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Models Of Business Cycles With Endogenous Technology

George Werner Stadler

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MODELS OF BUSINESS CYCLES WITH ENDOGENOUS TECHNOLOGY

by

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ABSTRACT

Traditionally, theories of the business cycle have assumed that technological change is exogenous to the economic process, although there is considerable evidence that changes in technology depend on economic factors. This thesis examines the implications of endogenous technology for business cycle theory. It constructs two basic general models of output fluctuations. In the first model technical knowledge advances through learning by doing and in the second case there exists an innovation production function and technical progress depends on R&D. Nested within the two general models are both real and monetary business cycle models, so enabling comparisons between monetary models with endogenous technology and conventional monetary models (with exogenous technology), between real business cycle models with endogenous technology and conventional real business cycle models, and between monetary models with endogenous technology and conventional real business cycle models.

The major findings are as follows. Firstly, there is a long-run non-neutrality of money in models with endogenous technology, because monetary innovations can alter technology and so permanently shift the time path of output. Secondly, money is not superneutral in the models with endogenous technology, nor is the conditional probability distribution of output invariant with respect to changes in the money supply rule. Thirdly, as regards the real business cycle models, if

technology is endogenous, even a temporary change in productivity can have permanent effects on output because it can change the level of technology. Finally, in both the real and monetary models with endogenous technology, output is non-stationary and exhibits a greater-than-unit root, implying that the growth rate of output increases over time. This stands in strong contrast to conventional monetary models (where the output process is stationary) and conventional real models (where output has at most a unit root). The thesis also considers the implications of wage indexation and of allowing technology to depreciate.

The conclusions of the thesis are that the influence of real and monetary shocks on the economy is very different when technology is endogenous than when it is treated as exogenous and it suggests that business cycle models that ignore the endogeneity of technology can give misleading results.

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CHAPTER 1

INTRODUCTION AND OVERVIEW

1.1 The Problem to be Addressed

The aim of this thesis is to synthesize a substantial microeconomic literature, that has shown that technology is endogenous, with macroeconomic theories of the business cycle. Thus it examines the implications of endogenous technology for business cycle theory. Its motivation is two-fold. Firstly, business cycle models to date have assumed that technical change follows some exogenous stochastic process, independent of economic conditions. There is, however, a substantial literature that suggests that such an approach is misleading, for most technological improvements stem from economic factors, with market conditions playing a crucial role. This means that technical change cannot be modelled realistically as independent of the economic process, but is an integral part of that process. The question that naturally arises is whether ignoring the endogeneity of technology has affected the predictions and results obtained from business cycle theory. It is therefore instructive to examine business cycle models that explicitly incorporate endogenous technology to see how such models compare with conventional ones.

The second motivation for this thesis arises from the dissatisfaction with existing monetary theories of the business cycle. It has become increasingly accepted that existing monetary theories do not provide adequate explanations of output fluctuations, and should be rejected in favour of real business cycle models.[1] One reason for this is that in monetary models, money is neutral in the long run. Therefore these models appear to be unable to account for the observed non-stationarity of output, that is, they cannot account for the evidence that some fraction of an innovation in output is permanent.[2] However, monetary models with endogenous technology exhibit properties that are markedly different from the monetary models of the Lucas-Barro imperfect information type or the Fischer-Taylor wage-contracting models. Thus, taking endogenous technology into account forces us to reassess both the real and the monetary models of the business cycle.

The outline of this chapter is as follows. The following section provides an overview of the current state of real and monetary business cycle theory. Section 1.3 discusses recent empirical work on the statistical characteristics of macroeconomic time series, together with the implications that this work has for the real and monetary business cycle theory. Section 1.4 surveys the literature on endogenous technology, while the final section presents an outline of the remainder of the thesis.

1.2 The Current State of Business Cycle Theory

There are currently two competing theories of output fluctuations.[3] One account is provided by the equilibrium real business cycle theory, that models output fluctuations as resulting from "real shocks" that impinge on the economy, i.e. sudden and unexpected changes in productivity and technology. A second account is furnished by the monetary business cycle theory that emphasizes the impact of unanticipated monetary changes on output. The monetary models in turn fall into two classes, namely the models first popularized by Lucas and Barro, that emphasize imperfect information, and secondly, the models originating with the work of Fischer and Taylor that have stressed price and wage rigidities.

We begin with the monetary models that feature imperfect information.[3] This work originated with Lucas, and sees the business cycle as the outcome of optimizing behaviour on the part of economic agents acting in a situation of incomplete information. Agents possess rational expectations and markets clear, but changes in nominal aggregate demand affect real activity because agents lack contemporaneous information about aggregate variables like the general price level or the money stock. Consequently, they cannot distinguish nominal from relative price movements. An increase in nominal demand is, to some degree, misperceived as real, and results in an increase in employment and output.

These models predict that only unanticipated monetary policy actions - or monetary "shocks" - should affect output; anticipated monetary changes are neutral and only result in prices being marked up. These predictions were tested, and the imperfect information - or New Classical models as they are also called - enjoyed initial empirical success. However, later work has cast doubt on their prediction that only unanticipated monetary changes have real effects.[4] A second, and more serious problem is that these models require that agents cannot observe certain aggregate variables except with a considerable lag. Yet in reality, these variables (the money stock, prices, nominal interest rates etc.) are available to the public with a very short lag in most developed economies. Thus, the imperfect information assumption as used by Lucas and others seems patently unrealistic in a modern economy.[5]

An alternative to the imperfect information models suggested itself in models that rely on nominal wage and price stickiness as the mechanism that enables monetary changes to have real effects. But these models, that originated with the work of Fischer and Taylor, have been criticized for resting on an ad hoc assumption: nominal wage or price inertia is not the outcome of a well-defined optimization problem on the part of economic agents. Models that emphasize risk or informational asymmetries or imperfections can explain the existence of contracts specified in real terms. They cannot explain why such contracts should be specified in money terms.

Two lines of defence exist against this criticism. Firstly, attention is drawn to the widespread existence of nominal wage contracts, and to the observation that prices seem to move sluggishly. A second justification is the argument by McCallum (1986,1987) that, for the individual agent, the benefits of indexation may be too small to justify the cost. Akerlof and Yellen (1987) propose a similar argument, that the costs of following rules of thumb, rather than strategies based on optimization, are second-order in magnitude. However, such "near-rational" behaviour can have a sizeable impact at the macro level. Yet it is clear that these arguments lack the rigour and appeal of choice functions derived from well articulated optimization problems set in models that have a clearly defined environment.

The real business cycle models do meet the aforementioned criterion of deriving choice functions from explicit optimization problems, and have aroused considerable interest.[6] This class of models, like the imperfect information models examined above, is based on the principle that aggregate fluctuations are consistent with the rational, maximizing behaviour of individual agents. Hence they start by postulating the optimization problem that an average, representative agent faces. The environments these agents inhabit are subject to random changes, in particular to temporary or permanent changes in productivity or in the technology at their disposal. These shocks to productivity and technology do not result from any actions of the part of economic agents, nor are they dependent upon any economic variables, but are exogenous to the economic system. However, as we shall see below, in reality most changes in technology do <u>not</u> come about through random changes in the underlying scientific base. This notwithstanding, real business cycle models have shown that models based on individual optimisation and subject to real shocks can mimic the time series movements observed in reality quite well.

A crucial feature of these real business cycle models is that they can imitate cyclical activity without introducing money. They explicitly or implicitly deny that monetary policy actions have a significant effect on real variables like output or employment. King and Plosser (1984) seek to explain the pro-cyclical movements in the money stock by a reverse-causation argument: a rise in output increases the demand for monetary and financial services, and so raises the quantity of (inside) money and credit.

Real business cycle theory is not without its critics. For example, Summers (1986) has raised the question of what the real-world counterpart of the technology disturbances, or shocks, that buffet the model economies, is. Prescott (1986a), amongst others, says that the technology shocks can be thought of as residuals from an aggregate production function - an approach first developed by Solow (1957). However, taking this approach ignores the long and heated debate on the theory of capital and the aggregate production function that occupied much of the 1950s and 1960s.[7] The relevance of the debate for real business cycle theory is that an aggregate production function only exists under stringent conditions. For example, Solow (1957) assumed constant-returns-to-scale, perfect competition, so that paying factors their marginal products exhausted the total product, and neutral technical progress that affects all capital goods equally (capital is malleable). These conditions do not hold in reality. In addition, obtaining a reliable time series for the annual flow of capital services is difficult, as Solow admits.[8] This means that the residuals Solow estimates may not be reflections of technical change but rather of errors that result from firstly, errors in the measurement of inputs and secondly, from imposing an inappropriate aggregate relationship on the data.[9] Clearly, residuals from an aggregate production function should be treated with due caution.

In conclusion, there exist two separate strands of theory capable of accounting for output fluctuations. The real business cycle theory focuses on exogenous disturbances to the supply-side of the economy as the impulse mechanism generating cycles. The monetary theory sees cycles resulting from innovations in aggregate demand, primarily from unanticipated changes in monetary policy to which prices do not adjust immediately. Recent empirical work on the properties of macroeconomic time series has resulted in the monetary theories falling into disfavour to some extent. In the next section we review this empirical work that has led to a re-evaluation of monetary (and real) business cycle theory.

1.3 Empirical Evidence on Output Fluctuations

The 1980s has seen the publication of a number of studies that have radically altered our view of the statistical characteristics of the major macroeconomic time series, and output in particular. Traditionally, it was common practice in macroeconomics to decompose real variables into a secular or growth component (dependent on real factors such as demographic changes, capital accumulation and technical change, and often modelled as a time trend) and a cyclical component that was assumed to be transitory (and hence stationary) and that depended on monetary, and to a lesser extent, real factors as primary causes. In this case output is described by a model like:-

$$(1.1) \qquad y_t = bt + \sum_{j=0}^{\infty} a_j v_{t-j}$$

where y_t is the logarithm of real GNP, bt describes the trend or secular component of output, and v_t is a random disturbance. The cyclical component is captured by $\Sigma a_j v_{t-j}$, which is a stationary stochastic process so that fluctuations in output are temporary. This requires that the a_j approach zero as j grows. This implies that shocks to output have transitory effects: a shock today has a declining impact on GNP in future periods and does not affect the trend of output. Thus, in model (1.1), y_t is called trend stationary.

In a seminal paper, Nelson and Plosser (1982) questioned this approach. They found that firstly, most macroeconomic time series are well characterized as non-stationary processes that have no tendency to return to a deterministic time path or trend line. Since the cyclical component is stationary (its effects are transitory and dissipate over time) the non-stationarity must be attributed to the secular component of the series. The simplest model that captures the non-stationarity of a variable like output is a random walk (with upward drift):-

$$(1.2) \quad y_t = b + y_{t-1} + v_t$$

Fluctuations in a random walk are permanent. If v_t takes on a positive value, then the entire future time profile of output rises by v_t, because in model (1.2)

$$y_{t+j} = y_{t-1} + (j+1)b + v_t + v_{t+1} + \dots + v_{t+j}$$

Shocks to output have (to some degree) permanent effects, and there is no tendency for output to revert to a trend line. A random walk is non-stationary, and in model (1.2) output has a unit root, while in model (1.1) it has a less-than-unit root. Thus the distinction between a stationary and non-stationary series can also be cast in terms of whether fluctuations in the series are transitory or permanent, or whether it has a less-than-unit root or a unit (or even greater-than-unit) root, and all these distinctions are used in the literature.

The central finding of Nelson and Plosser, that most macroeconomic time series are non-stationary stochastic processes, has received support from a number of other studies. Stulz and Wasserfallen (1985) carried out similar tests for data from the United Kingdom, France, Switzerland and West Germany and corroborated the results of Nelson and Plosser. Harvey (1985) estimated various structural time series models via the Kalman filter, and rejected a model with a deterministic trend in favour of one with a stochastic trend for five major U.S. macroeconomic time series. Stock and Watson (1986) used an improved test to re-estimate Nelson and Plosser's finding that real GNP is non-stationary (i.e. that it has a unit root) and confirmed the findings of Nelson and Plosser. Thus it appears that we cannot reject the proposition that output is non-stationary. This finding poses a problem for the conventional monetary models, for, while they can account for cyclical fluctuations, they cannot account for non-stationarity.

Secondly, Nelson and Plosser claimed that movements in output were dominated by movements in the secular component. The reason for this claim was that they found the autocorrelations in the first differences of output are positive at lag one and zero elsewhere. This implies that either the variance of the secular component of output is several times as large as that of the cyclical component, and/or the cyclical and secular components are strongly <u>negatively</u>

correlated. This led them to conclude that monetary factors seem to play a negligible role in explaining output, as most output movement seems to result from changes in the secular component, while monetary changes (which in conventional models are neutral in the long run) affect only the transitory component.[10]

However, Harvey (1985) and McCallum (1986) cast doubt on the above claim. Firstly, Harvey shows that the correlogram of the first differences of the log of real U.S. GNP is very different for the postwar period than it was for the prewar period. The autocorrelation in the first differences at lag one is <u>negative</u> for the sample period 1948-1970. Nelson and Plosser found a positive value using a sample extending from 1909 to 1970. Thus the proposition that the secular component of output dominates the cyclical finds no supporting evidence in the postwar period. Secondly, McCallum makes the point that finite-sample test procedures cannot distinguish conclusively between trend-stationary and difference-stationary series. Thus, the evidence of Nelson and Plosser is compatible with a stationary output process that has a coefficient of its own lagged value of close to (but less than) unity. If output consists of stationary movements around a deterministic trend, the proposition that the variation in the secular component of output dominates changes in the cyclical component is without foundation, even in light of Nelson and Plosser's estimate of the correlogram of the first differences of output.

Like Nelson and Plosser, Campbell and Mankiw (1986,1987) tried to assess whether changes in output were mainly permanent in nature, and estimated the persistence of innovations in output. They found that a positive innovation of one percent raises the level of real GNP by one percent over any foreseeable horizon. Other studies have found less persistence. For example, Watson (1986), using an unobserved components model found the long-run impact of a shock to be about 0.6 (i.e. a one percent innovation raises the level of GNP 0.6 percent in the long run). Cochrane (1987) finds even less persistence (the degree of persistence being less than 0.4). Indeed, Cochrane finds that the time series of output can be adequately modelled as a stationary AR(2) process with a deterministic trend. He argues that the estimation strategies of Nelson and Plosser and others, who have found strong evidence of non-stationarity and a high degree of persistence, are inappropriate. This notwithstanding, there appears to be considerable consensus that some fraction of an innovation in GNP is permanent - disagreement exists, however, as to the size of that fraction. This finding is seen as evidence in favour of real business cycle models and against monetary models, because monetary disturbances have transitory effects.

A different line of research is the vector autoregression studies of Sims (1980) and Litterman and Weiss (1985). These studies found that the fraction of output variability attributable to money stock innovations is small: money does not appear to account for much of the variance of output. This was seen as further evidence against monetary models. However, McCallum (1986) argues that given the monetary policies of the Federal Reserve Board during the period under investigation, the tests of Sims and Litterman and Weiss cannot establish that money only had a small impact on output, so that their conclusions are unwarranted.

A final word must be said about the work of Campbell and Mankiw and Harvey. If a large fraction of any innovation in output is permanent, then it is not clear that one can talk of business cycles, for a cycle is a temporary movement in a variable around an underlying (stationary or non-stationary) trend. In other words, a cycle consists of transitory deviations from an underlying trend. If most changes in a variable are permanent, then one cannot really talk of that variable as exhibiting cyclical behaviour. This conclusion receives some support from the work of Harvey, who finds that for the postwar period "a faint cycle can be detected in some series, but it is difficult to justify on statistical grounds the inclusion of a cyclical component anywhere in the model".[11]

This section has examined empirical work that poses problems for monetary business cycle theory. Monetary innovations are traditionally thought of as having transitory effects. In the long run, money is neutral.[12] Thus demand-side innovations cannot account for the non-stationarity of economic time series, nor for the permanence of (some fraction of) output innovations. Supply-side innovations that permanently alter an economy's productive capacity can account for these characteristics. Consequently, the empirical work surveyed above has been interpreted by some as a test that disqualifies monetary models in favour of real business cycle models. However, this thesis examines a class of models that shows that such an interpretation of the empirical evidence would be fallacious. If technology is endogenous, then demand-side disturbances can result in changes in technology, as is demonstrated in the following chapters. Demand and supply shocks are not independent in models with endogenous technology, so that even a purely monetary model can give rise to a non-stationary process for output, a process where some fraction of innovations in output is permanent. This leads us to the question of what the evidence in favour of endogenous technology is, a question we examine below.

1.4 Endogenous Technology: A Survey of the Evidence

The literature on endogenous technology falls into two distinct parts. The first part is theoretical, while the second part consists of empirical work that attempts to determine to what extent technical change depends on economic factors.

Arrow and Kaldor have both suggested that technical change is not independent of economic conditions. In a seminal paper, Arrow (1962) suggested a theory of endogenous changes in technical knowledge. The basic premise advanced by Arrow is that "learning is the product of experience ... and therefore only takes place during activity".[13] This implies that the level of economic activity is a determinant of changes in technical knowledge. Arrow assumed that learning is a function of cumulative gross investment. Other authors have assumed learning depends on cumulative output. In either case technical knowledge is not exogenous: it depends on economic conditions. Kaldor (1957) has suggested that the rate of technical progress will be positively related to the rate of capital accumulation, as captured by his "technical progress function".[14]

The second part of the literature on endogenous technology is empirical, and originated with the pioneering work of Schmookler (1966). Schmookler regarded an innovation or invention as essentially a response to profit opportunities. He also found the rate of technological opportunity - given by the underlying scientific base - to be of little significance, because chronologies of hundreds of inventions typically reveal the stimulus to be a technical problem or opportunity conceived largely in economic terms. There is little evidence of scientific discovery initiating an invention.

Rejecting the importance of technological opportunity, Schmookler argued that invention is largely an economic activity, pursued for profit. Using data from the railroad, building and petroleum refining industries, Schmookler found a strong positive correlation between investment and capital goods invention in each industry. Investment here acts as a proxy for expected future sales and thus expected future profits. Using deviations of a seven year moving average from a seventeen year moving average, he found that the turning points in patents granted and gross capital formation fall very close together, but with patents usually lagging behind slightly. Secondly, investment at any point in time is more highly correlated with patents issued in the near future, than patents issued in the immediate past, again suggesting that investment is the cause, not the effect, of innovation, Thirdly, interindustry differences in patenting are largely explained by interindustry differences in investment. Finally, an increase in investment was found to increase capital goods inventions in that industry by about the same proportion.

Schmookler concluded that "invention is governed by the extent of the market", and that "the belief that invention, or the production of technology generally, is in most instances a noneconomic activity, is false".[15]

Schmookler's views have been criticised, for example by Rosenberg (1974), not because demand conditions are seen as unimportant, but because Schmookler understated the role of the scientific base. As regards the relevance of demand conditions for innovation and invention, Schmookler's view has received some support from a number of other studies.[16] Utterback's 1974 survey is of particular interest. He finds that "Market factors appear to be the primary influence on innovation. From 60 to 80 percent of important innovations in a large number of fields have been in response to market demands and needs. The remainder have originated in response to new scientific or technological advances ...".[17] Furthermore, "Firms tend to innovate primarily in areas where there is a fairly clear, short-term potential for profit".[18]

As regards the form of the innovation production function, there is no doubt that a positive relationship exists between the application of resources to research and the emergence of inventions and improvements. More precise evidence is scant, but what evidence exists indicates that there are no economies of scale in the innovation process. Constant, or even diminishing returns appear to be the likely characteristic of the innovation production function.[19]

This section has shown that there is cogent evidence that technology is endogenous in that it depends on economic conditions. The work of Utterback suggests that most technological change is determined by economic factors, while only around 30 percent of new innovations depend on scientific or technical discoveries that could be modelled as exogenous to the economic system. Modelling technology as exogenous consequently only captures a small part of the process of technical change, and would appear to ignore the most important determinants of technical change.

1.5 An Outline of Chapters 2 to 4

This section gives an outline of the topics addressed by the remaining chapters. Chapters 2 and 3 examine two models of output fluctuations with endogenous technology. There are two ways of introducing endogenous technology. One approach is to define an innovation production function: innovation and invention are the outcome of the application of resources to research and development. An alternative approach is to introduce learning by doing. In this case productivity will depend on cumulative output or investment.

Both these approaches suggest that an increase in demand for the firm's output will affect technology. An increase in demand will make a cost-reducing innovation or invention more profitable. It will also increase the profitability of learning that results in higher productivity. Thus the returns to research and development or learning depend on the current and expected future levels of output, which in turn depend on demand. Consequently, the evolution of technology is no longer determined solely by supply-side considerations, because demand-side conditions also affect the profitability of a new or better technology.

Chapter 2 constructs a general model in which technology evolves through learning by doing, and it contains both real and monetary shocks. Within this general model are nested four sub-models: (1) a purely monetary model with no real shocks of any kind; (2) a model with real shocks but no endogenous technology, i.e. a typical real business cycle model; (3) a model with real shocks and endogenous technology; (4) a model with transitory real shocks. The properties of each of these models are examined in turn. This enables us to make a number of comparisons, firstly, between monetary models with endogenous technology and conventional monetary models; secondly, between monetary models with endogenous technology and conventional real business cycle models;[20] thirdly, between a conventional real business cycle model and a real business cycle model with endogenous technology. It emerges that in models with endogenous technology, output contains a greater-than-unit root, and the reason for this is considered. Chapter 2 also considers whether money is neutral or superneutral in a model with endogenous technology, and whether the conditional probability distribution of output is invariant with respect to the money supply rule followed.

Chapter 3 constructs a model that contains an innovation production function. The output path obtained from this model is surprisingly similar to that of the model of Chapter 2. Again, several sub-models can be identified as nested within the general model constructed in Chapter 3. As in Chapter 2, models with endogenous technology yield a greater-than-unit root. Chapter 3 discusses whether plausible restrictions could be placed on the commodity and innovation production functions that would result in a unit root for output. The chapter also examines how the properties of a conventional real business cycle model and of the general model would change if the models are perturbed by allowing technology to depreciate, given that there is some evidence that an existing technology can be rendered ineffective if the state of nature changes. Again, the questions whether money is neutral, superneutral, or whether policy invariance holds are examined.

In the models of Chapters 2 and 3, unanticipated changes in the money supply have real effects because labour enters into one period nominal wage contracts with firms. This naturally leads to the question whether the above conclusions would hold if wages were indexed to the price level, which is considered in Chapter 4 of the thesis. This chapter also addresses the question of whether indexation can reduce the variability of output, and suggests under what conditions unindexed nominal contracts might be preferred to indexed contracts.

The results obtained from models with endogenous technology are unconventional, because, as stated above, demand-side and supply-side innovations are not independent in such models. In models with endogenous technology, a demand-side innovation can elicit a supply-side innovation by changing the profitability of developing new techniques. These models thus contain features not found in conventional business cycle models.

Footnotes

1. Even Barro (1984) has suggested that the lasting contribution of the rational expectations revolution seems to lie in the real, not monetary, business cycle theory.

2. See, for example, Nelson and Plosser (1982, p.159), Campbell and Mankiw (1986, pp.20-21).

3. Zarnowitz (1985) provides a comprehensive survey of business cycle theory from the beginning of the century to the present day.

4. Initial support for the imperfect information story was provided by Barro (1977,1978) amongst others. Later papers that cast doubt on this theory were Boschen and Grossman (1982), Gordon (1981), Mishkin (1982) and Pesaran (1982). (This is not a complete listing, as these models spawned a vast empirical literature.)

5. It is of course conceivable that in much more complex models these aggregate variables may not convey much information that is of use to individual agents, so that imperfect information environments that cause monetary shocks to have real effects cannot be ruled out, except for those modelled as crudely as Lucas's and Barro's.

6. The most important real business cycle models are Kydland and Prescott (1982), Long and Plosser (1983) and King and Plosser (1984).

