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## ESSAYS IN INTERNATIONAL FINANCE

bу

Mary G. Finn

Department of Economics

Submitted in partial fulfilment of the requirements for the degree of Doctor of Philosophy

Faculty of Graduate Studies
The University of Western Ontario
London, Ontario
September, 1987

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The first essay evaluates the forecasting accuracy of monetary and random walk models of the exchange rate, using monthly data on the US and UK economies over the recent flexible exchange rate period. Instrumental-variable estimates of the simple monetary model are not supported by the data, while the full-information maximum-likelihood estimates of its rational-expectations counterpart are. The latter is found to forecast as well as the random walk model. Accordingly, in the context of the monetary model of the exchange rate, the explicit incorporation of the hypothesis of rational expectations permits a richer specification of the dynamics of the exchange rate process and thus an improvement in forecasting accuracy.

The second essay undertakes an econometric analysis of the exchange rate and current account of the balance of payments that seeks to establish whether the behaviour of these two variables can be explained by a small-scale choice-theoretic intertemporal general-equilibrium model in which both are endogenous. Quarterly data over the recent flexible exchange rate period serve as the case study. The model is found to be well-supported by the data and is capable of explaining a substantial proportion of exchange rate and current account movements. This suggests that the intertemporal general-equilibrium model constitutes an advancement in our ability to explain exchange rate behaviour over existing empirical exchange rate models.

The third essay constructs a stochastic intertemporal general-equilibrium model of savings and investment in a small open economy under conditions of perfect international capital mobility and examines, using simulation techniques, the predictions of the model for the dynamics of savings and investment in response to technological disturbances. The key findings of the study are that the cases of positively- (negatively-) autocorrelated disturbances to both domestic and foreign technology and of serially-uncorrelated disturbances to domestic (foreign) technology are characterized by a significantly positive (negative) relationship between saving and investment dynamics. These results suggest that high saving-investment correlations are not necessarily indicative of international capital immobility.

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#### CHAPTER I

# FORECASTING THE EXCHANGE RATE: A MONETARY OR RANDOM WALK PHENOMENON?

This paper evaluates the forecasting accuracy of monetary and random walk models of the exchange rate. Instrumental-variable estimates of the 'simple' monetary model are not supported by the data, while the full-information-maximum-likelihood estimates of its rational-expectations counterpart are. The latter is found to forecast as well as the random walk model. The rational-expectations monetary model is operationalized using the results of Hansen and Sargent (1980) and Flavin (1981) and Box-Jenkins time series techniques. Monthly data on the US and UK economies over the recent flexible exchange rate period serve as the case study.

The purpose of this paper is to undertake an empirical investigation of our ability to forecast the exchange rate. More precisely, the paper seeks to address the question as to whether future values of the exchange rate can be predicted more accurately on the basis of a monetary model of exchange rate determination or on that of a random walk model.

The motivation for the paper derives from two recent contributions to the literature: one, Meese and Rogoff (1983), analyzed the comparative forecasting accuracy of alternative time-series models and the following structural models of the exchange rate: the simple flexible-price and sticky-price monetary models and a sticky-price model which

incorporatés current account balances in an attempt to capture long-run real exchange rate changes. Their essential finding was startling, viz.: the random walk model outperformed all other models across all exchange rates and forecast horizons envisaged. Accordingly, their study calls for: '...a structural model which could perform substantially better than this,...' (Meese and Rogoff, 1983, p. 17). The second contribution, Hoffman and Schlagenhauf (1983), marks the first attempt to estimate a complete monetary model which explicitly incorporates the hypothesis of rational expectations. From their study, they conclude that both the parameter constraints associated with the monetary model and those implied by the rational expectations hypothesis are consistent with exchange rate behaviour. Therefore, in their words: '...our study can be considered part of the increasing amount of evidence which concludes that the monetary model of the exchange rate behaviour does have empirical content' (Hoffman and Schlagenhauf, 1983, p. 259). With view to these contributions, then, the question naturally arises as to whether the explicit incorporation of the hypothesis of rational expectations permits a richer specification of the dynamics of the exchange rate process and thus improved forecasting accuracy with structural economic models.

Here, this question is addressed in the context of the monetary model—more specifically, the simple flexible-price monetary model and its rational expectations extension. The US-UK exchange rate over the recent flexible exchange rate period serves as our case study.

The remainder of the paper is organized as follows: Section 1 specifies the monetary models which are employed in the empirical work; Section 2 contains some preliminary implementation considerations; Section 3 discusses the empirical findings for the US-UK exchange rate, and Section 4 concludes the paper.

## 1. MONETARY MODELS

The asset market approach to exchange rate determination views the exchange as being determined by the requirement of asset market equilibrium. One asset market model which has received much attention in the literature is the (flexible-price) monetary model.

The monetary approach<sup>2</sup> takes its point of departure from the assumption that domestic and foreign goods are perfect substitutes, so that abstracting from transportation costs and trade impediments, purchasing power parity is assumed to hold:

<.7

$$(1) S_t = P_t | P_t^*$$

where S denotes the domestic gurrency price of foreign exchange, and  $P(P^*)$  denotes the domestic (foreign) price level.

National price levels, in turn, are viewed as determined by the requirement of money market equilibrium:

$$P_{t}|P_{t}^{*} = M_{t}|M_{t}^{*}L^{*}()_{t}|L()_{t}$$

where M (M\*) denotes the domestic (foreign) money supply and L() (L\*()) the domestic (foreign) real demand for money function.

Specifying the real demand for money function as a Cagan-type function, we get:

(3) 
$$\frac{L^{+}()_{t}}{L()_{t}} = \frac{Y_{t}^{+} e^{-\varepsilon^{*}i\frac{t}{t}}}{Y_{t}^{n} e^{-\varepsilon i}t}$$

where Y (Y\*) denotes domestic (foreign) real income, I (i\*) the domestic (foreign) nominal interest rate, and  $\eta$  ( $\eta$ \*) and  $\varepsilon$  ( $\varepsilon$ \*) denote the domestic (foreign) income elasticity and interest semi-elasticity of the demand for money, respectively.

Using equations (1) to (3) we therefore obtain:

(4) 
$$s_t = m_t - m_t^* - ny_t + n^* y_t^* + \varepsilon i_t - \varepsilon^* i_t^*$$

where lower case letters are the logarithms of the corresponding capital letters, with the exception of interest rates which remain unchanged. This encapsulates the essence of the monetary approach view: the exchange rate is the relative price of two monies, determined by their relative demands and supplies. Equation (4) is what I refer to as the simple monetary (SM) model.

A further hallmark of the monetary approach is the assumption that domestic and foreign bonds are perfect substitutes, so that uncovered interest rate parity is assumed, 3 i.e.,

(5) 
$$i_t = i_t^* + t_{t+1} - s_t$$

where  $t_{t+1}$  denotes the value of  $t_{t+1}$  expected as of time t. Assuming rational expectations:

system are well fitted; and the DW/Dh statistics are consistent with the mull hypothesis of zero first-order autocorrelation.

Considering the estimation results for the unrestricted SM model, we note immediately that this model finds little empirical support in this study. All coefficient estimates are insignificantly different from zero and often incorrectly signed from the viewpoint of the monetary approach. It is therefore not meaningful to examine the forecasting performance of this model. 12

The difference between our findings for the REM and SM models is striking. The best vantage point from which to consider this difference is to compare the coefficients of the unrestricted REM and SM models. (This is so since the imposition of restrictions results in a gain in estimation efficiency, as noted above.) The coefficients on the corresponding variables in these models are, essentially, similar. Since the basic difference between the two models is whether or not the stochastic processes of the exogenous variables have been substituted for the domestic interest rate variable, it seems that the dynamics of exogenous variables are captured more accurately in the REM system. As a result, the REM 'structural estimates' (n,  $\varepsilon$ , and the unit coefficients of the money supplies) accord more closely to their theoretically-expected values than those of the SM model.

Finally, note that the results reported above (including the tests of coefficient restrictions) remained robust across the estimation subperiods used in the forecasting exercise. Therefore, we proceed to compare the forecasting performance of the restricted REM model vis-à-vis

Rogoff point out, while deviations from interest rate parity are significant they are small in magnitude and therefore, unlikely to explain the poor forecasting performance of structural models. Here, in attempting to explain the latter, attention is focussed on the explicit incorporation of the hypothesis of rational expectations.

#### 2. IMPLEMENTATION CONSIDERATIONS

#### 2.1 A Sample

The sample used in our study takes the USA as the domestic economy and the UK as the foreign economy; monthly data over the period 1974:5 to 1982:12 (after allowing for lagged variables) are employed. A complete description of the data and data sources is given in Appendix 1.

#### 2.2 Operationalizing the Monetary Models

First, with respect to the REM model (equation (7))--in order to operationalize this model it is necessary to obtain an observable expression for the expectational terms it contains. It is, therefore, necessary first to specify the stochastic processes of the exogenous variables. For this purpose the Box-Jenkins three-step, univariate time-series analysis of identification, estimation, and diagnostics was carried out. A prerequisite of this analysis is that the variable be stationary. First differencing was required to achieve this. Accordingly, the empirical counterpart of equation (7) will be expressed in first difference form for estimation purposes.

The following results were obtained: The identification step of the analysis suggested that  $\Delta y_t$ ,  $\Delta y_t^*$ , and  $\Delta i_t^*$  followed AR(1) processes, but was ambiguous with regard to the  $\Delta m_t$  and  $\Delta m_t^*$  processes. The estimation step suggested that  $\Delta m_t$  and  $\Delta m_t^*$  were adequately captured by AR(3) processes and confirmed that  $\Delta y_t$ ,  $\Delta y_t^*$ , and  $\Delta i_t^*$  followed AR(1) processes. We, therefore, have the following suggested specifications:

(8) 
$$\delta_1(L) \Delta m_t = v_{1t}, \quad \delta_1(L) = 1 - \delta_{11} \dot{L}^1 - \delta_{12} L^2 - \delta_{13} L^3$$

(9) 
$$\delta_2(L) \Delta m_t^* = v_{2t}, \qquad \delta_2(L) = 1 - \delta_{21} L^1 - \delta_{22} L^2 - \delta_{23} L^3$$

(10) 
$$\delta_3(L)\Delta y_t = v_{3t}, \quad \delta_3(L) = 1 - \delta_{31}L^{\frac{1}{2}}$$

(11) 
$$\delta_4(L)\Delta y_t^* = v_{4t}, \qquad \delta_4(L) = 1 - \delta_{41}^* L^{\frac{1}{2}}$$

(12) 
$$\delta_5(L)\Delta i_t^* = v_{5t}, \quad \delta_5(L) = 1 - \delta_{51}L^{\frac{1}{2}}$$

where, L denotes the lag operator,  $v_{it}$  is a white noise disturbance term (i = 1,...,5), and  $\delta_{ij}$  is a parameter (i = 1,...,5;j = 1,...,3).

The results of the Box-Jenkins estimation of equations (8)-(12) are reported in Table 1 (Appendix 3). Also reported are the Box-Pierce Q statistic and the critical value of the  $\chi^2$  for the Box-Pierce test. This test suggests, at reasonable levels of confidence, that the residuals have been reduced to white noise.

Next, equations (8) to (12) are used to generate observable expressions for the infinite discounted sum of (the first differences of) the expectational terms appearing in equation (7). In Appendix 2, we

establish the result: if any variable 1x, follows a covariancestationary AR(q) process with white noise disturbances, that may be represented by:

(13) 
$$\delta(L) \Delta x_t = v_t, \qquad \delta(L) = 1 - \delta_1 L^1 - \delta_2 L^2 - \dots - \delta_q L^q$$

where,  $S_i$  (i = 1,...,g) is a parameter. L the lag operator, and  $v_{\hat{t}}$  a white noise disturbance term; then

(14) 
$$\sum_{j=0}^{\infty} x^{j} \perp E \times_{t+j} = \delta(x)^{-1} [1 + \frac{q-1}{2} (\frac{q}{2} + \frac{k-j}{3} \delta_{k}) L^{j}] \Delta x_{t}$$

$$+ (x/1-x)\delta(x)^{-1} \delta(L) \Delta x_{t}$$

where is a scalar. As noted above,  $\Delta m_t$ ,  $\Delta m_t^*$ ,  $\Delta y_t$ ,  $\Delta y_t^*$  and  $\Delta i_t^*$  are stationary processes. We can, therefore, apply the foregoing result in our case. Doing this and rearranging, we get the following operationalized REM-exchange rate equation (in first difference form):

$$\Delta s_{t} = (1+\epsilon)^{-1} \delta_{1} (\epsilon/1+\epsilon)^{-1} ((1+\epsilon)\Delta m_{t} + [(\epsilon/1+\epsilon)\delta_{12} + (\epsilon/1+\epsilon)^{2} \delta_{13} + (\epsilon/1+\epsilon)^{2} \delta_{23} + (\epsilon/1+\epsilon)^{2} \delta_{23}$$

Turning, finally, to the SM model (equation (4)), this equation is estimated in first difference form in order to conform with the functional form of the REM equation (15). Therefore, the operationalized  $^{\circ}$  SM-exchange rate equation is:

(16) 
$$\Delta s_{\xi} = \Delta m_{\xi} - \Delta m_{\xi}^{*} - \eta \Delta y_{\xi} + \eta^{*} \Delta y_{\xi}^{*} + \varepsilon \Delta i_{\xi} - \varepsilon^{*} \Delta i_{\xi}^{*}$$

# 2.3 <u>Estimation Technique and Diagnostics</u>

'In deciding on estimation technique it is important to first carry. out a diagnostic check on the consistent residuals from the estimation of equations (15) and (16) (With unrestricted parameters) for possible problems such as: outliers; functional mis-specification and parameter instability; heteroscedasticity; multicollinearity; autocorrelation and non-normality of residuals. 6 Equation (15) is estimated consistently using ordinary least squares; while an instrumental variable technique is required to consistently estimate equation (16). This is due to the possible correlation between the domestic interest rate and the (omitted) disturbance term, precipitated by potential feedback from the exchange rate to the domestic interest rate, as stated by the interest rate parity hypothesis. The results of our diagnostic check suggested that none of the foregoing problems were present in the case of equation (15); whilst only (first-order) autocorrelation was detected in the case of equation (16). Accordingly, the REM system (equations (8) to (12) and (15)) is estimated using the full-information maximum likelihood (FIML) technique and the SM model (equation (16)) is estimated using Fair's (1970) instrumental variable (IV) technique (this corrects for first-order autocorrelation). In view of our diagnostic findings, these techniques yield consistent and asymptotically efficient coefficient estimates.

It is, fürthermore, noteworthy here that the REM and SM models are identified and the following tests of coefficients were undertaken.

First, with regard to the SM model: the 'monetary approach restrictions' (i.e., unitary elasticities with respect to domestic and foreign money supplies and equal cross-country real income and semi-interest elasticities) were tested by obtaining Fair's IV estimate of the restricted equation (16) and its unrestricted counterpart:

$$(16^{\circ})$$
  $\Delta s_{t} = \alpha_{1} \Delta m_{t} + \alpha_{2} \Delta m_{t}^{*} + \alpha_{3} \Delta y_{t} + \alpha_{4} \Delta y_{t}^{*} + \alpha_{5} \Delta i_{t}^{*} + \alpha_{6} \Delta i_{t}^{*}$ 

An F-test of the restrictions was then undertaken (see Chow, 1983, Ch. 2). Second, in relation to the REM model: the within- and cross-equation coefficient restrictions which were obtained above from the incorporation of the rational expectations hypothesis are tested jointly with the 'monetary approach restrictions' by obtaining the restricted and unrestricted FIML estimates of the REM system and carrying out a likelihood ratio test (see Harvey, 1981). The restricted REM system comprises equations (8) to (12) and (15) with the exception that the restrictions  $\eta=\eta^*$  and  $e=e^*$  are also imposed. The unrestricted FEM system, on the other hand, comprises equations (8) to (12) and the unrestricted counterpart of (15):

(15') 
$$\Delta s_{t} = a_{0} \Delta m_{t} + a_{1} \Delta m_{t-1} + a_{2} \Delta m_{t-2} + a_{3} \Delta m_{t-3}$$

$$+ b_{0} \Delta m_{t}^{*} + b_{1} \Delta m_{t-1}^{*} + b_{2} \Delta m_{t-2}^{*} + b_{3} \Delta m_{t-3}^{*}$$

$$+ c_{0} \Delta y_{t} + c_{1} \Delta y_{t-1}$$

$$+ d_{0} \Delta y_{t}^{*} + d_{1} \Delta y_{t-1}^{*}$$

$$+ e_{0} \Delta i_{t}^{*} + e_{1} \Delta i_{t-1}^{*} .$$

### 2.4 Forecasting Considerations

The forecasting exercise undertaken in this paper follows that of Meese and Rogoff (1983). This exercise involves using each exchange rate model (including the random walk model) to generate forecasts of the future exchange rate at one- to twelve-month horizons. The parameter estimates of the structural models are based on the most recent information available: at the time of a given forecast. This is achieved by reestimating the models each forecast period.

