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Decentralized Provision Of Public Goods

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DECENTRALIZED PROVISION OF PUBLIC GOODS

by

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ABSTRACT

When individuals provide noncooperatively many public goods, not only are contribution levels generally too low, but the composition of the contributions is also generally inefficient. Efficiency gains may then be obtained through institutions that constrain individuals' choices, either by increasing contribution levels and/or by improving the public goods mix.

A model of noncooperative public goods provision is set up in chapter 1. The presence of many public goods presents an additional difficulty because the theory of demand under rationing (instead of standard consumer theory) is needed to derive individuals' contribution functions. The impact of various institutions on individuals' choice sets is shown and a taxonomy is proposed.

In chapter 2, a United Fund is added as an autonomous player that collects charitable contributions and redistributes them to various charities. Necessary and sufficient conditions for contributing to the United Fund to be a Subgame Perfect Equilibrium strategy are found. It is also shown that contributions to a United Fund would likely be smaller than direct contributions to charities would have been, raising a mix-level dilemma for the Funds' administrators.

In chapter 3, a tax-earmarking scheme, where individuals must pay some tax but may earmark it to the provision of any public good, is analysed. It is shown that when there are only two public goods, the tax-earmarking outcome is particularly attractive since it is always unique, constrained pareto-efficient, and in the constrained core.

In chapter 4, subsidy schemes for private provision of public goods are analysed. It is shown that Lindahl equilibria are the only efficient allocations that may be supported as Nash equilibria where everyone contributes positively and that a uniform subsidy rate would generally be inefficient.

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CHAPTER 1 INTRODUCTION

The starting point for this thesis is a standard subscription model of public goods provision where individuals allocate non-cooperatively their exogenous income between private consumption and contributions to a continuously divisible public good, taking into account others' contributions.

1.1 Basic Model

Let $L=\{1,\dots,\ell\}$ be the set of private goods, $M=\{1,\dots,m\}$ be the set of public goods, and $N=\{1,\dots,n\}$ be the set of individuals. Superscripts refer to individuals and subscripts refer to goods, unless otherwise specified. Bold characters denote vectors. Let $\mathbf{w}'=(w^1,\dots,w^n)\in\mathbb{R}^n_+$ be the income distribution, exogenously determined.

Supply is assumed to be perfectly elastic at prices $\mathbf{p}_z$ and $\mathbf{p}_x$ for public and private goods respectively. $\mathbf{x}^i \in \mathbb{R}^\ell_+$ is a bundle of private goods consumed by individual i. c^i_h is individual i's contribution to public good h, and $z^i_h = c^i_h/p_{zh}$ is the quantity of public good h bought by him. $\mathbf{c}^i = (c^i_1,\dots,c^i_m) \in \mathbb{R}^m_+$ is i's vector of contributions and $\mathbf{z}^i = (z^i_1,\dots,z^i_m) \in \mathbb{R}^m_+$ is the bundle of public goods bought by him. $G^i=\sum_{j=1}^{m} c^i_j$ his total contribution.

$\mathbf{c}=(\mathbf{c}^1,\dots,\mathbf{c}^n)' \in \mathbb{R}^{m\times n}_+$ is a contributions matrix by all individuals and $G = \sum_{i=1}^{n}\sum_{h=1}^{m} c^i_h$ is total contributions by everyone. $\mathbf{c}^{-i} = (\mathbf{c}^1,\dots,\mathbf{c}^{i-1},\mathbf{c}^{i+1},\dots,\mathbf{c}^n)' \in \mathbb{R}^{m\times(n-1)}_+$ is the matrix $\mathbf{c}$ minus the ith row, and G^{-i} is total contributions by everyone except i. $z^{-i}_h = \sum_{j\neq i} c^j_h/p_{zh}$ is the quantity bought by everyone except i. $z_h = z^i_h+z^{-i}_h$ is the total quantity of public good h bought. $\mathbf{z}=(z_1,\dots,z_m) \in \mathbb{R}^m_+$ is the bundle of public goods consumed.

A.1 (Preferences): Let P^i be individual i's preference relation over consumption bundles (x^i,z) and assume that P^i is reflexive, connected, transitive, continuous, nonsatiated and strictly convex $\forall i \in N$. Let $U^i(x^i,z) \in \mathbb{R}$ be the utility function representing P^i. Under the above assumption, $U^i(x^i,z)$ is well-defined, continuous and strictly quasi-concave. Assume further, for convenience, that it is continuously differentiable.

Assumption 1 implies that individual i cares only about the actual public goods bundle consumed, and not about how it is arrived at, i.e. no "warm glow" from having done one's duty. Some individuals may not care about some components of z but no z_h is a "bad" for anyone.

A.2 (Complete Information): Everyone knows all the rules, endowments, and preferences, and this is common knowledge.

A.3 (Behaviour): Individuals seek to maximize their utility by choosing simultaneously a private goods consumption bundle and a vector of contributions to public goods, taking their income, the preferences and income of others and the prices of all goods parametrically.

This setting induces a game of complete information $\Gamma(N,w,p,U)$ with the individuals as players. Let $B(w^i) = \{(x^i,z^i) \in \mathbb{R}^{l+m}_+ ;\ p_x x^i + G^i \leq w^i \text{ and } c^i_h \geq 0\ \forall h \in M\}$ be individual i's choice set. Since $p_z z = G^i + G^{-i}$, the budget constraint may be written $p_x x^i + p_z z \leq w^i + G^{-i}$. The nonnegativity constraints may also be written $z - z^{-i} \geq 0$.

Figure 1 shows an individual's choice set (area FGH) given everyone else's contributions. x^i is private consumption and z_1 and z_2 are two public goods. In some cases, a clearer picture may be obtained by <u>projecting</u> the 3 dimensional budget constraint of Figure 1 onto the $z_1 z_2$ plane to obtain Figure 2. No relevant information is lost in this process since, given that w^i is fixed, any combination of c_1^i and c_2^i uniquely determines x^i. Browning(1975) discusses this type of representation. The intersection of i's three-dimensional indifference bowl with his budget constraint gives two-dimensional indifference curves that appear as concentric rings. As drawn, the individual would prefer bundle A but, since he cannot make negative contributions, he picks B on the boundary of his choice set.

Individual i's problem is to choose x^i and z^i to maximize $U^i(x^i,z)$ subject to $(x^i,z^i) \in B(w^i)$, given z^{-i}. The Lagrangean function for this problem is:

$$L^i = U^i(x^i,z) + \lambda^i[w^i+G^{-i}-p_x x^i-p_z z] + \mu^i(z-z^{-i})$$

where λ^i and μ^i are 1+m non-negative Lagrange multipliers.

The Kuhn-Tucker necessary conditions, assuming for simplicity, that $x^i>0$ at an optimum and that the budget constraint is binding, are:

(1.1.0.1) $$\frac{\partial L^i}{\partial x_h^i} = \frac{\partial U^i}{\partial x_h^i} - \lambda^i p_{x_h} = 0 \qquad h=1,\dots,\ell$$

(1.1.0.2) $$\frac{\partial L^i}{\partial z_h^i} = \frac{\partial U^i}{\partial z_h} - \lambda^i p_{z_h} + \mu_h^i = 0 \qquad h=1,\dots,m$$

(1.1.0.3) $$\frac{\partial L^i}{\partial \lambda^i} = w^i + G^{-i} - p_x x^i - p_z z = 0$$

(1.1.0.4) $$\frac{\partial L^i}{\partial \mu_h^i} = z_h - z_h^{-i} \geq 0 \qquad h=1,\dots,m$$

The complementary slackness conditions are:

(1.1.0.5) $$\mu_h^i(z_h - z_h^{-i}) = 0 \qquad h=1,\dots,m.$$

Given the quasi-concavity of $U^i(.)$, the 2nd order sufficient conditions would be satisfied, so that the solution of the first order conditions is a maximum. Assuming without loss of generality that the first $k \leq m$ quantity constraints are not binding, the solution of this system is a set of ℓ conditional demand functions for the private goods, and k conditional contribution functions to public goods:

$$x_h^i = \tilde{f}_h^i(p_x, p_z, w^i + G^{-i}, z^{-i}) \qquad h=1,..,\ell$$

$$c_h^i = p_{z_h} \cdot [\tilde{g}_h^i(p_x, p_z, w^i + G^{-i}, z^{-i}) - z_h^{-i}] \qquad h=1,..,k$$

$$c_h^i = 0 \qquad h=k+1,...,m$$

where $\tilde{f}^i(.)$ and $\tilde{g}^i(.)$ are conditional demand functions as defined by Pollak(1969).

A Nash equilibrium for this game is an allocation $(\hat{x},\hat{z})$ such that:

$$(\forall i \in N) \quad \hat{x}^i + \sum_{j=1}^{m} \hat{c}_j^i \leq w^i, \text{ and}$$

$$(\forall i \in N) \quad x^i + \sum_{j=1}^{m} c_j^i \leq w^i \Rightarrow U^i(\hat{x}^i,\hat{z}) \geq U^i(x^i, z^i + \hat{z}^{-i}).$$

Existence of a Nash equilibrium is guaranteed since the strategy sets are all compact and convex and the payoff functions are all continuous and strictly quasi-concave[1] by assumption.

The set of Nash equilibria will generally not be a singleton. For the special case m=1, Bergstrom, Blume and Varian(1986) have shown that normality of all goods is sufficient for uniqueness. For m>1 however, uniqueness does not obtain in general as Numerical Example 1 illustrates.

Multiplicity of equilibria raises a "coordination" problem in the sense that even perfect information about others' preferences and strategy sets is insufficient to guarantee that an equilibrium will be attained. Unfortunately, sufficient conditions for uniqueness of equilibrium in general non-cooperative games[2] do not

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[1] See Friedman(1986, p.39)

[2] See Friedman(1986, p.42)

Numerical Example 1

Let $n=2$, $m=3$, $w^i=(4,4)$, $U^1(.)=(z_1+3z_3)z_2+12z_1-z_1^2$ and $U^2(.)=(z_3+3z_1)z_2+12z_3-z_3^2$, where superscripts are exponents. Prices are constant and normalized to unity for all goods.

If both individuals determine simultaneously and independently their contributions to each public good, Nash equilibria for the resulting game would be:

c^1 =	2+c−a	2−c+a−b	b
c^2 =	a	1+c−a+b	3−c−b
z =	2+c	3	3−c

where $a,b,c \in [0,1]$; $a\neq 0$ only if $c=0$ and $b=0$; $b\neq 0$ only if $c=1$ and $a=0$.

There are an infinite number of Nash equilibria, each yielding $z\in\{\theta z^{1*}+(1-\theta)z^{2*};\theta\in[0,1]\}$, i.e. the set of Nash equilibrium bundles is the line segment joining $z^{1*}=(3,3,2)$ and $z^{2*}=(2,3,3,)$.

have obvious intuitive interpretation in this context.

1.2 Brief Overview of the literature

Early examples of this model can be found in Olson and Zeckhauser(1966), Buchanan(1967) and Breit(1968). A fundamental result is that the Nash equilibrium level of contributions to public goods will normally be inefficiently low. Cornes and Sandler(1986, p.80) have coined the expression "systemic easy riding" to describe this tendency for a public good to be underprovided in equilibrium. Subsequent discussions have dealt with some comparative static issues:

Chamberlin(1974,1976), McGuire(1974) and others analyze the effect of variations in the group's size. The central result is that if all goods are normal, total contributions would increase with group size but converge to a finite sub-optimal level.

Warr(1983), Bergstrom, Blume and Varian(1986), Bernheim(1986), Andreoni(1988) and others analyze the effects of a redistribution of income. The central result is a "neutrality" theorem that says essentially that within certain bounds, a redistribution of income among contributors to a public good would have no effect.

Weiss(1981,1986), Rose-Ackerman(1981), Schiff(1985), Steinberg(1987) and others discuss the effect of government provision. Dollar for dollar crowding out is predicted with the simplest specifications, but negative, partial, or super crowding out can also be obtained.

Hochman and Rogers(1977), Feldstein(1980), Young(1982), Strnad(1986), Roberts(1986,1987), Andreoni(1987), Boadway, Pestieau and Wildasin(1989) and others analyze the effect of subsidizing voluntary contributions through the tax system. Inquiries have ranged from characterization of efficient subsidy rates, to calculation of the welfare gains from financing public goods through tax expenditures, to discussion of the conditions under which subsidies would have no effect.

Sugden(1982) argues that the model yields predictions that are at variance with observed evidence about philanthropic contributions. Consequently, several papers have proposed modification of the model in various directions. For example, Hirshleifer(1983) and Andreoni(1989) consider impure public goods, Palfrey and Rosenthal(1984) and Hampton(1987) consider lumpy public goods, Cornes and Sandler(1984) consider the possibility that individuals make "non-Nash" conjectures, and Sugden(1984) adds ethical constraints to the individuals' preferences.

The literature on private provision of public goods far exceeds the references mentioned here, but a complete review of this literature is beyond the scope of this thesis. For a fuller discussion of the private provision model, see Cornes and Sandler(1985,1986).

With few exceptions, a common simplifying assumption in these papers is that there is only one public good. The advantage of this simplification is that boundary solutions may be, for the most part, neglected since if an individual contributes any amount to the public good, he must be at an interior solution, while individuals at boundary solutions contribute nothing. Standard consumer demand theory may then be used to describe contributors' responses to various changes in their environment.

When there are many public goods, an individual may contribute to one public good even if he is quantity constrained w'th regards to another public good. The theory of demand under rationing is then needed to describe individuals' contribution functions.

Aside from this technical difficulty, the presence of many public goods raises a substantive issue: not only is the <u>level</u> of contributions policy relevant, but so is its <u>composition</u>. Questions relating to the composition of contributions have not received much attention in the literature however. Fisher(1977) is, to my knowledge, the only published paper on this subject. In fact, the possibility that the Nash equilibrium composition of contributions would also generally be inefficient has so far been overlooked in the literature. Numerical example 2 illustrates.

Numerical Example 2

There are 2 individuals, each with an income of 66 2/3, and preferences $U^1 = x_1 z_1^2 z_2^2 z_3$ and $U^2 = x_2 z_1^2 z_2 z_3^2$, where superscripts are exponents. Prices are constant and normalized to unity for all goods.

A Nash equilibrium is: private goods demands of x_1 = 16 2/3 and x_2 = 16 2/3, and contributions vectors to each public goods of c^1 = (16 2/3, 33 1/3, 0) and c^2 = (16 2/3, 0,33 1/3) by individuals 1 and 2 respectively. The resulting public goods provision, z = (33 1/3,33 1/3,33 1/3), is inefficient not only because contributions are too low, but also because they are not well spent. For example, both individuals would strictly prefer $\hat{z}$ = (40,30,30) which is feasible without reducing either's private good consumption.

1.3 Public Goods Provision Institutions

Following Brennan and Buchanan(1978), we can identify three elements that determine the efficiency of any public expenditure institution: the *level*, the *composition* and the *disposition* of the budgetary outlay. Level refers to the relative withdrawal of resources from the private sector, composition refers to the allocation of the budget among several public commodities, and disposition refers to the proportion of revenues actually devoted to public goods provision (as opposed to provision of perquisites for politicians and bureaucrats or to fundraising expenses by charities).

Starting from a private provision setting, two distinct sources of efficiency gains may then be available: (i) Increase aggregate contribution level; (ii) Improve the public goods mix. To realize these gains, various institutions may be set up that constrain in some way individuals' choices.

1.3.1 *Taxation*

Taxation could increase the level of aggregate public goods provision: if individuals are unwilling to voluntarily contribute sufficiently to public goods, resources may simply be taxed away from them and used by a government to buy public goods. If the rationale for taxation is only to force individuals to contribute more to public goods than they would have done voluntarily, it does not follow that the taxing authority must also be given spending authority. *The tax could simply be a lower bound on an individual's contributions and he could be left free to buy the public goods bundle of his choice as long as he spent at least the required amount.*

Figure 3 illustrates i's choice set when he is subject to a public goods tax. Everyone else's contributions provide bundle F of public goods. He is required by law to contribute at least t^i to public goods provision. If he must pay this tax to the government, the composition of public outlay is determined by the government which chooses, say bundle D (assuming the government uses all its revenue to buy public goods). A lower-bound on his contributions (Tax-earmarking) would allow him to choose any bundle on line LM and, if the tax constraint was not binding, contributions over and above the tax would enable him to obtain any bundle in area GHLM.

1.3.2 *General Fund*

On the other hand, individuals may want to at least spend efficiently whatever money is contributed voluntarily even if the level of aggregate contributions remains too low. Institutions like the United Way, that have no taxing power but collect individual contributions and redirect them to various charities which provide public goods, may be able to perform this task.

Suppose individuals are not allowed to direct their voluntary contributions to their favourite public goods but may only contribute to a general fund. If the general fund's manager allocates money between z_1 and z_2 so that $z_2=(a_2/a_1)z_1$, along ray 0N in Figure 4, i's choice set would be reduced to line FN. In this situation, the individual could still choose his contribution level, but not its composition.

1.3.3 *Government provision*

Andreoni(1988, p.70) shows that when there is only one public good, "*the only way that a government can have any (significant) impact* [on the outcome], *is to completely crowd out private provision. Joint provision is a veil.*"

When there are many public goods, a government may still have a significant impact without completely crowding out private provision. The effect on the individual's choice set depends on whether he can take government's actions parametrically or not.

Suppose the government is itself constitutionally constrained to a fixed allocation rule, i.e. it must allocate a fixed proportion of its tax revenue to each public good regardless of private contributions. In this case, the individual would pay a tax to the government which would use it to buy, say, bundle D in Figure 5, but the individual would remain free to earmark any extra contributions. His choice set would then be area DEK instead of FGH. As drawn, he would choose B whereas he would have chosen B" in the absence of a government.

On the other hand, the government may be actively pursuing its own allocation objectives and may be able to "have the last word", i.e. play after everyone else and offset any individual earmarking. In Figure 5, the government wants z_1 and z_2 to be provided along ray ON. If it spends i's taxes in these proportions (bundle D), i would again choose B. By selecting D' instead, the government can ensure that i will choose B' on ray ON. If the individual anticipates this, he will behave as if his choice set was only line segment DN. In effect, the individual faces both an inequality constraint on his contribution level and an equality constraint on the composition of his contributions. As drawn, he would choose bundle D where he makes no voluntary contributions.

1.3.4 *Subsidies to private provision*

Even when the government is granted both taxing and spending authority, this power may be limited constitutionally. One such possible limitation is to require the government to finance public goods provision through tax expenditures. Several cases may be distinguished.

a. Tax expenditures only

Figure 6[3] illustrates an individual's choice set when public goods are financed entirely by tax expenditures and the government dissipates any surplus it collects in perquisites from office. If this individual makes no voluntary contributions, he gets bundle F since none of the taxes he pays would be used to buy

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[3] Projecting this figure on the z_1z_2 axis like Figure 2 may be confusing since the kink in the choice set would not appear.

public goods. Assuming the subsidy cannot exceed his tax bill, if he contributes enough that the subsidy he receives offsets his taxes completely, he gets a bundle in area GHJI. In this case, the marginal cost of a unit of a public good is still p_{z_n} because the marginal unit is not subsidized. If he chooses a bundle in area FIJ, the marginal cost is only $(1-r^i)p_{z_n}$ where r^i is the subsidy rate. Hinging on point F, a higher subsidy rate reduces the slope of plane FIJ, pulling line IJ up toward points M and L. For a subsidy rate of 100%, the choice set is again area GHLM (equivalent to the case of tax-earmarking discussed above).

b. Joint private and government provision

When the government also provides public goods, the individual must take into account the fact that the higher his subsidized contributions, the less net revenue remains available to the government to provide public goods. If there is only one public good, such a subsidy scheme would have no effect at all on the equilibrium [see Andreoni(1988)].

