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Two Essays In Real Business Cycle Theory

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by

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Abstract

In the first chapter, a real business cycle model with heterogeneous agents is parameterized, caiibrated, and simulated to see if it can account for some stylized facts characterizing postwar U.S. business cycle fluctuations, such as the countercyclical movement of labour's share of income, and the acyclical behaviour of real wages. There are two types of agents in the model, workers and entrepreneurs, who participate on an economy-wide market for contingent claims. On this market workers purchase insurance from entrepreneurs, through optimal labour contracts, against losses in income due to business cycle fluctuations. Optimal labour contracting is found to account, quantitatively, for the observed pattern of fluctuations in labour income. The insurance and savings components in measured labour income tend to move countercyclically over the business cycle, counterbalancing the strong procyclical behaviour of the marginal product of labour. The flow of transactions involving insurance against cyclical risk is measured to be about 1 to 4 percent of worker's wealth. The upper end of this range is obtained when workers are much more risk averse than entrepreneurs, and entrepreneurs and sufficiently numerous to allow a reasonably large insurance market to operate.

Results in Lucas (1987) suggest that if public policy can affect the growth rate of the economy, the welfare implications of alternative policies will be large. The question asked in the second chapter is whether large welfare costs are associated with inflation in an environment with endogenous growth. To answer this question, an economy with endogenous growth, arising through human capital accumulation, is examined. Money enters via a cash-in-advance constraint on purchases of the consumption good. In this setting, higher inflation lowers real growth through its effect on the return to working. However, the welfare cost of moderate inflation rates is found to be modest. Since households in the model only really care about the paths of consumption and leisure, the low welfare costs can be understood as

follows. First, a lower real growth rate means that less output needs to be devoted to physical capital accumulation. Consequently, the fall in consumption, in response to higher inflation, is small. Second, the existence of two productive activities, physical output and human capital, augments the responsiveness of leisure to changes in the rate of inflation. The net result trom these two effects is that small welfare costs are associated with moderate inflation rates.

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To my parents.

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Chapter 1: On the Cyclical Allocation of Risk[1]

1. Introduction

The cyclical behaviour of labour income has long been a topical question for macroeconomists. Two stylized facts concerning the movement of labour income over the business cycle are:

1. Labour's share of income is not constant, but rather moves countercyclically over the business cycle. Figure 1 shows the movement in labour's share of income using quarterly data over the 1954-1989 sample period.[2] In figure 2, detrended labour's share of income is plotted against detrended GNP; the correlation between these two series is -0.37. Detrended labour's share of income has a standard deviation of 0.8 percent. This compared to one of 0.9 percent for the ratio of total consumption to GNP.
2. The real wage does not appear to move systematically over the business cycle. The real wage's behaviour for the postwar period is shown in Figures 3 and 4. The correlation between the detrended real wage and GNP series plotted in Figure 4 is -0.40. This number should be viewed, however, with some caution. After examining data from 12 OECD countries, Geary and

[1] This chapter is based on joint work with Jeremy Greenwood

[2] For a complete description of the data used, see Section 7. The data were detrended using the Hodrick-Prescott (H-P) filter. Labour's share of income is taken to be compensation of employees divided by GNP, as given in the national income accounts. The real wage was defined as real compensation of employees divided by total hours worked. Also, the correlation between labour's share of income and GNP when the data is first differenced instead is -0.55. Furthermore, observe that over the 1954–1989 sample there is an upward trend in labour's share of income. At the same time proprietor's share of income fell over this period. This reflects a decline in the importance of sole proprietorships and partnerships relative to the corporate and government sectors. By netting proprietor's income out of GNP to control for this effect, the upward drift in labour's share of income can be removed. The correlation between labour's share of income and GNP now becomes -0.22 for H-P filtered data, and is -0.53 when the data is first differenced. In a similar vein (some fraction of) proprietor's income could be added to the compensation of employees when computing labour's share of income. Doing this does not significantly alter the correlations being reported.

Kennan (1982) conclude that real wages and employment are statistically independent of each other over the business cycle. Using disaggregated data for the U.S., Keane, Moffitt, and Runkle (1988) find that the real wage has a slight procyclical movement to it. The consensus seems to be that the real wage shows no strong cyclical pattern.

The purpose of this chapter will be to address these facts from the perspective of real business cycle theory as advanced by Kydland and Prescott (1982) and Long and Plosser (1983). Real business cycle models have been able to capture many features of the U.S. business cycle remarkably well, such as the postwar correlation structure between output, consumption, investment, hours worked, and productivity.[3] By and large, though, the real business cycle literature has been silent on the movement of labour income over the business cycle. Instead the prototypical real business cycle model has a representative agent with isoelastic preferences, defined over consumption and leisure, who produces output according to a Cobb-Douglas production technology that uses capital and labour. With this formulation, labour's share of income is constant over the business cycle. Given that the only source of fluctuations in the model is Hicks neutral technology shocks, the marginal product of labour is strongly procyclical. Skeptics have pointed to these facts when casting doubt on the utility of the real business cycle paradigm.[4]

The starting point for the current analysis is the observation that a given set of real allocations for an economy may be consistent with a wide variety of institutional arrangements. In particular, following Azariadis (1978) it will be assumed that built

[3] In addition to the path breaking papers by Kydland and Prescott (1982) and Long and Plosser (1983), see the work by Hansen (1985); Greenwood, Hercowitz, and Huffman (1988); King, Plosser and Rebelo (1988); and Cooley and Hansen (1989a). The prototypical real business cycle model is probably best described in the King, Plosser and Rebelo (1988) paper.

[4] To quote Summers (1986), this work "does not resolve—or even mention—the empirical reality ... that consumption and leisure move in opposite directions over the business cycle with no apparent procyclicality of real wages. It is finessed by ignoring wage data." (p. 25)

into labour income is an insurance component designed to provide workers with some degree of protection against business cycle fluctuations. (For a survey of the implicit contract literature see Rosen (1985).) This insurance component of labour income inserts a wedge between the marginal product of labour and measured real wages. As a theoretical proposition, it has been suggested by Wright (1988) and others that labour contracting may explain the apparent acyclical movement of real wages. It remains to be seen, however, whether the introduction of optimal labour contracting into a real business cycle model can account, quantitatively, for the observed pattern of fluctuations in labour income.

To conduct the analysis, a real business cycle model with heterogeneous agents is constructed.[5] Specifically, in the model there are two types of agents, viz workers and entrepreneurs. Agents have preferences that are formulated in line with Epstein's (1983) notion of stationary cardinal utility.[6] This allows (the deterministic version of) the model to possess a unique, invariant distribution of wealth across agents. In any given period, workers and entrepreneurs are free to transact on the economy-wide market for contingent claims. The quantity of contingent claims transacted in each possible state of the world is computed. This Arrow-Debreau equilibrium is consistent with many different trading arrangements. Attention is directed here toward an optimal labour contracting scheme that supports the Arrow-Debreau allocations. The optimal labour contract is priced, and its implications for the movement of labour income over the business cycle specified. Finally, the constructed model is parameterized, calibrated and simulated to see whether it can mimic the

[5] Rebelo (1988) discusses how heterogeneous agent economies can be computed for linear-quadratic settings. His technique involves finding the weights for a central planner's problem that will generate the competitive equilibrium under study. Presumably the trades in contingent claims, or other assets, that support the equilibrium could then be backed out. This is different that the tack taken here where the decentralized competitive equilibrium, including the flow of transactions undertaken on contingent claims markets, is solved for directly.

[6] Preferences of this type have also been used by Mendoza (1989) who simulates an open economy real business cycle model.

movements in labour income that are found in the data.[7]

The remainder of this chapter is organized as follows: In the next section an outline of the economic environment is presented. Then, in Section 3, the optimization problems faced by individual agents are cast and the economy's general equilibrium is characterized. Some optimal contract schemes that support the Arrow-Debreau equilibrium are discusses in Section 4. The algorithm used to solve the model is described in the following section. The model is parameterized and calibrated in Section 6, with the results from the simulation experiments performed being presented in Section 7. Finally, some concluding remarks are made in the last section.

[7] The current work complements two other papers on risk sharing by Cho and Rogerson (1987) and Danthine and Donaldson (1989). Both of these studies investigate the implication of labour contracting for the relative volatility of hours and productivity over the business cycle. Neither focus on the implications of labour contracting for the cyclical properties of labour income. Danthine and Donaldson (1989) construct a non-Walrasian real business cycle model of labour contracting that is substantially different from the equilibrium model being developed here. The model can mimic the relative volatilities of hours and productivity that are observed in the U.S. data. Cho and Rogerson (1987) examine the effects of risk sharing in a real business cycle model where entrepreneurs are risk neutral. The assumption of risk neutral entrepreneurs allows a simple aggregation procedure to be employed so that the model can be reformulated in terms of a representative agent's planning problem. They find that such a model is helpful in explaining the relative volatilities of hours and productivity, but that it mimics poorly the behaviour of other variables, such a consumption and investment, due to the risk neutrality assumption for entrepreneurs. The current study focuses on computing a fully decentralized Arrow-Debreau equilibrium where all agents are risk averse.

2. The Economic Environment

Consider a perfectly competitive economy inhabited by two types of agents, viz workers and entrepreneurs. There are n times more workers than entrepreneurs, with the number of entrepreneurs being normalized to one. Each entrepreneur operates a constant-returns-to-scale production process which produces output in period t, y_t, as specified by

$$y_t = F(k_t, n\ell_t, h_t; \lambda_t), \tag{2.1}$$

where k_t is his stock of capital in period t, $n\ell_t$ is the total amount of period-t labour services hired from workers, h_t is the entrepreneur's labour effort in this period, and λ_t is a technology shock. The technology shock λ_t evolves according to

$$\lambda_t = \lambda_{t-1}^{\rho}\epsilon_t, \quad 0 < \rho < 1, \tag{2.2}$$

where ϵ_t is drawn from the finite set $A = \{\epsilon^1, \epsilon^2, \ldots, \epsilon^q\}$ according to the distribution function $E(\epsilon_t)$. Observe that, conditional on a value for λ_{t-1}, λ_t will be drawn from the finite set $L_t = \{\lambda_t^1, \lambda_t^2, \ldots, \lambda_t^q\}$ where $\lambda_t^j = (\lambda_{t-1})^{\rho}\epsilon_t^j$. An entrepreneur's capital accumulation is governed by the law of motion

$$k_{t+1} = (1-\delta)k_t + i_t \tag{2.3}$$

where δ is the depreciation rate and i_t is gross investment at time t.

The representative entrepreneur in the model desires to maximize the expected value of his lifetime utility, $\underline{Z}$, as given by

$$\mathrm{E}[\underline{Z}] = \mathrm{E}\left\{\sum_{t=0}^{\infty}\left[\prod_{\tau=0}^{t-1} e^{-\phi\left(Z(x_\tau, 1-h_\tau)\right)}\right] Z\left(x_t, 1-h_t\right)\right\} \tag{2.4}$$

where x_t and $1-h_t$ represent his period-t consumption of goods and leisure. Here $Z\left(x_t, 1-h_t\right)$ represents the entrepreneur's momentary utility function for period t. It will be postulated that the function Z is negative, increasing and "more than strictly concave" in the sense that $\ell n(-Z)$ is strictly convex. The term $\prod_{\tau=0}^{t-1} e^{-\phi(Z(x_\tau, 1-h_\tau))}$ is the endogenous discount factor attached to the period-t momentary utility function. The function ϕ is assumed to be positive, increasing, and strictly concave. The conditions imposed on Z and ϕ guarantee that the agent's lifetime utility function, $\underline{Z}$, is both increasing and strictly concave.[8] Note that, by construction, an increase in the period-τ utility will cause the agent to discount future periods (all $t > \tau$) more heavily, or to become more impatient.

Similarly, the representative worker in the model desires to maximize his expected lifetime utility, $\mathrm{E}[\underline{U}]$, as given by

$$\mathrm{E}[\underline{U}] = \mathrm{E}\left\{\sum_{t=0}^{\infty}\left[\prod_{\tau=0}^{t-1} e^{-\nu\left(U(c_\tau,1-\ell_\tau)\right)}\right] U(c_t, 1-\ell_t)\right\} \tag{2.5}$$

The worker's period-t momentary utility function, $U(c_t, 1-\ell_t)$, is defined over his consumption, c_t, and leisure, $1-\ell_t$, in this period and is assumed to have the standard properties. The term $\prod_{\tau=0}^{t-1} e^{-\nu(U(c_\tau,1-\ell_\tau))}$ is the endogenous discount factor that the worker attaches to period-t utility. The functions U and ν are assumed to satisfy the same properties as Z and ϕ. Observe that workers and entrepreneurs can differ from one another in their attitudes toward both accumulation and risk.

Both workers and entrepreneurs may desire to insure themselves against aggregate fluctuations due to technology shocks. It will be assumed that all agents are free to participate on an economy-wide contingent claims market. Let s_t denote the state-of-the-world in period t, which is postulated to be governed by the transition function $S(s_t|s_{t-1})$—a precise definition of s_t and a justification for the assumed form of its law of motion will be provided later. Now let the price in period t of a unit of $t+1$ consumption contingent on the event λ_{t+1} occurring, given that the current state-of-the-world is s_t, be represented by $p_t(\lambda_{t+1}) \equiv p(\lambda_{t+1}; s_t)$. Note that, conditional upon the current state, s_t, the only source of uncertainty next period in the economy is from the technology shock, λ_{t+1}. The quantity of such claims purchased by

[8] This follows from a straightforward modification of the argument made in Epstein (1983), Lemma 1. Also, a minor notational point concerning the definition of the discount factor: Let $\prod_{\tau=0}^{t-1} \equiv 1$ for $t < 1$.

workers and entrepreneurs will be given by $b_{t+1} \equiv b_{t+1}(\lambda_{t+1})$ and $a_{t+1} \equiv a_{t+1}(\lambda_{t+1})$, respectively. Finally, define $\zeta(\lambda_{t+1}|s_t)$ to be the marginal density function for λ_{t+1}, conditional on s_t, that is associated with the transition function $S(s_{t+1}|s_t)$.

3. The Model's General Equilibrium

The decision-making of workers in competitive equilibrium is summarized by the outcome of the following dynamic programming problem

$$V\Big[b(\lambda); s\Big] = \max_{c,\, b'(\lambda'),\, \ell} \left\{ U\left(c, 1-\ell\right) + e^{-\nu(c,1-\ell)} \int V\Big[b'(\lambda'); s'\Big] \mathrm{d}S(s'|s) \right\} \tag{3.1}$$

subject to

$$c + \int p(\lambda') b'(\lambda') \mathrm{d}\lambda' \leq w\ell + b(\lambda) \tag{3.2}$$

where w is the spot market real wage rate. (To ease the burden on notation, time subscripts have been dropped in the standard fashion, and the function $\nu(U\left(c, 1-\ell\right))$ has been rewritten more compactly as $\nu(c, 1-\ell)$.)

The solution to the above programming problem is characterized by the two efficiency conditions shown below—in addition to (3.2):

$$\begin{aligned} p(\lambda') \Big[U_1\left(c, 1-\ell\right) - \nu_1(c, 1-\ell) e^{-\nu(c,1-\ell)} \int V\Big[b'(\lambda'); s'\Big] \mathrm{d}S(s'|s) \Big] \\ = e^{-\nu(c,1-\ell)} \Big[U_1\left(c', 1-\ell'\right) - \nu_1(c', 1-\ell') e^{-\nu(c',1-\ell')} \int V\Big[b''(\lambda''); s''\Big] \\ \times \mathrm{d}S(s'|s) \Big] \zeta(\lambda'|s), \quad \forall \lambda' \end{aligned} \tag{3.3}$$

$$\begin{aligned} w \Big[U_1\left(c, 1-\ell\right) - \nu_1(c, 1-\ell) e^{-\nu(c,1-\ell)} \int V\Big[b'(\lambda'); s'\Big] \mathrm{d}S(s'|s) \Big] \\ = \Big[U_2\left(c, 1-\ell\right) - \nu_2(c, 1-\ell) e^{-\nu(c,1-\ell)} \int V\Big[b'(\lambda'); s'\Big] \mathrm{d}S(s'|s) \Big]. \end{aligned} \tag{3.4}$$

Equation (3.3) is the optimality condition governing the worker's purchases of contingent claims. The left-hand side of this expression illustrates the marginal cost of purchasing a contingent claim today which pays one unit of consumption tomorrow if the state of technology then is λ'. Such a claim costs $p(\lambda')$ units of current consumption. This leads to losses of, first, $U_1\left(c, 1-\ell\right)$ in current utility, and second,

$\cdot\ \nu_1(c, 1-\ell)e^{-\nu(c,1-\ell)} \int V\Big[b'(\lambda'); s'\Big]\mathrm{d}S(s'|s)$ in discounted future expected utility, the latter effect due to the increase in the agent's subjective discount factor used to weight future expected lifetime utility (where the latter is a negative number). The right-hand side of (3.3) represents the marginal benefit of purchasing today a claim to a unit of consumption next period contingent on the occurrence of λ'. Conditional upon this state occurring, the worker would realize next period a gain in expected lifetime utility of $\Big[U_1(c', 1-\ell') - \nu_1(c', 1-\ell')e^{-\nu(c',1-\ell')} \int V\Big[b''(\lambda''); s''\Big]\mathrm{d}S(s''|s')\Big]$. The discounted, unconditional expected value of this gain is given by the right side of (3.3), where again $\zeta(\lambda'|s)$ is the marginal density function for λ', conditional on s, that is associated with the transition function $S(s'|s)$. Finally, equation (3.4) characterizes the worker's allocation of labour effort; it sets the marginal benefit from working equal to the marginal disutility of labour.

Similarly, the decision-making of entrepreneurs in competitive equilibrium is described by the solution to the dynamic programming problem shown below.[9]

$$J\Big[a(\lambda), k; s\Big] = \max_{x,\, k',\, a'(\lambda'),\, \ell,\, h} \Big\{Z(x, 1-h) + e^{-\phi(x,1-h)} \int J\Big[a'(\lambda'), k'; s'\Big]\mathrm{d}S(s'|s)\Big\} \tag{3.5}$$

subject to

$$x + k' + \int p(\lambda')a'(\lambda')\mathrm{d}\lambda' + wn\ell \le F(k, n\ell, h; \lambda) + (1-\delta)k + a(\lambda) \tag{3.6}$$

The upshot of the implied maximization routine is the following set of efficiency conditions:

$$\begin{aligned}&\Big[Z_1(x, 1-h) - \phi_1(x, 1-h)e^{-\phi(x,1-h)} \int J\Big[a'(\lambda'), k'; s'\Big]\mathrm{d}S(s'|s)\Big] \\ &\quad = e^{-\phi(x,1-h)} \int \Big[Z_1(x', 1-h') - \phi_1(x', 1-h')e^{-\phi(x',1-h')} \int J\Big[a''(\lambda''), k''; s''\Big] \\ &\qquad \times \mathrm{d}S(s''|s')\Big]\Big[F_1(k', n\ell', h'; \lambda') + (1-\delta)\Big]\mathrm{d}S(s'|s)\end{aligned} \tag{3.7}$$

[9] Again, to ease on notation, let the function $\phi(Z(x, 1-h))$ be expressed more compactly as $\phi(x, 1-h)$.

$$p(\lambda')\left[Z_1(x,1-h)-\phi_1(x,1-h)e^{-\phi(x,1-h)}\int J\left[a'(\lambda'),k';s'\right]\mathrm{d}S(s'|s)\right]$$
$$=e^{-\phi(x,1-h)}\left[Z_1(x',1-h')-\phi_1(x',1-h')e^{-\phi(x',1-h')}\int J\left[a''(\lambda''),k'';s''\right]\right. \tag{3.8}$$
$$\left.\times\, \mathrm{d}S(s''|s')\right]\zeta(\lambda'|s),\quad \forall\lambda'$$

$$F_2(k,n\ell,h;\lambda)=w \tag{3.9}$$

$$F_3(k,n\ell,h;\lambda)\left[Z_1(x,1-h)-\phi_1(x,1-h)e^{-\phi(x,1-h)}\int J\left[a'(\lambda'),k';s'\right]\mathrm{d}S(s'|s)\right]$$
$$=\left[Z_2(x,1-h)-\phi_2(x,1-h)e^{-\phi(x,1-h)}\int J\left[a'(\lambda'),k';s'\right]\mathrm{d}S(s'|s)\right] \tag{3.10}$$

The first equation, (3.7), characterizes optimal capital accumulation in the model. The next expression is the entrepreneur's efficiency condition governing purchases of contingent claims. The optimal employment of workers is regulated by (3.9). Finally, (3.10) specifies the amount of labour effort the entrepreneur will expend.

