \Delta_2 L^2)} g(Z_{t-1})$$

$$\alpha_t = \alpha + \ln(f\eta_t)$$

$$g(Z_t) = \theta Z_t + \gamma(|Z_t| - E|Z_t|)$$

$$Z_t = \sigma_t^{-1}\varepsilon_t$$

Model 4 EARCH model with no trading activity variable or ARCH-in-mean component;

$$R_t = \varepsilon_t$$

$$\ln(\sigma_t^2) = \alpha_t + \frac{(1+\Psi L)}{(1-\Delta_1 L - \Delta_2 L^2)} g(Z_{t-1})$$

$$\alpha_t = \alpha$$

$$g(Z_t) = \theta Z_t + \gamma(|Z_t| - E|Z_t|)$$

$$Z_t = \sigma_t^{-1}\varepsilon_t$$

EXHIBIT 1 (cont'd)

ARCH MODELS EXAMINED

Model 5 LL's GARCH model including trading activity variable:

$$R_t = \varepsilon_t$$

$$h_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \alpha_2 h_{t-1} + \alpha_3 \eta_t$$

Model 6 LL's GARCH model without trading activity variable:

$$R_t = \varepsilon_t$$

$$h_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \alpha_2 h_{t-1}$$

EXHIBIT 2

SPECIFICATION TESTS FOR EARCH AND GARCH MODELS. THE MODELS ARE FROM NELSON [1991] AND ARE BASED UPON ORTHOGONALITY CONDITIONS WHICH ARE OUTLINED IN NEWEY [1985]

1. $E(z_t)=0$

2. $E(z_t^2)-1=0$

3. $E[(z_t^2-1)(z_{t-1}^2-1)]=0$

4. $E[(z_t^2-1)(z_{t-2}^2-1)]=0$

5. $E[(z_t^2-1)(z_{t-3}^2-1)=0$

6. $E[(z_t^2-1)(z_{t-4}^2-1)]=0$

7. $E[(z_t^2-1)(z_{t-5}^2-1)]=0$

8. $E(z_t z_{t-1})=0$

9. $E(z_t z_{t-2})=0$

10. $E(z_t z_{t-3})=0$

11. $E(z_t z_{t-4})=0$

12. $E(z_t z_{t-5})=0$

where z_t is the normalized residual or $z_t = \varepsilon_t / \sigma_t$

TABLE 1

**CANADIAN SECURITIES EXAMINED IN STUDY.
THE TICKER SYMBOL AND NUMBER OF OBSERVATIONS AVAILABLE ARE LISTED.
ALL OF THE CANADIAN DATA IS TAKEN FROM TRADING ON THE TORONTO STOCK EXCHANGE BETWEEN 84-01-04 AND 91-07-31.**

Ticker Symbol	Company Name	Number Of Observations Available	Stock Exchange Traded On
AL	Alcan Aluminium Ltd.	1906	TSE
B	BCE Inc.	1906	TSE
BMS	Brunswick Mining & Smelting Corp. Ltd.	1787	TSE
BNS	Bank of Nova Scotia	1907	TSE
BVI	Bow Valley Industries Ltd.	1903	TSE
BWR	Breakwater Resources Ltd.	1906	TSE
IMS	Imasco Ltd.	1906	TSE
LBT	Labatt, John Ltd.	1907	TSE
MB	MacMillian Bloedel Ltd.	1906	TSE
MCL	Moore Corp. Ltd.	1907	TSE
MTT	Maritime Telegraph & Telephone Co. Ltd.	1903	TSE
N	Inco Ltd.	1906	TSE
NOR	Noranda Inc.	1907	TSE
NTL	Northern Telecom Ltd.	1907	TSE
RGO	Ranger Oil Ltd.	1907	TSE
RY	Royal Bank of Canada	1907	TSE
SPL	Scott Paper Ltd.	1813	TSE
TRP	TransCanada Pipelines Ltd.	1907	TSE
VO	Seagram Co Ltd.	1907	TSE
WN	Weston, George Ltd.	1890	TSE

TABLE 2

**AMERICAN SECURITIES EXAMINED IN STUDY.
THE TICKER SYMBOL AND NUMBER OF OBSERVATIONS AVAILABLE ARE LISTED.
ALL OF THE AMERICAN DATA IS TAKEN FROM TRADING ON THE NEW YORK STOCK EXCHANGE. THE TIME PERIOD FOR THE DATA IS 84-01-04 TO 91-07-31.**

Ticker Symbol	Company Name	Number Of Observations Available	Stock Exchange Traded On
AA	Aluminum Co. of America	1911	NYSE
C	Chrysler Corporation	1908	NYSE
DD	DuPont (E.I.) DeNemours & Company	1911	NYSE
DEC	Digital Equipment Corporation	1911	NYSE
DIS	Walt Disney Company	1911	NYSE
DOW	Dow Chemical Company	1908	NYSE
EK	Eastman Kodak Company	1910	NYSE
GM	General Motors Corporation	1908	NYSE
HAL	Halliburton Company	1908	NYSE
IBM	International Business Machines Corporation	1908	NYSE
IRC	Inspiration Resources Corp.	1908	NYSE
MCD	McDonald's Corporation	1907	NYSE
MER	Merrill Lynch & Co. Inc.	1908	NYSE
NSC	Norfolk Southern Corporation	1911	NYSE
OXY	Occidental Petroleum Corporation	1908	NYSE
RLM	Reynolds Metals Company	1910	NYSE
TDY	Teledyne Inc.	1909	NYSE
TGT	Tenneco Inc	1908	NYSE
VAT	Varity Corporation	1175	NYSE
XON	Exxon Corporation	1910	NYSE

TABLE 3

AVERAGE VALUES OF THE TRADING ACTIVITY VARIABLES TIME SERIES FOR THE CANADIAN SECURITIES

SECURITY	QUOTES	TRANSACTIONS	VOLUME
AL	150.8	169.9	162.6
B	133.6	373.3	78.5
BMS	10.1	5.8	19.6
BNS	103.4	221.0	142.4
BVI	26.6	38.5	153.1
BWR	35.0	44.6	191.9
IMS	44.4	63.2	57.7
LBT	32.5	39.2	66.8
MB	46.6	72.2	124.5
MCL	54.5	66.2	105.7
MTT	18.0	16.7	28.6
N	126.5	120.0	223.2
NOR	98.2	122.1	110.7
NTL	98.4	105.8	71.2
RGO	38.5	68.7	172.2
RY	85.9	158.5	118.3
SPL	9.2	5.1	28.0
TRP	57.3	106.9	86.6
VO	111.8	71.4	59.8
WN	15.2	11.2	22.3
AVG.	64.8	94.0	101.2

TABLE 4

AVERAGE VALUES OF THE TRADING ACTIVITY VARIABLES TIME SERIES FOR THE AMERICAN SECURITIES

SECURITY	QUOTES	TRANSACTIONS	VOLUME x 10^2
AA	43.9	130.6	4.2
C	54.3	302.0	3.7
DD	88.7	273.5	1.6
DEC	159.4	366.9	6.4
DIS	102.0	291.5	4.4
DOW	84.5	313.4	2.8
EK	75.2	386.6	3.0
GM	77.4	419.9	2.3
HAL	45.6	134.8	3.4
IBM	221.2	834.3	2.4
IRC	8.0	34.7	1.6
MCD	60.0	304.2	2.6
MER	53.6	212.0	4.7
NSC	38.9	98.9	1.6
OXY	34.0	408.7	3.4
RLM	42.9	90.3	4.6
TDY	77.3	100.8	2.5
TGT	51.1	207.0	2.7
VAT	3.7	104.6	2.7
XON	53.3	396.4	0.9
AVG.	68.8	270.6	3.08

TABLE 5

ARCH TEST FOR VARIABLES.
AVERAGE CHI-SQUARED STATISTIC FOR ALL TWENTY CANADIAN SECURITIES AND ALL TWENTY AMERICAN SECURITIES ARE GIVEN FOR THE TEST OF ARCH VERSUS THE NULL HYPOTHESIS OF NO ARCH EFFECTS UP TO LAG 6. THE TEST IS GIVEN IN ENGEL [1982]. THE TEST STATISTIC FOR ARCH LAGS UP TO ORDER 6 IS DISTRIBUTED AS χ^2_6.

ARCH Model Tested

$$Variable_t = \varepsilon_t$$

$$\varepsilon_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \alpha_2 \varepsilon_{t-2}^2 + \alpha_3 \varepsilon_{t-3}^2 + \alpha_4 \varepsilon_{t-4}^2 + \alpha_5 \varepsilon_{t-5}^2 + \alpha_6 \varepsilon_{t-6}^2$$

VARIABLE	CHI-SQUARED STATISTIC	
	TSE Listed Stocks	NYSE Listed Stocks
Returns	189	233
Quotes	801	764
Transactions	687	894
Volume	96	171

TABLE 6

MAXIMUM LIKELIHOOD RESULTS FOR MODEL 1 AND MODEL 2 FOR THE CANADIAN DATA. LOG LIKELIHOODS ARE GIVEN AT TOP OF COLUMNS AND t-STATISTICS ARE SHOWN BELOW THE PARAMETER ESTIMATES.

$$R_t = a + bR_{t-1} + c\sigma_t^2 + \varepsilon_t$$

$$\ln(\sigma_t^2) = \alpha + \ln(1 + \delta u_t + f\eta_t) + \frac{(1+\Psi L)}{(1-\Delta_1 L - \Delta_2 L^2)}[\theta Z_{t-1} + \gamma(|Z_{t-1}| - E|Z_{t-1}|)]$$

	AL				B			
η_t	Quotes	Transactions	Volume	- (Model 2)	Quotes	Transactions	Volume	- (Model 2)
	5342	5350	5321	5203	6768	6736	6749	6599
α	-12.10	-16.15	-11.32	-8.30	-21.94	-24.76	-24.21	-9.85
	-25.25	0.19	[illegible]	-56.29	-29.81	-46.11	-135.6	-84.08
δ	0.26	17.80	1.02	0.066	-0.25	1440	6957	0.33
	0.29	-0.012	0.94	1.65	-0.32	0.24	3.05	904.2
γ	-0.084	0.079	0.067	0.16	0.25	[illegible]	0.23	0.38
	4.40	3.57	3.14	[illegible]	3.98	4.85	[illegible]	5.58
Δ_1	-0.012	0.014	4.76e-03	1.77	1.60	[illegible]	1.62	1.49
	0.53	0.60	0.22	7.77	[illegible]	8.51	[illegible]	12.23
Δ_2	0.97	0.97	0.97	-0.77	-0.60	[illegible]	-0.62	-0.49
	43.82	42.76	44.95	-3.44	[illegible]	[illegible]	[illegible]	-4.15
ψ	1.41	1.44	1.56	-0.89	[illegible]	-0.91	-0.96	-0.94
	3.96	3.67	3.05	-6.95	-29.40	-17.23	-33.17	-26.47
θ	0.035	-0.034	-0.026	-0.040	-0.039	-0.044	-0.044	-0.078
	3.15	2.97	-2.29	-1.96	-1.18	-1.26	-1.30	-2.02
a	1.98e-04	-2.30e-04	-9.87e-04	1.04e-03	-2.74e-05	-5.25e-04	-3.17e-04	5.30e-03
	-0.40	-0.51	2.13	-1.41	[illegible]	-1.89	-1.43	3.30e-04
b	0.11	0.11	0.069	0.12	4.11e-03	-0.052	-0.042	[illegible]
	4.97	4.66	3.22	5.36	0.38	2.23	1.94	3.74e-03
c	1.25	1.34	4.51	4.28	1.46	15.12	11.00	5.45e-05
	0.59	0.64	2.22	1.54	0.55	2.91	2.62	4.68e-05
v	[illegible]	1.86	1.80	1.38	1.24	1.29	1.36	1.04
	27.80	23.54	22.94	30.48	22.64	22.57	21.05	28.00
f	0.30	[illegible]	[illegible]		5465	8226	24650	-
	2.45	0.012	1.12	-	1.53	1.88	12.05	-

TABLE 6 (cont'd)

MAXIMUM LIKELIHOOD RESULTS FOR MODEL 1 AND MODEL 2 FOR THE CANADIAN DATA. LOG LIKELIHOODS ARE GIVEN AT TOP OF COLUMNS AND t-STATISTICS ARE SHOWN BELOW THE PARAMETER ESTIMATES.

$$R_t = a + bR_{t-1} + c\sigma_t^2 + \varepsilon_t$$

$$\ln(\sigma_t^2) = \alpha + \ln(1 + \delta u_t + f\eta_t) + \frac{(1+\Psi L)}{(1-\Delta_1 L - \Delta_2 L^2)}[\theta Z_{t-1} + \gamma(|Z_{t-1}| - E|Z_{t-1}|)]$$

	BMS				BNS			
η_t	Quotes	Transactions	Volume	- (Model 2)	Quotes	Transactions	Volume	- (Model 2)
	4711	4642	4607	4591	5446	5437	5368	5373
α	-10.42	-8.77	-8.03	-7.65	-9.85	-17.94	-17.94	-8.38
	-24.27	-26.79	-30.81	-57.26	-26.34	-0.55	-0.77	-61.59
δ	-0.024	0.13	0.11	-4.20e-03	-0.058	348.7	355.9	0.026
	-0.083	0.89	1.15	-0.076	-0.31	0.031	0.043	0.46
γ	0.14	0.30	0.34	0.30	0.028	0.057	0.059	0.15
	3.88	4.50	4.64	4.20	1.79	3.70	4.46	3.13
Δ_1	1.87	1.81	1.78	1.73	0.032	5.65e-03	5.54e-03	1.81
	23.32	17.02	15.05	10.43	0.96	0.43	0.43	10.21
Δ_2	-0.87	-0.81	-0.79	-0.74	0.97	0.98	0.98	-0.81
	-11.01	-7.79	-6.81	-4.66	29.19	78.24	78.61	4.67
Ψ	-0.96	-0.95	-0.94	-0.93	2.24	1.44	1.46	-0.93
	-25.51	-18.84	-16.72	-10.47	1.46	3.60	4.17	10.31
θ	-0.078	-0.080	-0.058	-0.066	-0.021	0.034	0.033	-0.054
	-3.65	-2.04	-1.39	-1.63	-1.86	-3.36	-3.80	2.02
a	-1.94e-03	-1.46e-07	-6.75e-09	8.13e-08	9.48e-04	-7.04e-04	-8.46e-04	3.34e-06
	-3.84	-2.90e-04	-2.62e-05	1.21e-04	2.11	1.33	-2.84	3.47e-03
b	-0.12	-1.27e-05	-3.14e-08	-5.25e-06	-0.015	0.033	0.041	2.48e-05
	-5.93	-7.34e-04	-3.56e-06	-3.10e-04	0.68	1.41	2.24	1.11e-03
c	3.57	2.78e-04	2.70e-05	-2.75e-04	4.05	3.74	4.85	0.017
	2.20	2.44e-04	[illegible]66e-05	-1.84e-04	1.66	1.43	2.58	4.05e-03
v	1.56	0.97	0.90	0.95	1.80	1.81	1.81	1.16
	19.48	21.83	22.09	24.54	22.26	23.67	29.59	23.99
f	1.33	0.39	0.074		0.16	70.85	72.37	
	2.15	3.16	3.53		3.64	0.031	0.043	

TABLE 6 (cont'd)

MAXIMUM LIKELIHOOD RESULTS FOR MODEL 1 AND MODEL 2 FOR THE CANADIAN DATA. LOG LIKELIHOODS ARE GIVEN AT TOP OF COLUMNS AND t-STATISTICS ARE SHOWN BELOW THE PARAMETER ESTIMATES.

$$R_t = a + bR_{t-1} + c\sigma_t^2 + \varepsilon_t$$

$$\ln(\sigma_t^2) = \alpha + \ln(1 + \delta u_t + f\eta_t) + \frac{(1+\Psi L)}{(1-\Delta_1 L - \Delta_2 L^2)}[\theta Z_{t-1} + \gamma(|Z_{t-1}| - E|Z_{t-1}|)]$$

	BVI				BWR			
η_k	Quotes	Transactions	Volume	- (Model 2)	Quotes	Transactions	Volume	- (Model 2)
	5298	5245	5219	5140	3901	3892	3884	3657
α	-10.78	-10.18	-8.88	-8.32	-9.47	-8.87	-8.17	-7.87
	-38.85	-32.80	-43.22	-62.21	-29.77	-29.32	-43.37	-21.08
δ	-0.18	-0.25	-0.041	0.33	0.45	0.10	0.072	0.33
	-0.92	-1.54	-0.55	178.9	1.28	0.61	0.95	1654.
γ	0.13	0.14	0.19	0.29	0.14	0.072	0.096	0.17
	3.88	3.25	3.36	6.23	5.01	2.47	3.23	5.90
Δ_1	1.53	1.49	1.59	1.28	1.84	1.03	1.68	1.88
	4.30	3.41	5.45	4.16	21.06	0.69	5.00	54.26
Δ_2	-0.53	-0.50	-0.59	-0.29	-0.84	-0.034	-0.68	-0.88
	-1.50	-1.16	-2.06	-0.98	-9.67	-0.023	-2.05	-25.46
Ψ	-0.80	-0.76	-0.86	-0.72	-0.93	-0.30	-0.81	-0.99
	-5.45	-3.48	-7.75	-4.69	-22.66	-0.30	-3.99	-155.2
θ	-0.075	-0.070	-0.067	-0.055	-0.035	-0.071	-0.063	-0.057
	-3.38	-2.60	-2.02	-1.89	-2.26	-2.62	-3.17	2.90
a	-1.41e-03	-1.59e-03	-7.70e-08	9.49e-08	-5.06e-03	-4.65e-03	-5.94e-03	-1.31e-07
	3.17	-3.35	-1.64e-04	1.61e-04	-5.48	-5.52	-6.81	-1.50e-04
b	-0.053	-0.050	-1.60e-05	-7.80e-06	-0.13	-0.13	-0.13	-7.96e-05
	-2.54	-2.43	-8.23e-04	-3.78e-04	-5.82	-6.01	-6.14	-3.90e-03
c	4.68	5.31	6.48e-04	-4.11e-04	3.08	2.90	3.75	-1.88e-03
	2.48	2.79	4.16e-04	-2.66e-04	3.42	3.52	4.30	-2.74e-03
v	1.45	1.31	1.05	1.04	1.67	1.65	1.63	1.05
	25.00	25.40	25.74	33.60	23.04	21.55	21.91	32.02
f	0.35	0.16	8.73e-03	-	0.41	0.17	-0.018	-
	4.95	2.74	4.37	-	3.75	3.44	5.87	-

TABLE 6 (cont'd)

MAXIMUM LIKELIHOOD RESULTS FOR MODEL 1 AND MODEL 2 FOR THE CANADIAN DATA. LOG LIKELIHOODS ARE GIVEN AT TOP OF COLUMNS AND t-STATISTICS ARE SHOWN BELOW THE PARAMETER ESTIMATES.

$$R_t = a + bR_{t-1} + c\sigma_t^2 + \varepsilon_t$$

$$\ln(\sigma_t^2) = \alpha + \ln(1 + \delta n_t + f\eta_t) + \frac{(1+\Psi L)}{(1-\Delta_1 L - \Delta_2 L^2)}[\theta Z_{t-1} + \gamma(|Z_{t-1}| - E|Z_{t-1}|)]$$