7. Harcourt (1969) provides a survey of the debate.

8. See Solow (1957) p.314.

9. Jorgenson and Griliches (1967) argue that the finding that most of the rise in per capita output this century was due to technical change is the result of faulty measurement of the inputs in the aggregate production function.

10. See Nelson and Plosser, loc.cit.

11. Harvey (1985, p.225). Stulz and Wasserfallen find that "the business cycle does not persist beyond one year" (1985, p.10).

12. A class of models that exhibit long-run non-superneutralities are Tobin (1965) or Stockman (1981), where the rate of inflation affects the capital stock. In principle such models could account for non-stationarity if fluctuations in inflation rates lead to fluctuations in the capital stock. However if there are costs to adjusting the capital stock, then agents may not alter the capital stock in response to a change in inflation that is perceived as transitory, but only in response to an

expected permanent change in inflation. Froyen and Waud (1986) present a model in which money is non-neutral. They assume that the supply of labour depends on aggregate price variability. Changing to a different monetary regime can then have permanent effects on the labour supply and output.

13. Arrow, (1962), p.155.

14. A further branch of literature on endogenous technology that is of less interest to us is the work of Kennedy (1964) and von Weizsacker (1966) on the invention possibility frontier, that was developed to account for the long-run constancy of relative shares that appeared to be a stylized fact until the 1970s.

15. Schmookler (1966), p.208.

16. Kamien and Schwarz (1982) provide a survey, pp.58-64. See also Scherer (1982) and Utterback (1974).

17. Utterback (1974), p.621.

18. Utterback, ibid., (my emphasis).

19. See Kamien and Schwarz (1982), pp.55-58 and pp.64-70.

20. "Conventional" models denoting those with exogenous technology.

CHAPTER 2

MODELS WITH ENDOGENOUS TECHNOLOGY:

THE CASE OF LEARNING BY DOING

2.1 Introduction

This chapter examines the simplest case of a monetary model of output fluctuations in which technology depends on learning by doing. As Spence has said, "While learning is not an important structural feature of all industries, learning on both the demand and supply sides is believed to be a significant factor in many."[1] This chapter demonstrates that in the presence of learning, output depends on the accumulated volume of past output. It follows that in an economy characterized by learning, any rise in current output, whether originating on the demand side or on the supply side, will permanently affect the future path of output. In contrast to conventional monetary business cycle models, in models with endogenous technology the effects of expectational errors and misperceptions by households and firms are not transitory, but become imbedded in the history of the economy and have a lasting influence on the time paths of economic variables.

Once technology is endogenous, money is non-neutral, even in the long run. There is also a non-superneutrality of money in the model of this chapter, for we impose a cash-in-advance constraint on households and government. Finally, the probability distribution of output is not invariant with respect to the money supply rule followed.

It is also shown that the non-stationarity of the output process is independent of the inclusion of exogenous permanent shocks to technology. We also consider whether one could discriminate empirically between a learning model, where non-stationarity results from the endogenous nature of technology, and a model where non-stationarity results only from exogenous shocks.

The concept of technical change we adopt arises from the basic premise that learning is the product of experience and takes place during activity, especially when trying to solve a problem.[2] We follow Spence in positing that unit costs decline (productivity increases) with accumulated output or production. The rationale underlying this is that the firm operates with some resources that are fixed in the short and medium term. Higher levels of production result in increased efficiency, as this requires the fixed factors to be used more intensively, and so raises the incentive to eliminate waste, to develop cost-reducing innovations and to solve bottlenecks that might develop at higher levels of production, so that efficiency is a monotonically increasing function of the level of output. Furthermore, in discussing the literature on learning, Kennedy and Thirlwall (1972) conclude that "since product types are constantly changing it is probably safe to assume that in the aggregate there is no limit to the learning process."[3]

The following section lays out the basic framework of a simple one-commodity general equilibrium model. Section 2.3 examines the maximization problem of the representative household and derives the aggregate demand curve. Section 2.4 examines the optimising problem of the firm and derives the aggregate supply curve. Section 2.5 specifies the money supply process and solves the time path of output for this economy. The next section compares the various real and monetary business cycle models nested in our general model and shows that a purely monetary model will exhibit non-stationarity. At least four models can be identified as nested within the general model. The first is a purely monetary model with endogenous technology. The second is a "real" business cycle model - i.e. a model with real and monetary shocks but without endogenous technology. A third model contains transitory real and monetary shocks but also has no endogenous technology. A final model has only real shocks, but also has a learning curve. This enables us to compare a monetary model with endogenous technology to a real business cycle model with exogenous technology, and also to compare a real business cycle model with endogenous technology to a real business cycle model with exogenous technology. Section 2.7 examines the questions of neutrality, superneutrality and policy invariance in the context of the general model. In this chapter learning is treated as an externality. This results in a Pareto inefficiency, and section 2.8 considers whether this inefficiency could be corrected through government intervention. The final section contains the conclusions.

2.2 The Basic Framework

We consider a simple closed economy containing two markets: an economy-wide labour market and a commodity market. There are three kinds of agents in the economy: households, firms and the government.

The supply-side of the commodity market consists of a large number of competitive firms, all producing an identical good as output. Each firm is sufficiently small in relation to the size of the market that it can ignore the effect its output decision will have on its rivals, so allowing us to abstract from strategic and game-theoretic considerations, problems of entry, etcetera. The output price is taken as given by the firm.

Households provide labour, which is the only input into the production process. The household takes the current money wage as given. Households are the owners of firms and the recipients of firms' profits. Profits are distributed according to a pre-determined pattern (there is no equity market). Both households and the government face a cash-in-advance constraint, and this determines the households' demand for money.

The government produces no tangible commodity, but consumes some of the commodity output. The government's demand for the commodity is financed by a lump-sum tax levied on households each period, and by money creation.[4] We assume that the government adheres to a money supply rule that tries to ensure price stability. This rule is public knowledge. However, there is a random component to the money supply, as the monetary authority does not always achieve its monetary target.

We consider a representative household and firm, that is, a representative unit that conforms to the behaviour of the average of the atomistic units of its kind, so allowing us to avoid aggregation problems.

Trading arrangements are sequential, and each period is divided into two sub-periods. At the beginning of each period, during the first sub-period, denoted by t_0, households enter into a one-period nominal wage contract with firms. However, agents lack complete information at the time that the wage is set. In particular, they do not know the general price level that will prevail during the second sub-period when trading takes place.

Wages are not indexed to the price level. In Chapter 4 we consider how the conclusions we obtain here would be modified through the introduction of wage indexation, and under what circumstances wage indexation would reduce the variance of output.

In the second sub-period (denoted by t_1), the production process takes place, and households and government make their consumption decision. The output process experiences a composite productivity shock, F_t, each period, part of which is transitory and part of which is permanent. We assume that agents can distinguish the permanent from the transitory component, and denote the permanent component as $\bar{F}_t$. The value of this productivity shock is revealed at the start of the production process. The production and consumption decisions are made simultaneously, and the price level that clears the commodity market is determined.

The structure of the model and the information sets in each sub-period are shown in table 1.

TABLE 1: INFORMATION SETS

Sub period	Information	Decision Variable	Outcome
t_o	$I_{ot} = \{I_{t-1}, T_t\}$	W_t	W_t
t_1	$I_t = \{I_{ot}, P_t, M_t, \bar{F}_t, F_t, G_t\}$	L_t, C_t	C_t, Y_t, M_t^h

Notation is shown in Appendix 1. I_{ot} includes all past information, I_{t-1}, as well as current lump-sum taxes, T_t, which are announced by the government one period in advance. I_t includes the current output price, government spending, the level of the money stock and the current shock to technology, F_t, as well as its permanent component, $\bar{F}_t$.

The money wage is set on the basis of I_{ot}, but the production and consumption decisions are based on I_t. This attempts to capture the fact that money wages are generally pre-set for a fixed period of time. In reality, wage adjustments for inflation are usually made ex post, except in countries with severe inflations, where wage indexation is common, an alternative we examine in Chapter 4.

Table 2 shows the financial flows within the economy. It is assumed that profits are only distributed after the commodity market has closed, i.e. Π_{t-1} is distributed at time t_o. Taxes are also levied at the beginning of the period. Payment for labour services and for the purchases of consumption goods takes place during t_1. Households use their money balances to purchase a flow of commodities from the firm, and, at the end of the period, receive a wage income from the firm. There is no trading at false prices in the commodity market.

2.3 The Household

The representative household aims to maximize its discounted stream of utility. Each period it faces a two-stage decision problem. At the start of the period, during the first sub-period, it signs a nominal wage contract with the firm, basing its decision on its information set, I_{ot}. Employment is determined on the demand side of the labour market, and the household agrees to provide whatever quantity of labour the firm requires at that wage rate. The household's labour decision therefore depends on I_{ot}. During the second sub-period, t_1, the production process takes place, and the household makes its consumption and money-demand decisions based on its information set I_t.

We assume that the utility function has the form:

$$(2.1) \qquad U(C_t, L_t) = u_1 C_t - u_2 L_t \quad \text{for all } t.$$

This utility function ensures that the marginal rate of substitution between consumption and leisure is a constant. The assumption of a constant marginal utility of consumption eliminates wealth effects from the labour supply decision, and is a standard assumption. The constant marginal disutility of labour implies that the supply of labour will be infinitely elastic at a real wage equal to this marginal disutility.[5]

The nominal wage will be set such that

$$(2.2) \qquad W_t = AP_t^e E_{ot}[P_{t+1}/P_t](1 + \rho)$$

TABLE 2: FINANCIAL FLOWS WITHIN THE ECONOMY

Agent \ Time	t_o	t_1	t_{o+1}
Firms	Pay Π_{t-1} to households	(i) Receive P_tC_t from households and P_tG_t from government (ii) Pay W_tL_t to households	Pay Π_t to households
House-holds	Receive Π_{t-1} from firms. Pay T_t to government.	(i) Pay P_tC_t to firms (ii) Receive W_tL_t from firms.	Receive Π_t from firms Pay T_{t+1} to government
Govern-ment	Levies tax T_t on households.	Pays P_tG_t to firms.	Levies tax T_{t+1} on households.

where $P_t^e = E[P_t/I_{ot}]$ is the rational expectation of the price level and ρ is the rate of time preference. In a cash-in-advance economy, the nominal wage must be discounted by the rate of time preference and by the expected inflation rate, as the wage can only be spent the following period. We assume that the money supply rule is chosen from the set of money supply rules that ensures that $E_{ot}[P_{t+1}/P_t] = 1$, and for simplicity we set $A(1 + \rho) = 1$, so that, following Fisher (1977b) we can approximate (2.2) by:-

$$(2.3) \qquad W_t = P_t^e$$

This means that the labour supply is infinitely elastic at the agreed money wage. This utility function thus results in employment being solely demand determined. The household takes its labour income as given, and only faces a decision problem with respect to consumption and the demand for money.

The household faces a budget constraint:-

$$(2.4) \qquad (\Pi_{t-1}^h/P_t) + (M_{t-1}^h/P_t) + (W_t L_t^h/P_t) - C_t^h$$

$$- (T_t^h/P_t) - (M_t^h/P_t) = 0$$

and a cash-in-advance constraint:-

$$(2.5) \qquad M_{t-1}^h + \Pi_{t-1}^h - T_t^h > P_t C_t^h$$

The household's money holdings at the beginning of period t consist of money balances carried over from the previous period, M_{t-1}^h, plus Π_{t-1}^h, the profits firms have distributed to the household in sub-period t_o. We assume that (a) firms hold no money balances once they have paid out profits, and (b) the government carries no money over from the last period. This implies that, before paying taxes, households in aggregate hold all the money in this economy, i.e.

$$M_{t-1} = \sum_h (M_{t-1}^h + \Pi_{t-1}^h)$$

so that in aggregate the cash-in-advance constraint implies:-

$$\sum_h (M_{t-1}^h + \Pi_{t-1}^h - T_t^h) \geq \sum_h P_t C_t^h$$

or,

$$(2.6) \qquad M_{t-1} - T_t \geq P_t C_t$$

There is no equity or bond market and the only store of value is non-interest bearing cash balances. We assume throughout that the rate of deflation is always less than the rate of time preference, so that money always yields a real return less than the rate of time preference. This, coupled with the constant marginal utility of consumption, implies that the cash-in-advance constraint will hold with equality, and eliminates the margin of substitution between money and consumption, because all money holdings are consumed.

From (2.6) the aggregate consumption function is:-

$$(2.7) \quad P_tC_t = M_{t-1} - T_t$$

This implies that the household's money holdings at the end of the period consist only of its wage income:-

$$M_t^h = W_tL_t^h$$

Taxes follow an exogenous stochastic process. T_t, the nominal lump-sum tax the household faces at the start of period t, is announced one period in advance by the government, and hence is contained in I_{ot}.

Aggregate demand consists of the households' consumption demand, given by (2.7), and government's real demand for output, G_t. G_t is financed by taxes and money creation:-

$$G_t = \frac{T_t}{P_t} + \frac{M_t - M_{t-1}}{P_t}$$

Total demand, in real terms, is:-

$$Y_t^d = C_t + G_t$$

Substituting (2.7) for C_t, using the expression for G_t and simplifying, we obtain:-

$$(2.8) \quad Y_t^d = M_t/P_t$$

which is the aggregate demand function.

2.4 The Firm and Aggregate Supply

The representative firm's objective is to maximize its stream of discounted real profits:-[6]

$$V = \max_{\{L_t\}} E_t \sum_{j=0}^{\infty} \beta^j (Y^i_{t+j} - \frac{W_{t+j}}{P_{t+j}} L_{t+j})$$

subject to its production technology, which we assume is Cobb-Douglas:-

(2.9) $\quad Y^i_t = kL^{\alpha}_t Z^{1-\alpha}_t F_t \qquad 0 < \alpha < 1$

where F_t is a strictly positive stochastic shock to technology (to be specified below) and Z_t is a scale effect that captures the dependence of technical change on the level of aggregate output and evolves according to:-

(2.10) $\quad Z_t = Z_{t-1} Y^{\gamma}_{t-1} \qquad 0 < \gamma < 1$

We abstract from physical capital, though Z, representing technical knowledge, can be regarded as a kind of capital.[7] Z is disembodied knowledge, i.e. knowledge in books. It can consequently be thought of as a stock of non-depreciating capital. (Chapter 3 examines the implications of allowing knowledge to depreciate.)

Z_t depends on aggregate output: as the firm is small, we postulate that it ignores the effect of its own output decision on aggregate output. The learning process can be thought of as a by-product of the output process: it is not necessarily a variable under the conscious control of management, but rather a side-effect of the process of production. Furthermore, as labour moves between firms, learning skills become dispersed (one learns from one's colleagues) so that aggregate output rather than the output of any specific firm determines learning and technical knowledge.

An alternative (but more complex) approach would regard technical knowledge available to a particular firm as determined only by the history of that firm. Here the firm must clearly take account of learning, and the learning decision becomes a type of investment decision. This case is dealt with in Appendix 2. Qualitatively, it yields a similar result to the simpler case considered here.

The assumption that the evolution of technical knowledge is external to the firm allows us to treat the firm's problem like a one-period problem. The firm's only decision variable is L_t, the employment of labour. Substituting (2.9) into the objective function and taking the first-order-condition for labour yields:-

$$\alpha k L_t^{\alpha-1} Z_t^{1-\alpha} F_t = \frac{W_t}{P_t}$$

Letting lower-case letters denote natural logarithms, we can solve for the firm's demand for labour:-

$$(2.11)\qquad l_t = \frac{\ln(\alpha k)}{1-\alpha} + \frac{1}{1-\alpha}(p_t - p_t^e) + z_t + \frac{f_t}{1-\alpha}$$

where we have substituted from (2.3) for w_t. As employment is determined on the demand side of the labour market, we can substitute (2.11) into a logarithmic version of the production function to obtain the representative firm's supply of output function:-

$$(2.12) \qquad y_t^{s,i} = b_o + b_1(p_t - p_t^e) + z_t + b_2 f_t$$

$$b_o = \ln(k) + (\alpha/(1-\alpha))\ln(\alpha k); \quad b_1 = \alpha/(1-\alpha);$$

$$b_2 = 1/(1-\alpha)$$

Supply depends positively on the productivity shock, f_t, on the level of technical knowledge, z_t, and on the wedge between actual and expected prices. The greater the actual price level relative to the expected price level, the lower is the real wage, and this stimulates employment and output.

The aggregate supply function is obtained by aggregating over the number of firms in the economy:-

$$(2.13) \qquad y_t^s = q + b_o + b_1(p_t - p_t^e) + z_t + b_2 f_t$$

where q is the natural logarithm of the number of firms in the economy.

We assume that the exogenous shock f_t, has two components:- (i) a permanent component, ξ_t, that can be thought of as a shock to technology, and (ii) a temporary shock to productivity, η_t, so that:-

$$f_t = \bar{f}_t + \eta_t$$

$$\bar{f}_t = \bar{f}_{t-1} + \xi_t$$

ξ_t and η_t are stationary stochastic processes with zero mean, constant variances σ_ξ^2 and σ_η^2 respectively, and zero covariance. This implies that

$$(2.14) \quad f_t = \bar{f}_0 + \eta_t + \sum_{j=1}^{t} \xi_t$$

where $\bar{f}_0$ is a reference value in the past.

The economy presented in this and the following sections has a growth rate that increases over time. In general, models of this kind do possess a competitive equilibrium solution if the growth process is external to the firm. In the model presented here, growth depends on aggregate output, not on the output of the individual firm. Thus, the economy has a determinate equilibrium each period because, in addition, the production technology is Cobb-Douglas and satisfies the first and second-order-conditions for a maximum. The production technology determines the demand for labour and hence determines employment and wage income. Secondly, money is the only store of value - we have suppressed the equity market. Finally, consumption is determined by money holdings through the cash-in-advance constraint. Money balances depend on wage and profit incomes received from the previous period and thus on the level of output in the last period which depended on the production technology and last period's demand for output.

2.5 The Money Supply Process, Prices and Output

2.5.1 The Money Supply Process

The primary aim of the monetary authority in our model economy is to stabilize prices. This captures the concern about inflation that characterizes the policies adopted by many Central Banks in reality.

The monetary authority achieves price stability by accommodating the needs of trade - i.e. by expanding the money supply in response to expected increases in real output.[8]

Price stability requires that $p_t = p_{t-1}$. Aggregate demand is given by (2.8), and, in natural logarithms is:-

$$(2.15) \qquad y_t = m_t - p_t$$

If p_t is to equal p_{t-1}, then the target money stock, m_t^*, must be set so that:-

$$(2.16) \qquad m_t^* = p_{t-1} + y_t = p_t + y_t$$

Because

$$(2.17) \qquad p_{t-1} = m_{t-1} - y_{t-1}$$

the target growth of the money stock is given by:-

$$(2.18) \quad m_t^* - m_{t-1} = y_t - y_{t-1}$$

where we have substituted (2.17) for p_{t-1} into (2.16).

However, in reality the government lacks full contemporaneous information about the value of y_t (there is a lag in gathering statistics). To ensure price stability, the monetary authority must consequently forecast the level of output in period t and increase or decrease the money supply in proportion to the expected increase or decrease in output. We assume that the government determines the money supply once the nominal wage, W_t, has been set, but before the production process begins.[9] The Central Bank's information set is: $I_t^g = I_{ot}$, and $E_t^g X_t$ will be used to denote an expectation formed by the Central Bank with respect to any variable X (to distinguish it from expectations formed by the general public).

It follows from (2.18) that the rule for setting the target money stock is:-

$$m_t^* = m_{t-1} + E_t^g y_t - y_{t-1}$$

where $E_t^g y_t = E[y_t/I_t^g]$ and is the Central Bank's forecast of output. However, the monetary authority does not always achieve its target; in reality the money supply process is subject to a random error, ϵ_t, so that the actual money stock will be:-

$$m_t = m_t^* + \epsilon_t \qquad \epsilon_t \sim iid(0,\sigma_\epsilon^2)$$

or

(2.19) $m_t = m_{t-1} + E^g_t y_t - y_{t-1} + \epsilon_t$

2.5.2 The Public's Price Expectations

The household is aware of the money supply rule followed by the monetary authority, and utilizes this information in forming its prediction of the price level, p^e_t, at the start of the period. Taking the expectation of (2.15) with respect to the household's information set in sub-period t_o and re-arranging:-

(2.20) $p^e_t = m^e_t - y^e_t$

where the superscript 'e' denotes an expectation with respect to I_{ot}.

Taking the expectation of the money supply process, (2.19), based on I_{ot}, we obtain:-

$$m^e_t = m_{t-1} + E[E^g_t y_t - y_{t-1} + \epsilon_t)/I_{ot}]$$

$$\Rightarrow \quad m^e_t = m_{t-1} + y^e_t - y_{t-1}$$

because $E[E^g_t y_t/I_{ot}] = E[y_t/I_{ot}] = y^e_t$ and $E[\epsilon_t/I_{ot}] = 0$. Substituting this into (2.20) yields the public's rational expectations of the price level in time sub-period t_o, that is the expectation that is used in establishing the wage contract:-

(2.21) $p_t^e = m_{t-1} - y_{t-1}$

$\Rightarrow \quad p_t^e = p_{t-1}.$

Thus, given the money supply process (2.19), the public has a rational expectation of price stability, expected inflation is zero, for the monetary authority is expected to be successful in stabilizing the price level. Any unexpected changes in the price level are caused either by the failure of the monetary authority to hit its target money stock, or by real shocks that cause unanticipated changes in output.

2.5.3 Equilibrium Output and Prices

Expected market clearing implies that $y_t^d = y_t^s = y_t$. Thus the monetary authority can use the aggregate supply function (2.13) to form its expectation of output. As the government expects to hit its target (ϵ_t is random and $E_t^g \epsilon_t = 0$), it expects to be able to stabilize prices. Hence the monetary authority expects that $E_t^g p_t = p_{t-1} = p_t^e$. If this failed to hold, then the authority must not only be in possession of information which implied that its money stock would be off target, but the authority must also fail to react to such information by failing to correct the level of the money stock. This implies that $E_t^g(p_t - p_t^e) = 0$. The monetary authority consequently uses the following modification of the output supply function for forecasting:-

$$(2.22) \quad E^g_t y_t = q' + b_o + z_t + b_2 \bar{f}_{t-1}$$

Note that $E^g_t f_t = \bar{f}_{t-1}$, because $E^g_t \xi_t = E^g_t \eta_t = 0$.

$E^g_t y_t - y_t$ is obtained by subtracting (2.13), the actual output supply function, from (2.22), the central bank's forecast of supply:-

$$(2.23) \quad E^g_t y_t - y_t = - b_1(p_t - p^e_t) - b_2(\xi_t + \eta_t)$$

The actual price level that will obtain is given by substituting the money supply equation (2.19) into (2.15), and rearranging:-

$$(2.24) \quad p_t = m_{t-1} + (E^g_t y_t - y_t) - y_{t-1} + \epsilon_t$$

Subtracting (2.21) from (2.24) and substituting in (2.23) for $(E^g_t y_t - y_t)$ yields the expression for the error in the household's (and government's) price expectation:-

$$(2.25) \quad p_t - p^e_t = \frac{1}{1+b_1} (\epsilon_t - b_2(\xi_t + \eta_t))$$

The deviation of actual from expected (targeted) prices depends on the stochastic error component in the money supply process and on the unanticipated exogenous shocks. Substituting (2.25) into the aggregate supply function yields the equilibrium level of output:-

$$(2.26) \quad y_t = q + b_o + \frac{b_1}{1+b_1} (\epsilon_t - b_2(\xi_t + \eta_t)) + z_t + b_2 f_t$$

Output depends positively on the level of technical knowledge and on unexpected increases in the money supply. It also responds positively to positive exogenous productivity or technology shocks. Note, however, that exogenous shocks are partly offset through price effects. A positive value of ξ_t or η_t raises output; however, at the expected price level this results in an excess supply of commodities, and an excess demand for money. The actual price level must fall to clear the market, but in falling below its expected level it raises the real wage (the nominal wage being fixed). The rise in the real wage causes a fall in employment and output, so counteracting the productivity shock to some extent. However, the net effect of a positive real shock is still positive because $b_1/(1+b_1) < 1$. This raises the question of wage indexation, for if wages are partly or fully indexed to the price level, the impacts of real and monetary shocks on output will be different from those in this model. This question is taken up in Chapter 4.

