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Essays On Hysteresis In Trade And Exchange Rate Pass-through

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by

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Abstract

Chapter 1 of this thesis derives the long run general equilibrium of a home economy in a two-country overlapping generations model where agents' participation in production and the exchange rate are endogenous. We find that the presence of multiple equilibria is crucial for the existence of hysteresis defined as a change in the market structure of import or export markets, e.g. a change in the number of incumbent producers. With the presence of a positive network externality, the model will guarantee the possibility of hysteresis, i.e. multiple equilibria. Moreover, welfare is unambiguously increasing in the number of producers.

Chapter 2 applies the model developed in Chapter 1 to study exchange rate pass-through issues under different exogenous shocks. In additon, we discuss the hysteresis effects of temporary tariff protection. Exchange rate pass-through will generally be complete (incomplete) with (without) the entry or exit of foreign firms. Numerical examples are constructed to illustrate the predictions of the model. These examples demonstrate how hysteresis emerges and the related equilibrium responses of export prices, and the exchange rate.

Chapter 3 extends Chapter 1 by introducing the government into the model. We also study the impacts of a monetary shock to the economy. Similarly, the monetary shock (a shock to monetary growth rate) can cause hysteresis in this economy. We also construct numerical examples to compare the case of a monetary shock with the case of a real shock.

Chapter 4 contains an empirical test of Baldwin's hypothesis of hysteresis by using data on Canadian exports. We estimate the markup coefficient for the exports of each of five industries and test the hypothesis that hysteresis is present by examining the stability of these markup coefficients. Since we cannot reject the null hypothesis that the markup coefficient is stable for each industry, we find no evidence of hysteresis related to the exports of these Canadian industries.

I would like to dedicate this thesis to my late father, Chow, Wing-Lam from whom I got love and support throughout my youthful days.

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Any errors or omissions are of course mine.

Contents

Chapter 1

Can There Be Hysteresis in a General Equilibrium Model of Trade and the Exchange Rate ?

1.1 Introduction

The fact that the US trade deficit was slow to respond to the fall of the U.S. dollar in 1985 has motivated studies of the pass-through of exchange rates to domestic prices. The incomplete pass-through phenomenon is common among manufactured products where an imperfectly competitive market structure is a major feature. In the exchange rate pass-through literature, one phenomenon called 'hysteresis' gives fundamental reasons for the presence of incomplete pass-through. 'Hysteresis' is generally defined as the loss of capacity of a system to return to its original state. In the economics literature, Phelps (1979) defines the concept of 'hysteresis' to be "the dependence between two variables upon the past behavior of one or both of them"[1]. It is unpleasant that the equilibrium of the economy will depend on the initial conditions of the economy. It directly suggests the presence of instability. On the other hand, the presence of hysteresis in trade suggests that sufficiently large *temporary* exchange rate changes can have persistent real effects in the form of *permanent* changes in the market structure following an increase or decrease in

[1]See p. 104 of Phelps (1979).

the number of incumbent foreign suppliers.

Baldwin (1986, 1988a,b) develops the idea that a temporary swing in the exchange rate can cause hysteresis in trade when there are sunk costs in market entry. In his papers, a temporary rise in the value of domestic currency induces foreign firms to enter the domestic market. After the domestic currency returns to its original value, not all foreign firms will leave the domestic market. Therefore, the market structure will change altering the relationship between the exchange rate and import prices. This theory provides a testable hypothesis; we can apply a simple test (i.e. a Chow test), to identify a structural break in the parameter of profit margins if the number of foreign firms is not explicitly incorporated in the model. Such a break in profit margins is due to the permanent change in the number of incumbent suppliers. Baldwin (1988) finds that structural changes do exist in U.S. import markets.

Dixit (1989a,b) studies a similar problem borrowing tools like option pricing, and stochastic optimal control from financial economics[2]. He derives a partial equilibrium analysis of the industry with the presence of hysteretic bands. Dixit (1989b) and Baldwin (1988) suggest that outside firms are holding options to enter the market, equivalent to holding call options in acquiring the status of incumbent firms. If the exchange rate rises above the critical value for entry, the outside firms will exercise their call options. Similarly, the incumbent firms are also holding options to exit the market, similar to put options of relinquishing the status of incumbent firms and becoming outsiders. If the exchange rate falls below the critical value for exit, the incumbents will exercise their put options.

The models of Dixit (1989a, b), and Baldwin (1988a, b) are basically partial equilibrium models and the exchange rate is given exogenously. To remedy this shortcoming, Baldwin and Krugman (1989) consider the feedback from hysteresis in

[2]For details of such techniques, refer to Merton (1971, 1973), Black and Scholes (1973), especially Brennan and Schwartz (1985), and Constantinides (1986). Actually, Dixit's approach is not much different from Baldwin's. The Dixit model is in continuous time, and the Baldwin model is in discrete time.

trade to the exchange rate. However, the basic model that they use is still a partial equilibrium model. Hysteresis will arise when the exchange rate shocks which are created by the inflows and outflows of capital are sufficiently large. The changes in market structure resulting from more foreign firms entering the domestic market will tend to create a deficit in the current account. The presence of hysteresis also explains why the US trade deficit was so persistent in the eighties. The changes in the current account deficit will finally feed back to the exchange rate and bring the economy to a different equilibrium level. This gives us a prediction about the time path of the exchange rate: A large capital flow will produce a temporary movement of the exchange rate in one direction, followed by a *shift of the mean* of the exchange rate in the opposite direction[3]. As the purpose of Baldwin and Krugman (1989) is to explain the persistence of the trade deficit in the U.S., their model is of short or medium run nature. They assume the exchange rate is determined in a Mundell-Fleming type of balance of payments equilibrium.

Another line of research can be traced in the relevant literature. Dornbusch (1976) concludes that the exchange rate will be volatile under the assumption of sticky prices. The theory of incomplete exchange rate pass-through is appropriate here because it explains the incomplete feedback process of the exchange rate to prices given the volatility of the exchange rate. Thus, sticky prices is not an assumption but an endogenous outcome. As a result, Baldwin and Lyons (1988, 1989), and Krugman (1989) offer the theory of incomplete exchange rate pass-through resulting from the presence of hysteresis as an another reason for sticky prices. In contrast, Dornbusch's (1976) sticky price model is valid only in the short or medium run. In actual economies, individual prices may be fixed for substantial periods. Carlton (1986), using Stigler and Kindahl data, finds that industrial prices are generally sticky from 6 months to 18 months. As prices begin to adjust, exchange rate over-

[3]It is because the permanent structural change in the domestic market will affect the equilibrium import prices and the exchange rate. Therefore, the mean level of exchange rate is shifted.

shooting will disappear. Thus, prices should be flexible in the long run. If so, will hysteresis in trade and exchange rate still be possible? Baldwin and Lyons (1988a,b) suggest that hysteresis in the long run is difficult to obtain.

Since most hysteresis models consider only the short run and partial equilibrium, this chapter will consider the long run general equilibrium analysis of this problem. We show that hysteresis can exist in a general equilibrium model of trade and the exchange rate. We interpret the long run situation as being one in which prices are flexible, and the exchange rate is determined by purchasing power parity (PPP). In this chapter, we adopt Chatterjee and Cooper's (1989) overlapping generations model with entry and exit. Theirs is originally a closed economy model with the presence of a participation externality, i.e. the gains to participation depend on the number of other agents participating. They use the model to investigate the macroeconomic consequences of market participation decisions of imperfectly competitive firms. We shall extend their model to derive the general equilibrium of a home economy in a two-country world by introducing the endogenous determination of the exchange rate. There are two reasons for choosing this framework: (1) General equilibrium with exchange rate determination can be easily derived with a commonly used overlapping generations (OG) model. (2) The number of active agents will be endogenously determined, which is crucial for deriving the hysteresis result.

The setup of our model is somewhat simpler than the model of Baldwin and Krugman (1989) because foreign trade is always balanced. Therefore, hysteresis is simply captured as the endogenous change in the market structure and its impact on the long run equilibrium relationship between import prices and the exchange rate. Unlike Baldwin and Krugman (1989), we cannot explain the impact of hysteresis on the persistence of the trade deficit as Baldwin and Krugman (1989) did. Instead, our paper will explain the hysteretic movements of import prices and the exchange rate in a long run equilibrium perspective.

Instead of defining the number of producers on a real number line, we restrict the number of producers in our model to be a positive integer. The integer assumption is necessary for generating the result that several equilibria appear side by side when we incorporate a positive network externality in the model shown later in section 3. As a result, we have indivisibility in our model. The reason for this is due to the definition of the market structure, which requires the number of firms in the market to be a finite positive integer. Moreover, we assume that each agent has to incur his fixed cost during production. Since we allow the fixed cost measured in disutility units of the agent to be a decreasing function of the level of participation, the hysteresis effect in trade is a possible outcome as long as there is a permanent increase (decrease) in the number of incumbent suppliers in the market resulting from a temporary shock. This result follows directly from the presence of multiple equilibria.

Multiple equilibria will turn out to be a dominant feature of our model. Since hysteresis is defined as a permanent change in the equilibrium of the economy resulting from a sufficiently large temporary shock, it is obvious that the equilibrium will not be globally stable; but it may be locally stable. Thus, our model rules out the unique equilibrium case because a unique equilibrium always implies global stability with the usual concavity assumptions imposed. It is well known that the dynamics of a multiple equilibrium model will depend on the initial conditions; the past will affect the future. As a result, we explain the hysteresis effect in the context of multiple equilibria. Our model derives the result of hysteresis which is similar to Kemp and Wan's (1974) type of hysteresis with several equilibria emerging contiguously, i.e. $\hat{N}$, $\hat{N}+1$, ... $\hat{N}+k$ are all equilibria. Since a temporary shock moves the economy from one equilibrium to another, it is therefore observed that there is a permanent increase (decrease) in the number of incumbent suppliers. Also, the dependence of the fixed cost on the level of participation biases the system towards remaining in the after-shock state rather than the before-shock state even when the shock is removed.

Baldwin and Lyons (1988, 1989) acknowledge the inability of their model to produce welfare implications about the effects of hysteresis, but our model has the ability to Pareto-rank the equilibria in terms of the number of agents producing. The welfare of the economy varies positively with the rate of participation. Our model also produces clear-cut policy prescriptions for the economy; it requires an exogenous push in order to escape from the low level equilibrium, and the economy can sustain itself in a higher level equilibrium even if the policy is removed.

This chapter is divided into five sections. Section 2 describes the model in more detail than the introduction and derives its general equilibrium results. Section 3 discusses the hysteresis effect in trade and its effect on the exchange rate. Section 4 presents a numerical example of the hysteresis effect in trade and its impact on import prices and the exchange rate. The final section is the summary and conclusion of this chapter.

1.2 An Overlapping Generations Model with Entry and Exit

We consider a two-country world, where all the variables in the rest of the world or the foreign country will be indexed with (*). Each economy lasts indefinitely with time indexed by t=1,2,3,... There are $\tilde{N}$ (integer) identical agents born each period, and they survive two periods in the home economy. Each agent produces when young and consumes when old. Moreover, agents only consume the import good from the rest of the world and produce the export good for sale in the rest of the world. Therefore, consumption equals imports, and production equals exports.

A young agent chooses whether to activate a linear technology which transforms labour into output contemporaneously. This is the participation decision suggested by Chatterjee and Cooper (1989). All young agents are endowed with one unit of leisure time. Each agent's cost of activating the technology is k_i, $i = 2, 3, ..., \bar{N}$ measured in units of utility.[4] These costs are distributed across the population of young agents according to a time invariant cumulative distribution, $H(k)$, which is a right-hand continuous step function. If a youth decides to participate in production, he must decide the amount of output to produce for export to the old agents in the foreign country[5].

The product market is of Cournot type. After the sale, he/she converts the foreign currency into domestic currency and retains it until he/she consumes when old[6]. Money only plays the role of store of value in this model, and it has no other particular function. The details of the trading procedures are shown in figure 1.1. Thus, he/she holds $e_t R(y_t)$ units of nominal money balances at the end of period t, where e_t is the exchange rate defined to be the domestic currency price of the foreign currency in period t, and $R(y_t) = P_t^* y_t$ is the revenue the young agent receives by selling the export good in period t. If an agent does not participate in production, he will obtain zero utility and not enter the transactions of the economy. It is assumed that the participation and output decisions are made sequentially within a period. As a result, when making their participation decisions, agents anticipate that the other firms will select their output levels in an optimal fashion, ensuring that output levels will always be optimal given the level of participation.

As there are two stages of decisions, we apply dynamic programming and solve

[4]We do not assume that the fixed cost or market entry cost is different from the fixed cost or network maintenance cost as assumed by Baldwin et. al. The fixed cost is measured in term of units of utility because it is easier to derive the equilibrium number of active agents, N, later in this section.

[5]There is no labour market in this economy because the firm with the lowest cost can monopolize the market by offering a high enough wage to drive all the other high cost firms out of the competition.

[6]This is equivalent to a legal restriction that all savings must be in local assets and no savings in foreign assets. This assumption is crucial for the determination of the exchange rate. Without it, we shall have the indeterminacy of the exchange rate studied by Kareken and Wallace (1981).

the problem backwards. We assume that there are N_t young agents participating in production in period t, for $t = 1, 2, ...$, and solve for the supply decision of the firms. Later, we verify that the hypothesized participation is consistent with individual optimization. Now, we consider a young agent participating in the export market in period t. The agent solves the following problem:

$$\max_{y_t} \; V = -v(y_t) + \beta u(\frac{e_t R(y_t)}{P_{t+1}})$$

Where e_t is the exchange rate defined to be the home currency price of one unit of foreign currency, $R(y_t)$ is the revenue obtained by selling the output in the foreign market, P_{t+1} is the price level at home in period $t+1$; it is basically the money price of imports, y^*_{t+1}, $0 < \beta \leq 1$ is the discount factor of the agent, the function $u(\cdot)$ is increasing and strictly concave, $v(\cdot)$ is the disutility from working; it is increasing and strictly convex[7].

The agent, acting as a seller, has market power and thus has some influence over the money price of exports (i.e. foreign imports), P^*_t. He/she takes as given the current values of the exchange rate, e_t, the domestic price level next period, P_{t+1}, the number of participating agents, N_t, N^*_t, N^*_{t+1} and the output of all the other firms, and solves the above problem by choosing y_t. The first order condition is:

$$v'(y_t) = \beta u'(\frac{e_t R(y_t)}{P_{t+1}})\frac{e_t R'(y_t)}{P_{t+1}}$$

Now $R(y_t) = P^*_t y_t$, and $P^*_t = M^*/(y_t + Y_t)$, where Y_t is the output produced by all other domestic firms, and M^* is the constant amount of money stock existing in the foreign country. We set $M = M^* = 1$ for simplicity. Thus, we obtain

$$R'(y_t) = \frac{1}{y_t + Y_t}(1 - \frac{y_t}{y_t + Y_t})$$

We focus on the symmetric Nash Equilibrium (SNE) in period t. The first order condition becomes

[7] We further define $W(x) = xu'(x)$ and $G(x) = xv'(x)$. We assume that $u(\cdot)$ will not be too concave so that $W(\cdot)$ is an increasing function of x. By the convexity of $v(\cdot)$, $G(\cdot)$ is increasing in x.

$$v'(y_t) = \beta u'(\frac{e_t R(y_t)}{P_{t+1}}) e_t \frac{N^*_{t+1} y^*_{t+1}}{N_t y_t} (1 - \frac{1}{N_t})$$

Where $P_{t+1} = 1/N^*_{t+1} y^*_{t+1}$, N^*_{t+1}, y^*_{t+1} are the number and individual outputs of foreign producers in period t+1.

Since each producer has some market power to affect the output price, P^*_t, which will affect the exchange rate, e_t, through the purchasing power parity, each producer has some influence over the exchange rate. However, we still assume that each producer takes the exchange rate as given because the stylised fact is that purchasing power parity only holds in the long run and not in the short or medium run. The exchange rate is usually determined in the asset markets and not in the markets for traded goods, and the asset market is beyond the control of individual producers. Therefore, individual producers usually have no influence on the exchange rate. Since the producers are making output decisions which are usually short run or medium run decisions, they will take the exchange rate as given. We close the model with the imposition of purchasing power parity because purchasing power parity holds in the long run. By imposing purchasing power parity we can obtain a long run equilibrium feature from the model.

We assume that the equilibrium exchange rate is determined by purchasing power parity in the long run (i.e. the exchange rate is the relative price of consumption goods in both countries):

$$e_t = \frac{P_t}{P^*_t} = \frac{1/N^*_t y^*_t}{1/N_t y_t} = \frac{N_t y_t}{N^*_t y^*_t}$$

Notice that the exchange rate is simply the inverse of the terms of trade of the home country, P^*_t/P_t. Substituting this relationship into the first order condition,

$$v'(y_t) = \beta u'(\frac{N^*_{t+1} y^*_{t+1}}{N^*_t y^*_t} y_t) \frac{N^*_{t+1} y^*_{t+1}}{N^*_t y^*_t} (1 - \frac{1}{N_t})$$

Now let $a^*_{t+1} = N^*_{t+1} y^*_{t+1} / N^*_t y^*_t$. This is the gross growth rate (i.e. one plus the rate of growth) of foreign aggregate income and also the return on holding money, P_t/P_{t+1}.

It summarizes all the factors external to the home economy. We let $\eta_t = 1 - 1/N_t$; it summarizes the factors internal to the production activity at home. Then, the following equilibrium condition is obtained:

$$v'(y_t)y_t = \beta u'(a^*_{t+1}y_t)a^*_{t+1}y_t\eta_t \tag{1.1}$$

Which can be expressed as:

$$G(y_t) = \beta W(a^*_{t+1}y_t)\eta_t \tag{1.2}$$

The equation above describes the equilibrium output of each firm in the economy. η_t is a measure of market power which will increase as N_t, the number of competitors, increases.

We consider the steady state where $N_t = N$ and $\eta_t = 1 - 1/N$. The following result is obtained:

Proposition 1.2.1 *Given the assumption (A1) $u(\cdot)$ is increasing and concave and $v(\cdot)$ is increasing and convex, (A2) $d[xu'(x)]/dx > 0 \;\; \forall\, x \geq x$, $\lim_{x \to 0} xu'(x) > 0$, and a constant elasticity type utility function, there will be a unique steady state level of $y(N) > 0$ that solves*[8]

$$G(y) = \beta W(a^* y)\eta. \tag{1.3}$$

If the agents produce, there must be a unique $y > 0$ associated with that steady state equilibrium.

Now we can proceed to the participation decision by putting the optimal y into the utility function. First we show that all producing agents will be better off if more agents participate in production. Here we assume that the utility function is continuous and differentiable in N.

Proposition 1.2.2 *If $\eta < 1$, the level of utility of an agent, $V(N)$, is increasing in the number of participating agents, N* [9].

[8] The proof of this proposition and all the other assumptions can be examined in appendix 1.A.
[9] See appendix 1.A for proof.

$V(N)$ is the indirect utility function when there are N agents participating with a^* suppressed. With the presence of Cournot market, output will be less than the socially optimal level. An increase in the number of producers will enable the market to approach perfect competition, increasing welfare. As a result, this proposition is the participation externality which indicates that a participating agent is better off if there are more agents participating in production and exporting activities[10]. The presence of an imperfectly competitive market is crucial to generate this proposition. If the market is perfectly competitive, $\eta = 1$ and the value function, $V(\cdot)$, will not be affected by N.

Proposition 1.2.3 *If the utility function is of constant elasticity type and* $\epsilon_W/\epsilon_G < 4/5$ *where* $\epsilon_G = \partial \log G(x)/\partial \log x$, $\epsilon_W = \partial \log W(x)/\partial \log x$, *then* $V(N)$ *will be concave in* N[11].

Now we can demonstrate that there exists at least one equilibrium with N ($\bar{N} > N$) agents participating in the production of exports. An agent will participate in the production if the utility derived from participation and production is greater than or equal to the cost of participation, k_i. No inactive agent will gain from participation. Thus, active agents remain active, and inactive agents remain inactive. Now we list the agent's participation costs in ascending order: $k_1 \leq k_2 \leq k_3, ... \leq k_i, ... \leq k_{\bar{N}}$, which are equally spaced, and the following existence result is obtained:

Proposition 1.2.4 *If agents are ranked according to their participation costs,* $k_2 \leq ... \leq k_i \leq ... \leq k_{\bar{N}}$ *where* $i = 2, ..., \bar{N}$, $V(2) > k_2$ *and* $V(\bar{N}) < k_{\bar{N}}$, *then there exists a steady state equilibrium* $(\hat{N}, \hat{k})$ *satisfying* $k_{\hat{N}+1} > V(\hat{N}+1) \geq V(\hat{N}) \geq k_{\hat{N}}$[12].

Therefore, the equilibrium number of participating agents is determined by the cost and benefit of market participation. Moreover, this model has a participating ex-

[10]This is a sort of prisoner's dilemma situation; each agent is better off exercising his/her market power, but they would all be better off if they agreed to behave more competitively.

[11]See appendix 1.A for proof.

[12]See appendix 1.A for proof.

ternality, i.e. the gains to participation depend on the number of other agents participating as well. These are the main characteristics of Chatterjee and Cooper (1989). There may be multiple equilibria in this economy because the agent's utility, $V(N)$ is increasing in N. It is possible for the graph of $V(\bar{N}H(k))$ to intersect the 45° line more than once[13]. In addition, those multiple steady state equilibria can be Pareto-ranked because, from proposition 1.2.2, the welfare of active agents is increasing in the participation rate, N. In figure 1.2a, there are two steady state equilibria: The second step counting from the bottom step, k_3, and the third step counting from the top step, k_{13}[14]. Since the number of equilibria in this model depends on the shape of the $V(\cdot)$ and $H(k)$, it is possible to have several equilibria. To simplify the analysis, we shall focus on the case where $H(k)$ is a uniform distribution throughout the whole chapter. However, with this assumption, we may have the following result.

Proposition 1.2.5 *If the conditions of proposition 1.2.3 and 1.2.4 hold, and $H(k)$ is a uniform distribution so that $V(\bar{N}H(k))$ is concave in k, then the steady state equilibrium will be unique*[15].

The condition that the utility function is concave in N is not sufficient for the uniqueness of equilibrium. However, if the utility function is concave in k, it will be a sufficient condition for uniqueness. Figure 1.2b clearly illustrates this unique equilibrium outcome of proposition 1.2.5.

In this simple economy, fluctuations in output and prices in period t will depend on changes in the exogenous factor, a^{*}_{t+1} which serves as the shift parameter that exerts impact on the economy. Since the equilibrium is unique, the present model is

[13] $H(k)$ gives us the percentage of population with the value of fixed cost which is less than or equal to k, thus $\bar{N}H(k)$ gives the value of N with the fixed cost which is less than or equal to k.

[14] Usually, there will be three equilibria in the continuous case, two of them are stable, and the other is unstable. However, in the discrete case, the middle and unstable equilibrium will not exist. This is because the agent next to the agent associated with the unstable equilibrium will always find it profitable to participate in production, and it will upset the middle equilibrium.

[15] See appendix 1.A for proof.

not sufficient to generate hysteresis if proposition 1.2.5 holds. In the next section, we shall consider how hysteresis in trade emerges.

1.3 Hysteresis in Trade and the Exchange Rate

As pointed out in Baldwin (1988a,b), and Dixit (1989a,b), the hysteresis effect depends crucially on the presence of asymmetric responses of the system to temporary exogenous shocks. The hysteresis effect can be viewed as a phenomenon arising from a change in $\hat{N}$ (equilibrium number of participating agents) which results in a change in market structure. When the economy is disturbed by some sufficiently large temporary exogenous shock, entries or exits occur which result in a structural change in import and export markets. As a result, the temporary shock causes the appearance of hysteresis which resembles Kemp and Wan's (1974) type of hysteresis because several equilibria will appear side by side. Kemp and Wan (1974) consider the continuous case, resulting in a continuum of equilibria. For the case of a continuum of equilibria, the outcome will be quite unpredictable and depend on the initial position.

Now we will demonstrate that hysteresis is a possible outcome in our model. In the previous section, we assumed that k_i, $i = 2, ..., \hat{N}$ is constant. In contrast, we now assume that the agent-specific fixed cost, k_i, which is the per firm share of the cost of maintaining the sales network in the foreign country is a function of N.

$$k_i = k_i(N), \qquad \begin{cases} -1 < k_i'(N) < 0 & \text{if } N \leq N_* \\ k_i'(N) > 0 & \text{if } N > N_* \end{cases} \qquad i = 2, .., \hat{N}$$

Each producing agent takes as given N, total number of producing agents. There are two cases for discussion[16]. We discuss the first case: If the equilibrium, $\hat{N} \leq N_*$, we shall have a positive network externality which is similar to the thin market externality discussed in Diamond (1982), and Howitt (1985). When there are more exporters selling their goods in foreign countries, the per firm share of maintenance

[16]This is true provided $\hat{N} > N_*$. If $\hat{N} \leq N_*$, we only have a positive externality case.

cost will be lower. This is similar to a network externality[17] such that the more people are in the network, the less costly it is to operate. Actually, it does resemble some market setup costs e.g. shipping, insurance, shopping facilities, etc. These are parts of the sales network, and they all have some increasing returns characteristic such that the cost will be lower the larger the industry. An example is the shopping center where competing stores are operating close to one another. In 1951 there were only four shopping centers in Canada; in 1987, a small town would have that many. Acheson and Ferris (1988) suggest that shopping centers at least offer the following advantages: (1) Coordination of activities within the shopping center, e.g. the shopping center manager will lobby for the interests of the group with respect to zoning, studying traffic flows within the complex, etc. (2) Internalizing externalities through better coordination. (3) Reducing search costs of the clients[18]. These advantages lead to an efficiency gain or a decrease of the selling cost of each store in the shopping center where more stores are operating. To simplify the analysis, we continue to maintain the assumption that $H(k)$ is a uniform distribution, so the probability density function, $H'(k) = 1/\bar{N}\ \forall\ i$. Moreover, the shape of the cumulative distribution function, $H(k)$ will be as follows: Since $H(k)$ is a step function and $k \geq 0$, the entire graph of $H(k)$ will be shifting to the left if N is increasing.

After introducing the dependence of k_i on N, we find that not all of the propo-

[17]The network externality here differs from the Katz and Shapiro (1985), Farrell and Saloner (1985) study. They focus more on the compatibility decisions of the diffusion path of the chosen network and the influences of the existence of network externalities which is characterized by complementarity in consumption or production. The reason for the sales network externality discussed in this chapter is not due to the compatibility decisions but rather the presence of market setup costs and integer assumption. If we keep adding sellers to the network, the per-firm share of the setup costs will be declining. When there are too many sellers, there will be congestion in the network thus raising the cost of selling.

[18]In Ottawa the downtown Rideau Center has three department stores, Eaton's, the Bay and Ogilvy's and all regional center in the area have at least two. The Yorkdale complex which opened in Toronto in 1964 was the first in which Simpsons and Eaton's stores appeared in the same center. Eaton's had owned the land on which the complex was built. Mr. Kinnear, the CEO of Eaton's at that time, said, 'It's an axiom of retailing that two big stores near each other attract more than twice as many customers as they would singly.'

sitions in previous sections will apply. For the existence of equilibrium, proposition 1.2.4 will be modified to:

Corollary 1.3.1 *If agents are ranked according to their participation costs such that* $k_2 \leq ... \leq k_i \leq ... \leq k_{\bar{N}}$ *where* $i = 2, ..., \bar{N}$, *and if* $V(2) > k_2(2)$ *and* $V(\bar{N}) < k_{\bar{N}}(\bar{N})$, *then there exists a steady state equilibrium* $(\hat{N}, \hat{k}(\hat{N}))$ *satisfying* $k_{\hat{N}+1}(\hat{N}) > V(\hat{N} + 1) \geq V(\hat{N}) \geq k_{\hat{N}}(\hat{N})$.

The only difference between proposition 1.2.4 and corollary 1.3.1 is the change in boundary conditions: $V(2) > k_2(2)$, since $k_2(2) > k_2(3) > ... > k_2(\bar{N})$, so $V(2) > k_2(i)$ is satisfied for all $i = 2, ..., \bar{N}$. Similarly, $k_{\bar{N}}(2) > .... > k_{\bar{N}}(\bar{N})$, so, if $k_{\bar{N}}(\bar{N}) > V(\bar{N})$, then $k_{\bar{N}}(i) > V(\bar{N})$ for all $i = 2, ..., \bar{N}$. If these two boundary conditions hold, the proof of the existence of equilibrium will follow exactly as the proof in proposition 1.2.4. However, proposition 1.2.5 will not hold even if the conditions of proposition 1.2.5 are satisfied with the presence of a positive network externality. This is because proposition 1.2.5 assumes the absence of a positive externality. Actually, it is the presence of proposition 1.2.5 that rules out the possibility of hysteresis in this model. Without proposition 1.2.5, hysteresis will emerge under a positive network externality. As we consider the presence of a sales network externality, $k_i = k_i(N)$ and $k_i'(N) < 0$, it will raise $k_{\hat{N}}(\hat{N} - 1)$ and lower $k_{\hat{N}+1}(\hat{N} + 1)$ when $\hat{N}$ is the initial equilibrium. This introduces the possibility that $k_{\hat{N}-1}(\hat{N} - 1) < V(\hat{N} - 1) < V(\hat{N}) < k_{\hat{N}}(\hat{N} - 1)$ and $k_{\hat{N}+1}(\hat{N} + 1) < V(\hat{N} + 1)$, and makes the neighbours of $\hat{N}$, namely $\hat{N} - 1$ and $\hat{N} + 1$ also be equilibria. Therefore, it is possible to produce a result that the model will always have multiple equilibria which appear side by side.

The integer assumption on N is important here. If N is continuous, the upper bound, $k_{\hat{N}+1}(\hat{N})$, and lower bound, $k_{\hat{N}}(\hat{N})$, will merge together and the equilibrium will be $k_{\hat{N}}(\hat{N}) = V(\hat{N})$. There will be only one equilibrium if we assume a uniform distribution and the value function is concave in N. The positive network externality widens the gap between $k_{\hat{N}+1}(\hat{N})$ and $k_{\hat{N}}$ when $k_N(N)$ is decreasing in N. That is why the positive externality is able to make the neighbouring $\hat{N}-1$ and $\hat{N}+1$ also be equilibria. Therefore, the integer assumption on N and a positive network externality together produce the result that the equilibria appear side by side. Since the equilibria lie side by side, we do not need too great a shock to generate hysteresis. The multiple equilibria case shown in figure 1.2a where the two equilibria are separated requires a much greater shock to move it from one equilibrium to another. Therefore, we focus on the case of multiple equilibria lying side by side rather than the case of separated multiple equilibria shown in figure 1.2a. In the numerical example shown in the next section, we can have equilibria at $N = 49$, 50, 51 at the same time that there is only one equilibrium at $N = 50$ in the absence of a network externality.

The number of equilibria depends on the shapes of the following functions, $k_i(N)$, $H(k)$ and $V(N)$. There is no general condition to determine the number of equilibria. For example, figure 1.3 shows the case where $V(.,.)$ is concave in N, k_i follows the uniform distribution, and each $k_i(N)$ is equally spaced. There are three equilibria: one is at $N = \hat{N}-1$ where $k_{\hat{N}}(\hat{N}-1) > V(\hat{N}) > V(\hat{N}-1) > k_{\hat{N}-1}(\hat{N}-1)$; the second is at $N = \hat{N}$ where $k_{\hat{N}+1}(\hat{N}) > V(\hat{N}+1) > V(\hat{N}) > k_{\hat{N}}(\hat{N})$; and the third at $N = \hat{N}+1$ where $k_{\hat{N}+2}(\hat{N}+1) > V(\hat{N}+2) > V(\hat{N}+1) > k_{\hat{N}+1}(\hat{N}+1)$. The proof that each equilibrium is locally stable is given in appendix 1.A . This is basically derived from the definition of equilibrium. In the next section, we will illustrate with a numerical example, and show how hysteresis in trade emerges from those multiple equilibria.

The appearance of hysteresis requires more than one equilibrium in the economy. With our uniform distribution assumption and the value function, $V(\cdot)$, being concave in N, we have to rely on the presence of a positive network externality to generate hysteresis. Certainly, the presence of a positive externality is not the only reason for hysteresis. We can produce the hysteresis result as long as there is more than one equilibrium.

The second case: If the equilibrium is such that, $\hat{N} > N_*$, we shall have a negative network externality. It is basically the congestion externality: There are so many sellers in the market that congestion is created. Under the congestion externality, we have the following result.

Proposition 1.3.1 *Under the condition of proposition 1.2.5, and the presence of the congestion externality, the equilibrium is always unique*[19].

This is not a surprising result because it is well known that a negative externality will not generate multiple equilibria. As a result, we shall concentrate on the case of a positive network externality.

Before we study the responses of a numerical economy when it is subject to some exogenous shock, a^*_{t+1}, we examine how the economy behaves when there is entry and exit. Let us consider the following result:

$$\frac{\partial V(\cdot)}{\partial a^*} = \beta u'(a^* y)y(1 + \frac{1}{N}\frac{\epsilon_W}{\epsilon_G - \epsilon_W}) > 0$$

This says that an increase in the growth rate of foreign output will increase the well-being of each participating agent. If we maintain the assumption that the utility function is of a constant elasticity type; then we can obtain the following result:

Proposition 1.3.2 *The derivative of payoff function $V(a^*, N)$ with respect to a^* is increasing in N*[20].

$$\partial^2 V(a^*, N)/\partial N \partial a^* > 0.$$

This result tells us that $\partial V(\cdot)/\partial a^*_{t+1}$ is increasing in N. Referring to figure 1.2a, and 1.2b, the higher the step is, the greater the response of $V(.,.)$ to changes in a^*_{t+1}. This

[19]See appendix 1.A for proof

[20]See appendix 1.A for proof.

suggests that a low level equilibrium with fewer participating agents is more robust to exogenous shock than a high level equilibrium with many participating agents. It is because the equilibrium at a higher level of participation is more sensitive to changes in a^*, it is easier to disequilibrate the economy[21].

1.4 A Numerical Example of Hysteresis in Trade

Before we consider the case of hysteresis, let us construct an example of multiple equilibria without any positive externality under proposition 1.2.4. We retain the assumption that the utility function is concave in N but the distribution function is no longer a uniform distribution. We consider the following utility function of a representative agent:

$$V(y) = -\frac{y^{1+\psi}}{1+\psi} + \beta\frac{(a^*y)^{1-\alpha}}{1-\alpha}$$

From the equilibrium condition, we obtain

$$y = (\beta\eta a^{*1-\alpha})^{1/(\alpha+\psi)} \qquad \eta = \frac{N-1}{N}$$

Substituting in the utility function, we derive

$$V(a^*, N) = -\frac{(\beta\eta a^{*1-\alpha})^{(1+\psi)/(\alpha+\psi)}}{1+\psi} + \beta\frac{(\beta\eta a^{*1+\psi})^{(1-\alpha)/(\alpha+\psi)}}{1-\alpha}$$

We set $\beta = 1$[22], $\alpha = \psi = 0.5$ and $\bar{N} = 100$. We let the discrete density function of fixed cost k_i as follows:

[21] For example, suppose a^* is a random shock with mean equal to one, and there are two equilibria $N = 49$ and $N = 51$. Under proposition 1.3.2, $\partial V(N = 51)/\partial a^* > \partial V(N = 49)/\partial a^*$. Suppose a^* realizes a value of 1.006 with an original value of 1.0. The change in $V(N = 51)$ is greater than the change in $V(N = 49)$. The increase in $V(N = 51)$ may be great enough to cause a new entry and the equilibrium number of suppliers becomes $N = 52$. However, the increase in $V(N = 49)$ may not be great enough to cause any new entry. Thus, for the same value of $a^* = 49$, the equilibrium of $N = 51$ is not sustainable but the equilibrium of $N = 49$ is sustainable.

[22] We assume there is not discounting of the future because the discount factor is not the crucial element for generating hysteresis.