When there are many public goods, subsidies to private provision do have an effect. Suppose the government would spend any tax revenue it collects from i along path FD in Figure 7 but that private contributions are subsidized at 50%. A contribution of $40 would reduce the government's public expenditures by $20 to D', but the $40 in contributions would place i somewhere on line segment IJ. In particular, point I is now available if the individual spends the whole $40 on z_2. Compared to the no-subsidies case, where i's choice set was DEK, subsidies have widened his choice set to DQH. As the subsidy rate increases, lines KD and ED rotate around D toward M and L respectively, until they coincide with LM at a subsidy rate of 100%. Subsidies to private provision financed out of tax revenue relax the constraint on the mix of contributions by allowing the individual some earmarking opportunities at the cost of higher contributions.

1.3.5 *Taxonomy*

These institutions may be classified according to whether the *level* or the *composition* of contributions is constrained. Table I summarizes this taxonomy.

Table I

		Constraint on level of contributions	
		Yes	No
Constraint on mix of contributions	Yes	Government provision	United Fund
	No	Tax Earmarking	Voluntary Subscription

In this thesis, I examine the properties of semi-centralized institutions for public goods provision like United Funds and Tax Earmarking. I also examine the characteristics of efficient subsidies to private provision of public goods.

In chapter 2, a United Fund is added to the basic model, as an autonomous player that collects charitable contributions and redistributes them to various charities. Necessary and sufficient conditions for contributing to the United Fund to be a Subgame Perfect Equilibrium strategy are found. It is also shown that contributions to a United Fund would likely be smaller than direct contributions to charities would have been, raising a mix-level dilemma for the Funds' administrators.

In chapter 3, a tax-earmarking scheme, where individuals must pay some tax but may earmark it to the provision of any public good, is analyzed. It is shown that when there are only two public goods, the tax-earmarking outcome is particularly attractive since it is always unique, constrained pareto-efficient, and in the constrained core.

In chapter 4, subsidy schemes for private provision of public goods are analyzed. It is shown that Lindahl equilibria are the only efficient allocations that may be supported as Nash equilibria where everyone contributes positively and that a uniform subsidy rate would generally be inefficient.

CHAPTER 2 VOLUNTARY CONTRIBUTIONS TO UNITED CHARITIES

Since non-cooperation would generally result in relatively too little of some public services and relatively too much of some others, we might expect cooperation to emerge over the years, embodied in institutions charged with coordinating dispersed philanthropic efforts. In particular, cooperation could take the form of individuals relinquishing to a central body like the United Way, the task of allocating more efficiently their contributions.

This chapter examines United Funds' performance as a response to the problem of inefficient composition. In sections 1-3, a model of philanthropic contributions as a non-cooperative game in which a United Fund is an additional participant who plays after everyone else is set up. In section 4, it is shown that "everyone contributing only to the United Fund" can be a Subgame Perfect equilibrium and that this outcome is robust to a number of defections, if the United Fund is able to offset direct contributions to charities. In section 5, it is shown that contributions to the United Fund would likely be lower than direct contributions to charities would have been. In section 6, "optimal grants policies" for the United Fund are discussed since welfare maximization may require trading-off an inefficient mix of services for higher total contributions.

2.1 The institutional setting

Maintaining all the assumptions of the standard subscription model, charities are assumed to be the only producers of public goods. Without loss of generality, we can assume that each public good is produced by a different charity and that each charity produces only one public good.

A United Fund is added as an autonomous participant (player 0). The United Fund does not produce any goods itself but redistributes all the funds it collects to charities. For simplicity, assume all charities are eligible for United Fund grants.

To account for this new player, one column (i's contribution to the United Fund, denoted c_0^i) and one row (the United Fund's grant to charity h, denoted c_h^0) are added to contribution matrix $c \in \mathbb{R}_+^{(m+1)\times(n+1)}$.

Individuals may still buy directly a quantity $z_h^i = c_h^i/p_{zh}$ of a public good by contributing c_h^i to charity h. $z_h^{-i} = \sum_{j\neq i} c_h^j/p_{zh}$ is the quantity bought directly by everyone except i. $z_h^{-0} = z_h^i + z_h^{-i}$ is the quantity of public good h bought directly by individuals.

Individuals may also buy public goods indirectly by contributing to the United Fund. In this case, the mix of public goods their contribution ultimately purchases depends on the Fund's grants policy. Let $z_h^0 = c_h^0/p_{zh}$ be the quantity of public good h bought by the Fund. Then $z_h = z_h^0 + z_h^{-0}$ is the quantity of public good h ultimately consumed.

$G^i = \sum_{h=0}^{m} c_h^i$ is i's total (direct and indirect) contribution to charity, G^{-i} is total direct and indirect contributions by everyone except i, and $G = \sum_{i=1}^{n}\sum_{h=0}^{m} c_h^i$ is total direct and indirect contributions by everyone.

A.4 (United Fund's preferences): Let V(z) be the United Fund's objective function and assume it is continuous and quasi-concave. For simplicity, assume V(z) is continuously differentiable.

A.4 just means that the United Fund's preferences are defined over aggregate public goods bundles being provided, and that, if decision-making within the Fund is made by committee, any intransitivities have been resolved to yield a nice preference ordering. For our purposes, it does not matter how this ordering is arrived at.

The allocation of resources is now determined in two steps:

Step 1: Individuals choose simultaneously a vector of private goods and contributions to charities and/or to the United Fund.

Step 2: Given individual contributions, the United Fund distributes the funds it collects to charities.

A.5 (United Fund's behaviour): The United Fund seeks to maximize its objective function by choosing a vector of grants to charities, taking the prices of all goods and the individuals' contributions as given.

A.3a (Individuals' Behaviour, supplementary): Each individual anticipates the United Fund's reaction to his contributions.

This setting induces a game of complete information $\Gamma(N+1,w,p,U,V)$ with the individuals and the United Fund as players but not the charities themselves, where the United Fund gets to play after observing everyone else's choices.

2.2 The United Fund's grants function

A strategy for the United Fund is a grants function, $z^o(\sum_{i=1}^{n} c_o^i, z^{-o}) \in S^o$, where $S^o = \{ z^o(.): \mathbb{R}_+^{m+1} \rightarrow \mathbb{R}_+^m \mid \sum_{j=1}^{m} c_j^o = \sum_{i=1}^{n} c_o^i$ and $c_h^o \geq 0\ \forall h \in M, \forall (z^{-o}, c_o) \}$ is the United Fund's strategy space. S^o is not a parameter, but depends on how much individuals' contribute to the United Fund and directly to charities, since it cannot grant more than it receives, and it cannot make negative grants. $c_j^o = p_{z_j} z_j^o$ is the amount the United Fund would grant to charity j if individuals contribute a total of G dollars to charity, of which bundle z^{-o} has been bought directly by earmarked contributions.

Let $z^{o*}(\sum_{i=1}^{n} c_o^i, z^{-o}) \in \text{argmax}\{ V(z^{-o}+z^o) \mid z^o(.) \in S^o \}$ be the United Fund's optimal grants function[4]. This grants function may be described in more detail, using differential calculus. Since $\sum_{j=1}^{m} c_j^o = p_z z^o = p_z z - p_z z^{-o}$, and $\sum_{i=1}^{n} c_o^i = G - p_z z^{-o}$, its budget constraint may be written $G = p_z z$. We may also write the nonnegativity constraint as $z - z^{-o} \geq 0$. The Lagrangean function is then:

$L^o = V(z) + \lambda^o[G - p_z . z] + \mu^o(z - z^{-o})$ where λ^o and μ^o are 1+m nonnegative Lagrange multipliers.

The Kuhn-Tucker necessary conditions are:

$$\frac{\partial L^o}{\partial z_j^o} = \frac{\partial V}{\partial z_j} - \lambda^o p_{z_j} + \mu_j^o = 0 \qquad j=1,\dots,m$$

$$\frac{\partial L^o}{\partial \lambda^o} = G - p_z z = 0$$

$$\frac{\partial L^o}{\partial \mu_j^o} = z_j - z_j^{-o} \geq 0 \qquad j=1,\dots,m$$

The complementary slackness conditions are:

$$\mu_j^o(z_j - z_j^{-o}) = 0 \qquad j=1,\dots,m$$

Given the quasi-concavity of V(z), the 2nd order sufficient conditions would be satisfied, so that the solution of the first order conditions is a maximum.

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[4] Technically, $z^{o*}(.)$ could be a correspondence. For simplicity, $z^o(.)$ will be taken to be a function in the remainder.

Assuming without loss of generality that the first $k \leq m$ quantity constraints are not binding, the solution of this system is a set of k conditional grants functions:

(2.2.0.1) $\quad z^{o*}_j(\sum_{i=1}^{n} c_o^i, z^{-o}) = \hat{g}^o_j(p_z, G, z^{-o}) - z_j^{-o} \qquad j=1,\ldots,k$

(2.2.0.2) $\quad z^{o*}_j(\sum_{i=1}^{n} c_o^i, z^{-o}) = 0 \qquad j=k+1,\ldots,m$

In the special case where V(z) is homothetic, the United Fund wishes each charity to receive a fixed share of total contributions, i.e.

$$\hat{g}^o_j = (\frac{a_j}{p_{z_j}})G \qquad j=1,\ldots,m$$

where $\sum_{j=1}^{m} a_j = 1$. Substituting for $\hat{g}^o_j$ in (2.2.0.1) and using the fact that $p_{z_j} z_j^{-o} = \sum_{i=1}^{n} c_j^i$ we obtain:

(2.2.0.3) $\quad c^{o*}_j = a_j G - \sum_{i=1}^{n} c_j^i \qquad j=1,\ldots,m$

Proposition 1 shows that in general, if no quantity constraints are binding on the United Fund, then the United Fund's optimal policy would be to offset any redirection of individuals' contributions to some charity by reducing its grants to that charity by the same amount.

Proposition 1: $\forall (G, z^{-o}) \in \mathbb{R}_+^{m+1}$ such that $z^{-o} \leq z^{o*}(G,0)$,
$z^{o*}(G - p_z z^{-o}, z^{-o}) = z^{o*}(G,0) - z^{-o}$

Proof:

Let $z_h^o = z_h^{o*}(G,0) - z_h^{-o}$ $\forall h$. By assumption, $z_h^o \geq 0$ and $p_z z^o = G - p_z z^{-o}$, so z^o is a feasible solution to the United Fund's problem.

By way of contradiction, suppose z^o is not optimal, i.e. $\exists \tilde{z}^o \geq 0$ such that $V(z^{-o}+\tilde{z}^o) > V(z^{-o}+z^o)$ and $p_z \tilde{z}^o = G - p_z z^{-o}$. But $z^{-o}+z^o = z^{o*}(G,0)$ by assumption, hence $V(z^{-o}+\tilde{z}^o) > V(z^{o*}(G,0))$. This contradicts the definition of $z^{o*}(.)$. QED

2.3 The Individuals' contributions

Individual i's problem is now to choose x^i, z^i and c_o^i to maximize his utility subject to his budget and nonnegativity constraints, given others' contributions and given the United Fund's grants function. Let $(x^{i*},z^{i*},c_o^{i*}) \in \operatorname{argmax}\{ U^i(x^i, z^i+z^{-i}+z^o(c_o^i+\sum_{j\neq i} c_o^j,z^i+z^{-i})) : (x^i,z^i) \in B(w^i) \text{ and } c_h^i \geq 0\ \forall h=0,..,m \}$ be individual i's utility maximizing private and public goods purchases, and contributions to the United Fund.

Using differential calculus, the individual's best response can again be described in more detail. The Lagrangean function for the individual's maximization problem is now:

$$L^i = U^i(x^i,z) + \lambda^i[w^i+G^{-i}-p_x x^i-p_z z] + \mu^i(z-z^{-i}-z^o(.))$$

the terms $\partial z^o(.)/\partial z_k^i$ now enter the individual's calculus because he anticipates that the United Fund's grants to charities will be influenced by his contributions. (1.1.0.2, 1.1.0.4 and 1.1.0.5) are replaced by:

$$\text{(2.3.0.1)} \quad \frac{\partial L^i}{\partial z_k^i} = \frac{\partial U^i}{\partial z_k} + \sum_{j=1}^{m} \frac{\partial U^i}{\partial z_j}\frac{\partial z_j^o}{\partial z_k^i} - \lambda^i p_{z_k} + \mu_k^i p_{z_k} = 0 \qquad k=1,\dots,m$$

$$\text{(2.3.0.2)} \quad \frac{\partial L^i}{\partial \mu_h^i} = z_h-z_h^{-i}-z_h^o \geq 0 \qquad h=1,\dots,m$$

$$\text{(2.3.0.3)} \quad \mu_h^i(z_h-z_h^{-i}-z_h^o) = 0 \qquad h=1,\dots,m.$$

The solutions of this system are now:

$$x^{i*}_j(z^{-i},\sum_{j\neq i} c_o^j,z^o(.)) = \tilde{f}_j^i(p_x,p_z,w^i+G^{-i},z^{-i}+z^o(.)) \qquad j=1,..,\ell$$

and either

$$z^{i*}_j(z^{-i},\sum_{j\neq i} c_o^j,z^o(.)) = \tilde{g}_j^i(p_x,p_z,w^i+G^{-i},z^{-i}+z^o(.)) - z_j^{-i} - z_j^o(.) \qquad j=1,..,h$$

$$z^{i*}_j(z^{-i},\sum_{j\neq i} c_o^j,z^o(.)) = 0 \qquad j=h+1,\dots,m$$

$$c^{i*}_o(z^{-i},\sum_{j\neq i} c_o^j,z^o(.)) = 0$$

if the individual earmarks his contributions, or

$$z^{i*}_j(z^{-i},\sum_{j\neq i} c_o^j,z^o(.)) = 0 \qquad j=1,\dots,m$$

$$c^{i*}_{o}(\bar{z}^{-i}, \sum_{j \in J} c^{i}_{o}, \bar{z}^{o}(\cdot))$$

if he contributes to the United Fund[5]

Clearly, individuals could never do better by contributing to a general fund that may not spend their money as they would have themselves than by contributing directly to their favourite charities. "Contributing nothing to the United Fund" is always a (weak) best response to any level and mix of contributions by others. Since individuals are not constrained to contribute to the Fund, we need to explain why individuals would ever give to the Fund anyway.

Fisher(1977) notes that when some merged fund drives allow donors to specify how their gifts are to be allocated, large number of donors do not avail themselves of this opportunity. He further remarks that one reason for the failure to earmark may be that a dollar of direct contribution to a charity may result in a dollar less from the combined fund so that earmarking does not affect the ultimate composition of services.

Proposition 1 has shown however that the United Fund's optimal policy would indeed be to offset earmarked contributions. In this case, a reallocation by any i of his contributions to charities, which leaves x^i unchanged, would be meaningless since an extra dollar to charity h instead of j would simply induce the United Fund to grant one dollar less to charity h and one more to charity j. For example, suppose that $\bar{z}^{-i}=0$, and V(z) is homothetic, the United Fund's optimal response would then be to choose $c^{o*}_{j} = a_j(\bar{G}^{-i}+G^{i}) - c^{i}_{j}$ $j=1,\ldots,m$. If $c^{i*}_{j} \leq a_j(\bar{G}^{-i}+G^{i})$ $(\forall j \in M)$, no quantity constraints are binding on the United Fund. Then $z_j=(c^{o}_{j}+c^{i}_{j})/p_{Z_j} = [a_j(\bar{G}^{-i}+G^{i})-c^{i}_{j}+c^{i}_{j}]/p_{Z_j} = a_j(\bar{G}^{-i}+G^{i})/p_{Z_j}$, i.e. only i's aggregate contribution level

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[5] For simplicity, the case where the individual both earmarks and contributes to the United Fund is neglected

matters. Hence, i cannot be better off by earmarking his contributions than by contributing to the United Fund.

Since they have complete information about the United Fund's preferences, individuals would anticipate this when deciding on their contributions, and know that they have nothing to gain by earmarking them. They would then be indifferent between earmarking or not, so that the strategy of giving only to the United Fund is no longer strictly dominated.

The ability by the United Fund to offset earmarked contributions has the effect of transforming the m charities into one composite public good and reducing the individual's problem to simply choosing the size of his gift to the United Fund. Whether he earmarks or not, individual i will contribute until the weighted sum of the marginal utilities from public goods is equal to the marginal utility from private goods, where the weights are determined by the United Fund's grants policy (if he contributes at all).

Proposition 2: If $G^i>0$ and $V(z)$ is homothetic, then i will choose G^i such that

$$\sum_{j=1}^{m} \frac{a_j}{p_{z_j}}\left(\frac{\partial U^i}{\partial z_j}\right) = \frac{\partial U^i/\partial x_h}{p_{x_h}}$$

Proof:

Differentiating (2.2.0.3) with respect to z^i_k, $\frac{\partial z^o_j}{\partial z^i_k} = a_j p_{z_k}/p_{z_j}$ and $\frac{\partial z^o_j}{\partial z^i_k} = a_j - 1$ so substituting these into (2.3.0.1):

$$\sum_{j=1}^{m} \frac{a_j}{p_{z_j}}\left(\frac{\partial U^i}{\partial z_j}\right) + \mu^i_k = \lambda^i \qquad k=1,\ldots,m$$

Hence $\mu^i_1=\ldots=\mu^i_m=\mu^i$, and dividing by (1.1.0.1) to eliminate λ^i:

$$\sum_{j=1}^{m} \frac{a_j}{p_{z_j}}\left(\frac{\partial U^i}{\partial z_j}\right) + \mu^i = \frac{\partial U^i/\partial x_h}{p_{x_h}}$$

If the quantity constraint is binding, i will not contribute anything. If it is not binding, $\mu^i=0$ and this reduces to:

$$\sum_{j=1}^{m} \frac{a_j}{p_{z_j}}\left(\frac{\partial U^i}{\partial z_j}\right) = \frac{\partial U^i/\partial x_h}{p_{x_h}}$$ QED

2.4 Non-cooperative equilibria

A *Nash equilibrium* for this game is a vector of private consumption and contributions to charities and to the United Fund by individuals and a grants policy by the United Fund $[(\hat{x},\hat{z}^{-o},\hat{c}_o), \hat{z}^o(.)]$ such that:

$\hat{z}^o(.) = z^{o*}(\sum_{i=1}^{n} \hat{c}_o^i,\hat{z}^{-o})$, and

$(\forall i \in N)\ (\hat{x}^i,\hat{z}^i,\hat{c}_o^i) = (x^{i*},z^{i*},c_o^{i*})$

A problem with this concept is that, given the sequential nature of this game, some unreasonable allocations could be supported as equilibria. For example, the United Fund could threaten to give all its money to a little known and relatively useless charity unless the Fund receives a given level of contributions. To the extent that contributors took this threat seriously, they may be induced to increase their contributions to the Fund, in which case the threat would not have to be carried out. The fact that this threat may not be credible can be taken into account by refining the equilibrium concept to rule out such equilibria. A more appropriate equilibrium concept would be that of Subgame Perfect Equilibrium.