In the model's general equilibrium, both the markets for goods and contingent claims must clear. This necessitates that the two conditions below must hold:

$$nc+x+k'=F(k,n\ell,h;\lambda)+(1-\delta)k \tag{3.11}$$

and

$$a'(\lambda')+nb'(\lambda')=0,\quad \forall\lambda' \tag{3.12}$$

The formal characterization of the model's general equilibrium is now almost complete.

Note that equations (3.11) and (3.12) can be used to solve out for x, a', a'' in equations (3.5), (3.7), (3.8) and (3.10). Similarly, (3.2) can be used to eliminate c and c' in (3.1), (3.3) and (3.4). Also, (3.9) allows for w to be substituted out for in (3.6). Finally, observe that equations (3.3) and (3.8) hold for each λ' in the set $L'=\left\{\lambda^{1'},\lambda^{2'},\ldots,\lambda^{q'}\right\}$. Having done this, it is easy to deduce that (3.1), (3.3), (3.4), (3.5), (3.7), (3.8) and (3.10) represent a system of functional equations implicitly defining solutions for the equilibrium value functions V and J, the policy rules $\mathbf{b}'\equiv\left[b'\left(\lambda^{1'}\right),\ldots,b'\left(\lambda^{q'}\right)\right]$, ℓ, k', and h, and price functions $\mathbf{p}\equiv\left[p\left(\lambda^{1'}\right),\ldots,p'\left(\lambda^{q'}\right)\right]$.

Denote the solutions for the policy rules and price functions by $\mathbf{b}' = \mathbf{b}'(s)$, $\ell = \ell(s)$, $k' = k'(s)$, $h = h(s)$, and $\mathbf{p} = \mathbf{p}(s)$. Let the ith components of the vector functions $\mathbf{b}'(s)$ and $\mathbf{p}(s)$ be represented by $b'(\lambda^{i'}; s)$ and $p(\lambda^{i'}; s)$.

The state variable, s, remains to be specified. From the analysis above it is probably clear that s is given by the triplet $\left[b(\lambda), k, \lambda\right]$, representing the distribution of wealth between workers and entrepreneurs, the capital stock, and the state of technology. Given the current-state-of-the-world, s, next period's state, s', is given by $s' = \left[b'(\lambda'; s), k'(s), \lambda'\right]$. The conditional distribution governing next period's state, s', can easily be seen to be

$$\begin{aligned} S(s'|s) &= \text{prob}\left[\tilde{b}'(\tilde{\lambda}'; s) \le b', \tilde{k}'(s) \le k', \tilde{\lambda}' \le \lambda' \middle| \tilde{b}(\tilde{\lambda}) = b, \tilde{k} = k, \tilde{\lambda} = \lambda\right] \\ &= \int^{\lambda'/\lambda^{\rho}} I\left[\tilde{b}'(\lambda^{\rho}\tilde{\epsilon}'; s) - b'\right] I\left[\tilde{k}'(s) - k'\right] \mathrm{d}E(\tilde{\epsilon}') \end{aligned} \tag{3.13}$$

where $I(z) = 1$ if $z \le 0$ and $I(z) = 0$ if $z > 0$. Last, the equilibrium value functions for workers and entrepreneurs can be written more compactly, with some abuse of notation, as $V\left[b(\lambda); b(\lambda), k, \lambda\right] = V\left[b(\lambda), k, \lambda\right] = V(s)$, and $J\left[-nb(\lambda), k; b(\lambda), k, \lambda\right] = J\left[b(\lambda), k, \lambda\right] = J(s)$.

4. Market Structure

Many different structures for financial markets are consistent with the real allocations generated by the competitive equilibrium modeled above. To begin with, note that the assumption that workers cannot hold physical capital is innocuous. A unit of capital purchased today pays off the return $\left[F_1\left(k'(s), n\ell(s'), h(s'); \lambda'\right) + (1-\delta)\right]$ next period. While prohibited from holding physical capital, a worker could buy a portfolio of contingent claims mimicking this return. This portfolio would cost $\int p(\lambda'; s)\left[F_1\left(k'(s), n\ell(s'), h(s'); \lambda'\right) + (1-\delta)\right] \mathrm{d}\lambda'$ units of current consumption. Using (3.7) and (3.8) it is easy to see that $\int p(\lambda'; s)\left[F_1\left(k'(s), n\ell(s'), h(s'); \lambda'\right) + (1-\delta)\right] \mathrm{d}\lambda' = 1$, implying that in a competitive equilibrium this portfolio costs the same as a unit of capital.

Clearly, whether or not workers and entrepreneurs trade contingent claims on a separate financial market or instead do so through the structure of a firm should be immaterial for the model's real allocations. It will be material for the measurement of prices, however, such as wages and the return to capital. Now, envision an environment where the contingent claims desired by workers are loaded directly into the wage packages paid by firms. Denote the measured real wage in this setting by $\hat{w}$. From the worker's budget constraint, (3.2), it is apparent that measured labour income in this economy, $\hat{w}\ell$, would read as

$$\hat{w}(s)\ell(s) = w(s)\ell(s) + b(\lambda) - \int p(\lambda'; s)b'(\lambda'; s)\mathrm{d}\lambda', \tag{4.1}$$

with the measured real wage being given by

$$\hat{w}(s) = w(s) + \frac{b(\lambda) - \int p(\lambda'; s)b'(\lambda'; s)\mathrm{d}\lambda'}{\ell(s)}. \tag{4.2}$$

Additionally, one could assume that not all workers are covered by the above labour contract. Specifically, assume that the fraction u is. Measured labour income would now be $\left[u\hat{w}(s) + (1-u)w(s)\right]\ell(s)$.

Alternatively, one can imagine a setting where workers channel their savings through a bond market to entrepreneurs who use the funds to accumulate physical capital. Insurance against cyclical fluctuations could be provided by firms for workers as part of their wage package. Specifically, let the market price today of a bond paying back one unit of output next period be $1/r(s)$; arbitrage dictates that the gross interest rate, $r(s)$, will be given by $1/r(s) = \int p(\lambda'; s)\mathrm{d}\lambda'$. In the general equilibrium modeled above, workers expect to have $\bar{b}'(s) = \int b'(\lambda^{\rho}\epsilon'; s)\mathrm{d}E(\epsilon')$ units of wealth next period. Now, suppose that workers save through the bond market to attain this target level of wealth. The worker's budget constraint, (3.2), can accordingly be rewritten as

$$c(s) + \int p(\lambda'; s)\bar{b}'(s)\mathrm{d}\lambda' + \int p(\lambda'; s)\left[b'(\lambda'; s) - \bar{b}'(s)\right]\mathrm{d}\lambda' \leq w(s)\ell(s) + \bar{b} + \left[b(\lambda) - \bar{b}\right].$$

Here, the term $\int p(\lambda'; s)\overline{b}'(s) d\lambda'$ represents the worker's savings in bonds which will yield the return $\overline{b}'(s)$ in principal and interest next period, while $\int p(\lambda'; s) \left[b'(\lambda'; s) - \overline{b}'(s)\right] d\lambda'$ is the amount paid in premiums for insurance paying off $\left[b'(\lambda'; s) - \overline{b}'(s)\right]$ units of output next period contingent on the event λ' occurring. If firms provide the insurance against cyclical fluctuations as part of the worker's wage package, then measured labour income in the economy, $\tilde{w}\ell$, would be given by

$$\tilde{w}(s)\ell(s) = w(s)\ell(s) + \left[b(\lambda) - \overline{b}\right] - \int p(\lambda'; s) \left[b'(\lambda'; s) - \overline{b}'(s)\right] d\lambda', \tag{4.3}$$

with the measured real wage, $\tilde{w}$, being correspondingly defined as

$$\tilde{w}(s) = w(s) + \frac{\left[b(\lambda) - \overline{b}\right] - \int p(\lambda'; s) \left[b'(\lambda'; s) - \overline{b}'(s)\right] d\lambda'}{\ell(s)}. \tag{4.4}$$

5. Solution Algorithm

Let the system of equations (3.1), (3.3), (3.4), (3.5), (3.7), (3.8), and (3.10) defining a solution to the model be more compactly represented by—remember that there are q copies of each of equations (3.3) and (3.8)

$$\Delta\left[b(\lambda), k, V, J, \mathbf{b}', k', \ell, h, \mathbf{p}; \lambda\right] = \int \Omega\left[b'(\lambda'), k', V', J', \mathbf{b}'', k'', \ell', h', \mathbf{p}'; \lambda, \lambda'\right] d\lambda'. \tag{5.1}$$

Here, $\Delta : \mathrm{R}^{2q+8} \rightarrow \mathrm{R}^{2q+5}$ and $\Omega : \mathrm{R}^{2q+8} \rightarrow \mathrm{R}^{2q+5}$. In order to simulate the model, a set of value functions, policy-rules, and price functions of the form $V = V\left[b(\lambda), k, \lambda\right]$, $J = J\left[b(\lambda), k, \lambda\right]$, $\mathbf{b}' = \mathbf{b}'\left[b(\lambda), k, \lambda\right]$, $k' = k'\left[b(\lambda), k, \lambda\right]$, $\ell = \ell\left[b(\lambda), k, \lambda\right]$, $h = h\left[b(\lambda), k, \lambda\right]$, and $\mathbf{p} = \mathbf{p}\left[b(\lambda), k, \lambda\right]$ must be found. Note that $\mathbf{b}'$ and $\mathbf{p}$ are vector functions whose ith components read, respectively, as $b'(\lambda^{i'}) = b'\left[\lambda^{i'}; b(\lambda), k, \lambda\right]$ and $p(\lambda^{i'}) = p\left[\lambda^{i'}; b(\lambda), k, \lambda\right]$. To do this, an algorithm developed by Coleman (1989) will be employed that approximates the true solution functions over a grid using a multilinear interpolation scheme.[10]

[10] Coleman's (1989) technique is related to one developed by Baxter (1988) and Danthine and Donaldson (1988). The principle difference between Coleman's (1988) on the one hand and Baxter's (1988) and Danthine and Donaldson's (1988) on the other, is that the latter restrict the range of the functions describing the laws of motion for the state variables to lie on a grid, while the former does not.

To begin, restrict the permissible range of values for the capital stock, holdings of contingent claims, and the technology shock to be in the closed intervals $\left[b_1, b_m\right]$, $\left[k_1, k_n\right]$, and $\left[\lambda_1, \lambda_r\right]$ respectively, and let $B = \left\{b_1, b_2, \ldots, b_m\right\}$, $K = \left\{k_1, k_2, \ldots, k_n\right\}$ and $L = \left\{\lambda_1, \lambda_2, \ldots, \lambda_r\right\}$ represent sets of monotonically increasing grid points that span these intervals. Next, make an initial guess for the value of the function $z = z\left[b(\lambda), k, \lambda\right]$, for $z = V$, J, $b'\left(\lambda^{1'}\right), \ldots, b'\left(\lambda^{q'}\right)$, k', ℓ, h, and $p\left(\lambda^{1'}\right), \ldots, p\left(\lambda^{q'}\right)$, at each of the mnr points in the set $B \times K \times L$. Denote the value for the initial guess of the function z at the point $\left[b_h, k_i, \lambda_j\right]$ by $z^0(b_h, k_i, \lambda_j)$. A guess for z at other points in its domain $\left[b_1, b_m\right] \times \left[k_1, k_n\right] \times \left[\lambda_1, \lambda_r\right]$ is then constructed through multilinear interpolation (see Press, et. al. (1986)). Specifically, take some point $\left[b(\lambda), k, \lambda\right] \in \left[b_1, b_m\right] \times \left[k_1, k_n\right] \times \left[\lambda_1, \lambda_r\right]$. The value of the function z^0 at the point $\left[b(\lambda), k, \lambda\right]$ or $z^0\left[b(\lambda), k, \lambda\right]$ is defined as follows:

$$\begin{aligned} z^0\left[b(\lambda), k, \lambda\right] = {} & (1-u)(1-v)(1-w)z^0\left[b_h, k_i, \lambda_j\right] + u(1-v)(1-w)z^0\left[b_{h+1}, k_i, \lambda_j\right] \\ & + uv(1-w)z^0\left[b_{h+1}, k_{i+1}, \lambda_j\right] + (1-u)v(1-w)z^0\left[b_h, k_{i+1}, \lambda_j\right] \\ & + (1-u)(1-v)wz^0\left[b_h, k_i, \lambda_{j+1}\right] + u(1-v)wz^0\left[b_{h+1}, k_i, \lambda_{j+1}\right] \\ & + uvwz^0\left[b_{h+1}, k_{i+1}, \lambda_{j+1}\right] + (1-u)vwz^0\left[b_h, k_{i+1}, \lambda_{j+1}\right] \end{aligned} \tag{5.2}$$

where the weights u, v, and w are given by

$$u = \frac{b(\lambda) - b_h}{b_{h+1} - b_h}, \quad v = \frac{k - k_i}{k_{i+1} - k_i}, \quad \text{and} \quad w = \frac{\lambda - \lambda_j}{\lambda_{j+1} - \lambda_j}$$

and the grid points b_h, b_{h+1}, k_i, k_{i+1}, λ_j, and λ_{j+1} being chosen such that

$$b_h \leq b(\lambda) \leq b_{h+1}, \quad k_i \leq k \leq k_{i+1}, \quad \lambda_j \leq \lambda \leq \lambda_{j+1}$$

Thus, the interpolated value of z^0 at $\left[b(\lambda), k, \lambda\right]$ is simply taken to be a weighted average of its values at the eight nearest grid points. Note that the interpolated function z^0 is continuous on $\left[b_1, b_m\right] \times \left[k_1, k_n\right] \times \left[\lambda_1, \lambda_r\right]$.

Given initial guesses for the functions V, J, $\mathbf{b}'$, k', ℓ, h, and $\mathbf{p}$, denoted by V^0, J^0, $\mathbf{b}^{0'}$, $k^{0'}$, ℓ^0, h^0, and $\mathbf{p}^0$ respectively, it is straightforward to compute revised guesses V^1, J^1, $\mathbf{b}^{1'}$, $k^{1'}$, ℓ^1, h^1, and $\mathbf{p}^1$. Note that the q components of the vector functions $\mathbf{b}^{0'}$ and $\mathbf{p}^0$ are simply $b^{0'}(\lambda^{1'}), \ldots, b^{0'}(\lambda^{q'})$, and $p^0(\lambda^{1'}), \ldots, p^0(\lambda^{q'})$. Now for each point $(b_h, k_i, \lambda_j) \in B \times K \times L$ values for $V^1(b_h, k_i, \lambda_j)$, $J^1(b_h, k_i, \lambda_j)$, $\mathbf{b}^{1'}(b_h, k_i, \lambda_j)$, $k^{1'}(b_h, k_i, \lambda_j)$, $\ell^1(b_h, k_i, \lambda_j)$, $h^1(b_h, k_i, \lambda_j)$, and $\mathbf{p}^1(b_h, k_i, \lambda_j)$ can be computed by solving the nonlinear system of equations shown below for V, J, $\mathbf{b}'$, k', ℓ, h, and $\mathbf{p}$,

$$\begin{aligned}\Delta\left[b_h, k_i, V, J, \mathbf{b}', k', \ell, h, \mathbf{p}; \lambda_j\right] \\ = \sum_{d=1}^{q} \Omega\left[b'(\lambda^{d'}), k', V^{0'}, J^{0'}, \mathbf{b}^{0''}, k^{0''}, \ell^{0'}, h^{0'}, \mathbf{p}^{0'}; \lambda_j, \lambda^{d'}\right].\end{aligned} \tag{5.3}$$

where, for instance, $V^{0'} = V^0\left(b'(\lambda^{d'}), k', \lambda^{d'}\right)$ and $\mathbf{b}^{0''} = b^{0'}\left(b'(\lambda^{d'}), k', \lambda^{d'}\right)$. This represents a system of $2q + 5$ equations in $2q + 5$ unknowns which can be solved numerically using Newton's method. In practice a generalized secant method is employed first to obtain good initial guesses for Newton's procedure (see Ortega and Rheinboldt (1970) for more details). Given values for k', b', and λ' at each of the mnr grid points in $B \times K \times L$, the functions V^1, J^1, $\mathbf{b}^{1'}$, $k^{1'}$, ℓ^1, h^1, and $\mathbf{p}^1$ can be extended over the entire domain $\left[b_1, b_m\right] \times \left[k_1, k_n\right] \times \left[\lambda_1, \lambda_r\right]$ via interpolation, as was done previously. The functions V^1, J^1, $\mathbf{b}^{1'}$, $k^{1'}$, ℓ^1, h^1, and $\mathbf{p}^1$ are then used as guesses on the next iteration, with the whole procedure being repeated until the decision-rules have converged.

Once the decision-rules have been obtained the model can be simulated and various sample statistics for variables of interest computed. This is discussed in further detail later on.

6. Model Parameterization and Calibration

To begin with, let tastes and technology be specified in the following way:

$$U(c, 1-\ell) = \frac{[c^{\omega}(1-\ell)^{1-\omega}]^{1-\gamma}}{1-\gamma}, \quad 0 \le \omega \le 1, \quad \gamma > 1 \tag{6.1}$$

$$Z(x, 1-h) = \frac{\left[x^{\theta}(1-h)^{1-\theta}\right]^{1-\varphi}}{1-\varphi}, \quad 0 \le \theta \le 1, \quad \varphi > 1 \tag{6.2}$$

$$\nu(c, 1-\ell) = \ell n\left[1 + \eta c^{\omega}(1-\ell)^{1-\omega}\right], \quad \eta \ge 0 \tag{6.3}$$

$$\phi(x, 1-h) = \ell n\left[1 + \mu x^{\theta}(1-h)^{1-\theta}\right], \quad \mu \ge 0 \tag{6.4}$$

$$F(k, n\ell, h; \lambda) = \lambda k^{\alpha}\left[\iota(n\ell)^{\kappa} + (1-\iota)h^{\kappa}\right]^{(1-\alpha)/\kappa}, \quad 0 \le \alpha, \iota \le 1, \quad \kappa \le 1 \tag{6.5}$$

Observe that the functions U and Z are negative and have the property that $\ell n(-U)$ and $\ell n(-Z)$ are strictly convex.

