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Asymmetric Information And Union Strike Behavior In Competitive Industries

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ASYMMETRIC INFORMATION AND UNION STRIKE BEHAVIOR IN COMPETITIVE INDUSTRIES

by

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Chapter 1

INTRODUCTION AND SUMMARY

1.1 INTRODUCTION

Strike behavior on the part of unions has attracted a great deal of attention in the union and industrial relations literature. Empirical work in this area has had little guidance from economic theory. Recently, formal models of strikes based on the behavior of rational agents have been developed. Tracy (1987) uses a simple N-round bargaining model in which strikes can result as a consequence of each party following a rational bargaining strategy. Hayes (1984) uses a model of imperfect information in which strikes are used as an information-revealing device. Informational asymmetry in Hayes' model occurs because the firm knows more about the state of output demand than the union does. The union proposes either a high wage and no strike or a lower wage after a strike of some duration. These proposals are designed so that a firm facing a good state of output demand will accept the high wage in order to produce immediately, whereas a firm facing a bad state will accept the strike to obtain the lower wage. If the probability of the good state of output demand is small, the union would offer a single contract with no strike.

In Hayes' model, relatively few predictions can be obtained. This criticism can be applied to much of the union literature. Very few testable propositions have been established (Pencavel, 1984). MacDonald and Robinson (1986)

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ABSTRACT

When firms have private information a union may use a strike to reveal information about the firm. This thesis has asymmetrically-informed agents in a competitive industry in which union and nonunion firms may exist. This environment makes it possible to derive a wide class of prediction concerning union strike behavior. Many of the new predictions arise from the possibility of less than 100% unionization and the implied demand independence results. Also, multiple firm types may exist at any one time in an industry. This leads to a strong result concerning the joint distribution of wages and strikes within an industry, which is fundamental to the view of strikes as a mechanism for eliciting information on firm types.

Firms differ in the level of a cost parameter which can take on a high or low value. If there is perfect information (that is, all agents know the value of any particular firms's cost parameter), then there will be no strike in equilibrium. In the case of imperfect information, there are two types of equilibria: a pooling in which there will be no strikes, and a separating equilibrium in which strikes will occur. In the separating equilibrium the union makes two offers. One offer consists of a high wage and no strike. The other consists of a lower wage but after a strike of a chosen duration. The offers are designed to be incentive compatible.

Besides the wage rates and the strike length, the union

also chooses the number of firms that it organizes. The union's costs of operating increases with the number of firms organized and this may result in the union choosing to organize less than 100% of the firms operating in the industry. A robust prediction is that within a competitive industry, wages rates and strike lengths are inversely related. Also within an industry, the wage rates of firms that settle at the beginning of the contract period should be above those of firms that settle after a strike. A number of comparative static results are also generated. Predictions about strike behavior over the business cycle are presented.

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INTRODUCTION AND SUMMARY

1.1 INTRODUCTION

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In Hayes' model, relatively few predictions can be obtained. This criticism can be applied to much of the union literature. Very few testable propositions have been established (Pencavel, 1984). MacDonald and Robinson (1986) make headway in forming testable hypotheses on union behavior and in placing restrictions on covariations of endogenous variables. In MacDonald and Robinson, the behavior of a monopoly union is analyzed in an environment of competitive firms and union costs. The fact that the union faces a large number of small firms gives it the ability to use these firms to extract rents from the consumers. This ability to extract rents is, however, constrained by the fact that the union uses resources in its union activities. The presence of these costs yields the main innovation in MacDonald and Robinson, an equilibrium outcome in which less than 100% of an industry is unionized. When the industry is less than 100% unionized, the union's optimal choices are independent of shifts in consumer demand.

1.2 SUMMARY AND PRINCIPAL RESULTS

This thesis combines the asymmetrically-informed agents of Hayes with the framework of competitive firms and union cost of MacDonald and Robinson. This environment makes it possible to derive a considerably wider class of prediction concerning union strike behavior than was possible in Hayes. As in MacDonald and Robinson, many of the new predictions arise from the possibility of less than 100% unionization and the implied demand independence results. However, an additional source of predictions, not available in Hayes, is the existence of many firms. Hence, multiple firm types can exist at any one time in an industry. This leads to a strong result concerning the joint distribution of wages and strikes within an industry, which is fundamental to the view of strikes as a mechanism for eliciting information on firm types.

In Chapter 2 a model is presented in which the behavior of the agents (consumers, workers, firms and union) is analysised in an environment in which firms can be of two types. Firms differ in the level of a cost parameter which can take on a high or low value. If there is perfect information (that is, all agents know the value of any particular firm's cost parameter), then there will be no strike in equilibrium. In the case of imperfect information, there are two types of equilibria: a pooling equilibrium in which there will be no strikes, and a separating equilibrium in which strikes will occur. In the separating equilibrium, as in Hayes, the union makes two offers. However, in this case there are many firms which consider these offers. One offer consists of a high wage and no strike. The other consists of a lower wage but after a strike of a chosen duration. The offers are designed to be incentive compatible; that is, the firms with the low cost parameter find it more profitable to pay the high wage and get workers immediately. The firms with the high cost parameter find it more profitable to endure the strike and pay the lower wage.

Besides the wage rates and the strike length, the union also chooses the number of firms that it organizes. The union's costs increase with the number of firms organized and this may result in the union choosing to organize less than 100% of the firms operating in the industry. There would then exist non-union firms in the industry. How many non-union firms depends on the location of the product demand curve. Variations in the product demand curve affect non-union firms if the industry is less than 100% unionized.

Chapter 3 discusses the conditions under which the pooling and separating equilibria will occur, as well as the predictions of the model. A robust prediction is that within a competitive industry, wage rates and strike lengths are inversely related. Also within an industry, the wage rates of firms that settle at the beginning of the contract period should be above those of firms that settle after a strike. A number of comparative static results are also generated. For example, changes in the alternative wage available to workers have a positive effect on wage rates in both struck and non-struck firms. The alternative wage has a negative effect on the number of firms organized

Chapter 4 contains a literature survey and a discussion of how the model in Chapter 2 can be extended to obtain across industry predictions. Also, predictions about strike behavior over the business cycle are presented. If the alternative wage and the price of capital goods reflect movements along the business cycle, then the model predicts pro-cyclical movements in the incidence of strikes and counter-cyclical movements in the duration of strikes

Chapter 2

A MODEL OF UNION BEHAVIOR

2.1 Introduction

This chapter outlines the basic model. The behavior of the four agents of the model is specified. These agents are consumers, workers, firms and a union.

Firms are competitive and identical except that upon entering the industry, they learn the value of a cost parameter. This parameter may have either a high or low value. The union has monopoly rights over sales of labor in the industry. Workers are considered homogeneous and have a competitive wage alternative available to them. Consumer behavior is summarized by a demand curve.

The model is examined under cases of perfect and imperfect information. In the case of perfect information, the union charges the same wages to both types of firms and no strikes will occur. This case reduces to the model of MacDonald and Robinson (1986). With imperfect information, several propositions on the nature of the equilibrium are proved. In this case, there can exist two distinct equilibria: a pooling equilibrium and a separating equilibrium.

In a pooling equilibrium the union charges a single wage to both types of firms and no strike occurs. This equilibrium is the same as that of MacDonald and Robinson. Also, a separating equilibrium can exist in which the union separates the two different types of firms - firms with the low cost parameter will not endure a strike, but will pay a high wage, and firms with the high cost parameter will undergo a strike, but pay a lower wage.

2.2 FIRMS

The firms have access to a common ("daily") production function.

1) $X=f(L),\ f'>0,\ f''<0$

where X is daily output and L is daily labor input. Firms differ in type in a simple way. In order to produce, firms are required to purchase a "capital good" that has two quality levels, good (g) and bad (b). The quality level determines a daily fixed cost of operation. Good quality capital implies a daily fixed cost of c_g and bad quality capital results in cost c_b, where $c_b > c_g$. The length of the period in the model is T days and is determined by the length of time for which the difference in capital goods matters. This periodicity could come from a harvest cycle (where land, the capital good, is leased for an agricultural season) or a "model year" or a "fashion cycle", etc.

The capital good is assumed to be purchased in a competitive market at price K. It is common knowledge that the good comes in the types, g and b, and that the fraction of g types is θ. These types, however, are not apparent until the production process is started. Even then types are only observable to the owner. Firms contemplating entry into this industry know there is a union, know the union's objective function, and know the union's commitment powers. As in most monopoly union models, the firms play a follower's role in optimizing, given the union's offers. The union makes its offers taking into account the best response behavior of the firms

Suppose a firm enters the industry by purchasing the capital good, ie. paying the gate fee K[1]. The daily profits for the firm of type j would be:

2) $\Pi_j = pf(L) - wL - c_j, \quad j=g,b$

where p is the product price and w is the wage rate. Taking p and w as given, the firm chooses L to maximize daily profits,[2] which require $pf'(L)=w$, so,

3) $L(w/p;j) = L(w/p) = f'^{-1}(w/p)$

The daily demand function for labor does not depend on firm type, since the difference across firms is in the fixed costs.

2.3 UNION

Consider now the union that has monopoly rights over the sale of labor to firms in the industry. It is assumed that the union has the power to either require the firm to purchase its labor from the union or to permit it to purchase directly from the competitive labor market. As in MacDonald and Robinson, a simple form of objective function is adopted for the union in order to derive explicitly a large set of results. Many of the major results, however, would follow from a wider class of objective functions. The union is assumed to maximize the excess of revenues (union member earnings) over costs (alternative labor earnings), plus costs of operating the unions. These union profits rely on excluding some potential non-union firms from the industry. In the standard monopoly model, there are no costs to operating the union. Thus, it pays for the union to organize all the firms in the industry and to threaten potential entrants with certain unionization (and zero profits) in order to protect the union firms from non-union competition. In MacDonald and Robinson, where it may pay to permit some non-union firms to operate in the industry, it is still necessary to limit the number of non-union firms. This is done by credibly threatening the excess over the desired number of non-union firms with certain unionization -- that is, zero profits.

The union initially designates two classes of firms. One class is threatened with certain unionization and, in equilibrium, will not enter. The other class is a group of firms among which the union will choose N firms to be unionized, while the rest remain non-union firms which are permitted to operate in the industry and hire labor from the competitive labor market. The union is committed to this strategy for the second class of firms, and the firms take this into account when deciding on whether or not to pay the gate fee. In equilibrium, all of these firms enter the industry.

The union's problem is to maximize profits given its cost and the best response behavior of the firms. It does not have information on firm types. Its only instruments for deciding this information are commitments to wage rate and strike length contracts as in Hayes (1984). Truthful revelation requires that the firms not be able to earn higher profits by misrepresenting their type. The union must therefore ensure that the wage rates and strike lengths offered to the firms satisfy the following incentive compatibility constraints:

4) $(T-s_g)\Pi_g(w_g) \geq (T-s_b)\Pi_g(w_b)$ and

5) $(T-s_b)\Pi_b(w_b) \geq (T-s_g)\Pi_b(w_g)$,

where (w_g, s_g) and (w_b, s_b) are the wage-strike length combinations offered to the firms who declare themselves type g and b, respectively. These conditions are the same as in Hayes (1984) which immediately implies the first proposition.[3]

PROPOSITION 1

The wage offer must be a decreasing function of strike length for incentive compatibility.

Proof

If the union offers the same strike length with two different wage rates, the unionized firms will always accept the contract with the lower wage rate.

If $w_1 > w_2$ and $s_1 = s_2$, then

$\Pi_j(w_1) < \Pi_j(w_2)$, where $j = g, b$

then $(T-s_1)\Pi_j(w_1) < (T-s_2)\Pi_j(w_2)$, and incentive compatibility will be violated. Therefore the wage offers cannot be make independent of strike length. Suppose wage rates were a non-decreasing function of strike length, so for $w_2 > w_1$, $s_2 \geq s_1$. Then for a firm with capital of type j,

$\Pi_j(w_1) > \Pi_j(w_2)$ and $(T-s_1) \geq (T-s_2)$.

Therefore $(T-s_1)\Pi_j(w_1) > (T-s_2)\Pi_j(w_2)$, $j = g, b$,

and thus the combination (w_2, s_2) would never be chosen and one of the incentive compatibility constraints would necessarily be violated.

Q.E.D

The incentive compatibility constraints and the way that the firms' profit functions differ result in the second proposition.

PROPOSITION 2

For incentive compatibility, the strike length for b-type firms must be greater than or equal to the strike length for the g-type firm, ie. $s_b \geq s_g$.

PROOF

Since $\Pi_j(w) = pf(L(w/p)) - wL(w/p) - c_j$, where $j = g,b$

then $\Pi_g(w) - \Pi_b(w) = c_b - c_g$ and

$(T-s_b)\Pi_g(w_b) = (T-s_b)\Pi_b(w_b) + (T-s_b)(c_b-g)$,

then 4), the first incentive compatibility constraint becomes:

$(T-s_g)\Pi_g(w_g) \geq (T-s_b)\Pi_b(w_b) + (T-s_b)(c_b-c_g)$.

Using 5), the second incentive compatibility constraint the above becomes:

$(T-s_g)\Pi_g(w_g) \geq (T-s_g)\Pi_b(w_g) + (T-s_b)(c_b-c_g)$.

Therefore,

$(T-s_g)\{\Pi_g(w_g)-\Pi_b(w_g)\} \geq (T-s_b)(c_b-c_g)$,

hence $(T-s_g)(c_b-c_g) \geq (T-s_b)(c_b-c_g)$.

Therefore $(T-s_g) \geq (T-s_b)$;

therefore $sb \geq s_g$.

Q.E.D.

Given Proposition 1, Proposition 2 has the following corollary.

Corollary

The wage offered to g-type firms must be greater than or equal to the wage offered to b-type firms for incentive compatibility, ie. $w_g \geq w_b$.

The union sells labor to N firms on a daily basis; hence the union's total revenue over the entire period is:

$$(T-s_g)\theta NL(w_g)w_g + (T-s_b)(1-\theta)NL(w_b)w_b$$

where in the labor demand function, L, the price of output has been suppressed. The union uses real resources in its operations. This is modelled by a cost function specified below. The union also has to pay its workers their opportunity wage $\bar{w}$. The total cost to the union in the presence of strikes depends on whether these costs are incurred during the entire period T, or during some proportion of the time union members are on strike, or simply on the days during which production takes place, T-s. Let α be the proportion of time that the union must incur the cost of its operation when its members are on strike. The union's profit function may now be written as

$$\begin{aligned} 6) \qquad \Pi = {} & (T-s_g)\theta NL(w_g)w_g + (T-s_b)(1-\theta)NL(w_b)w_b \\ & - (T-(1-\alpha)s_g)\theta NL(w_g)\bar{w} - (T-(1-\alpha)s_b)(1-\theta)NL(w_b)\bar{w} \\ & - \theta(T-(1-\alpha)s_g)C(N,L(w_g)) - (1-\theta)(T-(1-\alpha)s_b)C(N,L(w_b)) \end{aligned}$$

where $C(N,L(w_j))$ is the operating cost to the union per day of servicing firms that employ $L(w_j)$ workers.[4] When $\alpha=1$, the union incurs the costs of its operation during the entire period T. When $\alpha=0$, the union only incurs the costs when production is taking place. If workers could easily obtain employment in the competitive sector during the strike, then the latter specification would be appropriate. This is the specification that is adopted. When $\alpha=0$, the union can avoid the costs of dealing with a particular type of firm by striking for the entire period, that is, setting $s = T$. In this case, if the profits to the union from dealing with one type of firm is negative, the union can avoid that type. This eliminates the possibility of the union earning negative profits in total. Therefore the union's profit function now has the form:

$$7) \quad \Pi = (T-s_g)\theta NL(w_g)(w_g-\bar{w}) + (T-s_b)(1-\theta)NL(w_b)(w_b-\bar{w}) - \theta(T-s_g)C(N,L(w_g)) - (1-\theta)(T-s_b)C(N,L(w_b)).$$

The formulation of the daily union cost $C(N,L(w_j))$ is a particularly useful one and can be developed as follows. If one lets $C(N, l_1, \ldots l_N)$ be the daily cost of a union which has unionized N firms, the ith of which employs l_i workers, a standard but strong, separability assumption allows this to be written as

$$C(N, l_1, \ldots l_N) = \sum_{i=1}^{N} \bar{C}(N, l_i).$$

Then a union which has unionized N_i firms who hire l_i

workers (for $i = 1, 2$) will have a total daily cost of $N_i C(N_1 + N_2, l_i)$ attributable to those firms. In the present case, with two possible levels of employment $L(w_g)$ and $L(w_b)$ and with $N_g = \theta N$ and $N_b = (1-\theta)N$, defining $C(N,l) = N\bar{C}(N,l)$ gives the specification of total union cost above.[6]

The union's problem is to maximize Π with respect to its choice variables of w_g, w_b, s_g, s_b and N. It is constrained by the incentive compatibility requirements 4) and 5). The union must also ensure that the firms it wishes to exclude are credibly threatened and stay out, and that the firms that it wishes to include are willing to pay the gate fee, knowing the union's problem. In order for the threat to a potential firm to be effective and not function as an invitation to enter requires:

$$8) \qquad K \geq \theta(T - s_g)\Pi_g(w_g) + (1-\theta)(T - s_b)\Pi_b(w_b).$$

The gate fee must be at least as large as the expected total daily profits for a unionized firm. For firms designated by the union as the ones to enter, their expected profits must be at least zero, where the expectation is taken over both firm types and union status. This requires

$$9) \qquad K \leq \gamma[\theta(T - s_g)\Pi_g(w_g) + (1 - \theta)(T - s_b)\Pi_b(w_b)] + (1-\gamma)[\theta T\Pi_g(\hat{w}) + (1-\theta T\Pi_b(\hat{w})]$$

where γ is the fraction of firms unionized. Note that 9) is automatically satisfied if 8) holds as an equality. In fact, the union can always make itself better off by having 8) hold as an equality.

PROPOSITION 3

$$K = \theta(T-s_g)\Pi_g(w_g) + (1-\theta)(T-s_b)\Pi_b(w_b)$$

PROOF

Suppose w_g^*, w_b^*, N^*, s_g^*, s_b^* are the optimum values for the union and γ^* is the optimum fraction of firms unionized Where γ^* is determined by the number of non-unionized firms the union allows into the industry. Suppose that at these values,

$$K > \theta(T-s_g^*)\Pi_g(w_g^*) + (1-\theta)(T-s_b^*)\Pi_b(w_b^*)$$

The union could reduce the number of non-unionized firms in the industry which would reduce total output and, with downward slope demand, would increase the product price. The union could then increase the wage rates by w_g and w_b to hold the level of employment constant. This would increase the level of union profit and the profit for each type of firm[7]. The union would keep reducing the number of non-union firms until either K equalled the expected profits of the union firms or until there were no non-union firms left. If there were no non-union firms, then $\gamma=1$ and 9) then becomes

$$K \leq \theta(T-s_g^*)\Pi_g(w_g^*) + (1-\theta)(T-s_b^*)\Pi_b(w_b^*).$$

This combined with 8) implies that the expected union profit will equal K.