A *Subgame Perfect Equilibrium* for this game is a vector of private consumption and contributions to charities and to the United Fund by individuals and a grants policy by the United Fund, such that the choices of all individuals are optimal, and where the United Fund is required to allocate the money it receives optimally, whether individuals' contributions were equilibrium choices or not. Formally, a Subgame Perfect Equilibrium is a $[(\hat{x},\hat{z}^{-o},\hat{c}_o),\ \hat{z}^o()]$ such that:

$\forall(x,z^{-o},c_o),\ \hat{z}^o(.) = z^{o*}(\sum_{i=1}^{n} c_o^i, z^{-o})$, and

$(\forall i \in N)\ (\hat{x}^i,\hat{z}^i,\hat{c}_o^i) = (x^{i*},z^{i*},c_o^{i*})$

To determine when "everyone failing to earmark" (i.e. all charitable contributions going to the United Fund) could be a Subgame Perfect equilibrium, it remains to formalize the condition that quantity constraints not be binding on the United Fund.

Consider an alternative institutional setting in which individuals are constrained to either contribute to a United Fund or not at all. The total amount contributed to charity, G, is now equal to the United Fund's receipts. Let $\Gamma(N+1,w,p,U,V)$ be the noncooperative game induced.

The United Fund's problem is still to choose a grants function to maximize its objective function, but only the constraint that it cannot grant more than it receives is binding. Individual i's problem is now to choose x^i and c_o^i to maximize his utility subject to his budget and nonnegativity constraints, given others' contributions to the United Fund and given the United Fund's grants function. Let $(\bar{x}^{i*},\bar{c}_o^{i*}) \in \text{argmax}\{\, U^i(x^i, z^o(c_o^i + \sum_{j \neq i} c_o^j, 0)) \,;\, (x^i,c_o^i) \in B(w^i) \}$ be individual i's utility maximizing private consumption and contributions to the United Fund, where $B(w^i) = \{(x^i,c_o^i) \,;\, x^i + c^i{}_o \leq w^i \text{ and } c_o^i \geq 0 \}$. The "bar" indicates that i's choice is subject to this additional "no-earmarking" constraint.

Now remove this "no-earmarking" constraint and return to the original setting. Could any individual improve his payoff by unilaterally changing his plans and earmarking some or all of his contributions ? Only if the United Fund is not able to offset what he does, i.e. only if he would choose to contribute more to any charity than the United Fund would have granted it if the Fund had continued to collect all contributions.

Proposition 3: Let $[(\bar{x},\bar{c}_o), \hat{z}^o(.)]$ be a Subgame Perfect Equilibrium for $\Gamma(.)$. Then $[(\bar{x},0,\bar{c}_o), \hat{z}^o(.)]$ is a Subgame Perfect Equilibrium for $\Gamma(.)$ iff $(\forall i\in N)$ $(\forall h\in M)$ $z_h^{i*}(0,\sum_{j\neq i}\bar{c}_o^j,z^o(.)) \leq z_h^{o*}(\sum_{j\neq i}\bar{c}_o^j + c_o^{i*} + p_m z^{i*}, 0)$

Proof:

Necessity: By way of contradiction, suppose $[(\bar{x},0,\bar{c}_o), \hat{z}^o(.)]$ is not a Subgame Perfect Equilibrium for $\Gamma(.)$. Then it must be that $\exists i\in N$ such that $U^i(x^i, z^i+z^{o*}(\sum_{j\neq i}\bar{c}_o^j+c_o^i,z^i)) > U^i(\bar{x}^i, \hat{z}^o(\sum_{i=1}^{n}\bar{c}_o^i,0))$.

Now in any Subgame Perfect Equilibrium of $\Gamma(.)$, $\hat{z}^o(G-p_m z^{-o},z^{-o}) = z^{o*}(G-p_m z^{-o},z^{-o})$ by definition. Further, $z^{i*}(0,\sum_{j\neq i}\bar{c}_o^j,z^o(.)) \leq z^{o*}(\sum_{j\neq i}\bar{c}_o^j + c_o^{i*} + p_m z^{i*}, 0)$ by assumption. Hence, by proposition 1, $z^{o*}(\sum_{j\neq i}\bar{c}_o^j + c_o^{i*}, z^{i*}) = z^{o*}(\sum_{j\neq i}\bar{c}_o^j + c_o^{i*} + p_m z^{i*}, 0) - z^{i*}$. So $U^i(x^i, z^{o*}(\sum_{j\neq i}\bar{c}_o^j + c_o^{i*} + p_m z^{i*}, 0)) > U^i(\bar{x}^i, \hat{z}^o(\sum_{i=1}^{n}\bar{c}_o^i,0))$, contradicting that $[(\bar{x},\bar{c}_o), \hat{z}^o(.)]$ is a Subgame Perfect Equilibrium for $\Gamma(.)$.

Sufficiency: Suppose $z_h^{i*}(0,\sum_{j\neq i}\bar{c}_o^j,z^o(.)) > z_h^{o*}(\sum_{j\neq i}\bar{c}_o^j + c_o^{i*} + p_m z^{i*}, 0)$ for some i. Then the United Fund would not be able to offset completely i's earmarked contributions. But by definition of $z^{i*}(.)$, it must be that $U^i(x^i, z^{i*}+z^o(\sum_{j\neq i}\bar{c}_o^j+c_o^{i*},z^{i*})) > U^i(\bar{x}^i, \hat{z}^o(\sum_{i=1}^{n}\bar{c}_o^i,0))$. This contradicts that $[(\bar{x},0,\bar{c}_o), \hat{z}^o(.)]$ is a Subgame Perfect Equilibrium for $\Gamma(.)$. QED

If some individuals care about their own contributions (e.g. pro-life members may be specially concerned that none of their contributions goes to Planned

Parenthood), they may always prefer to earmark, even when such earmarking would be completely offset by the United Fund. Depending on intensity of preferences and income distribution, voluntary centralization could however be robust enough to accommodate these defections without unravelling. As long as no group wants to provide by itself more of any public good than the United Fund would provide if it collected all contributions, direct contributions by some individuals leave it a Subgame Perfect equilibrium strategy for everyone else to continue giving only to the United Fund or not at all. This would be specially likely if the economy is composed of many small donors.

Proposition 4: Let $[(\bar{x},0,\bar{c}_o), \hat{z}^O(.)]$ be a Subgame Perfect Equilibrium. Then $[(\bar{x},\hat{z}^J,\bar{c}_o^{-J}), \hat{z}^O(.)]$ is also a Subgame Perfect Equilibrium only if for some $J \subseteq N$ contributing $\hat{z}^J \neq 0$ and $\hat{c}_o^J = 0$, $\exists i \notin J$ such that for some $h \in M$:

$$z_h^{i*}(\hat{z}^{-i}, \sum_{k \in J} \bar{c}_0^k, z^0(.)) + \sum_{j \in J} z_h^{j*}(\hat{z}^{-j}, \sum_{k \in J} \bar{c}_0^k, z^0(.)) > z_h^{0*}(\sum_{k \notin J\, k \neq i} \bar{c}_0^k + p_z z^{j*} + p_z z^{i*}, 0)$$

Proof:

<u>Necessity</u>: By way of contradiction, suppose $[(\bar{x}, \hat{z}^J,\bar{c}_o^{-J}), \hat{z}^O(.)]$ is not a Subgame Perfect Equilibrium. Then it must be that $\exists i \notin J$ such that

$U^i(x^i, z^i+\hat{z}^J+z^{O*}(\sum_{i \notin J, k \neq i}\bar{c}_o^k+c_o^i,\hat{z}^J+z^i)) > U^i(\bar{x}^i, \hat{z}^J+\hat{z}^{O*}(\sum_{i \notin J}\bar{c}_o^k,\hat{z}^J))$. Now since

$$z_h^{i*}(\hat{z}^{-i}, \sum_{k \notin J} \bar{c}_0^k, z^0(.)) + \sum_{j \in J} z_h^{j*}(\hat{z}^{-j}, \sum_{k \notin J} \bar{c}_0^k, z^0(.)) \leq z_h^{0*}(\sum_{k \notin J\, k \neq i} \bar{c}_0^k + p_z z^{j*} + p_z z^{i*}, 0) \ \forall h \in M$$

by assumption, the United Fund is able to offset all of group J's earmarking as well as i's earmarked contributions. Hence, by proposition 1:

$$z^{0*}(\sum_{k \notin J\, k \neq i} \bar{c}_0^k+c_0^{i*}, \hat{z}^J+z^{i*}) = z^{0*}(\sum_{k \notin J\, k \neq i} \bar{c}_0^k +c_0^{i*} +p_z z^{J*} +p_z z^i, 0) - \hat{z}^J - z^{i*}, \quad \text{and}$$

$$z^{0*}(\sum_{k \notin J} \bar{c}_0^k, \hat{z}^J) = z^{0*}(\sum_{k \notin J} \bar{c}_0^k+p_z \hat{z}^J, 0) - \hat{z}^J$$

So $U^i(x^i, z^{0*}(\sum_{k \notin J\, k \neq i} \bar{c}_0^k +c_0^{i*} +p_z z^{i*}, 0)) >$

$$U^i(\bar{x}^i, z^{0*}(\sum_{k \in J} \bar{c}_0^k + p_t x^J, \theta)) - U^i(\bar{x}^i, z^0(\sum_{i=1}^{n} \bar{c}_0^i, \theta)),$$

contradicting that $[(\bar{x}, 0, \bar{c}_0), \bar{z}^0(.)]$ is a Subgame Perfect Equilibrium. QED

In Numerical Example 3, contributing to the United Fund is still a Subgame Perfect equilibrium strategy for everyone else even if a total of up to 14.1 is earmarked by others toward charities 2 or 3.

2.5 The level of contributions

If the United Fund's objective function is a Paretian Social Welfare Function, individuals can ensure that their philanthropic contributions will be efficiently spent by giving to the United Fund instead of directly to their favourite charities. Delegating spending authority to a central institution may not be welfare improving however, since total contributions to the United Fund may be lower than total direct contributions would have been. Since the United Fund operates in a second best environment, removing one source of inefficiency may reduce rather than increase every donor's welfare.

In example 3, the United Fund's policy is derived from a Paretian SWF and $\bar{z}$ is an efficient mix of services in the sense that no other way of spending $\bar{G}$=1091/11 pareto-dominates it. However, in the absence of a United Fund, contributions would have been higher (G=120). It can easily be verified that when preferences are as defined in the example, everyone is better off if all contributions are earmarked even though the public goods mix is inefficient.

Figure 4 (page 68) compares i's behaviour in three situations:

(A) individual i is the only contributor to the public goods and has exogenous income equal to $w^i + G^{-i}$. He then chooses point A where his utility is maximal.

Numerical Example 3

Let $n=6$, $m=3$, $U^i=x_i z_1^2 z_2^2 z_3$ $i=1,2,3$ $U^i=x_i z_1^2 z_2 z_3^2$ $i=4,5,6$ $w=[40,40,40,40,40,40]$ $V(z)=x_i x_j z_1^4 z_2^3 z_3^3$, where superscripts are exponents, and normalize all prices to 1.

Subgame Perfect equilibria where everyone earmarks are:

	x=		c=			
			-	0	0	0
x=	20	c=	0	20-a-b	a+b	0
	20		0	a	20-a	0
	20		0	b	20-b	0
	20		0	20-c-d	0	c+d
	20		0	c	0	20-c
	20		0	d	0	20-d
			z=	40	40	40

where $0 \le a,b,c,d \le 20$ and $a+b, c+d \le 20$. Hence $z = [40,40,40]$.

If the United Fund chooses: $c_1^0 = -.6\sum_i c_1^i + .4\sum_i (c_0^i+c_2^i+c_3^i)$, $c_2^0 = -.7\sum_i c_2^i + .3\sum_i (c_0^i+c_1^i+c_3^i)$, and $c_3^0 = -.7\sum_i c_3^i + .3\sum_i (c_0^i+c_1^i+c_2^i)$, another Subgame Perfect equilibrium would be:

			——	43 7/11-a	32 8/11-b	32 8/11-c
x=	21 9/11	ĉ=	18 2/11-a-b-c	a	b	c
	21 9/11		18 2/11	0	0	0
	21 9/11		18 2/11	0	0	0
	21 9/11		18 2/11	0	0	0
	21 9/11		18 2/11	0	0	0
	21 9/11		18 2/11	0	0	0

where $0 \le a \le 18\ 2/11$, $0 \le b,c \le 14.1$ and $a+b+c \le 18\ 2/11$. Hence $\hat{z} = [43\ 7/11, 32\ 8/11, 32\ 8/11]$.

If individuals 2 to 6 all contribute 18 2/11 to the United Fund, then individual 1 is indifferent between giving 18 2/11 to the United Fund or earmarking the same amount among the three charities. Even if individual 1 earmarked his contribution, it would remain an equilibrium strategy for the others to give only to the United Fund, provided he did not contribute more than 14.1 directly to charities 2 or 3.

(B) everyone else is earmarking, and their aggregate contributions is some G^{-i}. i would then be able to choose a point in area FHG and chooses bundle B since z_2 is oversupplied as far as i is concerned, but the level of z_2 provided by others is a binding quantity constraint.

(C) no one else is earmarking, others' contributions to the United Fund sum to the same G^{-i}, and the United Fund is able to offset any earmarked contributions by i. If V(.) is homothetic, the United Fund would allocate grants between z_1 and z_2 so that $z_2=(a_2/a_1)z_1$, along ray 0N. i would then be able to choose a point on line FN.

Fisher(1977) defines i's preferences in such a way that point A is always feasible and shows that $\partial G^i/\partial(a_1/a_2)$ may be non-zero at that point for some specific utility function, so i's contributions could increase or decrease if charities denied him the opportunity to earmark his contributions toward specific programs or services by merging their fundraising drives. When i is not the only contributor to the public goods however, point A may be outside his possibility set, and the relevant comparison would in general involve points far from the optimum, e.g. points B and C . Proposition 5 shows that if V(.) is homothetic and $U^i(.)$ is Cobb-Douglas, i will always contribute more in case B than in case C.

Proposition 5: If $U^i(.)$ is Cobb-Douglas and V(.) is homothetic, $\tilde{G}^{i*}>\bar{G}^{i*}$, where $\tilde{G}^{i*}$ and $\bar{G}^{i*}$ are i's aggregate contributions in situations (B) and (C) respectively.

Proof:

Proof:

Consider an individual with preferences $U^i=b_x\log(x^i)+b_1\log(z_1)+b_2\log(z_2)$ where $b_x+b_1+b_2=1$

<u>Situation A</u>:

If the quantity constraints are not binding, i would choose:

(2.5.0.1) $x^i=(b_x/p_x)(w^i+G^{-i})$

(2.5.0.2) $z_1=(b_1/p_1)(w^i+G^{-i})$

(2.5.0.3) $z_2=(b_2/p_2)(w^i+G^{-i})$

<u>Situation B</u>:

Now leaving G^{-i} constant, increase z_2^{-i} and decrease z_1^{-i} so that the quantity constraint on z_2 becomes strictly binding. i will now choose:

(2.5.0.4) $\tilde{x}^i=(b_x/p_x)(w^i+p_1.z_1^{-i})/(b_1+b_x)$

(2.5.0.5) $\tilde{z}_1=(b_1/p_1)(w^i+p_1.z_1^{-i})/(b_1+b_x)$

To compare x^i and $\tilde{x}^i$, substitute for $p_1.z_1^{-i}$ in (2.5.0.4) from the definition of G^{-i}:

$\tilde{x}^i=(b_x/p_x)(w^i+G^{-i}-p_2.z_2{}^{-i})/(b_1+b_x)$, but from (2.5.0.3):

$z_2{}^{-i}>(b_2/p_2)(w^i+G^{-i})$ since the quantity constraint is strictly binding, so:

$\tilde{x}^i<(b_x/p_x)(w^i+G^{-i}-b_2(w^i+G^{-i}))/(b_1+b_x)$

Rearranging and using the fact that $b_x+b_1+b_2=1$:

$\tilde{x}^i<(b_x/p_x)(w^i+G^{-i})=x^i$.

Since private consumption decreases, it must be that i's contributions to charities have increased since his income is unchanged. Hence $\tilde{G}^i>G^i$.

Situation C:

If the United Fund would grant to charities so that $(z_1/z_2)=(a_1/a_2)$ and i contributes to the Fund instead of earmarking, we can write his utility function as $U^i=b_x\log(x^i)+(b_1+b_2)\log(z_2)+a$ where $a=b_1\log(a_1/a_2)$ so that i would choose $\hat{x}^i=(b_x/p_x)(w^i+G^{-i})$ just as in A. Hence $\tilde{G}^i=G^i$.

It then follows that $\tilde{G}^i>\bar{G}^i$ so that i's contributions would be smaller to a United Fund than he would have contributed directly to charities. QED

In Figure 8, as the quantity constraint on z_1 or z_2 is varied, the locus of equilibria for individual i, corresponding to situation B is MM'. As the ratio constraint is varied, the locus of equilibria for individual i, corresponding to situation C is LL'. In the Cobb-Douglas case, locus LL' is a straight line of slope p_{z_2}/p_{z_1} so that any ratio of a_1/a_2 results in the same level of contributions as at A. In situation B however, i's contributions increase as the quantity constraint is tightened. It must be then, that no matter what ratio the United Fund chooses or how tight the quantity constraint, i's contributions to the United Fund would be less than his earmarked contributions would have been.

In general, convexity of preferences implies that MM' must lie everywhere above LL' (except at A where they coincide). It thus seems likely then that i's contributions to a United Fund would be smaller. Proposition 5 does not hold in general however as can be seen from Figure 9 and Figure 10.

In Figure 9, both MM' and LL' are convex to the origin, so that i's contributions would increase both as the ratio constraint and the quantity constraint are tightened. The ratio constraint is much tighter however and i's contributions are higher at C than at B. In Figure 10, both MM' and LL' are concave to the origin so that i's contributions would decrease both as the ratio and the quantity constraints are tightened. The quantity constraint is much tighter and i's contributions are higher at C than at B. So in general, whether i's contribution to the United Fund would be higher or lower is ambiguous and depends on the United Fund's target and i's actual preferences.

Proposition 5 compares the best response of a single individual and is not a comparison of the equilibria with and without a United Fund. It would be interesting to determine whether anything specific could be said about the aggregate equilibrium levels of contributions by all individuals, but this is a topic for further research. If, as seems likely however, the aggregate contributions to a United Fund were lower, the welfare effect of the United Fund's activity would be ambiguous even in the best circumstances where it seeks an efficient mix of services, the more efficient mix having a positive effect but a lower level of contributions having a negative effect.

2.6 Optimal grant policy

If aggregate contributions would decrease enough when the United Fund follows a policy of offsetting direct contributions so that $V(\hat{z})<V(z)$, then a welfare maximizing United Fund would prefer not to follow this policy. In fact, following a simple rule of thumb like a "fixed share of receipts" rule may be more efficient overall even though it would entail an inefficient mix of services.