More precisely, the structural models in our study are initially estimated using data through to the first forecasting period, 1979:12. The choice as to where to begin forecasting was predicated on the desire to have sufficient degrees of Freedom for the estimation of the REM model. Forecasts are then generated at one-, six-, and twelve-month horizons. Next, data for 1980:1 are added to the sample and each model is re-estimated. New forecasts are then generated at the one-, six-, and twelve-month horizons. This process is continued through to the last forecast period, 1982:11.

The structural exchange rate models require forecasts of their explanatory variables in order to generate the forecasts of the exchange rate. In the tase of the REM model, its FIML estimates are used in equations (8) to (12) to forecast the explanatory variables. With regard to the SM model, Box-Jenkins estimates of equations (8) to (12) are used to forecast its exogenous explanatory variables; while the domestic interest rate variable is predicted from a least squares regression of this variable on the relevant exogenous variables. Note, furthermore, here that (due to the first difference specification) the structural models, in forecasting the future exchange rate, include its one-period lagged value on the right-hand side of the equation. This value is generated by the model's previous prediction.

Finally, note that forecasting performance is measured by the mean absolute error (MAE) and the root mean square error (RMSE). These statistics are defined as follows:

(17) MAE = 
$$\sum_{s=0}^{N_k-1} |F(t+s+k) - A(t+s+k)|/N_k$$

(12) RMSE = 
$$\left\{ \sum_{s=0}^{N_k-1} [F(t+s+k) - A(t+s+k)]^2 / N_k \right\}^{1/2}$$

where k = 1, 6, 12 (the forecast step);  $N_k$  is the number of forecasts throughout the entire forecasting period;  $^9$  F(p) and A(p) denotes the forecasted and actual values of the exchange rate for period 'p'. Forecasting begins from time t.

# 3. EMPIRICAL FINDINGS FOR THE US-UK EXCHANGE RATE

## \*3.1 <u>Estimation Results</u>

The FIML estimates of the restricted and unrestricted REM systems are presented in Table 2 (Appendix 3); and the unrestricted estimate of the SM model in Table 3 (Appendix 3). These estimation results are based on the entire sample period of our study: 1974:5 through 1982:12.

It is noteworthy at the outset that the test of coefficient restrictions, discussed in the previous section, could not be rejected (at the 5 per cent significance level) in the case of the REM model, but was strongly rejected for the SM model. These tests are very important—insuring, as Hoffman and Schlagenhauf (1983) point out, that the efficiency gain realized by structural parameter estimates does not come at the expense of incorporating a priori information that is inconsistent with the data.

Examining the estimation results for the restricted REM system, a number of points are in order:

First, the coefficients of the time-series processes of the exogenous variables are similar in magnitude to the corresponding Box-Jenkins (Table 1) estimates; they, furthermore, accord in significance. Second, the key structural parameters of interest (n and  $\epsilon$ , the real income elasticity and interest-rate semi-elasticity) assume plausible magnitudes in light of studies on the demand for money (see Laidler, 1977); further, they are both significantly different from zero. Third, the 'fit' statistics (SSR and SER) indicate that the equations of the

system are well fitted; and the DW/Dh statistics are consistent with the mull hypothesis of zero first-order autocorrelation.

Considering the estimation results for the unrestricted SM model, we note immediately that this model finds little empirical support in this study. All coefficient estimates are insignificantly different from zero and often incorrectly signed from the viewpoint of the monetary approach. It is therefore not meaningful to examine the forecasting performance of this model. 12

The difference between our findings for the REM and SM models is striking. The best vantage point from which to consider this difference is to compare the coefficients of the unrestricted REM and SM models. (This is so since the imposition of restrictions results in a gain in estimation efficiency, as noted above.) The coefficients on the corresponding variables in these models are, essentially, similar. Since the basic difference between the two models is whether or not the stochastic processes of the exogenous variables have been substituted for the domestic interest rate variable, it seems that the dynamics of exogenous variables are captured more accurately in the REM system. As a result, the REM 'structural estimates' (n,  $\varepsilon$ , and the unit coefficients of the momey supplies) accord more closely to their theoretically-expected values than those of the SM model.

Finally, note that the results reported above (including the tests of coefficient restrictions) remained robust across the estimation subperiods used in the forecasting exercise. Therefore, we proceed to compare the forecasting performance of the restricted REM model vis-à-vis

that of the random walk (RW).

#### 3.2 Forecasting Results

The summary statistics on forecasting performance (RMSE and MAE) for the restricted REM and RW models over one-, six-, and twelve-month horizons are listed in Table 4 (Appendix 3). As noted earlier, the forecasting period embraces 1980:1 through 1982:12.

On inspection it is clear that for both criteria, the two models are very closely ranked for the one- and six-month forecasts--the RW model performing marginally better at the one-month horizon and the REM model marginally better at the six-month horizon. For the twelve-month horizon forecasts, both models are also closely ranked--the RW model performing marginally better on the MAE criterion and better by approximately 1.1 percentage point on the RMSE criterion.

In light of these results it seems fair to conclude that the REM model forecasts as well as the RW model.

#### 4. CONCLUSION

This paper sought to address the question as to whether the exchange rate can be predicted more accurately by a monetary model of exchange rate determination or the RW model in the case of the US-UK exchange rate. The results of the study suggest that the simple flexible-price monetary model is not supported by the data while its rational expectations counterpart is. Using the latter to generate one-, six- and

twelve-month ahead forecasts, we found that this model performs as well as the RW model.

In conclusion, then, it would seem that the explicit incorporation of the hypothesis of rational expectations permits the dynamics of the forces influencing the exchange rate process to be captured more accuarately and therefore, our study too: '...can be considered part of the increasing amount of evidence which concludes that the monetary model of the exchange rate behaviour does have empirical content' (Hoffman and Schlagenhauf, 1983, p. 259).

#### CHAPTER 1 - NOTES

- The simple flexible-price monetary model is based on Frenkel (1976) and Bilson (1978), the sticky-price monetary model on Dornbusch (1976) and Frankel (1979), and the sticky-price model incorporating current account affects on Hopper and Morton (1982).
- The interested reader is referred to Mussa (1979) for a discussion of the asset market approaches and how they differ from the flow market approach.
- 3. More precisely, note that the Fisher equations for nominal interest rates give:

$$\dot{r}_t = r_t \bullet_{t+1} - p_t$$

$$i_{t}^{*} = r_{t}^{*} + p_{t+1}^{*} - p_{t}^{*}$$

where r (r\*) denotes the domestic (foreign) real interest rate, p (p\*) the logarithm of the domestic (foreign) price level and  $t^p_{t+1}$  ( $t^p_{t+1}^*$ ) denotes the value of  $t^p_{t+1}$  ( $t^p_{t+1}^*$ ) expected as of time t. The assumption that bonds are perfect substitutes implies that

and the assumption of (ex ante) purchasing power parity implies that

$$t^{s}_{t+1} - s_{t} = (t^{p}_{t+1} - p_{t}) - (t^{p}_{t+1} - p_{t}^{*}).$$

Both of these assumptions, together with the Fisher equations, imply uncovered interest rate parity (equation (5)).

- 4. This ensures that all variables are measured in the same units for the joint estimation reported in Section 3. Note, furthermore, that any sign that overdifferencing had occurred as a result of this first-differencing transformation would be revealed by diagnostic testing for functional misspecification and autocorrelation (for details see note 6). No such evidence was obtained in the present case study.
- 5. See Harvey (1981) for the details of the Box-Pierce test.
- 6. The procedures employed are briefly reported here:
  - (a) A time-series plot of the residuals was examined for the presence of outliers.
  - (b) Scatter diagrams of the residuals and fitted values of the dependent variables were examined for systematic relationships, which would imply inconsistency of the estimation technique.
  - (c) Scatter diagrams of the residuals and each explanatory variable were also examined for systematic relationships, which would imply functional misspecification or parameter instability. This diagnostic check was supplemented by a Chow test for stability and misspecification (see Chow, 1983, Ch. 2).

- (d) Scatter diagrams of the squared residuals and fitted dependent variables, as well as of the squared residuals and various. lagged values thereof, were also checked for systematic patterns; which would imply the presence of heteroscedasticity. This check was supplemented by the White and Bartlett tests (see White, 1980 for a description of the former and Pindyck and Rubinfeld, 1976 in relation to the latter).
- (e) Simple correlation coefficients were computed to check the correlation between the explanatory variables.
- (f) An LM test was used to check for the presence of autocorre
  lated residuals (see Pagan and Hall, 1983).
- (g) The Jarque-Bega test for normality of the residuals was undertaken (see Jarque and Bera, 1980).
- 7. More precisely, it is the future logarithm of the exchange rate that is forecasted. This is so since the benchmark of comparison is the random walk model which is in logarithmic form.
- 8. That is, the least squares estimate of:

$$\Delta i_t = \beta_1 \Delta m_t + \beta_2 \Delta m_t^* + \beta_3 \Delta y_t + \beta_4 \Delta y_t^* + \beta_5 \Delta i_t^* + \beta_6$$

where,  $\beta_i$  = parameters; is used to forecast future values of  $\Delta i_t$ , in conjunction with the forecasts of its explanatory variables, which were obtained as described in the text. This specification of the  $\Delta i_t$  equation is consistent with the instrumental variable technique used to estimate the SM model.

- 9. For  $k = 1, 6, 12 N_k = 36, 31, 25$ , respectively.
- 10. The restricted estimate of the SM model was omitted as it was rejected by the data.
- ll. This latter result is not—surprising in respect of: (a) the timeseries equations, since their parameters are close to those of the
  Box-Jenkins regressions, whose residuals were checked for white
  noise properties; (b) the exchange rate equation, in view of the
  diagnostic check on its consistent residuals, reported above (indeed higher order autocorrelation was also tested). Finally, note
  that the Dh was not defined for the foreign interest rate equation.
- More precisely, one would expect this model's forecasting performance to closely match that of the random walk model, since the coefficient estimates of the former are insignificant and small in magnitude. On checking, this was indeed the case. However, this... finding is meaningless since the estimates do not support the theoretical underpinnings of the SM model.

# Appendix 1

# DATA AND DATA SOURCES

S	US dollar price of one UK pound; end of period;
	seasonally adjusted.
M (M±)	US (UK) M1 plus quasi-money; end of period; seasonally
	adjusted.
Y (Y*)	US (UK) index of industrial production; seasonally
	adjusted.
i (i*)	US (UK) 3-month treasury bill rate; end of period.

Source: 0.E.C.D., <u>Main Economic Indicators: Historical Statistics</u>
<u>1964-83</u>.

#### Appendix 2

The purpose of this appendix is to obtain an observable expression for:

$$\int_{j=0}^{\infty} x^{j} \leq \frac{E}{t} \times x^{+j}$$

where  $\cdot$  is a scalar,  $\triangle$  is the first-difference operator. E is the expectations operator conditioned on information at time t, and x is a variable.

By definition:

(A1) 
$$\frac{1}{t} = \frac{E}{t} = \frac{X}{t+j} + \frac{(E-E)x}{t-1} + \frac{(E-E)x}{t+j-1}$$

therefore:

(A2) 
$$= \sum_{j=0}^{\infty} x^{j} \Delta E x_{t+j} = \sum_{j=0}^{\infty} x^{j} E \Delta x_{t+j} + \sum_{j=0}^{\infty} x^{j} (E - E) x_{t+j-1}$$

Next, note the Hansen and Sargent (1980) result: If  $\Delta x_t$  follows a covariance-stationary AR(q) process with white noise disturbances, that may be represented by:

(A3) 
$$\delta(L)\Delta x_t = v_t, \quad \delta(L) = 1 - \delta_1 L^{\frac{1}{2}} - \delta_2 L^{\frac{2}{2}} - \dots - \delta_q L^{\frac{q}{q}}$$

where,  $\delta_i$  (i = 1,...,q) is a parameter, L is the lag operator,  $v_t$  is a white noise disturbance term; then:

(A4) 
$$\sum_{j=0}^{\infty} \lambda^{j} \underset{t}{E} \Delta x_{t+j} = \delta(\lambda)^{-1} \begin{bmatrix} 1 & q-1 & q \\ 1 & z & (\sum_{j=1}^{\infty} \lambda^{k-j} \delta_{k}) L^{j} \end{bmatrix} \Delta x_{t}$$

With regard to the second term on the right-hand side of equation (A2)

notice:

(A5) 
$$\sum_{j=0}^{\infty} \lambda^{j} (E - E) \times_{t+j-1} = \sum_{j=1}^{\infty} \lambda^{j} (E - E) \times_{t+j-1}$$

since  $(E - E)x_{t-1} = 0$ . Letting s = j-1, we may therefore write:

(A6) 
$$\sum_{j=0}^{\infty} \lambda^{j} (E - E) x_{t+j-1} = \lambda \sum_{s=0}^{\infty} \lambda^{s} (E - E) x_{t+s}.$$

Flavin (1981) shows that when  $x_{t}$  follows an ARMA (p\*,q\*) given by:

(A7) 
$$\rho(L) \times_{t} = \phi(L) v_{t}, \quad \rho(L) = 1 - \rho_{1} L^{1} - \rho_{2} L^{2} - \dots - \rho_{p} L^{p} L^{p}$$

$$\phi(L) = 1 + \phi_{1} L^{1} + \phi_{2} L^{2} + \dots + \phi_{q} L^{q} L^{q}$$

where,  $\rho_i$  (i = 1,...,p\*) and  $\phi_i$  (i = 1,...,q\*) are parameters, L is the lag operator and  $v_t$  is again a white noise disturbance term; then:

(A8) 
$$\lambda \sum_{s=0}^{\infty} \frac{s(E-E)x}{t-1} x_{t+s} = \lambda \phi(x) \rho(\lambda)^{-1} \phi(L)^{-1} \rho(L) x_{t}$$

So in our case where  $x_t$  follows a first-differenced AR(q) process,  $\delta(L)\Delta x_t = v_t$ , note we may set  $\rho(L) = \delta(L)(1-L)$ ,  $\rho(\lambda) = \delta(\lambda)(1-\lambda)$ ,  $\phi(L) = \phi(\lambda) = 1$ . Therefore, requation (A8) becomes:

(A9) 
$$\lambda \sum_{s=0}^{\infty} \lambda^{s} (E - E) x_{t+s} = \lambda (1-\lambda)^{-1} \delta(\lambda)^{-1} \delta(L) \Delta x_{t}.$$

Finally, substituting equation (A4) into equation (A2) and using equations (A9) and (A6) in equation (A2), we obtain the following observable expression:

(A101° 
$$\sum_{j=0}^{\infty} \lambda^{j} \Delta E \times_{t+j}^{-1} = \delta(\lambda)^{-1} [1 + \sum_{j=1}^{q-1} (\sum_{k=j+1}^{q} \lambda^{k-j} \delta_{k}) L^{j}] \Delta x_{t}$$

$$+ \lambda (1-\lambda)^{-1} \delta(\lambda)^{-1} \delta(L) \Delta x_{t}.$$

Appendix 3

Table 1: Box-Jenkins Estimation and Diagnostics (Sample Period: 1974:5 to 1982:12).

Dep. Var.	••		, Parameter Estimates		0(20)	~×
t E	11 <sub>0</sub>	0.35 (3.83)	612 0.18 (1.85)	6 <sub>13</sub> 0.42 (4.37)	13.24	28.87
Δm* t	621	0.33 (3.59)	622 0.17 (1.79)	δ <sub>23</sub> 0.28 (2.96)	32.10	31.53*
Δyt	631	0.72 (10.48)		•	14.25	31.41
Δyt	6. 41	-0.22 (2.38)		٠	28.23	31.41
Δi‡	δ <sub>51</sub>	. 0.16 (1.62)			33.92	34.17*

O.E.C.D., Main Economic Indicators: Historical Statistics 1964-83 Source:

i) t values are reported in parentheses.