Q.E.D.

Having paid the gate fee, unionized firms of both types would only employ positive amounts of labor if their daily profits are non-negative. Therefore, the union must ensure that the wage offers obey individual rationality constraints.

10) $\Pi_g(w_g) \geq 0$

11) $\Pi_b(w_b) \geq 0$

Finally, the total output from the union firms must not exceed the consumer demand for output. Consumer behavior is represented simply by the following demand function

12) $Q=Q(p),\ Q'<0$

The equilibrium price, p, is implied by 8), given the wage rates and strike lengths. The demand constraint the union faces is:

13) $Q(p) \geq \theta N(T-s_g)f(L(w_g)) + (1-\theta)N(T-s_b)f(L(w_b))$

The other constraints the union faces are non-negativity of w_g, w_b, s_g, s_b, N and the strike lengths may not exceed the period length T.

The union's problem may be written:

$$
\begin{aligned}
14)\quad \max_{w_g, w_b, s_g, s_b, N} \quad L = \Pi &+ \mu[K - \theta(T-s_g)\Pi_g(w_g) - (1-\theta)(T-s_b)\Pi_b(w_b)] \\
&+ \psi_1[(T-s_g)\Pi_g(w_g) - (T-s_b)\Pi_g(w_b)] \\
&+ \psi_2[(T-s_b)\Pi_b(w_b) - (T-s_g)\Pi_b(w_g)] \\
&+ \psi_3\Pi_g(w_g) + \psi_4\Pi_b(w_b) \\
&+ \psi_5[Q(p) - \theta N(T-s_g)f(L(w_g)) \\
&\qquad - (1-\theta)(T-s_b)f(L(w_b))] \\
&+ \psi_6 w_g + \psi_7 w_b + \psi_8 s_g + \psi_9 s_b + \psi_{10} N \\
&+ \psi_{11}(T-s_g) + \psi_{12}(T-s_b).
\end{aligned}
$$

The presence of the union cost function implies that the quantity constraint 13) may not be binding at the optimal wage-strike length packages. The difference between total market demand and the union output is made up by the production of non-union firms. This is the case when there is less than 100% unionization of an industry. MacDonald and Robinson use this case of incomplete union coverage to provide new predictions. Clear predictions are easier to obtain in this case as compared to the usual complete coverage case with a monopoly union because of the separation from market demand considerations. For these reasons the focus of the rest of the thesis will be on the incomplete coverage case when the quantity constraint does not bind and the Lagrange multiplier, ψ_5 equals zero.

The first order conditions for a maximum in the incomplete coverage case are:

15) $\partial L/\partial w_g = \partial \Pi/\partial w_g - \mu\theta(T-s_g)\partial \Pi_g/\partial w_g + \psi_1(T-s_g)\partial \Pi_g/\partial w_g - \psi_2(T-s_g)\partial \Pi_b/\partial w_g + \psi_3 \partial \Pi_g/\partial w_g + \psi_6 = 0.$

16) $\partial L/\partial w_b = \partial \Pi/\partial w_b - \mu(1-\theta)(T-s_b)\partial \Pi_b/\partial w_b - \psi_1(T-s_b)\partial \Pi_g/\partial w_b + \psi_2(T-s_b)\partial \Pi_b/\partial w_b + \psi_4 \partial \Pi_b/\partial wb + \psi_7 = 0.$

17) $\partial L/\partial s_g = \partial \Pi/\partial s_g + \mu(1-\theta)\Pi_b(w_b) - \psi_1 \Pi_g(w_b) + \psi_2 \Pi_b(w_b) + \psi_8 - \psi_{11} = 0.$

18) $\partial L/\partial s_b = \partial \Pi/\partial s_b + \mu(1-\theta)\Pi_b(w_b) + \psi_1 \Pi_g(w_b) - \psi_2 \Pi_b(w_b) + \psi_9 - \psi_{12} = 0.$

19) $\partial L/\partial N = \partial \Pi/\partial N + \psi_{10} = 0$

where all the ψ_i satisfy complimentary slackness.

2.4 PERFECT INFORMATION

Suppose the union learns the firm's type at the same time that it is revealed to the firm. Suppose further that F, c_g and c_b are such that both types of firm's non-negative daily profit constraint were non-binding. Then the only binding constraint would be the zero expected unionized firm profit constraint, 8). In this case 15) and 16) become

20) $(T-s_g)\theta N\ \partial L(w_g)/\partial w_g\ (w_g-w) + (T-s_g)\theta N L(w_g)$
$- \theta(T-s_g)\ C_2\ \partial L(w_g)/\partial w_g - \mu\theta(T-s_g)\ \partial\Pi_g/\partial w_g = 0;$

21) $(T-s_b)(1-\theta)N\ \partial L(w_b)/\partial w_b\ (w_b-\bar{w}) + (T-s_b)(1-\theta)N L(w_b)$
$- (1-\theta)(T-s_b)\ C_2\ \partial L(w_b)/\partial w_b - \mu(1-\theta)(T-s_b)\ \partial\Pi_b/\partial w_b = 0.$

Let 20) and 21) be denoted by $\partial\bar{\Pi}(w_g)/\partial w_g = 0$ and $\partial\bar{\Pi}(w_b)/\partial w_b = 0$, respectively, where $\bar{\Pi} = \Pi + \mu[K-\theta(T-s_g)\Pi_g(w_g)+(1-\theta)(T-s_b)\Pi_b(w_b)]$. Since the firms' profit functions only differs by a daily fixed cost, $\partial\Pi_j(w_k)/\partial w_k = -L(w_k)$, where $j = g, b$, and $k = g, b$. If 20) is divided by $\theta(T-s_g)$ and 21) is divided by $(1-\theta)(T-s_b)$, then these are the same equations. Hence 20) and 21) imply

$w_g = w_b = w$. That is, the same value of w solves both 20) and 21). Moreover, given positive union profits, Π is decreasing in s_g and s_b so that under perfect information s_b and s_g equal zero. There would be no strikes, and since particular differences between firm types does not result in differences in labor demand, union profits would be maximized by a single wage offered to both types. Thus the problem with different firm types is reduced to the problem in MacDonald and Robinson (1986) and the same analysis follows.

2.5 IMPERFECT INFORMATION

As in Hayes (1984), the use of strikes as the mechanism for eliciting infotmation on firm types implies several restriction on the structure of the union's problem. As

10) $\partial\Pi/\partial s_g = -[\theta NL(w_g))(w_g-\hat{w}) - \theta C(N,L(w_g))]$

$\partial\Pi/\partial s_b = -[(1-\theta)NL(w_b))(w_b-\hat{w}) - (1-\theta)C(N,L(w_b))]$

These are the union profits from dealing with each type of firm. If either of these were negative, the union could set the strike length to T for that type and would not deal with the firm. If only one of the types of firms gives negative profits to the union, the other firms' strike length would be set to zero. The wage charged would be the wage that maximizes

$TN(Lw)(w-\hat{w}) - TC(N,L(w))$,

subject to the total profits of the firms equalling the gate fee. If both types of firms are unprofitable for the union, the union would choose not to operate.

11) See Appendix 4 for proof that the Kuhn-Tucker conditions are appropriate.

compatibility constraint binds, or $\Pi_g(w_b^*) < 0$, which implies $w_b^* > w_g^*$ and this contradicts the corollary to Proposition 2.

By the corollary to Lemma 2,[9] the above are the only two sign combinations of $\partial\bar{\Pi}(w_b^*)/\partial w_b$ and $\partial\bar{\Pi}(w_g^*)/\partial w_g$ that exist in equilibrium.

Q.E.D.

Corollary

Only if $\partial\bar{\Pi}(w_b^*)/\partial w_b > 0$ and $\partial\bar{\Pi}(w_g^*)/\partial w_g > 0$ can the first incentive compatibility constraint binds alone.

PROPOSITION 5

If only the first incentive compatibility constraint binds, then the second individual rationality constraint 11) binds.

PROOF

If the first incentive compatibility constraint binds alone, then from the corollary of Proposition 4, $\partial\bar{\Pi}(w_b^*)/\partial w_b > 0$. By complementary slackness $\psi_2 = 0$. Then 16) implies $\psi_4 > 0$, and therefore the second individual rationality constraint binds; $\Pi_b(w_b) = 0$.

Q.E.D.

Proposition 5 just states that in an equilibrium where the incentive compatibility constraint binds only on the g-type firms, then the individual rationality constraint on the b-type firms must bind. This is the single crossing property of Cooper (1984).

PROPOSITION 6

The optimal strike length for g-type firms is zero ie. $s_g^* = 0$.

PROOF

From Proposition 5 either the g-type firms' incentive compatibility constraint binds alone or both incentive compatibility constraints bind. Suppose $s_g^* > 0$.

If $(T-s_g^*)\Pi_g(w_g^*) = (T-s_b^*)\Pi_g(w_b^*)$ and

$(T-s_b^*)\Pi_b(w_b^*) = (T-s_g^*)\Pi_b(w_g^*)$,

then from the proof of Proposition 2, $s_g^* = s_b^*$ hence $w_g^* = w_b^*$. Lowering both s_g^* and s_b^* satisfies these constraints and raises the union profit.[10]

If $(T-s_g^*)\Pi_g(w_g^*) = (T-s_b^*)\Pi_g(w_b^*)$ and

$(T-s_b^*)\Pi_b(w_b^*) > (T-s_g^*)\Pi_b(w_g^*)$,

then by Proposition 5, $\Pi_b(w_b^*) = 0$. Therefore for the second incentive compatibility constraint to be satisfied, $\Pi_b(w_g^*) < 0$, which implies $w_g^* > w_b^*$ hence $s_b^* > s_g^*$. Rewriting the first incentive compatibility constraint yields

$(T-s_b^*)/(T-s_g^*) = \Pi_g(w_g^*)/\Pi_b(w_b^*)$.

Lowering s_b^* and s_g^* to keep the above equality satisfied until $s_g^* = 0$ will raise union profits. If $s_g^* > 0$ the union can earn higher profits by reducing the strike length for g-type firms to zero.

Q.E.D.

Propositions 1 to 6 imply that there are two types of equilibria. In the first, a pooling equilibrium, the union sets $w_g = w_b$ and $s_g = s_b = 0$, and so doesn't differentiate between the two types of firms. The second is a separating equilibrium where $w_g > w_b$, $s_b > s_g = 0$. The firms select between operating for the entire period T and paying a high wage, or obtaining a lower wage but are only able to operate for some fraction of the period. In the separating equilibrium, the cost advantage for the firms with the good type is such that the advantages from producing for the entire period outweighs the disadvantages of the higher wage that has to be paid.

The conditions that determine which of the two equilibria occur may be derived as follows. By Proposition 5, the incentive compatibility constant for low cost firms must bind and the individual rationality for the high cost firms must bind in the separating equilibrium.[11]

Thus,

22) $T\Pi_g(w_g^*) = (T-s_g)\Pi_g(w_b^*)$ and $\Pi_b(wb^*) = 0$.

Since $\Pi_g(w_b^*) = \Pi_b(w_b^*) + (c_b-c_g) = (c_b-c_g)$;
therefore, 22) implies $T\Pi_g(w_g^*) = (T-s_g^*)(c_b-c_g)$.
In addition, 8), the ex ante zero union profit constraint, given $\Pi_b(w_b^*) = 0$, implies

23) $K = \theta T\Pi_g(w_g^*) - \theta(T-s_b^*)(c_b-c_g)$.

Therefore the strike length in the separating equilibrium is

24) $s_b^* = T - K/\theta(c_b-c_g)$,

Thus, the optimal strike length will be non-zero, ie a separating equilibrium will occur if

25) $K/\theta T < c_b-c_g$

The optimal strike length is therefore

26) $s_b^* = 0$ if $K/\theta T \geq c_b-c_g$

$= T - K/\theta(c_b-c_g)$ if $K/\theta T < c_b-c_g$

2.6 POOLING EQUILIBRIUM

In the case of the pooling equilibrium, this model can be written in the same form as the MacDonald and Robinson (1984) model. Since $w_g = w_b$ and $s_g = s_b = 0$, the union's problem is:

$$\max_{w,N} \Pi = TN(w)(w-\bar{w}) - TC(N, L(w))$$

subject to $K = \theta T\Pi_g(w) + (1-\theta)T\Pi_b(w)$

and $Q(p) \geq TNf(L(w))$

Each unionized firm faces the same labor demand function and the same wage; therefore, both types of firms produce the same level of output and, on average, earn total profits equal to the gate fee. The results of MacDonald and Robinson (1986) follow from this model.

2.7 SEPARATING EQUILIBRIUM

In the separating equilibrium, $w_g > w_b$, $s_b > 0$, $s_g = 0$ and $\Pi_b(w_b) = 0$. The expected profit constraint reduces to $K = \theta T\Pi_g(w_g)$ and the (binding) incentive compatibility constraint reduces to $T\Pi_g(w_g) = (T-s_b)\Pi_g(w_b) = (T-s_b)(c_b-c_g)$. These may be combined into a single equality constraint:

27) $K = \theta(T-s_b)(c_b-c_g)$

or

28) $s_b = T - K/\theta\delta$, where $\delta = (c_b-c_g)$.

The union's problem is then reduced to:

29) $$\max_{w_g, w_b, N} \Pi = \theta NTL(w_g)(w_g-\hat{w}) + (K/\theta\delta)(1-\theta)NL(w_b-\hat{w}) - \theta TC(N,L(w_g)) - (1-\theta)(K/\theta\delta)C(N,L(w_b)),$$

subject to

$$Q(p) \geq \theta NTf(L(w_g)) + (1-\theta)N(T-s_b)f(L(w_b)).$$

There are two cases to be considered in the solution of 29): either the demand constraint binds, in which case all firms in the industry are unionized; or it does not bind and incomplete union coverage results. Since the incomplete coverage case in MacDonald and Robinson (1986) yields the strongest prediction, this is the case on which attention is focused.

When the demand constraint is not binding, the first order conditions are:

$$30)\quad \partial\Pi/\partial w_g = \theta NT\, \partial L(w_g)/\partial w_g\ (w_g - \hat{w}) + \theta NTL(w_g) - \theta TC_2(N, L(w_g))\ \partial L(w_g)/\partial w_g = 0,$$

$$31)\quad \partial\Pi/\partial w_b = (1-\theta)NT\, \partial L(w_b)/\partial w_b\ (w_b - \hat{w}) + (1-\theta)NTL(w_b) - (1-\theta)TC_2(N, L(w_b))'\, \partial L(w_b)/\partial w_b = 0, \text{ and}$$

$$32)\quad \partial\Pi/\partial N = \theta TL(w_g)(w_g - \hat{w}) + (1-\theta)(K/\theta\delta)L(w_b)(w_b - \hat{w}) - \theta TC_1(N, L(w_g)) - (1-\theta)(K/\theta\delta)C_1(N, L(w_b)) = 0$$

The optimal strike length, s_b^*, follows directly from the constraint 28) and hence may be solved independently from the equations 30) - 32), which determine the optimal values of w_g, w_b, and N. The explicit exogenous parameters in the model are T, K, θ, δ and $\hat{w}$. There are also implicit exogenous parameters that characterize the local behavior of the labor demand and cost functions.

FOOTNOTES

1) In principle, a "firm" could purchase several capital goods and operate many "plants". This would not alter the basic analysis. The distinction would, however, have some implications for the interpretation of the results with respect to "firm" size.

2) Permitting firms to choose the quantity of labor prevents the industry from becoming, in effect a producer's cartel.

3) This is Proposition 1 in Hayes (1984)

4) $C(N, L(w_j))$ is assumed to have the following properties

$$\partial C/\partial N = C_1 > 0, \quad \partial C/\partial L = C_2 > 0,$$

$$\partial^2 C/\partial N^2 = C_{11} > 0, \quad \partial^2 C/\partial L^2 = C_{22} > 0,$$

5) For simplicity, the union and firms are assumed not to discount future profits. Adding discounting does not change the results. In Appendix 2 a model is presented with continuous time and discounting. The major result is that discounting ensures the strike occurs at the beginning of the period if the union discounts the future by less then the firms do.

6) Note that with this development, $C(N,1) = NC(N,1)$ is interpretable as the daily cost attributable to having organized N firms, all of whom employ 1 workers.

7) Note that holding the level of employment constant means holding w/p constant as p increases (ie. $w/p = a$, then $dQ/dp = a$; therefore, $d\Pi_j/dp = f - aL > 0$, where $j = g, b$).

8) The complementary slackness conditions are:

$$\phi_1[(T-s_g)\Pi_g(w_g) - (T-s_b)\Pi_g(w_b)] = 0,$$

$$\phi_2[(T-s_b)\Pi_b(w_b) - (T-s_g)\Pi_b(w_g)] = 0,$$

$$\phi_3\Pi_g(w_g) = 0, \quad \phi_4\Pi_b(w_b) = 0,$$

$$\phi_5[Q(p) - \theta N(T-s_g)f(L(w_g)) - (1-\theta)N(T-s_b)f(L(w_b))] = 0,$$

$$\phi_6 w_g = 0, \quad \phi_7 w_b = 0, \quad \phi_8 s_g = 0, \quad \phi_9 s_b = 0, \quad \phi_{10} N = 0,$$

$$\phi_{11}(T-s_g) = 0, \quad \phi_{12}(T-sb) = 0.$$

9) See Appendix 3 for the proof of this lemma.

10) $\partial\Pi/\partial s_g = -[\theta NL(w_g))(w_g-\hat{w}) - \theta C(N,L(w_g))]$

$\partial\Pi/\partial s_b = -[(1-\theta)NL(w_b))(w_b-\hat{w}) - (1-\theta)C(N,L(w_b))]$

These are the union profits from dealing with each type of firm. If either of these were negative, the union could set the strike length to T for that type and would not deal with the firm. If only one of the types of firms gives negative profits to the union, the other firms' strike length would be set to zero. The wage charged would be the wage that maximizes

$TN(Lw)(w-\hat{w}) - TC(N,L(w)),$

subject to the total profits of the firms equalling the gate fee. If both types of firms are unprofitable for the union, the union would choose not to operate.

11) See Appendix 4 for proof that the Kuhn-Tucker conditions are appropriate.

Chapter 3

PREDICTIONS

3.1 INTRODUCTION

This chapter looks into the conditions under which a pooling or a separating equilibrium, described in Chapter 2, results. The implication of these conditions on firms' profits is also investigated. This chapter also slackens some of the behavioral restrictions on the firms and union as discussed in Chapter 2 and shows that these restrictions were not binding. Comparative statics are generated and a large number of predictions are formed.