There is some evidence, that United Funds have shown some concern for developing broadly defined efficient allocation procedures. Martin(1985, p.219) notes that "as the movement grew, spenders were merged with fundraisers to provide more community coordination and professional social workers were united with volunteer solicitors in an attempt to reconcile the community's social welfare needs with its free-will resources". Rose-Ackerman(1980) also mentions attempts to use "priority planning[6]" techniques that try to take community needs into account when allocating funds among agencies.

Rose-Ackerman(1980)'s evidence however, tends to suggest that in the main, the United Way does not allocate its funds more efficiently than noncooperative individuals would have, but allocates resources according to simple rules of thumb, like giving each member agency a fixed share of funds collected. Given the problem of aggregating preferences and the more basic difficulty of measuring marginal benefits from such services as the Salvation Army, the Boy Scouts or Planned Parenthood, it could be argued that any well motivated social planner would face the same difficulty and might be tempted to resort to simple rules of thumb too. On the other hand, the possibility that contributions would decrease when the United Fund follows a policy of offsetting direct contributions may explain why United Ways are lukewarm to "priority planning" policies.

The optimal grant policy by the United Fund would be a 2nd best policy akin to optimal tax rules, where the United Fund may have to trade-off an inefficient mix of services for higher contributions. The United Fund could implement this by committing itself to such a policy before individuals make their decisions. It would be interesting to derive what such a rule would look like, although doing so is made rather complicated by the fact that contributing to the United Fund need no longer be a Subgame Perfect Equilibrium strategy in this case, and is beyond the scope of this thesis.

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[6] Rose-ackerman(1980) reports that there are many local variations to this technique but that they share some common features, i.e. trying to assess community needs and set priorities in order to rank services from the most to the least needed.

Summary of results obtained from the model

1- Necessary and sufficient conditions for contributing only to a United Fund to be a Subgame Perfect Equilibrium strategy.

2- Contributing to the United Fund is a robust strategy.

3- If the United Fund's objective is a Paretian Social Welfare Function, contributing to the United Fund ensures an efficient mix of services.

4- The United Fund is not necessarily welfare improving since contributions could decrease.

5- If an individual has Cobb-Douglas preferences, his contributions to the United Fund would be smaller than direct contributions to charities would have been, regardless of United Fund's objective.

6- Explanation of United Funds' reticence to use "priority planning".

CHAPTER 3 TAX-EARMARKING[7]

Centralization of public expenditures through a general fund could be justified on efficiency grounds if the composition of the public goods bundle demanded by non-cooperative individuals was inefficient. It was suggested in chapter 2 that such centralization need not be legislated but that even non-cooperative individuals could be induced under some conditions to contribute to a general fund instead of earmarking their contributions. As was pointed out however, an inefficient level of voluntary contributions may often be a more severe problem.

In this chapter, the performance of institutions that require individuals to contribute some minimum level but do not constrain the composition of their contributions is examined. I show that in a non-negligible class of circumstances, inefficient composition is not a problem, so that a government could merely set the contribution levels through some tax, but leave the spending decisions in the hands of individuals.

3.1 The institutional setting

Maintaining all the assumptions of the basic model set up in chapter 1, assume that an inequality constraint is placed on the level of contributions to public goods.

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[7] Tax-earmarking has been analyzed in the context of voting models by Buchanan(1963) and Goetz(1968). Brennan and Buchanan(1978) also suggest tax-earmarking as a way to solve the disposition problem. In these papers, tax earmarking refers to tying a given tax instrument to the provision of a particular good, e.g. gasoline tax for road building, etc... In contrast, tax earmarking refers here to a situation where each taxpayer can assign the taxes they pay to a particular use.

Constitutional rule 3.1: **All individuals are assessed a nonnegative lump-sum tax t^i. They may either pay it to a general fund or earmark part or all of it among any of the m public goods.**

Let $t=(t^1,\ldots,t^n)'\in T$ be the tax structure, where $T = \{t\in \mathbb{R}_+^n : t^i \leq w^i \ \forall i\in N\}$ is the set of possible tax structures.

For simplicity units of measurement are normalized so that the price of a unit of any public good is unity. Assume also for simplicity that no one will contribute to the general fund. Notwithstanding the results of chapter 2, this is a reasonable assumption since earmarking remains a (weakly) dominant strategy for everyone. This allows us to neglect the general fund's role in the game, since it collects no taxes and hence has nothing to spend.

Assumption 1a(Preferences, Supplementary): Preferences are such that individuals' best response implies $G^i \leq t^i \ \forall i\in N$.

A.1a just means that the tax constraint is binding on everyone, i.e. individuals do not want to contribute more to public goods provision than they are constrained to by the tax structure. This is a reasonable assumption given the systemic tendency of voluntary contributions to be low. Andreoni(1988) shows for example that only a small fraction of individuals in a heterogeneous population would make positive voluntary contributions at the Nash equilibrium when there are no taxes. *A fortiori*, the number of individuals making voluntary contributions larger than the tax threshold would be even smaller and would fall to zero for a sufficiently high tax. Together with rule 3.1, A.1a implies that i would never pick a point above LM in Figure 11.

This setting induces a game of complete information $\Gamma(N,w,t,p,U,)$ among the taxpayers. Individual i's problem is now to choose x^i and c^i so that x^i and z maximize $U^i(x^i,z)$ given c^{-i} and subject to the constraints that $(x^i,c^i)\in B^i$ where $B^i=\{(x^i,c^i)\in R_+^{l+m}; p_m.x^i+G^i\leq w^i, c_h^i\geq 0\ \forall h\in M, \text{and } G^i\geq t^i\}$ is his budget set.

Let $A(G)=\{(x,z) ; (x^i,c^i)\in B^i\ \forall i\in N, \text{and } \sum_{h=1}^{m} z_h = G\}$ be the set of possible consumption bundles. A.1a implies that aggregate public expenditure is uniquely determined by the exogenous tax structure, i.e. $A(t) = \{(x,z); p.x^i = w^i-t^i\ \forall i\in N, \text{and } G=\Sigma t^i\}$. We are interested in how these expenditures would be allocated among the various public goods under a tax earmarking scheme.

A Nash equilibrium for this game is an allocation $(\hat{x},\hat{z})$ such that $\forall i\in N$ and $\forall (x^i,c^i)\in B^i$, $U^i(\hat{x}^i,\hat{z})\geq U^i(x^i,z^i+\hat{z}^{-i})$. Let $NE(t) = \{(x,z)\in A(t); (x,z) \text{ is a NE}\}$ be the set of Nash Equilibria for this game.

Let $\tilde{B}^i(c^i)=\{x^i;(x^i,c^i)\in B^i\}$ be i's private goods consumption set, conditional on his contributions c^i. Rule 3.1 and A.1a also imply that $\tilde{B}^i(c^i)$ remains the same no matter how he chooses to allocate his contributions.

3.2 Properties of the equilibria

In general, at any Nash equilibrium, although it is possible that some individuals could contribute positively to all goods, most individuals in a heterogeneous population would find themselves at boundary solutions with regards to their choice set and contribute nothing to some good(s). Proposition 6 shows that no individual would contribute positively to all goods in two Nash equilibria with $\bar{z}\neq\hat{z}$. Intuitively, since an individual who contributes positively to all goods is getting exactly the bundle of public goods he prefers, strict convexity of preferences implies that if a different bundle is also an NE, this individual must be at a boundary solution in this other NE and contribute nothing to some good(s).

Proposition 6: $[\bar{Z}\neq\hat{Z}$ and $(\bar{Z},\bar{X}),(\hat{Z},\hat{X}) \in NE(t)] \Rightarrow \nexists j\in N$ for which $\bar{Z}^j>0$ and $\hat{Z}^j>0$.

Proof[*]**:**

By way of contradiction, suppose $(\bar{Z},\bar{X}),(\hat{Z},\hat{X}) \in NE(t)$ where $\tilde{M} = \{h\in M : \bar{Z}_h\neq\hat{Z}_h\} \neq \emptyset$, and $\exists j$ for which $\bar{Z}^j>0$ and $\hat{Z}^j>0$.

Suppose that $U^j(\bar{X}^j,\bar{Z}) \geq U^j(\hat{X}^j,\hat{Z})$. Then by strict convexity, we have that for any $\lambda \in \,]0,1[$, $U^j(\lambda\bar{X}^j+(1-\lambda)\hat{X}^j,\ \lambda\bar{Z}+(1-\lambda)\hat{Z}) > U^j(\hat{X}^j,\hat{Z})$

In particular, choose $\lambda^* = \min\{\ \tfrac{1}{2},\ \min_{h\in\tilde{M}}\{\frac{\hat{z}^j_h}{|\bar{Z}_h - \hat{Z}_h|}\}\ \}$

Then $\hat{Z}^j > 0$ implies $\lambda^* \in \,]0,1/2]$

$$\text{Now, } \lambda\bar{Z} + (1-\lambda)\hat{Z} = \hat{Z} + \lambda(\bar{Z} - \hat{Z})$$
$$= \hat{Z}^{-j} + [\hat{Z}^j + \lambda(\bar{Z} - \hat{Z})]$$

Let $z^j(\lambda) = [\hat{Z}^j + \lambda(\bar{Z} - \hat{Z})]$, i.e. $z^j(\lambda)$ is the allocation of j's tax share that would enable him to obtain $\lambda\bar{Z}+(1-\lambda)\hat{Z}$ if everyone else continued to provide $\hat{Z}^{-j}$. Then for any h, $z^j_h(\lambda^*) = \hat{Z}^j_h + \lambda^*(\bar{Z}_h - \hat{Z}_h) > 0$ if $\bar{Z}_h \geq \hat{Z}_h$,

and if $\bar{Z}_h < \hat{Z}_h$, then $z^j_h(\lambda^*) = \hat{Z}^j_h - \lambda^*|\bar{Z}_h - \hat{Z}_h|$

$$\geq \hat{Z}^j_h - \left(\frac{\hat{Z}^j_h}{|\bar{Z}_h - \hat{Z}_h|}\right)|\bar{Z}_h - \hat{Z}_h| = 0$$

i.e. $z^j(\lambda^*)$ does not require "negative" contributions to any goods. Hence, since $\Sigma z^j_h(\lambda^*) = \Sigma\, \hat{Z}^j_h = t^j$ by assumption, $z^j(\lambda^*)$ is a feasible vector of contribution by j. Given $\hat{Z}^{-j}$, j could then obtain $\lambda^*\bar{Z} + (1-\lambda^*)\hat{Z}$, which, coupled with $\lambda^*\bar{X}^j+(1-\lambda^*)\hat{X}^j$, is strictly preferred by j to $(\hat{X}^j,\hat{Z})$, contradicting $(\hat{X},\hat{Z})$ being a N.E.

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[*]This proof was suggested to me by my thesis supervisor, prof. A. Slivinski

If, alternatively, $U^i(\hat{x}^i,\hat{z}) > U^i(\bar{x}^i,\bar{z})$, then a similar argument, using the fact that $\bar{x}^i > 0$, shows that $(\bar{x},\bar{z})$ could not be a N.E. QED

The next proposition shows that when there are only two public goods, all Nash equilibria yield the same consumption bundles. Although this does not solve the coordination problem arising from the multiplicity of equilibria, the public goods mix is uniquely determined in equilibrium.

Proposition 7: Let m=2, then $(x,z),(\hat{x},\hat{z}) \in NE(t) \Rightarrow z=\hat{z}$.

Proof:

By way of contradiction assume $z \neq \hat{z}$. Without loss of generality assume $z_1 > \hat{z}_1$. A.1a implies that $z_2 < \hat{z}_2$ and that $\tilde{B}^i(c^i) = \tilde{B}^i(\hat{c}^i)$ $\forall i$. Then, by definition of z_1, $I = \{i \in N \mid c_1^i > \hat{c}_1^i\} \neq \emptyset$. That is, there must be at least one individual who contributes strictly more to good 1 in allocation c than in $\hat{c}$. For any $i \in I$, $U^i(x^i,z) \leq U^i(\hat{x}^i,\hat{z})$ contradicts $(x,z) \in NE(t)$ since the convex combination $(\bar{x}^i,\bar{z}) = a[x^i,z]+(1-a)[\hat{x}^i,\hat{z}]$ is strictly preferred to (x^i,z) by the strict convexity of preferences assumption and is feasible unilaterally by reducing c_1^i by some $\varepsilon \leq (c_1^i - \hat{c}_1^i)$ and increasing c_2^i by the same amount, and reallocating private consumption appropriately. Similarly, $U^i(x^i,z) \geq U^i(\hat{x}^i,\hat{z})$ contradicts $(\hat{x},\hat{z}) \in NE(t)$ since $(\tilde{x}^i,\hat{z}) = b[x^i,z]+(1-b)[\hat{x}^i,\hat{z}]$ is strictly preferred to $(\hat{x}^i,\hat{z})$ and is also feasible unilaterally for i by reducing $\hat{c}_2^i$ by some $\varepsilon \leq (\hat{c}_2^i - c_2^i)$ and increasing $\hat{c}_1^i$ and reallocating private consumption appropriately.

QED

Not surprisingly, decentralizing public goods expenditures decisions down to individual taxpayers would give rise to an inefficient composition problem: If individuals do not cooperate, they would generally obtain an inefficient mix of public goods.

Define the *constrained pareto-preference relation* P(t) by:

$(x,z)\ P(t)\ (\hat{x},\hat{z}) \iff (x,z),(\tilde{x},\tilde{z}) \in A(t)$

$(x^i,z)\ R_i(\tilde{x}^i,\tilde{z})\ \forall i$

$(x^i,z)\ P_i(\tilde{x}^i,\tilde{z})$ for at least one i

Define the constrained Pareto-set,

$O(t) = \{(x,z) \in A(t) \mid [\nexists\ (\tilde{x},\tilde{z}) \in A(t)] : (\tilde{x},\tilde{z})P(t)(x,z)\}$

A consumption bundle is thus constrained pareto-efficient if the composition of a given level of public expenditures is pareto-efficient. i.e. given the arbitrary tax structure, any other bundle of public goods costing the same amount would reduce the welfare of at least one individual. Of course, Pareto-improvements might still be obtained by changing the tax structure.

In the special case where some individual contributes positively to all goods in equilibrium, the consumption bundle made available will be constrained Pareto-efficient. Intuitively, an individual who contributes to all goods is getting exactly the consumption bundle that maximizes his utility. Therefore, any reallocation of public expenditures would make him worse off. Proposition 8 formalizes this.

Proposition 8: If $(\tilde{x},\tilde{z}) \in NE(t)$ and $(\exists j \in N)$: $\tilde{z}^j > 0$, then $(\tilde{x},\tilde{z}) \in O(t)$

Proof:

Suppose, bwoc, that $(\bar{x},\bar{z})\ P(t)\ (\tilde{x},\tilde{z})$ and $\tilde{z}^j > 0$ for some NE, $(\tilde{x},\tilde{z})$. It follows that $\bar{z} \neq \tilde{z}$, since there is a strict preference for <u>some</u> i.

Then $U^j(\lambda\bar{x}^j+(1-\lambda)\hat{x}^j,\ \lambda\bar{z}+(1-\lambda)\hat{z}) > U^j(\hat{x}^j,\ \hat{z}^j)$ for all $\lambda \in]0,1[$ by strict convexity and the fact that $U^j(\bar{x}^j,\bar{z}) \geq U^j(\hat{x}^j,\ \hat{z}^j)$.

The rest of the proof is as in prop. 6 above, i.e.

choose $\lambda^* = \min\{1/2,\ \min_{h\in\tilde{M}}\{\hat{z}^j_h/|\bar{z}_h-\hat{z}_h|\}\}$ where $\tilde{M} = \{h\in M : \bar{z}_h \neq \hat{z}_h\}$

Then $\hat{z}^j > 0$ implies $\lambda^* \in]0,1/2]$

Now, $\lambda\bar{z} + (1-\lambda)\hat{z} \quad = \hat{z}^{-j} + [\hat{z}^j + \lambda(\bar{z} - \hat{z})]$

Let $z^j(\lambda) = [\hat{z}^j + \lambda(\bar{z} - \hat{z})]$. Then for any h, $z^j_h(\lambda^*) = \hat{z}^j_h + \lambda^*(\bar{z}_h-\hat{z}_h) > 0$ if $\bar{z}_h \geq \hat{z}_h$, and if $\bar{z}_h < \hat{z}_h$, then $z^j_h(\lambda^*) = \hat{z}^j_h - \lambda^*|\bar{z}_h - \hat{z}_h|$

$$\geq \hat{z}^j_h - \frac{\hat{z}^j_h}{|\bar{z}_h-\hat{z}_h|}|\bar{z}_h-\hat{z}_h| = 0$$

Hence, $z^j(\lambda^*)$ is a feasible vector of contribution by j. Given $\hat{z}^{-j}$, j could then obtain $(\lambda^*\bar{z}+(1-\lambda^*)\hat{z},\ \lambda^*\bar{x}^j+(1-\lambda^*)\hat{x}^j)$ which is strictly preferred by j to $(\hat{x}^j,\hat{z})$, contradicting $(\hat{x},\hat{z})$ being a NE. QED

In the special case of only two public goods the tax-earmarking equilibrium becomes particularly attractive because then, strict convexity ensures that all preferences are single peaked in the decision variables space and all Nash equilibria are constrained Pareto-efficient.

Proposition 9: Let m=2, then $NE(t) \subseteq O(t)$

Proof:

By way of contradiction suppose $(\hat{x},\hat{z}) \in NE(t)$ but $\exists\ (x,z) \in A(t)$ such that $(x,z)P(t)(\hat{x},\hat{z})$. Without loss of generality assume $z_1>\hat{z}_1$. A.1a implies that $z_2<\hat{z}_2$ and that $\tilde{B}^i(c^i)= \tilde{B}^i(\hat{c}^i)\ \forall i$. Let $I=\{i\in N : c^i_1>\hat{c}^i_1\}$ be the subset of individuals whose contribution to z_1 would increase in allocation (x,z). For any $i\in I$, some convex combination $(\bar{x}^i,\bar{z})=a[x^i,z]+(1-a)[\hat{x}^i,\hat{z}]$ would be feasible unilaterally by reducing $\hat{c}^i_2$

and increasing $\hat{c}_1^i$, and reallocating private consumption appropriately. By A.1, $U^i(\bar{x}^i,\bar{z})>U^i(\hat{x}^i,\hat{z})$. This contradicts that $(\hat{x},\hat{z}) \in NE(t)$. QED

The relationship between the tax earmarking equilibria and the core is also of interest. However, even formulating a <u>definition</u> of core allocations in a public goods economy is problematic [see Starrett(1973)]. There is no 'best' definition.