Q(20) denotes the Box-Pierce Q statistic for 20 autocorrelations of the residuals. (ii)

A.star (\*) denotes critical values at the 97.5 per cent confidence level; all others are at the 95 per cent level. (1111)

2 E

Table'2; FIM Estimates of Restricted and Unrestricted REM Systems (Sample Period: 1974:5 to 1982 12)

٠,

			11		::	• • • • • • • • • • • • • • • • • • • •	: ,		٠.٧		•			•	
1		Parameter Estimates	dame te	o. 18(2.31)		0.38(4.07)			L)kelihood 1924: 31	(1) - 1/6.	eq (	SSR 0.154	SER 0.038	UN7Uh 2.08	
Restricted	ر د د	0.37(5.83)	51. 523	522 0.15(1.76)	ξ. (γ)	0.32(2.76)		•		] w√ (::)	eq.: (	0.003	0.005	1.98/0.13	
System	دي در	0.72(12.42)	27 8	541 -0.19(3.57)	, <sub>(5</sub> ,	0.17(1.54)				) mc/(111)	ed.:. (	0.014	210.0	12.1/81.5	
	ء د	0.71(4.48)	;	2.97(3.81)	ή.	•				(14) Ay	) <del>1</del> 00	0.007	ж <b>о</b> о о	1.88/0.76	
						•			•	(v) · ./y	6d: (	0.030	0.017	2.09/0 4B	
				-	:			٠.		(11) 20	eq.: (	0.008 - 0.009		. 90.2	
-		6,, 0.34(5,49)	612 (	0.19(1.81)		0.39(2.41)		,	59 HZ61	(i) 88 <sub>t</sub>	eħ	970.0	0.027	1.96	
	ري -	37(4.41)	δ <sub>22</sub>	0.14(1.39)	6 <sub>2</sub> 3	0.29(2.07)				73 mg; (11)	eq.: (	0.003		2.00/0	
Unrestricted 633 0.	ا م	0.72(11.04)	γ (γ γ	-0.20(2.80)	, <sub>2</sub> ,	0.15(1.28)				, Jur/(111)	) ∵ba	0:014	0.012	2.11/1.57	
375 (6	ر م	-1.08(1.15)	· •	0.10(0.10)	, <sub>6</sub>	0.34(0.31)	.ه	0.38(0.46)	•	۱۸) ۲۸ (۱۲)	ed (	0.00	Q.008	1.84/0.82	
•		-0.001(0.002)	َم َ	-0.03(0.09)	, <del>4</del> ,	-0.13(0.41)	` ه	-0.13(0.45)	,.	λς (Λ)	eq (	0.030	0.017	2.07/0.45	
	ۍ د	-0.20(0.38)	· ;-	-0.42(0.21)		,	,			(v1) A1	eq.: (	300.0	0.009	2.01	
	<b>*</b> -	0.03(0.13)	-گ	0.23(1.26)		•				٠					
	0	-0.85(2.58)		0.44(1.03)		:				,					

Source: 0.E.C.D., Main Economic Indicators: Historical Statistics 1964-83.

(i) t values are reported in parentheses.

(11) Equation-specific statistics are listed in the 3 rightmost columns, viz. the sum of squared residuals (SSR), the standard error of the regression (SER), the Durbin-Watson statistic (DW) and/or the Durbin-h statistic (DM).

Table 3: IV Estimate of SM Model (Sample Period: 1974:5 to 1982:12)

٥	1		072	
			.0	
3			2.00	•
SSR SER UM			0.028	
SSR			0.074	
	A THE RESIDENCE OF THE PROPERTY OF THE PROPERT		$\alpha_{\rm c}$ 0.16(0.47) $\alpha_{\rm d}$ 0.01(0.04) $\alpha_{\rm c}$ -0.33(0.42) $\alpha_{\rm b}$ -0.60(1.80) 0.074 0.028 2.00 0.072	)
			ag -0.33(0.42)	•
			a4 0.01(0.04)	•
			$\alpha_2$ -0.16(0.47)	·
		•	α, 0.004(0.02)	, <b>,</b>
,			-0.50(1,30)	٠
	- [		2	

O.E.C.D., Main Economic Indicators: Historical Statistics 1964-83. Source:

i) t values are reported in parentheses.

Equation-specific statistics are listed in the 3 rightmost columns, viz.: the sum of squared residuals (SSR); the standard error of the regression (SER); the Durbin-Watson statistic (DW) and the first-order autocorrelation coefficient associated with the disturbance term (ρ).  $\Xi$ 

Table 4: Forecasting Performance (Forecast Period: 1980:1 to 1982:12

Part A: RI		MSE*	Part B: MAE*		
Horizon	- REM	RW	Horizon	REM	RW
1 month	3.13	3.12	1 month	2.53	2.47
6 months	9.74	9.89	6 months	7.67	7.94
12 months	18.00	16.90	12 months	15.39	14.96

<sup>\*</sup> These statistics are approximately in percentage terms.

## CHAPTER 2

## AND CURRENT ACCOUNT

### INTRODUCTION

The purpose of this paper is to undertake an econometric investigation of the exchange rate and the current account of the balance of payments, that seeks to establish whether the behaviour of these two variables can be explained by a small-scale choice-theoretic intertemporal general equilibrium model in which both are endogenous.

The motivation for the study derives from two considerations:

Firstly, existing empirical exchange rate models have been unsuccessful compared with the random walk model. For example, Meese and Rogoff (1983) analyzed the comparative forecasting accuracy of alternative time series and a variety of structural models of the exchange rate: a simple flexible-price monetary model, a sticky-price monetary model and a sticky-price model which incorporates current account balances in an attempt to capture long-run real exchange rate changes. All were outperformed by a random walk model across all the exchange rates and forecast horizons envisaged. Also, Backus (1984) undertook a regression analysis of the random walk model, the simple monetary model and its rational expectations extension, and various versions of sticky-price monetary models and portfolio balance models. Once again, empirical support for any of the structural models was weak, while the random walk

model provided a reasonably good approximation to observed exchange rate dynamics. These studies leave open the question as to why the random walk model does so well and establish it as the benchmark against which to evaluate the empirical performance of structural exchange rate models.

Secondly, no study has yet empirically examined the <u>simultaneous</u> determination of the exchange rate and current account based on the intertemporal general equilibrium (GE) model, despite its growing preponderance in the theoretical international finance literature. Within this framework, stemming from its intertemporal nature, current and expected future real opportunities and monetary policies play a key role in both the money demand decision that governs the evolution of the exchange rate and the consumption-savings-investment decision that determines the current account. Empirically examining the predictions of such models for exchange rate and current account behaviour permits not only a test of the validity of the framework but also a gain in estimation efficiency in exploiting its simultaneous nature. Accordingly, it will be of interest to compare this model's performance to that of the random walk.

It is, furthermore, of interest to note here that the intertemporal. GE model encapsulates the monetary model (flexible price) of exchange rate determination, while embracing (at least potentially) various views on the relationship between the exchange rate and current account which have been advanced in the earlier literature. More specifically:

(1) the Kouri-Branson view, which focuses on the effect of current account imbalances on net holdings of foreign-denominated assets, which

in turn effects the risk premium on the domestic asset, thereby inducing portfolio shifts that affect, inter alia, the demand for domestic money and thus the exchange rate; (2) the Dornbusch-Fisher view. which emphasizes the wealth of current account imbalances on the demand for money and thereby on the exchange rate; (3) the Stockman-Mussa view. which argues that real shocks may cause current account deficits (surpluses) and real exchange rate depreciations (appreciations) and to the extent nominal exchange rate movements accommodate the latter, may lead to nominal exchange rate depreciations (appreciations); thereby creating a statistical correlation between the current account and (nominal) exchange rate. The fundamental point of the intertemporal GE model is, however, that none of the aforementioned links are hard and fast--the exchange rate and current account are both endogenous variables; accordingly, the relationship between their movements depends crucially on the nature of the predominating exogenous shock to the economy and, in particular, on whether it is perceived as a transitory or permanent disturbance. feature is highlighted in Sachs (1981) and Greenwood (1983).

The remainder of the paper is organized as follows: Section 2 sets out the model which serves as the framework for our empirical analysis; Section 3 contains some preliminary implementation considerations; Section 4 discusses the empirical findings in the case of the UK; Section 5 concludes the paper.

#### 2. THE MODEL

The model describes a small open economy that is inhabited by identical, infinitely-lived agents and that operates under a flexible exchange rate. Agents have perfect foresight and maximize utility from consumption and real money balances over an infinite horizon subject to an intertemporal budget constraint. One good exists in this world, so that domestic and foreign goods are perfect substitutes; therefore, abstracting from transportation costs and trade impediments, purchasing power parity holds. Agents are (exogenously) endowed with "y" units of the good each period. The domestic government taxes agents and issues a currency which is held only by nationals. No foreign money is held by domestic residents. Finally, domestic residents (agents and government) can freely participate on an international bond market at a constant real interest rate, r\*, and interest rate parity (real and uncovered nominal) holds.

Three comments on this proposed structure are in order:

First, it is clear that the framework of analysis is a simple one. Some may fault, in particular, the assumptions of purchasing power parity, uncovered interest rate parity and real interest rate constancy, in view of substantial empirical evidence to the contrary [Frenkel (1980), Cumby and Obstfeld (1982), Mishkin (1984)]. The viewpoint adopted here is, nonetheless, that it is best to proceed (at least initially) from the simplest possible framework.

Second, with respect to the assumption of perfect foresight--this raises the issue of using a deterministic framework as a basis for empirical work. Clearly, a stochastic framework would be preferable. However, experimentation with: alternative functional forms (for the momentary utility function); solving the Euler equations forward [as in

Sargent (1984)]; and imposing restrictions on the joint distribution of consumption and asset returns [following Hansen and Singleton (1983)] failed to yield a closed form solution amenable to econometric estimation. Accordingly, I work with the deterministic solution, replacing future variables with their rationally-expected values.

finally, it is noteworthy here that motivating money via the utility function turned out to be more convenient/interesting than by alternative means. More specifically, motivating money via a transactions costs specification as in Greenwood (1983), lead to a similar framework but requires a data series on transactions costs for empirical purposes. Moreover, via a cash-in-advance constraint also leads to a similar framework, but one with a more restrictive specification of the demand for money than either of the above alternatives, since the velocity of circulation is fixed.

The model may be more formally described as follows: `The representative agent solves the following optimization problem:

maximize 
$$\sum_{t=1}^{\infty} \beta^{t-1} \left[ u(C_t) + v(\frac{M_t}{P_t}) \right]$$
 subject to: 
$$b_t^p = y_t - \tau_t - C_t - (\frac{M_t - M_{t-1}}{P_t}) + (\frac{1+r^*}{P_t}) b_{t-1}^p$$
 with  $b^p = 0$ ,

where,  $\beta$  is the subjective discount factor,

C is real consumption,

M is desired nominal money holdings,

P is the nominal price level,

u( ), v( ) are the momentary utility functions,

b<sup>p</sup> is the private agent's one-period real bond purchases.

y is real income,

T is the real, lump-sum tax,

r\* is the foreign real rate of interest.

The problem may be rewritten as:

$$V(M_{t-1}) e^{b_{t-1}^{p}}, P_{t}) = \frac{1}{2}$$

$$\max_{C_{t}, b_{t}^{p}, M_{t}} \{u(C_{t}) + v(\frac{M_{t}}{p}) + \beta V(M_{t}, b_{t}^{p}, P_{t+1})\}$$

$$S.t. b_{t}^{p} = y_{t} - \tau_{t} - C_{t} - (\frac{M_{t} - M_{t-1}}{P_{t}}) - y_{1} + r^{*}) b_{t-1}^{p}, b_{0}^{p} = 0$$

The first order conditions may be expressed as:

(1) 
$$u'(C_t) = S(T+r^*)u'(C_{t+1})$$

(2) 
$$v'(\frac{M_t}{P_t}) = u'(C_t) - 3 \frac{u'(C_{t+1})}{(1+t)}, \pi_t = \frac{P_{t+1} - P_t}{P_t}$$

The government's budget constraint is:

$$b_t^g + g_t = \tau_t + (1+r^*)b_{t-1}^g + (\frac{M_t^s - M_{t-1}^s}{P_t}), b_0^g = 0$$

where, b<sup>g</sup> denotes government one-period real bond purchases,

g denotes government, purchases of the one good,

MS denotes the nominal money supply.

Assuming  $M_t = M_t^S$  for all t, and substituting the government budget constraint into the individual's budget constraint gives:

(3) 
$$b_t = y_t - C_t - g_t + (l+r*)b_{t-1}, b_0 = 0, b = b^p + b^g.$$

This equation implies that the economy faces an intertemporal budget constraint of the form:

$$\frac{T}{z} \frac{C_{t}}{(1+r^{*})^{t-1}} = \frac{T}{z} \frac{(y_{t}-g_{t})}{(1+r^{*})^{t-1}} - \frac{b_{T}}{(1+r^{*})^{T-1}}$$

As  $T + \infty$ , we impose the transversatility condition:  $\lim_{T \to \infty} \frac{D_T}{(1+r^*)^{T+1}} = 0$ , so that the budget constraint may be rewritten as:

(4) 
$$\sum_{t=1}^{\infty} \frac{c_t}{(1+r^*)^{t-1}} = \sum_{t=1}^{\infty} \frac{(y_t^{-g})}{(1+r^*)^{t-1}}.$$

It is important to impose the transversality condition in the present setting of perfect capital mobility, for it rules out the possibility that the household can attain unbounded utility by borrowing arbitrarily large sums in the world capital market and meeting all interest payments through further borrowing.

Finally, rearranging the first order conditions and using (4), it is clear that the general equilibrium solution of the model involves the simultaneous solution of:

(1) 
$$u'(C_t) = \beta(1+r^*)u'(C_{t+1})$$

(5) 
$$v'(\frac{M_t^S}{P_t}) = u'(C_t) \frac{i_t}{(1+i_t)}$$

(4) 
$$\frac{\sum_{t=1}^{\infty} \frac{C_t}{(1+r^*)^{t-1}}}{\sum_{t=1}^{\infty} \frac{(y_t - g_t)}{(1+r^*)^{t-1}}} \equiv W_1$$

where  $i_t$  is the nominal rate of interest earned on a bond held-from period t to period t+1, and  $W_1$  is real wealth as of period one. In order to solve the model explicitly, some specific assumptions about functional forms are required.

## 2.1 Specific Functional Forms

Assume  $u(C_t) = \frac{C_t^{(1-\gamma)}}{(1-\gamma)}$ , = the coefficient of relative risk aversion with respect to consumption

$$u'(C_{t}) = C_{t}^{-\gamma}$$

Substituting this result in (1) and rearranging gives:

(6) 
$$C_t = q^{(t-1)}C_1$$
, for all t, where  $q = [\beta(1+r^*)]^{1/\gamma}$ 

. Substituting (6) into (4) and rearranging gives:  $^7$ 

$$[C_1 = mW_1, \text{ where } m = [1 - \frac{1}{(1+r^*)} \{\beta(1+r^*)\}^{1/\gamma}]$$

which, together with (6) -

$$C_t = q^{(t-1)} mW_1$$
, for all t.

It is straightforward to show that:

$$q^{(t-1)}W_1 = W_t \equiv \sum_{s=0}^{\infty} \frac{1}{(1+r^*)^s} [y_{t+s} - g_{t+s}] + (1+r^*)b_{t-1}$$

where  $W_{\underline{t}}$  is real wealth as of period  $\underline{t}$ .

We may, therefore, write:

(7) 
$$C_t = mW_t$$
, for all t

Next, assume  $v(\frac{M_t}{P_t}) = \frac{(\frac{M_t}{P_t})}{(1-\delta)}$ ,  $\delta$  = the coefficient of relative risk aversion with respect to real balances.

$$v'(\frac{M_t}{P_t}) = (\frac{M_t}{P_t})^{-\hat{0}}$$

Using this in (4) above, while noting  $u'(C_t) = C_t^{-\gamma}$ , gives:

$$\left(\frac{M_{t}}{P_{t}}\right)^{-\delta} = C_{t}^{-\gamma} \frac{i_{t}}{(1+i_{t})}$$

Using the approximation  $i_t/(1+i_t) = 1$  above, taking logarithms of the resulting expression and rearranging gives:

(8) 
$$\log P_t = \log M_t - \frac{\gamma}{\delta} \log C_t + \frac{1}{\delta} \log i_t$$

Next, taking a first-order Taylor series expansion of  $\log x_t$  (about  $i_t = T$ ) gives:

$$\log i_t = \log(1) + \frac{1}{1}(i_t-1)$$
, where  $1 = \text{mean of } i_t$ 

Substituting this approximation, the uncovered interest rate parity and purchasing power parity conditions, i.e.,

into (8) and rearranging gives:

Ž.

(9) 
$$e_{t}^{i} = M_{t}^{i} - P_{t}^{*i} - \frac{\gamma}{5} C_{t}^{i} + \frac{1}{s_{1}^{*}} (1_{t}^{*} + e_{t+1}^{i} - e_{t}^{i}) + \frac{1}{5} \log(\tilde{1}) - \frac{1}{5}$$

where, a prime denotes the logarithm of a variable,

- i\* denotes the foreign nominal interest rate,
- e denotes the domestic currency price of foreign exchange,
- ${\sf P^{*}}$  denotes the foreign price level.