The output process of equation (2.26) is non-stationary, and displays no tendency to return to a deterministic trend or time path once it has been disturbed. This can be seen more clearly if we take the natural logarithm of (2.10) and substitute backwards to obtain:-

$$(2.10a) \quad z_t = z_o + \gamma \sum_{j=0}^{t-1} y_j$$

where z_o is a reference point in the past. Substituting (2.10a) and (2.14) into (2.26):-

$$(2.27)\quad y_t = q + b_0 + \frac{b_1}{(1+b_1)}[\epsilon_t - b_2(\xi_t + \eta_t)] + z_0 + \gamma\sum_{j=0}^{t-1} y_j + b_2[\bar{f}_0 + \sum_{j=1}^{t} \xi_j + \eta_t]$$

Equation (2.27) demonstrates that output depends on the sum of past levels of output and on the accumulation of exogenous shocks to technology. Output also depends on the monetary error term and on the transitory shock to productivity. These 'transitory' shocks will also have a permanent effect on output. The reason is that learning acts as a propagation mechanism for any disturbance to output (the past values of the ϵ's and η's are contained in z_t) and causes all disturbances to have permanent effects. This is more apparent if we take the first difference of (2.27) and solve it backwards to obtain:-

$$(2.28)\quad y_t = (1+\gamma)^t y_0 + [b_1/(1+b_1)]\epsilon_t + [b_2/(1+b_1)](\eta_t+\xi_t) + [b_1\gamma/(1+b_1)]\sum_{j=0}^{t-2}(1+\gamma)^j\epsilon_{t-j-1} + b_2\gamma\sum_{j=0}^{t-2}(1+\gamma)^j\eta_{t-j-1} + \left[\frac{b_2(1+\gamma)+b_2b_1}{1+b_1}\right]\sum_{j=0}^{t-2}(1+\gamma)^j\xi_{t-j-1}$$

where y_0 is a reference value at time zero, and we have assumed that $\eta_0 = \xi_0 = \epsilon_0 = 0$. Thus output depends on accumulations of errors in price expectations and accumulations of technology shocks. Through learning, transitory errors in expectations and transitory shocks to productivity have a permanent impact on output: as (2.28) demonstrates, their impact does not disappear over time. Instead, these shocks permanently shift the economy's time path.

To put it differently, the output process described by (2.28) is non-stationary. The trend of output is not a deterministic function of time, but dependent upon stochastic shocks. The trend value of a non-stationary time series can be regarded as the expected future value, conditional on current information. If we update (2.28) and take expectations, we obtain:-

$$(2.28') \quad E_t y_{t+i} = (1+\gamma)^{t+i} y_0 + b_2\gamma \sum_{j=0}^{t-1} (1+\gamma)^{j+i-1} \eta_{t-j}$$

$$+ [b_1\gamma/(1+b_1)] \sum_{j=0}^{t-1} (1+\gamma)^{j+i-1} \epsilon_{t-j}$$

$$+ \left[\frac{b_2(1+\gamma)+b_2 b_1}{(1+b_1)}\right] \sum_{j=0}^{t-2} (1+\gamma)^{j+i-1} \xi_{t-j}$$

where $E_t\eta_{t+i} = E_t\xi_{t+i} = E_t\epsilon_{t+i} = 0,\ i > 0.$

Thus the trend, or expected future value of output, is a function of historical events - it depends on accumulations of stochastic error terms. Because these errors are premultiplied by $(1+\gamma)^j$, a term that increases exponentially over time, these accumulations are non-stationary processes of infinite variance, even though the errors themselves are random drawings from zero-mean stationary distributions.

In this model, learning acts as a propagation mechanism for both demand-side and supply-side shocks that would otherwise only have a transitory impact on output. Once we recognize that technology is -

at least to some degree - endogenous, we must also recognize that monetary shocks will change the trend value of output, as will temporary changes in productivity.

2.6 Real and Monetary Business Cycle Models

One can clearly distinguish four different models that are imbedded in the aforegoing general model of equation (2.27). The first model is a purely monetary model in which there are no real shocks of any kind. The second model contains permanent and transitory exogenous shocks but does not contain a learning curve. A third model contains transitory exogenous disturbances to productivity but no learning curve. The fourth model has permanent and transitory real shocks, but also contains a learning curve, although it contains no monetary shocks and is thus a pure real business cycle model with endogenous technology.

2.6.1 The Monetary Model

The purely monetary model is one in which $F_t = 1$ for all t (i.e. $\xi_t = \eta_t = 0$ for all t), so that (2.9) is replaced by

(2.29) $Y_t^i = kL_t^{\alpha}Z_t^{1-\alpha}$

In this case (2.25) is replaced by:-

$$(2.30)\quad (p_t - p_t^e) = \frac{1}{(1+b_1)}\epsilon_t$$

and the aggregate supply function is:-

$$(2.31)\quad y_t^s = q + b_o + b_1(p_t - p_t^e) + z_t$$

so that (2.27) reduces to:-

$$(2.32)\quad y_t = q + b_o + [b_1/(1+b_1)]\epsilon_t + z_o + \gamma\sum_{j=0}^{t-1} y_j$$

This is clearly a non-stationary process, as output still depends on accumulations of past levels of output, and the long-term forecast of output will always be influenced by historic events. Thus the inclusion or exclusion of exogenous shocks to technology is unnecessary to the non-stationarity of the output process in an economy with learning by doing.

Taking the first difference of (2.27a) we obtain:-

$$(2.33)\quad y_t = (1+\gamma)y_{t-1} + \frac{b_1}{1+b_1}(\epsilon_t - \epsilon_{t-1})$$

y_t can be written as an explosive moving average process of infinite order:-

$$y_t = [b_1/(1+b_1)]\epsilon_t + [b_1\gamma/(1+b_1)]\sum_{j=0}^{\infty}(1+\gamma)^j\epsilon_{t-j-1}$$

Alternatively, a more sensible expression for y_t is:-

$$(2.34)\quad y_t = (1+\gamma)^t y_o + \frac{b_1}{(1+b_1)}\epsilon_t$$

$$+ \frac{b_1\gamma}{(1+b_1)} \sum_{j=0}^{t-2} (1+\gamma)^j \epsilon_{t-j-1}$$

where y_0 is the value of income when the money supply rule under consideration was introduced, and $\epsilon_0 = 0$ for simplicity.

Equation (2.30) demonstrates that output depends on accumulated errors in price expectations (which within this framework amount to errors in expectations of the money supply: if we relax the cash-in-advance constraint and allow for velocity shocks, these would be an additional cause of non-stationarity in the output process). Furthermore, future output will also depend on these expectational errors, so that the effects of expectational errors are not transitory phenomena as is the case in the Lucas-Barro models. One cannot distinguish between real and monetary disturbances in the general model on the basis of their transitoriness or permanence. Neither can one distinguish them in terms of their impact on technological knowledge - for both the technological and monetary shocks influence technology in a broad sense by influencing learning. Monetary shocks therefore have permanent effects because they influence technology. To put it differently, learning, captured by z_t, depends on past levels of output. It is unimportant whether a rise in output came about through a demand-side shock, such as ϵ_t, or a supply-side shock such as ξ_t or η_t, although the impact of demand-side and supply side shocks on prices and wages will differ.

Empirical work has found that shocks to output are persistent (hence output is non-stationary). However, there is disagreement about the degree of persistence of shocks to output, as was noted in

Chapter 1. As can be seen from (2.28) or (2.34), monetary innovations have a persistent impact on output. However, the fraction of a rise in output caused by the monetary shock that is permanent depends on the horizon from which the shock is viewed. From (2.30) we see that a positive monetary shock will raise output by $[b_1/(1+b_1)]\epsilon_t$. The next period it raises output by $[\gamma b_1/(1+b_1)]\epsilon_t$, and in the period after that by $(1+\gamma)[\gamma b_1/(1+b_1)]\epsilon_t$. In general, j periods ahead it will have raised output by $(1+\gamma)^{j-1}[\gamma b_1/(1+b_1)]\epsilon_t$. Thus the impact of a monetary innovation increases as time goes by. . The reason is that a monetary (or real) innovation raises output, but the higher level of output raises technical knowledge through learning, which in turn raises output further, which raises technology further, and so on.

2.6.2 <u>A Real Business Cycle Model with Permanent Shocks</u>

An alternative model that could account for non-stationarity is one with exogenous shocks, i.e. without a learning curve, so that $Z_t = 1$ for all t, and (2.9) becomes:-

$$(2.35)\qquad Y_t^i = kL_t^{\alpha}F_t$$

In this instance (2.27) reduces to:-

$$(2.36)\qquad y_t = q + b_0 + \frac{b_1}{1+b_1}(\epsilon_t - b_2(\xi_t + \eta_t))$$

$$+ b_2(\bar{f}_o + \eta_t + \sum_{j=1}^{t} \xi_j)$$

which is also non-stationary with output dependent upon an accumulation of stochastic shocks. Although we have called this a real business cycle model, it is really a hybrid model in which both monetary and real innovations affect output. However, it is similar to conventional real business cycle models since technology is exogenous. Consequently, the effect of monetary shocks is transitory. In the absence of endogenous technology, only real shocks have a permanent impact on output. The non-stationary of (2.36) is solely due to the permanent real shocks impinging on this model economy - the ξ's.

There are therefore two alternative models that can account for non-stationarity: a monetary model with endogenous technology, or a model with permanent exogenous shocks to technology. In order to discriminate between these two models, notice that the first difference of (2.32), given by (2.33), contains a greater-than-unit root. By contrast, the first difference of (2.36) is:-

$$(2.37) \quad y_t = y_{t-1} + \frac{b_1}{(1+b_1)}(\epsilon_t - \epsilon_{t-1})$$

$$\left[\frac{b_2}{1+b_1}\right](\eta_t - \eta_{t-1} + \xi_t + b_1\xi_{t-1})$$

This model does not contain a greater-than-unit root.

The question that naturally arises is why the learning process should result in a greater-than-unit root for output. The answer is that in a learning model of the kind we specify, the change in output depends on learning, and thus on the level of output in the last period:-

$$y_t - y_{t-1} = \gamma y_{t-1} \quad (\text{if } \epsilon_t = \eta_t = \xi_t = 0 \text{ for all } t)$$

With a real business cycle model, the change in output depends only on the real stochastic shock:

$$y_t - y_{t-1} = \frac{b_1}{1+b_1}(\xi_t + b_1\xi_{t-1}) \quad (\text{if } \eta_t = \epsilon_t = 0 \text{ for all } t)$$

In the former case, the output process itself causes learning and creates technical knowledge, so increasing future productivity. The growth of output depends on the level of output. In the latter case, the increase in output is independent of its level: the growth in output depends only on the exogenous shock.

Two assumptions that have been made are important when considering the reasons for the manifestation of a greater-than-unit root. The first assumption is that there is an infinitely elastic supply of labour: the household will supply any amount of labour the firm requires at an agreed money wage that is set with the aim of keeping the expected real wage constant. This implies that any increase in the demand for labour that arises from an increase in knowledge (that increases the marginal product of labour) can always be met. Clearly this is only realistic in context of a society in which the labour supply is growing at least as fast as labour demand. The second assumption is that the growth of technical knowledge depends on the level of output.

Either of these assumptions could by itself generate the explosive growth process implied by the greater-than-unit root.[10] Even if labour supplies are limited, modelling the evolution of knowledge by (2.10) would still give the result that the change in output from one period to the next depends on the level of output in the previous period and thus output would still manifest a greater-than-unit root. If, instead of (2.10), we had assumed that the growth of knowledge depends, for example, on labour input into the production process, but retained the assumption that labour supply is infinitely elastic, a greater-than-unit root would still obtain.

However, if we assumed that firstly, the growth of technical knowledge is governed by labour input <u>and</u> secondly that the labour supply was fixed within certain limits, then output growth would not be explosive. The growth of knowledge would then be restricted by the limited availability of labour; the existence of limited labour supplies would act as a brake on the growth process, and the growth of technical knowledge would depend on population growth. However, even if these alternative assumptions had been employed, the basic results of this chapter would still hold. Even if technical knowledge depends on labour input, it is obvious from (2.11) that monetary shocks would alter employment and thus technology. Hence monetary shocks would still have permanent effects, and the output process would still be non-stationary.

2.6.3 A Real Business Cycle Model with Transitory Shocks[11]

A third model that deserves our consideration is one in which there are productivity shocks, that is, a model that contains transitory shocks to technology but has no learning curve. In this instance $Z_t = 1$ for all t, and $F_t = e^{\eta t}$ and (2.9) becomes:-

$$Y_t^i = kL_t^{\alpha} e^{\eta t} \tag{2.38}$$

Output will be governed by:-

$$y_t = q + b_o + \frac{b_1}{(1+b_1)} (\epsilon_t - b_2 \eta_t) + b_2 \eta_t \tag{2.39}$$

The trend, or expected future value of output is:-

$$E_t y_{t+i} = q + b_o; \qquad i \geq 1.$$

It is at once apparent that such a model cannot account for non-stationarity, for output will only vary about the point $(q + b_o)$. It is therefore necessary, in order for a real business cycle model to account for non-stationarity, that η_t be a non-stationary process, in which case the model becomes similar to a model with permanent exogenous shocks. Note that in this model money is neutral in the long run. Without endogenous technology, monetary shocks will not cause the output process to exhibit non-stationarity.

2.6.4 A Real Business Cycle Model with Endogenous Technology

A final model that is of interest, though not on account of its realism, is a pure real business cycle model with endogenous technology, i.e. model in which money has no impact on economic activity. This result can be obtained from the general model by assuming that not only the velocity but also the quantity of money are constant, so that the level of nominal aggregate demand is constant. In this case $\epsilon_t = 0$ for all t, and changes in the price level come about through changes in productivity and technology that alter real output. As is well known, in such a model the price level will be countercyclical. The model is therefore similar to that of Long and Plosser, for example, where money plays no role in determining output. If $\epsilon_t = 0$, then (2.28) reduces to:

$$(2.40) \quad y_t = (1+\gamma)^t y_o + (b_2/(1+b_1))[\eta_t + \xi_t] + b_2\gamma \sum_{j=0}^{t-2} (1+\gamma)^j \eta_{t-j-1}$$

$$+ \left\{\frac{b_2(1+\gamma) + b_2 b_1}{1 + b_1}\right\} \sum_{j=0}^{t-2} (1+\gamma)^j \xi_{t-j-1}$$

The output process of this model is non-stationary, but what is of interest is that output still has a greater-than-unit root (of size $(1+\gamma)$). Secondly, through learning, the transitory productivity shocks, the η's have a permanent effect on output, in contrast to the real business cycle model (2.36) without endogenous technology.

The above conclusion implies that if technology is endogenous, a univariate time series analysis of output cannot distinguish a real from a monetary business cycle model, as both predict a greater-than-unit root. However, the size of the root of output can in theory be used to discriminate between the class of models developed here that contain endogenous technology and conventional real business cycle models, such as (2.36), for the latter generally display a unit-root output process. However, this is likely to be difficult, if not impossible. While reliable tests for unit roots are gradually being developed, it is not clear how one could distinguish a unit-root model from one that has only a slightly greater-than-unit root, for γ may be small. The contribution of learning to the growth rate of output is given by $(1-\alpha)\gamma$. γ is the output elasticity of knowledge, i.e. the percentage change in Z divided by the percentage change in Y. If output doubles, and technical knowledge available to society rises by 10 percent, then $\gamma = 0.10$. In small samples the time series movements suggested by a model with a unit root, and one with a root slightly greater than unity are likely to be so similar that acceptance of one model would not enable us to reject the alternative. Finite-sample test procedures simply cannot distinguish between a unit root and one that is very close to unity.

2.7 Neutrality, Superneutrality and Policy Invariance

In the conventional New Classical and New Keynesian monetary business cycle models, monetary policy is non-neutral in the short run because of imperfect information or gradual wage and price adjustment. However, these effects are transitory: in the long run, monetary policy is neutral.

Secondly, money is also superneutral in these models.

anticipated inflation is incorporated into wage and price expectations, and thus does not influence output. A permanent increase in inflation can only affect output in the short run, until the public learns that a new money supply rule has come into effect, but this is a transitory phenomenon.

Thirdly, the probability distribution of output is invariant with respect to the money supply rule followed; only the stochastic, unpredictable component of the rule influences output. The predictable components of alternative money supply rules are public knowledge and hence exert no influence on output. Changing to a new rule has only transient effects on output, and only if the public is imperfectly informed or locked into nominal contracts.

None of these features apply to the monetary models with endogenous technology developed in this thesis. In these models money is non-neutral and non-superneutral in the short and long run, and the output process changes permanently in response to a shift in monetary policy.

2.7.1 Neutrality

Models with endogenous technology exhibit a long-run non-neutrality of money. This is confirmed by an inspection of (2.27) or (2.30): the influence of monetary innovations - the ϵ's - does not diminish over time, but actually increases. Expectational errors about monetary growth have permanent effects and shift the long-run output path of the economy. An unexpected monetary innovation has as lasting an impact on output as does a technological shock. In this respect models with endogenous technology depart radically from conventional monetary models.

2.7.2 Superneutrality

Money is not superneutral in the model we have developed. A change in the anticipated rate of inflation affects employment and output because, in a cash-in-advance constrained model, inflation acts as a tax on labour income. If expected inflation is non-zero, then, from (2.2) we obtain that the nominal wage will be set according to:-

$$(2.41)\qquad W_t = P^e_t . E_{ot}[P_{t+1}/P_t]$$

and we continue to assume that $A(1+\rho) = 1$. Ignoring Jensen's inequality, we can take the following logarithmic approximation of (2.41):-

$$(2.42)\qquad w_t = p^e_t + (p^e_{t+1} - p^e_t)$$

where, as before, a superscript "e" denotes an expectation taken with respect to information set $I_{\rho t}$.

We now assume that the money supply rule contains a growth component, so that instead of (2.19), the money supply evolves according to:-

$$(2.43)\qquad m_t = m_{t-1} + \mu + E^g_t y_t - y_{t-1} + \epsilon_t$$

where μ is a constant and is contained in the public's information set I_{ot}. As previously, the public's price expectation is given by (2.20). Proceeding as before, but using the money supply equation (2.43) instead of (2.19) we obtain:-

$$(2.44) \quad p_t^e = m_{t-1} - y_{t-1} + \mu$$

Recalling that $p_{t-1} = m_{t-1} - y_{t-1}$, we can write (2.44) as:-

$$(2.45) \quad p_t^e = p_{t-1} + \mu$$

Updating this equation by one period and taking expectations over the information set I_{ot}:-

$$p_{t+1}^e = p_t^e + \mu$$

or

$$(2.46) \quad p_{t+1}^e - p_t^e = \mu .$$

Thus the expected inflation rate is just equal to μ, the constant in the money supply equation. Substituting (2.46) into (2.42) yields:-

$$(2.47) \quad w_t = p_t^e + \mu$$

Due to the assumption that profits realized at the end of period t are only distributed to households at the beginning of period t+1, the firm's optimal behaviour consists of maximising the expected value of real profits deflated by the price level that will prevail in period t+1. Thus, in the case of non-zero inflation, the firms value function is:-

$$V = \max_{\{L_t\}} E_t \sum_{j=0}^{\infty} \beta^j \left[\frac{P_{t+j}}{P_{t+j+1}} Y_t^i - \frac{P_{t+j}}{P_{t+j+1}} \frac{W_{t+j}}{P_{t+j}} L_{t+j}\right]$$

and the first-order-condition for labour is:-

$$E_t[\alpha k L_t^{\alpha-1} Z_t^{1-\alpha} F_t.(P_t/P_{t+1}) - (W_t/P_t).(P_t/P_{t+1})] = 0$$

Ignoring Jensen's inequality and taking a logarithmic approximation results in the following demand for labour function:-

$$(2.48) \quad l_t = \frac{\ln(\alpha k)}{1-\alpha} + z_t + \frac{1}{1-\alpha}(p_t - w_t) + \frac{f_t}{1-\alpha}$$

This is essentially the same demand for labour function as (2.11). Substituting (2.47) into (2.48) for w_t, substituting the result into a logarithmic form of the production function and aggregating gives the aggregate supply function:-

$$(2.49) \quad y_t^s = q + b_o + b_1(p_t - p_t^e) - b_1\mu + z_t + b_2 f_t$$

where the b's are defined as for equation (2.12). The only difference between (2.49) and (2.13), the previous aggregate supply function, is that the term $-b_1\mu$ enters (2.49). Anticipated and unanticipated inflation affect output differently in this model. As before, a rise in unanticipated inflation (i.e., a positive value of $(p_t-p_t^e)$) raises output. But anticipated inflation exerts a negative effect on output. The higher the expected inflation rate is, the higher must be the money (and real) wage that is paid at time t, so as to compensate agents for the reduction in the value of the money wage when they spend it at time t+1. Forced to pay a higher real wage at time t, firms will reduce employment and output will fall accordingly. This is a conventional feature of cash-in-advance economies (for example, see Aschauer and Greenwood (1983)).

The equilibrium level of output will be:-

$$(2.50) \qquad y_t = q + b_o + \frac{b_1}{1+b_1}[\epsilon_t - b_2(\xi_t + \eta_t)] + z_t - b_1\mu + b_2 f_t$$

It is clear that money is not super-neutral in this model. If the anticipated inflation rate is changed from μ to μ' and $\mu \neq \mu'$, then from (2.50) it can be seen that the equilibrium level of output will change.

Not only does output depend on the current anticipated inflation rate, but also on the history of anticipated inflation rates. Assume that the monetary authority has allowed μ to vary over time, and denote the value of μ at time t as μ_t. For simplicity it is assumed that changes in μ are announced one period in advance, so that the aforegoing analysis still holds. In this case, using (2.10a), we can write (2.50) as:-

$$\begin{aligned}(2.51) \qquad y_t = {} & q + b_o + \frac{b_1}{1+b_1}[\epsilon_t - b_2(\xi_t + \eta_t)] \\ & - b_1\mu_t + b_2 f_t + z_o + \gamma(q + b_o)t \\ & + \gamma\sum_{j=0}^{t-1}\left[\frac{b_1}{1+b_1}\{\epsilon_j - b_2(\xi_j + \eta_j)\} + z_j \right. \\ & \left. - b_1\mu_j + b_2 f_j\right]\end{aligned}$$

The presence of the $\Sigma b_1\mu_j$ term in (2.51) demonstrates that output depends on both the current expected inflation rate as well as on past expected inflation rates. Endogenous technology acts as a propagation mechanism not only for shocks, but also for the effects of past expected inflation rates.

Money is non-superneutral in this model because money is introduced through a cash-in-advance constraint. If, by contrast, we had modelled money as entering the utility function, in the spirit of Sidrauski, then, provided we continue to eliminate wealth effects from the labour supply decision, a change in inflation would leave employment and consumption unchanged (although it would alter households' utility and money balances). Thus non-superneutrality arises in our model from the way in which we have introduced money.

2.7.3 Policy Invariance

Output is not invariant with respect to changes in the systematic component of the money supply rule. To see this, consider an alternative money supply rule by assuming that the government attempts to offset the real shocks hitting the economy. As was stated below, the monetary authority may lack precise contemporaneous information about these shocks and hence about the value of output. However, if it postpones its money-supply decision until the trading and production processes have actually begun, it will obtain some indication of the value of f_t. (In reality such indicators are available - changes in the oil price or input prices generally are known to government before they feed into the production process.) We therefore assume that the monetary authority has access to a noisy indicator of f_t, namely $\tilde{f}_t$, where $\tilde{f}_t = f_t + e_t$, e_t being a zero mean, stationary stochastic process with variance σ_e^2.