One possibility is to assume the worst for the dissenting coalition, i.e. that others would reduce their contributions to zero (or that it would be somehow excluded from consuming any public goods provided by the others). For any $K \subseteq N$ and t, let $B(K,t) = \{(x,z) \mid (\forall i \in K) : px^i = w^i - t^i, \text{ and } \sum_{i \in N}\sum_{h \in M} z_h^i = \sum_{i \in K} t^i \}$ be the consumption bundles available to dissenting coalition K. An allocation $(\tilde{x},\tilde{z})$ <u>dominates</u> the allocation $(\bar{x},\bar{z})$, for K if it can improve the payoff of at least one member without reducing the payoff of anyone else in the coalition. More formally, define the domination relation $\tilde{D}(K,t)$ as follows:

$(\tilde{x},\tilde{z})\ \tilde{D}(K,t)\ (\bar{x},\bar{z}) \iff$ (i) $(\tilde{x},\tilde{z}_1) \in B(K,t)$

(ii) $(\forall i \in K)$: $U^i(\tilde{x}^i,\tilde{z}) \geq U^i(\bar{x}^i,\bar{z})$

(iii) $(\exists j \in K)$: $U^j(\tilde{x}^j,\tilde{z}) > U^j(\bar{x}^j,\bar{z})$

Define the *constrained core* as:

$\tilde{C}o(t) = \{(x,z) \in A(N,t) \mid (\forall K \subseteq N)\ (\nexists(\bar{x},\bar{z}) \in B(K,t)) : (\bar{x},\bar{z})\tilde{D}(K,t)(x,z)\}$

A consumption bundle (x,z) belongs to $\tilde{C}o(t)$ if no coalition can improve on it when it is constrained to continue paying the same level of taxes while others reduce theirs to zero.

A result that Nash Equilibrium outcomes are actually in the core would only be noteworthy if the core is not huge. However, using the above definition, the fact that a dissenting coalition must buy $\tilde{z}$ only with its own resources makes it very difficult for any coalition to block and so, makes the set of core allocations rather large.

Consider the following alternative core definition:

Let $A(K,t) = \{(x,z) \mid (\forall i \in K) : px^i = w^i - t^i, \text{ and } \sum_{i \in K} \sum_{h \in M} z_h^i = \sum_{i \in K} t^i \}$ be the consumption bundles available to dissenting coalition K.

Define $(\tilde{x},\tilde{z})\ D(K,t)\ (\bar{x},\bar{z}) \iff$ (i) $(\tilde{x},\tilde{z}) \in A(K,t)$

(ii) $(\forall i \notin K)$: $(\tilde{x}^i,\tilde{z}^i) = (\bar{x}^i,\bar{z}^i)$

(iii) $(\forall i \in K)$: $U^i(\tilde{x}^i,\tilde{z}) \geq U^i(\bar{x}^i,\bar{z})$

(iv) $(\exists j \in K)$: $U^j(\tilde{x}^j,\tilde{z}) < U^j(\bar{x}^j,\bar{z})$

(ii) of the definition of the domination relation requires that $i \notin K$ not <u>change</u> their $(\bar{x}^i,\bar{z}^i)$ from what they are in the allocation to be blocked. This may be naive, but it also makes it relatively easy to block a given allocation, and so makes the core smaller. It also conveys the idea being considered by a blocking coalition, as "could we as a group, agree to collectively change our earmarking plans to the advantage of all in the group, given that those outside will continue as before?"

Define the *constrained core* as:

$Co(t) = \{(x,z) \in A(N,t) \mid (\forall K \subseteq N)\ (\exists (\bar{x},\bar{z}) \in A(K,t)) : (\bar{x},\bar{z}) D(K,t)(x,z)\}$

A consumption bundle (x,z) belongs to Co(t) if no coalition can improve on it when it is constrained to continue paying the same level of taxes, while others do not change their contributions.

Proposition 10: Let $m=2$, then $NE(t) \subseteq Co(t)$

Proof:

By way of contradiction, suppose $(\hat{x},\hat{z}) \in NE(t)$ but $\notin Co(t)$, so $\exists K \subseteq N$ such that $(x,z)\ D(K,t)\ (\hat{x},\hat{z})$ for some $(x,z) \in A(K,t)$. Then $U^i(x^i,z) > U^i(\hat{x}^i,\hat{z})$ for at least one $i \in K$ and by the strict convexity assumption, $U^i(\lambda x^i+(1-\lambda)\hat{x}^i,\ \lambda z+(1-\lambda)\hat{z}) > U^i(\hat{x}^i,\hat{z})$ for any $\lambda \in]0,1[$.

Suppose $z = \hat{z}$, then $U^i(\lambda x^i+(1-\lambda)\hat{x}^i,\ \lambda z+(1-\lambda)\hat{z}) = U^i(\lambda x^i+(1-\lambda)\hat{x}^i,\ \hat{z})$. But since i is constrained to continue paying the same taxes, $p_x[\lambda x^i+(1-\lambda)\hat{x}^i] = p_x\hat{x}^i$, so $(\lambda x^i+(1-\lambda)\hat{x}^i,\ \hat{z})$ is a feasible deviation for i, contradicting that $(\hat{x},\hat{z}) \in NE(t)$.

Suppose $z \neq \hat{z}$. Since everyone is constrained to continue paying the same taxes, ($\hat{z}_1 > z_1$ and $\hat{z}_2 > z_2$) and ($\hat{z}_1 < z_1$ and $\hat{z}_2 < z_2$) are impossible, so without loss of generality, suppose $\hat{z}_1 > z_1$ and $\hat{z}_2 < z_2$. Since $z^j = \hat{z}^j$ for all $j \notin K$, it must be that $\exists i \in K$ for whom $\hat{z}_1^i > z_1^i$.

For any h, $\lambda z_h+(1-\lambda)\hat{z}_h = \hat{z}_h+\lambda(z_h-\hat{z}_h) = \hat{z}_h^{-i}+\hat{z}_h^i+\lambda(z_h-\hat{z}_h)$. For any $\lambda>0$, $\hat{z}_2^i + \lambda(z_2-\hat{z}_2) > 0$ since $\hat{z}_2 > z_2$ and $\hat{z}_2^i \geq 0$. Also, for any $0 < \lambda < \frac{-\hat{z}_1^i}{z_1 - \hat{z}_1}$, $\hat{z}_1^i + \lambda(z_1-\hat{z}_1) > 0$ since $z_1 < \hat{z}_1$ and $\hat{z}_1^i > 0$. Further, $\hat{z}_1^i+\lambda(z_1-\hat{z}_1)+\hat{z}_2^i+\lambda(z_2-\hat{z}_2) = \hat{z}_1^i+\hat{z}_2^i+\lambda(z_1+z_2-\hat{z}_1-\hat{z}_2) = t^i$. So given $\hat{z}^{-i}$, the allocation $(\lambda x^i+(1-\lambda)\hat{x}^i,\ \lambda z+(1-\lambda)\hat{z})$ is attainable unilaterally by i for any $0 < \lambda < \frac{-\hat{z}_1^i}{z_1 - \hat{z}_1}$ and is strictly preferred to $(\hat{x},\hat{z})$, contradicting that $(\hat{x},\hat{z},) \in NE(t)$. QED

The tax earmarking outcome is stable in the sense that no coalition of any size could do better on its own if it must continue paying the same amount towards public goods.

As a decision-making process, tax-earmarking is the antipode of a unanimity voting rule and involves no cooperation whatsoever. Everyone allocates their taxes as they please without seeking agreement from anyone else. Yet under mild conditions, the tax-earmarking outcome in the two goods case has the same efficiency and stability properties as unanimity voting. By comparison, under the same circumstances the median voter would also pick a unique constrained pareto-efficient bundle in a simple majority vote but the two outcomes differ with regards to stability. Tax earmarking guarantees that the outcome would be in the constrained core, majority voting does not.

Propositions 7, 9 and 10 above can be extended to any number of public goods under the following restriction:

Assumption 1b: It is possible to partition the set of consumers into subsets $N_1, N_2, \ldots, N_J$ and the set of public goods into subsets $M_1, M_2, \ldots, M_J$ such that $\forall j \in N_i$, utility functions are of the form $U^j(x^j, z_h, z_k, G)$ where $h, k \in M_i$ and $|M_i| = 1$ or 2, i.e. all goods not in M_i may enter the utility functions only as an aggregate.

Although this restriction is rather severe, it may plausibly be satisfied if public goods have a local component. For example, suppose $z_1, \ldots, z_m$ are police stations. Individuals may care about how much is spent on police in their community, and about overall crime protection in the country, but not specifically about how much is spent in each of the other communities. If there were significant spill-overs from one neighbouring community to another, individuals may care specifically about levels of protection there. A.1b allows at most one such (reciprocal) spill-over so that the game can be effectively partitioned into separate subgames, each of which involves independent allocation of funds between only two public goods at most, and such that the outcome of one game has no bearing on the payoff of those who did not participate in it.

Proposition 11: If A.1, A.2, A.1a and A.1b are satisfied then

1) $(x,z),(\hat{x},\hat{z}) \in NE(t) \Rightarrow z=\hat{z}$;

2) $NE(t) \subseteq Co(t) \subseteq O(t)$.

Proof:

A.1b implies that we can split the game Γ into J separate games, each of which involves independent allocation of funds between only two public goods at most, and such that the outcome of one game has no bearing on the payoff of those who did not participate in it. For any such game Γ_i, if m=2 then P.7, P.9 and P.10 hold as before, and if m=1 the earmarking game is degenerate and P.7, P.9 and P.10 hold trivially. QED

Proposition 11 establishes that under certain restrictive but not implausible conditions, the allocation of taxes among public goods could be completely decentralized, without the adverse consequence of an inefficient mix.

3.3 Application: Public and separate school financing

An interesting application concerns the allocation of taxes to school boards. In Canada, Catholics were able to secure constitutional guarantees for the existence of separate school boards and the opportunity to earmark their property taxes to either the public or separate boards[9]. Other ethnic or religious groups may also

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[9] Usually the larger board announces its mill rate first and the other matches it so that taxpayers contribute the same amount whether they elect to support the public or the separate school system. Taxpayers must then make the discrete choice of paying all their taxes to one board or the other. Our model assumes that the strategy sets are convex, i.e. that taxpayers could split their taxes in any way between the two boards. In practice, this discrepancy is not of grave concern since any Nash equilibrium bundle in the two goods case could be obtained with an

demand this privilege. Should it be granted? The results above help answer this question.

Schools are a private good so far as pupils and to some extent their parents are concerned. Exclusion is possible at low cost and consumption is rival once the classroom is full. Taxpayers on the other hand, do not consume schools directly, but they benefit from knowing that children are being given an education they approve of. This benefit is neither excludable nor rival, so that schools may be considered pure public goods to taxpayers in general. Suppose each taxpayer distinguishes between two kinds of schools: (i) The ones attended by his children or those of relatives, friends or members of the same ethnic or religious group; and (ii) schools attended by other children. It is reasonable to expect that over a large range of allocations, taxpayers would feel differently about education in schools of the first kind than of the second even if they feel a positive externality from education of all children. In this case, generalized tax-earmarking to any number of school boards would yield a unique, efficient and core outcome[10].

3.4 Tax-Earmarking and Constitutional Choice

When tax-earmarking is efficient, whether or not to use it as a decision rule is a strictly constitutional question. In general, although everyone would like the

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allocation such that at least n-1 taxpayers allocate all their taxes to only one good. In effect, the discreteness constraint may be binding for at most one taxpayer, so that our model is still a close approximation of the actual process when there are many small consumers.

[10]. This abstracts from any supply side considerations (returns to scale, X-efficiency, etc...). Bagnoli and McKee(1988) argue that measures to increase the competition between the public and separate school boards in Ontario reduce the ability of administrators to divert funds for bureaucratic perquisites. These supply side effects reinforce our demand side argument.

opportunity to earmark their own taxes, many would not want to extend this opportunity to others if they fear a worse outcome than could be secured through some other voting process[11].

If constitutional choice is made behind a "veil of ignorance", a tax-earmarking constitutional proposal could always receive unanimous approval. With perfect information on the other hand, everyone could predict the tax-earmarking equilibrium outcome and compare it to the median voter's preferred bundle. Hence if constitutional rules can be imposed by a simple majority, tax earmarking could always be defeated by <u>some</u> general fund financing proposal, unless the minority is able to trade votes over some other issue. This may explain why such schemes are not more widespread.

The following special case of 2 homogeneous groups illustrates this.

Suppose $|N_1|$ individuals have preferences $U^i(z)=z_1^a z_2^{1-a}$ where $a>1/2$, while $|N-N_1|$ others have preferences $U^i(z)=z_1^b z_2^{1-b}$ where $b<1/2$. z_1 and z_2 can be thought of as 2 kinds of schools as described above. Suppose $\sum_{i\in N1} t^i=G_1<G$ and that $|N_1|>|N_2|$ is the majority group.

Under general fund financing, group N_1 would be able to use its majority to allocate all taxes regardless of N_2's preferences. Let $z^{i*}=\text{argmax}_{z\in A(t)}\ {}^i(x^i,z)$ be the public goods bundle that maximizes consumer i's payoff. z^{i*} is the bundle consumer i would choose if he was a dictator. By A.1, z^{i*} is unique for each consumer. Here, the majority would choose its favourite allocation $z^{1*}=(aG,(1-a)G)$.

Under tax-earmarking, the equilibrium depends on the distribution of tax shares. Suppose $G_1\geq aG$, then group N_1 can assure itself of its favourite allocation z^{1*} by cooperating with each other to allocate $(\sum_{i\in N_1} c_1^i,\sum_{i\in N_1} c_2^i)=(aG,G_1-aG)$ since in

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[11] In the words of J.M. Buchanan(1975, p.92): "Any person's ideal situation is one that allows him full freedom of action and inhibits the behaviour of others so as to force adherence to his own desires."

this case N_2's best reply is $(\sum_{i \in N_2} c_1^i, \sum_{i \in N_2} c_2^i) = (0, G-G_1)$. In this case, the majority is indifferent between tax-earmarking and general fund financing. Whenever $G_1 < aG$ however, the majority group would oppose tax-earmarking since this would not assure it of z^{1*}.

Summary of results obtained from the model

1- Sufficient conditions for the tax-earmarking outcome to be:

a) Unique; b) Efficient; c) In the core.

2- There is a relevant class of circumstances for which these conditions are met and decentralizing public decisions to individuals is an attractive policy option, e.g. school financing.

3- Unless constitutional choice is made behind a "veil of ignorance", tax-earmarking could always be defeated by some general fund financing proposal in a majority vote.

CHAPTER 4 EFFICIENT NON-COOPERATIVE PROVISION OF PUBLIC GOODS

Besides assuming only one public good, another simplification sometimes made is assuming that everyone is a contributor to the public good. This rules out boundary solutions and makes characterization of the equilibrium easy. This is for example, the assumption made by Boadway, Pestieau and Wildasin(1989).

Boundary solutions are however the norm in models of this type as shown by Andreoni(1988). In this chapter it is shown just how restrictive the assumption that "everyone is a contributor" is, and consequences of boundary solutions for the characterization of efficient subsidy rates are outlined. For simplicity it is assumed initially that there is only one public good. The analysis is then extended to the case of many public goods in the last section.

4.1 The Institutional Setting

Maintaining the assumptions and notation of the standard subscription model (Chap. 1), it is assumed that all individuals must pay a tax, but may earn tax credits for voluntary contributions to the public good.

Let $t' = (t^1,...,t^n) \in \{t \in R^n_+ \mid t^i \leq w^i \ \ \forall i \in N\}$ be the tax structure. Let $r = (r^1,...,r^n)$ be the subsidy rates structure, where $0 \leq r^i < 1$. Let d^i be i's <u>voluntary</u> contribution and let $s^i = r^i d^i$ be the subsidy he receives[12].

[12] An alternative definition would be $s^i = \min\{t^i, r^i d^i\}$ if the subsidy cannot exceed i's tax bill. If $r^i \leq 1$ was allowed, some such upper bound on the subsidy would be needed to ensure that individuals' choice sets are not unbounded. This would create a kink in the budget constraints however and unnecessarily complicate the analysis.

Taxes are assumed to be high enough that the government's budget constraint, $\sum_{i=1}^{n} r^i d^i \le \sum_{i=1}^{n} t^i$, is respected in equilibrium.

Constitutional rule 4.1: Individuals are assessed a lump-sum tax t^i, but may make voluntary contributions d^i to the public good and earn tax credits at rate r^i.

When there is only one public good, if the government spent all its net revenue on the public good, individuals would realize that the subsidy they receive just reduces government provision by the same amount so that the effective price of the public good would still be p_z [see Andreoni(1988)]. If subsidies are to have any effect on the equilibrium, it is then necessary to assume that the amount spent by the government on the public good is independent the subsidies it pays. Any surplus collected by the government must be dissipated.

Constitutional rule 4.2: The government does not provide directly any public goods.

This assumption could be relaxed by assuming instead that the government spends some fixed positive amount on the public good. We could also assume that it spends a fraction $\alpha<1$ of net revenue on public good provision. In this case, the effective subsidy rate would be $(1-\alpha)r^i$. In either case, the model would then need to be augmented to include the government as an additional player.

This setting induces a game of complete information among the individuals. Individual i's problem is to choose x^i and d^i to maximize $U^i(x^i,z)$ subject to $p_x.x^i+d^i \le w^i-t^i+s^i$ and $d^i \ge 0$. Rewriting the budget constraint as: $p_x.x^i+(1-r^i)p_z z \le w^i-t^i+(1-r^i)p_z z^{-i}$, the Lagrangean function is $L^i = U^i(x^i,z) + \lambda^i\{w^i-t^i+(1-r^i)p_z z^{-i}-p.x^i-(1-r^i)p_z z\} + \mu^i(z-z^{-i})$, where λ^i and μ^i are Lagrange multipliers.

The Kuhn-Tucker necessary conditions, assuming that $x^i>0$ at an optimum for simplicity, are:

$$\frac{\partial L^i}{\partial x_h^i} = \frac{\partial U^i}{\partial x_h^i} - \lambda^i p_{x_h} = 0 \qquad h=1,\ldots,\ell$$

$$\frac{\partial L^i}{\partial d^i} = \frac{\partial U^i}{\partial z} - \lambda^i(1-r^i)p_z + \mu^i = 0$$

$$\frac{\partial L^i}{\partial \lambda^i} = w^i-t^i+(1-r^i)p_z z^{-i}-px^i-(1-r^i)p_z z \geq 0$$

$$\frac{\partial L^i}{\partial \mu^i} = z-z^{-i} \geq 0$$

The complementary slackness conditions are:

$$\lambda^i\{w^i-t^i+(1-r^i)p_z z^{-i}-px^i-(1-r^i)p_z z\} = 0$$

$$\mu^i(z-z^{-i}) = 0$$

The solutions of this system are:

$$x_j^i = \tilde{f}_j^i(p_x, (1-r^i)p_z, w^i-t^i+(1-r^i)p_z z^{-i}, z^{-i}) \qquad j=1,\ldots,\ell$$

$$d^i = \max\{p_z[g^i(p_x, (1-r^i)p_z, w^i-t^i+(1-r^i)p_z z^{-i}) - z^{-i}], 0\}$$

Note that $g^i(.)$ is an ordinary demand function and not a conditional demand function thanks to the fact that there is only one public good and no rationing on the private goods.

A Nash equilibrium for this game is an allocation $(\hat{x},\hat{z})$ such that $\forall i\in N$ and $\forall(x^i,d^i)$ within i's budget set $U^i(\hat{x}^i,\hat{z}) \geq U^i(x^i,z^i+\hat{z}^{-i})$.

4.2 Efficiency of Equilibria

Proposition 12 shows that Lindahl equilibria are <u>the only</u> efficient allocations that may be supported as Nash equilibria where everyone is at an interior solution.