Solving (9) for e<sub>t</sub> gives:

$$e_{t}^{i} = \frac{5\overline{i}}{(1+\delta\overline{i})} \left[M_{t}^{i} - P_{t}^{*i}\right] - \frac{\gamma\overline{i}}{(1+\delta\overline{i})} C_{t}^{i} + \frac{1}{(1+\delta\overline{i})} \left[i_{t}^{*} + e_{t+1}^{i}\right] + s,$$

$$s = \frac{\overline{i}}{(1+\delta\overline{i})} \left[\log \overline{i} - \overline{i}\right]$$

Finally, solving this equation forward:

(10) 
$$e'_{t} = \frac{\delta \vec{1}}{(1+\delta \vec{1})} \sum_{j=0}^{\infty} \left( \frac{1}{1+\delta \vec{1}} \right)^{j} \left[ M'_{t+j} - P''_{t+j} \right] - \frac{\gamma \vec{1}}{(1+\delta \vec{1})} \sum_{j=0}^{\infty} \left( \frac{1}{1+\delta \vec{1}} \right)^{j} C'_{t+j} + \sum_{j=0}^{\infty} \left( \frac{1}{1+\delta \vec{1}} \right)^{j+1} i^{*}_{t+j} + s \sum_{j=0}^{\infty} \left( \frac{1}{1+\delta \vec{1}} \right)^{j}$$

Or, noting from (6) we have  $C_{t+j} = q^{j}C_{t}$ , (10) may be rewritten as

(11) 
$$e_{t}^{i} = \frac{\delta \tilde{1}}{(1+\delta \tilde{1})} \sum_{j=0}^{\infty} \left(\frac{1}{1+\delta \tilde{1}}\right)^{j} \left[M_{t+j}^{i} - P_{t+j}^{*i}\right]$$

$$- \frac{\gamma \tilde{1}}{(1+\delta \tilde{1})} \sum_{j=0}^{\infty} \left(\frac{1}{1+\delta \tilde{1}}\right)^{j} \log(q^{j}C_{t})$$

$$+ \sum_{j=0}^{\infty} \left(\frac{1}{1+\delta \tilde{1}}\right)^{j+1} i_{t+j}^{*} + s \sum_{j=0}^{\infty} \left(\frac{1}{1+\delta \tilde{1}}\right)^{j}$$

In summary, then, equations (7), (11) and the identity: 8

(12)  $CB_t \equiv y_t - C_t - g_t$ , where CB denotes the current account balance, from the basis of our study.

Before discussing some implementation considerations, a few comments must be made.

First, (7) is a statement of the permanent income hypothesis of consumptions.

Second, (11) is, in essence, a statement of the monetary approach view of exchange rate determination for more precisely of its rational expectations extension once the future values of the exogenous variables are replaced by their expected values, conditioned on all available current information—see Bilson (1978)]. This is, perhaps all the more evident if (7) is used to replace the consumption variable by permanent income and the parameter  $\frac{1}{\delta^{\frac{1}{1}}}$  is interpreted as the interest semi-elasticity of the demand for money.  $\frac{10^{\frac{1}{1}}}{\delta^{\frac{1}{1}}}$ 

In view of the foregoing points, it is clear that our empirical study constitutes a joint test of the permanent income hypothesis and the monetary approach to exchange rate determination.

Finally, it may be of interest to clarify here, in an intuitive fashion, the characteristics of our framework. Firstly, in respect of the current account: The model predicts that the current account willexhibit little response to permanent disturbances, while, on the other hand, will be significantly affected by temporary disturbances. The reason is that, in the former case, both  $(y_t-g_t)$  and  $C_t$  adjust more or less equi-proportionately since C, moves proportionately with permanent (disposable) income. In the case of temporary shocks  $\mathcal{L}_{\mathsf{t}}$  remains largely unaffected, thus giving rise to significant current accounts effects. Secondly, in respect of the exchange rate: The response of the exchange rate to temporary and permanent shocks depends crucially on the value of  $\frac{1}{\epsilon^2}$  (which corresponds, as noted above, to the interest semi-elasticityof the demand for money). As Adams and Boyer (1985) point out, high values of this elasticity imply great weight is attached to future movements. 11 The converse is true for small values of the interest semielasticity. Thus, in the former case, the exchange rate will exhibit little response to temporary disturbances and a significant response to permanent disturbances. Contrarily, when the interest-semi-elasticity, is low, the exchange rate responds similarly to both types of disturbance.

## 3. IMPLEMENTATION CONSIDERATIONS

## 3.1 Sample

The sample used in our study takes the UK as the domestic economy and the U.S. as the foreign economy. We thus, expect the small open economy assumptions noted above to be satisfied.

The sample employs quarterly data over the period: 1974:1 to 1984:1 (after allowing for lagged variables). A detailed description of the data and data sources is given in Appendix 1.

## 3.2 Consumption Function

With respect to the consumption function [equation (7)] derived above: It is convenient to operationalize this function by exploiting a first-order difference equation for the human wealth component of total wealth, as in Hayashi (1982). This may be outlined as follows:

Define 
$$H_t = \sum_{s=0}^{\infty} \frac{\tilde{y}_{t+s}}{(1+r^*)^s}$$
, where  $\tilde{y}_{t+s} = y_{t+s} - g_{t+s}$ .

This is the human wealth component of  $W_t$ . It is easy to show that  $H_t$  may be rewritten as:

.(13) 
$$H_{t} = (1+r^{*})(H_{t-1} - \tilde{y}_{t-1}).$$

Using (7) to eliminate  $H_{t}$  and  $H_{t-1}$  from (13), noting the general equilibrium budget constraint--equation (3)--and rearranging gives: 12

(14) 
$$C_t = (1+r*)(1-m)C_{t-1}$$

It is clear that this equation is more convenient for estimation purposes than equation (7).

## 3.3 Exchange Rate Equation

With regard to the exchange rate equation [equation (11)] derived above: We operationalize this equation firstly, by replacing the future values of variables by their expected values, conditioned on current information and secondly, by obtaining observable expressions for the latter. In order to do this, we proceeded in the following manner:

First, an unconstrained vector autoregression was estimated, comprising of the three exogenous variables: M', P\*' and i\* and the two endogenous variables: e' and C'. 13 Three lags were employed on each variable and since the joint estimation revealed negligible correlation across the equations' residuals, each equation was estimated by OLS. A sequence of F-tests suggested: (1) The endogenous variables do not Granger-cause the exogenous variables and (2) Each exogenous variable is not Granger-caused by any other exogenous variable. 14 The former finding is a pre-condition for the applicability of the Hansen and Sargent (1980) formula employed below; while the latter finding suggests that, for forecasting purposes, univariate time series models of the exogenous variables are adequate.

Therefore, in order to obtain a parsimonious representation of the stochastic processes for the exogenous variables, the Box-Jenkins three-step, univariate, time series analysis of identification, estimation and diagnostics was carried out. A prerequisite of this analysis is that the variables be stationary. First differencing was required to achieve this. [Accordingly, the observable counterpart of equation (11) will be

expressed in first difference form for estimation purposes. 15]

The following results were obtained: The identification step of the analysis proved to be ambiguous. The estimation step suggested that  $\Delta P_{t}^{*}$  was adequately captured by an AR(3) process, while  $\Delta M_{t}^{\prime}$  and  $\Delta i_{t}^{*}$  seemed to follow AR(2) processes. We, therefore, have:

(15) 
$$\alpha_{1}(L)\Delta M_{t}' = V_{lt}, \alpha_{1}(L) = 1 - \alpha_{11}L - \alpha_{12}L^{2}$$

(16) 
$$\alpha_2(L)\Delta P_t^* = V_{2t}, \alpha_2(L) = 1 - \alpha_{21}L - \alpha_{22}L^2 - \alpha_{23}L^3$$

(17) 
$$\alpha_3(L)\Delta i_t^* = V_{3t}, \alpha_3(L) = 1 - \alpha_{31}L - \alpha_{32}L^2$$

where, L denotes the lag operator,

 $V_{it}$  is a white noise disturbance term; i = 1, 2, 3 $\alpha_{ij}$  is a parameter, i = 1, 2, 3, j = 1, 2, 3.

The results of the Box-Jenkins estimation of (15)-(17) are reported in Table 1 (Appendix 2). Also reported are the individual autocorrelations of residuals, the Box-Pierce Q statistic and the critical value of the  $\chi^2$  for the Box-Pierce test. This test suggests that the residuals have been reduced to white noise. A further diagnostic check was undertaken by comparing the autocorrelation functions of the original and simulated time series. This suggested also that the specifications adopted were adequate [see Pindyck and Rubinfeld (1976) for further details].

Finally, equations (15)-(17) are used to generate observable expressions for the infinite discounted sum of (the first differences of) expectational terms in:

(11') 
$$2e_{t}^{i} = \frac{\delta \overline{i}}{(1+\delta \overline{i})}, \quad \frac{2}{5} \left(\frac{1}{1+\delta \overline{i}}\right)^{j} 2E[M'_{t+j} - P^{*'}_{t+j}]$$

$$+ \frac{1}{(1+\delta \overline{i})}, \quad \frac{2}{5} \left(\frac{1}{1+\delta \overline{i}}\right)^{j} 2E[i^{*}_{t+j}]$$

$$- \frac{\gamma \overline{i}}{(1+\delta \overline{i})}, \quad \frac{2}{5} \left(\frac{1}{1+\delta \overline{i}}\right)^{j} 2\log(q^{j}C_{t})$$

More specifically, this equation may now be operationalized using the . following result:

$$\frac{2}{j=0} \int_{t}^{j} \Delta E^{x} t + j = \frac{2}{j=0} \int_{t}^{j} E^{j} x t + j + \frac{2}{j=0} \int_{t}^{j} (E - E)^{x} t + j - 1$$

$$= \alpha(\lambda)^{-1} [1 + \frac{2}{2} (\frac{2}{2} + k - j \alpha_{k}) L^{j}] \Delta x_{t}$$

$$+ \lambda(1 - \lambda)^{-1} \alpha(\lambda)^{-1} \alpha(L) \Delta x_{t}$$

where,  $\Delta x_{t}$  is a covariance-stationary process:

$$\alpha(L)\Delta x_t = v_t, \ \alpha(L) = 1-\alpha_1 L^1 - \dots - \alpha_q L^q$$

where,  $v_t$  is a white noise disturbance,  $\lambda$  and  $\alpha_i$  are parameters. This result constitutes an extension of Hansen and Sargent's (1980) usage of the Weiner-Kolmogorov prediction formula in operationalizing infinite-discounted sums of expectational terms. [For further details see Finn

(1986).] Since  $(\Delta M_t')$ ,  $\Delta P_t^*$  and  $\Delta i_t^*$  are stationary processes we may directly apply the above result to equation (11'). This gives the operationalized exchange rate equation: 17

(18) 
$$\Delta e_{t}^{i} = \left(\frac{\delta \overline{i}}{1 + \delta \overline{i}}\right) \left[\alpha_{1} \left(\frac{1}{1 + \delta \overline{i}}\right)\right]^{-1} \left\{\left[1 + \sum_{j=1}^{1} \left(\frac{2}{s} \left(\frac{1}{1 + \delta \overline{i}}\right)^{k - j} \alpha_{1k}\right) L^{j}\right] \Delta M_{t}^{i} \right.$$

$$+ \frac{1}{\delta \overline{i}} \alpha_{1} (L) \Delta M_{t}^{i} \right\}$$

$$- \left(\frac{\delta \overline{i}}{1 + \delta \overline{i}}\right) \left[\alpha_{2} \left(\frac{1}{1 + \delta \overline{i}}\right)\right]^{-1} \left\{\left[1 + \sum_{j=1}^{2} \left(\sum_{k=j+1}^{3} \left(\frac{1}{1 + \delta \overline{i}}\right)^{k - j} \alpha_{2k}\right) L^{j}\right] \Delta P_{t}^{i} \right]$$

$$+ \frac{1}{\delta \overline{i}} \alpha_{2} (L) \Delta P_{t}^{i} \right\}$$

$$+ \left(\frac{1}{1 + \delta \overline{i}}\right) \left[\alpha_{3} \left(\frac{1}{1 + \delta \overline{i}}\right)\right]^{-1} \left\{\left[1 + \sum_{j=1}^{2} \left(\sum_{k=j+1}^{2} \left(\frac{1}{1 + \delta \overline{i}}\right)^{k - j} \alpha_{3k}\right) L^{j}\right] \Delta I_{t}^{i} \right\}$$

$$+ \frac{1}{\delta \overline{i}} \alpha_{3} (L) \Delta I_{t}^{i} \right\}$$

$$= \gamma / \delta \Delta C_{t}^{i}.$$

## 3.4 Estimation Technique and Diagnostics

Bringing together the various components of our solution, then, we have the following system of equations.

$$\begin{array}{lll} \Delta e_{t}^{i} &=& \displaystyle (\frac{\delta \overline{i}}{1+\delta \overline{i}}) \left[ \alpha_{1} \left( \frac{1}{1+\delta \overline{i}} \right) \right]^{-1} \left\{ \left[ 1 + \sum_{j=1}^{2} \left( \frac{2}{1+\delta \overline{i}} \right)^{k-j} \alpha_{1k} \right) L^{j} \right] \Delta M_{t}^{i} \\ &+& \displaystyle \frac{1}{\delta \overline{i}} \alpha_{1} (L) \Delta M_{t}^{i} \right\} \\ &-& \displaystyle \left( \frac{\delta \overline{i}}{1+\delta \overline{i}} \right) \left[ \alpha_{2} \left( \frac{1}{1+\delta \overline{i}} \right) \right]^{-1} \left\{ \left[ 1 + \sum_{j=1}^{2} \left( \frac{3}{2} \left( \frac{1}{1+\delta \overline{i}} \right)^{k-j} \alpha_{2k} \right) L^{j} \right] \Delta P_{t}^{*} \right] \\ &+& \displaystyle \frac{1}{\delta \overline{i}} \alpha_{2} (L) \Delta P_{t}^{*} \right\} \\ &+& \displaystyle \left( \frac{1}{1+\delta \overline{i}} \right) \left[ \alpha_{3} \left( \frac{1}{1+\delta \overline{i}} \right) \right]^{-1} \left\{ \left[ 1 + \sum_{j=1}^{2} \left( \frac{2}{2} \left( \frac{1}{1+\delta \overline{i}} \right)^{k-j} \alpha_{3k} \right) L^{j} \right] \Delta I_{t}^{*} \right. \\ &+& \displaystyle \frac{1}{\delta \overline{i}} \alpha_{3} (L) \Delta I_{t}^{*} \right\} \\ &-& \displaystyle \gamma/\delta \Delta C_{t}^{i} \,. \end{array}$$

(15) 
$$\Delta M_t' = \alpha_{11} \Delta M_{t-1}' + \alpha_{12} \Delta M_{t-2}' + v_{1t}$$

(16) 
$$\Delta P_{t}^{*} = \alpha_{21} \Delta P_{t-1}^{*} + \alpha_{22} \Delta P_{t-2}^{*} + \alpha_{23} \Delta P_{t-3}^{*} + v_{2t}$$

(17.) 
$$\Delta i_t^* = \alpha_{31} \nabla i_{t-1}^* + \alpha_{32} \Delta i_{t-1}^* + v_{3t}$$

(14') 
$$\Delta C_{t} = \alpha \Delta C_{t-1}, \quad \alpha \equiv (1+r^{*})(1-m)$$

Notice that (14) has been entered also in first difference form. Again, this was necessary in order to obtain a stationary process for consumption, which is important to ensure accurate system-coefficient estimation.

This simultaneous system of equations is estimated below using the full-information maximum likelihood technique (FIML). All coefficients are identified with the exception of m and r\*'in equation (14')--only the "product" parameter \alpha is identified. Furthermore, the within- and cross-equation coefficient restrictions are tested by obtaining both the restricted and unrestricted FIML estimates and carrying out a likelihood ratio test. The unrestricted system is as above, except that (18) is replaced by:

(18') 
$$\Delta e_{t}^{i} = C_{1}\Delta M_{t}^{i} + C_{2}M_{t+1}^{i} + C_{3}\Delta M_{t-2}^{i}$$

$$+ C_{4}\Delta P_{t}^{*} + C_{5}\Delta P_{t-1}^{*} + C_{6}\Delta P_{t-2}^{*} + C_{7}\Delta P_{t-3}^{*}$$

$$+ C_{8}\Delta i_{t}^{*} + C_{9}\Delta i_{t-1}^{*} + C_{10}\Delta i_{t-2}^{*} + C_{11}\Delta C_{t}^{i}$$

Finally, we examine the model's within-sample tracking ability with respect to exchange rate and current account movements. The former is compared with the random walk model's tracking performance. In view of the sample size, it was thought best to use all observations for estimation purposes; accordingly, we do not examine the model's forecasting ability. This remains a task for future research as more observations become available.

