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Multi-sector Labour Market Search: Interactions Between Unemployment Duration And Sectoral Shocks

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by

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ABSTRACT

This thesis comprises five chapters which are all concerned with the general theme of sectoral shocks and unemployment duration. The first chapter motivates the issues addressed in the thesis by establishing their relevance and originality. The chapter begins by showing that the interaction between sectoral shocks and unemployment duration has largely been ignored in the sectoral shocks debate. The importance of unemployment duration at the theoretical level is demonstrated and data are presented to establish the empirical relevance of the theory.

The second chapter constructs and analyses a two-sector model of labour market search in which unemployment duration is an endogenous variable. Structure is imposed on the search process by adopting the "matching function" framework. Probabilities of exiting unemployment are related to variation in the arrival rate of job offers rather than just the offer rejection rate. The third chapter tests this relationship between labour market tightness and the transition probability into employment using monthly data for three Canadian provinces. Testing methods use flexible spline regression and isotonic regression methods to avoid the imposition of extraneous restrictions. General concordance between theoretical assumptions and empirical facts is found, and non-linearity in the estimated relationship is evidence against a simple queuing model.

In the fourth chapter, a stochastic version of the model of chapter two is developed and applied to divergent patterns of unemployment durations in Alberta and Ontario. An objective test of the pertinence of the model is conducted using a Markov switching-regression estimation method. The estimation method recovers parameters of a recruitment intensity function, Markov transition probabilities, and probabilities over states. The results support the hypothesis that sectoral shocks have affected unemployment durations.

The subject of the final chapter is the behaviour of the unemployment rate fol-

lowing the 1982 recession. Two stylized facts are examined: the greater persistence in Canada relative to the United States and in Alberta relative to Ontario. The contributions of unemployment insurance policy and sectoral shocks to these patterns is evaluated and some support for the sectoral shocks explanation is obtained.

To Tina

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1.1 Introduction

Without doubt, a central axis of debate in macroeconomics is the interpretation of, and explanation for, the existence of unemployed labour. The status of macroeconomics as a distinct discipline is fundamentally linked to the question of whether decentralised 'micro' level decisions lead to well-coordinated aggregate outcomes. Some hold that unemployment is *prima facie* evidence that labour markets are poorly coordinated. Others argue that unemployment is simply a necessary adjunct to productive activity so that the unemployment rate should arouse no more moral debate than does the utilization rate of physical capital. These two polar views continue to coexist largely due to a paucity of empirical evidence that can be used to discriminate between the views.

Recently, an important contribution to this debate was made by David Lilien. Lilien suggested that fluctuations in the aggregate unemployment rate may be due to time-consuming labour reallocation caused by sector-specific shocks. Building on concepts that date back (at least) to David Ricardo, this paradigm merges real shocks and imperfect contemporaneous coordination of activity, and purports to explain fluctuations in unemployment. If the model is as empirically important as its supporters claim, it has significant policy implications since it admits no role for activist demand-management policy but emphasises the importance of policies which facilitate structural adjustment.

At present, the debate continues about the relevance of this view of unemployment. Empirical work has been conducted by researchers both in favour of, and opposed to, this view, with no clear result yet obtained. This thesis is designed to contribute to the debate by extending the analysis into a direction which is currently under-expored. The innovation is the analysis of sectoral and regional differences in the duration of unemployment. Unemployment duration should be a key element of the sectoral shift view because aggregate unemployment exists only if resource allocation is time-consuming. To see this, consider briefly the following points:

(i) If sectoral shocks can cause differences in unemployment duration, and mobility costs are high, then sectoral shocks can affect the aggregate unemployment rate even if there is not a great amount of sectoral reallocation of labour.

(ii) Sectoral differences in unemployment durations can lead to aggregation bias in national measures of unemployment duration. This bias tends to create the impression that long spells of unemployment are more important than is truly the case.

(iii) There are significant differences in the time-series behaviour of unemployment duration across regions and sectors. Points (i) and (ii) establish that the interaction of sectoral shocks and unemployment duration is important at the theoretical level. The consideration of unemployment duration effects contributes to two current areas of concern in the sectoral shocks view: the stylized facts that intersectoral labour flows are relatively small and that recent unemployment rate rises are largely attributable to the long-term unemployed. A sectoral shocks model which pays attention to unemployment duration has the potential to account for these facts. Moreover, point (iii) strongly suggests that these theoretical arguments are likely to be empirically relevant and not simply theoretical possibilities.

The argument of this chapter is presented as follows. The chapter begins by briefly reviewing the state of the sectoral/aggregate shock debate. The importance of considering unemployment duration when evaluating the sectoral shock theory is established by looking at two key interactions between these two. Following this, stylized facts of the regional variation of unemployment duration are examined in order to show that this phenomena is empirically important. Next, two sections consider the empirical and theoretical research programs which are needed to establish these insights. Related to this, a section is included that documents available data sources for applied work. A brief final section concludes and points toward the desired next step.

1.2 The Sectoral/Aggregate Shocks Debate: A Brief Summary

The modern genesis of the sectoral shocks debate was the publication of the article "Sectoral Shocks and Cyclical Unemployment" by David Lilien in 1982. As Lilien himself has noted, the idea that resources are constantly subject to reallocation is not itself novel. The modern resurgence of the question can be traced to several factors: the sectoral effects of important oil price shocks in the 1970's and 1980's, the apparent success of the original empirical work by Lilien, and the swing in the economics profession from nominal explanations for business cycles to explanations grounded in real factors.

The theoretical underpinnings of the Lilien argument are quite intuitive. In a multi-sector economy, idiosyncratic shocks to relative prices, relative product demands, input prices, and so on, prompt the reallocation of factors of production such as labour from one occupation to another. Concrete instances of this are the movement of labour from manufacturing to services following oil price shocks, and from agriculture to manufacturing during the early development of an industrial economy. Such reallocation cannot be accomplished without the passage of some time and factors of production must accordingly spend some time idle. Several theoretical explanations for such idleness of factors are possible, although concave adjustment cost functions and imperfect information about job opportunities are the explanations which enjoy the greatest currency.

Certainly, there is little reason to doubt Lilien's view from a theoretical perspective. The crucial question is empirical in nature. It is necessary to measure the response of the economy to sectoral shocks before one can conclude that such shocks are important sources of business cycle fluctuations. Lilien himself began this process in his original article. This work centred upon regression results intended to decompose movements in aggregate unemployment into those due to sectoral shocks and those due to aggregate demand disturbances. The basis of these tests was a regression for the unemployment rate (U_t):

$$U_t = \alpha + \beta \mathrm{SIG}_t + \gamma \mathbf{Z}_t + \epsilon_t.$$

Here SIG_t is a constructed measure of dispersion in hiring rates across sectors and is roughly a standard deviation of cross-sectional employment growth. The vector $\mathbf{Z}_t$ contains monetary and fiscal policy variables which may also be thought to affect the unemployment rate. Lilien obtained a positive sign on his estimate of β which was of such a magnitude that a large amount of the movement in the aggregate unemployment rate was explained by his measure of sectoral dispersion. The work of Lilien was extended and applied to the case of Canada by Samson (1984). Using the same definition of dispersion, Samson concluded that sectoral shocks were important in Canada.

While these initial empirical results seemed very positive, it now seems that there may have been some problems with the method. In particular, the SIG variable can confound the impact of sectoral and aggregate shocks. Abraham and Katz (1986) discussed plausible scenarios in which aggregate shocks create dispersion in sectoral hiring rates. These effects can produce a relation between SIG and aggregate unemployment even if sectoral shocks do not affect unemployment in any significant structural way. In an attempt to disentangle the effects of sectoral and aggregate shocks, Abraham and Katz looked at the relationship between the job vacancy rate and SIG. It was their contention that vacancies should fall with SIG if SIG captures aggregate effects but rise if it is measuring sectoral effects. In fact, the latter case holds and this led Abraham and Katz to conclude in favour of aggregate shocks. Enlightened by the Abraham and Katz analysis, Neelin (1987) re-examined the Canadian evidence regarding sectoral shocks with an adjusted SIG variable which was constructed to be purged of aggregate effects. Neelin concluded that while regional shocks have had a significant effect on the Canadian unemployment rate, there is no evidence to suggest that pure sectoral shocks have had similar effects.

Other authors have also assembled evidence that questions the empirical relevance of the Lilien hypothesis. First, Murphy and Topel (1987) conducted a detailed analysis of unemployment using panel data for the U.S. economy. The following facts emerged from their work:

(i) The increase in the unemployment rate during the 1970's was largely attributable to an increase in the number of agents experiencing spells of unemployment much longer than the average spell,

(ii) total mobility in the economy is procyclical,

(iii) industry changers account for a minor and relatively constant proportion of total unemployment.

These results are damaging for several reasons. First, the increased incidence of long spells is often linked with the emergence of a class of long term unemployed who have lapsed into a state which is almost indistinguishable from the category labelled 'not in the labour force'. This stylised fact is a challenge because it implies that most sectoral reallocation is accomplished quickly and thus is unlikely to cause persistent changes in the natural rate of unemployment. A corollary of this is that changes in the unemployment rate are due to worker-related characteristics rather than sector-specific shocks.

The second fact, namely that sectoral mobility is greatest during good times, casts doubt upon the belief that reallocation is associated with significant losses in production down to resources being in transit. Finally, the third assertion of Murphy and Topel is that reallocation between sectors is neither quantitatively important nor cyclically sensitive.

At present, the issues of the quantity of reallocation and the cyclical nature of these flows is still under debate. Some of these results have been confirmed by other research. Baldwin and Gorecki (1990), using Canadian data, concluded that sectoral reallocation is small relative to total mobility. Furthermore, these movements tend to be procyclical in Canada. On the other hand, work by Loungani and Rogerson (1990) showed that there are fairly large flows between sectors and between '2-digit' industries in the United States. Also, flows from services to manufacturing are pro-cyclical while those from manufacturing to services are counter-cyclical. Lastly, Loungani and Rogerson focus on permanent sector-switching while Murphy and Topel also include temporary movements between sectors. Permanent movements are more meaningful because they represent true sectoral reallocation and it is interesting that these flows are found to be mildly counter-cyclical by Loungani and Rogerson.

These favourable results are supported by the work of Davis and Haltiwanger (1991) which studied job creation and destruction using a very rich set of data for U.S. manufacturing. Davis and Haltiwanger found that a great deal of job reallocation occurs in the U.S.. The bulk of this reallocative flow was attributed to idiosyncratic shocks, however, rather than sectoral or aggregate shocks. This latter view is consistent with a model in which unemployment results from the reallocation of labour but is not traced to sectoral shocks of the type Lilien had in mind. What is interesting about the Davis and Haltiwanger work is that they fail to find evidence of certain patterns which we expect to see in the data. It would seem reasonable that, a *priori*, oil price shocks should induce flows between certain sectors. This does not seem to occur. Notice, however, that the work of Davis and Haltiwanger is for manufacturing only. The important sectoral shocks of the 1970's and 1980's may have been flows between manufacturing and sectors such as primary resource extraction and services. This would reconcile the Davis-Haltiwanger results with those of Loungani and Rogerson.

A final criticism of the sectoral shocks view is expressed by Blanchard and Diamond (1989). This work implements a method of identifying the provenance of shocks based on restrictions on dynamic relationships. In particular, it is assumed that sectoral shocks cause vacancies and unemployment to move in the same direction while aggregate shocks cause them to move in the opposite direction. The authors apply this identifying restriction to U.S. data and conclude that aggregate shocks were more important than sectoral shocks. Several problems with this reasoning are apparent. First, the identifying assumption was not derived from an explicit theoretical model. Second, the approach does not account for observed sectoral and regional variation in variables such as unemployment duration.

The bottom line is thus that the empirical relevance of the sectoral shock view has yet to be determined. To accomplish this, it would be desirable to extend the debate beyond reduced-form regressions for the unemployment rate and vacancy rate. Clearly, it is important to specify a fuller model of an economy with sectoral shocks in order to break the current stalemate in the analysis of the empirical relevance of sectoral shocks. The next sections of this chapter will show two ways of doing this.

1.3 Theoretical Implications of Regional Variation in Unemployment Duration

Heterogeneous Unemployment Duration and Aggregate Unemployment

The summary in section 1.2 explained that an important criticism of the sectoral shocks approach to unemployment fluctuations is that a large amount of sectoral migratio is a necessary condition for substantial aggregate effects of sectoral shocks. Some authors have concluded that such migration is not observed and have argued from this that sectoral shocks are of relatively low importance. The flaw in this syllogism is that sectoral shocks can have an effect on aggregate unemployment even if there is no migration. To see why this is so, it is necessary to consider the determinants of unemployment duration in a simple two sector search model.

The simplest model of search considers a case where unemployed workers find a job with a constant probability λ per period. One can simplify the analysis greatly by assuming that this unemployment escape rate can differ between sectors but varies directly with sectoral productivity. [1] Also, it is necessary to generate unemployment by assuming that there is an exogenous job separation process that leads to a constant job loss probability of s. In this environment, the steady state unemployment rate is given by:

$$\bar{u} = \frac{s}{s + \lambda}.$$

It is possible to insert plausible values into this formula to see how the unemployment rate in a two-sector model is affected by sectoral shocks. Consider an economy in which the separation probability is 0.09. Suppose that there are two sectors which each contain one thousand workers and the employment probability is 0.70 in each sector. In this case, the steady state unemployment rate is 11.4 per cent and there are 114 workers unemployed in each sector. If a sectoral shock occurs which raises the matching rate to 0.9 in one sector but lowers it to 0.5 in the other (a seemingly symmetrical shock), then steady-state unemployment rises to 153 in the bad sector and falls to 91 in the good sector. The resulting aggregate unemployment rate is 244/2000 or 12.2 per cent. In short, the change in employment probabilities raise the economy-wide unemployment rate from 11.4 per cent to 12.2 per cent.

A few comments can be made regarding this result. First, the intuitive expla-

[1] These simplifying assumptions are not without justification. Chapter Two of this thesis shows that they come out of an optimising model.

nation for the rise in unemployment is that raising the matching rate has a lower absolute effect than lowering it because, as the matching rate rises. unemployment falls so that fewer searchers exist to be affected by the higher matching rate. Of course, the opposite effect is true when the matching rate falls. Next, this example assumed that (i) neither sector experiences further separations and (ii) a pure sectoral shock has symmetric effects on match probabilities. If the bad sector experienced a series of lay-offs, there would be a much larger increase in unemployment during the adjustment to the steady state. Also, the empirical results of Chapter Three of this thesis suggest that a concave relationship exists between measured recruitment activity and matching probabilities. This additional non-linearity further amplifies the impact of the asymmetry analysed here. Lastly, the steady state argument does not consider what will happen during the adjustment period. Further theoretical work is clearly needed to resolve this issue. In passing, note the curious result that if the unemployed can migrate in this model they will move from a low match probability sector to a high probability sector. This migration would tend to reduce the impact of this sectoral shock on aggregate unemployment rather than magnify it as the conventional wisdom suggests.

(ii) Unemployment Duration Heterogeneity and 'Long' Spells of Unemployment

It has long been known that unobserved heterogeneity within a population can produce the *appearance* of a non-constant hazard rate for a population, even if individuals all have a constant hazard. For example, suppose that the population consists of two groups, both of which have constant but distinct per period probabilities of escaping unemployment. If the two groups are indistinguishable to the observer, there will seem to be an inverse relationship between elapsed search time and the escape probability for the population as a whole. This is because, as search time increases, any group of searching agents contains proportionally more of the low escape probability type. This type of pattern to escape rates can create the perception of "too long" spells.

A numerical example is useful to illustrate how important this heterogeneity can be. An example is constructed by drawing two random samples from distributions of job search times in which the two samples have different probabilities of finding a job. It will then be possible to quantitatively evaluate the effect heterogeneity can have on the shape of unemployment spell distributions. To operationalise the example, suppose that an economy exists in which one group has an expected duration of seven weeks while the other sector has an expected duration of fifteen weeks.[2]

Suppose that in each sector the conditional probability of leaving unemployment after searching for τ periods, the "hazard rate", does not vary with τ and is chosen to yield the average durations listed above. Waiting times (t) are exponentially distributed and described by the following density function:

$$f(t|\lambda_i) = \lambda_i \exp - (\lambda_i t) .$$

The expected waiting time for an exponential distribution is $1/\lambda_i$ and this allows for parameterisation of the example. Two samples of 1000 observations each can be drawn from exponential(1/7) and exponential(1/15) distributions. The series thus obtained can be used to look at the apparent distribution of spells when the two sectors are combined.

A qualitative presentation of the results from this experiment is provided by Figure One. This shows the hazard rate for leaving unemployment for a mixture of exponential distributions with expected durations of seven and fifteen weeks. The graph shows that the hazard rate for the mixture distribution falls with time spent searching. In other words, it seems that agents face a probability of leaving unemployment that declines with time spent searching. As a matter of fact, however, each of the two groups of agents has a constant hazard rate. The dotted line in the graph

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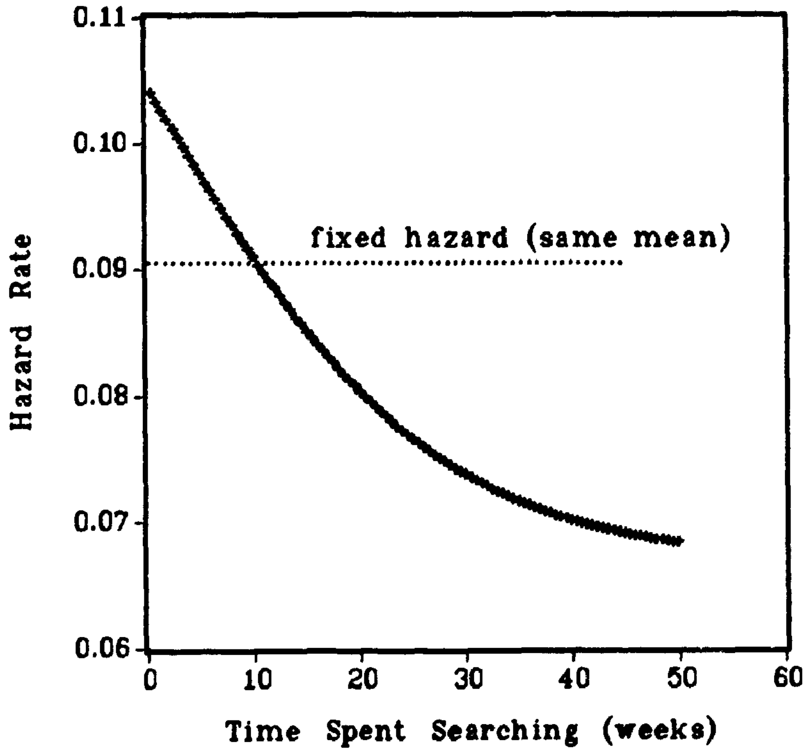
[2] Incidentally, this was basically the situation for Alberta and Ontario, respectively, during the mid-1970's.

is for comparison purposes. It shows the hazard rate for a homogeneous population with an exponential distribution for search time and an expected duration of 11 weeks. This population has the same expected duration as the mixture distribution but instead has a constant hazard rate. Long spells seem to be more important in the mixture distribution because individuals who have been unemployed for a long time have a higher probability of remaining unemployed.

Figure One: Hazard Rate for Mixture of Exponential R.V.'s



To quantitatively assess the degree to which regional heterogeneity can increase the importance of long spells, a tool used by Beach and Kaliski (1984) to examine unemployment duration is helpful. Beach and Kaliski use 'duration shares' as a measure of how important long spells of unemployment have been. Essentially, these shares measure the proportion of total unemployment time that is accounted for by each of ten ranges of spell lengths. To construct duration share measures, the following method is employed:

i.) Write down the number of weeks of unemployment, d, for each spell observed in a long list.

ii.) Sort the elements from smallest to largest.

iii.) Next, divide the sorted list into ten equal groups $\mathcal{G}_i, i = 1, 2, \ldots, 10$.

iv.) Create the decile sums $S_i = \sum_{d \in \mathcal{G}_i} d$ where the summation sign is over durations that are elements of the group $\mathcal{G}_i$.

v.) Finally, calculate the duration shares for each group. These shares are defined by $\mathrm{DS}_i = \mathrm{S}_i / \sum_1^{10} \mathrm{S}_i$.

These duration shares have been calculated for each of the three samples in the numerical experiment and are presented in Table One. In the numerical example, the two sub-samples have lists of 1,000 elements while the combined sample has a list of 2,000. The equal groups (deciles) created in step (iii) each have 200 elements for the combined sample and 100 for the two sub-samples. An interesting feature of Table One is that the low and high hazard samples have roughly the same distribution of total days unemployed between the decile ranges. As expected, the 100 longest spells account for the highest proportion of the total weeks unemployed. What is notable is that, when these two samples are combined, the percentage of time accounted for by the longest spells, those in the tenth decile, increases from 32 or 33 per cent to 37 per cent. Departures from the homogeneous case of this magnitude were reported by Beach and Kaliski, although not for the Ontario and Alberta case. It is also interesting that there is no enormous change for the lower decile shares.

The numerical example makes it clear that with plausible amounts of heterogeneity in a sample, there is a noticeable increase in the importance of long unemployment spells. Granted, it is clearly necessary to use empirical methods to examine the sources of hazard rate heterogeneity. The theoretical analysis above cannot discriminate between heterogeneity due to sectoral shocks versus low hazard individuals scattered throughout the economy. Simply, the point established in this example is that heterogeneity *between* regions rather than *within* them might be the important factor in explaining longer spells. Finally, notice that sectoral shocks of this type cause skewness toward long spells even if there is no sectoral migration.

Table One: Results of the Numerical Experiment

(i) Combined Sample

Total Weeks:	88.14	Share:	0.41
Total Weeks:	286.31	Share:	1.32
Total Weeks:	524.06	Share:	2.42
Total Weeks:	795.35	Share:	3.67
Total Weeks:	1149.11	Share:	5.30
Total Weeks:	1569.27	Share:	7.24
Total Weeks:	2115.68	Share:	9.76
Total Weeks:	2883.47	Share:	13.30
Total Weeks:	4130.13	Share:	19.04
Total Weeks:	8145.39	Share:	37.56

Total Weeks Spent Unemployed: 21686.9

(ii) Sample with Expected Duration of Seven Weeks

Total Weeks:	35.93	Share:	0.55
Total Weeks:	115.34	Share:	1.77
Total Weeks:	200.71	Share:	3.08
Total Weeks:	289.99	Share:	4.45
Total Weeks:	394.24	Share:	6.05
Total Weeks:	528.14	Share:	8.10
Total Weeks:	680.04	Share:	10.43
Total Weeks:	911.60	Share:	13.99
Total Weeks:	1260.14	Share:	19.33
Total Weeks:	2102.12	Share:	32.25

Total Number of Weeks Spent Unemployed: 6518.2

(iii) Sample with Expected Duration of Fifteen Weeks

Total Weeks:	58.18	Share:	0.38
Total Weeks:	201.23	Share:	1.33
Total Weeks:	402.34	Share:	2.65
Total Weeks:	663.84	Share:	4.38
Total Weeks:	907.39	Share:	5.98
Total Weeks:	1217.90	Share:	8.03
Total Weeks:	1593.49	Share:	10.51
Total Weeks:	2111.96	Share:	13.92
Total Weeks:	2921.23	Share:	19.26
Total Weeks:	5091.10	Share:	33.56

Total Number of Weeks Spent Unemployed: 15168.7

1.4 Selected Stylized Facts in Regional Unemployment Duration Data

To show that unemployment duration has exhibited significant differences, a graphical analysis is revealing. Figures Two through Five present monthly time series observations on provincial average unemployment duration series as measured by the average elapsed duration variable provided in the Labour Force Survey. Each graph pairs two series so that differences in behaviour are easily discerned. While neither exhaustive nor rigorous, this approach has the virtue of allowing the reader to interpret for herself the meaning of the differences.

The first graph shows series for Ontario and Quebec. There are clearly common movements shared between the series, the most notable being the rise in unemployment duration during the 1982 recession. Equally clear are divergences. Between 1977 and 1980, there is a striking divergence of behaviour: unemployment duration trended down in Ontario but up in Quebec. During the period of recovery between 1983 and 1987, unemployment duration fell steadily in Ontario while it was very much constant in Quebec.

Figure Three compares series in Ontario and British Columbia. This graph shows a phenomenon that recurs several times in this analysis. The unemployment series in these two provinces behave quite similarly between 1975 and 1982. During 1983 and 1984, however, the fall in duration in Ontario seems to coincide with a rise in B.C.. After 1984, the series move again in the same direction but with something of a level shift.

In Figure Four an interesting reversal of fortunes is revealed. During the 1980's, the average duration of unemployment was higher in Newfoundland than in Quebec. This pattern reverses itself following the 1982 recession. Beginning in 1983, the average duration of unemployment is about five weeks shorter in Newfoundland. Figure Five reveals a similar change of fortunes for Alberta and Newfoundland. Unemployment duration was far lower in Alberta than Newfoundland during the 1970's and early 1980's whereas, following the recession, the two series were at about the same levels.

A few key facts can be synthesised from these graphs. First, regional differences in average unemployment duration are large enough to be quite relevant. In the early 1970's, for example, the average unemployment spell in Ontario was fifteen weeks while it was only seven weeks in Alberta. This is an important difference. Next, the average duration of unemployment can change fairly quickly over time. The response at the time of the 1982 recession reveals this clearly. Finally, regional differences in unemployment duration are not constant over time. The relative positions of Ontario and Alberta reversed following the 1982 recession. The Ontario and British Columbia series were very close until after 1982. A similar pattern emerges when comparing Ontario and Quebec. Thus, regional differences in duration of unemployment are both large enough and variable enough to provoke interest and motivate research which can account for them.

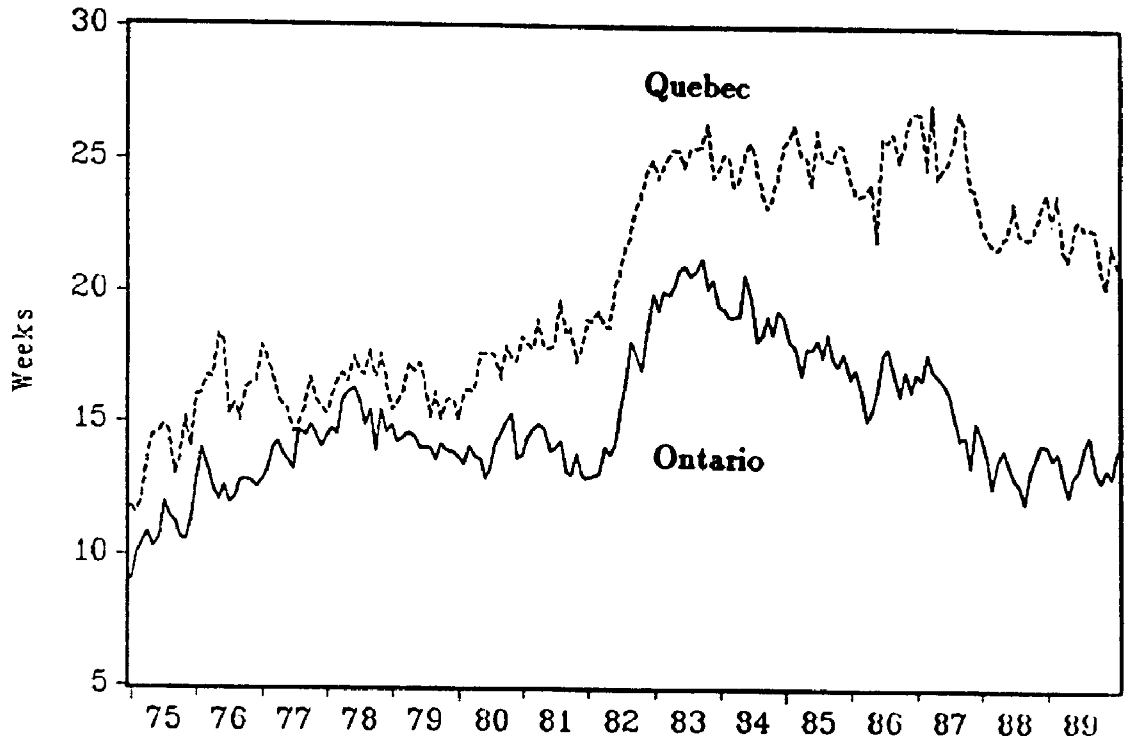


Figure Two: Unemployment Duration in Quebec and Ontario

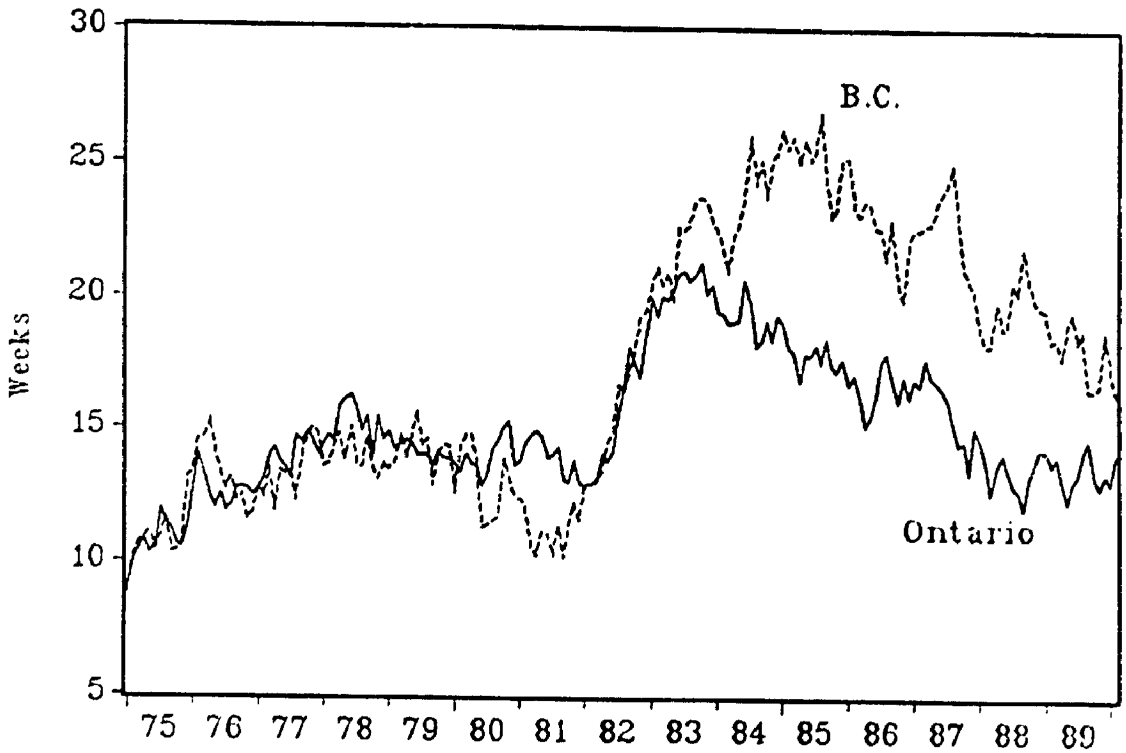


Figure Three: Unemployment Duration in Ontario and British Columbia

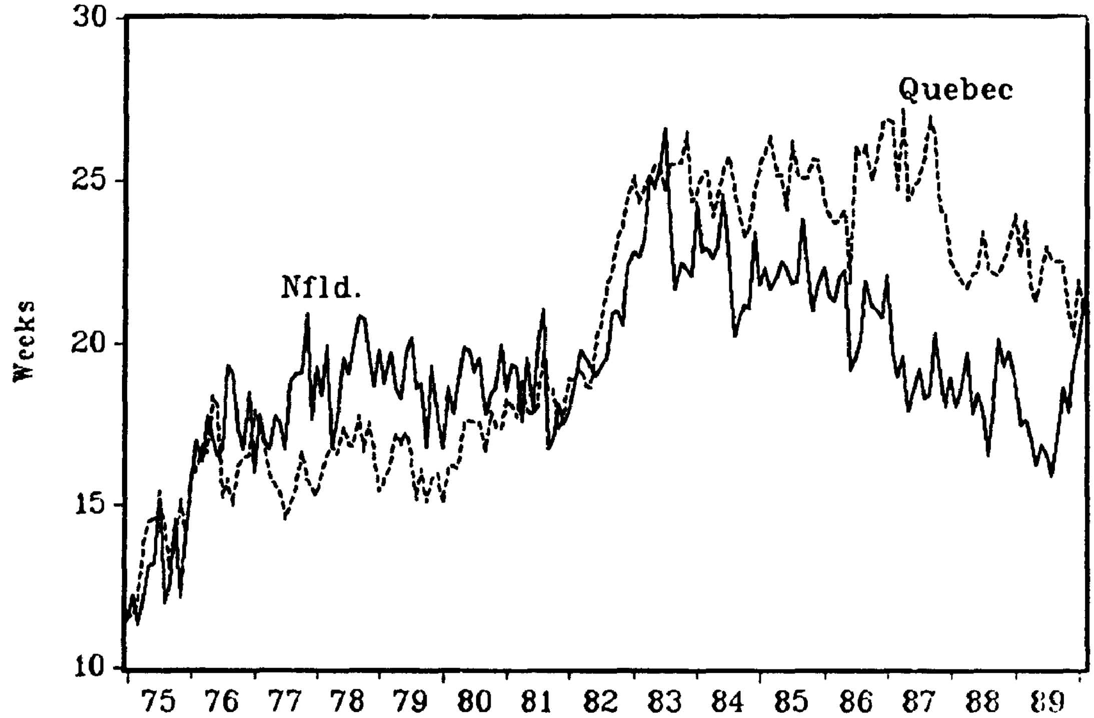


Figure Four: Unemployment Duration in Newfoundland and Quebec

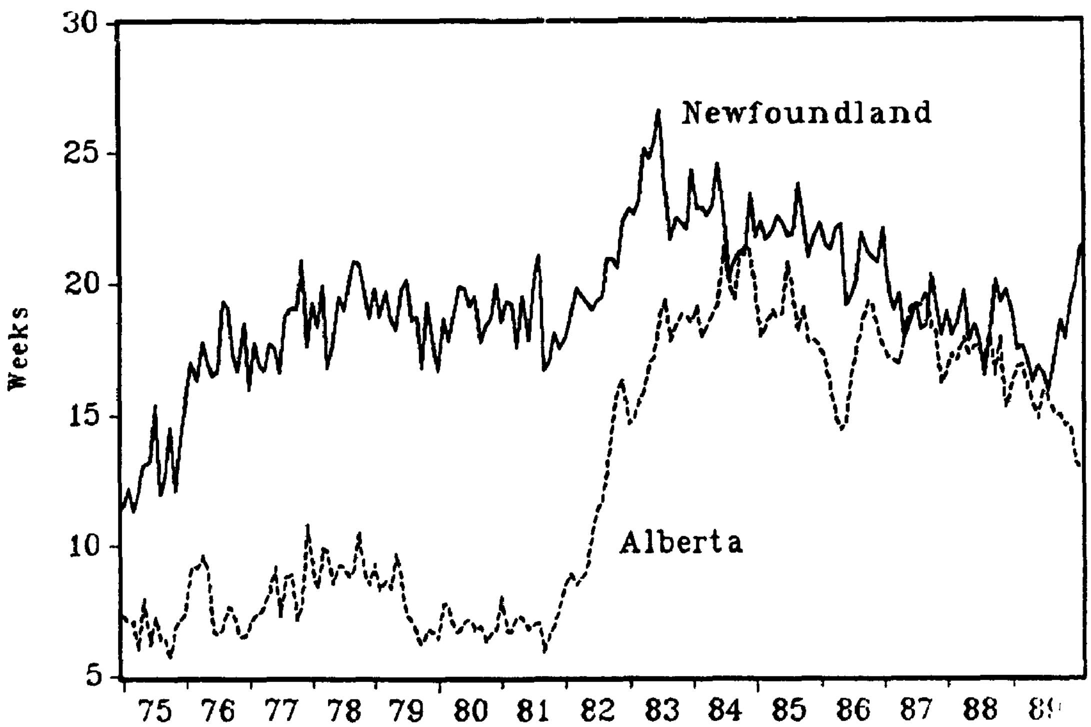


Figure Five: Unemployment Duration in Newfoundland and Alberta

1.5 A Survey of Existing Empirical Work on Unemployment Duration

Researchers in diverse fields are concerned with statistical models of duration. Biostatisticians study lengths of recovery periods or the time before a patient succumbs to a fatal condition. Reliability analysts concern themselves with the distribution of failure times for machinery and equipment. In economics, the length of time spent searching for employment and the duration of employment have been the topics of both theoretical and empirical work. An excellent survey of the variety of statistical methods which can be used to study the duration of unemployment is provided by Kiefer (1988).

The existing empirical literature on unemployment duration can be divided into three general categories: cross-sectional studies of the shape of the duration distribution, analysis of the pattern of unemployment duration over the cycle, and the effect of policies such as unemployment insurance on unemployment duration. The issue of sectoral and regional differences in unemployment duration seems to have eluded direct analysis by researchers. Some studies have included regional or sectoral variables as explanatory variables but none have directly attempted to determine how recruitment activity in markets has affected employment probabilities.

An interesting study by Corak (1990) of duration in Canada uncovered effects due to age and education but failed to find much evidence of differences in duration between provinces. The exceptions were between Quebec and Ontario and Newfoundland and Ontario. Two possible explanations for this are possible. Firstly, the use of a dummy variable technique in a parametric duration model will only reveal differences between the reference province and others directly. This work does not seem to have conducted tests of differences between durations in, say, Saskatchewan and Nova Scotia. Of course, Ontario does seem to provide a good base case.

More significantly, the work of Corak included other determinants of duration such as age and education level. The differences in duration in the monthly time series data may reflect the impact of omitting these effects. This would provide an interesting explanation of observed differences in duration. This seems unlikely in light of more recent (unpublished) work by Corak which extends the original analysis but uses the 1986 Labour Market Activity Survey data[3]. These results shows significant provincial effects in a broad range of provinces, in spite of the inclusion of other variables.

One important line of inquiry concerns the effects of heterogeneity on observed distributions of unemployment duration. As the example of section 1.3 above illustrated, cross-sectional distributions of waiting times can be affected by heterogeneity within the population of searchers. In particular, falling hazard rates and increased importance of long spells can be produced by a sample with unobserved heterogeneity. This type of feature has been of interest to researchers since Clark and Summers began to talk about the role of the long-term unemployed in unemployment fluctuations.

Work by Sider (1985) for the U.S. and Hasan and DeBroucker (1982) for Canada examine the extent to which a small sub-group of the unemployed account for a disproportionately large amount of search time. These studies confirm that there is heterogeneity with regard to escape rates from unemployment. This heterogeneity manifests itself by producing hazard rates which appear to fall with time spent searching and by adjustments in the shape of the duration density function which attribute greater weight to longer spells of unemployment. Recall that this should be expected given the results reported by Corak in his more recent paper. In addition to the regional heterogeneity mentioned above, Corak also finds considerable evidence in favour of the hypothesis that older workers have lower probabilities of leaving unemployment. An interesting exercise would be the use of the Corak results to see which of these effects would produce more pronounced 'long spell' effects.

Finally, there is a recent contribution to this literature which provides an important note of caution for the field in general. A recent study by Addison and Portugal

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[3] These results were presented at the 1990 meetings of the Canadian Economics Association in Victoria, British Columbia.

(1987) estimated a general parametric model which has the virtue of nesting many of the distributions commonly used in applied work. For instance, the Weibull and exponential distributions result when certain parameter restrictions are imposed. The work finds that conclusions about the dependence of hazard rates on time can be overturned when different restrictions are imposed. Also, a key finding is that shapes of distributions are often different for recipients of unemployment insurance benefits versus non-recipients. One clear implication of this work is that some commonly used distributions are too restrictive. Also, heterogeneity is again confirmed, this time due to a different source.

The second group of empirical studies are concerned with the cyclical properties of duration. Dynarski and Sheffrin (1982) discuss the conventional wisdom on this question and note that there have been conflicting results. In some cases, higher unemployment rates have been associated with shorter durations of unemployment (i.e. Solon (1985)) while in other instances individual durations have been found to rise with the aggregate unemployment rate (i.e. Katz (1985)). Dynarski and Sheffrin note that this is consistent with a reservation-wage model in which the cycle produces changes in wage distributions that have ambiguous effects. Note also that the sectoral aggregate shock case could matter here. A sectoral shock may produce 'in transit' workers but the duration of unemployment could be short as a result of intense recruitment by expanding firms. The conclusions of this work are that individual duration does increase with the aggregate unemployment rate and that, during periods of recovery, the greatest rise in employment probability is for those unemployed for long periods. The paper uses individual-level data and this leads to the conclusion against the Darby, Haltiwanger and Plant (1986) theory of cyclical variation in duration based on sorting effects.

Finally, some researchers have analysed how unemployment duration is affected by government policy with regard to entitlement periods for unemployment insurance. Meyer (1990) and Ham and Rea (1987) analyze this for the U.S. and Canada respectively. Generally, this work confirms the prediction from search theory that hazard rates rise toward the end of benefit periods.

It is not easy to draw conclusions from this fairly diverse body of work. Several comments are in order, however. First, there is evidence that observable factors, such as regions and sectors, affect unemployment duration. Nevertheless, it seems unlikely that the factors identified thus far are able to explain observed movements in regional unemployment data. In particular, the rapid changes in absolute and relative levels of these series defy explanation based on demographic factors. The age and skill profile of a province changes only but slowly and such explanations for heterogeneity in provincial unemployment duration seem lacking; hence it is desirable to consider that region or industry specific shocks mat be the source of regional unemployment rate heterogeneity.

1.6 Theoretical Models of Regional Unemployment Duration

This literature seems to suffer from a double-coincidence of wants. Models with more than one sector do not have variable unemployment duration and likewise multi-sector models of search and unemployment do not have endogenous unemployment durations. Taking the latter class of models first, island-economy search models such as those of Lucas and Prescott (1974) and King (1990) assume that search continues for a fixed period of time only and are thus unable to address the observed facts. A two-period model by Rogerson (1987) also presents a model of sectoral reallocation with search but the dynamic properties are not well established. Other models of unemployment, such as the lottery models of Hansen (1985), have variable unemployment duration but lack the concept of vacancies. Wright (1986) studies variation of duration in a signal-extraction model with search but sectoral differences are not possible with such aggregate shocks. In summary, no model currently exists which accounts for sectoral differences in duration within a model of reallocation.

A natural way to bridge this gap is to use the matching function approach to endogenize unemployment duration. The use of stochastic matching functions is largely due to the work of Mortenson (1982) which examined models in which the number of matches is a function of the number of participants on either side of the market. Since then, Pissarides (1987) has used this tool to provide a theoretical explanation for stylized facts of the time series behaviour of vacancies and unemployment (the 'Beveridge curve'). Blanchard and Diamond (1989) have calculated empirical estimates for a parametric matching function using U.S. data and have used them to study the relative importance of sectoral versus aggregate shocks. To date, this approach has not been adopted in other multi-sector models of adjustment, however. A multi-sector model in the matching function vein is desirable because it contains all the variables currently examined in the sectoral shocks debate (unemployment, vacancies, wages) as well as unemployment duration.

1.7 A Description of Data Available for Empirical Testing

To deal with the issue of unemployment duration and sectoral shocks it is vitally important to assemble data for empirical testing. Some sources have been used in the analyses of unemployment and vacancies by Lilien and Abraham and Katz. Also, Blanchard and Diamond (1989) used data to calibrate a matching function in their 'minimalist' model of labour market dynamics. Nevertheless, there is not an abundance of readily available data, especially for unemployment duration and for Canada. Accordingly, it is useful to list and describe sources of data which may be of use to future researchers.

Measuring Recruitment Effort by Firms

Ideally, the argument in the matching function which captures the firm's input to the matching process should be some measure of total hiring intensity. Unfortunately, such data cannot be collected directly so that researchers must make do with proxies such as numbers of job vacancies or outstanding offers. Several sources of such data are available: counts of newspaper "Help Wanted" advertisements, vacancy surveys from employers, and records of posted job vacancies from Employment and Immigration Canada. This section discusses the availability and suitability of these various data sources.

First, the Help Wanted Index (HWI) collected by Statistics Canada provides a monthly measure of job advertisements. The methodology underlying the index changed somewhat in 1989 and so the series has to be described in two steps. Originally, between 1962 and 1988, these measures were based on counts of help-wanted ads in 18 major Canadian newspapers. As well as a national series, regional series for British Columbia, the Prairie Provinces, Ontario, Quebec and Atlantic Canada are available. Historical series, both adjusted for seasonal variation and unadjusted, are available in the 1988 issue of Statistics Canada Publication 71-204.

Changes to the Help Wanted Index were made in 1988 with the intention of removing certain problems with the index. Problems were encountered when newspapers changed type sizes which created spurious variations in the HWI. To avoid this problem, a new index was begun which simply counted ads rather than measuring column inches. Unfortunately, historical values for the new series are only available beginning in 1981. This is a problem because the old index was not calculated after the end of 1988. This means that there are no contiguous series which are both up to date and available for a lengthy historical period. The severity of this problem may be mitigated, however, by the fact that analysis shows no evidence of significant differences between the behaviour of the old and new series. For example, Statistics Canada reports that the correlation between the new and old series over the period 1981-88 was 0.97.[4]

Conceptually, the Vacancy Survey is the most desirable data source because it

[4] See Haggar-Guenette, Cynthia, "The 'Old' and the 'Revised' Help-wanted Index: A Comparison", Statistics Canada Publication 71-204, 1988.

measures recruitment intentions directly rather than through advertising activity. It seems likely that such a measure would be unaffected by changes in prices of Help Wanted advertisements or market structure in the newspaper industry. Of course, this type of survey is subject to all the possible shortfalls of surveys in general. The incentive for firms to truly reveal their recruitment intentions may be weak. Furthermore, there is ambiguity concerning the concept of a vacancy. Firms could make statements to the effect that 'there's always room for a good, committed worker' but this cannot really be construed as a vacancy. The advantage of help wanted advertising is that it is costly to the firm and therefore reveals a true desire to recruit a worker.