Observe that equations (6.4) and (6.5) imply

$$e^{-\nu(c,1-\ell)} = \frac{1}{1 + \eta c^{\omega}(1-\ell)^{1-\omega}} \tag{6.6}$$

and

$$e^{-\phi(x,1-h)} = \frac{1}{1 + \mu x^{\theta}(1-h)^{1-\theta}} \tag{6.7}$$

Consequently, constant discount rates are a special case in which $\eta = \mu = 0$.[11] Recalling that the time-varying discount rate reflects an impatience effect, it can be seen that the larger is $c^{\omega}(1-\ell)^{1-\omega}$, the smaller is $e^{-\nu(c,1-\ell)}$, and so workers will in fact discount future utility more heavily. Similarly so for the entrepreneur.

The stochastic process (2.2) governing the technology shock will now be parameterized. In particular, suppose that the λ-process's innovation, ϵ, is described by a two-state Markov process. Specifically, ϵ is assumed to have a value lying in the time-invariant two-point set

$$A = \left\{e^{\xi}, e^{-\xi}\right\}$$

[11] Given the current forms of (6.6) and (6.7), the entrepreneurs' and workers' discount factors converge to unity as η and μ approach zero. Observe that any values for the limiting discount factors can be obtained by setting the numerators of these expressions to the desired numbers.

with the following probabilities:

$$\text{prob}\left\{\epsilon = e^{\xi}\right\} = \frac{1}{2} \text{ and } \text{prob}\left\{\epsilon = e^{-\xi}\right\} = \frac{1}{2}.$$

With this parameterization (the logarithm of) the λ-process has a standard deviation given by $\sigma = \xi/\sqrt{(1-\rho^2)}$ and a serial correlation coefficient of ρ.

In order to simulate the model, values must be assigned to the parameters shown below:

Utility-Workers:	η, ω, γ
Utility-Entrepreneurs:	μ, θ, φ
Technology	α, κ, ι, δ, n, σ, and ρ

So as to impose some discipline on the simulation experiments being conducted, the calibration procedure advanced by Kydland and Prescott (1982) is adopted. In line with this approach as many model parameter values as possible are set in advance based upon either (a) a priori information about their magnitudes, or (b) so that in the model's deterministic steady-state, values for various endogenous variables assume their average values for the postwar U.S. economy, based upon quarterly data for the 1954–1989 sample period.

To begin with, capital's share of income, or α, was set to 0.36, its average quarterly value over the 1954–1989 sample period. The depreciation rate, δ, was chosen to be 0.025, the value used by Kydland and Prescott (1982). The parameters κ and ι specify how the two types of labour input are aggregated in the production function. A value of 0.5 was picked for κ and ι. This configuration of parameter values implies that in the model's steady-state, the value of an entrepreneur's time is about ten times as high as a worker's. Unfortunately, no evidence was found to check on the validity of this choice of values for κ and ι. The parameters σ and ρ specifying the λ-process's standard deviation and autocorrelation can be determined by computing the Solow residuals for an aggregate production function from the U.S. data. Using quarterly data for the postwar period, Prescott (1986) reports values for σ and ρ of 0.0244 and 0.95, respectively. These numbers were used here to calibrate the process governing technical changes.

Next, the parameters ω, η, μ, and θ were chosen so that the model's deterministic steady-state satisfies four restrictions. The first two restrictions constrain the ratio of working to total hours for workers and entrepreneurs to be 0.24. This number corresponds to the average ratio of hours worked to total nonsleeping hours of the working age population observed in the U.S. data. The third restriction sets the steady-state real interest rate (at the quarterly level) to be 0.01. Finally, the last restriction specifies that the wealthiest 1 percent of the population holds 25 percent of aggregate wealth. This number conforms with statistics on this century's wealth distribution for the U.S.—see Wolff and Marley (1989). If entrepreneurs are viewed as comprising the upper 1 percent of the wealth distribution, then n should equal 99. Again, the values for ω, η, μ, and θ were picked so that the model's steady-state satisfied these four restrictions.

Specifically, given the current parameterization for tastes and technology the steady-state analogues to equations (3.2), (3.3), (3.4), (3.7), (3.8), (3.10), and (3.11) are

$$p = \frac{1}{1+\eta c^{\omega}(1-\ell)^{1-\omega}} \qquad (6.8)\ [cf.(3.3)]$$

$$(1-\alpha)k^{\alpha}\left[\iota(n\ell)^{\kappa}+(1-\iota)h^{\kappa}\right]^{(1-\alpha-\kappa)/\kappa}\iota(n\ell)^{\kappa-1}\omega(1-\ell)=(1-\omega)c \qquad (6.9)\ [cf.(3.4)]$$

$$c+[p-1]b=(1-\alpha)k^{\alpha}\left[\iota(n\ell)^{\kappa}+(1-\iota)h^{\kappa}\right]^{(1-\alpha-\kappa)/\kappa}\iota n^{\kappa-1}\ell^{\kappa} \qquad (6.10)\ [cf.(3.2)]$$

$$1=\frac{\alpha k^{\alpha-1}\left[\iota(n\ell)^{\kappa}+(1-\iota)h^{\kappa}\right]^{(1-\alpha)/\kappa}+(1-\delta)}{1+\mu x^{\theta}(1-h)^{1-\theta}} \qquad (6.11)\ [cf.(3.7)]$$

$$p=\frac{1}{1+\mu x^{\theta}(1-h)^{1-\theta}} \qquad (6.12)\ [cf.(3.8)]$$

$$(1-\alpha)k^{\alpha}\left[\iota(n\ell)^{\kappa}+(1-\iota)h^{\kappa}\right]^{(1-\alpha-\kappa)/\kappa}(1-\iota)h^{\kappa-1}\theta(1-h)=(1-\theta)x \qquad (6.13)\ [cf.(3.10)]$$

$$nc+x+\delta k=k^{\alpha}\left[\iota(n\ell)^{\kappa}+(1-\iota)h^{\kappa}\right]^{(1-\alpha)/\kappa} \qquad (6.14)\ [cf.(3.11)]$$

The four restrictions from the long-run data discusses above imply

$$\ell = 0.24 \tag{6.15}$$

$$h = 0.24 \tag{6.16}$$

$$p = 1/1.01 \tag{6.17}$$

$$k - 99pb = 0.25k \tag{6.18}$$

Given values for α and δ this system of eleven equations in eleven unknowns can be thought of as determining a solution for the eleven unknowns c, ℓ, b, x, k, h, p, ω, η, μ, and θ. The parameter values obtained for ω, η, μ, and θ are 0.26, 0.0195, 0.0095 and 0.31.[12]

Two parameters remain to be specified: the coefficients of relative risk aversion, γ and φ, for workers and entrepreneurs. The value of this parameter is somewhat controversial, but Prescott (1986) suggests that the weight of the evidence places it not too far from 1.0. In line with this, a value of 1.5 was picked for both entrepreneurs and workers.

[12] A few words on the steady-state distribution of wealth between workers and entrepreneurs might be in order. Note that given values for the parameters η, ω, γ, μ, θ, φ, α, κ, ι, δ, and n, the system of seven equations (6.8) to (6.14) determines a solution for the seven unknowns c, x, k, ℓ, h, p and b. Now, instead, consider a version of the model where agents have a constant discount factor, β. In this case (6.11) reads $1/\beta = \alpha k^{\alpha-1}[\iota(n\ell)^{\kappa} + (1-\iota)h^{\kappa}]^{(1-\alpha)/\kappa} + 1 - \delta$. Equations (6.8) and (6.12) both collapse to $p = \beta$ so that one of them, say (6.8), can be discarded. The rest of the system remains the same. Here the system of six equations (6.9) to (6.14) determine a solution for the six variables c, x, k, ℓ, h and p, *given a value for* b. Thus, it is easy to see that when discount factors are constant, the deterministic version of the model does not possess a unique, invariant steady-state distribution of wealth across entrepreneurs and workers. The long-run distribution of income will depend upon the initial distribution of wealth. The endogenous discount factor allows convergence to a unique steady-state wealth distribution in the following way: When an agent has a level of wealth below (above) his steady-state level, his discount factor is high (low). This encourages (discourages) savings so that his asset holdings increase (decrease) over time to their long-run level. This lets the model possess a stable long-run distribution of wealth across agents, as is observed in the U.S. data.

7. Results

The model's implications for the cyclical pattern of comovements among macroaggregates will now be investigated. The variables targeted for study are output, consumption, investment, hours, productivity, labour's share of income, and wages. Table 1 presents some stylized facts that characterize U.S. business cycles for the 1954–1989 sample period. The statistics reported are based on quarterly data which has been detrended using the Hodrick-Prescott (H-P) filter. The corresponding statistics for the model are shown in Table 2. The model's statistics were constructed in the following manner: First, the equilibrium decision rules for both the entrepreneur and worker were computed using the algorithm described in Section 5. Next, 5,000 artificial samples of 144 observations each (the number of quarters in the [illegible] 1989 sample period) were generated by simulating these decision-rules. The data collected from each sample was then detrended using the H-P filter. Finally, these sample moments were averaged over the 5,000 simulations undertaken.

The functions characterizing the model's general equilibrium were interpolated over grids with five points for each of the three state variables: capital, claims, and the technology shock.[13] Figure 5 illustrates the value function for the entrepreneur, which is drawn in the capital-claims space holding fixed the value of the technology shock at one. This value function is increasing in the amount of capital owned by entrepreneurs and decreasing in the amount of debt owed to workers; it is also strictly concave. The worker's value function is shown in Figure 6. It is increasing in his level of wealth (claims). Somewhat surprisingly, however, it is not increasing in the capital stock. Neither is it strictly concave. This can be readily explained as follows:

[13] Each of the reported $5 \times 5 \times 5$ grid experiments took about 250 minutes of CPU time on an Amdahl 580 computer (Federal Reserve Bank of Minneapolis). Some rough comparisons suggested that this machine is an order of magnitude faster than either a Cray-2 (Institute for Empirical Macroeconomics) or a VAX 3900 (University of Western Ontario) for problems of this type. Running the model over finer grid significantly increases the (already high) computational time involved. Some limited testing suggested, however, that further refinements of the grids would not materially affect the moments being reported here.

First, recall that the capital stock is not a decision variable for the worker. Thus, there should be no presumption that his value function should be jointly concave in capital and claims. It is concave in his holdings of claims, however, as should be expected. Second, imagine increasing the economy's capital stock, holding fixed the state of technology and the quantity of claims held by workers. This has two opposing effects on the worker's welfare. One the one hand, his welfare increases since labour's marginal product rises. On the other hand, his welfare falls since a lower return is now earned on savings. The net effect is ambiguous.

A loose indication of the model's ability to mimic the observed pattern of postwar U.S. business cycle fluctuations can be obtained by comparing Tables 1 and 2. It should be said, however, that it would be unrealistic to place high expectations on such a patently simplistic abstraction. The first thing to note is that in the model, macroaggregates tend to vary too little. This is fairly typical of real business cycle models that calibrate the technology shock to the observed sample moments for Solow residuals. Clearly, there have been factors other than technology shocks that have affected macroaggregates in the postwar period. In the U.S. data, investment is much more volatile than output, and consumption much less so. The model mimics this feature but quantitatively exaggerates it. Another feature of the data is that investment is more highly correlated with output than is consumption. The model shares this feature, too.

Turning now to the behaviour of labour income over the business cycle, it can be seen in Table 2 that both measures of labour's share of income move countercyclically, i.e. are negatively correlated with output. The first measure, based on equation (4.2), includes workers' savings and tends to be far too countercyclical. Here, the correlation between labour's share of income and output is -0.98 as opposed to the value of -0.37 that characterizes the data. The second measure is constructed using (4.3) and attempts to net out workers' savings. Now, the correlation between

Table 1: Quarterly U.S. Data (1954–1989)

	Standard Deviation	First-order Autocorrelation	Correlation with Output
Output	1.7	0.85	1.00
Consumption	0.8	0.84	0.72
Investment	8.3	0.91	0.90
Hours	1.8	0.89	0.87
Productivity	0.9	0.70	0.06
Labour's Share of Income	0.8	0.70	-0.37
Real Wage	0.6	0.67	-0.40

Note: The data for all the series reported was logged and detrended using the Hodrick-Prescott filter. The original data used was expressed in 1982 dollars and deflated by the 16+ population. GNP, the GNP deflator, consumption (nondurable goods plus services), and gross investment were taken from the national income accounts. Labour's share of income was computed by dividing the compensation of employees by GNP, again both series being taken from the national income accounts. The hours data was constructed by multiplying total employment by average weekly hours, the later series being obtained from the *Current Population Survey*. Productivity is defined as output divided by hours, while the real wage was computed by dividing the compensation of employees by hours. The data series were taken from the *Fame Economic Database* of the Board of Governors of the Federal Reserve System.

Table 2: Benchmark Model

		Standard Deviation	First-order Autocorrelation	Correlation with Output
Output		1.62	0.69	1.00
Consumption		0.42	0.86	0.75
Investment		5.43	0.68	0.99
Hours		1.00	0.68	0.98
Productivity		0.67	0.75	0.95
Labour's Share of Income	$\hat{w}\ell/y$	1.33	0.68	-0.98
	$\tilde{w}\ell/y$	1.42	-0.08	-0.43
Real Wage	$\hat{w}$	0.82	0.72	-0.81
	$\tilde{w}$	1.39	0.16	0.02

labour's share of income and output drops to -0.43, which is more in line with the evidence. Not surprisingly, then, the first measure of labour income implies a more countercyclical real wage than the second one does; the correlations with output are -0.81 and 0.02, respectively. If one takes the commonly held position that the real wage exhibits no strong cyclical pattern, then the second measure also performs better in this dimension. It may be the case that not all workers in the economy are covered by implicit labour contracts. Suppose, for example, that half are, with the rest selling their labour services on a spot market. Then the correlations between the real wage and output rise to -0.15 and 0.40, with those between labour's share of income and output remaining the same.

In at least one respect the implicit labour contracting story does not perform that well. For instance, by either measure labour's share of income and the real wage are a bit too volatile. To understand why, note that implicit labour contracts are designed not to stabilize the real wage paid to workers but instead to smooth out fluctuations in worker's utility. Now, suppose that labour income was held constant over the business cycle. Then labour's share of income would have the same percentage variability as output (1.62 in the model) while the real wage would vary as much as hours (1.00).[14] Additionally, when the second measure is used for labour income, the real wage rate and labour's share of income display no serial correlation, in contrast with the data. Here labour income has a large insurance component in it, which by its very nature varies unpredictably.

The role that the endogenous discount factor plays in the model will now be investigated. To do this, the baseline model will be rerun with less variable discount factors. Observe, from expressions (6.6) and (6.7), that as η and μ approach 0 the model collapses to the constant discount factor case. For the model to keep the same

[14] Let labour income be constant, say at some value C. Then labour's share of income would then be given by C/y while the real wage would read $C/n\ell$. Consequently, $\text{var}(\ell n(C/y)) = \text{var}(\ell n(y))$ and $\text{var}(\ell n(C/n\ell)) = \text{var}(\ell n(n\ell))$.

real interest rate in its deterministic steady state, the numerators of these expressions must also be adjusted simultaneously so that they approach 0.99. Values of 0.000975 and 0.000473 were chosen for η and μ, and the numerators of (6.6) and (6.7) were set to 0.9905.[15] This configuration of parameter values preserves the steady-state equilibrium outlined in Section 6.

The results of this simulation exercise are reported in Table 3. Most macroaggregates now become less volatile. For instance, the standard deviations of output, investment, and hours fall from 1.62, 5.43, and 1.00 percent to 1.47, 4.45, and 0.78. The variability of consumption, however, rises from 0.42 to 0.50. Also notice that the correlation of consumption with output increases from 0.75 to 0.93. In short, the effects of this experiment on macroaggregates is similar to a cut in the elasticity of intertemporal substitution in the standard real business cycle model. In other words, endogeneity in the discount factor operates here to increase the amount of intertemporal substitution in the model.[16]

Table 3: Discount Factor Experiment

		Standard Deviation	First-order Autocorrelation	Correlation with Output
Output		1.47	0.69	1.00
Consumption		0.50	0.76	0.93
Investment		4.45	0.68	0.99
Hours		0.78	0.68	0.98
Productivity		0.72	0.72	0.98
Labour's Share of Income	$\hat{w}\ell/y$	1.02	0.68	-0.98
	$\tilde{w}\ell/y$	0.03	-0.08	-0.34
Real Wage	$\hat{w}$	0.44	0.77	-0.70
	$\tilde{w}$	0.71	0.74	0.98

[15] More accurately the numerators of (6.6) and (6.7) were set to 0.9905 with the values for η and μ of 0.000975 and 0.000473 then being backed out from the calibration procedure that was described in Section 6.

[16] Also, as the discount factor becomes less variable the marginal density function for claims, or b, as measured by a histogram, spreads out. The mean of this distribution begins to drift away from the value of b found in the model's deterministic steady-state. See footnote 11.

It is easy to understand why endogeneity in the discount factor increases the amount of intertemporal substitution in the model. In Figure 7 the behaviour of the worker's discount factor over the business cycle is plotted for one simulation run. As can be seen, the discount factor moves procyclically. Thus, in times when the technology shock is good, the discount factor is high. This works to entice investment and labour effort in booms, and to dissuade consumption. The reverse happens in slumps. It may seem somewhat surprising, though, that the worker's discount factor moves countercyclically. Recall that the worker's discount factor is a decreasing function of his momentary utility. Consequently, in order for the worker's discount factor to be procyclical, it must be the case that his momentary utility is countercyclical. Figure 8 shows the countercyclical movement in momentary utility. In booms the gain in momentary utility from increased consumption is more than offset from the loss due to increased labour effort. Also, note in Figure 9 that the worker's expected lifetime utility (his value function) moves procyclically. Therefore in booms the worker sacrifices utility today for expected utility tomorrow.

One last observation on the role of the endogenous discount factor. A puzzle in the data is that hours tend to vary twice as much as productivity over the business cycle. As has been noted in Hansen (1985), in real business cycle models hours tend to fluctuate about the same amount as productivity. Hansen (1985) resolves this puzzle by introducing an indivisibility into agents' decisions about how much to work. This amplifies the responsiveness of hours worked to a shock in the economy. In fact, in Hansen's model indivisible labour makes hours too variable in the sense that they fluctuate two-and-a-half times as much as productivity. In the baseline version of the model, the variation in hours is one-and-a-half times that of productivity. Observe that when the degree of endogeneity in the discount factor is cut, this ratio falls to unity.[17] An endogenous discount factor enhances the responsiveness of hours worked

[17] The model performs poorly on one dimension here. In particular, the correlation between output and productivity is too high (0.95 versus 0.06 in the data).

to the current state of technology in much the same way as the nonseparability in preferences (for leisure across time) used in Kydland and Prescott (1982).