Before presenting the empirical findings, it is important to note that a diagnostic check on the consistent (OLS) residuals from the estimation of equations (18') and (14') was undertaken for possible problems such as outliers, functional mis-specification and parameter instability,

heteroscedasticity, multicollinearity, autocorrelation and non-normality of residuals. <sup>18</sup> The results of this diagnostic check suggested that none of the aforementioned problems were present. Accordingly, estimation of the above-joint system by FIML is expected to yield consistent and asymptotically efficient coefficient estimates.

## 4. EMPIRICAL FINDINGS FOR THE UK

The FIML estimates of the restricted and unrestricted systems in the case of the UK over the period 1974:1 to 1984:1 are presented in Table 2 (Appendix 2). 19,20,21

It is noteworthy at the outset that the likelihood ratio test could not reject the restrictions at the 5% level of significance. More specifically, the test result is:

$$-2 \log[L_R/L_{UR}] = 8.77 < \chi^2(9) = 16.92$$

where,  $L_R$  = the value of the restricted likelihood, and  $L_{IIR}$  = the value of the unrestricted likelihood.

A number of comments may be made about the coefficients of the resstricted system:

Firstly, the coefficients of the time series processes are similar both in magnitude and in significance to the corresponding Box-Jenkins estimates (in Table 1). In regard to the key structural parameters, viz.  $\gamma$  and  $\delta$ , the coefficients of relative risk aversion attached to consumption and real money balances, respectively; these are plausible

in magnitude and are very significant. More specifically, these values seem plausible in view of other estimates of  $\gamma$  which have been obtained in the literature: Hansen and Singleton (1983) found values of  $\gamma$  that typically were between zero and two, while Grossman and Shiller (1981) found that stock prices could be explained by values of  $\gamma$  in the range of four, Finally, notice that the estimate of  $\alpha$ --the consumption function coefficient--is insignificantly different from zero. This finding is consistent with Hall's (1982) findings, in suggesting that consumption follows a random walk (in levels).

Next, consider the explanatory power of the model: the rightmost columns of Table 2 list the sum of squared residuals and standard error of the regression for each of the system's equations. These suggest that the equations are well fitted. We also examine the model's tracking ability in respect of exchange rate and current account movements. Time series plots of the actual and reduced-form (of the restricted system) estimated values of the exchange rate and the current account indicate that the model tracks very well. Of particular interest in this regard is the fact that the model's tracking performance closely corresponds to that of the random walk model. Table 3 (Appendix 2) summarizes this performance using the mean absolute error (MAE) and root mean square error (RMSE) statistics. These statistics indicate that both the intertemporal general equilibrium and the random walk model track exchange rate movements well--the former having a slight edge over the latter. This result is interesting since, although the intertemporal general equilibrium model contains more information than the random walk

model (given that the former's estimates are based on the entire sample period), the Backus (1984) study found the random walk model to be superior in terms of within-sample explanatory power than the existing exchange rate models.

## 5. CONCLUSION

The small-scale intertemporal general equilibrium model finds a good deal of empirical support in this study. The key structural parameter estimates (of  $\gamma$  and  $\delta$ ) have plausible magnitudes and are significantly different from zero; highly non-linear within- and across-equation coefficient restrictions are accepted at a reasonable level of confidence and the explanatory power of the model is very good, with the model closely mimicking the observed random walk behaviour of exchange rates.

Therefore, while the intertemporal general equilibrium analytical framework employed here has maintained some strong assumptions—such as purchasing power parity, uncovered interest rate parity, real interest rate constancy—assumptions shared by most existing empirical exchange rate models; the evidence suggests that the framework constitutes an advancement in our ability to explain exchange rate behaviour over the latter, specifically by modelling current account behaviour simultaneously and by taking explicit account of the restrictions imposed by forward-looking rational economic behaviour.

## CHAPTER 2 - NOTES

- 1. The simple flexible-price monetary model is based on Frenkel (1976) and Bilson (1978); the sticky-price monetary model on Dornbusch (1976) and Frankel (1979) and the sticky-price model incorporating current account effects on Hooper and Morton (1982).
- The sticky-price monetary models were based on Dornbusch (1976), Frankel (1979) and Driskill (1981); and the portfolio balances models on Branson et al. (1977), Dornbusch and Fischer (1980), Kouri (1976) and Obstfeld (1982).
- 3. See Kouri (1976) and Branson et al. (1977).
- 4. See Dornbusch and Fischer (1980).
- 5. See Stockman (1980) and Mussa (1980).
- 6. This is not available on the quarterly basis required for our case study.
- 7. In solving for  $C_1$  we require  $\left|\frac{\left[\beta(1+r^*)\right]^{1/\gamma}}{(1+r^*)}\right| < 1$ . This condition unambiguously holds for  $\gamma \ge 1$ .
- 8. Note, this identity is consistent with the theoretical framework for our study. For empirical purposes, y is defined as GDP plus net factor income from abroad plus net unilated transfers from abroad minus investment. For more precise definitions on these and other variables see Appendix 1. Notice, furthermore, that since consumption is the only endogenous variable in the current account

identity, the current account <u>equation</u> contains only the parameters of the consumption equation—therefore, the current account identity plays no role in the system estimation below.

- 9. This is, perhaps, clearer to see when one notes that wealth and permanent income bear a proportionate relationship to one; another, at a constant interest rate,  $\underline{viz}$ .  $W_t^* = \frac{(1+r^*)}{r^*} y_t^p$ , where  $y^p$  denotes permanent (disposable) income.
- 10. To see this latter point clearly, note that a typical expression of the monetary approach is:

$$e'_{t} = M'_{t} - P''_{t} - ny_{t}^{0} + \epsilon(i''_{t} + e'_{t+1} - e'_{t}) + a$$

where the new notation is: .5.

n = the real income elasticity of money demand,

 $\varepsilon$  = the interest semi-elasticity of money demand,

a = constant term

 $y^0$  = real income measure (current/permanent income).

This equation compares directly with equation (9) in the text.

- 11. This is clearer when we note that  $(\frac{1}{1+\delta \overline{i}}) = \frac{\varepsilon}{(1+\varepsilon)}$  and  $\frac{\delta \overline{i}}{(1+\delta \overline{i})}$   $= \frac{1}{(1+\varepsilon)} \text{ in equations (10) and (11) of the text, where } \varepsilon \text{ is the interest semi-elasticity of the demand for money.}$
- 12. The details of this derivation are as follows:

$$\tilde{y}_{t}$$
 (i)  $H_{t} = \frac{\tilde{y}_{t+s}}{s=0} - \frac{\tilde{y}_{t+s}}{(1+r^{*})^{s}} = \tilde{y}_{t} + \frac{\tilde{y}_{t+1}}{(1+r^{*})} + \frac{\tilde{y}_{t+2}}{(1+r^{*})^{2}} + \dots$ 

(ii) 
$$H_{t-1} = \sum_{s=0}^{\infty} \frac{\tilde{y}_{t+s-1}}{(1+r^*)^s} = \tilde{y}_{t-1} + \frac{\tilde{y}_{t}}{(1+r^*)} + \frac{\tilde{y}_{t+1}}{(1+r^*)^2} + \dots$$

Equations (i) and (ii)

(iii) 
$$H_{t} = (1+r^{*})(H_{t-1} - \tilde{y}_{t-1})$$

From (7) in the text and the definition of  $H_{\hat{t}}$  we have:

$$C_{t} = mW_{t} = m[H_{t} + (1+r*)b_{t-1}]$$

+ (iv) 
$$H_t = \frac{1}{m} C_t - (1+r*)b_{t-1}$$

+ (v) 
$$H_{t-1} = \frac{1}{m} C_{t-1} - (1+r^*)b_{t-2}$$

Substituting (iv) and (v) into (iii) gives:

(vi) 
$$\frac{1}{m} C_t - (1+r^*)b_{t-1} = (1+r^*)(\frac{1}{m} C_{t-1} - (1+r^*)b_{t-2} - \tilde{y}_{t-1})$$

(vii) 
$$C_t = m(1+r^*)b_{t-1} + (1+r^*)C_{t-1} - m(1+r^*)^2b_{t-2} - m(1+r^*)\tilde{y}_{t-1}$$

But, from equation (3) in the text:

$$c_{t-1} = -b_{t-1} + \tilde{y}_{t-1} + (1+r^*)b_{t-2}$$

+ 
$$-m(1+r^*)C_{t-1} = m(1+r^*)b_{t-1} - m(1+r^*)\tilde{y}_{t-1} - m(1+r^*)^2b_{t-2}$$

Substituting this into (vii) gives equation (14) of the text, i.e.,

(14) 
$$C_t = (1+r^*)(1-m)C_{t-1}$$

~ 4

Notice, furthermore, equation (6) in the text directly implies equation (14), i.e.,

(6) 
$$C_{t} = q^{(t-1)}C_{1}$$

-

+ 
$$C_{t} = q C_{t-1}, q = [\tilde{B}(1+r^{*})]^{1/\gamma} = (1+r^{*})(1-m)$$

But, the reason the more lengthy derivation above is noted is to show the consistency of equation (7)--the permanent income hypothesis, which is more general than equation (6) in imposing the present-value budget constraint [equation (4)]--with equation (14).

- 13. Note that all variables were entered in percentage-change format (except for the interest rate, which was entered in first differences); since in (11) they enter logarithmically (the interest rate in levels) and first differencing was necessary to achieve stationarity.
- 14. A similar testing procedure was undertaken by Rotemberg and Giovannini (1984).
- This, together with the first-differencing of the consumption function, ensures that all variables—not just the exogenous variables—are measured in similar units for the joint-FIML estimation of the exchange rate, consumption and time—series equations. (This is important to ensure accurate system—coefficient estimation.)

  Also, first-differencing of all the variables in a model is desirable, as Granger and Newbold (1974) point out, so as to avoid the problem of spurious correlations which are likely to occur when both

•••

the dependent and the explanatory variables are non-stationary.

Finally, as Nelson and Plosser (1982) point out: First-differencing avoids the spurious periodicity of trend residuals.

- 16. Note that since the stochastic processes for the exogenous variables are modelled as autoregressive processes, all current period shocks decay through time at a speed determined by the magnitude of the autoregressive parameters. The larger are the latter, the more permanent are the effects of current period shocks.
- 17. Notice that in going from equation (11') to equation (18) we have also used the following result:

$$\frac{-\gamma \overline{1}}{(1+\delta \overline{1})} \sum_{j=0}^{\infty} \left(\frac{1}{1+\delta \overline{1}}\right)^{j} \Delta \log(q^{j}C_{t}) = -\frac{\gamma}{\delta} \Delta \log C_{t}.$$

- 18. The procedures employed and results are reported here:
  - (a) A time-series plot of the residuals indicated that outlyingresiduals were not present.
  - (b) Individual scatter diagrams of the residuals and each explanatory variable exhibited no systematic relationship, suggesting that there is no functional mis-specification or parameter instability. This diagnostic check was supplemented by a Chow test for stability and mis-specification, which confirmed the findings of the former check [see Chow (1983) for details].
  - (c) Scatter diagrams of the squared residuals and fitted dependent variables, as well as of the squared residuals and various lagged values thereof, indicated no evidence of

heteroscedasticity--time-varying or otherwise. White's test [see White (1980)] and Bartlett's test [see Pindyck and Rubinfeld (1970)] for heteroscedasticity confirmed the latter finding. Note that only an approximation to the White test was undertaken for the exchange rate equation since the large number of bivariate combinations of the explanatory variables required exhausted our degrees of freedom. This is why the Bartlett test was also undertaken.

- (d) As a check for the presence of multicollinearity, a simple cross-correlation matrix on all of the explanatory variables entering the model was examined. In no case was evidence of high correlation found, suggesting that multicollinearity is not a problem for our case study.
- (e) Scatter diagrams of the residuals and their lagged values indicated autocorrelation was not present. This was confirmed by an LM test [see Pagan and Hall (1983) for details].
- (f) Finally, the Jarque-Bera test for the normality of residuals was not rejected for the equations [see Jarque and Bera (1980) for details].
- 19. It is noteworthy that the estimates proved robust to alternative starting values. This is important as it suggests that the likelihood function is well-behaved and that we are at the global-maximum point. The actual starting values used for the results reported in Table 2 are:
  - (a) for the restricted systems: the Box-Jenkins estimates of the time series parameters,  $\alpha = 0.5$ ,  $\gamma = 2$ ,  $\delta = 2$ .

- (b) for the unrestricted system: the Box-Jenkins estimates of the time series parameters,  $\alpha$  = .5 and OLS coefficient estimates of the exchange rate equation.
- 20. We also estimated a version of the model in which we admitted the possibility of government (consumption) expenditure serving as a substitute for private consumption, following Aschauer (1985). It proved insignificant and, is, thus, omitted from consideration here.
- 21. The estimation was also undertaken using an alternative, broader measure of money--namely a measure of M2. Similar results were obtained.

# Appendix 1 . Data and Data Sources

- e: UK pound per US doller; end of period; seasonally adjusted; source 1.
- M: UK M1; end of period; seasonally adjusted; source 1.
- P\*: US GNP Price Deflator; 1980 = 100; seasonally adjusted; source 1.
- i\*: US 3-month treasury bill rate; end of period; source 1.
- C: UK Consumer Expenditure on Nondurable Goods and Services at the 1980 prices; seasonally adjusted; source 2.
- CB: UK Current Account deflated by GDP deflator (1980 = 100); seasonally adjusted; source 2.
- y: UK GNP <u>plus</u> net unilateral transfers from abroad <u>less</u> gross domestic fixed capital formation <u>less</u> increases in stocks <u>less</u> consumer expenditure on durables, at 1980 prices; seasonally adjusted; source 2.
- g: UK government consumption expenditures; seasonally adjusted: source 2.
- Source:1: 0.E.C.D. Main Economic Indicators, Historical Statistics, 1964-1983.
- Source 2: CSO (UK) Economic Trends, various issues.

Appendix 2

Table 1: Box-Jenkins Estimation and OTagnostics

Equation:	V	ΔM'		.**		A1*
Coefficients (t'Values)	$a_{11} = 0.32$ (2.8)	$\alpha_{12} = 0.58$ (5.0)	$a_{21} = 0.51  a_{22}$	0.10 (0.0)	$= 0.39  \alpha_{31} = -0.33  (2.7)$	α32 * -0.30 (2.0)
Autocorrelations	1-10	11-20	1-10	11-20	1-10	11-20
of residuals	0.01 -0.19		0.09	-0.11	0.04	-0.18
	-0.07		0.20	-0.16	0.03	-0.19 -0.03
	-0.17		-0.12 0.12	. 0.06 0.05	0.05	-0.03
	0.12		-0.21	-0.11	0.01	-0.15
•	-0.04		-0.22 -0.12	-0.12	-0.12	90.0 <del>1</del> 0
	0.22 -0.09	-0.04	<b>6</b> €	-0.16 -0.01	0.23 -0.10	-0.02 -0.10 -0.13
Box-Pierce Q statistic	. 22.2		. 19,1		15.5	-
Critical Value of $\chi^2_{m-q}$	28.87	,	. 56.30	·	. 28.87	

Table 2: FIML Estimates of Restricted and Unrestricted Systems Sample Period: 1974:1 to 1984:1

$a_{11}$ 0.43 (3.40) $a_{12}$ 0.55 (3.86) $a_{21}$ 0.44 (2.25) $a_{21}$ 0.41 (2.24) $a_{22}$ 0.11 (0.45) $a_{23}$ 0.44 (2.25) $a_{31}$ -0.35 (2.88) $a_{32}$ -0.31 (1.93) $a_{32}$ -0.31 (1.93) $a_{32}$ 0.43 (1.93) $a_{32}$ 0.12 (0.81) $a_{32}$ 0.51 (2.85) $a_{23}$ 0.38 (1.93) $a_{21}$ 0.49 (1.82) $a_{22}$ 0.69 (0.29) $a_{23}$ 0.38 (1.93) $a_{31}$ -0.36 (1.77) $a_{32}$ -0.32 (1.50) $a_{23}$ 0.55 (0.61) $a_{22}$ 0.65 (0.61)	541.84 (1) Aet eq.: 0.344	\$\$0.0 \$\$0.0
1 (2.24)		1000
5 (2.88)		0.067
1 (3.88) 6 2.01 (7.93) 2 (0.81) 3 (2.01) $a_{12}$ 0.51 (2.85) 6 (1.82) $a_{22}$ 0.03 (1.93) 6 (1.77) $a_{32}$ -0.32 (1.50) 5 (0.11) C 0.25 (0.61) C 0.06 (0.16)	(111) AM1 eq.: 0.017	0.020
2 (0.81)  0 (2.01)	(1v) AP* eq. 0.001	0.005
0 (2.01) $a_{12}$ 0.51 (2.85) 9 (1.82) $a_{22}$ 0.09 (0.29) $a_{23}$ 0.38 (1.93) 6 (1.77) $a_{32}$ -0.32 (1.50)	(v) Air eq.: 0.011	910,0
0.40 (2.01) $a_{12}$ 0.51 (2.85) 0.49 (1.82) $a_{22}$ 0.09 (0.29) $a_{23}$ 0.38 (1.93) -0.36 (1.77) $a_{32}$ -0.32 (1.50)		
0.49 (1.82) $a_{22}$ 0.09 (0.29) $a_{23}$ -0.36 (1.77) $a_{32}$ -0.32 (1.50)	546.225 (1) Aet eq.: 0.050	0.035
-0.36 (1.77) a <sub>32</sub> -0.32 (1.50)	(11) AC <sub>t</sub> eq.: 0.031	0.027
, C 0.25 (0.61) C	(111) AM1 eq.: 0.017	0.020
2 (20.0) 23.	(1y) AP*' eq.: 0.001	0.005
c <sub>5</sub> -0.59 (0.28) c <sub>6</sub> 0.47 (0.26) c <sub>7</sub>		910.0
<b>C<sub>8</sub> 0.53 (0.80)</b> · C <sub>9</sub> -0.46 (p.99) C <sub>10</sub> -0.31 (0.70) C <sub>11</sub> -1.34 (1.22)	1.34 (1.22)	
0.10 (0.60)		

		. MAE	RMSE
(i)	- Intertemporal General Equilibrium Model		
	(a) Exchange Rate	0.0354	0.0446
	(b) Current Account	0.0207	0.0274
a(ii)	Random Walk Model		
•	(a) Exchange Rate	0.0388	0.0486

## CHAPTER 3

# ON SAVINGS AND INVESTMENT DYNAMICS IN A SMALL OPEN ECONOMY

## 1. INTRODUCTION

The purpose of this essay is to construct a stochastic intertemporal general equilibrium (GE) model of savings (S) and investment (I) in a small open economy (SOE) under conditions of perfect international capital mobility and to examine, using simulation techniques, the predictions of the model for the dynamics of S and I behaviour in response to technological disturbances.