On a practical level, the survey is a rich source of information about vacancies in various regions and sectors. Data are disaggregated by province, by occupation category, and by industry. Provincial industry data are also available. In addition, separate series are collected for long-term versus short-term vacancies. This latter distinction may allow for some indication of which vacancies are really perpetual openings for skills in short supply. Unfortunately, the quantity of information available from this source is limited by a quarterly frequency and the cancellation of the vacancy survey in 1978.

Given the limitations of the vacancy survey data, it is to be hoped that the HWI performs well as a proxy. To investigate this question, it is useful to compare the behaviour of these series over the periods for which both were available. The monthly HWI series can be converted to a quarterly basis by taking averages of the three quarterly values. Regional comparisons can be made directly for British Columbia, Ontario and Quebec because HWI series are available for these three provinces. In addition, the Alberta vacancy series could be compared with the HWI for the Prairie Provinces because Alberta likely receives a high weight in this series and because Alberta is a particularly interesting province due to its reliance on natural resource extraction.

Graphs of these pairs of series are presented as Figures Six through Nine. In the graphs for Ontario and Alberta, the coherence between the two series is striking (note that two scales are used to account for level differences). Furthermore, correlation coefficients are around 0.9 for Ontario and 0.84 for Alberta. The lower value for Alberta may reflect the use of the HWI for Alberta, Saskatchewan and Manitoba to proxy that for Alberta but could also be due to sampling error in the correlations.

For British Columbia, the behaviour of the two series differ much more. There seems to be a break in the relationship between the vacancy series and the HWI during the middle of 1974. This break is more in the nature of a level shift than a change in dynamic behaviour so the usefulness of the British Columbia HWI data is also confirmed, although with some reservations. It is in the Quebec series that a strong difference in behaviour emerges. Reported vacancies from Quebec rose through 1973 and then fell back to approximately the level of 1971 by 1978. The HWI, on the other hand, rose through 1974 but then stayed constant for the remainder of the sample. No immediate resolution of this puzzle seems to be available. One possibility is related to the period of political uncertainty in Quebec before and after the election of the Parti Quebecois government in 1976. It may be that employers in English Canada placed advertisements in Quebec newspapers in order to attract valued workers who might be inclined to leave Quebec. Whatever the explanation, this puzzle suggests that care must be exercised when using the Quebec HWI and that it may provide an interesting topic for future research.

The results provide strong evidence in favour of the usefulness of the HWI for Ontario and Alberta. Clearly, if either the HWI or vacancy survey is bad, then both are. It would be surprising that both series are flawed in the same way since the methodologies underlying the two series are quite different: the HWI is derived from counts of newspaper advertisements while the vacancy survey was a direct survey of employers. For British Columbia this conclusion is weaker, while for Quebec it seems that at least one of the measures of recruitment activity has problems. It is possible to conclude from this that, at a minimum, the HWI was a good indicator of movements in recruitment activity during the 1970's in certain key provinces.

The broader question of whether any of the HWI series continue to perform well is more difficult to answer. On the one hand, a close link between the HWI and Vacancy Survey data in the 1970's is cause for optimism during the 1980's and 1990's. A more pessimistic note is sounded by the work by Katherine Abraham (Brookings Papers, 1987:1) which uncovered evidence of a gap between the U.S. HWI and vacancy surveys which began in 1981. This problem was attributed to changes in employment mixes and declines in competition in the newspaper industry. Clearly, both of these phenomena manifested themselves in Canada as well as the U.S..

Abraham conducted research to correct for these factors but, to date, this has not been done for Canada. Nevertheless, pending the completion of such research, it should be noted that the correction Abraham applies is a "drift" factor which corrects for a monotonic shift in the HWI over time. To the extent that this effect always goes in one direction, the HWI remains useful outside of the 1970's as a measure of the trends in recruitment intensity.

Some hope for the future resolution of this problem is fostered by the existence of vacancy data kept for administrative purposes by Employment and Immigration Canada. District-level statistics on posted vacancies by occupation code are available and these should give some more direct indication of recruitment activity than is possible from newspaper advertisements. Unfortunately, these data are in hard-copy form and have not been collated so that the creation of a data-base will be a time-consuming task for future research. The appendix to this chapter provides further information regarding the disposition of this information.

It is to be noted that two other sources of vacancy data are available, although neither adds significantly to those discussed above. First, the Technical Service Council, a private consulting and placement service located in Toronto provides a quarterly survey of job vacancies for professionals such as accountants, engineers and executives.

This organization is industry-sponsored and seems to be largely designed to find new employment for persons laid-off by member firms. Nevertheless, the report provides some anecdotal evidence with explanations that focus on specific industry developments. The usefulness of this is hindered by the lack of data for non-professional categories.

A second measure of job vacancies is calculated by the Canadian Labour Market and Productivity Research Centre. These are based on observed movements in the Help Wanted Index with units applied based on a methodology explained in a paper by Kapsalis (1988). Essentially, it is assumed that the number of frictional unemployed equals the number of vacancies and that all unemployment in 1966 was frictional. After this, the Help Wanted Index is used to build up a current vacancy series. This seems to add little to the HWI save units and the vacancy series provides a natural and simpler way to attribute units.

Measures of the Stock of Searching Workers

The Labour Force Survey which is conducted by Statistics Canada on a monthly basis provides information about three stocks: the unemployed, the employed, and those not in the labour force. These data are available in various disaggregations: by province, by age group, and by sex. The data also exist for a fairly lengthy period of time.

Clearly, all three types of agents may potentially take a job vacancy at a firm. The unemployed are most likely candidates for searchers since they are, by definition, desiring of work. Blanchard and Diamond have documented flows from out of the labour force directly into employment, however, indicating that this is also a source of matches. Finally, it is clear that employed workers may engage in on-the-job search to find better jobs. All of this means that it is far from straightforward to determine how the pool of searchers should be measured.

It is clear that the appropriate choice of a searching pool is an empirical rather than theoretical issue. Along with the Labour Force stocks, data is also collected on gross flows between the three states. These flows can be used to compare the flow probabilities into employment from each of the three labour market states. This work has been partly undertaken for the United States by Blanchard and Diamond and could be done for Canada given the availability of similar Canadian data starting in 1976. This exercise has not yet been done here and so the current maintained hypothesis will be that the reported unemployed is the appropriate search pool on the workers' side.

An alternative source of data is the Labour Market Activity Survey (LMAS) which was conducted by Statistics Canada as an add-on to the January and February Labour Force Surveys (LFS). A sub-sample of LFS respondents are asked to provide detailed retrospective information on a weekly basis regarding labour force status and methods of job search. The potential advantage of this is to identify more clearly the willingness of workers to take a job and the intensity with which they searched. A major drawbacks is that, as Jones and Riddell (1991) document, the form of the LMAS questionnaire actually destroys some of this useful information about unemployment and movement out of the labour force. Also, the data are available only for, at most, a two-year longitudinal survey period which precludes the assembly of a long time-series account of individual search behaviour.

Measures of Unemployment Duration/Matching Probabilities

One direct method of unemployment duration is provided directly through the Labour Force Survey. An 'average duration of unemployment' variable is collected which takes total elapsed time spent unemployed for all those currently unemployed and divides it by the size of the unemployed stock. This variable is available for each of the provinces in Canada. Problems with the variable are that it (i) only measures elapsed time (i.e. spells are right censored) and (ii) averages over all agents so that the average could change whenever a non-representative group of individuals exit unemployment.

A second measure of unemployment transition probabilities is yielded by gross flows data. It is possible to take the flows out of each labour force state and divide them by the corresponding stocks to yield measures of instantaneous transition probabilities. While also subject to heterogeneity bias, these measures do not suffer from truncation problems or the problem that they are complex functions of past hazard rates. These measures of probabilities have been calculated from available gross flow data and are used in Chapters Three and Five of this thesis.

Finally, the Labour Market Activity Survey has measures of weekly labour force status and can thus be used to determine the length of unemployment spells. Of course, the problems mentioned above with the design of the questionnaire limit the value of these data but there is, nevertheless, a great deal of individual specific data on the duration of non-employment spells (although not specifically unemployment) available from this source.

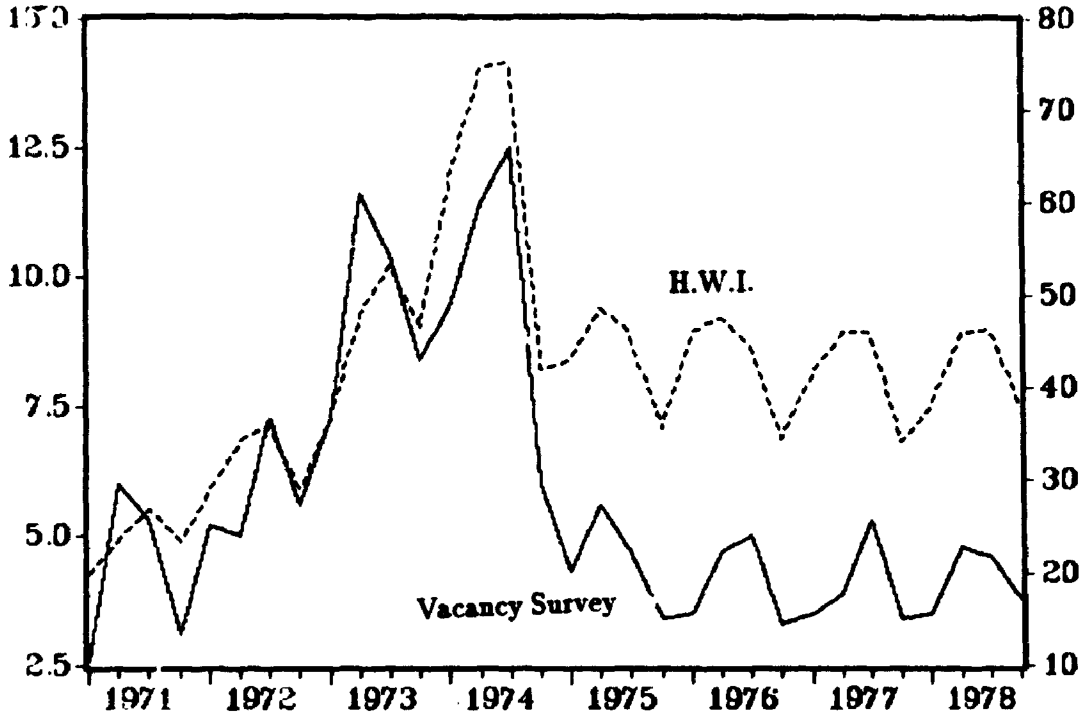

Figure Six: Comparison of Vacancy Survey and HWI: B.C.

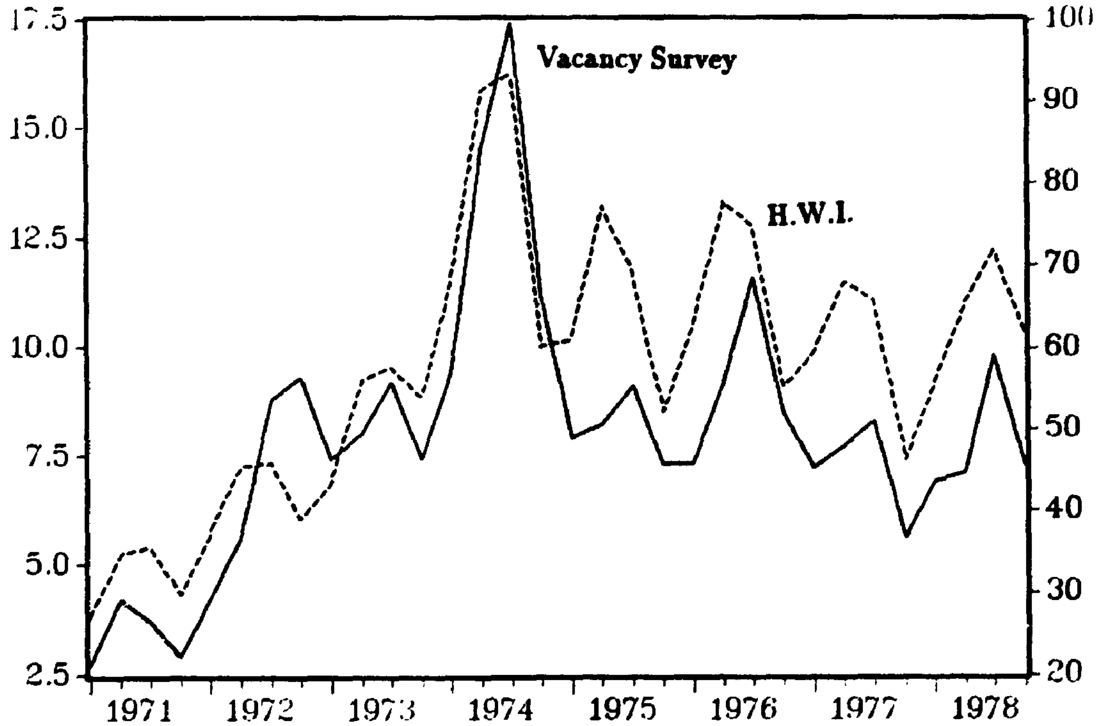

Figure Seven: Comparison of Vacancy Survey and HWI: Alberta

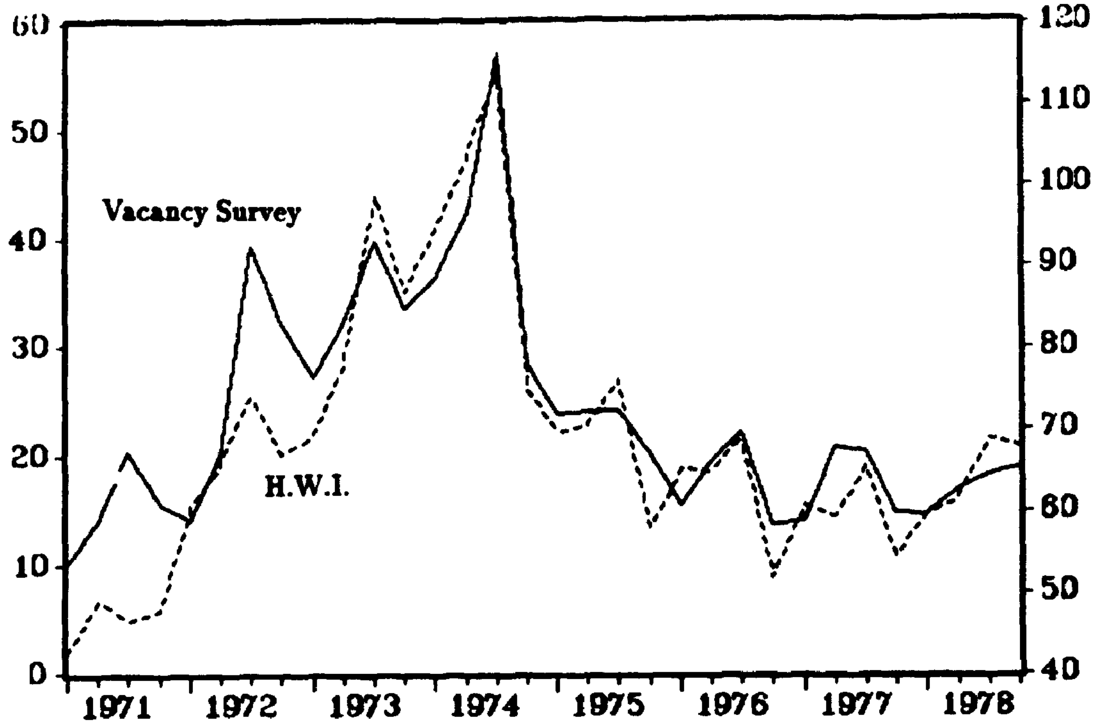

Figure Eight: Comparison of Vacancy Survey and HWI: Ontario

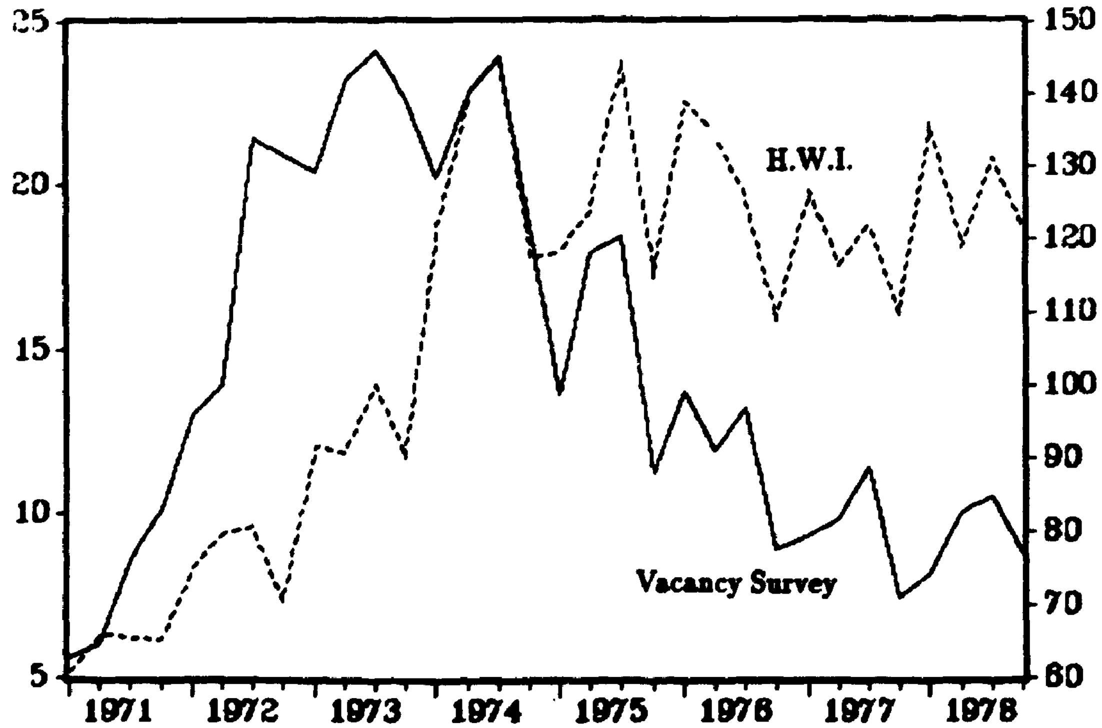

Figure Nine: Comparison of Vacancy Survey and HWI: Quebec

1.8 Conclusions

A summary of the various currents and eddies discussed in this chapter is now in order. First, recall that the sectoral shocks debate strikes to the heart of macroeconomics since it concerns the interpretation of unemployment. Next, at a theoretical and empirical level, it is important to broaden this debate to include a consideration of unemployment duration as well as the unemployment rate. It was shown that, presently, there is no comprehensive account of both observed phenomenon and their relationship with theory. It was also shown, however, that the lack of empirical testing does not reflect a scarcity of data.

Given the above, there is a clear need to undertake research in the area of search, unemployment duration and sectoral shocks. One important need is for a multi-sector model of unemployment with endogenous duration. This allows for the testing of the sectoral shocks model and the more careful formulation of identifying assumptions of the type used by Blanchard and Diamond. Next, attempts must be made to empirically determine the source of unemployment duration heterogeneity. This will help in the accounting for the aggregate unemployment rate. Finally, empirical work which looks at the contribution of unemployment rate heterogeneity to aggregate unemployment rates and skewness in distributions is needed. Several of these avenues of investigation are the subject of other chapters in this thesis. Others will form the basis of planned future research.

Chapter Two:

Unemployment Duration Dynamics with Sectoral Reallocation and Equilibrium Search

2.1 Introduction

The level of the unemployment rate is a perennial magnet for research in both theoretical and applied economics. Of late, however, increased attention has also be accorded to the duration of unemployment – that is, the amount of time the unemployed take to find a job. This interest in unemployment duration can be attributed to several factors. First, modern choice-theoretic macroeconomics treats the labour market state of an individual as the outcome of a decision-making process given the economic environment. The duration of unemployment thus reveals information that is interesting from the viewpoint of economic analysis. Likewise, empirical research has lately fixed upon interesting puzzles apparent in unemployment duration data. Among these are the concentration of unemployment time in spells of long duration and the divergent records of regions of Canada with regard to unemployment duration. Together, these factors have served to increase the need for research into the determinants of unemployment duration.

This chapter carefully constructs and analyses a two-sector model of labour market search in which unemployment duration is an endogenous variable. The inclusion of two sectors allows for the derivation of testable implications regarding unemployment duration both within and between sectors. One virtue of the endogenisation of unemployment duration is the improvement it represents over previous multi-sector models of labour markets which have imposed a fixed duration of search. These past analyses have excluded a potentially important variable when studying sectoral reallocation. The inclusion of unemployment duration may ultimately aid in the decomposition of unemployment rate movements into aggregate and sectoral factors.

Structure is imposed on the search process by adopting the "matching function" framework wherein the number of successful matches within a market is related to the number of searchers of each type. A virtue of this approach is that transition probabilities from unemployment to employment are affected more by variation in the arrival rate of job offers than by the offer rejection rate. This accords well with recent empirical studies such as Jones (1988) and Holzer (1988) which suggest that offer acceptance probabilities tend to be close to one. Finally, in contrast to reservation wage models, this method of endogenising duration does not rely on the confusion of relative and nominal price changes. This is fortunate because the profession views the empirical validity of such missperception with mounting skepticism.

Given the approach of the chapter, the empirical predictions that it yields help to provide ways to conduct general tests of matching function models. Pending the outcome of such tests, it is difficult to determine the practical relevance of theoretical work in this paradigm. Recently, papers by Davidson, Martin, and Matusz (1988) and Hosios (1990) have explored the implications for multi-sector trade theory of matching rate models. A byproduct of this chapter will be suggestions for global empirical tests of these types of models.

The heart of the chapter is a model of a simple economy with two spatially separated sectors or regions. A certain proportion of the labour force in each sector is mobile and can migrate between sectors. In contrast to the unspecialised labour of the model, capital is completely specific to its home sector. Workers and firms who wish to effect an exchange do not have access to a centralised spot market for labour. Instead, there is a time-consuming search process whereby unemployed workers and job vacancies are brought together. Moreover, this process is stochastic so that there is uncertainty regarding the amount of time that will elapse until a match is consummated. In this framework, there are two decisions which agents must make. First, an unmatched capitalist must determine whether it pays to expend recruitment effort by opening a job vacancy. Second, mobile workers must commit to searching in one of the two sectors and so have to consider the return to migrating between sectors.

The first section of the chapter following this introduction describes the environment of the model in some detail. Next, optimal decision-making is described in a general stochastic framework and the concept of an equilibrium is defined. The remainder of the chapter then restricts the environment to one in which shocks are perceived to be perfectly persistent. This allows for analysis of the steady state of the model and adjustment following 'rare' shocks such as the initial OPEC oil price rise. A subsequent section summarises the results and focusses on the empirical predictions of the model. The final section of the chapter considers possible extensions to the model.

2.2 The Environment of the Model

This chapter examines the properties of a simple two-sector model in which labour market participants have to engage in search to find a match. At each date t, there is a constant aggregate labour force of size L which is allocated endogenously between two sectors in quantities $L_{1,t}$ and $L_{2,t}$. In each sector, the labour force can be separated into those working ($N_{i,t}$) and those searching for employment ($U_{i,t}$). Working and searching are mutually exclusive activities.

Search is two-sided in this economy since firms must search for workers in order to commence production. A position in search of a worker is called a 'vacancy' or 'posted offer' and is analogous to an unemployed worker. The number of vacancies in section i at time t is denoted by $O_{i,t}$[1]. No upper limit is placed on the number of potential vacancies in a sector; this is in contrast to unemployment which is limited by the size of the labour force.

Firms in the model make decisions about whether to search for a worker or whether a producing firm should be closed. Workers will decide whether they should leave their current sector to search in the other sector. The only factor which flows between sectors is labour. This means that the model accords particularly well with agricultural or resource economies in which products can only be grown in certain regions.

To fix ideas, output in the model may be thought of as the fruit of a tree so that the capitalists are owners of the trees and the land they grow upon. The workers are then paid to pick the fruit of the trees. Search arises naturally in this environment since workers have to find a tree which is in need of husbandry. Search costs for capitalists could reflect the fact that effort is expended to attract workers or the fact that a tree yields no recreational value (i.e. shade) if it is prepared for tending. Finally, while fruit-pickers may move between apple orchards and orange groves to exploit favourable differentials, the trees are themselves fixed in place.

Preferences

All agents in the model are assumed to maximise expected discounted utility. Preferences are additively separable over time and the period utility function is linear in consumption. This rules out the question of risk pooling since all agents are risk neutral. Only consumption enters the utility function, and it is assumed that all current income is consumed since no opportunities for intertemporal exchange are allowed. Given this environment, the objective is equivalent to the maximisation

[1] O for 'offers' is used rather than V to avoid confusion with value functions.

of expected discounted income streams. All agents are assumed to face the same discount factor, β, which is parametric to the model.

Production Technology and Product Market Structure

Two sectors each produce different goods although, within each sector, the product is homogeneous across firms. Production combines labour with an input owned by the capitalists. All workers are identical with regard to productivity both within and across sectors. Firms in the model are single worker enterprises. Each sector faces a downward sloping Marshallian demand schedule for its product. Output at a particular location is characterised by a constant marginal product of labour.

Production equals employment in this environment so that market and technological factors can be summarised by a single average revenue product function:

$$y_{i,t} = F(N_{i,t}, \mu_{i,t}) \tag{2.2–1}$$

where: $N_{i,t}$ is total employment in sector i,

$y_{i,t}$ is the average revenue per worker in sector i,

$\mu_{i,t}$ is a demand shock to sector i.

Assume:

ASSUMPTION 2.1 *F is increasing in μ and decreasing in N. Futhermore, $F(N, \mu)$ is convex in N.*

This structure of shocks is much the same as that used by Lucas and Prescott (1974) in their multi-sector search model. The term $\mu_{i,t}$ may reflect relative demand shocks in world markets for the sector-specific goods. These shocks need not be independent across sectors. With a slight modification the shocks could be due to changes in sectoral productivity. Prices are expressed in terms of some numeraire rather than either sectoral good.

In order to rule out a steady state in which all mobile workers are located in one sector, make the assumption:

ASSUMPTION 2.2 $\lim_{N_i \to 0} \partial F(N_i, \mu_i)/\partial N_i = -\infty$ *for all* μ_i.

This assumption is a boundary condition which states that the return to keeping some production in each sector is very high. An analogous but weaker assumption was made by Davis (1985) in his study of sectoral reallocation. Unemployment is of variable duration in the current model and this stronger boundary condition is needed to make proofs of existence of equilibria more tractable.

Search Technology

To this point, the algebra of the model is fairly conventional. The remainder of this section introduces an important departure from the standard microeconomic paradigm. Instead of frictionless transacting between labour market participants, a model of real-time search is considered. The search process is modelled in a discrete time framework to make explicit the passage of time and the sequence of events within the model. The model assumes that there is a basic unit of work and search, which may be a day, week, or month, depending on the context. It seems most natural to begin with a view that one period equals one day.

At the beginning of each day, each worker is assumed to know whether she has a job for that day. Similarly, capitalists know whether a worker will be reporting to their plot or if a vacancy must be posted to locate a labourer. All agents are also aware of current labour force and employment conditions in both sectors as well as the current state of the shock terms μ_1 and μ_2. Given this information, several actions can be taken before work or search activities commence. First, employed or unemployed persons may decide to migrate to search in the other sector. Also, vacancies may be created or withdrawn by owners of capital. Finally, some firms that are known to have a match with a worker could decide that the shadow value of the capital exceeds the return to producing.

The decisions made at dawn are acted upon before production or search begins. If a worker chooses to migrate, she does so between 5:00am and 7:00am in a fashion that entails neither pecuniary nor other costs. Similarly, sufficient time is available to post vacancies before 7:00am. During the day proper, successfully matched pairs produce fruit while unemployed workers and owners of vacant trees engage in search activity. By the end of the day, it is known which searchers have succeeded in contacting partners. However, it is also true that some pairs that were previously matched will discover that they have suffered an exogenous separation shock.

The sequence of events in the economy is conveniently summarised in the following 'daily agenda' for a day labelled t:

5:00am All agents know if they are matched or not for the current day. The value of the stochastic shock terms $\mu_{1,t}$ and $\mu_{2,t}$ are also revealed at this point. Given this information, employed workers decide whether to quit while all workers have the option of leaving their current sector. Matched capitalists can cease to operate while unmatched capitalists may post or withdraw job offers.

7:00am Matched pairs begin producing. Unmatched agents search for a match. Those workers who migrated earlier in the day start searching now in the new sector.

5:00pm Productive activities cease. Matched pairs learn if they have suffered a separation. The outcome of search activity is revealed.

While engaged in search, flow costs/benefits accrue to participants in the search process. Unemployment benefits for searchers are denoted by b and are net of the direct costs of searching. A positive benefit may capture a pecuniary-equivalent value of searching rather than working. Capitalists must pay a per-period cost of γ which could represent either the cost of effort expended to attract a worker or the foregone shadow value of the capital resulting from its preparation for use.

The probability that any searching worker receives a job offer is assumed to be some function of $\theta_{i,t}$, the ratio of the number of vacant firms ($O_{i,t}$) to the number of searching workers ($U_{i,t}$) (i.e. $\theta_{i,t} = O_{i,t}/U_{i,t}$). This relationship is derived from a function that gives the total number of matches ($x_{i,t}$) as a function of $O_{i,t}$ and $U_{i,t}$:

$$x_{i,t} = X_i(U_{i,t}, O_{i,t}). \tag{2.2-2}$$

Such a 'matching function' figures prominently in recent work such as Pissarides (1987) and Blanchard and Diamond (1989) which have analysed labour market dynamics. Like a production function or utility function, the matching function *represents* a more fundamental feature of the economic environment. Using such a function is more expedient than modelling the underlying physical facts of the search process. The following assumptions are generally made concerning the matching function:

ASSUMPTION 2.3 *$X_i(U_{i,t}, O_{i,t})$ is increasing in both U_i and O_i and is concave.*

ASSUMPTION 2.4 *$X_i(U_{i,t}, O_{i,t})$ is homogeneous of degree one.*

Much ease of exposition is obtained by Assumption 2.4 since it implies that the number of matches per searching worker can be written as a function of a single variable, $\theta_{i,t}$:

$$x_{i,t}/U_{i,t} = X_i(U_{i,t}, O_{i,t})/U_{i,t} = X_i(1, \theta_{i,t}).$$

The probability that any given worker is matched is precisely this ratio of matches to searching workers. This probability will be denoted by the function $p_i(\theta_{i,t}) = X_i(1, \theta_{i,t})$. The matching function is increasing in both of its arguments by Assumption 2.3 and this means that the matching probability for a worker rises with θ_i. Intuitively, θ_i is a measure of the 'tightness' of a labour market and high values of θ_i indicate that jobs are relatively abundant in sector i.

Of course, the matching function also determines the number of vacancies that are matched in each period. The ratio of matches to vacancies is:

$$x_{i,t}/O_{i,t} = X_i(U_{i,t}, O_{i,t})/O_{i,t} = X_i(1/\theta_{i,t}, 1).$$

This ratio, which also depends on θ_i, gives the probability that a vacancy is matched, denoted $q_i(\theta_{i,t})$. Assumption 2.3 implies that $q'(\theta_i) < 0$.

In order to conduct the analysis of this chapter, the following additional assumptions are necessary:

ASSUMPTION 2.5 *$p_i(\theta_i)$ is strictly increasing in θ_i for $i = 1,2$ and $0 < p_i(\theta_i) < 1 \;\forall\; 0 < \theta < \infty$.*

ASSUMPTION 2.6 *$q_i(\theta_i)$ is strictly decreasing in θ_i for $i = 1,2$ and $0 < q_i(\theta_i) < 1 \;\forall\; \infty > \theta_i > 0$.*

These assumptions mean that no matter how unbalanced an economy becomes, no worker or firm is ever matched with certainty. This makes clear the fact that employment comes as the outcome of search rather than a rationing process where one side of a market can be fully accommodated.

The final detail to be specified in the economy is the process by which workers and firms suffer involuntary separations:

ASSUMPTION 2.7 *In each sector a match dissolves with probability s per period.*

This assumption may seem extreme but it allows for the analysis to highlight changes in rates of flow out of unemployment. In terms of the model, it could reflect the effects of localised disasters which render certain locations permanently non-productive.[2] This assumption is necessary because with a constant group of infinitely lived agents there would be zero unemployment in the limit if no separations occur. Growth of the economy-wide labour force through new entrants to unemployment is ruled out because this leads to a declining steady-state y_i.

Rewards for Achieving a Match

In a search model like this, a surplus generally accrues to agents who succeed in finding one another. Those who are fortunate enough to be matched can do no worse than receive the return available from outside pursuits since they can always decide

[2] Such an exogenous separation assumption has been used by Lucas (1987) and Pissarides (1987). Within the abstract agricultural environment it is quite appropriate. In general, it would be desirable to endogenise separations. This could be done with a model of match specific and derived-demand shocks such as formulated by Bull and Jovanovic (1988).

to pursue this alternative occupation. In fact, the matched pairs will likely earn more than their outside opportunity because the search process presents a barrier to entry. Given the likely existence of economic rents, it is necessary to specify some mechanism for allocating this surplus. Following Howitt (1988), it is assumed that:

ASSUMPTION 2.8 *A worker receives a wage which is a constant fraction k of her output.*

This is a pie-splitting rule in which workers and capitalists get arbitrary shares k and $1 - k$ of production, respectively. Any such split is a Nash equilibrium to a one-shot pie-sharing game and no attempt is made to endogenise the sharing rule or refine the equilibrium concept.

Dynamic Evolution of the Economy

In a model such as this, where migration occurs *within* a period, the concept of unemployment is ambiguous. This is because the beginning and within period pools of unemployed workers differ by the amount of immigration or emigration. Accordingly, it is important to be precise and to distinguish between beginning-of-period (5:00am) unemployment, $U^I_{i,t}$, and within-period (7:00am - 5:00pm) unemployment, $U^P_{i,t}$. It is this latter quantity which matters from the point of view of the matching technology. With the distinctions noted above, the evolution of the state of the labour market at the beginning of each period can be described by the following difference equations:

$$\begin{aligned} N_{i,t} &= N_{i,t-1} + p(\theta_{i,t-1})U^P_{i,t-1} - sN_{i,t-1} - Q_{i,t} - R_{i,t}, \\ U^P_{i,t} &= (1 - p(\theta_{i,t-1}))U^P_{i,t-1} + sN_{i,t-1} + M_{i,t} + Q_{i,t} + R_{i,t}. \end{aligned} \tag{2.2–3}$$

Employment can fall due to quits ($Q_{i,t}$) and redundancies ($R_{i,t}$). Similarly, the pool of unemployed may change due to quits, redundancies, or migration ($M_{i,t}$). By convention, positive values of $M_{i,t}$ indicate immigration. Note that a quit in sector i may be followed by immediate emigration to the other sector so that a quit may have no net effect on unemployment in a sector.

Stochastic Structure

Finally, the role of uncertainty in this model requires some explanation. It is assumed that the vector stochastic process (μ_1, μ_2) follows a finite state first-order Markov process. There seems to be little to gain from assuming a countable infinite number of states rather than an arbitrary finite number of states since both are approximations. In general, no restriction is placed on the correlation structure between shocks. Restrictions on this structure will characterise economy-wide shocks as aggregate, sector-specific, or a combination of these two. It is also possible that the shocks are perfectly persistent. When making decisions under uncertainty, agents form expectations rationally.

2.3 Decision-Making in the Search Model

Decisions are made based on an information set that can be summarised by using the following pair of state vectors:

$$\mathbf{z}_{i,t} = \{N_{i,t}, \mu_{i,t}, L_{i,t}, N_{j,t}, \mu_{j,t}, L_{j,t}\},$$

$$\mathbf{z}_{j,t} = \{N_{j,t}, \mu_{j,t}, L_{j,t}, N_{i,t}, \mu_{i,t}, L_{i,t}\}.$$

The convention is that, in each sector, the home-sector values of the shock, employment and labour force enter the state vector first. Here, $L_{i,t}$ is the beginning of period labour force ($= N_{i,t} + U^I_{i,t}$). This initial labour force differs from the within-period labour force because of the possibility of migration. Based on the information in $\{\mathbf{z}_{i,t}; \mathbf{z}_{j,t}\}$, agents will act in such a manner that the values of the market tightness variable, θ, and the levels of migration, M, will be decided for each sector. These relationships will be described by the stationary rules:

$$\theta_{i,t} = \Theta(\mathbf{z}_{i,t}),$$

$$M_{i,t} = \Omega(\mathbf{z}_{i,t}).$$

In any equilibrium, it must be true that if agents form their expectations assuming some particular form for these rules, then the decisions formulated under these expectations must, in turn, imply the assumed rules. At the point of making their decisions, agents take as given arbitrary values of θ and M. In equilibrium, however, these values must be consistent with the decisions taken by agents.

Firms

The entry decision of an unmatched capitalist requires that she compare the value of posting a job offer versus the value of leaving capital non-productive and enjoying some alternative shadow return. This involves computation of the value of running a producing firm at time t. Conditional on fixed values of $\theta_{i,t}$ and $M_{i,t}$, the maximised value of a producing firm is described by the recursive equation:

$$V_i^I(\mathbf{z}_{i,t}) = \max\Big\{0, (1-k)F(N_{i,t}, \mu_{i,t}) + \beta\sum_{l=1}^{n}\Big[(1-s)V_i^P(\mathbf{z}_l) + sV_i^O(\mathbf{z}_l)\Big]\Pr(\mathbf{z}_l|\mathbf{z}_{i,t})\Big\}, \tag{2.3-1}$$

where $V_i^O(\mathbf{z}_{i,t})$, the maximised value of a vacancy, comes from:

$$V_i^O(\mathbf{z}_{i,t}) = \max\Big\{0, -\gamma+\beta\sum_{l=1}^{n}\Big[(1-q_i(\theta_{i,t}))V_i^O(\mathbf{z}_l)+q_i(\theta_{i,t})V_i^P(\mathbf{z}_l)\Big]\Pr(\mathbf{z}_l|\mathbf{z}_{i,t})\Big\}. \tag{2.3-2}$$

In these two equations, the value functions do not need to be subscripted by sector if the matching function has the same form in each sector.

Workers

To make the migration decision, a worker must compare the reward to searching in her current sector against the value of leaving to search elsewhere. Given the structure of the economy, it is possible to write the maximised value of being employed in a sector i as:

$$V_i^W(\mathbf{z}_{i,t}) = \max\Big\{V_j^U(\mathbf{z}_{j,t}), kF(N_{i,t}, \mu_{i,t}) + \beta\sum_{l=1}^{n}\Big[(1-s)V_i^W(\mathbf{z}_l) + sV_i^U(\mathbf{z}_l)\Big]\Pr(\mathbf{z}_l|\mathbf{z}_{i,t})\Big\}, \tag{2.3-3}$$

while the maximised value of being unemployed is:

$$V_i^U(\mathbf{z}_{i,t}) = \max\Big\{V_j^U(\mathbf{z}_{j,t}), +\beta\sum_{l=1}^{n}\Big[(1-p_i(\theta_{i,t}))V_i^U(\mathbf{z}_l) + p_i(\theta_{i,t})V_i^W(\mathbf{z}_l)\Big]\Pr(\mathbf{z}_l|\mathbf{z}_{i,t})\Big\}. \tag{2.3-4}$$

These two equations determine the discounted present values of being either an employed or unemployed worker in sector one or sector two beginning at time t, assuming optimal action now and in the future. A worker would compute these at 5:00am on day t to decide whether to migrate before beginning to work or search.

The value function for a worker currently employed reflects a binary choice decision. The first term is the value of quitting in order to move and begin searching in the other sector. The second argument of the max operator is the value of remaining in the home sector and working. This argument is itself composed of two terms. A current wage term, $ky_{i,t}$, is the wage earned in period t. The second element of this value of staying is the expected discounted value of participating in the sector in the future. This expression reflects the possibility of a separation. The move/stay decision of a worker matched at t is made to maximise this value function.

There is also a binary decision facing workers currently destined to search in sector i. If the worker departs for the other sector, the value of being unemployed in that sector is immediately obtained. If the worker does not migrate, a current period return of b is received and the worker also has a discounted expected value of searching in the future. To maximise this expression, the unemployed also make a move/stay decision. Notice that, with no migration costs in the model, there is really no distinction between unemployment in either sector prior to the resolution of the migration decision. It is only after migration decisions are acted upon that the unemployed are differentiated since the probability of transit to the employed state varies between sectors.

Equilibrium Conditions

It is assumed that:

ASSUMPTION 2.9 *An abundant supply of unmatched capital exists in the economy and there are no barriers-to-entry in the market for posted vacancies.*

Consequently, the entry and exit decisions of those possessing unmatched capital force the value of a vacancy into equality with the shadow value of capital. This shadow value is normalised to zero. Free entry and exit will accordingly require that $V_i^O(\mathbf{z}_{i,t}) \equiv 0$ for any time t and sector i.[3]

Imposing this condition in the definition of $V_i^O(\mathbf{z}_{i,t})$ gives:

$$0 \equiv -\gamma + \beta \sum_{l=1}^{n} q_i(\theta_{i,t}) V_i^P(\mathbf{z}_l) \Pr(\mathbf{z}_l | \mathbf{z}_{i,t}). \tag{2.3-5}$$

The only variable which can adjust here is θ because the number of vacancies is a free variable. The state variables and the number of unemployed are given at time t. Accordingly, this condition will determine the number of vacancies given the state of the economy.

Equilibrium in the labour market requires that all favourable migration opportunities be exploited. Algebraically, this means that if:

$$\sum_{l=1}^{n} \Big[(1 - p_i(\theta_{i,t})) V_i^U(\mathbf{z}_l) + p_i(\theta_{i,t}) V_i^W(\mathbf{z}_l) \Big] \Pr(\mathbf{z}_l | \mathbf{z}_{i,t})$$
$$> \sum_{h=1}^{n} \Big[(1 - p_j(\theta_{j,t})) V_i^U(\mathbf{z}_h) + p_j(\theta_{j,t}) V_i^W(\mathbf{z}_h) \Big] \Pr(\mathbf{z}_h | \mathbf{z}_{j,t}).$$

then either (i) there are no mobile agents in sector j or (ii) there is migration occurring from j to i. The analogous condition holds if the inequality is reversed. This condition simply means that, for mobile agents to stay in their sector, the return to searching must be the same in both sectors. Note that equalization of search values does not imply no migration because the distribution of mobile agents across sectors is not pinned down when agents are indifferent between sectors. For now, it is assumed that no quits or redundancies occur so that equilibrium conditions with respect to these decisions are non-binding.

[3] Actually, this will only hold with equality when there is a strictly positive number of vacancies. If there are no vacancies, the value can be negative. Implicit in the analysis above is the assumption that production is always profitable enough to make it worthwhile for someone to post a vacancy. In other words, it is assumed that $\beta \sum_{l=1}^{n} q_i(\theta_{i,t}) V_i^P(\mathbf{z}_l) \Pr(\mathbf{z}_l | \mathbf{z}_{i,t}) > \gamma$ always.

2.4 Stationary State when Shocks are Perfectly Persistent

An important feature of the labour market dynamics in this model is the existence of a stationary point to the employment transition equation. In other words, there is a level of employment $\bar{N}$ which solves $\bar{N} = (1-s)\bar{N} + p(\theta)(L - \bar{N})$ for a pair of market tightness and labour force values $\{\theta, L\}$. This stationary state employment level is important because it represents a 'balance point' for the economy where inflows to employment are exactly balanced by outflows from employment. Given the special nature of the stationary state, it is interesting to see how labour market dynamics interact with firm behaviour to determine the form of the stationary state. It is furthermore helpful to study the case of permanent shock variables μ rather than a general stochastic structure because the stationary state has some likelihood of being approached with permanent shock variables.

Single-Sector Steady-State Conditions

Prior to discussing the steady state for the two-sector economy, it is useful to reflect upon the determinants of a single-sector steady state. This can be determined by looking at the steady-state versions of three key equations: the law of motion for employment in (2.2–3) and the two equations (2.3–1) and (2.3–2) which define V^P and V^O. In a single sector economy the value functions for workers are of no interest. Without a migration decision, workers simply spend their time searching or working as their employment state dictates.