Finally, some observations will be made about the cyclical allocation of risk. While it is difficult to quantify the amount of cyclical risk that gets shifted in competitive equilibrium from workers to entrepreneurs, an attempt will be made to do so anyway. As a starting point, consider a world where all agents are identical. Here, no insurance against cyclical risk would be bought or sold. Each agent would own an amount of capital equal to the per capita stock of capital. Next, suppose that in the current model no insurance gets traded in competitive equilibrium between workers and entrepreneurs. Then each worker would buy from an entrepreneur a portfolio of contingent claims that exactly replicates the return on some quantity of capital that the worker desires to hold. For example, suppose that the worker desires to purchase today a portfolio of contingent claims that will generate the same return tomorrow as $Q'(s)$ units of capital. This can be done by arranging his asset holdings such that $b'(\lambda'; s) = \left[F_1\left(k'(s), n\ell(s'), h(s'); \lambda'\right) + 1 - \delta\right] Q'(s)$. As was shown in Section 4, this portfolio costs $Q'(s)$ units of output, the same price the worker would pay if he purchased the physical capital directly himself. How much capital would each worker hold? A reasonable working hypothesis might be that he holds $Q'(s) = (f/n)k'(s)$ units of capital, where f represents the long-run fraction of the economy's wealth held by workers and n is the number of workers. Under this assumption each worker would hold a portfolio of contingent claims, $b^{*\prime}(\lambda'; s)$, such

As Christiano and Eichenbaum (1990) note, this is a feature of real business cycle models that rely solely on technology shocks as a source of fluctuations. Given the standard specification of technology, average labour productivity is proportional to the marginal product of labour, with the latter moving procyclically by construction. Here, technology shocks can be thought of as affecting the demand side of the labour market. This problem can be resolved by adding other shocks, such as innovations to government spending or shifts to tastes, into the model. These shocks operate on the supply side of the labour market. Including such shocks here would increase the computational burden of a model which is already computer-intensive— see footnote 12. In the standard real business cycle paradigm with such shocks, however, labour's share of income would still be constant over the business cycle.

that $b^{*\prime}(\lambda';s) = \left[F_1\left(k'(s), n\ell(s'), h(s'); \lambda'\right) + 1 - \delta\right](f/n)k'(s)$. The above discussion suggests that a useful metric of the amount of insurance traded in equilibrium might be

$$\int \left| \frac{b^{*\prime}(\lambda';s) - b'(\lambda';s)}{[b^{*\prime}(\lambda';s) + b'(\lambda';s)]\,\zeta(\lambda'|s)} \right| \mathrm{d}\lambda' \mathrm{d}S(s),$$

where $S(s)$ represents the long-run distribution governing the state variables; i.e., $S(s)$ solves $S(s') = S(s'|s)\mathrm{d}S(s)$. This metric can be thought of as measuring the value of insurance transactions against cyclical risk as a fraction of the worker's wealth.

In the baseline model, insurance transactions against cyclical risk amount to about 0.8 percent of worker's wealth. This number is small, but that should not be surprising for three reasons: First, the variability in macroaggregates is small. For example, output and hours have standard deviations of 1.62 and 1.00 percent, while consumption's variability is even smaller, as reflected by its standard deviation of only 0.42. Second, both sets of agents share the same coefficient of relative risk aversion. This limits the amount of equilibrium risk shifting. The value of 1.5 chosen for the coefficient of relative risk aversion makes agents only moderately risk averse, in the sense that their momentary utility functions are only slightly more concave than the logarithmic case. Third, the amount of risk shifting possible is limited by the fact that entrepreneurs constitute only a small proportion of the population.

Two additional experiments were run to see how these factors affect the volume of insurance transactions. In the first, the coefficient of relative risk aversion for workers was increased to 10. At the same time the standard deviation for the innovation to the technology shock was increased from 0.77 to 0.94 percent; this was done to prevent the variability of output from falling.[18] With this new configuration of parameter values,

[18] As was mentioned above, as the coefficient of relative risk aversion is increased (or equivalently the elasticity of intertemporal substitution decreased) fluctuations in macroaggregates diminish. This can be compensated for by increasing the amount of volatility in the technology shock. There are limits on this process since eventually the standard deviation for the technology shock will depart too far from what is

output's standard deviation remained the same at 1.62 percent, while consumption's and productivity's rose to 0.74 and 0.94, and the ones for investment and hours fell to 4.18 and 0.68. The volume of insurance transactions rose from 0.8 to about 2.9 percent of worker's wealth. In the second experiment, entrepreneurs were taken to comprise the upper 5 percent of the wealth distribution, rather than the top 1 percent. In line with the stylized facts on the distribution of wealth in the U.S., it was assumed that this segment of the population holds 50 percent of aggregate wealth. (Again see Wolff and Marley (1989)). Once again, workers were assumed to have a coefficient of relative risk aversion of 10. Not surprisingly, the variability of output, investment, and hours all rose slightly while that of consumption fell. This follows because the average coefficient of relative risk aversion across agents falls, since entrepreneurs are now more numerous. The value of insurance transactions in this case amounted to 3.8 percent of worker's wealth.

observed in the data. Also, Mehra and Prescott (1985) suggest that 10 is the upper bound on reasonableness for the coefficient of relative risk aversion.

8. Conclusions

A dynamic, stochastic general equilibrium model with heterogeneous agents was constructed to study the allocation of risk and the distribution of income over the business cycle. Specifically, in the model formulated there were two types of agents, namely workers and entrepreneurs. Entrepreneurs provided workers with insurance against cyclical risk. Agents had preferences in line with Epstein's (1983) notion of stationary cardinal utility. This allowed the model to possess a unique, invariant long-run distribution of wealth across agents. The constructed model was parameterized, calibrated, and simulated to see whether it could mimic some stylized facts of the postwar U.S. economy such as the countercyclical movement in labour's share of income over the business cycle and the acyclical behaviour of real wages.

The findings can be summarized as follows: It was found that optimal labour contracting could account, quantitatively, for the observed pattern of fluctuation in labour income. Measured labour income includes insurance and savings components that tend to move countercyclically over the business cycle, and which operate to counterbalance the procyclical movement in the marginal product of labour. The flow of transactions involving insurance against cyclical risk were measured to be about 1 to 4 percent of worker's wealth. The size of this number is limited by the fact that the amount of observed cyclical variability in macroaggregates for the postwar U.S. economy is small. The upper end of this range occurred when workers were made considerably more risk averse than entrepreneurs, and entrepreneurs were taken to be numerous enough to allow for a reasonably large insurance market to operate. Finally, the variable discount factor worked to increase the volatility in macroaggregates. The procyclical movement in the endogenous discount factor increased the amount of intertemporal substitution in the model by enticing investment and work effort in booms and discouraging them in slumps.