Table 1.1 The discrete density distribution of Fixed cost

Fixed cost	Agents	density function
k_1=1.00	1-2	0.02
k_2=1.20	3-4	0.02
k_3=1.30	5-6	0.02
k_4=1.32	7-8	0.02
k_5=1.325	9-11	0.03
k_6=1.330	12-50	0.39
k_7=1.335	51-70	0.2
k_8=1.340	71-85	0.15
k_9=1.345	86-95	0.1
k_{10}=1.350	96-100	0.05

The second column means agents 1 and 2 have k_1 as their fixed cost. Similarly, the fixed costs of agents 3 and 4 are k_2. We have $V(2) = 1.17851 > k_1$, the fixed cost of agent 2, and $V(100) < k_{10}$, the fixed cost of agent 100. We have two equilibria in this case: (1) The first equilibrium is at $N = 11$, the equilibrium condition is

$$k_5 = 1.325 < V(11) = 1.329100 < V(12) = 1.32976 < k_6 = 1.330.$$

The agents 11 and 12 have fixed costs equal to $k_5 = 1.325$ and $k_6 = 1.330$ respectively. (2) The second equilibrium is at $N = 50$, the equilibrium condition is

$$k_6 = 1.330 < V(50) = 1.333132 < V(51) = 1.333139 < k_7 = 1.335.$$

The agents 50 and 51 have fixed costs equal to $k_6 = 1.330$ and $k_7 = 1.335$ respectively. This example shows that we can easily construct a case of multiple equilibria as shown in figure 1.2a under proposition 1.2.4.

Now, let us consider the case of hysteresis by constructing another example in which the utility function is concave in N and the distribution of k_i is an uniform distribution. We introduce a positive externality as discussed in section 3. We demonstrate the hysteresis effect by showing how the economy move from initial equilibrium to its neighbouring equilibrium. We assume another schedule of the fixed costs which is as follows:

$$k_i(N) = \bar{k}_i - 0.0006N \qquad \bar{k}_i = 1.3147 + 0.001(i-2) \qquad i = 2, 3, ..., \bar{N}$$

The person-specific fixed cost, $\bar{k}_i$ are evenly spaced across the population. In this numerical economy, we have three equilibria, $N = 49$, $N = 50$, and $N = 51$. Now, suppose $a^* = 1$ initially, and assume that the initial equilibrium is at $N = 50$. The equilibrium condition becomes:

$$\begin{aligned} k_{51}(50) &= 1.33370 > V(1,51) = 1.3331398 > V(1,50) = 1.333132 > \\ k_{50}(50) &= 1.33270 \end{aligned}$$

We further assume that $N^*_{+1}y^*_{+1} = N^*y^* = 50y(1,50)$ at the initial equilibrium, the foreign import price, $P^*(1,50) = 1/[50y(1,50)] = 0.020408$, and the exchange rate, $e(1,50) = 50y(1,50)/N^*y^* = 1.0$.

Now, let us illustrate how a temporary shock can lead to a permanent change in the market structure. Suppose a^* increases (decreases) from 1.0 to 1.001 (0.999) in period 1. The economy will move from equilibrium $N = 50$ to equilibrium $N = 52$ ($N = 48$). In period 2, a^* falls (rises) from 1.001 (0.999) back to 1.0. However, the economy will now settle down at equilibrium $N = 51$ ($N = 49$) instead of $N = 50$. Therefore, we have a case of hysteresis in trade. The corresponding equilibrium values, changes in foreign import price and exchange rate are presented in the following table:

Table 1.2 Equilibrium values and changes in P^* and e

Equilibrium	P^*	e	$\hat{P}^*(\%)$	$\hat{e}(\%)$
$a^* = .999, N = 48$	0.021287	0.957745	4.31	-4.23
$a^* = 1.00, N = 49$	0.020833	0.979592	2.08	-2.04
$a^* = 1.00, N = 50$	0.020408	1.000000	0.00	0.00
$a^* = 1.00, N = 51$	0.020000	1.020408	-2.00	2.04
$a^* = 1.001, N = 52$	0.019598	1.042378	-3.97	4.24

$\hat{P}^*$ and $\hat{e}$ are the percentage changes in the equilibrium values of relative variables compared to the initial equilibrium, $N = 50$. We can observe that a 0.1 percent temporary change in a^* can cause about a 2 percent permanent change in the import price and the exchange rate. As a result, the entry and exit of firms has a tremendous impact on the import price and the exchange rate by magnifying by about 20 times the original fluctuations of the exogenous shock. This is because the changes in the number of incumbent exporters is of first order effect. This effect dominates the changes in individual output, y which is of second order effect. From equation (1.2), $P^* = 1/Ny$ and $e = Ny/N^*y^*$, we can obtain the elasticity of P^* and e in response to changes in N^*y^* holding N constant.

$$\frac{\partial \log P^*}{\partial \log N^* y^*}|_N = \frac{\epsilon_W}{\epsilon_G - \epsilon_W}, \qquad \frac{\partial \log e}{\partial \log N^* y^*}|_N = -\frac{\epsilon_G}{\epsilon_G - \epsilon_W}$$

In this numerical example, $\partial \log P^*/\partial \log N^*y^*|_N = 0.5$ and $\partial \log e/\partial \log N^*y^*|_N = -1.5$. As there is 0.1% change in N^*y^*, it will cause only 0.05% and 0.15% changes in P^* and e respectively. But the change in N will be about 2% from 50 to 49 or 51. It is obvious that the change in N will completely dominate all other changes.

We further expand this example by letting a^* belong to a normal distribution with mean=1 and standard deviation=0.0005. One hundred random numbers are drawn from this distribution. At every value of a^*, we identify the new equilibrium and then calculate the equilibrium exchange rate e and foreign import price P^* for each period. Therefore, there are 100 periods altogether. We plot the simulation results in figures 1.4 to 1.7. The means of P^* and e are 0.020408 and 1.0 respectively. The economy starts out at equilibrium, $N = 50$ where P^* and e equal their means. Figures 1.4 and 1.5 show that the exchange rate, defined as $e = P/P^*$ and the foreign import price, P^*, abruptly move upward or downward and stay persistently away from the mean level. The import price moves to a new level only following a shock sufficiently large to break its persistence. Since the exchange rate, e, is positively correlated with N and the import price, P^*, is negatively correlated with N, higher (lower) e and lower (higher) P^* implies higher (lower) N. Therefore, we can observe that the economy is more persistent in staying at equilibrium with $N \leq 50$ than staying at equilibrium with $N > 50$. This is correctly predicted by proposition 1.3.2 which states that a lower level equilibrium is more robust to an exogenous shock than a higher level equilibrium.

If there is a sufficiently large negative (positive) shock to hit the economy, it will move to a lower (higher) level equilibrium. As a result, participation, aggregate income and demand, domestic price, P, and the exchange rate all decrease (increase). Any move from the present equilibrium requires a considerable shock. This suggests that there is potential for policy intervention to enable the economy to shift to or remain at a higher level equilibrium. Suppose we are now at $N = 49$. Should there be a social planner and an appropriate policy instrument to move the economy from $N = 49$ to, say, $N = 52$, this would result in a higher level of welfare for all participants. If the policy were removed later, the economy would return to $N = 51$ instead of $N = 49$. Thus, the economy can remain at a higher level equilibrium.

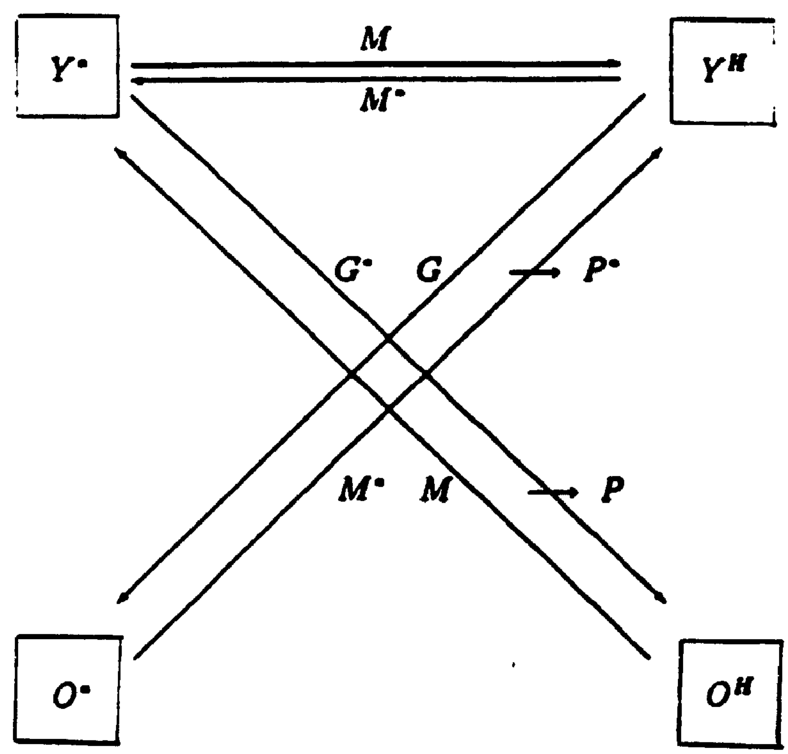
Figures 1.6 and 1.7 illustrate the magnitude of the fluctuations in e and P^*, which are greater than the fluctuations in a^*.

1.5 Conclusion

This chapter has studied the long run general equilibrium of a dynamic model where agents' entry and exit decisions are endogenous. We show that hysteresis can exist in a general equilibrium model of trade and the exchange rate, but it requires the presence of a positive network externality. In general equilibrium, some agents are participating and some are not. Those who do not participate will not enter the transactions of the economy. If we allow for the presence of a sales network externality, the fixed cost k will be a decreasing function of the level of participation N. Hysteresis in trade is a possible outcome following a sufficiently large temporary shock in the exchange rate because it causes a permanent increase (decrease) in the number of incumbent suppliers in the import market. It is the presence of $k(N)$ which biases the system to remain in the after-shock state and not to return to the before-shock state even when the shock is removed. Together with the assumption that the number of producing agents, N is indivisible, this model is able to produce the hysteresis result. Any departure from the present equilibrium requires a shock of sufficient magnitude. If there is a sufficiently large negative (positive) shock[23] to the economy, the economy will move to a lower (higher) welfare equilibrium. We obtain a lower (higher) participation, lower (higher) aggregate income and demand, a lower (higher) home consumption good price, P, and a lower (higher) exchange rate.

[23] Positive (negative) shock raises (lowers) the level of participation in the economy.

Figure 1.1 Active Agents' Activities at each period



Y, O are the coressponding young and old agents at each period,

H, $*$ corresponds the home and foreign,

G, G^* are the home and foreign goods produced correspondingly,

M, M^* are the home and foreign money stock,

P, P^* are the pri :es of G^* and G respectively determined in the trades.

Figure 1.2a Multiple equilibria when k_i is independent of N

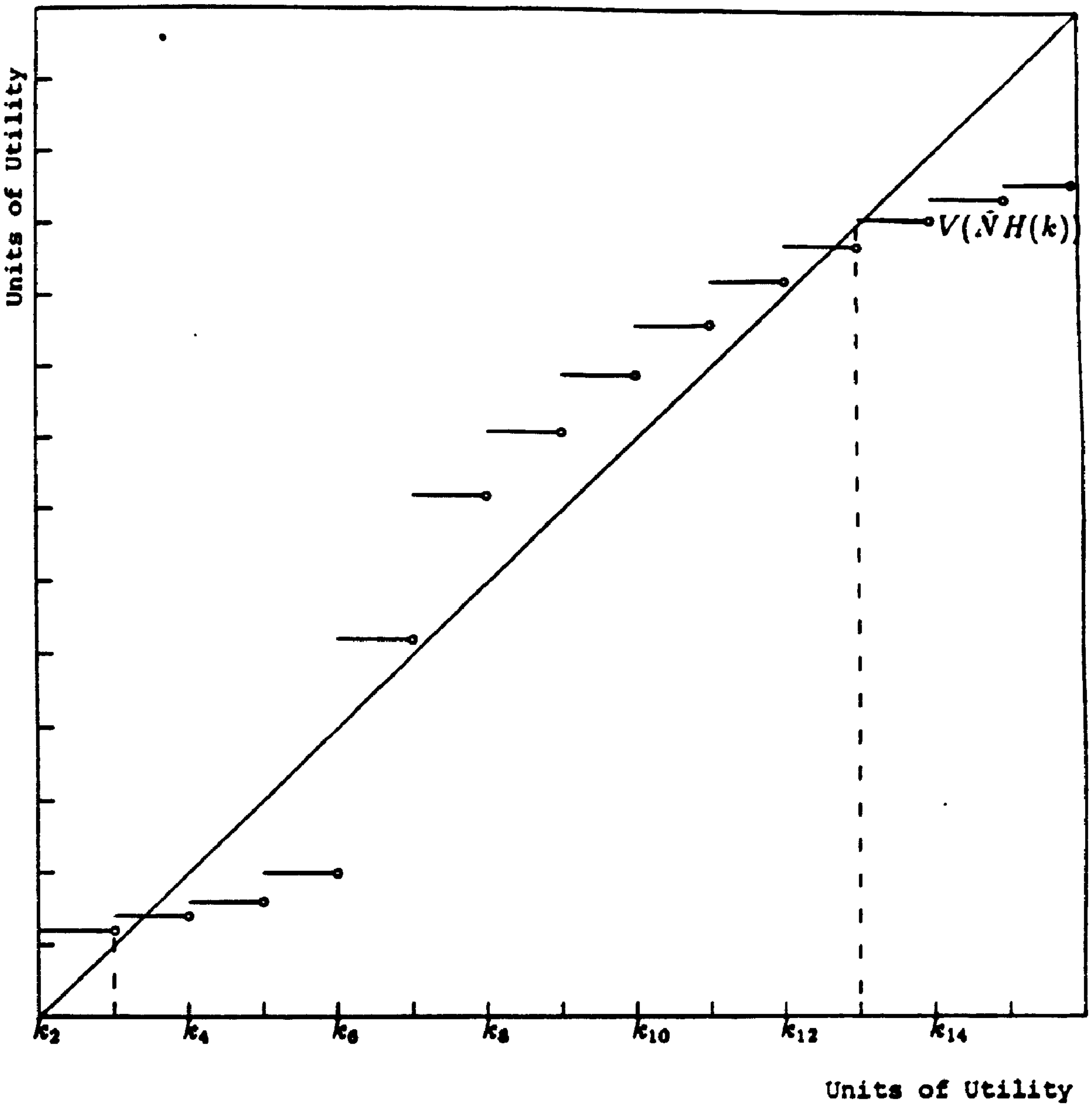


Figure 1.2b Unique equilibrium when k_i is independent of N

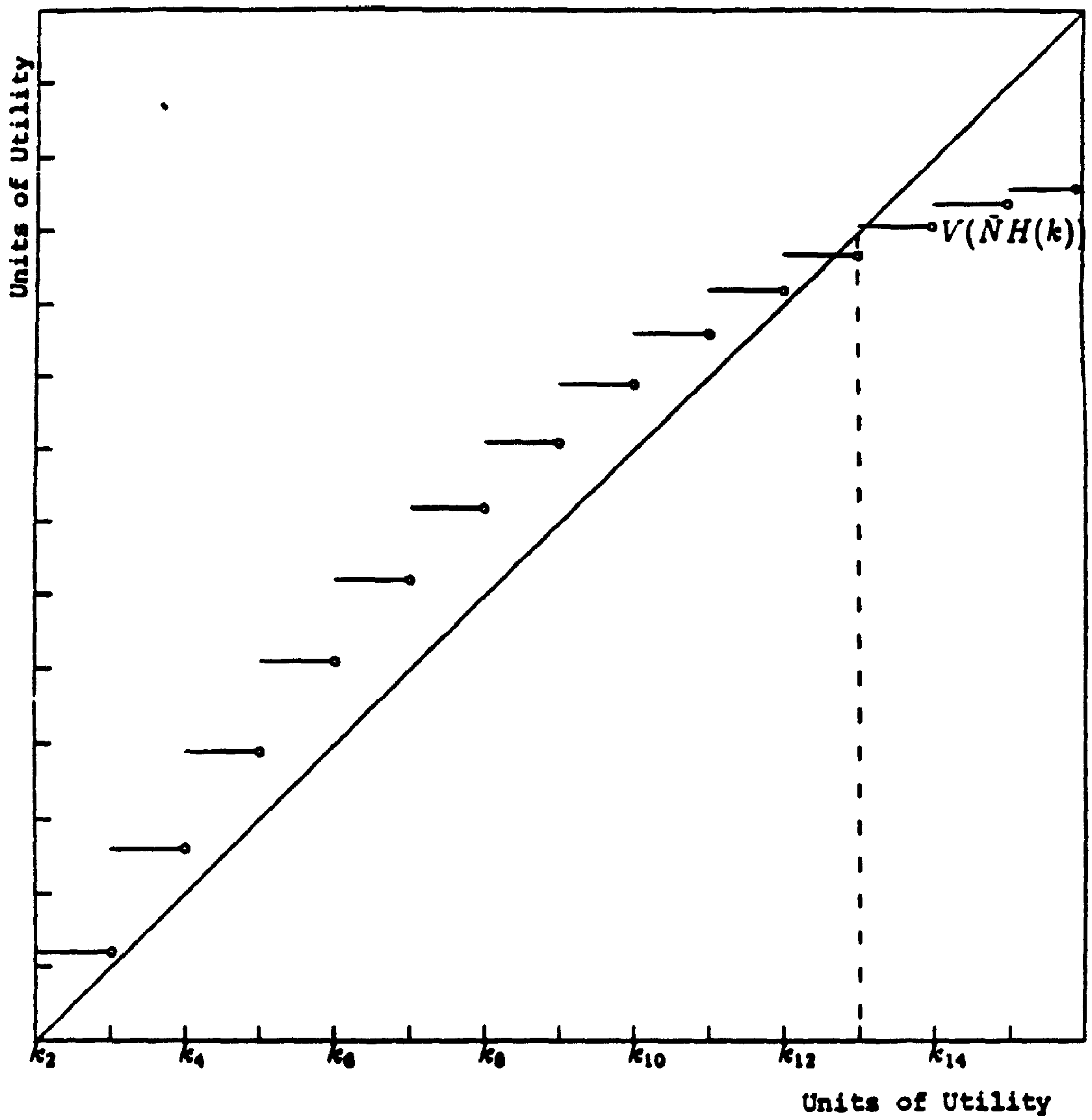


Figure 1.3 Multiple equilibria Case when k_i is function of N

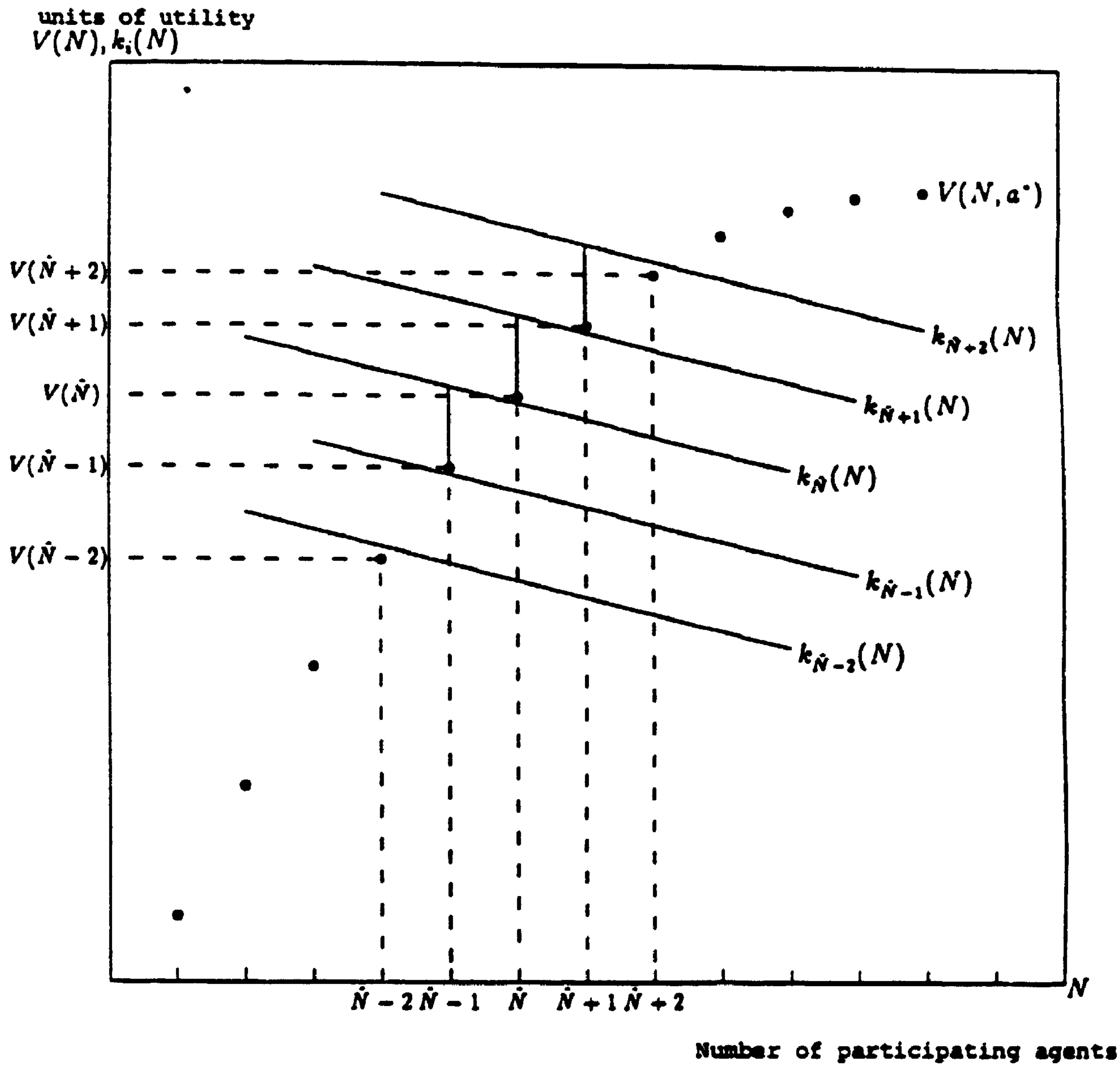


Figure 1.4 Simulated data of equilibrium exchange rate, e
(mean and S.D. of a^* are 1 and 0.0005)

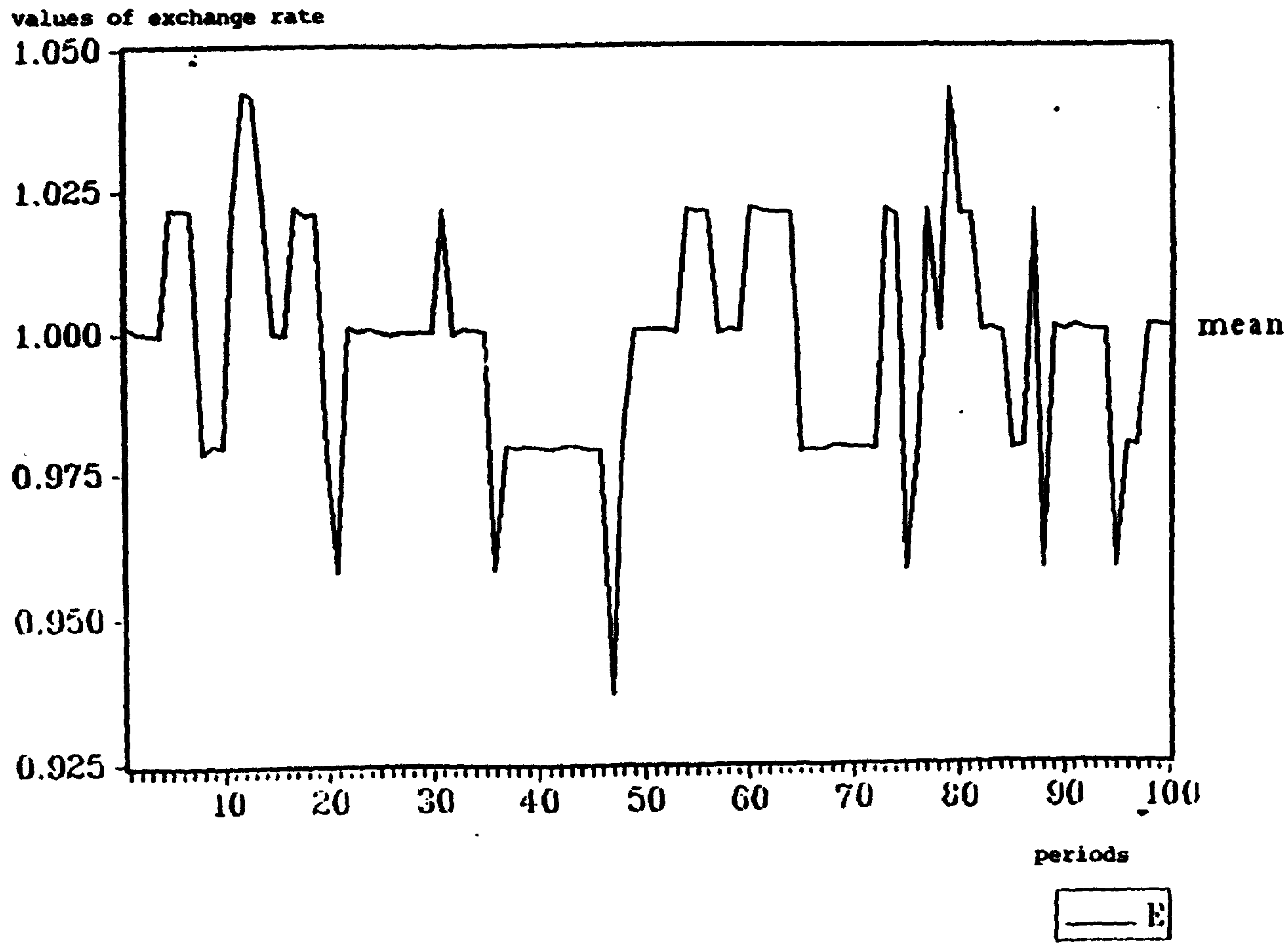


Figure 1.5 Simulated data of equilibrium foreign import price, P* (mean and S.D. of a* are 1 and 0.0005)

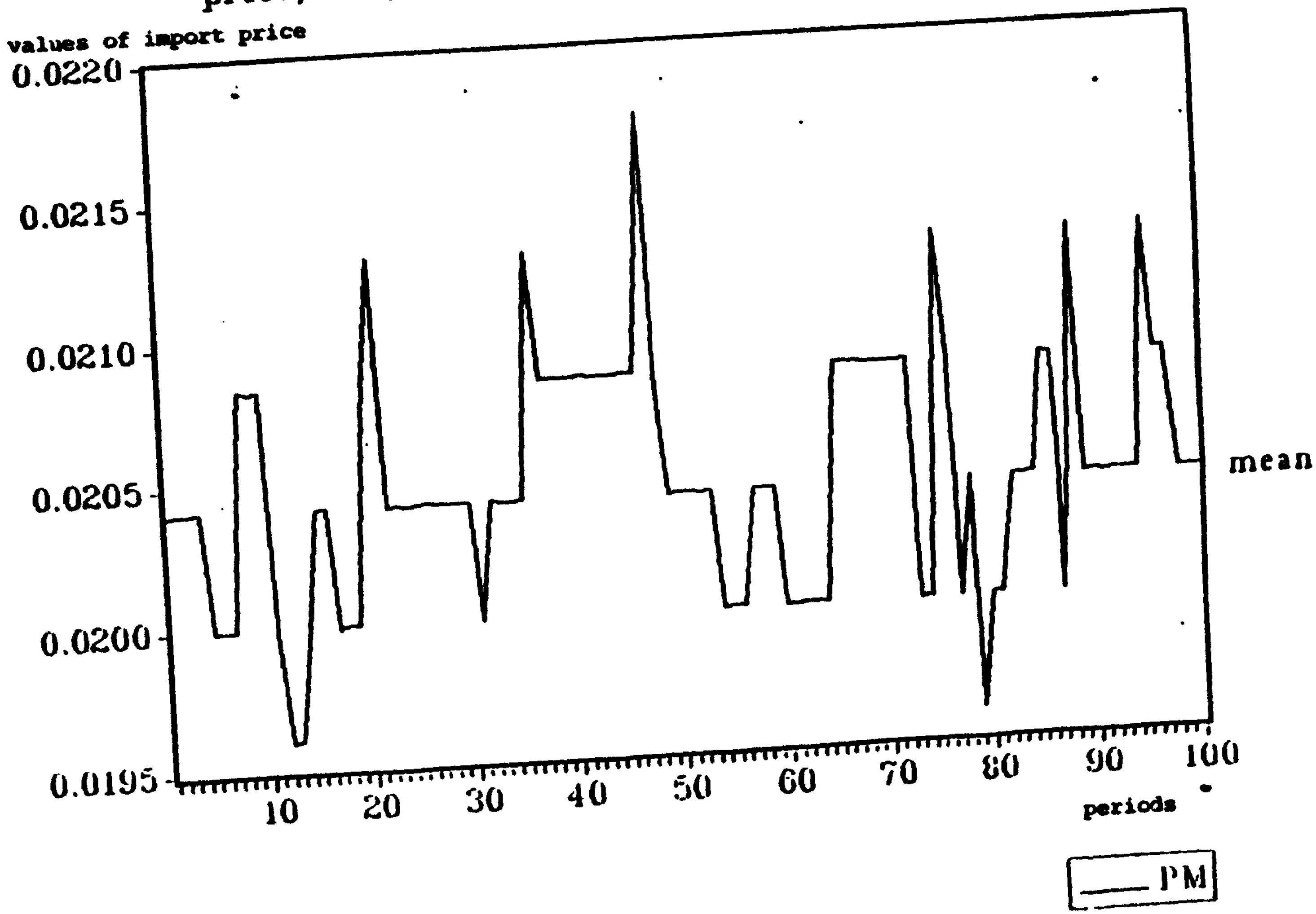


Figure 1.6 Percentage changes in foreign import price, P^* and a^* (mean and S.D. of a^* are 1 and 0.0005)

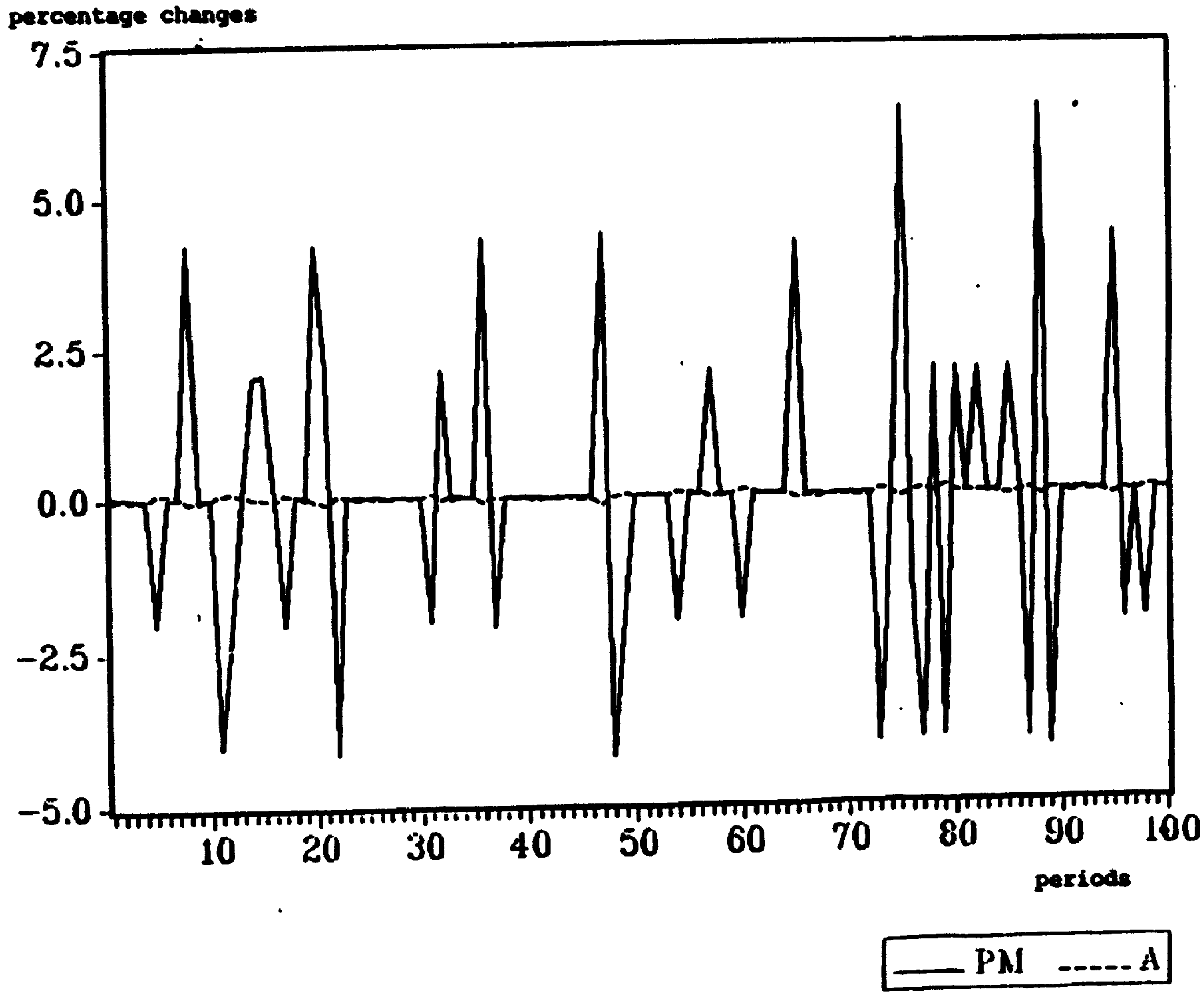
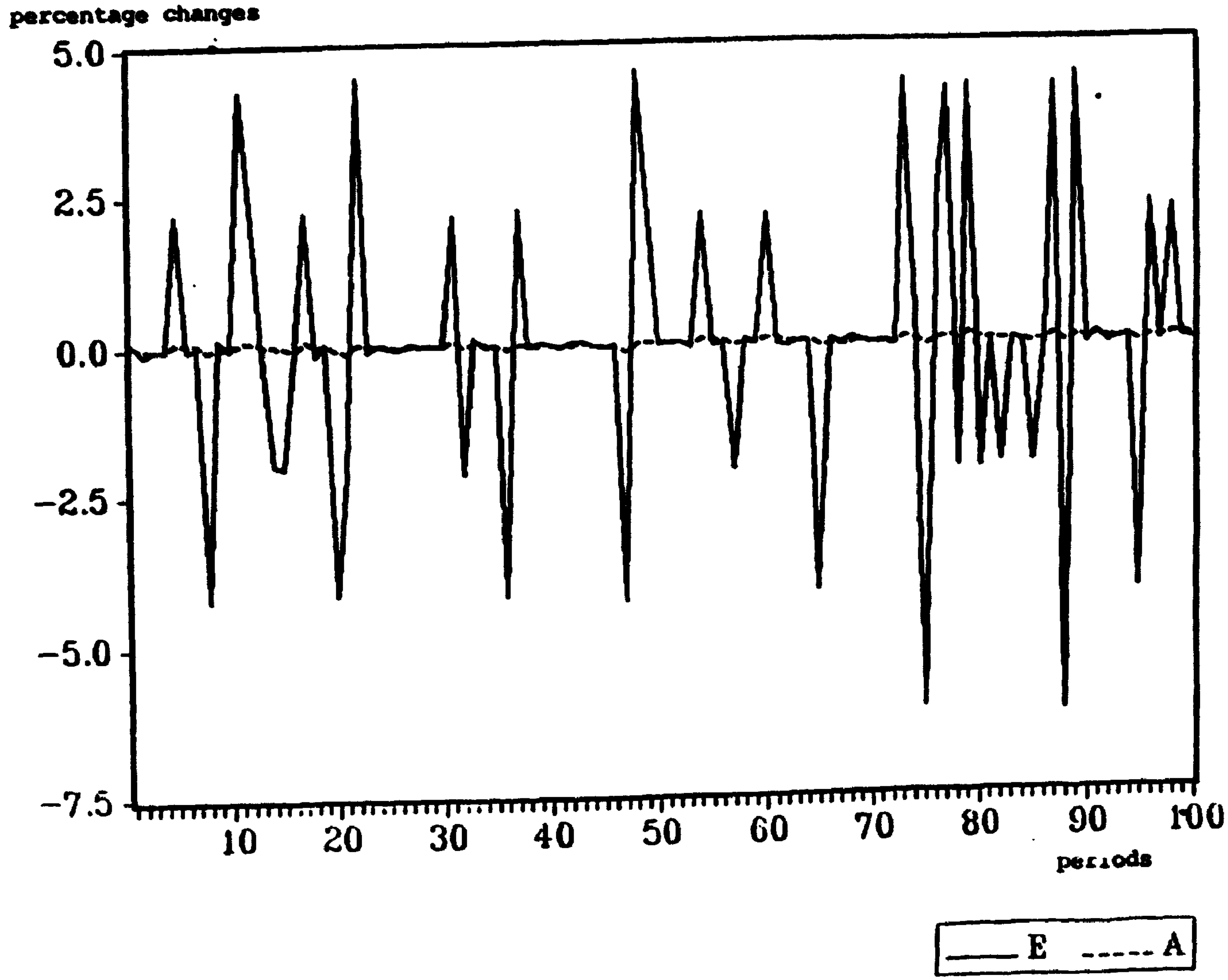


Figure 1.7 Percentage changes in exchange rate, e and a*
(mean and S.D. of a* are 1 and 0.0005)



Appendix 1.A

Proofs of the Propositions

We first state the assumptions on the utility function under which the analysis will be carried out:

(A1) $u(\cdot)$ is increasing and concave and $v(\cdot)$ is increasing and convex

(A2) $d[xu'(x)]/dx > 0 \ \forall \ x \geq 0$, $\lim_{x \to 0} xu'(x) > 0$

(A3) $\lim_{x \to 1} xv'(x) = \infty$

(A4) $\lim_{x \to 0} xv'(x) = 0$

(A5) $\epsilon_v < \epsilon_G$ where ϵ_v and ϵ_G are the elasticities of the $v(x)$ and $xv'(x)$ functions

Define ϵ_W and ϵ_G as the elasticity of $W(\cdot)$ and $G(\cdot)$, then we have

$$0 < \epsilon_W/\epsilon_G < 1$$

Since $W(x) = xu'(x)$, $\log W = \log x + \log u'(x)$, so $d\log W(x)/d\log x = 1 + u''(x)x/u'(x)$, by A.2, $0 < \epsilon_W < 1$. Similarly, $d\log G(x)/d\log x = 1 + v''(x)x/v'(x)$, by A.1, $\epsilon_G > 1$.