Using bars to denote steady-state values and defining $\bar{V}^P = V^P(\bar{\mathbf{z}})$, the relevant three equations are:

$$\bar{N} = \frac{p(\bar{\theta})}{s + p(\bar{\theta})} L, \tag{2.4–1}$$

$$\bar{V}^P = \frac{(1-k)}{(1-\beta(1-s))} F(\bar{N}, \bar{\mu}), \tag{2.4–2}$$

$$\gamma \equiv \beta q(\bar{\theta})\bar{V}^P. \tag{2.4-3}$$

These equations are non-linear so it is not obvious that there is a stationary-state equilibrium and uniqueness is far less immediately discernible. To demonstrate that an equilibrium exists, note that, for fixed values of L and μ, the state of the economy is fully characterised by two variables: employment (N) and market tightness (θ). In the steady state with permanent shocks, these variables are fixed by definition. Two equations in the two unknowns N and θ are obtained by solving (2.4–3) for $\bar{V}^P$ and substituting this into (2.4–2) to obtain:

$$q(\bar{\theta}) = \frac{\gamma(1-\beta(1-s))}{\beta(1-k)}\frac{1}{F(\bar{N},\bar{\mu})}. \tag{2.4-4}$$

The two equations (2.4–1) and (2.4–4) can be graphed in (N,θ) space to show which combinations of employment and market tightness are consistent with either steady-state condition. First, the difference equation for employment is plotted as the upward sloping curve in Figure One. The slope equals:

$$\frac{p'(\theta)L}{s+p(\theta)}\left[1-\frac{p(\theta)}{s+p(\theta)}\right] > 0, \qquad \text{since} \quad p'(\theta) > 0.$$

A second curve comes from the producer behaviour in equation (2.4–4). The slope along this curve is:

$$\frac{d\bar{\theta}}{d\bar{N}} = -\frac{\gamma(1-\beta(1-s))}{\beta(1-k)}\frac{1}{\left(F(\bar{N},\bar{\mu})\right)^2}\frac{F_N(N,\mu)}{q'(\theta)} < 0.$$

Thus, the labour market balance curve is *strictly* increasing while the producer equilibrium curve is *strictly* decreasing. It is the strict increasing and decreasing nature of these curves that allows for the statement of the following proposition:

Proposition 1: *There is a unique steady-state equilibrium in the single-sector search model with permanent shocks.*

Proof: See Appendix.

Intuitive interpretations of both curves are available. The labour market "rest-point" curve is analogous to an aggregate labour supply curve. While individual workers supply labour inelastically, the aggregate supply of labour (i.e. the total number of matches) depends on the matching probability $p(\theta)$. In this sense, the matching rate plays a role akin to the wage in a standard spot labour market. In this search framework, however, it is non-price mechanisms that equilibrate supply and demand. Note that this curve has a vertical asymptote at the point $N = (1/(1+s))L$. At this maximal employment level, all workers find jobs after searching for one period and employment falls short of the labour force only because a number of workers equal to $(s/(1+s))L$ are separated in every period.

There is an equally intuitive interpretation of the downward sloping curve as a steady-state demand for labour curve. More correctly, the curve is an economy-wide demand for total matches. There is an inverse relationship between θ and the amount of production in the economy because there is a downward sloping average revenue product schedule (see Assumption 2.1). Total production is proportional to total employment, N, in this economy so that the reward to filling a vacancy varies inversely with N. The expected return to filling a vacancy is zero in equilibrium so that $q(\theta)$ and N vary positively with one another. Finally, $q'(\theta) < 0$ so that there is a negative relationship between market tightness and employment implied by free entry to the market for vacancies.

The Steady State of the Two-Sector Economy

To move to the steady state of a two-sector economy, it is necessary to examine the decisions of workers as well as firms. The total labour force can now be endogenously allocated between two sectors. For a worker, the essential feature of any steady state is that no migration occurs in order that the labour force remain constant. This 'no migration' condition will only hold if the expected present discounted value of searching is the same in both sectors. To see how the conditions derived above must be modified, it is necessary to determine equilibrium conditions for a constant labour force.

From equation (2.3–4), the steady-state value of being unemployed in sector i is defined by:

$$\bar{V}_i^U = V_i^U(\bar{z}_i) = \frac{\beta}{r}\left[\frac{(r+s)b + p_i(\bar{\theta}_i)kF(\bar{N}_i, \bar{\mu}_i)}{r + p_i(\bar{\theta}_i) + s}\right] \quad (2.4\text{–}5)$$

where $r = 1/\beta - 1$. The no-migration condition implies that both sides of the max operator be equal in (2.3–4) and this simplifies the algebra. Recall that the equilibrium conditions for the labour market and vacancies within a sector were given by (2.4–1) and (2.4–4). The steady state is defined by the following six equalities:

Equalisation of the Value of Search–

$$\text{(i)}\quad \frac{(r+s)b + p_1(\bar{\theta}_1)kF(\bar{N}_1, \bar{\mu}_1)}{r + p_1(\bar{\theta}_1) + s} = \frac{(r+s)b + p_2(\bar{\theta}_2)kF(\bar{N}_2, \bar{\mu}_2)}{r + p_2(\bar{\theta}_2) + s};$$

Zero Expected Value of a Vacancy–

$$\text{(ii)}\quad q_1(\bar{\theta}_1) = \frac{\gamma(r+s)}{(1-k)F(\bar{N}_1, \bar{\mu}_1)},$$

$$\text{(iii)}\quad q_2(\bar{\theta}_2) = \frac{\gamma(r+s)}{(1-k)F(\bar{N}_2, \bar{\mu}_2)};$$

Sectoral Labour Flow Balance–

$$\text{(iv)}\quad \bar{N}_1 = \frac{p_1(\bar{\theta}_1)}{s + p_1(\bar{\theta}_1)}(\bar{N}_1 + \bar{U}_1),$$

$$\text{(v)}\quad \bar{N}_2 = \frac{p_2(\bar{\theta})}{s + p_2(\bar{\theta})}(\bar{N}_2 + \bar{U}_2);$$

Labour Force Accounting Identity–

$$\text{(vi)}\quad \bar{N}_1 + \bar{N}_2 + \bar{U}_1 + \bar{U}_2 = L.$$

Before proceeding further, it is necessary to ensure that there is, indeed, a steady-state equilibrium for the model. In this regard, the following proposition can be established:

Proposition 2: *There is a steady-state equilibrium allocation of labour in this two-sector search model; moreover, this equilibrium is unique.*

Proof: See Appendix.

To appreciate heuristically why there is a steady-state equilibrium, it is useful to collapse the condition for zero expected value of a vacancy and labour market balance to obtain a system involving only θ_i and L_i. This yields:

$$\gamma = \frac{1-k}{r+s} q_i(\bar{\theta}_i) F\left(\frac{p_i(\bar{\theta}_i)}{s+p_i(\bar{\theta}_i)} L_i, \mu_i\right).$$

For any given sectoral labour force, L_i, market tightness, θ_i, will adjust to ensure that this condition holds. The key property of this condition is that for low levels of the labour force, the value of output $F(\bar{N}_i, \mu_i)$ and matching probability $p_i(\bar{\theta}_i)$ are high while the converse is true for high values of the labour force (this is explained in more detail in the appendix).

The next link in the argument is the negative relationship between the labour force and the steady-state value of searching in a sector. This arises because $\bar{V}_i^U$ is increasing in both $F(\bar{N}_i, \mu_i)$ and $p_i(\bar{\theta}_i)$. By virtue of this relationship, it is possible to find a value of L_i so that the equality in (i) holds.

It is interesting to ask how quantities such as the labour force, employment and the matching probability can differ between sectors if features of the environment differ. First, consider what happens if the matching functions are equally productive in either sector but $\mu_1 > \mu_2$. In this case it must be true that:

$$\theta_1 = \theta_2$$

and

$$F(\mu_1, N_1) = F(\mu_2, N_2).$$

This is due to the fact that, with a common level of outside return to capital (γ) in the two sectors, the value of production must be equalised in steady state between the two sectors. When combined with the no-migration condition, this yields the following two steady-state equalities:

$$\frac{(r+s)b+p(\theta_1)kF(N_1,\mu_1)}{r+p(\theta_1)+s}=\frac{(r+s)b+p(\theta_2)kF(N_2,\mu_2)}{r+p(\theta_2)+s},$$

and

$$q(\theta_1)F(N_1,\mu_1)=q(\theta_2)F(N_2,\mu_2).$$

These two conditions will hold if the value of output and the matching probabilities are the same in either sector. Uniqueness ensures that these are the only conditions which yield an equilibrium. It is easy to see why uniqueness results here. If $F(N_1,\mu_1)\geq F(N_2,\mu_2)$, then $\theta_1\leq\theta_2$ for the first inequality to hold but $\theta_1\geq\theta_2$ is necessary for the second. Thus, any situation but $F(N_1,\mu_1)=F(N_2,\mu_2)$ and $\theta_1=\theta_2$ implies a contradiction.

A second interesting experiment is to assume that $\mu_1=\mu_2$ but that the sectors differ with regard to the efficiency of the matching process. A simple assumption is that, for the same numbers of vacancies and unemployed workers, sector one produces proportionally more matches than sector two. Suppose, for example, that $X_1(U,O)=\alpha X_2(U,O)$ where $\alpha>1$. In terms of the two match probability functions, this means that:

$$p_1(\theta)=\alpha p_2(\theta)\quad\text{and}\quad q_1(\theta)=\alpha q_2(\theta).$$

In other words, both matching probabilities rise by a factor α in the sector where the matching process is more efficient.

In general, it is not possible to determine the effect of this type of difference in the environment. This is because the condition for zero expected profits allows for the possibility that increased matching efficiency may lower θ. The expected value of a vacancy in steady state is:

$$\alpha\frac{\beta(1-k)}{1-\beta(1-s)}q_1(\theta_1)F\left(\frac{p(\theta)}{s/\alpha+p(\theta)}L_1,\mu_1\right),$$

where the expression for the steady-state level of employment has been substituted into the expression for the value of a vacancy.

More efficient matching affects this relationship in two ways. First, an increase in α raises the right-hand side in a proportional way for given employment levels. This is because a match becomes more likely. A second effect, however, is that, for any given value of θ, employment will rise because matching is more effective. This lowers the right-hand side of the relationship, other things equal. On balance, then, it is not clear if the expression above will rise or fall when α rises so that the adjustment to θ which is needed to maintain the value of a vacancy at zero is not clear.

Note, however, that F_N falls with N by the convexity assumption and that employment has an upper bound of $L/(1+s)$. This implies that, for suitably high levels of employment, the direct proportional effect of α will dominate. In this case, with high employment levels, it is to be expected that an increase i' the efficiency of the matching process would mean that $q(\theta)$ must fall or, alternatively, that θ rises. Note that this raises the interesting possibility that an economy with a low level of employment (say one experiencing a severe depression) may experience a perverse reaction if there is also a shock to the efficiency of matching.

A numerical example can be used to determine the likely effect of more efficient matching around "normal" employment levels. A model with a total labour force of 200 was specified and the following parameters were selected:

$$s = .15; \quad k = .6; \quad \beta = .99; \quad b = 4; \quad \gamma = 10.$$

The following functional form was chosen for the matching probabilities:

$$p_2(\theta) = 0.2828\theta^{1/2}; \quad q_2(\theta) = 0.2828/\theta^{1/2};$$

with $p_1(\theta) = 1.05p_2(\theta)$, $q_1(\theta) = 1.05q_2(\theta)$. Finally, for the average revenue product function the following specification was used:

$$F_i = 50 - 0.5N_i.$$

For this case, the steady-state equilibrium is described by the following:

$$\theta_1 = 1.33; \quad \theta_2 = 1.25; \qquad L_1 = 99; \quad L_2 = 101.$$

As predicted, the five per cent increase in matching efficiency produced a higher value of market tightness in sector one. More surprisingly, the labour force was lower in the sector with the more efficient matching process. This may seem undesirable on the grounds of social efficiency because it means that more labour is in the sector with more frictions in the labour market. Note that values of searching are equated because sector one has a higher match probability and thus higher employment, even given the fact that the labour force is higher in sector two. Finally, it is worth noting that, in the example here, the unemployment rate was approximately 30 per cent. This suggests that counter-intuitive effects of α on θ can only occur at remarkably high levels of unemployment.

This type of result can be varied if the shadow values of capital vary between sectors since the expected value of posting a vacancy is equated to the shadow value for each sector. At present, there is no reason to believe that this parameter should change between sectors.

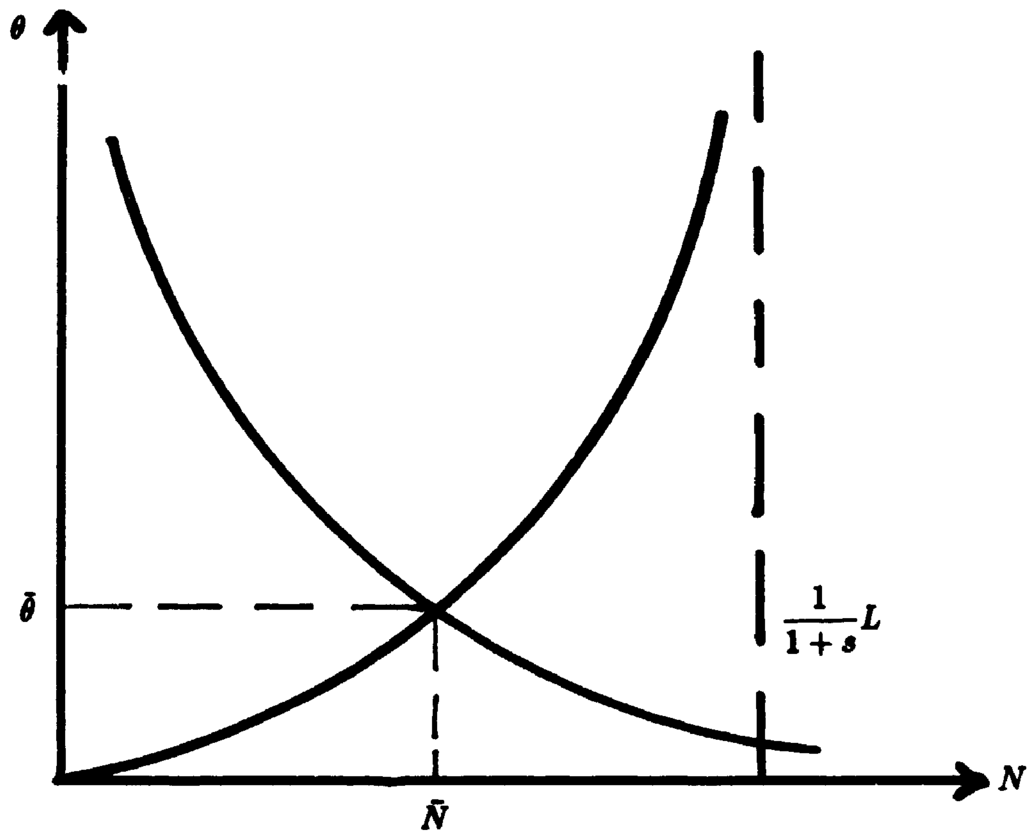

Figure One: Single-Sector Steady-State Equilibrium

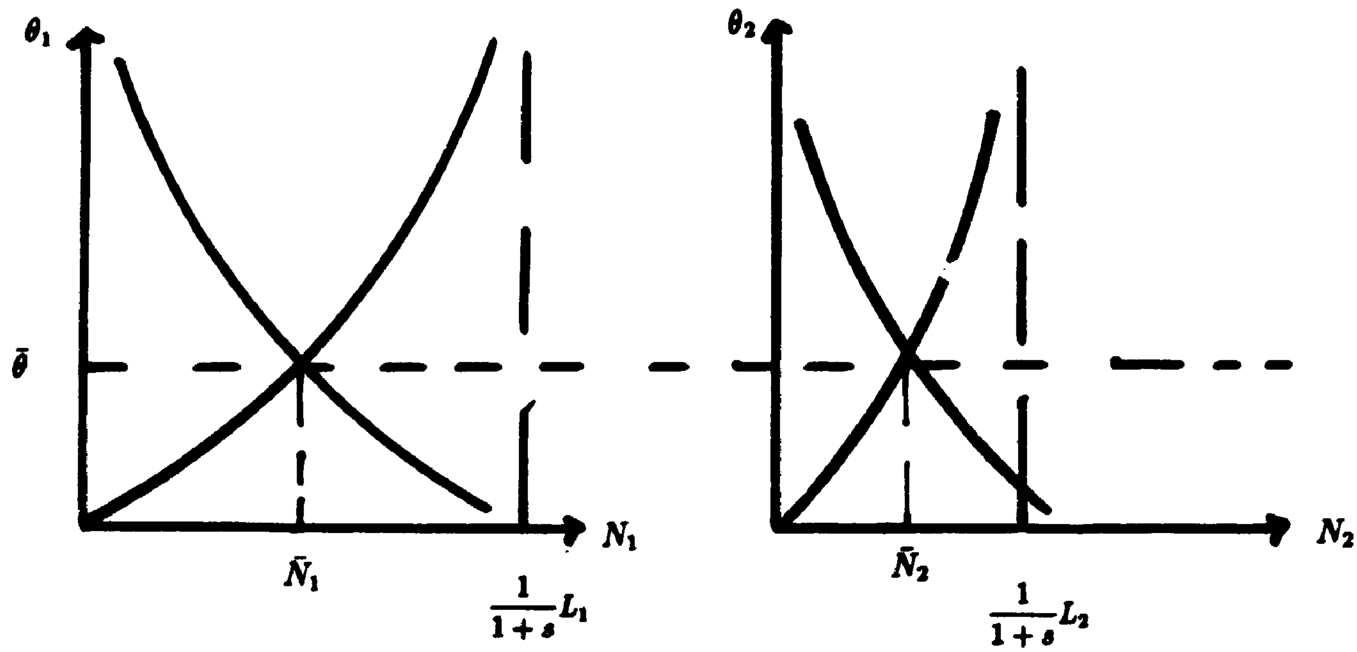

Figure Two: Two-Sector Steady-State Equilibrium

2.5 Adjustment to the Stationary State with Perfectly Persistent Shocks

In this section, the behaviour of the economy is examined during the period of adjustment toward the stationary states described above. The discussion focuses on the paths of the variables $(N_{1,t}, N_{2,t}, \theta_{1,t}, \theta_{2,t})$ which drive the model. These paths must meet two types of criteria to be equilibrium paths: they must obey the laws of the physical economic environment and involve optimal decision-making within this environment. The relevant features of the environment are given by equation (2.2–1) which determines sectoral output values and the equations of motion for employment and unemployment described in (2.2–3). Optimality requires that holders of capital must maximise (2.3–1) and (2.3–2) while workers act so as to maximise the value functions in (2.3–3) and (2.3–4).

To simplify the equilibrium conditions, the adjustment path is analysed for the case of two sectors with identical matching functions. It will become clear that, if the matching functions differ between sectors, this affects the nature of the steady-state equilibrium conditions only. The qualitative features of the adjustment path are unaffected by the relative efficiency of the matching process. Similarly, the shadow values of capital are also assumed to be the same across sectors and this does not affect the analysis of adjustment.

Two forces move the economy toward the steady state: the internal dynamics of each sector separately and the reallocation of labour between sectors. These two factors are isolated by examining instances in which both sectors are displaced from the steady state in symmetric ways as well as asymmetric departures from the steady state. With some slight abuse of language, the first case may be described as an aggregate shock since it could result if the economy had been moved from a rest point by an unanticipated change in μ_1 and μ_2 where $\Delta\mu_1\Delta\mu_2 > 0$. Sectoral shocks seems an appropriate name when at least one of μ_1 or μ_2 changes and $\Delta\mu_1\Delta\mu_2 \leq 0$. There are four cases for analysis: a symmetric aggregate shock, a simple sector-specific shock, an asymmetric aggregate shock and a pure sectoral shock. All but the first case involve some migration from a relatively less-favoured sector to a relatively more-favoured one. The analysis here looks at the nature of migration and within-sector adjustment for the first and second cases, and the analysis is easily extended to the others.

Adjustment with Perfectly Symmetric Displacement

Suppose that at some time t_0 the values of μ_1 and μ_2 are such that the relative positions of the two sectors, *vis a vis* the migration conditions, are identical. Migration will not occur because the two sectors are assumed to be subject to similar disturbances. Without migration, adjustment occurs entirely through natural dynamics of the search process within each sector.

During the period of adjustment, it will always be true that the expected return to posting a vacancy will be zero. This is because θ is a free variable which adjusts in each period to ensure that:

$$\gamma = \beta q(\theta_t)V^P(\mu, (1-s)N_t + p(\theta_t)(L - N_t))$$

in each sector. Note that the space of the state vector can be drastically shortened here because the analysis is essentially that of a single sector. Unlike market tightness, employment is not a jump variable and can only adjust through labour market dynamics. Adjustment in this economy will accordingly involve increases or decreases in the level of employment until an employment level is reached where $N_{t+1} = N_t$. When this labour market flow balance is achieved, it will also be true that expected profits from a vacancy are zero since this latter condition always holds. Thus, this final point will be the unique stationary equilibrium of the economy.

A convenient way of representing the adjustment path for this essentially single-sector economy is with a diagram of the type used in Figure One. The diagram in Figure Three below is slightly different from that in Figure One. The essence of the difference lies in the downward sloping 'labour demand' curve. This curve represents combinations of N_t and θ_t for which zero profits holds even outside of the steady state. The difference from Figure One is that $N_{t+1} = (1-s)N_t + p(\theta_t)(L - N_t)$ rather than $N_{t+1} = N_t$ is used to eliminate N_{t+1} in the zero-profit condition. This labour demand curve outside of the steady state is defined by:

$$\gamma = \beta \sum_{i=1}^{n} q(\theta_t) V^P(\mu_i, (1 - s - p(\theta_t))N_t + p(\theta_t)L)\Pr(\mu_i|\mu_t).$$

This condition must hold even when employment is changing over time so that the economy is always somewhere on the downward sloping curve. The "labour demand" curve still slopes downward, since total differentiation gives the slope of this curve as:

$$\frac{\partial \theta}{\partial N} = \frac{-q(\theta)\partial V^P/\partial N(1 - s - p(\theta))}{q'(\theta)V^P(\mu_i, N_{t+1}) + \partial V^P/\partial N q(\theta_t)[p'(\theta)(L - N_t)]}.$$

In addition to terms in the expression for the slope of the steady-state curve, there are additional complexities which reflect the dependence of N_{t+1} on N_t. One implication of this dependence is that it is no longer a simple matter to conclude that the slope of this curve is negative since the sign of the derivative of $V^P(N, \mu)$ with respect to employment is needed outside of the steady state. Fortunately, it is possible to establish the following proposition:

Proposition 3: *In a single sector economy with no migration the value of a producing firm , $V^P(N, \mu)$ is falling in employment (N).*

Proof: See Appendix.

Given this proposition, the slope of the 'labour demand' curve must be negative.

The dynamic adjustment path of the economy can be determined with the aid of this diagram and the law of motion for employment. First, the zero-profit condition always pins down θ, given N, so that it is clear that $N_t > \bar{N} \quad \Rightarrow \quad \theta_t < \bar{\theta}$ and vice versa. During the adjustment process, the paths of θ and N depend on whether $N_t > \bar{N}$ or $N_t < \bar{N}$. Taking the second case, the following is true:

$$N_t < \bar{N} \qquad \Rightarrow N_t < \frac{p(\bar{\theta})}{s + p(\bar{\theta})}L \quad < \quad \frac{p(\theta)}{s + p(\theta)}L$$

where the second inequality follows because $\theta > \bar{\theta}_t$ when $N_t < \bar{N}$, as established above. Thus, it is clear that movement will always be in the correct *direction* to take employment to the steady state.

What is less clear is whether the magnitude of the adjustment will be appropriate. It is possible that the adjustment will be sufficiently large to overshoot the state equilibrium point. To see this, consider the law of motion for employment again:

$$N_{t+1} = (1 - s)N_t + p(\theta_t)(L - N_t).$$

If $N_t > \bar{N}$, then the absolute value of $N_{t+1} - N_t$ is $sN_t - p(\theta_t)(L - N_t)$ while the absolute value of $N_t - \bar{N}$ is:

$$N_t - \frac{p(\bar{\theta})L}{s + p(\bar{\theta})}.$$

Accordingly, it will be true that overshooting happens only if:

$$\left(s + p(\bar{\theta})\right)\left[\frac{p(\theta_t)(L - N_t) - sN_t}{p(\bar{\theta}_t)(L - N_t) - sN_t}\right] > 1.$$

From this it is clear that overshooting is most likely to occur if θ is far from $\bar{\theta}$. Figure Three reveals that this is most likely to result if the labour demand curve is steep which, in turn, will happen if the derivative of the value of producing with respect to employment is large. It is also necessary to have $p'(\theta)$ fairly steep since, if the matching probability is flat in θ, monotone adjustment simply requires that $s + p(\bar{\theta}) \leq 1$. Notice that concavity of the matching probability implies that overshooting is more likely to occur for low rather than high values of market tightness θ_t. Finally, note that as $N_t \rightarrow L$ the effect of differential probabilities disappears and the conventional condition for monotonicity again is obtained.

Thus, in some region around full employment it is certainly true that monotone adjustment will occur. With higher unemployment rates, the potential for overshooting increases. Of course, the labour demand curve must be fairly responsive to changes in employment levels if differences in market tightness are to matter. The range of employment values over which monotone adjustment occurs increases as the labour demand curve becomes more flat and the matching probability function $p(\theta)$ become flatter. Note that this argument is equally applicable for movements in the other direction, that is, for the case when $N_t < \bar{N}$. Here the relevant inequality is:

$$\left(s + p(\bar{\theta})\right)\left[\frac{sN_t - p(\theta_t)(L - N_t)}{sN_t - p(\bar{\theta}_t)(L - N_t)}\right] > 1.$$

It is possible that, over some range, the adjustment in employment is sufficiently large that $|N_{t+1} - \bar{N}| > 2|N_t - \bar{N}|$. Clearly, if the right-hand side of the expression above exceeds two, then this type of overshooting will happen. If $|\theta - \bar{\theta}|$ is quite large and the slope of the $p(\theta)$ function is sufficiently steep, this could happen. In this case, the economy does not converge to the steady state and there are, instead, explosive oscillations between employment levels of zero and $L/(1 + s)$.

When considering these cases, it is important to realize that the possibility of negative adjustment is a feature of the discrete time framework. If the unit of time is made sufficiently small, the probability of a separation or a match can be made arbitrarily small. Thus, even if $p(\theta)$ differs from $p(\bar{\theta})$, the monotonicity condition will hold for all time horizons shorter than some critical level. Furthermore, the overlapping nature of individual decisions and actions when aggregated means that a close approximation to continuous-time dynamics actually holds.

Consequently, monotonic adjustment almost always characterizes this system. At the root of this result is the fact that the two-dimensional system for (n_t, θ_t) is really only one-dimensional. Market tightness is a jump-variable and is totally determined by the current state. This can be appreciated by first-differencing the zero-profit condition for vacancies to obtain the equation describing the locus of θ rest points:

$$V^P\left[(1 - s)N_t + p(\theta)(L - N_t), \mu\right] = V^P\left[(1 - s)N_{t+1} + p(\theta)(L - N_{t+1}), \mu\right].$$

This condition will hold only if $N_t = N_{t+1}$. In other words, the rest points for employment are the same as those for market tightness.

Given that the case of monotone adjustment seems to be the most reasonable, it is useful to summarise the features of this path:

- worker match probabilities move in the opposite direction to employment during the adjustment period,
- wages and worker matching probabilities move in the same direction through time during the adjustment process,
- the direction in which variables adjust depends on the relationship between initial N and $\bar{N}$.

Inter-Sectoral Adjustment with Value of Output Equalised

The analysis presented in the section above assumed that the two sectors were carbon copies of each other so that adjustment occurred completely within a sector. This will, of course, almost certainly never be the case in reality but the analysis does provide a useful base case for comparison. A more realistic scenario is one in which the economy begins with $F(N_1, \mu_1) = F(N_2, \mu_2)$ but $N_1 \neq \bar{N}_1$ so that there is need for adjustment of the labour force through natural dynamics combined with migration. An adjustment path is possible along which equality of the value of output is maintained, and migration serves to ensure that this equality holds throughout the adjustment period. The path of this economy can be usefully described by considering combinations of N_1 and N_2 which, for given μ_1 and μ_2, maintain equality of the value of output. Such combinations of values can be illustrated in the following diagram. The steady-state equilibrium pair of employment levels will necessarily lie somewhere along this curve. Adjustment involves movement from an arbitrary initial point on the curve to the appropriate equilibrium value.

Along the adjustment path the following laws of motion hold:

$$N_{1,t+1} = (1 - s)N_{1,t} + p(\theta_t)U_{1,t};$$

$$N_{2,t+1} = (1 - s)N_{2,t} + p(\theta_t)U_{2,t}.$$

There is a common probability of entering employment for the two sectors because the zero profit condition pins this down (given a path for output)and the paths of output are the same across sectors.

The stock of unemployed workers must be allocated between the two sectors in such a fashion that $F(N_{1,t+1}, \mu_1) = F(N_{2,t+1}, \mu_2)$. Note that if $F(N_1, \mu_1) \equiv F(N_2, \mu_2)$ then:

$$\frac{dN_1}{dN_2} = \frac{\partial F}{\partial N}\Big|_{N=N_1} \Big/ \frac{\partial F}{\partial N}\Big|_{N=N_2}.$$

Concavity of the average revenue product function with respect to employment means that this derivative will seldom equal one so that different adjustments to employment will be required each period in the two sectors. So long as the degree of curvature of $F(N, \mu)$ with respect to N is not too great, it will be possible to allocate the unemployed between sectors in the required ratio. Certainly, within some neighbourhood of the steady state, it is true that $(1 - s)N \approx p(\theta)U$ in both sectors so that sufficient flexibility exists to ensure that the economy remains on the path depicted above.

The allocation of searchers between sectors has no effect on the law of motion for aggregate employment, however, because matching probabilities are equalised across sectors throughout the adjustment period. Accordingly, $N_{t+1} = (1 - s)N_t + p(\theta)U_t$ in all periods. In other words, the total employment stock in the economy is following a simple non-homogeneous difference equation. The stationary point of this equation, assuming $\theta = \bar{\theta}$, is $\bar{N} = \frac{p(\bar{\theta})}{s+p(\bar{\theta})}$. It is necessary to determine whether the law of motion for aggregate employment implies convergence to this level.

To examine the issue of stability of the adjustment process, consider the properties possessed by a stationary state versus a general case in which output is equalised across sectors in all periods. In the latter instance, all that is lacking is the labour market balance condition since migration is non-optimal and the zero profit condition for vacancies holds. The situation is illustrated by the equality:

$$F\Big((1 - s + p(\theta_t))N_{1,t} + p(\theta_t)U_{1,t}, \mu_1\Big) = F\Big((1 - s)N_{1,t} + p(\theta_t)(L - N_{1,t}) - p(\theta_t)U_{1,t}, \mu_2\Big).$$

This condition must hold if the law of motion for employment is to be respected and output is to remain equalised between periods. Only one variable, $U_{1,t}$, can be adjusted through migration at time t to ensure that this happens. A unique value of $U_{1,t}$ solves this because the two sides of the equation are *strictly* decreasing and increasing, respectively, in $U_{1,t}$. Accordingly, it will only be by coincidence that the appropriate value of $U_{1,t}$ also keeps employment constant between periods in both sectors.

Fortunately, coincidence does not have to be appealed to when showing that the economy always moves toward a situation of labour balance. To see this, note that, if the condition above does not also satisfy labour market balance, it is because θ_t is either too high or too low. The direction of adjustment of θ can be determined from the following argument:

i.) Along the adjustment path, employment either falls or rises in *both* sectors. This is necessary for $F(N_{1,t}, \mu_1) = F(N_{2,t}, \mu_2)$ and $F(N_{1,t+1}, \mu_1) = F(N_{2,t+1}, \mu_2)$ to both hold.

ii.) If $\theta_t > \bar{\theta}$, then only a rise in employment in both sectors can occur on the adjustment path. This is because, if $\theta_t > \bar{\theta}$, then it is impossible to not have employment rising in at least one sector. It is possible that one sector could, in theory, have the direction of adjustment be incorrect because, as Figure Five shows, the 'labour demand' and 'labour supply' curves are displaced from their steady-state values because of the difference between actual and steady-state sectoral labour forces. However, it is not possible for adjustment to be perverse in both sectors because, if $L_t > \bar{L}$ in one sector, the opposite is true in the other. Since a rise in one sector and a fall in the other is precluded by i.), it must be that employment rises in both sectors and θ falls.

iii.) If $\theta_t < \bar{\theta}$, then only a fall in employment in both sectors can occur on the adjustment path. This is because, if $\theta_t < \bar{\theta}$, then it is impossible to not have employment falling in at least one sector. Since a rise in one sector and a fall in the other is precluded by i.), it must be that employment falls in both sectors and θ rises.

In summary, aggregate employment will rise when lower than its steady-state value and fall when above it. This ensures that the economy is moving toward a stationary point of the employment law of motion. It can now be shown that, when aggregate employment is at its stationary point, the two individual labour markets are also in balance. Note that when total employment converges the following is true:

$$N_1 + N_2 = \frac{p(\bar{\theta})}{s + p(\bar{\theta})} L,$$

$$F(N_1, \mu_1) = F(N_2, \mu_2),$$

$$q(\theta) = \frac{\gamma(r+s)}{(1-k)} \frac{1}{F(N_1, \mu_1}.$$

These three equations determine values for each of N_1, N_2, and θ such that employment is at its steady-state level in the aggregate, the value of a posted vacancy is zero, and the no-migration conditions hold. If these hold, then it must also be true that employment is at its steady-state value in each sector. To see why this latter condition is true, it is necessary to show that (i) there is a unique solution to the system above and that (ii) the steady-state values of N_1 and N_2 solve this system. It is then necessarily true that theses solutions are the same.

To establish uniqueness, obtain a system of two equations in N_1 and θ by using the first equation to eliminate N_2 from the second. Plotting the two equations in employment-market tightness space gives two curves with the properties shown below in Figure Six. The downward sloping curve is the steady-state version of the "labour demand" curve derived in the previous section.[4] The properties of this curve have already been established. The upward sloping curve depicts combinations of N_1 and θ for which total employment is at its steady-state level and the value of output is equalised across sectors. It is defined by:

$$F(N_1, \mu_1) = F\Big(\lambda(\theta)L - N_1, \mu_2\Big)$$

where $\lambda(\theta) \equiv \frac{p(\theta)}{s+p(\theta)}$. Totally differentiating this identity gives:

$$\frac{d\theta}{dN_1} = \frac{\left(\frac{\partial F}{\partial N}|_{N_1} + \frac{\partial F}{\partial N}|_{N_2}\right)}{L\lambda'(\theta)\frac{\partial F}{\partial N}|_{N_2}} > 0.$$

[4] Recall that the steady-state version of the equation imposes the condition that $N_t = N_{t+1}$.

This expression is strictly positive because of the assumptions on the average revenue function and the matching probability function. Accordingly, the two curves in Figure Six cross once, and only once, establishing uniqueness of the solution.

Given that only one combination of (N_1, N_2, θ) can solve the system above, it is clear that the fixed point of the dynamic adjustment path is given by any solution to the three equations. Note that the steady state of the two-sector economy has the second and third equations for the system above holding by definition. The first condition holds because:

$$\bar{N}_1 = \frac{p(\bar{\theta})}{s + p(\bar{\theta})} L_1 \quad \text{and} \quad \bar{N}_2 = \frac{p(\bar{\theta})}{s + p(\bar{\theta})} L_2.$$

Adding together these two equations gives the total employment stationarity condition.

Thus, the values of employment and market tightness which the economy adjusts toward are the steady-state values for the two-sector economy. Accordingly, this adjustment mechanism does converge to the desired steady-state conditions.

Adjustment with Initial Value-of-Output Differentials

This section examines an adjustment path which results when one sector is displaced from an initial steady state. Suppose that initially both sectors were in symmetric equilibrium with all parameters and functions equal. Imagine further that an unanticipated increase in μ_1 prompts adjustment to a new steady state. This adjustment path is the simplest type of shock with sectoral reallocation, although it cannot be viewed as a "pure" sectoral shock because it is accompanied by a change to steady-state employment. This simple scenario seems appropriate for sectors which are producing very different products, so that shocks to industry demand are uncorrelated. In certain cases, such as the candle and light bulb industries, this would clearly be an inappropriate assumption. The path is nevertheless interesting because it represents the simplest example of adjustment through both natural dynamics and the process of migration.

In general, the adjustment path will involve three distinct phases. The first phase is distinguished by migration from the relatively less-favoured sector to the more-favoured sector due to persistent incentives to do so. These incentives persist because employment can only adjust with time. A second phase is very short and involves "reverse migration" of some unemployed workers back to the original disfavoured sector. Finally, there is a phase of adjustment in which compensating differentials ensure that workers are indifferent between sectors. In this final phase, aggregate employment adjusts to its steady-state level according to the process outlined in the previous section.

Given that the last period of adjustment was described above, it is necessary here to describe the path of key variables in the first phase only. Specifically, the adjustment path involves the following paths for the key variables of the model:

- Unemployed workers will move toward the relatively 'good' sector during adjustment to a sectoral shock. As a consequence, employment will rise over time in the favoured sector and fall over time in the less-favoured sector.
- The value of output-per-worker will rise in the relatively favoured sector once the shock is recognised, and then fall over the adjustment period.
- The value of output-per-worker will rise over time in the relatively less-favoured sector.
- The hazard rate for matches will rise in a relatively favoured sector once the shock is recognised and then fall over the adjustment period.
- The hazard rate for matches will rise over time in the relatively less-favoured sector.
- There will be generally some 'reverse migration' at the end of an adjustment path. This will involve the departure of some unemployed workers from the favoured sector to reduce the labour force in that sector to its steady-state level.

To establish that this is, indeed, an equilibrium path consistency and optimality must be demonstrated. This is done by first recognising that upward displacement of

μ_1 from an initial steady state will induce migration into sector one. Optimality of this migration pattern will be established by referring to the value functions directly. Thus, after the displacement, all mobile unemployed agents from sector two will choose to transfer between the sectors to search. The effect of this on the model is the following:

Proposition 4: *As a result of migration during the adjustment period, employment falls in sector two and rises in sector one.*

Proof: See Appendix.

To support this path for employment and migration, it is necessary to verify that the assumed direction of migration is indeed optimal. By Assumption 2.1, as employment rises in sector one and falls in sector two, the value of output will rise in the bad sector and fall in the good. Accordingly, there will eventually be some time T at which equality of the value of output is achieved (i.e. $F(N_{1,T},\mu_1) = F(N_{2,T},\mu_2)$). From this point, adjustment occurs to ensure that equality of the value of output is maintained.

It is next useful to take the zero-profit condition for a vacancy and substitute-in the present value version of the value of matched capital to obtain:

$$\gamma = \beta q(\theta_{i,\tau})(1-k)\sum_{t=1}^{\infty} s^{\tau} F(N_{i,t},\mu_i,t).$$

By assumption, $F(N_{1,t},\mu_1) = F(N_{2,t},\mu_2) \forall t \geq T$ so that $\theta_{1,t} = \theta_{2,t} \forall t \geq T-1$. This makes analysis of the values of searching in periods prior to T fairly easy.

First, beginning in period $T-1$, it is possible to establish that the value of searching falls between $T-1$ and T in the good sector but rises in the bad sector. To see this, note that:

$$V^P(z_{1,T-1}) = (1-k)F(N_{1,T-1},\mu_1) + q(\theta_{1,T-1})\beta V^P(N_{1,T},\mu_1),$$

and

$$V^P(z_{2,T-1}) = (1-k)F(N_{2,T-1},\mu_1) + q(\theta_{2,T-1})\beta V^P(N_{2,T},\mu_2).$$

Now, the second term in each of these two expressions is the same given the paths of $F(N_i, \mu_i)$ and θ_i. Accordingly, the value functions for producers can only differ if the values of production differ in period $T-1$. This is the case because $N_{1,T-1} < N_{1,T}$ while $N_{2,T-1} > N_{2,T}$. Accordingly, the value of production is higher in sector one in period $T-1$, which implies that $V^P(\mathbf{z}_{1,T-1}) > V^P(N_{1,T-1}, \mu_1)$. Combining this with the zero profit condition for vacancies yields the condition that $\theta_{1,T-2} > \theta_{2,T-2}$.

Moving now to period $T-2$, note that the appropriate values for a producing firm are:

$$V^P(\mathbf{z}_{1,T-2}) = (1-k)F(N_{1,T-2}, \mu_1) + q(\theta_{1,T-2})\beta V^P(N_{1,T-1}, \mu_i),$$

and

$$V^P(\mathbf{z}_{2,T-2}) = (1-k)F(N_{2,T-2}, \mu_1) + q(\theta_{2,T-2})\beta V^P(N_{2,T-1}, \mu_2).$$

The second terms in these expressions are equal because of the zero profit condition and the assumption that there is no reason for the flow cost γ to vary between sectors [5]. Furthermore, this equality holds even in the case where the matching functions differ between sectors. Accordingly, it is only the current period return that differentiates the two sectors. Here, N_1 is larger than in $T-1$ while N_2 is smaller which means that $V^P(\mathbf{z}_{1,T-2}) > V^P(\mathbf{z}_{1,T-1})$ but $V^P(\mathbf{z}_{2,T-2}) < V^P(\mathbf{z}_{2,T-1})$. The same reasoning as above can be applied to the zero-profit condition to show that $\theta_{1,T-3} > \theta_{1,T-2}$ and $\theta_{2,T-3} < \theta_{2,T-2}$. Finally, this argument can be repeated back to the beginning of the adjustment period to show that $F(N_i, \mu_i)$ and θ_i are falling in sector one and rising in sector two due to the assumed adjustment path.

Given these paths for the driving variables of the model, it is possible to verify that migration from sector two to sector one is optimal. Note first that, from time T onward, conditions for searchers are identical in the two sectors so that all value functions take on the same values across sectors after time T. At time $T-1$, the

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[5] If $\gamma_1 \neq \gamma_2$, but the difference in the two flow-costs terms is a constant amount, then this analysis still goes through. If $\gamma_1 - \gamma_2$ varies over time, then the analysis becomes far more complex.

following holds:

$$(1-p(\theta_{1,T-1})V^U(z_{1,T})+p(\theta_{1,T-1})V^W(z_{1,T})$$
$$>(1-p(\theta_{2,T-1})V^U(z_{2,T})+p(\theta_{2,T-1})V^W(z_{2,T}).$$

This is because $\theta_{1,T-1} > \theta_{2,T-1}$ while all other terms in the expressions for the future period returns are identical across sectors. Given this, migration from sector two to sector one at time $T-1$ is a condition necessary for equilibrium behaviour along the adjustment path.

It is possible to move this analysis back now to period $T-2$. Note first that:

$$V^W(z_{1,T-1}) > V^W(z_{2,T-1})$$

because the current period return to working in sector one is higher at time $T-1$. This means that

$$(1-p(\theta_{1,T-2})V^U(z_{1,T-1})+p(\theta_{1,T-2})V^W(z_{1,T-1})$$
$$>(1-p(\theta_{2,T-2})V^U(z_{2,T-1})+p(\theta_{2,T-2})V^W(z_{2,T-1}).$$

This is true not only because the matching probability is higher in sector one but also because the return to getting matched is higher in sector two. Accordingly, migration from sector two to sector one is optimal at time $T-2$ also. Finally, note that this recursive argument can be extended back to the beginning of the adjustment period to prove that migration to sector one is optimal throughout the adjustment period.

Thus, until time T, migration occurs from sector two to sector one in response to pecuniary incentives. At T and beyond, unemployed mobile workers are indifferent between sectors so that migration occurs to maintain this situation. Labour market dynamics within each sector function in the manner described above to fulfill the second stage of the adjustment process. Of course, there is a razor's edge case in which employment is at its steady-state level at the end of the period of migration and output value differentials. In general, however, some adjustment along the curve in Figure Four is necessary.

Extension of this argument to the case where matching functions differ between sectors is not difficult. The point is that steady-state no-migration conditions impose equality of the value of searching between sectors. Prior to this, migration occurs and it is possible to demonstrate that market tightness and the value of output fall in sector one but rise in sector two. Migration in the assumed direction is thus optimal. A complication of this case is that the no-migration condition is not equivalent to a simple output equivalence condition when matching functions are different. A related, but less immediately clear, condition holds as an analogue to Figure Four.

Finally, the argument outlined above can be extrapolated to any of an infinity of combinations of sectoral shock configurations. The key elements in each such case are a period of migration from a more-favoured to a less-favoured sector. In this period, market tightness and the value of production tend to move in opposite directions. After this, a period of adjustment to the steady-state value of market tightness, θ, is observed and adjustment of this variable proceeds in the same direction in both sectors. Future analysis of two-sector adjustment can use the model here to examine a variety of interesting scenarios.