	IMS				LBT			
η_t	Quotes	Transactions	Volume	- (Model 2)	Quotes	Transactions	Volume	- (Model 2)
	5738	5860	5842	5773	5894	5806	5789	5766
α	-13.43	-16.69	-9.83	-8.74	-11.91	-10.75	-9.21	-8.79
	-41.42	-0.14	-46.95	-61.97	-23.25	-17.34	-56.77	-73.32
δ	-0.25	-0.25	-0.036	-0.12	-0.25	0.067	-0.049	-0.053
	-0.17	-4.48e-03	-0.38	-3.18	-0.34	-0.30	-0.81	1.36
γ	0.15	0.16	0.20	0.27	0.17	0.24	0.27	0.29
	3.78	3.35	3.45	3.92	3.26	3.68	4.01	4.18
Δ_1	0.61	0.61	0.35	0.88	1.73	1.61	1.63	1.60
	1.26	1.25	1.28	2.10	10.67	8.10	9.89	9.35
Δ_2	0.38	0.37	0.61	0.089	-7.29	-0.61	-0.64	0.60
	0.78	0.77	2.29	0.22	-4.50	-3.10	3.91	3.60
Ψ	-0.26	-0.28	-0.16	-0.45	-0.93	-0.91	-0.94	0.93
	-0.67	-0.70	-0.46	-1.59	-22.15	-14.78	20.41	-17.99
θ	-0.096	-0.097	-0.088	-0.080	-0.017	-0.021	-0.014	-5.62e-03
	-3.78	-3.41	-2.94	-2.69	-0.82	-0.70	0.40	-0.16
a	-4.92e-05	-9.30e-05	-1.53e-04	-6.85e-07	-7.68e-04	-4.95e-04	-4.68e-07	1.34e-08
	-0.16	-0.30	-0.44	-1.56e-03	-1.87	-1.13	-1.04e-03	-2.07e-05
b	0.011	0.011	7.07e-03	5.51e-05	3.57e-03	-1.63e-03	4.50e-06	9.41e-07
	0.48	0.50	0.34	2.58e-03	0.17	-0.074	2.13e-04	4.20e-05
c	0.58	1.07	1.65	5.54e-03	8.04	4.46	5.05e-03	2.32e-04
	0.26	0.45	0.67	1.95e-03	2.61	1.45	1.78e-03	5.54e-05
v	1.50	1.49	1.27	1.08	1.60	1.25	1.09	1.11
	23.32	23.16	23.11	35.27	28.39	23.77	28.93	34.71
f	2.28	42.62	0.035		1.10	0.17	9.38e-03	
	4.10	8.36e-03	3.72		1.94	1.44	3.55	

TABLE 6 (cont'd)

MAXIMUM LIKELIHOOD RESULTS FOR MODEL 1 AND MODEL 2 FOR THE CANADIAN DATA. LOG LIKELIHOODS ARE GIVEN AT TOP OF COLUMNS AND t-STATISTICS ARE SHOWN BELOW THE PARAMETER ESTIMATES.

$$R_t = a + bR_{t-1} + c\sigma_t^2 + \varepsilon_t$$

$$\ln(\sigma_t^2) = \alpha + \ln(1 + \delta u_t + f\eta_t) + \frac{(1+\Psi L)}{(1-\Delta_1 L - \Delta_2 L^2)}[\theta Z_{t-1} + \gamma(|Z_{t-1}| - E|Z_{t-1}|)]$$

	MB				MCL			
η_t	Quotes	Transactions	Volume	- (Model 2)	Quotes	Transactions	Volume	- (Model 2)
	5545	5466	5463	5343	5864	5808	5793	5734
α	-10.96	-13.25	-7.71	-10.72	-12.26	-16.37	-9.88	-8.80
	-21.63	-0.61	-29.42	-72.74	-24.72	-0.18	-51.28	-54.62
δ	-0.25	-0.25	0.019	0.048	0.40	31.29	0.14	0.028
	-0.50	-0.020	0.18	0.99	0.54	0.011	1.59	0.64
γ	0.16	0.19	0.16	0.48	0.20	0.19	0.26	0.29
	4.79	5.71	4.73	9.05	4.09	4.70	4.44	4.29
Δ_1	1.93	1.94	1.94	1.46	0.33	0.36	0.84	1.29
	55.56	100.4	1237.	17.33	1.34	1.50	2.69	5.80
Δ_2	-0.93	-0.94	-0.94	-0.45	0.65	0.62	0.15	-0.29
	-26.78	-48.63	-594.6	-5.41	2.64	2.61	0.48	-1.34
Ψ	-0.98	0.99	-1.01	-0.82	-0.17	-0.18	-0.60	-0.81
	-100.4	-269.3	-188.9	-38.46	-0.56	-0.66	-3.49	-9.44
θ	-0.020	-0.024	-0.018	-0.043	-0.044	-0.043	-0.053	-0.049
	-1.31	-1.47	-0.94	-1.54	-2.30	-2.51	-1.90	-1.69
a	-9.49e-04	-1.16e-03	-3.48e-04	-6.54e-08	2.51e-05	-1.09e-04	-7.40e-04	-1.24e-04
	-2.65	3.18	-1.10	-2.53e-04	0.078	-0.30	-1.91	-0.25
b	-9.92e-03	5.08e-03	3.34e-03	3.60e-06	0.031	0.030	0.030	0.029
	-0.44	0.23	0.18	1.81e-04	1.32	1.34	1.33	1.28
c	6.39	6.85	2.19	2.20e-04	2.82	3.69	7.89	2.08
	3.66	3.92	1.48	1.91e-04	1.15	1.36	2.89	0.66
v	1.53	1.38	1.12	1.00	1.66	1.64	1.39	1.20
	24.84	24.71	22.97	28.86	21.91	26.05	24.56	26.26
f	1.49	10.32	0.020	-	0.66	26.34	0.018	-
	2.20	0.046	3.74		2.16	0.011	3.84	-

TABLE 6 (cont'd)

MAXIMUM LIKELIHOOD RESULTS FOR MODEL 1 AND MODEL 2 FOR THE CANADIAN DATA. LOG LIKELIHOODS ARE GIVEN AT TOP OF COLUMNS AND t-STATISTICS ARE SHOWN BELOW THE PARAMETER ESTIMATES.

$$R_t = a + bR_{t-1} + c\sigma_t^2 + \varepsilon_t$$

$$\ln(\sigma_t^2) = \alpha + \ln(1 + \delta u_t + f\eta_t) + \frac{(1+\Psi L)}{(1-\Delta_1 L - \Delta_2 L^2)}[\theta Z_{t-1} + \gamma(|Z_{t-1}| - E|Z_{t-1}|)]$$

	MTT				N			
η_t	Quotes	Transactions	Volume	- (Model 2)	Quotes	Transactions	Volume	- (Model 2)
	6155	6120	6140	6048	4965	4969	4938	4810
α	-13.51	-13.51	-12.65	-13.34	-10.32	-15.71	-9.82	-7.82
	-27.10	-35.90	-44.48	-84.26	-34.60	-0.11	-31.05	-59.12
δ	2.22e-03	2.22e-03	-0.030	0.33	-0.21	-0.25	0.21	0.060
	8.92e-03	0.012	0.31	191.8	-0.50	-4.40e-03	2.40	1.26
γ	0.21	0.21	0.22	0.20	0.033	0.034	0.048	0.093
	4.53	4.66	4.07	8.03	1.02	1.12	1.25	4.17
Δ_1	1.68	1.68	1.6.	1.61	0.17	0.16	0.20	-5.37e-03
	16.05	1.42	14.02	14.75	1.20	1.27	1.09	-0.22
Δ_2	-0.68	-0.68	-0.67	-0.61	0.83	0.83	0.78	0.97
	-6.49	-5.73	-5.63	-5.59	5.98	6.55	4.22	40.41
Ψ	-0.85	-0.85	-0.85	-0.83	4.13	4.01	2.92	1.29
	-22.44	-19.08	-20.17	-24.11	0.89	0.96	1.05	4.12
θ	-0.18	-0.18	-0.19	-0.18	-0.017	-0.018	-0.026	-0.042
	-4.73	-4.64	-4.29	-7.70	-0.98	-1.07	-1.22	3.31
a	6.98e-10	2.98e-10	-9.61e-10	3.21e-09	-1.59e-03	1.63e-03	-1.83e-03	1.15e-03
	1.08e-05	-2.14e-06	-2.51e-05	7.99e-05	-2.76	-3.14	-3.71	1.16
b	-2.21e-07	-2.24e-09	3.25e-08	3.05e-07	0.070	0.067	0.051	0.11
	-3.47e-05	-3.18e-07	9.40e-06	1.07e-04	3.08	3.06	2.30	5.08
c	-4.31e-06	[illegible].06e-06	9.54e-06	-4.57e-05	3.84	3.83	4.39	3.49
	-8.54e-06	2.05e-05	3.62e-05	-1.64e-04	2.50	2.41	3.17	1.41
v	0.88	8.78	0.83	0.84	1.81	1.84	1.71	1.37
	20.27	21.76	23.01	30.62	20.37	19.76	19.86	22.30
f	0.31	3.06	0.045		0.22	23.20	0.047	
	1.85	2.56	3.40	-	5.06	7.077e-03	2.91	

TABLE 6 (cont'd)

MAXIMUM LIKELIHOOD RESULTS FOR MODEL 1 AND MODEL 2 FOR THE CANADIAN DATA. LOG LIKELIHOODS ARE GIVEN AT TOP OF COLUMNS AND t-STATISTICS ARE SHOWN BELOW THE PARAMETER ESTIMATES.

$$R_t = a + bR_{t-1} + c\sigma_t^2 + \varepsilon_t$$

$$\ln(\sigma_t^2) = \alpha + \ln(1 + \delta n_t + f n_t) + \frac{(1+\Psi L)}{(1-\Delta_1 L - \Delta_2 L^2)}[\theta Z_{t-1} + \gamma(|Z_{t-1}| - E|Z_{t-1}|)]$$

	NOR				NTL			
η_t	Quotes	Transactions	Volume	- (Model 2)	Quotes	Transactions	Volume	- (Model 2)
	5360	5329	5315	5228	5382	5355	5349	5235
α	-15.39	-18.28	-18.40	-8.32	-11.31	-15.09	-10.34	-8.24
	-1.03	-6.08	-11.21	-58.29	-42.23	-0.37	-29.77	-76.30
δ	28.82	771.1	1557.	0.039	0.043	12.26	0.18	0.014
	0.067	0.33	0.62	0.91	0.090	0.024	0.94	-0.33
γ	0.17	0.16	0.12	0.30	0.18	0.17	0.20	0.31
	5.09	4.28	3.48	6.91	3.57	3.62	3.74	4.78
Δ_1	1.70	1.66	1.70	1.64	0.68	0.66	0.59	0.72
	11.52	8.91	10.18	13.96	1.75	1.92	1.71	2.77
Δ_2	-0.70	-0.66	-0.70	-0.65	0.31	0.32	0.38	0.25
	-4.74	-3.57	-4.22	-5.62	0.80	0.93	1.12	1.00
Ψ	-0.86	-0.85	-0.87	-0.90	-0.51	-0.46	-0.44	-0.64
	-11.26	-9.65	-11.32	-15.67	-2.01	-1.79	-1.61	-4.05
θ	-0.081	-0.083	-0.092	-0.010	-0.067	-0.066	-0.036	-0.039
	-4.18	-3.97	-4.14	-0.42	-2.37	-2.61	-1.30	-0.10
a	-9.22e-04	-1.57e-03	-1.02e-03	-5.84e-04	-7.74e-04	-7.25e-04	-9.44e-04	-4.35e-04
	-2.51	-3.00	-3.03	-0.76	-1.40	-1.52	-2.04	-0.50
b	-0.062	0.057	0.018	0.071	0.017	0.012	2.54e-03	0.040
	2.64	2.38	0.83	2.92	0.68	0.52	0.11	1.74
c	2.13	5.09	1.71	2.35	4.06	4.06	5.15	2.45
	1.21	2.26	1.18	0.78	1.70	1.79	2.49	0.73
v	1.83	1.72	1.65	1.37	1.65	1.65	1.56	1.25
	21.83	22.68	21.50	22.73	20.13	22.04	22.12	23.80
f	27.79	195.7	473.5	-	0.20	6.98	0.10	-
	0.067	0.33	0.63	-	4.07	0.025	2.50	-

TABLE 6 (cont'd)

MAXIMUM LIKELIHOOD RESULTS FOR MODEL 1 AND MODEL 2 FOR THE CANADIAN DATA. LOG LIKELIHOODS ARE GIVEN AT TOP OF COLUMNS AND T-STATISTICS ARE SHOWN BELOW THE PARAMETER ESTIMATES.

$$R_t = a + bR_{t-1} + c\sigma_t^2 + \varepsilon_t$$

$$\ln(\sigma_t^2) = \alpha + \ln(1 + \delta u_t + f\eta_t) + \frac{(1+\Psi L)}{(1-\Delta_1 L - \Delta_2 L^2)}[\theta Z_{t-1} + \gamma(|Z_{t-1}| - E|Z_{t-1}|)]$$

	RGO			
η_t	Quotes	Transactions	Volume	- (Model 2)
	4517	4539	4518	4468
α	-14.17	-14.17	-13.25	-7.46
	-34.03	-12.03	-33.94	-64.78
δ	0.37	0.37	0.13	0.077
	0.86	0.48	0.75	1.43
γ	0.17	0.17	0.17	0.25
	5.06	3.11	3.25	3.73
Δ_1	1.10	1.10	1.03	1.53
	3.82	2.39	2.28	5.22
Δ_2	-0.10	-0.10	-0.034	-0.54
	-0.36	-0.22	-0.074	-1.90
Ψ	-0.58	-0.58	-0.53	-0.83
	-7.03	-2.48	-2.71	-5.87
θ	-0.11	-0.11	-0.11	-4.58e-03
	-3.80	-2.69	-2.74	-0.14
a	-2.67e-07	-1.05e-07	-3.10e-08	3.48e-07
	-6.31e-04	-2.44e-04	-1.16e-04	3.57e-04
b	-1.58e-05	-4.39e-06	7.60e-07	-1.36e-05
	-8.97e-04	-2.41e-04	-5.29e-05	-7.02e-04
c	-2.98e-06	1.12e-04	2.70e-05	6.92e-04
	-4.18e-06	1.58e-04	6.09e-05	-4.41e-04
v	0.98	0.98	9.44	1.00
	25.61	26.02	22.82	39.57
f	0.38	0.38	0.066	
	1.79	0.85	3.04	-

	RY			
η_t	Quotes	Transactions	Volume	- (Model 2)
	6255	6212	6212	6136
α	-12.93	-15.70	-10.65	-9.24
	-30.4	-1.24	-40.78	-78.86
δ	1.29	18.76	0.30	0.051
	1.45	0.080	2.10	0.95
γ	0.066	0.066	0.081	0.13
	3.68	3.86	3.59	3.68
Δ_1	1.97	1.97	1.93	9.45e-03
	189.8	253.9	30.85	0.13
Δ_2	-0.97	-0.97	-0.93	0.93
	-93.78	-125.6	-15.06	13.23
Ψ	-0.99	-0.99	-0.94	1.23
	-210.4	-273.9	-14.69	3.62
θ	-0.049	-0.050	-0.051	-0.039
	-4.68	-5.04	-3.69	-2.63
a	9.48e-05	2.07e-04	-3.47e-04	1.21e-04
	-0.29	-0.59	-1.07	0.25
b	0.082	0.083	0.076	0.10
	3.63	3.79	3.48	4.72
c	3.35	6.21	7.35	1.32
	0.94	1.47	1.98	0.27
v	1.81	1.75	1.61	1.36
	19.47	21.47	19.99	20.81
f	0.39	2.53	0.026	
	2.83	0.08	3.13	

TABLE 6 (cont'd)

MAXIMUM LIKELIHOOD RESULTS FOR MODEL 1 AND MODEL 2 FOR THE CANADIAN DATA. LOG LIKELIHOODS ARE GIVEN AT TOP OF COLUMNS AND t-STATISTICS ARE SHOWN BELOW THE PARAMETER ESTIMATES.

$$R_t = a + bR_{t-1} + c\sigma_t^2 + \varepsilon_t$$

$$\ln(\sigma_t^2) = \alpha + \ln(1 + \delta n_t + f\eta_t) + \frac{(1+\Psi L)}{(1-\Delta_1 L - \Delta_2 L^2)}[\theta Z_{t-1} + \gamma(|Z_{t-1}| - E|Z_{t-1}|)]$$

	SPL				TRP			
η_t	Quotes	Transactions	Volume	- (Model 2)	Quotes	Transactions	Volume	- (Model 2)
	5062	5045	5025	4810	5747	5704	5714	5643
α	-9.06	-9.06	-8.42	-8.21	-11.20	-18.30	-10.96	-8.70
	-30.78	-51.71	-53.25	-123.6	-24.55	-0.24	-21.49	-93.05
δ	-0.031	-0.031	-2.56e-03	0.33	0.064	233.8	0.29	0.027
	-0.23	-0.40	-0.040	6384.	0.15	0.013	1.00	0.49
γ	0.41	0.41	0.42	0.079	0.12	0.13	0.14	0.22
	4.86	6.23	5.77	4.74	2.77	3.09	3.56	4.00
Δ_1	1.46	1.46	1.33	-0.083	0.080	0.13	0.066	0.76
	6.45	7.76	5.43	-3.17	0.32	0.45	0.54	1.67
Δ_2	-0.47	-0.47	-0.35	0.91	0.91	0.86	0.92	0.20
	-2.19	-2.65	-1.54	34.93	3.67	3.02	7.60	0.46
Ψ	-0.85	-0.85	-0.80	1.13	0.63	0.48	0.55	-0.39
	-7.79	-9.01	-6.20	11.15	0.92	0.78	1.21	-1.10
θ	-0.033	-0.033	-0.049	-0.043	-0.023	-0.021	-0.016	-8.37e-03
	-0.65	-0.80	-1.10	-3.59	-1.09	-1.00	-0.74	-0.28
a	8.93e-09	1.04e-09	1.94e-08	-8.00e-08	2.33e-07	3.96e-08	-1.03e-08	4.14e-08
	5.32e-05	5.68e-06	1.52e-04	-1.94e-04	6.45e-04	9.98e-05	-3.50e-05	6.11e-05
b	2.37e-07	6.32e-09	-3.35e-07	-7.37e-07	[illegible].38e-05	-3.22e-06	-8.39e-09	2.97e-06
	-2.26e-05	9.79e-07	-3.50e-05	-5.31e-05	-2.66e-03	-1.60e-04	-4.82e-07	1.40e-04
c	-2.59e-05	-3.37e-06	-7.43e-05	3.36e-04	1.56e-03	8.60e-06	9.39e-05	9.02e-05
	-4.06e-05	-6.66e-06	-1.53e-04	2.56e-04	7.14e-04	3.63e-06	5.43e-05	2.34e-05
v	0.88	0.89	0.87	0.96	1.10	1.07	1.02	1.06
	22.18	26.19	24.25	32.92	26.95	27.34	30.12	[illegible].37
f	0.26	0.26	0.028	-	0.28	129.2	0.080	-
	2.65	3.94	3.71	-	3.86	0.013	2.19	-

TABLE 6 (cont'd)

MAXIMUM LIKELIHOOD RESULTS FOR MODEL 1 AND MODEL 2 FOR THE CANADIAN DATA. LOG LIKELIHOODS ARE GIVEN AT TOP OF COLUMNS AND t-STATISTICS ARE SHOWN BELOW THE PARAMETER ESTIMATES.

$$R_t = a + bR_{t-1} + c\sigma_t^2 + \epsilon_t$$

$$\ln(\sigma_t^2) = \alpha + \ln(1 + \delta u_t + f\eta_t) + \frac{(1+\Psi L)}{(1-\Delta_1 L - \Delta_2 L^2)}[\theta Z_{t-1} + \gamma(|Z_{t-1}| - E|Z_{t-1}|)]$$

	VO				WN			
η_t	Quotes	Transactions	Volume	- (Model 2)	Quotes	Transactions	Volume	- (Model 2)
	5831	5458	5706	5672	6060	5992	5956	5946
α	-12.71	-16.64	-20.46	-8.84	-12.23	-10.23	-9.72	-8.95
	-31.05	-1.00	-1.45	-50.38	-13.02	-43 15	-82.78	-68.24
δ	-0.18	32.99	155.2	0.019	-0.18	0.049	0.084	0.014
	-0.25	0.060	0.071	0.46	-0.30	0.31	1.02	0.28
γ	0.14	0.16	0.18	0.20	0.14	0.32	0.33	0.33
	5.46	20.68	5.94	5.15	5.00	4.60	5.27	5.71
Δ_1	1.95	1.79	1.80	1.84	1.94	0.13	0.24	1.76
	165.2	35.38	9.98	30.74	77.19	0.78	1.33	15.52
Δ_2	-0.95	-0.80	-0.80	-0.84	-0.94	0.65	0.63	-0.76
	-80.57	-16.74	-4.45	-14.21	-37 26	4.87	3.99	-7.01
ψ	-1.01	-0.79	-0.86	-0.96	-1.01	0.11	0.057	-0.91
	-207.2	-12.05	-7.19	-39.26	-168.8	0.37	0.23	-12.90
θ	-0.062	-0.12	-0.064	-0.57	-3.20e-03	0.030	0.015	-655.6
	-4.11	-21.55	-3.91	-2.43	-0.24	0.79	0.049	-0.024
a	1.43e-04	-3.23e-04	1.12e-04	7.37e-04	-4.31e-04	-4.64e-07	1.89e-08	3.56e-07
	0.41	-2.36	0.45	1.30	-1.69	-1.52e-03	7.18e-05	1.01e-03
b	0.095	0.097	0.097	0.12	-0.066	-3.52e-05	-4.95e-06	-1.88e-05
	4.14	8.32	4.76	5.31	-3.38	-1.70e-03	-2.62e-04	-8.68e-04
c	3.99	5.86	2.35	-0.85	7.13	4 27e-03	1.54e-04	-1.43e-03
	1.46	3.32	1.23	-0.25	2.61	1.70e-03	7.24e-05	-5.22e-04
v	1.71	2.78	1.44	1.27	1.39	1.05	1.02	1.05
	20.89	60.17	24.45	22.61	23.80	24.52	29.89	29.96
f	0.36	17.07	74.35	-	2.88	0.24	0.049	-
	2.59	0.060	0.071	-	1.02	3.12	4.44	-