The motivation for the paper derives from the following considerations:

the context of intertemporal GE models focus either exclusively on the savings side of current account determination (Greenwood (1983), Obstfeld (1981), Svensson and Razin (1983), Helpman and Razin (1984)) or model both S and I behaviour jointly in a two-period lived economy (Razin (1980), Sachs (1981), Svensson (1984)). In our view, it is of interest to examine S and I determination simultaneously, but in a model which does not restrict the lifetime of the economy to two periods—thereby permitting more interesting S and I dynamics.

Two recent studies do just that. More specifically, Persson and

Svensson (1985) use a deterministic, overlapping-generations (OLG), SOE model under conditions of perfect international capital mobility, to examine the dynamics of S, I and therefore the current account in response to static and intertemporal terms-of-trade disturbances. The model generally predicts S and I dynamics which move in opposite directions, thereby producing sizeable current account movements over time. Persson (1985) uses a deterministic, OLG model of a closed, SOE and large-open economy (under conditions of perfect capital mobility) to examine the effects of a one-period government budget deficit. The model predicts zero covariation between S and I in the SOE case, while some positive covariation between S and I obtains in the large open economy tase due to induced, equilibrating, world interest rate movements.

(2) There now exists a sizeable empirical literature (Feldstein and Horioka (1980), Feldstein (1983), Penati and Dooley (1984), Fieleke (1982), Caprio and Howard (1984)) which documents significantly-positive cross-sectional correlations—for a wide range of OECD countries—between S and I, averaged over periods varying in length between 5 and 20 years. 1,2 In addition, Obstfeld (1985) finds evidence—for seven OECD countries—of mostly significantly positive quarterly time series correlations between S and I and Frankel (1985) shows evidence of significantly positive decade average and annual time series correlations between S and I for the U.S. This evidence has, largely, been interpreted as being indicative that capital is strongly immobile for a wide range of countries. In the words of Feldstein and Horioka it is argued: "With perfect

world capital mobility, there should be no relation between domestic saving and domestic investment; saving in each country responds to the worldwide opportunities for investment and investment in that country is financed by the worldwide pool of capital" (Feldstein and Horioka (1980), p. 317). While it is recognized that common factors may affect saving and investment in the same direction even under perfect capital mobility, it is argued further that the above evidence places the burden of identifying these common factors on the proponents of the hypothesis that capital is internationally mobile. Obstfeld (1985) simulates a deterministic, OLG, SOE model with perfect capital mobility to show that effective labour-force growth is one such factor--capable of explaining the cross-sectional S and I correlations. The example is, however, qualified by the observation that labour-force growth is not an important determinant of real-world S and I rates. Obstfeld, furthermore, points out in the context of a deterministic, representative-agent, SOE model with perfect capital mobility that an unanticipated and sufficiently temporary domestic productivity shock in one period induces positive correlation in the impact response of S and I. He also notes that S and I must be expected to be more strongly (directly) related for larger economies than for smaller ones--since a country's influence on world real interest. rates is directly related to its size. Persson's (1985) results. referred to above, serve as a case in point.

With view to the points made in (1) and (2) above, this paper investigates, at a theoretical level, the question of the dynamics of S and I in response to technological disturbances under conditions of perfect capital mobility and, in particular, the question as to whether these dynamics will be characterized by significantly positive correlations between S and I. These questions are addressed in the context of a stochastic intertemporal GE model and dynamic simulation techniques are employed. Clearly, a stochastic framework is necessary to develop rigorously the implications of the model and, as will be indicated below, the stochastic framework differs fundamentally from a deterministic framework. Dynamic simulation techniques are used to generate the time series for S and I implied by the model under alternative specifications of the distribution of technological disturbances. (A similar simulation strategy is used in Huffman (1986).) The dynamics of S and I relations are then summarized by regressions and correlations between the simulated time series.

A few specific points on the analytical framework are in order here:

(1) The model is an OLG model of a 2-period lived agents. The main reason for choosing this framework is its analytical tractability in a stochastic environment with endogenous rates of return to investment. Indeed, by contrast, it can be shown that an infinitely-lived representative agent framework proves intractable in such an environment. Also, the assumption of infinitely-lived agents gives rise to a very high degree of consumption smoothing and intertemporal substitution. In this regard, Persson and Svensson (1985)

point out: "One could thus argue that a model with finite planning horizons seems to give rise to a more intuitively reasonable and even more realistic savings behavior". Furthermore, the OLG medel has nice stability properties—in particular, it does not require arbitrary restrictions on the rate of time preference in order to ensure stability of the steady state, as is the case for the infinite-horizon analysis of Obstfeld (1982) and Svensson and Razin (1983).

It is, finally, noteworthy here that before one compares statistics generated from simulations of the OLG model with documented statistics, one must decide on the length of the time period of the model --i.e., should the generated statistics be compared to monthly, quarterly or annual statistics, etc.? Some may argue that the OLG model is suitable only for analyzing low frequency S and I dynamics. However, as pointed out by Huffman (1986), the OLG model does not lend itself to an easy answer to this question and this problem is not specific to the OLG framework. Even if agents were infinitely lived there would still be ambiguity in matching the model with the data. This qualification must therefore be borne in mind in interpreting the results below.

(2) The prime concern of the analysis is in the dynamics of the S-I relationship in a SOE in response to shocks to technology. The focus on the case of an SOE is due to the fact that most of the empirical literature, referred to above, deals with the SOE.

Furthermore, one must automatically expect higher S-I covariations

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try's influence on world rates of return is directly related to its size. Accordingly, the SOE case is the most interesting.

The focus on technology shocks is due to the fact that expenditure disturbances have been analyzed by Persson (1985).

(3) As has been mentioned above, the analytical framework is stochastic—with uncertainty introduced alternatively via domestic or foreign technology shocks. A stochastic environment is necessary to develop rigorously the implications of the model. In addition, an interesting feature arises from the stochastic nature of the model: while the focus of attention is on an SOE, it is also necessary to model the rest-of-the-world (or foreign country) decision problem. The reason is that uncertainty and risk aversion on the part of agents prevents the international equalization of real returns on (real) investments—and this is so even—in a world of perfect capital mobility. Accordingly, especially for the purpose of the present study, the stochastic framework differs in an important respect from the deterministic framework.

The remainder of the paper is organized as follows: Section 2 outlines the model specification, Section 3 presents the simulation results and Section 4 concludes the paper.

#### 2. MODEL

The model describes a two-country (domestic and foreign-country) world in which agents live for two periods. A new generation of constant size is born in each country each period. The domestic country is assumed to be small in relation to the foreign country-this is reflected in population sizes.

In their first period of life, each individual in each country inelastically supplies one unit of labour to a firm located in their country. The firm combines this labour input with the (non-depreciating) capital invested in that country to produce output. Production functions are assumed to be of Cobb-Douglas form in each country and contain a multiplicative productivity (or technology) variable. Uncertainty is introduced into the model alternatively through stochastic variations in the domestic or foreign technology variable. The return to labour each period is its realized marginal product. Once incomes are received, individuals decide on their savings and its composition. Wealth may be held in the form of domestic and foreign physical capital and in domestic and foreign private consumption loans—i.e., perfect international mobility of both physical capital and consumption loans is assumed. The portfolio size and composition decisions are made in such a way that agents maximize expected utility.

Agents enter the second period of their lives as capitalists--owners of the firms. They now hire labour and pay each worker their marginal product. Their consumption is dictated by their portfolio choices when

they were young.

The model is formally described as follows. Table 1 (Appendix 1) lists all notation used in this description.

First, consider the case where uncertainty stems solely from stochastic variations in domestic technology. The representative young domestic agent solves the following optimization problem:

$$\max_{\mathbf{t}} E[\log(c_t) + \beta \log(d_{t+1}) = \log(c_t) + \beta \sum_{i} f_{ij} \log(d_{it+1})$$

where the choice variables are:  $k_{t+1}^d$ ,  $k_{t+1}^{*d}$ , and  $l_i^d(t)$ ; subject to:

(1) 
$$c_{t} = w_{t} - k_{t+1}^{d} - k_{t+1}^{*d} - \sum_{i} q_{i}(t) l_{i}^{d}(t)$$

(2) 
$$d_{it+1} = (1+r_{it+1})k_{t+1}^d + (1+r_{t+1}^*)k_{t+1}^d + 1_i^d(t)$$

where  $w_{t} = \theta_{jk} (1-\alpha) (\frac{\overline{K}_{t}}{N})^{\alpha}$  -- the marginal product of domestic labour

$$r_{it+1} \equiv \theta_{it+1} \alpha (\frac{\overline{K}_{t+1}}{N})^{\alpha-1}$$
 -- the marginal product of domestic capital

$$r_{t+1}^{*} \equiv \theta_{\star}^{*} \alpha^{*} (\frac{K_{t+1}}{N^{*}})^{\alpha^{*}-1}$$
 -- the marginal product of foreign capital

The domestic technology variable,  $\theta$ , is assumed to follow a first-order, two-state Markov process of the form:

$$\theta_{1} \text{ with probability } f_{11} \text{ if } \theta_{\text{it.}} = \theta_{1}$$

$$\theta_{2} \text{ with probability } f_{21} \equiv 1 - f_{11} \text{ if } \theta_{\text{it.}} = \theta_{1}$$

$$\theta_{1} \text{ with probability } f_{12} \equiv 1 - f_{22} \text{ if, } \theta_{\text{it.}} = \theta_{2}$$

$$\theta_{2} \text{ with probability } f_{22} \text{ if } \theta_{\text{it.}} = \theta_{2}$$

The state of nature, indexed by i, is identified with the realization of  $\theta_i$  (i = 1, 2).

Multiplying (2) by  $q_i(t)$  and summing over i gives:

Adding (1) and (3) gives:

(4) 
$$c_{t} + \sum_{i} q_{i}(t)d_{it+1} = w_{t} + \left[\sum_{i} q_{i}(t)(1+r_{it+1})-1\right]k_{t+1}^{d} + \left[\sum_{i} q_{i}(t)(1+r_{t+1}^{*})-1\right]k_{t+1}^{*d}$$

The absence of arbitrage opportunities in equilibrium implies that the following conditions must hold:

(5) 
$$[\sum_{i} q_{i}(t)(1+r_{it+1})-1] \leq 0 = 0 \text{ if } k_{t+1}^{d} > 0 \text{ (as will be assumed)}$$

(6) 
$$\left[\sum_{i} q_{i}(t)(1+r_{t+1}^{*})-1\right] \leq 0 = 0 \text{ if } k_{t+1}^{*d} > 0 \text{ (as will be assumed)}$$

otherwise, there would be an incentive to borrow infinite amounts and invest the proceeds in the domestic and/or foreign technologies.

Accordingly, (5) and (6) will be imposed on the agent's decision problem (see Sargent (1984)).

The first-order conditions are:

$$(7) \frac{-1}{\left[w_{t} - k_{t+1}^{d} - k_{t+1}^{\star d} - \sum_{i} q_{i}(t) l_{i}^{d}(t)\right]} + \beta \sum_{i} \frac{f_{ij}(1 + r_{it+1})}{\left[(1 + r_{it+1}^{\star}) k_{t+1}^{d} + (1 + r_{t+1}^{\star}) k_{t+1}^{\star d} + l_{i}^{d}(t)\right]} \le 0, = 0 \text{ if } k_{t+1}^{d} > 0$$

(8) 
$$\frac{-1}{\left[w_{t} - k_{t+1}^{d} - k_{t+1}^{\star d} - \sum_{i} q_{i}(t) l_{i}^{d}(t)\right]} + \beta \sum_{i} \frac{f_{ij}(1 + r_{t+1}^{\star})}{\left[(1 + r_{it+1})k_{t+1}^{d} + (1 + r_{t+1}^{\star})k_{t+1}^{\star d} + l_{i}^{d}(t)\right]} \le 0, = 0 \text{ if } k_{t+1}^{\star d} > 0$$

(9) 
$$\frac{-q_{i}(t)}{\left[w_{t} - k_{t+1}^{d} - k_{t+1}^{*d} - \sum_{i} q_{i}(t) 1_{i}^{d}(t)\right]} + \beta \frac{f_{ij}}{\left[(1 + r_{it+1})k_{t+1}^{d} + (1 + r_{t+1}^{*})k_{t+1}^{*d} + 1_{i}^{d}(t)\right]}$$

= 0 for i = 1, 2.

Assuming an interior solution for capital holdings, these first-order conditions may be rearranged to give:

(10) 
$$s_t = k_{t+1}^d + k_{t+1}^{*d} + q_1(t)1_1^d(t) + q_2(t)1_2^{*d(t)} = \frac{\beta}{(1+\beta)} w_t$$

$$- s_t \text{ is pre-determined at time t}$$

(11) 
$$\frac{f_{ij}[(1+r_{it+1})-(1+r_{t+1}^*)]}{[(1+r_{it+1})k_{t+1}^d+(1+r_{t+1}^*)k_{t+1}^*+1_i^d(t)]} = 0$$

(12) 
$$\frac{f_{2j}q_{2}(t)^{-1}}{[(1+r_{2t+1})k_{t+1}^{d}+(1+r_{t+1})k_{t+1}^{*d}+1]^{d}}]$$

$$-\frac{f_{1j}q_{1}(t)^{-1}}{[(1+r_{1t+1})k_{t+1}^{d}+(1+r_{t+1}^{*})k_{t+1}^{*d}+1]^{d}(t)]}=0$$

From (11) it is clear that under uncertainty and risk aversion on the part of agents, real rates of return to domestic and foreign investment are not equalized. Accordingly, it is not possible to solve for the domestic capital stock and, hence, domestic investment and saving without modelling the foreign country decision problem.