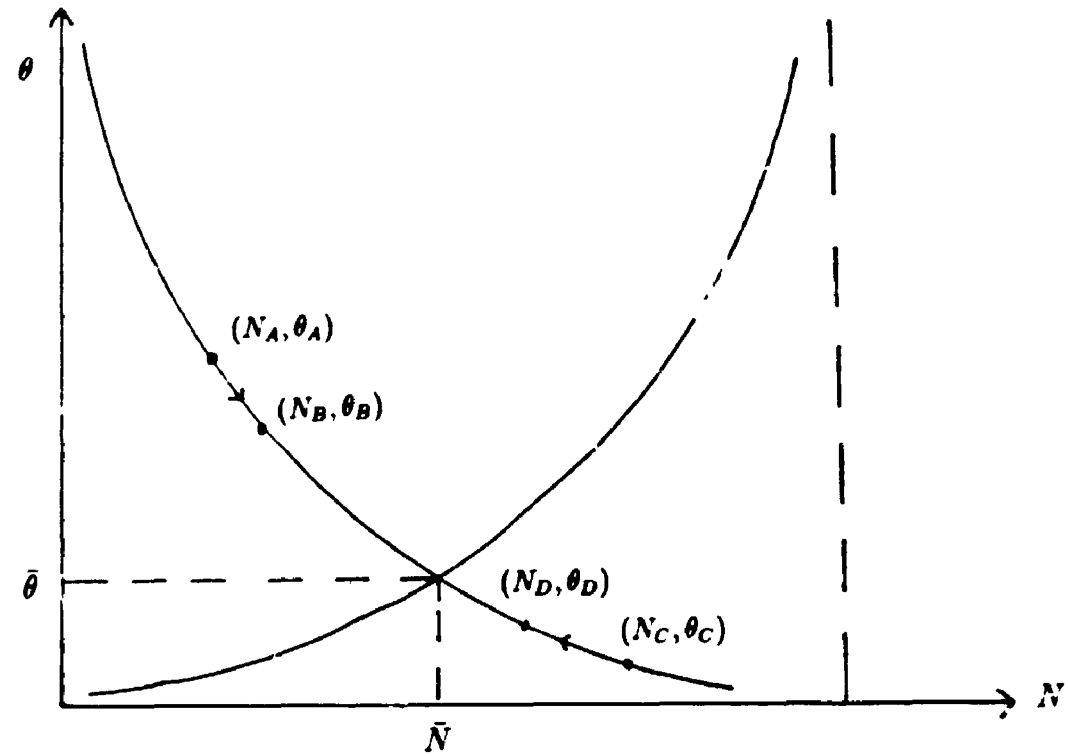

Figure Three: Dynamic Evolution of a One-Sector Economy

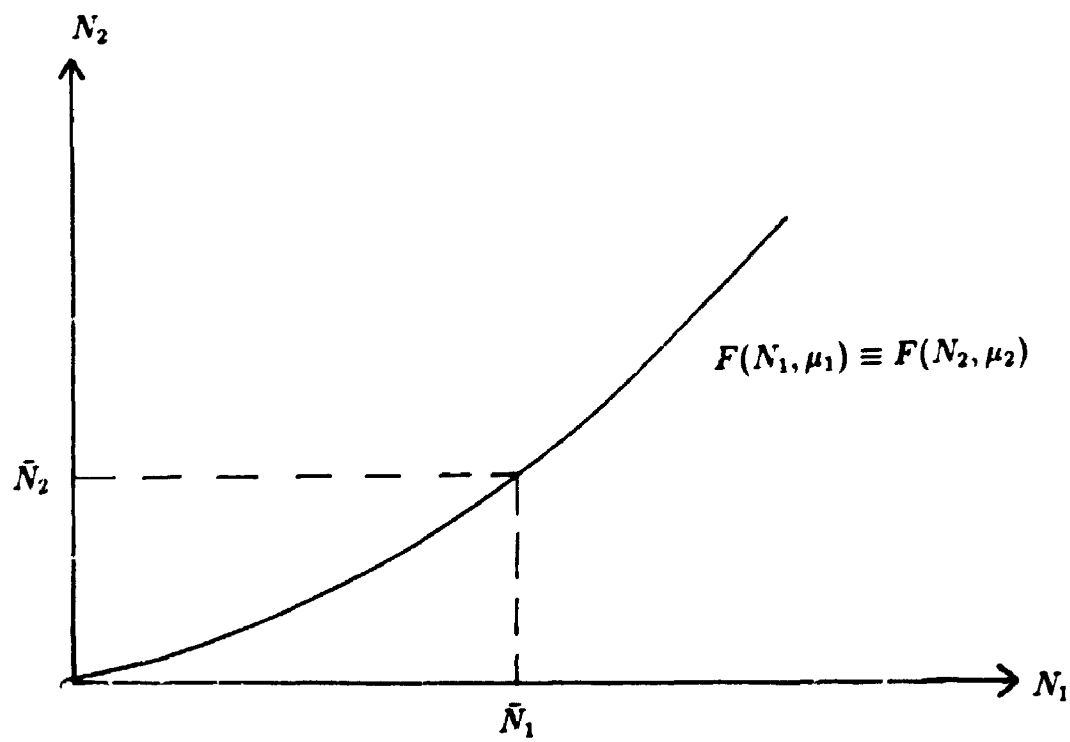

Figure Four: Employment Levels which Solve $F(N_1, \mu_1) = F(N_2, \mu_2)$

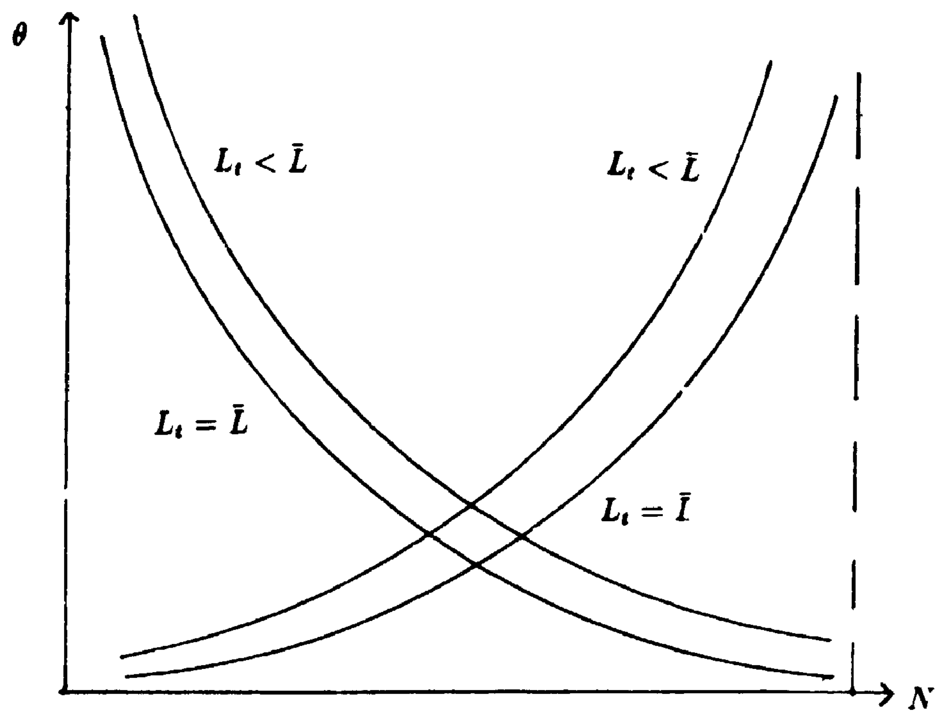

Figure Five: 'Labour Demand' and 'Labour Supply' when $L \neq \bar{L}$

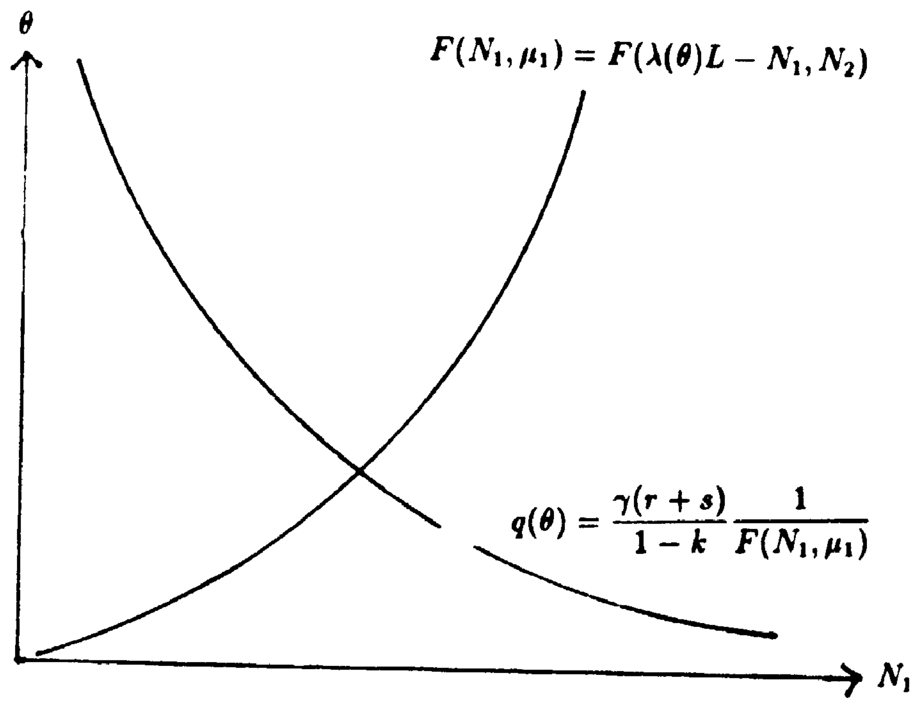

Figure Six: Two-Sector Equilibrium Conditions

2.6 Conclusions and Summary of Empirical Predictions

The model described in this chapter yields a number of interesting empirical predictions. Most significantly, it relates unemployment duration to sectoral productivity conditions so that the model has the potential to explain observed patterns of unemployment duration. Equally important, restrictions are placed on the types of outcomes which are to be expected conditional on certain observable variables. This allows researchers to devise applied tests which can determine whether the promise of the model at the theoretical level stands up to the rigours of empirical testing.

A fundamental implication of the matching function paradigm is that unemployment duration is positively related to the number of searching workers in a market but inversely related to the number of posted vacancies. Furthermore, the customary assumptions on the curvature of the matching function yield implications for the shape of the relationship between unemployment duration and tightness in the labour market. Given that measures of tightness are readily available, tests of these fundamental propositions are possible.

The model analysed here also yields predictions which do not flow directly from the assumptions of the matching function paradigm. For instance, the dynamic path of unemployment duration along the adjustment path to the steady state was examined. From this, predictions were obtained regarding the relationship between duration and unemployment over time. These implications can be tested if prior knowledge of sectoral fortunes is combined with econometric methods of inferring the realization of an unknown state variable. If the states of nature implied by theory and data correspond closely to prior knowledge of these states, then evidence in favour of the model is obtained.

An interesting feature of the model is that it offers an explanation for the duration of search unemployment that is not based on reservation wage decisions. This is desirable because empirical work such as Holzer (1988) and Jones (1988) place in doubt the ability of variation in reservation wages to explain variation in unemployment duration. From the viewpoint of empirical testing, there are several ways of differentiating the current model from one driven by reservation wage decisions. Most obviously, shocks to sectoral productivity are the key forcing variables in the present model while shifts in wage distributions, demographic factors, and policy variables (such as unemployment insurance eligibility periods) do most of the work in the reservation wage framework. While it is possible that both aspects of the search problem have some explanatory power, it is important to use structural models to identify the sources of unemployment duration shifts and thereby discriminate between the two approaches.

A final topical implication of the model is that sectors which experience relatively favourable shocks should have higher than average wages and unemployment spells that are shorter than average. This is interesting given the competing "queuing" view of the labour market which asserts that sector or industries with high wages have long entrance queues while low wage industries have short queues. Empirical work can easily be conducted, using large cross-sectional data sets, such as the Canadian Labour Market Activity Survey, to assess the relative merits of these two views.

2.7 Suggestions for Further Work

A number of directions for future research are suggested by the analysis presented above. The previous section outlined a series of possible empirical tests. In addition, the analysis exposed two promising avenues of theoretical enquiry. First, analysis of the law of motion for employment revealed that a rise in current employment could lower future employment if the impact of a fall in current market tightness were sufficiently large. While this seemed unlikely in periods of low unemployment, the theoretical possibility remained that employment dynamics were in some sense perverse for low levels of employment. Similarly, the theoretical possibility exists that, at low levels of employment, a shock to the efficiency of the matching process has an unusual impact on recruitment activity. These implications for models of severe recessions need to be further developed and tested using data from the Great Depression, for example.

Finally, while the work in this chapter represents an important foundation for analysis of sectoral adjustment with labour market search, there remain several important areas in which the work can be extended. Most importantly, the analysis of a model with non-permanent shocks is needed. This analysis is undertaken in Chapter Four using a restricted environment. Next, it would be interesting to see how the predictions of the model change if mobility costs, rather than declining sectoral average revenue, serve to equilibrate returns to searching in either sector. Related to this, a general analysis of the role of mobility costs in a model with non-permanent shocks would be interesting. Finally, the assumption that shadow values of capital are constant over time and between sectors merits some consideration given the alleged importance of cost of capital variations in economic fluctuations.

Chapter Three

An Empirical Investigation of the Relationship between Recruitment Activity and Employment Flows

3.1 Introduction

A recent theoretical innovation in macroeconomics is the explicit modelling of labour market search. As Pissarides describes in his recent book *Equilibrium Unemployment Theory*, a "second generation" of search models now exists which seeks to explicitly model the relationship between the rate at which searchers contact one another and features of the physical environment. Thus far, the aspect of the environment which has received the most attention has been the relative sizes of the search pool on either side of the market. Models in this tradition are currently being employed to explain a variety of macroeconomic phenomena and the goal of this chapter is to measure the empirical validity of the models as a natural complement to these theoretical exercises.

Incorporating search into macroeconomic models can produce significant and far-reaching results. Howitt (1988) shows that the dynamics of business cycles are modified considerably by the introduction of labour market search. Diamond (1982) began a tradition of modelling economies with multiple natural rates of unemployment which exist by virtue of strategic complementarities due to search-type transaction mechanisms. Davidson, Martin and Matusz (1988) and Hosios (1990) have shown that results in multi-sector trade theory are affected by the presence of search frictions. Finally, there is a growing literature, started by Pissarides(1987) which merges search and wage bargaining to examine cyclical properties of wages.

While search models of this new generation are clearly yielding a great deal of theoretical work, there is little direct testing of these models. Pissarides (1985) makes some broad statements about the accord between a search model and stylized facts of the business cycle. Blanchard and Diamond (1989) fit a Cobb-Douglas model of new hires as a function of labour market search pools but this exercise is only conducted to obtain parameters for a simulation exercise. Given the apparent growth of interest in this new aspect of search theory, there is clearly a need for empirical testing of the approach. The purpose of this chapter is to provide such tests.

This chapter examines the linkages between flows out of unemployment and observable measures of the size of the search pools on both sides of the labour market. This work facilitates tests of the notion that a "matching function" approach to labour market dynamics has significant explanatory power. The testing methods used in the chapter involve several innovations. First, transition probabilities are calculated from gross flows data rather than from the Labour Force Survey Average Duration of Unemployment series. This provides a measure of the instantaneous probability of leaving unemployment and eliminates right-censoring problems. Secondly, the analysis is conducted with an eye to avoiding the imposition of inappropriate functional forms. The spline estimation and inference techniques developed for this are interesting in their own right. Finally, the analysis uses pooled time-series cross-sectional data. This has not been possible in the past because provincial Help Wanted Indexes were not comparable by virtue of their lack of units. This problem is tackled by using observations for the (alas, discontinued) Quarterly Survey of Vacancies to normalize the HWI series.

The chapter begins by examining a theoretical model of search in order to identify propositions to be tested. The availability of data for these tests is then considered and the stylized facts revealed by the data are analyzed. Two following sections use monthly data for British Columbia, Ontario and Quebec to test the propositions identified in the second section. A separate section considers the use of pooled time-series and cross-sectional data. A final section of the chapter summarizes and evaluates the results.

3.2 The Relationship Between Employment Hazard Rates and Market Tightness: Theoretical Implications

To test the relationship between flows out of unemployment and market tightness implied by matching-function search models, it is necessary to first write down such a model. The general idea of the matching function approach is to explicitly recognize that in the "real world" workers and firms who wish to effect an exchange must do so without access to a centralised spot labour market. Rather, there is a time-consuming search process whereby unemployed workers and job vacancies are brought together. While this process may be stochastic at the individual level, it is assumed that a deterministic relationship exists between the total number of matches occurring in a market and the total number of searchers on either side of the market. It is this deterministic relationship which is called the matching function.

Such a 'matching function' figures prominently in recent work by Pissarides (1987) and Blanchard and Diamond (1989) which has analysed labour market dynamics. The base case of a matching function can be taken as:

$$x_t = X_i(U_t, O_t). \tag{3.2-1}$$

Here the total number of matches, x_t, is a function of the number of unemployed workers, U_t, and the number of job offers, O_t, posted by firms to fill vacancies. Like a production function or utility function, this deterministic relationship *represents* a more fundamental feature of the economic environment. Using such a function is more expedient than modelling the underlying physical facts of the search process in great detail every time a search model is used.

Following the theoretical work of Pissarides, the following assumptions are made here concerning the matching function:

ASSUMPTION 3.1 *$X_i(U_{i,t}, O_{i,t})$ is increasing in both U_i and O_i and is concave.*

The fact that the matching function is increasing in its argument is fundamental to the paradigm. This simply says that more searchers produce more matches. Concavity means that these increases happen at a decreasing rate (i.e. that there is non-increasing 'marginal productivity' of both vacancies and unemployment). Certainly, there is little reason to believe that the opposite will be true, namely that, as a market becomes more and more unbalanced, matches grow at an *increasing rate*. Increasing disparity in a market raises the probability of multiple contacts and likely causes other interference effects between searchers on the long side of the market. Not surprisingly, these arguments echo those for decreasing marginal productivity in an intermediate microeconomics text. Concavity is not a critical feature of this paradigm but is nevertheless important because it distinguishes matching function models of unemployment dynamics from those based on job queues. In the latter case, increasing the short side of the market with the other side held constant will create a proportional rise in matches. This property of queues is discussed in more detail below.

ASSUMPTION 3.2 *$X_i(U_{i,t}, O_{i,t})$ is homogeneous of degree one.*

Assumption 3.2 is imposed partly as a convenience because it implies that the number of matches per searching worker can be written as a function of a single variable, θ_t:

$$x_t/U_t = X_i(U_t, O_t)/U_t = X_i(1, \theta_t).$$

This convenience is justified by Blanchard and Diamond (1989) who found that the assumption of constant returns to scale could not be rejected for an aggregate U.S.

matching function. Some authors, such as Diamond and Fudenberg (1989) have considered increasing returns to scale because this allows for the interesting possibility of models with multiple natural rates of unemployment. There is little reason to prefer this assumption when testing the matching function approach since the assumption of increasing returns is usually more controversial than that of constant returns.

The procedure followed here will be to adopt the assumption of constant returns to scale in order to examine the more basic assumptions that the matching function is both increasing and concave. This allows for the analysis to be conducted in terms of a univariate relationship between the ratio of matches to searching workers and the tightness of a labour market, θ. Of course, this means that any rejection of Assumption 3.1 could be due to the inappropriate use of the homogeneity-of-degree-one assumption but, again, existing empirical work suggests that this will not be the case.

It is useful to recognise that the ratio of matches to searching workers can be interpreted as the probability that any particular worker is matched. While the matching function is deterministic at the aggregate level, there is uncertainty for an individual searcher. The probability that a worker is matched will be denoted by the function $p(\theta_t) = X(1, \theta_t)$. The matching function is increasing in both of its arguments by Assumption 3.1 and this means that the matching probability for a worker rises with θ_t. Intuitively, θ_t is a measure of the 'tightness' of a labour market and high values of θ_t indicate that jobs are relatively abundant.

The implications of Assumption 3.1 for the $p(\theta_t)$ function can be stated as follows:

$$p'(\theta) = \frac{\partial X}{\partial O} > 0,$$
$$p''(\theta) = \frac{\partial^2 X}{\partial O^2} \leq 0.$$

These sign and curvature restrictions will be examined in the remainder of the chapter.

3.3 Data for the Empirical Tests

Ideally, the empirical validity of the matching function approach would be examined by reference to unemployment probabilities and market tightness measures for labour markets covered by a single matching function. Unfortunately, while some cross-sectional data are available, they cannot readily be matched with vacancy and unemployment data. On balance, the best data source is provided by monthly provincial data on market tightness and unemployment transition probabilities [1]. With these data it is possible to analyze the structure of the matching process for three Canadian provinces: Quebec, Ontario and British Columbia. The choice of provinces is dictated by the availability of the monthly Help Wanted Index (HWI). The other provinces are grouped into two regions (the Prairie Provinces and Atlantic Canada) so that it is not possible to match vacancy data with the HWI at the provincial level for those seven provinces

To give units to the Help Wanted Series, the Quarterly Survey of Vacancies is used to normalize the Help Wanted Index. Normalised Help Wanted Index series were constructed which have the same 1976 annual mean as does the 1976 vacancy survey. These annual averages were approximately 4,000, 18,000, and 12,000 vacancies for British Columbia, Ontario and Quebec respectively. While the choice of base year is rather arbitrary, the choice of a year in the middle of the vacancy survey sample seems quite natural. With regard to the use of the HWI as a proxy for vacancies, the analysis of section 1.7 of this thesis suggests that the HWI performs well for all provinces but Quebec. Correlation coefficients ranged from 0.12 for Quebec to 0.69 for British Columbia and 0.90 for Ontario.

This is admittedly a simplistic way of attributing units to the Help Wanted Index. A more sophisticated method is advocated by Kapsalis (1988) based on an assumption that the number of frictional unemployed equals the number of vacancies and that all unemployment in 1966 was frictional. While there are virtues to this method,

[1] Chapter One of this thesis describes in detail the sources for these data.

there seems no compelling reason to prefer it to the simple use of the Vacancy Survey data. The tradeoff would appear to be between the potential problems in the Quebec series and the potential error in the identifying assumption used by Kapsalis. Here a preference for simplicity is followed. Any potential problems due to incorrect basing of the series will only affect the pooled results, however, and so less weight should be placed on these regressions.

The normalized series are presented together in Figure One. It is worth noting that the pooled regression provides only a slightly wider range of values than is yielded by the series for Ontario alone. In general, the degree of tightness has been roughly comparable in these three provinces. The two clear exceptions are the behaviour of the Ontario labour market during the recovery after the 1982 recession and the Quebec case during the early 1980's. The tests of the chapter will help to determine if these differences are also reflected in transition rates from unemployment.

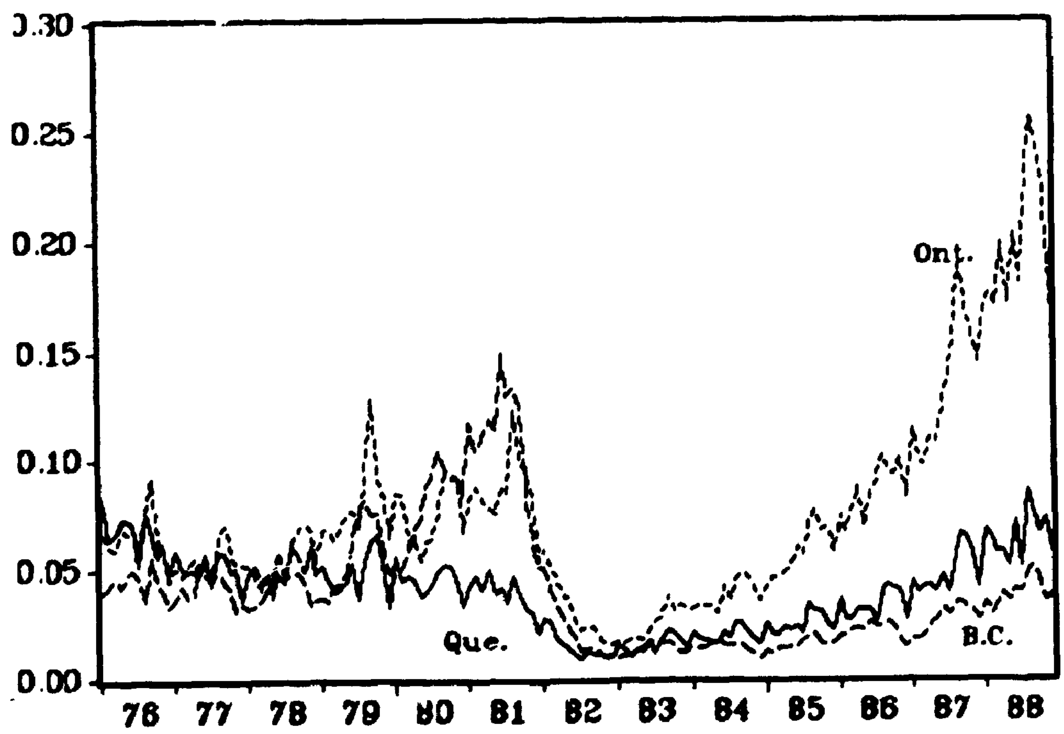

Figure One: Comparison of Normalized Market Tightness (θ) Series

Unemployment transition probabilities can be calculated from labour market gross flows data. Statistics Canada collects, but does not publish, estimates of the monthly flows between each of the following three labour market states: employment, unemployment, and non-participation. These estimates are derived from the portion of the Labour Force Survey sample that is retained in the sample between consecutive months. If the flow from unemployment to employment between month t and $t+1$ is divided by the stock of unemployed workers at time t, then a measure of the average instantaneous probability of moving from unemployment into employment at time t is obtained. Clearly, heterogeneity renders this average measure less than perfect but this estimated hazard rate is, nevertheless, a reasonable indicator of the transition probability $p(\theta_t)$ in a given labour market, particularly since work by Jones (1992) found little difference in the long-term average value of this flow probability for young versus old or men versus women.

Once constructed, the data above show strong seasonal patterns. A similar finding led Blanchard and Diamond (1990) to adjust their gross flows data for seasonal adjustment. It is not clear that this is desirable here, however, since the high degree of seasonal variation in hazard rates may provide information about the matching process. According to the theory, seasonality of hazard rates should reflect seasonality of recruitment intensity and thus market tightness. Indeed, analysis of seasonal factors from the X-11 adjustment procedure in Micro-TSP suggests that these season patterns are similar but not identical. The factors for adjustment with a multiplicative seasonal process are in Figures Two through Four. These graphs suggest that there may be value to conducting the analysis with data that are not adjusted for seasonality. For the sake of consistency, the present analysis will follow the lead of Blanchard and Diamond and use adjusted data to focus on longer-term fluctuations.

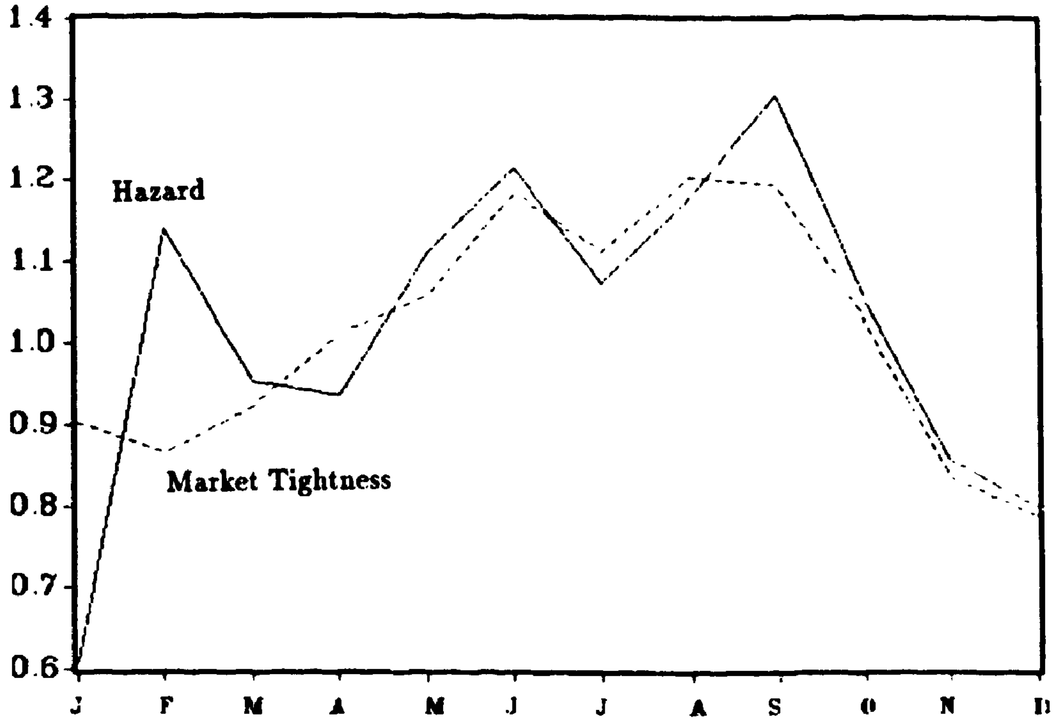

Figure Two: Seasonal Factors for British Columbia

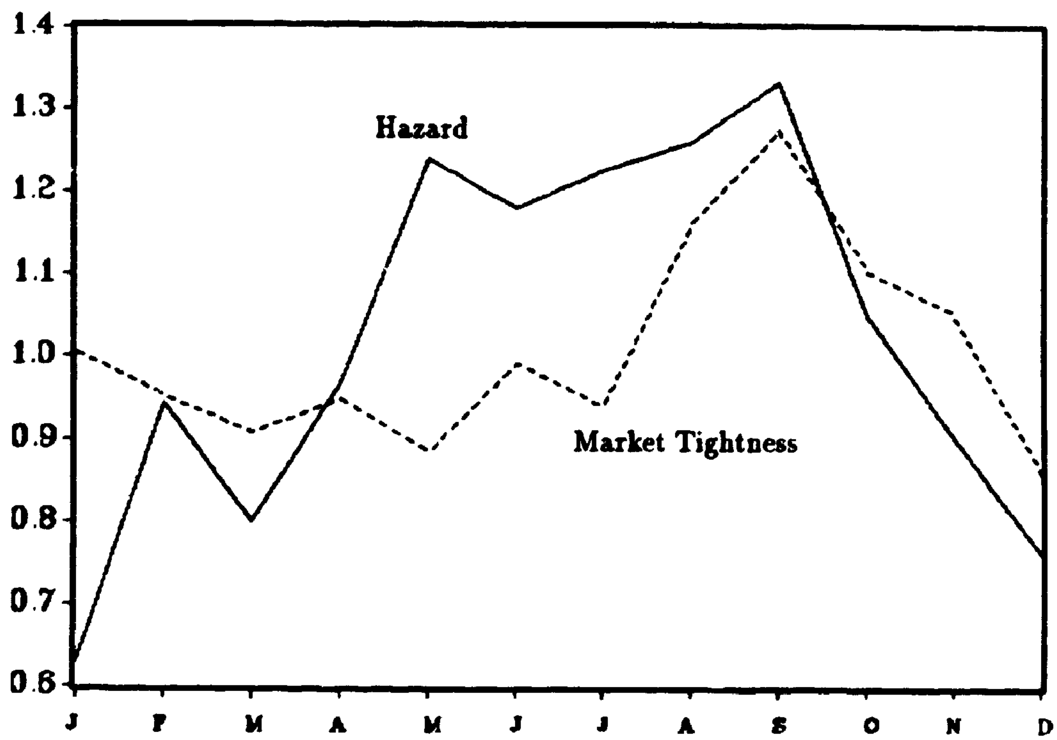

Figure Three: Seasonal Factors for Ontario

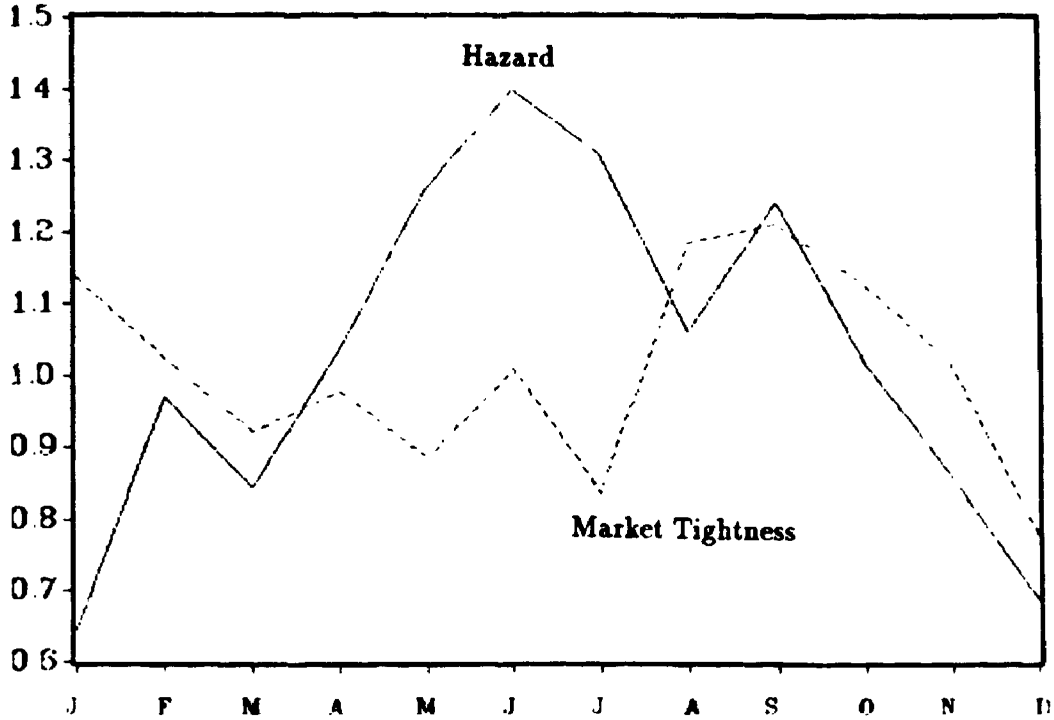

Figure Four: Seasonal Factors for Quebec

3.4 The Relationship Between Employment Hazard Rates and Market Tightness: The Stylized Facts

The validity of the matching function paradigm can be evaluated both formally and informally. Informal analysis is conducted by examining time series plots and scatter diagrams relating observed hazard rates to measured market tightness. Chronological graphs appear in Figures Five through Seven while Figures Eight through Ten are scatter-graphs. Unfortunately, a gap appears in the hazard rate series in late 1984 and early 1985 (six months in total). This reflects the fact that Statistics Canada did not calculate gross flow estimates during this period for technical reasons.[2]

The time series graphs reveal two general stylised facts. First, market tightness and hazard rates have seemed to move together over time. Market tightness and hazards fell during the 1982 recession in all provinces and the variation in post-recession hazard rates is also reflected in market tightness. It is nevertheless true that the hazard rate series is more variable than the market tightness series. This could reflect the fact that the sampling error is greater for the gross flows data or may indicate that the Help Wanted Index is overly smooth due to averaging. Interestingly, the graphs reveal that the divergent behaviour of market tightness in Quebec during 1980 and 1981, revealed in Figure One, is also reflected in the hazard rates.

It is easier to discern the form of the relationship between the hazard into employment and market tightness if these data are plotted on the y and x axes of a graph. This has been done in Figures Eight through Ten. In these graphs, small crosses represent actual observed data points. While all three graphs differ in several aspects, the general impression is of an upward sloping relationship with a possibly declining slope. Around this general impression there is a considerable amount of random variation.

[2] These 'holes' complicate the seasonal adjustment procedure. Adjustment was achieved by calculating seasonal factors for the contiguous observations between 1976 and 1983. These factors were then used to adjust all the observations. This procedure assumes that the seasonal pattern remained the same after the gap in the data.

To make any precise statement about the hazard rate/market tightness relationship will clearly require some method for separating allowable random deviations in the data and departures from theoretical shape restrictions.

The general statistical principal followed in this chapter is the use of flexible estimation methods that minimise the imposition of functional forms. It is not difficult to think of cases in which a non-monotonic regression might yield a positive slope globally due to the inappropriate choice of a linear functional form. Given this, two flexible estimation methods are used here. First, monotonicity is tested by fitting a linear spline function to the data and then fitting a restricted model in which the slope of the relationship is always positive.

A second functional form can be used for cases in which monotonicity cannot be rejected. Here, a curve can be fit through the data by minimising the sum of squared deviations from the curve *subject to* the restriction that the fitted curve be monotone non-decreasing and concave. This can be tested against a fitted curve which must only satisfy monotonicity. Implementing this model requires the use of numerical techniques for solving inequality-constrained quadratic programming methods. The monotonicity test could not use this same curve fitting method because the unrestricted case necessarily has an error variance of zero so that the variance of the error term could not be estimated. This prevents inference regarding whether the variance of the random noise term is large enough to explain departures from non-decreasing monotonicity in the data.

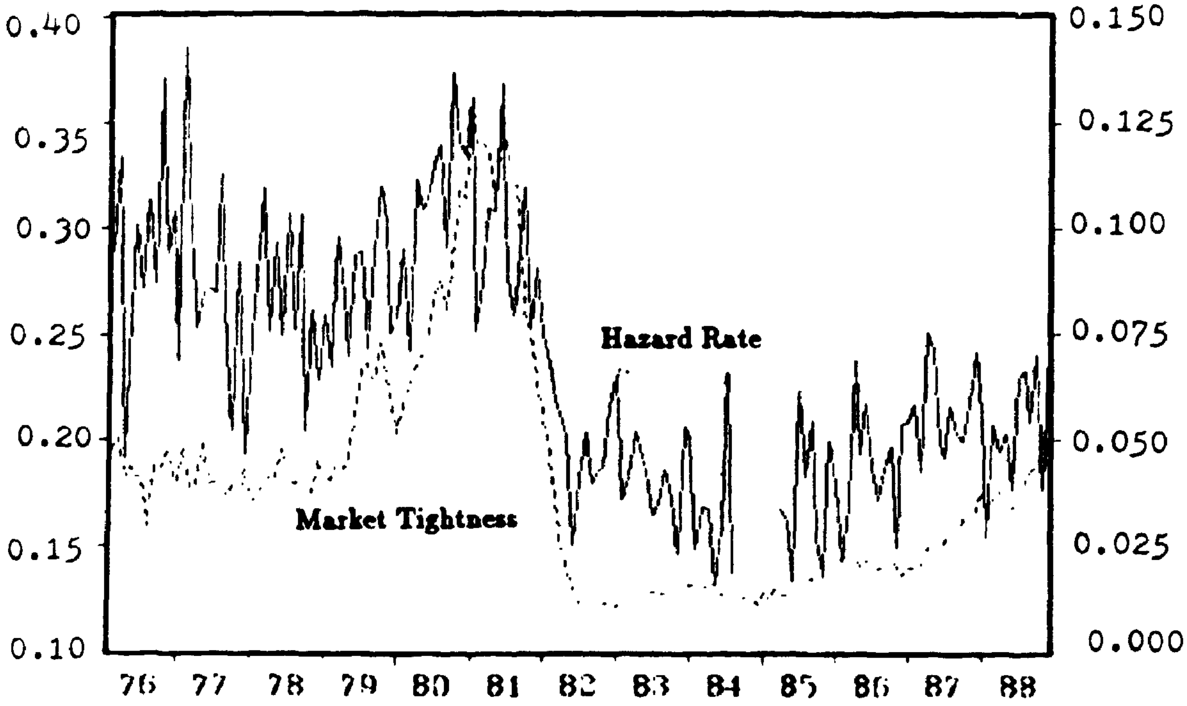

Figure Five: Market Tightness and Employment Hazard Rate: B.C.

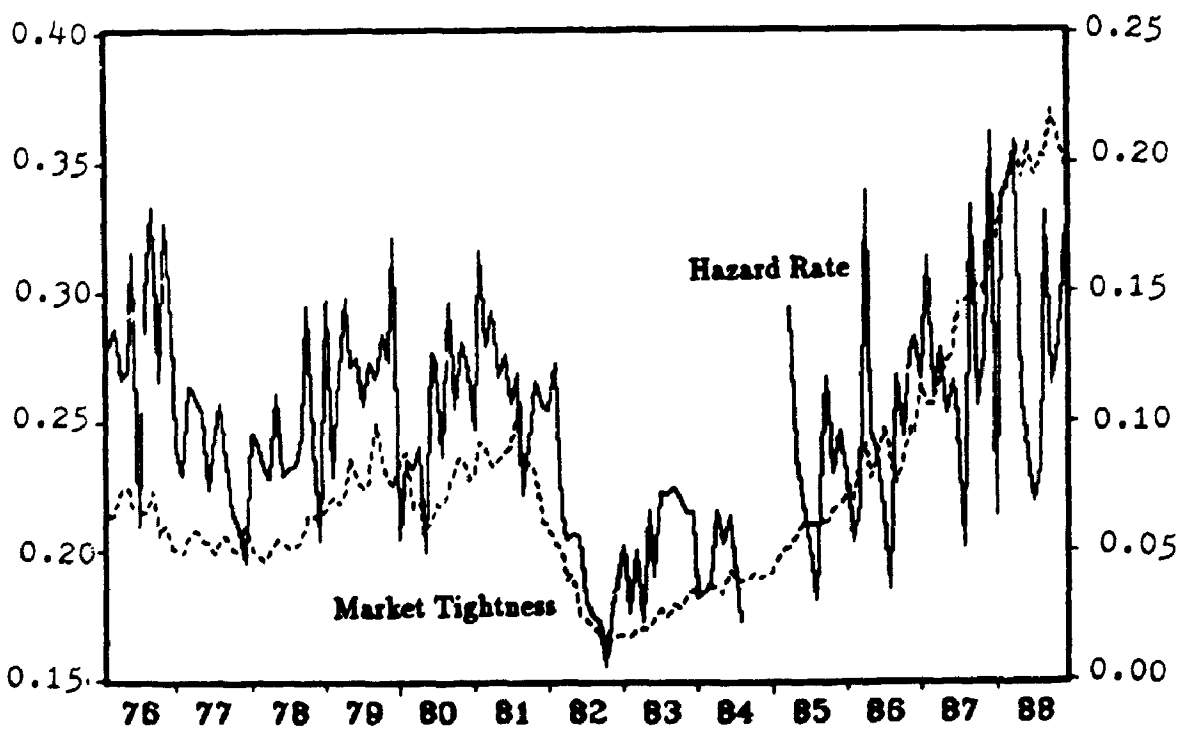

Figure Six: Market Tightness and Employment Hazard Rate: Ontario

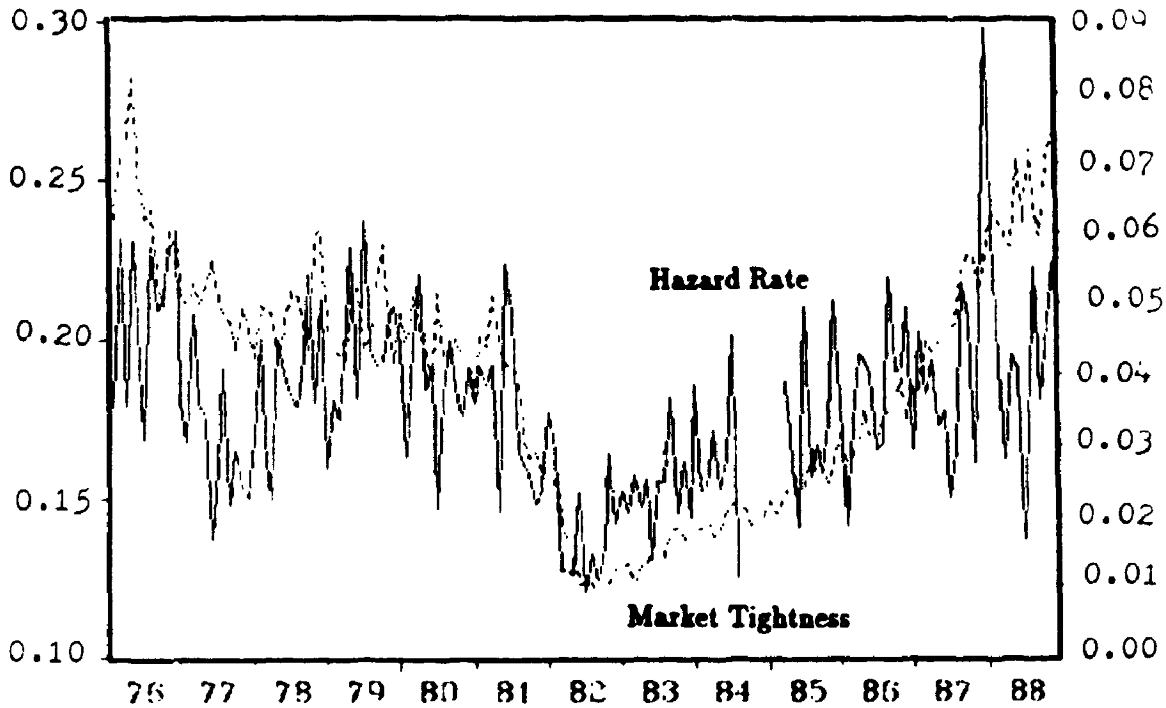

Figure Seven: Market Tightness and Employment Hazard Rate: Quebec

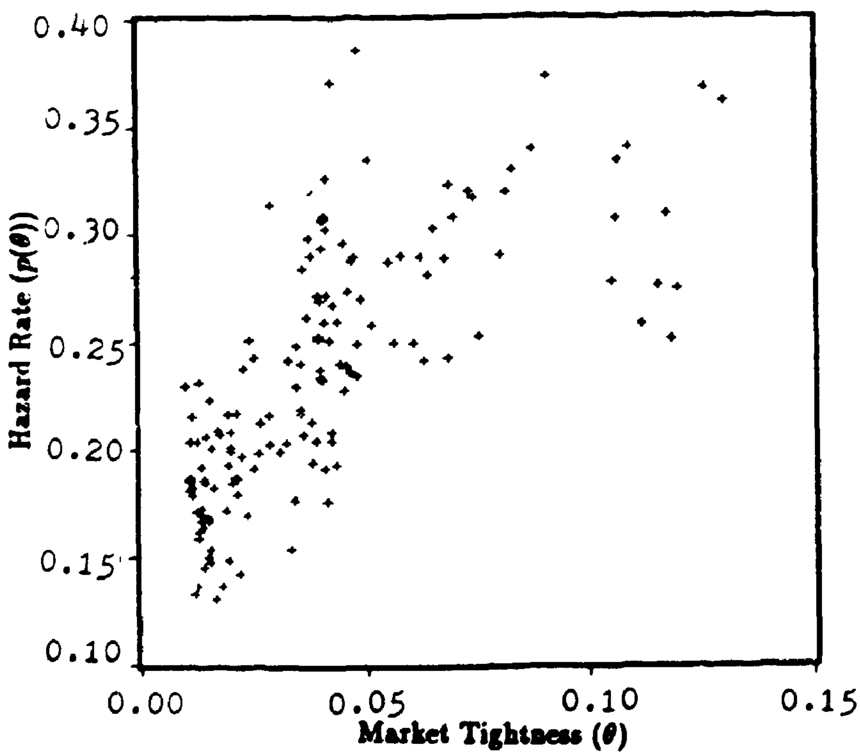

Figure Eight: Hazard-Market Tightness Scattergraph: B.C.