TABLE 7

MAXIMUM LIKELIHOOD RESULTS FOR MODEL 1 AND MODEL 2 FOR THE AMERICAN DATA. LOG LIKELIHOODS ARE GIVEN AT TOP OF COLUMNS AND t-STATISTICS ARE SHOWN BELOW THE PARAMETER ESTIMATES.

$$R_t = a + bR_{t-1} + c\sigma_t^2 + \varepsilon_t$$

$$\ln(\sigma_t^2) = \alpha + \ln(1 + \delta u_t + f\eta_t) + \frac{(1+\Psi L)}{(1-\Delta_1 L - \Delta_2 L^2)}[\theta Z_{t-1} + \gamma(|Z_{t-1}| - E|Z_{t-1}|)]$$

	AA				C			
η_t	Quotes	Transactions	Volume	- (Model 2)	Quotes	Transactions	Volume	- (Model 2)
	5252	5226	5227	5152	4719	4655	4592	4598
α	-11.16	-14.36	-11.19	-8.04	-11.16	-19.50	-18.30	-7.57
	-28.09	-1.19	-10.98	-40.59	-25.91	-18.72	-16.32	-52.87
δ	3.01	45.21	1.67	0.094	1.69	527.1	66.03	0.068
	2.21	0.083	0.86	1.76	1.62	1.38	0.93	14.07
γ	0.10	0.082	0.044	0.15	0.093	0.057	0.072	0.22
	2.72	2.54	0.85	3.23	6.19	4.80	9.28	3.52
Δ_1	1.89	1.90	1.15	1.80	1.97	1.92	1.94	1.32
	21.72	22.60	1.04	12.11	167.5	176.4	294.9	2.68
Δ_2	-0.89	-0.90	-0.16	-0.80	-0.97	-0.92	-0.94	-0.33
	-10.29	-10.76	-0.15	-5.41	-82.95	-84.31	-142.9	-0.68
Ψ	-0.96	-0.96	0.91	-0.92	-1.00	-1.00	-1.00	-0.65
	-29.24	-30.14	0.27	-14.29	-144.2	-485.8	-812.9	-2.18
θ	-0.05	-0.043	-9.93e-03	-0.035	-0.063	-0.082	-0.086	-0.043
	-2.46	-2.55	-0.73	-1.63	-5.94	-7.98	-12.85	-1.70
a	-1.22e-03	-1.27e-03	-1.02e-03	3.89e-04	-1.62e-03	-2.55e-03	-3.41e-03	1.58e-04
	-2.13	-2.20	-2.30	0.51	-2.36	-3.16	-7.93	0.15
b	0.12	0.11	0.084	0.14	-8.37e-03	-0.022	-6.70e-03	4.40e-03
	5.26	5.10	4.03	6.30	-0.37	-0.96	-0.38	0.19
c	5.40	5.60	4.43	-0.48	3.91	6.33	7.09	-0.38
	2.54	2.52	2.90	-0.18	2.40	3.57	55.70	-0.19
v	1.43	1.45	1.42	1.31	1.74	1.93	1.89	1.44
	30.23	28.27	31.51	29.14	26.71	27.85	32.85	24.66
f	0.42	3.13	4.95	-	0.49	41.55	184.2	-
	2.66	0.083	0.92	-	2.37	0.98	0.91	-

TABLE 7 (cont'd)

MAXIMUM LIKELIHOOD RESULTS FOR MODEL 1 AND MODEL 2 FOR THE AMERICAN DATA. LOG LIKELIHOODS ARE GIVEN AT TOP OF COLUMNS AND t-STATISTICS ARE SHOWN BELOW THE PARAMETER ESTIMATES.

$$R_t = a + bR_{t-1} + c\sigma_t^2 + \varepsilon_t$$

$$\ln(\sigma_t^2) = \alpha + \ln(1 + \delta x_t + f\eta_t) + \frac{(1+\Psi L)}{(1-\Delta_1 L - \Delta_2 L^2)}[\theta Z_{t-1} + \gamma(|Z_{t-1}| - E|Z_{t-1}|)]$$

	DD				DEC			
η_t	Quotes	Transactions	Volume	- (Model 2)	Quotes	Transactions	Volume	- (Model 2)
	5476	5471	5218	5401	4959	4993	4465	4885
α	-11.89	-14.69	-17.46	-8.45	-10.27	-13.92	-14.33	-7.86
	-14.69	-0.77	-3.47	-60.85	-21.67	-23.56	-5.64	-74.92
δ	4.79	52.93	28.53	0.063	4.31	98.38	80.93	0.070
	1.16	0.053	0.20	1.37	1.94	0.17	0.40	1.49
γ	0.15	0.16	0.26	0.16	0.14	0.12	0.29	0.21
	4.02	4.08	20.70	4.02	3.56	3.53	48.73	4.34
Δ_1	1.80	1.83	1.72	1.80	1.90	1.91	1.94	1.72
	13.54	16.91	51.01	9.69	44.95	53.19	275.9	11.55
Δ_2	-0.80	-0.83	-0.72	-0.80	-0.90	-0.91	-0.94	-0.73
	-6.13	-7.75	-21.43	-4.40	-21.24	-25.34	-133.9	-5.00
Ψ	-0.93	-0.95	-0.93	-0.91	-0.99	-0.99	-0.99	-0.90
	-15.02	-22.33	-232.7	-8.79	-203.0	-224.1	-3358	-12.36
θ	-0.082	-0.077	-0.12	-0.052	-0.051	-0.049	-0.073	-0.053
	-3.30	-3.43	-13.97	-2.42	-2.40	-2.52	-17.48	-2.20
a	-8.10e-04	-7.10e-04	-9.56e-04	-1.10e-04	-3.21e-03	-3.34e-03	-2.69e-03	-2.39e-03
	-1.70	-1.48	-10.80	-0.15	-4.26	-4.43	-17.19	-2.24
b	8.92e-03	4.24e-03	-3.14e-03	0.017	0.024	0.020	-0.011	0.049
	0.38	0.18	-0.23	0.74	1.02	0.86	-1.17	2.08
c	5.94	5.44	5.32	2.29	9.58	10.22	4.16	6.49
	2.47	2.31	3.81	0.68	4.48	4.57	3.70	2.31
v	1.45	1.43	1.80	1.35	1.39	1.45	2.25	1.35
	25.63	25.92	39.90	24.91	28.17	25.87	77.37	31.66
f	0.33	1.80	88.25	-	0.11	1.69	16.15	-
	1.18	0.053	0.20	-	2.15	0.17	0.39	

TABLE 7 (cont'd)

MAXIMUM LIKELIHOOD RESULTS FOR MODEL 1 AND MODEL 2 FOR THE AMERICAN DATA. LOG LIKELIHOODS ARE GIVEN AT TOP OF COLUMNS AND t-STATISTICS ARE SHOWN BELOW THE PARAMETER ESTIMATES.

$$R_t = a + bR_{t-1} + c\sigma_t^2 + \varepsilon_t$$

$$\ln(\sigma_t^2) = \alpha + \ln(1 + \delta u_t + f\eta_t) + \frac{(1+\Psi L)}{(1-\Delta_1 L - \Delta_2 L^2)} \cdot [\theta Z_{t-1} + \gamma(|Z_{t-1}| - E|Z_{t-1}|)]$$

	DIS				DOW			
η_t	Quotes	Transactions	Volume	- (Model 2)	Quotes	Transactions	Volume	- (Model 2)
	5177	5176	5176	5073	5339	5320	4878	5255
α	-17.24	-17.55	-20.72	-8.16	-14.27	-16.66	-17.31	-8.29
	-12.58	-13.47	-6.01	-46.17	-1.97	-1.61	-5.43	-63.84
δ	969.3	938.8	768.5	0.12	29.21	153.6	144.6	0.018
	0.72	0.76	0.32	2.37	0.14	0.095	0.32	0.43
γ	0.19	0.16	0.14	0.23	0.20	0.18	0.21	0.19
	7.50	5.57	4.43	7.14	5.61	5.48	32.35	5.33
Δ_1	1.84	1.89	1.63	1.87	1.79	1.81	1.82	1.81
	24.46	31.59	7.34	27.29	29.62	26.83	98.76	15.27
Δ_2	-0.84	-0.89	-0.63	-0.87	-0.79	-0.81	-0.82	-0.82
	-11.31	-14.96	-2.84	-12.91	-13.11	-12.22	-44.40	-7.02
Ψ	-0.95	-0.96	-0.62	-0.95	-0.97	-0.96	-0.94	-0.93
	-25.22	-35.45	-2.75	-21.92	-81.68	-39.00	-360.7	-13.46
θ	-0.054	-0.048	-0.014	-0.033	-0.15	-0.13	-0.13	-0.074
	-2.91	-2.73	-1.30	-1.84	-5.96	-5.51	-26.31	-3.60
a	-7.94e-05	-5.05e-05	-2.70e-04	6.48e-04	-9.36e-04	-8.66e-04	-5.10e-04	-6.11e-04
	-0.20	-0.12	-0.88	1.09	-2.30	-1.77	-3.41	-0.90
b	0.041	0.055	.042	0.085	0.084	0.071	0.068	0.068
	1.95	2.26	1.90	3.90	3.47	2.92	6.81	2.87
c	3.30	2.77	3.73	0.099	4.76	4.75	3.13	4.79
	2.16	1.77	2.80	0.053	2.67	2.24	2.21	1.82
v	1.50	1.52	1.53	1.26	1.52	1.54	2.14	1.35
	32.65	29.81	31.89	30.98	23.26	22.85	58.57	26.41
f	70.97	33.99	1011	-	5.39	12.51	101.9	-
	0.73	0.75	0.32	-	0.14	0.096	0.31	-

TABLE 7 (cont'd)

MAXIMUM LIKELIHOOD RESULTS FOR MODEL 1 AND MODEL 2 FOR THE AMERICAN DATA. LOG LIKELIHOODS ARE GIVEN AT TOP OF COLUMNS AND t-STATISTICS ARE SHOWN BELOW THE PARAMETER ESTIMATES.

$$R_t = a + bR_{t-1} + c\sigma_t^2 + \varepsilon_t$$

$$\ln(\sigma_t^2) = \alpha + \ln(1 + \delta u_t + f\eta_k) + \frac{(1+\Psi L)}{(1-\Delta_1 L - \Delta_2 L^2)}[\theta Z_{t-1} + \gamma(|Z_{t-1}| - E|Z_{t-1}|)]$$

	EK				GM			
η_k	Quotes	Transactions	Volume	- (Model 2)	Quotes	Transactions	Volume	- (Model 2)
	5388	5336	5370	5293	5405	5393	4971	5311
α	-12.39	-16.06	-18.45	-8.38	-15.98	-15.64	-17.18	-8.36
	-11.72	-0.62	-3.16	-74.50	-3.86	-1.54	-4.54	-74.51
δ	11.74	1115.4	51.33	0.15	330.7	161.8	31.52	0.019
	0.91	0.039	0.17	2.59	0.24	0.099	0.27	0.45
γ	0.15	0.17	0.20	0.27	0.13	[illegible]	0.21	0.16
	2.65	3.88	6.87	4.96	3.54	3.94	19.69	5.06
Δ_1	1.71	1.73	1.95	1.07	1.86	1.86	1.80	1.84
	8.90	19.18	129.0	1.79	14.94	18.07	73.20	17.36
Δ_2	-0.72	-0.73	-0.95	-0.11	-0.86	-0.86	-0.80	-0.84
	-3.77	-8.25	-62.89	-0.19	-6.99	-8.52	-32.53	-8.10
ψ	-0.92	-0.93	-0.99	-0.44	-0.95	-0.96	0.94	0.95
	-13.80	-23.09	-162.4	-0.92	-15.7	-19.48	-316.5	-19.22
θ	-0.069	-0.068	-0.068	-0.051	-0.058	-0.092	-0.14	-0.064
	-2.22	-3.47	-3.61	-1.97	-3.16	-4.14	-19.38	-3.74
a	-1.21e-03	-1.08e-03	-4.88e-04	-4.73e-04	-1.59e-03	-1.48e-03	-1.13e-03	-1.69e-03
	-2.58	-2.50	-1.78	-0.80	-2.93	-3.11	-7.75	-2.03
b	-7.88e-03	-0.011	-0.037	-17.03	[illegible]	5.63e-03	0.037	0.018
	-0.36	-0.48	-1.63	-0.078	0.72	0.23	4.09	0.75
c	6.95	7.10	2.78	2.31	8.72	6.70	4.95	8.14
	3.24	3.50	1.91	0.99	3.38	3.17	3.64	2.28
v	1.33	1.38	1.51	1.20	1.52	1.49	1.96	1.42
	30.87	29.80	32.72	31.17	26.38	25.26	52.22	26.67
f	0.60	4.51	58.43	-	2.16	3.29	51.72	
	0.93	0.039	0.17	-	0.24	0.10	0.27	

TABLE 7 (cont'd)

MAXIMUM LIKELIHOOD RESULTS FOR MODEL 1 AND MODEL 2 FOR THE AMERICAN DATA. LOG LIKELIHOODS ARE GIVEN AT TOP OF COLUMNS AND t-STATISTICS ARE SHOWN BELOW THE PARAMETER ESTIMATES.

$$R_t = a + bR_{t-1} + c\sigma_t^2 + \varepsilon_t$$

$$\ln(\sigma_t^2) = \alpha + \ln(1 + \delta u_t + f\eta_t) + \frac{(1+\Psi L)}{(1-\Delta_1 L - \Delta_2 L^2)}[\theta Z_{t-1} + \gamma(|Z_{t-1}| - E|Z_{t-1}|)]$$

	HAL				IBM			
η_t	Quotes	Transactions	Volume	- (Model 2)	Quotes	Transactions	Volume	- (Model 2)
	4934	4915	4736	4833	5656	5692	5471	5624
α	-20.84	-19.36	-19.14	-7.81	-15.72	-14.97	-18.16	-8.79
	-36.07	-43.77	-28.28	-70.14	-1.44	-0.87	-4.24	-99.98
δ	1077.	1208.	215.1	-4.39e-04	186.2	135.4	43.68	0.20
	2.40	4.86	1.35	-9.30e-03	0.093	0.058	0.23	3.57
γ	0.084	0.073	0.068	0.23	0.089	0.17	0.19	0.17
	5.35	5.83	14.48	4.73	3.40	3.23	17.03	4.76
Δ_1	1.87	1.96	1.95	0.95	1.82	1.74	1.83	1.81
	67.53	143.8	397.6	1.74	15.45	8.72	67.03	15.07
Δ_2	-0.87	-0.96	-0.95	0.015	-0.83	-0.74	-0.83	-0.81
	-31.39	-70.35	-194.0	0.029	-7.22	-3.78	-30.40	-6.95
Ψ	-0.98	-0.99	-1.00	-0.45	-0.94	-0.89	-0.89	-0.94
	-173.5	-294.6	-588.3	-1.18	-13.68	-9.03	-161.6	-14.11
θ	-0.12	-0.075	-0.077	-0.054	-0.082	-0.057	-0.067	-0.047
	-7.28	-6.09	18.17	-2.17	-3.79	-2.39	-16.38	-2.49
a	-1.14e-03	-1.85e-03	-2.76e-03	-3.53e-05	-7.26e-04	-1.28e-03	-1.16e-03	-1.93e-03
	-2.31	-2.83	-8.94	-0.037	-1.50	-2.78	-10.69	-2.75
b	0.018	0.016	0.011	8.00e-04	-0.018	-0.042	-0.041	-0.020
	0.90	0.73	0.73	0.036	-0.81	-1.76	-2.64	-0.89
c	3.27	5.27	7.11	0.097	5.46	8.45	5.12	12.29
	2.26	2.88	5.69	0.041	1.69	2.90	2.98	2.94
v	1.48	1.46	1.65	1.22	1.38	1.43	1.75	1.38
	29.00	29.73	46.26	26.72	36.74	27.60	44.08	34.25
f	325.3	159.8	211.9	-	3.94	0.53	52.99	-
	1.77	2.59	1.39	-	0.093	0.058	0.23	

TABLE 7 (cont'd)

MAXIMUM LIKELIHOOD RESULTS FOR MODEL 1 AND MODEL 2 FOR THE AMERICAN DATA. LOG LIKELIHOODS ARE GIVEN AT TOP OF COLUMNS AND t-STATISTICS ARE SHOWN BELOW THE PARAMETER ESTIMATES.

$$R_t = a + bR_{t-1} + c\sigma_t^2 + \varepsilon_t$$

$$\ln(\sigma_t^2) = \alpha + \ln(1 + \delta x_t + f\eta_t) + \frac{(1+\Psi L)}{(1-\Delta_1 L - \Delta_2 L^2)}[\theta Z_{t-1} + \gamma(|Z_{t-1}| - E|Z_{t-1}|)]$$

	IRC				MCD			
η_t	Quotes	Transactions	Volume	- (Model 2)	Quotes	Transactions	Volume	- (Model 2)
	4149	4102	4098	4072	4359	5344	4994	5294
α	-10.65	-10.65	-10.65	-6.80	-21.62	-16.72	-19.05	-8.46
	-20.18	-14.49	-30.59	-24.99	-21.42	-2.08	-3.39	-60.22
δ	-0.064	-0.064	-0.064	-0.029	287.3	38.0?	40.15	0.13
	-0.44	-0.16	-0.99	-0.54	1.08	0.12	0.18	2.58
γ	0.060	0.060	0.060	0.18	0.072	0.076	0.12	0.25
	2.65	3.10	3.71	3.58	12.88	8.18	9.14	4.87
Δ_1	1.97	1.97	1.97	0.26	1.98	1.97	1.86	1.29
	111.3	119.8	117.2	0.89	8042.	159.6	119.0	3.62
Δ_2	-0.97	-0.97	-0.97	0.72	-0.98	-0.97	-0.86	-0.30
	-55.81	-58.99	-57.76	2.44	-3888.	-78.66	-55.17	-0.86
Ψ	-0.98	-0.98	-0.98	0.044	-1.02	-0.99	-0.97	-0.71
	-78.73	-90.53	-77.52	0.10	-438.7	-182.0	-140.5	-4.17
θ	-0.069	-0.069	-0.069	-0.070	-0.077	-0.072	-0.14	-0.025
	-3.22	-3.61	-3.89	-2.48	-13.92	-8.94	-10.90	-1.12
a	-1.65e-08	-3.82e-09	-1.40e-09	8.83e-09	-7.41e-04	6.14e-04	6.27e-04	1.15e-03
	-2.99e-05	-6.59e-06	-2.47e-06	1.27e-05	-3.23	1.45	2.30	1.57
b	-8.49e-07	-1.33e-07	-3.26e-08	-6.21e-08	4.33e-03	0.012	0.031	0.033
	-4.86e-05	-7.49e-06	-2.02e-06	-4.09e-06	0.40	0.58	3.87	1.42
c	1.43e-06	1.77e-06	9.95e-07	-1.04e-05	7.22	-0.81	-0.019	-1.77
	2.39e-06	3.27e-06	1.81e-06	-1.53e-05	14.37	-0.42	-0.012	-0.56
v	0.98	0.98	0.98	0.94	1.87	1.83	1.81	1.50
	23.04	26.18	24.61	25.89	26.61	32.41	32.13	24.14
f	0.49	0.49	0.49	-	27.06	4.93	44.14	-
	2.79	1.37	3.33	-	1.05	0.12	0.18	-

TABLE 7 (cont'd)

MAXIMU * LIKELIHOOD RESULTS FOR MODEL 1 AND MODEL 2 FOR THE AMERICAN DATA. LOG LIKELIHOODS ARE GIVEN AT TOP OF COLUMNS AND t-STATISTICS ARE SHOWN BELOW THE PARAMETER ESTIMATES.