3.2 CONDITIONS FOR EQUILIBRIUM

In the pooling equilibrium the ex ante expected profit condition is:

$$K = \theta T\Pi_g(w^*) + (1-\theta)T\Pi_b(w^*)$$

Since the only difference between the two profit functions is the fixed costs, the above condition may be written as:

$$K = T\Pi_g(w^*) - \delta(1-\theta)T \text{ , where } \delta = (c_b - c_g).$$

This allows one to determine the daily profit levels for the two types of unionized firms in the pooling equilibrium. They are as follows:

$$\Pi_g(w^*) = K/T + (1-\theta)\delta;$$

$$\Pi_b(w^*) = K/T - \theta\delta.$$

K/T is the average daily profits of the unionized firms and, on average, the unionized firms earn zero profits over the production period. If the fraction of firms having the good capital is high (θ close to one), then these low cost firms do not earn much over the average level of daily profits. On the other hand, those firms unlucky to enough have the high cost, c_b, earn less than the average level of profits. This is a result of the union choosing w^*, so that the high cost firms are penalized more severely when the probability of being a high cost firm is low. The opposite occurs when the probability of being a low cost firm is low. In this case, the most common type of firm will be the b-type and the union will have to select the wage, w^*, so that the daily profit level of this type of firm is close to K/T and the low cost g-type firms will be earning close to the difference in costs, δ, more than this.

In the separating equilibrium, the incentive compatibility constraint for the low cost firms bind and the individual rationality constraint for the high cost firms bind. That is, high cost firms earn zero daily profits Therefore the expected profit constraint is:

$$K = \theta T \Pi_g(w_g^*)$$

so that the daily profit level for the g type firm is

$$\Pi_g(w_g^*) = K/\theta T.$$

From the binding incentive compatibility constraint, the strike length for b-type firms is derived

$$s_b^* = T - K/\theta\delta.$$

Since $s_b^* > 0$ requires $K/T < \theta\delta$,

therefore if $K/\theta T \not< \delta$, a pooling equilibrium results, and

if $K/\theta T < \delta$, a separating equilibrium results.[1]

The total profits, including the gate fee, may be written out for each type of firm in the different equilibria.[2]

Pooling equilibrium

Total profits of b-type $= K - \theta\delta T - K = -\theta\delta T$

Total profits of g-type $= K + (1-\theta)\delta T - K = (1-\theta)\delta T$

Separating equilibrium

Total profits of b-type $= 0 - K = -K$

Total profits of g-type $= K/\theta - K = ((1-\theta)/\theta)K$

Notice that in either equilibrium the high cost firms earn negative ex post profits, but ex ante the expected total profit is zero. Also note that, in the separating equilibrium, total profits of both types of unionized firms are independent of T, the length of the production period, and δ, the difference between the two types of firms. Whereas in the pooling equilibrium, total profits are independent of K, the cost of capital.[3]

In Chapter 2, it was assumed that once a firm had purchased its capital good, the firm had no choice but to keep that good. If this was relaxed, would a firm that received the bad type capital wish to throw that capital good away and repurchase? In the pooling equilibrium, where the wage rate paid by unionized firms is independent of their capital type, a unionized firm with the b-type capital would not wish to repurchase if the cost of repurchasing was greater than or equal to the expected benefit of repurchasing. The cost of repurchasing is K plus the foregone daily profits that the firm could have earned with its original b-type capital. The expected benefit of repurchasing is the expected profit of a capital good of unknown type. Therefore a unionized b-type firm would not wish to repurchase if:

$$T\Pi_b(w) + K \geq \theta T\Pi_g(w) + (1-\theta)T\Pi_b(w)$$

which reduces to

$$K \geq \theta T[\Pi_g(w) - \Pi_b(w)];$$

that is, $K \geq \theta T\delta$, where δ is the difference in the levels of profit between the two types of firms. However this is just the condition needed for the pooling equilibrium; therefore, the union chooses a pooling equilibrium when it would not be worthwhile for b-type unionized firms to repurchase capital.

In the separating equilibrium where unionized b-type firms undergo a strike and wage rates are dependent on the firm type, a unionized b-type firm would not wish to repurchase capital if:

$$(T-s_b)\Pi_b(w_b) + K \geq \theta T\Pi_g(w_g) + (1-\theta)(T-s_b)\Pi_b(w_b).$$

But, in the separating equilibrium, $\Pi_b(w_b) = 0$ and $\Pi_g(w_g) = K/\theta T$.

Therefore the above condition reduces to $K \geq K$ and so the unionized b-type firms could expect no gain from repurchasing capital. Since unionized g-type firms earn greater daily profit in both the pooling and separating equilibrium than the b-type firms the unionized g-type firms will never wish to repurchase capital.

If non-union b-type firms could repurchase capital, under the pooling equilibrium they wouldn't, for exactly the same reason union b-types wouldn't. Under the separating equilibrium, b-type non-union firms would repurchase. The cost from doing so is less than the expected returns. However, this assumes that non-union firms that repurchase get to retain this status.[4] In this model the union does not care if non-union firms repurchase since the union is committed to dealing with the unionized firms and b and g-type firms have the same labor demand curve. This means that b and g-type firms that face the same wage produce the same level of output. Therefore, if a non-union b-type repurchases capital and becomes a g-type firm, the total output would remain the same and there would be no effect on the unionized firms and no effect on the union.

Would the union wish to give up one of the unionized b-type firms to organize a non-union g-type firm? If the union could do this unexpectedly (ie break the commitment to deal with a firm that has been selected as a union firm), then the union would. However, if firms knew that this was a possible union action, firms would have an incentive to lie about their type. A g-type unionized firm would have an incentive to lie, and hope the union would release them to the non-union sector. If the union could break its commitment, this would effect the non-union b-type firm's decision as to whether or not to repurchase capital. A model could be formulated giving the union this power to switch firms, but that would involve including the number of non-union firms in the incentive compatibility constraint (since it enters in the probability of being unionized), and hence introduces the product demand schedule into the union's problem.

3.3 PREDICTIONS

If one aspect of strikes is their use as a mechanism for eliciting information from firms, then a central prediction of such models that follows directly from the incentive compatibility constraints is that strike lengths and wage rates are inversely related. In the environment of a competitive industry, this implies a correspondence between the distribution of firm types, the distribution of wage rates and the distribution of strike lengths. Since firm "type" may be difficult to observe, within an industry the most readily testable prediction is that the distribution of strike lengths should be the same as the distribution of wage rates. Further, within an industry, the wage rates prevailing in firms where the wage settlement follows a strike should be below those in firms that settled at the beginning of the period.

The existence of union's costs yield a further robust prediction that strike behavior within the union sector will be unaffected by changes in the product demand, provided union coverage is incomplete. This is an extension to strike behavior of the result of MacDonald and Robinson that all union behavior is independent of product demand shifts in the incomplete union coverage case. It follows from 30)-32) of Chapter 2 that this result continues to hold for union wages, w_g and w_b and the number of union firms, N, in the presence of multiple firm types and imperfect information regarding firm types.

Since only T, K, θ, and δ appear in the determination of strike length, these are the only factors that can affect the optimal strike length. In particular, the optimal strike length does not depend on the alternative wage rate, $\bar{w}$. This result does not rely on any of the assumptions about union costs, ie. whether they occur during a strike or not. It is, however, sensitive to the form in which the firms have been made different - a difference in daily fixed costs.

The effect of exogenous parameters, other than product demand shifts, on w_g, w_b, and N follow in the standard way from differentiating the system of first order conditions. The effects of an increase in the alternative wage rate, $\bar{w}$, on the union's good and bad state wage rates and the number of union firms are unambiguous, given a minor restriction on the union cost function.

By Cramer's rule (and noting that $\Pi_{gb} = \partial(\partial\Pi/\partial w_g)/\partial w_b = 0$),

$$dw_g/d\bar{w} = (1/\Delta)[-\Pi_{gw}(\Pi_{bb}\Pi_{NN} - \Pi_{bN}^2) + \Pi_{gN}(-\Pi_{bN} + \Pi_{Nw} - \Pi_{bb})]$$

where $\Pi_{gw} = \partial(\partial\Pi/\partial w_g)/\partial\bar{w} = -\theta NT\ \partial L(w_g)/\partial w_g > 0$,

$\Pi_{bw} = \partial(\partial\Pi/\partial w_b)/\partial\bar{w} = -(K/\theta\delta)(1-\theta)N\ \partial L(w_b)/\partial w_b > 0$,

and $\Pi_{Nw} = \partial(\partial\Pi/\partial N)/\partial\bar{w} = -\theta TL(w_g) - (K/\theta\delta)(1-\theta)L(w_b) < 0$.

The second order conditions that ensure a maximum require:

$$\Delta = \begin{vmatrix} \Pi_{gg} & \Pi_{gb} & \Pi_{gN} \\ \Pi_{bg} & \Pi_{bb} & \Pi_{bN} \\ \Pi_{Ng} & \Pi_{Nb} & \Pi_{NN} \end{vmatrix} < 0$$

Since $\Pi_{gb} = 0$, these conditions may be written as $\Pi_{gg} < 0$, $\Pi_{bb} < 0$ and $\Delta = \Pi_{gg}\Pi_{bb}\Pi_{NN} - \Pi_{gg}\Pi_{Nb}{}^2 - \Pi_{bb}\Pi_{gN}{}^2 < 0$ which imply $\Pi_{NN} < 0$, and $(\Pi_{bb}\Pi_{NN} - \Pi_{bN}{}^2) > 0$ and $(\Pi_{gg}\Pi_{NN} - \Pi_{gN}{}^2) > 0$. Thus $(1/\Delta)[-\Pi_{gw}(\Pi_{bb}\Pi_{NN} - \Pi_{bN}{}^2) > 0$. Therefore sufficient conditions for $dw_g/d\hat{w} > 0$ are $\Pi_{gN} < 0$ and $\Pi_{bN} \leq 0$.

Differentiating 29) yields:

$$\Pi_{gN} = \theta T(w_g - \hat{w})\ \partial L(w_g)/\partial w_g + \theta TL(w_g) - \theta TC_{12}(N, L(w_g))$$

Substituting from 29), ie. evaluating at the optimum

$$\Pi_{gN} = (\theta T/N)[C_2(N, L(w_g)) - NC_{12}(N, L(w_g))]\ \partial L(w_g)/\partial w_g$$

Thus $\Pi_{gN} < 0$ provided $C_{12}(N, L(w_g))$ is not "too large". The same condition yields $\Pi_{bN} < 0$ and hence $dw_g/d\hat{w} > 0$. Under these conditions it also follows that

$$dw_b/d\hat{w} > 0 \text{ and } dN/d\hat{w} < 0.$$

An increase in the alternative wage rate raises the wage the union sets for both types of firms, but reduces the number of union firms. The alternative wage acts as a factor price to the union. An increase in this factor price results in the union selling less labor in two ways: by selling less labor to each firm and by reducing the number of firms the union sells to. The output price also increases, $dp/d\hat{w} > 0$, and the amount of output produced each day by each firm declines as the alternative wage increases, since less labor is sold to each type of firm.

The level of profit that each firm earns in this separating equilibrium is independent of the alternative wage. Since the labor to output ratio is larger for b-type firms (the production function is concave), it takes a larger increase in w_g than in w_b for the two types to remain at these profit levels. Therefore, a relative size prediction is possible.

$$dw_g/d\hat{w} > dw_b/d\hat{w} > 0$$

As noted earlier, because of the "fixed cost" difference between firm types, the strike length is not affected by $\hat{w}$. Covariation of strike length and other aspects of union behavior are therefore not produced by variation in $\hat{w}$.[5]

Strike length decreases as K, the cost of the capital good, or gate fee, increases. The wage rates w_g, w_b and the price of output also decrease as this gate fee rises whereas the number of firms the union organizes increases.[6] That is,

$$ds_b/dK < 0,\ dw_g/dK < 0,\ dw_b/dK < 0,\ dp/dK < 0, \text{ and}$$
$$dN/dK > 0, \text{ and } dw_g/dK < dw_b/dK < 0.$$

The change in the wage rate for the low cost firms has a larger absolute value. This is because an increase in the sunken cost of capital results in the wage rate and price of output declining. These changes must be such that the daily profit of high cost firms remains at zero. Due to the concavity of $f(L(w))$, since the output per worker is larger in g-type firms, for a given price decrease, a larger decline in the wage for these type of firms is needed. Also, as K increases, the equilibrium daily profit of g-type firms is increased. Therefore even a larger decrease in w_g is needed.

These comparative statics are all consistent in the sense that the union finds it optimal to sell more labor when the cost of the capital good increases. The strike length is shorter, so union labor is active longer, the wage rate is lower and thus more union labor is hired, and the number of firms the union is selling to is larger.

Note that the wage difference between the two types of unionized firms declines. This is because w_g falls more then w_b. One is moving closer to the pooling equilibrium where the wage rate is the same for both types of firms. Recall that for a separating equilibrium, $K < \theta T\delta$. As K increases, one moves closer to the condition that results in a pooling equilibrium where the wage rates would be equal.

Comparative statics are also available for θ, the probability of a firm receiving the low cost capital. The strike length and both wage rates increase with θ. That is:

$ds_b/d\theta > 0$, $dw_g/d\theta > 0$, $dw_b/d\theta > 0$.

Also, the wage rate for g-type firms increases by more than the wage rate for b-type firms:

$dw_g/d\theta > dw_b/d\theta$.

This is because of the concavity of $f(L)$ and that the daily profit level that g-type firms earn in equilibrium is decreasing as θ increases. Also, the number of firms that the union organizes decreases, $dN/d\theta < 0$, and the product price increases, $dp/d\theta > 0$. The union is contracting its operations when the probability of being a g-type firm increases. Since g-type firm profits decrease as θ increases, it takes a longer strike to ensure incentive compatibility. As θ increases $K < \theta T\delta$ becomes stronger and one moves further away from the condition for a pooling equilibrium.

Comparative statics for T are like those for θ. Increasing the periodicity of the industry will result in longer strikes, $ds_b/dT > 0$. In fact, $ds_b/dT > 1$, but this is due to the simple way in which the profits of the two types of firms differ. The wage rates and the price of output increase with the length of the production period. Again a relative size prediction is possible.

$dw_g/dT > dw_b/dT > 0$, and $dp/dT > 0$.

The number of firms the union organizes decreases with T:

$dN/dT < 0$.

Increasing T alone decreases the daily profit which g-type firms earn in equilibrium. This allows the union to increase the wage rate for those firms and results in increased product price. To ensure that b-type firms still earn zero profits, the union raises w_b. Due to the larger output per worker in the g-type firm, w_g is raised by more than w_b.

Predictions as to the effect of the daily cost of production are also attainable. Consider c_g, the daily cost when the capital good is of type g. An increase in this daily cost results in the strike endured by b-type firms decreasing, $ds_b/dc_g > 0$. The product price decreases.

$dp/dc_g > 0$, as do both wage rates $dw_g/dc_g > 0$, $dw_b/dc_g > 0$. Also, the wage offered to g-type firms decreases by more than the wage offered to b-type firms $dw_g/dc_g < dw_b/dc_g$.

Increases in c_b, increases strike length and decreases the wage rates and the output price.[7]

$ds_b/dc_b > 0$, $dw_g/dc_b < 0$, $dw_b/dc_b < 0$, $dp/dc_b < 0$.

There is no relative size prediction in this case.

Increases in either c_g or c_b increase the number of firms organized $dN/dc_j > 0$, where $j = g, b$.

Note that movements in the exogenous parameters result in wage rates and strike length moving in the same direction and the number of firms organized moving in the opposite direction.

If movement from a pooling equilibrium to a separating equilibrium is caused by a change in K, θ, or T, then $w_g^* > w_b^* > w^*$, where w^* is the wage rate for both types of firms in the pooling equilibrium. The g-type firms wage, w_g^*, is more responsive to the exogenous variables than w_b^*. If one looked at the set of firms that had the same δ, and if the exogenous parameters are independently distributed, this model predicts there would be a larger variation in the wages of non-struck firms then for struck firms. If the only change that results in moving from a pooling to a separating equilibrium is a change in δ, which is due to an increase in c_b then g-type firms would earn higher profits in the separating equilibrium and pay a lower wage than w^*, and b-type firms would earn a lower profit and pay a lower wage than w^*. If the change in δ is due to a lowering of c_g, then both types of firms would pay higher wages in the separating equilibrium, but g-type firms would earn more than in the pooling equilibrium.[8]

There have been various measures of strike activity in the literature. For example, the number of strikes, the ratio of the number of strikes to the number of workers involved, the fraction of unionized firms that go on strike, the mean days lost, the mean length, the number of workers involved in strikes per labor force, the number of strikes per labor force, the time lost due to strikes, and the fraction of man-days lost have all been used as measures of strike activity. This model can make predictions concerning the directions in which these measures of strike activity move when there are changes in the exogenous variables. Below are the definitions of these various strike measures in terms of the model and how they would respond to a change in the alternative wage.

1) number of strikes = $(1-\theta)N$,

$$\frac{d(1-\theta)N}{d\bar{w}} = (1-\theta)\frac{dN}{d\bar{w}} < 0$$

2) number of workers involved = $(1-\theta)NL(w_b)$,

$$\frac{d(1-\theta)NL(w_b)}{d\bar{w}} = (1-\theta)\frac{dN}{d\bar{w}}L(w_b) + (1-\theta)N\frac{dL(w_b)}{d\bar{w}} < 0$$

3) $\dfrac{\text{number of strikes}}{\text{number of workers}} = \dfrac{(1-\theta)N}{(1-\theta)NL(w_b)} = \dfrac{1}{L(w_b)}$,

$$\frac{d}{d\bar{w}}\frac{1}{L(w_b)} = -\frac{1}{L(w_b)^2}\frac{d(L(w_b))}{d\bar{w}} > 0$$

4) fraction of unionized firms that go on strike

$$= \frac{(1-\theta)N}{(1-\theta)N + \theta N} = 1-\theta, \qquad \frac{d}{d\bar{w}}(1-\theta) = 0$$

5) mean days lost = total days lost/total number of firms

$$= \frac{s_b(1-\theta)NL(w_b)}{(1-\theta)N + \theta N} = s_b(1-\theta)L(w_b).$$

$$\frac{d}{d\bar{w}} s_b(1-\theta)L(w_b) = s_b(1-\theta)\frac{dL(w_b)}{d\bar{w}} < 0$$

6) $\dfrac{\text{total days lost}}{\text{total number of firms struck}} = \dfrac{s_b(1-\theta)NL(w_b)}{(1-\theta)N} = s_b L(w_b)$

$$\frac{d}{d\bar{w}} s_b L(w_b) = s_b \frac{dL(w_b)}{d\bar{w}} < 0$$

7) $\dfrac{\text{number of workers on strike}}{\text{Labor force}} = \dfrac{(1-\theta)NL(w_b)}{(1-\theta)NL(w_b) + \theta NL(w_g)}$

$$\frac{d}{d\bar{w}} \frac{(1-\theta)NL(w_b)}{(1-\theta)NL(w_b) + \theta NL(w_g)} > 0$$

8) $\dfrac{\text{number of strikes}}{\text{Labor force}} = \dfrac{(1-\theta)N}{(1-\theta)NL(w_b) + \theta NL(w_g)}$

$$\frac{d}{d\bar{w}} \frac{1-\theta}{(1-\theta)L(w_b) + \theta L(w_g)}$$

$$= \frac{-(1-\theta)}{[(1-\theta)L(w_b) + \theta L(w_g)]^2}\left[(1-\theta)\frac{dL(w_b)}{d\bar{w}} + \theta\frac{dL(w_g)}{d\bar{w}}\right] > 0$$

9) time loss $= \dfrac{s_b(1-\theta)N}{TN} = \dfrac{s_b(1-\theta)}{T}$.

since $\dfrac{ds_b}{d\bar{w}} = 0$, $\dfrac{d}{d\bar{w}}\left(\dfrac{s_b(1-\theta)}{T}\right) = 0$.