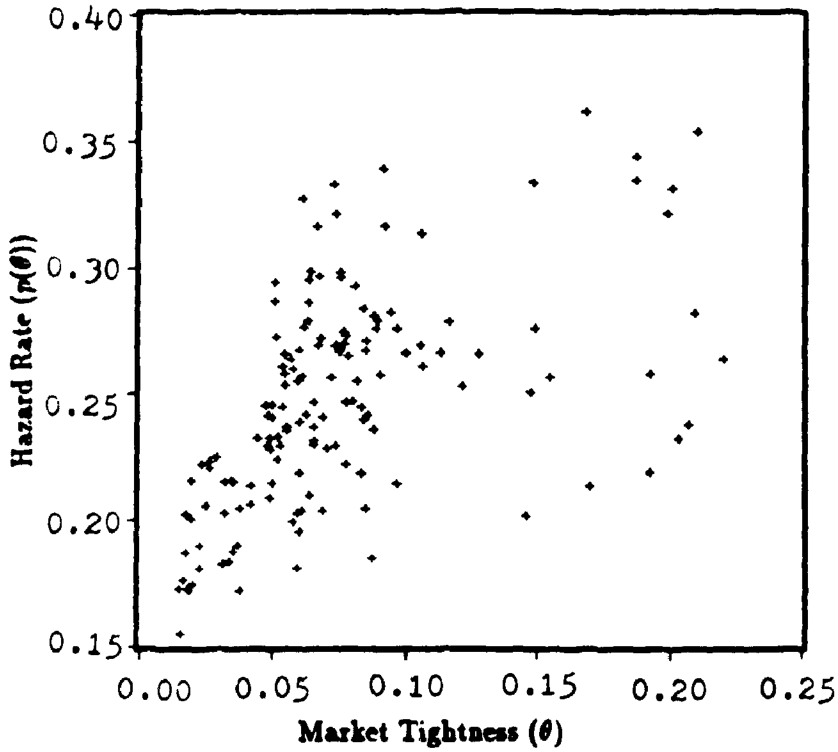

Figure Nine: Hazard-Market Tightness Scattergraph: Ontario

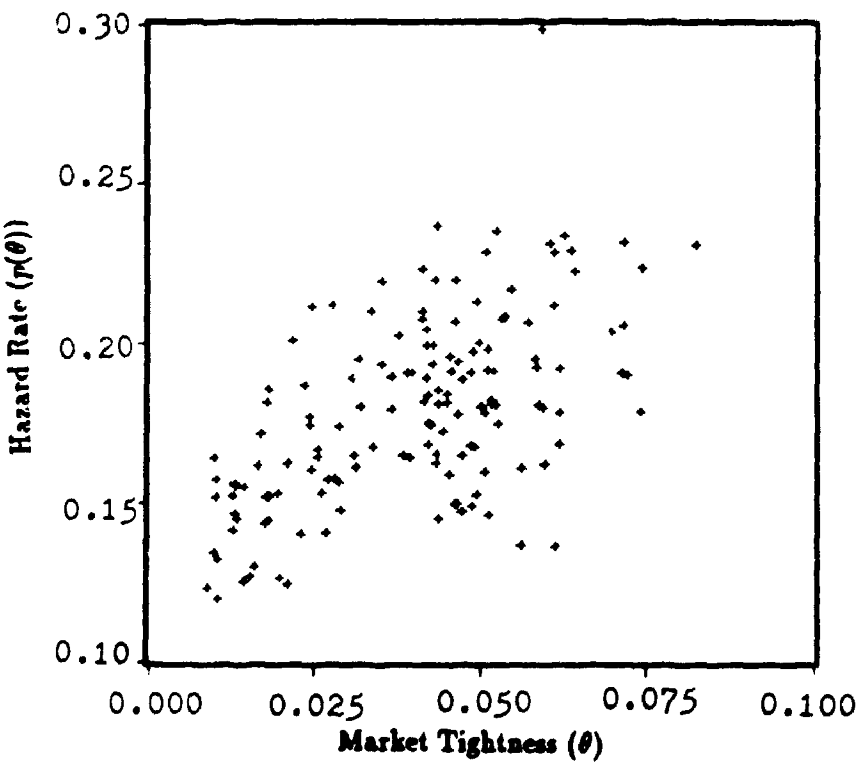

Figure Ten: Hazard-Market Tightness Scattergraph: Quebec

3.5 Testing for Monotonicity

To test the validity of the theoretical restrictions in the matching paradigm a natural approach is suggested by the tradition of imposing inequality constraints in a curve fitting exercise. This is done by first solving a problem of minimising the sum of squared differences between actual and fitted hazard-rate points, where the fitted points are required to fall on a piece-wise linear approximation (i.e. a linear spline function). This is then repeated subject to monotonicity constraints. Tests of the monotonicity condition are obtained by simulating the likelihood ratio statistic to determine a critical value.

To estimate a linear spline, the following regression function is fit:

$$\smile = \alpha + \sum_{i=1}^{M} \beta_i \mathbf{dum}_i + \mathbf{u}.$$

There are M piece-wise linear segments which partition the values of the market tightness variable into intervals $[0, \theta_1), [\theta_1, \theta_2), \ldots, [\theta_{M-1}, \infty)$. In the chapter, ten intervals are allowed and the intervals are chosen so that roughly the same number of data points fall in each interval. The $\mathbf{dum}_i$ variables are designed so that each β_i coefficient gives the slope of the i^{th} segment.[3] Within this framework, negative monotonicity is tested by examining the restriction that none of the β_i are greater than zero.

This investigation proceeds by estimating unrestricted and restricted versions of the regression equation. The unrestricted regression requires no more than the use of OLS techniques. Estimation of the model subject to inequality restrictions requires the use of a quadratic programming method and the constrained least-squares estimation routine BCLSJ from the IMSL library was chosen for the numerical work.

Unrestricted regression results for the three provinces are provided in Tables One, Two and Three. These regression results are further illustrated in Figures Eleven, Twelve, and Thirteen which show the piece-wise linear nature of the fitted curves.

[3] For details of how these dummy variables are constructed see Appendix B to this chapter.

The appropriateness of the spline specifications is confirmed by the R^2 fit measure and the Durbin-Watson diagnostic statistic. The second columns in Tables One through Three are restricted regression results. A test statistic is provided which is calculated as follows:

$$\text{stat} = \frac{SSR_R - SSR_{UR}}{SSR_{UR}/(N-M-1)}.$$

This statistic is analogous to the normal F-statistic for linear restrictions. The critical modification is that the numerator is not divided by the number of restrictions because the number of binding restrictions is unknown. Any of zero through M restrictions could be binding. Thus, rather than the familiar asymptotic χ^2 distribution, the statistic here is a mixture of chi-square variables with degrees of freedom ranging from zero to the number of linear segments. Conditional on the number of binding constraints, the statistic is chi-square with degrees of freedom equal to the number of constraints that bind in the data. Note that the probability that no constraints bind is non-zero with inequality constraints; thus, there is a mass of probability on a value of zero for the statistic.

In order to calculate critical values of the test statistic analytically it is necessary to know the probabilities of having zero through M constraints binding. These probabilities are difficult to compute. Fortunately, it is possible to obtain the distribution of the test statistic by using a numerical simulation. This method involves generating 148 realizations of a random variable with the same distribution as ϵ. Then, dependent variables are obtained for the case where all slope coefficients are zero. This represents the case of a positive monotone function that is on the boundary of the set of such functions. Then, the design matrix of θ variables is used to estimate the restricted and unrestricted spline estimates. This is repeated 500 times to determine critical values for the test statistic described above.

In parentheses under each value of the test statistic, probability values are provided for the likelihood of incorrectly rejecting the hypothesis that the restrictions are true based on the calculated value. These are calculated for the case of $H_o : \beta_i = 0 \ \forall \ i$ (the so-called 'least-favourable configuration'). In other words, it is supposed that the restrictions are "just" true; that is, the borderline case of a monotone non-decreasing relationship does, in fact, hold. The simulation then determines the distribution of the test statistic described above for this least-favourable case. The computed realised value from the spline regressions can then be compared against this simulated distribution.

The test procedure involves the choice of a critical value for the test statistic. The null hypothesis of non-decreasing monotonicity will be rejected if the realised value of the statistic exceeds this critical value. Clearly, this can happen because the null is false or because random variation in the statistic produced a large value, even though the null is true. When doing statistical inference, we wish the latter possibility to be very unlikely. A bound on the probability of making this type I error is provided by the simulations. If the simulated value exceeds the realised value x per cent of the time, then this is the probability of incorrectly rejecting the null hypothesis of non-decreasing monotonicity when the borderline case is true. This yields an upper bound on the case where some of the coefficients are non-zero because, in this case, the distribution of the test statistic will be closer to the origin.

In each of the three cases here, the probability of a Type-I error would be unacceptably high if the null of non-decreasing monotonicity were rejected. The probability ranges from 0.74 for Ontario up to 0.98 for British Columbia. Again, these are lower bounds on the probability because they are calculated under the least-favourable boundary assumption. The inference is therefore that in no case can the null hypothesis of non-decreasing monotonicity be rejected. This provides some objective support for the informal impression that an increasing trend exists in the data. Given this result, a second step can proceed to examine the assumption of concavity given that monotonicity holds.

Table One: Spline Function Regression Results for British Columbia

	Unrestricted	Restricted
Constant	0.30 (0.10)	0.18 –
Slope Coefficient for the Interval:		
0 –0.013	-10.13 (8.37)	0.00 –
0.013–0.018	0.63 (3.71)	0.00 –
0.018–0.025	6.24 (3.26)	5.13 –
0.025–0.032	-1.64 (3.86)	0.00 –
0.033.2–0.036	7.11 (6.59)	5.16 –
0.036–0.040	6.05 (4.53)	6.37 –
0.040–0.046	0.75 (2.47)	0.72 –
0.046–0.080	1.31 (0.52)	1.31 –
0.080+	0.04 (0.46)	0.04 –
R^2	0.63	0.63
DW	2.19	2.17
Stat (P–value)		1.88 (96.0 %)

(Standard errors are in parentheses below parameter estimates)

Table Two: Spline Function Regression Results for Ontario

	Unrestricted	Restricted
Constant	0.13	0.13
	(0.05)	-
Slope Coefficient for the Interval:		
0 -0.025	3.04	3.02
	(2.21)	-
0.025-0.045	0.57	0.60
	(1.03)	-
0.045-0.055	3.40	3.29
	(1.81)	-
0.055-0.065	0.23	0.55
	(1.47)	-
0.065-0.075	2.52	0.83
	(1.43)	-
0.075-0.085	-3.06	0.00
	(1.56)	-
0.085-0.095	3.21	1.05
	(1.70)	-
0.095-0.150	-0.15	0.00
	(0.35)	-
0.150+	0.33	0.28
	(0.32)	-
R^2	0.45	0.44
DW	2.05	1.99
Stat		3.88
(P-value)		(74.0 %)

(Standard errors are in parentheses below parameter estimates)

Table Three: Spline Function Regression Results for Quebec

	Unrestricted	Restricted
Constant	0.12 (0.03)	0.12 –
Slope Coefficient for the Interval:		
0 –0.016	1.99 (2.39)	1.93 –
0.016–0.020	-0.10 (4.37)	0.26 –
0.020–0.024	5.75 (4.05)	4.11 –
0.024–0.029	-2.41 (3.01)	0.00 –
0.029–0.035	5.12 (2.64)	2.60 –
0.035–0.041	0.29 (2.46)	0.00 –
0.041–0.044	-5.49 (4.36)	0.00 –
0.044–0.048	0.76 (2.48)	0.00 –
0.048+	1.20 (0.39)	1.00 –
R^2	0.39	0.38
DW	2.18	2.13
Stat (P–value)		4.08 (78.0 %)

(Standard errors are in parentheses below parameter estimates)

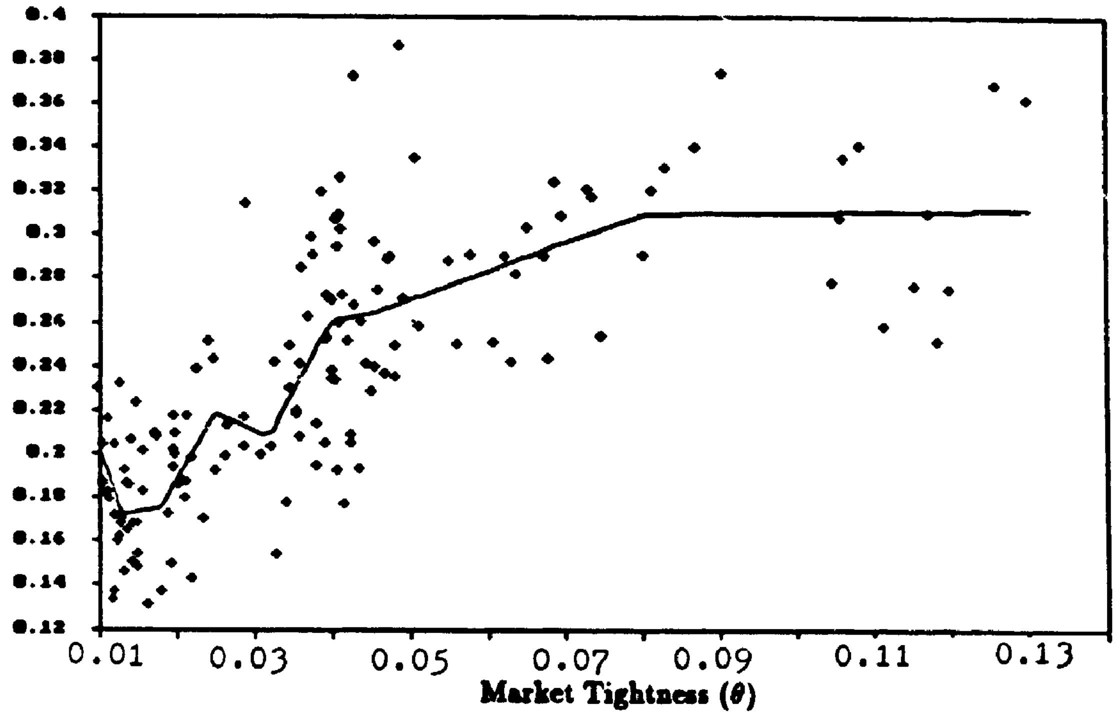

Figure Eleven: Unrestricted Spline Relationship: B.C.

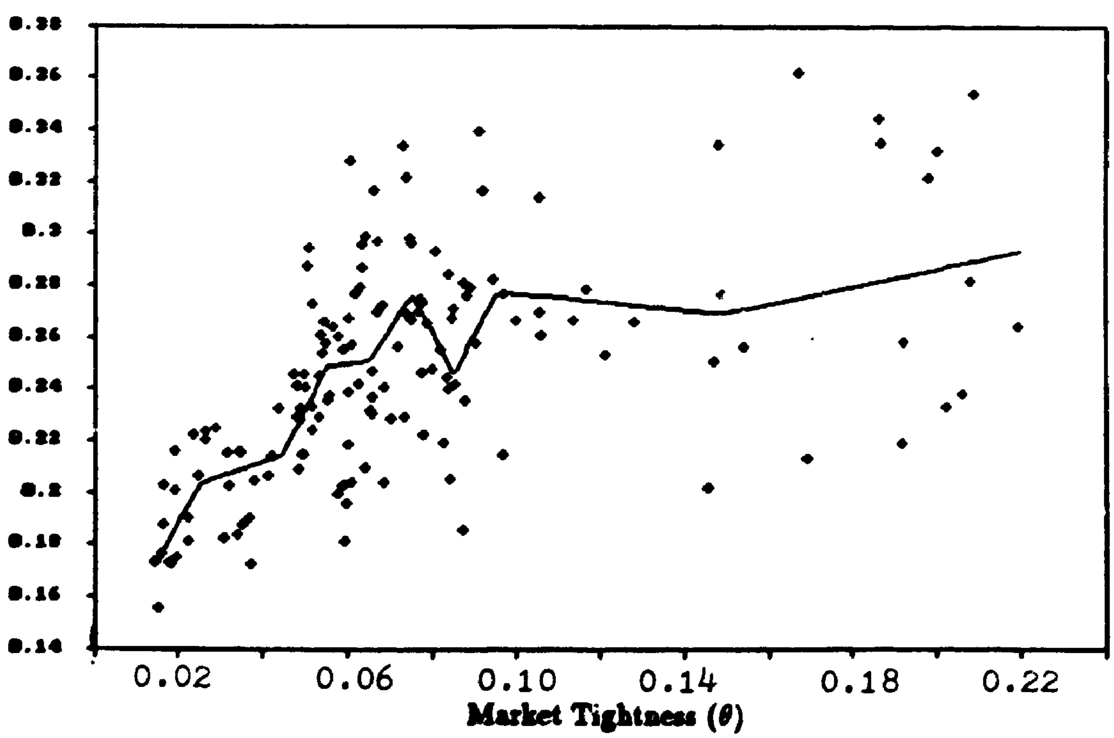

Figure Twelve: Unrestricted Spline Relationship: Ontario

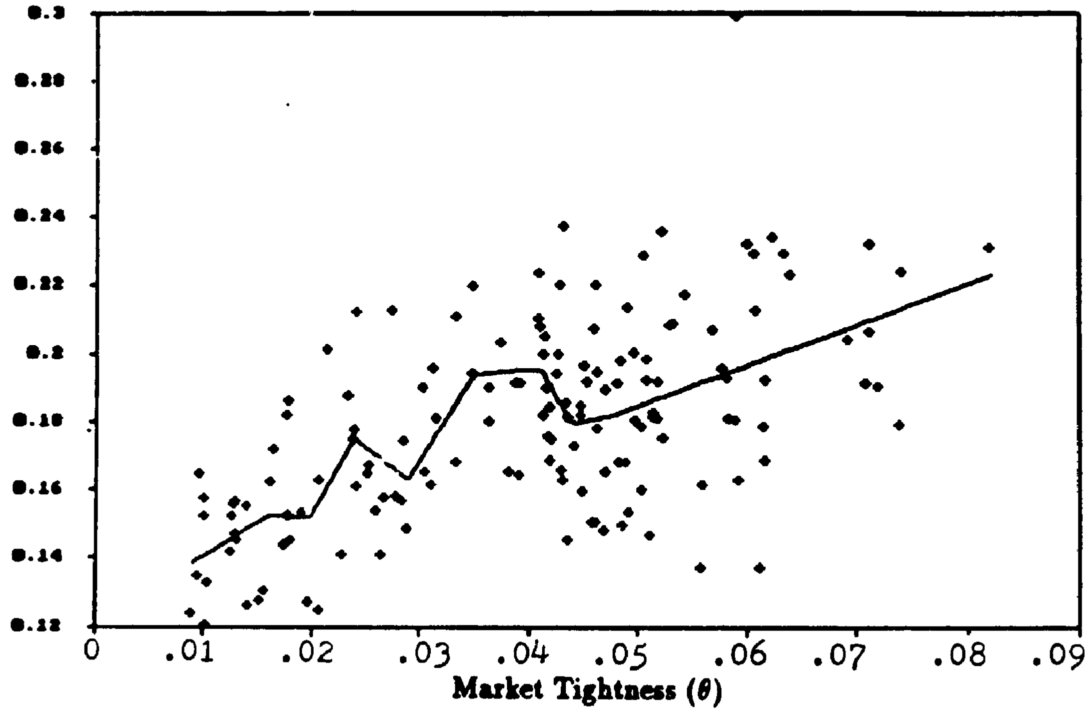

Figure Thirteen: Unrestricted Spline Relationship: Quebec

Preferences are additively separable over time and the period utility function is linear in consumption. This rules out the question of risk pooling since all agents are risk neutral. Only consumption enters the utility function and it is assumed that all current income is consumed since no opportunities for intertemporal exchange are allowed. Given this environment, the objective is equivalent to the maximisation of expected discounted income streams. All agents are assumed to face the same discount factor, β, which is parametric to the model.

Production Technology and Product Market Structure

Production combines labour with an input owned by the capitalists. All workers are identical with regard to productivity and firms in the model are single worker enterprises. At the industry level, there is a downward sloping Marshallian demand schedule for total output. Output at a particular location is characterised by constant marginal product of labour.

Production equals employment in this environment so that market and technological factors can be summarised by a single average-revenue-product (y_t) function:

$$y_t = F(N_t, \mu_t) \tag{3-1}$$

where: N_t is a total employment,

μ_t is a demand shock.

Assume:

ASSUMPTION 3.1 *F is increasing in μ and decreasing in N. Furthermore, F is concave in N.*

The term μ_t may reflect relative demand shocks in world markets for the sector-specific goods. These shocks need not be independent across sectors. With a slight modification, the shocks could be due to changes in sectoral productivity. Prices are expressed in terms of some numeraire rather than either sectoral good.

Table Four: Test Results with Concavity Imposed

Province	Statistic	P-value
B.C.	0.163	0.00
Ontario	0.106	0.00
Quebec	0.101	0.00

The meaning of these results is that it is possible to reject the additional assumption of global concavity. Clearly, enough non-concave regions exist to suggest that the underlying relationship is not concave. This means that the theory fails on one level. In another way, however, the theory is a success since the graphs suggest a departure from linearity in favour of concavity. This is confirmed by the following simple regression of the hazard rates on a quadratic in market tightness:

Table Five: Test Results with Concavity Imposed

(Standard Errors in Parentheses)

Province	Constant	θ	θ^2
B.C.	0.131 (0.007)	3.44 (0.28)	-17.28 (2.28)
Ontario	0.164 (0.008)	1.35 (0.16)	-3.91 (0.62)
Quebec	0.128 (0.008)	2.32 (0.53)	-18.73 (8.17)

These results show that it is possible to reject the assumption of a constant slope in the relationship between market tightness and the hazard rate for flows into employment. This means that the relationship is not linear and, in the simple quadratic example given here, the curve is concave. The isotonic results in Table Four suggest that a model exists with sufficiently many changes in curvature to lead us to reject the constant concavity assumption.

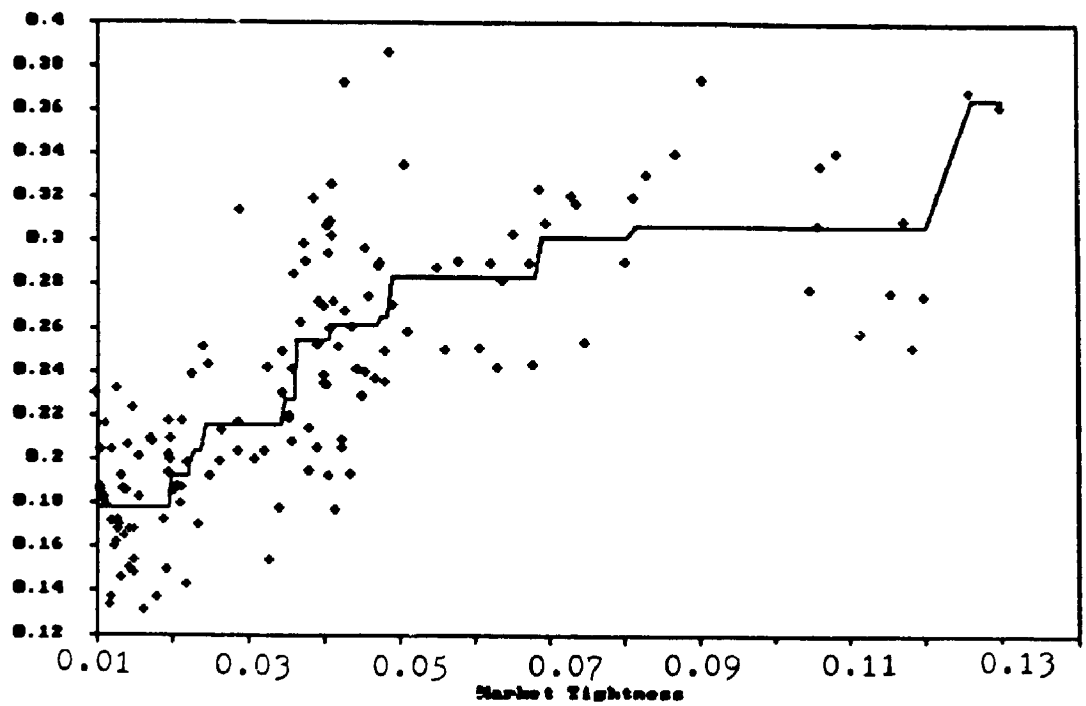

Figure Fourteen: Fitted Line with Monotonicity: B.C.

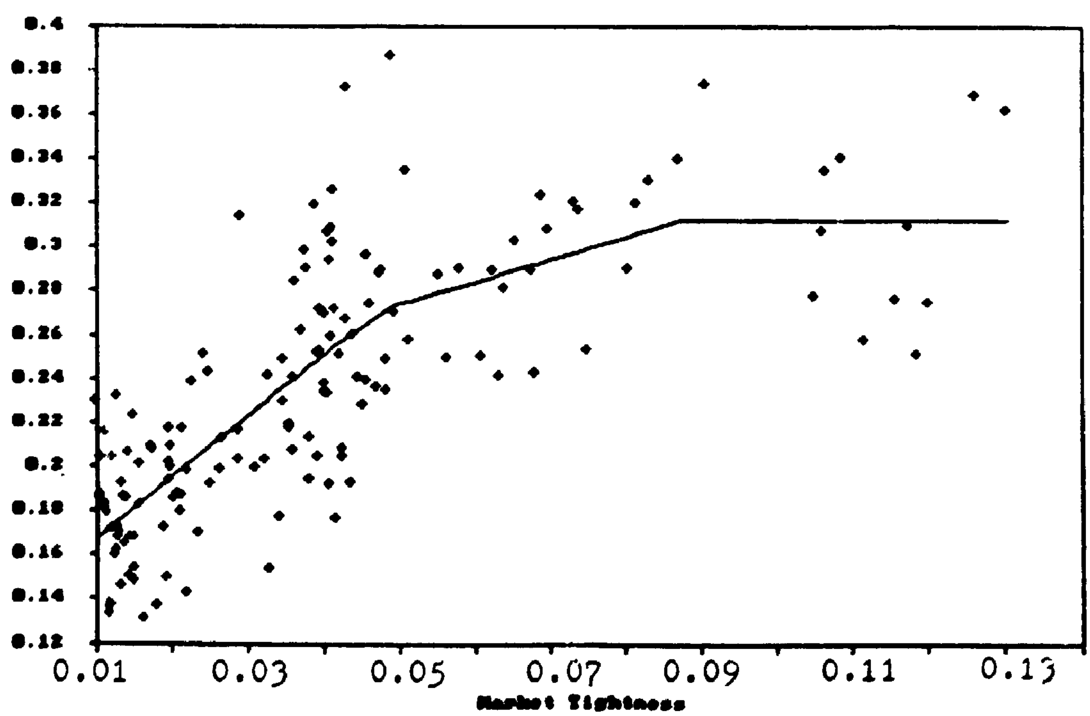

Figure Fifteen: Fitted Line with Concavity: B.C.

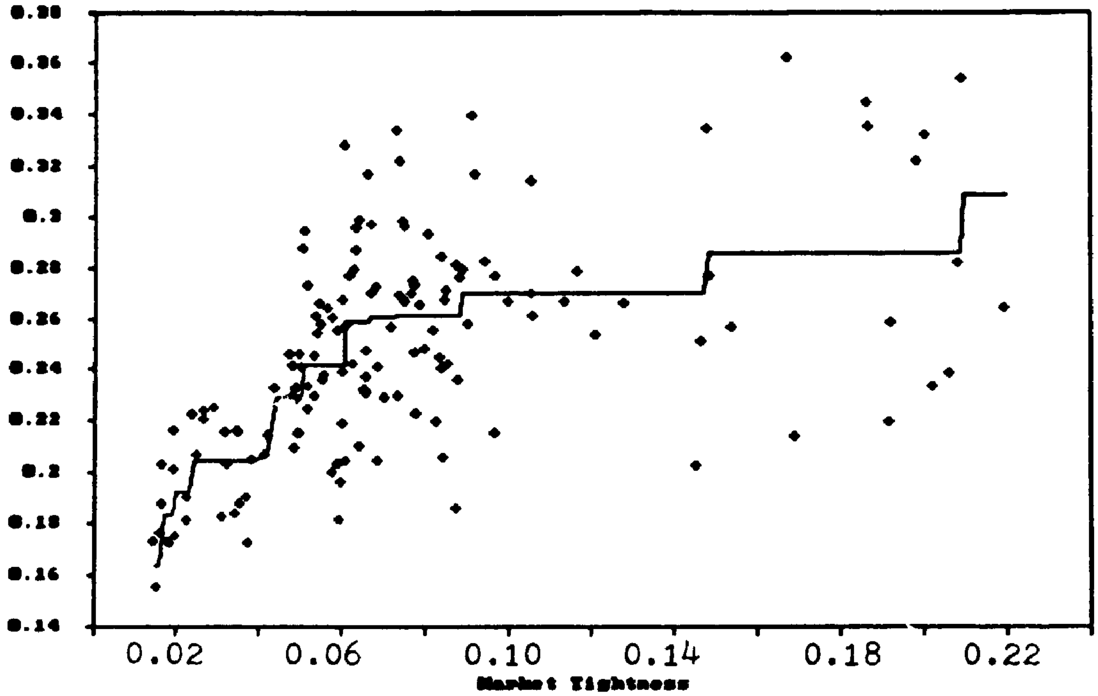

Figure Sixteen: Fitted Line with Monotonicity: Ontario

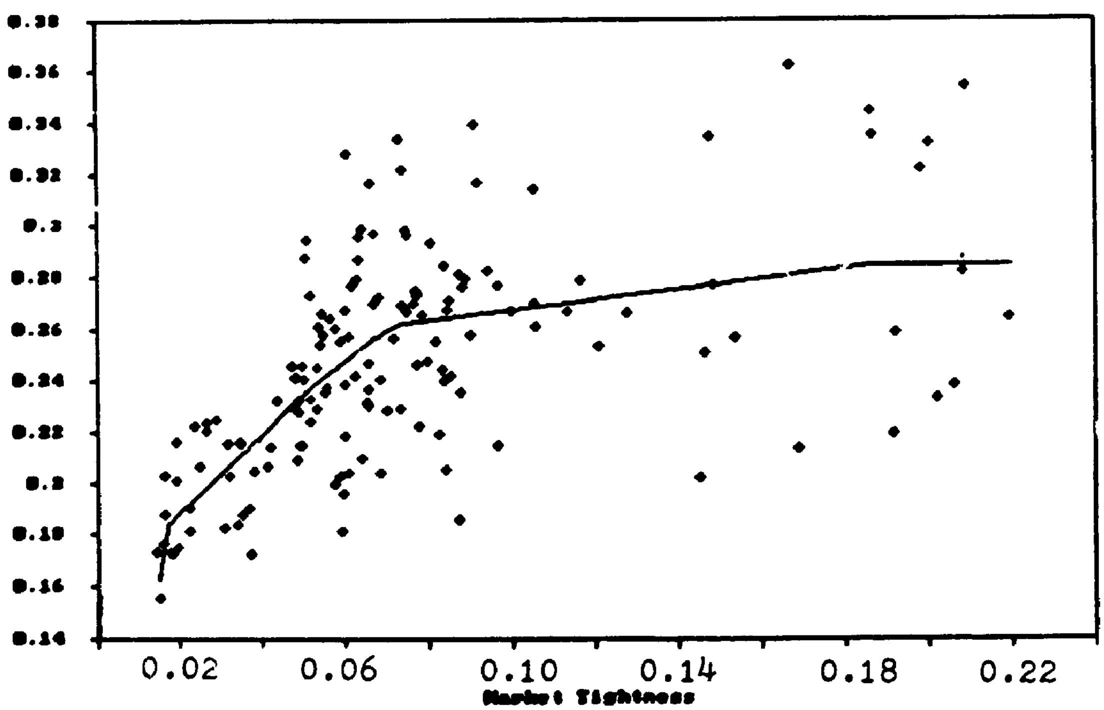

Figure Seventeen: Fitted Line with Concavity: Ontario

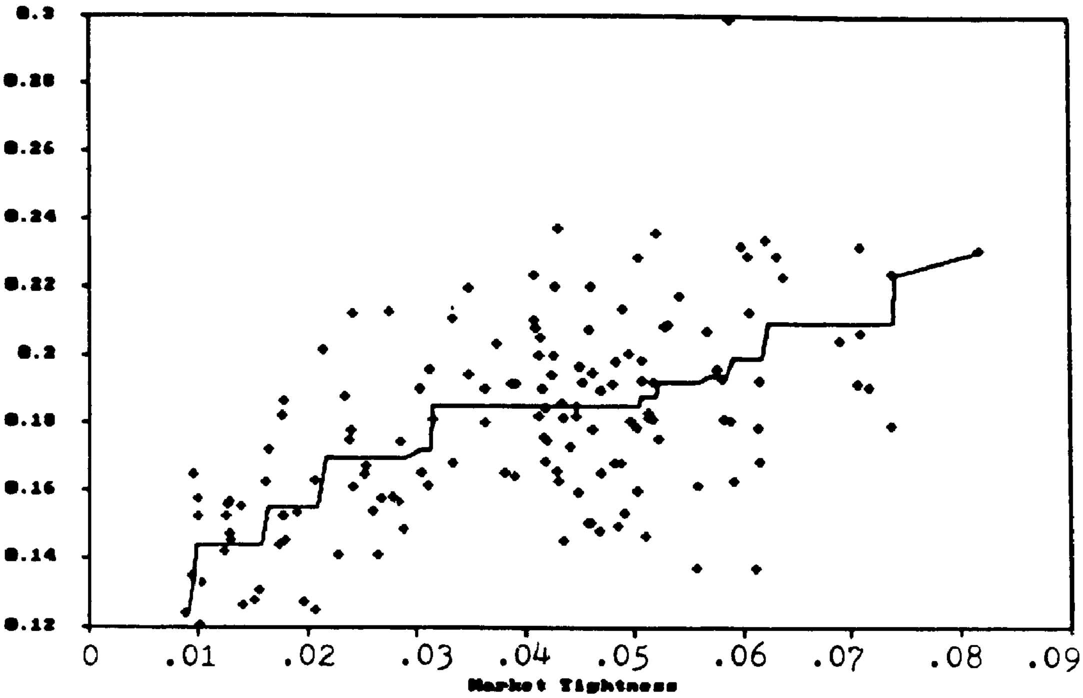

Figure Eighteen: Fitted Line with Monotonicity: Quebec

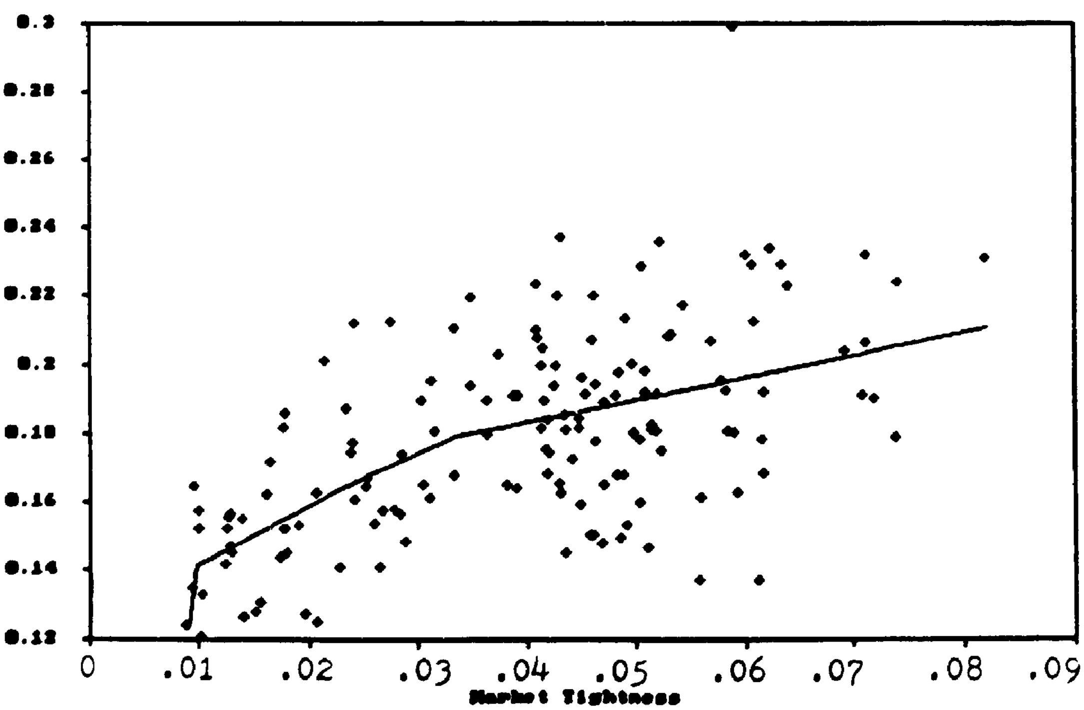

Figure Nineteen: Fitted Line with Concavity: Quebec

3.7 Pooled Regressions

This study has used a normalized Help Wanted Index series in the construction of the market tightness variable θ. For the analysis thus far, this has no importance because a scaling factor will not affect the test for non-increasing monotonicity or concavity so long as all the data have been subjected to the same re-scaling. The virtue of the normalization is that it allows for comparisons between values of market tightness in different provinces. In particular, it is possible to pool observations over time and across provinces to examine the issue of whether a stable matching relationship holds across provinces as well as over time.

In this section, the steps conducted above for each province are repeated for the combined data. A note of caution is sounded in advance: measurement error is potentially a problem if the normalization used here is not correct. The possibility that this may be the case is suggested by the finding (evidenced by the low correlation coefficient above) that the Vacancy series and Help Wanted Index series diverged significantly in Quebec. Nevertheless, subject to this caveat, the pooled analysis has some value.

Figure Twenty shows a scatter diagram for market tightness and the transition rate to employment using 444 pooled time-series/cross-section observations. As before, a general increasing pattern with a possibly declining slope is revealed. The sign of the relationship is tested by using a spline regression. The results of this method are presented in Table Six and the fitted relationship from the unrestricted spline regression is shown in Figure Twenty-One.

The probability value for this test strongly suggests that the sign restrictions will hold. Rather than continue with concavity tests, it seems preferable to leave this for future work which will investigate functions of both the vacancy and unemployment series to exploit the fact that observations are plentiful in the pooled regressions. Given the results above, it seems likely that the pooled regressions will reject global concavity while still favouring concavity over linearity.

Table Six: Pooled Spline Function Regression Results

	Unrestricted	Restricted
Constant	0.143 (0.01)	0.13 –
Slope Coefficient for the Interval:		
0 -0.025	2.15 (0.72)	2.22 –
0.025–0.045	1.58 (0.46)	1.44 –
0.045–0.055	-0.58 (1.04)	0.00 –
0.055–0.065	3.10 (1.32)	2.62 –
0.065–0.075	1.69 (1.45)	1.68 –
0.075–0.085	-0.36 (1.67)	0.00 –
0.085–0.95	2.96 (1.84)	2.22 –
0.095–0.150	-0.21 (0.42)	0.00 –
0.150+	0.12 (0.39)	0.00 –
R^2	0.43	0.43
DW	1.97	1.97
Stat (P–value)		0.59 % (99.6 %)

(Standard errors are in parentheses below parameter estimates)

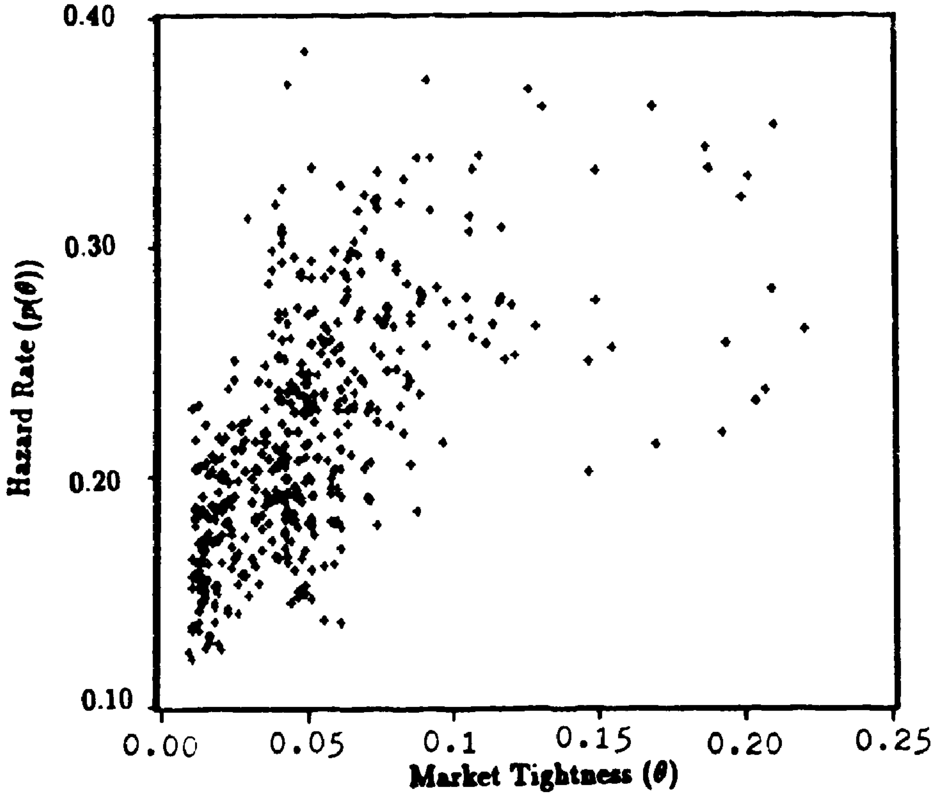

Figure Twenty: Hazard-Market Tightness Scatter-Graph: Pooled Data

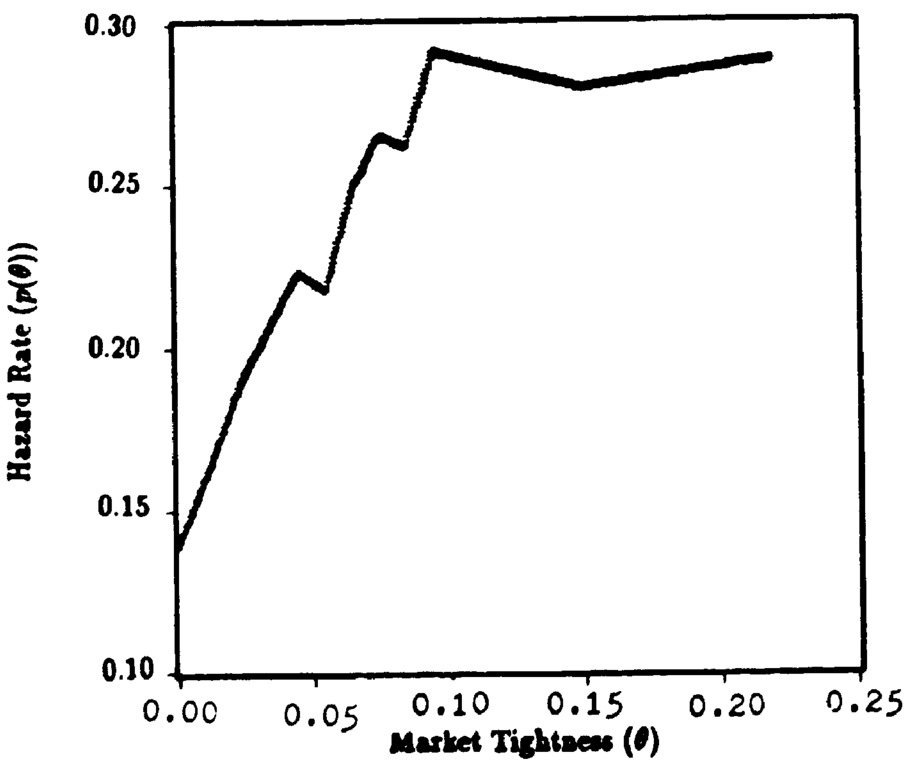

Figure Twenty-One: Unrestricted Spline Relationship: Pooled Data

3.8 Conclusions

The general conclusion of this chapter is that the matching function approach imposes restrictions that are reasonably consistent with observed empirical regularities. Monotonicity assumptions were accepted by the data, in a piece-wise context, but concavity was rejected in the more demanding context of concavity at all observed points. However, the data strongly suggest that the relationship is more nearly concave than linear. The fit of the spline models was reasonably close, and the regressions typically accounted for just under a half of the variation of the hazard series about their mean. This is interesting given the amount of high-frequency noise in the hazard series, much of which Blanchard and Diamond (1990) attributed to measurement error. The pooled regressions support these results for individual provinces.