Figure 1
Labour's Share of GNP

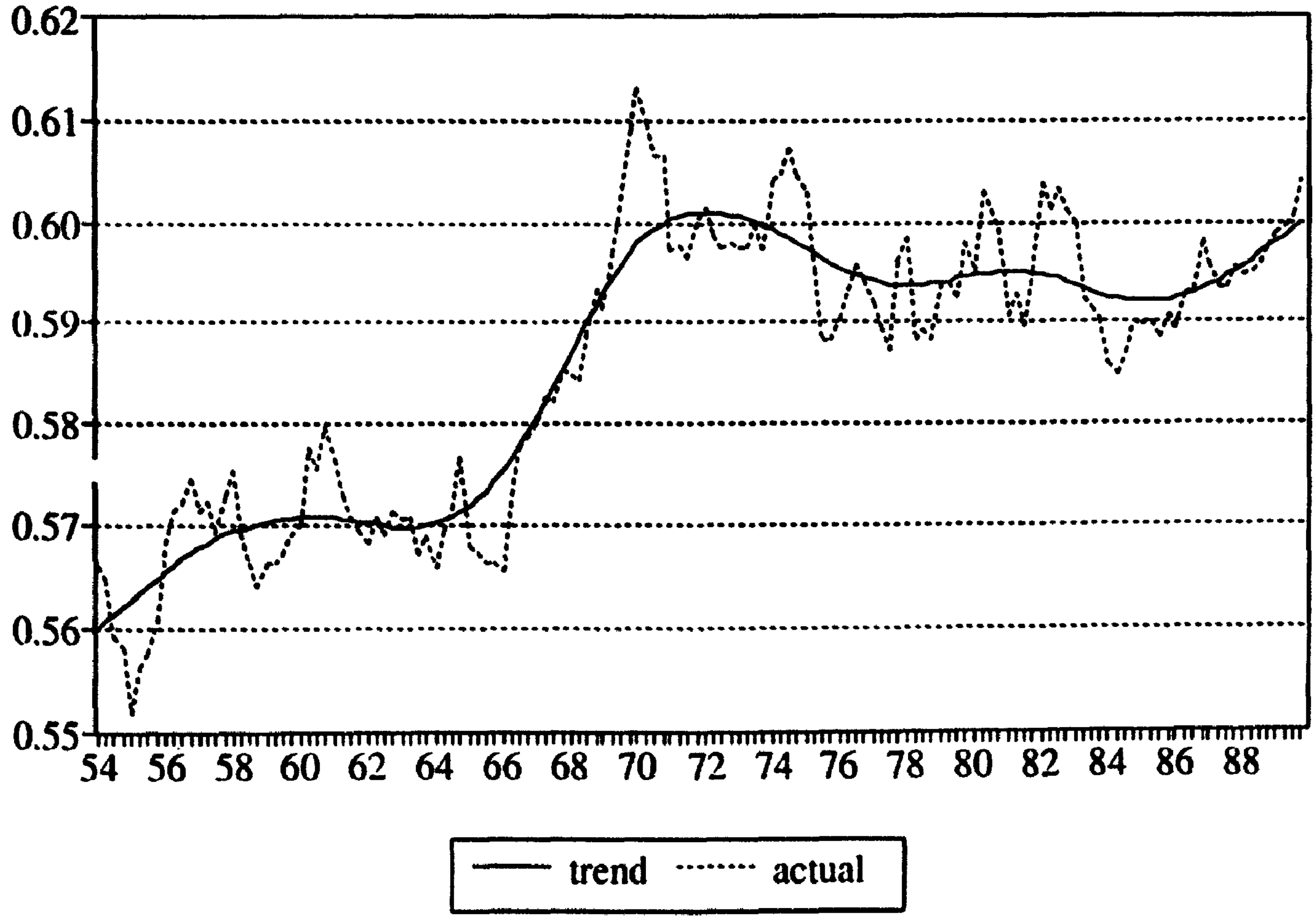


Figure 2

GNP and Labour's Share of GNP

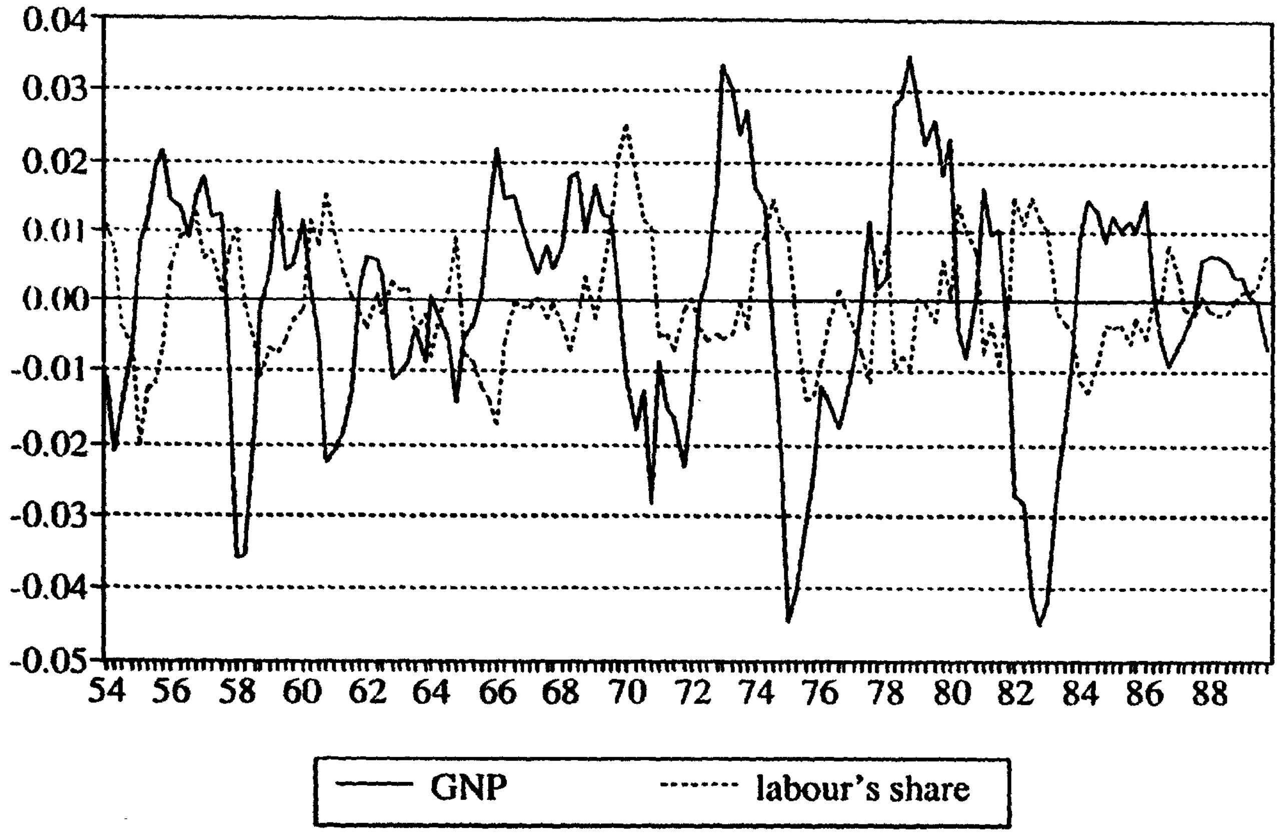


Figure 3
Real Wage

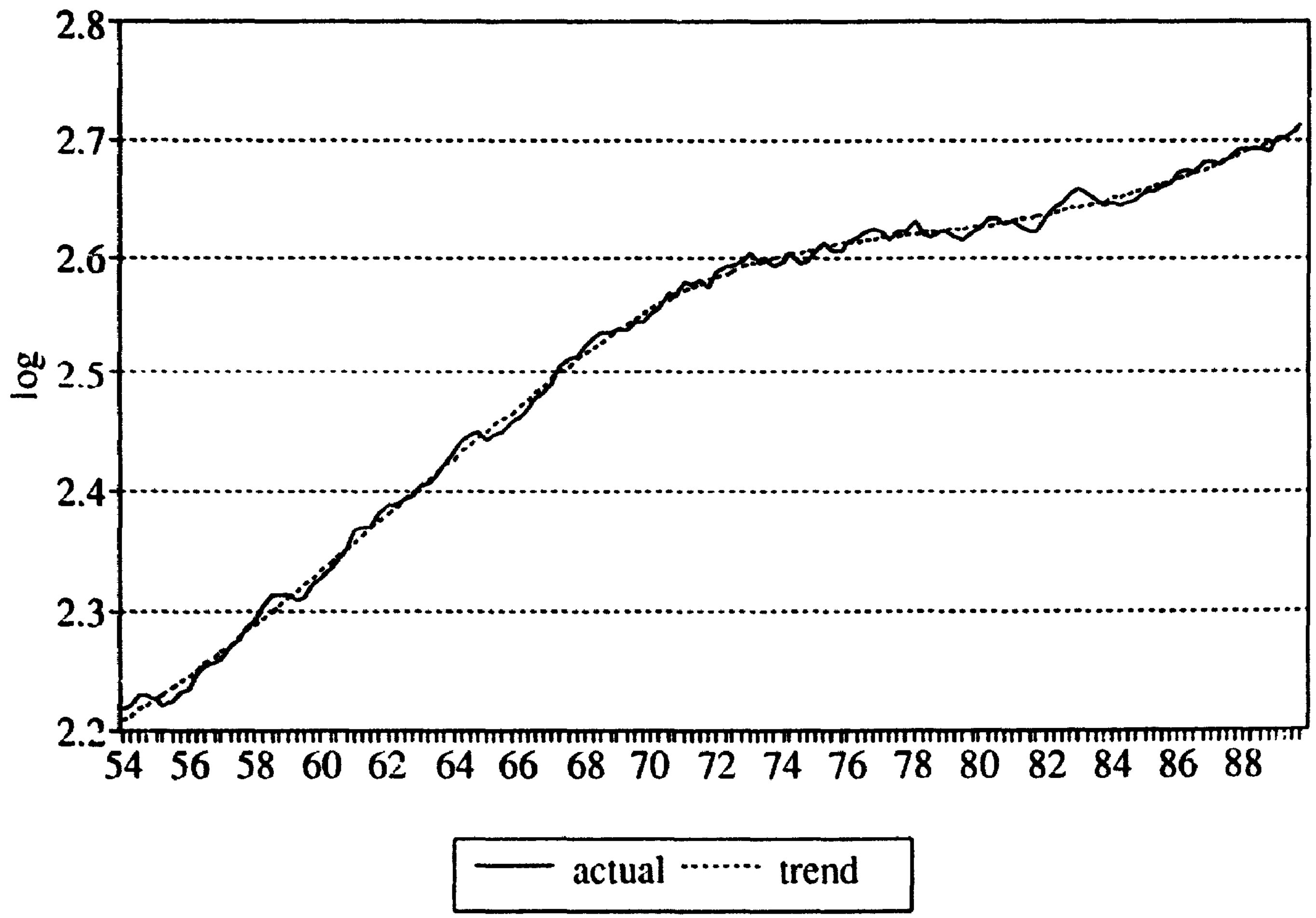


Figure 4
GNP and the Real Wage: Detrended

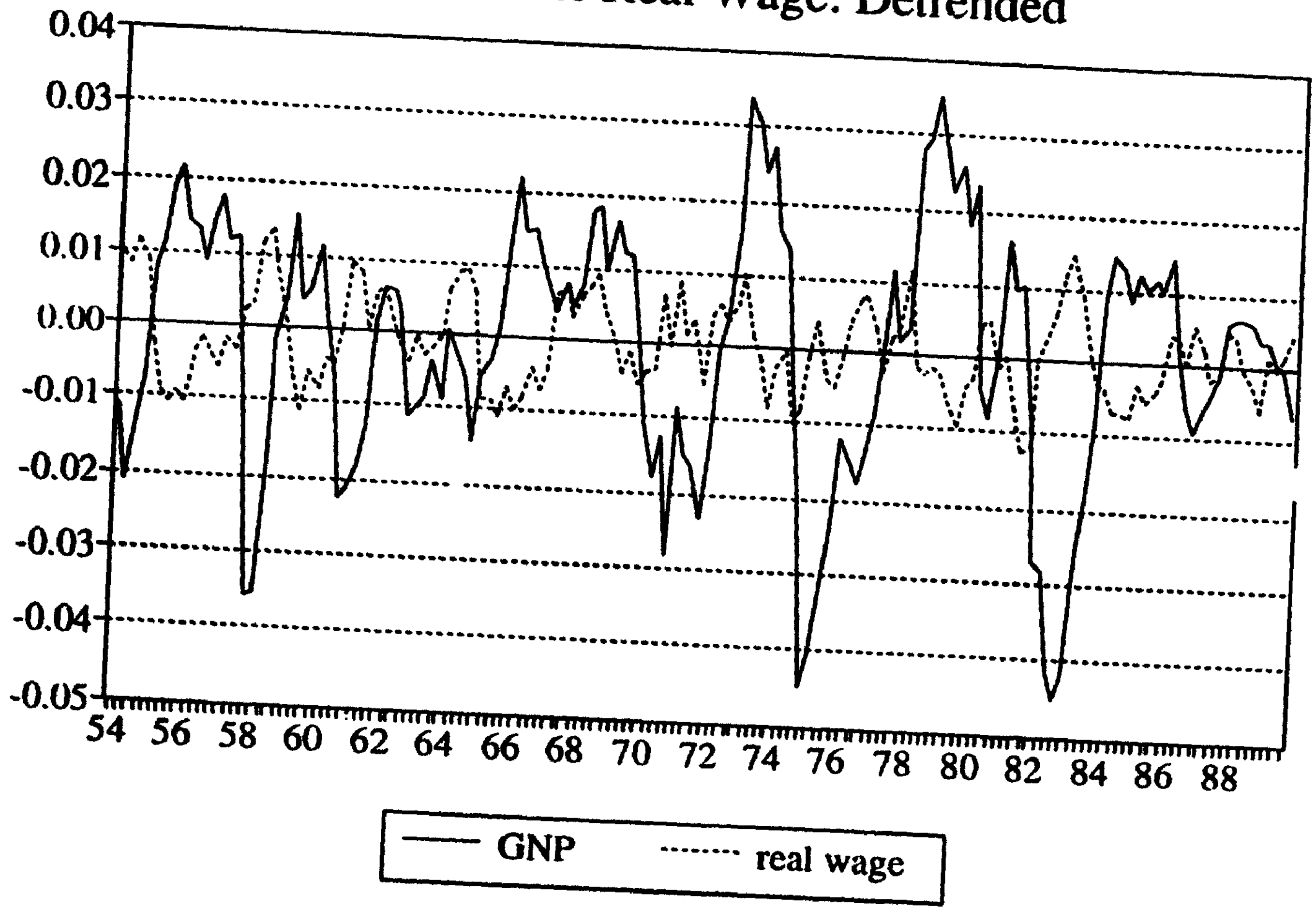
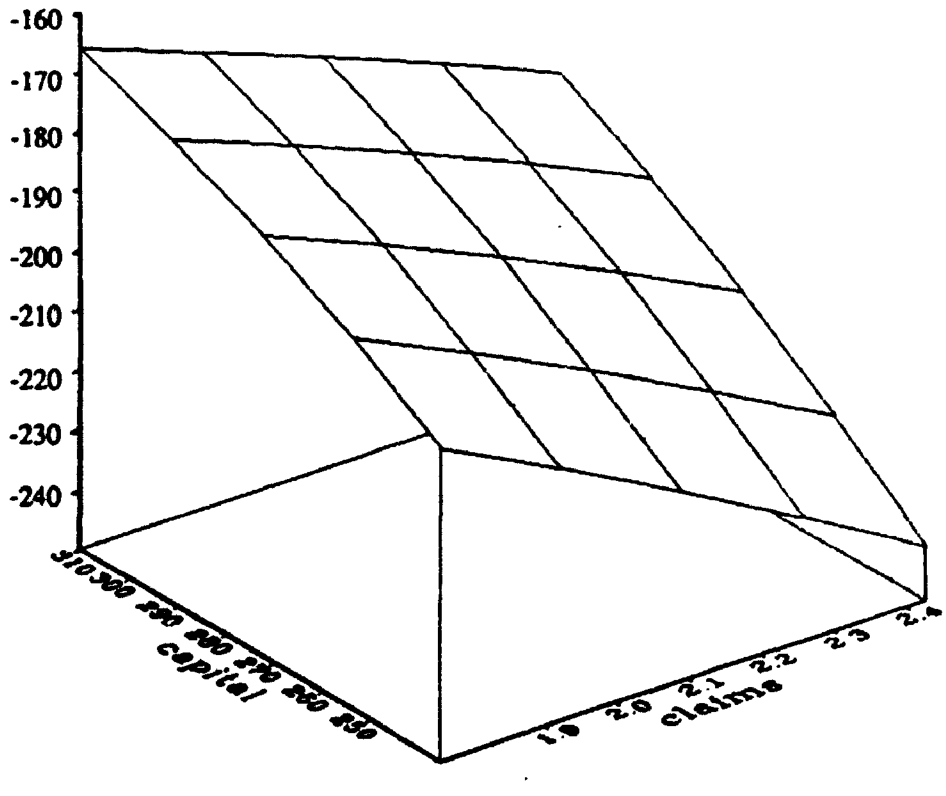
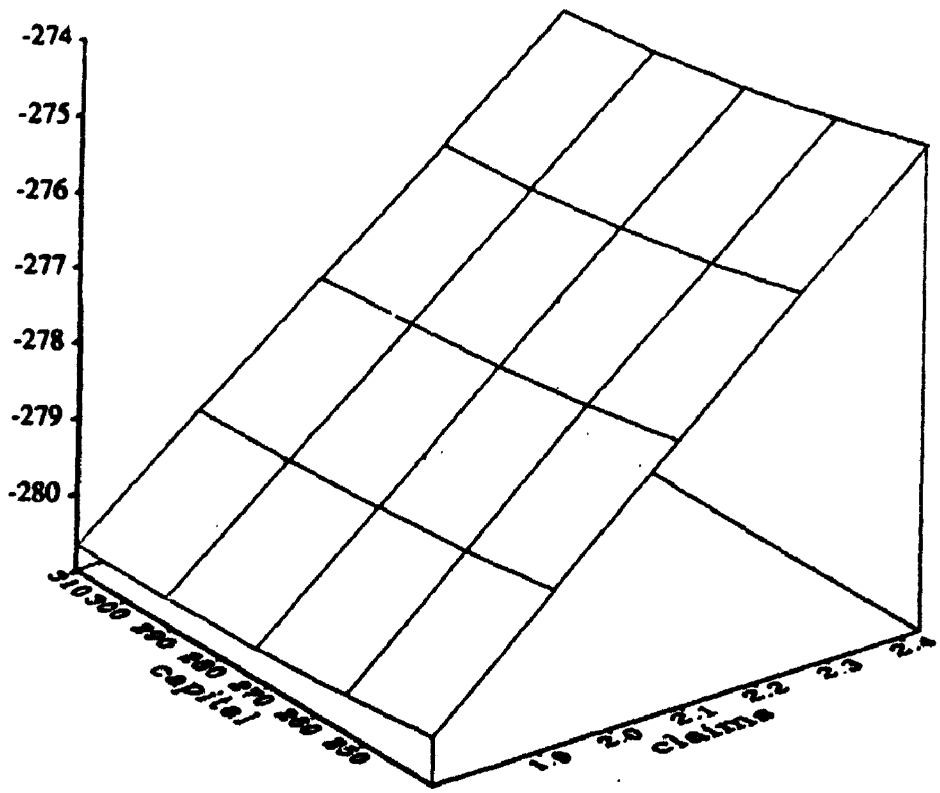


Figure 5
Entrepreneur's Value Function



(Technology Shock: λ = 1)

Figure 6
Worker's Value Function



(Technology Shock: λ = 1)

Figure 7
Worker's Discount Factor

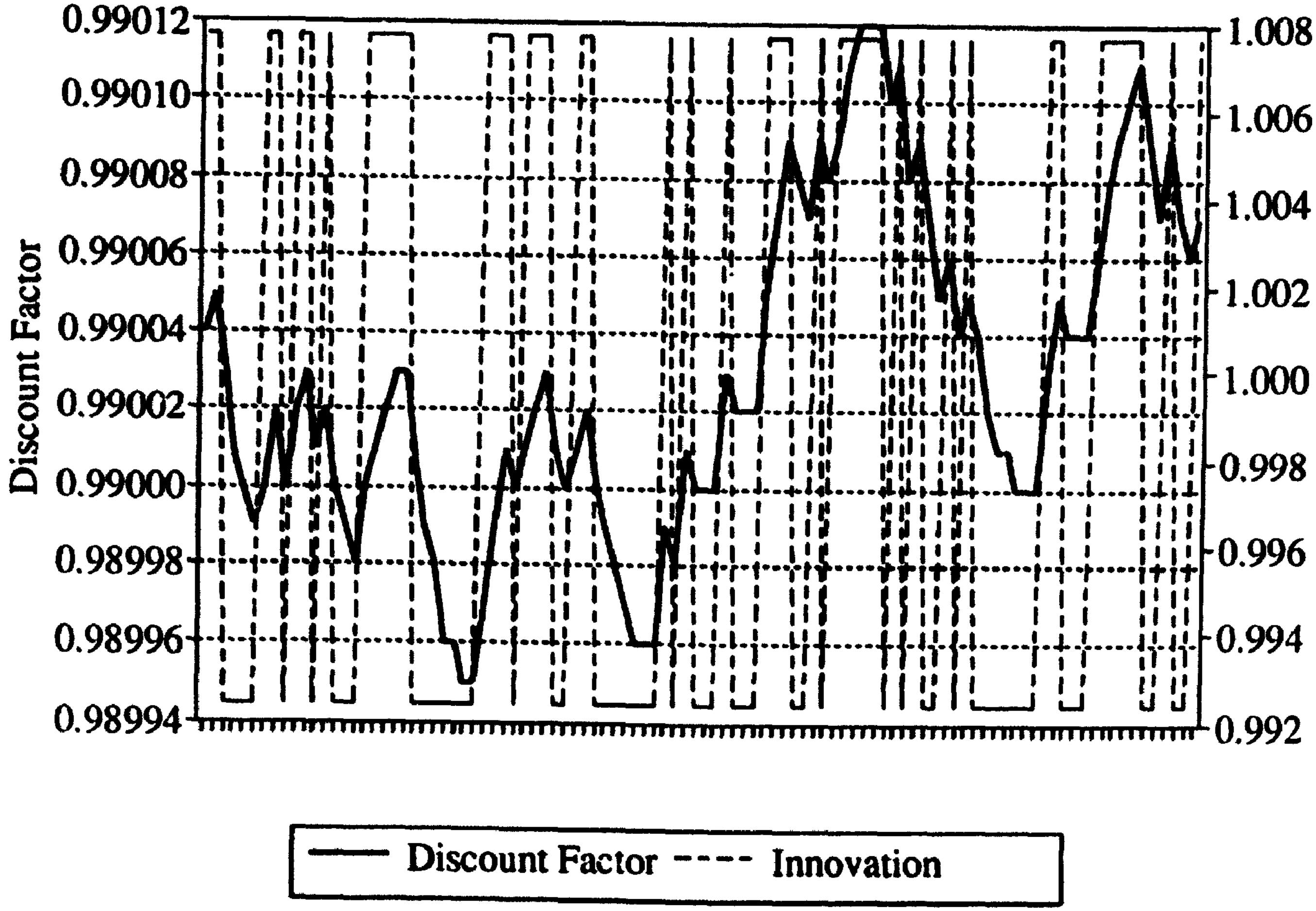


Figure 8

Worker's Momentary Utility

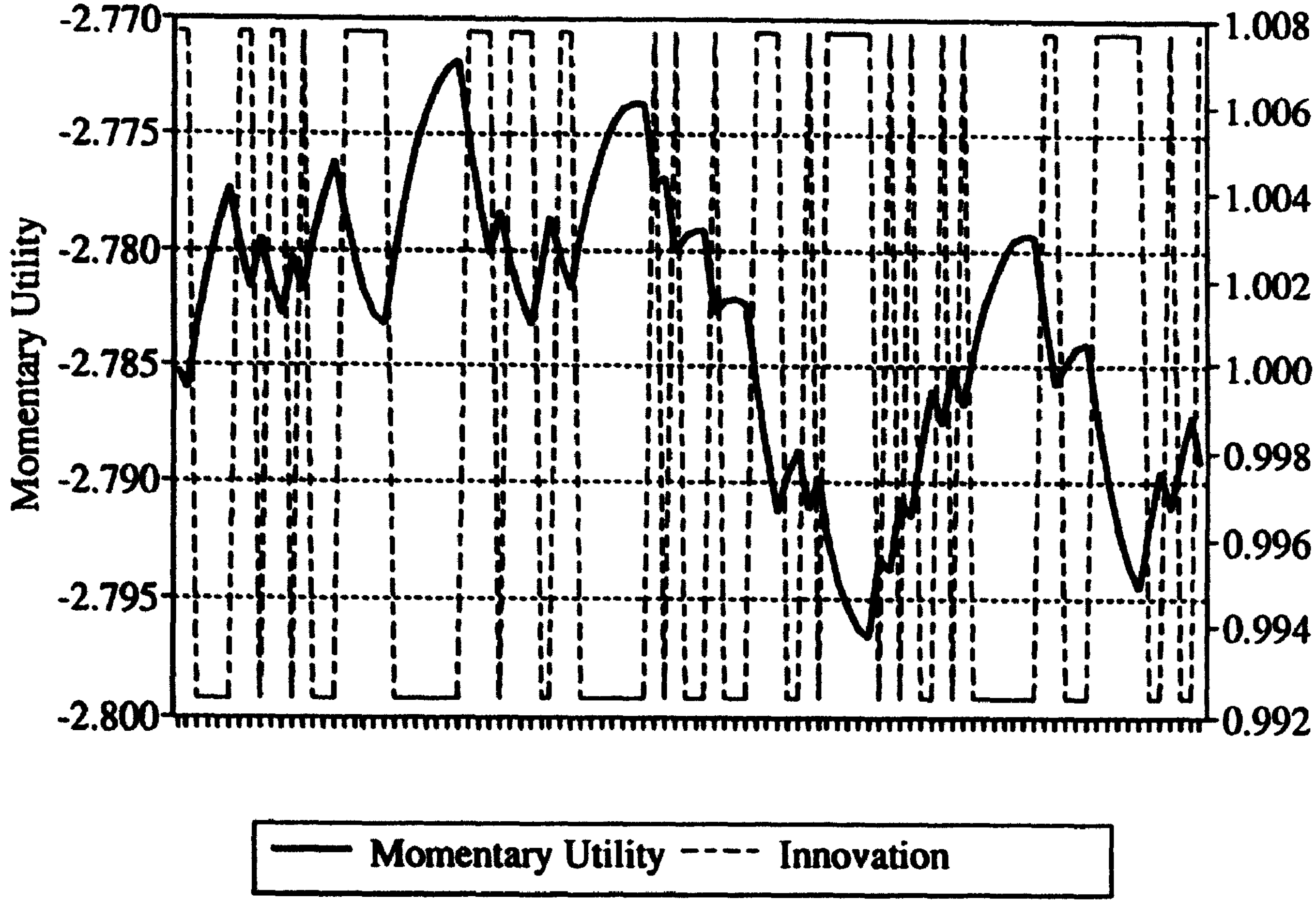
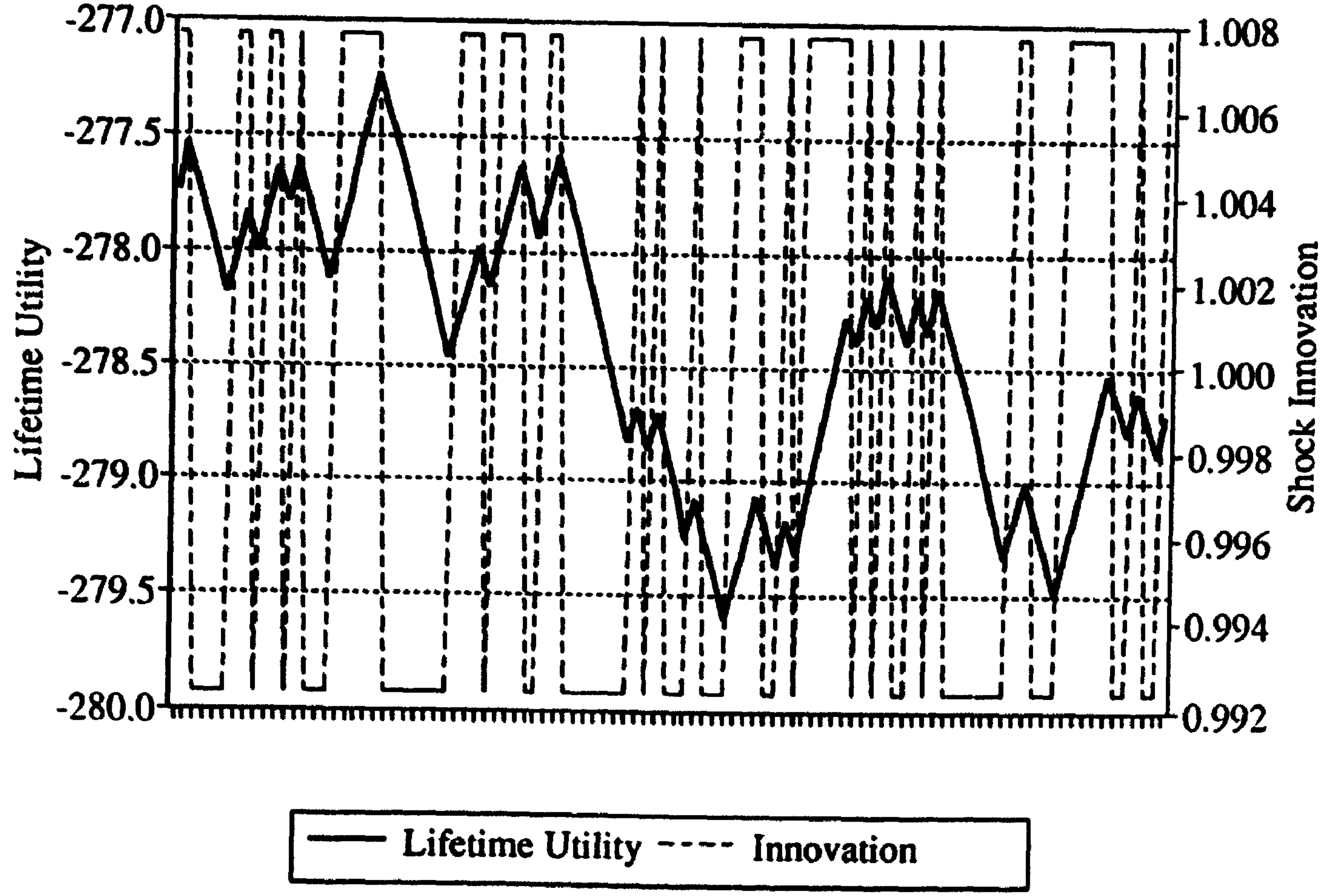


Figure 9
Worker's Lifetime Utility



Chapter 2: Money and Growth Revisited

1. Introduction

A widely held belief in economics is that if public policy can affect the economy's underlying growth rate, then alternative public policies will have large welfare implications. For example, Lucas (1987) estimates that consumers would be willing to part with up to 17% of their consumption (forever) to raise its growth rate from 3% to 4% per annum. King and Rebelo (1990) have used a real business cycle model with endogenous growth to analyze the effects of changes in the income tax rate. Raising this tax rate from 20% to 30%, roughly from the average Japanese tax rate from 1965 to 1975 to the average U.S. tax rate over the same period, results in a welfare loss in excess of 60% of consumption. Almost all of this welfare cost can be traced to the effects of the tax on growth. These numbers are large when compared with estimates of the losses arising from business cycle fluctuations. For example, Lucas (1987) calculates the gains from eliminating the cyclical variability in consumption to amount to no more than 0.1% of consumption, while Greenwood and Huffman (1991) place the potential benefits of business cycle stabilization at 0.5% of GNP. The question asked in this chapter is whether large welfare costs result from higher rates of inflation. Below, an endogenous growth model is presented in which higher long run inflation lowers growth, yet the welfare costs of inflation are very small.

International time series data provides some insight into the relationship, if any, between inflation and real growth. In Table 1, 62 of 82 countries exhibit a negative correlation between inflation and per capita real output growth.[19] This evidence is consistent with results in Backus and Kehoe (1989) showing that inflation is counter-

[19] Data for Table 1 was obtained from the International Financial Statistics (IFS) tape. Countries were included in Table 1 according to availability of the following data: real output, nominal output, the consumer price index, and population. Some countries were dropped due to very short time series. Grenada was removed due to a recorded population of zero; this is a limitation of the IFS data available on tape.

cyclical in the post-World War II period for the ten countries they consider.[20] While correlations do not imply causality, theories of inflation and real growth must at some point address the predominantly negative correlation seen in the international data.