Proceeding as in Section 2.5 below, the monetary authorities forecast of output is, instead of (2.22):-

$$(2.52) \quad E_t^g y_t = q + b_o + z_t + b_2(f_t + e_t)$$

so that we obtain

$$p_t - p_t^e = \frac{1}{1+b_1}(\epsilon_t + b_2 e_t)$$

and

$$(2.53) \quad y_t = q + b_o + \frac{b_1}{1+b_1}(\epsilon_t + b_2 e_t) + z_t + b_2 f_t$$

The variance of the one-period ahead forecast of output is:-

$$\sigma^2_{y/E_t^g f_t = \tilde{f}_t} = E_t[y_{t+1} - E_t y_{t+1}]^2$$

$$= \left[\frac{b_1}{1+b_1}\right]^2 \sigma_\epsilon^2 + \left[\frac{b_1 b_2}{1+b_1}\right]^2 \sigma_e^2 + b_2^2(\sigma_\xi^2 + \sigma_\eta^2)$$

In the case where the government does not take $\tilde{f}_t$ into account but sets $E_t^g f_t = \bar{f}_{t-1}$, the variance of output from (2.26) is:-

$$\sigma^2_{y/E_t^g f_t = \bar{f}_{t-1}} = \left[\frac{b_1}{1+b_1}\right]^2 \sigma_\epsilon^2 + \left[\frac{b_2}{1+b_1}\right]^2 (\sigma_\eta^2 + \sigma_\xi^2)$$

and.

$$\sigma^2_{y/E^g_t f_t - \bar{f}_{t-1}} \neq \sigma^2_{y/E^g_t f_t - \bar{f}_t}$$

Thus each of these money supply rules results in a different variance of output. Smoothing prices in the face of exogenous shocks by incorporating $\bar{f}_t$ into the output forecast in fact increases the variance of output. A positive exogenous shock requires an increase in the money supply to keep prices stable. If the money supply remains unchanged, the price level falls, so raising the real wage and offsetting the positive shock.

Secondly, changing the monetary rule will change the trend value, or expected future value, of output (conditional on current information). Under the old money supply rule, the expected future value of output is given by (2.28'):-

$$(2.28') \quad E_t y_{t+i} = (1+\gamma)^{t+i} y_0 + b_2\gamma \sum_{j=0}^{t-1} (1+\gamma)^{j+i-1} \eta_{t-j}$$

$$+ [b_1\gamma/(1+b_1)] \sum_{j=0}^{t-1} (1+\gamma)^{j+i-1} \epsilon_{t-j}$$

$$+ \left[\frac{b_2(1+\gamma)+b_2b_1}{1+b_1}\right] \sum_{j=0}^{t-2} (1+\gamma)^{j+i-1} \xi_{t-j}$$

Under the alternative money supply rule we have considered, output is given by (2.53). Taking the first difference of (2.53), solving it backwards, updating the result and taking expectations yields the expected future value of output under the alternative money supply process:-

$$(2.54) \quad E_t y_{t+i} = (1+\gamma)^{t+i} y_o + b_2 \sum_{j=0}^{t-1} (1+\gamma)^{j+i-1} \xi_{t-j}$$

$$+ [\gamma b_2 b_1/(1+b_1)] \sum_{j=0}^{t-1} (1+\gamma)^{j+i-1} e_{t-j}$$

$$+ [\gamma b_1/(1+b_1)] \sum_{j=0}^{t-1} (1+\gamma)^{j+i-1} \epsilon_{t-j}$$

$$+ \gamma b_2 \sum_{j=0}^{t-1} (1+\gamma)^{j+i-1} \eta_{t-j}$$

Again $E_t\xi_{t+i} = E_t\eta_{t+i} = E_t\epsilon_{t+i} = E_t e_{t+i} = 0$, $i > 0$.

A comparison of (2.28') and (2.54) demonstrates that expected future time path of output changes when the money supply rule changes. A new cumulative sum of errors - the e's - enters into (2.54). Furthermore, the weights with which the permanent shocks - the ξ's - enter into (2.54) has changed. Changing the money supply rule can lead to changes in the trend of output in models with endogenous technology. Consequently there is no policy invariance in models with endogenous technology: a change in the money supply rule can change both the trend and variance of output.

2.8 A Role for Government Intervention

In the model developed in this chapter, increases in output increase the future productivity of labour through the learning mechanism. Production is therefore a form of investment. However, individual firms fail to take this mechanism into account, as it depends on total output and is consequently external to the firm. Consequently, the individual firm underestimates the true marginal product of labour. If it took account of these "investment in learning" effects, and realized that today's output raises tomorrow's productivity, it would increase employment and output (and increase the real wage, if we used a more general utility function than (2.1)).

The question arises whether monetary policy can be used to correct this inefficiency by raising output to the level it would attain if firms took learning effects into account.

A monetary policy that resulted in a constant rate of deflation would have the desired effect. As we are dealing with a cash-in-advance economy, a deflation increases the real purchasing power of wages. It is obvious from (2.47) that the lower μ, the expected rate of inflation is, the lower in the money wage firms have to pay in period t. Thus a lowering of μ lowers the firms' money (and real) wage costs, and will lead to an increase in employment and output. This can also be seen from (2.50), the equation for output: the lower the expected inflation rate is, the higher is the equilibrium level of output. There should exist a level of μ that is sufficiently low (i.e. negative) so as to increase employment and output to their Pareto-efficient levels.[12]

An alternative correction is available through fiscal policy. This can be accomplished without imposing additional distortions by (a) raising lump-sum taxes, and (b) using tax revenues to subsidise output. The negative output tax will raise the marginal revenue product of labour, so raising employment and output to their efficient levels.

2.9 Conclusions

This chapter has demonstrated that there are basically two phenomena that can account for non-stationarity: endogenous technology, or, alternatively, permanent exogenous shocks to technology. A simple model containing both these phenomena was constructed, and it was demonstrated that in an economy characterized by learning, output will follow a non-stationary process with no tendency to return to a deterministic trend or time path. This is true even if there are no exogenous shocks to technology, provided there are disturbances to the demand side of the economy, such as monetary shocks. It therefore demonstrates that monetary models of output fluctuations cannot be dismissed as incapable of accounting for non-stationarity.

This model stands in strong contrast to other "monetary misperception" models of the business cycle pioneered by Lucas and Barro, for, in the Lucas-Barro models, the influence of an expectational error or misperception is transitory. In models with endogenous technology, the effects of these expectational errors are not transitory: their effect is cumulative and these errors alter the economy's long-run growth path. There is a long-run non-neutrality of money in models with endogenous technology. Money is also non-superneutral, as we impose a cash-in-advance constraint. There is also no policy invariance in this model - both the trend and the variance of output may change when the money supply rule changes. What are conventionally considered "nominal" shocks have a permanent effect on technology through learning, so that the conventional distinction between "real" and "nominal" shocks becomes blurred.

The chapter also considered a pure real business cycle model with endogenous technology. In contrast with the conventional real business cycle models with exogenous technology, the model with endogenous technology given by (2.40) has a greater-than-unit root output process. Furthermore, transitory changes in productivity have a permanent effect on output in this model. In a conventional model like (2.36), output has a unit root and transitory productivity shocks have no long-run effect. Thus, this chapter has shown that both real and monetary models yield significantly different results once the endogeneity of technology is taken into account.

Footnotes

1. Spence (1981), p.68 (italics original).

2. The seminal paper on learning by doing is Arrow (1962). Eltis (1971) provides a useful perspective.

3. Kennedy and Thirlwall (1972), p.39.

4. The assumption that the government buys output with the money it issues is not a crucial assumption. It could also make lump-sum transfers to the public during the trading process. Reductions in the money supply are brought about when the level of lump-sum taxes exceeds the value of government spending.

5. This results in the labour supply function assumed by Fischer (1977b).

6. For simplicity we assume that the relative price of the firm's output is unity. However, from the perspective of the owners of the firm, who only receive profits the following period, the relevant relative price is P_t/E_tP_{t+1}, which is only equal to unity under certain conditions, a necessary condition being that the money supply rule implies zero expected inflation. The money supply rule we adopt ensures that $P_t = E_tP_{t+1}$.

7. This characterization is similar to assuming, as Arrow did, that learning depends on accumulated investment, provided that investment is (approximately) proportional to the level of output.

8. As we impose a cash-in-advance constraint on the household and government, velocity is invariably equal to unity.

9. An alternative monetary strategy - discussed in section 2.7.3 - is that the Central Bank determines the money supply during the trading and production process, so that it has some - but not necessarily complete - information about the technology shock, f_t.

10. If all shocks take their mean values of zero then the logarithm of the output process can be represented as a continuous growth function:

$$y = (1+\gamma)^t y_o$$

and y_o is a reference value in the past. In this case the instantaneous rate of growth of the logarithm of output is a constant equal to $\ln(1+\gamma)$. However, in natural levels, we have

$$Y = Y_o^{(1+\gamma)^t}$$

and the instantaneous growth rate of oputput is $re^{rt}\ln Y_o$ where $r = \ln(1+\gamma)$. Thus the rate of growth of output is an increasing function of time. Romer (1986) finds that there is empirical support for the hypothesis that long run growth rates have a positive trend, and his empirical findings could be regarded as indirect verification of the hypothesis that output has a greater-than-unit root.

11. Again, this is a hybrid model in which both real and monetary shocks affect output.

12. If the rate of deflation is greater than the rate of time preference, the cash-in-advance constraint will no longer hold with equality and the household's problem may not have a solution in the case of the simple utility function specified in this chapter.

CHAPTER 3

MODELS WITH ENDOGENOUS TECHNOLOGY: THE CASE OF DETERMINISTIC INNOVATION

3.1 Introduction

This chapter presents a model with a different formulation of endogenous technology: there is a well-defined innovation production function, and technology advances through the application of resources to research and development. In addition, we assume that there is a random component to the production of technology, that captures the fact that technology does, to a limited degree, depend on serendipitous scientific discoveries (the technology "shocks" of real business cycle theory). This chapter demonstrates that even if we assume that these exogenous shocks to technology are zero, so that technology is completely deterministic, and the only disturbances to the system originate on the monetary side of the economy, the properties output displays are similar to those found in a conventional real business cycle model. In particular, a monetary model with a deterministic innovation production function is characterized by a non-stationary output process, and by the fact that monetary shocks have a permanent impact on technology and thus on the time profile of output.

Furthermore, the implications of allowing technology to depreciate are also examined. The effects of depreciation are considered in several different models, including models with endogenous technology and a real business cycle model with exogenous technology. It emerges that the properties displayed by output in these models may change significantly if technology depreciates.

3.2 The Basic Framework

The framework adopted in this chapter is very similar to the one adopted in the previous chapter. In particular, the demand side of the model is identical to that of Chapter 2. Again we consider an economy consisting of a large number of competitive firms, producing a single commodity, that take the market price as given. Trading is sequential, with the labour market opening before the market for commodities has opened, so that nominal wage contracts are signed before the current period's price level is known.

As before, a greater-than-expected money stock raises the price level and reduces the real wage. This has two effects. Firstly, the fall in the real wage results in increases in output, and makes innovation more profitable, for, the greater output is, the more profitable is a cost-reducing (or factor augmenting) innovation. Secondly, the decline in the real wage will make research less costly (the real cost of hiring researchers has fallen) and this also stimulates innovation. In this way monetary shocks can change the rate of innovation and thus the rate of technological advancement.

The following section introduces the technological constraints faced by firms, and formulates the innovation production function for the individual firm, as well as the evolution of technology for the economy as a whole. We consider the simplest case that enables us to treat the firm's problem like a one-period problem. Section 3.4 derives the supply function of the representative firm and the output solution for the economy. In general the output process contains a greater-than-unit root. Section 3.5 shows how Nelson and Plosser's finding that output has a unit root can be obtained by placing certain restrictions on the commodity and innovation production functions. The plausibility of these restrictions is also discussed. Section 3.6 examines the effect of perturbing the model by allowing technology to "depreciate". It emerges that if technology depreciates, the output process may be stationary, and, in a purely monetary model, one can obtain an aggregate supply curve very similar to the Lucas aggregate supply curve. A real business cycle model with depreciating exogenous technology is also examined, so allowing us to compare the properties of models with endogenous vis-a-vis exogenous technology if technology depreciates. The next section considers whether the model exhibits neutrality and superneutrality of money and policy invariance, and concludes that none of these properties hold. The final section contains a summary and conclusions.

3.3 The Innovation Production Function and the Evolution of Technology

The representative firm faces the following production technology:-

$$(3.1) \qquad Y_t^i = kL_t^{\alpha}X_t^{1-\alpha} \qquad 0 < \alpha < 1 \quad ,$$

where X_t is an index of the technology available to the individual firm that augments the productivity of labour (for simplicity we continue to abstract from capital). The technology available to the firm depends on (i) the level of generally available scientific knowledge within the community, $\bar{X}_t$, and on (ii) the resources devoted to research and development by the representative firm, N_t. Specifically, we assume that N_t consists of labour devoted to research activities, and that this labour receives the same remuneration as labour producing the final good. X_t will also depend on the length of time the firm can retain any advantage it gains from its research expenditures.

We shall consider the simplest possible case, for this captures the essence of the implications of deterministic endogenous technology. Specifically, it is assumed that the firm can retain an innovational advantage for only one period, that is, only for the period within which the research is undertaken.[1] Thereafter the innovation becomes public knowledge, and is contained in $\bar{X}$. The innovation production function is:-[2,3]

$$(3.2) \qquad X_t = N_t^r \bar{X}_t \qquad 0 < r < 1$$

This requires the firm to employ some resources before it can utilize any public knowledge. It can also augment public knowledge by setting $N_t^r > 1$.

We further assume that the firm takes $\bar{X}_t$ as well as future values of $\bar{X}$ as exogenously given; it ignores the influence its own research activity will have on the general level of technical knowledge in the economy, and that its own research activities may (infinitesimally) raise future values of $\bar{X}$. This can be justified by arguing that the firm's contribution is small relative to the level of research in the economy as a whole, and it is the latter that determines $\bar{X}_t$.

Finally, we must consider how $\bar{X}$, the economy's technical knowledge, evolves. Clearly $\bar{X}$ will depend on the research efforts of all individual firms and on increases in scientific knowledge that come about through non-commercial research, for example through research in universities or by government bodies. We postulate that, in log-linear form,

$$(3.3) \qquad \bar{x}_t = \bar{x}_{t-1} + rn_{t-1} + h_t \qquad h_t \sim iid(0, \sigma_h^2)$$

where lower-case letters denote natural logarithms. h_t is a random variable that captures the influence of non-commercial research and of exogenous changes in the scientific base on the level of technology that is available to all firms. It can be thought of as the economy-wide shock to technology that characterises some real business cycle models. n_{t-1} is the natural logarithm of the labour devoted to research by the average, representative firm at time (t-1).

3.4 The Firm's Problem and Equilibrium Output

The representative firm maximizes its expected discounted profit stream:-

$$V = \max_{\{N_t, L_t\}} E_t \sum_{j=0}^{\infty} \beta^j \left[Y^i_{t+j} - \frac{W_{t+j}}{P_{t+j}} (N_{t+j} + L_{t+j}) \right]$$

$$0 < \beta < 1$$

subject to the technology given by (3.1) and (3.2). The expectations operator E_t is based on I_t, the information set available during the production process as in Chapter 2.

As in the previous chapter, we assume that wages are set by one-period nominal wage contracts. These contracts are signed at the start of the period, before the current period's prices or the technology shock is known. The wage is:-

$$W_t = E[P_t/I_{ot}] = P^e_t.$$

where I_{ot} is the information set at the start of the period (as defined in the previous chapter).

The firm has two decision variables, L_t and N_t, $\bar{X}_t$ being taken as given for all t. The two first-order conditions are:-

(3.4) $$\alpha k L_t^{\alpha-1} [N_t^r \bar{X}_t]^{1-\alpha} = P^e_t/P_t$$

(3.5) $$r(1-\alpha) k L_t^{\alpha} \bar{X}_t^{1-\alpha} N_t^{r(1-\alpha)-1} = P^e_t/P_t$$

(3.4) and (3.5) represent two equations in two unknowns. Taking natural logarithms, one can solve for l_t and n_t to obtain:-

$$(3.6) \qquad l_t = a_o + a_1(p_t - p_t^e) + a_2 x_t$$

$$(3.7) \qquad n_t = a_o^* + a_1(p_t - p_t^e) + a_2 x_t$$

$$a_o = [\ln(\alpha k)(1-r(1-\alpha)) + (1-\alpha)r\ln(r(1-\alpha)k)]$$

$$\div [(1-\alpha)(1-r)]$$

$$a_o^* = [\ln(r(1-\alpha)k) + \{\alpha\ln(\alpha k)/(1-\alpha)\}]/(1-r)$$

$$a_1 = 1/(1-\alpha)(1-r)$$

$$a_2 = 1/(1-r)$$

Substituting (3.2) into (3.1) and taking the natural logarithm yields:-

$$y_t^i = \ln(k) + \alpha l_t + (1-\alpha)x_t + r(1-\alpha)n_t$$

Substituting into the above equation from (3.6) and (3.7) for l_t and n_t yields the representative firm's supply function:-

$$(3.8) \qquad y_t^{s,i} = b_o + b_1(p_t - p_t^e) + b_2 x_t$$

$$b_o = \ln(k) + \alpha a_o + r(1-\alpha)a_o^*$$

$$b_1 = [\alpha + r(1-\alpha)]/(1-\alpha)(1-r); \quad b_2 = 1/(1-r)$$

The aggregate supply function is:-

$$(3.9)\quad y_t^s = q + b_o + b_1(p_t - p_t^e) + b_2\bar{x}_t$$

where q is the natural logarithm of the number of firms in the economy.

As in the previous chapter, we assume that aggregate demand is given by (2.15):-

$$y_t^d = m_t - p_t$$

We further assume that the Central Bank follows the same money supply rule as in Chapter 2, and that this rule is public knowledge. However, in forming its forecast of output, the Bank will now use the aggregate supply function (3.9) instead of (2.13). We assume that the current exogenous technology shock, h_t, is not contained in the Bank's information set. Consequently, its expectation of $\bar{x}_t$ is:-

$$E_t^g\bar{x}_t = \bar{x}_{t-1} + rn_{t-1} \quad \text{since } (E_t^g h_t = 0)$$

$$= \bar{x}_t - h_t$$

This means that (2.22) is replaced by:-

$$E_t^g y_t = q + b_o + b_2(\bar{x}_t - h_t)$$

Proceeding as before, we obtain the deviation of actual from expected prices:-

$$(3.10) \quad p_t - p_t^e = [1/(1+b_1)](\epsilon_t - b_2 h_t)$$

Substituting (3.10) into (3.9) provides the equation for the time path of output:

$$(3.11) \quad y_t = q + b_o + [b_1/(1+b_1)](\epsilon_t - b_2 h_t) + b_2 \bar{x}_t$$

Lagging (3.7) one period, substituting it into (3.3) and using (3.10) yields the following expression or $\bar{x}_t$:-

$$(3.12) \quad \bar{x}_t = d_o + d_1 \bar{x}_{t-1} + d_2 \epsilon_{t-1} + h_t - d_3 h_{t-1}$$

$$d_o = ra_o^*$$

$$d_1 = (1 + ra_2) > 1$$

$$d_2 = ra_1/(1 + b_1)$$

$$d_3 = ra_1 b_2/(1 + b_1)$$

Solving (3.12) backwards:-

$$(3.13) \quad \bar{x}_t = d_1^t \bar{x}_o + d_{ot}^* + h_t + \sum_{j=0}^{t-1} d_1^j d_2 \epsilon_{t-j-1}$$

$$+ \sum_{j=0}^{t-2} d_1^j (d_1 - d_3) h_{t-j-1}$$

where $d^*_{ot} = \sum_{j=0}^{t-1} d_o d_1^j$ is a deterministic exponential time trend, $\bar{x}_o$ is a reference value in the past at time zero, and we have assumed $h_o = 0$ for simplicity.

Substituting (3.13) into the equation for output (3.11):-

$$(3.14) \quad y_t = b^*_{ot} + b^*_1 \epsilon_t + b^*_2 h_t + b^*_3 \sum_{j=0}^{t-1} d_1^j \epsilon_{t-j-1} + b^*_4 \sum_{j=0}^{t-2} d_1^j h_{t-j-1}$$

$$b^*_{ot} = q + b_o + b_2(d^*_{ot} + d_1^t \bar{x}_o)$$

$$b^*_1 = b_1/(1 + b_1); \qquad b^*_2 = b_2/(1 + b_1)$$

$$b^*_3 = b_2 d_2; \qquad b^*_4 = b_2(d_1 - d_3)$$

This equation demonstrates that output depends on monetary and on real exogenous shocks. A positive monetary surprise ($\epsilon_t > 0$) will raise output; similarly, an exogenous increase in technology ($h_t > 0$) raises output. However, output is also a non-stationary process, since it depends on non-stationary cumulative sums of past monetary and technology shocks. Because $d_1 > 1$, the influence of monetary shocks does not dissipate over time. Stationarity would require that $d_1 < 1$ so that the influence of the real and monetary shocks dissipates over time.

Output is not difference-stationary either, as can be seen by taking the first difference of output:-

$$(3.15) \quad y_t - y_{t-1} = (b^*_{ot} - b^*_{ot-1}) + b^*_1(\epsilon_t - \epsilon_{t-1})$$

$$+ b^*_2(h_t - h_{t-1}) + b^*_3\epsilon_{t-1} + b^*_4 h_{t-1}$$

$$+ b^*_3 \sum_{j=0}^{t-2} d_1^j(d_1 - 1)\epsilon_{t-j-2}$$

$$+ b^*_4 \sum_{j=0}^{t-3} d_1^j(d_1 - 1)h_{t-j-2}$$

and $(b^*_{ot} - b^*_{ot-1}) = b_2\bar{x}_o(d_1^t - d_1^{t-1}) + b_2 d_o d_1^{t-1}$.

Thus both output, the first difference of output, and higher-order differences, are non-stationary moving average processes. In fact, output has a greater than unit root in this model, which can be seen by writing (3.15) as an ARMA process:-

$$(3.16) \quad y_t = d_1 y_{t-1} + [(q+b_o)(1-d_1) + b_2 d_o] + b^*_1\epsilon_t$$

$$+ (b^*_3 - d_1 b^*_1)\epsilon_{t-1} + b^*_2 h_t + (b^*_4 - d_1 b^*_2)h_{t-1}$$

Thus output can be represented by an ARMA(1,1) model that contains a greater-than-unit root ($d_1 > 1$).

Money is not neutral in this model; the effect of a monetary shock does not diminish over time, but grows over time as the level of technology rises. The reason is that a monetary shock that raises technology today also raises the productivity of all future research and hence the future level of technology. It also raises the productivity of labour that produces commodities. Thus a monetary innovation raises or lowers the entire time profile of output and permanently raises or lowers expected future output.

The above conclusions also pertain to a purely monetary model that contains no real shocks of any kind. To transform the model we have developed into a purely monetary model, we set $h_t = 0$ for all t. (3.14) then reduces to:-

$$(3.17) \quad y_t = b_{ot}^* + b_1^* \epsilon_t + b_3^* \sum_{j=0}^{t-1} d_1^j \epsilon_{t-j-1}$$

As in Chapter 2, in a purely monetary model, output depends on the non-stationary cumulative sum of monetary innovations. This demonstrates that even in a purely monetary model output is still non-stationary and non-difference stationary, and contains a greater-than-unit root, none of these properties depending on the inclusion of exogenous real shocks.

3.5 A Model with a Unit Root

The question that naturally arises is why one should obtain a greater-than-unit root in the model developed in this chapter. The answer is the same as for the model in Chapter 2: in this model, as in the model of Chapter 2, the rate of growth of output depends on the <u>level</u> of output.