Of these monotonicity and concavity results, the first is less interesting because a number of theories suggest a relationship between market tightness and hazard rates. Perhaps only equilibrium models of unemployment which ignore recruitment activity (such as lottery or optimal leisure models) are not at ease with this finding. The concavity result is more interesting because it is a more precise implication and is thus more readily falsifiable and less amenable to other theories. Concavity results in an environment with costly search where it is assumed that physical search activity has diminishing marginal returns to activity on either side of the market. This may be contrasted with an extreme "job queue" model in which the unemployed simply wait their turn for a job. Here, a doubling of the number of vacancies on the short side of the market would simply double. This suggests that search activity is of some empirical relevance in labour markets.

A simple queuing model can be expressed in terms of the following matching technology:

$$x_t = \min(U_t, O_t).$$

In other words, the short side of the market always gets matched and those on the other side of the market are rationed. If the assumption is made that there are always more unemployed workers than vacancies, as the observed θ values suggest, then the ratio of matches to unemployed is just U/O i.e. $p(\theta) = \theta$. It is clear that this implies a linear relationship between hazard rates and market tightness which is not found in the data.

Of course, a strict concave relationship was also not found in the data. This may reflect the fact that the theory is only approximately true or be due to the fact that testing was done under the 'least-favourable configuration', as is necessary when testing inequality constraints. This method can serve to show when a restriction is not falsifiable for a chosen test size but cannot definitively reject an hypothesis. The message from this chapter is that the matching paradigm has empirical virtues as well as theoretical. Given this, future work should continue to elaborate on the state of knowledge regarding the features of such models.

Chapter Four

Estimating a Markov Switching Regression Model of Recruitment Intensity and Market Tightness with Sectoral Shocks

4.1 Introduction

An important focus of debate in macroeconomics is the contribution of sectoral or regional shocks to aggregate unemployment rate fluctuations. The debate was born when Lilien (1982) advanced the hypothesis that fluctuations in the pace of labour reallocation can produce a time-varying rate of frictional unemployment. Lilien supported this argument with empirical results intended to show that the magnitude of these fluctuations in the "natural rate" of unemployment is large relative to observed cyclical variation in unemployment. In fact, the bulk of recent fluctuations in the rate of unemployment are attributed to frictional unemployment in this empirical analysis.

Some authors have subsequently criticised this work on several fronts. First, it has been argued that the sectoral shock measure used by Lilien can also measure aggregate shocks. Abraham and Katz (1986) describe two plausible scenarios where this is true. Secondly, Abraham and Katz have also argued that the empirical relationship between vacancies and unemployment ("the Beveridge curve") would be positive if Lilien-type sectoral shocks are affecting the economy but that this is not consistent with an observed negative slope. Finally, Blanchard and Diamond (1989) use the Abraham and Katz dichotomy based on the slope of the Beveridge curve to identify sectoral shocks and aggregate shocks in U.S. data. The authors conclude that aggregate shocks account for the bulk of fluctuations.

Several responses to these criticisms are available. First, authors such as Samson (1985) have worked with adjusted measures of sectoral reallocation that are designed to be orthogonal to observed measures of aggregate shocks. Samson found evidence of large sectoral shock effects although Neelin (1987) did not. Next, models can be constructed in which sectoral shocks affect aggregate unemployment even in the absence of large inter-sectoral flows (see Rogerson (1990) or Chapter One of this thesis). Finally, models such as Hosios (1991) or Andolfatto (1991) reveal that the Beveridge curve may have a negative slope even if sectoral shocks dominate in the data. This last point is particularly relevant because it also tends to negate the Blanchard and Diamond work which uses the slope of the Beveridge curve to uncover underlying shocks.

Given this work, the current state of thinking on the sectoral shocks question is rightly viewed as a debate in which neither side is able to formulate a definitive argument. Further empirical work is thus needed to increase understanding of this important question. It is precisely such work which is presented in this chapter. An estimation procedure is outlined which can assess the importance of sectoral and regional shocks in the determination of unemployment duration. The duration of unemployment is a key variable of interest because the analysis in Chapter One identified endogenous unemployment duration as an avenue by which sectoral shocks can affect aggregate unemployment even in the absence of labour flows.

Briefly, the strategy in the chapter is as follows. An example of a useful data set is provided and a procedure for applying the Hamilton (1989) method to extract information from these data is outlined. This procedure requires some economic structure so a theoretical model is then developed which relates unemployment duration to recruitment intensity and the mix of sectoral and aggregate shocks. This model provides regime-dependent equations which determine the optimal recruitment intensity. The empirical part of the chapter applies the Hamilton technique to identify sectoral and aggregate shock regimes and estimate regime-dependent recruitment intensity rules. The chapter ends with a discussion of the results and a consideration of promising future avenues of research.

4.2 A Data Set and Estimation Method

An interesting feature of unemployment duration time series in Canada is a marked divergence in fortunes between Ontario and Alberta. As Figure One below shows, the late 1970's and early 1980's saw a much shorter average duration in Alberta than in Ontario. While unemployment duration rose in both provinces in 1982, this rise was far more pronounced in Alberta. During the post-1982 recovery, unemployment duration fell rather rapidly in Ontario but showed some persistence in Alberta. As a result, the late 1980's were characterised by a reversal of provincial fortunes. It is only in 1990 that there is an apparent equalisation of unemployment duration between the two provinces. This pattern strongly resembles the timing and direction of oil price shocks which occurred during the 1970's and 1980's. Oil price rises in 1973 and 1979 worked in the favour of Alberta relative to Ontario while the price collapse in 1986 had the opposite effect.

Resource price shocks are the most obvious sources of sectoral shocks during this period and it seems that this graph reveals the potential influence of sectoral shocks on unemployment. Extended to the Canadian economy as a whole, this episode suggests that fortunes within the country might be well-represented by two states: one in which the resource-rich provinces do well and the manufacturing provinces do poorly and the other in which these positions are reversed. There are several reasons why such a characterisation would be relevant if true. First, if oil-price shocks affect unemployment duration then this provides information about the process governing duration. Secondly, in a time of constitutional debate, it is interesting to see how political union can allow for diversification of regional risks. Finally, even work which is unfavorable to the pure sectoral shocks explanation for aggregate unemployment, such as the work of Neelin(1987), does allow that regional shocks may have a significant effect on unemployment that does not proxy for aggregate shocks.

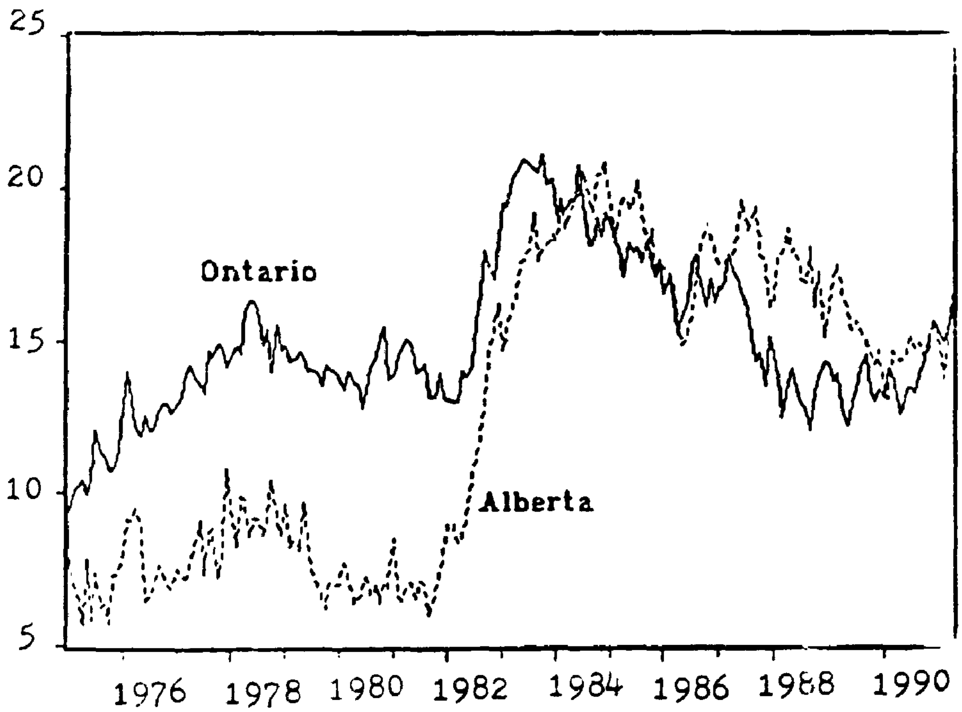

Figure One: Average Duration of Unemployment

To extract information from these data, an econometric method developed by Hamilton(1989) is adapted. The Hamilton algorithm can be applied to a model in which there is an unobserved state variable which can take on values from within a small set of discrete values. In particular, Hamilton assumes that the state variable evolves over time according to a Markov process. Recently, this method has been used to estimate the parameters of a process for real GNP (Hamilton, 1989), real interest rates (Garcia and Perron, 1990) and real dividends and real consumption (Bonomo and Garcia, 1990). Here it is proposed that the method be applied to recruitment intensity data which underlie the unemployment duration data in Figure One.

The method basically applies the switching-regression maximum likelihood methods of Goldfeld and Quandt and applies them to a situation in which a regime indicator variable follows a Markov process. Unlike traditional methods, this approach does not assume that all states are permanent. One output from the estimation procedure is the transition matrix for the Markov process. Examination of the transition probabilities allows inference regarding the frequency of switches between states. This analysis is useful because multi-sector search behaviour is much less difficult to analyze in a restricted version of the model in which states change with only very low probability. In addition to estimating the parameters of the process, estimates of the probability that the regime is in either of the states are also obtained. This will allow for a comparison of regime changes in the model with those known to have occurred in resource markets. If the model finds that the most likely shift points between regimes coincide with prior information, then support for the model grows.

4.3 The Environment of the Model

This analysis of a two-sector economy ignores migration as a variable of interest so that, while labour market participants have to engage in search to find a match, labour and capital are unable to leave their own sector. In effect, each sector is analysed in isolation so that only a single sector theoretical model need be constructed. This structure is obviously counter-factual and ignores fairly complex adjustment dynamics due to migration; however ignoring migration yields much simplicity and it can be argued that the costs are second order in magnitude. For example, Baldwin and Gorecki (1990) suggest that sectoral flows in Canada are not quantitatively important. Next, authors such as Murphy and Topel (1987) and Blanchard and Fischer (1989) have claimed that inter-sectoral flows do not have the cyclical properties dictated by the sectoral shifts paradigm. Recall that this chapter seeks to respond to this by considering the possibility that sectoral shocks can affect aggregate unemployment even if flows are zero and a model without migration is entirely appropriate to this task.

This chapter examines the properties of a simplified environment in which labour market participants have to engage in search to find a match. At each date t, there is a constant aggregate labour force of size L which is allocated endogenously between those working (N_t) and those searching for employment (U_t). Working and searching are mutually exclusive activities. Search is two-sided in this economy since firms must search for workers in order to commence production. A position in search of a worker is called a 'vacancy' or 'posted offer' and is analogous to an unemployed worker. The number of vacancies at time t is denoted by O_t. No upper limit is placed on the number of potential vacancies, in contrast to unemployment which is limited by the size of the labour force. Firms in the model make decisions about whether to search for a worker or whether a producing firm should be closed. Workers have no decision to make since they either work or search depending on their exogenously determined state.

The model accords fairly well with agricultural or resource economies in which products can only be grown in certain regions. To fix ideas, output in the model may be thought of as the fruit of a tree so that the capitalists are owners of the trees and the land they grow upon. The workers are then paid to pick the fruit of the trees. Search arises naturally in this environment since workers have to find a tree which is in need of husbandry. Search costs for capitalists could reflect the fact that effort is expended to attract workers or the fact that a tree yields no recreational value (i.e. shade) if it is prepared for tending.

Preferences

All agents in the model are assumed to maximise expected discounted utility.

Preferences are additively separable over time and the period utility function is linear in consumption. This rules out the question of risk pooling since all agents are risk neutral. Only consumption enters the utility function and it is assumed that all current income is consumed since no opportunities for intertemporal exchange are allowed. Given this environment, the objective is equivalent to the maximisation of expected discounted income streams. All agents are assumed to face the same discount factor, β, which is parametric to the model.

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Production combines labour with an input owned by the capitalists. All workers are identical with regard to productivity and firms in the model are single worker enterprises. At the industry level, there is a downward sloping Marshallian demand schedule for total output. Output at a particular location is characterised by constant marginal product of labour.

Production equals employment in this environment so that market and technological factors can be summarised by a single average-revenue-product (y_t) function:

$$y_t = F(N_t, \mu_t) \tag{3-1}$$

where: N_t is a total employment,

μ_t is a demand shock.

Assume:

ASSUMPTION 3.1 *F is increasing in μ and decreasing in N. Furthermore, F is concave in N.*

The term μ_t may reflect relative demand shocks in world markets for the sector-specific goods. These shocks need not be independent across sectors. With a slight modification, the shocks could be due to changes in sectoral productivity. Prices are expressed in terms of some numeraire rather than either sectoral good.

Search Technology

To this point, the algebra of the model is fairly conventional. The remainder of this section introduces an important departure from the standard microeconomic paradigm. Instead of frictionless transacting between labour market participants, a model of real-time search is considered. The search process is modelled in a discrete time framework to make explicit the passage of time and the sequence of events within the model. The model assumes that there is a basic unit of work and search, which may be a day, week or month depending on the context. It seems most natural to begin with a view that one period equals one day.

At the beginning of each day, each worker is assumed to know whether she has a job for that day. Similarly, capitalists know whether a worker will be reporting to their plot or if a vacancy must be posted to locate a labourer. All agents are also aware of current labour force and employment conditions as well as the current state of the shock term μ_1. Given this information, two actions can be taken before work or search activities commence. First, vacancies may be created or withdrawn by owners of capital. Also, some firms that are known to have a match with a worker could decide that the shadow value of the capital exceeds the return to producing and some matched workers could quit. These latter two possibilities will not be analyzed since it will be assumed that it is always worthwhile to remain matched.

The decisions made at dawn are acted upon before production or search begins. Sufficient time is available to post vacancies before 7:00 am. During the day proper, successfully matched pairs produce fruit while unemployed workers and owners of vacant trees engage in search activity. By the end of the day, it is known which searchers have succeeded in contacting partners; however, it is also true that some pairs that were previously matched will discover that they have suffered an exogenous separation shock.

The sequence of events in the economy is conveniently summarised in the following 'daily agenda' for a day labelled t:

5:00 am All agents know if they are matched or not for the current day. The value of the stochastic shock terms μ_t is also revealed at this point. Given this information, unmatched capitalists may post or withdraw job offers.

7:00 am Matched pairs begin producing. Unmatched agents search for a match.

5:00 pm Productive activities cease. Matched pairs learn if they have suffered a separation. The outcome of search activity is revealed.

Flow costs/benefits arise in the model when in the searching state. Unemployment benefits for searchers are denoted by b and are net of the direct costs of searching. A positive benefit may capture a pecuniary-equivalent value of searching rather than working. Capitalists must pay a per-period cost of γ which could represent either the cost of effort expended to attract a worker or the foregone shadow value of the capital resulting from its preparation for use.

The probability that any searching worker receives a job offer is assumed to be some function of θ_t, the ratio of the number of vacant firms (O_t) to the number of searching workers (U_t) (i.e. $\theta_t = O_t/U_t$). This relationship is derived from a function that gives the total number of matches (x_t) as a function of O_t and U_t:

$$x_t = X(U_t, O_t). \tag{3-2}$$

Such a 'matching function' figures prominently in recent work such as Pissarides (1987) and Blanchard and Diamond (1989) which have analysed labour market dynamics. Like a production function or utility function, the matching function *represents* a more fundamental feature of the economic environment. Using such a function is more expedient than modelling the underlying physical facts of the search process. The following assumptions are generally made concerning the matching function:

ASSUMPTION 3.2 *$X(U_t, O_t)$ is increasing in both U_t and O_t and is concave.*

ASSUMPTION 3.3 *$X(U_t, O_t)$ is homogeneous of degree one.*

Much ease of exposition is obtained by Assumption 3.3 since it implies that the number of matches per searching worker can be written as a function of a single variable, θ_t:

$$x_t/U_t = X(U_t, O_t)/U_t = X(1, \theta_t).$$

The probability that any given worker is matched is precisely this ratio of matches to searching workers. This probability will be denoted by the function $p(\theta_t) = X(1, \theta_t)$. The matching function is increasing in both of its arguments by Assumption 3.2 and this means that the matching probability for a worker rises with θ. Intuitively, θ is a measure of the 'tightness' of a labour market and high values of θ indicate that jobs are relatively abundant in a sector.

Of course, the matching function also determines the number of vacancies that are matched in each period. The ratio of matches to vacancies is:

$$x_t/O_t = X(U_t, O_t)/O_t = X(1/\theta_t, 1).$$

This ratio, which also depends on θ gives the probability that a vacancy is matched, denoted $q(\theta_t)$. Assumption 3.2 implies that $q'(\theta) < 0$.

In order to conduct the analysis, the following additional assumptions are necessary:

ASSUMPTION 3.4 *$p(\theta)$ is strictly increasing in θ for $i = 1,2$ and* $0 < p(\theta) < 1 \;\; \forall \;\; 0 < \theta < \infty$.

ASSUMPTION 3.5 *$q(\theta)$ is strictly decreasing in θ for $i = 1,2$ and* $0 < q(\theta) < 1 \;\; \forall \;\; \infty > \theta > 0$.

These assumptions mean that no matter how unbalanced an economy becomes, no worker or firm is ever matched with certainty. This makes clear the fact that employment comes as the outcome of search rather than a rationing process where one side of a market can be fully accommodated. These two assumptions are needed to ensure that the steady state of the economy is well defined.

The final detail to be specified in the economy is the process by which workers and firms suffer involuntary separations:

ASSUMPTION 3.6 *In each sector, a match dissolves with probability s per period.*

This assumption may seem extreme but it allows for the analysis to highlight changes in rates of flow out of unemployment. In terms of the model, it could reflect the effects of localised disasters which render certain locations permanently non-productive.[1] This assumption is necessary because with a constant group of infinitely lived agents there would be zero unemployment in the limit if no separations occur. Growth of the economy-wide labour force through new entrants to unemployment is ruled out because this leads to a declining steady state y_i.

Rewards for Achieving a Match

In a search model like this, a surplus generally accrues to agents who succeed in finding one another. Those who are fortunate enough to be matched can do no worse than receive the return available from outside pursuits since they can always decide to pursue this alternative occupation. In fact, the matched pairs will likely earn more than their outside opportunity because the search process presents a barrier to entry. Given the likely existence of economic rents, it is necessary to specify some mechanism for allocating this surplus. Following Howitt (1988), it is assumed that:

ASSUMPTION 3.7 *A worker receives a wage which is a constant fraction k of her output.*

This is a pie-splitting rule in which workers and capitalists get arbitrary shares k and $1-k$ of production, respectively. Any such split is a Nash equilibrium to a one-shot pie-sharing game and no attempt is made to endogenise the sharing rule or to refine the equilibrium concept.

[1] Such an exogenous separation assumption has been used by Lucas(1987) and Pissarides(1987). Within the abstract agricultural environment, it is quite appropriate. In general, it would be desirable to endogenise separations although the focus of this chapter is on effects due to changes in the rate of escape from unemployment; so, nothing is lost by assuming that matches dissolve at a constant rate.

Dynamic Evolution of the Economy

The evolution of the state of the labour market at the beginning of each period can be described by the following difference equations:

$$\begin{aligned} N_t &= N_{t-1} + p(\theta_{t-1})U_{t-1} - sN_{t-1}, \\ U_t &= (1 - p(\theta_{t-1}))U_{t-1} + sN_{t-1}. \end{aligned} \tag{2-4}$$

Stochastic Structure

Finally, the role of uncertainty in this model requires some explanation. It is assumed that the shock variable μ_1 follows a first-order n-state Markov process. The following assumption is made with regard to serial correlation of the shocks:

ASSUMPTION 3.8 *The conditional probability $Pr(\mu_{t+1} \geq x|\mu_t)$ is non-decreasing in μ_t for any given value of x.*

This restriction means that if the current value of the shock variable increases, then it becomes more likely that the shock will be high in the future. In this single sector framework, there is no distinction between aggregate and sectoral shocks since all shocks are aggregate to the sector. Once estimation is undertaken with a two sector model it will be possible to describe certain shocks as aggregate and others as sectoral.

4.4 Decision-Making in the Model

Decisions are made based on an information set that can be summarised by the state variables $\{N_t, \mu_t\}$. Based on this information, agents will act in such a manner that the value of the market tightness variable, θ, will be decided. This relationship will be described by the stationary rule:

$$\theta_t = \Theta(N_t, \mu_t).$$

In any equilibrium, it must be true that agents form their expectations of future values given these rules and that the decisions formulated under these expectations, in turn, give rise to the rules above. At the point of making their decisions, agents take as given arbitrary values of θ. In equilibrium, however, these values must be consistent with the decisions taken by agents.

The entry decision of an unmatched capitalist requires that she compare the value of posting a job offer versus the value of leaving capital non-productive and enjoying some return from it. This calculation will also involve determining the value of running a producing firm beginning at time t. Conditional on fixed values of θ_t, the maximised values of a producing firm and a job offer, V_t^P and V_t^O, are described by the recursive equations:

$$\begin{aligned} V^P(N_t, \mu_t) = \max\Big\{0, (1-k)F(N_t, \mu_t) \\ + \beta \sum_{i=1}^{n} \Big[(1-s)V^P(N_{t+1}, \mu_t) + sV^O(N_{t+1}, \mu_t)\Big] \Pr(\mu_i|\mu_t)\Big\}, \end{aligned}$$

and:

$$\begin{aligned} V^O(N_t, \mu_t) = \max\Big\{0, -\gamma + \beta \sum_{i=1}^{n} \Big[(1-q(\theta_t))V^O(N_{t+1}, \mu_t) \\ + q(\theta_t)V^P(N_{t+1}, \mu_t)\Big] \Pr(\mu_i|\mu_t)\Big\}. \end{aligned}$$

It will be assumed that

ASSUMPTION 4.1 *An abundant supply of unmatched capital exists in the economy and there are no barriers-to-entry.*

Accordingly, the entry and exit decisions of holders of unmatched capital will force the value of a vacancy into equality with the shadow value of capital. This shadow value is normalised to zero. Free entry and exit will accordingly require that $V_t^O = 0$ for any time t[2]. Substituting this into the equation for V^0 gives:

$$0 = -\gamma + \beta \sum_{i=1}^{n} q(\theta_t) V^P(\mu_i, N_{t+1}) \Pr(\mu_i | \mu_t).$$

This condition will determine the number of vacancies given the conditions within a market. This expression for the value of a producing firm is also simplified by this assumption and can now be written thus:

$$V^P(N_t, \mu_t) = (1-k)F(N_t, \mu_t) + \beta \sum_{i=1}^{n} (1-s) V^P(N_t + 1, \mu_i') \Pr(\mu' | \mu_t).$$

Given this equilibrium condition the following proposition can be established:

Proposition 1: *The value function $V^P(N_t, \mu_t)$ is increasing in μ_t and decreasing in N_t.*

Proof: See Appendix.

Following from this is the following:

Proposition 2: *The market tightness function, $\Theta(N_t, \mu_t)$, is increasing in μ_t and decreasing in N_t.*

Proof: See Appendix.

4.5 Fluctuations in the Economy

To examine how the economy behaves, it is useful to consider two relationships that drive the model. First, it is true that in equilibrium expected profits from opening a

[2] Actually, this will only hold with equality when there is a strictly positive number of vacancies. If there are no vacancies the value can be negative.

vacancy must be zero. This relationship must hold at all times. Note that the labour market dynamic equation:

$$N_{t+1} = (1-s)N_t + p(\theta_t)(L - N_t),$$
$$= (1 - s - p(\theta_t))N_t + p(\theta_t)L$$

can be used to eliminate N_{t+1} in the zero profit condition. This yields a relationship between N_t and θ_t *conditional on the state* μ_t.

$$\gamma = \beta \sum_{i=1}^{n} q(\theta_t) V^P \left[\mu_i, (1 - s - p(\theta_t))N_t + p(\theta_t)L\right] \Pr(\mu_i|\mu_t).$$

For given μ_t, this is a negative relationship between N_t and θ_t. This is shown by totally differentiating the expression above to obtain the following expression for $\frac{\partial\theta_t}{\partial N_t}$:

$$\frac{-\sum_{i=1}^{n} q(\theta_t)\left[1 - s - p(\theta)\right] \partial V^P(\mu_i, N_{t+1})/\partial N_t \Pr(\mu_i|\mu_t)}{\sum_{i=1}^{n} \left\{q'(\theta) V^P(\mu_i, N_{t+1}) + q(\theta_t)\left[p'(\theta_t)(L - N_t)\right] \partial V^P(\mu_i, N_{t+1})/\partial N_t\right\} \Pr(\mu_i|\mu_t)}.$$

Given the signs of $p'(\theta)$, $q'(\theta)$, and the fact that the value function falls in current employment, this derivative is negative. This curve can be viewed as a demand for labour curve in which lower values of θ are associated with greater production. For a two-state example, this relationship is shown as the two negatively sloped curves in Figure Two where different values of μ_i displace the curve. The higher curve corresponds to a higher value of μ given the positive serial correlation entailed by Assumption 3.8.

A second curve can be used to link N and θ through the conditions for labour market adjustment. The law of motion for employment has a fixed point for any given value of θ. This point, where $N_t = N_{t-1}$, is defined by:

$$\bar{N} = \frac{p(\theta)}{s + p(\theta)}.$$

For higher values of θ, N will be higher. This curve is analogous to a labour supply curve. Even though there is no labour supply decision at the individual level, there is a technological supply of matches relationship which works through the search process. The relationship implied by this curve is shown as the second curve in Figure Two.

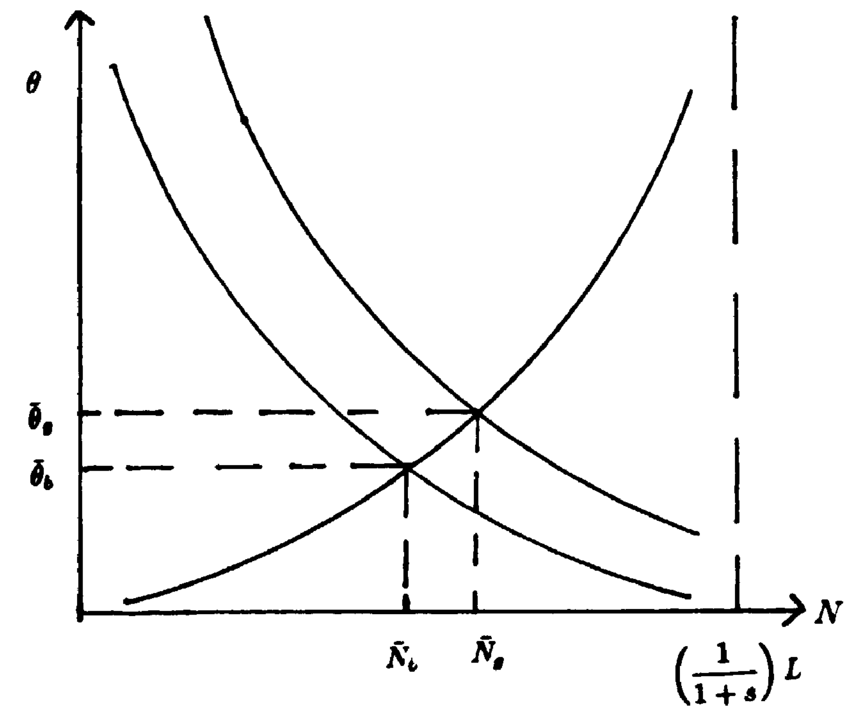

Figure Two: Fluctuations in Levels of Market Tightness and Employment

It is clear from Figure Two how employment and the market tightness variable will fluctuate as the shocks change. Employment either rises or falls smoothly over time depending on whether it is above or below the steady stationary point of employment given μ. The market tightness variable will move smoothly with N to stay on the zero profit line but may also jump either up or down as the shocks variable μ changes.

4.6 Estimating the Stochastic Search Model

Estimation of this model will focus on the curves in Figure Two and will essentially try to recover the shapes of the 'labour demand' curves. In order to accomplish this, several restrictions are placed upon the model through the following assumptions:

ASSUMPTION 6.1 *There are two aggregate states of the economy, one 'good' and the other bad.*

Clearly, such an assumption about the underlying structure of the economy is very restrictive. In spite of this (or perhaps because of this), there is a long tradition of such modelling. Hamilton (1989) considers a two-state model of growth in U.S. output and relates this to business cycle periods of negative and positive average growth rates. In any case, the structure of the present model is made more rich by the following assumption:

ASSUMPTION 6.2 *The economy is subject to a regional shock variable which will favour one region relative to the other. In the estimation, this yields two states: one in which Alberta is favoured relative to Ontario and one in which the opposite holds.*

This assumption attempts to capture the fact that the dominant sectoral shocks of recent history have been related to natural resource prices. To the extent that Alberta is a net producer of natural resources while Ontario is a net user, these shocks have opposite effects in the two provinces. Clearly, there are examples of sectoral shocks which could affect both provinces in similar ways but these will be labelled aggregate shocks by the model.

A final assumption is needed to obtain a functional form for the 'labour demand' curves:

ASSUMPTION 6.3 *The 'labour demand' curves for the two sectors are adequately represented by a linear relationship.*

Here it is assumed that departures from linearity are not sufficiently severe to seriously bias the results. This assumption simply makes explicit the implicit assumption of the bulk of applied econometric work.

Given this assumption, estimation of linear versions of the two downward sloping curves from Figure Two is possible since the economy is always on one of these two curves. A pair of curves is estimated for each of Alberta and Ontario. Algebraically, the model to be estimated is the following:

$$\theta_{A,t} = \alpha_0 + \alpha_1 S_t^A + \alpha_2 S_t^R + \alpha_3 N_{A,t} + \epsilon_{A,t},$$

$$\theta_{O,t} = \beta_0 + \beta_1 S_t^A + \beta_2(1 - S_t^R) + \beta_3 N_{O,t} + \epsilon_{A,t},$$

where:

$$\epsilon_{i,t} \sim \mathrm{N}(0, \sigma_i),$$

$$S_t^A = \begin{cases} 0, & \text{if the aggregate state is bad at } t, \\ 1, & \text{if the aggregate state is good at } t; \end{cases}$$

$$S_t^R = \begin{cases} 0, & \text{if Alberta is favoured at } t; \\ 1, & \text{if Ontario is favoured at } t. \end{cases}$$

Essentially, this is a four-state Markov model defined on the state-space $\{0,1\} \times \{0,1\}$ (i.e. the product space of the state-space for the aggregate and regional shock variables). Corresponding to this is a state S_t defined as follows:

$$S_t = \begin{cases} 1, & \text{if } s_t^A = 0 \text{ and } s_t^R = 0; \\ 2, & \text{if } s_t^A = 0 \text{ and } s_t^R = 1; \\ 3, & \text{if } s_t^A = 1 \text{ and } s_t^R = 0; \\ 4, & \text{if } s_t^A = 1 \text{ and } s_t^R = 1. \end{cases}$$

This four-state Markov process has a transition matrix P defined as follows:

$$P = \begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{24} \\ p_{31} & p_{32} & p_{33} & p_{34} \\ p_{41} & p_{42} & p_{43} & p_{44} \end{bmatrix}.$$

where $p_{i,j} = \Pr[S_t = s_j | S_{t-1} = s_i]$. Restrictions on the transition probabilities above imply that there are only four free parameters in the Markov process. There are the probabilities of staying in the good or bad aggregate states which are labelled p and q respectively. The other two parameters are the probabilities of remaining in the state favourable to Alberta (r) or Ontario (s). Given this, it is possible to calculate the probability of going from a state which is good and favourable to Alberta to one bad and favourable to Ontario (i.e. p_{32}) as $(1-p)(1-r)$.

In this specification, the intercepts of the two provincial labour-demand curves will vary with the state. For example, the intercept for Alberta takes on each of the following values in the four states respectively:

$$\alpha_0, \qquad \alpha_0 + \alpha_1, \qquad \alpha_0 + \alpha_2, \qquad \alpha_0 + \alpha_1 + \alpha_2.$$

The slopes are assumed to remain constant.

Estimation of the parameters of this model would be quite straightforward if the realizations of the state variable where observed. In this case estimation would fall into the class of OLS estimators which allow for regime-dependent means using dummy variable techniques. Hamilton's contribution consists of the formulation of an algorithm for obtaining maximum likelihood estimates of the parameters in this model based on *observed* θ_t values only. Clearly, the joint likelihood function for the θ_t and s_t random variables is of little use when the states are unobserved. Hamilton shows, however, that the marginal likelihood function for the observed data alone can be calculated and this likelihood allows for identification of all parameters of interest.

The Hamilton algorithm first numerically computes the value of the marginal likelihood function for the observed data given the parameters of the deterministic 'labour demand' curves and the Markov process. Since this likelihood can be computed for any vector of parameters, it can be submitted to a numerical optimization program [3] to obtain maximum likelihood estimates of the parameter vector. The method of numerically calculating the likelihood function is somewhat involved and is usefully described in the following steps.

Denoting the entire provincial parameter vectors by α and β, the sample marginal log-likelihood function that is to be maximised is:

$$\log f(\theta_{A,T}, \theta_{O,T}, \theta_{A,T-1}, \theta_{O,T-1}, \ldots, \theta_{A,1}, \theta_{O,1} | \alpha, \beta, \sigma_A, \sigma_O, p, q, r, s)$$

[3] Hamilton uses GQOPT with the Davidson-Fletcher-Powell option and this same choice is used here.

Under the assumption of independent observations over time, the contribution for each time period is:

$$\log f(\theta_{A,t}, \theta_{O,t} | \alpha, \beta, \sigma_A, \sigma_O, p, q, r, s).$$

To calculate the individual contributions to the likelihood an iterative technique is applied. To start up the iterative calculation, an initial estimate of the state at time zero is needed. Two ways of obtaining this are available. First, it is possible to assume that the stationary distribution of the Markov process has been achieved at time zero. This is equivalent to stating that the initial state of the economy at the beginning of time is unknown but irrelevant because, by time zero, repeated application of the ergodic Markov process has removed the memory from the state vector. Secondly, the value of the state could be estimated as an additional parameter. This would require the estimation of four additional parameters and an attendant increase in computation time. Thus far, authors applying the Hamilton algorithm have not estimated the start-up probabilities although in the current application, where recovering information about the state variable is of some interest, this would seem to be potentially more useful. In practice, estimating the initial state seemed not to matter much in estimation of a simplified version of the model so that results are presented using the established procedure of starting with long-run probability distributions.

For all periods but the very first, the calculation of the period contribution to the sample likelihood follows the four steps below. For the first period, the first step is skipped because $\Pr[S_1 = s_1 | \theta_0]$ is undefined and the unconditional distribution $\Pr[S_1 = s_1]$ described above is used in its place. After time $t = 1$, the calculation proceeds as follows:

(i) Given the distribution over states at time $t-1$ conditional on observed data through $t-1$, apply the transition probabilities to get the distribution at time t. Letting $\theta_t = (\theta_{A,t}, \theta_{O,t})$ and denoting the first observed θ value by θ_0, this distribution is defined by:

$$\Pr[S_t = s_t|\theta_{t-1}, \theta_{t-2}, \ldots, \theta_0] =$$
$$\sum_{s_{t-1}=1}^{4} \Pr[S_t = s_t|S_{t-1} = s_{t-1}]\Pr[S_{t-1} = s_{t-1}|\theta_{t-1}, \theta_{t-2}, \ldots, \theta_0].$$

In all but the first period, the probability $\Pr[S_{t-1} = s_{t-1}|\theta_{t-1}, \theta_{t-2}, \ldots, \theta_0]$ will be obtained from step (v) at the previous pass through the algorithm.

(ii) Use the distribution in (i) above and the conditional distribution of θ_t given the state to calculate the joint pdf of the current θ_t and the unobserved state.

Note that under the assumptions of normality and zero correlation between $\epsilon_{A,t}$ and $\epsilon_{\gamma,t}$, the conditional pdf of $(\theta_{A,t}, \theta_{O,t}$ given the current state is:

$$f(\theta_{A,t}, \theta_{O,t}|s_t^A, s_t^R, \alpha, \beta, \sigma_A, \sigma_O) =$$
$$\frac{1}{\sigma_A \sigma_O 2\pi} \exp - \left[\left(\frac{\theta_{A,t} - \alpha_0 - \alpha_1 s_t^A - \alpha_2 s_t^R - \alpha_3 N_t^A}{2\sigma_A^2} \right)^2 + \left(\frac{\theta_{O,t} - \beta_0 - \beta_1 s_t^A - \beta_2(1 - s_t^R) - \beta_3 N_t^O}{2\sigma_O^2} \right)^2 \right].$$

(iii) The joint distribution for θ_t and s_t is then obtained from the probability function calculated in (i) and the pdf in (ii).

(iv) The marginal likelihood to be estimated is obtained by summing over all probabilities for the unobserved state variable in the joint pdf from (iii) above. This depends on the parameters of the conditional distribution of θ_t given the state and the Markov transition process through step (i). Accordingly, maximization of this pdf value with respect to the parameters will yield maximum likelihood estimates of the parameters of interest.

For the next iteration, an updated estimate of the distribution over states is needed. The next step provides this:

(v) The pdf of the unobserved state variable given the sequence of θ values through to time t is obtained from:

$$\Pr[S_t = s_t|\theta_t, \theta_{t-1}, \ldots, \theta_0] = \frac{f(\theta_t, S_t = s_t|\theta_{t-1}, \theta_{t-2}, \ldots, \theta_0)}{f(\theta_t|\theta_{t-1}, \theta_{t-2}, \ldots, \theta_0)}.$$

This follows because the joint distribution of θ_t and S_t is the product of the two marginal distributions (where all three of these distributions are conditional on the data observed through period t).

This iterative procedure will compute the value of the log-likelihood function for any given set of values for the parameter vector. This can be used by a program such as GQOPT which will optimize by evaluating the function to calculate numerical derivatives.

4.7 Estimation Results

The Markov switching regression model was applied to market tightness data for Alberta and Ontario. These data are shown in Figure Three. Logarithms of the data were taken to convert a series on $[0, \infty)$ into one which can take values on the interval $(-\infty, \infty)$. This avoids restrictions on the process caused by non-negativity. Results of the exercise are first presented in two tables which give the parameter estimates for the deterministic part of the regression and the parameters of the Markov processes. The regression function actually estimated for the deterministic part of the model was:

$$\theta_{A,t} = \alpha_0 + \exp(\alpha_1)S_t^A + \exp(\alpha_2)S_t^R - \alpha_3 * \alpha_3 N_{A,t},$$

$$\theta_{O,t} = \beta_0 + \exp(\beta_1)S_t^A + \exp(\beta_2)(1 - S_t^R) - \beta_3 * \beta_3 N_{O,t}.$$

Here several non-negativity restrictions are imposed on the model in order to identify sectoral and aggregate shocks. In addition, an over-identifying assumption implied by the model is imposed by restricting the coefficients on employment to be non-positive. Estimates for the unrestricted case are also reported but the analysis focusses on the restricted version of the model because this assumption is critical to the theory. It is worth noting that the employment level data have a secular trend, as Figure Four reveals. In future work some attempt to remove this trend may be useful.

Table One: Estimation of the Deterministic Equations

	Restricted N	Unrestricted N
Parameter Estimates for Alberta		
α_0	-2.63 (0.17)	-1.28 (0.04)
α_1	0.42 (0.04)	0.17 (0.03)
α_2	-30.34 ($2x10^5$)	-2.17 (0.50)
α_3	-0.02 (0.003)	$-1.6x10^{-3}$ ($7.4x10^{-5}$)
σ_A	0.44 (0.0246)	0.38 (0.02)
Parameter Estimates for Ontario		
β_0	-3.37 (0.04)	-4.03 (0.13)
β_1	-0.26 (0.05)	-0.30 (0.05)
β_2	-0.22 (0.04)	-0.04 (0.04)
β_3	0.00 (0.002)	$1.7x10^{-4}$ ($3.2x10^{-5}$)
σ_O	0.26 (0.01)	0.25 (0.01)
LLF	291.2	344.7

(Estimated Standard errors are in parentheses below parameter estimates)

Table Two: Implied Intercepts Expressed in Levels
(Restricted Model)

	Alberta	Ontario
Bad and Alta.	0.070	0.034
Bad and Ont.	0.070	0.077
Good and Alta.	0.330	0.074
Good and Ont.	0.330	0.165

Table Three: Estimation of the Markov Persistence Parameters

	Restricted	Unrestricted
Parameter Estimates for the Aggregate Shocks		
Pr(bad | bad)	0.981	0.977
Pr(good | good)	0.996	0.990
Parameter Estimates for the Sectoral Shocks		
Pr(Alt. | Alt)	0.982	0.989
Pr(Ont. | Ont.)	0.986	0.995

Of these results, Table Two summarises best the implications of the estimation. For Alberta, the change in the intercept due to sectoral shocks is very small while the change due to aggregate shocks is striking. In Ontario, on the other hand, aggregate and sectoral shocks have had about the same size of effect. The other key finding of the results is that all shocks are quite persistent. Table Three reveals that the least persistent state is the bad aggregate state and it is still true that the probability of staying in this state for thirty-four consecutive months is slightly greater than one half. In other words, it is most likely that there will be no change in the aggregate state for almost three years.

A curious feature of these results is that the restriction on the sign of the employment variable changes the interpretation of these results. If this restriction is not imposed, the Ontario slope is positive and significantly different to zero. As a result, the sectoral effect increase in Alberta and the impact of the aggregate shock falls. In Ontario the same occurs. This suggests that the use of the restriction has the effect of decreasing the apparent relevance of sectoral shocks for changes in recruitment intensity and thus unemployment duration.

The final step of each iteration of the calculation algorithm is a probability distribution over the states at time t given data observed through t. This can be used as an indication of the state of the two provincial economies at each period. Graphs of the probability of being in the good state are provided in Figures Five and Six. The first graph shows the probabilities of being in each of the four states over the period 1966 to 1975. The second graph continues this from 1976 through 1988. This second graph is most closely matched to the duration series in Figure One.

The graphs show that over the 1966 to 1975 period almost all changes in the state were due to regional shocks within the good state. This could reflect the fact that the main negative aggregate shock was the fall in external demand (mainly from the United States) following the 1973 oil price shock and this may have had little negative effect on Alberta. Interestingly, the state is in the favour of Ontario until late in 1974, having switched from the favour of Alberta in early 1973. This is puzzling given that world oil prices began to rise in 1973 with the first OPEC shock. This likely reflects the fact that prices paid to domestic Canadian producers did not rise until after

1974 due to government "made in Canada" energy pricing policies. It is accordingly supportive of the model that the state shifts to the favour of Alberta late in 1974. The shift to the favour of Ontario between 1973 and late 1974 is puzzling but may reflect the relatively greater sensitivity of Ontario to the stimulative policies of that period.

Throughout the late 1970's, a good aggregate shock with Alberta favoured prevailed. This switched to a bad aggregate shock with Alberta favoured after the 1982 recession. By 1985, there was an indication of a bad aggregate state in the favour of Ontario while, by 1987, the aggregate state was good again but in the favour of Ontario. This timing makes sense given knowledge of events in this period. Oil price rises throughout the late 1970's worked in the favour of Alberta. The main negative shock was the contractionary monetary policy of 1982 which clearly appears in the data. Falls in oil prices occurred after the recession as drops in world energy demand and other factors undermined the discipline of the OPEC cartel. This is reflected in the shift to a state in which Ontario is favoured beginning in 1985.