Proof of Proposition 1.2.1

Since an active agent's optimal choice will satisfy

$$W(a^* y)\eta = G(y)$$

and

$$\begin{aligned} \frac{\partial W(a^* y)}{\partial y} &= u'(a^* y)a^* \eta (1 + \frac{u''(a^* y)a^* y}{u'(a^* y)}) \\ &= u'(\cdot)a^* \eta \epsilon_W > 0 \\ \frac{\partial^2 W(a^* y)}{\partial y^2} &= u''(a^* y)a^{*2}\eta < 0 \end{aligned}$$

Thus, $W(\cdot)$ is increasing and concave, now we check $G(\cdot)$

$$\begin{aligned}\frac{\partial G(y)}{\partial y} &= v'(y)[1+\frac{yv''(y)}{v'(y)}] \\ &= v'(y)\epsilon_G > 0 \\ \frac{\partial^2 G(y)}{\partial y^2} &= v''(y)\epsilon_G > 0\end{aligned}$$

$G(\cdot)$ is increasing and convex, by A.2 to A.4, we will have only a unique positive y to satisfy the equation. □

Proof of Proposition 1.2.2

We differentiate (1) with respect to η_t, we can obtain

$$\frac{\partial y(N)}{\partial \eta} = \frac{\beta u'(a^* y)a^*}{v''(y) - \beta a^{*2}\eta u''(a^* y)} > 0$$

This follows as $v'' > 0$ and $u'' < 0$. Then

$$\begin{aligned}\frac{\partial V(N)}{\partial N} &= [-v'(y) + \beta u'(a^* y)a^*]\frac{\partial y(N)}{\partial N} \\ &= (1-\eta)\beta u'(a^* y)a^* \frac{\partial y(N)}{\partial N} \\ Since\ \frac{\partial y}{\partial N} &= \frac{\partial y}{\partial \eta}\frac{\partial \eta}{\partial N} \\ &= \frac{\partial y}{\partial \eta}\frac{1}{N^2} > 0 \\ \Longrightarrow \quad \frac{\partial V(N)}{\partial N} &> 0\end{aligned}$$

□

Proof of Proposition 1.2.3

Since

$$\begin{aligned}\frac{\partial V(\cdot)}{\partial N} &= (1-\eta)\beta u''(a^* y)a^* \frac{\partial y}{\partial N} \\ &= \frac{1}{N}\beta u'(a^* y)a^* \frac{\partial y}{\partial N} \\ &= \frac{1}{N^2(N-1)}\beta u'(a^* y)a^* y \frac{1}{\epsilon_G - \epsilon_W}\end{aligned}$$

Where $\partial \log y / \partial \log N = 1/[(N-1)(\epsilon_G - \epsilon_W)]$ derived by taking logarithm on equation (2), then we take derivative with respect to $logN$. We further differentiate it

$$\begin{aligned}
\frac{\partial^2 V(\cdot)}{\partial N^2} &= \frac{\beta}{\epsilon_G - \epsilon_W}[-\frac{N(3N-2)}{[N^2(N-1)]^2}W(a^*y) + \frac{1}{N^2(N-1)}\frac{\partial a^* y u'(a^* y)}{\partial a^* y} \times \\
& a^* \frac{\partial y}{\partial N}] \\
&= \frac{\beta}{\epsilon_G - \epsilon_W}[-\frac{N(3N-2)}{[N^2(N-1)]^2}W(a^*y) + \frac{1}{N^2(N-1)}\epsilon_W u'(a^* y) a^* \frac{\partial y}{\partial N}] \\
&= \frac{\beta}{\epsilon_G - \epsilon_W}[-\frac{N(3N-2)}{[N^2(N-1)]^2}W(a^*y) + \frac{1}{N^2(N-1)}\epsilon_W \frac{W(a^* y)}{N} \times \\
& \frac{1}{(N-1)(\epsilon_G - \epsilon_W)}] \\
&= \frac{\beta W(a^* y)}{(\epsilon_G - \epsilon_W)[N^3(N-1)^2]}[-(3N-2) + \frac{\epsilon_W}{\epsilon_G - \epsilon_W}]
\end{aligned}$$

As the first term is positive, We only have to consider the values of the two terms inside the square bracket. Since $N \geq 2$, $\partial^2 V(\cdot)/\partial N^2 < 0$ if

$$\begin{aligned}
-(3(2)-2) + \frac{\epsilon_W}{\epsilon_G - \epsilon_W} &< 0 \\
\frac{-4(\epsilon_G - \epsilon_W) + \epsilon_W}{\epsilon_G - \epsilon_W} &< 0 \\
\Longrightarrow \quad -4\epsilon_G + 5\epsilon_W &< 0 \\
\frac{\epsilon_W}{\epsilon_G} &< \frac{4}{5}
\end{aligned}$$

Together with proposition 1.2.2, the concavity will follow. □

Proof of Proposition 1.2.4

Let us imagine that we are in a square box where axes is k from zero to $k_{\bar{N}}$. $V(\cdot)$ is continuous and increasing in N, $H(k)$ is a step function, so $V(\bar{N}H(k))$ is an increasing step function. Since $V(2) > k_2$ and $V(\bar{N}) < k_{\bar{N}}$, it implies the first step of $V(\cdot)$ is completely above the 45° line and the last step of $V(\cdot)$ is completely below 45° line. If there does not exist a k^* such that $V(\bar{N}H(\hat{k})) = \hat{k}$, it implies that there is no step crossing the 45° line. Since the first step is above the 45° line and $V(\cdot)$ is increasing in k, it implies the 2nd step is above 1st step and not intersecting 45° line, and 3rd step is above 2nd step and not intersecting 45° line, it continues and implies the $\bar{N}$th, last step is above and not intersecting 45° line and it contradicts the fact that the last step is below 45° line. So, there exist a $\hat{k}$ such that $V(\bar{N}H(\hat{k})) = \hat{k}$. Given there is a $\hat{k}$, we prove $k_{\hat{N}+1} > V(\hat{N}+1) \geq V(\hat{N}) \geq k_{\hat{N}}$ is true, this statement implies that the $\hat{N}$th step cross 45° line and the $\hat{N}$ + 1st step is above the $\bar{N}$th step but below the 45° line. Suppose not, the presence of a fixed point $\hat{k}$ implies that $k_{\hat{N}+1} > V(\hat{N}) > \hat{k}$, and $k_{\hat{N}+1} \leq V(\hat{N}+1)$, so that the $\hat{N}$ + 1st step is still intersecting or above the 45°, as the next step is also above the $\hat{N}$ + 1st step and with $k_{\hat{N}+i} \leq V(\hat{N}+i)$, it implies the $\hat{N}+i$th step is still intersecting or above the 45° line. We can continue until the last $\bar{N}$th step is still intersecting or above the the 45° line, it contradicts the fact that the last step is completely below the 45°. From this proposition, the $\hat{N}$ + 1th agent will not participate in production because his/her cost of participation is higher than the payoff of it. Those participating agents with $V(i) > k_i$, $i = 2, ..., \hat{N}$, will like to continue producing. If $k_{\bar{N}} < V(\bar{N})$, there exists an equilibrium that all agents will join the production team. □

Proof of Proposition 1.2.5

Given the assumptions of proposition 1.2.4, $V(\cdot)$ is concave in k, it implies that the difference in the height of each pair of steps is declining as we are moving up the steps. Suppose there is an equilibrium at $k_{\hat{N}}$, the next step will be completely below the 45° line, i.e. $k_{\hat{N}+1} - V(\hat{N}+1) > 0$, as k is on the 45° line, if we proceed the next step, $V(\hat{N}+2) - V(\hat{N}+1) < V(\hat{N}+1) - V(\hat{N})$ by concavity of $V(N)$. We must have $k_{\hat{N}+2} > V(\hat{N}+2)$. By continuity and concavity, the rest of steps will always be below 45° line and never intersect it again.□

Proof of local stability of hysteretic equilibria

The definition of Takayama (1985): An equilibrium $\hat{x}$ is locally stable if there exist a closed ball $B_\delta(\hat{x})$ about $\hat{x}$ with radius $\delta > 0$ such that $x^0 \in B_\delta(\hat{x})$, $\phi(t; x^0, t^0) \rightarrow \hat{x}$ i.e. pick any x^0 in $B_\delta(\hat{x})$ other than $\hat{x}$,it will converge to $\hat{x}$. To check the local stability, we consider equilibrium $(V(\hat{N}), y)$. From proposition 2.1, there is always a unique $y(\hat{N})$ that solves the output decision and the second order condition will be satisfied with our assumptions imposed on $u(\cdot)$ and $v(\cdot)$. Thus, $(V(\hat{N}), y(\hat{N}))$ will be locally stable given $\hat{N}$. Since our equilibrium condition is inequality condition, $k_{\hat{N}}(\hat{N}) < V(\hat{N}) < V(\hat{N}+1) < k_{\hat{N}+1}(\hat{N})$, we are always possible to construct an interval about $V(\hat{N})$ with tiny distance δ which does not upset the relation $< V(\hat{N}+1) < k_{\hat{N}+1}(\hat{N})$. Thus, for any value, V^0 in that interval, V^0 will always converge to $V(\hat{N})$ because y is unique and second order condition of output decision holds given $\hat{N}$. Thus, $V(\hat{N})$ is locally stable, the same argument also works for equilibria $V(\hat{N}-1)$ and $V(\hat{N}+1)$. □

Proof of Proposition 1.3.1

Suppose not. Besides the equilibrium $\hat{N}$ with equilibrium condition $k_{\hat{N}+1}(\hat{N}) > V(\hat{N}+1) > V(\hat{N}) > k_{\hat{N}}(\hat{N})$, there is another equilibrium $\hat{N}+1$. So, we have $k_{\hat{N}+2}(\hat{N}+1) > V(\hat{N}+2) > V(\hat{N}+1) > k_{\hat{N}+1}(\hat{N}+1)$. Since $k'_{\hat{N}+1}(N) > 0$, $k_{\hat{N}+1}(\hat{N}+1) > k_{\hat{N}+1}(\hat{N})$. Together with the equilibrium condition of $\hat{N}$, $k_{\hat{N}+1}(\hat{N}) > V(\hat{N}+1)$, we have $k_{\hat{N}+1}(\hat{N}+1) > V(\hat{N}+1)$. It is a contradiction. The same is true for $N = \hat{N}+j$, with $k_{\hat{N}+j}(\hat{N}+j) > k_{\hat{N}+j}(\hat{N}+j-1) > V(\hat{N}+j)$. By induction, there will be no equilibrium for $N = \hat{N}+1, ..., j, ..., \bar{N}$. Now consider there is an equilibrium for $\hat{N}-1$, so we have $k_{\hat{N}}(\hat{N}-1) > V(\hat{N}) > V(\hat{N}-1) > k_{\hat{N}-1}(\hat{N}-1)$. Since $k'_{\hat{N}}(N) > 0$, $k_{\hat{N}}(\hat{N}) > k_{\hat{N}}(\hat{N}-1)$. Together with the equilibrium condition of $\hat{N}$, $k_{\hat{N}}(\hat{N}) < V(\hat{N})$, we have $V(\hat{N}) > k_{\hat{N}}(\hat{N}-1)$. It is a contradiction. Similarly, the same will be true for $N = \hat{N}-j$ with $k_{\hat{N}-j}(\hat{N}-j-1) < k_{\hat{N}-j}(\hat{N}-j) < V(\hat{N}-j)$. By induction, there is no equilibrium for $N = 2, ..., \hat{N}-j, ..., \hat{N}-1$. The equilibrium is unique at $N = \hat{N}$. □

Proof of Proposition 1.3.2

We differentiate $\partial V(N, a^*)/\partial a^*$ with respect to N under constant elasticity utility assumption,

$$
\begin{aligned}
\frac{\partial V(N,a^*)}{\partial a^*} &= \beta u'(a^* y) y [1 + \frac{1}{N}\frac{\epsilon_W}{\epsilon_G - \epsilon_W}] \\
\frac{\partial^2 V(\cdot)}{\partial N \partial a^*} &= \frac{\beta}{a^*}\frac{\partial a^* y u'(a^* y)}{\partial a^* y}\frac{\partial a^* y}{\partial N}(1 + \frac{1}{N}\frac{\epsilon_W}{\epsilon_G - \epsilon_W}) + \\
&\quad \beta u'(\cdot) y (-\frac{1}{N^2}\frac{\epsilon_W}{\epsilon_G - \epsilon_W}) \\
&= \beta u'(\cdot)\epsilon_W \frac{y}{N}\frac{N}{y}\frac{\partial y}{\partial N}(1 + \frac{1}{N}\frac{\epsilon_W}{\epsilon_G - \epsilon_W}) - \\
&\quad \frac{\beta u'(\cdot) y}{N^2}\frac{\epsilon_W}{\epsilon_G - \epsilon_W} \\
&= \beta u'(\cdot)\frac{y}{N}[\frac{\epsilon_W}{(N-1)(\epsilon_G - \epsilon_W)}(1 + \frac{1}{N}\frac{\epsilon_W}{\epsilon_G - \epsilon_W}) - \frac{1}{N}\frac{\epsilon_W}{\epsilon_G - \epsilon_W}] \\
&= \beta u'(\cdot)\frac{y}{N}[\frac{\epsilon_W}{\epsilon_G - \epsilon_W}(\frac{1}{N-1} - \frac{1}{N} + \frac{1}{N(N-1)}\frac{\epsilon_W}{\epsilon_G - \epsilon_W})] \\
&= \beta u'(\cdot)\frac{y}{N}[\frac{\epsilon_W}{\epsilon_G - \epsilon_W}\frac{1}{N(N-1)}(1 + \frac{\epsilon_W}{\epsilon_G - \epsilon_W})] \\
&= \beta u'(\cdot)\frac{y}{N}\frac{\epsilon_W}{\epsilon_G - \epsilon_W}\frac{1}{N(N-1)}\frac{\epsilon_G}{\epsilon_G - \epsilon_W} > 0 \qquad \square
\end{aligned}
$$

Appendix 1.B

GAUSS Program for Computing the Numerical Example

```
output reset;
outwidth 130;
let x[101,4]=0;
let z[101,5]=0;
let beta=1;
let psi=.5;
let alpha=.5;
i=1;
n=50;
r=rndn(100,1);
/* Generating random number from N(1, 0.00000025) */
do while i<=100;
a=r[i];
a=a*0.0005+1;
do while n<=100;  /* set Nbar=100 */
kbar=1.3147+0.001*(n-2);
kbar1=1.3147+0.001*(n-1);
k=kbar-0.0006*n;
k1=kbar1-0.0006*n;
eta=(n-1)/n;
v=beta*((beta*eta*a^(1+psi))^((1-alpha)/(alpha+psi)))/
  (1-alpha)-((beta*eta*a^(1-alpha))^((1+psi)/
  (alpha+psi)))/(1+psi);
eta1=n/(n+1);
v1=beta*((beta*eta1*a^(1+psi))^((1-alpha)/(alpha+psi)))/
   (1-alpha)-((beta*eta1*a^(1-alpha))^((1+psi)/
   (alpha+psi)))/(1+psi);
if k>v;
 n=n-1;
 continue;
 endif;
if v1 <= k1;
    goto label;
    endif;
 n=n+1;
endo;
label: /* we set N_{+1}^{*}y_{+1}^{*}=N^{*}y^{*}=Ny
and a=1 at the initial state */
nyfor=50*(beta*((50-1)/50)*1^(1-alpha))^(alpha+psi);
y=(beta*eta*a^(1-alpha))^(alpha+psi);
x[1,3]=1/nyfor;
x[1,4]=1;
x[i+1,1]=n;
x[i+1,2]=a;
x[i+1,3]=1/(n*y);
x[i+1,4]=(n*y)*(a/nyfor);
z[i+1,1]=n;
```

```
z[i+1,2]=(a-1)*100;
z[i+1,3]=100*(x[i+1,3]/x[i,3]-1);
z[i+1,4]=100*(x[i+1,4]/x[i,4]-1);
z[i+1,5]=z[i+1,3]/z[i+1,4];
i=i+1;
endo;
print "1st col is number of exporters, 2nd col is
value of a, 3rd col is equilibrium value of import
price, 4th col is equilibrium exchange rate"
x[2:101,.];
print "1st col is number of exporters, 2nd col is
percentage change in a, 3rd col is percentage
change in import price, 4th col is percentage
change in exchange rate, 5th col is the exchange
rate pass-through coeff" z[2:101,.];
output off;
end;
```

Chapter 2

Hysteresis in Trade, Exchange Rate Pass-Through and Temporary Protection

2.1 Introduction

Besides the persistence of the US trade deficit in the eighties mentioned in Chapter 1, it was also observed that import prices did not fall with the continuous appreciation of the US dollar in the early eighties. Mann (1986), Krugman and Baldwin (1987), Froot and Klemperer (1989), have provided thorough discussions of this phenomenon. They suggest that foreign producers prefer to shelter import prices from the continual fluctuations of the exchange rate by absorbing those fluctuations into their profit margins.

'Exchange rate pass-through' is defined as the degree of import or export price change in response to exchange rate changes holding other factors constant. Pass-through is said to be complete when any change in the exchange rate is reflected entirely in the local prices of import goods in the destination countries. We measure the degree of exchange rate pass-through by the elasticity of import prices with respect to the exchange rate. Knetter (1989) and Krugman (1987), take a cross-country point of view to analyse the phenomenon of incomplete exchange rate pass-through. They examine whether foreign producers have different pricing policies for the US market compared to the rest of the world. Krugman (1987) terms this as 'Pricing to Market'. It is a kind of price discrimination adopted by the foreign producers across export destinations. They both find evidence of 'Pricing to Market' in the early eighties in some import markets e.g., automobiles. The German and Japanese producers would especially like to maintain stable import prices in the USA when the exchange rate fluctuates because they would like to maintain the market shares in the USA. As a result, their actions produce a lower exchange rate pass-through in comparison to the import prices in the rest of the world.

There are at least two reasons for incomplete exchange rate pass-through: (1) The foreign producers have an incentive to preserve their market share over time. Dohner (1984), and Gottfries (1986) have addressed the importance of maintaining market share when the importers or exporters are making pricing decisions in an intertemporal framework. (2) There are endogenous changes in imperfectly competitive market structures due to any prolonged overvaluation or undervaluation of the exchange rate, the so-called hysteresis in trade. For examples, see Baldwin (1988a,b,c), Baldwin and Krugman (1989), Baldwin and Lyons (1988, 1989), Dixit (1989a,b), Dornbusch (1987), etc. We shall focus on the incomplete exchange rate pass-through due to the presence of hysteresis.

As mentioned in Chapter 1, the hysteresis approach basically requires the presence of entry costs, e.g., sunk costs or exit costs for the generation of hysteretic effect. This view suggests that the number of foreign incumbent firms is changing as there are entries or exits due to the exchange rate changes. In addition, the number of firms will not return to the original number even when the exchange rate disturbance is removed. Dixit (1989a, b) uses an option pricing approach to study the relationship between hysteresis and the exchange rate pass-through in a partial equilibrium framework. The entry and exit of each firm in an export industry is treated as an option. When the exchange rate is high enough, it creates sufficiently large operating profits to cover the sunk costs. The foreign firms can enter the domestic markets. The complete process is equivalent to exercising options at the profitable moment. Dixit's (1989b) analysis is at the industry level, the exchange rate is specified exogenously, and the response of imports and prices in this industry is derived. Through a numerical computation, Dixit (1989b) obtains a result that the pass-through coefficient during the entry and exit phase will be higher than that without entry or exit. Actually, the calculated coefficients during the entry and exit phase are close to one; those during no entry or exit approach zero.

Baldwin and Lyons' (1988, 1989) results and all those partial equilibrium models are of short or medium run, as prices are sticky. The stickiness causes the exchange rate to fluctuate beyond its fundamental value. As Baldwin and Lyons (1989) state in their paper, it is more difficult to use the usual exchange rate determination mechanism e.g., purchasing power parity (PPP), to generate hysteresis results.

This chapter attempts to tackle the problem posed above and to provide a long run general equilibrium analysis of hysteresis in trade and the exchange rate pass-through. The long run situation is that prices are flexible and purchasing power parity (PPP) holds. The basic setup is quite similar to Dixit (1989b) except the exchange rate is determined endogenously by the purchasing power parity in our model. Dixit (1989b) assumes that the exchange rate is exogenous and follows a geometric Brownian motion. We shall directly apply the model developed in Chapter 1 to study the pass-through problem. We have derived the following results: (1) If there is no entry or exit, pass-through will generally be incomplete and it is determined by the preferences and the type of shock to the economy. (2) When there is entry or exit, the pass-through will be complete because the change in the number of exporters will dominate the individual output response of each producing exporter. This result turns out to be similar to what Dixit (1989b) obtained in his partial equilibrium model. Moreover, we have extended the analysis to study temporary protection. If tariff changes are sufficiently large, entry and exit of firms will occur, leading to hysteresis.

2.2 Exchange Rate Pass-Through

In this chapter, we shall utilize the general equilibrium model developed in chapter 1 to study the exchange rate pass-through relationship in an economy where the exporters can enter and exit the market. We conduct a number of comparative statics exercises on this economy. Since the economy exhibits multiple equilibria, the comparative statics become more difficult and ambiguous. We follow Chatterjee and Cooper (1989) and focus mainly on the shock of a^{*}_{t+1} to the equilibrium. The analysis starts from an equilibrium and examines the change in the incentives of a representative agent to enter or exit after a positive shock to the economy. If all the existing firms still have incentives to produce, the economy will move to a higher level equilibrium[1]. In principle, the economy can move to any one of several equilibria following a shock. Here we focus on the closest one with more (less) participation after the shock. Finally, we compare the old equilibrium with the new equilibrium. This will be illustrated more clearly in the numerical example later.

We define $V(a^{*}_{t+1}, N_t)$ as the indirect utility function, or the value function of participation in production of the active agent, when there are N_t incumbent producers in the export market in period t. The derivative of $V(a^{*}_{t+1}, N_t)$ with respect to a^{*}_{t+1} is interpreted as the change in the utility of the active agent corresponding to a change in a^{*}_{t+1} in all periods with the number of incumbents, N_t, held fixed.

Proposition 2.2.1 *Given the current number of incumbents, N_t, the individual output and utility is increasing in a^{*}_{t+1}.*

$$\begin{aligned} \partial V(a^{*}_{t+1}, N_t)/\partial a^{*}_{t+1} &> 0, \\ \partial \log y(a^{*}_{t+1}, N_t)/\partial \log a^{*}_{t+1} &= \epsilon_W/(\epsilon_G - \epsilon_W) > 0. \end{aligned}$$

[1] A higher level equilibrium is a equilibrium with higher level of participation.

See appendix 2.A for proof[2]. Basically, a^{*}_{t+1} is the return to saving, the equilibrium interest rate. An increase in the interest rate is equivalent to an increase in the marginal benefit of working.

Proposition 2.2.2 *Given a^{*}_{t+1}, any increase in the current number of competitors in the export market (N_t) raises the individual output y_t and the utility of the producing agent.*

$$\begin{aligned} \partial \log y(a^{*}_{t+1}, N_t)/\partial \log N_t &= 1/[(N_t - 1)(\epsilon_G - \epsilon_W)] > 0, \\ \partial V(a^{*}_{t+1}, N_t)/\partial N_t &> 0, \ \ \textit{and} \\ \partial \log N_t y(a^{*}_{t+1}, N_t)/\partial \log N_t &= 1 + [1/(N_t - 1)(\epsilon_G - \epsilon_W)] > 0 \end{aligned}$$

See appendix 2.A for proof. The presence of this feature contrasts with the closed economy model analysed by Chatterjee and Cooper (1989). In their model, the utility of all firms will be reduced if there are more entries of foreign firms in the current period holding fixed the future entry levels. The individual output response to N is ambiguous. The reasons are as follows: In a closed economy, the return to holding money is $P_t/P_{t+1} = N_{t+1}y_{t+1}/N_t y_t$. Money only plays the role of 'store of value', and it is the means of savings. As N_t increases given all future variables, it will have two effects on the individual choice of the output: One is that η_t rises with N_t, therefore raising the output. However, the increase in N_t raises the total output, $N_t y_t$ which lowers P_t/P_{t+1} and discourages current output production. Thus, individual output response is ambiguous in Chatterjee and Cooper's closed economy model. The individual is worse off given the future variables because more competitors reduce the current profit, lowering the utility for each firm. In my model, although new entries lower P^{*}_t, they simultaneously raise e_t. This compensates for the fall in revenue due to the fall in P^{*}_t by raising revenue denominated in the domestic currency. Thus, only the effect of the increase of η_t is observed as N_t increases. It raises individual output and welfare in equilibrium.

[2] $\partial V(a^{*}_{t+1}, N_t)/\partial a^{*}_{t+1} > 0$ is already stated in chapter 1.

In performing the comparative statics exercises in response to the changes in real shocks, $N_t^* y_t^*$ or $N_{t+1}^* y_{t+1}^*$, we shall only consider the temporary changes because the hysteresis effect is due to temporary instead of permanent changes in exogenous variables. As a result, when we consider the shift in current foreign aggregate income, $N_t^* y_t^*$, the future foreign aggregate income, $N_{t+1}^* y_{t+1}^*$ is held fixed and vice versa.

Now we attempt to compute the exchange rate pass-through coefficient in this model, which we denote by $\Delta \log P_t^* / \Delta \log e_t$, under general equilibrium. The exogenous shock we consider is a change in current foreign aggregate income, $N_t^* y_t^*$. We first study the case without entry or exit. Since e_t and P_t^* are continuous, differentiable and monotonic functions of $N_t^* y_t^*$, the elasticity of the exchange rate in response to the change in $N_t^* y_t^*$ becomes:

$$\begin{aligned}
\log e_t &= \log N_t y_t - \log N_t^* y_t^* \\
\frac{\partial \log e_t}{\partial \log N_t^* y_t^*}|_N &= \frac{\partial \log N_t y_t}{\partial \log a_{t+1}^*} \frac{\partial \log a_{t+1}^*}{\partial \log N_t^* y_t^*} - 1 \\
Where \ \frac{\partial \log a_{t+1}^*}{\partial \log N_t^* y_t^*} &= -1 \\
\frac{\partial \log N_t y_t}{\partial \log a_{t+1}^*} &= \frac{\epsilon_W}{\epsilon_G - \epsilon_W}
\end{aligned}$$

$$\frac{\partial \log e_t}{\partial \log N_t^* y_t^*}|_N = -(1 + \frac{\epsilon_W}{\epsilon_G - \epsilon_W}) = -\frac{\epsilon_G}{\epsilon_G - \epsilon_W} < 0 \tag{2.1}$$

The subscript 'N' on the derivative means that the derivative is evaluated holding N constant. Now we consider the elasticity of the export price P_t^* in response to the change in $N_t^* y_t^*$,

$$\begin{aligned}
\frac{\partial \log P_t^*}{\partial \log N_t^* y_t^*}|_N &= \frac{\partial \log P_t^*}{\partial \log N_t y_t} \frac{\partial \log N_t y_t}{\partial \log a_{t+\cdot}^*} \frac{\partial \log a_{t+1}^*}{\partial N_t^* y_t^*} \\
&= (-1)(\frac{\epsilon_W}{\epsilon_G - \epsilon_W})(-1) \\
&= \frac{\epsilon_W}{\epsilon_G - \epsilon_W}
\end{aligned}$$

The major component of the elasticities of the exchange rate, e_t and export price, P_t^* is $\partial \log N_t y_t / \partial \log a_{t+1}^*$ which is simply $\partial \log y_t / \partial \log a_{t+1}^*$ as N_t is held constant in this case. $\partial \log y_t / \partial \log a^*_{t+1}$ is the output elasticity in response to the exogenous variable, a^*_{t+1}. Moreover, there are two components affecting the elasticity of the exchange rate but only one component affecting the elasticity of export price, P^*_t. Since both e_t and P^*_t are endogenous, they all respond to the shift in the exogenous variable, $N^*_t y^*_t$. At equilibrium, the relative movement of P^*_t to e_t is by the ratio of $\partial \log P^*_t / \partial \log N^*_t y^*_t$ to $\partial \log e_t / \partial \log N^*_t y^*_t$:

$$-1 < \frac{\Delta \log P^*_t}{\Delta \log e_t}|_N = -\frac{\epsilon_W}{\epsilon_G} < 0 \tag{2.2}$$

ϵ_W and ϵ_G are the elasticities of the marginal benefit function, $W(\cdot)$ and the marginal cost function, $G(\cdot)$ of producing y_t. This is not the exchange rate pass-through coefficient, $\partial \log P^*_t / \partial \log e_t$, obtained in most partial equilibrium models in which the exchange rate is assumed to be exogenous. However, at equilibrium, we shall be able to approximate the pass-through coefficient by the relative movement of P^*_t to e_t by $\Delta \log P^*_t / \Delta \log e_t$. Therefore, the pass-through coefficients in this model have a different interpretation compared to those obtained in partial equilibrium model in which the exchange rate is completely exogenous. The exchange rate pass-through coefficient at a symmetric Nash equilibrium is independent of the individual output level, the number of incumbents and the market structure. Even if we set $\eta = 1$, the perfectly competitive case, the same result will be derived. The change in $N^*_t y^*_t$, foreign production, directly affects e_t as $e_t = N_t y_t / N^*_t y^*_t$ and indirectly affects it through $N_t y_t$, home production, and both move the exchange rate in the same direction. However, the export price, P^*_t is only affected by $N_t y_t$ because $P^*_t = 1/N_t y_t$. This causes the percentage change in the exchange rate to exceed the percentage change in the import price in this economy and is independent of the market structure. This point is usually overlooked when we treat the exchange rate as exogenous. This result differs from Dornbusch's (1987) result that the market structure is a major reason for the incomplete exchange rate pass-through. The pass-through coefficient is always less than one when there is no entry or exit. Suppose

$\epsilon_G = 1.5$, and $\epsilon_W = .5$, the absolute value of pass-through coefficient is 1/3. If we assume $u(c) = \log c$, and $\epsilon_W = 0$, the measured exchange rate pass-through will be zero without entry and exit.

Now we consider the exchange rate pass-through in the case with entry or exit. In this case, there are two effects operating: One is the effect of the changes in $N_t^* y_t^*$ on P_t^* holding N_t constant which has already been examined in the previous case without entry or exit; and the other one is the effect of changing N_t on the P_t^* which we will now examine. In this analysis, we encounter the problem that the change in N is discrete where N takes an integer value in reality, making impossible differential calculus. For example, a_{t+1}^* has to increase to a certain value so as to induce one more entry, otherwise no entry will be observed. As a result, the ratio of the change in N to the change in a_{t+1}^* should be $\Delta N_t / \Delta a_{t+1}^*$, approximated by the derivative $\partial N_t / \partial a_{t+1}^*$. In general, we are approximating this discrete effect by a continuous one.

$$
\begin{aligned}
\frac{\partial \log P_t^*}{\partial \log N_t} &= \frac{\partial \log P_t^*}{\partial \log N_t y_t} \frac{\partial \log N_t y_t}{\partial \log N_t} \\
&= -(1 + \frac{1}{(N_t - 1)(\epsilon_G - \epsilon_W)}) \\
\frac{\partial \log P_t^*}{\partial \log N_t^* y_t^*}|_E &= \frac{\partial \log P_t^*}{\partial \log N_t^* y_t^*}|_N + \frac{\partial \log P_t^*}{\partial \log N_t} \frac{\partial \log N_t}{\partial \log N_t^* y_t^*}
\end{aligned}
$$

The subscript 'E' on the derivative means that the derivative allowing entry and exit. The elasticity of P_t^* in response to the exogenous change of $N_t^* y_t^*$ with entry or exit can be decomposed into two effects: The first term is the elasticity of P_t^* in response to the exogenous change in $N_t^* y_t^*$ with N_t held fixed; the second term captures the effect of entry or exit on P_t^*. Define

$$
\frac{\partial \log N_t}{\partial \log a_{t+1}^*} = \gamma_t > 0 \qquad \frac{\partial \log N_t}{\partial \log N_t^* y_t^*} = -\gamma_t < 0
$$

They will be derived in the Appendix.

$$
\frac{\partial \log P_t^*}{\partial \log N_t^* y_t^*}|_E = \frac{\epsilon_W}{\epsilon_G - \epsilon_W} + \gamma_t (1 + \frac{1}{(N_t - 1)(\epsilon_W - \epsilon_W)}) \tag{2.3}
$$

Now we consider the elasticity of the exchange rate, e_t in response to the changes in $N_t^* y_t^*$:

$$\frac{\partial \log e_t}{\partial \log N_t^* y_t^*}|_E = \frac{\partial \log e_t}{\partial \log N_t^* y_t^*}|_N + \frac{\partial \log e_t}{\partial \log N_t y_t}\frac{\partial \log N_t y_t}{\partial \log N_t}\frac{\partial \log N_t}{\partial \log N_t^* y_t^*}$$

Similarly, the elasticity of e_t in response to the changes in $N_t^* y_t^*$ can be decomposed into two effects: The first one is the effect of $N_t^* y_t^*$ on e_t without entry or exit. The second one is the response of e_t to entry or exit.

$$\frac{\partial \log e_t}{\partial \log N_t^* y_t^*}|E = -[(\frac{\epsilon_G}{\epsilon_G - \epsilon_W}) + \gamma_t(1 + \frac{1}{(N_t - 1)(\epsilon_G - \epsilon_W)})] < 0 \tag{2.4}$$

Comparing the cases of with and without entry or exit, we can derive the following results ($|.|$ is the absolute value of the relevant variables):

Proposition 2.2.3

$$|\frac{\partial \log P_t^*}{\partial \log N_t^* y_t^*}|_E > |\frac{\partial \log P_t^*}{\partial \log N_t^* y_t^*}|_N,$$
$$|\frac{\partial \log e_t}{\partial \log N_t^* y_t^*}|_E > |\frac{\partial \log e_t}{\partial \log N_t^* y_t^*}|_N$$

Proof The proof is straight forward because both the absolute values of the two elasticities under the case of entry or exit have a second term that is positive, and the inequalities follow immediately. □

Obviously, P_t^* and e_t are more sensitive to the shifts in the exogenous variable when there are entries and exits in the market. Such entries and exits tend to amplify the responses of these two variables to $N_t^* y_t^*$. Dixit (1989b) also gives a similar prediction in his partial equilibrium model where the exchange rate is an exogenous stochastic process. The import price is less sensitive to the changes in the exogenous shock (the exchange rate) when there is no entry and exit. The entries and exits cause the import prices to fluctuate much more in response to the exchange rate shock. Moreover, the response of the exchange rate to the shift in $N_t^* y_t^*$ in general exceeds the response of import price. This provides us the incomplete pass-through outcome. If we try to obtain the pass-through coefficient when there is entry or exit, the following relationship is obtained,

$$\frac{\Delta \log P_t^*}{\Delta \log e_t}|_E = \frac{\partial \log P_t^* / \partial \log N_t^* y_t^*}{\partial \log e_t / \partial \log N_t^* y_t^*}$$

$$\frac{\Delta \log P_t^*}{\Delta \log e_t}|_E = -\frac{\epsilon_W/(\epsilon_G - \epsilon_W) + \gamma_t[1 + 1/(N_t - 1)(\epsilon_G - \epsilon_W)]}{\epsilon_G/(\epsilon_G - \epsilon_W) + \gamma_t[1 + 1/(N_t - 1)(\epsilon_G - \epsilon_W)]} \tag{2.5}$$

As we compare the absolute value of the coefficient above with the absolute value of the one without entry or exit, we obtain the following proposition

Proposition 2.2.4 *In general equilibrium, the exchange rate pass-through coefficient under the case of entry or exit will exceed that without entry or exit*[3].