10) fraction of man-days lost $= \dfrac{s_b(1-\theta)NL(w_b)}{T[(1-\theta)NL(w_b) + \theta NL(w_g)]}$

$$\frac{d}{dw}\left[\frac{s_b(1-\theta)L(w_b)}{(1-\theta)NL(w_b) + \theta NL(w_g)}\right] > 0$$

Therefore, different measures of strike activity respond differently to changes in the alternative wage.[9]

Recall that product demand shifts have no effect on the union's behavior. However, product demand shifts may influence some measures of strike activity. In particular, if the fraction of man-days lost was taken with respect to the total number of man-days in the industry (that is, both union and non-union), then product demand variations would affect this measure. This occurs because product demand variations affect the number of non-union firms in the industry. Therefore, if the product demand curve shifted out, more non-union firms would exist in the industry and the total number of man-days in the industry would increase and the fraction of man-days lost to strikes would decrease.

Finally, there are implicit exogenous variables in the union costs of operating. Changes in these implicit variables which increase the cost of operating the union will act like a change in the alternative wage. The union will wish to reduce the scale of its operations by increasing the wage rates, hence, lowering the amount of labor sold to each firm; and reduce the number of firms that it sells labor to. Again, given the way in which firms' profits have been made different, there would not be a change in the strike length.

FOOTNOTES

1) The g-type firms earn less than the difference in daily cost between the two types of firms in the separating equilibrium, but more than the difference in daily cost in the pooling equilibrium.

In the pooling equilibrium $K/\theta T \geq \delta$, g-type firm daily profits are:

$$\Pi_g(w^*) = K/T + (1-\theta)\delta,$$

and b-type firm daily profits are:

$$\Pi_b(w^*) = K/T + \theta\delta > 0.$$

Therefore $K/T - \theta\delta + \delta > \delta$,

and thus $\Pi_g(w^*) = K/T + (1-\theta)\delta > \delta$.

See Footnote 2.

2) Note that at least one of δ, θ, K, and T must be different between the pooling and separating cases.

3) If $K > T\delta$, there can never be a separating equilibrium If $\theta = 1$, there exist only g-type firms and these firms get total profits equal to zero. If $\theta = 0$, there exist only b-type firms and they earn total profits equal to zero.

If $K < T\delta$, then the type of equilibrium obtained depends on the value of θ. If $K > \theta T\delta$, given the restriction on K, T, and δ above, there are not enough g-types to make it worthwhile for the union to separate them out Given K, T, δ there will be some θ^* such that, for $\theta < \theta^*$, there will only be a pooling equilibrium.

4) If a non-union firm faced certain unionization if it repurchased capital, then a non-union b-type firm would not repurchase if:

$$T\Pi_b(\tilde{w}) + K \geq \theta T\Pi_g(w_g) + (1-\theta)(T-s_b)\Pi_b(w_b).$$

That is, $T\Pi_b(\tilde{w}) + K \geq K$, hence $T\Pi_b(\tilde{w}) > 0$ Since unionized b-type firms earn zero daily profits in the separating equilibrium, the non union firms who pay a lower wage must earn positive profits and hence would not repurchase.

5) For co-variation in other aspects of union behavior apart from strikes, this model produces results that generalize MacDonald and Robinson (1986) to multiple firms types

6) See Appendix 5 for details of these comparative statics.

7) These predictions on the effect of changes in c_b need another condition. See Appendix 5.

8) To see this, suppose $\delta_s > \delta_p$, where $K/\theta T < \delta_s$ and $K/\theta T > \delta_p$. The daily profits for a g-type firm in the pooling equilibrium is:

$$\begin{aligned} K/T + (1-\theta)\delta_p &< K/T + (1-\theta)K/\theta T \\ &= K/T + K/\theta T - K/T \\ &= K/\theta T \end{aligned}$$

and $K/\theta T$ is the daily profit that g-type firms earn in the separating equilibrium.

9) 7) and 10) need restrictions on the labor demand curve to obtain these predictions. See Appendix 6.

Chapter 4

LITERATURE REVIEW AND CROSS-INDUSTRY PREDICTIONS

4.1 INTRODUCTION

This chapter first reviews the literature on strikes. Then the model of Chapter 2 is expanded to deal with different industries, and predictions for cross-industry data are formed. How the model deals with the "stylized facts" is also discussed.

The literature on strikes can be divided into five areas. The first is that strikes are a result of irrational behavior on the part of workers. Workers may overestimate the wage that firms can pay and a strike is needed to reduce worker expectations. This is the model of Ashenfelter and Johnson (1969). There the firm maximizes the present value of its profits subject to a union concession curve. That is, a curve showing how the wage workers will accept declines as the duration of a strike increases. Ashenfelter and Johnson (1969) and Farber (1978) examined how the union concession curve influences strikes. A pro-cyclical relationship was found between strikes and the unemployment rate. Pencavel (1970) and Abbott (1984) found the same result.

Another area of the literature may be called pareto optimal tendencies. Reder and Neumann (1980) and Kennan (1980) have proposed that strikes are inversely related to the joint cost of strikes. That is, a tendency toward the pareto optimal exists, and this tendency increases as the strikes in their empirical work and these measures are treated as equivalent. The model in Chapter 2 gives specific predictions on how these various measures of strike activity respond to changes in the exogenous variables. How these various measures of strike activity respond to changes in the alternative wage were presented in Chapter 3. These measures do not all change in the same direction when the alternative wage increases. In this simple model, even though the alternative wage does not affect the strike length itself, measures of strike activity such as the fraction of man-days lost, the fraction of unionized workers on strike, and the number of strikes divided by the number of workers involved, increase as the alternative wage increases. But other measures of strike activity such as the number of strikes, the number of workers involved, the mean days lost, and the total days lost divided by the number of firms struck, decline as the alternative wage increases. If one treated increases in the alternative wage as an indicator of an upswing in the general level of activity, then depending on the measure of strike activity used, one can get a pro-cyclical reaction of strikes.

4.2 CROSS-INDUSTRY PREDICTIONS

Since much of the empirical work uses ~~cross industry data~~, the model of Chapter 2 needs to be structured to accommodate different industries. One way of doing this is to change the production technology of Chapter 2. Let $f(l, \beta)$ be the

(1982).

Finally, strikes can be viewed as an information-revealing mechanism. In the literature, this approach has been taken by Hayes (1984) and Tracy (1987). If there exists an informational asymmetry between the union and the firm, then a strike can be used to reveal this information.

Tracy (1987) uses a bargaining model in which the union continues to make wage demands until a settlement is reached. A strike takes place whenever this process continuous beyond the expiration of the current contract. Tracy makes predictions on how industry-wide shocks and local labor market shocks affect strike activity. He finds that increases in the industry employment conditions which are above trend, reduce the strike probability. Also, increases in worker experience results in a reduction in the unconditional strike length, but this effect results from a reduction in the probability of a strike and not from a reduction the conditional strike length. Tracy uses changes in the residual from a local employment trend regression as local labor shocks. The effect of an increase in this shock results in a strong increase in the probability of a strike, but this is largely offset by a large decrease in the conditional strike duration.

Tracy's model predicts that the probability of a strike and its expected unconditional duration are positively related to the degree of uncertainty the union faces and the value of the opportunities the union has during a strike.

Also, the probability of a strike and the expected unconditional duration are negatively related to the total expected rents of the firm and union.

Kennan (1986) has reviewed the literature on strikes[1] and states:

> "It is difficult to assess the extent of empirical knowledge on economic aspects of strikes. We do know that strikes are rare,... There is persuasive although not conclusive evidence that the frequency and (more importantly and more doubtfully) the incidence of strikes are positively related to general cyclical movements in the economy. There is also some recently developed evidence that strike duration is negatively related to the cycle."[2]

Kennan (1985) and Harrison and Stewart (1985) have found a statistically significant counter-cyclical variation in the duration of strikes.

There is a large amount of literature relating strike activity to various economic variables such as unemployment rates, inflation rates, and the rate of change in real wages. These studies show that strikes negatively related to unemployment and real wage changes and positively related to the inflation rate. See Paldem and Pedersen (1982). Strikes have been shown to be relatively rare, about 15% of all contract negotiations lead to strikes. See Gunderson, Kervin and Ried (1985) and Swidinsky and Vanderkamp (1982).

Many of the studies on strikes use various measures of strikes in their empirical work and these measures are treated as equivalent. The model in Chapter 2 gives specific predictions on how these various measures of strike activity respond to changes in the exogenous variables. How these various measures of strike activity respond to changes in the alternative wage were presented in Chapter 3. These measures do not all change in the same direction when the alternative wage increases. In this simple model, even though the alternative wage does not affect the strike length itself, measures of strike activity such as the fraction of man-days lost, the fraction of unionized workers on strike, and the number of strikes divided by the number of workers involved, increase as the alternative wage increases. But other measures of strike activity such as the number of strikes, the number of workers involved, the mean days lost, and the total days lost divided by the number of firms struck, decline as the alternative wage increases. If one treated increases in the alternative wage as an indicator of an upswing in the general level of activity, then depending on the measure of strike activity used, one can get a pro-cyclical reaction of strikes.

4.2 CROSS-INDUSTRY PREDICTIONS

Since much of the empirical work uses cross industry data, the model of Chapter 2 needs to be restructured to accommodate different industries. One way of doing this is to change the production technology of Chapter 2. Let f(l) β_i be the production function, where β is an industry specific parameter. Introducing this parameter will result in different industries having different labor demand functions. In this way, if industries had the same set of exogenous variables, there would still be a distribution of union wage rates and union employment levels.

Other ways in which industries vary would be in the values of the exogenous parameters of the model. That is, different industries could have different K, the sunk cost of purchasing capital, θ, and the probability of getting low daily cost, c_g. The daily costs c_g and c_b which might be thought of as maintenance costs, may vary among industries. Also, industries might vary in the alternative wage that they faced. In order for the alternative wage to vary across industries, one might think of different industries requiring different skill levels in their workers. Each of these skill levels would have its own market which generated an alternative wage for each industry. One might think of variation in the cost of the capital good as a result of some industries having technologies which use a more costly capital good. Similarly, different capital goods may have different probabilities of needing a high or low maintenance cost, and these fixed daily costs could vary with different technologies.

All of the exogenous parameters, except c_b, affect the number of firms organized, N, and hence the number of strikes, $(1-\theta)N$, and the length of a strike, s_b, in opposite directions. Therefore, this model gives the possibility of the incidence and duration of strikes moving in opposite direction over the business cycle. Kennan (1986) has pointed out there is some evidence of incidence being pro-cyclical and duration counter-cyclical.

In Chapter 3, it was found that as K increased, this resulted in lower wage rates, prices, and an increased number of strikes, but with shorter durations. Increasing $\bar{w}$ raises the wage rates and prices, but lowers the number of strikes and has no affect in this simple model on the strike length.[3] Increasing both c_g and c_b results in lower wage rates, and increased number of strikes. However increasing c_g causes a shorter duration, whereas increasing c_b causes a longer duration of strikes. If c_g gets closer to c_b, then δ decreases and results in shorter strikes. As θ, the probability of being a g-type firm, increases, there is an increase in wage rates and prices and a lower number of strikes[4] with an increase in strike duration. Increasing the periodicity has the same effect as θ. Therefore most of the parameters move strike length and wage rates in the same direction.

If the parameters were distributed independently across industries, then a positive correlation between wage rates and strike lengths would hold in industry level data. Also, there would be a negative correlation between the duration of a strike and the number of firms that were subject to a strike. A negative correlation between the number of firms unionized and the wage rates paid would also exist. This last prediction does not say that the wage rates paid in an industry are negatively correlated to the fraction of unionized firms in an industry. Since the number of non-unionized firms is determined by the product demand schedule, the fraction of the industry unionized will, given the exogenous parameters, be determined by the product demand schedule. It is possible in this model to have two different industries, differentiated by the exogenous parameters, which have the same output demand schedule in which the industry with the larger fraction of firms organized pays a higher or lower wage than the other industry.[5]

If the parameters, w, θ, and T are positively correlated across industries and K, c_g and c_b are positively correlated, and these two groups are negatively correlated, then the wage rates and strike lengths are positively related. The number of firms organized and strike lengths will be negatively correlated. This is because w, θ, and T all move the wage rates in the same direction and K, c_g and c_b move w_g and w_b in the opposite direction of the first group.

These, however, are not the most natural correlations between the parameters across industries. For example, it may be that industries with a long periodicity may require more costly capital. This would mean that K would be positively correlated to T. In the two firm type model of

Chapter 2, a separating equilibrium, in which strikes occur, results if $K/\theta T < \delta$. If K/T increases as T increases across industries, and θ and δ are independently distributed, then one would be more likely to find no strikes (ie. a pooling equilibrium) in those industries with a long periodicity. Or, if there were strikes, they would be shorter than in industries with shorter periodicities.

The model in Chapter 2 has only two types of firms. Suppose, instead, there are n types of firms. The union then must select a set of wage-strike combinations that satisfy a set of incentive compatibility constraints for each type of firm. The optimum policy for the union may involve a complete separating equilibrium in which all n firms reveal their type, or a pooling equilibrium in which one wage is offered and no strike is taken on, or some combination of these where some firm types are separated out and other types are pooled.[6]

In the case where strikes occur, there will be a negative correlation between the wage rates and the strike lengths within an industry. Looking at cross-industry data, changes in the exogenous variables move wage rates and strike lengths in the same direction. If the exogenous parameters are independently distributed across industries, the model predicts that there would be a positive correlation between average wage rates and average strike lengths.

Other predictions are possible from this model if one looks at the effect of different types of shocks. One could look at the effect of industry specific shocks. Suppose that for a particular industry there was a reduction of the daily fixed costs c_g and c_b, but the difference between these costs stayed the same. This would be a positive shock to the industry. The model predicts a reduced number of strikes and no effect regarding the duration. There would be an increase in equilibrium wage rates and product price. If the difference in costs, δ_o decreases, there would be a decrease in the duration of strikes. Suppose there was a reduction in the cost of the capital good, K, for an industry. This would be a positive shock that would result in increased wage rates and product price as well as a decreased number of strikes and increased duration. These give the counter-cyclical results on incidence that Tracy (1987) found; positive industry shocks reduce strike incidence.

One may view θ as a measure of union uncertainty. The closer θ is to zero or one, the more certain the union is as to the type of any particular firm. Since strikes are rare events, in this model that means that θ is close to one. Reduction in θ therefore increases union uncertainty. This results in decreased wages and reduced strike duration. It also increases the number of strikes in the model. Decreasing θ also reduces the expected value for the union. If one were to decrease θ and decrease $\hat{w}$ to keep union profits constant, then one just gets an increase in uncertainty and holds the expected value the same. This results in an increase in the number of strikes and reduces duration.

The last type of shock to be considered is the effect of aggregate or business cycles. If increases in general economic activity can be reflected in increases in the alternative wage and the price of the capital good, then this would result in a reduced duration of strikes and an ambiguous change in the number of strikes in an industry. This conforms with Kennan's (1986) interpretation of the literature.

Finally, note that increases in K, c_g, and c_b reduce the rents that are available to the union and the firm. The union responses by reducing the strike length and organizing more firms.

4.3 CONCLUSIONS

This thesis models strikes as a solution to an information problem on the part of a monopoly union. The setting for the union behavior is sufficiently rich to permit predictions about a wide variety of union activities including strikes. The major predictions follow directly from the role of strikes as an information-eliciting mechanism. These deal primarily with the relationship between strike distributions and wage distributions within industries. Stronger predictions follow from the very different behavior in the model of industries that contain a non-union sector in equilibrium than from those that are completely unionized. In particular, union strike behavior is independent of product demand shifts in the former case. Finally, the model permits predictions regarding alternative measures of strike activity and cross-industry correlations.

FOOTNOTES

1) Hirsch and Addision (1986) also review the literature on strikes.

2) Kennan (1986)

3) If firms are not differentiated by a simple difference in costs, then, in the first incentive compatibility constraint, the substitution for $\Pi_g(w_b)$ can not take place. But changing the way firms differ will not effect the result that, in the separating equilibrium the second individual rationality constraint will bind, ie. $\Pi_b(w_b) = 0$. Therefore, by the zero expected profit constraint $\Pi_g(w_g) = K/\theta T$. Using this in the first incentive constraint yields:

$$T\ K/\theta T = (T - s_b)\Pi_g(w_b)$$

Therefore, $s_b = T - (K/\theta)\ 1/\Pi_g(w_b)$.

Hence the optimum strike length is now affected by the b-type firm wage rate, and so will be affected by changes in the alternative wage.

$$\frac{ds_b}{dw} = \frac{-K}{\theta}\ \frac{1}{\Pi_g^{\,2}(w_b)}\ \frac{\partial \Pi_g(w_b)}{\partial w_b}\ \frac{dw_b}{dw}$$

Since increasing w_b will decrease $\Pi_g(w_b)$, the strike length will respond in the same direction as the wage rate for b-type firms.

4) Note

$$\frac{d(1-\theta)N}{d\theta} = -N + (1-\theta)\frac{dN}{d\theta} < 0, \text{ since } \frac{dN}{d\theta} < 0$$

5) This would depend on the elasticity of output demand. A less elastic demand would imply a higher percentage unionized in the low wage industry than would occur with a more elastic demand. It could be possible that the percentage unionized in the low wage industry for a give output demand would be higher than the percentage unionized in the high wage industry with the same output demand.