Proposition 12: If (x^*,z^*) is an efficient Nash equilibrium with subsidy rates r, where everyone is at an interior solution, then (x^*,z^*) is also attainable as a Lindahl equilibrium with prices $\phi^i = 1-r^i$ $\forall i$.

Proof:

(W.l.o.g. all prices normalized to 1 for simplicity)

a) Unanimity: B.w.o.c. let $g^j(.)\neq g^i(.)$ for some $i,j\in N$. By assumption that both are at an interior solution, $d^i=g^i(.)-z^{-i,j}-d^j$ and $d^j=g^j(.)-z^{-i,j}-d^i$. Combining and simplifying, we get: $0=g^i(.)-g^j(.)$ which is a contradiction. Hence it must be that $g^i(.)=g^j(.)$ $\forall i,j\in N$.

b) Prices sum to 1: Since everyone is at an interior solution, it must be that $MRS^i_{z,x^i}=(1-r^i)$ $\forall i\in N$. Summing over i: $\sum MRS^i_{z,x^i}=\sum(1-r^i)$. But Samuelson's condition requires $\sum MRS^i_{z,x^i}=1$, so at an efficient Nash equilibrium $\sum r^i=n-1$, hence $\sum(1-r^i)=1$, i.e. $(1-r^i)$ is a Lindahl price for individual i. QED

Young(1982) has shown that if taxes and subsidy rates are set appropriately, a Lindahl equilibrium could be supported as a Nash equilibrium[13]. Proposition 13 shows the converse and thus establishes a 1 to 1 correspondence between efficient Nash equilibria and Lindahl equilibria[14]. That is, for a given after-tax income distribution, the set of efficient Nash equilibria coincides with the set of Lindahl equilibria.

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[13] An earlier geometric demonstration can be found in Hochman and Rodgers(1977).

[14] Boadway, Pestieau and Wildasin(1989) claim to show this correspondence. However after restating Young(1982)'s proof that Lindahl equilibria can (with an appropriate choice of subsidy rates) be supported as Nash equilibria, their "converse" proposition merely restates the well known fact that any efficient allocation can be achieved as a Lindahl equilibrium with a suitable distribution of income.

Proposition 13: If (x^*,z^*) is an efficient Nash equilibrium, then (x^*,z^*) is also attainable as a Lindahl equilibrium for the same income distribution.

Proof:

(W.l.o.g. all prices normalized to 1 for simplicity)

Let (x^*,z^*) be an efficient Nash equilibrium. If $MRS^i_{z,x^i} = (1-r^i)\ \forall i \in N$, then by Proposition 12, (x^*,z^*) is also attainable as a Lindahl equilibrium by setting $\phi^i = 1-r^i\ \forall i$. If $\exists i \in N$ such that $MRS^i_{z,x^i} < (1-r^i)$, it is possible to increase r^i until $1-\tilde{r}^i$ just equals MRS^i. Since this is not enough for the individual to become a contributor, the allocation would not be changed. Proposition 12 then implies that (x^*,z^*) is also attainable as a Lindahl equilibrium by setting $\phi^i = 1-\tilde{r}^i\ \forall i$. QED

Boadway, Pestieau and Wildasin(1989) suggest an easy way to determine whether an equilibrium is efficient without even knowing individuals' preferences. All that is required is to verify that: (i) the government's budget constraint is tight; (ii) everyone contributes (assuming the commodity is a good and not a bad for anyone); and (iii) subsidy rates sum to n-1. These three conditions together are *sufficient* for Pareto-efficiency[15].

Conditions (ii) and (iii) are not *necessary* however, since for a given after tax income distribution, many efficient allocations where some individuals do not contribute could be supported as Nash equilibria. Even though proposition 13 implies that any efficient allocation that can be obtained with $\sum r^j < n-1$ can also be obtained with $\sum r^j = n-1$, efficiency does not require that subsidy rates necessarily sum to n-1.

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[15] Condition (ii) could be replaced by the weaker condition that everyone be at an interior solution with regards to their choice set. This is not readily observable however.

Proposition 14: If (x^*,z^*) is an efficient Nash equilibrium, then $\sum r^i \leq n-1$.

Proof:

At a Nash equilibrium it must be that $MRS^i_{z,x^i} \leq (1-r^i)\ \forall i \in N$. Summing over i: $\sum MRS^i_{z,x^i} \leq \sum (1-r^i)$. But Samuelson's condition requires $\sum MRS^i_{z,x^i} = 1$, so at an efficient Nash equilibrium $\sum (1-r^i) \geq 1$ and hence $\sum r^i \leq n-1$. QED

Note that $\sum r^i \leq n-1$ is in turn a necessary but not sufficient condition for an efficient Nash equilibrium[16]. Since $r^i < 1$ by definition and hence $\sum r^i < n$, this formula narrows down slightly (if n is small) the range of efficient subsidy rates but does not permit a conclusive verification of the efficiency of an allocation when preferences are not known.

In practice, taxes may have been set constitutionally or the government may be politically constrained to offer a given uniform subsidy rate across individuals. Another issue is whether a uniform subsidy rate could support an efficient Nash equilibrium where everyone contributes. Proposition 14 shows that this would not be the case in general.

Proposition 15: For an arbitrary income distribution, a uniform subsidy would support an efficient Nash equilibrium where everyone is at an interior solution iff preferences were such that Lindahl prices would also be uniform.

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[16] If the alternate definition of s^i was used (see fn.13), this condition would not even be necessary.

Proof:

Follows immediately from the correspondence between Efficient Nash Equilibria where everyone is at an interior solution and Lindahl Equilibria. QED

Lindahl prices would be uniform if all individuals were identical or if the individuals who like the public good more had just enough lower income to maintain equality of MRS's in equilibrium, a condition that is not likely to be satisfied in general. If a uniform subsidy rate yielded an efficient Nash equilibrium, this would not generally be a Lindahl equilibrium and some individuals would be at boundary solutions.

4.3 Second-Best subsidy rates

Efficiency of the equilibrium depends not only on the subsidy rates but also on the taxes being set at a level such that the government's budget constraint is "just" binding. The government would not likely have ex-ante the information necessary to set the subsidy rates efficiently or to adjust taxation levels so as to equilibrate its budget perfectly either.

In these circumstances, it would not be surprising if after a first round of contributions, the government's budget constraint was found to be slack. As postulated, any surplus would then be dissipated. If the taxation level remained fixed, it would then be optimal from society's point of view to increase the subsidy rates enough to wipe out this surplus regardless of the level of provision of the public good.

Proposition 16: If the government's budget constraint is slack, increasing the subsidy rate will always result in an efficiency gain, regardless of the level of provision of the public good.

Proof:

Let $U^{i*}(r)$ be the maximum value of any individual i's utility function when the subsidy rate is r. By the envelope theorem,

$$\frac{\partial U^{i*}}{\partial r} - \frac{\partial U^{i*}}{\partial z}\frac{\partial z^{-i}}{\partial r} + \lambda^{i*}d^{i} \geq 0$$

where * indicates optimal values. Since this is true for all individuals whether they are at an interior or a boundary solution, then increasing the subsidy rate would result in a Pareto-improvement. QED

There is nothing very surprising about this result. If the public commodity in question is a good, then everyone would benefit if resources that would otherwise be wasted were used to provide more of it. Proposition 15 serves a cautionary note however against seeing too much in proposition 13. Even subsidy rates that sum to more than n-1 could be "efficient" in a second-best sense if taxes are too high.

Of course, when the government's budget constraint is slack, efficiency gains could also be had by lowering the taxes (since $\partial U^{i*}/\partial t^{i} = -\lambda^{i*} < 0\ \forall i$), or any combination of lower taxes and higher subsidy rates reducing the government's surplus.

4.4 Many public goods

To accommodate many public goods, the institutional setting is modified as follows:

Let the nXm matrix $r=(r^1,\ldots,r^n)$ be the subsidy rates structure. The subsidy rate r^i_h may vary across individuals as when contributions are deductible from taxable income, and may vary across public goods, as when each public good is subsidized at a different rate.

Let $d^i=(d^i_1,\ldots,d^i_m)$ be i's <u>vector</u> of voluntary contributions and let $s^i=\sum_{h=1}^{m} r^i_h d^i_h$ be the subsidy he receives. Taxes are assumed to be high enough that the government's budget constraint, $\sum_{i=1}^{n}\sum_{h=1}^{m} r^i_h d^i_h \le \sum_{i=1}^{n} t^i$, is respected.

Constitutional rule 4.1b: Individuals are assessed a nonnegative lump-sum tax t^i, but may make voluntary contributions d^i_h to any of the m public goods and earn tax credits at rate r^i_h.

This setting induces a game of complete information $\Gamma(N,w,t,r,p,U)$ among the individuals. Individual i's problem is to choose x^i and d^i so that x^i and z maximize $U^i(x^i,z)$ subject to the constraints that $p_x.x^i+\sum_{h=1}^{m} d^i_h \le w^i-t^i+s^i$ and $d^i_h \ge 0$ $\forall h \in M$. The budget constraint may be written $p_x x^i+[(1-r^i)p_z]z \le w^i-t^i+[(1-r^i)p_z]z^{-i}$ and the Lagrangean function is:

$$L^i = U^i(x^i,z) + \lambda^i\{w^i-t^i+[(1-r^i)p_z]z^{-i}-px^i-[(1-r^i)p_z]z\} + \mu^i(z-z^{-i})$$

where λ^i and μ^i_h's are 1+m nonnegative Lagrange multipliers.

The Kuhn-Tucker necessary conditions, assuming that $x^i>0$ at an optimum for simplicity, are:

$$\frac{\partial L^i}{\partial x^i_h} = \frac{\partial U^i}{\partial x^i_h} - \lambda^i p_{X_h} = 0 \qquad h=1,\ldots,\ell$$

$$\frac{\partial L^i}{\partial d^i_k} = \frac{\partial U^i}{\partial z_k} - \lambda^i(1-r^i_k)p_{Z_k} + \mu^i_k = 0 \qquad k=1,\ldots,m$$

$$\frac{\partial L^i}{\partial \lambda^i} = \ldots^i-t^i+[(1-r^i)p_z]z^{-i}-px^i-[(1-r^i)p_z]z \ge 0$$

$$\frac{\partial L^i}{\partial \mu_h} = z_h - z_h^{-i} \geq 0 \qquad h=1,\ldots,m$$

The complementary slackness conditions are:

$$\lambda^i\{w^i - t^i + [(1-r^i)p_z]z^{-i} - px^i - [(1-r^i)p_z]z\} = 0$$

$$\mu_h^i(z_h - z_h^{-i}) = 0 \qquad h=1,\ldots,m.$$

The solutions of this system are:

$$x_j^i = \hat{f}_j^i(p_x, (1-r^i)p_z, w^i-t^i+[(1-r^i)p_z]z^{-i}, z^{-i}) \qquad j=1,\ldots,\ell$$

and assuming that the first h quantity constraints are not binding:

$$d_j^i = p_{zj}[\hat{g}_j^i(p_x,(1-r^i)p_z,w^i-t^i+[(1-r^i)p_z]z^{-i},z^{-i})-z_j^{-i}] \qquad j=1,\ldots,h$$

$$d_j^i = 0 \qquad j=h+1,\ldots,m$$

A Nash equilibrium for this game is an allocation $(\hat{x},\hat{z})$ such that $\forall i \in N$ and $\forall (x^i,d^i)$ within i's budget set $U^i(\hat{x}^i,\hat{z}) \geq U^i(x^i,z^i+\hat{z}^{-i})$.

Young(1982)'s demonstration that Lindahl equilibria may be supported as Nash equilibria is easily generalized to m goods since at a Lindahl equilibrium no quantity constraints would be binding on anyone. If we set the subsidy rates so that $(1-r^i)=\phi^i$ $\forall i$, where ϕ^i is a vector of Lindahl prices for individual i, and the taxes $t^i = \sum_{h=1}^{m} r_h^i(1-r_h^i)\hat{z}_h$ $\forall i$ where $\hat{z}$ is a Lindahl level of public goods provision, then $d^i = (1-r^i)\hat{z}'$ is a Nash equilibrium vector of contributions. Example 4 illustrates.

Lindahl equilibria typically require personalized prices across individuals and across goods. Differentiated subsidy rates may not however be politically feasible so it would be interesting to know if a uniform subsidy rate could still support an efficient Nash equilibrium.

When there is only one public good, a uniform subsidy rate could support an efficient Nash equilibrium, although generally only a subset of the individuals would make positive contributions. An additional complication when there are many public goods is that a subsidy rate that would support an efficient level of aggregate

Numerical Example 4

Let n=2, m=3, w=(110,130), $U^1=x_1z_1^2z_2^2z_3$ and $U^2=x_2z_1^2z_2z_3^2$, where superscripts are exponents. Using Young's result, the Lindahl equilibrium $(\hat{x},\hat{z})$=(20,20,80,60,60) could be obtained as a Nash equilibrium if a public goods tax t=(46 2/3,46 2/3) and a redistributive tax τ=(-10,10) is imposed, and subsidy rates are r^1=(1/2,1/3,2/3)and r^2=(1/2,2/3,1/3).

In this case, a Nash equilibrium would have d^1=(40,40,20) and d^2=(40,20,40). Contributions are just high enough that subsidies exactly offset the public goods taxes and the government collects no net revenue so there is no waste. Given the redistribution of income τ, the allocation $(\hat{x},\hat{z})$ is both a Lindahl equilibrium and a Nash equilibrium.

contributions would not generally yield an efficient mix of public goods. Example 5 illustrates.

Numerical Example 5

Let n=2, m=3, w=(110,130), $U^1=x_1z_1^2z_2^2z_3$ and $U^2=x_2z_1^2z_2z_3^2$, where superscripts are exponents. The set of pareto-optima is the set of allocations (x^*,z^*)=(40-a,a,80,80-a,40+a) where $0\leq a\leq 40$.

If the government provides public goods directly and offers no subsidies to private provision, the symmetric efficient allocation $\hat{x}$=(20,20), $\hat{z}$=(80,60,60) can be obtained with the tax structure t=(90,110). At this level of taxation, all voluntary contributions are crowded out. The government simply levies sufficiently high taxes and spends this revenue efficiently.

If instead, private provision is subsidized at a uniform rate r=.5 up to t=(45,55), a Nash equilibrium would be: x_1=20, d^1=(23 1/3,66 2/3, 0) and x_2=20, d^2=(43 1/3, 0,66 2/3), yielding x=(20,20) and z=(66 2/3,66 2/3,66 2/3). Since G=200 for both schemes, the level of aggregate provision is still efficiently high but its composition is not Pareto-efficient since both individuals strictly prefer $\hat{z}$=(80,60,60).

Proposition 17: A uniform subsidy rate will not generally support efficient non-cooperative equilibria

Proof:

Let Kj be the set of contributors to public good j. For this group, $\sum_{i\in K} MRS^i_{Z_j,X^i} = k_j(1-r)p_{Z_j}/p_{X_i}$ at a Nash equilibrium where k_j is the number of individuals in K_j. Efficiency requires that r be set so that $\sum_{i\notin K} MRS^i_{Z_j,X^i} = [1-k_j(1-r)]p_{Z_j}/p_{X_i}$ j=1,..,m. Since the optimizer controls only one instrument (r), it will generally not be able to satisfy all m conditions. QED

The fact that an efficient mix of public goods generally requires different subsidy rates for each public good is important in light of demonstrations by Feldstein(1980) and Roberts(1987) that financing <u>a</u> public good through tax expenditures would, subject to caveats mentioned in Roberts(1986), be more efficient than direct government provision. In a nutshell, the minimum taxes required to provide a level G of a public good must sum to G with direct government provision, but only to rG if tax expenditures are used. The government may thus set lower rates of taxation when using tax expenditures (provided r<1), thereby incurring a smaller deadweight loss if lump-sum taxes are unavailable. If a uniform subsidy rate is used however, the welfare loss from the inefficient composition of private contributions may offset or outweigh the potential gain from financing public goods through tax expenditures.

There may be other advantages to financing public goods through tax expenditures. Hochman and Rodgers(1977) argue that subsidies to private provision may be necessary to attain an efficient allocation of resources when a majority ignores the demand of the minority. Martin(1985) suggests that more generous tax credits for voluntary provision of public goods would enhance taxpayers' incentives to take personal responsibility for public goods provision. Welfare gains might also be available on the supply side when the monopoly of public bureaus is broken and public goods producers must minimize bureaucratic excesses to compete for taxpayers' contributions[17].

[17] See Bagnoli and McKee(1988) for a discussion of the effects of competition among bureaus.

Summary of results obtained from the model

One public good model:

1- Lindahl equilibria are the only efficient allocations that may be supported as Nash equilibria where everyone is at an interior solution.

2- This, together with a theorem of Young(1982), establishes a one to one correspondence between efficient Nash equilibria where everyone is at an interior solution and Lindahl Equilibria.

3- A uniform subsidy rate would support an efficient Nash equilibrium where everyone is at an interior solution iff preferences were such that Lindahl prices would be uniform.

4- A necessary condition for an efficient Nash equilibrium is that the sum of subsidy rates be smaller or equal to n-1.

5- If the government's budget constraint is slack, increasing the subsidy rate will always result in an efficiency gain, regardless of the level of provision of the public good.

Many public goods model:

1- A uniform subsidy rate will not generally support efficient non-cooperative equilibria.

2- Efficiency generally requires setting different subsidy rates for each public good.