Thus, we obtain a general equilibrium result consistent with Dixit's (1989b) partial equilibrium analysis: The exchange rate pass-through with entry or exit in the market is greater than the one without any entry or exit. The exact values of the pass-through coefficient during entry or exit must be evaluated numerically. This evaluation will be done in the next section.

Now we consider the case where the underlying exogenous shock involves anticipated changes in future foreign aggregate income, $N_{t+1}^* y_{t+1}^*$, and see how this type of shock affects the relationship between e_t and P_t^*. This case resembles the case of change in $N_t^* y_t^*$, and the only differences are

$$\begin{aligned}
\frac{\partial \log a_{t+1}^*}{\partial \log N_{t+1}^* y_{t+1}^*} &= 1 \\
\frac{\partial \log e_t}{\partial \log N_{t+1}^* y_{t+1}^*}|_N &= \frac{\partial \log N_t y_t}{\partial \log a_{t+1}^*} \frac{\partial \log a_{t+1}^*}{\partial \log N_{t+1}^* y_{t+1}^*} \\
&= \frac{\epsilon_W}{\epsilon_G - \epsilon_W}
\end{aligned}$$

Then the elasticities under the case of no entry and exit will be

$$\frac{\partial \log P_t^*}{\partial \log N_{t+1}^* y_{t+1}^*}|_N = \frac{\partial \log P_t^*}{\partial \log N_t y_t} \frac{\partial \log N_t y_t}{\partial \log a_{t+1}^*} \frac{\partial \log a_{t+1}^*}{\partial \log N_{t+1}^* y_{t+1}^*} = -\frac{\epsilon_W}{\epsilon_G - \epsilon_W}$$

[3]See appendix 2.A for proof.

$$\frac{\Delta \log P_t^*}{\Delta \log e_t}|_N = -1$$

Under the purely anticipated change in $N_{t+1}^* y_{t+1}^*$, there is complete pass-through. Now, let us consider the elasticities under the case of entry and exit:

$$\begin{aligned}
\frac{\partial \log P_t^*}{\partial \log N_{t+1}^* y_{t+1}^*}|_E &= \frac{\partial \log P_t^*}{\partial \log N_{t+1}^* y_{t+1}^*}|_N + \frac{\partial \log P_t^*}{\partial \log N_t}\frac{\partial \log N_t}{\partial \log N_{t+1}^* y_{t+1}^*} \\
Where \ \frac{\partial \log N_t}{\partial \log N_{t+1}^* y_{t+1}^*} &= \frac{\partial \log N_t}{\partial \log a_{t+1}^*} = \gamma_t > 0, \\
Thus \ \frac{\partial \log P_t^*}{\partial \log N_{t+1}^* y_{t+1}^*}|_N &= -[\frac{\epsilon_W}{\epsilon_G - \epsilon_W} + \gamma_t(1 + \frac{1}{(N_t - 1)(\epsilon_G - \epsilon_W)})] \\
\frac{\partial \log e_t}{\partial \log N_{t+1}^* y_{t+1}^*}|_E &= \frac{\partial \log e_t}{\partial \log N_{t+1}^* y_{t+1}^*}|_N + \frac{\partial \log N_t y_t}{\partial \log N_t}\frac{\partial \log N_t}{\partial \log N_{t+1}^* y_{t+1}^*} \\
&= \frac{\epsilon_W}{\epsilon_G - \epsilon_W} + \gamma_t(1 + \frac{1}{(N_t - 1)(\epsilon_G - \epsilon_W)})
\end{aligned}$$

The proposition 2.2.3 is still valid that both the absolute values of the elasticities under the case of entry or exit are larger than those under the case of no entry and exit. However, the measured exchange rate pass through coefficients will be the same.

Proposition 2.2.5 *If the underlying exogenous shock is an anticipated changes in* $N_{t+1}^* y_{t+1}^*$, *the exchange rate pass-through coefficient will equal minus one.*

Proof

$$\frac{\Delta \log P_t^*}{\Delta \log e_t}|_E = \frac{\partial \log P_t^* / \partial \log N_{t+1}^* y_{t+1}^*|_E}{\partial \log e_t / \partial \log N_{t+1}^* y_{t+1}^*|_E} = -1 \qquad \square$$

It is now clear that the measured pass-through coefficients depends upon the nature of the underlying shock. An anticipated future changes in foreign aggregate income will not cause a difference in the measured pass-through coefficient whether there is entry or exit. Moreover, the pass-through is complete. Only if the underlying exogenous shock involves the changes in $N_t^* y_t^*$ will there be incomplete exchange rate pass-through in this model.

Now we consider a different kind of exogenous change which is the shift in the parameter of disutility of working. With this exogenous variable, the agent's utility function will be

$$V(y_t, c_t) = -\psi_t v(y_t) + \beta u(c_t)$$

The equilibrium condition will be

$$\psi_t G(y_t) = W(a^{*}_{t+1} y_t)\eta_t \tag{2.6}$$

We examine the comparative statics properties of this equilibrium. If we apply the logarithmic transformation and take the derivative of the equation above with respect to $\log \psi_t$, we obtain

$$\frac{\partial \log y_t}{\partial \log \psi_t} = -\frac{1}{\epsilon_G - \epsilon_W} < 0$$

The elasticity of export price P^{*}_t in response to ψ_t when there is no entry and exit

$$\frac{\partial \log P^{*}_t}{\partial \log \psi_t}|_N = \frac{\partial \log P^{*}_t}{\partial \log N_t y_t}\frac{\partial \log N_t y_t}{\partial \log \psi_t} = \frac{1}{\epsilon_G - \epsilon_W} > 0$$

The elasticity of the exchange rate in response to ψ_t when there is no entry and exit

$$\frac{\partial \log e_t}{\partial \log \psi_t}|_N = \frac{\partial \log N_t y_t}{\partial \log \psi_t} = -\frac{1}{\epsilon_G - \epsilon_W}$$

The measured exchange rate pass-through coefficient under the case of no entry and exit is

$$\frac{\Delta \log P^{*}_t}{\Delta \log e_t}|_N = \frac{\partial \log P_t / \partial \log \psi_t}{\partial \log e_t / \partial \log \psi_t} = -1 \tag{2.7}$$

Thus, there is a complete pass-through case. Now we consider the case of entries and exits,

$$\frac{\partial \log P^{*}_t}{\partial \log \psi_t}|_E = \frac{\partial \log P^{*}_t}{\partial \log \psi_t}|_N + \frac{\partial \log P^{*}_t}{\partial \log N_t y_t}\frac{\partial \log N_t y_t}{\partial \log N_t}\frac{\partial \log N_t}{\partial \log \psi_t}$$

$$Let \quad \frac{\partial \log N_t}{\partial \log \psi_t} = -c_t < 0$$

$$\frac{\partial \log P^{*}_t}{\partial \log \psi_t}|_E = \frac{1}{\epsilon_G - \epsilon_W} + c_t(1 + \frac{1}{(N_t - 1)(\epsilon_G - \epsilon_W)})$$

$$\frac{\partial \log e_t}{\partial \log \psi_t}|_E = \frac{\partial \log e_t}{\partial \log \psi_t}|_N + \frac{\partial \log e_t}{\partial \log N_t y_t}\frac{\partial \log N_t y_t}{\partial \log N_t}\frac{\partial \log N_t}{\partial \log \psi_t}$$

$$= -[\frac{1}{\epsilon_G - \epsilon_W} + c_t(1 + \frac{1}{(N_t - 1)(\epsilon_G - \epsilon_W)})]$$

The derivation of $\partial \log N_t / \partial \log \psi_t = -c_t$ is shown in the appendix 2.A. Proposition 2.2.3 still holds. The absolute value elasticities under the case of entries and exits exceed those under the case of no entry and exit. The exchange rate pass-through coefficient under the case of entry and exit is

$$\frac{\Delta \log P_t^*}{\Delta \log e_t}|_E = -1$$

Similarly, we also obtain a complete pass-through result.

Proposition 2.2.6 *When the underlying exogenous shock involves a shift in parameter of disutility in working, ψ_t, the exchange rate pass-through is complete and does not depend on the condition of entry or exit.*

Now we consider the shift in the parameter of foreign disutility, ψ_t^* on e_t and P_t^*, for the case that there is no entry and exit:

$$\begin{aligned}
\frac{\partial \log P_t^*}{\partial \log \psi_t^*}|_N &= \frac{\partial \log P_t^*}{\partial \log N_t y_t} \frac{\partial \log N_t y_t}{\partial \log a_{t+1}^*} \frac{\partial \log a_{t+1}^*}{\partial \log N_t^* y_t^*} \frac{\partial \log N_t^* y_t^*}{\partial \log \psi_t^*} \\
&= (-1)\frac{\epsilon_W}{\epsilon_G - \epsilon_W}(-1)\frac{-1}{\epsilon_G^* - \epsilon_W^*} \\
&= -\frac{\epsilon_W}{(\epsilon_G - \epsilon_W)(\epsilon_G^* - \epsilon_W^*)} < 0 \\
\frac{\partial \log e_t}{\partial \log \psi_t^*}|_N &= \frac{\partial \log N_t y_t}{\partial \log a_{t+1}^*} \frac{\partial \log a_{t+1}^*}{\partial \log N_t^* y_t^*} \frac{\partial \log N_t^* y_t^*}{\partial \log \psi_t^*} - \frac{\partial \log N_t^* y_t^*}{\partial \log \psi_t^*} \\
&= [1 + \frac{\epsilon_W}{\epsilon_G - \epsilon_W}]\frac{1}{\epsilon_G^* - \epsilon_W^*} \\
&= \frac{\epsilon_G}{(\epsilon_G - \epsilon_W)(\epsilon_G^* - \epsilon_W^*)} > 0
\end{aligned}$$

The exchange rate pass-through coefficient is

$$\frac{\Delta \log P_t^*}{\Delta \log e_t}|_N = -\frac{\epsilon_W}{\epsilon_G} > -1$$

This result is exactly the same as what we obtained by shifting $N_t^* y_t^*$. The reason is that $N_t y_t$ in this model is only affected by a_{t+1}^*, the external shift parameter in the model. The shift in ψ_t^* goes through the same channel to change $N_t^* y_t^*$ which shifts a_{t+1}^*. Thus, the analysis of the effect of the shift in ψ_t^* on the home economy is equivalent to the analysis of the shift of $N_t^* y_t^*$, and we shall not repeat it here.

Proposition 2.2.7 *With shifts in the foreign parameter of disutility in working, ψ_t^*, the exchange rate pass-through is incomplete when there is no entry or exit. It is complete when there is entry or exit.*

Finally, we try to consider the dynamic adjustment of the economy when it is subject to some exogenous shock, a_{t+1}^*. We examine how the economy behaves when there are entries and exits. Previously, we derived that

$$\frac{\partial V(\cdot)}{\partial a_{t+1}^*} = \beta u'(a_{t+1}^* y_t) y_t (1 + \frac{1}{N_t} \frac{\epsilon_W}{\epsilon_G - \epsilon_W})$$

We maintain the assumption that the utility function is of constant elasticity type, and we obtain the following result:

Proposition 2.2.8 *The derivative of the value function $V(a_{t+1}^*, N_t)$ with respect to a_{t+1}^* with respect to a_{t+1}^* is increasing in N_t.*

$$\partial^2 V(a_{t+1}^*, N_t)/\partial N_t \partial a_{t+1}^* > 0.$$

See appendix 2.A for proof[4]. This result indicates that as N_t increases, $\partial V(\cdot)/\partial a_{t+1}^*$ becomes larger than before. Thus, it is easier to remain at the equilibria with a lower number of suppliers persistently than than those with a higher number of suppliers. The equilibrium at a high number of suppliers is more sensitive to the changes in a_{t+1}^* and it becomes easier to move the economy to another equilibrium[5].

On the other hand, if $k_i'(N) < 0$, $\partial N_t/\partial a_{t+1}^*$ will not be the same, since k_i depends on N_t. The new $\partial \log N_t/\partial \log a_{t+1}^*|_{k(N)}$ (the subscript, $|_{k(N)}$ means that the derivative is taken under the condition that k_i is no longer a constant but a function of N) is

$$\frac{\partial \log N_t}{\partial \log a_{t+1}^*}|_{k(N)} = \frac{\bar{N} H'(k)}{1 + \bar{N} H'(k) k'(N_t)} \frac{\beta u'(a_{t+1}^* y_t) a_{t+1}^* y_t}{N_t} (1 + \frac{1}{N_t} \frac{\epsilon_W}{\epsilon_G - \epsilon_W}) > 0$$

The details of this derivation is in appendix 2.A. We replace

[4]It is similar to the proposition 1.3.2. The slight difference is proposition 1.3.2 is for steady state situation but proposition 2.2.8 is not for the steady state.

[5]For an example, please see section 3 of chapter 1.

$$\partial \log N_t/\partial \log a^*_{t+1} \text{ by } \partial \log N_t/\partial \log a^*_{t+1}|_{k(N)} = \gamma_t$$

Since $-1 < \bar{N}H'(k)k'(N_t) < 0$, we have

Corollary 2.2.1 *The derivative of* $\log N_t$ *with respect to* $\log a^*_{t+1}$ *under the presence of a positive network externality is greater than the one without any positive network externality.*

$$\partial \log N_t/\partial \log a^*_{t+1}|_{k(N)} > \partial \log N_t/\partial \log a^*_{t+1}.$$

Thus, it is more likely to have entries and exits due to shifts in a^*_{t+1}. In the next section, we shall study the pass-through coefficients during entries and exits by considering a numerical economy.

To sum up, the exchange rate pass-through will be incomplete under the cases of current aggregate income shock, $N^*_t y^*_t$, and shifts in foreign parameter of disutility in working ψ^*_t. For all the other shocks, the pass-through is complete.

2.3 A Numerical Example of Hysteresis and Exchange Rate Pass-through

We consider the following utility function of a representative agent:

$$V(y) = -\frac{y^{1+\psi}}{1+\psi} + \beta\frac{(a^* y)^{1-\alpha}}{1-\alpha}$$

From the equilibrium condition, we have

$$y = (\beta\eta a^{*1-\alpha})^{1/(\alpha+\psi)} \qquad \eta = \frac{N-1}{N}$$

Substituting in the utility function, we have

$$V(a^*, N) = -\frac{(\beta\eta a^{*1-\alpha})^{(1+\psi)/(\alpha+\psi)}}{1+\psi} + \beta\frac{(\beta\eta a^{*1+\psi})^{(1-\alpha)/(\alpha+\psi)}}{1-\alpha}$$

We set $\beta = 1$[6] and $\alpha = \psi = 0.5$, we assume the sunk cost is as follows:

[6] We assume there is not discounting of the future because the discount factor is not the crucial element for generating hysteresis.

$$k_i(N) = \bar{k}_i - 0.0006N \qquad \bar{k}_i = 1.3147 + 0.001(i-2) \qquad i = 2, 3, ..., \bar{N}$$

Here the personal specific sunk costs, $\bar{k}_i$ are evenly spaced across the population. Now suppose $a^* = 1$ initially. As we have shown in the previous chapter, there will be three equilibria in this economy: $N = 49, 50$ and 51. The corresponding equilibrium values and changes in the foreign import price and the exchange rate are presented as follows:

Table 2.1 Equilibrium values and changes in P^* and e

Equilibrium	P^*	e	$\hat{P}^*(\%)$	$\hat{e}(\%)$
$a^* = 1.00, N = 49$	0.020833	0.979592	2.08	-2.04
$a^* = 1.00, N = 50$	0.020408	1.000000	0.00	0.00
$a^* = 1.00, N = 51$	0.020000	1.020408	-2.00	2.04

$\hat{P}^*$ and $\hat{e}$ are the percentage changes in the equilibrium values of relative variables compared to the initial equilibrium.

Initially, we assume the economy satisfies the following equilibrium condition:

$$\begin{aligned} k_{51}(50) &= 1.33370 > V(1,51) = 1.3331398 > V(1,50) = 1.333132 > \\ k_{50}(50) &= 1.33270 \end{aligned}$$

so the equilibrium is at $N = 50$ We assume $N^*_{+1}y^*_{+1} = N^*y^* = 50y(1,50)$, so the foreign import price, $P^*(1,50) = 1/[50y(1,50)] = 0.020408$ and the exchange rate, $e(1,50) = 50y(1,50)/N^*y^* = 1.0$ at the initial equilibrium. This is the original point of reference for the economy. Whenever there is a change in a^*, it is due to the change in N^*y^* holding $N^*_{+1}y^*_{+1}$ constant.

In order to ascertain the pass-through relationship during the cases of entry or exit, and no entry or exit, we further extend this numerical model to consider 100 random numbers of a^* drawn from a normal distribution with mean and standard deviation equal one and 0.0005. For each shock, we search for the corresponding equilibrium and calculate the equilibrium exchange rate, import price and the exchange rate pass-through coefficient, $(\Delta P^*_t/P^*_{t-1})/(\Delta e_t/e_{t-1})$ where $\Delta x_t = x_t - x_{t-1}$

in each period. Therefore, there are 100 periods altogether. The starting values of the exchange rate and import price are $e_0 = 1.0$ and $P_0^* = 0.020408$ which are also the means of the corresponding distributions of e_t and P^*. We plot the simulated observations in figures 2.1 to 2.5. If there is no hysteresis effects, we should observe that e_t and P^* fluctuate around their means, and there should be no persistent deviations from their mean levels. From figures 2.1 and 2.2, e and P^* remain persistently away from their mean levels, moving to a new level only when the economy experiences a considerable shock which will break the persistence. Figures 2.3 and 2.4 illustrate that the magnitude of fluctuations in e and P^* are much larger than the fluctuations in a^*. Figure 2.5 plots the calculated exchange rate pass-through coefficients, which mainly concentrate on two values, -1/3, and -1. The value of -1/3 is predicted in section 2.2 where $\Delta \log P_t^* / \Delta \log e_t|_N = -\epsilon_w/\epsilon_G$. In this example, $\epsilon_W = 0.5$ and $\epsilon_G = 1.5$, so the coefficient equals -1/3. If we superimpose figures 2.1 or 2.2 on figure 2.5, it is found that -1 occurs when the firms are entering or leaving, -1/3 occurs when there is no entry or exit. This result is similar to what Dixit (1989b) obtained in his numerical partial equilibrium model. There are more cases of -1/3 because the pass-through coefficients equal -1 only when there are entries and exits i.e., N moves from 50 to 52 or from 51 to 49, etc. We discussed in chapter 1 that N will change only when there is a sufficiently large shock hitting the economy. When the shock is not large enough, the economy will stay in the last period equilibrium. The pass-through coefficient will equal -1/3. How many pass-through coefficients equal -1 or -1/3 depends on the variance of the shock that we set. If we choose a shock with a greater variance, the chance of getting larger shocks is bigger. We shall observe more entries or exits and more cases of pass-through coefficients being -1.

2.4 Tariff Effects

Feenstra (1989) suggests that exchange rate changes and tariff changes will have identical effects on the exporters' decisions when analysed in a partial equilibrium framework. The reason is that e_t and the tariff function like a revenue tax (subsidy) as perceived by exporters in a partial equilibrium model. We examine this argument in our general equilibrium model. We assume that the foreign country levies a tariff, τ_t on imports and redistributes the tariff revenue in a lump sum to the foreign citizens who are producing. The exporter's revenue after tariff becomes

$$e_t R(y_t) = e_t \frac{P_t^*}{(1+\tau_t)} y_t$$

The equilibrium condition under the tariff will be

$$v'(y_t)y_t = \beta u'(a_{t+1}^{**} y_t) a_{t+1}^{**} y_t \eta_t \qquad (2.8)$$

Where

$$a_{t+1}^{**} = \frac{N_{t+1}^* y_{t+1}^*}{N_t^* y_t^*} \frac{1}{(1+\tau_t)}$$

We compare the effect of a change in the tariff to the effect of a change in e_t under the case of no entry and exit,

$$\frac{\partial \log P_t^*}{\partial \log(1+\tau_t)}|_N = \frac{\partial \log P_t^*}{\partial \log a_{t+1}^{**}} \frac{\partial \log a_{t+1}^{**}}{\partial \log(1+\tau_t)} = \frac{\epsilon_W}{\epsilon_G - \epsilon_W}$$

$$\frac{\partial \log P_t^*}{\partial \log \tau_t}|_N = \frac{\tau_t}{1+\tau_t} \frac{\epsilon_W}{\epsilon_G - \epsilon_W} \qquad (2.9)$$

In the previous section, if the change in a_{t+1}^* was caused by $N_t^* y_t^*$

$$\frac{\Delta \log P_t^*}{\Delta \log e_t}|_N = -\frac{\epsilon_W}{\epsilon_G}.$$

If the change is caused by $N_{t+1}^* y_{t+1}^*$

$$\frac{\Delta \log P_t^*}{\Delta \log e_t}|_N = -1.$$

Under the case of entry or exit,

$$\begin{aligned}\frac{\partial \log P_t^*}{\partial \log(1+\tau_t)}|_E &= \frac{\partial \log y_t}{\partial \log(1+\tau_t})|_N + \\ &\quad \frac{\partial \log P_t^*}{\partial \log N_t y_t}\frac{\partial \log N_t y_t}{\partial \log N_t}\frac{\partial \log N_t}{\partial \log a_{t+1}^{**}}\frac{\partial \log a_{t+1}^{**}}{\partial \log(1+\tau_t)} \\ &= [\frac{\epsilon_W}{\epsilon_G-\epsilon_W} + \gamma_t(1+\frac{1}{(N_t-1)(\epsilon_G-\epsilon_W)})]\end{aligned}$$

$$\frac{\partial \log P_y^*}{\partial \log \tau_t}|_E = \frac{\tau_t}{1+\tau_t}[\frac{\epsilon_W}{\epsilon_G-\epsilon_W} + \gamma_t(1+\frac{1}{(N_t-1)(\epsilon_G-\epsilon_W)})] \tag{2.10}$$

If the change in a_{t+1}^* is caused by $N_t^* y_t^*$ or disutility parameter, ψ_t

$$\frac{\Delta \log P_t^*}{\Delta \log e_t}|_E = -\frac{\epsilon_W/(\epsilon_G-\epsilon_W)+\gamma_t[1+1/(N_t-1)(\epsilon_G-\epsilon_W)]}{\epsilon_G/(\epsilon_G-\epsilon_W)+\gamma_t[1+1/(N_t-1)(\epsilon_G-\epsilon_W)]}$$

If the change in a_{t+1}^* is caused by $N_{t+1}^* y_{t+1}^*$

$$\frac{\Delta \log P_t^*}{\Delta \log e_t}|_E = -1$$

We observe that the measured effect of a change in the tariff on P_t^* is quite different from the measured exchange rate pass-through coefficient at general equilibrium. In a general equilibrium model, the exchange rate is endogenous. However, in partial equilibrium model, the exchange rate is exogenous. The tariff is always exogenous in both cases. Therefore, we observe the difference between the tariff effect on P_t^* and the exchange rate pass-through effect. Actually, a change in the tariff is similar to a change in a_{t+1}^* studied in the previous section as both are exogenous variables. A tariff is a common instrument used to limit the entries of foreign firms. Any rise in tariffs imposed by a foreign country will have the same effect as a drop in a_{t+1}^*, therefore influencing foreign firms to leave by lowering their $V(\cdot)$ so that they are less than their sunk utility costs.

The presence of hysteresis effects will have further implications on a temporary protection policy like a tariff. Baldwin and Green (1988) mention the possibility of hysteresis effect in temporary protection in discussing Baldwin's (1986a) paper. As the changes in τ_t have similar effects on the endogenous variables compared to the changes in $N_t^* y_t^*$ or $N_{t+1}^* y_{t+1}^*$, the same analysis on hysteresis can be applied to a temporary protection policy. For example, a foreign country may like to protect their import competing industries by temporarily raising tariffs and driving the home country exporters out of the markets. With N being discrete, the tariff must be raised beyond the threshold level which will drive the exporters out of the markets. Otherwise, the change in the tariff may have little effect in reducing the volume of imports and foreign exporters. After a few years, the foreign country may like to remove the tariff to raise its nation's welfare. However, after removal of the tariff, not all foreign exporters will return and the hysteresis effect appears. This further adds a potential cost and more uncertainty to temporary protection.

2.5 Conclusion

This chapter studies the general equilibrium of a dynamic model where agents' entry and exit decisions and the exchange rate are endogenous. In general equilibrium, some agents participate in production and some do not. Under shifts in the current foreign aggregate income, the exchange rate pass-through is incomplete when there is no entry or exit. The value of the pass-through coefficient depends on the elasticity (concavity) of the agent's utility function. When there are entries or exits, the pass-through coefficients exceed the pass-through coefficients under the case of no entry and exit. In a numerical example, the coefficient is close to -1 (complete pass-through) during the entry and exit phase. This result is consistent with Dixit's (1989) numerical results that the exchange rate pass-through during the entry and exit phases is close to -1 which is higher than the one during no entry and exit. However, if there are shifts in the domestic disutility parameter, ψ_t and anticipated shifts in the foreign future aggregate income, the pass-through coefficients all equal minus one (complete pass-through) whether there are entries and exits or not.

In this general equilibrium model, a tariff has a different effect from an exchange rate change because the exchange rate is endogenous and the tariff is exogenous to the domestic exporters in this model. This result is different from Feenstra's (1989) partial equilibrium result that the tariff and the exchange rate have identical effects on the exporters. In addition, the presence of hysteresis in trade adds some potential costs and uncertainty to temporary protection policy.

Figure 2.1 Simulated data of equilibrium foreign import price, P^* (mean and S.D. of a^* are 1 and 0.0005)

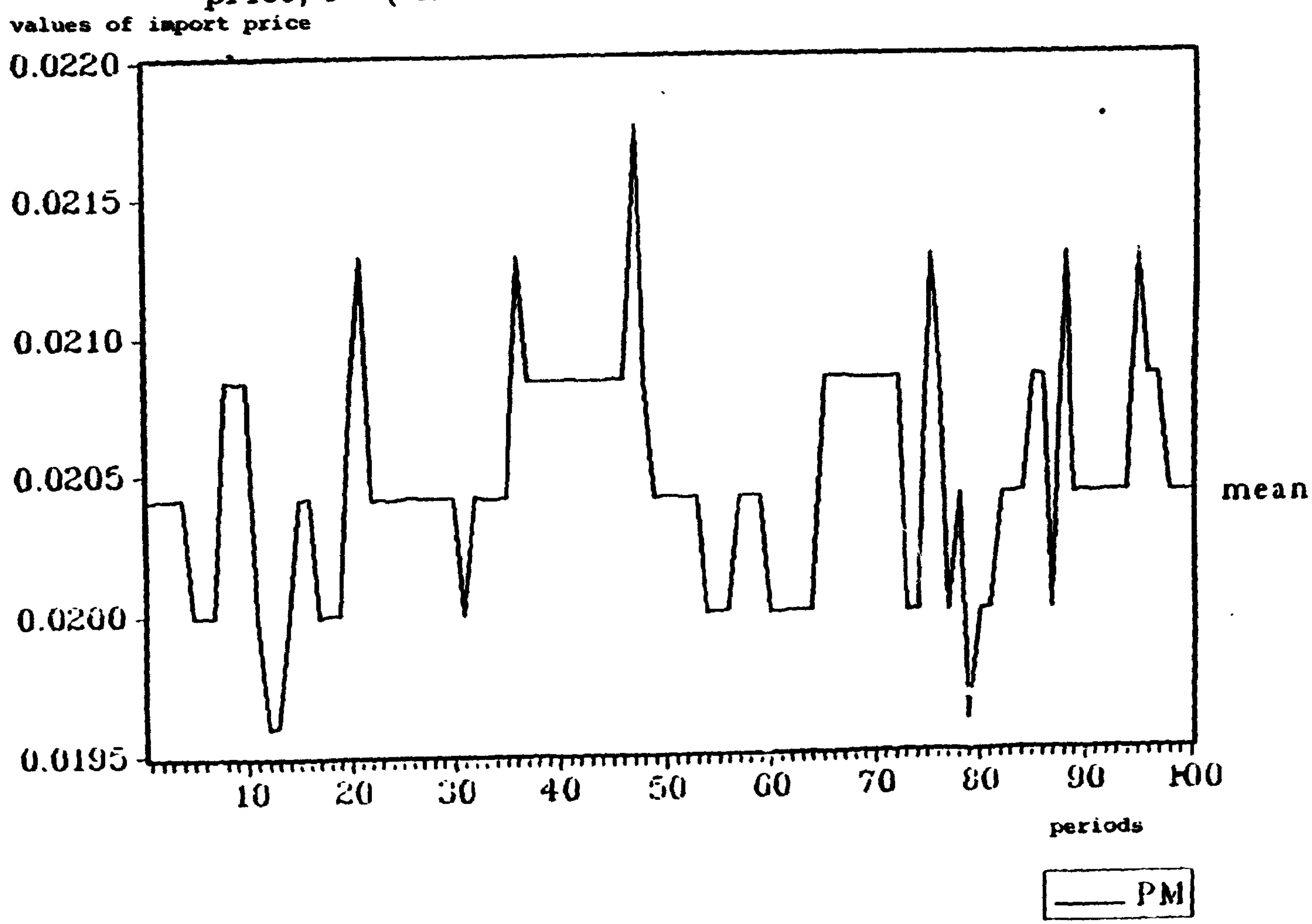


Figure 2.2 Simulated data of equilibrium exchange rate, e
(mean and S.D. of a* are 1 and 0.0005)

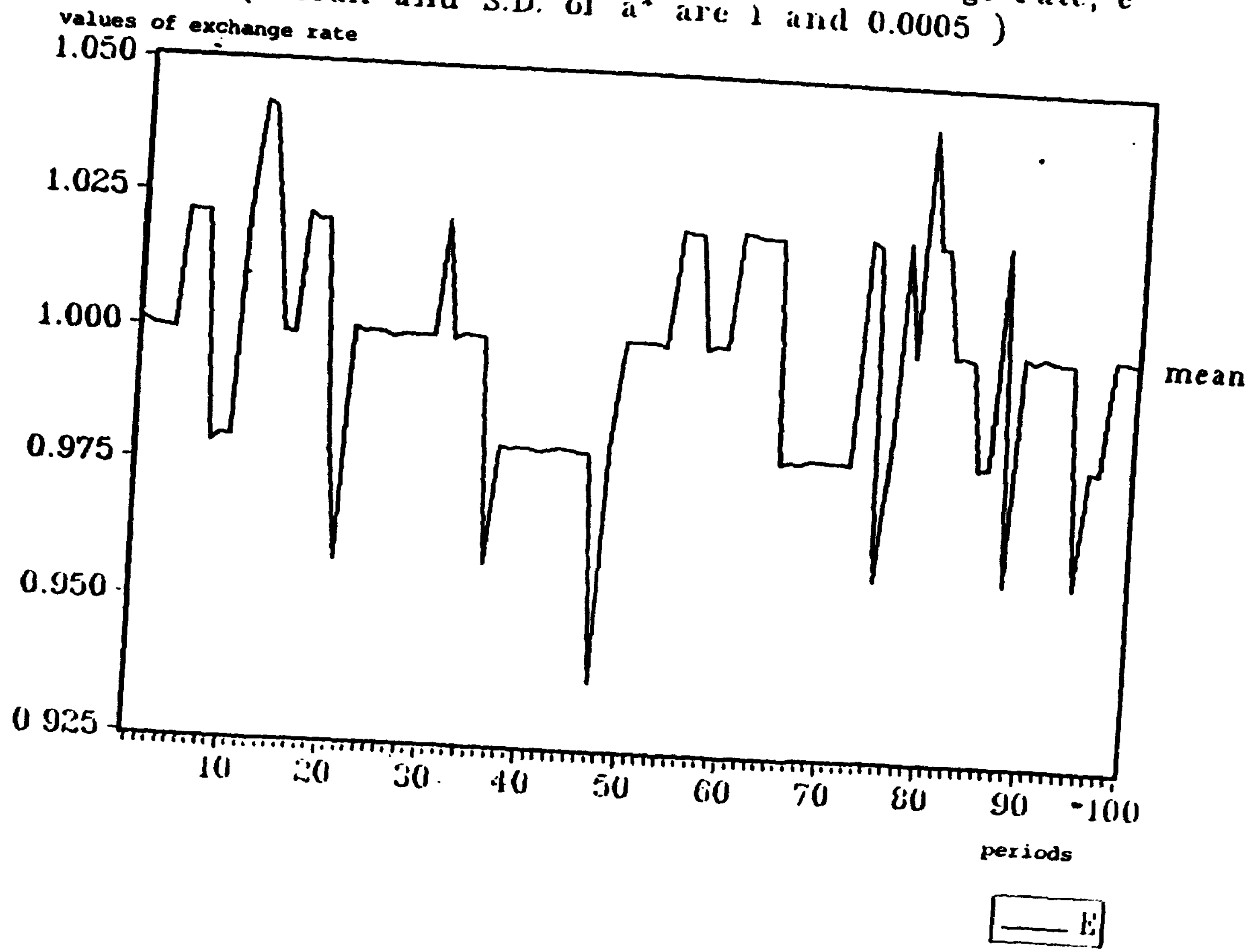


Figure 2.3 Percentage changes in foreign import price, P^* and a^* (mean and S.D. of a^* are 1 and 0.0005)

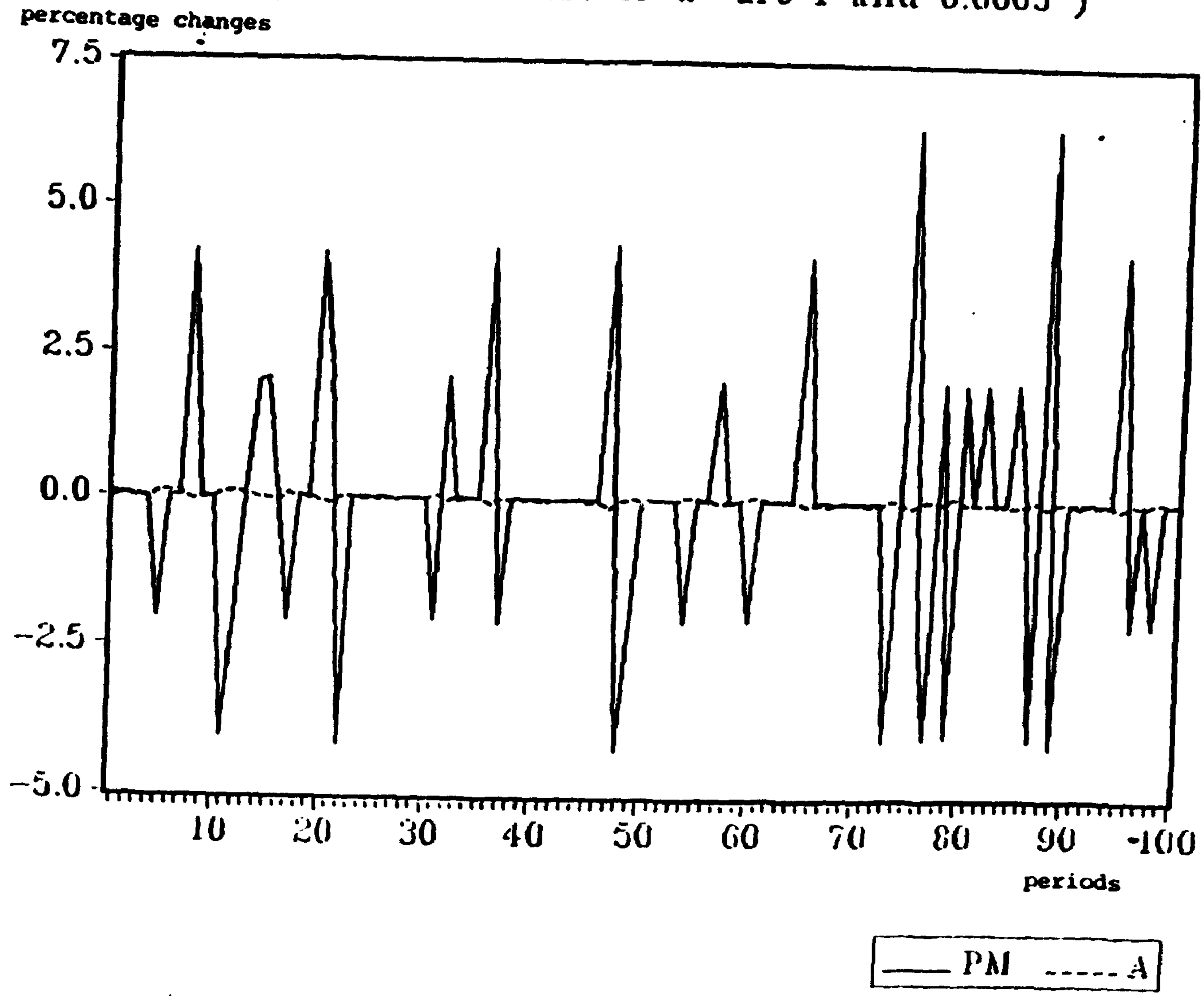


Figure 2.4 Percentage changes in exchange rate, e and a*
(mean and S.D. of a* are 1 and 0.0005)