There is a large literature incorporating money into the neoclassical growth model. In Tobin (1965), money competes with capital for a place in the portfolios of households. One prediction from Tobin's model is that money growth and capital are ***positively*** correlated. Sidrauski (1967), using a model with money in the utility function, develops long run superneutrality results. Stockman (1981) presents a model in which money growth and capital are ***negatively*** related when a cash-in-advance constraint applies to both consumption and investment. Money is superneutral in Stockman (1981) when consumption alone is subject to the cash-in-advance constraint.

In real business cycle models, as advanced by Kydland and Prescott (1982) and Long and Plosser (1983), money typically plays no role. An exception is Cooley and Hansen (1989a) who introduce money through a cash-in-advance constraint on consumption in an effort to assess the welfare costs of inflation. Higher inflation has the effects typically associated with a cash-in-advance constraint--see, for example, Aschaeur and Greenwood (1983) and Carmichael (1989). Specifically, higher inflation reduces the effective return to working since income earned in the current period cannot be spent until the next. This leads households to substitute leisure for labour, consequently reducing output and consumption.[21]

In steady state, Cooley and Hansen (1989a) report that a 10% inflation rate

[20] Backus and Kehoe (1989) examine business cycle behaviour of ten countries for which at least a century of data is available. The primary focus of their paper is on moments from Hodrick-Prescott filtered data, although results for growth rate filtering are included. Inflation is predominantly countercyclical in the pre-World War I and Interwar periods.

[21] These effects subsume the taxation effect of inflation emphasized by, for example, Stockman (1981).

Table 1: Inflation–Per Capita Real Growth Rate Correlations International Time Series Evidence

Country	Period	Correlation
Argentina	1959–1989	-0.05275
Australia	1950–1989	-0.35413
Austria	1976–1988	0.12366
Bahrain	1976–1988	0.11883
Bangladesh	1974–1988	0.34065
Belgium	1954–1988	-0.27764
Bolivia	1961–1984	-0.38552
Botswana	1975–1989	-0.48713
Brazil	1964–1988	-0.33534
Burundi	1971–1989	-0.15403
Cameroon	1970–1985	0.04952
Canada	1949–1988	-0.17897
Chile	1964–1989	-0.44053
Columbia	1969–1988	-0.28983
Costa Rica	1961–1989	-0.55981
Cyprus	1961–1988	-0.25148
Denmark	1951–1989	-0.45390
Dominican Republic	1964–1988	-0.13504
Ecuador	1966–1989	-0.23851
El Salvador	1952–1989	-0.42507
Fiji	1970–1988	0.16429
Finland	1961–1987	-0.43244
France	1951–1989	-0.38174
Germany	1961–1989	-0.31514
Ghana	1965–1988	-0.13037
Greece	1950–1988	-0.64105
Guatemala	1952–1989	-0.18438
Guyana	1961–1988	-0.20401
Haiti	1967–1987	0.39248
Honduras	1951–1989	-0.06100
Hungary	1973–1988	-0.36113
Iceland	1951–1988	-0.25263
India	1961–1988	0.06931
Indonesia	1965–1989	-0.36907
Iran	1965–1987	-0.35374
Ireland	1949–1988	-0.09517
Israel	1965–1988	-0.14150
Italy	1961–1989	-0.33788
Japan	1961–1988	-0.32264
Jamaica	1953–1988	-0.47148
Jordan	1970–1988	0.04949
Kenya	1967–1988	-0.26887
Korea	1967–1986	-0.57646
Kuwait	1973–1988	-0.39039

Table 1 (continued)

Country	Period	Correlation
Liberia	1966–1986	-0.23082
Luxembourg	1951–1986	-0.04201
Malasia	1981–1989	0.39854
Malta	1971–1988	0.38392
Malawi	1955–1988	0.18086
Mauritius	1964–1987	-0.02026
Mexico	1949–1986	-0.65904
Morocco	1965–1988	0.24233
Myanmar	1968–1988	-0.34465
Nepal	1965–1989	0.04275
Netherlands	1957–1989	0.10029
New Zealand	1973–1987	-0.22765
Nicaragua	1974–1988	-0.08691
Nigeria	1962–1989	-0.48619
Norway	1955–1989	-0.24359
Panama	1957–1989	-0.00449
Pakistan	1951–1989	-0.03473
Paraguay	1960–1989	0.13211
Philippines	1950–1989	-0.53678
Portugal	1967–1986	-0.49788
Saudi Arabia	1968–1988	0.30833
Singapore	1961–1989	0.05194
Spain	1955–1989	-0.52442
St. Vincent	1977–1985	-0.30949
Sweden	1978–1986	-0.60975
Swaziland	1951–1989	0.08122
Switzerland	1949–1989	-0.24902
Syrian Arab Republic	1961–1988	-0.20626
Tanzania	1966–1988	-0.55888
Togo	1971–1986	-0.29603
Trinidad and Tobago	1967–1987	0.19769
Tunisia	1969–1988	-0.41738
Turkey	1958–1988	-0.43260
United Kingdom	1949–1989	-0.46028
United States	1956–1989	-0.22931
Uruguay	1949–1989	0.18278
Venezuela	1958–1989	-0.56122
Yugoslavia	1969–1988	-0.67549

Source: International Financial Statistics tape. Inflation is measured by the percentage change in the consumer price index. Real output is typically measured by real GDP (gross domestic product) or real GNP (gross national product).

results in a welfare cost of about 0.4% of income relative to an optimal monetary policy. This is somewhat smaller than the 0.8% and 0.5% figures calculated by Fischer (1981) and Lucas (1981), respectively, using the "traditional" welfare triangle analysis associated with Bailey (1956).[22] Imrohoroglu (1990), using a model in which optimizing households hold money to insure against unemployment, suggests that welfare triangles may underestimate the true costs of inflation by a factor of three or more. Along the steady state, balanced growth path of the endogenous growth model analyzed below, a welfare cost of less than 0.03% of income results from a 10% money growth rate (8.7% inflation rate)—an order of magnitude smaller than Cooley and Hansen (1989a)!

Growth theory typically assumes that long run growth occurs at some exogenous rate. For many issues, this supposition is likely innocuous. However, when considering public policy questions this may be a poor assumption, as King and Rebelo (1990) have shown in the context of income taxation. While Howitt (1990) considers a model in which money can affect the economy's long run growth rate, he does not quantify this effect, nor the implications for welfare.

In the model developed here, endogenous growth arises through human capital accumulation as suggested by Lucas (1988). Rebelo (1990) has examined some of the theoretical properties of such models, and King and Rebelo (1990) have used such a model to analyze the welfare effects of income taxation. There are two productive activities in the model: market or physical output production, and new human capital production. While each activity is constant-returns-to-scale in physical capital and human capital-augmented labour effort, there are increasing-returns-to-scale at the economy level to the three inputs, physical capital, labour effort and human capital. It is in this way that perpetual growth is feasible.

Money enters the model via a cash-in-advance constraint. As in Cooley and

[22] The experiment considered by Fischer (1981) and Lucas (1981) is to lower the inflation rate from 10% to 0%.

Hansen (1989a), higher money growth-cum-inflation reduces the return to working. However, here there are two channels of effect since there are two productive activities. In equilibrium, the wage rate must be equalized across the two sectors since labour is freely mobile . As a result, not only does market output fall, but so does human capital production. It is this latter avenue through which inflation affects long run growth in this economy.

Since households fundamentally care only about the paths of consumption and leisure, it is sufficient to consider what happens with these variables to understand the welfare results presented below. Through its effect on the path of consumption, households also care about the real growth rate of the economy. Higher money growth lowers the real growth rate, making households worse off. There is, however, an offsetting effect: lower growth means that less output needs to be devoted to capital accumulation (to maintain the new, lower real growth rate) and more can be allocated to consumption. In the steady state, balanced growth calculations below, consumption relative to human capital falls only slightly for moderate money growth rates.

As mentioned above, labour effort in both the market sector and human capital production activity fall in the face of higher money growth. This serves to make leisure *more* responsive to changes in the money growth rate. This also helps to ameliorate the negative effects which lower real growth has on household welfare. Rather than increasing the costs of inflation, endogenous growth, through its effects on consumption and leisure, serves to reduce the welfare costs of moderate inflations.

The remainder of the chapter is organized as follows. In Section 2, the physical environment is presented, household and firm problems cast, competitive equilibrium defined, and the balanced growth path transformation performed. The model is parameterized, calibrated and simulated in Section 3. Welfare results are found in Section 4 for both the steady state, balanced growth path and the stochastic version of the model. Section 5 concludes.

2. The Model

2.1 The Economic Environment

The representative household maximizes the expected value of a discounted stream of utility given by:

$$E_0 \sum_{t=0}^{\infty} \beta^t U(c_t, \ell_t), \quad 0 < \beta < 1 \tag{2.1}$$

where: c_t is consumption at date t, and ℓ_t is leisure in period t. The household's time endowment is normalized to one so that ℓ_t is the fraction of time the household allocates to leisure. In addition to the usual properties, it is assumed that the utility function can be written as $U(c, \ell) = u(c)v(\ell)$ where $u(c)$ is homogeneous of degree $1 - \sigma$. This assumption is essentially the same as that made in King, Plosser and Rebelo (1988), and similar to that in Greenwood and Hercowitz (1991). As in King, Plosser and Rebelo, such a specification for utility simultaneously allows for (positive) growth of consumption and zero growth in leisure (the household's time endowment is fixed at unity).

The timing of transactions within a period proceeds as follows. The typical household enters period t with physical capital, k_t, human capital, h_t, and nominal cash balances, m_t. At the start of the period, the state-of-the-world is revealed; in particular, the current period market sector productivity shock, z_t, and gross per capita growth rate of money, g_t, are revealed. The government makes a transfer to the household, v_t, in the form of nominal balances. Taking as given the rental price of physical capital, r_t, and the wage rate paid on human capital-augmented labour effort, w_t, the household chooses ϕ_t, the fraction of physical capital allocated to the market sector, and n_t, the fraction of time devoted to the market sector. Time and physical capital not allocated to the market are used to produce new human capital as described below.

The representative household finances its purchases of the consumption good through beginning-of-period cash balances which are the sum of balances from from the previous period, m_t, and transfers from government, v_t. That is, the typical household faces a cash-in-advance constraint of the form,

$$p_t c_t \leq m_t + v_t \tag{2.2}$$

where p_t is the price level in period t. Investment can be thought of as a credit good while consumption is a cash good.

At the end of the period, the household receives from firms factor payments for capital and labour. These payments, in nominal terms, are $p_t r_t \phi_t k_t$ and $p_t w_t n_t h_t$, respectively. Along with any unspent cash balances, the household allocates its earnings between purchases of the physical investment good, $p_t i_t$ in nominal terms, and the accumulation of nominal cash balances to take into next period, m_{t+1}. The household's budget constraint can now be written as:

$$c_t + i_t + \frac{m_{t+1}}{p_t} \leq r_t \phi_t k_t + w_t n_t h_t + \frac{m_t + v_t}{p_t} \tag{2.3}$$

A quantity of physical capital, $(1 - \phi_t)k_t$, and human capital-augmented labour effort, $(1 - \ell_t - n_t)h_t$, were not allocated to the market sector and are used instead to produce new human capital. The evolution equation for human capital is given by:

$$h_{t+1} = F^h\Big[(1 - \phi_t)k_t, (1 - n_t - \ell_t)h_t\Big] + (1 - \delta_h)h_t \tag{2.4}$$

where $F^h(\cdot)$ is homogeneous of degree one in physical capital and human capital-augmented labour effort, and δ_h is the depreciation rate of human capital. Notice that an allocation of time to market or human capital production implies an allocation of human capital to these activities as well.

A number of institutional arrangements can support the real allocations analyzed below. Here, it is convenient to think of human capital accumulation as a "household" activity. Alternatively, human capital could be produced in an "education" sector with a price attached to human capital, as in King and Rebelo (1990). Here, the price of human capital is a shadow price.

The law of motion for physical capital is:

$$k_{t+1} = (1 - \delta_k)k_t + i_t \tag{2.5}$$

where δ_k is the depreciation rate of physical capital.

Firms have access to a constant-returns-to-scale production function which produces output, y_t, according to:

$$y_t = F^m(\phi_t k_t, n_t h_t; z_t) \tag{2.6}$$

where z_t is a productivity shock, assumed to evolve as:

$$z_t = \rho z_{t-1} + \epsilon_t \tag{2.7}$$

Output can be divided between consumption and physical investment,

$$y_t = c_t + i_t \tag{2.8}$$

Finally, government's actions are taken to be exogenous. Government finances its transfer to households through the creation of money, facing the per capita budget constraint,

$$v_t = (g_t - 1)m_t \tag{2.9}$$

where the gross growth rate of money, g_t, evolves according to:

$$\ell n g_t = \psi \ell n g_{t-1} + (1 - \psi)\ell n \overline{g} + \xi_t \tag{2.10}$$

where $\overline{g}$ is the long run, average rate of money growth and ξ_t is a random shock.

2.2 Competitive Equilibrium

Denote the state by $s = (k, h, m, z, g)$ where time subscripts have been dropped in the usual fashion. Suppose that prices and the government transfer can be written as functions of the state; viz, $p = P(s)$, $r = R(s)$, $w = W(s)$ and $v = \Upsilon(s)$. Further suppose that the laws of motion for k, h and m are described by $K(s)$, $H(s)$ and $M(s)$, respectively. Write the law of motion for the productivity shock as $z' = Z(s,\epsilon) \equiv \rho z + \epsilon'$, and for money growth as $g' = G(s,\xi') \equiv \exp\{\psi g + (1-\psi)\bar{g} + \xi'\}$. Now, s evolves according to $s' = S(s, \epsilon', \xi'; K, H, M) \equiv S\big(K(s), H(s), M(s), Z(s,\epsilon'), G(s,\xi')\big)$.

The problem faced by the representative household is to choose consumption, $\tilde{c}$, an allocation of time to leisure and market activity, $\tilde{\ell}$ and $\tilde{n}$, stocks of physical capital, human capital and cash balances, $\tilde{k}'$, $\tilde{h}'$ and $\tilde{m}'$, and a division of physical capital between the market sector and human capital production, $\tilde{\phi}$, which solve the following dynamic programming problem:

$$V(\tilde{k}, \tilde{h}, \tilde{m}; s) = \max_{\tilde{c}, \tilde{\ell}, \tilde{n}, \tilde{\phi}, \tilde{k}', \tilde{h}', \tilde{m}'} \left\{ U(\tilde{c}, \tilde{\ell}) + \beta E V(\tilde{k}', \tilde{h}', \tilde{m}'; s') \right\} \qquad (P1)$$

subject to

$$\tilde{c} + \tilde{k}' + \frac{\tilde{m}'}{P(s)} \leq R(s)\tilde{\phi}\tilde{k} + W(s)\tilde{n}\tilde{h} + (1-\delta_k)\tilde{k} + \frac{\tilde{m} + \Upsilon(s)}{P(s)} \qquad (2.11)$$

$$P(s)\tilde{c} \leq \tilde{m} + \Upsilon(s) \qquad (2.12)$$

$$\tilde{h}' = F^h\left[(1-\tilde{\phi})\tilde{k}, (1 - \tilde{n} - \tilde{\ell})\tilde{h}\right] + (1-\delta_h)\tilde{h} \qquad (2.1$$

and

$$s' = S(s, \epsilon', \xi'; K, H, M) \qquad (2.14)$$

The problem of a typical firm is to maximize period profits through its choice of $\tilde{\phi}\tilde{k}$ and $\tilde{n}\tilde{h}$:

$$\max_{\tilde{\phi}\tilde{k}, \tilde{n}\tilde{h}} \left\{ F^m(\tilde{\phi}\tilde{k}, \tilde{n}\tilde{h}; z) - R(s)\tilde{\phi}\tilde{k} - W(s)\tilde{n}\tilde{h} \right\} \qquad (P2)$$

Since $F^k(\cdot)$ is constant-returns-to-scale, in equilibrium zero profits are earned and it is not necessary to account for distributed profit income in the household's problem.

Definition: A competitive equilibrium consists of policy functions, $c = C(s)$, $\ell = L(s)$, $n = N(s)$, $\phi = \Phi(s)$, $h' = H(s)$, $k' = K(s)$ and $m' = M(s)$, pricing functions $p = P(s)$, $r = R(s)$ and $w = w(S)$, and a transfer function $v = \Upsilon(s)$ such that:

(i) Households solve (P1) taking as given the state-of-the-world, $s = (k, h, m, z, g)$ and the functions $R(s)$, $W(s)$, $P(s)$, $K(s)$, $H(s)$, $M(s)$ and $\Upsilon(s)$, with the solution to this problem being $\tilde{c} = C(s)$, $\tilde{\phi} = \Phi(s)$, $\tilde{\ell} = L(s)$, $\tilde{n} = N(s)$, $\tilde{k}' = K(s)$, $\tilde{h}' = H(s)$, and $\tilde{m}' = M(s)$.

(ii) Firms solve (P2), given s and the functions $R(s)$ and $W(s)$, with the solution having the form $\tilde{\phi}\tilde{k} = \Phi(s)k$ and $\tilde{n}\tilde{h} = N(s)h$.

(iii) Goods and money markets clear:

$$c + k' = F^m(\phi k, nh; z) + (1 - \delta_k)k \tag{2.15}$$

and

$$m' = m + v \tag{2.16}$$

Assuming that the household's constraints hold with equality (the budget constraint will hold with equality due to non-satiation while the cash-in-advance constraint will hold with equality for sufficiently rapid money growth), the definition of a competitive equilibrium implies that the allocation rules for c, ϕ, ℓ, n, k', h', m' and the pricing function p are implicitly defined by the market clearing conditions, (2.15) and (2.16), and the following:

$$\frac{U_2(c, \ell)}{hF_2^m(\phi k, nh; z)} = \beta \mathrm{E}\left\{\frac{U_1(c', \ell')}{p'/p}\right\} \tag{2.17}$$

$$\frac{U_2(c, \ell)}{hF_2^m(\phi k, nh; z)} = \beta \mathrm{E}\left\{\frac{U_2(c', \ell')}{h'F_2^m(\phi' k', n'h'; z)}\left[F_1^m(\phi' k', n'h'; z) + 1 - \delta_k\right]\right\} \tag{2.18}$$

$$\frac{F_2^m(\phi k, nh; z)}{F_2^h\left[(1-\phi)k, (1-n-\ell)h\right]} \times \frac{U_2(c,\ell)}{hF_2^m(\phi k, nh; z)} =$$
$$\beta \mathrm{E}\left\{\frac{U_2(c',\ell')}{h'F_2^m(\phi' k', n'h'; z)}\left[(1-\ell')F_2^m(\phi' k', n'h'; z) + \frac{F_2^m(\phi' k', n'h'; z)}{F_2^h\left[(1-\phi')k', (1-n'-\ell')h'\right]}(1-\delta_h)\right]\right\} \quad (2.19)$$

$$\frac{F_1^m(\phi k, nh; z)}{F_2^m(\phi k, nh; z)} = \frac{F_1^h\left[(1-\phi)k, (1-n-\ell)h\right]}{F_2^h\left[(1-\phi)k, (1-n-\ell)h\right]} \quad (2.20)$$

$$h' = F^h\left[(1-\phi)k, (1-n-\ell)h\right] + (1-\delta_h)h \quad (2.21)$$

$$pc = m + v \quad (2.22)$$

Equation (2.17) illustrates how money distorts decisions in this environment. Under an optimal monetary policy (in which case the cash-in-advance constraint does not bind), (2.17) would look, instead, like:

$$U_2(c,\ell) = hF_2^m(\phi k, nh; z)U_1(c,\ell) \quad (2.23)$$

In (2.23), as in (2.17), the marginal utility of leisure is equated to the marginal return to working, evaluated in terms of utility. However, the cash-in-advance constraint introduces a wedge of inefficiency in (2.17) since money earned in the current period cannot be spent until the next. Consequently, the left-hand side of (2.17) represents the utility cost of accumulating the last unit of nominal cash balances while the right-hand side gives the return, evaluated in terms of current-period utility. The gross inflation rate, p'/p, is the return earned on money. Thus, even if perfectly anticipated, inflation erodes the value of cash balances and so affects real variables in the model economy. This last effect is the taxation aspect of inflation emphasized by Stockman (1981).