$$R_t = a + bR_{t-1} + c\sigma_t^2 + \varepsilon_t$$

$$\ln(\sigma_t^2) = \alpha + \ln(1 + \delta u_t + f\eta_t) + \frac{(1 + \Psi L)}{(1 - \Delta_1 L - \Delta_2 L^2)}[\theta Z_{t-1} + \gamma(|Z_{t-1}| - E|Z_{t-1}|)]$$

	MER				NSC			
η_t	Quotes	Transactions	Volume	- (Model 2)	Quotes	Transactions	Volume	- (Model 2)
	4735	4779	4819	4698	5543	5530	5532	5475
α	-17.30	-16.53	-11.56	-7.59	-15.15	-16.95	-10.91	-8.52
	-2.49	-3.76	-16.05	-46.67	-0.78	-2.02	-15.38	-44.43
δ	113.9	91.00	1.63	0.077	95.77	273.1	0.75	0.019
	0.14	0.23	1.41	1.36	0.052	0.12	0.92	0.38
γ	0.095	0.060	0.073	0.27	0.19	0.17	0.25	0.38
	5.82	4.36	3.88	5.05	4.63	4.43	4.92	7.01
Δ_1	1.90	1.95	1.98	1.46	1.77	1.81	0.41	0.97
	86.85	117.0	241.4	6.74	16.21	15.77	2.30	4.62
Δ_2	-0.91	-0.95	-0.98	-0.46	-0.77	-0.81	0.55	0.015
	-41.25	-57.11	-119.3	-2.16	-7.25	-7.24	3.15	0.073
Ψ	-0.98	-0.99	-1.01	-0.84	-0.92	-0.92	-0.33	-0.69
	-146.5	-158.0	-277.7	-9.55	-16.77	-13.85	-1.82	-6.89
θ	-0.12	-0.069	-0.054	-0.066	-0.11	-0.085	-0.11	-0.075
	-6.82	-5.30	-4.51	-2.50	-4.37	-3.51	-3.12	-2.18
a	-8.08e-04	-1.13e-03	-2.36e-03	-7.64e-05	-3.60e-04	-2.08e-04	-2.83e-04	2.72e-04
	-1.62	-1.85	-3.98	-0.080	-0.96	-0.55	-0.76	0.61
b	-0.030	-0.045	-0.062	-8.73e-03	3.01e-03	7.80e-03	-1.60e-04	0.011
	-1.52	-2.01	-2.71	-0.37	0.14	0.35	-7.26e-03	0.49
c	1.42	2.58	4.76	-0.011	5.10	3.70	3.50	-0.20
	1.10	1.67	3.38	-5.43e-03	2.51	1.88	1.91	-0.098
v	1.57	1.53	1.44	1.27	1.45	1.40	1.29	1.20
	37.33	30.90	2.97	26.23	29.06	27.71	23.76	24.18
f	9.98	3.97	2.97	-	16.16	40.68	6.51	-
	0.14	0.23	1.50	-	0.052	0.12	1.27	

TABLE 7 (cont'd)

MAXIMUM LIKELIHOOD RESULTS FOR MODEL 1 AND MODEL 2 FOR THE AMERICAN DATA. LOG LIKELIHOODS ARE GIVEN AT TOP OF COLUMNS AND t-STATISTICS ARE SHOWN BELOW THE PARAMETER ESTIMATES.

$$R_t = a + bR_{t-1} + c\sigma_t^2 + \varepsilon_t$$

$$\ln(\sigma_t^2) = \alpha + \ln(1 + \delta n_t + f n_t) + \frac{(1 + \Psi L)}{(1 - \Delta_1 L - \Delta_2 L^2)}[\theta Z_{t-1} + \gamma(|Z_{t-1}| - E|Z_{t-1}|)]$$

	OXY				RLM			
n_t	Quotes	Transactions	Volume	- (Model 2)	Quotes	Transactions	Volume	- (Model 2)
	5278	5203	5003	5150	5032	5026	4882	4986
α	-17.07	-21.03	-21.00	-7.69	-10.54	-11.32	-12.85	-7.96
	-2.17	-11.52	-2.77	-36.75	-43.42	-10.43	-68.36	-40.46
δ	29.68	129.4	54.41	-0.052	0.22	0.99	0.29	0.063
	0.13	0.26	0.13	-1.18	0.95	0.76	2.91	1.37
γ	0.075	0.092	0.11	0.40	0.027	0.032	0.13	0.29
	4.87	7.06	5.59	7.16	1.84	2.68	11.73	5.14
Δ_1	1.93	1.94	1.83	1.59	1.98	1.98	1.98	1.58
	141.1	127.6	67.44	14.43	382.9	356.8	410.0	10.58
Δ_2	-0.93	-0.93	-0.83	-0.59	-0.98	-0.98	-0.98	-0.58
	-67.87	-61.79	-30.57	-5.41	-189.6	-176.2	-203.2	-3.92
Ψ	-0.99	-0.98	-0.91	-0.92	-1.02	-1.02	-1.01	-0.90
	-213.4	-126.5	-43.94	-25.99	-356.2	-400.8	-281.6	-17.99
θ	-0.10	-0.12	-0.13	-0.10	-0.055	-0.067	-0.13	-0.014
	-6.10	-8.33	-6.67	-2.73	-5.58	-5.47	-16.73	-0.44
a	-1.11e-04	1.17e-04	2.39e-04	1.78e-06	-8.02e-04	-9.74e-04	3.26e-04	8.60e-04
	-0.39	0.31	1.03	3.66e-03	-2.74	-3.67	0.83	1.09
b	-0.049	-0.042	-0.021	-1.85e-04	0.12	0.12	0.092	0.15
	-2.74	-2.21	-1.73	-8.43e-03	5.51	6.31	5.90	6.05
c	0.79	-0.86	-1.41	-0.010	5.12	5.49	-0.17	-0.59
	0.59	-0.56	-1.43	-6.66e-03	5.34	7.78	-0.13	-0.26
v	1.34	1.30	1.33	1.07	1.54	1.52	1.83	1.42
	38.03	39.00	30.44	31.16	23.56	24.20	32.18	25.01
f	22.76	307.4	172.6	-	0.16	0.33	0.64	-
	0.13	0.55	0.13	-	5.45	0.89	5.27	-

TABLE 7 (cont'd)

MAXIMUM LIKELIHOOD RESULTS FOR MODEL 1 AND MODEL 2 FOR THE AMERICAN DATA. LOG LIKELIHOODS ARE GIVEN AT TOP OF COLUMNS AND t-STATISTICS ARE SHOWN BELOW THE PARAMETER ESTIMATES.

$$R_t = a + bR_{t-1} + c\sigma_t^2 + \varepsilon_t$$

$$\ln(\sigma_t^2) = \alpha + \ln(1 + \delta n_t + f n_t) + \frac{(1+\Psi L)}{(1-\Delta_1 L - \Delta_2 L^2)}[\theta Z_{t-1} + \gamma(|Z_{t-1}| - E|Z_{t-1}|)]$$

	TDY				TGT			
η_t	Quotes	Transactions	Volume	- (Model 2)	Quotes	Transactions	Volume	- (Model 2)
	5575	5628	5603	5485	5532	5508	5540	5445
α	-12.23	-19.37	-19.97	-8.31	-10.67	-14.63	-12.13	-8.55
	-15.27	-13.79	-1.23	-25.15	-59.30	-0.51	-7.02	-52.63
δ	1.26	1922.	126.3	-0.035	0.28	42.64	1.72	-0.048
	0.99	0.82	0.062	-0.77	0.94	0.035	0.50	-1.28
γ	0.25	0.29	0.38	0.28	0.16	0.23	0.17	0.27
	5.68	6.53	6.78	6.77	3.34	4.86	3.38	5.04
Δ_1	1.78	1.77	1.17	1.76	1.68	1.75	1.67	1.65
	18.87	21.04	5.21	18.16	8.38	18.27	6.16	9.88
Δ_2	-0.78	-0.77	-0.17	-0.76	-0.68	-0.76	-0.68	-0.65
	-8.30	-9.24	-0.76	-7.91	-3.47	-8.09	-2.55	-3.98
Ψ	-0.92	-0.93	-0.57	-0.92	-0.89	-0.93	-0.84	-0.87
	-21.53	-25.06	-5.18	-19.80	-10.79	-21.08	-5.58	-12.39
θ	-0.070	-0.066	-0.036	-0.054	-0.095	-0.12	-0.097	-0.074
	-3.09	-2.82	-1.60	-2.51	-2.86	-3.72	-3.21	-2.46
a	-5.25e-05	-4.02e-04	-3.84e-04	-2.21e-05	-2.98e-04	-9.69e-05	-2.35e-04	-2.14e-05
	-0.22	-1.58	-1.82	-0.073	-0.74	-0.27	-0.67	-0.044
b	0.013	4.11e-03	-6.42e-03	0.031	0.024	0.041	0.012	0.053
	0.64	0.19	-0.30	1.53	1.08	1.79	0.55	2.36
c	0.20	1.36	0.27	-0.33	3.82	2.84	2.58	2.86
	0.16	0.98	0.24	-0.24	2.02	1.69	1.54	1.26
v	1.15	1.30	1.34	1.06	1.33	1.36	1.46	1.28
	40.58	31.99	29.31	39.92	29.39	27.80	31.15	30.89
f	0.40	375.8	703.6	-	0.14	1.86	15.16	-
	1.57	0.71	0.062	-	5.08	0.035	0.56	-

TABLE 7 (cont'd)

MAXIMUM LIKELIHOOD RESULTS FOR MODEL 1 AND MODEL 2 FOR THE AMERICAN DATA. LOG LIKELIHOODS ARE GIVEN AT TOP OF COLUMNS AND t-STATISTICS ARE SHOWN BELOW THE PARAMETER ESTIMATES.

$$R_t = a + bR_{t-1} + c\sigma_t^2 + \varepsilon_t$$

$$\ln(\sigma_t^2) = \alpha + \ln(1 + \delta a_t + f\eta_t) + \frac{(1+\Psi L)}{(1-\Delta_1 L - \Delta_2 L^2)}[\theta Z_{t-1} + \gamma(|Z_{t-1}| - E|Z_{t-1}|)]$$

	VAT				XON			
η_t	Quotes	Transactions	Volume	- (Model 2)	Quotes	Transactions	Volume	- (Model 2)
	1985	2298	1533	2760	5655	5616	5230	5579
α	-14.70	-18.08	-18.04	-10.29	-11.24	-14.43	-18.78	-8.71
	-25.71	-23.50	-231.2	-31.80	-27.41	-0.88	-6.32	-96.42
δ	-0.25	-0.25	-0.25	0.13	2.09	45.89	124.1	0.061
	-0.069	-2.47e-04	-0.085	1.32	1.92	0.061	0.34	1.34
γ	0.057	0.055	0.065	1.08	0.16	0.23	0.21	0.19
	16.48	4.27	48.62	6.60	4.52	5.26	19.64	5.91
Δ_1	1.96	1.96	1.97	1.28	1.77	1.84	1.92	1.79
	440.1	95.38	920.1	12.35	10.86	28.14	177.5	17.93
Δ_2	-0.96	-0.96	-0.97	-0.28	-0.78	-0.84	-0.92	-0.79
	-221.1	-47.27	-460.9	-2.61	-4.90	-12.99	-85.13	-8.24
Ψ	-0.98	-0.98	-0.98	-0.84	-0.92	-0.96	-0.92	-0.94
	-186.7	-61.56	-428.8	-23.47	-10.15	-39.73	-488.8	-17.48
θ	-0.065	-0.088	-0.10	-0.16	-0.10	-0.081	-0.016	-0.099
	-15.18	-3.73	-41.56	-1.54	-3.53	-3.30	-4.82	-4.19
a	-1.31e-04	-2.36e-03	-2.36e-04	-1.47e-09	-2.71e-04	-2.02e-05	-4.36e-04	-1.04e-03
	-0.27	-1.42	-1.17	-4.88e-03	-0.63	-0.043	-7.74	-1.65
b	-0.011	-0.085	-0.060	-8.46e-09	-0.042	-0.056	-0.13	-0.047
	-0.94	-3.24	-9.09	-1.56e-03	-1.75	-2.34	-13.05	-1.99
c	0.64	1.51	3.53	1.98e-07	6.74	5.80	6.64	11.42
	0.64	1.15	4.63	9.46e-03	2.66	2.13	4.39	3.31
v	1.42	1.44	1.54	0.50	1.47	1.43	1.71	1.36
	46.11	25.29	89.07	21.23	26.25	26.58	45.42	27.71
f	174.4	852.1	1115.	-	0.21	0.64	374.7	-
	1.62	1.38	14.12	-	2.27	0.061	0.34	-

TABLE 8

MAXIMUM LIKELIHOOD RESULTS FOR MODEL 3 AND MODEL 4 FOR THE CANADIAN DATA. LOG LIKELIHOODS ARE GIVEN AT TOP OF COLUMNS AND t-STATISTICS ARE SHOWN BELOW THE PARAMETER ESTIMATES.

$$R_t = \varepsilon_t$$

$$\ln(\sigma_t^2) = \alpha + \ln(f\eta_t) + \frac{(1+\Psi L)}{(1-\Delta_1 L-\Delta_2 L^2)}[\theta Z_{t-1} + \gamma(|Z_{t-1}| - E|Z_{t-1}|)]$$

	AL				B			
η_t	Quotes	Transactions	Volume	- (Model 4)	Quotes	Transactions	Volume	- (Model 4)
	5330	5343	5309	5190	6771	6759	6743	6758
α	-11.83	-19.07	-10.80	-8.23	-24.03	-32.17	-18.40	-8.48
	-34.66	-0.69	-22.67	-52.77	-63.62	-14.31	-0.10	-23.61
γ	0.087	0.065	0.063	0.22	0.26	0.35	0.23	0.73
	5.00	3.41	3.04	4.12	3.61	5.31	3.97	3.83
Δ_1	5.76E-03	5.83E-03	7.36E-03	1.10	1.60	1.42	-0.14	1.31
	0.31	0.34	0.34	3.07	8.42	6.58	-0.88	2.67
Δ_2	0.98	0.98	0.97	-0.11	-0.60	-0.42	0.78	-0.37
	51.47	58.97	43.96	-0.31	-3.14	-1.93	5.66	-0.86
Ψ	1.26	1.38	1.62	-0.62	-0.94	-0.76	1.04	-0.69
	4.68	3.47	2.93	-2.96	-26.11	-8.26	6.87	-1.90
θ	-0.033	-0.025	-0.025	-0.033	-0.046	-0.084	-0.043	-0.035
	-3.27	-2.71	-2.42	-1.28	-1.17	-1.38	-1.35	-0.34
v	1.80	1.89	1.79	1.33	1.06	0.50	1.12	0.50
	28.15	24.52	23.25	32.87	23.45	27.08	22.92	27.41
f	0.25	276.2	0.078	-	4.57e+06	3.45e+04	81.03	-
	3.52	0.04	1.97	-	13.62	0.45	0.01	-

TABLE 8 (cont'd)

MAXIMUM LIKELIHOOD RESULTS FOR MODEL 3 AND MODEL 4 FOR THE CANADIAN DATA. LOG LIKELIHOODS ARE GIVEN AT TOP OF COLUMNS AND t-STATISTICS ARE SHOWN BELOW THE PARAMETER ESTIMATES.

$$R_t = \epsilon_t$$

$$\ln(\sigma_t^2) = \alpha + \ln(f\eta_t) + \frac{(1+\Psi L)}{(1-\Delta_1 L-\Delta_2 L^2)}[\theta Z_{t-1} + \gamma(|Z_{t-1}| - E|Z_{t-1}|)]$$

	BMS				BNS			
η_t	Quotes	Transactions	Volume	- (Model 4)	Quotes	Transactions	Volume	- (Model 4)
	4855	4288	4231	4752	5445	5435	5434	5468
α	-19.36	-19.36	-19.36	-6.54	-9.86	-20.36	-9.41	-6.81
	-27.24	-142.1	-150.2	-19.32	-26.02	-2.23	-54.40	-12.00
γ	0.45	0.45	0.45	0.67	0.029	0.062	0.053	0.21
	2.88	26.59	26.76	4.50	1.43	3.48	2.87	2.09
Δ_1	1.73	1.73	1.73	-0.16	0.049	8.65e-03	2.53e-03	-0.033
	5.74	141.0	81.52	-2.30	1.02	0.46	0.11	-0.86
Δ_2	-0.74	-0.74	-0.74	0.80	0.95	0.98	0.97	0.94
	-2.52	-60.85	-35.46	12.03	20.14	54.50	43.23	24.64
Ψ	-0.89	-0.89	-0.89	0.89	2.72	1.41	1.34	1.36
	-5.10	-299.8	-83.39	8.10	1.17	3.55	2.91	2.74
θ	-0.073	-0.073	-0.073	-6.56e-03	-0.016	-0.039	-0.034	-0.069
	-0.86	-8.31	-8.05	-0.08	-1.44	-3.27	-2.79	-1.33
v	0.50	0.50	0.50	0.50	1.71	1.72	1.65	0.50
	25.35	70.67	70.07	27.24	22.49	22.18	22.64	24.39
f	7.05e+06	100.0	100.0	-	0.16	881.1	0.014	-
	7.37	12.46	13.51	-	3.55	0.11	4.09	-

TABLE 8 (cont'd)

MAXIMUM LIKELIHOOD RESULTS FOR MODEL 3 AND MODEL 4 FOR THE CANADIAN DATA. LOG LIKELIHOODS ARE GIVEN AT TOP OF COLUMNS AND t-STATISTICS ARE SHOWN BELOW THE PARAMETER ESTIMATES.

$$R_t = \varepsilon_t$$

$$\ln(\sigma_t^2) = \alpha + \ln(f\eta_t) + \frac{(1+\Psi L)}{(1-\Delta_1 L-\Delta_2 L^2)}[\theta Z_{t-1} + \gamma(|Z_{t-1}| - E|Z_{t-1}|)]$$

	BVI				BWR			
η_t	Quotes	Transactions	Volume	- (Model 4)	Quotes	Transactions	Volume	- (Model 4)
	5293	5338	5318	5285	3869	3865	3850	3836
α	-11.26	-9.51	-7.23	-6.84	-8.91	-9.80	-8.78	-5.12
	-24.79	-9.65	-9.31	-18.03	-24.00	-23.32	-22.35	-9.11
γ	0.094	0.22	0.23	0.34	0.19	0.052	0.075	0.54
	2.82	1.61	1.65	3.40	4.07	3.09	3.01	3.46
Δ_1	1.78	1.38	1.50	0.036	1.03	1.96	1.90	0.69
	6.71	1.16	1.64	0.27	2.04	69.44	21.48	1.35
Δ_2	-0.78	-0.40	-0.51	0.86	-0.39	-0.96	-0.90	0.28
	-2.95	-0.34	-0.56	7.08	-0.08	-34.02	-10.19	0.56
Ψ	-0.89	-0.64	-0.73	0.54	-0.59	-0.97	-0.94	-0.40
	-7.00	-0.85	-1.49	1.22	-2.48	-42.96	-16.50	-1.04
θ	-0.073	-0.13	-0.092	-0.018	-0.036	-0.063	-0.058	0.051
	-2.92	-1.42	-1.25	-0.31	-1.42	-4.41	-3.35	0.58
v	1.19	0.50	0.50	0.50	1.53	1.53	1.46	0.50
	25.24	26.42	27.39	30.07	21.71	21.05	21.73	28.00
f	0.43	0.36	0.013	-	0.25	0.14	0.014	-
	3.95	1.04	3.02	-	2.84	3.33	5.22	-

TABLE 8 (cont'd)

MAXIMUM LIKELIHOOD RESULTS FOR MODEL 3 AND MODEL 4 FOR THE CANADIAN DATA. LOG LIKELIHOODS ARE GIVEN AT TOP OF COLUMNS AND t-STATISTICS ARE SHOWN BELOW THE PARAMETER ESTIMATES.

$$R_t = \varepsilon_t$$

$$\ln(\sigma_t^2) = \alpha + \ln(f\eta_t) + \frac{(1+\Psi L)}{(1-\Delta_1 L - \Delta_2 L^2)}[\theta Z_{t-1} + \gamma(|Z_{t-1}| - E|Z_{t-1}|)]$$