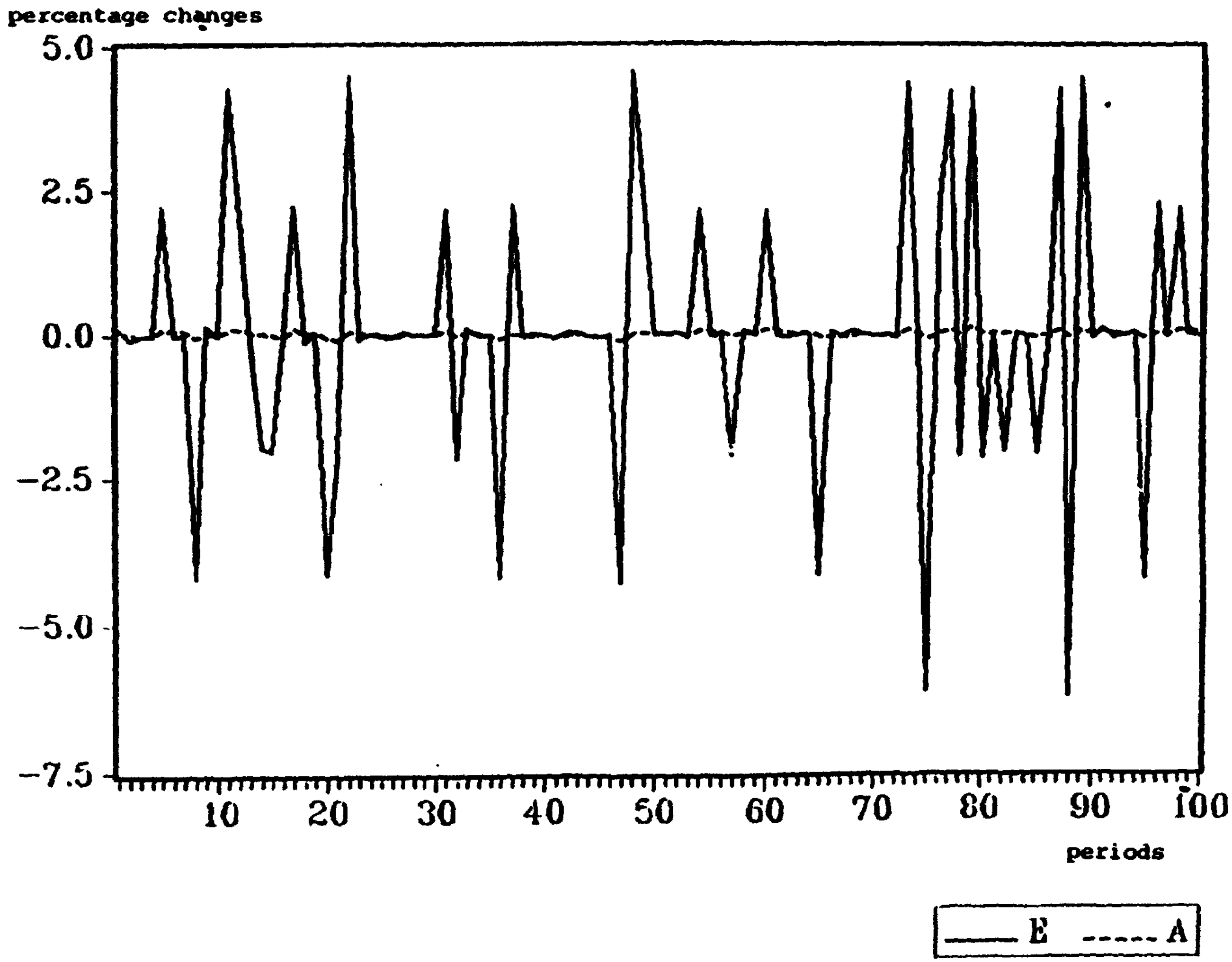
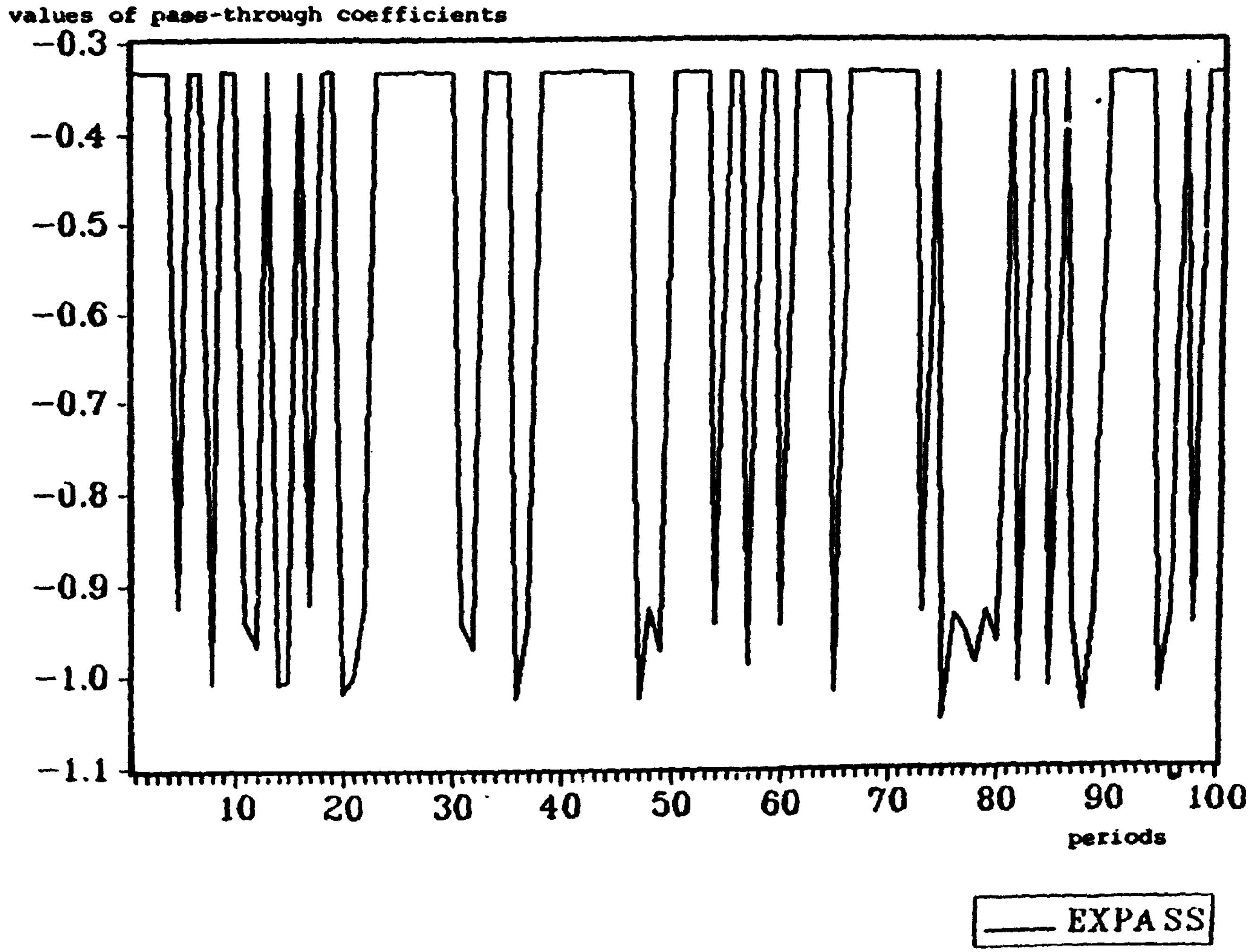


Figure 2.5 Exchange rate pass-through coefficient calculated at each equilibrium (mean and S.D. are 1 and 0.0005)



Appendix 2.A

Proofs of the Propositions

We first state the assumptions on the utility function under which the analysis will be carried out:

(A1) $u(\cdot)$ is increasing and concave and $v(\cdot)$ is increasing and convex

(A2) $d[xu'(x)]/dx > 0 \ \forall \ x \geq 0$, $\lim_{x \to 0} xu'(x) > 0$

(A3) $\lim_{x \to 1} xv'(x) = \infty$

(A4) $\lim_{x \to 0} xv'(x) = 0$

(A5) $\epsilon_v < \epsilon_G$ where ϵ_v and ϵ_G are the elasticities of the $v(x)$ and $xv'(x)$ functions

Define ϵ_W and ϵ_G as the elasticity of $W(\cdot)$ and $G(\cdot)$, then we have

$0 < \epsilon_W/\epsilon_G < 1$

Since $W(x) = xu'(x)$, $\log W = \log x + \log u'(x)$, so $d\log W(x)/d\log x = 1 + u''(x)x/u'(x)$, by A.2, $0 < \epsilon_W < 1$. Similarly, $d\log G(x)/d\log x = 1 + v''(x)x/v'(x)$, by A.1, $\epsilon_G > 1$.

Proof of Proposition 2.2.1

We differentiate equation (1) with respect to a^*_{t+1}

$$\beta\eta_t a^*_{t+1} y_t u''(\cdot)[y_t + a^*_{t+1}\frac{\partial y_t}{\partial a^*_{t+1}}] + \beta u'(\cdot)\eta_t[y_t + a^*_{t+1}\frac{\partial y_t}{\partial a^*_{t+1}}] = \frac{\partial y_t}{\partial a^*_{t+1}}[v'(\cdot) + y_t v''(\cdot)]$$

Define $\Delta_t = a^*_{t+1} y_t$

$$\beta\Delta_t u'(\Delta_t)\eta_t[1 + \frac{\Delta_t u''(\Delta_t)}{u'(\Delta_t)}] = \frac{\partial y_t}{\partial a^*_{t+1}}\frac{a^*_{t+1}}{y_t}[y_t v'(1 + \frac{y_t v''}{v'}) - \eta_t\beta u'(\Delta_t)\Delta_t(1 + \frac{\Delta_t u''}{u'})]$$

From equilibrium condition, it can be simplified to be

$$\begin{aligned}\epsilon_W &= \frac{\partial \log y_t}{\partial \log a^*_{t+1}}(\epsilon_G - \epsilon_W)\\ \Longrightarrow \frac{\partial \log y_t}{\partial \log a^*_{t+1}} &= \frac{\epsilon_W}{\epsilon_G - \epsilon_W} > 0\end{aligned}$$

As $V(a^*_{t+1}, N_t) = \beta u(a^*_{t+1}y(\cdot)) - v(y(\cdot))$,

$$\begin{aligned}\frac{\partial V(\cdot)}{\partial a^*_{t+1}} &= \beta u'(\cdot)[y_t + a^*_{t+1}\frac{\partial y_t}{\partial a^*_{t+1}}] - v'(\cdot)\frac{\partial y_t}{\partial a^*_{t+1}}\\ &= \beta u'(\cdot)y_t + [\beta u'(\cdot)a^*_{t+1} - v'(\cdot)]\frac{\partial y_t}{\partial a^*_{t+1}}\\ &= \beta u'(\cdot)y_t + (1-\eta_t)\beta u'(\cdot)a^*_{t+1}\frac{\partial y_t}{\partial a^*_{t+1}}\\ &= \beta u'(\cdot)y_t[1 + \frac{1}{N_t}\frac{a^*_{t+1}}{y_t}\frac{\partial y_t}{\partial a^*_{t+1}}]\\ &= \beta u'(\cdot)y_t[1 + \frac{1}{N_t}\frac{\epsilon_W}{\epsilon_G - \epsilon_W}] > 0 \qquad \square\end{aligned}$$

Proof of Proposition 2.2.2

$$\beta\eta_t a^*_{t+1}u''(\cdot)(a^*_{t+1}\frac{\partial y_t}{\partial N_t}) + \beta a^*_{t+1}u'(\cdot)\eta_t\frac{\partial y_t}{\partial N_t} + \frac{\beta a^*_{t+1}y_t u'(\cdot)}{N^2} = \frac{\partial y_t}{\partial N_t}[v'(\cdot) + y_t v''(\cdot)]$$

Define $\Delta_t = a^*_{t+1}y_t$

$$\begin{aligned}\frac{\beta\Delta_t u'(\Delta_t)}{N_t} &= \frac{\partial y_t}{\partial N_t}\frac{N_t}{y_t}[y_t v'(\cdot)(1 + \frac{y_t v''}{v'}) - \eta_t\Delta_t u'(\Delta_t)(1 + \frac{\Delta_t u''}{u'})]\\ \frac{y_t v'}{\eta_t N_t} &= \frac{\partial y_t}{\partial N_t}\frac{N_t}{y_t}(\epsilon_G - \epsilon_W)y_t v'\\ \Longrightarrow \frac{\partial y_t}{\partial N_t}\frac{N_t}{y_t} &= \frac{1}{(N_t - 1)(\epsilon_G - \epsilon_W)} > 0\end{aligned}$$

Since $V(a^*_{t+1}, N_t) = \beta u(a^*_{t+1}y(\cdot)) - v(y(\cdot))$,

$$\frac{\partial V(\cdot)}{\partial N_t} = (1-\eta_t)\beta u'(\cdot)a^*_{t+1}\frac{\partial y_t}{\partial N_t} > 0$$

$$\begin{aligned}\frac{\partial N_t y_t}{\partial N_t} &= y_t(1 + \frac{\partial y_t}{N_t}\frac{N_t}{y_t})\\ \frac{\partial N_t y_t}{\partial N_t}\frac{N_t}{N_t y_t} &= 1 + \frac{1}{(N_t - 1)(\epsilon_G - \epsilon_W)} > 0 \qquad \square\end{aligned}$$

Derivation of $\partial \log N_t / \partial \log N_t^* y_t^* < 0$

It can be derived by considering the fixed point relationship

$$V(\bar{N}H(k_t)) = k_t, \qquad \bar{N}H(k_{N_t}) = N_t$$

from the fixed point relationship, we have

$$\begin{aligned}
\frac{\partial N_t}{\partial a_{t+1}^*} &= \bar{N}H'(k_t)\frac{\partial k_t}{\partial a_{t+1}^*} \\
Since\ \frac{\partial V(\cdot)}{\partial a_{t+1}^*} &= \frac{\partial k_t}{\partial a_{t+1}^*} \\
\frac{\partial N_t}{\partial a_{t+1}^*} &= \bar{N}H'(k_t)\frac{\partial V(\cdot)}{\partial a_{t+1}^*} \\
\frac{a_{t+1}^*}{N_t}\frac{\partial N_t}{a_{t+1}^*} &= \frac{a_{t+1}^*}{N_t}\bar{N}H'(k)\frac{\partial V(\cdot)}{\partial a_{t+1}^*} \\
&= \beta \bar{N}H'(k)\frac{u'(a_{t+1}^* y_t)a_{t+1}^* y_t}{N_t}[1 + \frac{1}{N_t}\frac{\epsilon_W}{\epsilon_G - \epsilon_W}] = \gamma_t > 0
\end{aligned}$$

where $\partial V(\cdot)/\partial a_{t+1}^* > 0$ and $H'(k_t)$ is density function which is positive. Thus, we arrive at

$$\frac{\partial \log N_t}{\partial \log N_t^* y_t^*} = \frac{\partial \log N_t}{\partial \log a_{t+1}^*}\frac{\partial \log a_{t+1}^*}{\partial \log N_t^* y_t^*} = -\frac{\partial \log N_t}{\partial \log a_{t+1}^*} < 0 \qquad \square$$

Proof of proposition 2.2.3

We consider the difference of the absolute values of them

$$\left|\frac{\Delta \log P_t^*}{\Delta \log e_t}\right|_E - \left|\frac{\Delta \log P_t^*}{\Delta \log e_t}\right|_N = \frac{\gamma_t[1 + 1/(N_t - 1)(\epsilon_G - \epsilon_W)]}{\Delta} > 0$$

Since the denominator is

$$\Delta = [\frac{\epsilon_G}{\epsilon_G - \epsilon_W} + \gamma_t(1 + \frac{1}{(N_t - 1)(\epsilon_G - \epsilon_W)})]\epsilon_G > 0 \qquad \square$$

Derivation of $\partial \log N_t / \partial \log \psi_t$

From the result of deriving $\partial \log N_t / \partial \log N_t^* y_t^*$, we have

$$\begin{aligned}
\frac{\partial V(\cdot)}{\partial \psi_t} &= \frac{\partial k_t}{\partial \psi_t} \\
\frac{\partial N_t}{\partial \psi_t} &= \bar{N} H'(k_t) \frac{\partial k_t}{\partial \psi_t} \\
&= \bar{N} H'(k_t) \frac{\partial V(\cdot)}{\partial \psi_t} \\
\frac{\partial V(\cdot)}{\partial \psi_t} &= (1 - \eta_t) \beta u'(\cdot) a_{t+1}^* \frac{\partial y_t}{\partial \psi_t} \\
\frac{\partial N_t}{\partial \psi_t} &= \frac{\bar{N}}{N_t} H'(k_t) \beta u'(\cdot) a_{t+1}^* \frac{\partial y_t}{\partial \psi_t} \\
\frac{\partial \log N_t}{\partial \log \psi_t} &= \frac{\bar{N}}{N_t^2} H'(k_t) \beta u'(\cdot) a_{t+1}^* y_t \frac{\psi_t}{y_t} \frac{\partial y_t}{\partial \psi_t} \\
&= -\frac{\bar{N}}{N_t^2} H'(k_t) \beta u'(\cdot) a_{t+1}^* y_t \frac{1}{\epsilon_G - \epsilon_W} = -c_t < 0 \qquad \Box
\end{aligned}$$

Proof of Proposition 2.2.6

We differentiate $\partial V(N_t, a_{t+1}^*)/\partial a_{t+1}^*$ with respect to N_t under constant elasticity utility assumption,

$$\begin{aligned}
\frac{\partial V(N_t, a_{t+1}^*)}{\partial a_{t+1}^*} &= \beta u'(a_{t+1}^* y_t) y_t [1 + \frac{1}{N_t} \frac{\epsilon_W}{\epsilon_G - \epsilon_W}] \\
\frac{\partial^2 V(\cdot)}{\partial N_t \partial a_{t+1}^*} &= \frac{\beta}{a_{t+1}^*} \frac{\partial a_{t+1}^* y_t u'(a_{t+1}^* y_t)}{\partial a_{t+1}^* y_t} \frac{\partial a_{t+1}^* y_t}{\partial N_t} (1 + \frac{1}{N_t} \frac{\epsilon_W}{\epsilon_G - \epsilon_W}) + \\
&\quad \beta u'(\cdot) y_t (-\frac{1}{N_t^2} \frac{\epsilon_W}{\epsilon_G - \epsilon_W}) \\
&= \beta u'(\cdot) \epsilon_W \frac{y_t}{N_t} \frac{N_t}{y_t} \frac{\partial y_t}{\partial N_t} (1 + \frac{1}{N_t} \frac{\epsilon_W}{\epsilon_G - \epsilon_W}) - \\
&\quad \frac{\beta u'(\cdot) y_t}{N_t^2} \frac{\epsilon_W}{\epsilon_G - \epsilon_W} \\
&= \beta u'(\cdot) \frac{y_t}{N_t} [\frac{\epsilon_W}{(N_t - 1)(\epsilon_G - \epsilon_W)} (1 + \frac{1}{N_t} \frac{\epsilon_W}{\epsilon_G - \epsilon_W}) - \\
&\quad \frac{1}{N_t} \frac{\epsilon_W}{\epsilon_G - \epsilon_W}] \\
&= \beta u'(\cdot) \frac{y_t}{N_t} [\frac{\epsilon_W}{\epsilon_G - \epsilon_W} (\frac{1}{N_t - 1} - \frac{1}{N_t} + \frac{1}{N_t(N_t - 1)} \frac{\epsilon_W}{\epsilon_G - \epsilon_W})] \\
&= \beta u'(\cdot) \frac{y_t}{N_t} [\frac{\epsilon_W}{\epsilon_G - \epsilon_W} \frac{1}{N_t(N_t - 1)} (1 + \frac{\epsilon_W}{\epsilon_G - \epsilon_W})] \\
&= \beta u'(\cdot) \frac{y_t}{N_t} \frac{\epsilon_W}{\epsilon_G - \epsilon_W} \frac{1}{N_t(N_t - 1)} \frac{\epsilon_G}{\epsilon_G - \epsilon_W} > 0 \qquad \Box
\end{aligned}$$

Derivation of $\partial \log N_t / \partial \log a^*_{t+1}|_{k(N)}$

The change in N_t in response to change in a^*_{t+1} is equal to the total displacement in $V(N_t, a^*_{t+1})$ and $k_i(N)$ caused by a^*_{t+1} and multiplied by $\bar{N}H'(k)$. If $H(k)$ is a continuous uniform distribution, its density function will be $1/k_{\bar{N}}(N_t)$ for $k \in [0, k_{\bar{N}}(N_t)]$, so $\bar{N}H'(k) = \bar{N}/k_{\bar{N}}(N_t)$. We are always possible to choose a scale for N, e.g. $N \in [0, k_{\bar{N}}(\bar{N})]$ so that $\bar{N}H'(k) \leq 1$. In discrete uniform distribution, the probability density function will be $1/\bar{N}$ as there are $\bar{N}$ agents in the economy, and $\bar{N}H'(k) = 1$, so we have

$$\begin{aligned}
\frac{\partial \log N_t}{\partial \log a^*_{t+1}}|_{k(N)} &= \frac{a^*_{t+1}}{N_t}\bar{N}H'(k)(\frac{\partial V(\cdot)}{\partial a^*_{t+1}} - \frac{\partial k(N_t)}{\partial N_t} \times \\
&\quad \frac{\partial N_t}{\partial a^*_{t+1}}) \\
(1 + \bar{N}H'(k)k'(N_t))\frac{\partial \log N_t}{\partial \log a^*_{t+1}}|_{k(N)} &= \bar{N}H'(k)\frac{\beta u'(a^*_{t+1}y_t)a^*_{t+1}y_t}{N_t}[1 + \\
&\quad \frac{1}{N_t}\frac{\epsilon_W}{\epsilon_G - \epsilon_W}] \\
\textit{Since } \bar{N}H'(k) \leq 1, \quad -1 < k'(N_t) < 0 & \\
\frac{\partial \log N_t}{\partial \log a^*_{t+1}}|_{k(N)} &= \frac{\bar{N}H'(k)}{1 + \bar{N}H'(k)k'(N_t)}\frac{\beta u'(a^*_{t+1}y_t)}{N_t} \\
&\quad a^*_{t+1}y_t[1 + \frac{1}{N_t}\frac{\epsilon_W}{\epsilon_G - \epsilon_W}] > 0 \quad \square
\end{aligned}$$

Chapter 3

Hysteresis in Trade in a Monetary Shock Model

3.1 Introduction

In Chapter 1, we considered an overlapping generation (OG) model in which the agent's participation decision in production was endogenous. Only the active agents were producing and the inactive agents were abstaining from production. There was no government activity in that model. In this chapter, I shall formally introduce the government which is mainly responsible for the redistribution of income in the economy. As a significant share of government expenditures in the real world is transfer payments, it is reasonable to focus on that role. The government finances its expenditure by taxing the active young agents and printing new money. We study how the random monetary growth rate leads to hysteresis in trade in this simple monetary model.

This chapter is an extension of chapter 1, so some of the results which have already been derived in chapter 1 will not be repeated here. Let us briefly describe the rest of this chapter. In section 2, we discuss the model setup and the results derived. Section 3 contains a numerical example to demonstrate the results obtained in section 2.

3.2 Model

Basically, we adopt the same structure of the overlapping generation (OG) model that we have used in chapter 1. The agents live for two periods and there are two generations co-existing in every period. Only the active agents are producing when they are young. All goods produced are exported and all goods consumed are imported. We assume that the government plays the role of costlessly re-distributing income from the active young agents to the inactive young agents. The government can finance the transfer payments by income taxation on the active young agents or printing new money. In this economy, the evolution of money is

$$M_t = M_{t-1} + \sum_{i=1}^{\bar{N}-N_t} T_t^i = (1+\theta)M_{t-1}$$

Where M_t is the aggregate money stock in period t. N_t is the number of active young agents in the economy in period t; so $\bar{N} - N_t$ is the number of the inactive young agents born in the economy in period t. T_t^i is the transfer payment which is financed by printing new money received by the inactive agent i. θ is the monetary growth rate in the economy.

Now, let us consider the decision of the inactive young agent i. He receives the transfer payment and saves it until consumption when he is old. Therefore, his consumption is represented by $C_{t+1}^i = (T_t^i + tr_t^i)/P_{t+1}$. Where tr_t^i is the transfer payment financed by income taxation on the active young agents in period t. P_{t+1} is the consumption good price in period t+1. As a result, the utility is derived by consuming all of the income received from the government.

$$V^{in} = \beta u[\frac{P_t}{P_{t+1}}(\frac{T_t^i + tr_t^i}{P_t})]$$

For the active young agent i's problem, we also follow the method used in chapter 1 that the active young agent's decisions are divided into two stages: The first stage is the participation decision; the second stage is the output decision. We apply dynamic programming and solve the problem backwards. (1) The active agent i's output decision: He solves the following problem given N_t, N_t^*, N_{t+1}, N_{t+1}^*, $\bar{t}$, P_t, P_{t+1}, and $\bar{t}$ is the income tax rate.

$$\max_{y_{it}} V = -v(y_{it}) + \beta u[\frac{(1-\bar{t})e_t R(y_{it})}{P_{t+1}}]$$

The first order condition is

$$v'(y_{it}) = \beta u'[\frac{(1-\bar{t})e_t R(y_{it})}{P_{t+1}}](1-\bar{t})\frac{e_t}{P_{t+1}}R'(y_{it})$$

Since $R(y_{it}) = P_t^* y_{it}$ and the goods market equilibrium in the foreign country is $P_t^* \sum_{i=1}^{N_t} y_{it} = M_t^*$, we have $P_t^* = M_t^*/(y_{it} + Y_t)$, $Y_t = \sum_{j \neq i}^{N_t} y_{jt}$, the total output produced by the rest of the firms in the market. At the Symmetric Nash Equilibrium (SNE), $y_{it} = y_{jt} = y_t \ \forall i \neq j$,

$$v'(y_t) = \beta u'[\frac{(1-\bar{t})e_t R(y_t)}{P_{t+1}}](1-\bar{t})\frac{e_t}{P_{t+1}}\frac{M_t^*}{N_t y_t}\eta_t, \qquad \eta_t = 1 - \frac{1}{N_t} < 1$$

Imposing the equilibrium condition, $e_t = P_t/P_t^*$ (the exchange rate is determined by the relative price of consumption goods in both countries.), $P_t^* N_t y_t = M_t^*$, and $P_t N_t^* y_t^* = M_t$ (foreign and domestic market clearing conditions), we derive

$$v'(y_t) = \beta u'[(1-\bar{t})\frac{P_t}{P_{t+1}}y_t](1-\bar{t})\frac{P_t}{P_{t+1}}\eta_t$$

Since $P_t = M_t/(N_t^* y_t^*)$, we obtain

$$\begin{aligned}
v'(y_t) &= \beta u'[(1-\bar{t})\frac{N_{t+1}^* y_{t+1}^* M_t}{N_t^* y_t^* M_{t+1}}y_t](1-\bar{t})\frac{N_{t+1}^* y_{t+1}^* M_t}{N_t^* y_t^* M_{t+1}}\eta_t \\
&= \beta u'[(1-\bar{t})\frac{a_{t+1}^*}{1+\theta}y_t](1-\bar{t})\frac{a_{t+1}^*}{1+\theta}\eta_t, \quad where\ a_{t+1}^* \frac{N_{t+1}^* y_{t+1}^*}{N_t^* y_t^*} \\
y_t v'(y_t) &= \beta u'[(1-\bar{t})\frac{a_{t+1}^*}{1+\theta}y_t](1-\bar{t})\frac{a_{t+1}^*}{1+\theta}y_t\eta_t
\end{aligned}$$

$$G(y_t) = \beta W[(1-\bar{t})\frac{a_{t+1}^*}{1+\theta}y_t](1-\bar{t})\eta_t \tag{3.1}$$

Where $G(x) = xv'(x)$ and $W(x) = xu'(x)$. This is the optimality condition for the active agents' output decisions. The left hand side of the equation is the marginal cost of producing output, y_t; the right hand side is the marginal benefit of producing y_t. At optimality, they are equal.

We now discuss the inactive agents' payoff. The inactive agents receive two types of transfers from the government: The first type is financed by the revenue from income taxation; the second type is financed by printing new money.

$$G_t = (\bar{N} - N_t)(T_t^i + tr_t^i) = M_t - M_{t-1} + \bar{t}N_t e_t R(y_t) \qquad \forall\ t = 1, 2, \ldots$$

The total transfers financed by printing money:

$$\begin{aligned} (\bar{N} - N_t)\frac{T_t^i}{P_t} &= \frac{M_t - M_{t-1}}{P_t} = \frac{\theta M_{t-1}}{P_t} = \frac{\theta}{1+\theta}\frac{M_t}{P_t} \\ \frac{T_t^i}{P_t} &= \frac{\bar{N}}{\bar{N} - N_t}\frac{\theta}{1+\theta}\frac{M_t}{\bar{N}P_t} = \frac{\bar{N}}{\bar{N} - N_t}\frac{\theta}{1+\theta}m_t \end{aligned}$$

Where $m_t = M_t/(\bar{N}P_t)$ is the per capita real money balance in home country. Notice that the transfers financed by printing money is effectively the inflation tax levied on all agents. The total transfers financed by the income taxes are:

$$\begin{aligned} (\bar{N} - N_t)tr_t^i &= \bar{t}N_t e_t R(y_t) \\ tr_t^i &= \frac{N_t}{\bar{N} - N_t}\bar{t}P_t y_t \quad Since\ e_t = \frac{P_t}{P_t^*},\ R(y_t) = P_t^* y_t \\ \frac{tr_t^i}{P_t} &= \bar{t}\frac{N_t}{\bar{N} - N_t}y_t \end{aligned}$$

Notice that the transfers financed by income taxation are levied on active agents only. The payoff of the inactive agent is

$$\begin{aligned} V^{in} &= \beta u[\frac{P_t}{P_{t+1}}(\frac{\bar{N}}{\bar{N} - N_t}\frac{\theta}{1+\theta}m_t + \bar{t}\frac{N_t}{\bar{N} - N_t}y_t)] \\ &= \beta u[\frac{a_{t+1}^*}{1+\theta}(\frac{\bar{N}}{\bar{N} - N_t}\frac{\theta}{1+\theta}m_t + \bar{t}\frac{N_t}{\bar{N} - N_t}y_t)] \end{aligned}$$

Now, we explore the relationship between y_t and m_t at equilibrium. The total nominal revenue generated by the sales of exports will be converted into domestic currency and is held until consumption when they are old. $N_t e_t R(y_t) =$

$N_t(P_t/P_t^*)P_t^* y_t = N_t P_t y_t$

$$
\begin{aligned}
N_t P_t y_t &= M_t - \sum_{i=1}^{\bar{N}-N_t} T_t^i = M_{t-1} = \frac{M_t}{1+\theta} \\
y_t &= \frac{1}{1+\theta}\frac{M_t}{N_t P_t} = \frac{\bar{N}/N_t}{1+\theta}\frac{M_t}{\bar{N}P_t} = \frac{\bar{N}}{N_t}\frac{m_t}{1+\theta} \\
Then\ m_t &= \gamma_t(1+\theta)y_t \qquad where\ \gamma_t = \frac{N_t}{\bar{N}}
\end{aligned}
$$

Substituting in $V^{in}(\cdot)$, we obtain

$$
\begin{aligned}
V^{in} &= \beta u[\frac{a_{t-1}^*}{1+\theta}(\frac{\bar{N}}{\bar{N}-N_t}\theta\gamma_t y_t + \bar{t}\frac{N_t}{\bar{N}-N_t}y_t)] \\
&= \beta u[\frac{a_{t-1}^*}{1+\theta}(\frac{\gamma_t}{1-\gamma_t})(\theta+\bar{t})y_t]
\end{aligned}
$$

Where $\bar{N}/(\bar{N}-N_t) = 1/(1-\gamma_t)$, and $N_t/(\bar{N}-N_t) = (N_t/\bar{N})/[(\bar{N}-N_t)/\bar{N}] = \gamma_t/(1-\gamma_t)$. The payoff, $V^{in}(\cdot)$ is the equilibrium payoff that the inactive agent receives. At steady state, $y_t = y$, $N_t = N$, $\eta_t = \eta$ and our optimality condition becomes:

$$G(y) = \beta W[(1-\bar{t})\frac{a^*}{1+\theta}y](1-\bar{t})\eta \tag{3.2}$$

This equation determines the optimal allocations of resources in the second stage of the decision problem.

We will consider some of the comparative statics results by studying the response of y to the shift in parameters. From equation (3.2), we obtain

Proposition 3.2.1 *The output, y is decreasing in the monetary growth rate, θ, and the income tax rate, $\bar{t}$ but increasing in the number of active agents, N, and a^*.*

$$
\begin{aligned}
&\partial \log y/\partial \log \theta = -[\theta/(1+\theta)]/(\epsilon_G - \epsilon_W) < 0, \\
&\partial \log y/\partial \log \bar{t} = -[\bar{t}/(1-\bar{t})]/(\epsilon_G - \epsilon_W) < 0, \\
&\partial \log y/\partial \log a^* = \epsilon_W/(\epsilon_G - \epsilon_W) > 0, \ and \\
&\partial \log y/\partial \log N = 1/[(N-1)(\epsilon_G - \epsilon_W)] > 0.
\end{aligned}
$$

See appendix 3.A for proof. In this economy, only money has a store of value. Thus, agents must put all their savings in the form of money. If the government raises the transfers purely by printing more money, it will raise the monetary growth rate θ, and inflation. As money is the only form of savings, anyone who holds money will be taxed by inflation. Inflation lowers the return on money which is savings. Therefore, it immediately reduces the payoff to production because the revenue from the sales of goods is saved in the form of money, represented by $\partial \log y / \partial \log \theta < 0$. This causes a substitution of nontaxable leisure for taxable labour. Similarly, any rise in the income tax rate $\bar{t}$ lowers the individual output, y. The results of $\partial \log y / \partial \log a^* > 0$ and $\partial \log y / \partial \log N > 0$ have already been discussed in section 2 of chapter 2; therefore I shall not repeat them here.

We proceed backwards to the first stage of the problem, the participation decision. From equation (3.1), we can solve for the optimal output, $y = y(N, \theta, a^*, \bar{t})$, and substitute into the utility function to obtain the value function of working.

$$V(N,\theta,a^*,\bar{t}) = -v[y(N,\theta,a^*,\bar{t})] + \beta u[(1-\bar{t})\frac{a^*}{1+\theta}y(N,\theta,a^*,\bar{t})] \qquad (3.3)$$

The payoff function of any inactive agent is

$$V^{in}(N,\theta,a^*,\bar{t}) = \beta u[\frac{a^*}{1+\theta}(\frac{\gamma}{1-\gamma})(\theta+\bar{t})y] \qquad (3.4)$$

Before we state the equilibrium of this model, let us consider the responses of the value functions, $V(N,\theta,a^*,\bar{t})$ and $V^{in}(N,\theta,a^*,\bar{t})$ to the shifts in the parameters.