Equation (2.18) governs the accumulation of physical capital.[23] The term in square brackets on the right-hand side is the return, in consumption units, earned

[23] Lucas (1990) provides an alternative method to interpret accumulation equations like (2.18) and (2.19).

by holding the last unit of capital acquired for one period. Since capital is mobile within a period, the rental price of capital in market and human capital production must be the same in equilibrium. In terms of current period utility gain, this return must just equal the cost of acquiring that last unit of physical capital, which is given by the left-hand side of (2.18).

Human capital accumulation is governed by (2.19). Since labour can costlessly and instantaneously be switched between the market sector and production of human capital, it follows that the return earned by labour must be equalized across the two sectors. Since $(1-\ell)$ is the fraction of time allocated to working in a period, the term in square brackets in (2.19) is the return, in consumption units, to the last unit of human capital accumulated. Notice that $F_2^m(\cdot)/F_2^h(\cdot)$ is the shadow price of human capital (in units of consumption). On the margin, the last unit of human capital acquired must generate a benefit which just equals its cost, given by the left-hand side of (2.19).

(2.20) is an efficiency condition which arises since both labour effort and physical capital are freely mobile across sectors within a period. Equations (2.20) and (2.21) can be thought of as determining the allocation of physical capital and non-leisure time between the market sector and human capital production.

Finally, (2.22) reproduces the cash-in-advance constraint.

2.3 Balanced Growth

To facilitate the use of computational techniques, it is convenient to consider the balanced growth path for the economy. Recalling that $U(c,\ell) = u(c)v(\ell)$ where $u(c)$ is homogeneous of degree $1-\sigma$, it follows that $U_1(c,\ell)$ is homogeneous of degree $-\sigma$ in c while $U_2(c,\ell)$ is homogeneous of degree $1-\sigma$ in c. Since $F^m(\cdot)$ and $F^h(\cdot)$ are each homogeneous of degree one in their two arguments, their partial derivatives are homogeneous of degree zero. Consequently, from the system of equations implicitly defining the allocation functions and pricing function, (2.15) (2.22):

(a) the allocation functions $C(s)$, $L(s)$, $N(s)$, $\Phi(s)$, $K(s)$ and $H(s)$ are each homogeneous of degree one in (k,h) and homogeneous of degree zero in m;

(b) the function governing money accumulation, $M(s)$, is homogeneous of degree zero in k and h, and homogeneous of degree one in m; and

(c) the pricing function, $P(s)$, is homogeneous of degree zero in k, homogeneous of degree -1 in h, and homogeneous of degree one in m.

Now, define $\hat{c} = c/h$, $\hat{k} = k/h$, $\hat{p} = ph/m$ and $\hat{s} = (\hat{k},1,1;z,g)$. Then the functions $\hat{c} = C(\hat{s})$, $\ell = L(\hat{s})$, $n = N(\hat{s})$, $\phi = \Phi(\hat{s})$, $\hat{k}' = K(\hat{s})$, $h'/h = H(\hat{s})$, $\hat{m}' = M(\hat{s})$ and $\hat{p} = P(\hat{s})$ are implicitly defined by:

$$\hat{c} + \left(\frac{h'}{h}\right)\hat{k}' = F^m(\phi\hat{k},n;z) + (1-\delta_k)\hat{k} \tag{2.24}$$

$$\hat{m}' = g \tag{2.25}$$

$$\frac{gU_2(\hat{c},\ell)}{\hat{p}F_2^m(\phi\hat{k},n;z)} = \beta\left(\frac{h'}{h}\right)^{1-\sigma} \mathrm{E}\left\{\frac{U_1(\hat{c}',\ell')}{\hat{p}'}\right\} \tag{2.26}$$

$$\frac{U_2(\hat{c},\ell)}{F_2^m(\phi\hat{k},n;z)} = \beta\left(\frac{h'}{h}\right)^{-\sigma} \mathrm{E}\left\{\frac{U_2(\hat{c}',\ell')}{F_2^m(\phi'\hat{k}',n';z)}\left[F_1^m(\phi'\hat{k}',n';z) + 1 - \delta_k\right]\right\} \tag{2.27}$$

$$\frac{U_2(\hat{c},\ell)}{F_2^h\left[(1-\phi)\hat{k},1-n-\ell\right]} = \beta\left(\frac{h'}{h}\right)^{-\sigma} \mathrm{E}\left\{U_2(\hat{c}',\ell')\left[1-\ell'+\frac{1-\delta_h}{F_2^h\left[(1-\phi')\hat{k}',1-n'-\ell'\right]}\right]\right\} \tag{2.28}$$

$$\frac{F_1^m(\phi\hat{k},n;z)}{F_2^m(\phi\hat{k},n;z)} = \frac{F_1^h\left[(1-\phi)\hat{k},1-n-\ell\right]}{F_2^h\left[(1-\phi)\hat{k},1-n-\ell\right]} \tag{2.29}$$

$$\hat{p}\hat{c} = g \tag{2.30}$$

$$\frac{h'}{h} = F^h\left[(1-\phi)\hat{k},1-n-\ell\right] + (1-\delta_h) \tag{2.31}$$

3. Model Parameterization and Calibration

There are two tasks undertaken in this section. The first is to provide specific forms for the utility and production functions used and assign values to the various parameters in the model. The second is to compare the model against the U.S. economy.

3.1 Model Parameterization

The period utility function is parameterized as:

$$U(c,\ell) = \frac{[c^{\omega}\ell^{1-\omega}]^{1-\gamma}}{1-\gamma}, \quad 0<\gamma<1,\ \gamma>1 \tag{3.1}$$

The production functions are specified as:

$$F^m(\phi k, nh; z) = A_m e^z(\phi k)^{\alpha}(nh)^{1-\alpha} \tag{3.2}$$

and

$$F^h\Big[(1-\phi)k,(1-n-\ell)h\Big] = A_h\Big[(1-\phi)k\Big]^{\theta}\Big[(1-\ell-n)h\Big]^{1-\theta} \tag{3.3}$$

Innovations to the market productivity shock, ϵ_t, are assumed to lie in a two point set,

$$\epsilon_t \in \{-\varphi,\varphi\} \tag{3.4}$$

These innovations are assumed to be equally likely:

$$\text{prob}\,\{\epsilon_t=-\varphi\} = \text{prob}\,\{\epsilon_t=\varphi\} = \frac{1}{2} \tag{3.5}$$

Likewise, the innovations to money growth, ξ_t, are assumed to lie in a two point set,

$$\xi_t \in \{-\zeta,\zeta\} \tag{3.6}$$

and

$$\text{prob}\,\{\xi_t=-\zeta\} = \text{prob}\,\{\xi_t=\zeta\} = \frac{1}{2} \tag{3.7}$$

The innovations to productivity and money growth are assumed to be independent.

To solve and simulate the model, the following parameters must be assigned values:

Preferences:	ω, γ, β, σ
Market Production:	A_m, α, δ_k, ρ, φ
Human Capital Production:	A_h, θ, δ_h
Government:	ϕ, ζ, $\overline{g}$

As in the seminal work of Kydland and Prescott (1982), as much discipline as possible is imposed by choosing parameter values based on either micro evidence, or to obtain long run averages observed in the data.

As noted by Davies and Whalley (1989) and King and Rebelo (1990), there is little evidence to guide the choice of parameters for the human capital production function. To minimize discretion, the market production function and physical capital are used as guides in the choice of human capital parameters. The capital share parameters, α and θ, are set equal to 0.36, capital's average share of GNP for the U.S. economy in the post-Korean War period.[24] The scale parameters, A_m and A_h, also share the same value, 0.105, which is chosen to achieve a steady state growth rate of 0.3542, the average quarterly growth rate of per capita U.S. GNP over the period 1954Q1–1989Q4. From the homogeneity results in Section 2.3, the model's results are insensitive to normalizing A_m to unity and allowing A_h to adjust to achieve the target growth rate. Conceptually, this would be equivalent to changing the units in which h, the stock of human capital, is measured.

The model is compared with quarterly data. Kydland and Prescott (1982) suggest an annual depreciation rate for capital of 10%. Restricting the depreciation rates, δ_k and δ_h, to have a common value, this corresponds to setting each to 0.025. The discount factor, β, is chosen such that in steady state, a real return of 1% is earned on physical capital. Evaluating (2.27) in steady state, β is chosen to satisfy:

$$1 = \beta \left(\frac{h'}{h} \right)^{-\sigma} \times 1.01 \tag{3.8}$$

where h'/h is the steady state real growth rate. This implies a value of 0.9954 in the benchmark economy.

[24] King and Rebelo (1990) consider a smaller capital share parameter for the human capital sector since this reduces the sensitivity of growth to changes in the income tax rate in their model.

The key parameters governing the stochastic process of the productivity shock are its autocorrelation coefficient, ρ, and its variability which is governed by φ. The value for ρ is 0.95 as suggested by Prescott's (1986) analysis of the properties of Solow residuals for the U.S. economy. However, since human capital plays no role Prescott's work, it would be inconsistent to use his estimate of the variance of the Solow residuals to fix the variance of the productivity shock in this model. Instead, the value of φ was chosen such that the standard deviation of the growth rate of output from the model matches that of U.S. GNP.[25] This implies a value of 3.6952×10^{-4} for φ.

Mehra and Prescott (1985) cite micro evidence on the coefficient of relative risk aversion, and suggest that it has a value between 1 and 2. Recalling that $U(c,\ell) = u(c)v(\ell)$ where $u(c)$ is homogeneous of degree $1-\sigma$, the evidence in Mehra and Prescott guides the choice of the parameter σ. For the purposes of the benchmark model, setting σ to 1.5 seems reasonable.

The parameter ω, which governs the importance of consumption relative to leisure in the period utility function, is chosen such that in steady state, households allocate 24% of their time to market production. This fraction corresponds to the per capita fraction of time spent working by the U.S. working age population. The value of ω is thus 0.2281. Notice that, with σ, this leads to a value of 3.1922 for γ.

Finally, parameters describing government's actions must be chosen. $\bar{g}$, the average quarterly growth rate of money, is 1.014%, the observed quarterly growth rate of per capita U.S. M1 over 1959Q2–1989Q4.[26] The autoregressive coefficient of money growth, ψ, and the variance of its innovations are obtained by estimating a first-order autoregressive process to money growth. The resulting values are 0.5814 and

[25] Hansen (1985) and Greenwood, Hercowitz and Huffman (1988) perform similar exercises.

[26] The database used has U.S. M1 starting in 1959Q1; one quarter is lost in calculating the growth rate.

8.2357×10^{-3}, respectively, the last of which is also the value of ζ.

3.2 Model Results

Two sets of tables are presented for the U.S. economy and the model. The first set consists of moments based on quarterly growth rates (first difference in logs), while the second consists of moments for Hodrick-Prescott filtered data. In typical real business cycle exercises—see, for example, Kydland and Prescott (1982) and Hansen (1985)—the model abstracts from growth and Hodrick-Prescott filtering is used as an "agnostic" means of detrending the data. Since the model presented above explicitly incorporates endogenous growth, it seems appropriate to base the comparison of the model with the U.S. data on growth rate filtered data. Moments for Hodrick-Prescott filtered data are provided, however, to facilitate comparisons with studies of other real business cycle models. Emphasis in the presentation will, however, be placed on the moments for the growth rate filtered data. Table 2 presents selected growth rate filtered moments for the U.S. economy while Table 4 provides the same moments for data logged and Hodrick-Prescott filtered.

In matching the model up with U.S. macroaggregates, the following assumptions have been made. First, consumption in the model is associated with consumption of non-durables and services in the U.S. National Accounts. Second, investment is taken to be measured by fixed investment. Finally, as noted above, M1 is the monetary aggregate chosen to match up with money in the model, although summary statistics for other aggregates are provided.

The balanced growth version of the model is solved using a procedure suggested by Coleman (1989). Essentially, this algorithm seeks policy and pricing functions which satisfy the Euler equations and constraints. For details on implementing the algorithm, see Coleman (1989) or Gomme and Greenwood (1990). A key feature of this algorithm, exploited here, is that it can be used to seek non-pareto optimal equilibria.[27]

Moments for the model are obtained by simulating the functions thus obtained, taking care to transform variables from their balanced growth values. Since the number of observations affects the degree of smoothing achieved by the Hodrick-Prescott filter, 50 sets of 144 observations, the number of quarters from 1954Q1 to 1989Q4, were generated. The averages of the moments across the 50 sets are presented in Table 3 for growth rate filtered data, and Table 5 for Hodrick-Prescott filtered data.[28]

Concentrating on the growth rate filtered data (Tables 2 and 3), it can be seen that the model does well in replicating the core U.S. business cycle facts that consumption varies less than output while investment varies more, although in the model investment varies too much relative to the U.S. economy. The model has problems capturing the magnitude of the correlation between consumption and output exhibited by the U.S. data, and generates a negative correlation between productivity and output where this is positive in the data.

For the real variables (that is, excluding money and the price level), the model uniformly delivers negative first-order autocorrelations which stands in contrast with the positive correlations seen in the U.S. data. This is likely due to the assumption that in the model both labour and physical capital are perfectly mobile within a period. Introducing adjustment costs to human capital, as in King and Rebelo (1990), or to physical capital may help on this dimension. Alternatively, the allocation of physical capital between the two sectors could be set prior to the realization of the productivity and money growth shocks.

Turning to the behaviour of the nominal variables, it should not be surprising

[27] Cooley and Hansen (1989a) use a modified linear-quadratic procedure; see Hansen and Prescott (1991) for details. King and Rebelo (1990) use an alternative linear-quadratic technique; see King, Plosser and Rebelo (1988) for particulars.

[28] With the exception of currency, the moments reported for the monetary aggregates for the U.S. economy are based on data from 1959Q1. No attempt was made to shorten the simulated samples for money in the moments reported for the model.

Table 2: United States, 1954Q1–1989Q4
Growth Rate Filtered

	Growth Rate	Standard Deviation	First-order Autocorrelation	Correlation with Output
Output	0.35	1.01	0.33	1.00
Consumption	0.40	0.52	0.25	0.46
Investment	0.43	2.61	0.49	0.70
Hours	-0.09	0.93	0.52	0.70
Productivity	0.44	0.75	0.03	0.47
Currency	1.05	0.80	0.90	0.08
Base	1.11	0.64	0.78	0.10
M1	1.01	0.99	0.58	0.13
M2	1.56	0.77	0.61	0.15
M3	1.74	0.75	0.78	0.15
L	1.69	0.64	0.80	0.14
GNP Deflator	1.11	0.67	0.74	-0.26
CPI	1.07	0.85	0.86	-0.27

All variables except the price indexes have been deflated by the 16+ population. All variables except the monetary aggregates and price indexes are expressed in constant 1982 dollars. Output is measured by gross national product; consumption by consumption of non-durables and services; investment by gross fixed investment; and hours by total hours of persons in the business sector (establishment survey). Productivity is defined by output divided by hours. Moments for the monetary aggregates (base, M1, M2, M3 and L) are based on data over 1959Q2–1989Q4. CPI denotes the consumer price index (1982–1984=100). The GNP deflator is the implicit GNP deflator (1982 base).

Table 3: Selected Model Moments
Growth Rate Filtered

	Growth Rate	Standard Deviation	First-order Autocorrelation	Correlation with Output
Output	0.35	1.01	-0.12	1.00
Consumption	0.35	0.48	-0.22	0.16
Investment	0.35	6.17	-0.53	0.74
Labour: Market	0.00	1.73	-0.49	0.82
Labour: Human Capital	0.00	1.95	-0.49	-0.82
Leisure	0.00	0.04	-0.21	-0.29
Productivity	0.35	1.07	-0.65	-0.38
ϕ	0.00	1.71	-0.49	0.82
Capital Stock	0.35	0.11	-0.20	0.93
Money	0.98	0.95	0.54	-0.06
Price Level	0.63	1.25	0.22	-0.11

Table 4: United States, 1954Q1–1989Q4
Hodrick-Prescott Filtered

	Standard Deviation	First-order Autocorrelation	Correlation with Output
Output	1.70	0.85	1.00
Consumption	0.85	0.84	0.75
Investment	5.35	0.85	0.89
Hours	1.77	0.88	0.88
Productivity	0.85	0.67	0.16
Currency	0.71	0.88	0.21
Base	0.84	0.88	0.41
M1	1.63	0.87	0.31
M2	1.48	0.89	0.46
M3	1.50	0.92	0.48
L	1.09	0.91	0.58
GNP Deflator	0.89	0.91	-0.55
CPI	1.41	0.94	-0.57

All variables except the price indexes have been deflated by the 16+ population. All variables except the monetary aggregates and price indexes are expressed in constant 1982 dollars. Output is measured by gross national product; consumption by consumption of non-durables and services; investment by gross fixed investment; and hours by total hours of persons in the business sector (establishment survey). Productivity is defined by output divided by hours. Moments for the monetary aggregates (base, M1, M2, M3 and L) are based on data over 1959Q2–1989Q4. CPI denotes the consumer price index (1982–1984=100). The GNP deflator is the implicit GNP deflator (1982 base).

Table 5: Selected Model Moments
Hodrick-Prescott Filtered

	Standard Deviation	Autocorrelation	Correlation with Output
Output	0.88	0.35	1.00
Consumption	0.45	0.42	0.24
Investment	3.93	0.23	0.71
Labour: Market	1.18	-0.06	0.85
Labour: Human Capital	1.33	-0.07	-0.84
Leisure	0.04	0.44	0.34
Productivity	0.63	-0.44	-0.20
ϕ	1.17	-0.07	0.85
Capital Stock	0.11	0.49	0.78
Money	1.81	0.89	-0.05
Price Level	1.96	0.81	-0.10

that the behaviour of money in the model closely matches that observed in the U.S. economy the parameters governing money growth were chosen such that this should be true. However, the inflation rate is not high enough in the model; using a broader definition of money would help since the broad aggregates grow faster than M1 in the U.S. economy.[29] Also, inflation is too variable and not as highly autocorrelated as observed in the U.S. data; the comments in the previous paragraph may be relevant here as well.

As in King, Plosser and Rebelo (1988), the model makes strong predictions regarding growth rates. For example, the model restricts the growth rate of hours worked to be zero since the household's time endowment is fixed while the U.S. data shows a modest decline.[30] Some likely explanations for the decline in hours in the U.S. are: average full-time hours of employees has been declining, there are more part-time workers, and the unemployment rate has an upward trend. These effects are partially offset by the increased participation rate.

The model also restricts output, consumption and investment to grow at a common rate while the U.S. data shows that consumption and investment have grown faster than output.

The effects money has on the benchmark economy are explored in Table 6. Here, standard deviations for macroaggregates (growth rate filtered) are provided for the U.S. economy, the benchmark economy, and a non-monetary version of the model.[31] The most striking result is the effect money has on consumption. In the benchmark

[29] In the model, the inflation rate is given by the growth rate of money less the real growth rate of the economy. Calibrating to a higher long run money growth rate would, then, lead to a higher average inflation rate.