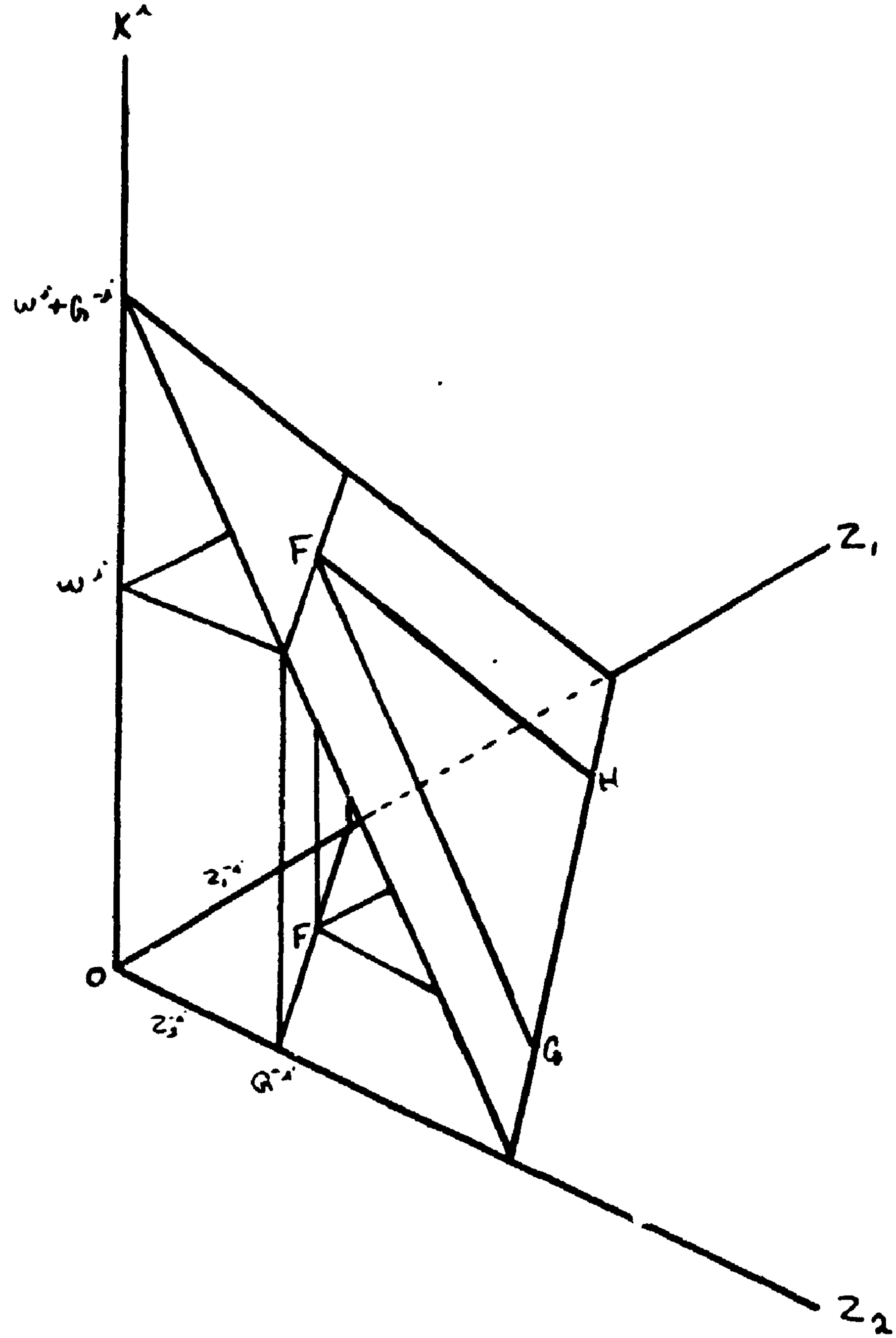

Figure 1: Individual's choice set

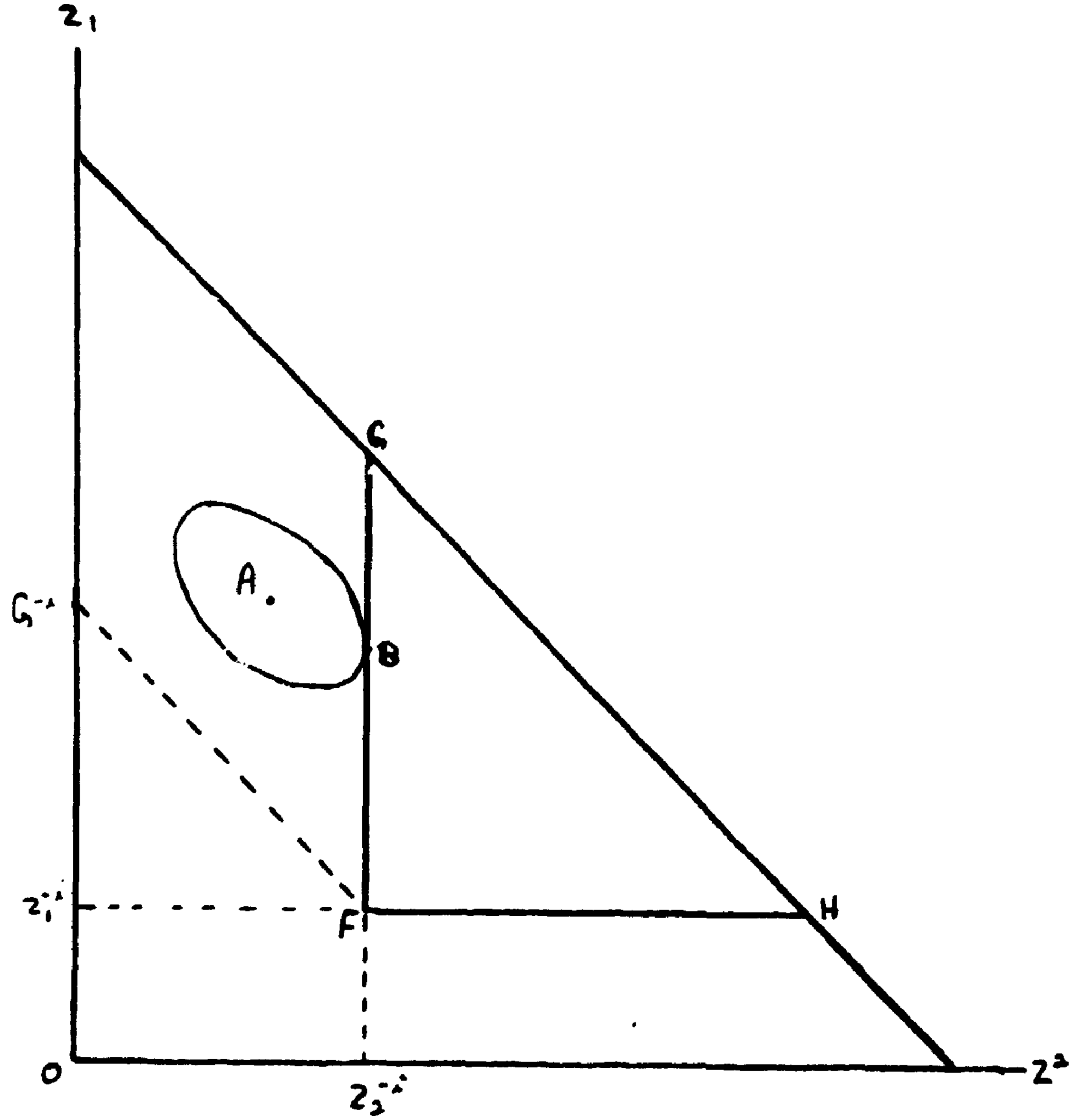

Figure 2: two–dimensional projection of i's choice set

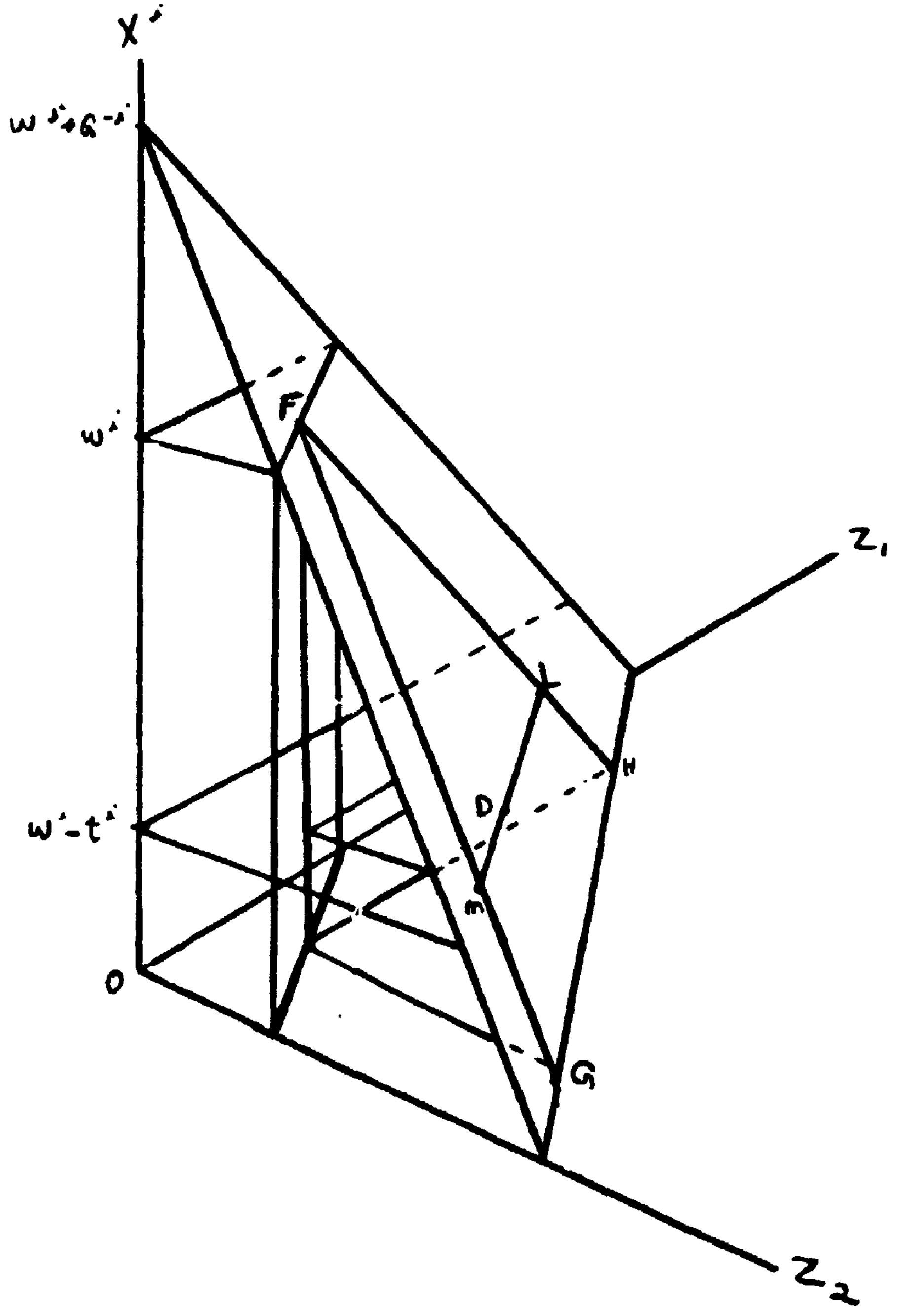

Figure 3: individual's choice set under a tax earmarking scheme

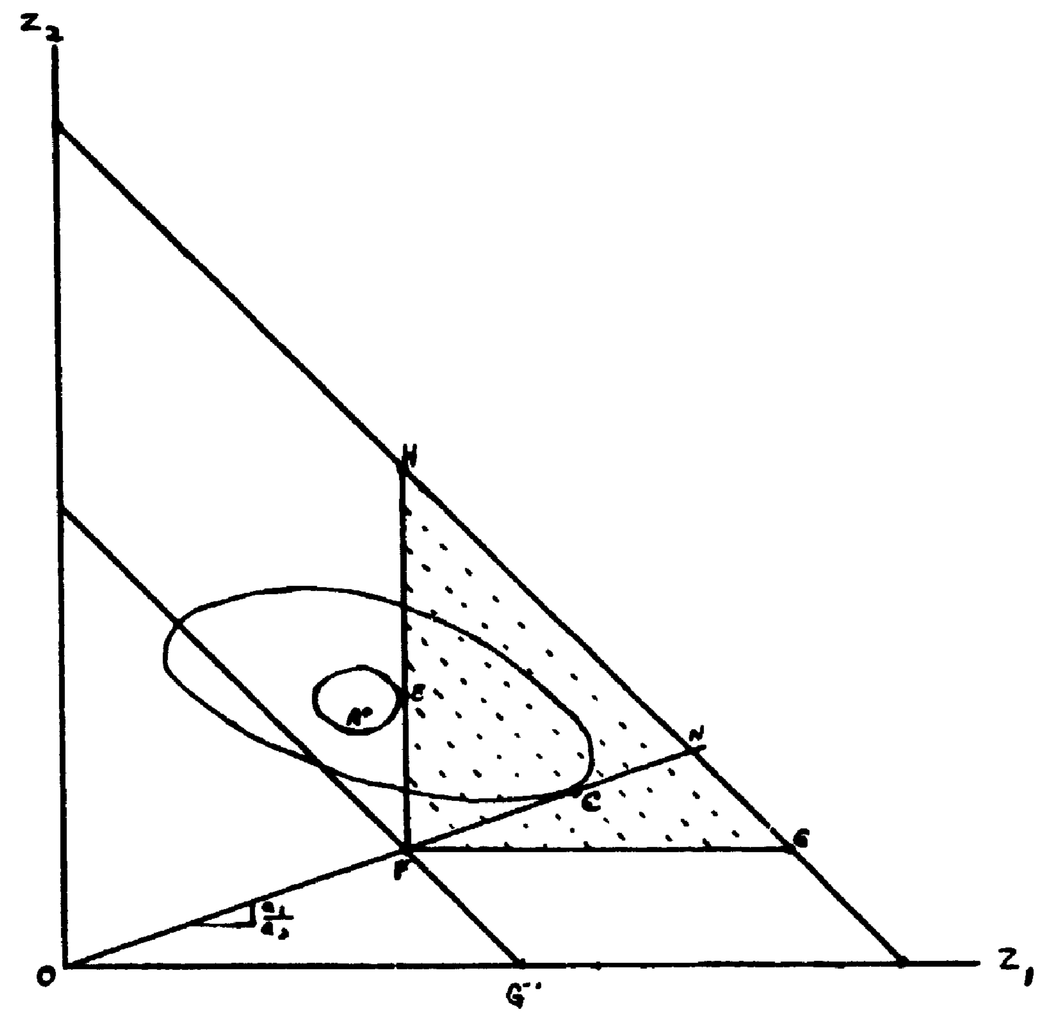

Figure 4: Choice set with a general fund

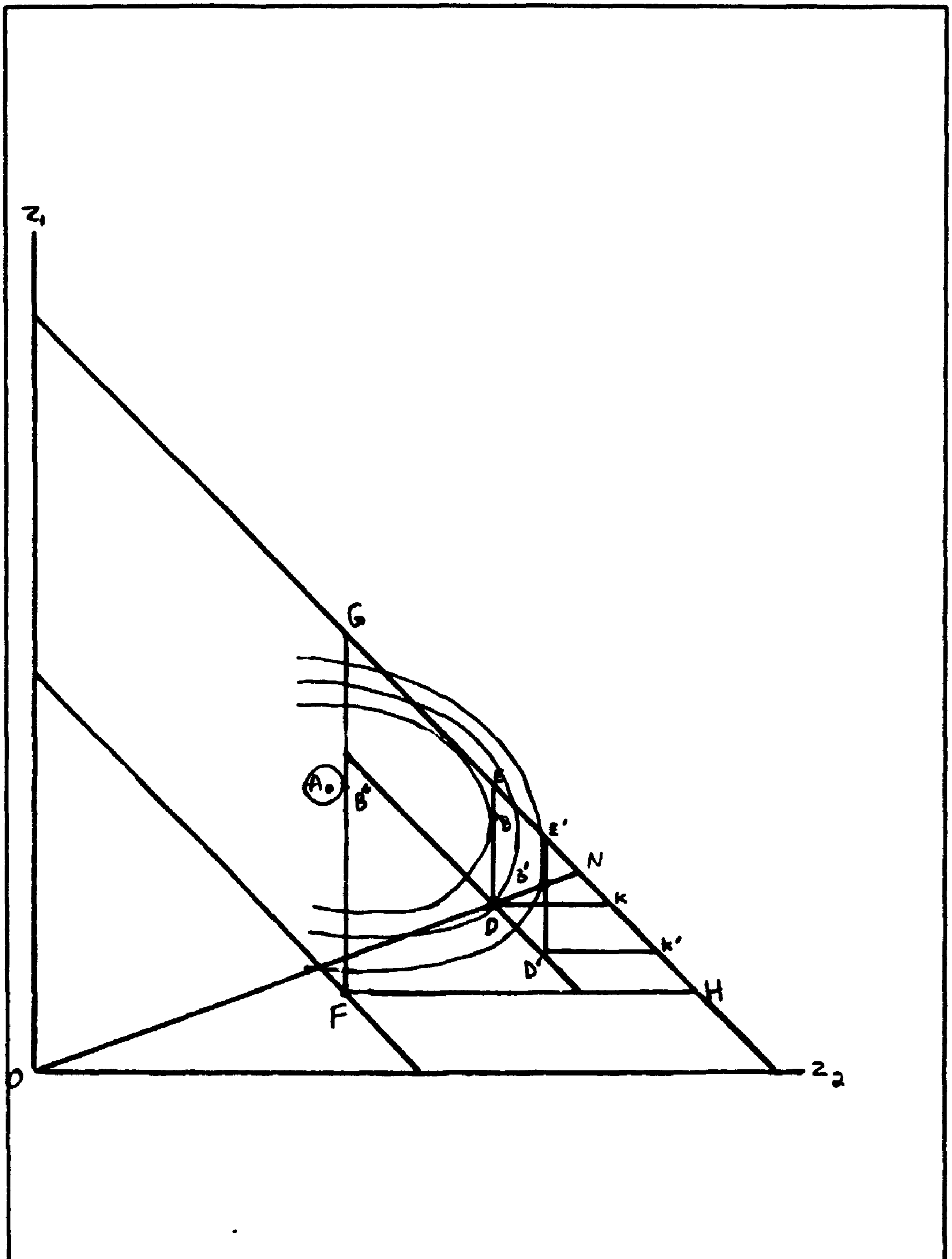

Figure 5: Choice set with government provision

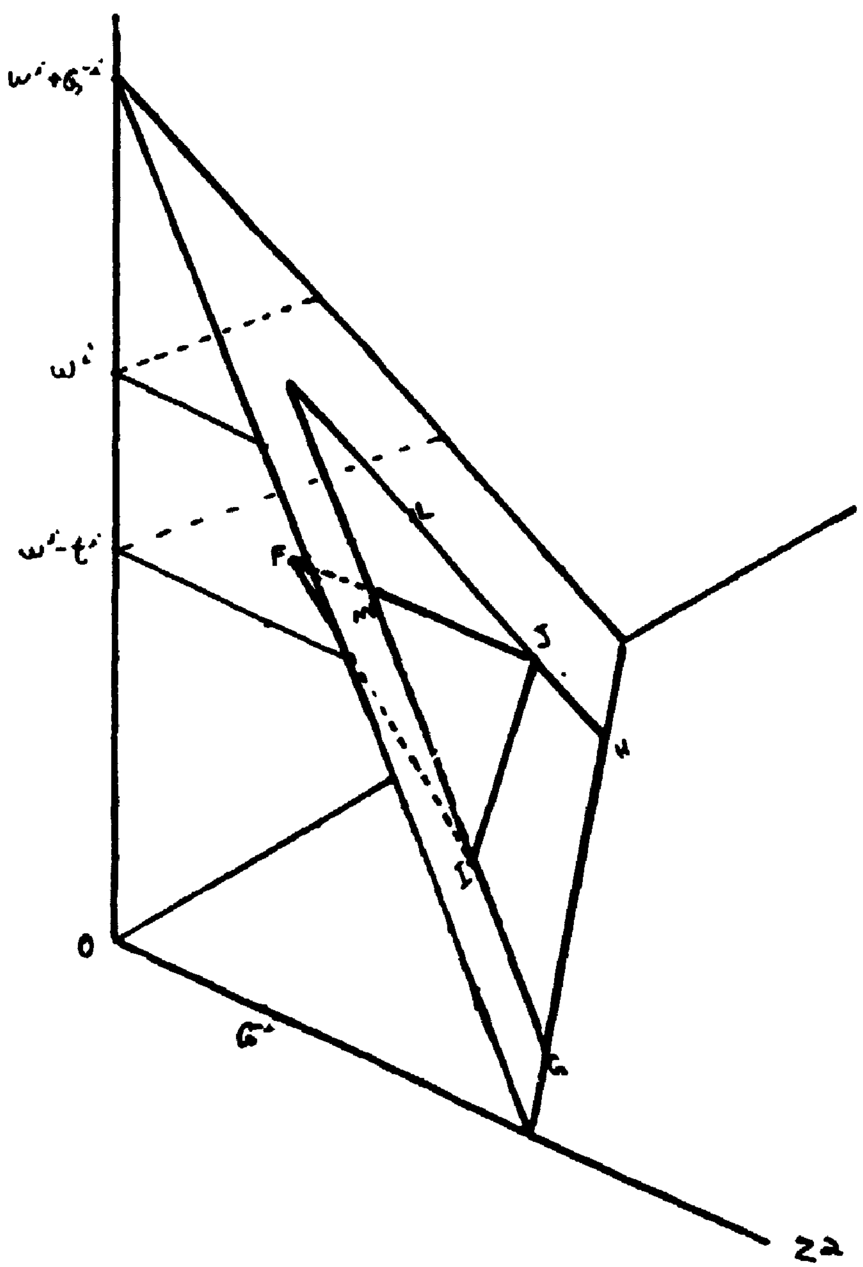

Figure 6: Choice set with subsidies and no government provision

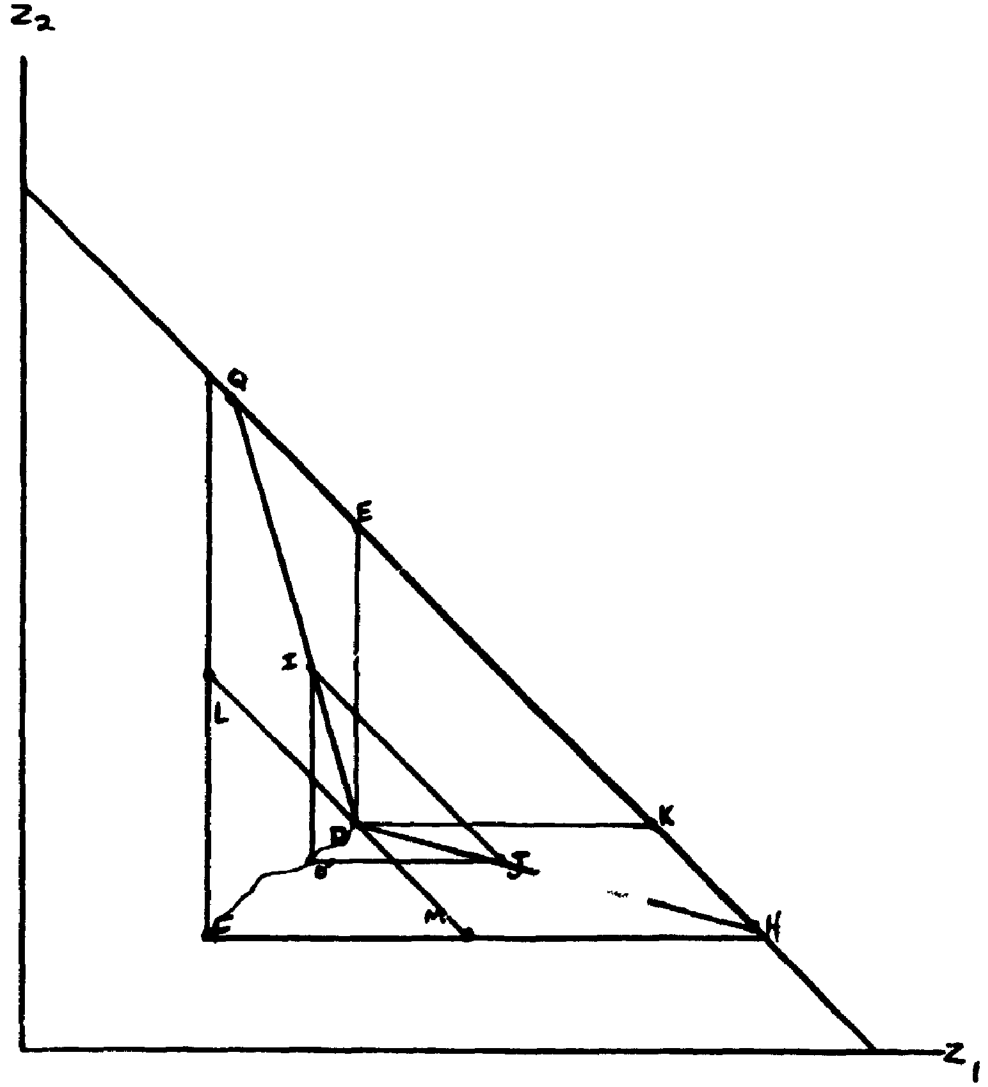