Proposition 3.2.2 *The value function of active agent, $V(N,\theta,a^*,\bar{t})$ is decreasing in the monetary growth rate, θ, and the income tax rate, $\bar{t}$ but increasing in the number of active agents, N, and a^*. The value function of inactive agent, $V^{in}(N,\theta,a^*,\bar{t})$ is ambiguous in the changes in monetary growth rate and income tax rate but unambiguously increasing in the number of active agents, and a^*.*
$\partial V(\cdot)/\partial \theta < 0$, $\partial V(\cdot)/\partial \bar{t} < 0$, $\partial V(\cdot)/\partial a^* > 0$, *and* $\partial V(\cdot)/\partial N > 0$.
$\partial V^{in}(\cdot)/\partial \theta \gtreqless 0$ *if* $|\partial \log y/\partial \log \theta| \lesseqgtr [\theta/(\theta+\bar{t})][(1-\bar{t})/(1+\theta)]$, $\partial V^{in}(\cdot)/\partial N > 0$,

$\partial V^{in}(\cdot)/\partial a^* > 0$, *and* $\partial V^{in}(\cdot)/\partial \bar{t} \gtrless 0$ *if* $|\partial \log y/\partial \log \bar{t}| \lessgtr 1/(\theta + \bar{t})$.

See appendix 3.A for proof. The value function of the active young agent must be decreasing in θ and $\bar{t}$ because both θ and $\bar{t}$ are the tax rates imposed on the active agent. The value function of the active agent is increasing in a^* and N, the results of which have already been discussed in the section 2 of chapter 1 or 2. The value function of the inactive agent is ambiguous in response to the changes in θ and $\bar{t}$ because the increases in θ and $\bar{t}$ mean that the government is transferring more resources to the inactive agents. These transfers should raise the welfare of the inactive agents holding other things constant. However, the increases in θ and $\bar{t}$ lower the total output in the economy and less national income becomes available for the redistribution. These two events combined produce ambiguous results. However, we shall concentrate the analysis on the case that $\partial V^{in}(\cdot)/\partial\theta > 0$ and $\partial V^{in}(\cdot)/\partial\bar{t} > 0$. We are assuming that the initial effect of the income transfer to the inactive agent dominates the subsequent output contraction effect. Thus, the inactive agent is better off. Finally, the value function of inactive agent is increasing in a^* and N.

Now let us describe the determination of equilibrium. We assume that each agent has an agent-specific cost in selling and producing the goods. If we rank the agent specific fixed cost in ascending order, we obtain $k_2 < k_3 < \ldots < k_{\bar{N}}$. However, k_i is not the only opportunity cost of producing. Let us consider the case where there are $\hat{N} - 1$ agents who are producing in the economy. If agent $\hat{N}$ does not participate in production, he receives $V^{in}(\hat{N} - 1, \theta, a^*, \bar{t})$. However, if he participates in production, he receives $V(N, \theta, a^*, \bar{t}) - k_{\hat{N}}$. Thus, his opportunity cost of participation is $k_{\hat{N}} + V^{in}(\hat{N} - 1, \theta, a^*, \bar{t})$. The equilibrium of this model exists by imposing the same conditions of proposition 1.2.4 in chapter 1.

Proposition 3.2.3 *If agents are ranked according to their participation costs,* $k_2 < k_3 < \ldots < k_{\bar{N}}$, *where* $i = 2, \ldots, \bar{N}$, $V(2) > k_2 + V^{in}(2)$ *and* $V(\bar{N}) < k_{\bar{N}} + V^{in}(\bar{N} - 1)$, *then there exists a steady state equilibrium* $(\hat{N}, \hat{k})$ *which satisfies*

$$k_{\hat{N}+1} + V^{in}(\hat{N}) > V(\hat{N}+1) > V(\hat{N}) \geq k_{\hat{N}} + V^{in}(\hat{N}) \tag{3.5}$$

The proof is similar to that of proposition 1.2.4 in chapter 1. Since $V^{in}(\cdot)$ is increasing in N, the marginal firm's opportunity cost of production activity is increasing in N. It is equivalent to the congestion externality that the cost of selling and producing is increasing as a result of too many sellers in the market. As discussed in chapter 1, there will be only one equilibrium under congestion externality[1]. This is not a surprising result because it is well known that negative externality will not generate multiple equilibria e.g. Cooper and John (1988). As a result, we shall concentrate on the case of positive network externality.

However, if we consider the presence of a positive network externality, $k_i = k_i(N)$, $k_i'(N) < 0$, $\forall\, i = 2, ..., \bar{N}$, it is possible to derive the presence of multiple equilibria. With the presence of a positive network externality, the equilibrium condition is as follows:

$$k_{\hat{N}+1}(N) + V^{in}(\hat{N}) > V(\hat{N}+1) > V(\hat{N}) \geq k_{\hat{N}}(N) + V^{in}(\hat{N}) \tag{3.6}$$

Now both $k_i(N)$ and $V^{in}(N)$ are functions of N,

$$\frac{\partial[k_i(N) + V^{in}(\hat{N})]}{\partial N} = k_i'(N) + \frac{\partial V^{in}(\cdot)}{\partial N}$$

Since $k_i'(N) < 0$ and $\partial V^{in}(\cdot)/\partial N > 0$, the derivative of this composite function is ambiguous. If $k_i'(N) + \partial V^{in}(\cdot)/\partial N > 0$, we obtain the same results as under the congestion externality i.e. only one equilibrium emerges. If $k_i'(N) + \partial V^{in}(\cdot)/\partial N < 0$, and the positive network externality dominates the other component, we shall have the same hysteresis result as in the real shock case in which only $k_i(N)$ (with $k_i'(N) < 0$) are present. We shall concentrate on the study of the hysteresis case i.e. the case of $k_i'(N) + \partial V^{in}(\cdot)/\partial N < 0$. In the next section, a numerical example of hysteresis is provided.

[1] Please see proposition 1.3.1 of chapter 1.

3.3 Numerical Example of Hysteresis

We consider the same utility function of the representative agent that we have used in chapter 1,

$$V(C,y) = -\frac{y^{1+\psi}}{1+\psi} + \beta\frac{C^{1-\alpha}}{1-\alpha}$$

At equilibrium, $C = (1-\bar{t})a^{*}y/(1+\theta)$ and

$$y = [\beta\eta((1-\bar{t})\frac{a^{*}}{1+\theta})^{1-\alpha}]^{1/(\alpha+\psi)} \qquad \eta = \frac{N-1}{N}$$

Substituting in the utility function, we get the value function of the active agent,

$$V(N,\theta,a^{*},\bar{t}) = -\frac{y^{1+\psi}}{1+\psi} + \frac{[(1-\bar{t})a^{*}y/(1+\theta)]^{1-\alpha}}{1-\alpha}$$

The payoff function of the inactive agent is

$$V^{in}(N,\theta,a^{*},\bar{t}) = \beta\frac{a^{*}\gamma(\theta+\bar{t})y/[(1+\theta)(1-\gamma)]^{1-\alpha}}{1-\alpha}$$

We assume the fixed cost is as follows:

$$k_i(N) = \bar{k}_i - 0.0025N \qquad \bar{k}_i = 0.9579 + 0.001(i-2) \qquad i = 2,3,\ldots,\bar{N}$$

We set $\beta = 1$, $\alpha = \psi = 0.5$ ($\epsilon_G = 1.5$, and $\epsilon_W = 0.5$), the 20% income tax rate $\bar{t} = 0.2$, $\bar{N} = 1000$, both currencies are growing at 5% $\theta = \theta^{*} = 0.05$, and $a^{*} = 1$. In this economy, there are five equilibria, N=48, 49, 50 , 51, and 52.

To illustrate the presence of hysteresis, we consider both the real shock a^{*} and the monetary shock θ. We let a^{*} and θ belong to a normal distribution with the same standard deviation equalling 0.002, with means equal to 1 and 0.05 respectively. We assume the economy starts from $N_0 = 50$, and $M_0 = M_0^{*} = 1$. We draw 100 random numbers from a standard normal distribution. We transform them to have a mean of 1 and a standard deviation of 0.002 for the real shock. The same set of random numbers are also transformed to have a mean of 0.05 and a standard deviation of 0.002 for the monetary shock. The shock is performed 100 times to find the corresponding equilibrium for each shock for each period. Therefore, there are 100 periods altogether. All the computations are done on GAUSS 2.0. We first consider the case of the real shock, a^*. Figures 3.1 to 3.3 illustrate the equilibrium number of active agents, the import price P^*, and the exchange rate e. P^* is increasing exponentially because the foreign money stock is increasing at a rate of 5% in each period. Figure 3.1 shows that the economy will move to a new equilibrium only if there is a considerable shock to the economy. For the monetary shock, θ, the results are shown in figures 3.4 to 3.6. We observe the outcomes similar to the case of the real shock. However, more fluctuations occur in the case of the monetary shock, because the economy is more sensitive to the monetary shock.

3.4 Conclusion

We have extended the model in chapter one to consider the presence of governmental activities like income taxation and money issue. We obtain the results that the monetary shock can also cause hysteresis under the same conditions assumed in chapter one. The hysteresis outcomes are similar to those generated under the real shock analysed in chapter one.

Figure 3.1 The equilibrium number of active agents in the economy under real shock with $N_0 = 50$

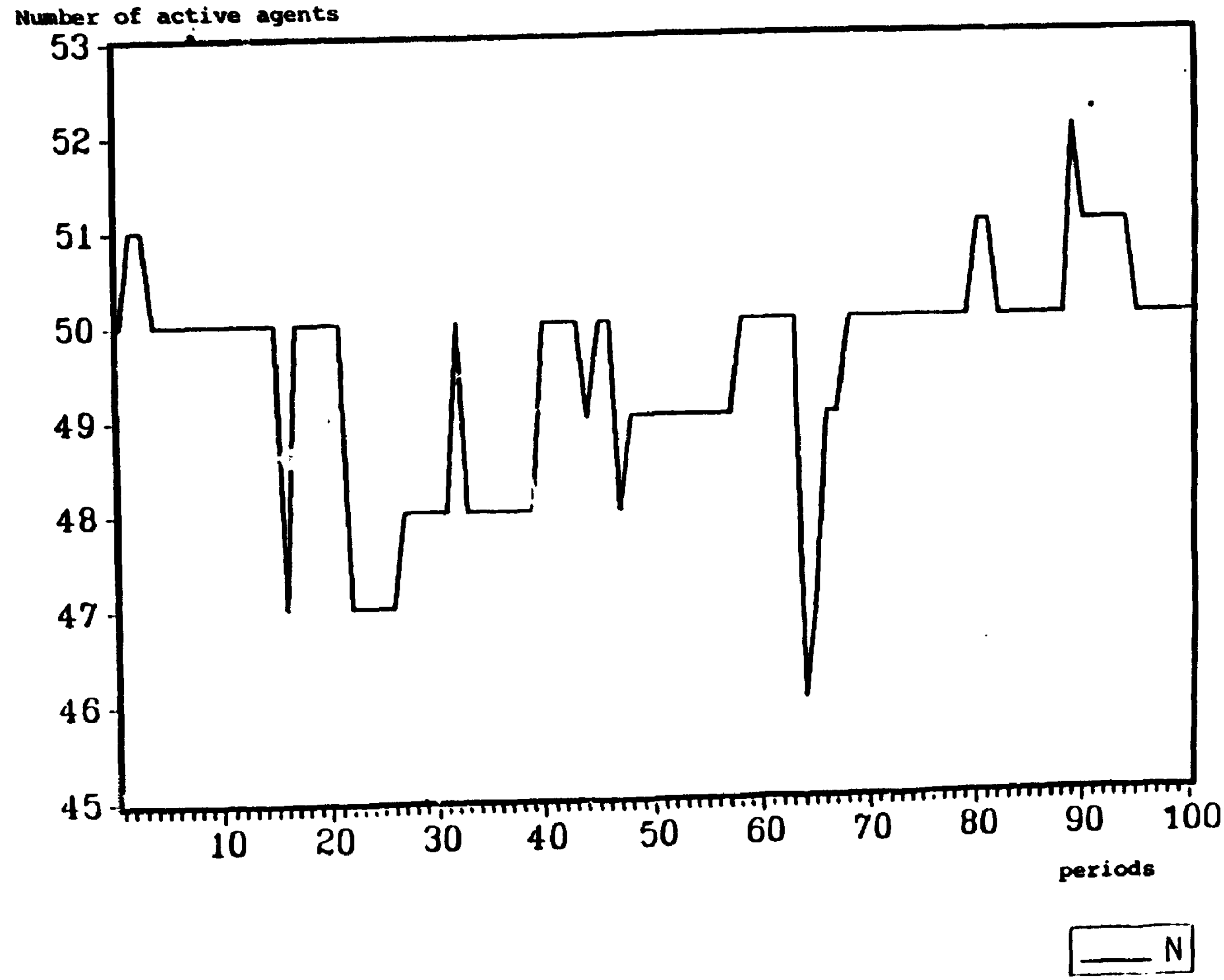


Figure **3.2** The equilibrium import price P^* under real shock with N_0=50

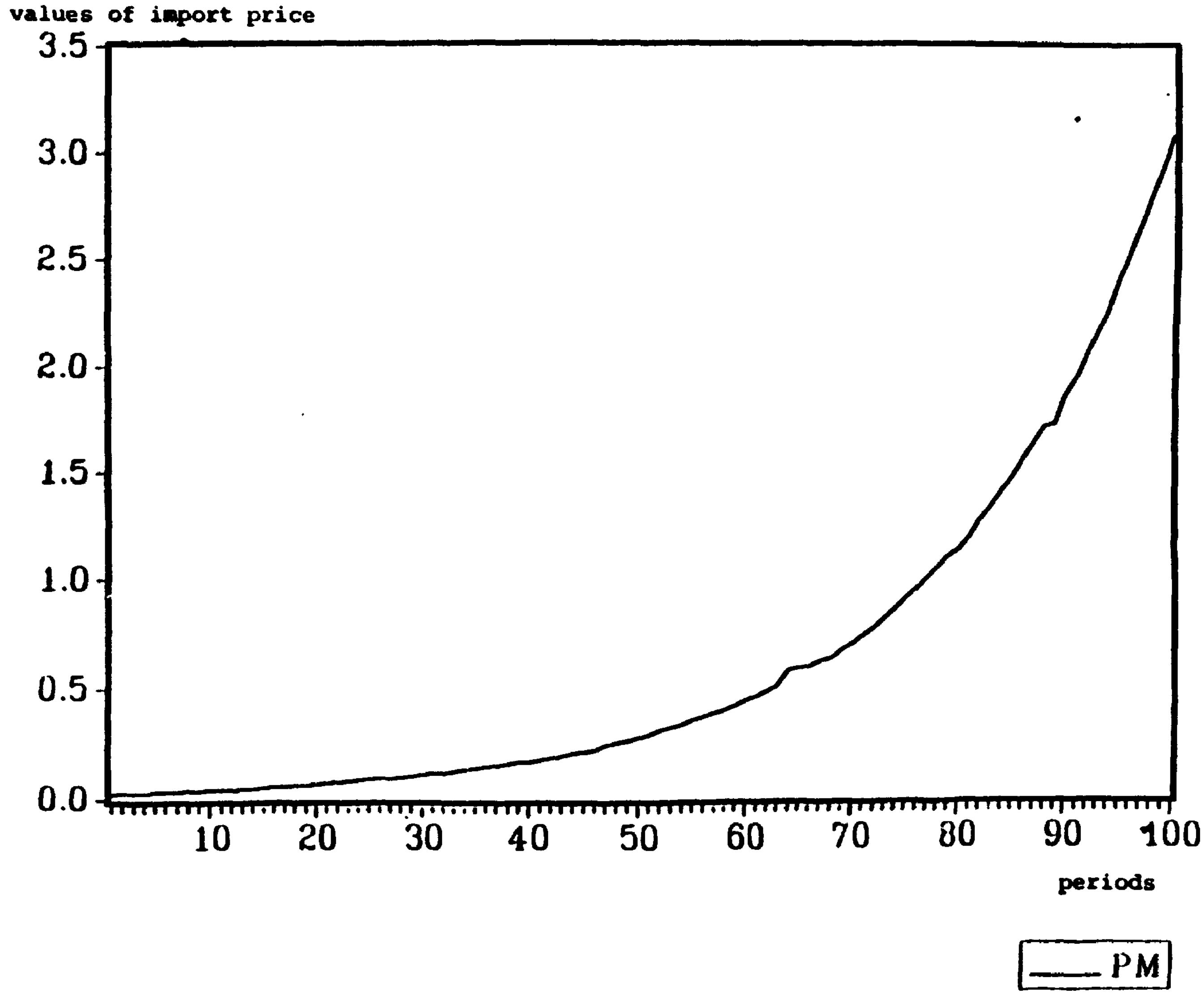


Figure 3.3 The equilibrium exchange rate e under real shock with $N_0=50$

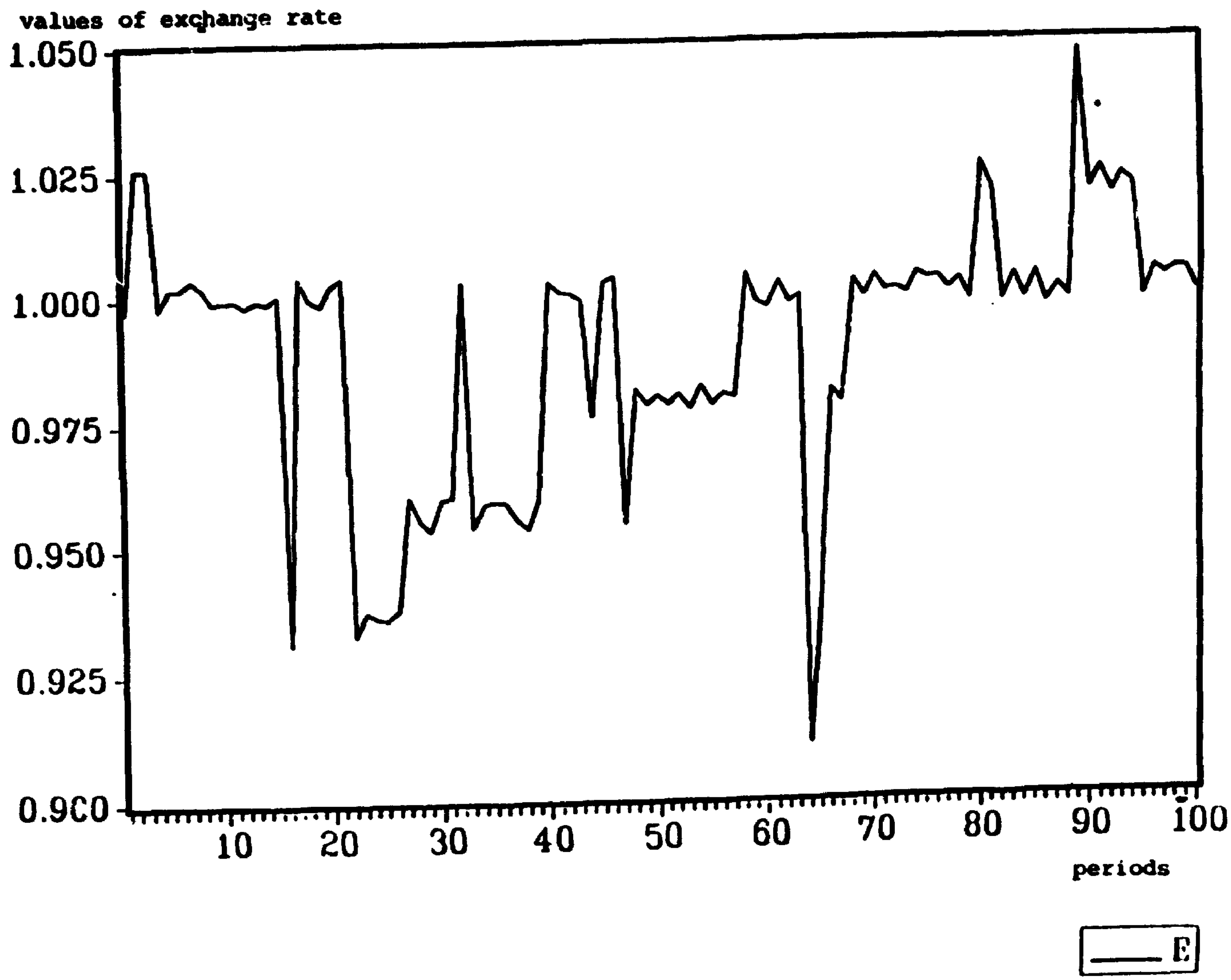


Figure 3.4 The equilibrium number of active agents in the economy under monetary shock with $N_0=50$

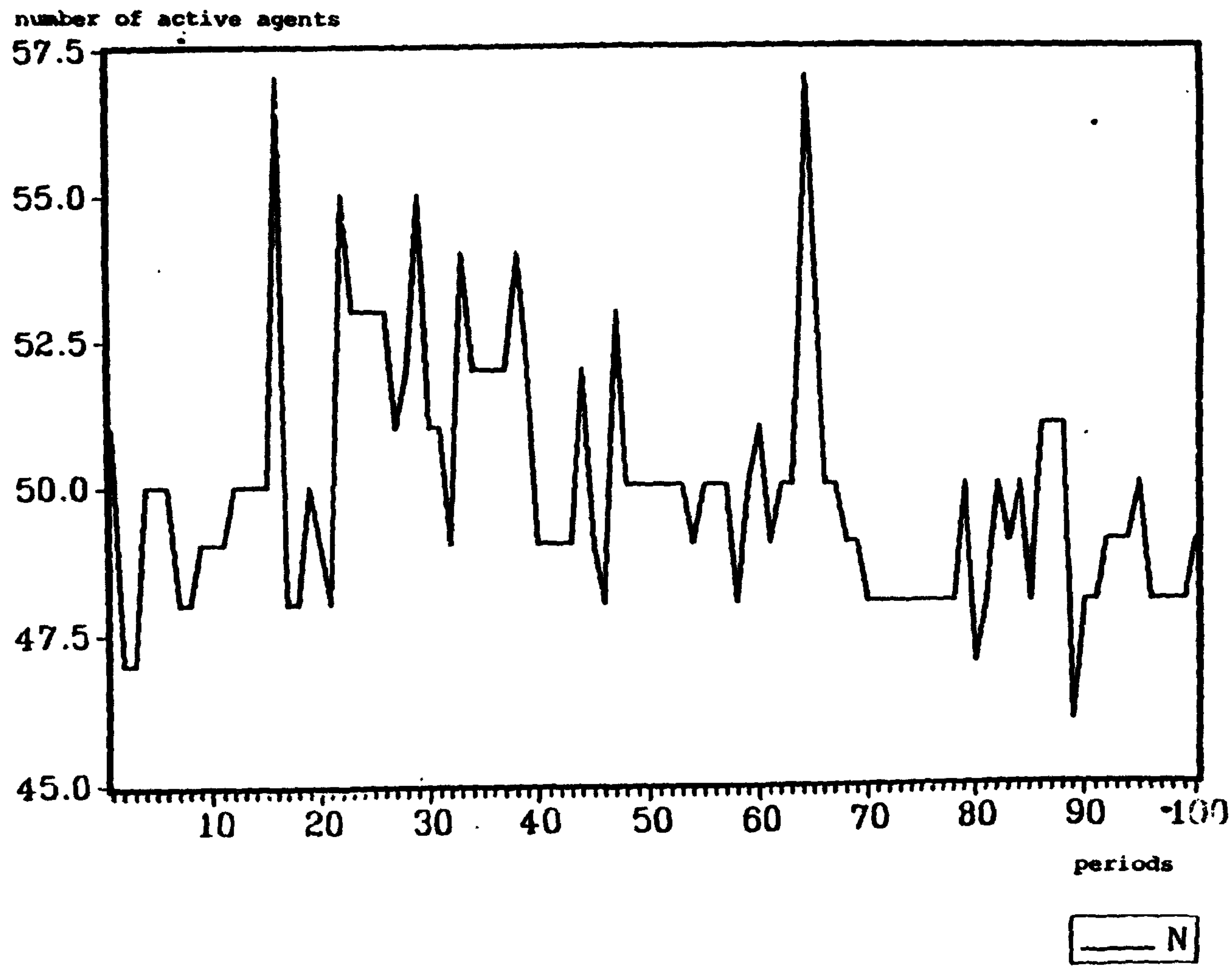


Figure 3.5 The equilibrium import price P^* under monetary shock with $N_0=50$

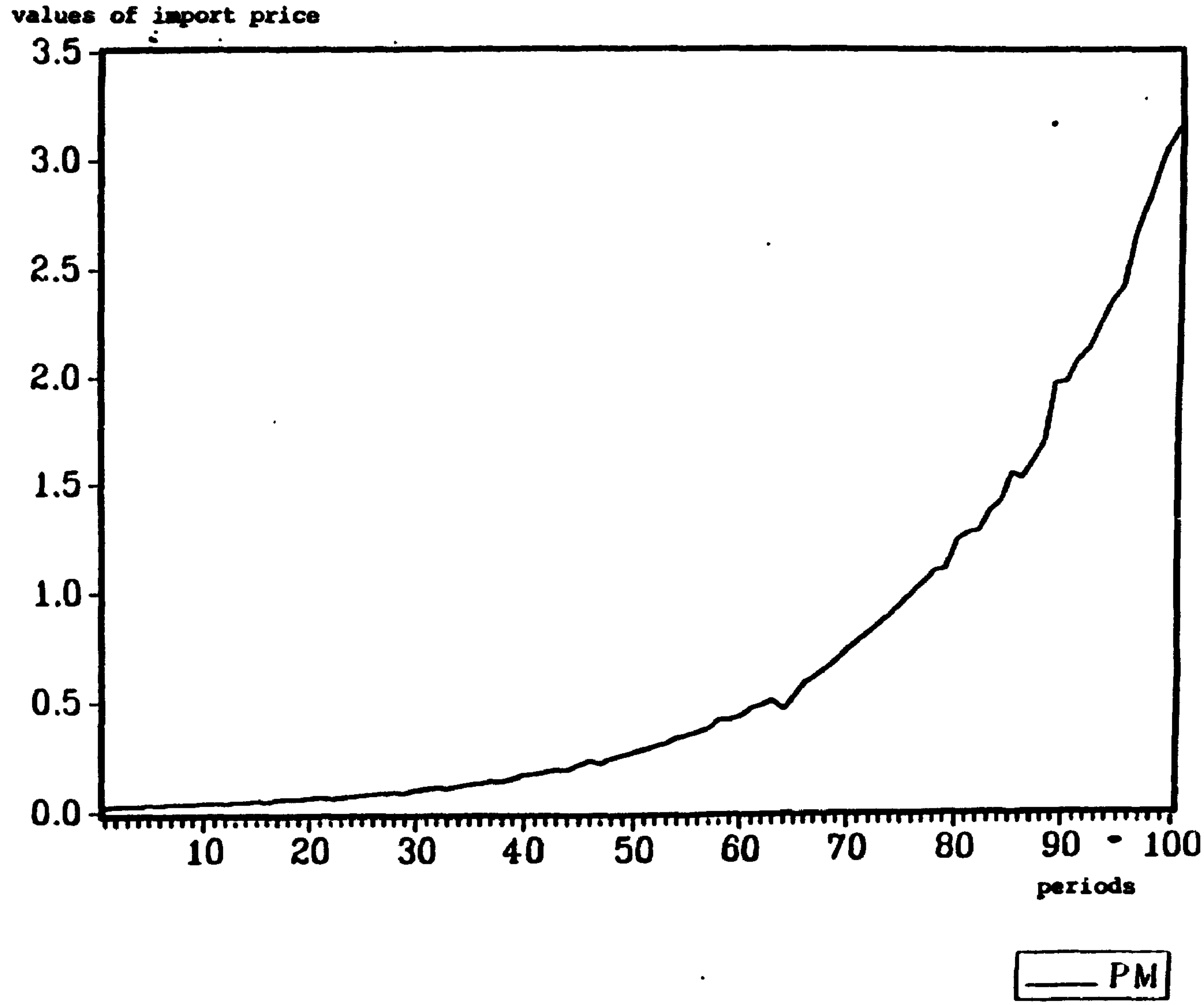
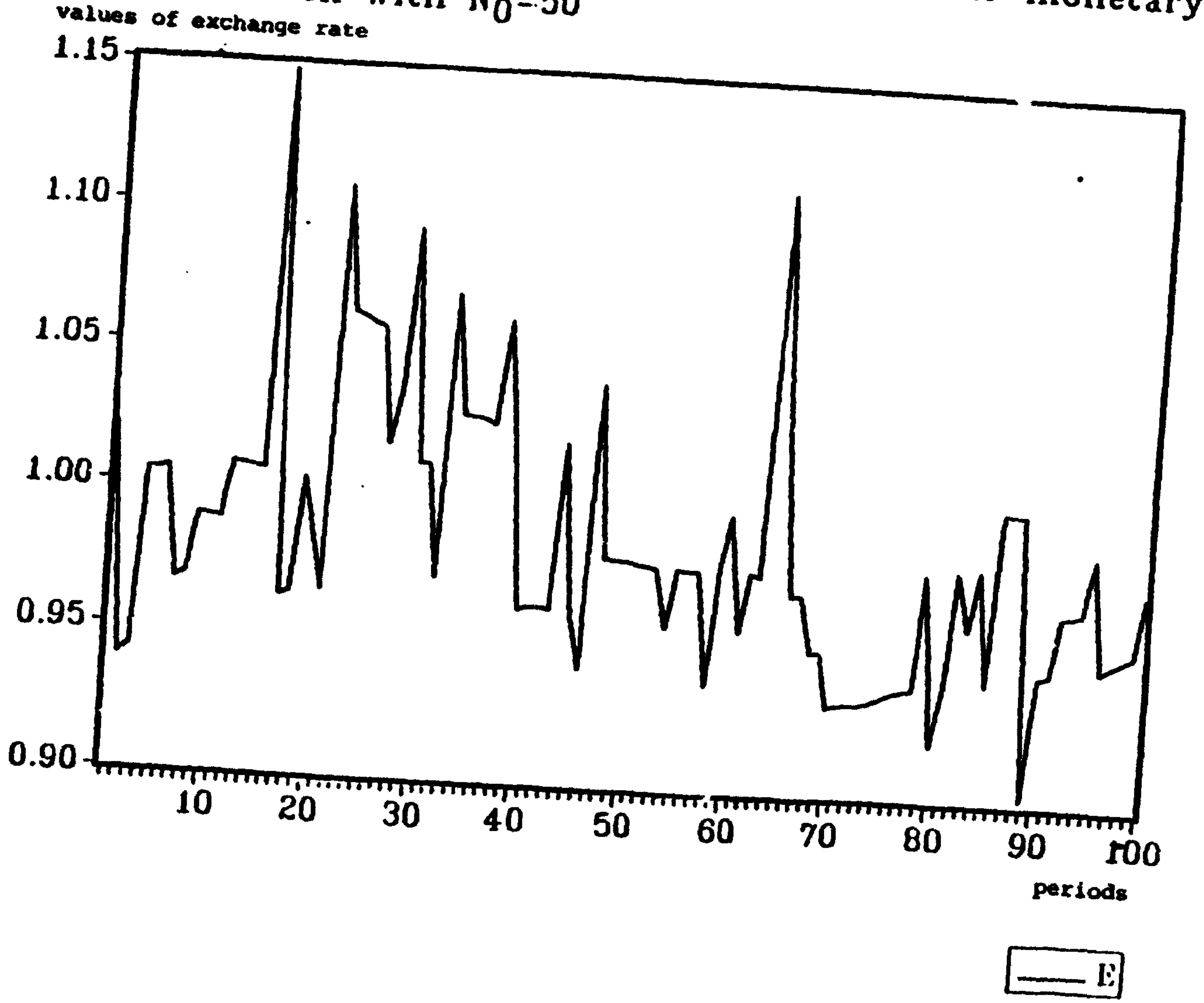


Figure 3.6 The equilibrium exchange rate e under monetary shock with $N_0=50$



Appendix 3.A

Proofs of the Propositions

Proof of Proposition 3.2.1 The proofs of $\partial \log y/\partial a^* > 0$ and $\partial \log y/\partial N > 0$ have already done in chapter 1. Thus, we shall not repeat here. We take logarithm of equation (1)

$$\log G(y) = \log \beta + \log W((1-\bar{t})\frac{a^*}{1+\theta}y) + \log(1-\bar{t}) + \log \eta$$

We differentiate the equation above with respect to $\log \theta$

$$\begin{aligned}
\frac{\partial \log G(y)}{\partial \log y}\frac{\partial \log y}{\partial \log \theta} &= \frac{\partial \log W((1-\bar{t})a^* y/(1+\theta))}{\partial \log((1-\bar{t})a^* y/(1+\theta))} \\
&\quad [\frac{\partial \log((1-\bar{t})a^* y/(1+\theta))}{\partial \log \theta}] \\
\epsilon_G \frac{\partial \log y}{\partial \log \theta} &= \epsilon_W[-\frac{\partial \log(1+\theta)}{\partial \log \theta} + \frac{\partial \log y}{\partial \log \theta}] \\
\frac{\partial \log y}{\partial \log \theta} &= -\frac{\theta/(1+\theta)}{\epsilon_G - \epsilon_W} < 0 \qquad \epsilon_G > 1,\ \ 1 > \epsilon_W > 0.
\end{aligned}$$

Next, We differentiate it with respect to $\bar{t}$

$$\begin{aligned}
\frac{\partial \log G(y)}{\partial \log y}\frac{\partial \log y}{\partial \log \bar{t}} &= \frac{\partial \log W((1-\bar{t})a^* y/(1+\theta))}{\partial \log((1-\bar{t})a^* y/(1+\theta))} \times \\
&\quad [\frac{\partial \log((1-\bar{t})a^* y/(1+\theta))}{\partial \log \bar{t}}] \\
\epsilon_G \frac{\partial \log y}{\partial \log \bar{t}} &= \epsilon_W[\frac{\partial \log(1-\bar{t})}{\partial \log \bar{t}} + \frac{\partial \log y}{\partial \log \bar{t}}] \\
(\epsilon_G - \epsilon_W)\frac{\partial \log y}{\partial \log \bar{t}} &= -\frac{\bar{t}}{1-\bar{t}} \\
\frac{\partial \log y}{\partial \log \bar{t}} &= -\frac{\bar{t}/(1-\bar{t})}{\epsilon_G - \epsilon_W} < 0 \qquad \square
\end{aligned}$$

Proof of Proposition 3.2.2

The value function of the active agent is as follows:

$$V(N,\theta,a^*,\bar{t}) = -v[y(N,\theta,a^*,\bar{t})] + \beta u[(1-\bar{t})\frac{a^*}{1+\theta}y(N,\theta,a^*,\bar{t})]$$

We differentiate the value function with respect to θ

$$\begin{aligned}\frac{\partial V(\cdot)}{\partial \theta} &= -v'(\cdot)\frac{\partial y}{\partial \theta} + \beta u'(\cdot)(1-\bar{t})[-\frac{a^*}{(1+\theta)^2}y + \frac{a^*}{1+\theta}\frac{\partial y}{\partial \theta}] \\ &= \beta u'(\cdot)\frac{a^*}{1+\theta}(1-\eta)\frac{\partial y}{\partial \theta}(1-\bar{t}) - \beta u'(\cdot)\frac{a^*}{(1+\theta)^2}(1-\bar{t})y \\ &= \beta u'(\cdot)\frac{a^*}{1+\theta}(1-\bar{t})[\frac{1}{N}\frac{\partial y}{\partial \theta} - \frac{y}{1+\theta}] < 0 \quad \frac{\partial y}{\partial \theta} < 0\end{aligned}$$

We differentiate $V(\cdot)$ with respect to N

$$\begin{aligned}\frac{\partial V(\cdot)}{\partial N} &= -v'(\cdot)\frac{\partial y}{\partial N} + \beta u'(\cdot)(1-\bar{t})\frac{a^*}{1+\theta}\frac{\partial y}{\partial N} \\ &= \frac{1}{N}\beta(1-\bar{t})\frac{a^*}{1+\theta}\frac{\partial y}{\partial N} > 0\end{aligned}$$