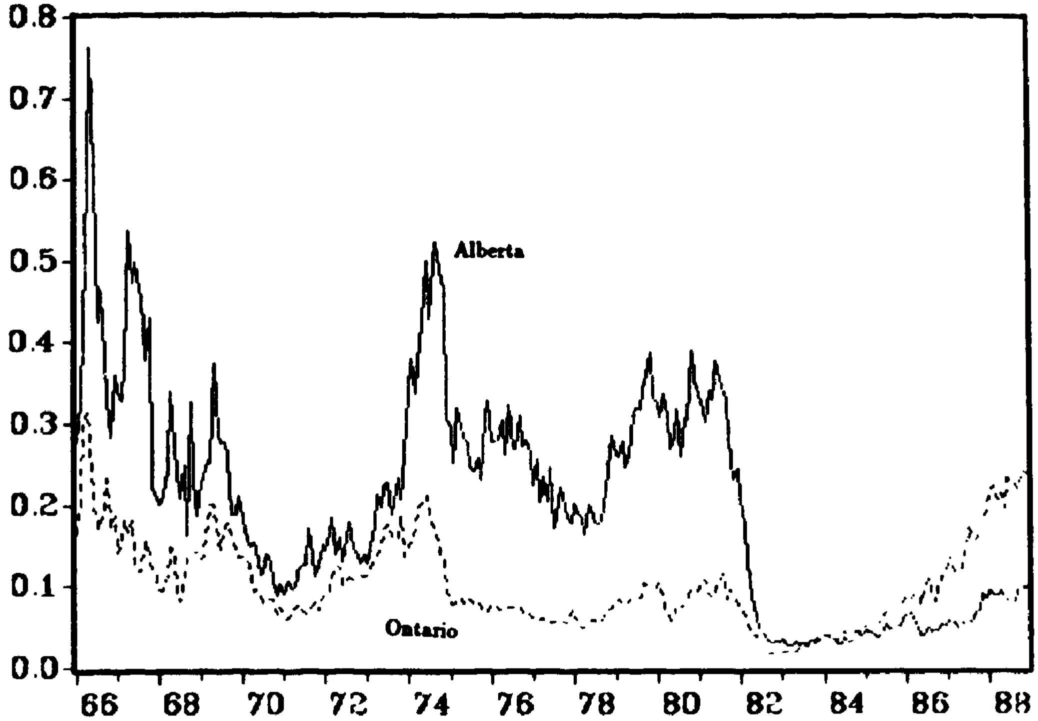

Figure Three: Labour Market Tightness in Alberta and Ontario

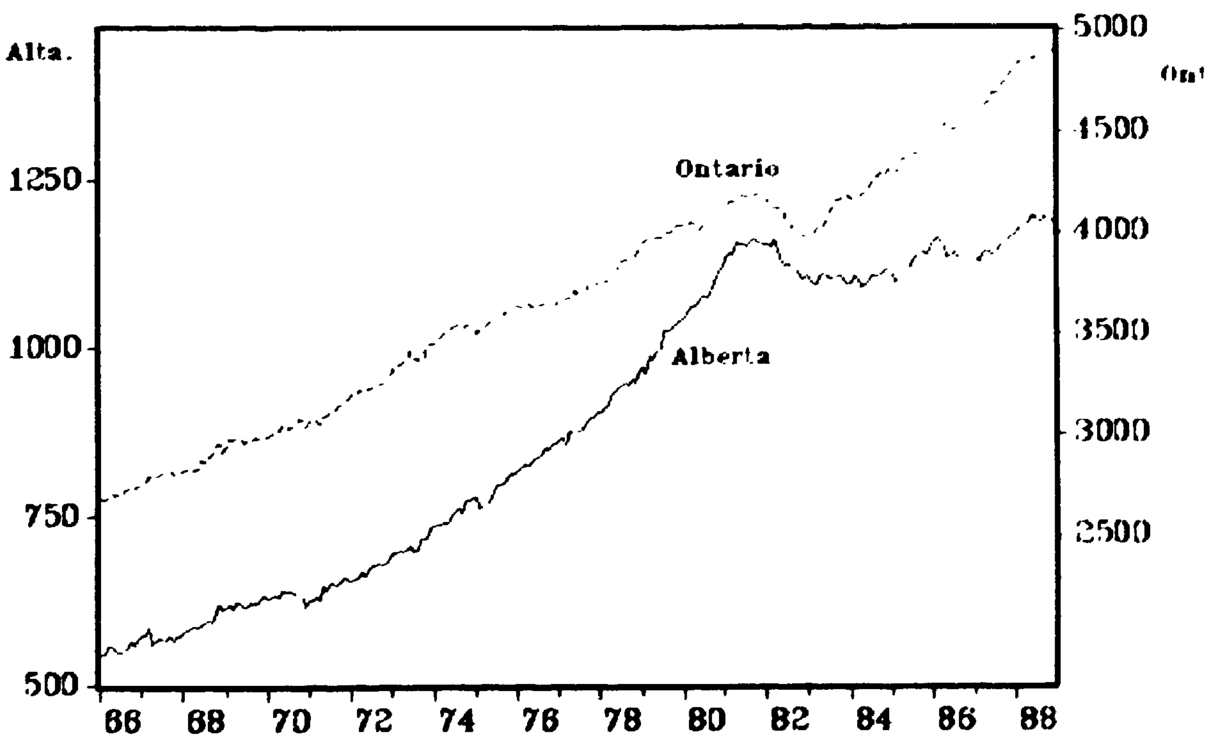

Figure Four: Employment Levels in Alberta and Ontario

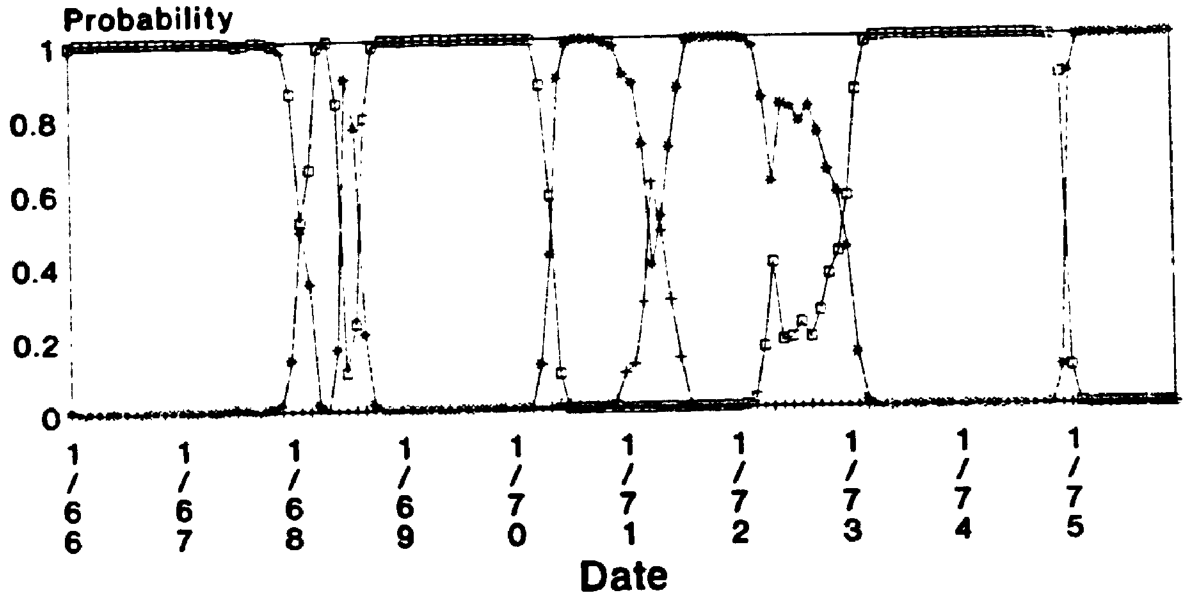

Figure Five: Probabilities over States: 1966 - 1975

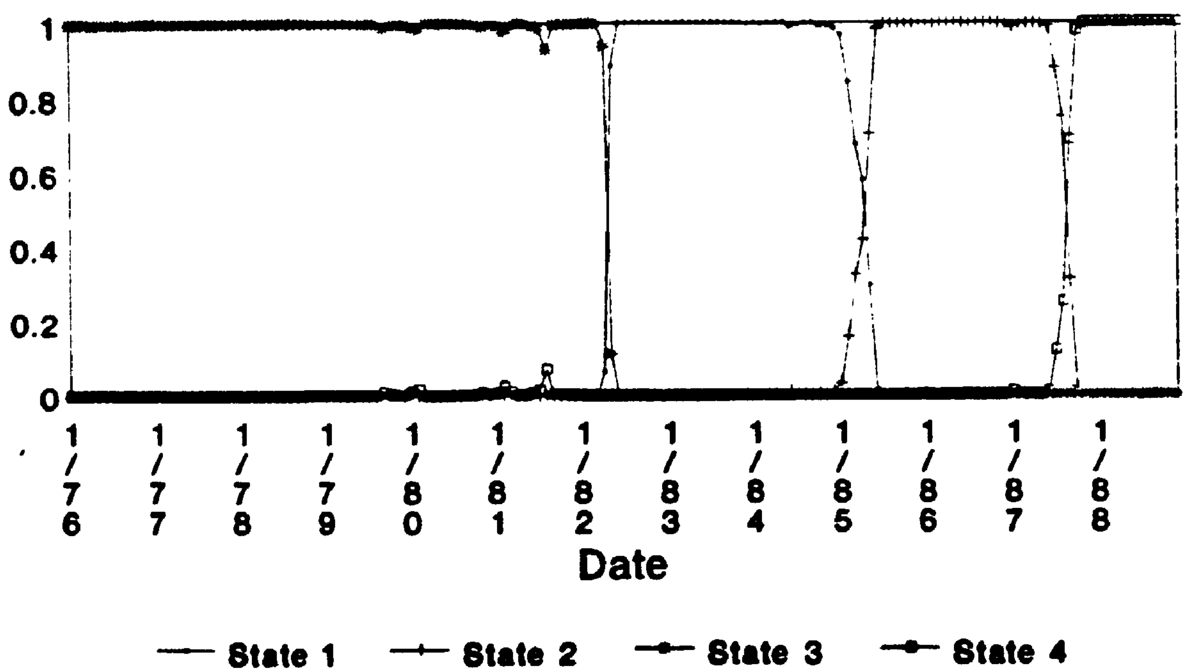

Figure Six : Probabilities over States: 1976 - 1988

While the model above is correct, based on direct application of the model, there is a potential problem because the theory used to derive the estimating equations applies to a situation with zero population growth. Accordingly, the counter-factual assumption that employment does not grow over time is imposed. Figure Four reveals, however, that this is not the case with the data used in the study. This trend growth in employment is likely due to growth in the labour force corresponding to population growth. Such growth is almost certainly linked to increases in demand which work to shift out the average revenue product function $F(N, \mu)$ over time due to general equilibrium effects. It is clear that the employment fluctuations that are pertinent to the model are higher frequency fluctuations that are not related to population growth.

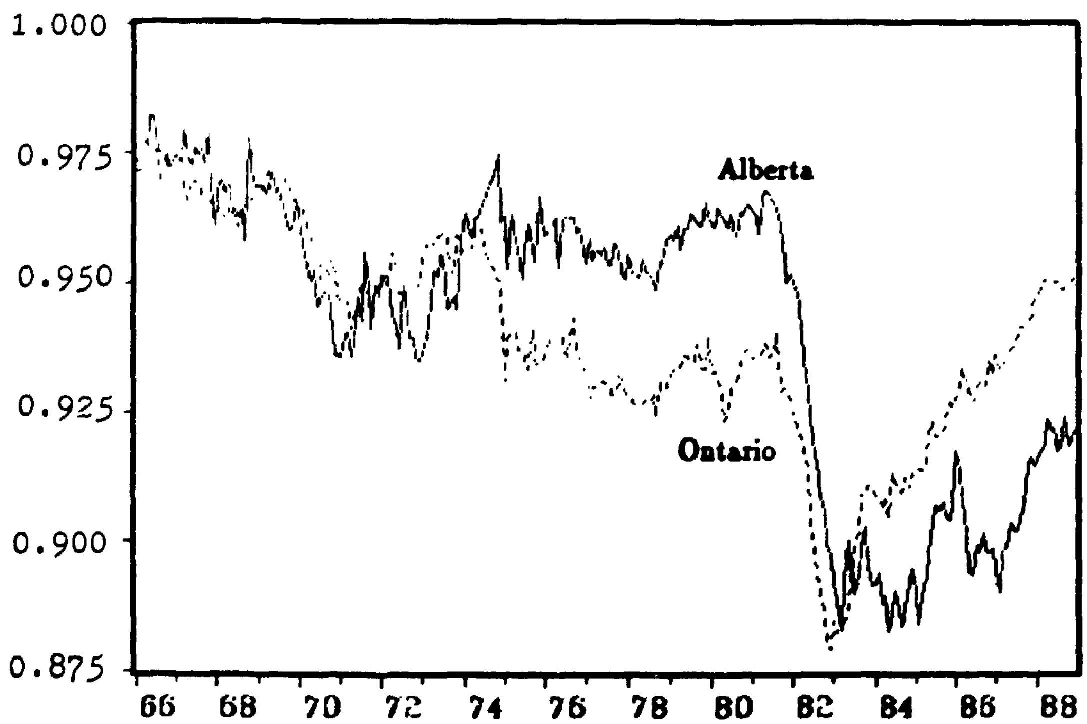

Figure Seven: Employment Rates, Alberta and Ontario

One way to incorporate this factor into the model is to use an employment variable that has been divided by the labour force. Under a particular assumption about the relationship between growth in the labour force and demand for industry output, this method of de-trending will be exactly correct. In general, use of the employment rate can be viewed as a first approximation to a correct measure of employment. Employment rate series were constructed by dividing the level of employment by the size of the labour force and these are shown in Figure Seven above.

Estimation results with the employment rate variable are presented in Tables Four , Five and Six. These are analogous to the earlier tables which used the level of employment. Several features of these tables are worthy of note. First, when employment is restricted to have a negative effect, the estimated employment coefficients are essentially zero. The remaining parameters are very similar to those for the restricted regression with employment in Table One. Similarly, the predicted state of the economy is almost exactly the same as that for the results with the level of employment included in the regression that were shown in Figure Five.

If negativitity restrictions are not imposed on the employment rate, the estimates for the parameters change dramatically. In the unrestricted regression, the fit of the model is greatly improved due to strong positive effects of the employment rate variables. Now, it is essentially true that the constant in the regression is zero in all states. This means that variation in market tightness is all completely derived from fluctuations in the employment rate. Another manifestation of this is a (slight) reduced persistence in the Markov processes and the changes observed in the probabilities of being in each of the four states. As Figures Eight and Nine show, there is now much more uncertainty regarding the state and more switching between the states.

Clearly, this model is flawed because a fundamental over-identifying restriction is overwhelmingly rejected given the drop in the likelihood function when a positive slope is imposed on the employment rate. In response to this, it is useful to reflect and consider what the source of rejection is since there are many assumptions underlying the model. A likely culprit is the fact that the theoretical model estimated here has a constant job destruction rate since the focus is on changes in unemployment duration due to sectoral shocks. This does not appear to conform well to observed facts. The employment rate graphs show that de-trended output frequently jumps to lower levels and this cannot be due to changes in the matching rate alone because such an adjustment path would be far more smooth. Instead, it is the job separation rate which is fluctuating with the state.

In effect, then, the employment rate proxies for the state of the economy and thus robs the state-dependent means of their effects. This does not mean that the results with a negativity constraint are invalid, however. This restriction simply identifies a model in which we seek to explain changes in unemployment through changes in the matching rate. The result does suggest that a Markov model of the law of motion for employment may provide better estimates of the underlying state of the economy. This would be part of an exercise purely designed to look at how sectoral and aggregate shocks affect employment dynamics and is thus separate from the current goal of examining the relation between unemployment duration and sectoral shocks.

Table Four: Deterministic Equation with the Employment Rate

	Restricted N	Unrestricted N
Parameter Estimates for Alberta		
α_0	-3.15 (0.05)	-30.01 (0.37)
α_1	0.51 (0.04)	-1.00 (0.06)
α_2	-85.5 (12.0)	-25.5 (1.16x10^5)
α_3	0.25x10^{-7} (0.15)	29.73 (0.39)
σ_A	0.44 (0.02)	0.14 (0.007)
Parameter Estimates for Ontario		
β_0	-3.33 (0.05)	-25.6 (0.62)
β_1	-0.32 (0.08)	-1.26 (0.08)
β_2	-0.20 (0.04)	-0.72 (0.07)
β_3	0.12x10^{-7} (0.33)	24.4 (0.66)
σ_O	0.27 (0.01)	0.16 (0.008)
LLF	286.8	738.6

(Estimated Standard errors are in parentheses below parameter estimates)

Table Five: Implied Intercepts Expressed in Levels

(Restricted Model)

	Alberta	Ontario
Bad and Alta.	0.043	0.036
Bad and Ont.	0.043	0.081
Good and Alta.	0.228	0.074
Good and Ont.	0.228	0.169

Table Six: Estimation of the Markov Persistence Parameters

	Restricted	Unrestricted
Parameter Estimates for the Aggregate Shocks		
Pr(bad | bad)	0.982	0.987
Pr(good | good)	0.996	0.957
Parameter Estimates for the Sectoral Shocks		
Pr(Alt. | Alt)	0.982	0.952
Pr(Ont. | Ont.)	0.986	0.953

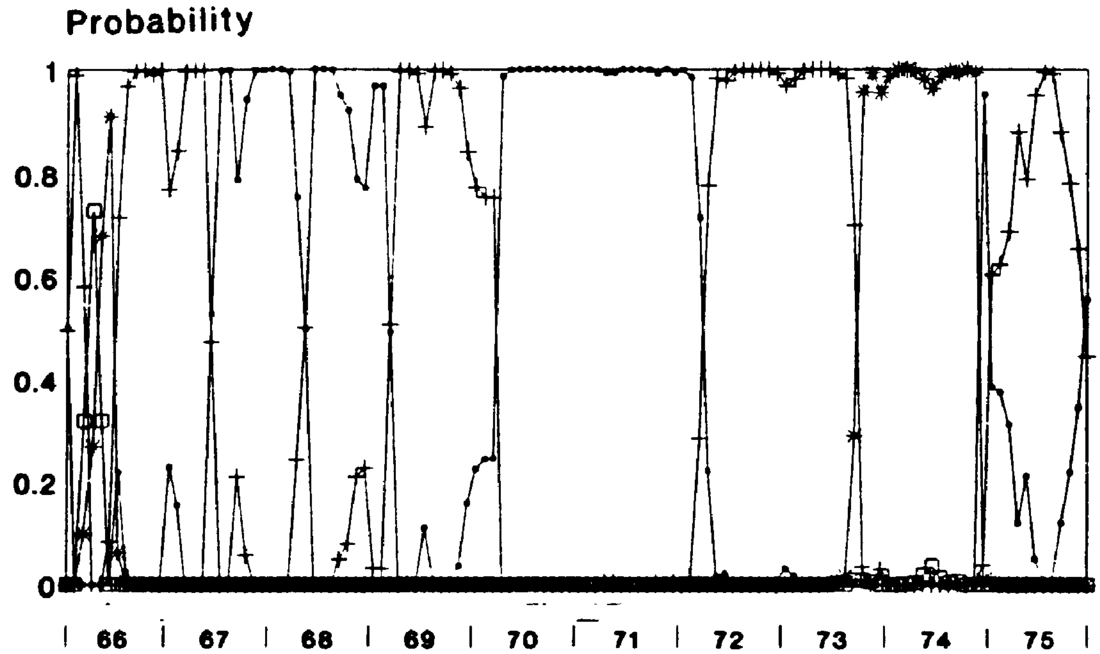

Figure Eight: Probabilities over States: 1966 - 1975

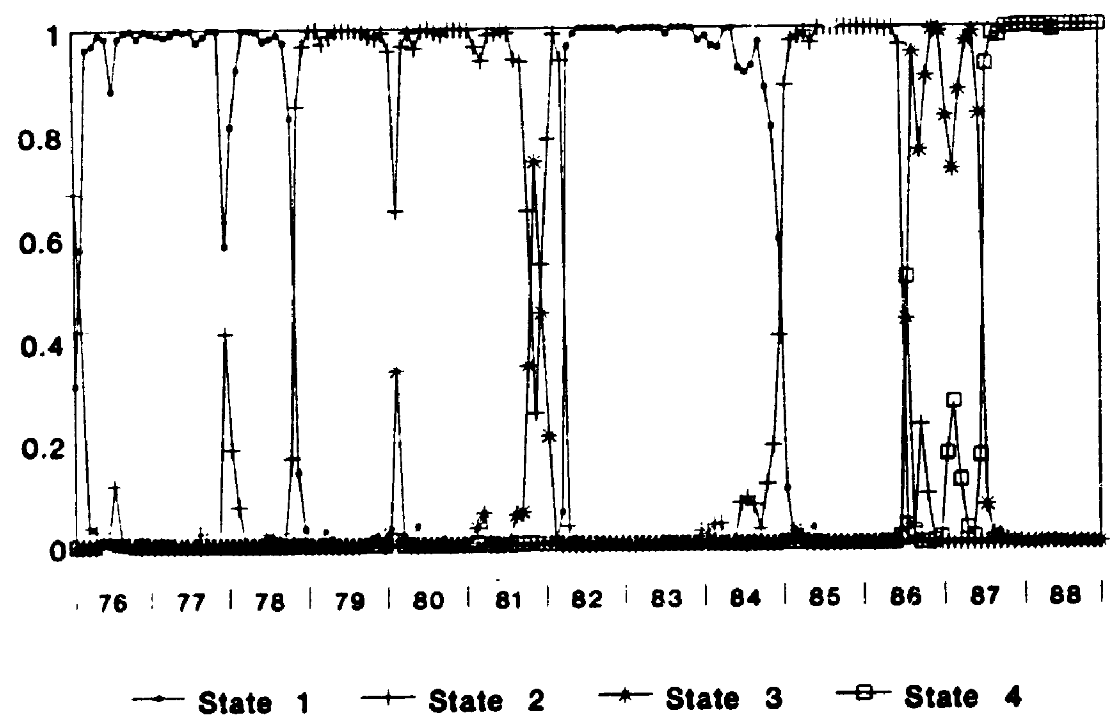

Figure Nine: Probabilities over States: 1976 - 1988

4.8 Discussion and Conclusions

Several basic conclusions can be drawn from this work. First, there is some evidence that shocks to recruitment activity are quite persistent. For example, the probability of remaining in the good state in Ontario is 0.984. This implies that the forecast probability of staying in the good state for 43 consecutive months is approximately 0.5. In other words, the odds are that things will not change at all for almost four years. This can be taken as some justification for the use of perfect foresight analysis in sectoral shift models.

A second basic conclusion is that sectoral fortunes with regard to recruitment activity have differed in several important ways. More importantly, these differences ocurred in the mid to late 1970's, when resource prices were high, and after 1985, when resource prices collapsed. In these two periods, the relative positions of Alberta and Ontario were reversed. Given that Chapter Three of the thesis has established a close link between recruitment activity and unemployment duration, a strong argument can be made that resource price shocks account for much of the variability observed in provincial unemployment duration rate series. This said, however, there are signs that the distinction between regional and aggregate shocks can be hazy since almost any aggregate shock imaginable has differential regional impacts. This may explain why the state switched from the favour of Alberta to that of Ontario during 1973 and part of 1974.

For the Ontario economy, the results suggest that sectoral shocks have an impact on recruitment intensity that is roughly equal to that of aggregate shocks. For Alberta, the effect of sectoral shocks is quite small unless the slopes of the labour demand curves are not restricted. Changes in regional fortunes in Ontario could raise the market tightness variable by 0.043. Evidence on the relationship between market tightness and hazard rates in Chapter Three suggests that, depending on the initial level of market tightness, this could change the hazard rate rather substantially. At an initial level of .02, this implies a rise from a hazard rate of 0.18 to one of around 0.25 using monthly data which translates into a change in expected duration from 4.5 months to 3 months. This effect is quantitatively significant but is lower for higher initial values of market tightness due to apparent concavity of the market tightness-recruitment intensity relationship.

Finally, the evidence presented here suggests that the nature of the shocks hitting the Alberta economy is indeed quite different from that for Ontario. This suggests that, so long as Canadians are risk averse, there may be gains from trade in state-contingent contracts between Albertans and Ontarians. Of course, one way to exploit these gains is to internalise insurance through the use of a federal system of government. Analysis in the Hamilton approach could be used to price insurance for various provinces so that federal transfers could be placed on a firm economic ground. Of course, provinces with very similar shocks (perhaps Ontario and Quebec or Nova Scotia and New Brunswick) would gain less from such an incentive to federate. Future work with other regional data might establish this more precisely.

Chapter Five

Persistent Unemployment, Sectoral Shocks, and the 1982 Recession

5.1 Introduction

The recession of 1982 was notable largely because of the steep rise in the Canadian unemployment rate from approximately seven percent in 1978 to almost thirteen percent. It has become increasingly clear that this rise in unemployment was also marked by a degree of persistence that was unprecedented in post-war business cycles. By 1987, for example, the national unemployment rate had only fallen to nine percent but had been fairly constant around a level of seven percent between 1975 and 1981.

As is documented in a recent paper by Milbourne, Purvis and Scoones (1991), this persistence in Canada contrasts markedly with the experience in the United States. While the U.S. unemployment rate also rose sharply in 1982, it had fallen back to its pre-recession level by 1984. This divergence in post-recession unemployment rate dynamics points to underlying structural differences in the U.S. and Canadian economies. Interestingly, the most obvious candidate, differences in real output performance, can be ruled out. There is no apparent difference in the behaviour of aggregate real output between Canada and the U.S. during the recovery period.

5.2 The Role of Unemployment Insurance

If real output cannot explain differences in unemployment behaviour, economists must search elsewhere for an explanation for differences in unemployment rate dynamics between Canada and the United States. A potential explanation is suggested by Milbourne, Purvis and Scoones (henceforth MPS) which is based upon differences in unemployment insurance programmes. Beginning in 1977, the Canadian unemployment insurance system was changed and a new "Regional Extended Benefits" scheme was instituted. Under this system, the number of weeks of unemployment insurance benefits to which a worker was entitled was made a non-decreasing function of the unemployment rate in the worker's area. Prior to 1977, regional and national extended benefits existed. The regional extended benefit allowance was based on a differential between national and regional unemployment rates as opposed to the absolute level of the regional rate. The national extended benefits were available whenever the national rate exceeded four per cent.

Under the post-1977 scheme, sectoral shocks would have little effect on the average number of extended benefit weeks in Canada as a whole. Increases in weeks of benefits in bad regions would offset decreases in good ones. With an aggregate shock, however, unemployment rates would increase everywhere and so the national average benefit eligibility period would rise. This type of national effect was absent in the U.S. and consequently the presence of such a policy in Canada has the potential to explain differences in unemployment dynamics, both relative to the past and relative to the U.S..

Empirical work with micro-data such as Ham and Rea (1987) has supported the theoretical prediction that, ceteris paribus, when benefit eligibility periods are increased, reservation wages rise and unemployment spells become longer on average. If benefit eligibility periods increased throughout Canada, then search times would be expected to increase everywhere, thereby making unemployment more persistent. MPS present empirical results which support their contention that the unemployment insurance scheme is responsible for making the unemployment rate more persistent.

The heart of the MPS analysis is a regression of the national unemployment rate on (i) a lagged dependent variable, (ii) a monthly real gdp measure,and (iii) contemporaneous and lagged indicators of the number of benefit weeks available to a minimally qualified unemployment insurance claimant. The net effect of these last two variables is shown to have had a significant impact on the degree of persistence of the unemployment rate. This latter fact is established through the use of a counterfactual simulation of the unemployment rate with unemployment benefits determined according to the pre-1977 regime. This analysis apparently shows that unemployment dynamics are far more history-dependent under the post-1978 system.

5.3 Some Doubts Concerning this Explanation

A curious anomaly which is problematic for the MPS explanation is the divergent behaviour of unemployment rates in different regions of Canada. Roughly, the provinces can be divided into two groups. The first includes Ontario, and the Atlantic provinces and is distinguished by a fairly rapid decline in the unemployment rate following the recession. The second group comprises the Prairie provinces and British Columbia as well as Quebec. For this latter group, there has been little tendency for the unemployment rate to fall until very recently. The divergence between these two groups casts in doubt an explanation for persistence based on a policy administered uniformly across the country.

To further illustrate the point, consider the unemployment rate and unemployment duration series in Figures One and Two which display data for Alberta (solid lines) and Ontario (broken lines). These provinces well represent the two groups into which Canadian provinces can be divided. First, note that the unemployment rate for Ontario fell consistently beginning in 1983 and indeed dropped quite sharply during 1983. The rate for Alberta, in contrast, is quite constant through 1983 and 1984 and only drops slightly in 1985 before remaining flat again until 1987.

These differences are also apparent in data on the average duration of unemployment. This series is important because unemployment insurance is often thought to affect the unemployment rate through its impact on the length of search spells. It is evident that the Regional Extended Benefits program had different effects on unemployment duration in these two provinces, if, indeed, it had any effect at all. The series in Ontario falls steadily after 1982 while unemployment duration remains high throughout the 1980's in Alberta.

These stylized facts bring to mind an alternate explanation for some of the persistence in the unemployment rate. This explanation comes from a search paradigm which explains variations in offer arrival rates as well as offer rejection probabilities. Figure Three shows a measure of labour market tightness – the ratio of job vacancies to searching workers – for Ontario and Alberta. This variable shows a strong rise in Ontario during the period of recovery to the 1982 recession. Indeed, the variable reaches a higher level than that prevailing before the recession. On the other hand, such a rebound is totally absent in Alberta. The analysis of Chapter Two of this thesis demonstrates that this pattern is consistent with differential offer arrival probabilities in the face of negative shocks to resource and agricultural prices. The competing explanation of MPS is that the maximum weeks of benefits series shown in Figure Four explain the change in the unemployment rate.

The argument advanced in this chapter is that some of the measured increase in persistence in aggregate unemployment may be due to persistent shocks impacting upon the resource-oriented areas of Canada. The aggregate effect does not even out because there is less of a positive effect on offer probabilities in eastern Canada than there is a negative effect in the West. This chapter will examine this issue by attributing unemployment rate fluctuations to changes in offer arrival rates and offer acceptance rates. The former reflects changes in recruitment intensity by firms while the latter could be due to changes in reservation wages induced by unemployment insurance.

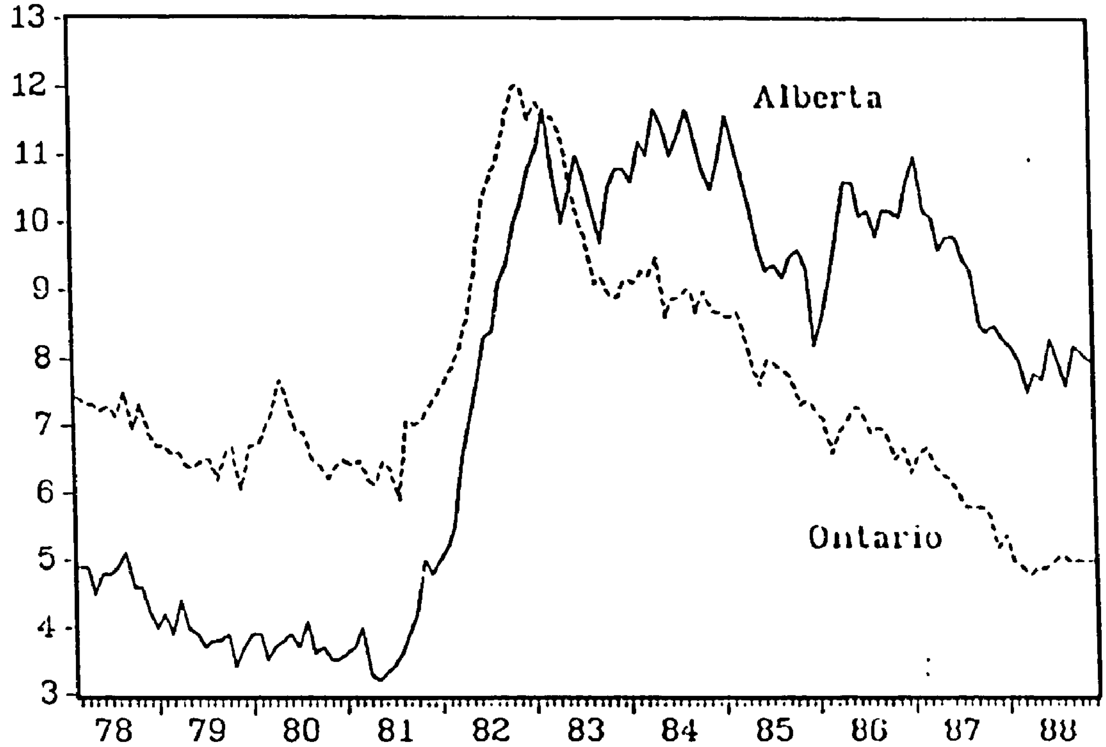

Figure One: Unemployment Rate in Alberta and Ontario

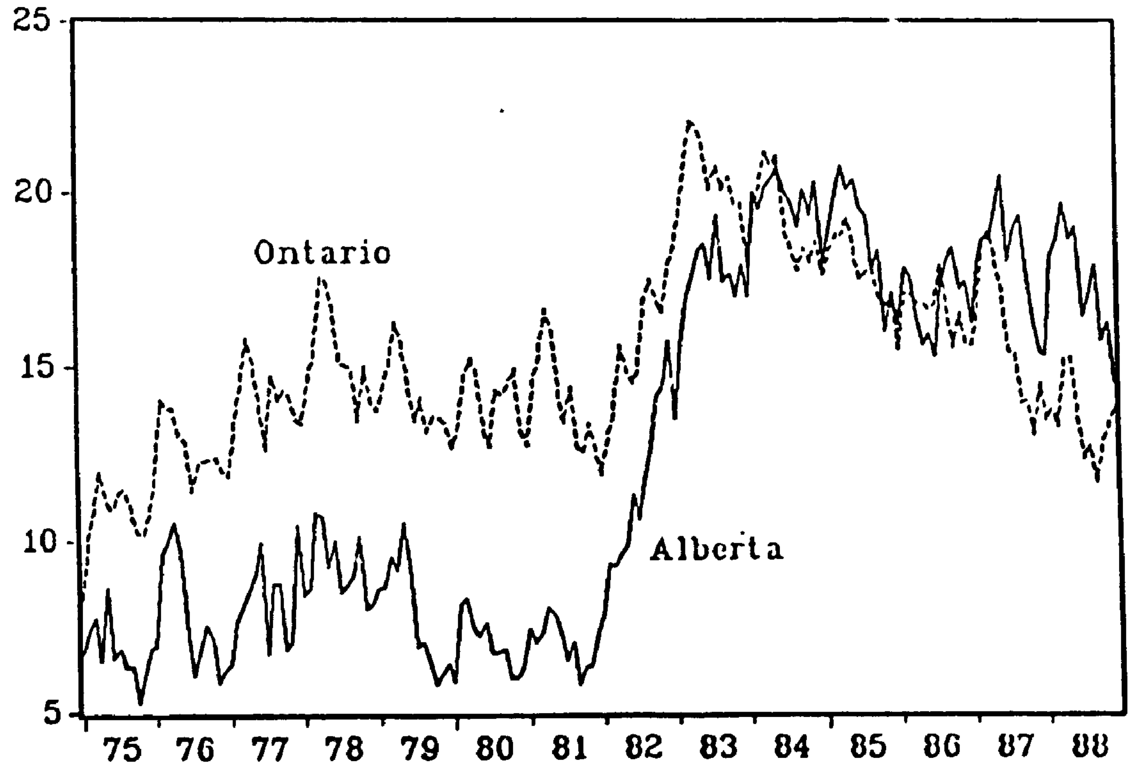

Figure Two: Average Unemployment Duration in Alberta and Ontario

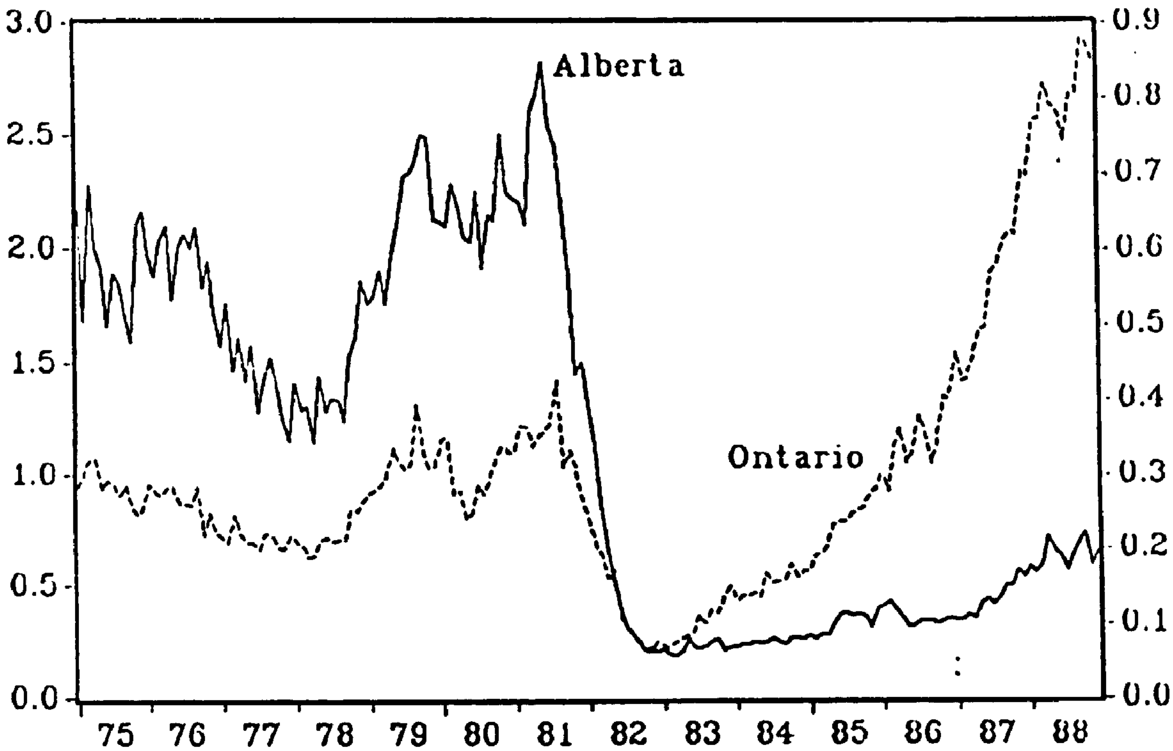

Figure Three: Labour Market Tightness in Alberta and Ontario

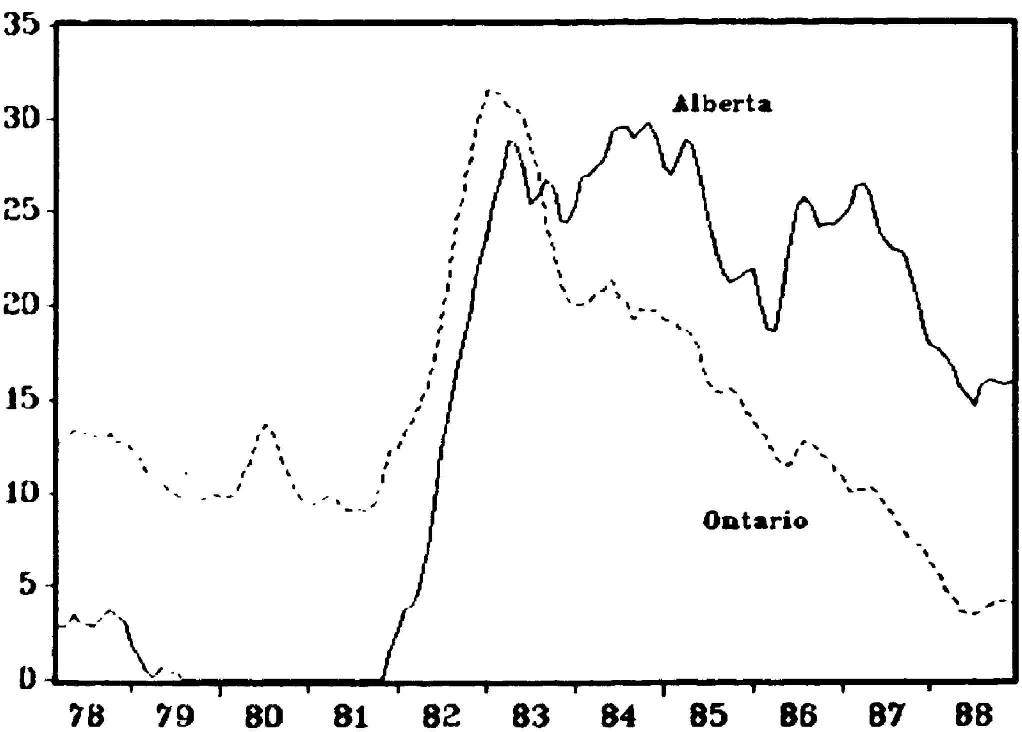

Figure Four: Regional Extended Benefit Weeks in Alberta and Ontario

5.4 An Empirical Account of Unemployment Rate Persistence

It is clear from the preceding analysis that disaggregation of Alberta and Ontario should be useful. This will be done here to assess the impact of regional extended benefits on the unemployment rate in Alberta and Ontario. To this end, the following reduced-form regression is useful:

$$UR_{i,t} = \alpha + \beta UR_{i,t-1} + \gamma_1 NB_{i,t} + \gamma_2 NB_{i,t-1} + \theta[HWI_{i,t}/U_{i,t-1}] + \epsilon_{i,t}.$$

This equation is closely related to the one used by MPS to test their theory. The dependent variable is the unemployment rate in province i. A lagged value is included to account for natural dynamics. The current and once-lagged values of the supplemental benefits variable, NB, are included to allow UIC policy to affect unemployment dynamics[1]. The market tightness measure is included here while MPS use a measure of monthly GDP to capture variations in economic activity. The market tightness measure seems to be preferable because it directly measures conditions in the provincial labour markets. The denominator of the tightness variable is lagged by one month to avoid possible simultaneity problems[2]. Finally, all variabies were adjusted for seasonal variation by Statistics Canada.

The regression above was estimated for Ontario and Alberta over the June 1978 to March 1988 sample period used by Milbourne, Purvis and Scoones to create their Table 2. The regression above is an "encompassing" model in that it allows for offer

[1] The number of weeks of total benefits is calculated from the formula:

$$NB = N + \max\{0, \min[32, 4(\bar{U} - 4)]\}$$

where $\bar{U} = (UR_{t-1} + UR_{t-2} + UR_{t-3})/3$. Technically, this is the number of benefit weeks for a minimally qualified unemployment insurance recipient. In the regression, N is not specified because it is absorbed into the regression constant α.

[2] The theory developed by Storer(1991) implies that the market tightness ratio HWI/U should respond only to the exogenous shocks. This relies upon an assumption of free entry to the posting of vacancies and constant returns to scale in the matching technology. In any case, the lagged value of the unemployment rate is used to avoid problems.

arrival, job rejection, and quit effects. Two restricted versions of this model are nested within this general specification. The first, with the restriction $\gamma_1 = \gamma_2 = 0$, is a pure offer arrival rate model. The second model only considers unemployment benefit effects and results when the restriction $\theta = 0$ is imposed. These two restrictions were tested for the two regressions. The regression results are presented in Table One.

These regression results show that the market tightness variable has a significant effect in both Alberta and Ontario. Only in Ontario, on the other hand, does the unemployment insurance variable show statistical significance. This means that the significant persistence observed in Alberta is largely due to offer arrival rate effects which can be linked to long-term resource price shocks. It is interesting to note the results in Table Two. Here the market tightness variable is removed from the long-sample regressions. In both provinces this raises the coefficient on the lagged unemployment rate. In Alberta, this increase is so large that the series appears to be non-stationary when the market tightness variable is ignored. It is worth stressing that this non-stationarity occurs *in spite of* the presence of the Regional Extended Benefits variable. While the Ontario regressions show that there is an unemployment insurance policy effect, the economic relevance of this is diminished by the fact that the Ontario unemployment rate does not display 'undue' persistence relative the the experience in the United States.

Finally, it is useful to obtain some measure of the quantitative significance of the two sets of variables. This is accomplished in Figures Five and Six which simulate the values of the provincial unemployment rate with either the market tightness or the benefit weeks series constrained to be at its sample mean. The first panel looks at Alberta and shows that holding the number of benefit weeks fixed moves the series further away from the actual than holding market tightness constant does. Note, however, that the effect on persistence is not strong. In Ontario, a similar pattern emerges. Here it is clear, too, that the benefit weeks series is more quantitatively important, even though it was judged to be less important on statistical grounds.

Table One: Encompassing Model Regression Results

	Alberta	Ontario
α	1.63	2.59
	(0.89)	(0.95)
β	0.84	0.50
	(0.18)	(0.27)
γ_1	0.02	0.31
	(0.09)	(0.11)
γ_2	-0.02	-0.20
	(0.05)	(0.07)
θ	-0.48	-0.88
	(0.14)	(0.26)
R^2	0.983	0.963
DW	1.81	2.21

χ^2 Test Statistics for Restrictions

	Alberta	Ontario
$H_o{:}\gamma_1 = \gamma_2 = 0$	0.22	11.12
P-value	(0.90)	(0.004)
$H_o{:}\theta = 0$	11.00	5.64
P-value	(0.001)	(0.02)

Table Two: Regressions with $\theta = 0$

	Alberta	Ontario
α	-0.59	1.75
	(0.61)	(0.90)
β	1.18	0.58
	(0.16)	(0.22)
γ_1	-0.07	0.30
	(0.09)	(0.12)
γ_2	0.02	-0.20
	(0.06)	(0.07)
R^2	0.980	0.960
DW	1.88	2.22

Note: Standard errors are in parentheses below parameter estimates.