	IMS				LBT			
η_t	Quotes	Transactions	Volume	- (Model 4)	Quotes	Transactions	Volume	- (Model 4)
	5885	5864	5841	5772	5868	5752	4874	5817
α	-11.09	-18.26	-9.83	-8.69	-14.99	-16.01	-19.88	-7.74
	-38.45	-0.23	-49.75	-63.14	-77.14	-0.52	-46.56	-29.96
γ	0.21	0.17	0.20	0.29	0.086	0.087	0.11	0.49
	3.34	3.25	3.50	3.66	6.06	4.73	32.30	2.85
Δ_1	0.63	0.52	0.36	0.83	-3.74e-03	-3.66e-03	1.83e-03	-0.15
	1.28	1.32	1.30	1.99	-0.69	-0.90	3.11	-0.51
Δ_2	0.36	0.46	0.61	0.14	0.99	0.99	1.00	0.71
	0.75	1.20	2.27	0.35	200.4	259.0	1749.0	2.89
Ψ	-0.30	-0.30	-0.18	-0.49	0.61	0.61	0.77	1.05
	-0.81	-0.87	-0.53	-1.75	4.22	4.24	46.48	4.10
θ	-0.079	-0.093	-0.090	-0.090	-0.016	-0.016	-9.54e-03	5.99e-04
	-2.89	-3.15	-2.96	-2.40	-4.04	-1.71	-5.55	0.01
v	1.27	1.38	1.24	0.93	1.68	1.68	2.49	0.50
	24.96	22.77	23.19	32.24	23.98	22.29	165.33	28.33
f	0.21	192.4	0.036	-	17.62	199.8	4.26e+05	-
	5.21	0.01	3.94	-	8.39	0.03	2.65	-

TABLE 8 (cont'd)

MAXIMUM LIKELIHOOD RESULTS FOR MODEL 3 AND MODEL 4 FOR THE CANADIAN DATA. LOG LIKELIHOODS ARE GIVEN AT TOP OF COLUMNS AND t-STATISTICS ARE SHOWN BELOW THE PARAMETER ESTIMATES.

$$R_t = \varepsilon_t$$

$$\ln(\sigma_t^2) = \alpha + \ln(f\eta_t) + \frac{(1+\Psi L)}{(1-\Delta_1 L-\Delta_2 L^2)}[\theta Z_{t-1} + \gamma(|Z_{t-1}| - E|Z_{t-1}|)]$$

	MB				MCL			
η_t	Quotes	Transactions	Volume	- (Model 4)	Quotes	Transactions	Volume	- (Model 4)
	5528	5452	5518	5469	5861	5809	5788	5733
α	-13.38	-17.15	-8.76	-6.22	-11.99	-13.42	-9.72	-8.75
	-26.13	-1.05	-13.67	-7.49	-27.47	-5.15	-51.64	-52.55
γ	0.11	0.11	0.23	0.59	0.19	0.18	0.26	0.29
	5.94	6.35	2.19	3.43	4.10	3.54	4.40	4.26
Δ_1	-4.01e-03	-4.45e-03	0.022	1.42	0.33	0.47	0.86	1.30
	-0.37	-0.31	0.25	4.34	1.34	1.10	2.53	5.54
Δ_2	1.00	1.00	0.93	-0.43	0.66	0.50	0.13	-0.30
	90.14	69.58	10.67	-1.33	2.67	1.21	0.38	-1.31
Ψ	0.95	0.93	1.45	-0.81	-0.17	-0.21	-0.59	-0.81
	24.17	10.24	2.14	-5.89	-0.57	-0.52	-3.09	-9.04
θ	-0.030	-0.031	-8.30e-03	0.049	-0.049	-0.055	-0.058	-0.047
	-2.95	-3.15	-0.25	0.58	-2.61	-2.33	-2.09	-1.58
v	1.42	1.41	0.50	0.50	1.64	1.41	1.34	1.14
	26.79	27.07	29.35	28.57	21.90	23.26	24.17	26.12
f	8.21	265.5	0.045	-	0.58	1.65	0.016	-
	5.28	0.06	2.36	-	2.54	0.38	4.09	-

TABLE 8 (cont'd)

MAXIMUM LIKELIHOOD RESULTS FOR MODEL 3 AND MODEL 4 FOR THE CANADIAN DATA. LOG LIKELIHOODS ARE GIVEN AT TOP OF COLUMNS AND t-STATISTICS ARE SHOWN BELOW THE PARAMETER ESTIMATES.

$$R_t = \varepsilon_t$$

$$\ln(\sigma_t^2) = \alpha + \ln(f\eta_t) + \frac{(1+\Psi L)}{(1-\Delta_1 L - \Delta_2 L^2)}[\theta Z_{t-1} + \gamma(|Z_{t-1}| - E|Z_{t-1}|)]$$

	MTT				N			
η_t	Quotes	Transactions	Volume	- (Model 4)	Quotes	Transactions	Volume	- (Model 4)
	6476	3192	6442	6428	4955	4959	4924	4798
α	-20.81	-11.91	-8.22	-7.88	-10.01	-19.71	-9.78	-7.71
	-31.21	-15.99	-24.31	-21.35	-28.26	-1.68	-24.97	-52.74
γ	0.80	0.08	0.76	0.74	0.028	0.048	0.087	0.14
	4.95	0.25	4.44	4.29	0.97	1.89	4.18	3.08
Δ_1	-0.38	0.27	0.72	0.70	0.16	0.12	9.28e-03	0.65
	-3.57	0.02	3.26	3.57	1.28	0.94	0.37	0.54
Δ_2	0.61	0.47	0.22	0.25	0.82	0.87	0.98	0.33
	5.77	0.03	1.16	1.41	6.55	6.97	39.42	0.28
Ψ	0.97	0.37	-0.76	-0.80	4.66	2.28	1.28	-0.19
	29.06	0.02	-4.29	-5.90	0.85	1.55	3.56	-0.02
θ	-0.049	5.56e-03	-0.086	-0.10	-0.015	-0.033	-0.045	-0.054
	-0.54	0.03	-0.84	-0.99	-0.93	-1.83	-3.45	-2.44
v	0.50	3.00	0.50	0.50	1.80	1.86	1.68	1.28
	27.62	1.16	27.14	27.38	20.46	19.49	20.66	22.62
f	1.99e+04	59.78	8.60e-03	-	0.21	1.37e+03	0.051	-
	1.59	1.68	1.78	-	5.36	0.09	2.49	-

TABLE 8 (cont'd)

MAXIMUM LIKELIHOOD RESULTS FOR MODEL 3 AND MODEL 4 FOR THE CANADIAN DATA. LOG LIKELIHOODS ARE GIVEN AT TOP OF COLUMNS AND t-STATISTICS ARE SHOWN BELOW THE PARAMETER ESTIMATES.

$$R_t = \varepsilon_t$$

$$\ln(\sigma_t^2) = \alpha + \ln(f\eta_t) + \frac{(1+\Psi L)}{(1-\Delta_1 L-\Delta_2 L^2)}[\theta Z_{t-1} + \gamma(|Z_{t-1}| - E|Z_{t-1}|)]$$

	NOR				NTL			
η_t	Quotes	Transactions	Volume	- (Model 4)	Quotes	Transactions	Volume	- (Model 4)
	5353	5047	4887	5225	5380	5355	5346	5234
α	-22.99	-23.39	-17.43	-8.27	-11.25	-12.26	-10.14	-8.22
	-57.69	-0.35	-72.31	-56.72	-46.28	-12.08	-34.21	-72.33
γ	0.11	0.13	5.45e-04	0.29	0.16	0.18	0.19	0.30
	5.78	18.49	0.03	6.40	3.37	3.81	3.73	4.77
Δ_1	3.09e-03	0.014	1.44	1.62	0.65	0.30	0.52	0.66
	0.35	4.06	28.01	11.12	1.57	1.37	1.59	2.61
Δ_2	0.99	0.99	-0.44	-0.62	0.34	0.65	0.46	0.31
	110.9	288.9	-8.56	-4.38	0.83	3.10	1.43	1.25
Ψ	0.91	0.77	101.1	-0.89	-0.46	-0.15	-0.37	-0.62
	8.02	15.28	0.03	-12.17	-1.62	-0.45	-1.26	-3.84
θ	-0.042	-0.054	-6.06e-05	-5.45e-03	-0.075	-0.044	-0.041	-0.035
	-4.22	-9.95	-0.03	-0.21	-2.70	-1.63	-1.52	-0.98
v	1.72	1.73	1.44	1.24	1.65	1.57	1.56	1.21
	22.43	30.27	26.95	23.42	20.37	20.77	22.42	24.14
f	4.74e+04	133.3	0.040	-	0.20	0.50	0.084	-
	5.15	0.02	3.61	-	5.05	0.97	2.89	-

TABLE 8 (cont'd)

MAXIMUM LIKELIHOOD RESULTS FOR MODEL 3 AND MODEL 4 FOR THE CANADIAN DATA. LOG LIKELIHOODS ARE GIVEN AT TOP OF COLUMNS AND t-STATISTICS ARE SHOWN BELOW THE PARAMETER ESTIMATES.

$$R_t = \epsilon_t$$

$$\ln(\sigma_t^2) = \alpha + \ln(f\eta_t) + \frac{(1+\Psi L)}{(1-\Delta_1 L - \Delta_2 L^2)}[\theta Z_{t-1} + \gamma(|Z_{t-1}| - E|Z_{t-1}|)]$$

	RGO				RY			
η_t	Quotes	Transactions	Volume	- (Model 4)	Quotes	Transactions	Volume	- (Model 4)
	4815	4783	4739	4398	6245	6286	6186	6130
α	-15.00	-15.00	-15.00	-7.49	-12.69	-18.42	-18.14	-9.10
	-48.14	-0.34	-0.36	-107.7	-35.00	-8.75	-2.69	-53.36
γ	0.063	0.063	0.063	0.19	0.061	0.046	0.083	0.21
	0.80	1.09	1.73	6.64	3.39	2.95	3.65	2.83
Δ_1	1.06	1.06	1.06	-0.082	1.97	1.98	1.96	1.13
	0.62	1.10	1.55	-3.93	202.0	225.57	52.17	1.02
Δ_2	-0.053	-0.053	-0.053	0.93	-0.97	-0.98	-0.96	-0.15
	-0.03	-0.05	-0.08	43.85	-99.81	-111.97	-25.72	-0.14
Ψ	-0.83	-0.83	-0.83	1.01	-0.99	-0.99	-0.97	-0.40
	-2.28	-3.18	-4.73	77.76	-242.4	-134.4	-24.75	-0.47
θ	-0.032	-0.032	-0.032	9.77e-04	-0.048	-0.045	-0.053	-0.057
	-0.67	-0.88	-1.34	0.05	-4.67	-4.46	-3.64	-1.98
v	0.50	0.50	0.50	1.24	1.71	1.56	1.34	1.04
	31.88	42.29	47.87	54.08	19.64	19.91	20.93	22.59
f	234.0	100.0	100.0	-	0.36	63.32	89.92	-
	2.79	0.02	0.02	-	3.45	0.45	0.15	-

TABLE 8 (cont'd)

MAXIMUM LIKELIHOOD RESULTS FOR MODEL 3 AND MODEL 4 FOR THE CANADIAN DATA. LOG LIKELIHOODS ARE GIVEN AT TOP OF COLUMNS AND t-STATISTICS ARE SHOWN BELOW THE PARAMETER ESTIMATES.

$$R_t=\varepsilon_t$$

$$\ln(\sigma_t^2)=\alpha+\ln(f\eta_t)+\frac{(1+\Psi L)}{(1-\Delta_1 L-\Delta_2 L^2)}[\theta Z_{t-1}+\gamma(|Z_{t-1}|-E|Z_{t-1}|)]$$

	SPL				TRP			
η_t	Quotes	Transactions	Volume	- (Model 4)	Quotes	Transactions	Volume	- (Model 4)
	5241	5848	4768	5233	5855	5831	5817	5791
α	-15.32	-15.32	-15.32	-6.90	-9.38	-11.75	-9.78	-7.29
	-28.24	-38.09	-45.24	-17.06	-11.90	-0.90	-4.77	-17.75
γ	0.21	0.21	0.21	0.61	0.15	0.14	0.14	0.26
	5.25	12.25	16.63	4.24	2.64	1.65	1.61	1.90
Δ_1	-0.018	-0.018	-0.018	1.54	-4.50e-03	0.055	0.042	0.33
	-1.66	-2.62	-4.50	3.63	-0.79	0.04	0.04	0.58
Δ_2	0.98	0.98	0.98	-0.56	0.99	0.93	0.94	0.63
	84.17	134.87	244.35	-1.48	161.57	0.70	1.01	1.15
Ψ	1.01	1.01	1.01	-0.81	0.86	0.87	0.87	-0.063
	20.08	76.92	101.84	-2.64	4.68	0.42	0.50	-0.08
θ	-0.15	-0.15	-0.15	0.017	-0.019	3.55e-03	3.17e-03	-8.23e-03
	-4.06	-10.96	-15.07	0.19	-0.47	0.10	0.07	-0.13
ν	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50
	36.88	47.99	52.62	28.46	27.23	28.73	26.80	29.32
f	767.3	25.00	25.00	-	0.51	1.06	0.62	-
	2.99	7.24	8.74	-	1.72	0.08	0.48	-

TABLE 8 (cont'd)

MAXIMUM LIKELIHOOD RESULTS FOR MODEL 3 AND MODEL 4 FOR THE CANADIAN DATA. LOG LIKELIHOODS ARE GIVEN AT TOP OF COLUMNS AND t-STATISTICS ARE SHOWN BELOW THE PARAMETER ESTIMATES.

$$R_t = \varepsilon_t$$

$$\ln(\sigma_t^2) = \alpha + \ln(f n_t) + \frac{(1+\Psi L)}{(1-\Delta_1 L-\Delta_2 L^2)}[\theta Z_{t-1} + \gamma(|Z_{t-1}| - E|Z_{t-1}|)]$$

	VO				WN			
n_t	Quotes	Transactions	Volume	- (Model 4)	Quotes	Transactions	Volume	- (Model 4)
	5886	5704	5740	5651	6049	6042	6029	6017
α	-12.67	-18.99	-18.94	-8.65	-12.35	-9.08	-8.11	-7.64
	-24.14	-6.73	-26.00	-93.24	-15.57	-17.53	-18.00	-17.00
γ	0.13	0.13	0.12	0.20	0.23	0.65	0.61	0.61
	3.48	3.92	4.49	3.92	4.44	3.32	3.53	3.49
Δ_1	0.20	1.86	1.96	0.25	0.15	0.45	0.66	0.29
	0.24	30.04	157.3	0.32	1.40	1.28	2.05	1.00
Δ_2	0.78	-0.86	-0.96	0.66	0.81	0.43	0.30	0.62
	0.93	-14.08	-77.03	0.91	8.10	1.44	0.98	2.35
Ψ	0.64	-0.96	-1.01	0.48	0.016	-0.27	-0.43	0.061
	0.60	-30.13	-199.2	0.52	0.06	-0.67	-1.48	0.16
θ	-0.060	-0.074	-0.041	-0.061	7.24e-03	0.039	0.084	7.94e-03
	-3.24	-3.59	-2.69	-2.60	0.30	0.37	0.96	0.10
ν	1.64	1.60	1.48	1.22	1.08	0.50	0.50	0.50
	21.72	20.16	20.92	22.80	23.92	26.65	29.10	28.71
f	0.57	398.1	409.1	-	2.10	0.25	0.011	-
	1.94	0.37	1.43	-	1.21	1.93	1.85	-

TABLE 9

MAXIMUM LIKELIHOOD RESULTS FOR MODEL 3 AND MODEL 4 FOR THE AMERICAN DATA. LOG LIKELIHOODS ARE GIVEN AT TOP OF COLUMNS AND t-STATISTICS ARE SHOWN BELOW THE PARAMETER ESTIMATES.

$$R_t = \varepsilon_t$$

$$\ln(\sigma_t^2) = \alpha + \ln(f\eta_t) + \frac{(1+\Psi L)}{(1-\Delta_1 L - \Delta_2 L^2)}[\theta Z_{t-1} + \gamma(|Z_{t-1}| - E|Z_{t-1}|)]$$

	AA				C			
η_t	Quotes	Transactions	Volume	- (Model 4)	Quotes	Transactions	Volume	- (Model 4)
	5229	4549	4672	5136	4490	4671	4430	4599
α	-10.36	-12.18	-24.72	-8.06	-14.31	-17.73	-19.28	-7.47
	-40.98	-0.26	-25.26	-68.59	-64.58	-0.11	-1.50	-68.25
γ	0.080	0.10	0.19	0.11	0.095	0.086	0.16	0.18
	5.04	5.79	14.86	6.55	6.53	4.12	11.33	5.79
Δ_1	2.38e-03	9.43e-04	0.14	-0.018	-0.013	5.47e-03	0.052	-0.038
	0.47	0.34	7.37	-2.48	-2.09	0.09	3.96	-3.20
Δ_2	0.99	0.99	0.86	0.98	1.00	0.96	0.95	0.96
	191.5	372.1	43.58	131.28	200.4	16.07	71.75	78.08
Ψ	0.64	0.24	0.21	0.72	1.01	0.82	0.81	0.87
	5.23	1.24	3.52	6.63	56.51	2.35	6.16	9.27
θ	0.021	-0.022	-0.021	-0.045	-0.060	-0.023	9.81e-03	-0.027
	-2.48	-1.05	-1.51	-4.19	-5.67	-1.81	1.14	-1.64
v	1.42	1.42	1.73	1.29	1.82	1.87	2.30	1.44
	26.70	20.15	104.7	30.80	38.95	33.79	61.21	24.87
f	0.19	240.6	1.13e+04	-	16.05	75.87	502.7	-
	4.98	0.02	1.04	-	7.61	0.01	0.08	-

TABLE 9 (cont'd)

MAXIMUM LIKELIHOOD RESULTS FOR MODEL 3 AND MODEL 4 FOR THE AMERICAN DATA. LOG LIKELIHOODS ARE GIVEN AT TOP OF COLUMNS AND t-STATISTICS ARE SHOWN BELOW THE PARAMETER ESTIMATES.

$$R_t = \epsilon_t$$

$$\ln(\sigma_t^2) = \alpha + \ln(f\eta_t) + \frac{(1+\Psi L)}{(1-\Delta_1 L-\Delta_2 L^2)}[\theta Z_{t-1} + \gamma(|Z_{t-1}| - E|Z_{t-1}|)]$$

	DD				DEC			
η_t	Quotes	Transactions	Volume	- (Model 4)	Quotes	Transactions	Volume	- (Model 4)
	5372	5460	5406	5399	4933	4925	4557	4880
α	-16.48	-16.04	-22.21	-8.36	-9.57	-19.53	-19.57	-7.88
	-127.2	-0.12	-9.56	-63.34	-36.61	-0.39	-0.81	-92.33
γ	0.030	0.12	6.71e-05	0.16	0.083	0.092	0.11	0.12
	2.89	4.29	0.20	4.00	3.12	3.52	7.36	2.32
Δ_1	1.99	4.85e-03	1.08	1.79	-0.036	2.44e-03	0.049	0.26
	568.4	0.11	4.02	8.69	-2.67	0.04	3.04	0.64
Δ_2	-0.99	0.96	-0.081	-0.80	0.97	0.95	0.95	0.65
	-282.9	22.75	-0.30	-3.93	68.85	17.47	59.01	1.69
Ψ	-1.01	0.81	1.97e+03	-0.90	1.05	0.75	1.12	1.39
	-268.4	2.68	0.19	-7.53	16.24	1.95	5.03	1.64
θ	-0.019	-0.049	-3.03e-06	-0.053	-0.048	-0.014	-1.23e-03	-0.029
	-3.19	-3.31	-0.20	-2.51	-3.07	-1.12	-0.22	-1.80
v	1.45	1.37	1.46	1.32	1.33	1.60	1.95	1.34
	43.69	26.73	29.37	24.97	33.22	54.43	76.60	32.02
f	29.44	8.66	3.69e+03	-	0.027	256.7	110.7	-
	10.39	0.01	0.44	-	3.10	0.02	0.04	-

TABLE 9 (cont'd)

MAXIMUM LIKELIHOOD RESULTS FOR MODEL 3 AND MODEL 4 FOR THE AMERICAN DATA. LOG LIKELIHOODS ARE GIVEN AT TOP OF COLUMNS AND t-STATISTICS ARE SHOWN BELOW THE PARAMETER ESTIMATES.

$$R_t = \varepsilon_t$$

$$\ln(\sigma_t^2) = \alpha + \ln(f\eta_t) + \frac{(1+\Psi L)}{(1-\Delta_1 L - \Delta_2 L^2)}[\theta Z_{t-1} + \gamma(|Z_{t-1}| - E|Z_{t-1}|)]$$