We differentiate the value function with respect to a^*

$$\begin{aligned}\frac{\partial V(\cdot)}{\partial a^*} &= -v'(\cdot)\frac{\partial y}{\partial a^*} + \beta u'(\cdot)(1-\bar{t})[\frac{y}{1+\theta} + \frac{a^*}{1+\theta}\frac{\partial y}{\partial a^*}] \\ &= \beta u'(\cdot)(1-\bar{t})\frac{a^*}{1+\theta}[\frac{1}{N}\frac{\partial y}{\partial a^*} + \frac{y}{a^*}] > 0 \quad \frac{\partial y}{\partial a^*} > 0\end{aligned}$$

We differentiate the value function with respect to $\bar{t}$

$$\begin{aligned}\frac{\partial V(\cdot)}{\partial \bar{t}} &= -v'(\cdot)\frac{\partial y}{\partial \bar{t}} + \beta u'(\cdot)[-\frac{a^*}{1+\theta}y + (1-\bar{t})\frac{a^*}{1+\theta}\frac{\partial y}{\partial \bar{t}}] \\ &= \beta u'(\cdot)(1-\bar{t})\frac{a^*}{1+\theta}(1-\eta)\frac{\partial y}{\partial \bar{t}} - \beta u'(\cdot)\frac{a^*}{1+\theta}y \\ &= \beta u'(\cdot)\frac{a^*}{1+\theta}[(1-\bar{t})\frac{1}{N}\frac{\partial y}{\partial \bar{t}} - y] < 0 \quad \frac{\partial y}{\partial \bar{t}} < 0\end{aligned}$$

The value function of the inactive agent's at steady state is

$$V^{in}(N,\theta,a^*,\bar{t}) = \beta u[\frac{a^*}{1+\theta}(\frac{\gamma}{1-\gamma})(\theta+\bar{t})y]$$

We differentiate $V^{in}(\cdot)$ with respect to θ,

$$\begin{aligned}\frac{\partial V^{in}(\cdot)}{\partial \theta} &= \beta u'(\frac{a^*}{1+\theta}(\theta+\bar{t})\frac{\gamma}{1-\gamma}y)[-\frac{a^*}{(1+\theta)^2}(\theta+\bar{t})\frac{\gamma}{1-\gamma}y + \\ &\quad \frac{a^*}{1+\theta}\frac{\gamma}{1-\gamma}y + \frac{a^*}{1+\theta}\frac{\gamma}{1-\gamma}(\theta+\bar{t})\frac{\partial y}{\partial \theta}] \\ &= \beta u'(\frac{a^*}{1+\theta}(\theta+\bar{t})\frac{\gamma}{1-\gamma}y)\frac{a^*}{1+\theta}\frac{\gamma}{1-\gamma}[-y\frac{\theta+\bar{t}}{1+\theta} + y + \\ &\quad (\theta+\bar{t})\frac{\partial y}{\partial \theta}]\end{aligned}$$

$$= \beta u'(\frac{a^*}{1+\theta}(\theta+\bar{t})\frac{\gamma}{1-\gamma}y)\frac{a^*}{1+\theta}\frac{\gamma}{1-\gamma}[y\frac{1-\bar{t}}{1+\theta} + (\theta+\bar{t})\frac{\partial y}{\partial\theta}]$$

$$If\ |\frac{\partial y}{\partial\theta}| \gtrless \frac{y}{\theta+\bar{t}}\frac{1-\bar{t}}{1+\theta} \Longrightarrow \frac{\partial V^{in}(\cdot)}{\partial\theta} \lessgtr 0$$

$$If\ |\frac{\theta\partial y}{y\partial\theta}| \gtrless \frac{\theta}{\theta+\bar{t}}\frac{1-\bar{t}}{1+\theta} \Longrightarrow \frac{\partial V^{in}(\cdot)}{\partial\theta} \lessgtr 0$$

We differentiate $V^{in}(\cdot)$ with respect to $\bar{t}$,

$$\frac{\partial V^{in}(\cdot)}{\partial\bar{t}} = \beta u'(\frac{a^*}{1+\theta}(\theta+\bar{t})\frac{\gamma}{1-\gamma}y)[\frac{a^*}{1+\theta}\frac{\gamma}{1-\gamma}y + \frac{a^*}{1+\theta}(\theta+\bar{t}) \times \frac{\gamma}{1-\gamma}\frac{\partial y}{\partial\bar{t}}]$$

$$= \beta u'(\frac{a^*}{1+\theta}(\theta+\bar{t})\frac{\gamma}{1-\gamma}y)\frac{a^*}{1+\theta}\frac{\gamma}{1-\gamma}\frac{y}{\bar{t}}[1 + (\theta+\bar{t})\frac{\bar{t}\partial y}{y\partial\bar{t}}]$$

$$If\ |\frac{\partial y}{\partial\bar{t}}| \gtrless \frac{1}{\theta+\bar{t}}\ \frac{\partial V^{in}(\cdot)}{\partial\bar{t}} \lessgtr 0$$

We differentiate $V^{in}(\cdot)$ with respect to N,

$$\frac{\partial V^{in}(\cdot)}{\partial N} = \beta u'(\frac{a^*}{1+\theta}(\theta+\bar{t})\frac{\gamma}{1-\gamma}y)[\frac{a^*}{1+\theta}(\theta+\bar{t})\frac{1}{\bar{N}}\frac{1}{(1+\gamma)^2}y + \frac{a^*}{1+\theta}(\theta+\bar{t})\frac{\gamma}{1-\gamma}\frac{\partial y}{\partial N}]$$

$$Since\ \frac{\partial\gamma/(1-\gamma)}{\partial N} = \frac{(1-\gamma)/\bar{N} - \gamma/\bar{N}}{(1-N/\bar{N})^2} = \frac{1}{\bar{N}}\frac{1}{(1-\gamma)^2}$$

$$\frac{\partial V^{in}(\cdot)}{\partial N} = \beta u'(\frac{a^*}{1+\theta}(\theta+\bar{t})\frac{\gamma}{1-\gamma}y)\frac{a^*}{1+\theta}(\theta+\bar{t})\frac{y}{N}[\frac{\gamma}{(1-\gamma)^2} + \frac{\gamma}{1-\gamma}\frac{N\partial y}{y\partial N}] > 0$$

We differentiate $V^{in}(\cdot)$ with respect to a^*,

$$\frac{\partial V^{in}(\cdot)}{\partial a^*} = \beta u'(\frac{a^*}{1+\theta}(\theta+\bar{t})\frac{\gamma}{1-\gamma}y)[\frac{\theta+\bar{t}}{1+\theta}\frac{\gamma}{1-\gamma}y + \frac{a^*}{1+\theta}(\theta+\bar{t}) \times \frac{\gamma}{1-\gamma}\frac{\partial y}{\partial a^*}]$$

$$= \beta u'(\frac{a^*}{1+\theta}(\theta+\bar{t})\frac{\gamma}{1-\gamma}y)\frac{\theta+\bar{t}}{1+\theta}\frac{\gamma}{1-\gamma}y(1 + \frac{a^*\partial y}{y\partial a^*}) > 0 \qquad \square$$

Appendix 3.B

GAUSS Program for the Numerical Examples

```
/* GAUSS Program for the real shock case */
output reset;
outwidth 130;
let x[101,4]=0;
let z[101,5]=0;
let beta=1;
let psi=.5;
let alpha=.5;
let a=1;
let tau=.2;
let nbar=1000;
let m=1;
let m1=1;
let theta=.05;
let theta1=.05;
Eg=1+psi;
Ew=1-alpha;
i=1;
n=50;
seed1=397;
r=rndns(100,1,seed1);
 /* Generating real shock from N(1, 0.000004) */
do while i<=100;
a=r[i]*0.002+1;
do while n<=1000;  /* set Nbar=100 */
/* Calculating the fixed cost of production for
nth and n+1st agent */
kbar=0.9579+0.001*(n-2);
kbar1=0.9579+0.001*(n-1);
k=kbar-0.0025*n;
k1=kbar1-0.0025*n;
/* calculating the value function for the nth and
n+1st agent to be active */
eta=(n-1)/n;
eta1=n/(n+1);
eta2=(n-2)/(n-1);
y=(beta*eta*((1-tau)*(a/(1+theta)))^Ew)^(1/(alpha+psi));
y1=(beta*eta1*((1-tau)*(a/(1+theta)))^Ew)^(1/(alpha+psi));
y2=(beta*eta2*((1-tau)*(a/(1+theta)))^Ew)^(1/(alpha+psi));
v=beta*((1-tau)*(a/(1+theta))*y)^Ew/Ew-y^Eg/Eg;
/* calculating the payoff to n+1st and nth agent to be
inactive */
v1=beta*((1-tau)*(a/(1+theta))*y1)^Ew/Ew-y1^Eg/Eg;
g1=n/nbar;
vin1=(beta*(g1*(a/(1+theta))*(theta+tau)*y/(1-g1))^Ew)/Ew;
g=(n-1)/nbar;
vin=(beta*(g*(a/(1+theta))*(theta+tau)*y2/(1-g))^Ew)/Ew;
/* search for the equilibrium corresponding to the
```

```
monetary shock generated */
 if k+vin > v;
  n=n-1;
  continue;
  endif;
 if v1 <= k1+vin1;
     goto label;
     endif;
  n=n+1;
 endo;
label: /* we set N_{+1}^{*}y_{+1}^{*}=N^{*}y^{*}=Ny and
a=1, theta^{*}=theta=.05 at the initial state, domestic
money stock, m and foreign money stock, m1 are set to 1
in period zero.  Moreover, we set the foreign monetary
growth rate to 0.05. Thus, the domestic economy is
inflating but the rest of world keep the money stock
constant. The foreign national income is as follows */
nyfor=50*(beta*((50-1)/50)*((1-tau)*(1/(1+theta1)))^(1-
alpha))^(1/(alpha+psi));
m=(1+theta)*m;
m1=(1+theta1)*m1;
x[1,3]=1/nyfor;
x[1,4]=1;
x[i+1,1]=n;
x[i+1,2]=a;
x[i+1,3]=m1/(n*y);
x[i+1,4]=(m/(nyfor/a))/(m1/(n*y));
z[i+1,1]=n;
z[i+1,2]=(a-1)*100;
z[i+1,3]=100*(x[i+1,3]/x[i,3]-1-theta);
z[i+1,4]=100*(x[i+1,4]/x[i,4]-1);
z[i+1,5]=z[i+1,3]/z[i+1,4];
i=i+1;
endo;
print "Domestic economy is inflating at a random foreign
income growth rate, its mean is 1 and its standard
deviation is 0.002. 1st col is number of exporters, 2nd
col is value of theta, 3rd col is equilibrium value of
import price, 4th col is equilibrium exchange rate"
x[2:101,.];
print "Domestic economy is inflating at a random foreign
income growth rate, its mean is 1 and its standard
deviation is 0.002. 1st col is number of exporters, 2nd
col is percentage change in theta, 3rd col is percentage
change in import price, 4th col is percentage change in
exchange rate, 5th col is exchange rate pass-through
coeff" z[2:101,.];
output off;
end;
```

```
/* GAUSS Program for the monetary shock case */
output reset;
outwidth 130;
let x[101,4]=0;
let z[101,5]=0;
let beta=1;
let psi=.5;
let alpha=.5;
let a=1;
let tau=.2;
let nbar=1000;
let m=1;
let m1=1;
let theta1=.05;
Eg=1+psi;
Ew=1-alpha;
i=1;
n=50;
seed1=397;
r=rndns(100,1,seed1);
/* Generating domestic monetary shock from
N(.05, 0.000004) */
do while i<=100;
theta=r[i]*0.002+0.05;
do while n<=1000;  /* set Nbar=100 */
/* Calculating the fixed cost of production for nth
and n+1st agent */
kbar=0.9579+0.001*(n-2);
kbar1=0.9579+0.001*(n-1);
k=kbar-0.0025*n;
k1=kbar1-0.0025*n;
/* calculating the value function for the nth and
n+1st agent to be active */
eta=(n-1)/n;
eta1=n/(n+1);
eta2=(n-2)/(n-1);
y=(beta*eta*((1-tau)*(a/(1+theta)))^Ew)^(1/(alpha+psi));
y1=(beta*eta1*((1-tau)*(a/(1+theta)))^Ew)^(1/(alpha+psi));
y2=(beta*eta2*((1-tau)*(a/(1+theta)))^Ew)^(1/(alpha+psi));
v=beta*((1-tau)*(a/(1+theta))*y)^Ew/Ew-y^Eg/Eg;
/* calculating the payoff to n+1st and nth agent to be
inactive */
v1=beta*((1-tau)*(a/(1+theta))*y1)^Ew/Ew-y1^Eg/Eg;
g1=n/nbar;
vin1=(beta*(g1*(a/(1+theta))*(theta+tau)*y/(1-g1))^Ew)/Ew;
g=(n-1)/nbar;
vin=(beta*(g*(a/(1+theta))*(theta+tau)*y2/(1-g))^Ew)/Ew;
/* search for the equilibrium corresponding to the
monetary shock generated */
if k+vin > v;
 n=n-1;
 continue;
 endif;
if v1 <= k1+vin1;
```

```
    goto label;
    endif;
 n=n+1;
endo;
label: /* we set N_{+1}^{*}y_{+1}^{*}=N^{*}y^{*}=Ny and
a=1, theta^{*}=theta=.05 at the initial state, domestic
money stock, m and foreign money stock, m1 are set to 1
in period zero.  Moreover, we set the foreign monetary
growth rate to 0.05. Thus, the domestic economy is
inflating but the rest of world keep the money stock
constant. The foreign national income is as follows */
nyfor=50*(beta*((50-1)/50)*((1-tau)*(1/(1+theta1)))^(1-
alpha))^(1/(alpha+psi));
m=(1+theta)*m;
m1=(1+theta1)*m1;
x[1,3]=1/nyfor;
x[1,4]=1;
x[i+1,1]=n;
x[i+1,2]=theta;
x[i+1,3]=m1/(n*y);
x[i+1,4]=(m/nyfor)/(m1/(n*y));
z[i+1,1]=n;
z[i+1,2]=(theta-.05)*100;
z[i+1,3]=100*(x[i+1,3]/x[i,3]-1-theta);
z[i+1,4]=100*(x[i+1,4]/x[i,4]-1);
z[i+1,5]=z[i+1,3]/z[i+1,4];
i=i+1;
endo;
print "Domestic economy is inflating at a random
monetary growth rate, its mean is 5% and its standard
deviation is 0.002. 1st col is number of exporters,
2nd col is value of theta, 3rd col is equilibrium value
of import price, 4th col is equilibrium exchange rate"
x[2:101,.];
print "Domestic economy is inflating at a random
monetary growth rate, its mean is 5% and its standard
deviation is 0.002. 1st col is number of exporters,
2nd col is percentage change in theta, 3rd col is
percentage change in import price, 4th col is percentage
change in exchange rate, 5th col is exchange rate
pass-through coeff" z[2:101,.];
output off;
end;
```

Chapter 4

Testing Baldwin's Hypothesis of Hysteresis

4.1 Introduction

In the recent literature on exchange rate pass-through, Baldwin (1988a, b) proposed an interesting hypothesis called sunk cost hysteresis theory to explain the phenomenon. This hysteresis hypothesis suggests that temporary changes in the exchange rate can lead to permanent changes in the structure of import or export markets. Baldwin's hypothesis of hysteresis depends on the presence of sunk costs. The foreign firms have to incur these sunk costs whenever they enter the domestic market. The sunk costs[1] would disappear if the foreign firms left the domestic market. If the foreign firms re-entered the domestic market, they would have incur the sunk cost again. Therefore, these sunk costs act as the barrier to entry in the industry. An outside firm will only enter the foreign market when there is a sufficiently high profit margin to cover the sunk cost. In Baldwin (1988a, b), a temporary appreciation of the domestic currency provides a higher profit opportunity for foreign firms to enter the domestic market. After the shock is removed, not all foreign firms will leave the domestic market. This results in a change in the market structure that alters the relationship between the exchange rate and import prices.

[1]The sunk cost could be thought as a lump sum of money for establishing a sales network in the foreign country.

This hysteresis hypothesis is testable by applying a simple test, i.e. the Chow test in Chow (1960), to test for a structural break in the parameter of profit margins. Such a break in the profit margin will occur if there is a permanent change in the number of incumbent suppliers. Baldwin (1988c) investigated the presence of hysteresis in US aggregate non-oil import prices and detected the presence of a break in the profit margin parameter in a model of an exchange rate pass-through relationship. He found strong evidence that a shift did occur in the exchange rate pass-through relationship in the 1980s. Moreover, the nature of that shift was consistent with the hypothesis of hysteresis. However, Hooper and Mann (1989), and Melick (1989) did not find sufficient evidence for the presence of hysteresis: They could not reject the null hypothesis that the parameters of the exchange rate pass-through relationship were stable. Thus, it is controversial whether the hysteresis hypothesis is acceptable or not.

The purpose of this chapter is to re-examine the validity of Baldwin's hysteresis hypothesis by adopting a different approach. We do not attempt to test the theoretical results obtained in previous chapters. Instead, we shall follow Baldwin (1988c) and use a partial equilibrium approach to test his hypothesis of hysteresis. Most empirical studies on hysteresis and exchange rate pass-through concentrate on the import side and study the relationship between import prices and the exchange rate. Since Baldwin (1988c), Hooper and Mann (1989), and Melick (1989) obtained different results from testing the presence of hysteresis, it may be worthwhile to take a different approach that does not use the equation of exchange rate pass-through as the empirical model for studying hysteresis. Our approach is to concentrate on the exporters' side and test the hypothesis using Canadian industrial data. A prolonged real depreciation (appreciation) of the Canadian dollar provides a relatively high (low) profit opportunity for any potential Canadian exporter to enter (exit) the foreign markets given that the export prices denominated in the foreign currency are relatively stable. According to recent studies on exchange rate pass-through, e.g. Mann (1986), Dornbusch (1987), the prices of manufactured goods, denominated in local currency, are relatively stable in response to exchange rate fluctuations. When the Canadian dollar returns to its original level, not all new Canadian exporters leave (re-enter) the foreign market. The permanent change in the number of exporters results in a change in the market structure which leads to equilibrium hysteresis. Under imperfect competition and downward sloping market demand[2], the change in the number of exporters changes the profit margins permanently. Therefore, hysteresis implies a break in the profit margin which is equivalent to a break in the markup coefficient or Lerner Index, (P-MC)/P. We estimate the Lerner Index explicitly, using Domowitz, Hubbard and Petersen's (1988) method, and test for the presence of hysteresis for each export industry.

Some factors that we have not considered in this chapter may affect the estimation and testing of the markup coefficients. For example, the presence of multinational firms will affect the validity of the test. The major reason is that the subsidiaries in Canada may sell their output below or above the market price to their parents in the US. This will affect the computation of the actual Solow residual in some industry. Furthermore, labour hoarding, wage smoothing, and excess capacity utilization will all have considerable impacts on the markup coefficients.

Recently, Schembri (1989) studied the relationship between export prices and the exchange rate using Canadian industrial data. However, his aim was not to explain the phenomenon of hysteresis but the phenomenon of 'pricing to market'. This phenomenon has also been studied by Krugman (1987), Giovannini (1988), Knetter (1989), and Froot and Klemperer (1989) in detail[3]. We will discuss only

[2]We assume the Canadian producers cannot price discriminate between the local buyers and foreign buyers.

[3]'Pricing to Market' is not the issue that we wish to study here. So , we will only briefly discuss this issue. There are two key elements necessary to explain the 'Pricing to Market' phenomenon: (1) The exporting firm must be able to price discriminate across markets e.g. Knetter (1989). (2) The exporting firm must incur dynamic costs of adjustment on the supply side to be affected by

those parts of Schembri's (1989) results which are related to our study of hysteresis in the Canadian export industries. One result is that the markup coefficient is significantly greater than zero, at the one percent level, in both the US and Canadian markets. His result emphasizes the importance of the imperfectly competitive market structure faced by Canadian exporters. His conclusion is basically consistent with the methodology that we assume in our empirical analysis because it indicates the presence of imperfectly competitive markets faced by Canadian exporters.

In this study, we have chosen five Canadian manufacturing industries (SITC 2 digits): (1) The wood industry, (2) the paper and allied products industry, (3) the primary metal industry, (4) the machinery and equipment industry, and (5) the transport equipment industry[4]. They are chosen for our study because they have the highest export orientations compared to the rest of manufacturing industries. Export orientation is defined as the ratio of the value of export to the value of total output produced and shipped[5]. They are shown below in table 4.1.

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exchange rate changes.

[4]The wood industry includes: (1) Planing, shingle and saw mills, (2) Veneer and plywood mills, (3) Sash, door and millwork plants, (4) Sash, door and millwork plants, N.E.S., (5) Wooden box factories, (6) Coffin and casket, and (7) Misc. wood. The paper and allied products industry includes: (1) Pulp and paper mills, (2) Asphalt roofing, (3) Paper box and bag, (4) Misc. paper converter. The primary metal industry includes: (1) Iron and steel mills, (2) Steel pipe and tube mills, (3) Iron foundries, (4) Smelting and refining, and (5) Roll. cast. extruding (aluminium, copper, alloys, and N.E.S.). The machinery industry includes: (1) Agricultural implement, (2) Misc. machinery and Equipment, (3) Commercial refrigerators and air conditioning equipment, and (4) Office and store machinery. The transport equipment industry includes: (1) Aircraft and parts, (2) Motor vehicle, (3) Truck body and trailer, (4) Motor vehicles parts and accessory, (5) Railroad, rolling stock, (6) Shipbuilding and repairs, (7) Boatbuilding and repairs, and (8) Misc. vehicle.

[5]Source: Department of Regional Industrial Expansion, Manufacturing Trade and Measures 1985: 1966-1984, Ottawa: 1985. Although my data set is up to 1989, my data set contains the the most recent information about the export orientations of Canadian manufacturing industries.

Table 4.1 Export Orientations of Canadian Manufacturing Industries (66-84)

Industry	Export Orientation
Food and beverage	11.2%
Tobacco products	0.7%
Rubber and plastic products	11.7%
Leather	7.3%
Textiles	6.4%
Knitting	1.7%
Wood	48.4%
Furniture and Fixture	8.8%
Paper and allied products	54.9%
Printing, publishing and allied products	14.4%
Primary metal	48.8%
Metal fabricating	6.5%
Machinery	49.5%
Transport equipment	72.9%
Electrical products	20.2%
Non-metallic mineral products	9.7%
Petroleum and coal products	7.4%
Chemical and chemical products	23.1%
Misc. manufacturing	25.4%

Our plan is to estimate the markup coefficient and test for its stability using quarterly data for each industry. When we discuss the empirical model in section 2, heteroskedasticity is one of the problems that is likely to appear in our model even under the null hypothesis. Therefore, the conventional Chow test, or its Lagrangian Multiplier (LM) version, which relies on the conditional homoskedasticity assumption can result in inference with an asymptotically wrong size due to the second moment misspecification. This causes a loss in the power in testing. As a result, we adopt Wooldridge's (1990a, b, 1991) heteroskedasticity-serial-correlation-robust testing procedure to test for the presence of a structural break in the markup coefficient hypothesized by the hysteresis hypothesis. This procedure is appropriate and convenient because it is in the spirit of the LM test approach. Moreover, the tests can be computed by any standard regression package making this procedure attractive. Finally, we implement Wooldridge's (1990b) heteroskedasticity-serial-correlation-robust Chow tests for identifying any structural break that may be present in the parameters. We find that Baldwin's hypothesis of hysteresis is rejected for all the export industries that we have chosen. We ask the reader to interpret the result of the transport equipment industry with great caution because the industry is dominated by a few giant multinationals, like General Motors, Ford, and Chrysler. Some of the products, especially the parts produced by the subsidiaries in Canada are sold to the parent companies in the US below or above the market price. Changes in the markup coefficient in such an industry may just result in increased monopoly rents for the fixed set of producers.

Let us briefly describe the content of this chapter. Section 2 will describe the methodology and the empirical model that I adopt. Section 3 will discuss the data and the empirical results that I have obtained. The final section will be the conclusion of this chapter.

4.2 Methodology

As our study concentrates on individual Canadian manufacturing industries, we assume each industry has a general m-factor production function,

$$Q_t = A_t e^{\gamma t} F(v_{1t}, v_{2t}, ..., v_{mt}), \tag{4.1}$$

Q_t=output in period t,

γ=rate of Hicks-neutral disembodied technical progress,

A_t=random total productivity factor in period t,

v_{it}, $i = 1, ..., m$ is the services of the i^{th} factor used in the production in period t.

We take the derivative of (4.1) with respect to time t,

$$\frac{dQ_t}{dt} = \frac{dA_t}{dt} e^{\gamma t} F(\cdot) + \gamma A_t e^{\gamma t} F(\cdot) + \sum_{i=1}^{m} Q_t^i \frac{dv_{it}}{dt}, \qquad Q_t^i = A_t e^{\gamma t} \frac{\partial F}{\partial v_{it}}.$$

Dividing by Q_t, we obtain

$$\dot{Q}_t = \dot{a}_t + \gamma + \sum_{i=1}^{m} \frac{Q_t^i v_{it}}{Q_t} \dot{v}_{it} \tag{4.2}$$

Where $\dot{Q}_t = (dQ_t/dt)/Q_t$, $\dot{v}_{it} = (dv_{it}/dt)/v_{it}$, $\dot{a} = (dA_t/dt)/A_t$. $Q_t^i v_{it}/Q_t$ is the output elasticity of factor i.

Domowitz, Hubbard and Petersen (1988) have provided a method to estimate the markup coefficient from Solow residuals. By using the discrete approximation to equation (4.2), we have

$$\begin{aligned}\Delta \log Q_t &= \Delta \log A_t + \gamma + \sum_{i=1}^{m} \frac{Q_t^i v_{it}}{Q_t} \Delta \log v_{it} \\ &= \Delta a_{it} + \gamma + \sum_{i=1}^{m} \frac{Q_t^i v_{it}}{Q_t} \Delta \log v_{it} \qquad (4.2a)\end{aligned}$$

Where $\Delta \log x_t = \log x_t - \log x_{t-1}$. (4.2a) is another version of (4.2) in discrete approximation. The Solow residual is obtained from the following accounting relationship:

$$Solow\ \ residual = \Delta \log Q_t - \sum_{i=1}^{m} \frac{Q_t^i v_{it}}{Q_t} \Delta \log v_{it}$$

Solow (1957) first proposed that if price equalled marginal cost and the marginal product of labour equalled the real wage, the residual, which consists of the percentage change in A_{it} (Δa_{it}) plus γ, could be measured from the data[6].

Let us look at the following simple production function in the i^{th} industry in period t. Output Q is produced with constant returns to scale technology[7] from capital K, labour L and intermediate goods, e.g materials and energy, etc[8]. We simply call the intermediate goods 'materials', M. Thus, we consider an economy with n industries and each industry has a three-factor production technology ($v_1 = L$, $v_2 = K$, $v_3 = M$). Let $q = \log(Q/K)$, $l = \log(L/K)$, $m = \log(M/K)$, $\alpha_{Lit} =$

[6]If Δa_{it} is assumed to be normal, A_t/A_{t-1} is lognormally distributed.

[7]The assumption of constant returns to scale seems to be incompatible with imperfectly competitive market structure because there is no barrier to entry. However, the barrier to entry is not from the production technology but from the marketing technology. The presence of sunk costs in establishing sales networks in the foreign country will act as the barrier to entry.

[8]We are aggregating all intermediate goods into a single composite intermediate good. The condition for this to be valid are either that the prices of all intermediate goods are moving together so there is no relative price change (i.e. Hicks aggregation) or that the production function be weakly separable between intermediate goods an the other two primary inputs, capital and labour (Leontif aggregation).

$Q_{Lit}L_{it}/Q_{it}$, $\alpha_{Mit} = Q_{Mit}M_{it}/Q_{it}$, and $\alpha_{Kit} = Q_{Kit}K_{it}/Q_{it} = 1 - \alpha_{Lit} - \alpha_{Mit}$, where $Q_{Lit} = \partial Q_{it}/\partial L_{it}$, $Q_{Kit} = \partial Q_{it}/\partial K_{it}$ and $Q_{Mit} = \partial Q_{it}/\partial M_{it}$. Moreover, under constant returns to scale technology and perfect competition, the output elasticity of the factor equals the factor share of total revenue, i.e. $\alpha_{Lit} = Q_{Lit}L_{it}/Q_{it} = W_{it}L_{it}/P_{it}Q_{it}$, $\alpha_{Mit} = Q_{Mit}M_{it}/Q_{it} = P_{Mit}M_{it}/P_{it}Q_{it}$, and $\alpha_{Kit} = Q_{Kit}K_{it}/Q_{it} = P_{Kit}K_{it}/P_{it}Q_{it} = 1 - \alpha_{Lit} - \alpha_{Mit}$. W_{it}, L_{it}, P_{Mit}, M_{it}, P_{Kit}, K_{it}, P_{it}, and Q_{it} are the wage, labour input, price of materials, material inputs, price of capital, capital input, price of the output and output quantity respectively in the i^{th} industry in period t. The percentage change in A_{it} in i^{th} industry is as follows:

$$\Delta a_{it} = \Delta q_{it} - \alpha_{Lit}\Delta l_{it} - \alpha_{Mit}\Delta m_{it} - \gamma_i, \qquad i = 1, ..., n \tag{4.3}$$

Hall's (1987,1988) insight is that the measured growth of total factor productivity (Solow residual) will be positively correlated with output growth in the presence of market power. If price exceeds marginal cost, then the proper measure of the value of labour and materials should be calculated from the marginal cost. The labour share in cost, α^{*}_{Lit}, is equal to $(P_{it}/MC_{it})\alpha_{Lit}$; and the material share in cost, α^{*}_{Mit}, is equal to $(P_{it}/MC_{it})\alpha_{Mit}$. MC_{it} is the marginal cost of the i^{th} industry in period t. Consider an industry with identical firms. A representative profit-maximizing firm in the i^{th} industry will try to attain

$$P_{it}(1 - \frac{1}{n\bar{\eta}_i}) = MC_{it},\ i = 1, ..., n \Longrightarrow \frac{P_{it}}{MC_{it}} = \frac{1}{1 - 1/(n\bar{\eta}_i)} = \mu_i,\ i = 1, ..., n$$

where $\bar{\eta}_i$ $(n\bar{\eta}_i > 1)$ is the elasticity of market demand for the output of the i^{th} industry. Also, $P_{it} = P^{*}_{it}/e_t$, where P^{*}_{it} is the selling price denominated in the foreign currency and e_t is the foreign currency price of domestic currency. Under imperfect competition, e.g. Cournot market, and downward sloping market demand, the profit margin or markup coefficient of each firm depends on the number of firms, n. Therefore, any change in the number of producers due to hysteresis will be reflected in the markup coefficient of each firm[9]. The presence of a break in the markup coefficient is a necessary condition for the existence of hysteresis which is characterized by a permanent change in the number of producers after a considerable temporary shock. The above equation is true if there is no random disturbance to the demand or supply side.

In order to calculate the Solow residuals in the i^{th} industry, we need to measure the output elasticity of factors e.g. labour, $Q_{Lit}L_{it}/Q_{it}$. As price is not equal to marginal cost under imperfect competition, the proper measures of the values of labour and materials will be calculated from the marginal cost.

$$\frac{W_{it}}{MC_{it}} = \frac{W_{it}}{P_{it}/\mu_i}, \qquad \frac{P_{Mit}}{MC_{it}} = \frac{P_{Mit}}{P_{it}/\mu_i}, \qquad i = 1, ..., n.$$

Where $\mu_i = [1 - (1 - 1/n\bar{\eta}_i)] = P_{it}/MC_{it}$. On the other hand, with the assumption that the factor markets are perfectly competitive, $W_{it} = P_{it}Q_{Lit}/\mu_i$, the wage equals the marginal revenue product of labour in the i^{th} industry where $P_{it}/\mu_i = P_{it}(1 - 1/n\bar{\eta}_i)$, the marginal revenue of output. Similarly, $P_{Mit} = P_{it}Q_{Mit}/\mu_i$, the price of materials equals the marginal revenue product of material input in the i^{th} industry. Re-arranging leads to $Q_{Lit} = W_{it}/MC_{it} = \mu_i W_{it}/P_{it}$ and $Q_{Mit} = P_{Mit}/MC_{it} = \mu_i P_{Mit}/P_{it}$. When they are substituted in the formulae for output elasticities, we can explicitly measure the output elasticity of each input. Thus, the output elasticities of labour and materials in the i^{th} industry can be measured as

$$\alpha^{*}_{Lit} = \frac{Q_{Lit}L_{it}}{Q_{it}} = \mu_i\frac{W_{it}L_{it}}{P_{it}Q_{it}} = \mu_i\alpha_{Lit}, \quad \alpha^{*}_{Mit}\frac{Q_{Mit}M_{it}}{Q_{it}} = \mu_i\frac{P_{Mit}M_{it}}{P_{it}Q_{it}} = \mu_i\alpha_{Mit}.$$

Therefore, the unobservable α^{*}_{Lit} and α^{*}_{Mit} are linked with the observable α_{Lit} and α_{Mit} by the ratio of price to marginal cost, μ_i.

Since μ_i is the ratio of price to marginal cost, we further define $\mu_i = (1 - \beta_i)^{-1}$ assuming $\beta_i \neq 1$. β_i is the Lerner index, $(P_i - MC_i)/P_i$, which is the parameter

[9] I rely on the symmetrical equilibrium condition to obtain this result. Under the symmetry assumption, all firms are the same and the demand faced by each firm will be proportional to the market demand.

of interest in our model. Replacing α_{Lit} and α_{Mit} in equation (4.3) by α^{*}_{Lit} and α^{*}_{Mit}, we derive the following model in which Δa_{it} is the production shock in the i^{th} industry which is assumed to be random and treated as the error term in the regression model (4.5)[10].

$$\Delta a_{it} = \Delta q_{it} - (1-\beta_i)^{-1}\alpha_{Lit}\Delta l_{it} - (1-\beta_i)^{-1}\alpha_{Mit}\Delta m_{it} - \gamma_i \tag{4.4}$$

$$\Delta q_{it} - \alpha_{Lit}\Delta l_{it} - \alpha_{Mit}\Delta m_{it} = y_{it} = \gamma_i(1-\beta_i) + \beta_i \Delta q_{it} + (1-\beta_i)\Delta a_{it}. \tag{4.5}$$

The markup coefficient or Lerner index parameter, β_i, is the only parameter of interest, and the systematic component of the Solow residual. The parameter γ_i includes the constant rate of depreciation of capital in the i^{th} industry. Since we do not have any information on the rate of depreciation in each industry, we cannot derive an estimate the systematic component of the Solow residual, or equivalently the rate of Hicks neutral disembodied technical progress from an estimate of γ_i.

If we assume that P_{it}/MC_{it} deviates from μ_i by a white noise process, w_{it}[11], with mean zero and variance, σ^2_{wi}, we have

$$\frac{P_{it}}{MC_{it}} = \mu_i + w_{it},$$

$$\begin{aligned} \frac{Q_{Lit}L_{it}}{Q_{it}} &= (\mu_i + w_{it})\frac{W_{it}L_{it}}{P_{it}Q_{it}} = \mu_i\alpha_{Lit} + \alpha_{Lit}w_{it}, \\ \frac{Q_{Mit}M_{it}}{Q_{it}} &= (\mu_i + w_{it})\frac{P_{Mit}M_{it}}{P_{it}Q_{it}} = \mu_i\alpha_{Mit} + \alpha_{Mit}w_{it}, \quad i = 1,\ldots,n \end{aligned}$$

Substituting into (4.3), we obtain

$$\begin{aligned} \Delta a_{it} &= \Delta q_{it} - [(1-\beta_i)^{-1}\alpha_{Lit} + \alpha_{Lit}w_{it}]\Delta l_{it} - \\ &\quad [(1-\beta_i)^{-1}\alpha_{Mit} + \alpha_{Mit}w_{it}]\Delta m_{it} - \gamma_i \\ &= \Delta q_{it} - (1-\beta_i)^{-1}\alpha_{Lit}\Delta l_{it} - \alpha_{Lit}\Delta l_{it}w_{it} - \\ &\quad (1-\beta_i)^{-1}\alpha_{Mit}\Delta m_{it} - \alpha_{Mit}\Delta m_{it}w_{it} - \gamma_i \end{aligned}$$

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[10] In the literature, it is usually treated as the error term because it includes a large number of other factors that affect production. It may not be white noise.

[11] Such a disturbance is likely to be present because of the presence of other shocks such as demand shocks.