Specifically, the rate of growth of technology depends on n_t, the resources devoted to R&D. However, n_t in turn depends on l_t, the employment of labour that produces final goods, and on $\bar{x}_t$ the level of technology. As l_t and/or $\bar{x}_t$ rise, so do the marginal benefits of R&D, so that n_t increases. Thus an increase in the level of production (an increase in l_t) will raise n_t, and so stimulate the rate of growth of production. Similarly, a rise in $\bar{x}_t$ will raise n_t, and in so doing raise the rate of growth of $\bar{x}$. This can be seen by re-examining (3.1)-(3.3), (3.7) and (3.12). (3.12) can be written as:-

$$\bar{x}_t - \bar{x}_{t-1} = a_2 r \bar{x}_{t-1} + h_t + \ldots$$

If, on the other hand, technology is completely stochastic, as in a real business cycle model, a unit root would result in both the technology and output processes because, with exogenous technology, n_t would be zero for all t and one would obtain:-

$$\bar{x}_t = \bar{x}_{t-1} + h_t$$

It is worth considering the restrictions that must be placed on the commodity and innovation production functions (3.1) and (3.2), so that a model with an innovation production function would yield a unit root. If these restrictions are implausible, it implies that one cannot expect output to display a unit root in an economy with an innovation production function.

In order to obtain a unit root, it is sufficient that the marginal productivity of R&D (i.e. of n_t) be independent of <u>both</u> the level of output (i.e. of l_t) and of $\bar{x}_t$. Only under these conditions will the rates of growth of technology and output be independent of their levels. To ensure this, (3.1) and (3.2) must be linearized and replaced by

$$(3.1a) \quad Y_t^i = k_1 X_t + k_2 L_t^{\alpha}$$

$$(3.2a) \quad X_t = \bar{X}_t + k_3 N_t^r$$

The firm has the same value function as in Section 3.4 above, and the first order conditions are:-

$$\alpha k_2 L_t^{\alpha-1} = P_t^e / P_t$$

$$r k_1 k_3 N_t^{r-1} = P_t^e / P_t$$

Taking natural logarithms, the solutions for l_t and n_t are:-

$$(3.18) \quad l_t = [\ln(\alpha k_2)/(1-\alpha)] + [(p_t - p_t^e)/(1-\alpha)]$$

$$(3.19) \quad n_t = [\ln(r k_1 k_3)/(1-r)] + [(p_t - p_t^e)/(1-r)]$$

The production function is additive, and this requires that we take a log-linear <u>approximation</u> about a point:-

$$(3.20) \quad y_t^i = g_o + g_1 \bar{x}_t + g_2 r n_t + g_3 \alpha l_t$$

Substituting (3.18) and (3.19) into (3.20) and aggregating yields the aggregate supply function:-

$$(3.21) \quad y_t^s = c_o + c_1(p_t - p_t^e) + c_2\bar{x}_t$$

$$c_o = q + g_o + [g_2 r \ln(rk_1k_3)]/(1-r)$$

$$+ [g_3\alpha \ln(\alpha k_2)]/(1-\alpha)$$

$$c_1 = [g_2 r/(1-r)] + [g_2\alpha/(1-\alpha)]$$

$$c_2 = g_1$$

Proceeding as in the last section, we obtain a modified form of (3.10):

$$(3.22) \quad p_t - p_t^e = [\epsilon_t - c_2h_t]/(1 + c_1)$$

Substituting (3.22) into (3.21) we obtain the output path:-

$$(3.23) \quad y_t = c_o + [c_1/(1+c_1)](\epsilon_t - c_2h_t) + c_2\bar{x}_t$$

We continue to assume that the evolution of $\bar{x}$ is given by (3.3). Lagging (3.19) by one period and substituting the result into (3.3) for n_{t-1}, and using (3.22) yields:-

$$(3.24) \quad \bar{x}_t = \bar{x}_{t-1} + f_o + f_1\epsilon_{t-1} + h_t - f_2h_{t-1}$$

$$f_o = [r\ln(rk_1k_3)]/(1-r)$$

$$f_1 = r/(1+c_1)(1-r)$$

$$f_2 = rc_2/(1+c_1)(1-r)$$

Now n_t is no longer dependent on $\bar{x}_t$, (i.e. the resources devoted to R&D are independent of the level of technology). (3.24) contains a unit root for $\bar{x}_t$, so that the output equation will also contain a unit root. Solving (3.24) backwards, and substituting the solution into the equation for output, (3.23), yields:-

$$(3.25) \quad y_t = c^*_{ot} + c^*_1(\epsilon_t - c_2h_t)$$

$$+ c^*_2 \sum_{j=1}^{t} \epsilon_{t-j} + c^*_3 \sum_{j=1}^{t} h_{t-j}$$

$$c^*_{ot} = c_o + \bar{x}_o + tf_o; \quad c^*_1 = c_1/(1+c_1)$$

$$c^*_2 = c_2f_1; \quad c^*_3 = c_2(1-f_2)$$

and we have assumed $h_o = 0$ for simplicity. Output again follows a non-stationary process and consists of accumulations of monetary and technology shocks. However, this process is first difference stationary:-

$$(3.26) \quad y_t - y_{t-1} = f_o + c_1^*(\epsilon_t - c_2 h_t)$$

$$+ (c_2^* - c_1^*)\epsilon_{t-1} + (c_3^* + c_1 c_2^*)h_{t-1}$$

As the ϵ's and h's are independent random variables with zero mean, output can be written as an IMA(1,1) process, which is the form Nelson and Plosser argued best approximated the output process in reality.

Further, if we abstract from real shocks and set $h_t = 0$ for all t, and only allow for monetary shocks, output is still non-stationary in the levels, but contains a unit root and hence is difference stationary. Thus a monetary model with endogenous technology can yield an output equation that contains a unit root.

The question of the plausibility of these restrictions must be addressed. While one might argue that expenditure on R&D might be independent of the state of technology, as is suggested by (3.2a), it does <u>not</u> seem tenable to suggest that the marginal product of labour is unrelated to the level of technology, as (3.1a) does. However, even if we replace only (3.1a) by a multiplicative production function like (3.1), we will again obtain a greater-than-unit root.[4] Thus it would appear that a greater-than-unit root in the output process is a plausible outcome in models with an innovation production function.

As in the previous chapter, the presence of a greater-than-unit root in a model like (3.16) suggests this can be used to discriminate between monetary models with endogenous technology, and a real business cycle model, such as (2.36). However, in (3.16), like in the model of the previous chapter, discrimination is difficult. The reason is that d_1 may to be close to unity,[5] so that again the problem of distinguishing a model with a unit root from one with an only slightly greater-than-unit root arises. In small samples the time series movements suggested by these two models are likely to be so similar that acceptance of one model would not enable us to reject the alternative. Thus, the work of Nelson and Plosser and others who have been unable to reject the hypothesis that output has a unit root does not constitute a rejection of the models developed in this thesis that display greater-than-unit roots.

In conclusion this section has demonstrated that a monetary model with endogenous technology (and with or without exogenous shocks to technology) can yield an output equation that contains a unit root, provided certain restrictions are placed on the innovation and commodity production functions. However, the restrictions that are required are of questionable plausibility.

3.6 Depreciation of Technology

Unlike physical capital, scientific and technical knowledge are not generally regarded as subject to depreciation. Technical knowledge does not decrease as time goes by: once a new technology is adapted by society, it does not go back to using older, less efficient technology (except under abnormal circumstances such as times of war), even if very little is spent on R&D. However, one can think of certain technologies that become obsolete before replacements are found for them. For example, The Economist reports that "Bacteria are evolving resistance to antibiotics and insects resistance to pesticides faster than new chemicals are being invented. Before long, at this rate, medicine will revert to the days before penicillin and agriculture to the days before DDT".[6] Thus there is evidence that, at least in the fields of medicine and agriculture, the technologically useful knowledge society has at its disposal is depreciating quite rapidly. In general, a technology developed to cope with a particular state of nature may prove useless if the state of nature changes. But, while it is admittedly difficult to make a plausible general case for depreciation of technology, it is instructive to see what occurs if we perturb the model of Section 3.4 by allowing X to depreciate. To do this, we replace (3.3) by:-

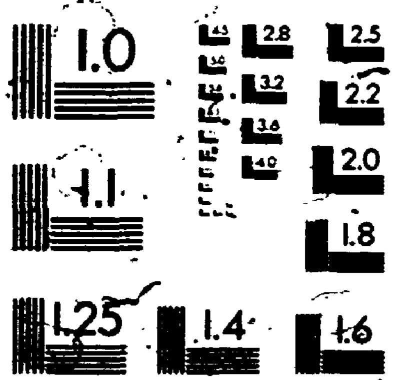
$$(3.3a) \quad X_t = (1 - \delta)X_{t-1} + rn_{t-1} + h_t \qquad 0 < \delta < 1$$

where δ is the rate of depreciation of technology. We now consider the impact of depreciating technology in three models. The first is the general model developed in Section 3.5 above. The second is a purely monetary model with endogenous technology. The third is a conventional real business cycle model, that is, a model with only exogenous shocks to technology and no innovation production function.

3.6.1 Depreciation in the General Model

Using (3.3a) instead of (3.3), and proceeding as before, we obtain the following expression for X_t:-

2 of/de 2



$$(3.27) \quad \bar{x}_t = d_o + \tilde{d}_1 \bar{x}_{t-1} + d_2 \epsilon_{t-1} + h_t - d_3 h_{t-1}$$

and $\tilde{d}_1 = (1 + ra_2 - \delta)$ while d_o, d_2 and d_3 are as defined above. Equation (3.27) is almost identical to (3.12), the only difference being that d_1 in (3.12) has been replaced by $\tilde{d}_1$. If we proceed as before, we can obtain an expression analogous to (3.16) for output:-

$$(3.28) \quad y_t = (1 + ra_2 - \delta) y_{t-1} + [(q + b_o)(1 - \tilde{d}_1) + b_2 d_o]$$

$$+ b_1^* \epsilon_t + (b_3^* - \tilde{d}_1 b_1^*) \epsilon_{t-1}$$

$$+ b_2^* h_t + [b_2(\tilde{d}_1 - d_3) - \tilde{d}_1 b_2^*] h_{t-1}$$

Recall that $ra_2 = r/(1-r)$. Thus, the output process represented by (3.28) will be non-stationary provided that $\{1-\delta + [r/(1-r)]\} > 1$, or if $r/(1-r) > \delta$, where δ is the depreciation rate. If δ is approximately equal to $r/(1-r)$, one would obtain a unit root (or a root insignificantly different from unity) without the restrictions that section 3.5 placed on technology. Obviously, the higher the rate of depreciation is, the less likely it is that the condition $r/(1-r) > \delta$ will be met. Thus, introducing depreciation does not guarantee that the output process will be stationary in an economy with endogenous technology. The value of the root of output is ambiguous in this model: it may be greater than, less than, or insignificantly different from unity. Thus, if technology depreciates, output may exhibit either stationarity or non-stationarity in a model with endogenous technology.

3.6.2 Depreciation in a Monetary Model with Endogenous Technology

This section examines a purely monetary model that contains no exogenous real shocks, so that $h_t = 0$ for all t, and (3.3a) reduces to:-

(3.29) $\bar{x}_t = (1 - \delta)\bar{x}_{t-1} + rn_{t-1}$

Lagging (3.7) and substituting it into (3.29) for n_{t-1} yields:-

(3.30) $\bar{x}_t = (1-\delta + ra_2)\bar{x}_{t-1} + ra_o^* + ra_1(p_t - p_t^e)$

Even if there are no real shocks impinging on $\bar{x}$, the aggregate supply function derived for the general model (equation (3.9)) is still applicable. Writing (3.9) as an ARMA process, and using (3.30), yields, after some manipulation:-

(3.31) $y_t^s = \lambda_o + \lambda_1(p_t - p_t^e) + \lambda_2(p_{t-1} - p_{t-1}^e) + \lambda_3 y_{t-1}^s$

$\lambda_o = (q + b_o)(\delta + ra_2) + b_2 ra_o^*$

$\lambda_1 = b_1 + b_2 ra_1$

$\lambda_2 = - b_1(1 - \delta + ra_2)$

$\lambda_3 = (1 - \delta + ra_2)$

Notice that if $\lambda_3 < 1$, then (3.31) is a stationary process. Furthermore, if λ_3 is less than unity, (3.31) bears a striking resemblance to the aggregate supply function that Lucas (1973) first proposed. The sole difference is that (3.31) contains a lagged price-surprise term as well. Thus, if depreciation of technology is sufficient to render the output process stationary, then a monetary model with endogenous technology provides a microeconomic foundation for the Lucas aggregate supply curve. The Lucas aggregate supply curve thus emerges as a special case of the general model considered in this chapter.

3.6.3 A Real Business Cycle Model with Depreciating Technology

This section examines a conventional real business cycle model with endogenous technology. The model is conventional in so far as technology is exogenous, and money has no effect on real income. We therefore assume that there are no monetary innovations in this model economy: either the money supply is constant, or all changes in the money supply are correctly forecast and take no one by surprise. As in the previous chapter, velocity is equal to unity through the cash-in-advance constraint, so that there are no velocity shocks. Since technology is exogenous, there is no innovation production function, so that

$$x_t = \bar{x}_t$$

and the technology available to the firm is that which is publicly available. Furthermore, the evolution of technology is determined by random shocks:-

$$\bar{x}_t = (1 - \delta)\bar{x}_{t-1} + h_t \tag{3.32}$$

and, as previously, δ is the rate of depreciation of technology. (3.32) can be written as:-

$$\bar{x}_t = \sum_{j=0}^{\infty} (1 - \delta)^j h_{t-j}$$

Since $(1-\delta) < 1$, the impact of a shock h_t will diminish over time. Thus, assuming technology depreciates is equivalent to assuming that the impact of the real shocks hitting the economy diminishes over time. The representative firm's only decision variable is L_t, its labour input and its value function is:-

$$V = \underset{\{L_t\}}{\text{Max}} \; E_t \sum_{j=0}^{\infty} \beta^j [Y^i_{t+j} - \frac{W_{t+j}}{P_{t+j}} L_{t+j}]$$

and

$$Y^i_t = kL^{\alpha}_t \bar{X}^{1-\alpha}_t .$$

Proceeding as previously, we take the first-order-condition for labour, derive the demand for labour function and thus the aggregate supply function:-

$$y^s_t = q + \bar{b}_o + \bar{b}_1(p_t - p^e_t) + \bar{x}_t$$

$$\bar{b}_o = \ln(k) + \alpha\ln(\alpha k)$$

$$\bar{b}_1 = \alpha/(1-\alpha)$$

As there are no monetary shocks, price surprises are only due to the real shock that causes unanticipated changes in output, and (3.10) must be amended to read:-

$$(p_t - p_t^e) = -[1/(1+\bar{b}_1)]h_t$$

Substituting the above expression into the aggregate supply function provides the equilibrium value of output:-

$$(3.33) \quad y_t = q + \bar{b}_o - [\bar{b}_1/(1+\bar{b}_1)]h_t + \bar{x}_t$$

Using (3.32), we can write (3.33) as an ARMA process:-

$$(3.34) \quad y_t = (1-\delta)y_{t-1} + \delta(q+\bar{b}_o) + [1/(1+\bar{b}_1)]h_t - \left[\frac{(1-\delta)\bar{b}_1}{(1+\bar{b}_1)}\right]h_{t-1}$$

Because $(1-\delta) < 1$, the output process of (3.34) is stationary. Note that if technology did not depreciate, (i.e. $\delta = 0$), then the process captured by (3.34) would display a unit root and would be non-stationary. However, if $\delta > 0$, and technology depreciates, then a conventional real business cycle model like (3.34) cannot account for non-stationarity. Conventional real business cycle models result in stationary output processes if technology depreciates. Thus, if technology is generally subject to depreciation, conventional real business cycle models are incapable of accounting for the properties output displays.

3.7 Neutrality, Superneutrality and Policy Invariance

We now consider whether the model developed in this chapter exhibits the properties of neutrality, superneutrality and policy invariance.

3.7.1 Neutrality

Money is not neutral in this model. Indeed, there is a long-run non-neutrality of money, as can be seen by examining (3.14) or (3.25). Both of these equations show that output depends on accumulations of monetary shocks. The effect of a monetary surprise does not dissipate over time: it has a permanent effect and changes the long-run path of output.

3.7.2 Superneutrality

Secondly, money is not superneutral. This results, as in the model of Chapter 2, from our imposition of a cash-in-advance constraint, and is not a feature of models of endogenous technology *per se*. Had money been introduced differently, superneutrality might hold.

To demonstrate this, we allow for a non-zero rate of inflation. As was shown in Chapter 2, the equation for the money wage then becomes

$$(2.47) \quad w_t = p_t^e + \mu$$

where μ is the expected rate of inflation. Using this expression for wages, one obtains the following equations for l_t and n_t instead of (3.6) and (3.7):-[7]

$$(3.35) \quad l_t = a_o + a_1(p_t - p_t^e) + a_2\bar{x}_t - a_1\mu$$

$$(3.36) \quad n_t = a_o^* + a_1(p_t - p_t^e) + a_2\bar{x}_t - a_1\mu$$

This results in the aggregate supply function:-

$$(3.37) \quad y_t^s = q + b_o + b_1(p_t - p_t^e) - b_1\mu + b_2\bar{x}_t$$

This demonstrates that aggregate supply depends on both the expected change in prices, μ, and on the unexpected change in prices, $(p_t - p_t^e)$. Substituting (3.10) for the price surprise into (3.37) gives us the value of output:-

$$(3.38) \quad y_t^s = q + b_o + \frac{b_1}{1+b_1}(\epsilon_t - b_2h_t) - b_1\mu + b_2\bar{x}_t$$

Output depends on anticipated inflation and on unanticipated inflation (the price surprise). However, anticipated and unanticipated inflation affect output differently. A positive expected inflation rate has a negative impact on output for the reasons set out in the previous chapter; a positive price surprise (caused, for example, by a positive value of ϵ_t, the money supply shock) has a positive effect on output. It follows that a change in the anticipated inflation rate will alter output, so that money is not superneutral in the general model developed in this chapter.

Output in the current period will depend not only on the current anticipated inflation rate, but on past values of the anticipated inflation rate as well. This can be seen from (3.36): n_t, the resources devoted to R&D, depend on π, the expected inflation rate. Inflation affects n_t through two channels. Firstly, the rise in the money (and real) wage caused by an anticipated inflation rate that is greater than zero will make research more expensive. As the wages of researchers rise, so must their marginal product, and because research yields diminishing returns, the quantity of research undertaken will fall. Secondly, the rise in money and real wages in the current period caused by a positive expected inflation rate will also reduce the employment of labour that produces the final good. A lower level of output will however reduce the benefits of research; the marginal product of n_t falls as l_t falls. The level of publicly available technical knowledge, $\bar{x}_t$, depends on past values of n_t and will therefore be a function of past values of the expected inflation rate. Consequently output will also depend on the history of anticipated (and unanticipated) inflation.

3.7.3 Policy Invariance

Thirdly, the probability distribution of output is not policy invariant. To illustrate this we change the money supply rule slightly, by changing the monetary authorities information set. Specifically, we assume that the monetary authority has access to a noisy indicator of h_t, the exogenous real shock, and utilizes this information when making its money supply decision.[8] We denote the indicator by $\tilde{h}_t$, and $\tilde{h}_t = h_t + e_t$, where e_t is a zero mean, normally distributed random variable with constant variance, σ_e^2. This new information means that the Central Bank's output forecast becomes:-

$$E_t^g y_t = q + b_o + b_2(\bar{x}_t + e_t)$$

because now $E_t^g \bar{x}_t = \bar{x}_{t-1} + rm_{t-1} + (h_t + e_t)$.

Proceeding as before, we obtain the price surprise as:-

$$(3.39) \quad (p_t - p_t^e) = \frac{1}{1+b_1}\left[\epsilon_t + b_2 e_t\right]$$

so that the output equation changes to:-

$$(3.40) \quad y_t = q + b_o + \frac{b_1}{1+b_1}\left[\epsilon_t + b_2 e_t\right] + b_2 \bar{x}_t$$

The variance of output conditional upon its expected value one period ahead is:-

$$\sigma^2_{y/E_t^g h_t - \tilde{h}} = E_t[y_{t+1} - Ey_{t+1}]^2$$

$$= \left[\frac{b_1}{1+b_1}\right]^2(\sigma_\epsilon^2 + b_2^2\sigma_e^2) + b_2^2\sigma_h^2$$

Under the old money supply rule, output is given by (3.11):-

$$(3.11) \quad y_t = q + b_o + \frac{b_1}{1+b_1}(\epsilon_t - b_2 h_t) + b_2 \bar{x}_t$$

The conditional variance of output under the old rule is:-

$$\sigma^2_{y/E^g_t h_t=0} = [b_1/(1+b_1)]^2\sigma^2_\epsilon + [b_2/(1+b_1)]^2\sigma^2_h$$

and $\sigma^2_{y/E^g_t h_t=0} \neq \sigma^2_{y/E^g_t h_t=\tilde{h}_t}$

Thus the variance of output changes if the money supply rule changes.

Furthermore, the trend value of output also changes if the money supply rule changes. As output is non-stationary, the mean, or trend value of output is not deterministic, but stochastic. It can be thought of as the expected future value of output, conditional on information available at time t. The mean, or expected value, will consequently depend on accumulations of shocks, as can be seen by updating (3.14) and taking expected values:-

$$(3.41) \quad E_t y_{t+i} = b^*_{ot+i} + b^*_3 \sum_{j=0}^{t} d_1^{j+i-1}\epsilon_{t-j} + b^*_4 \sum_{j=0}^{t-1} d_1^{j+i-1} h_{t-j}$$

and $E_t h_{t+i} = E_t \epsilon_{t+i} = 0$ for $i > 0$ since these variables are random with a zero mean.

For the new money supply rule that results in the output process (3.40), the trend value can be obtained by lagging (3.7) one period, substituting into (3.39) for n_{t-1}, and using (3.39) derive an expression for $\bar{x}_t$:-

$$(3.42) \quad \bar{x}_t = d_0 + d_1\bar{x}_{t-1} + d_2 \epsilon_{t-1} + d_3 e_{t-1} + h_t$$

where the d's are those defined in (3.12) above. Solving (3.42) backwards and substituting the result into the output equation derived under the new money supply rule, (3.40):-

$$(3.43) \quad y_t = b_{ot}^* + b_1^*\epsilon_t + b_2^*e_t + b_3^* \sum_{j=0}^{t-1} d_1^j \epsilon_{t-j-1}$$

$$+ b_2 d_3 \sum_{j=0}^{t-1} d_1^j e_{t-j-1} + b_2 \sum_{j=0}^{t-1} d_1^j h_{t-j}$$

and b_{ot}^*, b_1^*, b_2^* and b_3^* are as defined in (3.14) below. Updating (3.43) and taking expectations gives us the trend or expected future value:-

$$(3.44) \quad Ey_{t+i} = b_{ot+i}^* + b_3^* \sum_{j=0}^{t} d_1^{j+i-1} \epsilon_{t-j}$$

$$+ b_2 d_3 \sum_{j=0}^{t} d_1^{j+i-1} e_{t-j} + b_2 \sum_{j=0}^{t-1} d_1^{j+i-1} h_{t-j}$$

and again $E_t h_{t+i} = E_t \epsilon_{t+i} = E_t e_{t+i} = 0$. A comparison of (3.41) and (3.44) indicates that the mean, or trend value of output is different under different money supply rules. Firstly, a new cumulative sum of shocks, $\Sigma d_1^{j+i-1} e$, enters (3.44). Secondly, the coefficient premultiplying the sum of exogenous real shocks has changed, so that the exogenous real shocks have a different impact on the trend value of output under the new money supply rule. It is worth noting that in (3.14) and (3.43) (as well as in (3.41) and (3.44)) the shocks of each period are premultiplied by a term that grows exponentially over time $d_1^j > 1$, $j > 0$ - so that the cumulative sums in these equations are non-stationary processes and do not have a zero mean, although theshocks themselves are random drawings from zero mean distributions.