6) See Appendix 7 for a discussion of the multiple firm type case.

Appendix 1.

SUMMARY OF NOTATION

$f(L)$ - production function

$L(w)$ - labor demand function

K - cost of capital good

w_g - wage rate paid by firms with the g-type capital

w_b - wage rate paid by firms with the b-type capital

c_g - daily fixed cost of g-type capital

c_b - daily fixed cost of b-type capital

θ - probability of receiving the g-type capital

$\bar{T}$ - total number of days available for production

s_g - strike length for firms with g-type capital

s_b - strike length for firms with b-type capital

N - number of firms organized by the union

$\Pi_i(w_j)$ - daily profit of an i-type firm paying w_j

Π - total profit of the union

$C(N,L(w))$ - daily cost attributable to having organized N firms, each employing $L(w)$ workers

Appendix 2

CONTINUOUS TIME MODEL

This appendix outlines a continuous time version of the model in the text in which both the firms and union maximize discounted future profits. The union must decide how much of the entire production period to work. It could do this in several ways. The union could choose to not work for some length of time at the beginning, or not work at some point in the middle of the period, or work at some fraction of full-time for some or all of the period. The union must decide this for each type of firm. Let α_j be the fraction of each period that the union works in a j-type firm and $\alpha_j \in [0,1]$, where $j = g, b$.

The union's problem is then

$$\max_{\substack{\alpha_g, \alpha_b, \\ w_g, w_b, N}} \int_0^T \{\alpha_g[\theta N L(w_b)(w_g - \bar{w}) - \theta C(N, L(w_g))] + \alpha_b[(1-\theta) N L(w_b - \bar{w}) - (1-\theta) C(N, L(w_b))]\} e^{-\rho t} dt$$

Where ρ is the discount rate of the union. The constraints the union faces are the zero ex ante profit condition for union firms,

1) $$K = \int_0^T \alpha_g \theta \Pi_g(w_g) e^{-rt} dt + \int_0^T \alpha_b (1-\theta) \Pi_b(w_b) e^{-rt} dt$$

where $\Pi_j(w_j)$ is the instantaneous profit for a j type firm and r is the discount rate for the firm, and the incentive compatibility constraints

2) $$\int_0^T \alpha_g \Pi_g(w_g) e^{-rt} dt \geq \int_0^T \alpha_b \Pi_g(w_b) e^{-rt} dt,$$

3) $$\int_0^T \alpha_b \Pi_b(w_b) e^{-rt} dt \geq \int_0^T \alpha_g \Pi_b(w_g) e^{-rt} dt,$$

and the individual rationality constraints $\Pi_g(w_g) \geq 0$, and $\Pi_b(w_b) \geq 0$.

The problem is now an optimal control problem with isoperimetric constraints. The present value Hamiltonian is:

$$\begin{aligned} H = \; & \alpha_g[\theta NL(w_g)(w_g - \bar{w}) - \theta C(N, L(w_g))]e^{-\rho t} \\ & + \alpha_b[(1-\theta)NL(w_b - \bar{w}) - (1-\theta)C(N, L(w_b))]e^{-\rho t}dt \\ & + \lambda_0[\alpha_g \theta \Pi_g(w_g)e^{-rt} - \alpha_b(1-\theta)\Pi_b(w_b)e^{-rt}] \\ & + \lambda_1[\alpha_g \Pi_g(w_g)e^{-rt} - \alpha_b \Pi_g(w_b)e^{-rt}] \\ & + \lambda_2[\alpha_b \Pi_b(w_b)e^{-rt} - \alpha_g \Pi_b(w_g)e^{-rt}]. \end{aligned}$$

Since the Hamiltonian is linear in the controls α_g and α_b, this will have a "bang-bang" solution. The union will set α_j equal to one or zero at any point in time. The union will either be working full-time or not working at all, at any point in time. Also, the union would switch from either one to zero, or zero to one at most once, at some critical time, t^*. The length of time that the union sets $\alpha_j = 0$ will be the strike at the j-type firms. As in the text, the incentive compatibility constraints imply the strike at g-type firms will be shorter than the strike at b-type firms. Hence w_g will be greater than or equal to w_b.

The union's problem can now be written as the following

Lagrangian (the non-negativity of wage rates, the number of firms, and the firms' daily profits will be assumed):

$$L = H + \mu_1\alpha_g + \mu_2(1-\alpha_g) + \eta_1\alpha_b + \eta_2(1-\alpha_b)$$

where μ_1, μ_2, η_1, η_2 satisfy complementary slackness. The first order conditions with respect to α_g and α_b are:

$$4)\quad L_{\alpha g} = [\theta NL(w_g)(w_g-\hat{w}) - \theta C(N,L(w_g))]e^{-\rho t} + \lambda_0\theta\Pi_g(w_g)e^{-rt} + \lambda_1\Pi_g(w_g)e^{-rt} - \lambda_2\Pi_b(w_g)e^{-rt} + \mu_1 - \mu_2 = 0$$

$$5)\quad L_{\alpha b} = [(1-\theta)NL(w_b)(w_g-\hat{w}) - (1-\theta)C(N,L(w_g))]e^{-\rho t} + \lambda_0\theta\Pi_b(w_b)e^{-rt} - \lambda_1\Pi_g(w_b)e^{-rt} + \lambda_2\Pi_b(w_b)e^{-rt} + \eta_1 - \eta_2 = 0.$$

The terms $\theta NL(w_g)(w_g-\hat{w}) - \theta C(N,L(w_g))$ and $(1-\theta)NL(w_b)(w_g-\hat{w}) - (1-\theta)C(N,L(w_g))$ are the union profits from g-type and b-type firms respectively. If either one of these terms were negative the union would not work in that type of firm. The union would set the α parameter equal to zero throughout the period. Each of these profit terms will be assumed to be positive. The other discounted terms in 4) and 5) are profits earned by the two types of firms, depending on which wage is paid. The non-negative daily profit constraints and $w_g \geq w_b$ ensure $\Pi_g(w_b) \geq 0$.

If $\Pi_g(w_b)$ is large enough, so at $t = 0$, the sum of the first four terms in 5) is negative, then η_1 must be greater than zero and by complementary slackness $\alpha_b = 0$. If $\rho < r$, that is the union discounts the future at a lower rate than firms, then this negative term in 5) would shrink in absolute value faster then the discounted union profit of dealing with b-type firms. This may, at some t^*, cause the sum of these four terms to become positive and the union would then switch to $d_b = 1$. The strike at b-type firms would then occur at the beginning of the production period.

$\Pi_b(w_g)$ may be of either sign. If $\Pi_b(w_g) \leq 0$, then the sum of the first four terms of 4) are positive for all t and so $\mu_2 > 0$; hence, $\alpha_g = 1$ for all t. Therefore the union would not strike the g-type firms. If $\Pi_b(w_g) > 0$ and is large enough, the same argument as for $\Pi_g(w_b)$ holds. That is, if $\rho < r$, then the strike will appear at the beginning of the production period.

Appendix 3

PROOF FOR CHAPTER 2

This appendix proves the union's profits, when constrained only by the zero ex ante firm profit constraint, is non-decreasing in the wage rates at equilibrium.

Let $\tilde{\Pi} = \hat{\Pi} + \mu[K - \theta(T-s_g)\Pi_g(w_g) - (T-s_b)\Pi_b(w_b)]$, which is the first two terms of the union's maximization problem. Differentiating $\tilde{\Pi}$ with respect to w_g and w_b results in:

$$\begin{aligned}\partial\tilde{\Pi}/\partial w_g = {} & \theta N(T-s_g)\ \partial L(w_g)/\partial w_g \cdot (w_g-\bar{w}) \\ & + \theta N(T-s_g)L(w_g) - \theta(T-s_g)C_2(N,L(w_g))\ \partial L(w_g)/\partial w_g \\ & - \mu\theta N(T-s_g)\ \partial\Pi_g(w_g)\ /\partial w_g;\end{aligned}$$

$$\begin{aligned}\partial\tilde{\Pi}/\partial w_b = {} & (1-\theta)N(T-s_b)\ \partial L(w_b)/\partial w_b \cdot (w_b-\bar{w}) \\ & + (1-\theta)N(T-s_b)L(w_b) \\ & - (1-\theta)(T-s_b)C_2(N,L(w_b))\ \partial L(w_b)/\partial w_b \\ & - \mu(1-\theta)N(T-s_b)\ \partial\Pi_b(w_b)/\partial w_b.\end{aligned}$$

Since the firms' profit function only differ by a daily fixed cost,

$$\partial\Pi_j(w_i)/\partial w_j = -L(w_i),\ \text{where } j = g,b,\ \text{and } i = g,b.$$

Setting the above two expressions equal to zero yields 20) and 21) of the text. These are the first order conditions in the case of perfect information where the only binding constraint is the zero total expected profits for unionized firms. These imply $w_g = \tilde{w}_b = \tilde{w}$.

Suppose w_g^*, w_b^*, s_g^*, s_b^*, N^* are the argument maximums to the union's problem 14). The first order conditions of 14) with respect to w_g and w_b will be repeated here for convenience.

15) $\partial L/\partial w_g = \partial \Pi/\partial w_g - \mu\theta(T-s_g)\ \partial \Pi_g/\partial w_g + \psi_1(T-s_g)\ \partial \Pi_g/\partial w_g$
$\quad - \psi_2(T-s_g)\ \partial \Pi_b/\partial w_g + \psi_3\ \partial \Pi_g/\partial w_g + \psi_6 = 0;$

16) $\partial L/\partial w_b = \partial \Pi/\partial w_b - \mu(1-\theta)(T-s_b)\ \partial \Pi_b/\partial w_b$
$\quad + \psi_1(T-s_b)\ \partial \Pi_g/\partial w_b - \psi_2(T-s_b)\ \partial \Pi_b/\partial w_b$
$\quad + \psi_4\ \partial \Pi_b/\partial w_b + \psi_7 = 0.$

Define $\hat{w}_g$ and $\hat{w}_b$ as the wage rates which result in the non-negative daily profit constraints binding, given the above argument maximums.

ie. $\Pi_j(\hat{w}_j) = 0$, $j = g,b$,

Note $\hat{w}_g > \hat{w}_b$ since $c_b > c_g$.

In determining $\hat{w}_g$ and $\hat{w}_b$ the equilibrium price, p^*, is needed since labor demand is a function of the price. p^* is determined from 8), the zero total expected profit constraint, given w_g^*, w_b^*, s_g^*, and s_b^*.

The following two lemmas can now be proved.

Lemma 1

The first two terms of 15) and 16) are non-negative in equilibrium.

ie. $\partial\bar{\Pi}/\partial w_g \geq 0$, and $\partial\bar{\Pi}/\partial w_b \geq 0$ in equilibrium.

PROOF

Suppose $\hat{w}_g > \hat{w}_b > \bar{w}$.

Assume $\partial\bar{\Pi}(w_g^*)/\partial w_g < 0$, and $\partial\bar{\Pi}(w_b^*)/\partial w_b < 0$.

These imply $w_g^* > \bar{w}$ and $w_b^* > \bar{w}$, from the definition of $\bar{w}$.

The union's profit could be raised by setting $w_g = w_b = \bar{w}$ and $s_g = s_b$. This still satisfies the incentive compatibility constraints and the individual rationality constraints, and results in

$\partial\bar{\Pi}/\partial w_g = \partial\bar{\Pi}/\partial w_b = 0$.

Suppose, only one of $\partial\bar{\Pi}(w_g^*)/\partial w_g$ and $\partial\bar{\Pi}(w_b^*)/\partial w_b$ is negative.

Let $\partial\bar{\Pi}(w_j^*)/\partial w_j \geq 0$ and $\partial\bar{\Pi}(w_i^*)/\partial w_i < 0$, where $j = g,b$ and $i = g,b$ and $j \neq i$. Then $w_j^* < \bar{w} < w_i^*$ and $s_j > s_i$.

Since $\partial\bar{\Pi}(w_i^*)/\partial w_i < 0$, then by $\partial L/\partial w_i = 0$ the Lagrange multiplier on the term $(T-s_i)\ \partial\Pi_j(w_i^*)/\partial w_i$ must be greater the zero, and so the incentive compatibility constraint

$$1)\quad (T-s_j)\Pi_j(w_j) \geq (T-s_i)\Pi_j(w_i)$$

binds.

Since $\partial\bar{\Pi}/\partial s_j < 0$, as long as union profits are positive, then the union profits could be raised by lowering s_j and raising w_j to keep the above incentive compatibility constraint binding. This could continue until $\partial\bar{\Pi}/\partial w_j = 0$, or until $s_j = s_i$, which- ever happens first. At $s_j = s_i$, 1) implies $\Pi_j(w_j) = \Pi_j(wi)$, which in turn implies $w_j = w_i > \tilde{w}$ and this then reduces to the case above. At $\partial\bar{\Pi}/\partial w_j = 0$, $w_j^* \leq \tilde{w}$, if 1) is still binding, that is has a non-zero Lagrange multiplier, then by $\partial L/\partial w_j = 0$, the Lagrange multiplier on the other incentive compatibility constraint is positive. Thus the other incentive compatibility constraint would also bind and so $w_j^* = w_i^* = \tilde{w}$; hence, $\partial\bar{\Pi}/\partial w_i = 0$. If, when $\partial\bar{\Pi}/\partial w_j = 0$, the Lagrange multiplier on 1) becomes zero. Since, it is assumed $\hat{w}_g > \hat{w}_b > \tilde{w}$, then $\Pi_j(\tilde{w}) > 0$; hence the multipliers on the individual rationality constraints equal zero. Because $\partial L/\partial w_j = 0$, the multiplier on the other incentive compatibility constraint must be zero. Then $\partial L/\partial w_i = 0$ would mean that $\partial\bar{\Pi}/\partial w_i = 0$ because all of the Lagrange multipliers equal zero. Therefore $w_j^* = w_i^* = \tilde{w}$.

Suppose $\hat{w}_g > \tilde{w} \geq \hat{w}_b$, then $\partial\bar{\Pi}(w_b^*)/\partial w_b$ must be non-negative to satisfy the non-negativity constraint on $\Pi_b(w_b)$. Assume $\partial\bar{\Pi}(w_g^*)/\partial w_g < 0$. This implies $w_g^* > w_b^*$, since $\partial\bar{\Pi}(w_b^*)/\partial w_b \geq 0$ implies $w_b^* \leq \tilde{w}$. Therefore by Proposition 1, $s_b > s_g$.

If $\partial\bar{\Pi}(w_g^*)/\partial w_g < 0$, then from 15) $\psi_2 > 0$. This means the second incentive compatibility constraint binds:

2) $$(T-s_b)\Pi_b(w_b) \geq (T-s_g)\Pi_b(w_g).$$

The union could be made better off by lowering s_b and raising w_b so that $(T-s_b)\Pi_b(w_b)$ remains constant until $s_b = s_g$ or $w_b = \hat{w}_b$.

If $s_b = s_g$ then 2) implies $w_b = w_g$, which means $\partial\bar{\Pi}/\partial w_g = \partial\bar{\Pi}/\partial w_b \geq 0$. If $w_b = \hat{w}_b$ then the left hand side of 2) equals zero. Therefore either $s_g = T$ or $\Pi_b(w_g) = 0$. The union profit is higher if $\Pi_b(w_g) = 0$, in which case $w_g = w_b = \hat{w}_b$ which means

$\partial\bar{\Pi}/\partial w_g = \partial\bar{\Pi}/\partial w_b \geq 0$.

Suppose $\bar{w} \geq \hat{w}_g > \hat{w}_b$, then neither $\partial\bar{\Pi}(w_g^*)/\partial w_g$ nor $\partial\bar{\Pi}(w_b^*)/\partial w_b$ could be negative in equilibrium because that would violate the individual rationality constraints.

Therefore $\partial\bar{\Pi}(w_g^*)/\partial w_g$ and $\partial\bar{\Pi}(w_b^*)/\partial w_b$ are both non-negative in equilibrium.

Q.E.D.

Corollary

$\partial\bar{\Pi}(w_b^*)/\partial w_b = 0$ and $\partial\bar{\Pi}(w_g^*)/\partial w_g = 0$ if $\hat{w}_b > \bar{w}$.

Lemma 2

$\partial\bar{\Pi}(w_b^*)/\partial w_b > 0$ and $\partial\bar{\Pi}(w_g^*)/\partial w_g > 0$ if $\hat{w}_b < \bar{w}$.

Proof

If $\hat{w}_b < \bar{w}$, then $\partial\bar{\Pi}(w_b^*)/\partial w_b > 0$, ie. $w_b^* < \bar{w}$, for individual rationality to hold for b-type firms. Lemma 1 showed if $\hat{w}_b < \bar{w}$, then $\partial\bar{\Pi}(w_b^*)/\partial w_b$ can not be negative.

Assume $\partial\bar{\Pi}(w_g^*)/\partial w_g = 0$, then $w_g^* = \bar{w}$. Then by 15) either

i) $\psi_1 = \psi_2 = 0$ and the incentive compatibility constraint may or may not bind, or

ii) $\psi_1 = \psi_2 \neq 0$ and $\psi_3 = 0$, and the incentive compatibility constraint bind and $\Pi_g(\bar{w}) \geq 0$, or

iii) $\psi_2 > \psi_1 > 0$ and $\psi_3 > 0$, and both incentive compatibility constraints bind, and $\bar{w} = \hat{w}_g$, or

iv) $\psi_1 = 0$, $\psi_2 > 0$ and $\psi_3 > 0$, and the second incentive compatibility constraint binds while the first may or may not bind and $\bar{w} = \hat{w}_g$.

i) If $\psi_1 = \psi_2 = 0$, then 16) implies $\psi_4 > 0$ since $\partial\bar{\Pi}(w_b^*)/\partial w_b > 0$. $\psi_4 > 0$ means $\Pi_b(w_b^*) = 0$, hence the second incentive compatibility constraint implies either

a) $s_g^* = T$, which implies the zero total expected profit condition does not hold.

b) $\Pi_b(w_g^*) = 0$ so that $w_b^* = \bar{w}$, which violates the above $w_b^* < \bar{w}$.

c) $\Pi_b(w_g^*) < 0$, therefore $w_g^* > w_b^*$, but from 18) if $\partial\bar{\Pi}/\partial s_b < 0$. Ie. union profits from b-type firms are positive, then $\psi_9 > 0$ and so $s_b^* = 0$. However $w_g^* > w_b^*$, implies $s_b^* > s_g^*$, yet s_g^* can't be negative. Hence a contradiction.

ii) If $\psi_1 = \psi_2 \neq 0$ and $\psi_3 = 0$, then $\partial\bar{\Pi}(w_b^*)/\partial w_b > 0$ in 16) implies $\psi_4 > 0$ and hence $\Pi_b(w_g^*) = 0$ from the second incentive compatibility constraint. Hence, $s_g^* = T$, or $\Pi_b(w_g^*) = 0$ and a contradiction results.

iii) If $\psi_2 > \psi_1 > 0$ and $\psi_3 > 0$, then $\Pi_b(w_g^*) = 0$. By the first (binding) incentive compatibility constraint $(T-s_g)\Pi_b(w_b^*) = 0$ and either $s_b^* = T$, which violates the expected profits condition or $w_g^* = w_b^*$ so $\Pi_b(w_g^*) < 0$.