Correspondingly, the solution to the representative young foreign agent's decision problem involves the simultaneous solution of (5) and (6) and:

(10\*) 
$$s_t^* = k_{t+1}^f + k_{t+1}^{*f} + q_1(t)l_1^f(t) + q_2(t)l_2^f(t) = \frac{\beta^*}{(1+\beta^*)} w_t^*$$
  
-  $s_t^*$  is pre-determined at time t

(11\*) 
$$\sum_{i} \frac{f_{ij}[(1+r_{it+1})-(1+r_{t+1}^*)]}{[(1+r_{it+1})k_{t+1}^f+(1+r_{t+1}^*)k_{t+1}^f+1]^f(t)]} = 0$$

(12\*) 
$$\frac{f_{2j}q_{2}(t)^{-1}}{[(1+r_{2t+1})k_{t+1}^{f}+(1+r_{t+1}^{*})k_{t+1}^{*f}+1_{2}^{f}(t)]}$$

$$-\frac{f_{1j}q_{1}(t)^{-1}}{[(1+r_{1t+1})k_{t+1}^{f}+(1+r_{t+1}^{*})k_{t+1}^{*f}+1_{1}^{f}(t)]}=0$$

where  $w_t^* \equiv \theta_t^*(1-\alpha^*) \frac{\overline{K}_t^*\alpha^*}{N^*}$  -- the marginal product of foreign labour. The equilibrium conditions for the world economy are:

(13) 
$$\bar{K}_{t+1} = Nk_{t+1}^d + N*k_{t+1}^f \bullet$$

(14) 
$$\overline{K}_{t+1}^{*} = Nk_{t+1}^{*d} + N*k_{t+1}^{*f}$$

(15) 
$$N1_1^d(t) + N+1_1^f(t) = 0$$

(16) 
$$N1_2^d(t) + N*1_2^f(t) = 0$$

It is straightforward, although lengthy, to show that, noting the definitions of  $w_t$ ,  $w_t^{\frac{1}{2}}$ ,  $r_{t+1}^{\frac{1}{2}}$ , the equilibrium solution of the model (i.e., equations (5), (6), (10)-(12), (10\*)-(12\*), (13)-(16))

freduces to the solution of the following system of four equations in the four unknowns:  $\overline{K}_{t+1}$ ,  $\overline{K}_{t+1}^*$ ,  $s_t$ ,  $s_t^*$ :

(17) 
$$\overline{K}_{t+1}^* + \overline{K}_{t+1}^* = Ns_t \rightarrow N^*s_t^*$$

$$(18) \quad \overline{K}_{t+1}^{\overline{K}} \overline{K}_{t+1}^{*} = \\ \{[1 + e^{*} \alpha^{*} (\frac{\overline{K}_{t+1}^{*}}{N^{*}})^{\alpha^{*}-1}][e^{*} \alpha^{*} (\frac{\overline{K}_{t+1}^{*}}{N^{*}})^{\alpha^{*}-1} (1 + f_{1j}/f_{2j}) - e_{2} \alpha (\frac{\overline{K}_{t+1}^{*}}{N^{*}})^{\alpha^{-1}} - f_{1j}/f_{2j} e_{1} \alpha (\frac{\overline{K}_{t+1}^{*}}{N^{*}})^{\alpha^{-1}}]\}/\\ \{f_{1j}/f_{2j}[e_{1} \alpha (\frac{\overline{K}_{t+1}^{*}}{N^{*}})^{\alpha^{-1}} - e^{*} \alpha^{*} (\frac{\overline{K}_{t+1}^{*}}{N^{*}})^{\alpha^{*}-1}][1 + e_{2} \alpha (\frac{\overline{K}_{t+1}^{*}}{N^{*}})^{\alpha^{-1}}]$$

$$= [e^{*} \alpha^{*} (\frac{\overline{K}_{t+1}^{*}}{N^{*}})^{\alpha^{*}-1} - e_{2} \alpha (\frac{\overline{K}_{t+1}^{*}}{N^{*}})^{\alpha^{-1}}][1 + e_{1} \alpha (\frac{\overline{K}_{t+1}^{*}}{N^{*}})^{\alpha^{-1}}]\}$$

(19) 
$$s_{t} = \frac{\beta}{(1+\beta)} \theta_{jt} (1-\alpha) (\frac{\overline{K}_{t}}{N})^{\alpha}$$

(20) 
$$s_{t}^{*} = \frac{\beta^{*}}{(1+\beta^{*})} \theta^{*}(1-\alpha^{*})(\frac{\overline{K}_{t}^{*}}{N^{*}})^{\alpha^{*}}$$

The assumption of perfect international capital mobility combined with the assumption of identical relative risk aversion across domestic and foreign agents implies that agents are indifferent as to the composition of their portfolios between domestic and foreign physical capital holdings and consumption loans. The model is, therefore, indeterminate with regard to portfolio composition. For our purpose of determining

aggregate net investment and saving it is, of course, irrelevant as to who holds what asset. In particular, for example, the solution for  $\overline{K}_{t+1}$  and  $s_t^2$  from equations (17)-(20) are used in the following equations to solve for aggregate domestic net investment and saving:

$$(21) I_t = \overline{K}_{t+1} - \overline{K}_t$$

(22) 
$$S_t = N(s_t - s_{t-1})$$

An interesting extension of the model would be to allow for differences in relative risk aversion across domestic and foreign agents and develop and test its predictions for domestic and foreign portfolio composition. An attractive feature of the OLG framework is that such an extension is tractable. In contrast, the solution procedures for open-economy infinitely-lived representative agent models rely heavily on the assumption of an internationally-perfectly-pooled equilibrium.

Finally, the model is easily amended to deal with the case where uncertainty stems solely from stochastic variations in foreign technology. The equilibrium solution of the model reduces, in this case, to the solution of the following system of four equations in the four unknowns:  $\overline{K}_{t+1}$ ,  $\overline{K}_{t+1}^*$ ,  $s_t$ ,  $s_t^*$ :

$$(17') \qquad \overline{K}_{t+1} = \overline{K}_{t+1}^* = Ns_t + N*s_t^*$$

$$(18') \qquad \overline{K}_{t+1}^* / \overline{K}_{t+1} = \\ \{ [1 + \theta \alpha (\frac{\overline{K}_{t+1}}{N})^{\alpha-1}] [\theta \alpha (\frac{\overline{K}_{t+1}}{N})^{\alpha} - \frac{\overline{K}_{t+1}^*}{N} ]^{\alpha} - \frac{\overline{K}_{t+1}^*}{N^*} ]^{\alpha-1} - f_{1j}^* / f_{2j}^* \theta_{1}^* \alpha^* (\frac{\overline{K}_{t+1}^*}{N^*})^{\alpha^*-1} ] \} / \\ \{ f_{1j}^* / f_{2j}^* [\theta_{1}^* \alpha (\frac{\overline{K}_{t+1}^*}{N^*})^{\alpha^*-1} - \theta \alpha (\frac{\overline{K}_{t+1}^*}{N})^{\alpha-1}] [1 + \theta_{2}^* \alpha^* (\frac{\overline{K}_{t+1}^*}{N^*})^{\alpha^*-1}] \} / \\ - [\overline{\theta \alpha} (\frac{\overline{K}_{t+1}^*}{N})^{\alpha-1} - \theta_{2}^* \alpha^* (\frac{\overline{K}_{t+1}^*}{N^*})^{\alpha^*-1}] [1 + \theta_{1}^* \alpha^* (\frac{\overline{K}_{t+1}^*}{N^*})^{\alpha^*-1}] \}$$

(19') 
$$s_{t}^{*} = \frac{\beta^{*}}{(1+\beta^{*})} \theta_{jt}^{*} (1-\alpha^{*}) (\frac{\overline{K}_{t}^{*}}{N^{*}})^{\alpha^{*}}$$

(20') 
$$s_t = \frac{\beta}{(1+\beta)} \theta(1-\alpha) (\frac{\overline{K}_t}{N})^{\alpha}$$

The solution for  $\overline{K}_{t+1}$  and  $s_t$  from equations (17')-(20') can be used in equations (21) and (22) to solve for aggregate domestic net investment and saving.

#### 3. SIMULATION RESULTS

Tables 2-4 (5-7) (Appendix 1) report the results from simulating the model in the case where uncertainty stems solely from stochastic variations in domestic (foreign) technology. In particular, Tables 2, 3 and 4 (5, 6 and 7) deal, respectively, with the cases where the domestic (foreign) technology process exhibits positive, negative and zero autocorrelation. Each table lists: the parameter values used in the

process in question; and the results of the regressions and correlations between the simulated domestic savings (S) and investment (I) rates. 9

For the latter regressions, maximum likelihood estimates (corrected for serial correlation) are reported in addition to least-square estimates in the event of evidence of serial correlation of the error term.

Each simulation experiment was conducted as follows: The indicated parameter values on the probability distribution of  $\theta$  ( $\theta^*$ ) were used to generate one-thousand time series values for  $\theta$  ( $\theta^*$ ). The resulting  $\theta$  ( $\theta^*$ ) time series, together with the indicated parameter values of the model  $\theta^{10}$  were then used in equations (17)-(22)[(17')-(20'), (21)-(22)] to obtain a dynamic solution for S and I--one-thousand time series values for each. The solution technique used is a variant of Newton's iterative method for solving nonlinear simultaneous models. The deterministic steady state solution of the model was used to provide initial values for the domestic and foreign capital stocks.  $\theta^{12}$ 

First, we examine the results from simulating the model in the cases where uncertainty stems from stochastic variations in domestic technology. As is indicated in Table 2, the case where  $\theta$  follows a positively-autocorrelated stochastic process is characterized by a significantly positive relationship between S and I rate dynamics. In this case: the maximum-likelihood coefficient estimate of the S-I rate relationship is 0.69 and it is significantly different from zero; the simple correlation coefficient between S and I rates is 0.97. Table 3 shows that the case where  $\theta$  follows a negativeTy-autocorrelated stochastic process is

characterized by a significantly negative relationship between S and I rate dynamics. In this case: the maximum-likelihood coefficient estimate of the S-I rate relationship is -0.62 and it is significantly different from zero; the simple correlation coefficient between S and I rates is -0.98. Finally, Table 4 shows that the case of a serially uncorrelated 6 process is characterized by a significantly positive relationship between S and I rate dynamics. In this case: the maximum-likelihood coefficient estimate of the S-I rate relationship is 0.15 and it is significantly different from zero; the simple correlation coefficient between S and I rates is 0.95.

The aforementioned findings are qualitatively what one expects of the relationship between S and I dynamics stemming from stochastic variations in  $\theta$ . A change in  $\theta$  instantaneously drives S and I in the same direction when this change is expected to persist for some time, and a change in θ instantaneously drives S and I in the opposite direction when this change is expected to be reversed next period. Furthermore, note that the induced change in I affects the domestic capital \_stock and, therefore, the domestic wage one period later. Since S and wages are positively related, cycles in I lead to similar cycles in S with a one-period lag. This technology-induced lagged S and I-cycle relationship serves to reinforce the aforementioned instantaneous positive (negative) covariation between S and I in the positively- (negatively-) autocorrelated 0 cases. These considerations also suggest that the qualitative characteristics of the serially-uncorrelated  $\boldsymbol{\theta}$  case are theoretically ambiguous since this presents an intermediate case to the foregoing two.

Next, the following results were obtained from simulating the model in the cases where uncertainty stems from stochastic variations in foreign technology: As js-indicated in T-51e 5, the case where  $\theta^*$  follows a positively-autocorrelated stochastic process is characterized by a significantly positive relationship between S and I dynamics. In this case: the least-squares estimate of the S-I rate relationship is 0.11 and it is significantly different from zero; the simple correlation coefficient between S and I rates is 0.38. Table 5 shows that the case where 0\* follows a negatively-autocorrelated stochastic process is characterized by a significantly negative relationship between S and I Pate dynamics. In this case: the maximum-likelihood estimate of the S-I rate relationship is -0.14 and it is significantly different from. zero; the simple correlation coefficient between S and I rates is -0.71. Finally, Table 7 shows that the case of a serially uncorrelated  $\theta^*$  process is characterized by a significantly negative relationship between S and I rate dynamics. In this case: the least-squares coefficient estimate of the S-I rate relationship is -0.08 and it is significantly different from zero; the simple correlation coefficient between S and I rates is -0.30.

The above findings are also qualitatively what one expects of the relationship between S and I dynamics stemming from stochastic variations in  $\theta^*$ . A change in  $\theta^*$  has no effect on the instantaneous covariation between S and I but the lagged S and I-cycle relationship, explained above, operates to create some positive (negative) covariation between S and I in the positively- (negatively-) autocorrelated  $\theta^*$  cases. The

absence of an instantaneous-covariation effect implies a weaker relationship between S and I dynamics obtains in the case of autocorrelated- $\theta^*$  processes when compared to those obtaining in the corresponding autocorrelated- $\theta$  cases. The qualitative characteristics of the serially-uncorrelated  $\theta^*$  case are theoretically ambiguous since this represents an intermediate case to the positively- and negatively-autocorrelated cases.

The results for the positively-autocorrelated  $\theta$  and  $\theta^*$  cases are especially of interest in view of the documented evidence on significantly-positive S and I relationships, referred to above, as well as evidence suggesting that technology variables exhibit high persistence (see, e.g., Prescott (1986)). It is not, of course, to be claimed that the simulation results for the artificial economies studied, and summarized in Tables 2 and 5, therefore, offer an <u>explanation</u> of real world S and I behaviour. Clearly, estimation and econometric testing of the model using actual data would be necessary before deciding on the model's explanatory power. The results are, however, <u>suggestive</u> that persistent innovations in domestic and foreign technology at least play a partial role in explaining the high S and I covariations which have been observed to occur empirically and that these latter high covariations do not necessarily indicate international capital immobility.

#### 4. CONCLUSION

This paper developed—a stochastic intertemporal GE model of S and I in a small open economy, under conditions of perfect international

capital mobility, and examined, using simulation techniques, the predictions of the model for the dynamics of S and I behaviour in response to domestic technological disturbances.

Some key features of the analytical framework and suggestions for further research are:

- (1) It was shown that an overlapping generations structure with twoperiod lived agents permitted a tractable solution to an open economy
  model with stochastic, endogenous rates of return to investment.

  An interesting topic for future research would be to examine whether
  there are conditions under which the infinitely-lived representative
  agent framework would have a unique stationary competitive equilibrium solution in an open-economy set-up with stochastic, endogenous returns to investment—as has been shown for closed economies
  (see Lucas and Prescott (1971)). Suelf a framework would be useful
  not only in addressing current account issues but also in analyzing
  international business cycle behaviour.
- (2) An interesting feature arises from the stochastic nature of the model. While the focus of the analysis is on S and I behaviour in a SOE, it is shown that uncertainty and risk aversion on a part of agents prevents the international equalization of real returns to investment, even under perfect international capital mobility. The foreign country decision problem must therefore also be modelled. This feature marks a fundamental difference between a stochastic and deterministic framework for analyzing S and I in a SOE. In particular, in identifying common factors which cause positive S

and I covariation under conditions of perfect international capital mobility, it would seem important to conduct the analysis in an explicitly stochastic framework.

mobility, together with the assumption of perfect international capital mobility, together with the assumption of identical coefficients of relative risk aversion across domestic and foreign agents implies an indeterminancy of the composition of agents' portfolios. An interesting extension of the model would be to relax the assumption of identical relative risk aversion and develop and test the model's predictions in respect of portfolio composition in this case. An attractive feature of the OLG framework is the tractability of such an extension.

With regard to the simulation results: The model was simulated for six cases in which, alternatively, the domestic or foreign technology variable exhibited positive, negative and zero autocorrelation. A significantly positive relationship between S and I dynamics characterized the positively-autocorrelated domestic and foreign technology cases and the serially-uncorrelated domestic technology case. A significantly negative relationship between S and I dynamics characterized the negatively-autocorrelated domestic and foreign technology cases and the serially-uncorrelated foreign technology case. The finding of a significantly positive relationship between S and I dynamics generated by positively-autocorrelated technology processes, is interpreted as being suggestive that persistent innovations in technology play, at least, a partial role in explaining the high S and I correlations which have

been observed to occur empirically. In particular, it suggests that high S and I covariations are not necessarily indicative of international capital immobility.

#### CHAPTER 3 - NOTES

- More precisely, these studies document a significantly positive coefficient in least-square regressions of investment rates against savings rates.
- 2. Sachs (1981, 1983), on the other hand; finds evidence of a strong inverse cross-sectional relationship--again for a range of OECD countries--between the current-account and investment in apparent contradiction of the aforementioned findings. Penati and Dooley (1984) in an attempt to reconcile these findings suggests that Sachs's results are sensitive to outlier problems.
- 3. Exceptions to this interpretation are: Frankel (1985), who interprets his evidence as indicative of weakly integrated goods rather than financial markets; Obstfeld (1985) argues that explanations other than that of capital immobility are quite possible—such as labour-force growth, country size effects, temporary productivity shocks.
- 4. An interesting topic for future research would be to investigate this issue further—i.e., to examine whether there are conditions under which a two-country model with infinitely-lived representative agents in which uncertainty stems from shocks to technology would have a unique stationary competitive equilibrium solution. Such a model would constitute an open economy extension of the neoclassical growth paradigm used by Kydland and Prescott, amongstothers (see e.g., Prescott (1986) and the references therein), to

study business cycle behaviour in a closed economy. The extended model would be useful not only in examining current account behaviour, but also international business cycle behaviour.