	DIS				DOW			
η_t	Quotes	Transactions	Volume	- (Model 4)	Quotes	Transactions	Volume	- (Model 4)
	5165	5166	5208	5059	5327	5313	5337	5247
α	-14.12	-15.03	-15.84	-8.00	-10.54	-11.63	-11.77	-8.21
	-9.09	-0.49	-5.94	-51.81	-29.15	-5.95	-6.98	-63.08
γ	0.14	0.18	0.19	0.29	0.21	0.23	0.20	0.19
	4.48	4.66	5.94	5.90	5.93	5.25	5.08	5.06
Δ_1	1.92	1.91	1.96	1.43	1.77	1.78	1.79	0.47
	64.94	61.56	104.27	4.77	26.43	16.80	16.68	0.61
Δ_2	-0.92	-0.91	-0.96	-0.44	-0.77	-0.78	-0.79	0.49
	-31.18	-29.36	-50.94	-1.51	-11.57	-7.50	-7.45	0.65
Ψ	-0.99	-0.99	-0.99	-0.71	-0.95	-0.93	-0.92	0.14
	-206.0	-245.8	-228.1	-3.83	-46.68	-17.81	-18.24	0.20
θ	-0.084	-0.11	-0.076	-0.043	-0.14	-0.13	-0.12	-0.054
	-3.72	-3.99	-3.80	-1.96	-5.97	-4.67	-4.95	-2.89
v	1.26	1.26	1.33	1.21	1.48	1.44	1.57	1.31
	44.65	32.47	35.96	33.48	23.69	22.60	23.81	26.12
f	3.62	3.81	122.8	-	0.14	0.10	14.25	-
	0.65	0.03	0.38	-	2.89	0.49	0.57	-

TABLE 9 (cont'd)

MAXIMUM LIKELIHOOD RESULTS FOR MODEL 3 AND MODEL 4 FOR THE AMERICAN DATA. LOG LIKELIHOODS ARE GIVEN AT TOP OF COLUMNS AND t-STATISTICS ARE SHOWN BELOW THE PARAMETER ESTIMATES.

$$R_t = \varepsilon_t$$

$$\ln(\sigma_t^2) = \alpha + \ln(f\eta_t) + \frac{(1+\Psi L)}{(1-\Delta_1 L - \Delta_2 L^2)}[\theta Z_{t-1} + \gamma(|Z_{t-1}| - E|Z_{t-1}|)]$$

	EK				GM			
η_t	Quotes	Transactions	Volume	- (Model 4)	Quotes	Transactions	Volume	- (Model 4)
	5304	5225	5301	5288	5303	5300	5328	5309
α	-17.51	-14.28	-16.05	-8.29	-10.49	-15.90	-19.74	-8.30
	-87.23	-1.09	-3.27	-74.54	-41.67	-0.10	-75.27	-60.74
γ	0.10	0.14	0.11	0.25	0.13	0.15	0.085	0.16
	3.69	2.50	4.66	4.34	3.16	3.47	7.18	4.73
Δ_1	1.89	1.31	1.95	1.13	1.79	1.73	1.96	1.83
	45.78	1.39	156.95	1.51	8.27	8.24	233.4	15.13
Δ_2	-0.89	-0.33	-0.95	-0.16	-0.79	-0.73	-0.96	-0.83
	-21.48	-0.37	-76.57	-0.23	-3.70	-3.55	-114.5	-6.99
Ψ	-1.01	-0.57	-0.99	-0.42	-0.91	-0.88	-1.01	-0.94
	-194.3	-0.90	-328.0	-0.68	-8.95	-8.11	-251.6	-16.25
θ	-9.69e-03	-0.017	-0.044	-0.051	-0.080	-0.14	-0.091	-0.63
	-0.52	-0.93	-2.74	-1.97	-3.09	-3.86	-8.19	-3.63
v	1.46	1.58	1.67	1.17	1.43	1.34	1.74	1.40
	37.52	37.09	53.06	30.51	26.95	25.68	34.95	26.38
f	34.73	0.82	70.74	-	0.091	4.30	959.7	-
	17.65	0.08	0.20	-	3.77	0.01	3.58	-

TABLE 9 (cont'd)

MAXIMUM LIKELIHOOD RESULTS FOR MODEL 3 AND MODEL 4 FOR THE AMERICAN DATA. LOG LIKELIHOODS ARE GIVEN AT TOP OF COLUMNS AND t-STATISTICS ARE SHOWN BELOW THE PARAMETER ESTIMATES.

$$R_t = \varepsilon_t$$

$$\ln(\sigma_t^2) = \alpha + \ln(f\eta_t) + \frac{(1+\Psi L)}{(1-\Delta_1 L-\Delta_2 L^2)}[\theta Z_{t-1} + \gamma(|Z_{t-1}| - E|Z_{t-1}|)]$$

	HAL				IBM			
η_t	Quotes	Transactions	Volume	- (Model 4)	Quotes	Transactions	Volume	- (Model 4)
	4.34	4912	4910	4833	5340	5602	5710	5615
α	-12.26	-17.37	-9.36	-7.81	-17.89	-19.30	-18.66	-8.69
	-110.4	-0.07	-36.98	-73.75	-55.28	-1.64	-9.55	-101.8
γ	1.65	0.20	0.22	0.23	0.031	0.031	0.11	0.14
	98.83	4.94	4.12	4.98	2.91	2.67	2.85	4.07
Δ_1	0.60	0.23	0.95	0.94	1.94	1.94	0.064	0.052
	12.17	1.73	2.45	1.76	97.22	132.85	0.32	0.29
Δ_2	0.23	0.74	0.025	0.025	-0.94	-0.94	0.89	0.87
	5.89	5.69	0.07	0.05	-47.12	-64.46	4.50	5.29
Ψ	0.056	-0.058	-0.51	-0.45	-1.02	-1.02	0.60	0.65
	1.08	-0.25	-1.80	-1.18	-127.5	-152.9	1.08	1.50
θ	-0.15	-0.10	-0.10	-0.054	-0.049	-0.048	-0.066	-0.062
	-10.69	-5.08	-3.10	-2.21	-3.86	-4.35	-2.92	-3.42
ν	3.00	1.39	1.37	1.22	1.34	1.35	1.44	1.36
	79.55	29.25	28.11	26.80	30.49	40.13	34.97	34.16
f	4.89	86.98	1.06	-	16.06	23.79	8.64e+03	-
	27.66	0.00	3.23	-	4.89	0.09	0.51	-

TABLE 9 (cont'd)

MAXIMUM LIKELIHOOD RESULTS FOR MODEL 3 AND MODEL 4 FOR THE AMERICAN DATA. LOG LIKELIHOODS ARE GIVEN AT TOP OF COLUMNS AND t-STATISTICS ARE SHOWN BELOW THE PARAMETER ESTIMATES.

$$R_t = e_t$$

$$\ln(\sigma_t^2) = \alpha + \ln(f\eta_t) + \frac{(1+\Psi L)}{(1-\Delta_1 L-\Delta_2 L^2)}[\theta Z_{t-1} + \gamma(|Z_{t-1}| - E|Z_{t-1}|)]$$

	IBC				MCD			
η_t	Quotes	Transactions	Volume	- (Model 4)	Quotes	Transactions	Volume	- (Model 4)
	4353	4329	4325	4312	5412	5366	5371	5287
α	-9.49	-9.49	-9.49	4.93	-19.32	-20.38	-11.60	-8.34
	-14.90	-13.98	-15.97	-7.62	-1.36	-1.49	-7.59	-65.00
γ	0.36	0.36	0.36	0.33	0.10	0.12	0.15	0.24
	2.63	2.94	3.00	2.43	3.23	3.74	3.95	4.92
Δ_1	0.26	0.26	0.26	0.22	0.068	0.094	0.063	1.23
	0.89	0.95	0.97	0.54	0.53	0.75	0.49	3.27
Δ_2	0.76	0.76	0.76	0.74	0.87	0.90	0.88	-0.24
	2.59	2.77	2.83	1.86	7.07	7.19	7.15	-0.65
Ψ	0.076	0.076	0.076	0.21	0.50	0.42	0.54	-0.67
	0.15	0.16	0.16	0.31	1.23	1.02	1.45	-3.40
θ	-0.098	-0.98	-0.098	-0.057	-0.075	-0.041	-0.061	-0.36
	-1.60	-1.67	-1.71	-0.86	-4.34	-3.10	-3.42	-1.66
v	0.50	0.50	0.50	0.50	1.74	1.69	1.61	1.46
	27.90	28.47	30.74	26.82	24.67	28.13	27.57	24.66
f	0.35	0.10	4.00	-	973.2	962.9	9.59	-
	2.48	1.60	2.69	-	0.07	0.07	0.62	-

TABLE 9 (cont'd)

MAXIMUM LIKELIHOOD RESULTS FOR MODEL 3 AND MODEL 4 FOR THE AMERICAN DATA. LOG LIKELIHOODS ARE GIVEN AT TOP OF COLUMNS AND t-STATISTICS ARE SHOWN BELOW THE PARAMETER ESTIMATES.

$$R_t = \varepsilon_t$$

$$\ln(\sigma_t^2) = \alpha + \ln(f\eta_p) + \frac{(1+\Psi L)}{(1-\Delta_1 L-\Delta_2 L^2)}[\theta Z_{t-1} + \gamma(|Z_{t-1}| - E|Z_{t-1}|)]$$

	MER				NSC			
η_p	Quotes	Transactions	Volume	- (Model 4)	Quotes	Transactions	Volume	- (Model 4)
	4886	4785	4697	4697	5557	5529	5528	5474
α	-17.95	-17.72	-20.33	-7.56	-11.11	-10.82	-10.29	-8.46
	-0.40	-0.09	-0.94	-47.88	-18.35	-19.35	-26.02	-47.67
γ	0.16	0.10	0.15	0.28	0.30	0.31	0.27	0.38
	3.75	2.82	6.91	5.87	5.37	5.79	5.48	6.82
Δ_1	0.19	0.16	0.28	1.42	0.43	0.39	0.38	0.96
	1.95	1.66	1.63	6.93	3.10	2.85	2.43	4.47
Δ_2	0.79	0.82	0.72	-0.43	0.54	0.58	0.59	0.022
	8.10	8.61	4.20	-2.11	4.04	4.35	3.86	0.10
Ψ	-0.16	-0.11	0.16	-0.83	-0.43	-0.36	-0.32	-0.68
	-1.21	-0.69	0.63	-9.70	-3.10	-2.48	-1.92	-6.64
θ	-0.10	-0.099	-0.10	-0.071	-0.13	-0.13	-0.12	-0.079
	-4.02	-3.85	-6.92	-2.64	-3.70	-3.78	-3.65	-2.29
v	1.40	1.36	1.54	1.26	1.30	1.26	1.27	1.17
	26.93	31.04	33.00	26.28	23.69	23.78	23.76	24.36
f	516.7	101.6	614.6	-	0.34	0.11	3.67	-
	0.02	0.00	0.05	-	1.48	1.57	2.13	-

TABLE 9 (cont'd)

MAXIMUM LIKELIHOOD RESULTS FOR MODEL 3 AND MODEL 4 FOR THE AMERICAN DATA. LOG LIKELIHOODS ARE GIVEN AT TOP OF COLUMNS AND t-STATISTICS ARE SHOWN BELOW THE PARAMETER ESTIMATES.

$$R_t = e_t$$

$$\ln(\sigma_t^2) = \alpha + \ln(f\eta_t) + \frac{(1+\Psi L)}{(1-\Delta_1 L-\Delta_2 L^2)}[\theta Z_{t-1} + \gamma(|Z_{t-1}| - E|Z_{t-1}|)]$$

	OXY				RLM			
η_t	Quotes	Transactions	Volume	- (Model 4)	Quotes	Transactions	Volume	- (Model 4)
	4400	5194	5252	5117	5001	5000	4987	4907
α	-10.33	-18.94	-11.04	-7.85	-9.90	-17.15	-9.27	-7.85
	-165.02	-0.11	-13.66	-80.68	-39.43	-0.15	-39.39	-39.89
γ	1.45	0.54	0.36	0.39	0.15	0.17	0.23	0.29
	91.37	15.70	6.18	5.49	3.29	3.82	4.82	5.33
Δ_1	1.63	1.46	0.99	-0.26	0.27	0.27	1.68	1.52
	118.1	20.40	4.70	-4.13	1.35	1.63	11.13	9.06
Δ_2	-0.69	-0.46	-1.96e-03	0.75	0.72	0.72	-0.68	-0.52
	-63.67	-6.45	-0.01	12.28	3.64	4.38	-4.55	-3.14
Ψ	-0.79	-0.81	-0.75	0.98	-0.18	-0.18	-0.89	-0.88
	-49.18	-23.60	-8.27	19.81	-0.57	-0.72	-14.68	-15.56
θ	-0.17	-0.094	-0.16	-0.18	-0.072	-0.091	-0.081	-0.027
	-20.83	-3.84	-3.95	-4.32	-2.65	-3.24	-2.65	-0.98
v	3.00	1.23	1.32	0.85	1.42	1.38	1.50	1.36
	104.5	36.30	28.14	26.93	33.45	35.97	30.45	25.95
f	0.36	52.07	6.30	-	0.20	136.3	0.80	-
	20.81	0.01	1.16	-	5.76	0.01	3.70	-

TABLE 9 (cont'd)

MAXIMUM LIKELIHOOD RESULTS FOR MODEL 3 AND MODEL 4 FOR THE AMERICAN DATA. LOG LIKELIHOODS ARE GIVEN AT TOP OF COLUMNS AND t-STATISTICS ARE SHOWN BELOW THE PARAMETER ESTIMATES.

$$R_t = \varepsilon_t$$

$$\ln(\sigma_t^2) = \alpha + \ln(f\eta_t) + \frac{(1+\Psi L)}{(1-\Delta_1 L - \Delta_2 L^2)}[\theta Z_{t-1} + \gamma(|Z_{t-1}| - E|Z_{t-1}|)]$$

	TDY				TGT			
η_t	Quotes	Transactions	Volume	- (Model 4)	Quotes	Transactions	Volume	- (Model 4)
	5194	5626	5624	5405	5529	5506	5539	5437
α	-19.81	-18.95	-17.54	-8.35	-10.57	-15.86	-13.25	-8.42
	-53.67	-1.70	-11.03	-28.29	-59.29	-0.14	-14.00	-74.35
γ	0.15	0.28	0.31	0.40	0.18	0.23	0.11	0.13
	8.89	8.57	7.86	6.64	3.28	4.03	6.70	3.20
Δ_1	1.80	1.78	1.76	0.88	1.64	1.72	1.97	0.090
	33.22	24.29	21.85	5.78	7.69	11.89	389.6	0.44
Δ_2	-0.81	-0.78	-0.76	0.11	-0.64	-0.72	-0.97	0.85
	-14.88	-10.70	-9.45	0.75	-3.07	-5.11	-191.5	4.26
Ψ	-0.94	-0.94	-0.93	-0.62	-0.88	-0.91	-1.10	1.29
	-44.60	-28.85	-29.82	-6.49	-10.79	-13.87	-255.9	2.74
θ	-0.12	-0.066	-0.049	-0.066	-0.10	-0.12	-0.071	-0.053
	-8.60	-3.46	-2.40	-2.16	-2.88	-3.29	-3.77	-2.34
v	1.17	1.32	1.36	1.05	1.32	1.28	1.46	1.24
	38.97	38.39	33.41	39.34	34.42	29.62	30.56	30.52
f	1.04e+03	183.1	1.55e+03	-	0.14	8.43	8.85	-
	4.44	0.09	0.69	-	5.28	0.01	0.94	-

TABLE 9 (cont'd)

MAXIMUM LIKELIHOOD RESULTS FOR MODEL 3 AND MODEL 4 FOR THE AMERICAN DATA. LOG LIKELIHOODS ARE GIVEN AT TOP OF COLUMNS AND t-STATISTICS ARE SHOWN BELOW THE PARAMETER ESTIMATES.

$$R_t = \epsilon_t$$

$$\ln(\sigma_t^2) = \alpha + \ln(f\eta_t) + \frac{(1+\Psi L)}{(1-\Delta_1 L-\Delta_2 L^2)}[\theta Z_{t-1} + \gamma(|Z_{t-1}| - E|Z_{t-1}|)]$$

	VAT				XON			
η_t	Quotes	Transactions	Volume	- (Model 4)	Quotes	Transactions	Volume	- (Model 4)
	3002	2937	3012	2885	5644	5597	5652	5566
α	-14.44	-14.44	-14.44	-5.95	-11.04	-10.44	-14.94	-8.56
	-44.63	-9.97	-34.98	-19.95	-27.20	-14.91	-0.93	-101.2
γ	0.53	0.53	0.53	0.42	0.20	0.23	0.19	0.22
	15.83	13.12	13.51	3.80	5.97	5.99	6.40	5.48
Δ_1	0.30	0.30	0.30	0.053	0.029	0.068	0.017	0.25
	14.20	12.87	12.33	0.20	0.88	1.15	0.77	0.56
Δ_2	0.70	0.70	0.70	0.61	0.89	0.87	0.92	0.64
	33.77	30.58	29.26	2.56	33.66	16.38	52.97	1.52
Ψ	-0.47	-0.47	-0.47	1.15	0.37	0.38	0.46	0.41
	-11.99	-11.10	-10.70	3.57	2.29	1.82	3.75	0.70
θ	0.036	0.036	0.036	-0.077	-0.12	-0.10	-0.12	-0.099
	1.15	1.06	1.18	-1.18	-5.37	-4.63	-6.02	-4.11
v	0.50	0.50	0.50	0.50	1.51	1.42	1.55	1.36
	51.07	48.25	48.54	26.08	22.68	24.34	23.04	25.25
f	1.99	0.11	3.67	-	0.22	0.015	672.5	-
	2.29	0.59	2.38	-	2.26	1.21	0.06	-

TABLE 10

NUMBER OF SIGNIFICANT EARCH PARAMETERS Ψ, Δ_i, θ, AND γ IN MODEL 1. THE NUMBER OF THE EARCH PARAMETERS THAT ARE SIGNIFICANT IN MODEL 1 IS GIVEN FOR THE VARIOUS CASES WHERE THE TRADING ACTIVITY VARIABLE IS SIGNIFICANT, NOT SIGNIFICANT, OR NOT PRESENT (MODEL 2). THE LEVEL OF SIGNIFICANCE IS 0.05

Canadian Data

	Number of Significant EARCH Parameters	Quotes	Transactions	Volume	Total	Trading Activity Variable Not Present Model 2
Trading Activity Variable Significant	0	0	0	0	0	
	1	1	0	1	2	
	2	1	2	2	5	
	3	4	0	2	6	
	4	6	4	7	17	
	5	5	1	4	10	
Trading Activity Variable Not Significant	0	0	0	0	0	0
	1	0	1	0	1	0
	2	0	2	0	2	1
	3	0	1	0	1	1
	4	2	6	2	10	11
	5	1	3	2	6	7

American Data

	Number of Significant EARCH Parameters	Quotes	Transactions	Volume	Total	Trading Activity Variable Not Present Model 2
Trading Activity Variable Significant	0	0	0	0	0	
	1	0	0	0	0	
	2	0	0	0	0	
	3	0	0	0	0	
	4	0	0	0	0	
	5	8	1	4	13	
Trading Activity Variable Not Significant	0	0	0	1	1	0
	1	0	0	0	0	0
	2	0	0	0	0	0
	3	0	0	1	1	4
	4	0	0	1	1	5
	5	12	19	13	44	11

TABLE 11

NUMBER OF SIGNIFICANT EARCH PARAMETERS Ψ, Δ_ψ, θ, AND γ IN MODEL 3. THE NUMBER OF EARCH PARAMETERS THAT ARE SIGNIFICANT IN MODEL 3 IS GIVEN FOR THE VARIOUS CASES WHERE THE TRADING ACTIVITY VARIABLE IS SIGNIFICANT, NOT SIGNIFICANT, OR NOT PRESENT (MODEL 4). THE LEVEL OF SIGNIFICANCE IS 0.05

Canadian Data

	Number of Significant EARCH Parameters	Quotes	Transactions	Volume	Total	Trading Activity Variable Not Present Model 4
Trading Activity Variable Significant	0	0	1	0	1	
	1	3	1	2	6	
	2	3	0	2	5	
	3	3	0	3	6	
	4	6	0	4	10	
	5	3	3	4	10	
Trading Activity Variable Not Significant	0	0	1	1	2	0
	1	0	2	0	2	1
	2	1	3	1	5	6
	3	0	1	1	2	8
	4	1	5	0	6	5
	5	0	3	2	5	0

American Data

	Number of Significant EARCH Parameters	Quotes	Transactions	Volume	Total	Trading Activity Variable Not Present Model 4
Trading Activity Variable Significant	0	0	0	0	0	
	1	0	0	0	0	
	2	1	0	0	1	
	3	0	0	1	1	
	4	5	0	3	8	
	5	10	0	2	12	
Trading Activity Variable Not Significant	0	0	0	0	0	0
	1	0	1	1	2	0
	2	0	1	0	1	5
	3	1	4	3	8	3
	4	2	5	5	12	7
	5	1	9	5	15	5

TABLE 12

GARCH LOG-LIKELIHOOD RESULTS AND STATISTICS FOR SIGNIFICANCE OF COEFFICIENT OF TRADING ACTIVITY VARIABLES AND MODEL SELECTION FOR THE CANADIAN DATA. THE SIC VALUE IS LISTED FOR MODEL SELECTION PURPOSES. THE ROWS LABELLED SUM α'S GIVE THE SUM OF THE α_1 AND α_2 COEFFICIENTS IN THE GARCH MODELS.