The trend value of output is not fixed nor a deterministic function of time in models with endogenious technology, but depends on accumulations of stochastic shocks. Different money supply rules result in different trend values of output, as both the shocks and their impact on output can change when the money supply rule changes.

To conclude, money is not neutral in models with endogenous technology. Nor is money superneutral, because we have imposed a cash-in-advance constraint. Finally, both the conditional variance and trend of output can change when the money supply rule changes.

3.8 Summary and Conclusions

This chapter has set out an alternative model of endogenous technology. Technology was treated as (partly) deterministic, and an innovation production function was explicitly introduced. It was assumed that technology depends, through the innovation production function, on resources devoted to R&D. Technology was also allowed to depend on exogenous stochastic shocks. It was shown that output is non-stationary in such a model, and that the property of non-stationarity does not depend on the inclusion of exogenous real shocks: non-stationarity also arises in a purely monetary model that contains no real shocks, as (3.17) demonstrates.

It was also found that output contains a greater-than-unit root and is therefore non-difference stationary. The reason is that the growth of output depends on the level of output:- a monetary surprise that raises output also raises the profitability of research. However, a higher level of research results in a higher level of technology, which in turn raises the average and marginal productivity of labour, and so results in a permanent increase in output.

The restrictions that have to be placed on technology to yield a unit root were examined. In this case, a purely monetary model still produces a non-stationary output process but with a unit root. However, the necessary restrictions are not very plausible, so that a unit root is unlikely to occur in the output process of an economy with an innovation production function.

The effect of depreciation of technology was also examined in three models. In the general model it was found that depreciation of technology renders the value of the root of the output process ambiguous. Consequently, even if technology, like capital, depreciates, output may still be non-stationary in an economy with endogenous technology. A monetary model with endogenous technology that contained no real shocks was also examined. Again, allowing technology to depreciate makes the root of output uncertain. However, if the rate of depreciation is large enough to render the output process stationary, the aggregate supply function that the purely monetary model yields is very similar to the Lucas aggregate supply function. However, while there is some evidence for depreciation of technology in medicine and agriculture, it is difficult to make a plausible general case for depreciation of technology, on a significant scale, so that the Lucas-type supply curve we derive is only a theoretical curiosity. A final model that was considered was a typical real business cycle model, where technological change is driven by a sequence of stochastic shocks. It was demonstrated that if technology depreciates, such a model cannot account for the non-stationarity of output.

Finally, we addressed the question of whether money is neutral in this model, whether the model exhibits superneutrality of money and policy-invariance. As in the case of the model of the previous chapter, money is not neutral. The effect of a monetary surprise does not disipate over time; it has a permanent impact on output. This stands in contrast to the conventional monetary business cycle models in which money is neutral. Secondly, money is not superneutral either, because we impose a cash-in-advance constraint. Thirdly, both the conditional variance and the trend of output change if the money supply rule changes. This further emphasizes the distinction between these models and conventional monetary models.

Footnotes

1. A more general model, which allows the firm to retain an innovational advantage longer, is presented in Appendix 3.

2. Empirical work on the innovation production function shows a clear positive relationship between innovative output and inputs to the R&D process. See Kamien and Schwarz (1982), pp.64-70.

3. With respect to (3.2), it must be noted that Scherer (1982) found that there is "no lag between the emergence of demand pull influences and the time when invention occurs ... These results ... may reflect a tendency for corporate inventors to anticipate favourable demand conditions even before they materialize fully" (p.230). Hall et al. (1986) find that there is a strong contemporaneous relationship between patents and R&D, as did earlier work. If patents measure inventive output accurately, this implies that at least some of the results of R&D are available very quickly (using annual data, within one year at most). Thus (3.2) may not be a totally unrealistic representation.

4. This can be seen by substituting (3.2a) into (3.1). The marginal product of research (i.e. of N_t) will depend on L_t, which in turn depends on $\bar{X}_t$, so that we again obtain the result that increases in technology brought about by research will depend on the level of technology.

5. Note that $d_1 = (1 + ra_2) = (1 + r/(1-r))$. The "research elasticity" of output, i.e. $(\%\Delta Y/Y) = (\%\Delta N/N)$ is equal to $(1-\alpha)r/(1-r)$. This means that if a doubling of research expenditure leads to a 1% rise in output, then $(1-\alpha)r/(1-r) = 0.01$. As α, labour's share in output, is about 0.7, this suggests that $r/(1-r) = 0.033$, which is sufficiently close to zero to cause confusion between a unit root model and a model where $d_1 > 1$.

6. The Economist, (March 21, 1987), p.93.

7. As noted in Chapter 2, with a non-zero anticipated inflation rate the firm's expected profit stream should be deflated not by the current price level but by the next period's expected price level. However, if we ignore Jensen's inequality, (3.35) and

(3.36) can be regarded as approximations of the firm's decision rules for l_t and n_t.

8. We therefore assume that when the Central Bank determines the money stock it has more information than the household had when households and firms negotiated the money wage. The Central Bank's information set is now $I_t^g = [I_{0t}, \tilde{h}_t]$. The reason for the Bank's richer information set results from the time structure of the model: the Central Bank can wait until wage contracts have been signed and until it has some indication of productivity before determining its monetary target. At the beginning of the period, before wage contracts are signed, the Bank and households share the same information set I_{0t}.

CHAPTER 4

WAGE INDEXATION

4.1 Introduction

This chapter investigates the effect of wage indexation on the properties of output fluctuations in a monetary model with endogenous technology. In particular, it considers whether the results of the two previous chapters - that money is non-neutral and output is non-stationary in monetary models with endogenous technology - still hold if nominal wages are indexed to the price level. The general conclusion of this chapter is that if there is complete (or near-complete) indexation of wages, monetary models lose their ability to account for non-stationarity, and money is neutral.

The following section extends the model developed in Chapter 2 by assuming that the nominal wage contracts that firms and workers sign are (to some degree) indexed to the price level. It examines the effects of real and monetary shocks with and without indexation and reiterates some standard results. Indexation insulates the economy from nominal shocks, but increases the impact of real shocks.

The third section considers whether monetary models can still account for non-stationary once wages are indexed. It concludes that if there is full indexation, money is neutral. The extent of wage indexation in the United States is also considered.

The fourth section considers to which extent - if any - wage indexation might be optimal. While the framework developed in this thesis is not suited to addressing the question of the optimality of labour contracts per se, except in the most general terms, one can argue that, if there exist adjustment costs and/or agents are risk averse, an institution that minimizes the variance of output is preferable to one that does not. What degree of indexation minimizes the variance of output will depend on whether it is possible to index contracts to real shocks. If real shocks are unobservable, indexation of wages to the price level may not reduce the variance of output.

4.2 Wage Indexation, Real and Nominal Shocks

The model under consideration is basically the same as that developed in Chapter 2. The labour market opens before the commodity market, so that households enter into nominal wage contracts with firms before the current period's price level is known. The sole difference between the model of this chapter and that of Chapter 2 is that we assume wages are partly or wholly indexed to the price level. Thus the wage-setting equation (2.3) is replaced by:-

$$(4.1) \qquad w_t = \theta p_t + (1 - \theta)p_t^e \qquad 0 < \theta < 1$$

and lower-case letters denote natural logarithms. θ is the degree of indexation. If $\theta = 1$, there is complete indexation and $w_t = p_t$. If $\theta = 0$, there is no indexation and (4.1) reduces to (2.3).

If θ lies between zero and unity, there is partial indexation and the money wage will be determined partly by expected prices and partly by actual prices, the influence of actual relative to expected prices depending on the degree of indexation.

We continue to assume that employment is determined on the demand side of the labour market. The justification for this is that if employment is determined on the supply side, one obtains the result that a monetary surprise reduces output: a monetary surprise lowers the real wage, which reduces the supply of labour and thus employment and output.

The firm's problem is the same as that of Section 2.4, and the first-order-condition for labour is:-

$$(4.2) \qquad \alpha k L_t^{\alpha-1} Z_t^{1-\alpha} F_t = W_t/P_t$$

Substituting (4.1) into a logarithmic form of the above equation yields the firm's demand for labour function as:-

$$(4.3) \qquad l_t = \frac{\ln(\alpha k)}{1-\alpha} + \frac{(1-\theta)}{(1-\alpha)}(p_t - p_t^e) + z_t + \frac{f_t}{1-\alpha}$$

Substituting this into the production function and aggregating gives us the aggregate supply function:-

$$(4.4) \qquad y_t^s = q + b_0 + b_1(1-\theta)(p_t - p_t^e) + z_t + b_2 f_t$$

$$b_0 = \ln(k) + \alpha \ln(\alpha k)/(1-\alpha)$$

$$b_1 = \alpha/(1-\alpha); \qquad b_2 = 1/(1-\alpha)$$

Notice that if $\theta = 0$ (the case of no indexation) then (4.3) is identical to (2.13), the aggregate supply function of Chapter 2. As previously, output depends positively on f_t, the joint productivity and technology shock, on the level of technical knowledge, z_t, and on the wedge between actual and expected prices, $(p_t - p_t^e)$. The demand-side framework is exactly the same as in Chapter 2. However, with indexation equation (2.25) must be replaced by:-

$$(4.5) \qquad p_t - p_t^e = \frac{1}{1+b_1(1-\theta)}[\epsilon_t - b_2(\xi_t + \eta_t)]$$

Substituting (4.5) into (4.4) gives us the equilibrium value of output:-[1]

$$(4.6) \qquad y_t = q + b_0 + \frac{b_1(1-\theta)}{1+b_1(1-\theta)}[\epsilon_t - b_2(\xi_t + \eta_t)] + z_t + b_2 f_t$$

From (4.6) one obtains the conventional result that an unexpected rise in the money supply ($\epsilon_t > 0$) will raise output, as it raises the price level above the value expected and causes a fall in the real wage that increases demand for labour and employment. However, as is clear from the output equation (4.6), complete indexation ($\theta=1$) will insulate the economy from monetary shocks.

A negative shock to productivity ($\eta_t < 0$) or technology ($\xi_t < 0$) lowers demand for labour.[2] Employment and output will fall, causing an excess demand for output and an excess supply of money at the expected price level. The actual price level must rise above p_t^e to restore equilibrium. But, in rising, the price level reduces the real wage, so offsetting some of the impact of the decline in productivity on the demand for labour and employment. Inspecting

(4.6) we find that a negative value of η_t or ξ_t has a negative impact on output with coefficient b_2, but an offsetting positive impact with coefficient $[b_2b_1(1-\theta)/(1+b_1(1-\theta))] < b_2$, resulting from the reduction of the real wage caused by the rise in prices. If there is full indexation (θ=1) this offsetting effect vanishes, because indexation guarantees that as the price level rises above its expected value so does the nominal wage, leaving the real wage unchanged. With full indexation the impact of a real shock is b_2f_t, which is greater than the impact with less-than-full indexation. Furthermore, as indexation rises (θ rises) the impact of a real shock increases as this offsetting effect diminishes.

In the case of a positive shock to productivity, lack of indexation again dampens the effect on output. A rise in productivity raises demand for labour and employment, increasing output and causing an excess supply of commodities and an excess demand for money at the price level expected to prevail. To restore equilibrium the price level must fall below its expected value. This causes the real wage to rise, and produces a decline in the demand for labour that to some degree offsets the increase in demand for labour caused by the productivity shock, so dampening the rise in employment and output.[5] However, in the case of full indexation, the real wage remains constant, and the offsetting effect of a rising real wage is absent. The greater is the degree of indexation, the smaller is the offsetting rise in the real wage.

This section has recapitulated the standard results, that indexation reduces the effects of monetary shocks on output, and full indexation can eliminate their effect completely, but indexation increases the impact of real shocks.[3] Thus, whether indexation is optimal - and what degree of indexation is optimal - will depend on the relative variances of real and nominal shocks, and is discussed below.

4.3 Non-Stationarity and Indexation

The output process represented by (4.6) is non-stationary. This can be seen more clearly by taking the first difference of (4.6):-

$$(4.7) \qquad y_t = (1+\gamma)y_{t-1} + c_1(\epsilon_t - \epsilon_{t-1}) + c_2(\eta_t - \eta_{t-1}) + c_2\xi_t + c_3\xi_{t-1}$$

$$c_1 = b_1(1-\theta)/[1 + b_1(1-\theta)]$$

$$c_2 = b_2/[1 + b_1(1-\theta)]$$

$$c_3 = b_2 b_1(1-\theta)/[1 + b_1(1-\theta)]$$

(4.7) shows that output has a greater-than-unit root. Writing (4.7) as a moving average process yields:-

$$(4.8) \qquad y_t = (1+\gamma)^t y_o + c_1\epsilon_t + c_2(\eta_t + \xi_t) + \gamma c_1 \sum_{j=0}^{t-2}(1+\gamma)^j \epsilon_{t-j-1} + \gamma c_2 \sum_{j=0}^{t-2}(1+\gamma)^j \eta_{t-j-1} + [(1+\gamma)c_2 + c_3]\sum_{j=0}^{t-2}(1+\gamma)^j \xi_{t-j-1}$$

where y_o is a reference value in the past, and we have assumed $\epsilon_o = \eta_o = \xi_o = 0$. As in previous chapters, output depends upon non-stationary accumulations of real and monetary shocks. Output will be non-stationary even if there are no real shocks, i.e. even if $\xi_t = \eta_t = 0$ for all t, because it will still depend on non-stationary accumulations of monetary shocks.

This conclusion, however, does not hold if there is complete indexation. For, if there is complete indexation, the economy is completely insulated from monetary shocks. If $\theta = 1$, then $c_1 = 0$, $c_2 = b_2$ and $c_3 = 0$ and (4.8) reduces to:-

$$(4.9) \qquad y_t = (1+\gamma)^t y_o + b_2(\eta_t + \xi_t) + \gamma b_2 \sum_{j=0}^{t-2} (1+\gamma)^j \eta_{t-j-1} + (1+\gamma) b_2 \sum_{j=0}^{t-2} (1+\gamma)^j \xi_{t-j-1} .$$

This is still a non-stationary process with a greater-than-unit root, but it is a pure real business cycle model. If indexation is complete, money has no impact on output, and cannot account for any properties the output process might display. Thus, if there is complete indexation of wage contracts, only a real business cycle model can account for non-stationarity.

This raises the question whether in reality indexation is complete (or near-complete) so as to make the assumption $\theta = 1$ a realistic one. In practice, in most market economies only a fraction of wage agreements are covered by indexation, and even then indexation is only partial. In their study of wage indexation in the United States, Hendricks and Kahn (1985) found that a cost-of-living allowance (COLA) on average only produces a wage increase of 0.6% in response to a 1% rise in the cost of living. Furthermore, it appears that "at most 10 to 20 per cent of workers" are covered by COLAs. Thus, empirically, θ is probably quite small at an aggregate level in the U.S.A. Hence the case of complete indexation does not appear to be of practical relevance in most market economies.

4.4 The Optimality of Indexation

The models developed in this thesis cannot really address the question of under what conditions indexation is optimal, except in the most general terms.

Within the framework of the models we have developed, a variable income stream implies a variable consumption stream, because all output is consumed by households and government. We have assumed that the government's consumption is stochastic and independent of the level of output - it does not cushion household consumption by absorbing some of the variability in output. Thus, fluctuations in income translate into fluctuations in consumption. If agents prefer smooth consumption streams, a system of indexation will be preferred to non-indexation provided it reduces the variability of output (and consumption).

Secondly, in reality firm's may face costs in adjusting to different levels of output. In this case an institution that reduces the variance of output will also reduce adjustment costs, so economizing on scarce resources. Consequently, there are intuitively plausible reasons for believing that an output path with small variance may be more desirable to one with larger variance.

Three different informational situations exist. In the first case, the real shock is observable before any wage payments are made to labour, so that the wage can be indexed to both the real and the nominal shocks. In the second case, only a noisy indicator of the real shock is available to agents when wage payments are made: the exact value of the shock is observed with a lag. The third possibility is that the real shock is unobservable, so that wages can only be indexed to the price level. We now treat each of these cases in turn.

4.4.1 Observable Real Shocks

If the real shock hitting the economy is observable, then it is possible to eliminate output fluctuations by indexing the money wage not only to the price level, but also to the real shock. In this case (4.1) is amended to:-

$$(4.10) \qquad w_t = \theta p_t + (1-\theta)p_t^e + \psi f_t$$

The demand for labour is found by substituting (4.10) into a logarithmic version of (4.2):-

$$(4.11) \qquad l_t = \frac{\ln(\alpha k)}{(1-\alpha)} + z_t + \frac{(1-\theta)}{(1-\alpha)}(p_t - p_t^e) + \frac{f_t(1-\psi)}{1-\alpha}$$

Proceeding as before, the aggregate supply function is:-

$$(4.12) \quad y_t^s = q + b_0 + b_1(1-\theta)(p_t - p_t^e) + z_t + b_2^* f_t$$

$$b_2^* = \frac{1 - \alpha + \alpha(1-\psi)}{(1 - \alpha)}$$

In order to eliminate the effect of real shocks on output, ψ must take a value that makes the coefficient b_2^* equal to zero. This is achieved if $\psi = (1/\alpha)$. Secondly, full indexation to the price level ($\theta = 1$) provides insulation from nominal shocks. If $\psi = (1/\alpha)$ and $\theta = 1$, the equilibrium value of output is:-

$$(4.13) \quad y_t = q + b_0 + z_t$$

Thus, indexing the money wage to both the price level and the real shock results in a smooth, non-stochastic output path. The wage-setting equation that guarantees this is:-

$$w_t = p_t + \frac{1}{\alpha} f_t$$

Any change in prices results in an equal change in money wages. A rise or fall in productivity is offset by a rise or fall in the real wage, so that output does not fluctuate in response to real shocks.

4.4.2 <u>Partly Observable Real Shocks</u>

The real shock, f_t, is in reality a composite shock. It consists not only of changes in technique, but attempts to capture the exogenous factors that impinge on production, such as changes in the availability of natural resources, climatic factors and so on. The influences of these forces on production may only become clear after a considerable time. It is therefore more plausible to assume that agents have available a noisy indicator of the real shock, f_t, namely $\tilde{f}_t$, and

$$\tilde{f}_t = f_t + e_t$$

where e_t is a normally distributed random variable with constant variance σ_e^2 and zero mean.

The wage equation is now

$$(4.14) \quad w_t = \theta p_t + (1-\theta)p_t^e + \psi\tilde{f}_t$$

$$= \theta p_t + (1-\theta)p_t^e + \psi(f_t + e_t)$$

Substituting (4.14) into a logarithmic version of (4.2), the first-order-condition for labour, yields the demand for labour function:-

$$(4.15) \quad l_t = \ln(\alpha k) + z_t + \frac{(1-\psi)}{(1-\alpha)} f_t - \frac{\psi e_t}{1-\alpha}$$

$$+ \frac{(1-\theta)}{(1-\alpha)}(p_t - p_t^e)$$

Substituting (4.15) into the production function and aggregating gives the aggregate supply function:-

$$(4.16) \quad y_t^s = q + b_0 + b_1(1-\theta)(p_t - p_t^e) + z_t$$

$$+ b_2^* f_t + \left[\frac{\alpha\psi}{1-\alpha}\right] e_t$$

If we set $\theta = 1$ and $\psi = 1/\alpha$ then the equilibrium value of output is:-

(4.17) $$y_t = q + b_0 + z_t + [1/(1-\alpha)]e_t$$

and the variance of output conditional on information at time t is:-

$$\sigma_y^2 = \left[\frac{1}{(1-\alpha)}\right]^2 \sigma_e^2 .$$

In this case output fluctuations arise through misperceptions of the real shock caused by the use of a noisy indicator; $e_t = \tilde{f}_t - f_t$ and is the deviation of the indicator from the true value of the real shock. The variance of output is proportional to the variance of the error agents make in attempting to observe the real shock.

However, many writers have noted that indexed contracts of the kind expressed by (4.10) or (4.14) are not observed in reality. This may be due, for example, to negotiation costs, but it could also be due to the fact that the real shock is not readily observable, or if an indicator of the real shock is available, it may be so noisy as to render it useless. This implies that the more realistic case may be that examined in the following sub-section, where wages can only be indexed to the price level.

4.4.3 Unobservable Real Shocks

If the real shock is unobservable, wages can only be indexed to the price level. In this case, whether or not indexation reduces the variance of output will depend on the relative sizes of the real and nominal shocks. The conditional variance of the output process given by (4.6) is:-

$$(4.18) \quad \sigma_y^2 = \left[\frac{b_1(1-\theta)}{1+b_1(1-\theta)}\right]^2 \sigma_\epsilon^2 + \left[\frac{b_2}{1+b_1(1-\theta)}\right] (\sigma_\eta^2 + \sigma_\xi^2)$$

The value of θ that minimizes the variance of output is obtained by differentiating (4.18) with respect to θ and setting the partial derivative to zero and solving for θ. This yields the optimal value of θ as:-[4]

$$(4.19) \quad \theta = 1 - \{b_2^2(\sigma_\eta^2 + \sigma_\xi^2)/b_1\sigma_\epsilon^2\}$$

$$= 1 - \frac{1}{\alpha(1-\alpha)}\left[\frac{\sigma_\eta^2 + \sigma_\xi^2}{\sigma_\epsilon^2}\right]$$

As σ_ϵ^2, the variance of the nominal shock rises, so θ, the optimal degree of indexing rises. As the sum of the variances of the real shocks (recall footnote 2) rises, so θ, the optimal degree of indexing falls, and less indexation is optimal so that the real shocks can be offset by real wage changes.

Assuming that α, labour's share of output, is 0.7, then (4.19) says:-

$$(4.20) \quad \theta = 1 - 4.76\left[\frac{\sigma_\eta^2 + \sigma_\xi^2}{\sigma_\epsilon^2}\right]$$

For θ to be greater than zero, that is, for <u>any</u> degree of indexation to be optimal, the second term on the right-hand-side of (4.20) must be less than unity. This condition is met if

$$\sigma_{\epsilon}^2 > 4.76(\sigma_{\eta}^2 + \sigma_{\xi}^2)$$

Thus, in this model the variance of the nominal shock must be more than four times greater than the sum of the variances of the real shocks, before indexation will reduce the variability of output. Even if the variance of the nominal shock is ten times the size of the variance of the real shocks taken together, the optimal degree of indexing is only 0.53, less than that encountered in U.S. labour contracts that have COLAs.

This suggests one reason for the absence of widespread indexation: if it is not possible to index wages to real shocks, then indexing wages to the price level may increase the variability of output, especially if the variances of the real shocks are large vis-a-vis the variance of the nominal shock. Thus, if it is difficult to observe real shocks, the convention of nominal contracts that are not indexed to the price level may help to reduce the variability of output.

4.5 Conclusions

The models developed in this thesis all share a common characteristic: monetary changes have real effects on output because of the existence of nominal wage contracts. This raises the questions of whether the properties of the models of Chapters 2 and 3 would change if wage contracts were indexed to the general price level, and secondly, whether unindexed contracts are optimal under any circumstances.

This chapter therefore extended the model of Chapter 2 to incorporate indexed contracts. It was found that as the degree of indexation rose, the impact of monetary shocks fell. In the limit, with complete indexation, monetary shocks have no impact on output. With complete indexation money is neutral, and a purely monetary model can account neither for output fluctuations nor non-stationarity of output. Thus full indexation destroys some of the results of the previous chapters. However, the degree of indexation actually found in most Western economies is fairly small, so that the case of full indexation has little practical relevance in most economies.

The chapter also considered whether indexation would reduce the variance of output. It was found that indexing wages to both real shocks and the price level insulates the economy from real and nominal shocks and results in a non-stochastic output path. However, it is conceivable that not all real shocks are readily observable. If real shocks are difficult or impossible to observe, the indexation of wages to the price level may not reduce the variance of output. Within the model considered here, it appears that a degree of indexation is desirable only if the variance of nominal shocks is large relative to the variance of real shocks. Thus under certain circumstances unindexed contracts may minimize the variance of output. This suggests one reason why indexation to the price level is not more widespread: if real shocks are unobservable, no indexation may be optimal.