- 5. Note it is not entirely evident, though, that Persson's results remain robust in a stochastic setting.
- of course, the fact that uncertainty and risk aversion on the part of agents prevents the international equalization of real returns on bond investments has long been recognized in the literature dealing with foreign exchange market risk (see Obstfeld (1985) for a review of this literature). Given this, it is no surprise to find that real returns on real investments are not equalized under conditions of uncertainty and risk aversion. The point is worth emphasizing here, however, since no study (at least to the author's knowledge) models S and I behaviour in a stochastic SOE model and, as mentioned in the text, the implication is that the foreign country decision problem must also be modelled.
- 7. A version of the model incorporating domestic and foreign government bonds was also developed: It can be shown that the inclusion of such bonds is of interest only if one is concerned in addressing crowding-out issues. This version of the model is accordingly omitted from consideration here.
- 8. Recall that domestic and foreign agents are assumed to have logarithmic utility functions.

9. The regressions and correlations deal with S and I <u>rates</u> rather than  $\hat{S}$  and I <u>levels</u> since the empirical evidence referred to in the introduction pertains to rates. The S and I rates are defined by  $S_t/Y_t$  and  $I_t/Y_t$ , respectively, and note  $Y_t$  is also generated by the simulation of the model using the equation:

$$Y_t = \theta_t K_t N^{(1-\alpha)}$$

- 10. The parameter values for the probabilities describing the distribution of  $\theta$  ( $\theta^*$ ) are chosen to capture persistence or nonpersistence in the disturbances. The parameter values for  $\theta_1$ ,  $\theta_2$  and  $\theta^*$  ( $\theta_1^*$ ,  $\theta_2^*$ ,  $\theta$ ) are arbitrarily chosen for illustrative purposes. The parameter values for N and N\* are chosen to capture the assumed smallness of the domestic economy in relation to the foreign economy. The parameter values for  $\alpha$  and  $\alpha^*$  equal capital's share of output in the U.S. economy over the postwar period. Finally, the parameter values for  $\beta$  and  $\beta^*$  equal the Kydland and Prescott (1986) estimate of this coefficient (interpreted as a per-year coefficient) for the U.S. economy over the postwar period.
- from the IMSL package. The solution values for the domestic and foreign capital stocks each period are used to feed the lags on these variables, indicated in equations (17) and (18), used in next period's solution—hence, I refer to the solution as dynamic.
- 12. The steady state solution was also used to provide starting guesses for the iterative solution in each period.

# Appendix 1

# Table 1: Notation

c <sub>t</sub> (c <sub>t</sub> *):	Consumption when young at time t by the domestic (foreign) agent.
d <sub>it+1</sub> (d* <sub>it+1</sub> ):	Consumption when old at time t+l in state i by the . domestic (foreign) agent.
w <sub>t</sub> (w*):	Wage payment received when young at time t by the domestic (foreign) agent.
$k_{t+1}^{d}(k_{t+1}^{*d}):$	Desired holdings of domestic (foreign) capital at the beginning of time t+1 by the domestic agent young at time t.
k <sub>t+1</sub> (k <sub>t+1</sub> ):	Desired holdings of domestic (foreign) capital at the beginning of time t+1 by the foreign agent young at time t.
1 <sup>d</sup> (t)[1 <sup>f</sup> (t)]:	Number of loans made by the young domestic (foreign) agent at time t, each of which guarantee the delivery of one real unit in time t+1 if state i occurs.
<b>q</b> <sub>i</sub> (t):	Price at time t in terms of time-t goods of a consumption loan whose payoff occurs in state i.
$\overline{K}_{t+1}(\overline{K}_{t+1}^*):$	Aggregate domestic (foreign) capital stock-at the beginning of time t+1.
θ <sub>it</sub> (θ*):	Domestic (foreign) technology parameter in the case where uncertainty stems from stockastic variations in domestic technology. The former is time-varying depending on realizations of $\theta_i$ ( $i > 1, 2$ ).
θ*(θ):	Foreign (domestic) technology parameter in the case where uncertainty stems from stochastic variations in foreign technology. The former is time-varying depending on realizations of $\theta_i^*$ (i = 1, 2).
θ <sub>1</sub> , θ <sub>2</sub> : .	Comestic technology parameter in state i, i = 1, 2.
θ, θ.:	Foreign technology parameter in state i, i = 1, 2.
r <sub>it+1</sub> (r* <sub>t+1</sub> ):	Net real rate of return from investment in domestic (foreign) production between time t and time t+1 in the case where uncertainty stems from stochastic variations in domestic technology. The former depends on the state of nature at time t+1, identified with the realization

# Table 1 (continued)

s <sub>t</sub> .(s <sub>t</sub> *):	Saving when young at itime t by the domestic (foreign) agent.
I <sub>t</sub> :	Aggregate net investment during time t in domestic production.
s <sub>t</sub> :	Aggregate net saving during time t by the domestic country.
Υ <sub>t</sub> (Υ*):	Aggregate domestic (foreign) output at time t.
β(β*):	Domestic (foreign) agent's subjective rate of discount.
` <b>⊕</b> j∶	Probability of state i occurring next period given state j occurred this period (i,j = 1, 2), where states are identified with realizations of $\theta_i$ (i = 1, 2).
f*;: .	Probability of state i occurring next period given state j occurred this period (i,j = 1, 2), where states are identified with realizations of $\theta_i^*$ (i = 1, 2).
α(α*):.	Domestic (foreign) capital share of domestic (foreign) output.
N(N*): '	Size of domestic (foreign) population.

#### CHAPTER I

# FORECASTING THE EXCHANGE RATE: A MONETARY OR RANDOM WALK PHENOMENON?

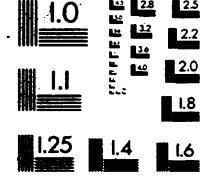
This paper evaluates the forecasting accuracy of monetary and random walk models of the exchange rate. Instrumental-variable estimates of the 'simple' monetary model are not supported by the data, while the full-information-maximum-likelihood estimates of its rational-expectations counterpart are. The latter is found to forecast as well as the random walk model. The rational-expectations monetary model is operationalized using the results of Hansen and Sargent (1980) and Flavin (1981) and Box-Jenkins time series techniques. Monthly data on the US and UK economies over the recent flexible exchange rate period serve as the case study.

The purpose of this paper is to undertake an empirical investigation of our ability to forecast the exchange rate. More precisely, the paper seeks to address the question as to whether future values of the exchange rate can be predicted more accurately on the basis of a monetary model of exchange rate determination or on that of a random walk model.

The motivation for the paper derives from two recent contributions to the literature: one, Meese and Rogoff (1983), analyzed the comparative forecasting accuracy of alternative time-series models and the following structural models of the exchange rate: the simple flexible-price and sticky-price monetary models and a sticky-price model which

# Of/de







$$\alpha = \alpha^* = 0.36$$
  $\beta = \beta^* = 0.96$   $\gamma = 100$   $\gamma = 15$   $\gamma = 15$   $\gamma = 10$   $\gamma = 15$   $\gamma = 10$ 

# Least-Squares Estimate of the Domestic Technology Process:

$$\theta_t = 4.97 + 0.50 \frac{\theta}{16.30} + 0.50 \frac{\theta}{18.10} + 0.50 \frac{\theta}{16.30}$$

$$R^2 = 0.25 \qquad SSR = 18.789.65 \qquad DW = 2.06$$

# Estimates of the Domestic S-I Relationship:

# (i) Least Squares:\_\_\_\_

$$\frac{I_{t}}{Y_{t}} = 0.01 + 0.76 \frac{S_{t}}{Y_{t}}$$

$$R^{2} = 0.93 \qquad SSR = 1.8616 \qquad DW = 2.62$$

# (ii) Maximum, Likelihood (Correcting for Autocorrelation):

$$\frac{I_{t}}{Y_{t}} = 0.002 + 0.69 \frac{S_{t}}{Y_{t}}$$

$$R^{2} = 0.94 \qquad SSR = 1.5025 \qquad DW = 2.01$$

$$\phi_{1} = -0.60 (18.70) \qquad \phi_{2} = -0.28 (7.59) \qquad \phi_{3} = -0.12 (3.35)$$

$$\phi_{4} = -0.05 (1.57)$$

# Correlation Between $I_t/Y_t$ and $S_t/Y_t$ : 0.97

- Notes: (1) tavalues are reported in parentheses
  - (2) R<sup>2</sup> denotes the coefficient of determination SSR denotes the sum-of-squared residuals DW denotes the Durbin-Watson statistic
    - $\phi_i$  denotes the autocorrelation coefficient of order i
  - (3) The number of observations used is one thousand

Table 3: The Case of a Negatively Autocorrelated Domestic Technology Process

$$\alpha = \alpha^* = 0.36$$
 $N = 100$ 
 $\theta_1 = 15$ 
 $\theta_2 = 5$ 
 $\theta_3 = 6^* = 0.96$ 
 $\theta_4 = 10$ 
 $\theta_5 = 10$ 
 $\theta_7 = 10$ 
 $\theta_8 = 10$ 
 $\theta_8 = 10$ 
 $\theta_8 = 10$ 

# Least-Squares Estimate of the Domestic Technology Process:

$$e_t = 14.75 - 0.47 e_{t-1}$$
  
 $R^2 = 0.22$  SSR = 19,422.39 DW = 1.95

# Estimates of the Domestic S-I Relationship:

# (i) Least Squares:

$$\frac{I_t}{Y_t} = 0.01 - \frac{0.62}{(170.21)} \cdot \frac{S_t}{Y_t}$$

$$R^2 = 0.97 \qquad SSR = 3.1893 \qquad DW = 2.48$$

(ii) Maximum Likelihood (Correcting for Autocorrelation):

$$\frac{I_{t}}{Y_{t}} = 0.01 - \frac{0.62}{(170.91)} \frac{S_{t}}{Y_{t}}$$

$$R^{2} = 0.97 \qquad SSR = 2.7079 \qquad DW = 2.0$$

$$\phi_{1} = -0.31 \quad (9.48) \quad \phi_{2} = -0.33 \quad (9.98) \quad \phi_{3} = -0.04 \quad (1.14) \quad \phi_{4} = -0.16 \quad (4.49)$$

$$\phi_{5} = -0.05 \quad (1.38) \quad \phi_{6} = -0.13 \quad (3.60) \quad \phi_{7} = -0.04 \quad (1.31) \quad \phi_{8} = -0.07 \quad (2.25)$$

Correlation Between  $I_t/Y_t$  and  $S_t/Y_t$ : -0.98

Notes: (1) tavalues are reported in parentheses

- (2) R<sup>2</sup> denotes the coefficient of determination
  SSR denotes the sum-of-squared residuals
  DW denotes the Durbin-Watson statistic
  of denotes the autocorrelation coefficient of order i
- (3) The number of observations used is one thousand

Table 4: The Case of a Serially Uncorrelated Domestic Technology

# Least-Squares Estimate of the Domestic Technology Process:

$$\theta_t = 9.76 + 0.003 \theta_{t-1}$$
  
 $R^2 = 0.00$  SSR = 24,932.67 DW = 2.0

# Estimates of the Domestic S-I Relationship:

# (i) Least Squares:

$$\frac{I_{t}}{Y_{t}} = -0.004 + 0.16 \frac{S_{t}}{Y_{t}}$$

$$R^{2} = 0.90 \qquad SSR = 0.25 \qquad DW = 1.79$$

(ii) Maximum Likelihood (Correcting for Autocorrelation):

$$\begin{split} \frac{I}{Y_t} &= -0.01 + 0.15 \frac{S_t}{Y_t} \\ R^2 &= 0.95 \\ \phi_1 &= 0.23(7.25) \\ \phi_4 &= -0.26(7.09) \end{split} \qquad \begin{aligned} &SSR &= 0.196 \\ \phi_2 &= -0.56(16.93) \\ \phi_5 &= -0.001(0.04) \end{aligned} \qquad \begin{aligned} &DW &= 2.02 \\ \phi_3 &= 0.10(2.71) \\ \phi_6 &= -0.11(3.55) \end{aligned}$$

# Correlation Between $I_{t}/Y_{t}$ and $S_{t}/Y_{t}$ : 0.95

Notes:

 (1) t-values are reported in parentheses
 (2) R<sup>2</sup> denotes the coefficient of determination SSR denotes the sum-of-squared residuals DW denotes the Durbin-Watson statistic

φ<sub>7</sub> denotes the autocorrelation coefficient of order i

(3) The number of observations used is one thousand

Table 5: The Case of a Positively Autocorrelated Foreign Technology Process

# Parameter Values:

$$\alpha = \alpha^* = 0.36$$
  
 $N = 100$   
 $\theta_1^* = 15$ 

θ **= 10** 

<u>Least-Squares Estimate of the Foreign Technology Process:</u>
(same as that of the domestic technology process in Table 2);

$$\theta_{t}^{*} = 4.97 + 0.50 \quad \theta_{t-1}^{*}$$

$$R^2 = 0.25$$

DW = 2.06

# <u>Least-Squares Estimate of the Domestic S-I Relationship:</u>

$$\frac{I_{t}}{Y_{t}} = 0.0002 + 0.11 \frac{S_{t}}{Y_{t}}$$

$$(1.28) (12.97) \frac{Y_{t}}{Y_{t}}$$

$$R^2 = 0.15$$

$$SSR = 0.0234$$

DW = 1.69

Correlation Between  $I_t/Y_t$  and  $S_t/Y_t$ : 0.38

- Notes: (1) tovalues are reported in parentheses
  - (2) R<sup>2</sup> denotes the coefficient of determination SSR denotes the sum-of-squared residuals DW denotes the Durbin-Watson statistic
  - (3) The number of observations used is one thousand

Table 6: The Case of a Negatively Autocorrelated Foreign Technology Process

$$\alpha = \alpha^* = 0.36$$
 $N = 100$ 
 $\theta^*_1 = 15$ 
 $\theta^*_2 = 0.25$ 
 $\beta = \beta^* = 0.96$ 
 $\beta = \beta^*_3 = 0.96$ 
 $\beta = \beta^*_4 = 0.96$ 
 $\delta = 0.96$ 

<u>Least-Squares Estimate of the Foreign Technology Process:</u>
(same as that of the domestic technology process in Table 3)

$$\theta_{t}^{*} = 14.75 - 0.47 \quad \theta_{t-1}^{*}$$

$$R^{2} = 0.22 \qquad SSR = 19.422.39 \qquad DW = 1.95$$

# Estimates of the Domestic S-I Relationship:

# (i) Least Squares:

$$\frac{I_t}{Y_t} = 0.004 - \frac{0.21}{(31.47)} \frac{S_t}{Y_t}$$

$$R^2 = 0.50$$

$$SSR = 1.2888$$

$$DW = 2.48$$

(ii) Maximum Likelihodd (Correcting for Autocorrelation):

$$\frac{I_{t}}{Y_{t}} = 0.01 - 0.14 \frac{S_{t}}{Y_{t}}$$

$$R^{2} = 0.22$$

$$\phi_{1} = -0.43 (4.48)$$

$$S_{t}$$

$$SSR = 1.1739$$

$$DW, = 2.22$$

Correlation Between  $I_c/Y_t$  and  $S_t/Y_t$ : -0.71

Notes: '(1)°t\_values are reported in parentheses

(2) R<sup>2</sup> denotes the coefficient of determination
SSR denotes the sum-of-squared residuals
DW denotes the Durbin-Watson statistic

\$\phi\_1\$ denotes the autocorrelation coefficient of order is

(3) The number of observations used is one thousand

# Table 7: The Case of a Serially Uncorrelated Foreign Technology Process

# Parameter Values:

$$\alpha = \alpha^* = 0.36$$
 $N = 100$ 
 $\theta^* = 15$ 
 $f^* = f^* = 0.5$ 

$$\beta = \beta^* = 0.96$$
 $N^* = 5,000$ 
 $\theta^* = 5$ 

$$-f_{11}^* = f_{22}^* = 0.5$$

<u>Least-Squares Estimate of the Foreign Technology Process:</u>
(same as that of the domestic technology process in Table 4)

$$\theta_{t}^{*} = 9.76 + 0.003 \quad \theta_{t-1}^{*}$$
(28.06) (0.102)

# Least-Squares Estimate of the Domestic S-I Relationship:

$$\frac{I_t}{Y_t} = 0.002 - 0.08 \frac{S_t}{Y_t}$$

$$R^2 = 0.09$$
 ,

$$SSR = 0.3581$$

# Correlation Between $I_{+}/Y_{+}$ and $S_{+}/Y_{+}$ : -0.30

(1) tavalues are reported in parentheses

- (2) R<sup>2</sup> denotes the coefficient of determination SSR denotes the sum-of-squared residuals DW denotes the Durbin-Watson statistic
- (3) The number of observations used is one thousand

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