		AL	B	BMS	BNS	BVI
Model 5		GARCH MODELS				
Quotes	Ln Li	5198.77	6581.79	4626.23	5343.26	5157.67
	f t-stat	20.17	1.38	20.92	1.98	24.20
	sum α's	0.09	0.23	0.09	0.77	0.23
	SIC	-10367.34	-13133.37	-9222.50	-10656.30	-10285.14
Trans	Ln Li	5246.64	6632.36	4479.21	5344.29	5140.98
	f t-stat	15.79	1.37	12.93	2.76	19.90
	sum α's	0.07	0.18	0.16	0.82	0.[illegible]6
	SIC	-10463.06	-13234.55	-8920.46	-10658.37	-10251.76
Volume	Ln Li	5237.53	6688.61	4430.81	5363.18	5113.91
	f t-stat	21.51	1.32	1.07	8.67	18.43
	sum α's	0.05	0.24	0.90	0.08	0.24
	SIC	-10444.84	-13347.01	-8831.66	-10696.15	-10197.61
Model 6						
	Ln Li	5123.84	6568.86	4430.62	5341.64	4973.11
	sum α's	0.98	0.89	0.91	0.89	0.81
	SIC	-10225.03	-13115.05	-8838.78	-10660.62	-9923.57

TABLE 12 (cont'd)

GARCH LOG-LIKELIHOOD RESULTS AND STATISTICS FOR SIGNIFICANCE OF COEFFICIENT OF TRADING ACTIVITY VARIABLES AND MODEL SELECTION FOR THE CANADIAN DATA. THE SIC VALUE IS LISTED FOR MODEL SELECTION PURPOSES. THE ROWS LABELLED SUM α'S GIVE THE SUM OF THE α_1 AND α_2 COEFFICIENTS IN THE GARCH MODELS.

		BWR	IMS	LBT	MB	MCL
Model 5				GARCH MODELS		
Quotes	Ln L_1	3750.28	5748.69	5795.51	5278.61	5726.16
	f t-stat	16.67	24.43	19.56	2.57	21.49
	sum α's	0.17	0.23	0.09	0.94	0.15
	SIC	-7470.35	-11467.16	-11560.80	-10527.02	-11422.11
Trans	Ln L_1	3672.75	5781.99	5721.15	5281.88	5704.79
	f t-stat	11.47	18.65	1.61	4.39	26.73
	sum α's	0.25	0.11	0.12	0.93	0.13
	SIC	-7315.28	-11533.76	-11412.09	-10533.55	-11379.37
Volume	Ln L_1	3710.39	5724.24	5679.79	5329.55	5665.17
	f t-stat	14.89	15.70	17.46	13.97	9.87
	sum α's	0.36	0.16	0.40	0.53	0.76
	SIC	-7390.56	-11418.28	-11329.37	-10628.88	-11300.14
Model 6						
	Ln L_1	3419.24	5606.12	5640.17	5277.18	5648.29
	sum α's	0.97	0.95	0.79	0.95	0.97
	SIC	-6815.81	-11189.58	-11257.68	-10531.70	-11273.92

TABLE 12 (cont'd)

GARCH LOG-LIKELIHOOD RESULTS AND STATISTICS FOR SIGNIFICANCE OF COEFFICIENT OF TRADING ACTIVITY VARIABLES AND MODEL SELECTION FOR THE CANADIAN DATA. THE SIC VALUE IS LISTED FOR MODEL SELECTION PURPOSES. THE ROWS LABELLED SUM α'S, GIVE THE SUM OF THE α_1 AND α_2 COEFFICIENTS IN THE GARCH MODELS.

		MTT	N	NOR	NTL	RGO
Model 5		GARCH MODELS				
Quotes	Ln Li	6054.15	4737.23	5173.12	5293.03	4253.67
	f t-stat	16.45	0.11	2.36	1.57	-5.49
	sum α's	0.22	0.98	0.75	0.14	0.99
	SIC	-12078.09	-9444.25	-10316.03	-10555.84	-8477.12
Trans	Ln Li	5986.83	4764.24	5182.84	5300.12	4399.95
	f t-stat	2.98	10.50	6.58	1.72	25.99
	sum α's	0.73	0.22	0.42	0.09	0.05
	SIC	-11943.46	-9498.26	-10335.47	-10570.03	-8769.68
Volume	Ln Li	5992.98	4762.31	5223.28	5271.18	4393.91
	f t-stat	5.08	11.66	9.99	1.46	23.03
	sum α's	0.69	0.17	0.12	0.09	0.11
	SIC	-11955.76	-9494.41	-10416.35	-10512.14	-8757.61
Model 6						
	Ln Li	5948.52	4737.23	5171.91	5153.10	3722.49
	sum α's	0.80	0.97	0.85	0.96	1.00
	SIC	-11874.39	-9451.80	-10321.15	-10283.54	-7422.32

TABLE 12 (cont'd)

GARCH LOG-LIKELIHOOD RESULTS AND STATISTICS FOR SIGNIFICANCE OF COEFFICIENT OF TRADING ACTIVITY VARIABLES AND MODEL SELECTION FOR THE CANADIAN DATA. THE SIC VALUE IS LISTED FOR MODEL SELECTION PURPOSES. THE ROWS LABELLED SUM α'S, GIVE THE SUM OF THE α_1 AND α_2 COEFFICIENTS IN THE GARCH MODELS.

		RY	SPL	TRP	VO	WN
Model 5				GARCH MODELS		
Quotes	Ln Li	6079.67	4870.71	5545.91	5632.17	5927.61
	f t-stat	7.27	15.31	19.27	14.29	27.86
	sum α's	0.13	0.54	0.15	0.15	0.12
	SIC	-12129.13	-9711.41	-11061.60	-11234.12	-11825.04
Trans	Ln Li	6090.90	4837.90	5557.61	5675.37	5913.53
	f t-stat	7.83	10.52	16.07	16.82	18.25
	sum α's	0.09	0.52	0.12	0.14	0.48
	SIC	-12151.59	-9645.78	-11085.01	[illegible]	-11796.87
Volume	Ln Li	6087.96	4816.83	5577.18	5643.71	5818.12
	f t-stat	7.30	5.34	20.58	16.25	4.53
	sum α's	0.20	0.62	0.12	0.15	0.94
	SIC	-12145.70	-9603.65	-11124.14	-11257.20	-11606.06
Model 6						
	Ln Li	6078.14	4808.47	5458.75	5579.84	5810.17
	sum α's	0.96	0.62	0.75	0.94	0.98
	SIC	-12133.62	-9594.43	-10894.85	-11137.02	-11597.71

TABLE 13

GARCH LOG-LIKELIHOOD RESULTS AND STATISTICS FOR SIGNIFICANCE OF COEFFICIENT OF TRADING ACTIVITY VARIABLES AND MODEL SELECTION FOR THE AMERICAN DATA. THE SIC VALUE IS LISTED FOR MODEL SELECTION PURPOSES. THE ROWS LABELLED SUM α'S GIVE THE SUM OF THE α_1 AND α_2 COEFFICIENTS IN THE GARCH MODELS.

		AA	C	DD	DEC	DIS
Model 5			GARCH MODELS			
Quotes	Ln Li	5090.33	4597.55	5398.66	4852.94	4999.89
	f t-stat	10.01	13.28	18.38	9.06	14.99
	sum α's	0.59	0.39	0.24	0.57	0.42
	SIC	-10150.44	-9164.88	-10767.10	-9675.66	-9969.56
Trans	Ln Li	5059.80	4613.48	5369.30	4891.01	4996.83
	f t-stat	3.68	11.37	13.98	13.14	22.95
	sum α's	0.96	0.39	0.21	0.4	0.29
	SIC	-10089.38	-9196.75	-10708.38	-9751.80	-9963.44
Volume	Ln Li	5069.85	4565.39	5392.10	4897.48	5090.05
	f t-stat	12.08	5.78	12.75	13.96	11.37
	sum α's	0.34	0.92	0.33	0.38	0.65
	SIC	-10109.48	-9100.56	-10753.98	-9764.74	-10149.88
Model 6						
	Ln Li	5057.33	4557.42	5329.22	4799.36	4954.12
	sum α's	0.98	0.98	0.97	0.84	0.98
	SIC	-10091.99	-9092.18	-10635.77	-9576.05	-9885.57

TABLE 13 (cont'd)

GARCH LOG-LIKELIHOOD RESULTS AND STATISTICS FOR SIGNIFICANCE OF COEFFICIENT OF TRADING ACTIVITY VARIABLES AND MODEL SELECTION FOR THE AMERICAN DATA. THE SIC VALUE IS LISTED FOR MODEL SELECTION PURPOSES. THE ROWS LABELLED SUM α'S GIVE THE SUM OF THE α_1 AND α_2 COEFFICIENTS IN THE GARCH MODELS.

		DOW	EK	GM	HAL	IBM
Model 5		GARCH MODELS				
Quotes	Ln Li	5232.27	5234.83	5329.62	4891.18	5581.76
	f t-stat	16.74	8.58	10.16	21.04	23.65
	sum α's	0.23	0.66	0.33	0.25	0.42
	SIC	-10434.32	-10439.44	-10629.02	-9752.14	-11133.30
Trans	Ln Li	5210.7	5214.74	5362.09	4821.52	5596.67
	f t-stat	13.43	15.13	12.32	20.8	10.08
	sum α's	0.34	0.5	0.18	0.26	0.5
	SIC	-10391.18	-10399.26	-10693.96	-9612.82	-11163.12
Volume	Ln Li	5221.74	5275.51	5344.59	4819.23	5618.15
	f t-stat	14.74	18.37	9.99	17.12	10.91
	sum α's	0.24	0.32	0.33	0.37	0.5
	SIC	-10413.26	-10520.80	-10658.96	-9608.24	-11206.08
Model 6						
	Ln Li	5187.54	5152.88	5256.92	4755.79	5534.59
	sum α's	0.98	0.84	0.91	0.88	0.89
	SIC	-10352.42	-10283.10	-10491.18	-9488.92	-11046.52

TABLE 13 (cont'd)

GARCH LOG-LIKELIHOOD RESULTS AND STATISTICS FOR SIGNIFICANCE OF COEFFICIENT OF TRADING ACTIVITY VARIABLES AND MODEL SELECTION FOR THE AMERICAN DATA. THE SIC VALUE IS LISTED FOR MODEL SELECTION PURPOSES. THE ROWS LABELLED SUM α'S GIVE THE SUM OF THE α_1 AND α_2 COEFFICIENTS IN THE GARCH MODELS.

		IRC	MCD	MER	NSC	OXY
GARCH MODELS						
Model 5						
Quotes	Ln Li	3999.01	5346.16	4718.85	5425.57	5232.96
	f t-stat	13.76	14.1	18.78	8.12	32.87
	sum α's	0.13	0.16	0.31	0.79	0.23
	SIC	-7967.80	-10662.11	-9407.48	-10820.92	-10435.70
Trans	Ln Li	3930.9	5259.19	4708.19	5404.36	4996.19
	f t-stat	9.74	4.99	13.88	11.53	7.76
	sum α's	0.36	0.81	0.28	0.68	0.76
	SIC	-7831.58	-10488.17	-9386.16	-10778.50	-9962.16
Volume	Ln Li	3944.89	5300.37	4706.4	5395.43	5105.4
	f t-stat	10.78	10.58	14.28	10.54	19.28
	sum α's	0.35	0.61	0.26	0.72	0.63
	SIC	-7859.56	-10570.53	-9382.58	-10760.64	-10180.58
Model 6						
	Ln Li	3913.69	5252.37	4583.47	5365.67	4982.17
	sum α's	0.96	0.97	0.61	0.93	0.82
	SIC	-7804.72	-10482.08	-9144.27	-10708.67	-9941.68

TABLE 13 (cont'd)

GARCH LOG-LIKELIHOOD RESULTS AND STATISTICS FOR SIGNIFICANCE OF COEFFICIENT OF TRADING ACTIVITY VARIABLES AND MODEL SELECTION FOR THE AMERICAN DATA. THE SIC VALUE IS LISTED FOR MODEL SELECTION PURPOSES. THE ROWS LABELLED SUM α'S GIVE THE SUM OF THE α_1 AND α_2 COEFFICIENTS IN THE GARCH MODELS.

		BLM	TDY	TGT	VAT	XON
GARCH MODELS						
Model 5						
Quotes	Ln Li	4887.17	5281.71	5439.61	2162.73	5597.58
	f t-stat	14.56	18.4	18.57	6.89	25.57
	sum α's	0.24	0.94	0.47	0.98	0.11
	SIC	-9744.12	-10533.20	-10849.00	-4297.18	-11164.94
Trans	Ln Li	4848.78	5360.63	5400.41	2196.4	5518.75
	f t-stat	0.37	27.57	14.01	0.97	2.54
	sum α's	0.99	0.71	0.49	0.98	0.87
	SIC	-9667.34	-10691.04	-10770.60	-4364.52	-11007.28
Volume	Ln Li	4885.04	5377.08	5441.17	2132.92	5545.24
	f t-stat	13.79	26.58	13.64	5.55	6.85
	sum α's	0.34	0.92	0.49	0.99	0.83
	SIC	-9739.86	-10723.94	-10852.12	-4237.56	-11060.26
Model 6						
	Ln Li	4848.74	5256.59	5339.9	2104.03	5516.84
	sum α's	0.99	0.99	0.94	1	0.9
	SIC	-9674.82	-10490.52	-10657.14	-4186.85	-11011.02

TABLE 14

EARCH LOG-LIKELIHOOD RESULTS AND STATISTICS FOR SIGNIFICANCE OF COEFFICIENT OF TRADING ACTIVITY VARIABLES AND MODEL SELECTION FOR THE CANADIAN DATA. THE SIC VALUE IS LISTED FOR MODEL SELECTION PURPOSES.

		AL	B	BMS	BNS	BVI
		EARCH MODELS				
Model 1						
Quotes	Ln Li	5342.19	6768.02	4710.51	5446.37	5298.05
	f t-stat	2.45	1.53	2.15	3.64	4.95
	SIC	-10583.76	-13445.40	-9331.16	-10802.11	-10505.46
Trans	Ln Li	5360.02	6736.29	4642.04	5437.07	5245.44
	f t-stat	0.01	1.88	3.16	0.03	2.74
	SIC	-10809.41	-13381.95	-91-4.22	-10783.50	-10400.27
Volume	Ln Li	5320.96	6749.03	4606.96	5367.57	5218.87
	f t-stat	1.12	1.21	3.53	0.04	4.37
	SIC	-10551.28	-13407.42	-9124.05	-10644.50	-10347.12
Model 2						
	Ln Li	5203.44	6586.75	4591.00	5373.27	5139.91
	SIC	-10323.80	-13114.43	-9099.63	-10663.44	-10196.75
Model 3						
Quotes	Ln Li	5329.70	6770.68	4854.89	5444.51	5292.95
	f t-stat	3.52	13.62	7.37	3.55	3.95
	SIC	-10598.96	-13480.94	-9649.87	-10828.60	-10525.49
Trans	Ln Li	5343.50	6759.39	4269.43	5434.86	5338.29
	f t-stat	0.04	0.45	12.46	0.11	1.04
	SIC	-10826.57	-13458.36	-8516.96	-10809.30	-10616.17
Volume	Ln Li	5309.46	6742.56	4231.43	5433.92	5317.60
	f t-stat	1.97	0.01	13.51	4.09	3.02
	SIC	-10558.53	-13424.70	-8402.94	10807.41	-10574.79
Model 4						
	Ln Li	5190.05	6757.62	4751.76	5467.59	5284.79
	SIC	-10327.24	-13462.37	-9451.10	-10882.31	-10516.73

TABLE 14 (cont'd)

EARCH LOG-LIKELIHOOD RESULTS AND STATISTICS FOR SIGNIFICANCE OF COEFFICIENT OF TRADING ACTIVITY VARIABLES AND MODEL SELECTION FOR THE CANADIAN DATA. THE SIC CRITERIA IS LISTED FOR MODEL SELECTION PURPOSES.

		BWR	IMS	LBT	MB	MCL
Model 1		EARCH MODELS				
Quotes	Ln Li	3900.95	5738.27	5893.97	5544.88	5863.78
	f t-stat	3.75	4.10	1.94	2.20	2.16
	SIC	-7711.27	-11385.90	-11697.29	-10999.13	-11636.92
Trans	Ln Li	3892.19	5860.62	5806.25	5466.2120	5808.14
	f t-stat	3.44	0.01	1.44	0.05	0.01
	SIC	-7693.74	-11630.60	-11521.87	-10841.79	-11525.65
Volume	Ln Li	3883.54	5841.66	5788.81	5462.65	5793.32
	f t-stat	5.87	3.72	3.55	3.74	3.84
	SIC	-7676.45	-11592.69	-11486.98	-10834.67	-11496.00
Model 2						
	Ln Li	3657.13	5772.85	5766.28	5342.63	5733.85
	SIC	-7231.19	-11462.62	-11449.47	-10602.17	-11384.61
Model 3						
Quotes	Ln Li	3869.35	5885.24	5868.34	5527.55	5861.02
	f t-stat	2.84	5.21	8.39	5.28	2.54
	SIC	-7678.28	-11710.05	-11676.25	-10994.68	-11661.61
Trans	Ln Li	3865.00	5863.85	5752.01	5451.80	5809.40
	f t-stat	3.33	0.01	0.03	0.06	0.38
	SIC	-7669.57	-11667.27	-11443.60	-10843.19	-11558.37
Volume	Ln Li	3849.54	5841.31	4874.12	5517.59	5787.62
	f t-stat	5.22	3.94	2.65	2.36	4.09
	SIC	-7638.65	-11622.20	-9687.82	-10974.76	-11514.82
Model 4						
	Ln Li	3836.23	5771.69	5817.05	5468.90	5733.10
	SIC	-7619.59	-11490.50	-11581.22	-10884.93	-11413.32

TABLE 14 (cont'd)

EARCH LOG-LIKELIHOOD RESULTS AND STATISTICS FOR SIGNIFICANCE OF COEFFICIENT OF TRADING ACTIVITY VARIABLES AND MODEL SELECTION FOR THE CANADIAN DATA. THE SIC CRITERIA IS LISTED FOR MODEL SELECTION PURPOSES.

		MTT	N	NOR	NTL	BGO
Model 1			EARCH MODELS			
Quotes	Ln Li	6155.20	4964.83	5359.77	5381.77	4516.69
	f t-stat	1.85	5.06	0.07	4.07	1.79
	SIC	-12219.78	-9839.04	-10628.91	-10672.89	-8942.75
Trans	Ln Li	6119.63	4968.55	5328.57	5354.82	4539.47
	f t-stat	2.56	0.01	0.33	0.02	0.85
	SIC	-12148.64	-9846.46	-10566.50	-10619.00	-8988.30
Volume	Ln Li	6139.74	4938.21	5314.90	5349.65	4517.80
	f t-stat	3.40	2.91	0.63	2.50	3.04
	SIC	-12188.87	-9785.79	-10539.16	-10608.66	-8944.97
Model 2						
	Ln Li	6048.44	4810.01	5228.35	5235.11	4467.70
	SIC	-12013.82	-9536.94	-10373.61	-10387.14	-8852.32
Model 3						
Quotes	Ln Li	6475.81	4954.89	5353.06	5380.34	4815.09
	f t-stat	1.59	5.36	5.15	5.05	2.79
	SIC	-12891.21	-9849.36	-10645.69	-10700.25	-9569.75
Trans	Ln Li	3192.24	4959.38	5047.31	5354.89	4783.48
	f t-stat	1.68	0.09	0.02	0.97	0.02
	SIC	-6324.07	-9858.34	-10034.19	-10649.36	-9506.54
Volume	Ln Li	6441.82	4923.78	4887.14	5346.06	4738.95
	f t-stat	1.78	2.49	3.61	2.89	0.02
	SIC	-12823.23	-9787.15	-9713.85	-10631.69	-9417.48
Model 4						
	Ln Li	6427.58	4798.21	5225.17	5233.95	4397.84
	SIC	-12802.29	-9543.54	-10397.48	-10415.04	-8742.80

TABLE 14 (cont'd)

EARCH LOG-LIKELIHOOD RESULTS AND STATISTICS FOR SIGNIFICANCE OF COEFFICIENT OF TRADING ACTIVITY VARIABLES AND MODEL SELECTION FOR CANADIAN DATA. THE SIC CRITERIA IS LISTED FOR MODEL SELECTION PURPOSES.