[30] In Tables 2 and 4, hours are measured by hours of all persons in the business sector. If, instead, hours are measured either by hours of all employees in the business sector or hours of all persons in the non-farm business sector, the growth in hours is close to zero, although still negative. The growth rate of hours is −0.04% per quarter using household data rather than −0.08% as reported in Table 2

[31] For the non-monetary version, the cash-in-advance constraint is removed. This is equivalent to an optimal monetary policy, in which case the cash-in-advance constraint does not bind.

economy, the standard deviation of consumption almost matches that found in the U.S. economy while in the non-monetary economy, the standard deviation of consumption is virtually zero. It would seem that money is important in generating plausible consumption variability for this model.

Table 6: Contribution of Money to the Model Standard Deviations, Growth Rate Filtered

	U.S.	Benchmark	Non-monetary
Output	1.01	1.01	1.00
Consumption	0.52	0.48	0.03
Investment	4.00	5.03	4.99
Labour: Market	0.93	1.75	1.73
Labour: Human Capital		1.59	1.58
Leisure		0.08	0.01
Productivity	0.75	1.09	1.08

The benchmark economy is compared with more standard (stationary) real business cycle models in Table 7. Results in Hansen (1985) for the divisible l bour case are taken to be representative of standard real business cycle models.[32] Results from Cooley and Hansen (1989a) are included to provide an assessment of the importance of endogenous growth. All moments are based on Hodrick-Prescott filtered data. The benchmark economy performs well with respect to the standard deviation (relative to output) of consumption and hours. The relative standard deviation of productivity is too high in the benchmark economy while the indivisible labour model (Cooley and Hansen (1989a)) produces numbers which are too low. The Cooley and Hansen model out-performs the benchmark economy with respect to the behaviour of the price level, although both exaggerate its variability and are not sufficiently negatively correlated with output, vis-à-vis the U.S. economy. The benchmark economy does poorly on the dimensions of the correlation of consumption with output and

[32] The results for Hansen's (1985) indivisible labour model are qualitatively similar to those reported in Cooley and Hansen (1989a)

the correlation of the capital stock with output. Arguably, the benchmark economy performs better with respect to the correlation between productivity and output: it delivers a small negative correlation where a small positive correlation is seen in the U.S. data while the other models produce large positive correlations.

Table 7: Comparisons with Other Models
Hodrick-Prescott Filtered

	U.S.		Hansen		Cooley-Hansen		Benchmark	
	(a)	(b)	(a)	(b)	(a)	(b)	(a)	(b)
Consumption	0.50	0.75	0.31	0.89	0.36	0.72	0.56	0.2[illegible]
Investment	5.25	0.89	3.14	0.99	3.29	0.97	4.46	0.71
Capital Stock	0.22†	0.28†	0.27	0.06	0.28	0.06	0.13	0.78
Hours	1.04	0.88	0.52	0.98	0.77	0.98	1.34	0.85
Productivity	0.50	0.16	0.50	0.98	0.29	0.87	0.72	-0.20
Price Level					0.98	-0.27	2.23	-0.10
CPI	0.83	-0.57						
GNP deflator	0.52	-0.55						

Results for Hansen (1985) are for the divisible labour case; the indivisible case is similar to the results for Cooley-Hansen.

Results for the Cooley and Hansen (1989a) model are for the autoregressive growth rate ($\overline{g} = 1.015$) case.

Column (a): standard deviation relative to the standard deviation of output.

Column (b): correlation with output.

† From Cooley and Hansen (1989a).

Finally, Hansen (1985) introduced indivisible labour to a real business cycle model to account for the ratio of the standard deviation of hours to standard deviation of productivity. Standard (divisible labour) models deliver a ratio of about one while the U.S. data exhibits a ratio of about two.[33] Hansen's indivisible labour model actually goes too far: it raises this ratio to 2.7. From Table 5, it can be seen that the benchmark economy gives a ratio of about 1.9, very close to that seen in the U.S. data, but without the indivisible labour assumption.

[33] This ratio is higher than reported in Hansen (1985), but is in line with the results reported in Kydland and Prescott (1990) for either the household or establishment surveys for hours, and to the figures presented in Cooley and Hansen (1989a)

4. Welfare Results

The task at hand is to provide a measure of the welfare costs of money growth-cum-inflation for the environment described above. Throughout, the functional forms and parameter values for the benchmark model have been used.

At some minor abuse of notation, the $t = 0$ value function for a household can be written as:

$$V^a(k_0, h_0, m_0; s, \lambda) = \mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t U(c_t^a + \lambda y_t^a, \ell_t^a) \tag{4.1}$$

where the a superscript denotes equilibrium allocation rules obtained for monetary regime a (assumed not to depend on λ), and $\lambda \hat{y}_t^a$ is a lump-sum equivalent variation payment made to households. Using the properties of $U(\cdot)$, (4.1) can be rewritten as:

$$V^a(k_0, h_0, m_0; s, \lambda) = h_0^{1-\sigma} \mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t U(\hat{c}_t^a + \lambda \hat{y}_t^a, \ell_t^a) \left[\prod_{\tau=0}^{t} \left(\frac{h_{\tau+1}^a}{h_\tau^a} \right) \right]^{1-\sigma} \tag{4.2}$$

This transformation is computationally convenient. The welfare cost of operating monetary regime a rather than regime b is measured by the unique value of λ satisfying $V^a(k_0, h_0, m_0; s, \lambda) = V^b(k_0, h_0, m_0; s, \lambda = 0)$.

In generating measures of the welfare costs of alternative monetary regimes, the experiment being considered is to face the representative household with a choice between two regimes, but the *same* initial conditions that is, the same initial physical and human capital stocks and nominal cash balances. Consequently, transition costs from one regime to another have been ignored.

Conceptually, this method of calculating welfare gains/losses is the same as the exercises conducted by, for example, Cooley and Hansen (1989a), Greenwood and Huffman (1991) and Lucas (1990).

4.1 Steady State Results

These experiments can be thought of as setting the variance of the innovations to the productivity shock and money growth equal to zero. There are two motives for starting with steady state, balanced growth path calculations of the welfare costs of inflation: first, they are directly comparable with the welfare costs reported in Cooley and Hansen (1989a); and second, the calculations are simple enough that they can be verified with a calculator.

In steady state, (4.2) can be written as:

$$V^a(k_0, h_0, m_0; s, \lambda) = \frac{h_0^{1-\sigma} U(\hat{c}^a + \lambda \hat{y}^a, \ell^a)}{1 - \beta \left[\left(\frac{h'}{h} \right)^a \right]^{1-\sigma}} \tag{4.3}$$

Denote the optimal monetary regime—in which case the cash-in-advance constraint does not bind - by an asterisk superscript. Then the cost of operating monetary policy a relative to the optimal policy is the unique, positive value of λ satisfying:

$$\frac{U(\hat{c}^*, \ell^*)}{1 - \beta \left[\left(\frac{h'}{h} \right)^* \right]^{1-\sigma}} = \frac{U(\hat{c}^a + \lambda \hat{y}^a, \ell^a)}{1 - \beta \left[\left(\frac{h'}{h} \right)^a \right]^{1-\sigma}} \tag{4.4}$$

Given the functional form for $U(\cdot)$, (4.4) can be solved directly for λ.

The behaviour of the model and welfare costs of alternative monetary policies are summarized in Table 8. Note, in particular, that the welfare costs of a 10% money growth rate (8.7% inflation rate) is less than 0.03% of income while Cooley and Hansen (1989a) report a cost of 0.4%. Some insight as to why the welfare costs of inflation are so modest can be culled from Table 8.

Note that while period utility, defined over consumption normalized by human capital and leisure, $U(\hat{c}, \ell)$, is *increasing* for moderate money growth rates, lifetime utility, $V(\hat{s}, \lambda)$, is monotonically decreasing.

Higher money growth has the expected effects: it lowers (normalized) consumption and real growth, and raises leisure. However, the decline in consumption is slight. The goods market clearing condition, reproduced below along the steady

Table 8: Steady State Welfare Results
Alternate Annual Money Growth Rates

	optimal	0%	5%	10%
$\hat{k}$	0.5625	0.5625	0.5625	0.5625
ϕ	0.4785	0.4783	0.4780	0.4777
ℓ	0.4947	0.4960	0.4984	0.5007
n	0.2418	0.2411	0.2398	0.2385
$\hat{c}$	0.0102	0.0101	0.0101	0.0100
$U(\hat{c}, \ell)$	-14.8842	-14.8414	-14.7616	-14.6869
$V(\hat{s}, \lambda = 0)$	-2310.5359	-2310.5685	-2310.8070	-2311.2498
$\hat{y}$	0.0263	0.0262	0.0261	0.0260
quarterly growth rate	0.3639	0.3602	0.3530	0.3463
annual inflation rate	-3.9532	-1.4329	3.5428	8.5198
welfare cost (%)	0.0000	0.0011	0.0091	0.0238
	100%	1,000%	10,000%	100,000%
$\hat{k}$	0.5625	0.5625	0.5625	0.5625
ϕ	0.4739	0.4617	0.4436	0.4240
ℓ	0.5296	0.6060	0.6873	0.7487
n	0.2229	0.1819	0.1387	0.1065
$\hat{c}$	0.0093	0.0074	0.0053	0.0035
$U(\hat{c}, \ell)$	-13.8384	-12.3412	-11.8418	-12.5634
$V(\hat{s}, \lambda = 0)$	-2335.6150	-2603.3832	-3508.2076	-5578.1379
$\hat{y}$	0.0249	0.0216	0.0179	0.0149
quarterly growth rate	0.2596	0.0211	-0.2527	-0.4772
annual inflation rate	98.2595	999.4913	10032.2465	100340.1116
welfare cost (%)	0.8198	9.2787	38.4535	113.8397

state balanced growth path, sheds some light on why this is so.

$$\hat{c} + \left(\frac{h'}{h}\right)\hat{k} = F^m(\phi\hat{k}, n, z = 0) + (1 - \delta)\hat{k} \tag{4.5}$$

The term $\hat{k}h'/h$ is the amount of capital households must take from a period to stay on the balanced growth path. Noticing from Table 8 that $\hat{k}$ is unaffected by the money growth rate, the fall in the real growth rate induced by increased money growth allows a reallocation of output from capital accumulation to consumption. That normalized consumption falls results from the negative effect money growth has on output.

As mentioned previously, increases in the growth of money lowers the return to working. Since labour is perfectly mobile within a period between the market sector and human capital production, in equilibrium the return to working in the sectors will be equalized. As a consequence, there are two productive activities from which labour is drawn into leisure rather than just one as in models which abstract from growth, like Cooley and Hansen (1989a). If households do not value leisure, it can be shown that along the steady state balanced growth path, changes in money growth have no real effects a result similar to that of Stockman (1981)—and consequently no welfare effects. It is the augmented response of leisure to increases in the money growth rate, relative to that found in Cooley and Hansen (1989a), which helps to compensate households for the fall in the real growth rate, and the slight decline in consumption.

Results for hyperinflations are included in Table 8 not in the belief that the model can be used to assess the effects of such high inflation rates on the U.S. economy—clearly, agents would change their behaviour in the face of such inflation rates and the transactions technology would be expected to change—but rather to verify that such monetary policies have drastic effects on the model economy. Surely enough, this is exactly what happens: consumption falls sharply, and for sufficiently high money growth, negative real growth is registered. As well, large welfare costs are associated with such policies.

To summarize, the contribution of growth is to actually reduce the costs of inflation vis-à-vis the Cooley and Hansen (1989a) results. Lower growth dampens the fall in consumption and increases the response of leisure to increases in the money growth rate. The net effect is that very small welfare costs of inflation are associated moderate money growth rates.

4.2 Welfare Results for the Stochastic Economy

The basic task here is the same as in the previous section: find the welfare costs of alternative monetary regimes. The primary difference between the two sections is that here the variance of the innovations to the productivity shock and money growth are set equal to their values in the benchmark economy. Endogenous growth complicates the task of calculating welfare costs somewhat. To start, define

$$J^a(\hat{s}_0, \lambda) = \mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t U(\hat{c}_t^a + \lambda \hat{y}_t^a, \ell_t^a) \left[\sum_{\tau=0}^{t} \left(\frac{h_{\tau+1}}{h_\tau} \right)^a \right]^{1-\sigma} \tag{4.6}$$

Clearly, $V(k_0, h_0, m_0; s_0, \lambda) = h_0^{1-\sigma} J^a(\hat{s}_0, \lambda)$. (4.6) can be rewritten in the form of a Bellman equation:

$$J^a(\hat{s}^a, \lambda) = U(\hat{c}^a + \lambda \hat{y}^a, \ell^a) + \beta \left[\left(\frac{h'}{h} \right)^a \right]^{1-\sigma} EJ^a(\hat{s}^{a\prime}, \lambda) \tag{4.7}$$

In comparing the welfare cost of monetary regime a relative to regime b, the task is to find the value of λ satisfying

$$\int J^a(\hat{s}^a; \lambda) \mathrm{d}\Gamma(\hat{s}^a) = \int J^b(\hat{s}^b; \lambda = 0) \mathrm{d}\Gamma(\hat{s}^b) \tag{4.8}$$

where $\Gamma(\hat{s})$ is the distribution function for $\hat{s}$.

$J(\hat{s}; \lambda)$ is obtained by iterating on the Bellman equation, (4.7). The integrals above are approximated by averaging observed values of $J(\hat{s}; \lambda)$ over arbitrarily long simulations. Letting T denote the length of the simulation,

$$\int J(\hat{s}; \lambda) \mathrm{d}\Gamma(\hat{s}) \approx \frac{1}{T} \sum_{j=1}^{T} J(\hat{s}_j; \lambda) \tag{4.9}$$

Equation (4.8) is now effectively a single equation in the unknown, λ.

The first set of welfare results are based on deviations from the benchmark economy. These experiments consist of, first, increases in the variability of money growth, and, second, increases in the average, quarterly growth rate of money.

Results for increased money variability are summarized in Table 9. The experiment conducted is to increase the standard deviation of ξ, the innovation to the money growth process, by some known factor. This leads to an equiproportional increase in the standard deviation of the money growth rate. In addition to welfare costs, Table 9 also includes the standard deviations of macroaggregates for the model economy based on growth rate filtered data—other moments are little affected by these experiments, and moments for Hodrick-Prescott filtered data shows qualitatively similar behaviour. One striking result in this table is that of the real variables, consumption is the only one which responds substantially to changes in the variance of money growth. This fits well with the results in Table 6 which compared the performance of the benchmark economy with a version of the model without the cash-in-advance constraint. In Table 6, it was seen that money plays an important role in generating plausible consumption variability.

Table 9: Money Variance Experiments
Stochastic Economy Welfare Results

	benchmark	+5%	+10%	+25%	+50%	+100%
Standard Deviation:						
Output	1.01	1.01	1.01	1.01	1.01	1.02
Consumption	0.48	0.50	0.52	0.60	0.72	0.95
Investment	5.03	5.03	5.03	5.03	5.03	5.03
Labour: market	1.75	1.75	1.75	1.76	1.76	1.78
Labour: human capital	1.59	1.59	1.59	1.59	1.59	1.59
Leisure	0.08	0.08	0.09	0.10	0.12	0.16
Productivity	1.09	1.09	1.09	1.09	1.09	1.10
Money	0.99	1.04	1.09	1.23	1.48	2.81
Price Level	1.28	1.35	1.41	1.60	1.92	1.97
Average Real Growth:	0.35	0.35	0.35	0.35	0.35	0.35
Welfare Cost (%, $\times 10^{-3}$):		0.05	0.11	0.29	0.64	1.54

Table 10 summarizes the results of the growth rate experiments. The columns denote the percentage point increase in the quarterly money growth rate. Of interest is the fact that the variability of macroaggregates increases only slightly in the face of higher money growth, with this effect noticeable for money growth rates which are outside the U.S. historical experience.

Turning now to the welfare costs in Tables 9 and 10, notice that the welfare

Table 10: Money Growth Experiments
Stochastic Economy Welfare Results

	benchmark	+0.25	+0.50	+1.00	+2.50	+10.00
Standard Deviation:						
Output	1.01	1.01	1.01	1.01	1.02	1.05
Consumption	0.48	0.48	0.48	0.48	0.48	0.48
Investment	5.03	5.03	5.03	5.04	5.07	5.20
Labour: market	1.59	1.75	1.76	1.76	1.77	1.83
Labour: human capital	1.75	1.59	1.59	1.59	1.60	1.64
Leisure	0.08	0.08	0.08	0.08	0.08	0.08
Productivity	1.09	1.09	1.09	1.09	1.10	1.14
Money	0.99	0.99	0.99	0.99	0.99	0.99
Price Level	1.28	1.28	1.28	1.28	1.28	1.28
Average Real Growth:	0.35	0.35	0.35	0.35	0.34	0.30
Welfare Cost (%, $\times 10^{-3}$):		2.45	5.21	11.63	38.04	314.38

costs of increased money growth variability are small when compared to the costs of increases in the growth rate of money: doubling the standard deviation of money growth is less costly in welfare terms than increasing the money growth rate by 0.25 percentage points per quarter.

The second set of welfare results duplicate the experiments conducted for the steady state, balanced growth path of the model economy; welfare results for both are included in Table 11 to facilitate comparison of the two. Qualitatively, the costs of inflation are similar to those seen in steady state. While the costs of money growth are higher for the stochastic version of the model, the increase in the cost is relatively small. The results in this table fit well with the conclusion above indicating that money growth per se is more important in welfare terms than variability of money growth.

Table 11: Alternate Annual Money Growth Rates
Stochastic Economy Welfare Results

Annual Money	Welfare Cost	
Growth Rate	Steady State	Stochastic
0%	0.0011	0.0542
5%	0.0091	0.0628
10%	0.0238	0.0782
100%	0.8198	0.8838
1,000%	9.2787	9.4230
10,000%	38.4535	39.3500
100,000%	113.8397	119.3031

5. Conclusions

The usual intuition, as exemplified by Lucas (1987), is that if public policy can affect an economy's real growth rate, then large welfare costs will result. This suggested that a model of endogenous growth would deliver large welfare costs of money growth-cum-inflation—certainly larger than found in the stationary environment of Cooley and Hansen (1989a). In the endogenous growth model examined above, increased money growth-cum-inflation has the expected effects of lowering consumption, real growth and labour effort, yet the welfare costs are smaller than obtained by Cooley and Hansen (1989a). This result can be traced to the effect which endogenous growth has in augmenting the response of leisure to changes in the money growth rate, and its role in lessening the decline in consumption.

The analysis above compares welfare *across* different monetary regimes. Lucas (1990) and King and Rebelo (1990) have pointed out the importance of transitional dynamics in considering policy changes. Accounting for transitional dynamics should *lower* the welfare costs calculated above. On the other hand, results in Imrohoroglu (1990) suggest that introducing heterogeneity would *increase* the costs of higher money growth.

Finally, government revenue requirements have been ignored in the computation of welfare results above. It may be interesting to think about the mix of government taxes by introducing labour and capital taxation. In considering a change from the U.S. tax structure to an optimal mix of labour and capital taxes, Lucas (1990) calculates the benefit to be about one percent of consumption. In a stationary environment, Cooley and Hansen (1989b) find the inflation tax to be an efficient means of raising government revenue relative to labour and capital taxes. The large welfare costs of income taxation computed by King and Rebelo (1990) in an endogenous growth model similar to the one analyzed above indicate that the results of Cooley and Hansen (1989b) may be strengthened by considering an endogenous growth model. However, the analysis above suggests that intuition cannot always be trusted.

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