Hence a contradiction is shown.

iv) If $\psi_1 = 0$, $\psi_2 > 0$ and $\psi_3 > 0$, then $\Pi_g(w_g^*) = 0$ and, if the first incentive compatibility constraint holds with equality, this case reduces to case iii). If the first incentive compatibility constraint doesn't bind, then this implies $\Pi_g(w_b^*) < 0$ and hence $w_b^* > w_g^*$. However $w_g^* \geq w_b^*$ from the corollary to Proposition 2, hence a contradiction is achieved.

Therefore $\partial\bar{\Pi}(w_b^*)/\partial w_b > 0$ and $\partial\bar{\Pi}(w_g^*)/\partial w_g > 0$ if $\hat{w}_b < \tilde{w}$.

Q.E.D.

Corollary

$$\partial\bar{\Pi}(w_b^*)/\partial w_b = \partial\bar{\Pi}(w_g^*)/\partial w_g = 0$$

or $$\partial\bar{\Pi}(w_b^*)/\partial w_b > 0 \text{ and } \partial\bar{\Pi}(w_g^*)/\partial w_g > 0$$

are the only sign combinations that exist in equilibrium.

Appendix 4

RANK CONDITION

This appendix shows that the Kuhn-Tucker conditions for determining the optimal values for the union's problem are appropriate.

Let x represent the vector $(w_g, w_b, s_g, s_b, N) \in R^N$.

The constraints the union faces are:

$$g_1(x) = \gamma[\theta(T-s_g)\Pi_g(w_g) + (1-\theta)(T-s_b)\Pi_b(w_b)] + (1-\gamma)[\theta T\Pi_g(\hat{w}) + (1-\theta)T\Pi_b(\hat{w})] - K \geq 0,$$

$$g_2(x) = K - [\theta(T-s_g)\Pi_g(w_g) + (1-\theta)(T-s_b)\Pi_b(w_b)] \geq 0$$

$$g_3(x) = (T-s_g)\Pi_g(w_g) - (T-s_b)\Pi_g(w_b) \geq 0,$$

$$g_4(x) = (T-s_b)\Pi_b(w_b) - (T-s_g)\Pi_b(w_g) \geq 0,$$

$$g_5(x) = \Pi_g(w_g) \geq 0,$$

$$g_6(x) = \Pi_b(w_b) \geq 0,$$

$$g_7(x) = (T-s_g) \geq 0,$$

$$g_8(x) = (T-s_b) \geq 0,$$

$$g_9(x) = Q(p) - \theta N(T-s_g)f(L(w_g)) - (1-\theta)(T-s_b)f(L(w_b)) \geq 0,$$

$$g_{10}(x) = w_g \geq 0,$$

$$g_{11}(x) = w_b \geq 0,$$

$$g_{12}(x) = s_g \geq 0,$$

$$g_{13}(x) = s_b \geq 0,$$

$$g_{14}(x) = N \geq 0.$$

The union's profit is:

$$\Pi(x) = (T-s_g)\theta NL(w_g)(w_g-\hat{w}) + (T-s_b)(1-\theta)NL(w_b)(w_b-\hat{w}) - \theta N(T-s_g)C(N,L(w_g)) - (1-\theta)N(T-s_b)C(N,L(w_b)).$$

If there exists an x^* such that $\Pi(x)$ has a local maximum at x^*, subject to $g_i(x) \geq 0$, $i = 1,\ldots,14$ and the matrix of partial derivatives $[\partial g_i(x^*)/\partial x_j]$ $i \in E$ has full rank, where E is the set of indices of effective distinct constraints at x^*, then there exists a vector ψ^* such that:

$\partial\Pi(x^*)/\partial x + \psi^* \partial g(x^*)/\partial x = 0$, $g_i(x^*) \geq 0$ and $\psi_i^* g_i(x^*) = 0$, where $i = 1,\ldots,14$.

If the matrix of partial derivatives of effective distinct constraints has linearly independent rows when evaluated at the optimum values, then the first order conditions are satisfied and the Lagrange multipliers satisfy complementary slackness. This is the rank condition of the Arrow-Hurwicz-Uzawa Theorem. See Takayama (1985).

For each of the equilibria, the above condition must be checked. In the case of incomplete union coverage and the separating equilibrium, the effective constraints are g_2, g_3, g_6 and g_{12}. The matrix of partial derivatives evaluated at this equilibrium is:

$$\begin{bmatrix} \theta NTL(w_g) & (1-\theta)N(T-s_g)L(w_g) & \theta N\Pi_g(w_g) & (1-\theta)N\Pi_b(w_b) & -\theta T\Pi_g(w_g) + (1-\theta)(T-s_b)\Pi_b(w_b) \\ -TL(w_g) & (T-s_g)L(w_b) & -\Pi_g(w_g) & \Pi_b(w_b) & 0 \\ 0 & -L(w_b) & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

This matrix has full rank. Therefore the procedure in the text is valid.

In the case of incomplete union coverage and the pooling equilibrium, the effective constraints are g_2, g_3, g_4, g_{12} and g_{13}; but g_3 and g_4 are not distinct. In the pooling equilibrium $w_g^* = w_b^* = w^*$.

The matrix to be checked is:

$$\begin{bmatrix} \theta NTL(w) & (1-\theta)NTL(w) & \theta N\Pi_g(w) & (1-\theta)N\Pi_b(w) & \begin{matrix} -\theta T\Pi_g(w) \\ -(1-\theta)T\Pi_b(w) \end{matrix} \\ -TL(w) & TL(w) & -\Pi_g(w) & \Pi_g(w) & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

This matrix has full rank. It is possible that at the wage w^*, the high cost firm would make zero daily profits. This would mean that g_6 held with equality, but this constraint would have a zero Lagrange multiplier because the constraint is not binding.

The cases of both a pooling equilibrium and a separating equilibrium under complete union coverage may also be checked. In these cases the first two constraints, g_1 and g_2 bind, but they are not distinct. In both cases, the matrix of partial derivatives has full rank. The pooling equilibrium under complete union coverage is the equilibrium of a monopoly union with costs of organizing a competitive industry. The separating equilibrium under complete unionization is a version of Hayes (1984), with free entry and multiple firms.

Appendix 5

COMPARATITIVE STATIC DERIVATIONS

This appendix derives the comparative statics results of Chapter 3.

Recall that the labor demand function, for both types of firms is:

$L(w) = f'^{-1}(w/p)$, where both w and p are endogenous. In order to remove p from the labor demand function, the following substitution is employed.

Let h(Q) be the amount of labor required to produce Q. Therefore $h(Q) = f^{-1}(Q)$, where $h' > 0$ and $h'' > \epsilon$, for some $\epsilon > 0$.

In the separating equilibrium the b-type firms earn zero profits, therefore b-type firms' output is defined by:

1) $$q(w_b, c_b) = \operatorname{argmin} \frac{c_b + w_b h(Q)}{Q}$$

the quantity which minimizes b-type firms average cost, since the firms are profit maximizing. The product price, p, equals the minimum of the b-type firms' average cost curve. Therefore,

2) $$p = \frac{c_b + w_b\ h(q(w_b, c_b))}{q(w_b, c_b)}$$

and since the firm is profit maximizing, price equals marginal cost. Also high state firms are profit maximizing, so $p = w_g h'(Q_g)$ and the level of profits for type g firms is known to be $K/\theta T$. Therefore:

3) $$w_g h'(Q_g) Q_g - w_g h(Q_g) - c_g + K/\theta T$$

defines the output level of type g firms. Call this $q(w_g, c_g, \theta, K/T)$ θ, K, T may be suppressed.

Now, one can write the labor demand for each type of firm as a function of the wage rate and daily cost.

4) $L(w_j) = h(q(w_j, c_j))$, where $j = g, b$.

Before moving to the comparative statics some preliminary definitions will be useful.

Let $A = N(w_g - \hat{w}) - C_2(N, L(w_g))$

and $B = N(w_b - \hat{w}) - C_2(N, L(w_b))$.

From the first order conditions 30) and 31) of Chapter 2, $A \geq 0$ and $B > 0$.

Let $D = L(w_g)(w_g - \hat{w}) - C_1(N, L(w_g))$

and $E = L(w_b)(w_b - \hat{w}) - C_1(N, L(w_b))$.

Define $\Pi^j = NL(w_j)(w_j - \hat{w}) - C(N, L(w_j))$. This would be the union's maximand if there were only j-type firms in the industry. The first order conditions for a maximum of this objective function are:

$$\frac{\partial \Pi^j}{\partial w_j} = N \frac{\partial L(w_j)}{\partial w_j}(w_j - \hat{w}) + NL(w_j) - C_2(N, L(w_j)) \frac{\partial L(w_j)}{\partial w_j} = 0$$

$$\frac{\partial \Pi^j}{\partial N} = L(w_j)(w_j - \hat{w}) - C_1(N, L(w_j)) = 0.$$

Since $C_{11} > 0$, then $\Pi^j_{NN} < 0$ and $\frac{\partial \Pi^j}{\partial N}$ is downward sloped.

Also, $\partial(\frac{\partial \Pi^j}{\partial w_j})/\partial N < 0$ as long as C_{12} is not too large.

Given $w_b < w_g$, the optimum number of firms organized if there were only g-type firms, N_g, would be smaller than the number of firms organized if there were only b-type firms.

But the union must choose N so that

$$\frac{\partial \Pi}{\partial N} = \theta T \frac{\partial \Pi^g}{\partial N} + (1-\theta) \frac{K}{\theta\delta} \frac{\partial \Pi^b}{\partial N} = 0.$$

N is chosen so that the negative value of $\theta T \frac{\partial \Pi^g}{\partial N}$ is equal to the negative of the positive value of $(1-\theta) \frac{K}{\theta\delta} \frac{\partial \Pi^b}{\partial N}$.

Therefore $D < 0$ and $E > 0$.

Let $G = C_2(N, L(w_g)) - NC_{12}(N, L(w_g)) > 0$

and $I = C_2(N, L(w_b)) - NC_{12}(N, L(w_b)) > 0$.

These are the restrictions that C_{12} not be too large.

Let $R = (w_g - \bar{w}) - C_{12}(N, L(w_g))$

$$= \frac{1}{N} [C_2(N, L(w_g)) - NC_{12}(N, L(w_g)) + N(w_g - \bar{w}) - C_2(N, L(w_g))]$$

$$= \frac{1}{N} [G + A] > 0.$$

Likewise $V = (w_b - \bar{w}) - C_{12}(N, L(w_b))$

$$= \frac{1}{N} [I + B] > 0.$$

Let $H = N - C_{22}(N, L(w_g)) \frac{\partial L}{\partial w_g}(w_g) > 0$

and $J = N - C_{22}(N, L(w_b)) \frac{\partial L}{\partial w_b}(w_b) > 0$.

To determine the comparative statics for K, some derivatives will be needed. Recall $q(w_g, c_g, \theta, K, T)$ is the quantity produced by g-type firms. This is determined by the equation:

5) $$w_g h'q - w_g h - c_g = \frac{K}{\theta T}$$

where h(q) is the amount of labor employed to produce q.

Differentiating 5) with respect to K yields:

6) $$\frac{\partial q}{\partial K} = \frac{1}{w_g h'' q \theta T} > 0,$$

and differentiating 5) with respect to w_g yields:

7) $$\frac{\partial q}{\partial w_g} = \frac{h - h'q}{w_g h'' q} = \frac{-(c_g + K/\theta T)}{w_g^2 h'' q} < 0.$$

Differentiating 6) with respect to w_g yields:

$$\frac{\partial^2 q}{\partial K \partial w_g} = -\frac{1}{(w_g h'' q \theta T)^2}[h'' q \theta T + w_g h''' \frac{\partial q}{\partial w_g} q \theta T + w_g h'' \frac{\partial q}{\partial w_g} \theta T]$$

and substituting in 7) yields:

8) $$\frac{\partial^2 q}{\partial K \partial w_g} = \frac{-\theta T}{w_g h'' q \theta T}[h'q + (h'''q + h'') \frac{(h - h'q)}{h'' q}].$$

Also, $$\frac{\partial L(w_g)}{\partial w_g} = h' \frac{\partial q}{\partial w_g} = h' \frac{(h - h'q)}{w_g h'' q} < 0,$$

$$\frac{\partial}{\partial K}(\frac{\partial L}{\partial w_g}) = \frac{\partial}{\partial w_g}(h' \frac{\partial q}{\partial K}) = h'' \frac{\partial q}{\partial w_g} \frac{\partial q}{\partial K} = h' \frac{\partial^2 q}{\partial w_g \partial K}$$

$$= \frac{h'}{w_g h'' q \theta T} (\frac{h - h'q}{w_g h'' q})(\frac{h''}{h'} - \frac{h'' q}{h - h'q} - \frac{h'''q + h''}{h'' q})$$

and $$\frac{\partial L(w_g)}{\partial K} = h' \frac{\partial q}{\partial K} = \frac{h'}{w_g h'' q \theta T} > 0.$$

Differentiating 7) with respect to w_g yields:

9) $$\frac{\partial^2 q}{\partial w_g^2} = -(\frac{h - h'q}{w_g h'' q})^2 [\frac{h'''q + h''}{h'' q} + \frac{2h'' q}{h - h'q}]$$

and so $$\frac{\partial}{\partial w_g}(\frac{\partial L}{\partial w_g}) = h''(\frac{\partial q}{\partial w_g})^2 + h' \frac{\partial^2 q}{\partial w_g^2},$$

using 7) and 9),

$$\frac{\partial}{\partial w_g}(\frac{\partial L}{\partial w_g}) = h'(\frac{h - h'q}{w_g h'' q})^2 (\frac{h''}{h'} + \frac{2h'' q}{h - h'q} - \frac{h'''q + h''}{h'' q}).$$

Note the signs of $\frac{\partial}{\partial w_g}(\frac{\partial L}{\partial w_g})$ and $\frac{\partial}{\partial K}(\frac{\partial L}{\partial w_g})$ are ambiguous. The following relationship will be useful.

$$10)\quad \frac{\dfrac{\partial^2 L}{\partial w_g^2}}{\dfrac{\partial L}{\partial w_g}} - \frac{\dfrac{\partial^2 L}{\partial K \partial w_g}}{\dfrac{\partial L}{\partial K}} = \frac{1}{w_g} - Z - \frac{1}{w_g}$$

since $\frac{\partial L}{\partial w_g} < 0$ and $\frac{\partial L}{\partial K} > 0$ then $\frac{\partial^2 L}{\partial w_g^2} - \frac{\dfrac{\partial L}{\partial w_g}}{\dfrac{\partial L}{\partial K}} \frac{\partial^2 L}{\partial K \partial w_g} - \frac{\partial L}{\partial w_g} \frac{1}{w_g}$.

This implies a limited set of sign combinations for these two second derivatives.

i) $\frac{\partial^2 L}{\partial w_g^2} < 0$ implies $\frac{\partial^2 L}{\partial K \partial w_g} > 0$

ii) $\frac{\partial^2 L}{\partial w_g^2} \geq 0$, $\frac{\partial^2 L}{\partial K \partial w_g} > 0$

iii) $\frac{\partial^2 L}{\partial K \partial w_g} \leq 0$ implies $\frac{\partial^2 L}{\partial w_g^2} > 0$

Comparative Statics for K

Using Cramer's rule,

$$\frac{dw_g}{dK} = \frac{1}{\Delta} \begin{vmatrix} -\Pi_{gK} & \Pi_{gb} & \Pi_{gN} \\ -\Pi_{bK} & \Pi_{bb} & \Pi_{bN} \\ -\Pi_{NK} & \Pi_{Nb} & \Pi_{NN} \end{vmatrix}$$

where $\Delta <$ for a maximum, and

$$\Pi_{gg} = \theta TA \frac{\partial_2 L}{\partial w_g^2} + \theta T(2N - C_{22} \frac{\partial L}{\partial w_g}) \frac{\partial L}{\partial w_g}$$

$$\Pi_{NN} = -\theta TAC_{11}(N, L(w_g)) - (1-\theta) \frac{K}{\theta\delta} C_{11}(N, L(w_b)),$$

$$\Pi_{gK} = \theta TA \frac{\partial_2 L}{\partial w_g \partial K} + \theta T(N - C_{22} \frac{\partial L}{\partial w_g}) \frac{\partial L}{\partial K}$$

$$\Pi_{bK} = 0$$

$$\Pi_{NK} = \theta TR \frac{\partial L}{\partial K} + \frac{1-\theta}{\theta\delta} E > 0,$$

$$\Pi_{gN} = \frac{\theta T}{N} G \frac{\partial L}{\partial w_g}$$

Therefore $\frac{dw_g}{dK} = \frac{1}{\Delta} [\Pi_{bb}(\Pi_{NK}\Pi_{gN} - \Pi_{gK}\Pi_{NN}) + \Pi_{gK}\Pi_{bN}^2]$.

$$\frac{dw_b}{dK} = \frac{1}{\Delta} \begin{vmatrix} \Pi_{gg} & -\Pi_{gK} & \Pi_{gN} \\ \Pi_{gb} & -\Pi_{bK} & \Pi_{bN} \\ \Pi_{gN} & -\Pi_{NK} & \Pi_{NN} \end{vmatrix}$$

$$= \frac{1}{\Delta} (\Pi_{gg}\Pi_{NK} - \Pi_{gK}\Pi_{gN})\Pi_{bN}$$

and $\frac{dN}{dK} = \frac{1}{\Delta} (\Pi_{gg}\Pi_{NK} - \Pi_{gK}\Pi_{gN})\Pi_{bb}$.

Since $\Pi_{bN} < 0$ and $\Pi_{bb} < 0$, a sufficient condition for

$\frac{dwb}{dK} < 0$ and $\frac{dN}{dK} > 0$ is.

11) $\Pi_{gg}\Pi_{NK} - \Pi_{gK}\Pi_{gN} < 0$.

The sign of Π_{gK} is ambiguous, but it will be shown that it does not matter. Since Π_{NK} is positive, one can see that 11) is satisfied immediately if $\Pi_{gK} \leq 0$. What follows shows that 11) holds even if $\Pi_{gK} > 0$.