Figure 7: subsidies widen individuals' choice set

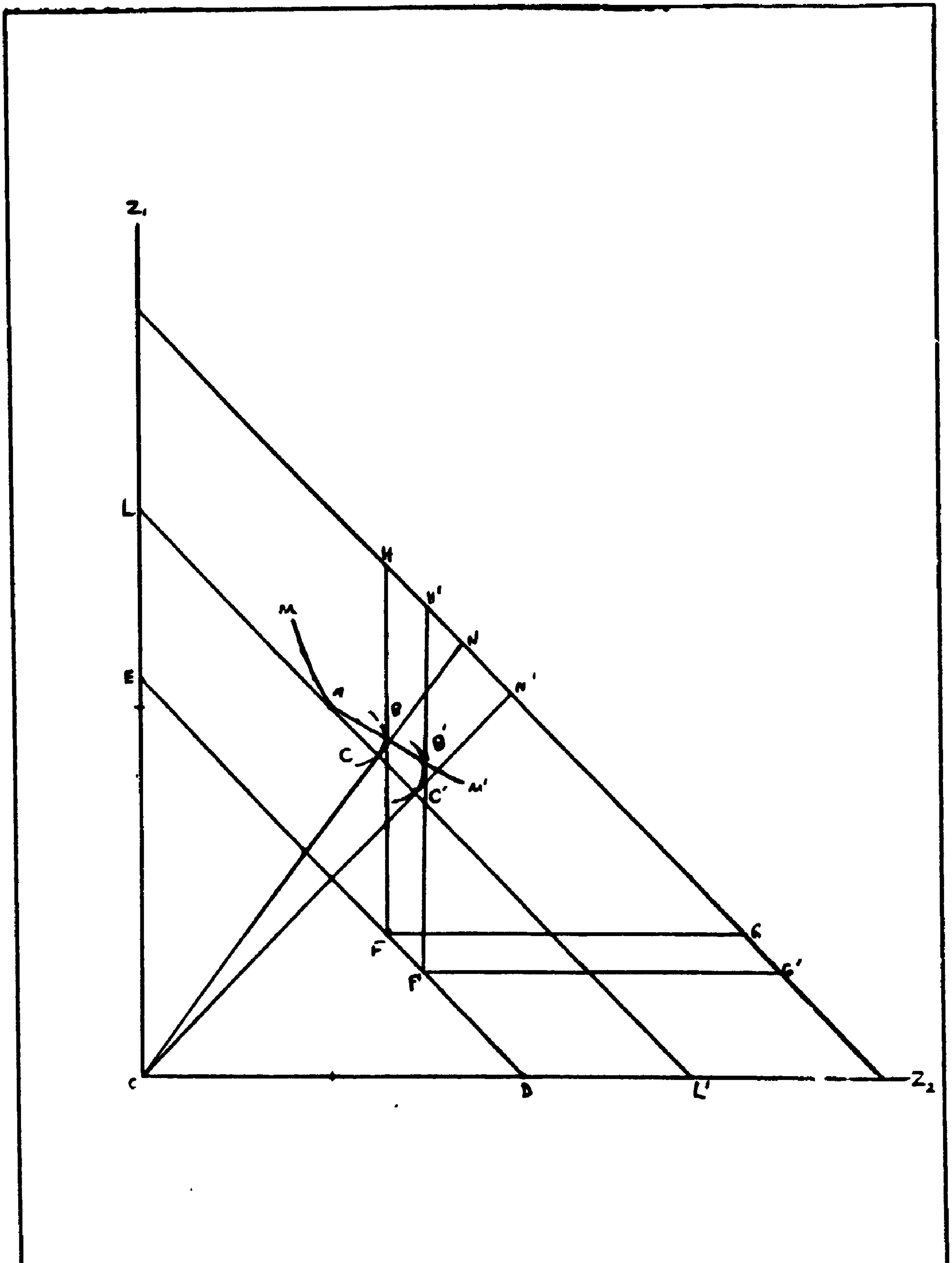

Figure 8: the level of contributions with and without a United Fund (Cobb-Douglas preferences)

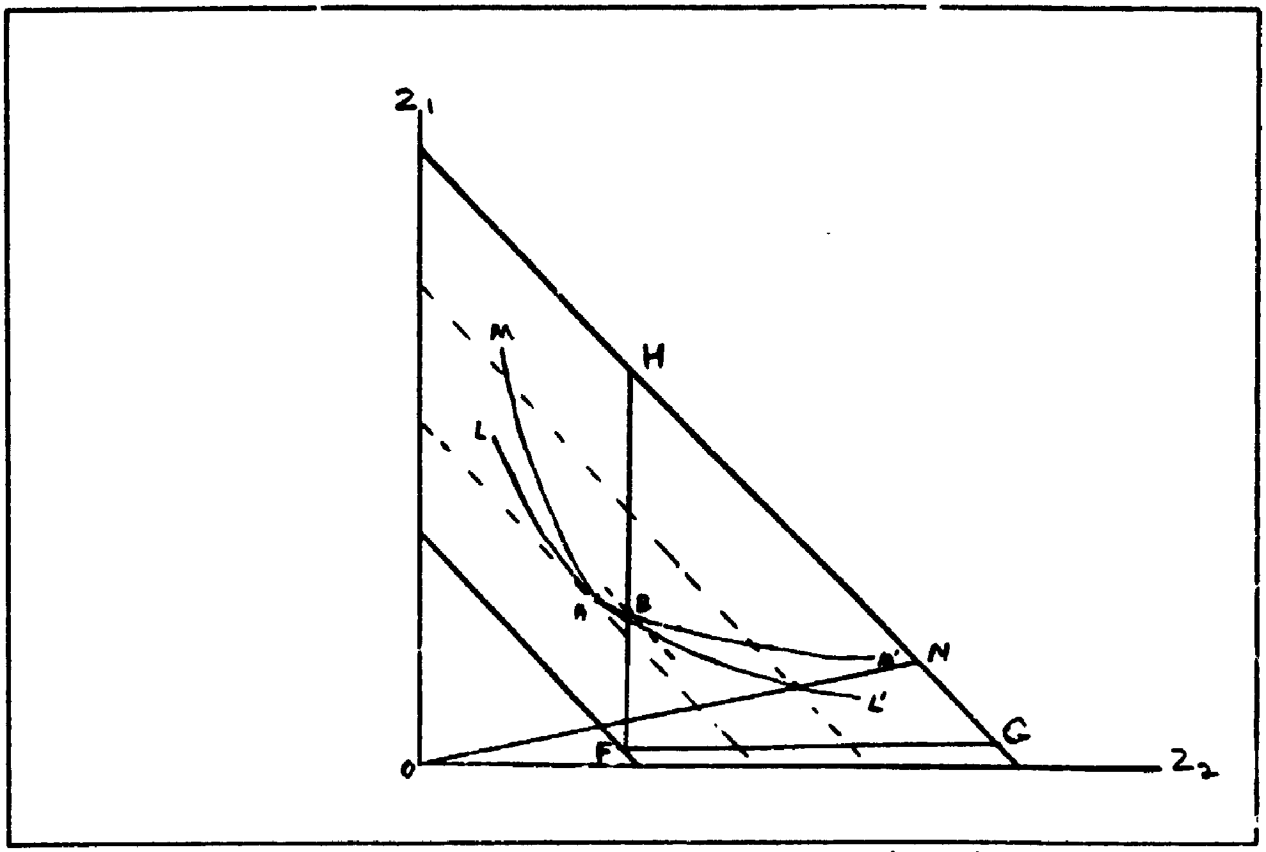

Figure 9: Special case (a) Higher contributions to a United Fund

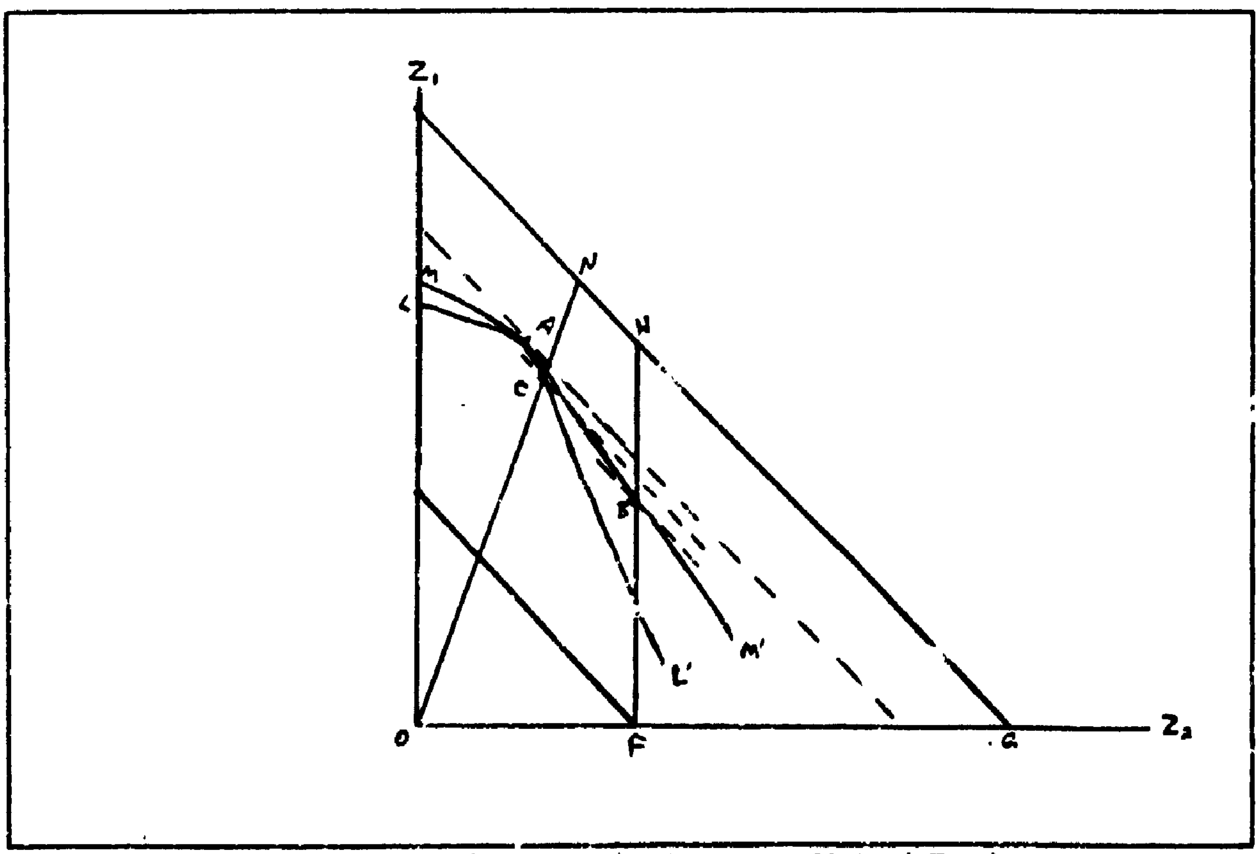

Figure 10: Special case (b) Higher contributions to a United Fund .

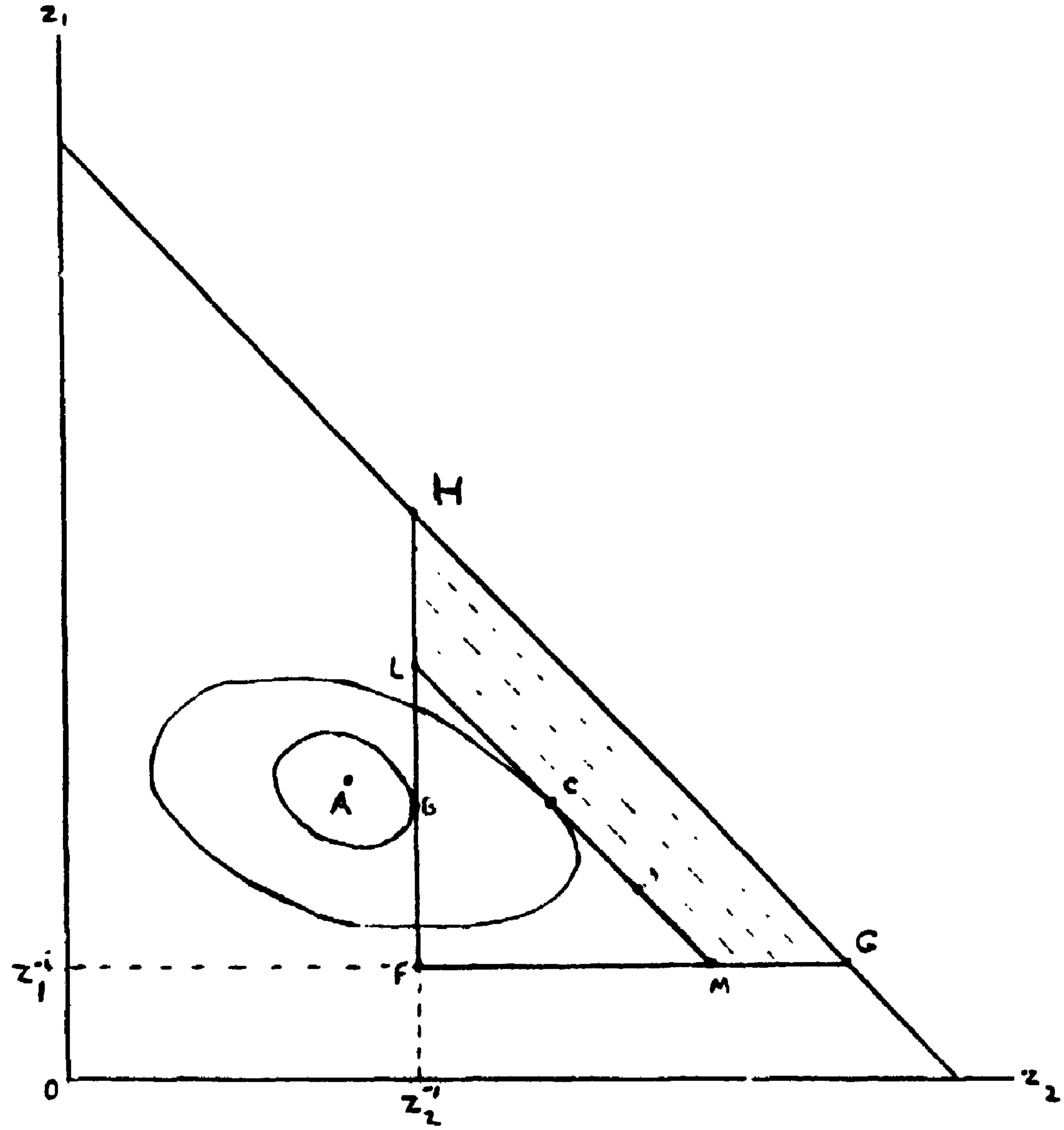

Figure 11: Two-dimensional projection of choice set under tax-earmarking

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