		RY	SPL	TRP	VO	WN
Model 1			EARCH MODELS			
Quotes	Ln Li	6254.71	5062.33	5747.21	5830.75	6060.25
	f t-stat	2.83	2.65	3.86	2.59	1.02
	SIC	-12418.77	-10034.63	-11403.77	-11570.86	-12029.97
Trans	Ln Li	6211.88	5044.75	5703.95	5458.08	5992.36
	f t-stat	0.08	3.94	0.01	0.06	3.12
	SIC	-12333.11	-9999.47	-11317.25	-10825.52	-11894.18
Volume	Ln Li	6212.42	5025.13	5714.13	5706.08	5955.92
	f t-stat	3.13	3.71	2.19	0.07	4.45
	SIC	-12334.20	-9960.23	-11337.63	-11321.52	-11821.32
Model 2						
	Ln Li	6135.86	4810.30	5642.97	5672.25	5945.67
	SIC	-12188.63	-9538.08	-11202.85	-11261.42	-11808.34
Model 3						
Quotes	Ln Li	6245.09	5241.32	5854.84	5802.99	6048.61
	f t-stat	3.45	2.99	1.72	1.94	1.21
	SIC	-12429.75	-10422.62	-11649.26	-11545.55	-12036.87
Trans	Ln Li	6205.97	5048.17	5830.74	5783.65	6042.00
	f t-stat	0.45	7.24	0.08	0.37	1.93
	SIC	-12351.52	-10036.31	-11601.06	-11506.88	-12023.64
Volume	Ln Li	6185.84	4768.37	5817.32	5749.22	6029.27
	f t-stat	0.15	8.74	0.48	1.43	1.85
	SIC	-12311.25	-9476.72	-11574.20	-11438.01	-11998.19
Model 4						
	Ln Li	6130.20	5233.46	5791.49	5651.20	6017.26
	SIC	-12207.53	-10414.40	-11530.11	-11249.54	-11981.71

TABLE 15

EARCH LOG-LIKELIHOOD RESULTS AND STATISTICS FOR SIGNIFICANCE OF COEFFICIENT OF TRADING ACTIVITY VARIABLES AND MODEL SELECTION FOR THE AMERICAN DATA. THE SIC VALUE IS LISTED FOR MODEL SELECTION PURPOSES.

		AA	C	DD	DEC	DIS
Model 1		EARCH MODELS				
Quotes	Ln Li	5252.10	4718.80	5476.46	4959.16	5176.57
	f t-stat	2.66	2.37	1.18	2.15	0.73
	SIC	-10413.54	-9346.95	-10862.26	-9827.66	-10262.48
Trans	Ln Li	5226.10	4655.24	5471.10	4993.3	5175.89
	f t-stat	0.08	0.98	0.05	0.17	0.75
	SIC	-10361.54	-9219.83	-10851.54	-9895.94	-10261.12
Volume	Ln Li	5227.22	4592.47	5217.66	4465.44	5176.31
	f t-stat	0.92	0.91	0.20	0.39	0.32
	SIC	-10363.78	-9094.29	-10344.66	-8840.22	-10261.96
Model 2						
	Ln Li	5152.52	4598.32	5400.77	4885.04	5072.59
	SIC	-10221.93	-9113.55	-10718.43	-9686.97	-10062.07
Model 3						
Quotes	Ln Li	5228.73	4489.72	5372.40	4933.21	5165.33
	f t-stat	4.98	7.61	10.39	3.1	0.65
	SIC	-10397.02	-8919.01	-10684.36	-9805.98	-10270.22
Trans	Ln Li	4548.94	4671.28	5460.26	4924.76	5165.04
	f t-stat	0.02	0.01	0.01	0.02	0.03
	SIC	-9037.44	-9282.13	-10860.08	-9789.08	-10269.64
Volume	Ln Li	4672.42	4419.98	5405.68	4557.39	5200.38
	f t-stat	1.04	0.08	0.44	0.04	0.38
	SIC	-9284.40	-8779.53	-10750.92	-9054.34	10340.32
Model 4						
	Ln Li	5136.03	4598.73	5398.85	4879.79	5059.4
	SIC	-10219.17	-9144.58	-10744.81	-9706.69	-10065.91

TABLE 15 (cont'd)

EARCH LOG-LIKELIHOOD RESULTS AND STATISTICS FOR SIGNIFICANCE OF COEFFICIENT OF TRADING ACTIVITY VARIABLES AND MODEL SELECTION FOR THE AMERICAN DATA. THE SIC VALUE IS LISTED FOR MODEL SELECTION PURPOSES.

		DOW	EK	GM	HAL	IBM
Model 1		EARCH MODELS				
Quotes	Ln L1	5339.42	5387.55	5404.56	4934.24	5656.28
	f t-stat	0.14	0.93	0.24	1.77	0.09
	SIC	-10588.19	-10684.44	-10718.47	-9777.83	-11221.91
Trans	Ln L1	5319.98	5335.64	5393.28	4914.83	5691.66
	f t-stat	0.096	0.04	0.01	2.59	0.06
	SIC	-10549.31	-10580.62	-10695.91	-9739.01	-11292.67
Volume	Ln L1	4877.68	5370.11	4970.72	4736	5471.03
	f t-stat	0.31	0.17	0.27	1.39	0.23
	SIC	-9664.71	-10649.56	-9850.79	-9381.35	-10851.41
Model 2						
	Ln L1	5255.11	5292.63	5311.22	4832.63	5624.33
	SIC	-10427.13	-10502.16	-10539.35	-9582.17	-11165.57
Model 3						
Quotes	Ln L1	5326.97	5204.37	5382.61	4033.93	5340.42
	f t-stat	2.89	17.65	3.77	27.66	4.89
	SIC	-10593.51	-10348.30	-10704.79	-8007.43	-10620.41
Trans	Ln L1	5313.32	5225.14	5380.11	4912.48	5602.05
	f t-stat	0.49	0.08	0.01	0	0.09
	SIC	-10566.21	-10389.84	-10699.79	-9764.53	-11143.67
Volume	Ln L1	5337.01	5301.03	5328.14	4909.99	5709.66
	f t-stat	0.57	0.2	3.58	3.23	0.51
	SIC	-10613.59	-10541.62	-10595.85	-9759.55	-11358.89
Model 4						
	Ln L1	5246.86	5267.73	5309.21	4832.62	5615.16
	SIC	-10440.84	-10522.58	-10565.54	-9612.36	-11177.44

TABLE 15 (cont'd)

EARCH LOG-LIKELIHOOD RESULTS AND STATISTICS FOR SIGNIFICANCE OF COEFFICIENT OF TRADING ACTIVITY VARIABLES AND MODEL SELECTION FOR THE AMERICAN DATA. THE SIC VALUE IS LISTED FOR MODEL SELECTION PURPOSES.

		IRC	MCD	MER	NSC	OXY
Model 1		EARCH MODELS				
Quotes	Ln Li	4148.93	4359.13	4735.36	5563.25	5277.68
	f t-stat	2.79	1.05	0.14	0.05	0.13
	SIC	-8207.21	-8627.62	-9380.06	-11035.84	-10464.71
Trans	Ln Li	4101.69	5344.12	4779.37	5529.82	5203.25
	f t-stat	1.37	0.12	0.23	0.12	0.55
	SIC	-8112.73	-10597.60	-9468.08	-10968.98	-10315.85
Volume	Ln Li	4097.89	4994.32	4818.53	5532.35	5002.8
	f t-stat	3.33	0.18	1.5	1.27	0.13
	SIC	-8105.13	-9898.00	-9546.40	-10974.04	-9914.95
Model 2						
	Ln Li	4071.88	5294.25	4698.48	5474.78	5150.4
	SIC	-8060.67	-10505.41	-9313.85	-10866.45	-10217.71
Model 3						
Quotes	Ln Li	4353.2	5412.46	4806.04	5556.69	4399.86
	f t-stat	2.48	0.07	0.02	1.48	20.81
	SIC	-8645.97	-10764.49	-9551.64	-11052.94	-8739.29
Trans	Ln Li	4328.68	5366.35	4785.3	5528.71	5194.23
	f t-stat	1.6	0.07	0	1.57	0.01
	SIC	-8596.93	-10672.27	-9510.16	-10996.98	-10328.03
Volume	Ln Li	4325.49	5370.58	4697.1	5528.41	5252.47
	f t-stat	2.69	0.62	0.05	2.13	1.16
	SIC	-8590.55	-10680.73	-9333.76	-10996.38	-10444.51
Model 4						
	Ln Li	4311.69	5286.65	4697.07	5474.26	5117.26
	SIC	-8570.50	-10520.43	-9341.25	-10895.63	-10181.64

TABLE 15 (cont'd)

EARCH LOG-LIKELIHOOD RESULTS AND STATISTICS FOR SIGNIFICANCE OF COEFFICIENT OF TRADING ACTIVITY VARIABLES AND MODEL SELECTION FOR THE AMERICAN DATA. THE SIC VALUE IS LISTED FOR MODEL SELECTION PURPOSES.

		RLM	TDY	TGT	VAT	XON
Model 1				**EARCH MODELS**		
Quotes	Ln Li	5031.57	5575.01	5531.89	1985.48	5654.57
	f t-stat	5.45	1.57	5.08	1.62	2.27
	SIC	-9972.48	-11059.36	-10973.13	-3886.13	-11218.48
Trans	Ln Li	5025.68	5628.4	5508.18	2298.1	5615.84
	f t-stat	0.89	0.17	0.04	1.38	0.06
	SIC	-9960.70	-11166.14	-10925.71	-4511.37	-11141.02
Volume	Ln Li	4882.14	5631.18	5539.95	1533.45	5220.06
	f t-stat	5.27	2.85	0.56	14.12	0.34
	SIC	-9673.62	-11171.70	-10989.25	-2982.07	-10349.46
Model 2						
	Ln Li	4926.27	5484.58	5445.24	2759.87	5578.75
	SIC	-9769.44	-10886.06	-10807.39	-5441.98	-11074.40
Model 3						
Quotes	Ln Li	5000.91	5194.03	5529.08	3001.8	5644.1
	f t-stat	5.76	4.44	5.28	2.29	2.26
	SIC	-9941.38	-10327.62	-10997.73	-5947.05	-11227.76
Trans	Ln Li	4999.51	5625.86	5505.92	2937.06	5596.99
	f t-stat	0.01	0.09	0.01	0.59	1.21
	SIC	-9938.58	-11191.28	-10951.41	-5817.57	-11133.54
Volume	Ln Li	4986.86	5623.98	5539.23	3012.15	5651.8
	f t-stat	3.7	0.69	0.94	2.38	0.06
	SIC	-9913.28	-11187.52	-11018.03	-5967.75	-11243.16
Model 4						
	Ln Li	4906.85	5485.18	5437.6	2884.85	5566.14
	SIC	-9760.82	-10917.48	-10822.32	-5720.22	-11079.40

TABLE 16

SIGNIFICANCE OF TRADING ACTIVITY VARIABLES. THE NUMBER OF SIGNIFICANT TRADING ACTIVITY VARIABLES IS SHOWN FOR THE VARIOUS MODELS. THE AVERAGE t-STATISTICS OF THE COEFFICIENTS OF THE VARIABLES IS ALSO SHOWN. THE LEVEL OF SIGNIFICANCE CHOSEN IS 5 PERCENT.

	Canadian Securities		American Securities	
	Number of times variable significant out of twenty cases	Average t-statistic	Number of times variable significant out of twenty cases	Average t-statistic
Model 1				
Quotes	17	2.73	8	1.62
Transactions	7	1.19	1	0.49
Volume	16	2.85	4	1.73
Model 3				
Quotes	18	4.38	16	6.42
Transactions	5	1.54	0	0.30
Volume	15	3.22	6	1.22
Model 5				
Quotes	17	12.62	20	15.88
Transactions	18	11.64	18	11.51
Volume	17	11.41	20	12.95

TABLE 17

SUMMARY OF LOG-LIKELIHOOD RESULTS

	Average Log-Likelihood Values		Number of Times Model Chosen Overall By SIC		Number of Times Trading Activity Variable Chosen Within Model By SIC	
	Canadian	American	Canadian	American	Canadian	American
Model 1						
Quotes	-10893	-9926	4	8	16	13
Transactions	-10806	-10031	0	1	4	4
Volume	-10800	-9645	0	0	0	5
Model 2	-10634	-9948	0	0	-	-
Model 3						
Quotes	-11027	-9877	12	4	17	8
Transactions	-10555	-10047	3	1	3	4
Volume	-10719	-10023	0	6	0	8
Model 4	-10846	-10000	1	0	-	-
Model 5						
Quotes	-10647	-9900	0	0	8	11
Transactions	-10641	-9846	0	0	8	3
Volume	-10623	-9883	0	0	4	6
Model 6	-10427	-9752	0	0	-	-

TABLE 18

t-STATISTICS FROM THE SPECIFICATION TESTS FOR MODEL 3. THE SPECIFICATION TESTS ARE LISTED IN EXHIBIT 2. THE TESTS FOR CANADIAN SECURITY MTT EXPLODED FOR MODEL 3 WITH TRANSACTIONS AS THE MIXING VARIABLE. THUS MTT IS LEFT OUT OF THE RESULTS BUT ONLY FOR THE MODEL 3 TRANSACTIONS SET. LIKEWISE THE AMERICAN SECURITY IRC GAVE VERY HIGH t-STATISTICS FOR MOST MODELS. THE AMERICAN RESULTS ARE PRESENTED BOTH WITH AND WITHOUT THE IRC RESULTS INCLUDED.

Model 3	Canadian Data	American Data With IRC	American Data Without IRC
Quotes			
1	1.40	1.23	1.20
2	5.71	2.79	0.89
3	3.51	2.19	1.66
4	3.39	2.88	1.65
5	3.13	2.52	1.30
6	3.28	1.98	0.90
7	3.53	2.00	0.87
8	3.47	2.36	2.27
9	1.18	1.28	1.27
10	1.26	1.01	1.06
11	0.89	1.14	1.15
12	0.86	0.52	0.52
Transactions			
1	1.08	1.19	1.14
2	5.82	3.07	1.76
3	3.51	1.16	0.99
4	3.59	2.66	1.73
5	[illegible]	2.14	1.31
6	3.17	2.12	1.34
7	3.16	2.13	1.36
8	3.04	2.21	2.19
9	1.10	1.24	1.23
10	1.15	0.86	0.89
11	0.89	1.26	1.28
12	0.82	0.55	0.56
Volume			
1	1.16	1.17	1.07
2	7.79	2.48	0.78
3	3.91	1.10	0.86
4	9.01	1.87	0.82
5	3.91	1.99	0.96
6	3.94	1.93	0.91
7	4.21	1.92	0.96
8	3.01	2.21	2.16
9	1.02	1.43	1.43
10	1.04	1.16	1.20
11	0.88	1.40	1.44
12	0.86	0.54	0.56

TABLE 19

t-STATISTICS FROM THE SPECIFICATION TESTS FOR MODEL 5. THE SPECIFICATION TESTS ARE LISTED IN EXHIBIT 2. THE AMERICAN SECURITY IRC GAVE VERY HIGH t-STATISTICS FOR MOST MODELS THUS THE AMERICAN RESULTS ARE PRESENTED BOTH WITH AND WITHOUT THE IRC RESULTS INCLUDED.

Model 5	Canadian Data	American Data With IRC	American Data Without IRC
Quotes			
1	1.43	1.28	1.28
2	0.32	1.48	1.56
3	0.71	1.21	1.23
4	1.55	1.43	1.32
5	1.61	1.54	1.41
6	1.74	1.46	1.51
7	1.49	1.47	1.42
8	3.46	2.67	2.51
9	1.19	1.35	1.33
10	1.30	1.33	1.37
11	0.87	1.12	1.12
12	0.94	0.58	0.61
Transactions			
1	1.29	1.35	1.36
2	0.54	0.27	0.28
3	0.62	1.02	1.06
4	1.56	1.27	1.26
5	1.77	1.17	1.07
6	1.51	1.30	1.31
7	1.53	1.24	1.23
8	3.19	2.54	2.44
9	1.28	1.35	1.34
10	1.32	1.22	1.24
11	0.84	1.14	1.13
12	0.90	0.61	0.63
Volume			
1	1.34	1.17	1.16
2	0.14	0.97	1.02
3	0.85	0.96	1.00
4	1.62	1.42	1.42
5	1.64	1.36	1.24
6	1.89	1.47	1.50
7	1.72	1.47	1.45
8	3.03	2.46	2.36
9	1.25	1.48	1.47
10	1.38	1.32	1.36
11	0.80	1.22	1.22
12	0.97	0.57	0.59

TABLE 20

MODEL RESULTS FROM MODEL 1 AND MODELS 5 AND 6 UTILIZING ONLY 750 DAYS OF DATA. THE RESULTS ARE FOR A SUB-SAMPLE OF THE AMERICAN STOCKS. THE LOG-LIKELIHOODS AND SIC VALUES ARE GIVEN.

Short Models		DD	IRC	MCD	NSC	OXY
Model 1						
Quotes	Ln Li	1478	1120	1521	1564	1334
	SIC	-2881.42	-2165.42	-2967.42	-3053.42	-2593.42
Trans	Ln Li	1512	1134	1530	1574	1340
	SIC	-2949.42	-2193.42	-2985.42	-3073.42	-2605.42
Volume	Ln Li	1381	703	1400	1087	1192
	SIC	-2687.42	-1331.42	-2725.42	-2099.42	-2309.42
Model 5						
Quotes	Ln Li	1529	1112	1531	1579	1328
	SIC	-3033.14	-2199.14	-3037.14	-3133.14	-2631.14
Trans	Ln Li	1512	1095	1535	1568	1339
	SIC	-2999.14	-2165.14	-3045.14	-3111.14	-2653.14
Volume	Ln Li	1513	1093	1525	1564	1338
	SIC	-3001.14	-2161.14	-3025.14	-3103.14	-2651.14
Model 6						
	Ln Li	1493	1079	1504	1558	1267
	SIC	-2967.36	-2139.36	-2989.36	-3097.36	-2515.36

TABLE 20 (cont'd)

MODEL RESULTS FROM MODEL 1 AND MODELS 5 AND 6 UTILIZING ONLY 750 DAYS OF DATA. THE RESULTS ARE FOR A SUB-SAMPLE OF THE AMERICAN STOCKS. THE LOG-LIKELIHOODS AND SIC VALUES ARE GIVEN.

Short Models		RLM	TDY	TGT	VAT	XON
Model 1						
Quotes	Ln Li	1310	1506	1524	803	1536
	SIC	-2545.42	-2937.42	-2973.42	-1531.42	-2997.42
Trans	Ln Li	1330	1044	1515	847	1506
	SIC	-2585.42	-2013.42	-2955.42	-1619.42	-2937.42
Volume	Ln Li	1088	-192	1515	804	1271
	SIC	-2101.42	458.5753	-2955.42	-1533.42	-2467.42
Model 5						
Quotes	Ln Li	1306	1421	1520	823	1557
	SIC	-2587.14	-2817.14	-3015.14	-1621 14	-3089.14
Trans	Ln Li	1309	1473	1499	859	1530
	SIC	-2593.14	-2921.14	-2973.14	-1693.14	-3035.14
Volume	Ln Li	1317	1499	1499	814	1537
	SIC	-2609.14	-2973.14	-2973.14	-1603.14	-3049.14
Model 6						
	Ln Li	1299	1364	1475	799	1515
	SIC	-2579.36	-2709.36	-2931.36	-1579.36	-3011.36

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