It will be assumed for the moment that $\frac{\partial^2 L}{\partial w_g^2} < 0$, this will be relaxed later.

Expanding $\Pi_{gg}\Pi_{NK} - \Pi_{gK}\Pi_{gN} = (\theta T)^2 AR \frac{\partial^2 L}{\partial w_g^2} \frac{\partial L}{\partial K} + \frac{\theta T(1-\theta) AE}{\theta\delta} \frac{\partial^2 L}{\partial w_g^2}$

$$+ (\theta T)^2 (N + H) R \frac{\partial L}{\partial w_g} \frac{\partial L}{\partial K} + \frac{\theta T(1-\theta)(N + H) E_g}{\theta\delta} \frac{\partial L}{\partial w_g}$$

$$- (\theta T)^2 \frac{GA}{N} \frac{\partial^2 L}{\partial w_g} \frac{\partial L}{\partial K} - (\theta T)^2 \frac{GH}{N} \frac{\partial L}{\partial w_g} \frac{\partial L}{\partial K}$$

The fourth and second terms in this expression are negative (given $\frac{\partial^2 L}{\partial w_g \partial K} < 0$).

Combining the third and sixth terms results in

$$(\theta T)^2 \frac{\partial L}{\partial w_g} \frac{\partial L}{\partial K} [(N+H)R - \frac{GH}{N}] = (\theta T)^2 \frac{\partial L}{\partial w_g} \frac{\partial L}{\partial K} [(NR + (R-G)H]$$

$$= (\theta T)^2 \frac{\partial L}{\partial w_g} \frac{\partial L}{\partial K} [NR + H((w_g - \hat{w}) - C_{12} - \frac{C_2}{N} + C_{12})]$$

$$= (\theta T)^2 \frac{\partial L}{\partial w_g} \frac{\partial L}{\partial K} [NR + \frac{HA}{N}] < 0.$$

Combining the first and fifth terms gives:

$$(\theta T)^2 [AR \frac{\partial^2 L}{\partial w_g^2} \frac{\partial L}{\partial K} - \frac{GA}{N} \frac{\partial^2 L}{\partial w_g \partial K} \frac{\partial L}{\partial w_g}]$$

$$= (\theta T)^2 A \frac{\partial L}{\partial K} \frac{\partial L}{\partial w_g} [R \frac{\frac{\partial^2 L}{\partial w_g^2}}{\frac{\partial L}{\partial w_g}} - \frac{G}{N} \frac{\frac{\partial^2 L}{\partial w_g \partial K}}{\frac{\partial L}{\partial K}}]$$

and substituting in 10), and noting $R = \frac{G}{N} + \frac{A}{N}$

$$= (\theta T)^2 A \frac{\partial L}{\partial K} \frac{\partial L}{\partial w_g} [(\frac{G}{N} + \frac{A}{N})(Z - \frac{1}{w_g}) - \frac{GZ}{N}]$$

note $Z > 0$ if $\frac{\partial^2 L}{\partial w_g^2} < 0$, by i).

$$= (\theta T)^2 A \frac{\partial L}{\partial K} \frac{\partial L}{\partial w_g} [\frac{AZ}{N} - \frac{1}{w_g} R]$$

adding back in the third and sixth terms yields:

12) $$= (\theta T)^2 A \frac{\partial L}{\partial K} \frac{\partial L}{\partial w_g} [\frac{A^2 Z}{N} + \frac{N}{w_g} \frac{AR}{N} + NR + \frac{HA}{N}]$$

$$= (\theta T)^2 A \frac{\partial L}{\partial K} \frac{\partial L}{\partial w_g} [\frac{A^2 Z}{N} + NR + \frac{A}{N}(H - \frac{N}{w_g} R)]$$

$$= (\theta T)^2 A \frac{\partial L}{\partial K}\frac{\partial L}{\partial w_g}\left[\frac{A^2 Z}{N} + NR + \frac{A}{N}\left(N - C_{22}\frac{\partial L}{\partial w_g} - N + \frac{N\bar{w}}{w_g} + \frac{NC_{12}}{w_g}\right)\right]$$

$$= (\theta T)^2 A \frac{\partial L}{\partial K}\frac{\partial L}{\partial w_g}\left[\frac{A^2 Z}{N} + NR + \frac{A}{N}\left(\frac{N\bar{w}}{w_g} + \frac{NC_{12}}{w_g} - C_{22}\frac{\partial L}{\partial w_g}\right)\right]$$

< 0 since $\partial L/\partial w_g < 0$.

If $\frac{\partial^2 L}{\partial w_g^2} \geq 0$ and if $\frac{\partial^2 L}{\partial w_g \partial K} \geq 0$,

then the second and fourth term sum

to $\frac{\theta T(1-\theta)}{\theta\delta} E\left[A \frac{\partial^2 L}{\partial w_g^2} + (N + H)\frac{\partial L}{\partial w_g}\right]$ which is less than zero,

since the term in square brackets is just Π_{gg} (which is less than zero by the second order conditions), and the first, third, fifth, and sixth terms of 11) still sum to less than zero as above.

If $\frac{\partial^2 L}{\partial w_g^2} > 0$ and if $\frac{\partial^2 L}{\partial w_g \partial K} < 0$, then there are two possible cases.

a) If $\frac{\partial^2 L}{\partial w_g \partial K} < 0$, by enough so that $\Pi_{gK} \leq 0$ then

$\Pi_{gg}\Pi_{NK} - \Pi_{gK}\Pi_{gN} < 0$.

b) If $\frac{\partial^2 L}{\partial w_g \partial K} < 0$ and $\Pi_{gK} > 0$, that is $A \frac{\partial^2 L}{\partial w_g \partial K} - H \frac{\partial L}{\partial K} > 0$,

then the first, third, fifth and sixth terms sum to

(from 12)

$$(\theta T)^2 \frac{\partial L}{\partial w_g}\left[\frac{A^2}{N}\frac{\partial^2 L}{\partial w_g \partial K} + NR\frac{\partial L}{\partial K} + \frac{A}{N}\left(\frac{N\bar{w}}{w_g} + \frac{NC_{12}}{w_g}\right)\frac{\partial L}{\partial K} - \frac{AC_{22}}{N}\frac{\partial L}{\partial w_g}\frac{\partial L}{\partial K}\right]$$

$$= (\theta T)^2 \frac{\partial L}{\partial w_g}\left[NR\frac{\partial L}{\partial K} + A\left(\frac{\bar{w}}{w_g} + \frac{C_{12}}{w_g}\right)\frac{\partial L}{\partial K} + \frac{A}{N}\left(A\frac{\partial^2 L}{\partial w_g \partial K} + C_{22}\frac{\partial L}{\partial w_g}\frac{\partial L}{\partial K}\right)\right]$$

Using $NR = G + A$

$$- (\theta T)^2 \frac{\partial L}{\partial w_g} \left[G \frac{\partial L}{\partial K} + A\left(\frac{\bar{w}}{w_g} + \frac{C_{12}}{w_g}\right) \frac{\partial L}{\partial K} + A\left(\frac{A}{N} \frac{\partial^2 L}{\partial w_g \partial K} + (N \cdot C_{22} \frac{\partial L}{\partial w_g}\right) \frac{\partial L}{\partial K} \right]$$

Which is less than zero, since $A \frac{\partial^2 L}{\partial w_g \partial K} - H \frac{\partial L}{\partial K} > 0$.

Therefore the first, third, fifth, and sixth terms sum to less than zero ($\partial L/\partial w_g < 0$).

Therefore $\Pi_{gg}\Pi_{NK} - \Pi_{gK}\Pi_{gN} < 0$, and so $\frac{dw_b}{dK} < 0$ and $\frac{dN}{dK} > 0$.

But what of $\frac{dw_g}{dK}$?

Recall $p = w_b h'(q(w_b, c_b))$.

Therefore
$$\frac{dp}{dK} = \frac{dw_b}{dK} h' + w_b h'' \frac{\partial q}{\partial w_b} \frac{dw_b}{dK}$$

$$= \left[h' + w_b h'' \frac{h - h'q}{w_b h'' q}\right] \frac{dw_b}{dK}$$

$$= \frac{h}{q} \frac{dw_b}{dK} < 0.$$

The product price falls as the gate fee rises.

Also $p = w_g h'(q(w_g, c_g, \theta, K, T))$

Therefore
$$\frac{dp}{dK} = \frac{dw_g}{dK} h' + w_g h''\left[\frac{\partial q}{\partial w_g} \frac{dw_g}{dK} + \frac{\partial q}{\partial K}\right]$$

$$= \left[h' + w_g h'' \frac{h - h'q}{w_g h'' q}\right] \frac{dw_g}{dK} + w_g h'' \frac{\partial q}{\partial K}$$

$$= \frac{h}{q} \frac{dw_g}{dK} + \frac{1}{q\theta T}$$

Let $q(w_b, c_b) = q_b$ and $q(w_g, c_g, \theta, K, T) = q_g$.

Combining the above yields:

$$\frac{h(q_b)}{q_b} \frac{dw_b}{dK} = \frac{h(q_g)}{q_g} \frac{dw_g}{dK} + \frac{1}{q\theta T}$$

Note $\frac{d}{dq}(\frac{h(q)}{q}) = \frac{qh' - h}{q^2} > 0$ (This follows from the profits of each firm in equilibrium) and $q_b > q_g$.

Therefore $\frac{h(q_b)}{q_b} > \frac{h(q_g)}{q_g}$.

There is a larger labor to output ratio in the b-type firms.

Therefore $\frac{dw_g}{dK} < \frac{dw_b}{dK} < 0$.

Since w_b falls as K rises, the product price must fall in order to maintain the zero profits of the b-type firms in the separating equilibrium. But the g-type firms must earn $K/\theta T$ in profits which rise as K rises. As the product price falls, the wage w_g must also fall, but by more than w_b.

Comparative Statics for θ

The profits for the g-type firms are:

$$w_g h'q - w_g h - c_z = \frac{K}{\theta T}.$$

Differentiating with respect to θ yields:

$$\frac{\partial q}{\partial \theta} = \frac{-K}{w_g h'' q \theta^2 T} < 0.$$

That is, g-type firm output goes down as the probability of being a g-type firm increases.

Differentiating again with respect to w_g yields:

$$\frac{\partial^2 q}{\partial \theta \partial w_g} = \frac{K\theta^2 T}{w_g h'' q \theta^2 T}[h'q + (h'''q + h'')\frac{(h - h'q)}{h''q}].$$

Also,

$$\frac{\partial}{\partial \theta}(\frac{\partial L}{\partial w_g}) = \frac{\partial}{\partial w_g}(h'\frac{\partial q}{\partial \theta}) = h''\frac{\partial q}{\partial w_g}\frac{\partial q}{\partial \theta} + h'\frac{\partial^2 q}{\partial w_g \partial \theta}$$

$$= \frac{-Kh'}{w_g h'' q \theta^2 T} \left(\frac{h - h'q}{w_g h'' q}\right)\left(\frac{h''}{h'} - \frac{h''q}{h - h'q} - \frac{h'''q + h''}{h''q}\right)$$

and
$$\frac{\frac{\partial^2 L}{\partial\theta\partial w_g}}{\frac{\partial L}{\partial\theta}} = \left(\frac{h - h'q}{w_g h'' q}\right)\left(\frac{h''}{h'} - \frac{h''q}{h - h'q} - \frac{h'''q + h''}{h''q}\right)$$

which equals $\dfrac{\frac{\partial^2 L}{\partial K \partial w_g}}{\frac{\partial L}{\partial K}}$.

Using Cramer's rule,

$$\frac{dw_b}{d\theta} = \frac{1}{\Delta} \cdot \begin{vmatrix} \Pi_{gg} & -\Pi_{g\theta} & \Pi_{gN} \\ 0 & -\Pi_{b\theta} & \Pi_{bN} \\ \Pi_{gN} & -\Pi_{N\theta} & \Pi_{NN} \end{vmatrix}$$

where

$$\Pi_{g\theta} = \theta TA \frac{\partial^2 L}{\partial w_g \partial\theta} + \theta TH \frac{\partial L}{\partial\theta},$$

$$\Pi_{b\theta} = 0,$$

$$\Pi_{N\theta} = TD - \frac{K}{\theta^2\delta} E + \theta TR \frac{\partial L}{\partial\theta}.$$

Note $\dfrac{\partial\Pi}{\partial N} = \theta TD + \dfrac{(1-\theta)K}{\theta\delta} E = 0.$

Therefore, $\Pi_{N\theta} = \dfrac{-(2-\theta)K}{\theta^2\delta} E + \theta TR \dfrac{\partial L}{\partial\theta} < 0.$

Since $\Pi_{b\theta} = 0$, $\dfrac{\partial w_b}{\partial\theta}$ may written as:

$$\frac{\partial w_b}{\partial\theta} = \frac{1}{\Delta} (\Pi_{gg}\Pi_{N\theta} - \Pi_{g\theta}\Pi_{gN})\Pi_{bN}.$$

A sufficient condition for $\dfrac{\partial w_b}{\partial\theta} > 0$ is:

$$\Pi_{gg}\Pi_{N\theta} - \Pi_{g\theta}\Pi_{gN} > 0.$$

If $\Pi_{g\theta} \geq 0$, then the above condition holds immediately. In the same way as for the comparative statics for K, it can be shown that the above condition holds for $\Pi_{g\theta} < 0$. The difference between this case and the case for K is that in $\Pi_{N\theta}$ there is a coefficient $\frac{-(2-\theta)K}{\theta^2\delta}$ as opposed to $\frac{1-\theta}{\theta}$ in Π_{NK},

and that $\frac{\partial L}{\partial \theta} < 0$ and

whereas $\frac{\partial L}{\partial K} > 0$ and $\dfrac{\frac{\partial^2 L}{\partial w_g \partial \theta}}{\frac{\partial L}{\partial w_g}} = \dfrac{\frac{\partial^2 L}{\partial K \partial w_g}}{\frac{\partial L}{\partial K}}$.

Therefore, $$\frac{\partial^2 L}{\partial w_g^2} = \dfrac{\frac{\partial L}{\partial w_g}}{\frac{\partial L}{\partial \theta}} \frac{\partial^2 L}{\partial \theta \partial w_g} - \frac{\partial L}{\partial w_g} \frac{1}{w_g}.$$

This implies a limited set of sign combinations for these two second derivatives.

i) $\frac{\partial^2 L}{\partial w_g^2} < 0$ implies $\frac{\partial^2 L}{\partial \theta \partial w_g} < 0$

ii) $\frac{\partial^2 L}{\partial w_g^2} \geq 0$, $\frac{\partial^2 L}{\partial \theta \partial w_g} < 0$

iii) $\frac{\partial^2 L}{\partial \theta \partial w_g} \geq 0$ implies $\frac{\partial^2 L}{\partial w_g^2} > 0$

This is now equivalent to the problem of determining the sign of $\frac{dw_b}{dK}$. But since $\frac{\partial L}{\partial \theta}$ is of the opposite sign to $\frac{\partial L}{\partial K}$, the sign of $\Pi_{gg}\Pi_{N\theta} - \Pi_{g\theta}\Pi_{gN}$ will be positive. Therefore $\frac{dw_b}{d\theta} > 0$.

Also the same condition is sufficient for $\frac{dN}{d\theta} < 0$.

Since $\frac{dw_b}{d\theta} > 0$ then,

$$\frac{dp}{d\theta} > 0 \text{ and } \frac{h(q_b)}{q_b}\frac{dw_b}{d\theta} - \frac{h(q_g)}{q_g}\frac{dw_g}{d\theta} - \frac{K}{q_g\theta^2 T},$$

since $\frac{h(q_b)}{q_b} > \frac{h(q_g)}{q_g}$, then $\frac{dw_g}{d\theta} > \frac{dw_b}{d\theta} > 0$.

The wage rate for the g-type firms increases by more than the wage rate for the b-type firms.

Comparative Statics for T

Given 3), one can determine how the output of g-type firms varies with T,

$$\frac{\partial q}{\partial T} = \frac{-K}{w_g h'' q \theta T^2} < 0.$$

Also $$\frac{\partial^2 q}{\partial T \partial w_g} = \frac{K\theta T^2}{(w_g h'' q \theta T)^2}\left[h''q + (h'''q + h'')\frac{(h - h'q)}{h''q}\right]$$

and $$\frac{\partial L(w_g)}{\partial T} = h'\frac{\partial q}{\partial T} = \frac{-h'K}{w_g h'' q \theta T^2} < 0$$

and $$\frac{\partial^2 L}{\partial T \partial w_g} = \frac{-Kh'}{w_g h'' q \theta T^2}\left(\frac{h - h'q}{w_g h'' q}\right)\left(\frac{h''}{h'} - \frac{h''q}{h - h'q} - \frac{h'''q + h''}{h''q}\right).$$

Therefore, the following second derivatives can be determined:

$$\Pi_{gT} = \theta TA \frac{\partial^2 L}{\partial w_g \partial T} + \theta TH \frac{\partial L}{\partial T},$$

$$\Pi_{bT} = 0,$$

$$\Pi_{NT} = \theta D + \theta TR \frac{\partial L}{\partial T}.$$

Since, $\theta TD + \frac{(1-\theta)}{\theta\delta} K_i E = 0.$

$$\Pi_{NT} = -\frac{(1-\theta)}{\theta\delta}\frac{K}{T}E + \theta TR\frac{\partial L}{\partial T} < 0.$$

Now,

$$\frac{dw_b}{dT} = \frac{1}{\Delta}\begin{vmatrix} \Pi_{gg} & -\Pi_{gT} & \Pi_{gN} \\ 0 & 0 & \Pi_{bN} \\ \Pi_{gN} & -\Pi_{NT} & \Pi_{NN} \end{vmatrix}$$

$$= \frac{1}{\Delta}(\Pi_{gg}\Pi_{NT} - \Pi_{gT}\Pi_{gN})\Pi_{bN}$$

Therefore $\Pi_{gg}\Pi_{NT} - \Pi_{gT}\Pi_{gN} > 0$ is sufficient for $\frac{dw_b}{dT} > 0$.

This case is equivalent to the case for θ. The same procedure ensures that $\frac{dw_b}{dT} > 0$, $\frac{dN}{dT} < 0$, and $\frac{dp}{dT} > 0$.