Footnotes

1. Again, (4.5) reduces to (2.25) and (4.6) reduces to (2.26) if $\theta = 0$.

2. Recall that $f_t = \bar{f}_{t-1} + \eta_t + \xi_t$, and that η_t is a transitory shock to productivity and ξ_t is a permanent shock to productivity (a technology shock).

3. See for example Gray (1976). Hendricks and Kahn (1985) Ch.4 provide a survey of the economic effects of wage indexation.

4. This value of θ also satisfies the second-order-condition for a minimum.

CHAPTER 5

CONCLUSIONS

5.1 The Implications of Endogenous Technology

The aim of this thesis was to examine the implications of endogenous technology for both real and monetary business cycle theory. Two alternative frameworks were adopted. In Chapter 2 a general model was developed in which technical change depends on learning by doing, and thus on output. Chapter 3 examined a model containing an innovation production function. Within the general model of each chapter are nested a number of alternative models, such as a purely monetary model or a real business cycle model without endogenous technology. These show that the properties of business cycle models change significantly once the endogeneity of technology is taken into account: endogenous technology has important implications for both real and monetary business cycle theory.

As was mentioned in Chapter 1, conventional monetary models, that implicitly assume technology is exogenous, have fallen into disfavour. The main reason for this appears to be the inability of these models to account for the non-stationarity of output, that is, they cannot account for the finding that some fraction of the innovation in real GNP is permanent. This has led to the growing belief that monetary models should be rejected as unable to provide a satisfactory account of output fluctuations in favour of real business cycle models.

However, if one compares the conventional monetary models to monetary models with endogenious technology, one discovers several significant differences between the two. Firstly, there is a long-run non-neutrality of money in models with endogenous technology: monetary innovations have persistent effects, because a monetary innovation can elicit a supply-side innovation by changing technology. Thus, in contrast to traditional monetary models, in the models considered in this thesis monetary innovations change the trend, or expected future value of output. Secondly, the conditional probability distribution of output is not invariant with respect to the money supply rule followed. A change in the money supply rule can change both the donditional variance of output and the trend, or expected future value of output. In the models considered, output is non-stationary, exhibiting a stochastic trend. In the models developed in Chapters 2 and 3, output exhibited a greater-than-unit root (although this is not a feature of models with endogenous technology generally: alternative frameworks could result in a unit root for output). All this stands in strong contrast to conventional monetary models, where money is neutral in the long run, invariant with respect to changes in the money supply process, and output has a less-than-unit root in such models.

A comparison was also made between real business cycle models with endogenous technology and real business cycle models with exogenous technology. It was found that within the frameworks adopted in Chapters 2 and 3, real business cycle models with endogenous technology display greater-than-unit roots in the output process, while models with exogenous technology typically produce unit roots. Secondly, endogenous technology acts as a propagation mechanism for disturbances that would otherwise only have transitory effects. In a model with endogenous technology, a temporary rise or fall in productivity can change the rate of accumulation of technical knowledge and thus have a permanent impact on output. With exogenous technology, a temporary change in productivity would only have a temporary effect on output.

Monetary models with endogenous technology were also compared to conventional real business cycle models (with exogenous technology). These two classes of models were found to have several features in common. In both types of models output innovations are, to some extent, persistent, so that both classes of models exhibit a non-stationary output process. Thus, one cannot distinguish between these two classes of models on the basis of whether they account for non-stationarity. Nor can one clearly distinguish between them on the grounds that one relies solely on demand-side disturbances as an impulse mechanism, while the other relies solely on supply-side innovations, for, if technology is endogenous, demand-side and supply-side innovations will not be independent of each other.

This means that a major reason for the dissatisfaction with monetary models - that they conventionally produce stationary output processes - does not apply to models with endogenous technology. This implies that the current debate between real and monetary business cycle theory must be re-assessed: monetary models <u>per se</u> cannot be dismissed as incapable of accounting for the movements of the non-stationary output paths that we appear to observe in reality.

5.2 Depreciation of Technical Knowledge

There is some evidence, mainly in the fields of medicine and agriculture, that technologically useful knowledge is decreasing as the state of nature changes. Consequently, the implications of depreciating technology were considered. It emerged that the output process may be stationary if endogenous technology depreciates - the value of the root of output becomes uncertain. However, if one introduces depreciation into a real business cycle model with exogenous technology, one obtains a stationary output process. This implies that if depreciation of technology is a fairly general phenomenon, conventional real business cycle models simply cannot account for non-stationarity. Thus, if technology depreciates, only models with endogenous technology can produce a non-stationary output process.

5.3 Implications for Testing

The way endogenous technology is modelled in this thesis results in a greater-than-unit root in the output process. In Chapter 3 in particular it was argued that an economy with an innovation production function is likely to have a greater-than-unit-root output process. This suggests a potential test to discriminate between models with endogenous vis-a-vis exogenous technology: if output does not contain a greater-than-unit root, then the models developed in this thesis must be rejected. (This would not, however, be a rejection of endogenous technology as such, but only a rejection of the way it has been modelled here.) Thus, testing the size of the root of output would provide one way of falsifying the models presented here. However, it was pointed out that finite-sample test procedures cannot distinguish between a unit root and one only slightly greater than unity, so that existing test porcedures may not provide an unambiguous answer. Hence these models are only potentially falsifiable in this respect.

A second prediction of models with endogenous technology is that demand-side and supply-side innovations are related; a demand-side shock elicits a supply-side shock by changing technology. This suggests a second potential test of these models. Provided reliable data on demand-side and supply-side disturbances can be found, there should be some correlation between the two. Absence of any relationship between demand and supply shocks would amount to a rejection of the models developed here.

5.4 Concluding Comments

There is no doubt that technology is - to some extent - endogenous. This thesis has examined models that explicitly take this endogeneity into account. It has shown that the effects of both monetary and real shocks are very different if technology is endogenous than if it is treated as exogenous. In general, if technology is endogenous, any disturbance to output, whether emanating from the demand side or the supply side, can change technology and thus shift the long-run path of output. It is consequently important that models of output fluctuations take this endogeneity into account, for it alters our view of the effects that the real and monetary shocks that impinge on market economies have.

APPENDIX 1: NOTATION

The following notation is used throughout. In general, capital letters denote natural levels, and lower-case letters denote natural logarithms.

C_t	is real consumption expenditure
F_t	is an exogenous shock to productivity (Ch.2 & Ch.4)
G_t	is real government expenditure
h_t	is an exogenous shock to technology (Ch.3)
L_t	is labour input into the commodity production process
M_t	is the money supply
N_t	is labour input into R and D
P_t	is the general price level
q	is the natural logarithm of the number of firms in the economy
T_t	is nominal lump-sum taxes paid by households
W_t	is the money wage rate
X_t	is an index of technology available to the firm (Ch.3)
$\bar{X}_t$	is an index of publicly available technology (Ch.3)
Y_t	is aggregate output
Y_t^d	is real aggregate demand
Y_t^s	is aggregate supply
Y_t^i	is the output of the representative firm
Z_t	is an index of technical knowledge (Ch.2 & Ch.4)
Π_{t-1}	is equity income distributed to households at the beginning of period t (in nominal terms)

ρ	is the rate of time preference.
β	$= (1/(1+\rho))$
[illegible]$_t$	is a monetary shock
η_t	is a transitory shock to productivity (Ch. 2 & Ch.4)
ξ_t	is a permanent shock to productivity (Ch.2 & Ch.4)
E_{ot}	is the expectations operator with respect to information set I_{ot}
E_t	is the expectations operator with respect to information set I_t

A superscript "e" also denotes an expectation taken over information set I_{ot}.

A superscript "h" denotes the representative household, for example, C_t^h is the consumption of the representative household as opposed to aggregate consumption, and M_t^h is the money holdings of the representative household at the beginning of period t (i.e. carried over from period t-1).

APPENDIX 2:- THE CASE OF LEARNING BY DOING: A GENERALISATION

A2.1 Introduction

This appendix considers two generalisations of the model of Chapter 2. In both cases it is assumed that the firm takes into account that its current production decision will affect future output, so that the learning process becomes internalized. This means that its current output decision becomes a type of investment decision, and its future expected levels of output are among the determinants of current output. In Chapter 2 it was assumed that learning is external to the firm and depends on aggregate output. As the output of the individual firm forms only a tiny fraction of aggregate output, it was argued that the firm can ignore the (negligible) impact its own output decision has on learning. In this appendix we assume that the technical knowledge available to the firm depends not on aggregate output, but on the accumulated output of the particular firm. Thus the firm can no longer ignore the influence of its current output decision on future output.

A2.2 The Infinite-Horizon Case

We first consider the case where the firm has an infinite time horizon. In this case the representative firm's value function is:-

(A1) $$V = \max_{(L_t)} E_t \sum_{j=0}^{\infty} \beta^j [Y_{t+j} - \frac{W_{t+j}}{P_{t+j}} L_{t+j}]$$

As previously, its production technology is:-

(A2) $$Y_t^i = kL_t^{\alpha} Z_t^{1-\alpha} F_t \qquad 0 < \alpha < 1$$

Technical knowledge now evolves according to:-

(A3) $$Z_t = Z_{t-1}[Y_{t-1}^i]^{\gamma} \qquad 0 < \gamma < 1$$

and

(A4) $$W_t = P_t^e$$

where $P_t^e = E[P_t/I_{ot}]$.

To solve the problem we require two boundary conditions. One boundary condition is provided by the historically given initial value of Z_t. The second boundary condition is the transversality condition:-

$$\lim_{T\to\infty} \beta^T [\alpha k L_T^{\alpha-1} Z_T^{1-\alpha} F_T - \frac{W_T}{P_T}] \to 0$$

Sufficient conditions for the transversality condition to hold are - that the sequence of real wages $\{W_{t+j}/P_{t+j}\}_{j=0}^{\infty}$, the sequence of productivity shocks, $\{F_{t+j}\}_{j=0}^{\infty}$, and technical knowledge, $\{Z_{t+j}\}_{j=0}^{\infty}$, as well as the solution for the sequence $\{L_{t+j}\}_{j=0}^{\infty}$ should be of exponential order less than $1/\beta$. The discount factor β is equal to $1/(1+\rho)$, where ρ is the rate of time preference. Thus the transversality condition is satisfied if the above sequences are of exponential order less than $(1+\rho)$.[1] However, once learning is internalized, the transversality condition may not hold. From (A2) and (A3) we see that

$$Z_t = Z_{t-1}^{1+(1-\alpha)\gamma}[kL_{t-1}^{\alpha}F_{t-1}]^{\gamma}$$

so that, as in Chapter 2, the growth rate of Z will depend on its level. As the level of technical knowledge rises, the growth rate of both technology and output will increase. This means that the transversality condition may be violated, and that the value function given by (A1) may not be finite. This is analogous to a general problem arising in growth models with increasing returns. It is well known that if increasing returns are external to the firm, as in Chapter 2, a competitive equilibrium will generally exist. In Ch.2 the employment decision the firm makes each period is independent of future expected values of output. However, once learning is internalized, the employment decision depends on expected future levels of output. This raises the possibility that the output and consumption paths may grow so fast that the objective function is not finite.

There are two ways to circumvent this problem. The first is to assume that the discount rate, β, is not constant, but diminishes over time, so that as $t \to \infty$, $\beta \to 0$. This is the same as assuming that the rate of time preference is increasing over time.[2] In this case the transversality conditions can be satisfied even if the sequences of employment and technical knowledge are growing at an increasing rate. A second way of restricting the range of the value function is by assuming the firm has a limited time horizon. The simplest case that is more general than the model of Ch.2 is to assume that the firm has a two-period time horizon. This is discussed in section A2.3 below, but first we consider the infinite period case with a time-varying discount rate, β^t, that declines sufficiently over time to ensure that the value function is finite.

With a time-varying discount rate, the first-order-condition for L^t is:-

$$\text{(A5)} \qquad \alpha k L_t^{\alpha-1} Z_t^{1-\alpha} F_t + \beta_t E_t[\alpha\gamma(1-\alpha) L_t^{\alpha\gamma(1-\alpha)-1} k^{1+\gamma(1-\alpha)} \cdot L_{t+1}^{\alpha} F_{t+1} F_t^{\gamma(1-\alpha)} Z_t^{(1-\alpha)+\gamma(1-\alpha)^2}] - \frac{W_t}{P_t} = 0$$

where we have substituted an updated form of (A2) for Y_{t+1}^i. As previously $E_t F_{t+1} = \bar{F}_t$. Ignoring Jensen's inequality, we can take a log-linear approximation of (A5):-

$$\ln(\alpha k) + (1-\alpha) l_t + (1-\alpha) z_t + f_t$$
$$= g_0 + g_1(p_t^e - p_t) - \bar{g}_2\{\ln(\beta_t \alpha\gamma(1-\alpha) k^{1+\gamma(1-\alpha)}$$
$$+ (\alpha\gamma(1-\alpha)-1) l_t - [(1-\alpha) + \gamma(1-\alpha)^2] z_t - \bar{f}_t$$
$$- \gamma(1-\alpha) f_t - \alpha E_t l_{t+1}\}$$

where the g's are constants. Simplifying this expression we obtain:-

$$\text{(A6)} \qquad l_t = a_{0t} + a_1(p_t - p_t^e) + a_2 z_t + a_3 f_t + a_4 \bar{f}_t + a_5 E_t l_{t+1}$$

$$a_{0t} = [\ln(\alpha k) - g_0 + g_2 \ln(\beta_t \alpha\gamma(1-\alpha) k^{1+\gamma(1-\alpha)})]/\Omega$$

$$a_1 = g_1/\Omega$$

$$a_2 = (1-\alpha)(1 + g_2 + g_2\gamma(1-\alpha))/\Omega$$

$$a_3 = [1 + g_2\gamma(1-\alpha)]/\Omega$$

$$a_4 = g_2/\Omega$$

$$a_5 = \alpha g_2/\Omega$$

$$\Omega = (1-\alpha) - g_2(\alpha\gamma(1-\alpha) - 1)$$

and $a_1, a_2, \ldots, a_5 > 0$ because $\Omega > 0$ (α and γ are less than unity). (A6) demonstrates that output depends on the current price shock $(p_t - p_t^e)$ and technology shock f_t, and the level of knowledge, z_t, but also on the expected future technology shock $\bar{f}_t$ and on the expected future labour input, $E_t l_{t+1}$. Updating (A6) and taking expectations would show that $E_t l_{t+1}$ in turn depends on $E_t l_{t+2}$, and so on.

To solve the model we use undetermined coefficients, and conjecture the following solution for the decision rule for labour:-

$$\text{(A7)} \qquad l_t = \pi_0 + \pi_1(p_t - p_t^e) + \pi_2 z_t + \pi_3 f_t + \pi_4 \bar{f}_t$$

Updating (A7) and taking expectations over information set I_t, this implies:-

$$\text{(A8)} \qquad E_t l_{t+1} = \pi_0 + \pi_2 E_t z_{t+1} + (\pi_3 + \pi_4)\bar{f}_t$$

because $E_t(p_t - p_t^e) = 0$ and $E_t f_{t+1} = E_t \bar{f}_{t+1} = \bar{f}_t$.

Furthermore,

$$E_t z_{t+1} = z_t + \gamma y_t^i$$
$$= z_t + \gamma[\ln(k) + \alpha l_t + (1-\alpha)z_t + f_t]$$

=>

(A9) $$E_t z_{t+1} = z_t(1 + \gamma(1-\alpha)) + \gamma\ln(k) + \gamma\alpha l_t + \gamma f_t$$

Substituting (A9) into (A8):-

(A10) $$E_t l_{t+1} = \pi_0 + \pi_2(1 + \gamma(1-\alpha))z_t + \pi_2\gamma\ln(k) + \pi_2\gamma\alpha l_t + \pi_2\gamma f_t + (\pi_3 + \pi_4)\bar{f}_t$$

Substituting (A10) into (A6) on re-arranging terms:-

(A11) $$l_t(1 - a_5\pi_2\gamma\alpha) = a_{0t} + a_1(p_t - p_t^e) + a_2 z_t + a_3 f_t + a_4\bar{f}_t + a_5(\pi_0 + \pi_2\gamma\ln(k)) + a_5\pi_2(1 + \gamma(1-\alpha))z_t + a_5\pi_2\gamma f_t + a_5(\pi_3 + \pi_4)\bar{f}_t$$

Substituting (A7) into (A11) for l_t:-

(A12) $$\Delta\pi_0 + \Delta\pi_1(p_t - p_t^e) + \Delta\pi_2 z_t + \Delta\pi_3 f_t + \Delta\pi_4\bar{f}_t = a_{0t} + a_1(p_t - p_t^e) + a_2 z_t + a_3 f_t + a_4\bar{f}_t + a_5(\pi_0 + \pi_2\gamma\ln(k)) + a_5\pi_2(1 + \gamma(1-\alpha))z_t + a_5\pi_2\gamma f_t + a_5(\pi_3 + \pi_4)f_t$$

and $\Delta = (1 - a_5\pi_2\gamma\alpha)$.

The solutions for the π-coefficients in terms of the parameters of the model are:-

$$\pi_0 = \frac{a_{0t} + \pi_2 a_5\gamma\ln(k)}{1 - \pi_2 a_5\alpha\gamma - a_5}$$

$$\pi_1 = a_1/(1 - \pi_2 a_5 \alpha\gamma)$$

$$\pi_2 = \frac{-\Gamma \pm [\Gamma^2 - 4a_2 a_5 \alpha\gamma]^{\frac{1}{2}}}{-2a_5\alpha\gamma}$$

$$\Gamma = 1 - a_5(1 + \gamma(1-\alpha))$$

$$\pi_3 = \frac{a_3 + \pi_2 a_5 \gamma}{1 - \pi_2 a_5 \alpha\gamma}$$

$$\pi_4 = \frac{a_4 + \pi_3 a_5}{1 - \pi_2 a_5 \alpha\gamma - a_5}$$

The π-coefficients have multiple solutions because the expression for π_2 is a quadratic function. For π_2 to have real roots requires:-

(A13) $$1 + a_5^2[1 + \gamma(1-\alpha)]^2 > 2a_5(1 + \gamma(1-\alpha)) + 4a_2 a_5 \alpha\gamma$$.

Recall that

$$a_5 = \alpha g_2/[(1-\alpha) + g_2(1 - \alpha\gamma(1-\alpha))] .$$

As g_2 is a constant used to approximate an additive function by a multiplicative one, it is likely to be small (less than unity), so that a_5 is likely to be less than unity and the above condition likely to be met.

If both solutions for π_2 are positive, then we also require

(A14) $$1 > \pi_2 a_5 \alpha\gamma .$$

Failure of this to hold would result in $\pi_1 < 0$, so that the real wage is positively related to the employment of labour, which is irrational in the context of this model.

Furthermore, since π_0 depends on a_{0t}, it will not be constant but will vary over time as β_t and hence a_{0t} vary (for simplicity we have not given π_0 a subscript t). Thus the decision rule (A7) that the firm follows has a time-varying parameter, so that employment and output will vary as a result of changes in the discount rate.

Substituting (A7) into (A2) yields the supply function of the representative firm:-

$$\text{(A15)} \qquad y_t^{s,i} = \hat{b}_{0t} + \hat{b}_1(p_t - p_t^e) + \hat{b}_2 z_t + \hat{b}_3 f_t + \hat{b}_4 \bar{f}_t$$

$$\hat{b}_{0t} = \ln(k) + \alpha\pi_0$$

$$\hat{b}_1 = \alpha\pi_1; \qquad \hat{b}_2 = 1-\alpha + \alpha\pi_2$$

$$\hat{b}_3 = (1 + \alpha\pi_3); \qquad \hat{b}_4 = \alpha\pi_4$$

and $\hat{b}_0$ has been given a subscript t to denote that, because it depends on π_0, it is a time-varying parameter. Provided that condition (A13) holds, so that the expression for π_2 has real roots, and that (i) in the case of two positive values of π_2 at least one satisfies condition (A14), or (ii) in the case of one positive and one negative value for π_2, the positive one satisfies (A14) or the negative value satisfies the conditions

$$|\pi_2| < (1-\alpha)/\alpha \qquad \text{and} \qquad |\pi_2 a_5 \gamma| < a_3$$

then $\hat{b}_1, \hat{b}_2, \ldots, \hat{b}_4$ are positive.

The aggregate supply function is

$$\text{(A16)} \qquad y_t^s = q + \hat{b}_{0t} + \hat{b}_1(p_t - p_t^e) + \hat{b}_2 z_t + \hat{b}_3 f_t + \hat{b}_4 \bar{f}_t$$

and we assume that the average level of technical knowledge in the economy is equal to that of the representative firm. This is very similar to the aggregate supply function derived from the simple model of Chapter 2:-

$$(2.13) \quad y_t^s = q + b_0 + b_1(p_t - p_t^e) + z_t + b_2 f_t$$

The only differences are changes in the values of the b-coefficients and the inclusion of the term $\hat{b}_4\bar{f}_t$ in (A.16), and the fact that $\hat{b}_{0t}$ is not a constant but will vary over time.

This demonstrates that the simple one-period model yields very similar results to a more complex multi-period model. Thus the simple model of Chapter 2 can be regarded as a reasonable approximation of a more complex model. (A16) and (2.13) are sufficiently similar that none of the non-stationarity or non-neutrality results derived in Chapter 2 would be destroyed if (2.13) were replaced by (A16).

A2.3 A Two-Period Horizon

This section considers the case where the firm has a two-period horizon. This is more general than the case considered in Ch.2, but it avoids having to make the assumption that β decreases over time. The firm's value function is:-

$$V(t) = \max E_t[Y_t - \frac{W_t}{P_t} L_t + \beta(Y_{t+1} - \frac{W_{t+1}}{P_{t+1}} L_{t+1})]$$

The first-order-condition is identical to (A5) except that $\beta_t = \beta$.

As before, (A6) forms a valid approximation of (A5) but with a_{0t} being constant because β is constant over time:-

$$\text{(A6')} \quad l_t = a_0 + a_1(p_t - p_t^e) + a_2 z_t + a_3 f_t + a_4 \bar{f}_t + a_5 E_t l_{t+1}$$

and a_0 is the value taken by a_{0t} when β_t is a constant equal to β. Updating (A6) and taking expectations with respect to information set I_t:-

$$\text{(A17)} \quad E_t l_{t+1} = a_0 + a_2(z_t + \gamma y_t^i) + (a_3 + a_4)\bar{f}_t$$

Note that $E_t(p_{t+1} - p_{t+1}^e) = 0$, $E_t f_{t+1} = E\bar{f}_{t+1} = \bar{f}_t$, and $E_t l_{t+2} = 0$ by assumption of a two-period problem. Substituting a log-linear version of the production function into (A17) for y_t^i, and substituting the resulting expression into (A6') gives us the decision rule for labour:-

$$\text{(A18)} \quad l_t = a_0^* + a_1^*(p_t - p_t^e) + a_2^* z_t + a_3^* f_t + a_4^* \bar{f}_t$$

$$a_0^* = \{a_0(1+a_5) + a_2 a_5 \gamma \ln(k)\}/\Omega^*$$

$$a_1^* = a_1/\Omega^*$$

$$a_2^* = \{a_2 + a_5 a_2(1 + \gamma(1-\alpha))\}/\Omega^*$$

$$a_3^* = (a_3 + a_2 a_5 \gamma)/\Omega^*$$

$$a_4^* = \{a_4 + a_5(a_3 + a_4)\}/\Omega^*$$

$$\Omega^* = (1 - a_2 a_5 \alpha\gamma)$$

Each period the firm will use the decision rule (A18) to determine employment, so rolling its time horizon forward period by period.[3]

This means that at time t+1, l_{t+1} is not given by (A17), but by an updated form of (A18), because at time t+1 the firm's time horizon will extend one period further than it did at time t. One possible justification for using such a decision rule is that the future is uncertain, so that at any time point t, one can only make reliable forecasts for a limited period ahead; if one predicts too far ahead, the variances of the prediction errors are so large as to render the prediction useless. This is, however, not obvious from the simple framework employed here, but would be true in a more complex, general model, where, for example, the function generating f_t might be changing over time in a manner that such changes are not predictable long before they occur.

Substituting the decision rule for l_t into a log-linear version of the representative firm's production function yields the firm's output supply function:-

$$\text{(A19)} \qquad y_t^{s,i} = b_0^* + b_1^*(p_t - p_t^e) + b_2^* z_t + b_3^* f_t + b_4^* \bar{f}_t$$

$$b_0^* = \ln(k) + \alpha a_0^*; \qquad b_1^* = \alpha a_1^*$$

$$b_2^* = \alpha a_2^* + (1-\alpha); \qquad b_3 = 1 + \alpha a_3^*$$

$$b_4^* = \alpha a_4^*$$

Comparing this to (2.12), the representative firm's supply function derived in Ch.2:-

$$\text{(2.12)} \qquad y_t^{s,i} = b_0 + b_1(p_t - p_t^e) + z_t + b_2 f_t$$

Again (A19) and (2.12) are sufficiently similar so that none of the non-neutrality, non-superneutrality or policy invariance results of Ch.2 would be destroyed if (2.12) were replaced by (A19). Thus again the simple model of Ch.2 can be regarded as a fair approximation of a somewhat more general, two-period model.

Footnotes

1. If the transversality condition holds, it implies that the growth rate of output must be less than ρ, the rate of time preference. If this failed to hold, then the future return of producing more output today (through learning) would always exceed the rate of time preference. This is equivalent to a situation where the return on investment is higher than the rate of time preference, but is also exogenous and independent of the level of investment, so ruling out a determinate solution.

2. An increasing rate of time preference is difficult to justify on economic grounds. However, it might be argued that as the level of technology increases, so does the scope of accidents and disasters. This increases the uncertainty of life and may raise the return that the average household expects before it will save any income.

3. Ω^* will be positive for a wide range of values of the parameters, for example if $\alpha = \gamma = 0.7$ and $g_2 = 0.2$, $\Omega^* > 0$. This is a sufficient condition for $a_1^*, a_2^*, \ldots, a_4^*$ to be positive.

APPENDIX 3:- THE CASE OF DETERMINISTIC INNOVATION: A GENERALISATION

A3.1 Introduction

This appendix presents two generalisations of the model of Chapter 3. In Chapter 3 it was assumed that the firm only benefited from research for one period, i.e. the period wherein the research was undertaken. Thereafter the fruits of research become part of the general technology available to the community. If the individual firm's research is only a small fraction of the total research in the economy, it is plausible that the firm ignores the (insignificant) impact its research has on the general level of technology. The evolution of technology is external to the firm, and this enabled us to treat the firm's problem like a one-period problem.

In this appendix we assume that research confers a permanent benefit on the firm. The fruits of research do not become part of the general level of technology. Instead, the technology available to the firm depends on its history of research activity, and on an exogenous shock to technology. In this model, research activity does not confer only a one-period benefit, but permanently raises the firm's level of technology.

However, as was the case in Appendix 2, once the evolution of technology is fully internalized, there may not be a solution because the objective function may not be finite, except under special conditions. Two cases where a solution will exist are considered. In Section A3.2 it is assumed that the firm has an infinite horizon, but that the discount rate is not time invariant. We assume that the discount rate follows a time path that ensures the finiteness of the objective function. An alternative case that yields a determinate solution is where the firm has a finite time horizon. The simplest example of this that is more general than the model of Chapter 3 is if the firm solves a two-period problem. Thus the firm always looks one period ahead when making its labour and research decisions. This case is considered in Section A3.3.

Both these cases yield an output supply function with similar properties to that of the model of Chapter 3. Thus, this appendix demonstrates that the model of Ch.3 can be considered a fair approximation of a more general model.

A3.2 The Infinite-Horizon Case

This section considers the case where the firm has an infinite horizon. It emerges that there exist multiple solutions to the firm's problem. However, as in Ch.3, the output process is still non-stationary and has a greater-than-unit root.

The firm's value function is:-

$$V = \max_{(L_t,N_t)} E_t \sum_{j=0}^{\infty} \beta^j [Y^i_{t+j} - \frac{W_{t+j}}{P_{t+j}} (L_{t+j} + N_{t+j})]$$

The firm's production function is

(B1) $$Y^i_t = kL^{\alpha}_t X^{1-\alpha}_t \qquad 0 < \alpha < 1$$

But technology now evolves according to:-

(B2) $$X_t = X_{t-1} N_t^r H_t \qquad 0 < r < 1$$

where H_t is a strictly positive, stationary stochastic process with constant variance. H_t represents an exogenous real technology shock that impinges equally on all firms in the economy. Substituting (B2) into (B1):-

(B3) $$Y_t^i = kL_t^{\alpha} X_{t-1}^{1-\alpha} H_t^{1-\alpha} N_t^{r(1-\alpha)}$$

The first-order conditions for L_t and N_t are:-

(B4) $$\alpha k L_t^{\alpha-1} [N_t^r X_{t-1} H_t]^{1-\alpha} = \frac{W_t}{P_t}$$

(B5) $$r(1-\alpha)kL_t^{\alpha}[X_{t-1}H_t]^{1-\alpha} N_t^{r(1-\alpha)-1}$$

$$+ \beta E_t\{r(1-\alpha)kL_{t+1}^{\alpha}[X_{t-1}H_t H_{t+1}]^{1-\alpha}$$

$$N_t^{r(1-\alpha)-1} N_{t+1}^{r(1-\alpha)}\} - \frac{W_t}{P_t} = 0$$

As previously, $W_t = P_t^e = E[P_t/I_{ot}]$. Solution of the problem requires two boundary conditions. The first boundary condition is provided by the historically given initial value of X_{-1}. The second boundary condition is provided by the transversality conditions:-

$$\lim_{T\to\infty} \beta^T [\alpha k L_T^{\alpha-1} [N_T^r X_{T-1} H_T]^{1-\alpha} - (W_T/P_T)] = 0$$

$$\lim_{T\to\infty} \beta^T [r(1-\alpha)kL_T^{\alpha}[X_{T-1}H_T]^{1-\alpha} N_T^{r(1-\alpha)-1} - (W_T/P_T)] = 0$$

Sufficient conditions for the transversality conditions to hold are that the sequences of real wages, $\{W_{t+j}/P_{t+j}\}_{j=0}^{\infty}$, real shocks, $\{H_{t+j}\}_{j=0}^{\infty}$, the level of technology, $\{X_{t+j}\}_{j=0}^{\infty}$, and the sequences for the solutions $\{N_{t+j}\}_{j=0}^{\infty}$ and $\{L_{t+j}\}_{j=0}^{\infty}$ be of exponential order less than $1/\beta$, where $1/\beta = (1+\rho)$ and ρ is the rate of time preference. This is equivalent to the requirement that the growth rates of these sequences should be less than the rate of time preference. However, in this model (as in Ch.3) the rates of growth of output and technology will be increasing over time. To satisfy the transversality conditions, β, the discount rate, will have to decrease over time (as the rate of time preference increases) and be denoted as β_t.

Taking the logarithm of (B4):-

$$\text{(B6)} \qquad l_t = \frac{\ln(\alpha k)}{1-\alpha} + \frac{(p_t-p_t^e)}{1-\alpha} + rn_t + x_{t-1} + h_t$$

Taking a log-linear approximation of (B5), and ignoring Jensen's inequality:-

$$\begin{aligned}
&\ln(r(1-\alpha)k) + \alpha l_t + (1-\alpha)(x_{t-1} + h_t) \\
&\quad + [r(1-\alpha)-1]n_t = g_0 + g_1(p_t^e - p_t) \\
&\quad - g_2\{\ln(\beta_t kr(1-\alpha)) + [r(1-\alpha)-1]n_t \\
&\quad + (1-\alpha)(x_t + h_t + E_t h_{t+1}) + \alpha E_t l_{t+1} + r(1-\alpha)E_t n_{t+1}\}
\end{aligned}$$

Assuming for simplicity that h_t, the natural logarithm of the real technology shock H_t, has a zero-mean distribution, then $E_t h_{t+1} = 0$. The above equation can then be written more parsimoniously as:-

(B7) $$n_t = a_{0t} + a_1(p_t - p_t^e) + a_2(x_{t-1} + h_t)$$

$$+ a_3E_t l_{t+1} + a_4E_t n_{t+1}$$

and (B6) can be written as:-

(B8) $$l_t = a_{5t} + a_6(p_t - p_t^e) + a_7(x_{t-1} + h_t)$$

$$+ ra_3E_t l_{t+1} + ra_4E_t n_{t+1}$$

$$a_{0t} = \{\alpha\ln(\alpha k)/(1-\alpha) + \ln(r(1-\alpha)k) - g_0$$

$$+ g_2\ln(\beta_t kr(1-\alpha))\} - \chi$$

$$a_1 = [g_1 + 1/(1-\alpha)] - \chi$$

$$a_2 = [1 + g_2(1-\alpha)] - \chi$$

$$a_3 = g_2\alpha/\chi; \quad a_4 = g_2r(1-\alpha)/\chi$$

$$a_{5t} = ra_{0t} + \ln \alpha k/(1-\alpha)$$

$$a_6 = ra_1 + 1/(1-\alpha)$$

$$a_7 = 1 + ra_2$$

$$\chi = [1 - r + g_2(1 - r(1-\alpha))].$$

Since α, g_1, g_2 and r lie between zero and unity, $a_1, a_2, \ldots, a_4, a_6$ and a_7 will be positive. Both l_t and n_t now depend on future expected values of output and thus on expected future values of n_t and l_t. To solve the model we conjecture the following solutions:-

(B9) $$n_t = \pi_0 + \pi_1(p_t - p_t^e) + \pi_2x_{t-1} + \pi_3h_t$$

(B10) $\quad l_t = \pi_4 + \pi_5(p_t - p_t^e) + \pi_6 x_{t-1} + \pi_7 h_t$

Updating (B9) and taking expectations we obtain:-

$$E_t n_{t+1} = \pi_0 + \pi_2 E_t x_t$$

because $E_t(p_{t+1} - p_{t+1}^e) = E_t h_{t+1} = 0$. Taking the logarithm of (B2) and substituting into the above equation for x_t:-

(B11) $\quad E_t n_{t+1} = \pi_0 + \pi_2(x_{t-1} + rn_t + h_t)$

Proceeding in the same way to derive an expression for $E_t l_{t+1}$, we obtain

(B12) $\quad E_t l_{t+1} = \pi_4 + \pi_6(x_{t-1} + rn_t + h_t)$

Substituting (B9), (B11) and (B12) into (B7):-

(B13) $\quad \Delta\pi_0 + \Delta\pi_1(p_t - p_t^e) + \Delta\pi_2 x_{t-1} + \Delta\pi_3 h_t$

$$= (a_{0t} + a_3\pi_4 + a_4\pi_0) + a_1(p_t - p_t^e)$$

$$+ (a_2 + a_3\pi_6 + a_4\pi_2)(x_{t-1} + h_t)$$

$$\Delta = 1 - ra_3\pi_6 - ra_4\pi_2$$

The solutions for the π-coefficients in terms of the parameters of the model are:-

(B14) $\quad \pi_0 = \dfrac{a_{0t} + a_3\pi_4}{1 - a_4 - r(a_3\pi_6 + a_4\pi_2)}$

(B15) $\quad \pi_1 = \dfrac{a_1}{1 - r(a_3\pi_6 + a_4\pi_2)}$

For π_2 we obtain:-

(B16) $-ra_4\pi_2^2 + (1 - a_4)\pi_2 - a_2 - (ra_3\pi_2 + a_3)\pi_6 = 0$

(B17) $\pi_3 = \dfrac{a_2 + a_3\pi_6 + a_4\pi_2}{1 - r(a_3\pi_6 + a_4\pi_2)}$

Thus it is apparent that the coefficients of the conjectured solutions for l_t and n_t - equations (B9) and (B10) - are interdependent. Note that some of the parameters are not constant, but time-varying. Both π_0 and π_4 (see below) depend on a_{0t} and thus on the discount rate, β_t. For simplicity we do not add subscript t's to π_0 and π_4.

Substituting (B10) - (B12) into (B8) for l_t, $E_t l_{t+1}$ and $E_t n_{t+1}$ yields the following solutions for the π-coefficients:-

(B18) $\pi_4 = \dfrac{a_5 + ra_4\pi_0 + r^2\pi_0(a_3\pi_6 + a_4\pi_2)}{1 - ra_3}$

(B19) $\pi_5 = a_6 + \pi_1 r^2(a_3\pi_6 + a_4\pi_2)$

(B20) $\pi_6 = \dfrac{a_7 + ra_4\pi_2 + r^2a_4\pi_2^2}{1 - ra_3 - r^2a_3\pi_2}$

(B21) $\pi_7 = a_7 + (r + \pi_3 r^2)(a_3\pi_6 + a_4\pi_2)$

Substituting (B20) into (B16) provides an expression for π_2:-

$$- r(a_4 + ra_3)\pi_2^2 + (1 - a_4 - 2ra_3)\pi_2 - (a_2 + a_3) = 0$$

As the expression for π_2 is quadratic, it has two solutions:-

(B22) $$\pi_2 = \frac{-(1-\psi) \pm [(1-\psi)^2 - 4r(a_4+ra_3)(a_2+a_3)]^{\frac{1}{2}}}{-2r(a_4+ra_3)}$$

where $\psi = (a_4 + 2ra_3)$.

For the expression for π_2 to have real roots requires

$$(1 - a_4 - 2ra_3)^2 > 4r(a_4 + ra_3)(a_2 + a_3)$$

Because a_3 and a_4 are likely to be small, because g_2 - the constant used in approximating an additive function by a multiplicative function - is likely to be very small, the above condition for real roots is likely to be met. Furthermore, substituting in the values of the a's shows that, after some manipulation

$$1 - a_4 - 2ra_3 = 1 - r + g_2(1 - 2r)$$

This expression is greater than zero (but less than unity) for a wide range of values of g_2 and r (recall that r is less than unity). This means that if π_2 has real roots, the numerator of the expression for π_2 will be negative because $(1-\psi)^2 < (1-\psi)$. Thus, provided π has real roots, they will both be positive.

If π_2 is positive, then π_6 will be positive provided that

(B23) $$1 - ra_3 > r^2a_3\pi_2$$

Condition (B23) is likely to hold as $r < 1$ and a_3 is small and likely to be less than unity. π_3 and π_1 will be positive if

(B24) $$1 - r(a_3\pi_6 + a_4\pi_2) > 0.$$

Note if this does not hold, $\pi_1 < 0$ and the decision to hire labour for research purposes would be positively related to the cost of such labour, ($(p_t - p_t^e)$ being the inverse of the real wage). It can be argued that π_1 (and by the same logic π_5) must be positive. If $\pi_6 < 0$ i.e. if (B23) does not hold, then the condition (B24) is met more easily. We shall assume that one of the two values of π_2 provides a sensible solution such that π_1, π_2 and π_5 are all positive. Note that if (B24) holds, π_3 will be positive if (i) π_6 is positive or (ii) π_6 is negative but $|a_2 + a_4\pi_2| > |a_3\pi_6|$. π_7 will be positive or negative depending in sign on the signs of π_3 and π_6.

Substituting (B9) and (B10) - the solutions for l_t and n_t - into a logarithmic form of (B3) yields the supply function of the representative firm:

$$(B25) \quad y_t^{i,s} = \hat{b}_{0t} + \hat{b}_1(p_t - p_t^e) + \hat{b}_2 x_{t-1} + \hat{b}_3 h_t$$

$$\hat{b}_{0t} = \ln(k) + r(1-\alpha)\pi_0 + \alpha\pi_4$$

$$\hat{b}_1 = \alpha\pi_5 + r(1-\alpha)\pi_1$$

$$\hat{b}_2 = (1-\alpha) + \alpha\pi_6 + r(1-\alpha)\pi_2$$

$$\hat{b}_3 = \alpha\pi_7 + (1-\alpha) + r(1-\alpha)\pi_3$$

and $\hat{b}_0$ is given a subscript "t" because it depends on π_0 and π_4, both of which are time-varying parameters.

The aggregate supply function is:-

$$(B26) \quad y_t^s = q + \hat{b}_{0t} + \hat{b}_1(p_t - p_t^e) + \hat{b}_2 x_{t-1} + \hat{b}_3 h_t$$

where we assume that x_t, the level of technology of the representative firm, is equal to the average level of technology for the economy as a whole. Equilibrium output will be:-

$$y_t = q + \hat{b}_{0t} + \hat{b}_1\omega_t + \hat{b}_2 x_{t-1} + \hat{b}_3 h_t$$

where $\omega_t = (p_t - p_t^e)$, and is a random shock with zero mean (because agents form their price expectations rationally). ω_t will depend on unanticipated changes in output, in the quantity of money and in the velocity of money, as all these factors can cause unexpected changes in the general price level. Taking the logarithm of (B3), lagging it one period, and substituting in (B9):-

$$x_{t-1} = (1 + \pi_2)x_{t-2} + r\pi_0 + r\pi_1\omega_{t-1} + r\pi_3 h_{t-1}$$

Solving backwards:-

(B27) $$x_{t-1} = x_0(1 + \pi_2)^{t-1} + \sum_{j=0}^{t-2}(1+\pi_2)^j[r\pi_0 + r\pi_1\omega_{t-j-1} + r\pi_3 h_{t-j-1}]$$

Substituting (B27) into (B26) we obtain:-

(B28) $$y_t = q + \hat{b}_{0t} + \hat{b}_1\omega_t + \hat{b}_2 x_0(1 + \pi_2)^{t-1} + \hat{b}_2\sum_{j=0}^{t-2}(1+\pi_2)^j[r\pi_0 + r\pi_1\omega_{t-j-1} + r\pi_3 h_{t-j-1}]$$

Note that both solutions for π_2 are positive (if π_2 has real roots), so that $(1+\pi_2) > 1$, and the output process of (B28) is non-stationary. Indeed, the model of this appendix has yielded very similar results to the models of the previous chapters: output depends on accumulations of real shocks and price surprises (in the more precisely formulated models of the previous chapters the price surprises are due to monetary and real shocks). Again, even if the real shock, h_t, is set equal to zero for all t, output is still non-stationary because it will still depend on price surprises caused by unanticipated monetary shocks. The output process also has a greater-than unit root, the value of the root being $(1+r_2)$.

A3.3 The Two-Period Case

This section considers the case where the firm's time horizon is limited. In particular, the firm only looks one period ahead when making its labour and research decisions. In this case the objective function is finite and can be written as

$$\max_{\{L_t, N_t\}} E_t \{Y_t^i - \frac{W_t}{P_t}(L_t + N_t) + \beta[Y_{t+1}^i - \frac{W_{t+1}}{P_{t+1}}(L_{t+1} + N_{t+1})]\}$$

The production function is given by (B1) and the evolution of technology by (B2). Initially this model is similar to the one in the previous section. Substituting (B2) into (B1) and taking the first-order- conditions for L_t and N_t gives us (B4) as the first-order-condition for L_t and (B5) as the first-order-condition for N_t. Taking logarithmic approximations of these conditions and solving for l_t and n_t gives us the following equations:-

(B29) $n_t = a_0 + a_1(p_t - p_t^e) + a_2(x_{t-1} + h_t)$
$+ a_3E_tl_{t+1} + a_4E_tn_{t+1}$

(B30) $l_t = a_5 + a_6(p_t - p_t^e) + a_7(x_{t-1} + h_t)$
$+ ra_3E_tl_{t+1} + ra_4E_tn_{t+1}$

The only differences between (B29) and (B30) and (B7) and (B8) are that a_0 is the value a_{0t} takes on when β_t is a constant and equal to β, and a_5 is the value taken by a_{5t} when a_{0t} is equal to a_0.

Updating (B29) by one period and taking expectations:

$$E_tn_{t+1} = a_0 + a_2E_tx_t$$

because $E_t[p_{t+1} - p_{t+1}^e] = 0$, $E_th_{t+1} = 0$, and $E_tl_{t+2} = E_tn_{t+2} = 0$ because the firm has a limited time horizon and only looks one period ahead. At time t+1, n_{t+1} and l_{t+1} will not take on their expected values, E_tn_{t+1} and E_tl_{t+1}, because the time horizon will have been rolled forward one period, so that at time t+1 one looks ahead to period t+2. Thus the limited time horizon simplified the firm's decision problem.

From (B2) we obtain:-

(B31) $x_t = x_{t-1} + h_t + rn_t$

Substituting (B31) into the expression for E_tn_{t+1}:-

$$E_tn_{t+1} = a_0 + a_2(x_{t-1} + h_t + rn_t)$$

Proceeding in the same way, we update (B30) by one period, take expectations and using (B31) obtain:-

(B32) $$E_t l_{t+1} = a_5 + a_7(x_{t-1} + h_t + rn_t)$$

Substituting (B32) and (B31) into (B29) and (B30) yields, after some manipulation:-

(B33) $$n_t = a_0^* + a_1^*(p_t - p_t^e) + a_2^*(x_{t-1} + h_t)$$

(B34) $$l_t = a_3^* + a_4^*(p_t - p_t^e) + a_5^*(x_{t-1} + h_t)$$

$$a_0^* = [a_0(1 + a_4) + a_3a_5]/\Xi$$

$$a_1^* = a_1/\Xi$$

$$a_2^* = [a_2 + a_2a_4 + a_3a_7]/\Xi$$

$$a_3^* = [a_5(1 + ra_3) + a_0ra_4 + r^2a_0^*(a_2a_4 + a_3a_7)]$$

$$a_4^* = [a_6 + a_1^*r^2(a_3a_7 + a_2a_4)]$$

$$a_5^* = [a_7 + (r + r^2a_2^*)(a_2a_4 + a_3a_7)]$$

$$\Xi = [1 - r(a_2a_4 + a_3a_7)]$$

Provided $(1 - ra_2a_4 - ra_3a_7)$ is positive, a_1^* to a_5^* will be positive. This condition is satisfied for a range of values of α, r and g_2 (for example, if $\alpha = r = 0.7$ and $g_2 = 0.5 \Rightarrow \Xi > 0$).

(B33) and (B34) are the firm's decision rules for n_t and l_t. Taking the natural logarithm of the production function (B3):-

$$y_t^i = \ln(k) + \alpha l_t + (1 - \alpha)(x_{t-1} + h_t) + r(1 - \alpha)n_t$$

Substituting (B33) and (B34) into the above expression yields the representative firm's supply function:-

$$\text{(B35)} \quad y_t^{i,s} = \tilde{b}_0 + \tilde{b}_1(p_t - p_t^e) + \tilde{b}_2(x_{t-1} + h_t)$$

$$\tilde{b}_0 = \ln(k) + \alpha a_3^* + r(1 - \alpha)a_0^*$$

$$\tilde{b}_1 = \alpha a_4^* + r(1 - \alpha)a_1^*$$

$$\tilde{b}_2 = (1 - \alpha) + \alpha a_5^* + r(1 - \alpha)a_2^*.$$

This is very similar to (3.8), the representative firm's supply function in Chapter 3:-

$$\text{(3.8)} \quad y_t^{s,i} = b_0 + b_1(p_t - p_t^e) + b_2\bar{x}_t$$

Both (B35) and (3.8) are non-stationary processes because both x and $\bar{x}$, the levels of technology, are non-stationary. Furthermore, in both cases price surprises influence technology, as can be seen from (3.7) or (3.12) and by substituting (B33) into (B31) for n_t. Thus, this section has shown that if one internalises learning and allows the firm to solve a two-period problem, the result is very similar to the simple case considered in Chapter 3.

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