Multiplying by $1 - \beta_i$ leads to

$$\begin{aligned}\Delta q_{it} - \alpha_{Lit}\Delta l_{it} - \alpha_{Mit}\Delta m_{it} = y_{it} &= \gamma_i(1-\beta_i) + \beta_i \Delta q_{it} + (1-\beta_i)(\Delta a_{it} + \\ &\quad w_{it}\alpha_{Lit}\Delta l_{it} + w_{it}\alpha_{Mit}\Delta m_{it}), \qquad (4.5a)\end{aligned}$$

Equations (4.5), and (4.5a) are not yet valid regression models because Δq_{it} is correlated with the error term, $e_{it} = (1-\beta_i)[\Delta a_{it} + w_{it}(\alpha_{Lit}\Delta l_{it} + \alpha_{Mit}\Delta m_{it})]$. We must, therefore, apply Instrumental Variable (IV) estimation. The variance of e_{it} is conditional on y_{it}:

$$\begin{aligned}\sigma^2_{eit} &= (1-\beta_i)^2[\sigma^2_{ai} + (\alpha_{Lit}\Delta l_{it} + \alpha_{Mit}\Delta m_{it})^2\sigma^2_{wi} + \\ &\quad 2COV(\Delta a_{it}, w_{it}(\alpha_{Lit}\Delta l_{it} + \alpha_{Mit}\Delta m_{it}))] \\ &= (1-\beta_i)^2\sigma^2_{ai} + (1-\beta_i)^2\sigma^2_{wi}(\alpha_{Lit}\Delta l_{it} + \alpha_{Mit}\Delta m_{it})^2 + \\ &\quad 2(1-\beta_i)^2 E(\Delta a_{it} w_{it})(\alpha_{Lit}\Delta l_{it} + \alpha_{Mit}\Delta m_{it}) \\ &= \psi_{0i} + \psi_{1i}(\alpha_{Lit}\Delta l_{it} + \alpha_{Mit}\Delta m_{it})^2 + \psi_{2i}(\alpha_{Lit}\Delta l_{it} + \alpha_{Mit}\Delta m_{it})\end{aligned}$$

$\psi_{0i} = (1-\beta_i)^2\sigma^2_{ai}$, $\psi_{1i} = (1-\beta_i)^2\sigma^2_{wi}$, $\psi_{2i} = 2(1-\beta_i)^2 E(\Delta a_{it} w_{it})$. Thus, e_{it} is heteroskedastic. We shall discuss empirical testing later in this section.

To specify the test of Baldwin's hysteresis hypothesis formally, we consider the following null hypothesis, H_0 : There is no break in the coefficients β_i. Under the alternative hypothesis (Baldwin's hysteresis hypothesis), H_1 : β_i will exhibit a structural break after a prolonged real appreciation (depreciation) of the exporters' currency. As the hypothesis of interest is whether there is a structural break in the parameter, β_i. we re-write (4.5) or (4.5a) as

$$y_{it} = \theta_{0i} + \theta_{1i}\Delta q_{it} + e_{it}, \quad i = 1, ..., n, \quad t = 1, ..., T.$$

$$y_i = X_i\theta_i + e_i, \quad i = 1, ..., n \qquad (4.6)$$

y_i and e_i are $T \times 1$ vectors, X_i is a $T \times 2$ matrix, $\theta_i' = [\gamma_i(1-\beta_i), \beta_i]$ are 2×1 vectors. We want to test $H_0 : \theta_i$ is constant against $H_1 : \theta_i$ changes after prolonged real appreciation or depreciation of the Canadian dollar. As β_i appears in the conditional variance equation, heteroskedasticity will be present under the alternative hypothesis. What is more troublesome is the presence of w_t which causes heteroskedasticity even under the null hypothesis. The traditional econometric testing procedure requires that all the estimators used to compute the statistics are consistent under the null hypothesis. The standard Chow test of Chow (1960) or the LM version of the Chow test e.g. Pagan and Hall (1983), requires the consistency of estimators and the conditional homoskedasticity under the null hypothesis. If heteroskedasticity is present under the null hypothesis, the standard forms of the well known tests can have the wrong asymptotic size while having no asymptotic power due to the misspecification of the second moment.

To overcome the problem of heteroskedasticity, we must use a Chow-type test which is robust to the presence of heteroskedasticity. Although White (1980) provides heteroskedasticity-robust t-statistics, it is only a one degree of freedom test which is inconvenient in most of hypothesis testing situations involving more than one degree of freedom. To compute the statistics for tests with more than one degree of freedom requires the inversion of White's estimated covariance matrix which is used to compute the quadratic form of the heteroskedasticity-robust Wald test. This procedure is not robust to the misspecification of serial correlation. Wooldridge (1990a, b, 1991) proposes a robust, regression-based testing procedure which is in the spirit of the LM approach. The tests can be computed using any standard regression package.

Wooldridge (1990b) suggests the following procedure for the dynamically complete model which has no serial correlation under the null hypothesis: Given the model, $y_{it} = x_{it}\theta + e_{it}$, $i = 1, ..., n$, $t = 1, 2, ..., T$.

(1) Obtain $\hat{e}_t$ by applying the 2SLS regression of y_t on x_t using the instrument w_t. Compute the fitted values $\hat{x}_t$ from the first stage regression of x_t on w_t, and $\hat{\lambda}_t$, which are misspecification indicators. In our case, the misspecification indicators

are $(d_t, d_t\hat{x}_t)$ where d_t is a dummy variable equal to unity after the hypothesized break point.

(2) Obtain $\hat{r}_t$ as the $1 \times J$ (J is the dimension of $\hat{\lambda}_t$) vector of the residuals from the regression of $\hat{\lambda}_t$ on $\hat{x}_t$.

(3) Define $\hat{\epsilon}_t$ to be $1 \times J$ vector $\hat{\epsilon}_t = \hat{e}_t\hat{r}_t$. Use $TR_e^2 = T - SSR$ from the regression of 1 on $\hat{\epsilon}_t$. T is the actual number of observations used in the final regression and SSR is the sum of squared residuals. TR_e^2 is distributed as χ_J^2 asymptotically.

For the dynamically incomplete model which has serial correlation under the null hypothesis[12], Wooldridge (1990b) suggests a similar heteroskedasticity-serial correlation robust testing procedure: We retain procedure (1) and (2) and consider the (3').

(3') Define $\hat{\epsilon}_t$ to be $1 \times J$ vector $\hat{\epsilon}_t = \hat{e}_t\hat{r}_t$. Save the $1 \times J$ residuals $\hat{v}_t$ from the G^{th} order Vector Autoregression (VAR(G)) of $\hat{\epsilon}_t$ on $\hat{\epsilon}_{t-1}, ..., \hat{\epsilon}_{t-G}$. G is chosen to be $Int(T^{1/4})$ where $Int(\cdot)$ is the integer part of the argument.

(4) Use $TR_e^2 = T - SSR$ from the regression of 1 on $\hat{\epsilon}_t$. T is the actual number of observations used in the final regression and SSR is the sum of squared residuals. Asymptotically, TR_e^2 is distributed as χ_J^2 under H_0.

Note that, the standard errors of the estimated coefficients from equation (4.6) which has both heteroskedasticity and serial correlation are inconsistent. Although White (1980) provides a heteroskedasticity robust standard error, it is not robust to serial correlation. Wooldridge (1990b) provides a simple procedure to obtain heteroskedasticity-serial-correlation-robust standard errors. The procedure is as follows:

(i) Estimate $\hat{\theta}_j$ by 2SLS using instruments, w_t. This yields '$se(\hat{\theta}_j)$', $\hat{\sigma}$, and the 2SLS residuals ($\hat{e}_t$, i=1,...,T). Obtain the fitted values $\hat{x}_t$ from the first step

[12]The presence of serial correlation in the residuals is likely due to the use of capital which is measured at the end of the quarter. That capital will not be in operation until the next quarter.

regression of x_t on w_t.

(ii) Compute the residuals ($\hat{r}_{tj}$, i=1,...,T) from the regression of $\hat{x}_{tj}$ on $\hat{x}_{t1},...,$ $\hat{x}_{t,j-1}, \hat{x}_{t,j+1}, ..., \hat{x}_{tk}, t = 1,...,T$.

(iii) Set $\hat{\epsilon}_{tj} = \hat{r}_{tj}\hat{e}_t$ and run the regression of $\hat{\epsilon}_{tj}$ on $\hat{\epsilon}_{t-1,j}, ..., \hat{\epsilon}_{t-G,j}$, where G is, say, the integer part of $T^{1/4}$. Compute the following term

$$\hat{c}_j = (\frac{T}{T-k})\frac{\hat{\tau}_G^2}{(1 - \hat{a}_1 - \hat{a}_2 - ... - \hat{a}_G)^2}$$

where $\hat{a}_i$, $i = 1,...,G$ are the OLS coefficients from the regression (iii). $\hat{\tau}_G^2$ is the square of the standard error of the regression in (iii).

(iv) compute the $se(\hat{\theta}_i)$ from

$$se(\hat{\theta}_j) = ['se(\hat{\theta}_i)'/\hat{\sigma}]^2(T\hat{c}_j)^{1/2}.$$

The standard errors computed in this way can be used to construct t-ratios which are asymptotically normal under H_0.

In the next section, we shall discuss the results of this empirical study.

4.3 Empirical Results

For each industry, we compute the quarterly 'Solow residual'. Each set contains labour, capital and materials[13] as inputs. The quantity of output is obtained by deflating the total sales of goods and services by the industrial product price index for each industry. The industrial product price indices are the factory gate prices of domestic production[14]. The details of the procedure for obtaining the 'Solow residuals' are discussed in appendix 4.A. For the transport equipment industry, the sample length is from the 2nd quarter of 1981 to the third quarter of 1989[15], 34

[13] In appendix 4.A, we use the total raw material price index as the deflator to obtain the quantity of materials in use for each industry. Since each industry uses a different mix of raw materials and the prices of all raw materials do not move together, it is preferable to construct different price indices for each industry. Our single price index will be appropriate provided that different material prices are highly correlated. However, the data of those raw material prices are not available.

[14] Therefore, we assume that there is no price discrimination between the domestic buyers and foreign buyers.

[15] For the 77:1 to 80:4 data on the industrial product price index of transport equipment are secure data which are not available publicly. Therefore, the sample begins at 1981:2.

sample points in total. For the other four industries, the sample length is from the 2nd quarter of 1977 to the third quarter of 1989, 50 sample points in total. We follow Wooldridge's (1990b) robust regression-based testing procedure. We first test the models for the presence of 4th order serial correlation because we are dealing with quarterly data. The instruments that we use are: Federal defence spending, GDP at 1986 prices (Expenditure based), and the same variables lagged by one quarter. If any model is found to have serial correlated error, the lagged residuals obtained from the first stage regression can be used to predict the endogenous variables. Therefore, those lagged residuals should be used as instruments as well. All the computations are done by VAX TSP 4.1.

The statistics of the heteroskedasticity robust tests for 1st to 4th order serial correlations for each industry are obtained by implementing procedures (1), (2), and (3). The misspecification indicators, $\hat{\lambda}_{it}$ are the lagged residuals, $\hat{e}_{it-1}$, $\hat{e}_{it-2}$, $\hat{e}_{it-3}$, and $\hat{e}_{it-4}$. The test statistics computed will be asymptotically distributed as χ^2_i, i=1, 2, 3, 4 under the null hypothesis. They are shown in table 4.2 below[16].

[16] For the wood industry, paper and allied products industry, primary metal industry, and machinery industry, the sample lengths for testing 1st, 2nd, 3rd and 4th serial correlations are 77:3 to 89:3, 77:4 to 89:3, 78:1 to 89:3, and 78:2 to 89:3 respectively. For the transport equipment industry, the sample lengths are 81:3 to 89:3, 81:4 to 89:3, 82.1 to 89:3 and 82:2 to 89:3.

Table 4.2 Heteroskedasticity Robust Tests of 1st to 4th Order Serial Correlations

Industry	AR(1)	AR(2)	AR(3)	AR(4)
Wood	0.320	0.209	1.076	3.314
	(0.5716)	(0.9008)	(0.7829)	(0.5067)
Paper	1.466	1.578	1.826	3.999
	(0.2260)	(0.4543)	(0.6093)	(0.4061)
Primary Metal	0.929	0.999	0.1032	2.164
	(0.3351)	(0.6068)	(0.7935)	(0.7065)
Machinery	1.791	1.920	5.394	6.287
	(0.1808)	(0.3829)	(0.1451)	(0.1787)
Transport Equipment	4.107	5.161	7.232	6.407
	(0.0427)	(0.0757)	(0.0649)	(0.1701)

The values in parentheses are the corresponding p-values[17]. Only in the case of transport equipment was the p-value of AR(1) small enough to reject the null hypothesis of no serial correlation[18]. The test statistic for the transport equipment industry indicates the presence of 1st order serial correlation. All the other statistics computed are statistically insignificant at the 5 % level. As a result, the lagged residual $\hat{e}_{t-1}$ of transport equipment industry will be used as instrument to obtain $\hat{x}_{it}$ and the sample of transport equipment industry begins at 81:3 instead of 81:2.

To obtain the correct t-ratios of the estimated coefficients, we apply Wooldridge's (1990b) heteroskedasticity-serial-correlation-robust standard error[19]. As the parameter of interest in all these models is the estimated markup coefficient, (P-MC)/P, θ_1, we present the coefficients to equation (4.6), θ_1 estimated by 2SLS in table 4.3 below.

[17]The statistic of AR(3) is larger than AR(4) because the sample size is not the same. The test for AR(3) has one more observation than the test for AR(4). All the p-values are computed by the CDF procedure of MINITAB Release 7.2.

[18]Note that the sample size of the transport equipment is the smallest.

[19]There are 33 observations for the transport equipment industry (81:3 to 89:3), and 50 observations (77:2 to 89:3) for the other industries, so $G = Int(50^{1/4}) = Int(33^{1/4}) = 2$.

Table 4.3 Estimated Markup Coefficients in Each Industry

Industry	Markup	R^2	Adj. R^2
Wood	0.1916*	0.3397	0.3259
	(0.0849)		
Paper	0.3625**	0.5667	0.5577
	(0.0050)		
Primary Metal	0.4149**	0.6197	0.6118
	(0.0375)		
Machinery	0.1637**	0.3912	0.3785
	(0.0441)		
Transport Equipment	0.1687**	0.6691	0.6588
	(0.0342)		

The 'Markup' is the estimated markup coefficient. All the values in parentheses are the heteroskedasticity-serial-correlation-robust standard error[20]. '**' and '*' indicate that the coefficients are significantly different from zero at the 1 % and 5 % levels respectively. All industries have highly statistically significant markup coefficients which are positive. On the whole, the markup coefficients estimated are consistent with what Schembri (1989) obtains. He adopts a different approach and finds that the markup coefficient is also significantly greater than zero at the one percent level in both the US and Canadian markets. His result emphasizes the importance of the imperfectly competitive market structure that Canadian exporters are facing in the foreign market.

We are now ready to test Baldwin's hypothesis of hysteresis. To identify when the breaks take place, we examine the time series behavior of the Canadian exchange rate from 1977:2 to 1989:3. Figure 4.1 plots the behavior of the Canadian nominal exchange rate index (MERM) and the Canadian real exchange rate index (RERI). MERM is constructed from the IMF's Multilateral Exchange Rate Model. RERI is the real effective exchange rate index also constructed by the IMF[21]. Both indices are expressed in foreign currency per unit of Canadian dollar. The rise (fall) of

[20]The R^2 and adjusted R^2 are obtained from the instrumental variable estimation. Thus, their interpretation is not the same as for R^2's obtained using OLS.

[21]We do not use the G10 index of the Canadian dollar compiled by the Bank of Canada because Howitt (1985) suggests that it is too heavily weighted on the US dollar, at more than 80 %.

the indices implies real appreciation (depreciation) of the Canadian dollar. From figure 4.1, we can observe that both indices move closely together throughout the entire sample period. There are prolonged depreciations[22] during these two periods: 1977:1 to 1979:4, and 1983:3 to 1986:2; there are prolonged appreciations during the following two periods: 1980:1 to 1983:2, and 1986:3 to 1989:3. We assume the breaks appear at 1979:4, 1983:2, and 1986:2. Thus, we can divide the figure into 4 regions. These prolonged depreciations (appreciations) provide a relatively high (low) profit opportunity for any potential Canadian exporters to enter (exit) the foreign markets. We first apply the joint tests of all the possible break points to all the export industries. Unfortunately, the observations of the transport equipment industry begin at 1982:1[23], so it is not possible to test for the present of the break at 1980:1. Therefore, we test for the presence of only two break points (1983:3 and 1986:3) in the transport equipment industry. For the other industries, we test for the presence of these three break points using the sample from 1977:4 to 1989:3. Using Wooldridge's(1990b) heteroskedasticity-serial-correlation-robust testing procedure, we obtain the test statistic for transport equipment industry which is distributed as χ^2_4 distribution with 4 degrees of freedom. All the other statistics computed are distributed as χ^2_6 distribution with 6 degrees of freedom. The results are listed in table 4.4 below.

Table 4.4 Tests for Baldwin's Hypothesis of Hysteresis

Industry	Chow	p-value
Wood	3.383	0.7594
Paper	6.012	0.4217
Primary Metal	6.550	0.3643
Machinery	6.555	0.3638
Transport Equipment	2.470	0.6500

[22] 'Prolonged depreciation (appreciation)' means that the Canadian dollar is appreciating (depreciating) for at least 15% of the whole sample period.

[23] After step (3'): Running the VAR(2) system, the sample begins at 1982:1 instead of 1981:3 for the transport equipment industry. For the other industries, the samples begin at 1977:4 instead of 1977:2.

Chow is the test statistic computed. $\chi^2_{4,.05} = 9.49$ and $\chi^2_{6,.05} = 12.6$[24]. The high p-values suggest the absence of hysteresis in Canadian export industries.

We further test for the presence of each break individually to see whether any single event of hysteresis is present. We focus on the movements of the real effective exchange rate index (RERI). We label the breaks at 1980:1, 1983:3, and 1986:3 as break (1), break (2), and break (3) respectively.

To test for the presence of break (1), we use the subsample from 1977:4 to 1981:3 for the wood industry, paper and allied products industry, primary metal industry and machinery industry. The end point 1981:3 is chosen because the index value at 1981:3 is roughly equal to the index value at 1977:4. If hysteresis does appear due to prolonged real depreciation of the Canadian dollar, the number of exporters before the break at 1980:1 will not equal the number of exporters after the break. New exporters will enter the industry after incurring the sunk cost and not leave even when the exchange rate returns to its original value. As a result, the average profit margin before the break will not equal the average profit margin after the break. This is the hypothesis that we want to test. For the transport equipment industry, there is no data available for testing the presence of break (1). To test for the presence of break (2), we use the subsample from 1980:4 to 1986:2 for the wood industry, paper and allied products industry, primary metal industry and machinery industry. 1980:4 is chosen because its index value is roughly equal to the index value at 1986:2. For the transport equipment industry, we use the subsample from 1982:1 to 1985:4 because the sample begins at 1982:1. The end point 1985:4 is chosen because its index value is roughly equal to the index value at 1982:1. Similarly, we

[24]We also test the presence of the two break points (1983:2 and 1986:2) in the wood industry, paper and allied products industry, primary metal industry and machinery industry. The statistics computed are distributed as χ^2_4 distribution with 4 degrees of freedom. The results are as follows: For the wood industry, the chi-square statistic is 2.547 (0.6362). For the paper and allied products industry, the chi-square statistic is 2.382 (0.6659). For the primary metal industry, the chi-square statistic is 3.403 (0.4929). For the machinery industry, the chi-square statistic is 5.195 (0.2679). The values in parentheses are the corresponding p-values. The high p-values suggest the absence of hysteresis.

use the subsample from 1983:3 to 1988:2 to test for the presence of break (3) in all industries. The end point 1988:2 is chosen because its index value is roughly equal to the index value at 1983:3. The results are shown in Table 4.5 below.

Table 4.5 Tests for the Presence of individual Breaks

Industry	Chow(1)	Chow(2)	Chow(3)
Wood	3.095	2.210	2.472
	(0.2128)	(0.3312)	(0.2905)
Paper	5.423	1.319	1.293
	(0.0664)	(0.5171)	(0.5239)
Primary Metal	3.071	2.538	1.269
	(0.2153)	(0.2811)	(0.5302)
Machinery	2.629	1.390	2.973
	(0.2686)	(0.4991)	(0.2262)
Transport Equipment	N.A.	1.053	3.467
		(0.5906)	(0.1767)

N.A. means 'not available'. $\chi^2_{2,.05} = 5.99$. The values in parentheses are the corresponding p-values. Chow(1), Chow(2), and Chow(3) are the test statistics computed in correspondence to the presences of break (1), break (2) and break (3) respectively. Only in the case of paper and allied products would the p-value of Chow(1) be considered small but it is still above 0.05 which is the level of the significance in our test.

Since most of the output of the transport equipment industry goes to the US due to the Auto Pact, the real US-Canada exchange rate may be more relevant than the real effective exchange rate index which includes a number of countries other than the US. We construct the real US-Canada exchange rate by using the following formula

$$s_{US-Can} = e_{US-Can}\frac{P_{Can}}{P_{US}}.$$

e_{US-Can} is the market rate of US dollar price of Canadian dollar. P_{US} is the Consumer Price Index of the US. P_{Can} is the Consumer Price Index of Canada. s_{US-Can} is the real US-Canada exchange rate. They are all obtained from International Financial Statistics of IMF. We plot the real US-Canada exchange rate from 1980:1 to

1989:3 on figure 4.2. We can observe that there is a peak and a trough. The peak is at 1983:3 and the trough is at 1986:1. The peak in figure 4.2 happens at the same time as the peak in figure 4.1. Therefore, the break (2) observed in figure 4.1 is the same as the one in figure 4.2. The only difference is that the break (3) in figure 4.1 happens at 1986:3 but it may take place at 1986:1 according to figure 4.2. We repeat the tests on the transport equipment industry according to the information from figure 4.2. We obtain the following results: (1) The joint test statistics of two break points is 2.575 (0.6313). (2) The statistic of the break at 1983:3 is 1.172 (0.5565). (3) The statistic of the break at 1986:1 is 3.245 (0.1974). The values in parentheses are the corresponding p-values of those statistics. They all indicate that we cannot reject the null hypothesis at the 5 % level of significance. Therefore, for all the five industries, we cannot reject the null hypothesis that there is no structural break in the markup coefficients. Since the presence of a structural break in the markup coefficient is a necessary condition for hysteresis, the acceptance of the null hypothesis means that there is no hysteresis for the whole sample period.

4.4 Conclusion

In this chapter, we conduct an empirical test of the validity of Baldwin's hypothesis of hysteresis by using the data on Canadian manufacturing industries which display a high export orientation. We study the exporters' (suppliers') response to any large temporary swing in the exchange rate. We adopt Domowitz, Hubbard and Petersen's (1988) method to estimate the markup coefficient, $(P - MC)/P$ directly for each industry. Our approach is different from the existing empirical studies on hysteresis because these studies test for the presence of structural break in the exchange rate pass-through equation on the import side. Since heteroskedasticity and serial correlation may be present under the null hypothesis, we implement Wooldridge's (1990b) heteroskedasticity-serial-correlation-robust testing procedures to test for the presence of any structural break in the markup coefficient in each industry. From the five export industries that we have chosen, none of the industries display any hysteresis throughout the whole sample period. Since the transport equipment industry is dominated by a few multinationals like General Motors, Ford, and Chrysler, the subsidiaries in Canada may sell their products, especially, the parts to the parent companies in the US below or above the market prices. If this happens, it will affect the validity of the testing results in the transport equipment industry because we cannot compute the actual Solow residuals in such an industry.

Figure 4.1 The nominal exchange rate index (MERM) and the real effective exchange rate index (RERI) of Canada

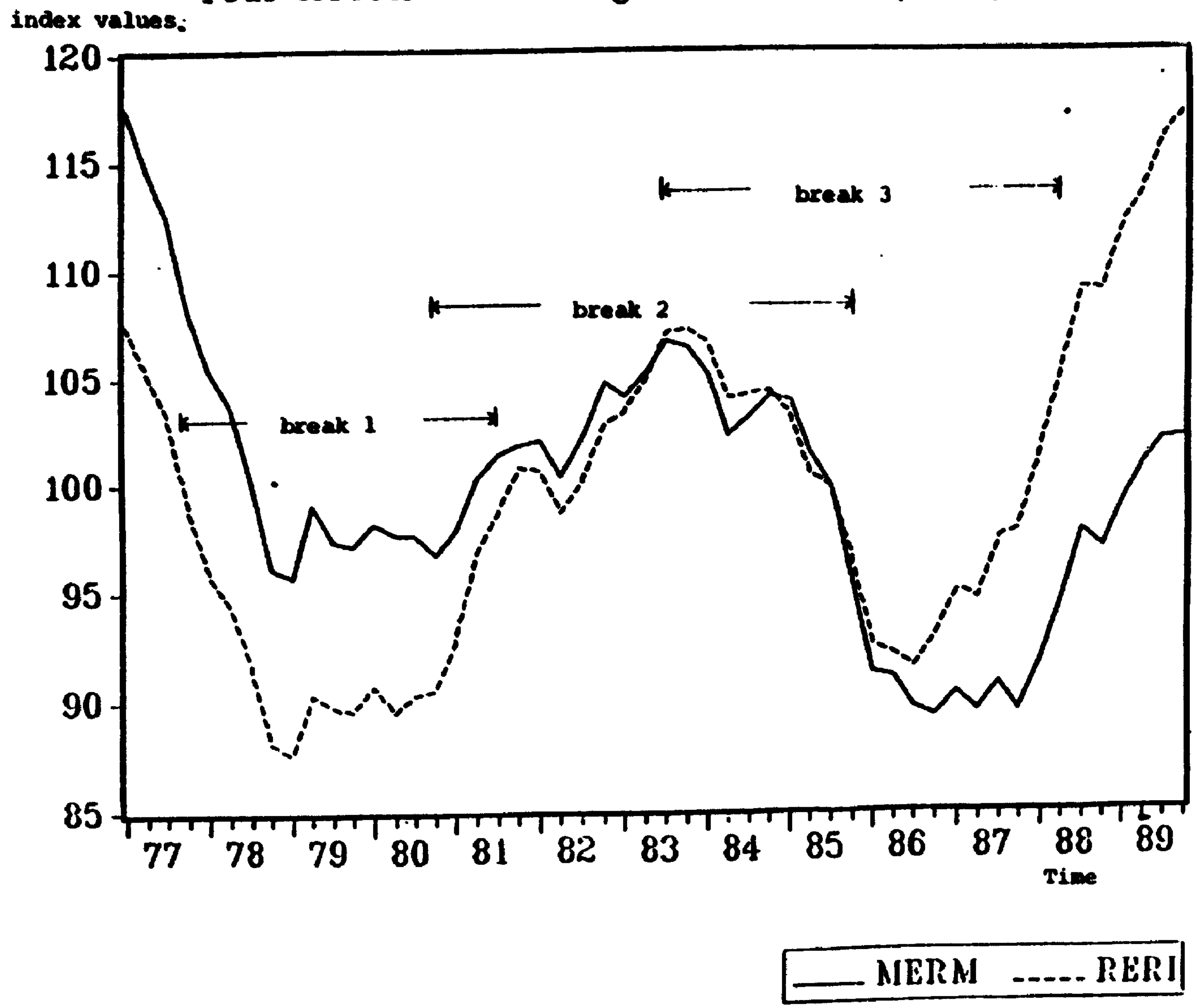
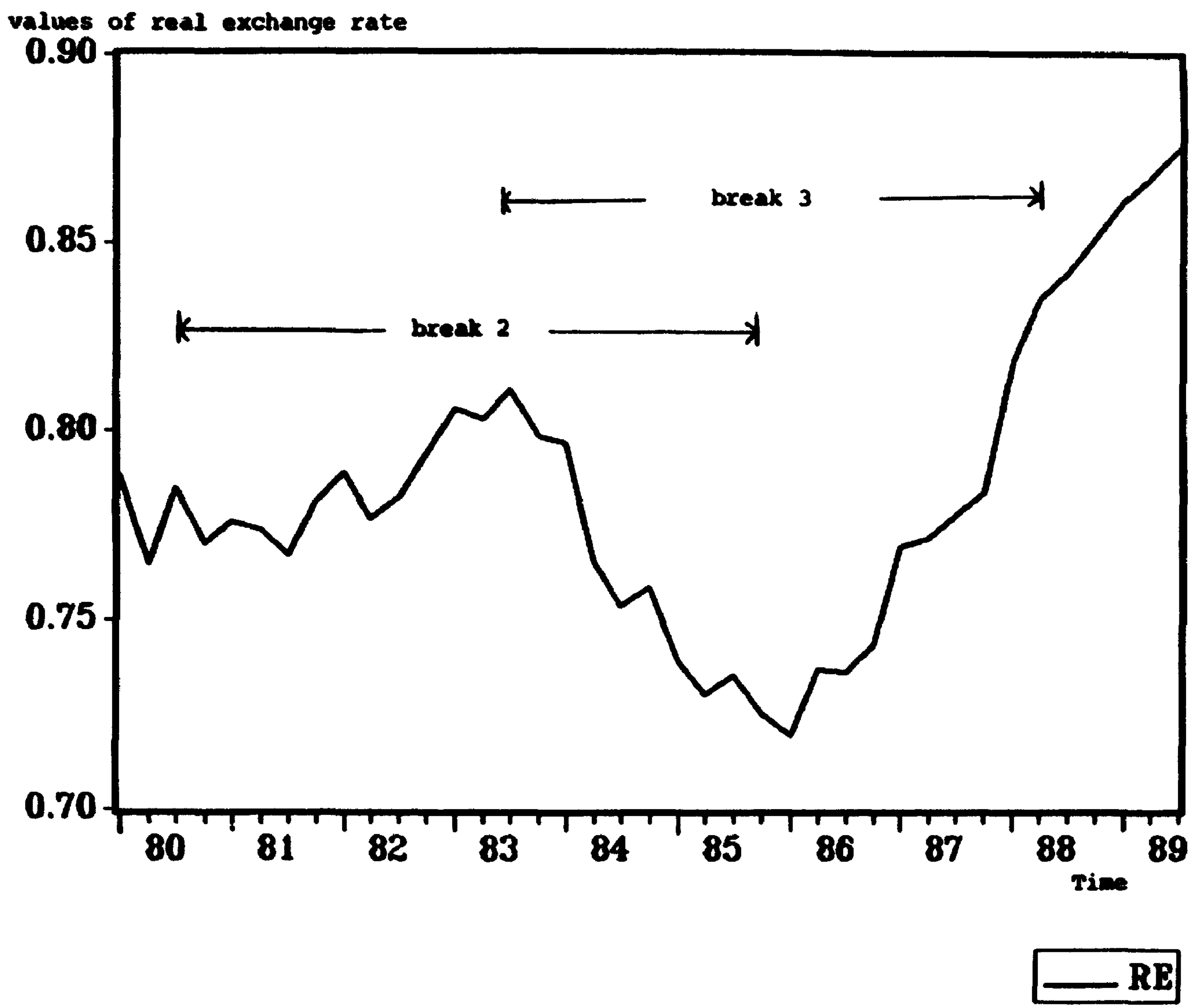


Figure 4.2 The real US–Canada exchange rate (US $/CAN $)



Appendix 4.A

Data Sources

'Solow residuals' are constructed from the following quarterly two digit SITC industrial data in CANSIM.

(1) Output is calculated by deflating the total sales of good and services in each industry by the corresponding industrial product price index (1981=100). This procedure gives us a measure of output quantity for each industry.

(2) Capital use is computed by adding up fixed assets, long term intercorporate investments in affiliated corporations and long term investments from 'Quarterly Statement of Assets, Liabilities and Shareholders' Equity - for companies having 10 millions or more of total assets' and deflating them by the GDP implicit price deflator of business investment in fixed capital to obtain the real value of capital in use.[25]

(3) Labour is measured in number of hours used by each industry computed as follows:

(3.1) Labour share is α_{Lit}=wage bill in i^{th} industry/sales in i^{th} industry. Wage bill=Average weekly earnings × number of workers × 13.

(3.2) Labour input from 77:1 to 82:4 is calculated by multiplying the number of workers employed in 1967 by the labour employment indexes (77:1-82:4) to get the number of workers employed in that period. The number of workers employed in manufacturing sectors in 1967: Wood, 85000, paper and allied products, 120100, primary metal, 120400, machinery, 65300, transportation equipment, 141800[26].

[25] We assume the depreciation rate is always constant for each industry. Since only $\Delta \log K_t$, the rate of change of capital, is relevant for calculating the Solow residual in the empirical model, the constant depreciation rate of capital will not add any information in explaining the variations of the Solow residual over time. Therefore, the estimated markup coefficient (slope estimate in the regression model) is not affected whether we include the constant depreciation rate or not.

[26] These estimates are given in 'Labour Costs in Manufacturing 1967, Occasional paper, Statistics Canada (72-506)'.

Then labour input=number of workers × 13 × (average weekly hours of hourly-rated workers + average weekly overtime hours).

(4) Materials and other intermediate goods, e.g. energy, used in each industry are calculated as follows: (Other operating expenses - wage bill)/total raw materials price index (1981=100)[27]. The share of materials = (Other operating expenses - wage bill)/Sales of goods and services.

(5) The left hand side variable (the Solow residual) in equation (4.6) = the rate of change in output - (labour share)× the rate of change in labour - (intermediate good share)× the rate of change in intermediate goods - (1 - labour share - intermediate good share)× the rate of change in capital.

The following table describes the exact locations of data used in CANSIM.

[27]Other operating expenses includes a lot of intermediate goods besides labour and materials. It includes energy cost, overheads, etc. Implicitly, we assume all five industries are using homogeneous intermediate goods as inputs because we are deflating the item, 'other operating expenses - wage bills' by the total raw material price index instead of the industry-specific raw material price indices which is not available.

Table 6 Databank number of the Five Industries in CANSIM

Variables	Wood	Paper	Primary Metal	Machinery	Transport.
Long-term Intercorporate Investment in aff. corp.	D81962	D82168	D82375	D82581	D82686
Long-term Investment	D81963	D82169	D82376	D82582	D82687
Fixed Assets	D81964	D82170	D82377	D82583	D82688
Sales of goods and services	D81998	D82205	D82411	D82617	D82722
Average weekly hours of hourly-rated wage-earners (77:1-82:4)	D705651	D705658	D705665	D705679	D705682
(83:1-89:3)	L4369	L4471	L4381	L4399	L4404
Average weekly overtime hours (83:1-89-3)	L4989	L5091	L5001	L5019	L5024
Average Weekly Earnings (77:1-82:4)	D703059	D703066	D703074	D703089	D703093
(83:1-89:3)	L1269	L1371	L1281	L1299	L1408
Employment Indexes (67:1-82:4)	D700158	D700165	D700174	D700189	D70012
Total number of Employees (83:1-89:3)	L29	L131	L41	L59	L64
Industrial product price Index	D614055	D614067	D614079	D614100	D614106

The industrial product price index of transport equipment is available from 81:1 to 89:3. The industrial product price indices are factory gate selling prices of domestic production. GDP implicit price deflator of business investment in fixed capital (1986=100, seasonally adjusted) (D20566) is from 77:1-89:3. The total raw material price index is in two parts: Total raw material price indices (D636141) (77:4-80:4) and total raw material price indices (D614316) (81:1-89:3). For Instruments, we use the following data from CANSIM: Defence spending (D459026), GDP at 1986 prices (Expenditure based) (D20463), GDP implicit price indexes (D20556). The Canadian nominal exchange rate index (MERM, CA00AMX) and the Canadian real effective exchange rate index (CA00REU) are obtained from *International Financial Statistics*. The (MERM) nominal exchange rate index (CA00AMX) is based on weights derived from the Fund's Multilateral Exchange Rate Model (MERM) which represent the model's estimate of the medium-term effect on a country's trade balance of a one percent change in the domestic currency price of each of the other currencies. The weights are estimated for 1977 and comprise traded and non-traded goods and certain feedback parameters derived from the MERM; they therefore take account of the size and direction of trade flows in the base year (1985), as well as the relevant price elasticities and effects of exchange rate changes on domestic costs and prices. The real effective exchange rate index (CA00REU) is calculated from the nominal effective exchange rate index (CA00NEU) and a cost indicator of relative normalized unit labour costs in manufacturing. The base year is also 1985. The US-Canada exchange rate (CA00RH) is the period averages of the market exchange rate for Canada quoting rates in US dollars per unit of Canadian dollar. The Canadian Consumer Price Index (CA64) and the US Consumer Price Index (US64) are calculated by using Laspeyres formula with base year 1985.

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