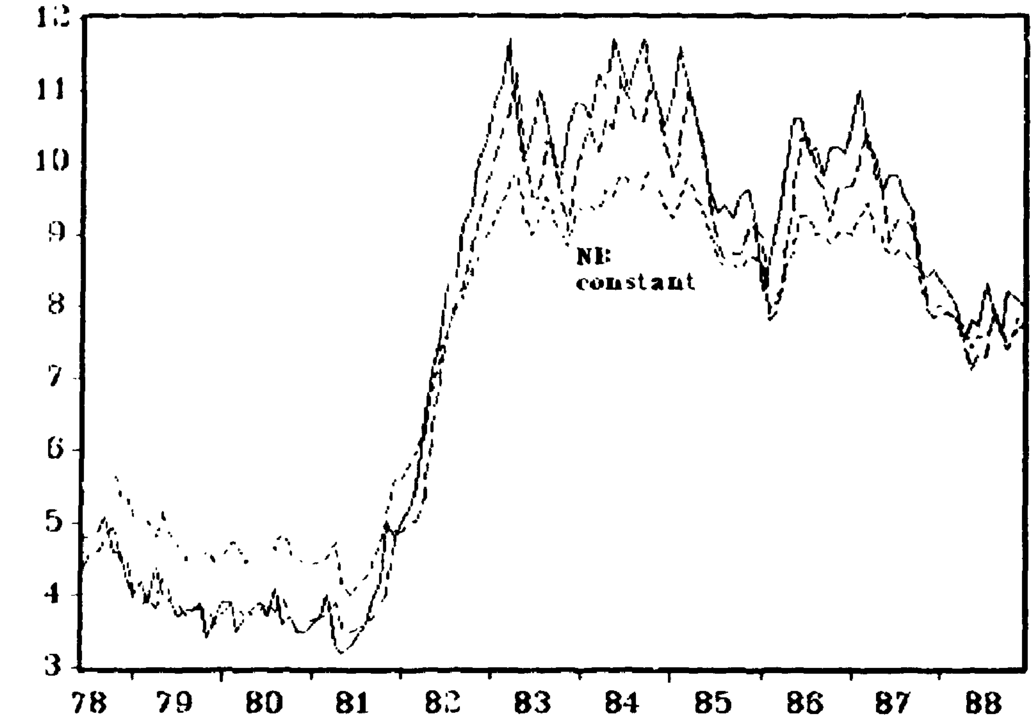

Figure Five: Simulated Unemployment in Alberta

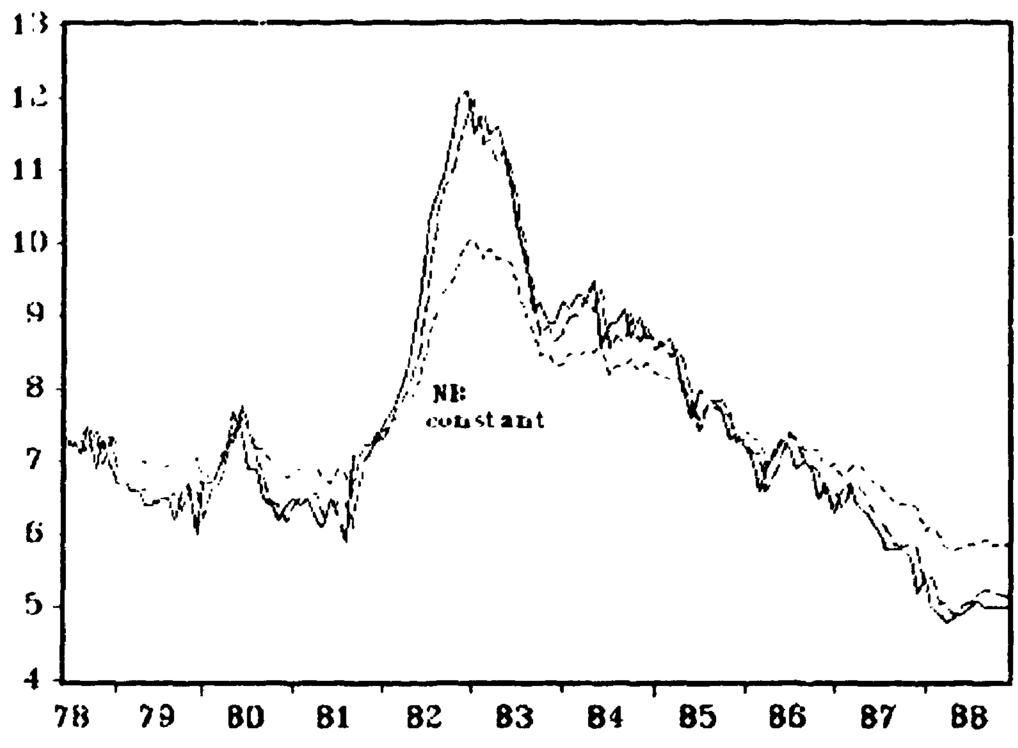

Figure Six: Simulated Unemployment in Ontario

5.5 The Information in Gross Employment Flows

To this point, the results of this chapter are unclear. The regressions suggest that market tightness accounts for more persistence but benefit weeks are quantitatively more important. To resolve this situation, it is useful to look at gross flows between labour market states for Alberta and Ontario. This is necessary to identify whether there have been differences in gross labour market flows for the two different areas. In particular, it is important to know whether the greater apparent degree of persistence in Alberta represents a steady flow into employment along with high flows out of employment or just slow rates of flow out of unemployment. It is then interesting to compare this with the experience in Ontario.

Figures Seven and Eight below show hazard rates calculated for the monthly flow from employment to unemployment and unemployment to employment for Alberta and Ontario. Hazard rates are calculated using the following formula:

$$HAZ_t^{EU} = \frac{EU_{t,t+1}}{E_t}; \qquad HAZ_t^{UE} = \frac{UE_{t,t+1}}{U_t},$$

where:

HAZ_t^{EU} is the average probability of going from employment at time t to unemployment at time $t+1$.

HAZ_t^{UE} is the average probability of going from unemployment at time t to employment at time $t+1$.

$EU_{t,t+1}$ is the total flow between employment at time t and unemployment at time $t+1$.

$UE_{t,t+1}$ is the total flow between unemployment at time t and employment at time $t+1$.

E_t is the total stock of employment at time t.

U_t is the total stock of unemployment at time t.

Monthly total flow data are estimates of labour force grows flows calculated by Statistics Canada using Labour Force Survey data. These estimates are available monthly from January 1976 until the present. There are two gaps in the data caused by interruptions to the data collection process at Statistics Canada. The calculated hazard rates are adjusted for seasonality (the raw data are highly seasonal) by estimating seasonal factors for 1976 to 1983 and applying these factors to the entire series. This method of adjustment is necessitated by the two gaps in the series.

Several interesting patterns emerge from these graphs. First, it is clear that since 1982 Alberta has suffered a persistent increase in the rate of flow out of employment and a decrease in the rate of flow into employment. In Ontario, the effect on these two series has been quite short-lived. The flow out of unemployment rose in 1982 but this rate was back at its previous level by 1984. A mirror image of this is presented by flow into employment which briefly fell but then rose back again.

Given these graphs it is clear that the dynamics of job creation and job destruction were quite different in these two provinces. Alberta seems to have suffered from permanent unfavorable shifts in these two series. Ontario, on the other hand, recovered very quickly from the shock of the recession. One way of interpreting these results is suggested by the effects of the oil price collapse that followed the 1982 recession. Weak world demand for energy during the recession lowered real oil prices following the 1982 recession and this may be thought of as a secondary wave of shocks from the recession. Alberta and Ontario are affected quite differently by such shocks. Alberta, as an oil producer, suffers when oil prices fall while the Ontario economy, which gives much more weight to manufacturing and the automobile industry, benefits from a fall in the price of oil.

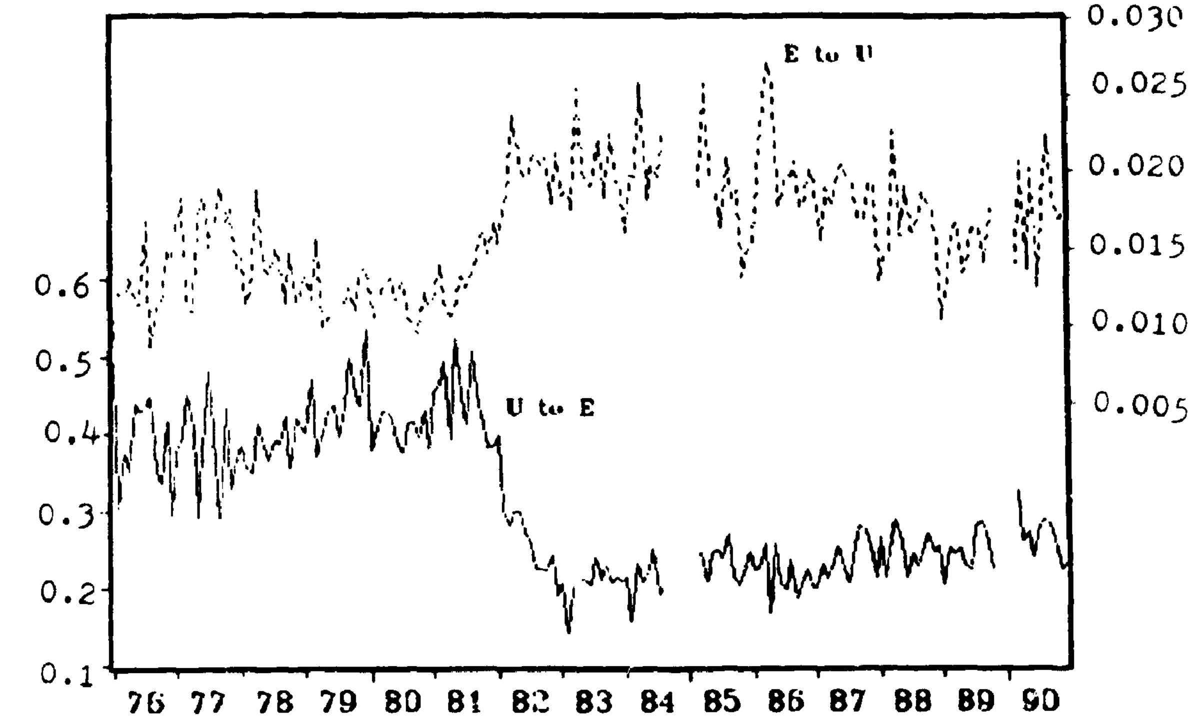

Figure Seven: Employment and Unemployment Hazards for Alberta

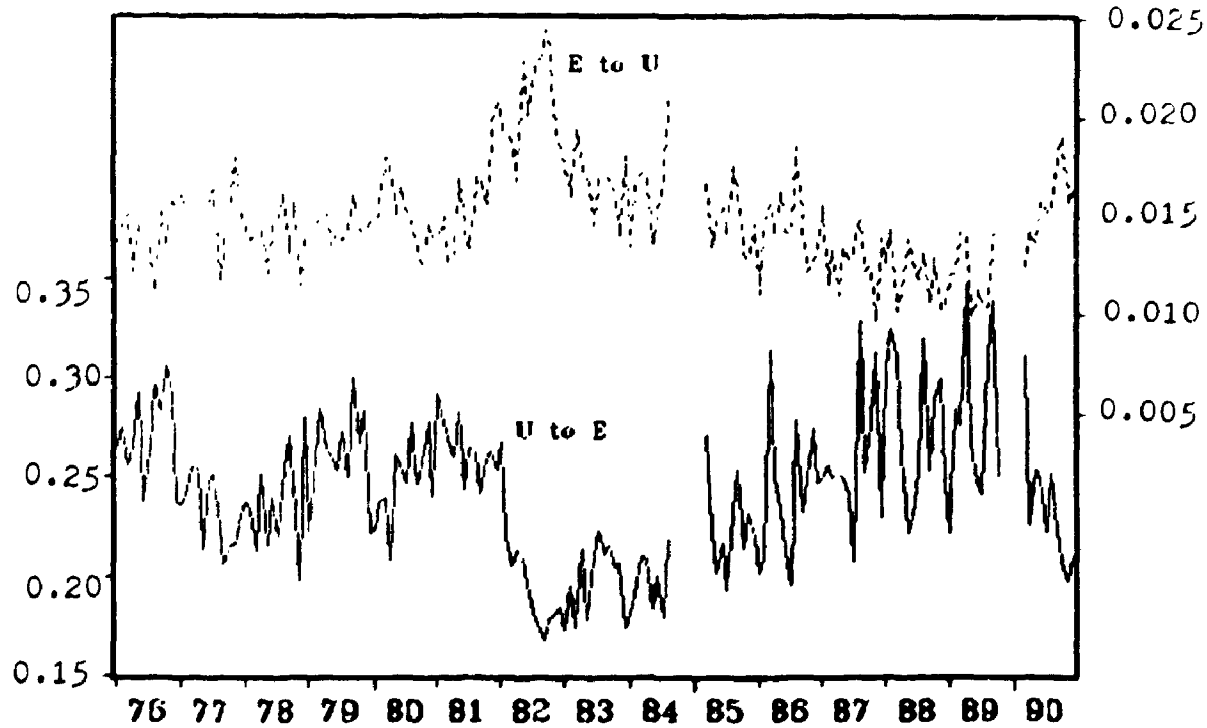

Figure Eight: Employment and Unemployment Hazards for Ontario

5.6 Determinants of Labour Market Transition Rates

The previous section revealed differences between the dynamics of labour-market hazard rates in Alberta and Ontario. Accordingly, instead of using the unemployment rate as the dependent variable in the regressions, it is useful to look at the direct linkages between benefit weeks, market tightness, and hazard rates for the flow from unemployment to employment and vice versa. The MPS model focusses on two routes by which an increase in benefit weeks can affect the unemployment rate: increase in unemployment due to a decline in transition probabilities (as documented by Ham and Rea) or flows into unemployment from those previously out of the labour force. The strength of this first effect can be examined directly using labour force gross flows data. A second possibility (mentioned by MPS in a footnote) is that flows from employment to unemployment might increase . This latter channel exists only when a waiting period is in place and there is no indication of whether currently observed waiting periods are long enough to matter.

It is clear that the hazard rate series shown in Figures Seven and Eight display patterns that are in reasonable accord with changes to the benefit week series. The rate of flow from unemployment to employment fell in both provinces in 1981-82 while the flow out of employment rose. This general impression is summarized by the top parts of the regression results in Tables Three and Four which explain the two hazard rates as functions of benefit weeks alone. Table Three looks at flows between unemployment and employment while Table Four looks at the reverse flow[3]. The net effect of benefit weeks is to lower escape rates from unemployment and raise job destruction rates in both provinces. It is important to further note, however, that the low Durbin-Watson statistics in these equations is suggestive of misspecification.

When these equations are modified by the inclusion of variables suggested by a

[3] These regressions were estimated over the period June 1978 – August 1984 plus March 1985 – December 1988. The gap in the sample is due to a hole in the gross flows series

matching function model the results change in a rather striking manner. In both provinces, the help-wanted index enters with a positive sign and the lagged unemployment rate enters with a negative sign. This is in accord with theoretical priors. The coefficients on the benefit weeks variables are perversely affected by the inclusion of market tightness variables, however. In the equations for the hazard rate into employment, benefit weeks have a net *positive* effect when market tightness variables are included in both Alberta and Ontario. Similarly, the flow out of employment is *decreased* by the benefits week series after controlling for recruitment activity variation.

These results suggest that the benefits weeks series may actually be proxying for recruitment and layoff effects in the regressions without market tightness variables. It may be that sectoral and aggregate shocks affect hiring and firing decisions first, that these then change employment levels, and that shortly afterward the benefit weeks series responds. An objective test of the timing in this story can be obtained by Granger causality tests in which vector auto-regressions of the following form are estimated:

$$HAZ = \alpha + A(L)HAZ + B(L)NB,$$

$$NB = \beta + C(L)HAZ + D(L)NB.$$

Here $A(L), B(L), C(L)$ and $D(L)$ are fourth-order polynomials in the lag operator and α and β are constants. The variable HAZ is either the employment to unemployment or unemployment to employment hazard rate. In other words, each variable in the vector autoregression is allowed to depend on its first four own lagged values and four lagged values of the other variable.

A variable x is said to be Granger caused by another variable y if lagged values of y provide information about the current value of x above that provided by lagged values of x itself. The idea is to test the null hypothesis that, for example, the employment to unemployment hazard is *not* Granger-caused by benefits weeks by testing the restriction that all the coefficients in $D(L)$ are zero. When these tests were applied, it was possible for both provinces to reject the null hypotheses that the benefits weeks are not Granger-caused by the two hazard rate series. On the other hand, for Ontario and Alberta it was impossible to reject the nulls that benefits weeks are not Granger-caused by either hazard rate. While it is always necessary to emphasise that Granger causality is not the same as structural causality, the timing revealed by these tests is consistent with a causal relationship form hazard rates to benefit weeks in preference to the reverse.

Table Three: Unemployment to Employment Hazard

a.) Regressions without Market Tightness Variables

	Alberta	Ontario
Constant	4.15×10^{-1}	2.90×10^{-1}
	(5.88×10^{-3})	(5.54×10^{-3})
NB	-1.76×10^{-2}	-9.43×10^{-3}
	(3.62×10^{-3})	(2.91×10^{-3})
NB(-1)	9.56×10^{-3}	5.70×10^{-3}
	(3.61×10^{-3})	(2.93×10^{-3})
$\bar{R}^2$	0.835	0.503
DW	1.30	1.80

b.) Regressions with Market Tightness Variables

	Alberta	Ontario
Constant	4.70×10^{-1}	3.36×10^{-1}
	(3.49×10^{-2})	(2.70×10^{-2})
NB	2.51×10^{-2}	3.41×10^{-3}
	(5.68×10^{-3})	(5.67×10^{-3})
NB(-1)	-1.36×10^{-2}	-6.20×10^{-5}
	(3.81×10^{-3})	(3.62×10^{-3})
HWI	1.67×10^{-3}	2.97×10^{-4}
	(2.7×10^{-4})	(1.2×10^{-4})
U(-1)	-4.75×10^{-3}	-5.07×10^{-4}
	(6.68×10^{-4})	(2.00×10^{-4})
$\bar{R}^2$	0.901	0.515
DW	2.14	1.97

Table Four: Employment to Unemployment Hazard

a.) Regressions without Market Tightness Variables

	Alberta	Ontario
Constant	1.29×10^{-2}	1.25×10^{-2}
	(4.01×10^{-4})	(4.24×10^{-4})
NB	9.11×10^{-4}	1.78×10^{-3}
	(4.01×10^{-4})	(2.22×10^{-4})
NB(-1)	-6.43×10^{-4}	-1.57×10^{-4}
	(2.47×10^{-4})	(2.24×10^{-4})
$\bar{R}^2$	0.557	0.502
DW	1.31	1.36

b.) Regressions with Market Tightness Variables

	Alberta	Ontario
Constant	1.19×10^{-2}	1.40×10^{-2}
	(2.84×10^{-3})	(1.97×10^{-3})
NB	-8.36×10^{-4}	9.60×10^{-4}
	(4.63×10^{-4})	(4.15×10^{-4})
NB(-1)	3.22×10^{-4}	-1.19×10^{-3}
	(3.11×10^{-4})	(2.65×10^{-4})
HWI	-7.90×10^{-5}	-3.45×10^{-5}
	(2.17×10^{-5})	(9.10×10^{-6})
U(-1)	1.84×10^{-4}	2.33×10^{-5}
	(5.44×10^{-5})	(1.47×10^{-5})
$\bar{R}^2$	0.621	0.562
DW	1.53	1.62

5.7 Conclusions

To summarize the results of this chapter, there is evidence that some regions of Canada experienced persistently high unemployment rates due to labour market tightness effects. These can be linked to resource price shocks. While there is evidence that unemployment rate policy has had an impact on unemployment rate dynamics, this does not seem to matter in the regions were persistence was most marked.

To conclude, there is strong evidence that shocks to certain regions of Canada have been persistent in nature and have had negative effects on the level of unemployment in Canada. Considering the lesser extent to which the United States is exposed to resource price shocks, it is no surprise that the unemployment rate has fallen more slowly in Canada. Note that Chapter Four of this thesis found that sectoral shocks between Ontario and Alberta are quite persistent; furthermore, the shock was in the favour of Ontario during much of the post-1982 recovery period. This lends credence to the belief that unfavorable shocks, rather than government policy account, for the persistence of the Canadian unemployment rate.

Appendix to Chapter One

Availability and Disposition of Data

Help Wanted Index: The old Help Wanted Index is available monthly from 1962 to 1988, inclusive. For data which are not adjusted for seasonal variation, it is necessary to consult the 1988 issue of Statistics Canada Publication 71-204. Data adjusted for seasonal variation are also available on CANSIM under the following identification numbers:

Atlantic Provinces	D736316
Quebec	D736317
Ontario	D736318
Prairie Provinces	D736319
British Columbia	D736320

Vacancy Survey (1972-78): Available in Statistics Canada Publication 71-203.

Canada Employment Centre Data: Currently only in hardcopy form on microfiche at the regional centre in Belleville.

Contact person: Mr. Dan Brady, MIS office, Belleville.

Unemployment Duration Data (as derived from the Labour Force Survey): Available in Statistics Canada Publication 71-001. Also available on CANSIM in unadjusted form under the following identification numbers:

Newfoundland	D773278
P.E.I.	D773279
Nova Scotia	D773280
New Brunswick	D773281
Quebec	D773282
Ontario	D773283
Manitoba	D773284
Saskatchewan	D773285
Alberta	D773286
British Columbia	D773287

Labour Market Activity Survey. Available on tape from Statistics Canada.

Appendix to Chapter 2

Proposition 1: *There is a unique steady-state equilibrium in the single-sector search model with permanent shocks.*

Proof: To formally establish existence and uniqueness, recall the implicit functions of θ in terms of N which were earlier depicted in Figure One:

$$\frac{p(\theta)}{s + p(\theta)} L = N,$$

and:

$$q(\theta) = \frac{\gamma(1 - \beta(1 - s))}{\beta(1 - k)} \frac{1}{F(N, \mu)}$$

Define $\theta^A(N)$ to be the value of θ which, for given N, satisfies the first equation. Similarly, let $\theta^B(N)$ be the value for the second relationship between employment and market tightness. The two functions $\theta^A(N)$ and $\theta^B(N)$ are continuous and defined over the domain $0 \leq N \leq (1/(1 + s)L$.

Next, define the difference function $D(N) = \theta^A(N) - \theta^B(N)$. It is possible to establish that $\lim_{N \to 0} D(N) = -\infty$ and $\lim_{N \to L/(1+s)} D(N) = \infty$. Continuity of the function $D(N)$ means that the intermediate value function can be invoked to prove that there exists a value of N such that $\theta^A(N) = \theta^B(N)$. This point is unique because the slopes determined above are never zero.

To establish that $\lim_{N \to 0} D(N) = -\infty$, notice that $\lim_{N \to 0} \theta^A(N) = 0$ because the steady state employment only approaches zero in the limit, by Assumption 2.5. On the other hand, $\lim_{N \to 0} \theta^B(N) = \infty$ because $\lim_{N \to 0} F(N) = \infty$ by Assumption 2.2. This means that $\lim_{N \to 0} q(\theta) = 0$ which occurs for $\theta = \infty$ by assumption 2.6.

A similar analysis shows that $\lim_{N \to L/(1+s)} D(N) = \infty$. First, note that:

$$\lim_{N \to L/(1+s)} \theta^A(N) = \infty$$

because Assumption 2.5 states that $p(\theta) < 1$ for all $\theta < \infty$. On the other hand,

$$0 < \lim_{N \to L/(1+s)} \theta^B(N) < \infty$$

by Assumption 2.6 and the fact that $0 < \frac{\gamma(1-\beta(1-s))}{\beta(1-k)} \frac{1}{F(N,\mu)} < 1$ in order for profits to be zero.

∎

Proposition 2: *There is a steady-state equilibrium allocation of labour in this two-sector search model. Moreover, this equilibrium is unique.*

Proof: To establish this proposition, it is easiest to examine two separate single-sector economies. This is a valid exercise since in the steady state of the two sector economy the labour force is constant in each sector. Consequently, an equilibrium exists if it is possible to find two single-sector economies with labour forces L_1 and L_2 such that (i) the economies are in single sector steady state, (ii) there is no incentive to migrate, and (ii) $L_1 + L_2 = L$.

The sense of this proof will be to show that if either sector gets almost all of the total labour force, then θ is very high in that sector and $F(\mu, N)$ is very low so that the expected discounted return to searching in that sector would be very high. The converse would. of course, be true in the other sector. In other words, there exists values of L_1 and L_2 such that migration is desirable in either direction. Continuity of the reward functions in the labour force then implies that there will be an interior allocation of the labour force where the sectoral returns to searching are equalised.

Consider first single-sector steady states with arbitrary labour forces L_1 and L_2. For any value of the sectoral labour force there are corresponding steady-state values of employment and market tightness which are appropriate for a single-sector economy. The effect of varying the labour force on this equilibrium manifests itself in Figure One as a shift in the vertical asymptote of the "labour supply" curve. Lowering the labour force moves the asymptote to the left while raising L shifts the asymptote out to the right. As the asymptote moves toward the origin employment approaches zero so that the value of production gets very large due to assumption 2.2. Zero profits for vacancies then implies that market tightness becomes very high by virtue of assumption 2.6 regarding the shape of $q(\theta)$. Conversely, as the asymptote moves away from the origin the point of intersection with the "labour demand" curve will be further to the right which implies a higher employment level and a lower level of market tightness. Summarising this:

$$\text{If} \quad L_i \to 0 \quad \text{then} \quad N_i \to 0 \quad \text{so that} \quad \theta_i \to \infty.$$

$$\text{Conversely, if} \quad L_i \to L \quad \text{then} \quad N \to N^+ \quad \text{so that} \quad \theta_i \to \theta^- < \infty.$$

Next, consider the equality which must hold for migration to not occur:

$$\frac{(r+s)b + p_1(\theta_1)kF(N_1, \mu_1)}{r + p_1(\theta_1) + s} = \frac{(r+s)b + p_2(\theta_2)kF(N_2, \mu_2)}{r + p_2(\theta_2) + s}.$$

This condition states that the values of remaining forever in each of the two sectors must be equalised if this type of immobility is to be optimal. It is useful to define this constant value for a given sector as:

$$\bar{V}_i^U = \frac{(r+s)b + p_i(\bar{\theta}_i)kF(\bar{N}_i, \bar{\mu}_i)}{r + p(\theta_i) + s}.$$

Now, $0 \leq p_i(\bar{\theta}_i) \leq 1$ so that as L_i varies between 0 and L, $p_i(\theta_i)$ ranges from one to some value less than one. Given this and Assumption 2.2, it is true that for any combination (μ_i, μ_j):

$$\bar{V}_i^U < \bar{V}_j^U \quad \text{as} \quad L_i \to L,$$

and:

$$\bar{V}_i^U > \bar{V}_j^U \quad \text{as} \quad L_j \to L.$$

Thus defining a function which measures departures from equality of values of searching in a sector:

$$\bar{D}(L_i, L_j) := \bar{V}_i^U(L_i) - \bar{V}_j^U(L_i),$$

it is clear that $\lim_{L_i \to L} D(L_i, L_j) > \lim_{L_i \to 0} D(L_i, L_j)$. The intermediate value theorem can be applied here because $D(L_i, L_j)$ is a continuous function so that there is a value of L_i (i.e. a partition of the labour force between sectors) such that there is no difference in search values. Finally, note that $D(L_i, L_j)$ is strictly decreasing in L_i so that this steady state equilibrium is unique.

■

Proposition 3: *In a single sector economy with no migration the value of a producing firm , $V^P(N,\mu)$ is falling in employment (N).*

Proof: The proof proceeds by noting that the system of equations for V^P and V^O the rule $\Theta(N,\mu)$ define an operator T_Θ which is a contraction mapping. Completeness of the metric space on which T_Θ operates is then invoked to argue that there is a unique fixed point to the operator and that iterations of T_Θ from any arbitrary starting vector will converge to this unique fixed point.

To establish this argument, note that T_Θ is a mapping from $C \times C$ to itself. For this space, the usual metric is the sup norm metric defined by $\rho(X,Y) = \sup|X - Y|$ Under this definition, it can be shown that (using the short hand notation: $z = (N,\mu)$):

$$T_\Theta V^P(z) = (1-k)F(z) + \beta \sum_{i=1}^{n} (1-s)V^P(z')\Pr(z'|z).$$

Given this,

$$\rho\left[T_\Theta X^P(z), T_\Theta Y^P(z)\right] = \beta(1-s)\sup_z\left\{\left|\sum_{i=1}^{n}\left(X^P(z') - Y^P(z')\right)\Pr(z'|z)\right|\right\}.$$

while:

$$\rho\left[X^P(z), Y^P(z)\right] = \sup_z |X^P(z) - Y^P(z)|.$$

It follows immediately then that

$$\rho\left[T_\Theta X^P(z), T_\Theta Y^P(z)\right] \leq \beta\rho\left[X^P(z), Y^P(z)\right].$$

where $\beta < 1$. By definition, the operator T_Θ is a contraction mapping.

Next, since C and the supnorm metric is a complete metric space, the contraction mapping theorem can be invoked to show that (i) there is a unique fixed point to the operator T_Θ, and (ii) for any arbitrary vector (X^P), repeated application of T_Θ will converge toward the fixed point. This is useful because it means that starting with a V^P function that meets the sign assumptions of the proposition does not prevent iterations of T_Θ from converging to the fixed point, regardless of the correctness of the original assumptions.

First it will be shown that:

i) The value of producing, $V^P(N, \mu)$, decreases with the level of employment, N. To show this it is necessary to consider, in turn, two exhaustive cases which may hold at each point (N, μ) in the domain of the value function.

Case a: When N rises N' falls.

To see how this could occur, consider the law of motion for employment in this economy:

$$N_{t+1} = (1-s)N_t + p(\Theta(N_t, \mu_t))(L - N_t). \tag{A-1}$$

Now, if an increase in N_t is to lower N_{t+1}it must be true that at least one of $s + \lambda(N_t, \mu_t) > 1$ or $\partial\Theta(N_t, \mu_t)/\partial N_t < 0$ holds. The first condition will not hold for a period of analysis that is sufficiently short and it is always possible to choose the basic unit of time accordingly.

To see when the second condition holds, consider that the zero-profit condition implies that the following expression equals zero:

$$\beta q(\theta) \sum_{i=1}^{n} V^P((1 - s - p(\theta_t))N_t + p(\theta_t)L, \mu_i)\Pr(\mu_i|\mu_t). \tag{A-2}$$

Given the signs of $p(\theta)$ and $q(\theta)$ it is clear that this will only occur if $V^P(N, \mu)$ is decreasing in N. In this case, the proposition is proved trivially.

Case b: When N rises N' rises.

In this case it is not possible to put restrictions of the relationship between $\Theta(N, \mu)$ and N. Instead, it is necessary to examine the value function directly. Writing this out in detail gives:

$$T_\Theta V^P(N_t,\mu_t) = (1-k)F(N_t,\mu_t) + \beta\sum_{i=1}^{n}(1-s)V^P\left[(1-s)N_t + \Theta(N_t,\mu_t)(L-N_t),\mu'_i\right]\Pr(\mu'|\mu_t). \tag{A-3}$$

In the case at hand, N_{t+1} rises with N_t regardless of the effect of N_t on $\Theta(N_t,\mu_t)$. It is thus possible to sign $V^P(N,\mu)$ by starting with an initial guess for $V^P(N,\mu)$ which is decreasing in N. Applying the operator T_Θ to this yields a function which is also decreasing in N because $F(N,\mu)$ is decreasing in N and the second term in (A–2) is decreasing in N by assumption. Repeated applications of T_Θ will also yield functions decreasing in N and so the limit of this procedure, the fixed point of T_Θ, will be $V^P(N,\mu)$ and it will be non-increasing in N. This argument holds for any arbitrary point in the domain of the value function and thus holds for all of them.

∎

Proposition 4: As a results of migration during the adjustment period, employment falls in sector two and rises in sector one.

Proof: For sector two, note that the law of motion is:

$$\Delta N_{2,t} = p_2(\theta_{2,t-1})U^P_{2,t-1} - sN_{2,t-1} \quad \forall \quad t \in (t_0, T].$$

Initially, before t_0, this expression is zero by the definition of a steady state. Once adjustment begins, the left-hand-side falls because migration shrinks U_2^P. This means that $\Delta N_{2,t} < 0$ after t_0. As employment falls, the rate of outflow from employment, sN_2, falls while θ_2 rises during the adjustment period. This means that the net change in employment is negative during the adjustment period but becomes progressively smaller in absolute value.

The law of motion for sector one employment is:

$$\Delta N_{1,t} = p_1(\theta_{1,t-1})N_{1,t-1} - sN_{1,t-}\ .$$

As for sector two, this expression is initially zero. At time t_0+1, however, the expression becomes positive because θ_1 rises at t_0 and U_1^P rises at t_0 due to migration. Thus, employment rises through the adjustment period in sector one.

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Appendix to Chapter Three

Appendix A: Data Definitions and Sources

1. Time Series Observations of Hazard Rates

Source: Statistics Canada.

Seasonal Adjustment: Not adjusted for seasonal variation by Statistics Canada. Data used for analysis in the paper were seasonally adjusted with the X-11 adjustment routine in MicroTSP. Seasonal adjustment was applied to the constructed hazard series rather than to the two original series separately. Due to the existence of gaps in the series, seasonal factors were first calculated for the longest contiguous series of observations and then used to adjust the entire series.

Two series were used to calculate the hazard rate:

The estimated flow from unemployment to employment from month t and month $t+1$ ($UE_{t,t+1}$).

The estimated stock of unemployed at time t (U_t).

From these, the hazard rates were calculated as:

$$HAZ_t = \frac{UE_{t,t+1}}{U_t}$$

Flows and stocks used were for all ages groups and for both men and women. It is worth noting that for the three provinces studied here these data are available broken down by sex and for workers in the two age groups 15 to 24 years and 25 years or over.

2. Time Series Vacancy Measures

Source: Statistics Canada.

Series Title: Help Wanted Index ("old" index), Monthly.

Seasonal Adjustment: Not adjusted for seasonal variation.

Data were transcribed from Tables 9, 10, and 12 of Statistics Canada publication 71-204.

Units were associated with these HWI series by using the numbers of total vacancies (i.e long and short term, for full and part-time jobs) indicated by the Vacancy Series data. The new series has the same monthly growth rates as the HWI series but has a multiplicative level adjustment so that the average level of the adjusted HWI series for 1977 is the same as that for the actual Vacancy Survey.

3. Time Series Unemployment Measures

Source: Statistics Canada.
Series Title: Unemployed, 15 years and over (in thousands).
Seasonal Adjustment: Not adjusted for seasonal variation.
Cansim series numbers:

British Columbia	D769170
Ontario	D768585
Alberta	D768415

Appendix B: Construction of Spline Regressors

As explained in the text, the linear spline functions fitted in section five allow for a piece-wise linear relationship between the average transition rate into employment and the tightness of recruitment in the labour market. To estimate such a regression it is necessary to ensure that the estimated relationship has a slope that may vary over the domain of the independent variable and is a continuous function. A simple dummy variable technique can meet the first requirement but care must be taken to ensure that continuity also holds.

To appreciate how this is done, consider a simple linear spline regression with a slope that can take on four values. The break points for the series will be at values X_1, X_2 and X_3. Dummy variables can be defined as follows:

$$D_2 = \begin{cases} 0, & \text{if } \; x < X_1; \\ x - X_1, & \text{otherwise.} \end{cases}$$

$$D_3 = \begin{cases} 0, & \text{if } \; x < X_2; \\ x - X_2, & \text{otherwise.} \end{cases}$$

$$D_4 = \begin{cases} 0, & \text{if } \; x < X_3; \\ x - X_3, & \text{otherwise.} \end{cases}$$

The desired result is obtained by running the following regression:

$$Y = \alpha + \beta_1 X + \beta_2 D_2 + \beta_3 D_3 + \beta_4 D_4$$

If the slopes over the four intervals are designated by S_1, S_2, S_3, S_4 then the following relationships hold

$$\beta_1 = S_1,$$
$$\beta_2 = S_2 - S_1,$$
$$\beta_3 = S_3 - S_2,$$
$$\beta_4 = S_4 - S_3,$$

The form of the D_i variables ensures that the fitted relationship is continuous. To appreciate why, consider the value of the fitted value at the point X_2. Just to the left of X_2, the definition of the fitted value is:

$$\begin{aligned} Y &= \alpha + \beta_1 X + \beta_2 D_2, \\ &= \alpha + \beta_1 X + \beta_2 (X - X_1) \\ &= (\alpha - \beta_2 X_1) + (\beta_1 + \beta_2) X. \end{aligned}$$

Just to the right of X_2 an additional term is non-zero so that the fitted value is:

$$\begin{aligned} Y &= \alpha + \beta_1 X + \beta_2 D_2 + \beta_3 D_3, \\ &= \alpha + \beta_1 X + \beta_2 (X - X_1) + \beta_3 (X - X_2) \\ &= (\alpha - \beta_2 X_1 - \beta_3 X_2) + (\beta_1 + \beta_2 + \beta_3) X. \end{aligned}$$

Now, the limit from the left as X approaches X_2 is just $\alpha + \beta_1 X_1 + (\beta_1 + \beta_2) X_2$ while the limit from the right is exactly the same because two $\beta_3 X_2$ terms cancel each other. Accordingly, the spline function is continuous at the point X_2. Indeed, the spline regressors were constructed to ensure this and the result accordingly also holds for other points where the slope changes.

A final note is that the regression specified here was modified slightly to make hypothesis testing easier. In particular, the regression was transformed so that the slope parameters for each region were estimated directly. This meant that simple sign restrictions could be used in the constrained least-squares routine DBCLSJ. The transformation uses the fact that, given the relationship between the slopes S_i's and the β_i's, the following holds:

$$\begin{aligned} Y &= \alpha + S_1 X + (S_2 - S_1) D_2 + (S_3 - S_2) D_3 + (S_4 - S_3) D_4, \\ &= \alpha + S_1 (X - D_2) + S_2 (D_2 - D_3) + S_3 (D_3 - D_4) + S_4 D_4. \end{aligned}$$

Estimation of this regression with transformed dummy variables allows for direct application of non-negativity constraints on the piece-wise linear regression.

Appendix C: Construction of Isotonic Regressions

The regressions with monotonicity and concavity imposed are estimated by solving the following problem:

$$\min_{\hat{y}}(y - \hat{y})'(y - \hat{y})$$

subject to :

$$A\hat{y} \geq 0$$

Here y is an n-vector of observations on an independent variable and $\hat{y}$ is the vector of fitted values for y. The matrix A is an ncon x n matrix which expresses the linear inequality constraints imposed on the estimation problem. For both monotonicity alone, and monotonicity plus concavity, ncon is $n-1$. While the number of constraints is the same in both cases it will always be the case that the constraints will be at least as active in the data when monotonicity and concavity are imposed

To impose the constraints, the pairs of (x_i, y_i) observations are sorted into increasing order according to the size of x_i. Monotonicity then implies that $\hat{y}_{i+1} \geq \hat{y}_i$ for $i = 1, \ldots, n-1$ (i.e. n-1 constraints). Accordingly, the first row of the A matrix is

$$(-1, \quad 1, \quad 0, \quad \ldots, \quad 0)$$

and the remaining n-1 rows are similarly determined.

For a concavity constrair' define the set of λ_i's defined by $x_{i+1} = \lambda_i x_i + (1 - \lambda_i)x_{i+2}$. Concavity then requires that $\hat{y}_{i+1} \geq \lambda_i \hat{y}_i + (1 - \lambda_i)\hat{y}_{i+2}$. If row 2 of the A matrix is:

$$(-\lambda_1, \quad 1, \quad -(1 - \lambda_1), \quad 0, \quad \ldots, \quad 0).$$

and the following n-3 rows of the matrix are determined in a similar fashion it is possible to also impose monotonicity with a single row of the matrix. This row is:

$$(0, \quad 0, \quad \ldots, \quad -1, \quad 1).$$

This first row imposes the condition that $\hat{y}_n \geq \hat{y}_{n-1}$ so that non-decreasing monotonicity holds at the end of the sample. Given that concavity will assure that the slope of the relationship is non-increasing this terminal condition ensures that the slope is non-negative for all x values. Thus, concavity and monotonicity are imposed with one row of monotonicity constraints and $n - 2$ rows of concavity constraints.

Appendix to Chapter Four

Proposition 1: *The value function V^P is increasing in μ, and decreasing in N.*

Proof: The proof begins by noting that the equation for V^P and the rule $\Theta(N,\mu)$ define an operator T_Θ which is a contraction mapping. Completeness of the metric space on which T_Θ operates is then invoked to argue that there is a unique fixed point to the operator and that iterations of T_Θ from any arbitrary starting vector will converge to this unique fixed point. This argument will hold for any arbitrary $\Theta(N,\mu)$ rule so that it can be applied without making advance assumptions on the properties of the market tightness rule.

To establish this argument, note that T_Θ is a mapping from C to itself. For this space, the usual metric is the sup norm metric defined by $\rho(X,Y) = \sup|X-Y|$ Under this definition, it can be shown that (using the short hand notation: $z = (N,\mu)$):

$$T_\Theta V^P(z) = (1-k)F(z) + \beta\sum_{i=1}^{n}(1-s)V^P(z')\Pr(z'|z).$$

Given this,

$$\rho\left[T_\Theta X^P(z), T_\Theta Y^P(z)\right] = \beta(1-s)\sup_z\left\{\left|\sum_{i=1}^{n}\left(X^P(z') - Y^P(z')\right)\Pr(z'|z)\right|\right\}.$$

while:

$$\rho\left[X^P(z), Y^P(z)\right] = \sup_z|X^P(z) - Y^P(z)|.$$

It follows immediately then that

$$\rho\left[T_\Theta X^P(z), T_\Theta Y^P(z)\right] \leq \beta\rho\left[X^P(z), Y^P(z)\right].$$

where $\beta < 1$. By definition, the operator T_Θ is a contraction mapping.

Next, since C and the supnorm metric is a complete metric space, the contraction mapping theorem can be invoked to show that (i) there is a unique fixed point to the operator T_Θ, and (ii) for any arbitrary vector (X^P), repeated application of T_Θ will converge toward the fixed point. This is useful because it means that starting with a V^P function that meets the sign assumptions of the proposition does not prevent iterations of T_Θ from converging to the fixed point, regardless of the correctness of the original assumptions.

First it will be shown that:

i) The value of producing, $V^P(N,\mu)$, decreases with the level of employment, N. To show this it is necessary to consider, in turn, two exhaustive cases which may hold at each point (N,μ) in the domain of the value function.

Case a: When N rises N' falls.

To see how this could occur, consider the law of motion for employment in this economy:

$$N_{t+1} = (1-s)N_t + p(\Theta(N_t, \mu_t))(L - N_t). \tag{A-1}$$

Now, if an increase in N_t is to lower N_{t+1}it must be true that at least one of $s + \lambda(N_t, \mu_t) > 1$ or $\partial\Theta(N_t, \mu_t)/\partial N_t < 0$ holds. The first condition will not hold for a period of analysis that is sufficiently short and it is always possible to choose the basic unit of time accordingly.

To see when the second condition holds, consider that the zero-profit condition implies that the following expression equals zero:

$$\beta q(\theta) \sum_{i=1}^{n} V^P((1 - s - p(\theta_t))N_t + p(\theta_t)L, \mu_i)\Pr(\mu_i|\mu_t). \tag{A-2}$$

Given the signs of $p(\theta)$ and $q(\theta)$ it is clear that this will only occur if $V^P(N, \mu)$ is decreasing in N. In this case, the proposition is proved trivially.

Case b: When N rises N' rises.

In this case it is not possible to put restrictions of the relationship between $\Theta(N, \mu)$ and N. Instead, it is necessary to examine the value function directly. Writing this out in detail gives:

$$T_\Theta V^P(N_t, \mu_t) = (1-k)F(N_t, \mu_t) \\ + \beta \sum_{i=1}^{n} (1-s) V^P \left[(1-s)N_t + \Theta(N_t, \mu_t)(L - N_t), \mu_i'\right] \Pr(\mu'|\mu_t). \tag{A-3}$$

In the case at hand, N_{t+1} rises with N_t regardless of the effect of N_t on $\Theta(N_t, \mu_t)$. It is thus possible to sign $V^P(N, \mu)$ by starting with an initial guess for $V^P(N, \mu)$ which is decreasing in N. Applying the operator T_Θ to this yields a function which is also decreasing in N because $F(N, \mu)$ is decreasing in N and the second term in (A-2) is decreasing in N by assumption. Repeated applications of T_Θ will also yield functions decreasing in N and so the limit of this procedure, the fixed point of T_Θ, will be $V^P(N, \mu)$ and it will be non-increasing in N. This argument holds for any arbitrary point in the domain of the value function and thus holds for all of them.

ii.) The value of producing, $V^P(N, \mu)$, increases with the current value of the shift term μ. To show, this, it is again useful to consider two cases.

Case a: When μ rises N' rises.

In this case it must be true that $\Theta(N, \mu)$ rose with μ since nothing else in the law of motion for employment (A-1) is affected by μ. In this case, $\Theta(N, \mu)$ must be rising in μ since the zero profit condition (A-3) implies this.

Case b: When μ rises N' falls.

In this case, it can be shown that the value function rises with μ through a contraction mapping argument. In equation (A-3), it is possible to take any arbitrary starting form for V^P and so one which is increasing in μ is legitimate. Application of the operator T_Θ yields a new function that is also increasing in μ because the current period return rises with μ and N' falls with μ and this raises the future period return by virtue of the property of the value function with respect to employment that was established above.

■

Proposition 2: *The market tightness function, $\Theta(\mathbf{z}_t; \mathbf{z}_{j,t})$ is increasing in μ_t and decreasing in N_t.*

Proof: Consider the expected return to posting a vacancy:

$$\beta q(\theta) \sum_{i=1}^{n} V^P(\mu_i, (1 - s - p(\theta_t))N_t + p(\theta_t)L_t, L_t + M_t)\Pr(\mu_i|\mu_t).$$

From this expression and the properties of V^P in Proposition 1, the following relationships can be established:

(i) expected returns to a vacancy rise with μ,

(ii) expected returns to a vacancy fall with N,

(iii) expected returns to a vacancy falls with θ. Now, (i) and (ii) imply that when μ rises, θ must fall for expected profits from posting a vacancy to remain constant. Next, (ii) and (iii) imply that n and θ must move in opposite directions to maintain the zero profit condition. This establishes the assertion of Proposition 2.

■

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