Again, a relative size prediction is possible for w_g:

$$\frac{dw_g}{dT} > \frac{dw_b}{dT} > 0.$$

Comparative Statics for c_g

Following along as before, differentiating 3) with respect to c_g yields $\frac{\partial q}{\partial c_g} = \frac{1}{w_g h'' q}$ and it can be shown

$$\frac{\partial L}{\partial c_g} > 0, \text{ and } \frac{\frac{\partial^2 L}{\partial w_g \partial c_g}}{\frac{\partial L}{\partial w_g}} = \frac{\frac{\partial^2 L}{\partial K \partial w_g}}{\frac{\partial L}{\partial K}}.$$

Also, $\Pi_{gc_g} = \theta TA\frac{\partial^2 L}{\partial w_g \partial c_g} + \theta TH\frac{\partial L}{\partial c_g}$,

$\Pi_{bc_g} = 0.$

$$\Pi_{Nc_g} = \frac{(1-\theta)}{\theta\delta^2}K E + \theta DR\frac{\partial L}{\partial c_g} < 0.$$

This case is equivalent to that of changes in K and, in the same fashion, it can be shown that:

$$\Pi_{gg}\Pi_{Nc_g} - \Pi_{gc_g}\Pi_{gN} < 0$$

which is sufficient for $\frac{dw_g}{dc_g} < 0$ and $\frac{dN}{dc_g} > 0$.

Also, $\frac{dp}{dc_g} = \frac{h(q_b)}{q_b}\frac{dw_b}{dc_b} < 0$, and a relative size prediction is possible: $\frac{dw_g}{dc_g} < \frac{dw_b}{dc_g} < 0$.

As the daily cost for g-type firms reises the output price falls. In order for g-type firms to earn $\frac{K}{\theta T}$ daily, the wage w_g must fall by more than w_b, since b-type firms remain earning zero daily profits and their daily cost has not changed.

Comparative Statics for c_b

This case is slightly different from the other cases and will be presented in more detail. The b-type firms output $q(w_b, c_b)$ is determined by:

$$w_b h'(q)q - w_b h - c_b = 0.$$

Differentiating this with respect to c_b yields

$$\frac{\partial q}{\partial c_b} = \frac{1}{w_b h'' q} > 0,$$

hence $\frac{\partial L}{\partial c_b} = h'\frac{\partial q}{\partial cb} > 0.$

Also, $$\frac{\partial^2 q}{\partial c_b{}^2} = \frac{1}{w_b h'' q}\{h''q + (h'''q + h'')\frac{(h - h'q)}{h''q}\}$$

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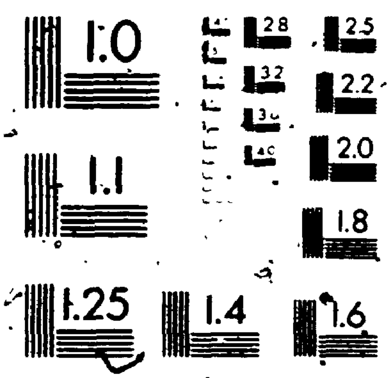

and $\frac{\partial^2 L}{\partial c_b \partial w_g} = \frac{h'}{w_g h'' q} \cdot (\frac{h - h'q}{w_g h'' q})(\frac{h''}{h'} - \frac{h''q}{h - h'q} - \frac{h'''q + h''}{h''q})$.

Therefore, $\frac{\frac{\partial^2 L}{\partial c_b \partial w_g}}{\frac{\partial L}{\partial c_b}} = (\frac{h - h'q}{w_b h'' q})(\frac{h''}{h'} - \frac{h''q}{h - h'q} - \frac{h'''q + h''}{h''q})$.

and $\frac{\partial^2 L}{\partial w_b^2} = \frac{h'(h - h'q)}{w_b h'' q} (\frac{h - h'q}{w_b h'' q})(\frac{h''}{h'} - \frac{2h''q}{h - h'q} - \frac{h'''q + h''}{h''q})$;

hence, $\frac{\frac{\partial^2 L}{\partial w_b^2}}{\frac{\partial L}{\partial w_b}} = \frac{\frac{\partial^2 L}{\partial c_b \partial w_b}}{\frac{\partial L}{\partial c_b}} - \frac{1}{w_b}$.

Therefore, $\Pi_{bc_b} = (1-\theta)\frac{K}{\theta\delta} B \frac{\partial_2 L}{\partial w_b \partial c_b} + (1-\theta)\frac{K}{\theta\delta}(N + J)\frac{\partial L}{\partial \theta}$

$\Pi_{gc_b} = 0$,

$\Pi_{Nc_b} = -(1-\theta)\frac{K}{\theta\delta^2} E + (1-\theta)\frac{K}{\theta\delta}(N + J)\frac{\partial L}{\partial \theta}$.

$\Pi_{bb} = (1-\theta)K \frac{B}{\theta\delta} \frac{\partial_2 L}{\partial w_b^2} + (1-\theta)\frac{K}{\theta\delta}(N + J)\frac{\partial L}{\partial w_b}$

By Cramer's rule,

$$\frac{dw_g}{dc_b} = \frac{1}{\Delta} \begin{vmatrix} 0 & 0 & \Pi_{gN} \\ -\Pi_{bcb} & \Pi_{bb} & \Pi_{bN} \\ -\Pi_{Ncb} & \Pi_{Nb} & \Pi_{NN} \end{vmatrix}$$

$$= \frac{1}{\Delta}(\Pi_{bb}\Pi_{Nc_b} - \Pi_{bc_b}\Pi_{bN})\Pi_{gN}.$$

Since $\Pi_{gN} < 0$ and $\Delta < 0$ the sign of $\frac{dw_g}{dc_b}$ equals

the sign of $\Pi_{bb}\Pi_{Nc_b} - \Pi_{bc_b}\Pi_{bN}$.

Expanding this term gives:

13) $\Pi_{bb}\Pi_{Nc_b} - \Pi_{bc_b}\Pi_{bN} = -(1-\theta)^2 \frac{K^2}{(\theta\delta)^2} \frac{E}{\delta} \left(B\frac{\partial^2 L}{\partial w_b^2} + (N+J)\frac{\partial L}{\partial w_b}\right)$

$$+ (1-\theta)^2 \frac{K^2}{(\theta\delta)^2} V \left(B\frac{\partial^2 L}{\partial w_b^2} + (N+J)\frac{\partial L}{\partial w_b}\right)$$

$$- (1-\theta)^2 \frac{K^2}{(\theta\delta)^2} \frac{I}{N} \partial L \left(B \frac{\partial^2 L}{\partial w_b \partial c_b} + J \frac{\partial L}{\partial w_b}\right)$$

Let $Y = \frac{\frac{\partial^2 L}{\partial w_b^2}}{\frac{\partial L}{\partial w_b}} = \frac{\frac{\partial^2 L}{\partial c_b \partial w_b}}{\frac{\partial L}{\partial c_b}} - \frac{1}{w_b}$;

and note $V = \frac{B}{N} + \frac{I}{N}$

Therefore 13) equals

$$-(1-\theta)^2 \frac{K^2}{(\theta\delta)^2} \frac{\partial L}{\partial w_b} \left[\frac{E}{\delta}(BY + N + J) - \left(\frac{B}{N} + \frac{I}{N}\right)(BY + N + J) \frac{\partial L}{\partial c_b} \right.$$

$$\left. + \frac{I}{N}\left(BY + \frac{B}{w_b} + J\right)\frac{\partial L}{\partial c_b}\right]$$

$$= -(1-\theta)^2 \frac{K^2}{(\theta\delta)^2} \frac{\partial L}{\partial w_b} \left[\left(\frac{E}{\delta} - \frac{B}{N}\frac{\partial L}{\partial c_b}\right)(BY + N + J) - \frac{I}{N}\left(\frac{N\hat{w}}{w_b} + \frac{C_2}{w_b}\right)\frac{\partial L}{\partial c_b}\right]$$

Note $\Pi_{bb} < 0$ implies $BY + N + J > 0$.

Therefore, if

14) $\frac{E}{\delta} < \frac{B}{N}\frac{\partial L}{\partial c_b}$

then $\Pi_{bb}\Pi_{Nc_b} - \Pi_{bc_b}\Pi_{bN} > 0$.

Condition 14) states that the union's marginal profit from a b-type firm, divided by the difference in costs, must be lower than the marginal profit of having an extra worker per firm in the b-type firms, multiplied by the increase in workers per firm from an increase in c_b. This condition is sufficient for $\Pi_{Nc_b} > 0$.

It does not seem clear what this condition means. I think the reason why it is needed is that one cannot be sure whether the union's marginal cost, $C_1(N, L(w_b))$, is greater or less than the average cost of dealing with b-type firms. This is not the case for the g-type firms and can be shown:

$$C_1(N, L(w_g)) > C(N, L(w_g))/N.$$

With condition 14), then $\frac{dw_g}{dc_b} < 0$ and $\frac{dN}{dc_b} > 0$.

Now $p = w_g h'(q(w_g, c_g, \theta, K, T)$ since the low cost firms are profit maximizing.

Therefore $\frac{dp}{dc_b} = \frac{dw_g}{dc_g} \frac{h(q_g)}{q_g}$

Since high cost firms are also profit maximizing, then $p = w_b h'(q(w_b, c_b))$ and hence

$$\frac{dp}{dc_b} = \frac{dw_b}{dc_b} \frac{h(q_b)}{q_b} + \frac{1}{q_b},$$

Therefore $$\frac{h(q_b)}{q_b} \frac{dw_b}{dc_b} + \frac{1}{q_b} = \frac{h(q_g)}{q_g} \frac{dw_g}{dc_b}$$

since $\frac{dw_g}{dc_b} < 0$ then $\frac{dw_b}{dc_b} < 0$.

No relative size prediction is available in this case.

Finally in order to generate the relative size prediction for $\hat{w}$, note $\frac{dp}{d\hat{w}} = \frac{h(q_b)}{q_b} \frac{dw_b}{d\hat{w}}$, and $\frac{dp}{d\hat{w}} = \frac{h(q_g)}{q_g} \frac{dw_g}{d\hat{w}}$.

Therefore $$\frac{h(q_b)}{q_b} \frac{dw_b}{d\hat{w}} = \frac{h(q_g)}{q_g} \frac{dw_g}{d\hat{w}}$$

and $\frac{h(q_b)}{q_b} > \frac{h(q_g)}{q_g}$; hence $\frac{dw_g}{d\hat{w}} > \frac{dw_b}{d\hat{w}}$.

APPENDIX 6

FRACTION OF MAN-DAYS LOST

This appendix generates the change in the fraction of man-days lost as a result of a change in the alternative wage.

The fraction of man-days lost is $\dfrac{s_b(1-\theta)L(w_b)}{T[(1-\theta)L(w_b) + \theta L(w_g)]}$

1) $$\frac{d}{d\bar{w}} \frac{[s_b(1-\theta)L(w_b)]}{T[(1-\theta)L(w_b) + \theta L(w_g)]}$$

$$= \frac{s_b(1-\theta)\theta}{T[(1-\theta)L(w_b) + \theta L(w_g)]^2}\left[L(w_g)\frac{dL(w_b)}{d\bar{w}} - L(w_b)\frac{dL(w_g)}{d\bar{w}}\right]$$

since $\dfrac{ds_b}{d\bar{w}} = 0$

Note that the derivative of the number of workers on strike divided by the unionized labor force, with respect to the alternative wage, reduces to a similar expression.

Using the notation of Appendix 5, $L(w_g) = h_g$ and $L(w_b) = h_b$ and $\dfrac{dL(w)}{d\bar{w}} = h' \dfrac{\partial q}{\partial w} \dfrac{dw}{d\bar{w}}$ then 1) equals:

2) $$\frac{s_b(1-\theta)\theta}{T[(1-\theta)L(w_b) + \theta L(w_g)]^2}\left[h_g h'_b \frac{\partial q_b}{\partial w_b}\frac{dw_b}{d\bar{w}} - h_b h'_g \frac{\partial q_g}{\partial w_g}\frac{dw_g}{d\bar{w}}\right],$$

since $\dfrac{dw_g}{d\bar{w}} = \dfrac{h_b}{q_b}\dfrac{q_g}{h_g}\dfrac{dw_b}{d\bar{w}}$ (see the end of Appendix 5).

Also, using the approximation:

$$0 = h(0) \approx h(q) - h'(q)q + h''(q)q^2,$$

then $\dfrac{\partial q}{\partial w} = \dfrac{-q}{w}$;

therefore, 2) equals:

3) $\frac{s_b(1-\theta)\theta}{T[(1-\theta)L(w_b)+\theta L(w_g)]^2}\frac{dw_b}{d\bar{w}}\frac{h_b^2}{w_g}\frac{h_g'}{h_g}\frac{q_g^2}{q_b}[1-\frac{h_g^2w_g}{q_g^2h_g'}\frac{h_b'q_b^2}{h_b^2w_b}]$.

Knowing how $\frac{h^2w}{q^2h'}$ changes as w increases will allow one to sign the term in the square brackets. Differentiating $\frac{h^2w}{q^2h'}$ with respect to $\bar{w}$ and using the above approximation,

$h'' \approx \frac{1}{q^2}(h - h'q)$,

yields: $\frac{\partial}{\partial w}(\frac{h^2w}{q^2h'}) = h'[\frac{2h}{qh'} - 4(\frac{h}{qh'})^2 + (\frac{h}{qh'})^3]$

which is greater than zero if $\frac{h}{qh'} < 2 - \sqrt{2}$.

Therefore $\frac{\partial}{\partial w}(\frac{h^2w}{q^2h'}) > 0$,

if the elasticity of labor demand is greater than $1 + 1/\sqrt{2}$.

If labor demand is elastic enough then, since $w_g > w_b$,

$$\frac{h_g^2w_g}{q_g^2h_g'} < \frac{h_b^2w_b}{q_b^2h_b'}$$

and hence 3) is greater than zero. The fraction of man-days lost increases with the alternative wage.

If the above approximation is not accurate, the actual expression of $\frac{\partial q}{\partial w}$ is $\frac{-(h - h'q)}{wh'q}$. Using this expression in 2), it is still possible to achieve a prediction on the fraction of man-days lost. It turns out that knowing how the expression,

4) $\frac{h'}{h^2}\frac{h - h'q}{wh''}$

changes as w changes will allow one to sign the derivative of man-day's lost.

4) increases with w if $\frac{h''}{h'} - \frac{2h'}{h'} - \frac{h'''}{h''} > 0$ and this is sufficient to ensure $\frac{d}{d\hat{w}} \frac{[s_b(1-\theta)L(w_b)]}{T[(1-\theta)L(w_b) + \theta L(w_g)]} > 0$.

Appendix 7

MULTIPLE FIRM TYPES

This appendix looks at the case where the union now faces three different types of firms.

Consider the tree firm type case, good, medium and bad, with fixed daily costs, $c_g < c_m < c_b$ and associated probabilities θ_g, θ_m, and θ_b.

There are six incentive compatibility constraints:

1) $(T-s_g)\Pi_g(w_g) \geq (T-s_m)\Pi_g(w_m)$,

2) $(T-s_g)\Pi_g(w_g) \geq (T-s_b)\Pi_g(w_b)$,

3) $(T-s_m)\Pi_m(w_m) \geq (T-s_g)\Pi_m(w_g)$,

4) $(T-s_m)\Pi_m(w_m) \geq (T-s_b)\Pi_m(w_b)$,

5) $(T-s_b)\Pi_b(w_b) \geq (T-s_g)\Pi_b(w_g)$,

6) $(T-s_b)\Pi_b(w_b) \geq (T-s_m)\Pi_b(w_m)$,

and three individual rationality constraints

$\Pi_i(w_i) \geq 0$, where $i = g, m, b$.

The ex ante zero profit condition is

$$K = \theta_g[(T-s_g)\Pi_g(w_g) + \theta_m(T-s_m)\Pi_m(w_m) + \theta_b(T-s_b)\Pi_b(w_b).$$

The incentive compatibility constraints that will bind in a separating equilibrium are 1) and 4) along with the individual rationality constraint on the b-type firms; also $s_g = 0$. Given the way that firms differ, using the above binding constraints and the ex ante zero profit constraint yields the combined constraint:

$$K = \theta_g[(T-s_b)(c_b-c_m) + (T-s_m)(c_m-c_g)] + \theta_m[(T-s_b)(c_b-c_m)].$$

This may be rearranged to give

$$K = T[(\theta_g+\theta_m)c_b - \theta_g c_g - \theta_m c_m] - s_b(\theta_g+\theta_m)(c_b-c_m) - s_m\theta_g(c_m-c_g).$$

Note, if $c_m = c_g$, the above reduces to

$$K = T(\theta_g+\theta_m)(c_b-c_m] - s_b(\theta_g+\theta_m)(c_b-c_m).$$

Let $\theta_g + \theta_m = \theta$, the probability of a low cost firm, then

$$K = (T-s_b)\theta(c_b-c_g),$$

which is the condition that determines strike length in the two firm type case.

Unlike the two firm type case, in the n-type case where $n > 2$, the strike length will no longer be independent of the alternative wage. This is because the combined constraint no longer determines the strike length uniquely. The union's maximization problem now determines what the strike lengths for the different firm types will be. Since the maximization problem is affected by the alternative wage rate, the optimal strike lengths will be affected by the alternative wage. The combined constraint now only puts a restriction on the relative size of the strike lengths.

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This appendix generates the change in the fraction of man-days lost as a result of a change in the alternative wage.

The fraction of man-days lost is $\frac{s_b(1-\theta)L(w_b)}{T[(1-\theta)L(w_b) + \theta L(w_g)]}$

1) $\frac{d}{d\hat{w}} \frac{[s_b(1-\theta)L(w_b)]}{T[(1-\theta)L(w_b) + \theta L(w_g)]}$

$$= \frac{s_b(1-\theta)\theta}{T[(1-\theta)L(w_b) + \theta L(w_g)]^2} [L(w_g) \frac{dL(w_b)}{d\hat{w}} - L(w_b) \frac{dL(w_g)}{d\hat{w}}]$$

since $\frac{ds_b}{d\hat{w}} = 0$.

Note that the derivative of the number of workers on strike divided by the unionized labor force, with respect to the alternative wage, reduces to a similar expression.

Using the notation of Appendix 5, $L(w_g) = h_g$ and $L(w_b) = h_b$ and $\frac{dL(w)}{d\hat{w}} = h' \frac{\partial q}{\partial w} \frac{dw}{d\hat{w}}$ then 1) equals:

2) $\frac{s_b(1-\theta)\theta}{T[(1-\theta)L(w_b) + \theta L(w_g)]^2} [h_g h'_b \frac{\partial q_b}{\partial w_b} \frac{dw_b}{d\hat{w}} - h_b h'_g \frac{\partial q_g}{\partial w_g} \frac{dw_g}{d\hat{w}}]$,

since $\frac{dw_g}{d\hat{w}} = \frac{h_b}{q_b} \frac{q_g}{h_g} \frac{dw_b}{d\hat{w}}$ (see the end of Appendix 5).

Also, using the approximation:

$$0 = h(0) \approx h(q) - h'(q)q + h''(q)q^2,$$

then $\frac{\partial q}{\partial w} = \frac{-q}{w}$;

therefore, 2) equals: