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Price And Output Fluctuations In An Economy With A Limited Capital Market

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PRICE AND OUTPUT FLUCTUATIONS IN AN ECONOMY WITH A LIMITED CAPITAL MARKET

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ABSTRACT

This thesis sets out a rational-expectations model of an economy in which firms are constrained to finance their advances to labor and their purchases of commodity inputs by borrowing from a domestic banking system, which in turn constitutes the entire financial system of the economy. The major implication of this "financial constraint" is that the supply of output comes to depend positively on the real value of the monetary base.

In the closed economy version of the model, a fully anticipated increase in the rate of growth of the monetary base or an anticipated temporary decrease in the level of the monetary base reduces the equilibrium level of output and affects the real wage rate and the real interest rate. Thus, money is not superneutral and the Fisher effect does not hold. In addition, permanent monetary shocks mistakenly viewed as temporary also have real effects.

The open economy version of the model distinguishes between tradable and nontradable goods. In this context, it is shown that the effect of changes in the exogenous variables on the levels of output can be decomposed into a "financial constraint effect," and a "relative price effect."

Under flexible exchange rates, the financial constraint effect always dominates the relative price effect. Hence, any monetary change that results in a decrease of output in the closed economy case, also results in a decrease of the levels of output of both goods in the flexible exchange rate case. In addition, a permanent monetary decrease

mistakenly viewed as temporary causes the exchange rate to "overshoot" relative to its full current information level.

Under fixed exchange rates, the output effects of an anticipated change in government debt, or in the price level of the tradable good or of a devaluation, depend on the importance of the financial constraint effect as compared to relative price effect. If the financial constraint effect is strong enough, an anticipated rise in the level of the government debt will increase the output of both commodities while an anticipated rise in the price level of the tradable good or an anticipated devaluation will reduce them. Finally, neither unanticipated changes in the price level of the tradable good nor an unanticipated devaluation affect the current level of output of either commodity. Both might lead to a contraction of output of both commodities in subsequent periods, however.

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Assumptions (1) and (2) together, imply that firms face the following "financial constraint:" they need to finance their advances by demanding credit solely from the banking sector. Thus, the availability of real credit places a constraint on the level of output. Since the financial structure of this economy consists of only a banking sector, the total supply of money in the economy will coincide with the total supply of credit (to the government and to firms).

3. All transactions are carried out with money in this economy. Just as firms need to finance their production process with money obtained from the banking system, households need money in advance to purchase commodities. Since households cannot engage in borrowing or lending activities (assumption 1), their consumption demand will be limited by receipts from their sales of labor services (advances from the firms), by their previous savings, and by net current transfers from the Government. That is, the transactions role of money is emphasized in this thesis, and money enters the model in the same way as in the models of Clower (1967), McKinnon (1973), Kapur (1976a), Mathieson (1979), Lucas (1980), Stockman (1980, 1981), and Helpman (1981).

The motivation to model an economy with the features described above comes from the seminal works of McKinnon (1973) and Shaw (1973) and later formalizations and extensions by Galbis (1977, 1979a, 1979b), Fry (1978, 1981), Kapur (1976a, 1976b) and Mathieson (1979, 1980). The McKinnon and Shaw hypothesis is that in less developed countries (LDCs) financial restrictions imposed by the domestic governments (what they call "financial repression") have resulted in a low level of savings,

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CHAPTER I

INTRODUCTION

The purpose of this thesis is to specify and analyze a model of price and output fluctuations for an economy characterized as follows:

1. The economy has a severely limited capital market. The chief limitations are the absence of a market for primary securities together with an absence of any type of financial intermediation other than commercial banking.

The assumption of a limited capital market will be taken to imply that:

(a) The government will not be able to borrow from the non-banking sector by selling bonds or bills (open market operations are ruled out) and hence, the government deficit will be financed entirely by the central bank.

(b) The only source of finance for firms is loans from domestic commercial banks. Sales of equities or bonds to domestic households or foreigners are ruled out.

(c) Imperfections in the capital market prevent households from obtaining bank loans. In particular, it will be assumed that information costs about the credit worthiness of individual households are so high that banks do not engage in making loans to households. Households will hold only one kind of deposits: noninterest bearing demand deposits.

2. Firms need to pay in advance for inputs of labor and commodities (working capital) that will be used in the production process.

Assumptions (1) and (2) together, imply that firms face the following "financial constraint:" they need to finance their advances by demanding credit solely from the banking sector. Thus, the availability of real credit places a constraint on the level of output. Since the financial structure of this economy consists of only a banking sector, the total supply of money in the economy will coincide with the total supply of credit (to the government and to firms).

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This thesis takes from McKinnon the assumption that firms need to finance working capital in advance, but relaxes the assumption of self finance by introducing a banking sector. The papers by Galvis, Fry, Mathieson, and Kapur follow this route, but the main contribution of this thesis is to derive a macromodel from the decision rules followed by rational agents who maximize utility or profits subject to the constraints imposed by the assumptions stated above. The thesis adds to previous work in the area in two ways: first, it derives, rather than impose the aggregate demand and supply functions that forms the relevant macro model, and second, the model is solved in a general equilibrium-rational expectations stochastic framework, rather than in the disequilibrium-adaptive expectations framework that has been used in previous work.

The remainder of the thesis is organized as follows:

Chapter II sets out and solves the model in a closed economy set up. The macroeconomic model is entirely derived from the maximizing behavior or rational economic agents (banks, firms, and households). Careful attention is paid to the organization of trade and payments that is consistent with the assumptions regarding the financial structure of the economy. To analyze the intertemporal maximizing behavior of households, the existence of overlapping generations at each moment is assumed (as in Samuelson (1958)). This framework is chosen for purposes of simplicity and because (as is shown in chapter II) it is consistent with the restrictions that are assumed in the capital market. The resulting macroeconomic model is then solved for the price and output levels as well as for the real interest rate and the real wage rate. The effects of temporary and permanent changes in the level of the monetary base, as well as variations in its rate of growth, are analyzed under two alternative assumptions regarding the information sets on which agents base their decisions.

It is shown that, as a result of the "financial constraint" assumption, and in contrast with models where money enters as a wealth variable, the supply of output depends positively on the real monetary base. In this context, a fully anticipated increase in the rate of growth of the monetary base or a fully anticipated temporary decrease in the monetary base causes a reduction in the level of output through the negative effects of expected inflation on the demand for money. Moreover, under the assumption that agents cannot distinguish between temporary and permanent monetary disturbances, even permanent shocks have real effects.

Chapter III extends the model of Chapter II to allow for international transactions in commodities. While zero capital mobility is assumed, the economy produces and consumes two kinds of goods: tradable and nontradable commodities. This chapter derives the optimal decision rules and the macroeconomic model relevant for an open economy with a limited capital market of the type already described.

Chapter IV solves the macroeconomic model set up in Chapter III for the flexible exchange rate case and Chapter V solves the macroeconomic model for the fixed exchange rate case. In both cases, the effects of changes in exogenous monetary variables and in the price level of the tradable good are analyzed. In conducting this analysis, emphasis is given to the role of the "financial constraint" assumption in obtaining the model's results. In addition, these results are contrasted with those obtained from alternative models that do not impose restrictions on the capital market.

In particular, in the flexible exchange rate case, it is shown that the monetary changes that result in a decrease of output in the closed economy also result in decreases of the output levels of <u>both</u> commodities in the open economy. This result holds in spite of the fact that such monetary variations in the open economy might also cause a relative price effect favoring an increase in the supply of one commodity and a decrease in the supply of the other. That is, the financial constraint <u>dominates</u> the relative price effect.

Also, flexible exchange rates insulate the domestic economy from variations in the world price level of the tradable good. This result holds because the exchange rate and the price level of the tradable good always enter together in the supply and demand functions of the model and in the alternative information sets assumed to solve the model. It is emphasized that the "insulation" effect does not depend on the degree of capital mobility or on assumptions regarding purchasing power parity. In fact, the maintenance of purchasing power parity following a monetary or a real shock is shown to depend on the relative elasticities of demand for both commodities.

In the fixed exchange rate case, the model is compared with the Dornbusch (1973) tradables-nontradables model. It is shown that the financial constraint assumption is the key element in yielding results different from those of the Dornbusch model. Specifically, while in the Dornbusch model, changes in the exogenous component of the money supply or in the price level of the tradable good results in a short-run expansion of the supply of one commodity and a reduction in the supply of the other, in the model developed in this thesis, the output's response critically depends on the importance of the financial constraint effect relative to the relative price effect.

Finally, Chapter VI provides a brief summary of the results obtained in this thesis.

CHAPTER II

A CLOSED ECONOMY MODEL WITH LIMITED CAPITAL MARKETS

1. Introduction

This chapter concentrates on a closed economy case in order to examine the effects on the behavior of prices and output of the assumption that the economy faces a severely limited capital market in the sense described in Chapter I. The assumption that firms must finance their advances to workers or their purchases of commodity inputs by using the banking sector as the only source of finance will be termed the "financial constraint." This constraint will give rise to an aggregate supply of output function that depends positively on the real amount of money of the economy. This feature contrasts with most other models in which real money is included as an argument in the short-run aggregate supply of output, because real money is usually assumed to affect the level of real output negatively. For instance, Barro (1976) introduces real money as a wealth variable that impinges positively on desired leisure and hence negatively on the supply of labor and on the supply of output. In the model of this thesis, the demand for credit by firms is completely motivated by the firm's need to finance their purchases of inputs of production; hence, the availability of real credit will constitute a constraint on the firms' production level. Since the financial structure of this economy consists only of a banking system, the total supply of money in the economy (the monetary base held by the public plus demand deposits) equals the total supply of credit (to the government and to firms). This equality is the origin of the positive dependence of output on real money.

It follows, of course, that if a rise in inflation induces a decrease in the real quantity of money (and credit), output and inflation will be negatively correlated. For related reasons, models in which the relevant constraint in the acquisition of factors of production is a strict cash-in-advance constraint (instead of the credit-in-advance constraint used in this chapter), have also found this negative correlation between output and inflation but as a steady-state result. Stockman (1981), for example, constructs a model in which the rate of inflation in a closed economy is inversely related to the steady-state capital stock. However, he obtained this result at the expense of accepting a constant and unitary income velocity of circulation of money. As it will be shown below, the model of this chapter will derive a velocity of money function that, like the level of output, is sensitive to variations in the inflation rate.

This chapter is organized as follows: Section 2 describes the basic characteristics of the economy, the organization of markets, and the timing of transactions. Section 3 derives the optimal decision rules (demand and supply equations) implied by the maximizing behavior of economic agents.

Section 4 obtains the solution for the price and output levels, for the nominal (and real) interest rate, and for the real wage rate, and discusses the implications for the economy of changes in the monetary base, and the economy's response to real shocks. The analysis is conducted in a rational expectations framework and assumes two alternative sets of information. In addition, changes in both the level and the rate of growth of the monetary base are analyzed separately and changes in the level of the monetary base are decomposed into temporary and permanent components. Section 5 extends the model to include investment and compares its implications with the model of the previous sections. Finally, Section 6 presents a brief summary of the results.

2. The model: Essentials

Consider an economy where all transactions are carried out with money. These transactions involve three markets: those for output, labor, and credit. The economy is inhabited by four kind of agents: firms, households, banks, and government.

Current output consists of a single good produced by firms with a common technology and purchased by households. In Sections 2 to 4, it will be assumed that output is produced using one variable factor of production, labor services provided by households. This assumption will be relaxed in Section 5, where in addition to labor, the production of output will be assumed to require a commodity input purchased by the firm at the end of the previous period. All other inputs are fixed in quantity, have no alternative use, and zero user cost.

The credit market involves the banking sector as the only supplier of credit and firms as the only agents demanding it. Firms demand credit from the commercial banks to finance their wage bill (and their purchases of investment goods in Section 5) and these bank loans constitute the only source of finance for firms. Specifically, firms are not allowed to finance production from undistributed profit since all profits are distributed to households, nor are they allowed to finance production through the sale of equities or bonds to households.

In addition, it will be assumed that information costs about the credit worthiness of individual households are so high that banks do not make loans to households. That is, imperfections in the capital market prevent households from obtaining banking loans. Households hold only noninterest bearing demand deposits.

Money is the only means of payment, the unit of account, and the only (tradable) store of value for households. As will be shown below, these assumptions about household behavior and the organization of trade and payments, prevent equities being a tradable stock. The transactions role of money is emphasized in this model. Households' purchases of goods or firms' purchases of inputs must necessarily be paid for by money obtained in advance of the purchases.

Firms maximize profits. They supply output and demand labor and credit in a competitive environment, such that every one takes as given the output price, the wage rate, and the interest rate. As stated above, in order to buy labor, firms are forced to borrow from the banking system to pay labor in advance. Profits are completely distributed to households who are the owners of the firms.

Households maximize expected utility over their lifetime. Each unit acts as a price taker with respect to the price level and the wage rate. We assume the existence of overlapping generations at each moment, as in Samuelson (1958). Each generation lives for two periods. When young, households supply labor and demand commodities and money. Money is held to buy commodities at the end of this first period or to be carried over into the second period as a store of value with which to pay for consumption during retirement. When old, households receive profits from the firms and commercial banks (they own both kind of businesses) and consume these and their previous savings. They hold money <u>during</u> the second period, but not at the <u>end</u> of it.

It is assumed that, in the initial period of the life of this economy, an old generation owned all relevant property rights. Since every individual is assumed to maximize utility over only his own lifetime, it would have been on the best interest of the old generation to sell these property rights at the beginning of this period, because they were to die at the end of it. However, in this economy, there is no demand for property rights at the beginning of any period. Other old people obviously do not want to buy them, and young people are not able to buy them. They begin their life with no money and are unable to borrow. Thus, there is no market for property rights and the only alternative left to the old generation is to pass them over to the next generation before dying. Hence, inheritances of property rights are consistent with the existence of overlapping generations and the timing of transactions assumed in this model. Inheritances of money, however, are ruled out; and, hence, it is in the best interest of the old generation to spend all their money balances during the period.

It is assumed for the sake of simplicity that households' desired currency-deposit ratio is equal to zero. That is, households hold all their money in the form of demand deposits, the only kind of deposits offered by the banking system. The old generation pays lump-sum taxes and receives lump-sum transfers from the government.

The banking system completely characterizes the financial sector of the economy. It is made up of a central bank and commercial banks. The central bank has two functions: to finance the government's net transfers through the issue of monetary base (high powered money) and to impose a reserve ratio on the commercial banks. The reserve ratio is the only "purely monetary" tool to control the money supply directly available to the monetary authorities. The central bank does not lend to the private sector either directly or indirectly through advances to the commercial banks.

Commercial banks hold reserves, supply credit to firms, and as we have said issue only one kind of deposit: demand deposits that do not pay interest. It is assumed that banks incur no labor costs in providing the services of intermediation. The limit to the nominal size of the banking system is given by the reserve ratio. Since the banks' holdings of required reserves do not yield interest payments, the net income generated by the banking process is equal to the interest payments on loans. These payments are distributed to the owners of the banks (households) as soon as they are realized. Also, it is assumed that bank loans have a maturity length of one period. That is, they have to be paid back (principal plus interest payments) after one period has elapsed. Finally note that, since households do not hold currency, all of the monetary base issued by the central bank is held as reserves by the commercial banks.

The government collects taxes and provides transfers to households, and finances all its debt with base money issued by the central bank. Interest bearing government bonds are assumed not to exist in this model.

When analyzing the behavior of private economic units: firms, banks, and households, the "representative unit" of each kind will be considered; that is, a unit that conforms to the behavior of the average of all identical atomistic units. This assumption allows aggregation problems to be evaded and eliminates the need explicitly to deal with the transition from microeconomic behavior to macroeconomic functions. 1/

In our economy, time is divided into discrete uniform intervals. A sequential trading arrangement 2/ in which labor and credit markets open and clear at the beginning of the period and the commodity market at the end, is assumed. 3/ The sequencing of market activity coincides with the sequencing of decisions made by economic agents. At the beginning of the period, firms decide on their demand for labor and credit, and households decide on their supply of labor. At the end of the period, household decide on their demand for consumption goods and money. 4/

To gain insight into the nature of the model, let us follow the activities of the economy during any given period t. To ease the exposition, the notation of the model is summarized in Appendix II.1, to which the reader will find it helpful to refer from time to time.

At the beginning of period t, firms distribute profits realized at the end of period t-1 to households, pay back past loans to the commercial banks, demand labor from households (the young generation), and demand loans from the banking system. Households (the old generation) receive profits from firms and from the banking system equal to interest payments on past period loans. Finally, the old generation receive a lump-sum transfer from the government and pay lump-sum taxes, and any increase in the monetary base arising from these activities is also assumed to occur at the beginning of the period. It will be equal to net governmental transfers (Tr_t). That is:

$$(2.1) \quad GC_t \equiv H_t \equiv R_t$$

and:

$$\Delta GC_t = GC_t - GC_{t-1} = Tr_t$$

where: H_t = monetary base

R_t = level of banks' reserves

GC_t = government's outstanding debt.

Given the level of banks' reserves, the supply of credit by the commercial banks is also determined at the beginning of the period. Once the credit market and the labor market have cleared, production begins and no further transactions occur until the end of the period when the commodity market opens. Then, the young generation decides on its current and future consumption plan, and thus, its demand for money (given that money is the only store of value). Both generations demand commodities and the equilibrium price of output is therefore determined.

The young generation carry over money from the end of the period to the beginning of the next period in the form of savings. 5/ Firms also carry over money in an amount equal to the value of the total output (P_tY_t). This will be used at the beginning of the next period to pay back banking loans and distribute profits. 6/ The old generation dies between periods and it is assumed that it has depleted its money balances on consumption before dying. At the start of t+1, a new generation is born and the whole process just described begins again.

Notice that this particular sequence of events highlights the importance of money as a medium of exchange because only money serves as a means of effective demand. Household first accumulate money (received from firms, banks, and the government) and then purchase consumption goods. Firms also need money to pay labor services in advance.

Table 2.1 sets out the sequence of household, firm, and commercial bank decisions described above. The balance sheet of the commercial banks summarizes the activities of the economy at the beginning and end of the period. Notice that firms do not hold money during the period. They only hold money at the end of every period and then completely deplete their holdings at the beginning of the next period. On the other hand, the young generation holds money both during the period and between periods, while the old generation depletes its holdings at the end of every period. Figure 1 shows the time profile of holdings of money by firms and households in this economy.

3. The maximizing behavior of the representative bank, firm, and households

Agents are rational, and maximizing behavior implies that they use all the available information relevant for their decisions. However, they do not have perfect foresight and, hence, have incomplete information about the relevant variables. Specifically, neither the representative household nor the representative firm observe the current period price of output when labor market decisions are carried out. Also, future periods' prices are not known when the household sets its current period demands for commodities and money. Hence, the household is forced to form expectations about these variables.

3.1 The bank

The objective of the bank is to maximize profits. As we have already said, the bank does not make loans to households; moreover, demand deposits are noninterest bearing but bank loans to firms are interest bearing. These assumptions imply that the representative bank's maximization problem is:

$$(2.2) \quad \underset{\{C_t^s\}}{\text{Max}} \frac{{}^b\pi_t}{{}_{at}E(P_{t+1})} = \frac{i_t}{{}_{at}E(P_{t+1})} C_t^s - 0$$

where: ${}^b\pi_t$ = nominal profits realized by the bank at the beginning of period t.

${}_{at}E(P_{t+1})$ = beginning of period expectations of the period t+1 price level. In general, the subscript "at" refers to "beginning of period t"

C_t^s = nominal amount of credit (loans) supplied by the representative bank

i_t = nominal interest rate

The bank maximizes "expected real profits" because it belongs to households who will only receive the bank's profits at the beginning of the subsequent period. The commercial bank faces the following set of constraints:

(2.3) $R_t \geq K\, D_t$

(2.4) $R_t + C_t^s = D_t$

(2.5) $R_t \equiv RR_t + RE_t$

(2.6) $R_t = H_t$

where: R_t = reserves in the vault of the commercial banks

D_t = level of demand deposits

K = reserve requirement ratio

RR_t = required reserves

RE_t = excess reserves

Equation (2.3) is the reserve requirement constraint imposed by the central bank. Equation (2.4) is the bank's balance sheet constraint, which requires that the total assets of the bank equal its total liabilities. Identity (2.5) defines the composition of the level of reserves and equation (2.6) follows from the assumption that the nonbanking sector holds its money only in the form of demand deposits.

The maximization procedure will result in the following Kuhn-Tucker conditions:

(2.7). $\dfrac{i_t}{a_t E(P_{t+1})} - K\lambda \leq 0$

(2.8) $R_t(1-K) - KC_t^s \geq 0$

(2.9) $[\dfrac{i_t}{a_t E(P_{t+1})} - K\lambda]\ C_t^s = 0$

(2.10) $[R_t(1-K) - KC_t^s]\lambda = 0$

Given zero costs, profit maximization implies a positive supply of credit; hence, according to equation (2.9), λ (the Kuhn-Tucker multiplier) is equal to:

$$\lambda = \frac{i_t}{K \ {}_{at}E(P_{t+1})}$$

As $\lambda \neq 0$ (because $i_t \neq 0$), equation (2.10) can be used to obtain the commercial bank's decision rule to supply credit:

$$(2.11) \quad C_t^s = \frac{1-K}{K} R_t$$

or using equation (2.6)'

$$(2.12) \quad C_t^s = \frac{1-K}{K} H_t$$

Equation (2.12) gives the commercial bank's supply of credit. It implies that the bank finds it optimal to supply as much credit as possible. The bound is, of course, given by the reserve requirement ratio and hence the optimal decision rule for the bank is to hold zero excess reserves. This result follows obviously from the assumption that the bank pays no interest on demand deposits, but it will also and equally obviously hold as long as the interest payments on deposits is lower than the interest charged on credit.

3.2 The firm

The objective of the firm is to maximize profits. In this section, commodities are assumed to be nonstorable. Hence, the firm does not hold inventories and profit maximization is a one period problem.

Profits (${}^{f}\pi_t$) equal the total revenue made by the firm minus total cost. The firm hires labor services as the only input that must be purchased and borrows bank credit to finance the purchase. Thus, profit is equal to:

$$(2.13) \quad {}^{f}\pi_t = P_t\, Y_t^s - W_t\, L_t^d - i_t\, C_t^d$$

where: P_t = price level of output

Y^s = output supplied by the firm

W_t = nominal wage rate

L_t^d = the firm's demand for labor

C_t^d = firm's demand for nominal bank loans.

Due to the assumption that the firm's profits realized at the end of period t are distributed to households at the beginning of period t+1, the firm's optimal behavior consists in maximizing the expected value (E) of real profits (deflated by the price level that will prevail in period t+1). Hence, its problem becomes:

$$(2.14) \quad \underset{L_t^d}{\text{Max}}\ {}_{a t}E\left[\frac{{}^{f}\pi_t}{P_{t+1}}\right] = \underset{L_t^d}{\text{Max}}\ {}_{a t}E\left\{\frac{P_t}{P_{t+1}}\, Y_t^s - \frac{W_t}{P_{t+1}}\, L_t^d - \frac{i_t}{P_{t+1}}\, C_t^d\right\}$$

In addition to the information set (which will be specified below), the maximization procedure is limited by two constraints:

$$(2.15)\quad Y_t^s = F(L_t^d)\,\phi_t \qquad \text{where } F'(L) > 0;\; F''(L) < 0$$

and

$$(2.16)\quad C_t^d = W_t L_t^d$$

Equation (2.15) is a stochastic production function where F exhibits positive and diminishing marginal product of labor. The production process is subject to a random term ϕ_t. The sequence $\{\phi_t\}$ is assumed to be a strictly positive stationary stochastic process. The distribution of ϕ_t will be specified in Section 4.

Equation (2.16) is the financial constraint on the firm. Given that the only reason for a firm to demand loans is to finance labor, this constraint has to be satisfied as an equality every period.

The information set that the firm uses to form its expectations (Ω_{at}) consists of the information on variables up to period t-1, plus the current wage rate, the interest rate, and the productivity term. (The information set at the beginning of the period also includes the random term μ_t which affects the household's utility function, and will be considered below). 7/ Thus:

$$(2.17)\quad \Omega_{at} = \Omega_{t-1},\, W_t,\, i_t,\, \phi_t,\, \mu_t$$

By substituting equations (2.15) and (2.16) into equation (2.14), equation (2.14) can be rewritten as:

$$(2.14'')\quad \underset{L_t^d}{\text{Max:}}\ [{}_{at}E\,(\frac{1}{1+\psi_t})]\,[F(L_t^d)\,\phi_t] - [{}_{at}E\,\frac{W_t}{P_t}\frac{(1+i_t)}{(1+\psi_t)}]$$

where: $1 + \psi_t = (P_{t+1}/P_t)$

The maximization procedure leads to the demands for labor and credit.

$$(2.18) \quad L_t^d = \rho\left[(1 + i_t)\ \frac{W_t}{{}_{at}EP_t}\ \frac{1}{\phi_t}\right]$$

where: ρ is a linear function relating labor to its marginal productivity, and $\rho' < 0$

$$(2.19) \quad C_t^d = (W_t)\ \rho\left[(1 + i_t)\ \frac{W_t}{{}_{at}EP_t}\ \frac{1}{\phi_t}\right]$$

where: $\delta C_t^d/\delta i_t < 0$

$$\delta C_t^d/W_t \lessgtr 0 \qquad \text{as } \frac{\rho}{\rho'} \lessgtr \frac{(1+i_t)}{{}_{at}EP_t}\ \frac{W_t}{\phi_t}$$

Notice that the demands for both labor and credit depend on the "effective real cost of labor:" $\dfrac{(1+i_t)W_t}{{}_{at}EP_t}$

Equations (2.18) and (2.19) should only be taken as approximations because:

$${}_{at}E\ [P_{t+1}/P_t] \neq [{}_{at}EP_{t+1}/{}_{at}EP_t]$$

3.3 Households

The objective of households is to maximize expected utility. The representative individual of the old generation obtains utility from consumption, but attaches no utility to holding money. Thus, he will supply his total money holdings in exchange for commodities at the end of the period. 8/ The budget constraint for the representative agent of the old generation is:

$$(2.20) \quad P_t \, {}^oY_t^d = {}^oD_{t-1} + Tr_t + {}^h\pi_t$$

where:

${}^oY_t^d$ = demand for output by the representative agent of the old generation.

${}^oD_{t-1}$ = demand deposits carried from period t-1 = savings of the representative member of the young generation during period t-1 $({}^yD_{t-1}^d)$

and

$${}^h\pi_t = {}^f\pi_{t-1} + i_{t-1} \, C_{t-1}$$

That is, the representative old individual starts the period with ${}^oD_{t-1}$ and at the beginning of the period receives net transfers (Tr_t) from the government and profits from the firm (${}^f\pi_{t-1}$) and the bank ($i_{t-1} \, C_{t-1}$). 9/

In contrast, the representative young individual faces a meaningful decision problem: he has to choose his supply of labor at the beginning of the period and his demand for commodities (current and future) at the end of the period. Because money serves as a store of value, the utility maximization problem faced by the young individual is an intertemporal one. To analyze his decision problem, let us use the subscript 1 to indicate the first period of his life and 2 to indicate the second and last period.

Consumption commodities and leisure are assumed to yield positive utility. Thus, utility depends inversely on time devoted to work. The utility function is assumed to be time separable and the young agent attempts to maximize utility over his planning horizon. That is, he tries to maximize the expected value of his lifetime utility function:

$$(2.21)\quad \text{Max}_{a1} E\left\{U(Y_1^d, L_1^s) + \beta V(Y_2^d)\right\} \qquad 0 < \beta < 1$$

where: Y_1^d and L_1^s refer to his demand for consumption goods and supply of labor respectively for the first period, and Y_2^d refers to his consumption demand for the second period.

The utility for period 1 is given by U and that for period 2 is V. β is the discount factor.

Before further discussing the properties of this utility function, the two budget constraints that the young individual faces need to be specified. In the first period, he works, consumes, and saves money. Thus:

$$(2.22)\quad P_1 Y_1^d + {}^yD_1^d = W_1 L_1^s$$

In his second period, he is old so his stochastic budget constraint is:

$$(2.23)\quad P_2 Y_2^d = {}^oD_1 + Tr_2 + {}^h\pi_2$$

where ${}^oD_1$ (stock of money held by the old individual from period 1) is equal to the individual's demand for money in period 1 = ${}^yD_1^d$.

Equations (2.22) and (2.23) lead to the lifetime budget constraint:

$$(2.24)\quad Y_2^d = \frac{W_1}{P_1}\frac{P_1}{P_2} L_1^s + \frac{Tr_2}{P_2} + \frac{{}^h\pi_2}{P_2} - \frac{P_1}{P_2} Y_1^d$$

The problem for the young agent is to maximize equation (2.21) subject to two constraints: the money exhaustion condition of equation (2.24) and the condition that his demand for money be non-negative. This last condition is represented by the following inequality constraint:

$$(2.25) \quad W_1 L_1^s - P_1 Y_1^d > 0$$

Given that the young agent starts life with no money, equation (2.25) implies that his saving cannot be negative (see footnote 5).

The agent's utility function is assumed to be:

$$(2.26) \quad [q\, Y_1^d - \frac{1}{2}(Y_1^d)^2]\, \mu_1 - j L_1^s - \frac{1}{2}(L_1^s)^2 + \beta\, Y_2^d$$

where q and j are positive constants and μ_1 is a random term affecting the utility derived from current consumption. That is, an increase in μ_1 represents a change in perferences that assigns a higher weight to current consumption relative to both current leisure and future consumption. As in the case of the shock affecting the production function, the sequence $\{\mu_t\}$ assumed to be a stationary stochastic process, which will be specified in Section 4.

This quadratic utility function 10/ is additively separable in consumption and labor, shows diminishing marginal utility of current consumption, negative and diminishing marginal utility of labor and positive and constant marginal utility of future consumption (assumed equal to one in this case). 11/ 12/

The young agent maximizes the expected value of equation (2.26) subject to the budget constraint (2.24) and to the inequality constraint (2.25). He faces a two-stage maximization problem: at the beginning of the period, he chooses his labor supply, taking into account the available and relevant information at his disposal at that moment. At the end of the period, the representative agent decides on his demand for commodities and money, taking into account the information available at that time. To illustrate the agent's maximization procedure, let us start at the end of the period, when he already knows his labor supply. Substituting equation (2.24) into equation (2.26) the agent problem becomes:

$$(2.27)\quad \underset{\{Y_1^d,\ \lambda'\}}{\text{Max}}\ {}_{b1}E\{(qY_1^d - \frac{1}{2}(Y_1^d)^2)\ \mu_1 - j\ L_1^s - \frac{1}{2}(L_1^s)^2 + \beta\,(\frac{W_1}{P_1}\frac{P_1}{P_2}L_1^s + \frac{Tr_2}{P_2} + \frac{{}^h\pi_2}{P_2} - \frac{P_1}{P_2}Y_1^d)\} + \lambda'\,(\frac{W_1}{P_1}L_1^s - Y_1^d)$$

subject to: $Y_1^d \geq 0$, $\lambda' \geq 0$

where λ' is the Kuhn-Tucker multiplier associated with the constraint (2.25).

The information set available to the agent at the end of period 1 is:

$$(2.28)\quad \Omega_{b1} = \Omega_{a1},\ P_1$$

where, Ω_{a1}, the information set at the beginning of period 1 is: 13/

$$(2.29)\quad \Omega_{a1} = \Omega_0,\ W_1,\ i_1,\ \phi_1,\ \mu_1$$

The maximization procedure leads to the following first order requirements (the Kuhn-Tucker conditions): 14/

$$(2.30) \quad (q - Y_1^d)\,\mu_1 - \beta \;{}_{b1}E\,(\frac{P_1}{P_2}) - \lambda' \leq 0$$

$$(2.31) \quad Y_1^d \,\{(q - Y_1^d)\,\mu_1 - \beta \;{}_{b1}E\,(\frac{P_1}{P_2}) - \lambda'\} = 0$$

$$(2.32) \quad \frac{W_1}{P_1} L_1^s - Y_1^d \geq 0$$

$$(2.33) \quad \lambda' \,\{\frac{W_1}{P_1} L_1^s - Y_1^d\} = 0$$

To allow consumption to be positive every period, the following ad-hoc condition is imposed:

$$q - \frac{\beta}{\mu_1} \;{}_{b1}E(P_1/P_2) > 0$$

Then, equation (2.30) holds as an equality to satisfy (2.31). Hence:

$$(2.30') \quad (q - Y_1^d)\,\mu_1 = \beta \;{}_{b1}E\,(\frac{P_1}{P_2}) + \lambda'$$

Equation (2.30') states that the marginal utility of current consumption is equal to its marginal cost, which in turn is the marginal (indirect) utility obtained by the young agent from holding an additional unit of real money. 15/

There are two possibilities: $\lambda' = 0$ or $\lambda' > 0$. If $\lambda' = 0$, the agent's demand for money will be positive and the decision rule for consumption demand will be:

$$(2.34) \quad Y_1^d = q - \frac{\beta}{\mu_1} \frac{P_1}{{}_{b1}EP_2}$$

At the end of period 1, P_1 is observed and the agent has only to form expectations on P_2. Equation (2.34) should be taken as an approximation 16/ and states that the demand for current commodities depends positively on expected inflation, 17/ implying that current consumption is a gross substitute for future consumption. The decision rule (2.34) also depends positively on the random term affecting utility.

If, on the other hand, $\lambda' > 0$, the consumer's demand for commodities would be:

$$(2.35) \quad Y_1^d = \frac{W_1}{P_1} L_1^s$$

In this case, all the money balances would be spent on commodities and the agent would demand no money at the end of the period. In this situation, the ratio of period 1 marginal utility of consumption to the discounted value of period 2 marginal utility is greater than the ratio of actual to expected future price levels, implying that the individual's desire to consume more of the current good faces a real money balances constraint.

An intuitive explanation of our results follow: the specification of the utility function (2.26) eliminates the income effect from the demand of current commodities. In such circumstances, the ratio of actual to expected future prices will determine the consumer's allocation of income between consumption and savings. However, suppose that the expected price level for period 2 relative to the price level in period 1 is high enough that the individual wishes to consume more than his level of income; this is not possible due to the inequality constraint (2.25);

thus, in such a case, his income will be fully spent and the consumer will not save. Notice that at the aggregate level the actual price level will adjust so that equilibrium in the commodity market (the agents' consumption demand is satisfied) will prevail.

For our purposes, we will assume that at every moment, $D_t^d > 0$, implying that $\lambda' = 0$ and that equation (2.34) is the relevant decision rule.

Now, let us consider the young agent's maximization problem at the beginning of the period. At that time, the individual's demand for the commodity is a stochastic variable. Let us define:

$$(2.36) \quad \tilde{Y}_1^d = q - \frac{\beta}{\mu_1} \frac{P_1}{{}_{b1}EP_2}$$

The agent's problem will now be:

$$(2.37) \quad \underset{\{L_1^s\}}{\text{Max}} \; [q \; {}_{a1}E \; \tilde{Y}_1^d - \frac{1}{2} ({}_{a1}E \; \tilde{Y}_1^d)^2] \; \mu_1 - j \; L_1^s - \frac{1}{2} (L_1^s)^2$$

$$+ \beta \; {}_{a1}E \; [\frac{W_1}{P_1} \frac{P_1}{P_2} L_1^s + \frac{Tr_2}{P_2} + \frac{{}^h\pi_2}{P_2} - \frac{P_1}{P_2} \tilde{Y}_1^d]$$

Notice that the inequality constraint does not apply at the beginning of the period, because the demands for consumption and money are made effective at the end of the period. The maximization procedure leads to the labor supply decision rule:

$$(2.38) \quad L_1^s = \beta \frac{W_1}{{}_{a1}EP_1} \frac{{}_{a1}EP_1}{{}_{a1}EP_2} - j$$

Both P_1 and P_2 are stochastic variables at the beginning of period 1. This equation (which is also an approximation) states that the labor supply depends positively on the expected real wage 18/ and negatively on expected inflation. This last feature implies that current leisure is a gross substitute for future consumption.

Before turning to the derivation of the demand for money; it is important to point out that the quadratic utility function assumed in equation (2.26) was chosen only for convenience. Another utility function that will give rise to similar consumption and labor supply decision rules is the following: 19/

$$(2.39) \quad U = (\ln Y_1)\mu_1 + j \ln(\overline{L} - L_1) + \beta Y_2$$

where: ln refers to natural logarithms and $(\overline{L} - L_1)$ is the level of leisure available to households after they have supplied labor. In fact, when dealing with the open economy case in Chapter III, equation (2.39) will prove to be a simplier and more convenient formulation.

Turning now to the demand for money, it can be obtained from equation (2.22) (the budget constraint in period 1). At the end of period 1, the labor supply decision has already been made; so substituting equation (2.34) into equation (2.22):

$$(2.40) \quad \frac{y_{D1}^{d}}{P_1} = \frac{W_1}{P_1} L_1 - q + \frac{\beta P_1}{\mu_1 b_1 E P_2}$$

Thus, the household demand for the real stock of money depends positively on real wage income and negatively on the expected inflation rate.

To get the total economy wide demand for consumption at the end of period t (the demands of the representative agents of both generations), equation (2.20) and (2.34) have to be added up. Thus:

$$(2.41)\quad Z_t = q - \frac{\beta}{\mu_t}\frac{P_t}{b_t EP_{t+1}} + \frac{{}^{o}D_{t-1}}{P_t} + \frac{Tr_t}{P_t} + \frac{{}^{h}\pi_t}{P_t}$$

where:

$$Z_t = {}^{y}Y_t^d + {}^{o}Y_t^d$$

The aggregate consumption function (Z_t), which in this model is also equal to the aggregate demand function, depends positively on expected inflation, on the stock of money held by the old generation at the beginning of the period, on a random term affecting utility, and on the real personal disposable income (${}^{o}Q_t$) received by the old generation:

$${}^{o}Q_t = \frac{{}^{h}\pi_t}{P_t} + \frac{{}^{o}Tr_t}{P_t}$$

Notice that another interpretation of Z_t is that it depends on the stock of money (with a unit coefficient) accumulated by the old generation and held by the firms at the end of the previous period. This is so because in this model all transactins involve the use of money.

4. Price and output equilibrium in the macro model

4.1 The aggregate supply function

The model presented in Section 3 plus the equilibrium conditions that both the labor and credit market clear at the beginning of the period ($L_t^d = L_t^s$ and $C_t^d = C_t^s$), will now be used to derive the economy's aggregate supply function. In order to achieve this, the model will be cast in log-linear terms. Lower case letters will be used to represent the log of the variable (with the exception of i_t, which will stand for the observed natural value of the interest rate). 20/ Based on equations (2.18), (2.38), (2.19), and (2.12) the following log-linear relationships are postulated:

$$(2.42) \quad l_t^d = \alpha_0 - \alpha_1(w_t - {}_{at}p_t^e + i_t - u_t)$$

$$(2.43) \quad l_t^s = \gamma_0 + \gamma_1(w_t - {}_{at}p_t^e) - \gamma_2({}_{at}p_{t+1}^e - {}_{at}p_t^e)$$

$$(2.44) \quad c_t^d = \beta_0 + \beta_1({}_{at}p_t^e - i_t + u_t) + (1 - \beta_1)w_t$$

$$(2.45) \quad c_t^s = (k + h_t)$$

where: $k = \log(1 - K)/K$; $\alpha_0, \alpha_1, \beta_0, \beta_1, \gamma_0, \gamma_1 > 0$

and the expectation operator E has been replaced by the superscript "e".

Notice that the theoretical model presented in Section 3 would imply the following restrictions on the parameters:

$$\alpha_1 = \beta_1$$
$$\gamma_1 = \gamma_2$$

These restrictions could be tested empirically. Also, note that equations (2.42) through (2.45) are log-linear approximations; because, for any variable x: $\log(Ex) \neq E(\log x)$.

The stochastic process governing the productivity shock ϕ_t should now be specified. It is assumed that ϕ_t is a serially independent distributed log normal random disturbance, such that:

$$u_t = \log \phi_t \sim N(0, \sigma_u^2)$$

Also, u_t is independent of other disturbances in the model.

Equations (2.42) through (2.45) and the market equilibrium conditions can now be used to solve for the equilibrium level of employment, the nominal wage rate, and the nominal interest rate as functions of the monetary base, expectations about prices and the random term affecting productivity.

To gain insight into the workings of the model, most of the steps involved in its solution will be presented, in addition to a detailed discussion of the results.

Equations (2.44) and (2.45) and the equilibrium condition: $c_t^d = c_t^s$ produce:

$$(2.46) \quad i_t = \frac{\beta_0 - k}{\beta_1} - \frac{1}{\beta_1} h_t + {}_{at}p_t^e + u_t + \left(\frac{1-\beta_1}{\beta_1}\right) w_t$$

Equations (2.42) and (2.43) and the equilibrium condition: $l_t^d = l_t^s$ produce:

$$(2.47) \quad w_t = \frac{\alpha_0 - \gamma_0}{\alpha_1 + \gamma_1} + {}_{at}p_t^e - \frac{\alpha_1}{\alpha_1 + \gamma_1} i_t + \frac{\alpha_1}{\alpha_1 + \gamma_1} u_t + \frac{\gamma_2}{\alpha_1 + \gamma_1} \left({}_{at}p_{t+1}^e - {}_{at}p_t^e\right)$$

The impact (or first round) effect of a change in ${}_{at}p_t^e$ on the wage rate and on the interest rate can now be analyzed. In fact, when ${}_{at}p_t^e$ increases, both the nominal wage rate and the interest rate rise in the same proportion. The explanation of this result is straightforward. In equation (2.46), the wage rate is treated as exogenous. Since the demand for credit depends on the "effective real cost of labor" $(w_t + i_t - {}_{at}p_t^e)$, and the supply of credit is exogenous, an increase in the current expected price level produces a proportionate increase in i_t in order to clear the credit market. Correspondingly, in equation (2.47), the interest rate is treated as exogenous. Since the demand for labor depends on $(w_t + i_t - {}_{at}p_t^e)$ and the supply of labor depends on $(w_t - {}_{at}p_t^e)$, an increase in the expected price level produces a proportionate increase in w_t in order to clear the labor market.

Substituting equation (2.47) into equation (2.46), the solution for the interest rate can be obtained:

$$(2.48) \quad i_t = \gamma_0' - \gamma_1' (h_t - {}_{at}p_t^e) + \gamma_2' ({}_{at}p_{t+1}^e - {}_{at}p_t^e) + u_t$$

where:

$$\gamma_0' = \frac{\gamma_1 \beta_0 + \alpha_1 \beta_0 - \gamma_0 + \gamma_0 \beta_1 + \alpha_0 - \alpha_0 \beta_1}{\alpha_1 + \gamma_1 \beta_1} - \gamma_1' k$$

$$\gamma_1' = \frac{\gamma_1 + \alpha_1}{\alpha_1 + \gamma_1 \beta_1}$$

$$\gamma_2' = \frac{(1-\beta_1)\, \gamma_2}{\alpha_1 + \gamma_1 \beta_1}$$

Several results follow from equation (2.48).

First, the nominal interest rate depends negatively on the expected <u>real</u> monetary base. To understand this result, equation (2.46) may be rewritten as:

$$(2.46') \quad k + h_t = \beta_0 + \beta_1 ({}_{at}p_t^e - i_t - w_t + u_t) + w_t$$

Consider a proportionate increase in both ${}_{a t}p_t^e$ and h_t. As stated above, when the expected current price level increases, the impact effect on the labor market is to increase the nominal wage proportionally, implying that the real wage does not change and the demand for labor remains constant. Since $c_t^d = w_t + l_t^d$, the effect of a rise in ${}_{a t}p_t^e$ on the demand for credit is to increase it proportionally. In addition, since the monetary base is exogenous, increase in its level will result, obviously enough, in the supply of credit to increase proportionally.

Thus, since an increase in both the monetary base and the expected current price level increase both the demand and supply of credit in the same proportion, the interest rate will remain unchanged. It follows then that for the interest rate to change, the real monetary base has to change.

Second, the expected inflation rate affects the nominal interest rate with a coefficient different from one, or to put the same point in another way, the expected inflation rate affects the real interest rate. This can be seen by rewriting equation (2.48) as:

$$(2.48') \quad i_t - ({}_{a t}p_{t+1}^e - {}_{a t}p_t^e) = \{\gamma_0' - \gamma_1' (h_t - {}_{a t}p_t^e) - (1- \gamma_2')({}_{a t}p_{t+1}^e - {}_{a t}p_t^e) + u_t\}$$

The pure Fisher effect does not hold in this model. This occurs both because of the assumption that firms need to finance labor in advance of actual sales of output, and of the assumption of imperfect capital markets (households are not allowed to engage in borrowing and lending activities).

Taken together, these assumptions (which constitute the "financial constraint" assumption) imply that the economy wide demand for credit is solely formed by the firms' demand for credit. This in turn is derived from the demand for labor and hence depends on the behavior of the wage rate. If β_1 were set equal to one (the demand for credit would be independent of the labor market) and if households were allowed to borrow (the demand for credit would depend on the real interest rate) the Fisher effect would hold.

In this model, the transmission mechanism through which the expected inflation rate affects the interest rate is as follows: when expected inflation increases, labor supply decreases (because of a substitution effect in favor of current leisure). To maintain equilibrium in the labor market, the nominal wage increases. The increase in the wage rate in turn affects the nominal demand for credit (it will increase if $\beta_1 < 1$) and hence affects the nominal interest rate. The sign of γ_2' is ambiguous, but if the sensitivity of the demand for credit to the interest rate (that is, β_1) is less than one, γ_2' will be positive.

Third, an exogenous increase in productivity will have as its only effect a proportional increase in the interest rate, because when u_t increases, its impact effect is an increase in the demands for both labor and credit. Given an exogenously determined supply of credit, the rise in the demand for credit will generate an increase in the interest rate in the same proportion as the increase in u_t. The resulting increase in the interest rate will in turn reduce the demand for labor (due to increased effective cost of labor) offsetting the initial impact of the productivity change on the demand for labor. Hence, the level of employment and the wage rate will remain unchanged. Notice that this result holds only because the total supply of credit is used to buy labor services. As will be shown in Section 5, the introduction of investment demand also financed with credit, will allow u_t to have positive effects on employment.

We can now turn to the determination of the wage rate. Substituting equation (2.46) in equation (2.47) we get:

$$(2.49)\quad w_t = {}_{at}P_t^e + \beta_0' + \beta_1' \left({}_{at}P_{t+1}^e - {}_{at}P_t^e\right) + \beta_2' \left(h_t - {}_{at}P_t^e\right),$$

where: $\beta_0' = \dfrac{\beta_1 \alpha_0 - \beta_1 \gamma_0 - \alpha_1 \beta_0 + \alpha_1 k}{\gamma_1 \beta_1 + \alpha_1}$

$$\beta_1' = \frac{\beta_1 \gamma_2}{\gamma_1 \beta_1 + \alpha_1}$$

$$\beta_2' = \frac{\alpha_1}{\gamma_1 \beta_1 + \alpha_1}$$

The results obtained from equation (2.49) are as follows:

First, an increase in the real monetary base will increase the real wage. This occurs because, as explained above, an increase in the real monetary base results in a decrease in the nominal interest rate; this, in turn, lowers the "effective" cost of labor, and hence, increases the demand for labor. Given that both the demand and supply of labor depend on the real wage, an increase in the demand for labor will increase the real wage.

Second, an increase in the expected inflation rate will cause the supply of labor to fall, and hence will increase the real wage rate. The magnitude of the wage rate response will be higher, the higher the absolute value of the elasticity of the supply of labor with respect to the expected inflation rate, and the lower the absolute value of the elasticity of the demand for labor with respect to the "effective" cost of labor.

Third, as explained above, a change in productivity does not affect the wage rate.

Next, the equilibrium level of employment can be obtained by substituting equations (2.48) and (2.49) into the demand for labor (equation (2.42)):

$$(2.50)\quad l_t = \alpha_0' + \alpha_1' (h_t - {}_{at}p_t^e) - \alpha_2' ({}_{at}p_{t+1}^e - {}_{at}p_t^e)$$

where:

$$\alpha_0' = \frac{\gamma_0 \alpha_1 + \gamma_1 \alpha_0 \beta_1 - \gamma_1 \beta_0 \alpha_1}{\gamma_1 \beta_1 + \alpha_1} + \alpha_1' k$$

$$\alpha_1' = \frac{\alpha_1 \gamma_1}{\gamma_1 \beta_1 + \alpha_1}$$

$$\alpha_2' = \frac{\alpha_1 \gamma_2}{\gamma_1 \beta_1 + \alpha_1}$$

The level of employment depends positively on the real monetary base, negatively on the expected inflation rate and is not affected by a change in productivity. These effects occur through the induced changes in the "effective" cost of labor (a change in w_t and/or i_t) already discussed above.

Substituting equation (2.50) into a logarithmic approximation to the output supply equation (a log approximation of equation (2.15)), the short-run aggregate supply function is obtained:

$$(2.51)\quad y_t^s = \bar{a}_0 + a_1(h_t - {}_{at}p_t^e) - a_2({}_{at}p_{t+1}^e - {}_{at}p_t^e) + u_t$$

Notice that although the random term affecting productivity does not affect the level of employment, it does affect the level of output because every unit of labor will now be more productive, and hence output will increase. Output supply does not depend on the actual price level, but on its expectation. Obviously, this happens because the output supply decision is made at the beginning of the period, before the commodity market has opened.

An important feature of the model is that the term $(h_t - {}_{at}p_t^e)$ is not a wealth variable, which affects desired leisure positively and hence the supply of output negatively as in Barro (1976). Rather its presence is due to the financial constraint faced by the firm in an environment of limited capital markets. Appendix II.2 shows that if we relax either of the components of the "financial constraint" assumption (that is, the requirement that firm's finance labor in advance, or the limited capital market postulate), the real monetary base term does not appear in the aggregate supply of output function. Equation (2.51) is an implication of the financial constraint assumption. The positive response of aggregate supply to an increase in the real monetary base will be bigger, the higher the absolute value of the elasticity of the supply of labor with respect to the real wage, and the higher the elasticity of the demand for labor with respect to its "effective" cost.

The negative response of aggregate supply to an increase in expected inflation will be bigger the higher the absolute value of both, the labor supply elasticity with respect to expected inflation and the labor demand elasticity with respect to the "effective" cost of labor. These results are suggested by our previous analysis. 21/

4.2 The aggregate demand function

Because of Walras' law, equilibrium in the commodity market at the end of the period implies equilibrium between the demand and supply of money. The demand for money will be used to represent aggregate demand.

At the end of the period, the money supply (D^s_t) is equal to the sum of the monetary base plus the commercial banks' loans ($D^s_t = H_t + C_t$). In equilibrium, firms' holdings of money equals the value of output, which according to equations (2.13) and (2.16) is:

$$(2.52) \quad {}^fD_t = P_tY_t = C_t\,(1+i_t) + {}^f\pi_t$$

Hence, the households' holdings of money at the end of the period equal:

$$(2.53) \quad {}^hD_t = D^s_t - {}^fD_t = H_t - i_tC_t - {}^f\pi_t$$

On the other hand, the households' demand for money is given by equation (2.40).

Equating equation (2.53) and (2.40) and using (2.52) will lead to:

$$(2.54) \quad \frac{H_t}{P_t} = Y_t - q + \frac{\beta\, P_t}{\mu_t\,({}_{bt}EP_{t+1})}$$

Equation (2.54) represents equilibrium in the money market. A log linear approximation of equation (2.54) is:

$$(2.54') \quad h_t - p_t = b_0 + b_1 y_t - b_2({}_{bt}p^e_{t+1} - p_t) - \epsilon_t$$

where ϵ_t is a serially independent distributed log normal disturbance which is also independent of the rest of disturbances of the model, and where:

$$\epsilon_t = \log \mu_t \sim N(0, \sigma^2_\epsilon)$$

4.3 Expectations and price and output equilibrium

The equations for the supply of output, the wage rate, and the interest rate derived in the previous section were not final solutions because price expectations were not treated as endogenous variables. To find the final solution for the price level, output, interest rate, and the real wage rate it will be assumed that expectations are formed rationally in the sense of Muth (1961). The relevant information sets were defined in Section 3 and will now be used.

The model can be completed by specifying the process that generates the monetary base. It is assumed that this process involves a constant trend growth rate, m, in addition to stochastic elements that make the growth rate fluctuate around m. Stochastic shocks will be of two kinds: permanent shocks; that is, shocks that cause a permanent adjustment in the level of the monetary base, or "temporary" shocks that cause only a temporary adjustment in that level.

Hence, h_t is generated in accordance with: 22/

$$(2.55) \quad h_t = h_{t-1} + m + v_t + x_t - x_{t-1}$$

The two random terms v and x are generated by white noise processes, that is:

$$v_t \sim N(0, \sigma_v^2)$$
$$x_t \sim N(0, \sigma_x^2)$$

and both v_t and x_t are serially independently distributed.

Two experiments will now be conducted. First, it will be assumed that the agents posess full current information, that is, they know (or have enough information to infer) all current values of the relevant variables that affect their decisions. In particular, not only do they know the value of the monetary base, but they can also distinguish between a permanent and a temporary shock. In that context, the effect of the anticipated components of the monetary base on the supply of output will be analyzed. Second, it will be assumed that the agent's information set is the one defined in Section 3: that is, even though economic agents can infer the value of h_t (through the observation of the nominal interest rate and the wage rate), they cannot distinguish between a current permanent or temporary monetary shock. The effect of such "lack of complete information" on the price level, the output level, the interest rate, and the real wage rate will be analyzed.

a. The full current information case

In this case, the current expectation of the price level is equal to its actual value. Equations (2.51) and (2.54') and the equilibrium condition $y_t^d = y_t^s$ will yield: 23/

$$(2.56)\quad p_t = \frac{1}{1 + b_2 - b_1a_1 + b_1a_2}\{(-b_0 - a_0b_1) + (1-b_1a_1)\, h_t + (b_2 + b_1a_2)\, {}_{t}p^e_{t+1} - (b_1)\, u_t + \varepsilon_t\}$$

The price level depends positively on the expected inflation rate and on the random term affecting the marginal utility of current consumption because an increase in either variable lowers the real demand for money. In addition, the price level depends negatively on the productivity term because this increases the aggregate supply of output. The monetary base afffects the price level positively only if $b_1a_1 < 1$; such a restriction will be assumed to hold in the rest of this chapter. A comment at this stage may be useful. The need to impose the restriction $b_1a_1 < 1$ arises because the output supply depends positively on the real monetary base to reflect the financial constraint faced by the firms in an environment of limited capital markets. Thus, when $(h_t - p_t)$ increases both the demand and supply of output increases and the price level will increase only if the response of the demand for output $(1/b_1)$ is greater than the response of the supply of output (a_1).

It is interesting to note that if the term $(h_t - p_t)$ stood for a wealth variable, the coefficient a_1 would be negative and the above restriction would not have been necessary. Hence, it is important to recall that the reasons for the presence of the real monetary base in the aggregate supply function may differ between models and generate very different results. The importance of the microfoundations supporting a macro model is, in this way, highlighted.

Using the method of undetermined coefficients (see Lucas (1972), Barro (1976, 1978)), it is conjectured that the solution for the full current information price level is:

$$(2.57)\quad p_t = \theta_0 + \theta_1 h_{t-1} + \theta_2 m + \theta_3 v_t + \theta_4 x_t + \theta_5 x_{t-1} + \theta_6 u_t + \theta_7 \varepsilon_t$$

where the θs are the unknown coefficients (Appendix II.3 shows that, for stability purposes, the forward solution implied by the use of the method of undetermined coefficients is, in fact, the solution to be chosen).

Updating equation (2.57) and taking expectations:

$$(2.58)\quad {}_{a t}p^{e}_{t+1} = \theta_0 + \theta_1 h_{t-1} + (\theta_1 + \theta_2) m + \theta_1 v_t + (\theta_1 + \theta_5) x_t - \theta_1 x_{t-1}$$

The solutions for the θs coefficients in terms of the parameters of the model are obtained by substituting equation (2.58) into equation (2.56) and by equating the resulting coefficients with those of equation (2.57):

$$(2.59)\quad \theta_0 = (-b_0 - a_0 b_1)/(1 - b_1 a_1)$$

$$(2.60)\quad \theta_1 = \theta_3 = -\theta_5 = 1$$

$$(2.61)\quad \theta_2 = (1 - b_1 a_1 + b_2 + b_1 a_2)/(1 - b_1 a_1)$$

$$(2.62)\quad \theta_4 = (1 - b_1 a_1)/(1 + b_2 - b_1 a_1 + b_1 a_2)$$

$$(2.63)\quad \theta_6 = (-b_1)/(1 + b_2 - b_1 a_1 + b_1 a_2)$$

$$(2.64) \quad \theta_7 = (1)/(1 + b_2 - b_1a_1 + b_1a_2)$$

Thus, the solution for the full curent information price level is:

$$(2.65) \quad p_t = \left[\frac{-b_0 - a_0b_1}{1-b_1a_1}\right] + h_{t-1} + \left[\frac{1 - b_1a_1 + b_2 + b_1a_2}{1 - b_1a_1}\right] m$$

$$+ v_t + \left[\frac{1 - b_1a_1}{1 + b_2 - b_1a_1 + b_1a_2}\right] x_t - x_{t-1}$$

$$- \left[\frac{b_1}{1 + b_2 - b_1a_1 + b_1a_2}\right] u_t + \left[\frac{1}{1 + b_2 - b_1a_1 + b_1a_2}\right] \varepsilon_t$$

The solution for the full current information output (y_t) is obtained by substituting equations (2.57) and (2.58) into the supply of output equation (equation (2.51)).

$$(2.66) \quad y_t = \left[\frac{a_0 + a_1b_0}{1 - b_1a_1}\right] - \left[\frac{a_2 + a_1b_2}{1-b_1a_1}\right] m + \left[\frac{a_2 + a_1b_2}{1 + b_2 - b_1a_1 + b_1a_2}\right] x_t$$

$$+ \left[\frac{1 + b_2}{1 + b_2 - b_1a_1 + b_1a_2}\right] u_t + \left[\frac{a_2 - a_1}{1 + b_2 - b_1a_1 + b_1a_2}\right] \varepsilon_t$$

Several results follow from equations (2.65) and (2.66). 24/

First, both the past value of the monetary base (h_{t-1}) and the permanent monetary shock (v_t) affect the price level with a unitary coefficient and do not affect the level of real output. This is because both variables increase the current <u>and</u> future price levels in the same proportion, leaving unchanged the expected inflation rate and the real monetary base. Hence, fully perceived permanent monetary changes are neutral.

Second, an increase in the rate of growth of the monetary base, m, even if fully anticipated, has a negative effect on real output. This result holds because when m rises, the expected inflation rate rises, generating a reduction in equilibrium real monetary balances and hence a decrease in the equilibrium levels of output and employment. Hence, money is not superneutral in this model. Even fully anticipated movements in the rate of growth of the money supply have negative effects on output. This result is due to the financial constraint faced by firms operating in a limited capital market.

Third, a temporary monetary change, even if fully anticipated, has a positive effect on real output (notice that the coefficient of x_t on the price equation is less than one, while the corresponding coefficient on the output equation is positive). The temporary shock is assumed to have a one period effect. This assumption implies that the current price level p_t will increase relative to ${}_{at}p^e_{t+1}$. That is, ceteris paribus, an increase in the monetary base due to a temporary monetary increase will result in expected deflation. The consequences will be opposite to those generated by an increase in the rate of growth of the monetary base. 25/ Expected deflation will increase the equilibrium level of real balances and, hence, real output will rise.

These results show that, even in the absence of long-term contracts (as in Taylor (1979)) or wage and price rigidities (as in Fischer (1977)), fully anticipated monetary changes can have real effects in a rational expectations framework. In this chapter, the imposition of a "financial" constraint faced by firms in an economy with limited capital markets, results in a fully anticipated temporary monetary change or a fully anticipated change in the rate of growth of the money supply, affecting the level of real output.

b. Incomplete current information

In this case, although the economic agents can infer, every period, the value of the current monetary base (observing the wage rate and the nominal interest rate), they cannot perceive the division of monetary shocks between permanent and temporary components. For simplicity, it will be assumed that there is one-period-lagged information on this division; that is, x_{t-1} will be assumed to belong to the current information set. Hence, knowledge of the current value of h_t implies that the sum of both kind of current money shocks (v_t and x_t) is also known:

$$(2.67) \quad v_t + x_t = h_t - h_{t-1} + x_{t-1} - m$$

The signal extraction problem in this case consists in forming expectations of v_t and x_t based on knowledge of the right hand side of equation (2.67).

Given the assumptions regarding the stochastic process governing the behavior of x_t and v_t, the expectations of both random terms can be formed by running their regressions on the observed sum $(v_t + x_t)$.

That is: $E(v_t) = f_1 (v_t + x_t)$

$E(x_t) = g_1 (v_t + x_t)$

where f_1 and g_1 (the regression coefficients) are:

$$f_1 = \frac{\sigma_v^2}{\sigma_v^2 + \sigma_x^2} \qquad g_1 = \frac{\sigma_x^2}{\sigma_v^2 + \sigma_x^2}$$

The higher the variance of v_t, the permanent shock, the higher the value of f_1, since a larger fraction of money shocks is attributable to permanent disturbances. In the same way, a higher variance of x_t implies a higher value of g_1.

Equations (2.51) and (2.54') and the equilibrium condition $y_t^d = y_t^s$ will now yield the following expression for the incomplete current information price level ($\hat{p}$):

$$(2.68) \quad \hat{p}_t = \frac{1}{1 + b_2} \{(-b_0 - a_0 b_1) + (1 - b_1 a_1)\, h_t + (b_1 a_1 - b_1 a_2)\, {}_{at}\hat{p}_t^e + (b_1 a_2)\, {}_{at}\hat{p}_{t+1}^e + (b_2)\, {}_{bt}\hat{p}_{t+1}^e - (b_1)\, u_t + \varepsilon_t\}$$

The conjectured solution of the price level (using the method of undetermined coefficients) is:

$$(2.69) \quad \hat{p}_t = \hat{\theta}_0 + \hat{\theta}_1\, h_{t-1} + \hat{\theta}_2\, m + \hat{\theta}_3\, (v_t + x_t) + \hat{\theta}_5\, x_{t-1} + \hat{\theta}_6\, u_t + \hat{\theta}_7\, \varepsilon_t$$

Notice that now, v_t and x_t enter as a sum into the solution of the price level. This is so because agents cannot infer the values of v_t and x_t separately.

Taking expectations of $\hat{p}_t$ conditional on the information available at the beginning of the period (the information set defined in Section 3, plus the addition of x_{t-1}), yields:

$$(2.70) \quad {}_{at}\hat{p}^{e}_{t} = \hat{\theta}_0 + \hat{\theta}_1 h_{t-1} + \hat{\theta}_2 m + \hat{\theta}_3 (v_t + x_t) + \hat{\theta}_5 x_{t-1} + \hat{\theta}_6 u_t + \hat{\theta}_7 \varepsilon_t$$

Updating equation (2.70) and taking expectations conditional to the information available at the beginning and end of the period respectively, we get:

$$(2.71) \quad {}_{at}\hat{p}^{e}_{t+1} = {}_{bt}\hat{p}^{e}_{t+1} = \hat{\theta}_0 + \hat{\theta}_1 h_{t-1} + (\hat{\theta}_1 + \hat{\theta}_2) m + \hat{\theta}_1 (v_t + x_t) - \hat{\theta}_1 x_{t-1} + \hat{\theta}_5 E(x_t)$$

The expectations of $\hat{p}_{t+1}$ formed at the beginning and at the end of period t are identical. This is because knowledge of the price level at the end of the period does not convey additional information which permits agents to infer the temporary monetary component. Also, the expectation at time t of prices at any time in the future will be the same as that given by equation (2.71); this result was first noted by Muth (1960).

Substituting equations (2.70) and (2.71) into equation (2.68), and equating the resulting coefficients with those of equation (2.69), the solution for the undetermined coefficients can be obtained. The solutions for $\hat{\theta}_0$, $\hat{\theta}_1$, $\hat{\theta}_2$, $\hat{\theta}_5$, $\hat{\theta}_6$, and $\hat{\theta}_7$ are identical to the correspond-ponding θ_s derived for the full current information case. The only difference in this case is that:

$$(2.72) \quad \hat{\theta}_3 = \frac{1-b_1 a_1 - g_1(b_1 a_2 + b_2) + b_1 a_2 + b_2}{1 + b_1 a_2 + b_2 - b_1 a_1}$$

The higher the value of g_1, the lower will be $\hat{\theta}_3$, because agents will attribute most of a monetary shock to a temporary change; hence, the price level will not increase in the same proportion as the monetary shock and the level of output will be affected. In the extreme case, when $g_1 = 1$ (that is, when the variance of the permanent shock is zero), the agents will attribute all the monetary change to a temporary shock. If on the other hand $g_1 = 0$ (the temporary shock has a zero variance), agents will attribute all the monetary changes to a permanent shock. In that case, $\hat{\theta}_3 = \theta_3 = 1$; the price level will change in the same proportion as the change in the monetary base and no real effects will occur.

Substituting the $\hat{\theta}$s values in equation (2.69), the final solution for the price level is:

$$(2.73) \quad \hat{p}_t = \left[\frac{-b_0 - a_0 b_1}{1 - b_1 a_1}\right] + h_{t-1} + \left[\frac{1 + b_2 - b_1 a_1 + b_1 a_2}{1 - b_1 a_1}\right] m + \left[\frac{1 - b_1 a_1 + b_1 a_2 + b_2 - g_1(b_1 a_2 + b_2)}{1 + b_2 - b_1 a_1 + b_1 a_2}\right] (v_t + x_t) - x_{t-1} - \left[\frac{b_1}{1 + b_2 - b_1 a_1 + b_1 a_2}\right] u_t + \left[\frac{1}{1 + b_2 - b_1 a_1 + b_1 a_2}\right] \epsilon_t$$

Also, the solution for the incomplete current information level of output ($\hat{y}_t$) is:

$$(2.74)\quad \hat{y}_t = \left[\frac{a_0 + a_1 b_0}{1 - b_1 a_1}\right] - \left[\frac{a_1 b_2 + a_2}{1 - b_1 a_1}\right] m$$

$$+ \left[\frac{g_1 (a_1 b_2 + a_2)}{1 + b_2 - b_1 a_1 + b_1 a_2}\right] (x_t + v_t)$$

$$+ \left[\frac{1 + b_2}{1 + b_2 - b_1 a_1 + b_1 a_2}\right] u_t + \left[\frac{a_2 - a_1}{1 + b_2 - b_1 a_1 + b_1 a_2}\right] \varepsilon_t$$

The main difference between equation (2.74) and equation (2.66) (the output solution in the full current information case) arises because confusion between temporary and permanent shocks can generate real effects, even in response to permanent monetary shocks, and these effects depend on the relative variance of permanent and temporary shocks. This result can be highlighted by considering the difference between the incomplete and complete current information levels of output:

$$(2.75)\quad \hat{y}_t - y_t = \left[\frac{g_1 (a_1 b_2 + a_2)}{1 + b_2 - b_1 a_1 + b_1 a_2}\right] v_t$$

$$+ \left[\frac{(a_1 b_2 + a_2)(g_1 - 1)}{1 + b_2 - b_1 a_1 + b_1 a_2}\right] x_t$$

The higher the relative variance of the temporary shock (the higher g_1), the lower the effect of a temporary shock and the higher the effect of a permanent shock on the deviation of the incomplete current information output from its full (current) information level.

4.4 Solutions for the real interest rate, the real wage, and the income velocity of money

The solutions for the real interest rate and the real wage rate in the incomplete current information case are derived and presented in Appendix II.4. As with the foregoing analysis, changes in the rate of growth of the monetary base affect both the real rate of interest and the real wage even if fully anticipated. From the analysis in Section 4.3, it also follows that a fully anticipated temporary monetary change affects both variables. This violation of the Fisher effect does not occur because of misperceptions regarding the price level. It occurs because of the assumptions regarding the nature of the financial constraint faced by the firms.

The confusion between permanent and temporary shocks in the short run leads to permanent monetary shocks having effects on the real interest rate and wage rate. The lower the variance of the temporary shock, the smaller the effect of these misperceptions.

Also, although the model predicts that an increase in the rate of growth of the money supply leads to a decrease in the level of output, while (given a value of g_1) a positive random monetary shock (that cannot be distinguished as permanent or temporary) causes an increase in the short-run level of output, the signs of the effects of $\bar{m}$ or $(v_t + x_t)$ on the real wage rate and nominal interest rate are ambiguous. They depend on the sensitivity of the supply of labor with respect to the expected inflation rate relative to the sensitivity of the demand for labor with respect to its "effective" cost. For example, setting $\gamma_2 = 0$ (the supply of labor does not depend on the expected inflation rate) leads to an increase in the rate of growth of the money supply causing a decrease in the real wage rate and an increase the nominal interest rate. It also leads to a positive unanticipated random monetary shock increasing the real wage rate and decreasing the nominal interest rate. That is, if the effects of inflation on the demand for labor predominate, the real wage will move in the same direction as the output level, while the nominal interest rate will move in the opposite direction.

Finally, note that since the income velocity of money is the ratio of nominal income (output) to money, equations (2.73), (2.74), (2.55), and (2.3) can be used to obtain the following solution for velocity (vel_t) in the incomplete current information case:

$$(2.76)\quad vel_t = \left[\frac{a_0 - b_0 - a_0 b_1 + a_1 b_1}{1 - b_1 a_1}\right] + k$$

$$+ \left[\frac{b_2 + b_1 a_2 - a_1 b_2 - a_2}{1 - b_1 a_1}\right] m$$

$$+ \left[g_1 \left(\frac{a_1 b_2 + a_2 - b_2 - b_1 a_2}{1 + b_2 - b_1 a_1 + b_1 a_2}\right)\right] (v_t + x_t)$$

$$+ \left[\frac{1 + b_2 - b_1}{1 + b_2 - b_1 a_1 + b_1 a_2}\right] u_t$$

$$+ \left[\frac{1 + a_2 - a_1}{1 + b_2 - b_1 a_1 + b_1 a_2}\right] \varepsilon_t$$

The two most important results from equation (2.76) are as follows: first, velocity is not a constant in this model; it is an endogenous variable depending on both nominal and real factors. 26/ However, as is common in models where the role money as a medium of exchange is emphasized, and transactions take place once per period, velocity per period is bounded to be less or equal to one. This feature is not clear in equation (2.76), but can be obtained by noticing that, in this model, the value of output is equal to the firms' demand for money (see Section 3). Thus, velocity in natural numbers can be expressed as: $Vel = {}^{f}D_t/D$. Since the total demand for money in the economy equals the firms' demand for money plus the households' demand for money, it is clear that velocity is less or equal to one. 27/ However, the velocity of the monetary base (H) is clearly not bounded to be less or equal to one. Since (from equation (2.3)), $H = KD$ and $K < 1$, the income velocity of the monetary base (Vel_H) can be expressed as $Vel_H = {}^{f}D_t/KD$. If K is low enough, velocity can be greater than one. This result contrasts with those standard cash-in-advance models, in which currency in circulation is the relevant monetary aggregate for the trading process.

Second, a permanent monetary shock affects the level of velocity in the incomplete current information case, but leaves velocity unchanged under full current information, because a fully anticipated permanent increase in money increases the level of prices proportionally ($\theta_3 = 1$) but leaves the level of output unchanged. Moreover, changes in the rate of growth of money affect velocity even if fully anticipated. The variability of velocity is thus highly dependent on the nature of the monetary shock.

5. The model with investment demand

5.1 Microeconomic implications

In contrast with the analysis of Section 3, we now assume that the production of output requires two variable inputs of production: labor services provided by households and a commodity input purchased by the firm at the end of the previous period. There is still but one kind of output being produced (Y_t^s) and every unit is sold at the same price (P_t). However, if households purchase it, the commodity will be considered a "consumer" good; if firms purchase it, the commodity will be treated as an "investment" good. The commodity is assumed to last only one period; thus, a firm will only purchase it to use it as an input in the next period's production process. It can be used neither to increase the stock of fixed capital nor to speculate on future price movements. Every unit of the commodity will enter the output market only once. In short, we have only one kind of investment: investment in "working" capital (as opposed to fixed capital).

In what follows, we retain all assumptions specified in Sections 2 and 3 concerning households, government, and banks' behavior. Thus, to analyze the implications of introducing investment into the model, we need to concentrate only on firms' behavior. The objective of the representative firm is still to maximize profits. At the beginning of the period, the firm demands labor and uses it, together with the previous period's investment, to produce output. Therefore, the firm's effective planning horizon includes two periods, because its purchases of labor affect its revenue on the same period, while its purchases of investment affects its revenue in the next period. It is now assumed that the firm borrows bank credit at the beginning of each period to finance both its purchases of labor and its purchases of investment at the end of the period.

Thus, the firm's maximization problem is:

$$(2.77) \quad \underset{(L^d_t,\ I^d_t)}{\text{Max}} \ {}_{at}E \left\{ \frac{P_t}{P_{t+1}} Y^s_t - \left(\frac{W_t}{P_{t+1}} L^d_t + \frac{P_t}{P_{t+1}} I^d_t \right)(1+i_t) + \beta\left[Y^s_{t+1} - \left(\frac{W_{t+1}}{P_{t+1}} L^d_{t+1} + I^d_{t+1} \right)(1+i_{t+1}) \right] \right\}$$

subject to two constraints:

$$(2.78) \quad Y^s_t = F\,(L^d_t,\ I_{t-1}) = F_1\,(L^d_t)\ \phi_t + F_2\,(I_{t-1})$$

where: $(\delta F_1 / \delta L_t) > 0;\ (\delta^2 F_1 / \delta L^2_t) < 0 \qquad \forall_t$

$(\delta F_2 / \delta I_{t-1}) > 0;\ (\delta^2 F_2 / \delta I_{t-1}{}^2) < 0 \qquad \forall_t$

and:

$$(2.79) \quad C^d_t = W_t\ L^s_t + ({}_{at}EP_t)(I^d_t)$$

Here I^d_t = real investment demand at the end of period t. The same discount factor (β) utilized by the households will be used here because the firms are owned by the households.

Equation (2.78) is a production function, additively separable in L_t and I_{t-1}. The marginal productivity of labor is affected by the random term ϕ_t. The separability condition implies that the marginal productivity of each input depends solely on the amount used of that input. Equation (2.79) is the financial constraint faced by the firm. Recall that, although the firm is a competitive unit facing exogenously given prices, it does not know (at the beginning of the period) the price that will prevail in the output market at the end of the period. Hence, its nominal demand of credit to finance investment equals the expected value of investment.

The maximization procedure leads to the following firm's demand for labor and investment:

$$(2.80) \quad L_t^d = \rho_1 \left[(1-i_t)\frac{W_t}{a_t EP_t}\frac{1}{\phi_t}\right] \qquad \text{where } \rho_1' < 0$$

and ρ_1 is a linear function relating the demand for labor to its marginal productivity. And:

$$(2.81) \quad I_t^d = \rho_2 \left[(1+i_t)\frac{a_t EP_t}{\beta_{a_t} EP_{t+1}}\right] \quad \text{where } \rho_2' < 0$$

and ρ_2 is a linear function relating the demand for working capital to its marginal productivity.

The firm's demand for labor only involves current expected prices because both the marginal cost and the marginal revenue associated with an extra unit of labor hired are realized in the current period, while the firm's demand for investment will involve current (marginal cost) and future (marginal revenue) expected prices. In fact, the first order condition from which equation (2.81) is obtained states that the firm demands investment goods up to the point where the marginal cost of investment $(1+i_t)({}_{at}EP_t)$ equals the actual value of the future expected marginal revenue obtained from current investment:

$$\beta({}_{at}EP_{t+1})\left(\frac{\delta F_{2(t+1)}}{\delta K_t^d}\right)$$

Also, from the financial constraint (2.79), the firm's demand for credit is:

$$(2.82)\quad C_t^d = (W_t)\ \rho_1\left[(1+i_t)\frac{W_t}{{}_{at}EP_t}\ \frac{1}{\phi_t}\right] + ({}_{at}EP_t)\ \rho_2\left[(1+i_t)\frac{{}_{at}EP_t}{\beta_{at}EP_{t+1}}\right]$$

where:

$$\frac{\delta C_t^d}{\delta i_t} < 0$$

$$\frac{\delta C_t^d}{\delta \phi_t} \gtrless 0 \text{ as } \frac{\rho_1}{\rho_1'} \gtrless \frac{(1+i_t)W_t}{({}_{at}EP_t)\ \phi_t}$$

$$\frac{\delta C_t^d}{\delta({}_{at}EP_t)} \gtrless 0 \quad \text{as:}$$

$$\rho_2 - W_t\ (\rho_1')\ \frac{(1+i_t)}{({}_{at}EP_t)^2}\ \frac{W_t}{\phi_t} \gtrless ({}_{at}EP_t)\ \frac{\rho_2'(1+i_t)}{({}_{at}EP_{t+1})}$$

The effect of an increase of the expected current price level on the nominal demand for credit is ambiguous. Although the nominal demand for credit to finance labor increases, there are two effects on the nominal demand for credit to finance investment: the expected price of investment goes up, but the demand for real investment decreases. If the elasticity of the demand for investment with respect to its "effective" real cost is less than one, the demand for nominal credit to finance investment would increase.

5.2 Derivation of the aggregate supply function

As in Section 4, the model will now be solved using a log-linear version based on the equations developed at the micro level. Given that neither the households' nor the banks' behavior has been modified, equations (2.42), (2.43), and (2.45) from Section 4 can be used here to derive the aggregate supply function. However, a different equation for the demand for credit needs to be specified since now credit-financed investment demand is included in the model. Based on equation (2.82), the following log-linear relationship will now be postulated.

$$(2.83)\quad c_t^d = \beta_0 + {}_{at}p_t^e + (1-\beta_1)(w_t - {}_{at}p_t^e) - (\beta_1 + \beta_2)i_t + \beta_2\,({}_{at}p_{t+1}^e - {}_{at}p_t^e) + \beta_1\, u_t'$$

Equations (2.42), (2.43), (2.45), and (2.83) yields the following equilibrium solution for employment:

$$(2.84)\quad l_t = \tilde{\alpha}_0' + \tilde{\alpha}_1'\,(h_t - {}_{at}p_t^e) - \tilde{\alpha}_2'\,({}_{at}p_{t+1}^e - {}_{at}p_t^e) + \tilde{\alpha}_3'\, u_t$$

where:

$$\tilde{\alpha}_0 = \frac{\alpha_1\gamma_0 - \alpha_1\gamma_1\beta_0 + \gamma_1\alpha_0(\beta_1+\beta_2) + \alpha_1\beta_2\gamma_0 + \tilde{\alpha}_1 k}{\gamma_1(\beta_1+\beta_2) + \alpha_1(1+\beta_2)}$$

$$\tilde{\alpha}_1 = \frac{\alpha_1\gamma_1}{\gamma_1(\beta_1+\beta_2) + \alpha_1(1+\beta_2)}$$

$$\tilde{\alpha}_2 = \frac{\alpha_1\gamma_2 + \alpha_1\beta_2\gamma_1 + \alpha_1\beta_2\gamma_2}{\gamma_1(\beta_1+\beta_2) + \alpha_1(1+\beta_2)}$$

$$\tilde{\alpha}_3 = \frac{-\alpha_1\gamma_1\beta_2}{\gamma_1(\beta_1+\beta_2) + \alpha_1(1+\beta_2)}$$

Notice that now the effect of expected inflation on employment depends not only on its intertemporal effect on the supply of labor (as in Sections 3 and 4), but also on its effect on the demand for nominal credit (which in turn depends on the sensitivity of investment to the inflation rate).

Notice also that, because the only difference between this analysis and that of Section 4 is the inclusion of the investment demand, setting $\beta_2 = 0$, would yield the same equation for the labor market as that derived in Section 4.

An important difference betwen equation (2.50) and (2.84) is that now, the random term affecting productivity affects the equilibrium level of employment. The reason for this is that the demand for credit is no longer completely tied to <u>only</u> the demand for labor. Assume that, <u>ceteris</u>

paribus, a positive value of u_t occurs. The demand for labor will rise and so will the demand for credit, increasing the interest rate. However, the interest rate will not increase in the same proportion as the increase in u_t because now the demand for credit depends on both the demand for labor and the demand for investment. Thus, the rise in the interest rate will not fully offset the increase in productivity (in contrast to Section 4), and the level of employment will increase. 28/

Solving the system for the nominal interest rate yields:

$$(2.85)\quad i_t = \tilde{\gamma}'_0 - \tilde{\gamma}'_1 (h_t - {}_{at}p^e_t) + \tilde{\gamma}'_2 ({}_{at}p^e_{t+1} - {}_{at}p^e_t) + \tilde{\gamma}'_3 u_t$$

where: $$\tilde{\gamma}'_0 = \frac{(\gamma_1+\alpha_1)\beta_0 - (1-\beta_1)\gamma_0 + (1-\beta_1)\alpha_0}{\gamma_1(\beta_1+\beta_2) + \alpha_1(1+\beta_2)} - \tilde{\gamma}'_1 k$$

$$\tilde{\gamma}'_1 = \frac{\gamma_1 + \alpha_1}{\gamma_1(\beta_1+\beta_2) + \alpha_1(1+\beta_2)}$$

$$\tilde{\gamma}'_2 = \frac{\beta_2(\gamma_1 + \alpha_1) + \gamma_2(1-\beta_1)}{\gamma_1(\beta_1+\beta_2) + \alpha_1(1+\beta_2)}$$

$$\tilde{\gamma}'_3 = \frac{\beta_1\gamma_1 + \alpha_1}{\gamma_1(\beta_1+\beta_2) + \alpha_1(1+\beta_2)}$$

As in the case of employment, setting $\beta_2 = 0$ would give the same equation derived in Section 4 for the interest rate. Again, the pure Fisher effect does not hold due to the financial constraint assumption. In addition, the coefficient affecting u_t ($\tilde{\gamma}'_3$) is less than one as discussed above.

Finally, based on the production function (equation (2.78)) and on equation (2.84), the following log-linear relationship for the short-run aggregate supply of output is postulated:

$$(2.86)\quad y_t^s = \tilde{a}_o + \tilde{a}_1 (h_t - {}_{at}p_t^e) - \tilde{a}_2 ({}_{at}p_{t+1}^e - {}_{at}p_t^e) + \tilde{a}_3 I_{t-1} + \tilde{a}_4 u_t$$

Because investment in period t-1 is part of the output level during period t-1, equation (2.86) implies persistence in any the output disturbances.

5.3 Derivation of the aggregate demand function

As stated in Section 4, equilibrium in the commodity market at the end of the period implies equilibrium between the demand and supply of money. The firm's holdings of money at the end of the period equals the value of output. Thus,

$$(2.87)\quad {}^fD_t = P_t Y_t^s = (W_t L_t + ({}_{at}EP_t) I_t^d)(1+i_t) + {}^f\pi_t$$

Equilibrium in the money market can therefore be obtained by equating equations (2.40) and (2.53) and using equation (2.87):

$$(2.88)\quad \frac{H_t}{P_t} = Y_t^s - \left[\frac{{}_{at}EP_t}{P_t}\right] I_t^d - q + \frac{\beta}{\mu_t} \frac{P_t}{({}_{bt}EP_{t+1})}$$

A log-linear approximation to equation (2.88) is:

$$(2.89)\quad h_t - p_t = \tilde{b}_0 + \tilde{b}_1 y_t^s - \tilde{b}_2 ({}_{at}p_t^e - p_t) - \tilde{b}_2 I_t^d$$

$$- \tilde{b}_3 ({}_{bt}p_{t+1}^e - p_t) - \epsilon_t$$

To avoid confusion with the nominal interest rate i_t, the remainder of this section will use I_t to represent the log of real investment demand.

A log-linear relationship for the investment demand, based on equation (2.81) is:

$$(2.90) \quad I_t^d = I_0 - \beta_2(i_t - {}_{at}p_{t+1}^e + {}_{at}p_t^e)$$

That is, real investment demand depends negatively on the real interest rate.

Substituting equation (2.85) into equation (2.90):

$$(2.91) \quad I_t^d = (I_o - \beta_2\tilde{\gamma}_o') + \beta_2\tilde{\gamma}_1' (h_t - {}_{at}p_t^e)$$

$$+ \beta_2 (1-\tilde{\gamma}_2')({}_{at}p_{t+1}^e - {}_{at}p_t^e) - \beta_2\tilde{\gamma}_3' u_t$$

Again, the assumption that firms need to finance their advances by borrowing in a limited capital market implies that expected inflation affects real investment. If the "financial constraint" assumption were removed, a rise in expected inflation would raise the nominal rate of interest such as to leave unchanged the real interest rate, and there would be no variation in real investment.

Equations (2.89) and (2.91) represent the aggregate demand side of the model.

5.4 The solution for the price and output levels

Since the objective of Section 5 is to analyze the differences in the solutions of major macroeconomic variables implied by the introduction of investment demand, rather than by alternative information sets, the rest of this section will concentrate only on the full current information case. In that context, ${}_{at}p^e_t = p_t$ and ${}_{at}p^e_{t+1} = {}_{bt}p^e_{t+1}$. Thus, by substituting equations (2.86) and (2.91) into equation (2.89), the following expression for the price level is obtained:

$$(2.92)\quad p_t = [1 + \tilde{b}_3 - \tilde{b}_1\tilde{a}_1 + \tilde{b}_1\tilde{a}_2 + \tilde{b}_2\beta_2 (1-\tilde{\gamma}_1' - \tilde{\gamma}_2')]^{-1}$$

$$[(\tilde{b}_o - \tilde{b}_1\tilde{a}_o + \tilde{b}_2 I_o - \tilde{b}_2\beta_2\tilde{\gamma}_o')$$

$$+ (1 - \tilde{b}_1\tilde{a}_1 + \tilde{b}_2\beta_2\tilde{\gamma}_1')\, h_t$$

$$+ (\tilde{b}_1\tilde{a}_2 + \tilde{b}_3 + \tilde{b}_2\beta_2 (1-\tilde{\gamma}_2'))\ {}_{at}p^e_{t+1}$$

$$- (\tilde{b}_2\beta_2\tilde{\gamma}_3' - \tilde{b}_1\tilde{a}_4)\, u_t$$

$$- (\tilde{b}_1\tilde{a}_3)\, I_{t-1} + \epsilon_t]$$

Appendix II.5 shows the method used to find the final solution for the price level.

As in the model without investment demand (Section 4), a fully anticipated permanent change in the level of the monetary base affects the price level proportionally and has no effect on output. Also, as in Section 4, an increase in the rate of growth of the money supply increases the level of prices and decreases the level of output, while a fully anticipated temporary monetary increase or an increase in productivity have the opposite effect on the levels of prices and output. However, the important difference as compared with Section 4 is that, now, all exogenous variables that affect the current period level of output (m, v_t, u_t, and ϵ_t) have persistent effects. This result occurs because both the supply of output and investment demand are positively affected by the real monetary base (see equations (2.86) and (2.91)). Thus, a change in an exogenous variable that results in an increase in the real monetary base will increase both current output and the level of investment. This increase in investment will, in turn, increase the next period's level of output. Hence, this model provides us with a framework where persistence in output fluctuations coexists with fully anticipated permanent monetary changes being neutral, and fully anticipated temporary monetary changes being non-neutral.

6. Summary

This chapter has presented a model of a closed economy facing a "financial constraint," in the sense that firms are required to finance labor in advance of sales by using a banking system as their only source of finance. It has been shown that such an assumption implies that the supply of output depends positively on the real monetary base, and that the real interest rate is affected by a change in expected inflation; that is, the pure Fisher effect does not hold. The positive dependence of the supply of output on the real monetary base contrasts with results obtained in models where real money enters as a wealth variable. There, real money affects current leisure positively and, hence, the output level negatively.

The model has been used to investigate the effects of alternative changes in monetary and real exogenous variables on output and the price level. In particular, it has been shown that anticipated changes in the rate of growth of the monetary base affect the level of output negatively because of their negative effect on the real monetary base. Thus, money is not superneutral in this model. In addition, it has been shown that a fully perceived permanent increase in the level of the money supply causes a proportional rise in the price level leaving output unchanged, while a fully perceived temporary increase in the level of the money supply causes an increase in the output level because of the induced increase in the real monetary base.

Under the assumption that agents cannot distinguish between current permanent and temporary components of the monetary base, even permanent monetary disturbances have real effects. The lower the relative variance of the transitory shock, the lower the effect of a permanent shock on output, real interest rates, and real wage rates. Thus, confusion between permanent and transitory shocks results in non-neutralities of permanent changes in the monetary base. Although a random monetary disturbance (that cannot be distinguished as permanent or transitory) always affects the level of output positively, the direction of the effect on the real interest rate, and on the real wage rate depends on the sensitivity of the supply of labor with respect to the expected inflation rate relative to the sensitivity of the demand for labor with respect to its "effective" cost.

Finally, it has been shown that, if a simple form of investment demand is introduced, the effects of changes in exogenous variables impinging on the output level will persist. These persistence results are obtained without otherwise modifying the nature of the model.

Table 2.1. The Economic Agents' Activities During Period t

		Firms			
$D^f_{t-1} = P_{t-1} Y_{t-1}$	Firms pay: a) $^f\Pi_{t-1}$ (to households) b) $(1 + i_{t-1})C_{t-1}$ (to banks) $(^fD_t = 0)$	Then borrow: C_t To pay $W_t L_t$ to the young households $(^fD_t \; 0)$	(Production process takes place)	Firms receive: $P_t Y^s_t = {}^fD_t$	
t-1		t			t+1
		Households			
$^YD^d_{t-1}$	The Old Generation receive: a) $^f\Pi_{t-1}$ (from firms) b) $i_{t-1} C_{t-1}$ (from banks) c) Tr_t (from government)	The Young Generation receive: $W_t L_t$ (from firms)	(Production process takes place)	Households spend $P_t \, {}^YY^d_t$	
	So, they have: $^oD_{t-1} + {}^f\Pi_{t-1} + i_{t-1}C_{t-1} + Tr_t$	So, they have: $W_t L_t$		So: Young: $^YD^d_t + P_t \, {}^YY^d_t = W_t L_t$ Old: $P_t \, {}^oY^d_t = {}^oD_{t-1} + {}^h\Pi_{t-1} + Tr_t$	

Commercial Banks

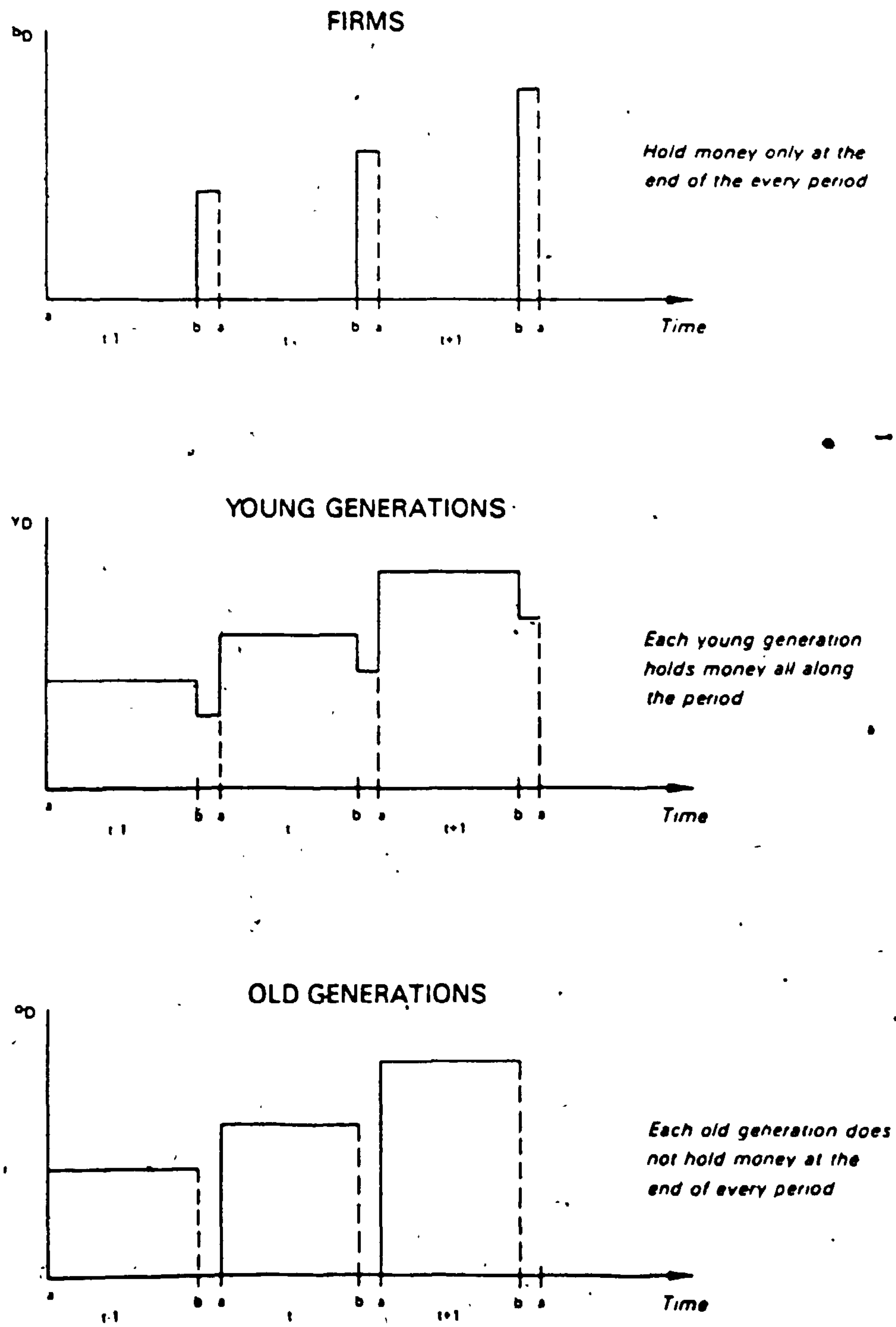
A	L
R_{t-1}	$^YD_{t-1}$
C_{t-1}	$^fD_{t-1}$
	D_{t-1}

A	L
R_t	$^oD_{t-1}$
C_t	$^{A}D_t = {}^f\Pi_{t-1} + i_{t-1} C_{t-1} + W_t L_t + Tr_t$

A	L
R_t	^YD_t
C_t	^fD_t
	D_t

Figure 1

AGENTS' HOLDINGS OF MONEY



FOOTNOTES TO CHAPTER II

1/ This implies that we are ruling out distributional effects.

2/ Wicksell (1936) deals with a sequential trading arrangement similar to ours.

3/ The assumption of discrete time allows us to guarantee that the economic agents will hold money during the period (households have to wait for the commodity market to open before they can spend their money); otherwise, if the transaction period would approach zero (a continuous time set up), we would have to introduce additional assumptions (such as transaction costs or money as an argument in the utility function) to guarantee the agents' holdings of money in the model.

The size of the transactions also depends on the length of the transaction period. For instance, if the length of the working period would increase, the total nominal income per period would also increase, rising the nominal demand for commodities and money. However, we will not deal here with this problem: several papers (Clower and Howitt (1978), Feige and Parkin (1971), Fried (1973)) provide a rationale for the length of the transaction period using an inventory approach. We will just assume exogenous the transactions period, although we recognize that it imposes limitations to the empirical application of the model.

4/ Output supply is determined at the beginning of the period because the equilibrium levels of labor and credit are determined at that moment.

5/ In this model, households' saving (a flow) equals their demand for money (D_t^d) (a stock) because, $Saving_t = D_t^d - D_{t-1}$ and, $D_{t-1} = 0$ because the generation that saves (the young generation) is born at the beginning of every period t.

6/ Even though some firms could be making losses, we are dealing with the "representative" unit and assuming that its net profits are positive.

7/ We could have assumed that ϕ_t is unknown at the beginning of the period, but that would complicate the solution of the model, since both P_t and Y_t would be unknown and we would not be able to take expectations of a product: $E[P_t Y_t] \neq (E P_t)(E Y_t)$. Also, μ_t is included in the firm's information set because all agents share the same information (also see footnote 13).

8/ As Lucas (1972) has pointed out, this implies that the old generation has a unit elastic demand for goods.

9/ Notice from equation (2.20) that the value of goods consumed by the old generation ($P_t \, {}^oY_t^d$) equals the current monetary base (H_t). This is so because the old generation consumes all the previous savings of the economy (savings of firms: ${}^{f}D_{t-1}$ plus savings of the young generation ${}^{y}D_{t-1}$) minus the value of previous period wage payments to labor (which equals the past period level of banking credit), plus the government's net transfers. Hence:

$$P_t \, Y_t^d = {}^{f}D_{t-1} + {}^{y}D_{t-1} - C_{t-1} + Tr_t$$

$$= D_{t-1} - C_{t-1} + Tr_t$$

$$= H_{t-1} + Tr_t$$

$$= H_t$$

10/ A quadratic utility function forces the consumption and labor decision rules derived from the households' utility maximization to be linear.

11/ The marginal utility of current consumption will be positive only if $Y_1^d < q$. This is obviously one important limitation of the quadratic utility function use.

12/ The assumption that current utility is additive separable and shows diminishing marginal utility in current consumption and labor supply guarantees that neither consumption nor leisure are inferior goods. The assumption of constant marginal utility of future consumption eliminates the income effect on the supply of labor and hence guarantees that the labor supply slopes to the right with respect to the real wage rate.

13/ Φ_t is included in the young generation information set because it is assumed that the relevant decision makers (firms and young consumers) share the same information.

14/ The second order condition for a maximum is also satisfied.

15/ In this model, the indirect utility function (U_{ind}) can be:

(a) if $\lambda' = 0$:

$$U_{ind} = q[q\mu_1 - \beta \, {}_{b1}E(\frac{P_1}{P_2})] - \frac{\mu_1}{2} [q - \frac{\beta}{\mu_1} \, {}_{b1}E(\frac{P_1}{P_2})]^2 - jL_1^s \frac{1}{2} (L_1^s)^2$$

$$+ \beta \, {}_{b1}E \, [\frac{W_1}{P_1} \frac{P_1}{P_2} L_1^s + \frac{Tr_2}{P_2} + \frac{{}^{h}\pi_2}{P_2} - \frac{P_1}{P_2} [q - \frac{\beta}{\mu_1} \, {}_{b1}E(\frac{P_1}{P_2})]$$

The real balances held by the young generation before the commodity market opens is: $[(W_1/P_1)L_1]$. Thus, the marginal (indirect) utility of real balances is:

$$\frac{\partial U_{ind}}{\partial(\frac{W_1}{P_1} L_1)} = \beta\ b_1 E(\frac{P_1}{P_2})$$

(b) if $\lambda' > 0$:

$$U_{ind} = q\mu_1 [\frac{W_1}{P_1} L_1^s] - \frac{\mu_1}{2} [\frac{W_1}{P_1} L_1^s]^2 - j\ L_1^s - \frac{1}{2} (L_1^s)^2$$

$$+ \beta [\frac{W_1}{P_1} \frac{P_1}{P_2} L_1 + \frac{Tr_2}{P_2} + \frac{h\pi_2}{P_2} - \frac{P_1}{P_2} [\frac{W_1}{P_1} L_1^s]]$$

In this case, the marginal (indirect) utility of real balances is:

$$\frac{\partial U_{ind}}{\partial(\frac{W_1}{P_1} L_1)} = \mu_1 (q - \frac{W_1}{P_1} L_1^s)$$

16/ This is so because: $E[P_1/P_2] \neq [EP_1/EP_2]$.

17/ Notice that the term "expected inflation" is used conditional to the available information.

18/ Notice that a change in the wage rate in this model should be considered as a transitory change from the point of view of the consumer, given that in the second period, when he is old, he will not receive any wage income.

19/ In fact, using equation (2.39) instead of equation (2.26) in the agent's maximization problem gives rise to the following decision rules:

$$Y_1^d = \frac{\mu_1}{\beta} \frac{b_1 EP_2}{P_1}$$

and

$$L_1^s = \frac{1}{\beta} \frac{W_1}{a_1 EP_1} \frac{a_1 EP_1}{a_1 EP_2}$$

Where, as in the decision rules of the main text, consumption demand is a positive function of expected inflation and a real shock to utility, and the supply of labor depends positively on the expected real wage and negatively on expected inflation.

20/ This is so because the log of (1+i) is approximately equal (using a Taylor's expansion) to i.

21/ Notice that in equation (2.51) $a_1 = a_2$ since $\gamma_1 = \gamma_2$ and $\alpha_1 = \beta_1$.

22/ This process is identical to the one presented in Barro (1978).

23/ Notice at ${}_{at}P^e_{t+1} = {}_{bt}P^e_{t+1}$ because it is assumed that complete current information is obtained at the beginning of the period.

24/ Notice that by strictly imposing the restrictions from the microfoundations ($\gamma_1 = \gamma_2$; $\alpha_1 = \beta_1$), $a_1 = a_2$ and hence a random term affecting utility has no effect on the level of real output. This result occurs because ε_t only affects the current price level but not the expected future price level; hence, if $a_1 = a_2$, the supply of output depends on $(h_t - {}_{at}P^e_{t+1})$ which is unaffected by ε_t.

25/ Notice that θ_2 is the inverse of θ_4.

26/ Notice that if the restriction $a_1 = a_2$ is imposed, the effect of an exogenous change in utility on velocity is $(1/(1 + b_2))$, which is unambiguously positive and less than one. This result follows because a change in ε_t leaves output unchanged and hence a change in velocity fully reflects the induced change in the price level.

27/ Households' demand for money cannot be negative (see equation (2.25)).

28/ The degree in which the increase in the interest rate offsets the initial rise in productivity depends on the value of β_2.

APPENDIX II:1

THE NOTATION OF THE MODEL

$^{f}\pi_{t-1}$ = nominal profits realized by the representative firm at the end of period t-1.

$^{b}\pi_{t}$ = nominal profits realized by the representative bank at the beginning of period t.

C_t^s = nominal amount of credit (loans) supplied by the representative bank at the beginning of period t.

C_t^d = nominal demand for bank loans by the representative firm at the beginning of period t.

L_t^d = the firm's demand for labor at the beginning of period t.

L_t^s = the household's supply of labor at the beginning of period t.

P_t = price level of output at period t.

W_t = nominal wage rate during period t.

i_t = nominal interest rate.

Tr_t = government net transfers received by the old generation at the beginning of period t.

GC_t = total assets of the central bank = government's outstanding debt in period t.

H_t = monetary base in period t.

R_t = reserves in the vault of the commercial bank during period t (equals reserve requirements (RR_t) plus excess reserves (RE_t)).

K = reserve requirement ratio (assumed to be constant).

Y_t^s = supply of output at the end of period t.

$^{y}Y_t$ ($^{o}Y_t$) = demand for output by the representative individual of the young (old) generation at the end of period t.

${}^{f}D_t^d$ = nominal stock of money (demand deposits) demanded by the firm at the end of period t.

${}^{y}D_t^d$ = nominal demand for money by the young generation at the end of period t = nominal demand for money by the household = ${}^{h}D_t^d$.

${}^{o}D_{t-1}$ = demand deposits carried over from period t-1 and held at the beginning of period t by the old generation = savings of the young generation during period t-1: ${}^{y}D_{t-1}^d$.

D_t^s = total supply of money at the end of period t

$$D_t^s = R_t + C_t = (1/K)\ H_t = (1/K)\ (H_{t-1} + Tr_t)$$

D_t^d = total demand for money at the end of period t

$$D_t^d = {}^{Y}D_t^d + {}^{f}D_t^d$$

Ω_{t-1} = information on all variables up to period t-1.

Ω_{at} = information set available and relevant to the firm's and young generation's decisions at the beginning of period t.

Ω_{bt} = information set available to the young generation's decisions at the end of period t.

E = the expectations operator. It will be used with a variable to indicate its expected value and will be accompanied by two subscripts. The first refers to the time when expectations are formed: at the beginning of the period (a) or at the end of the period (b). The second refers to the period in which it applies: current period (t) or next period (t+1).

For instance, ${}_{bt}EP_{t+1}$ refers to the expectations of P_{t+1} based on the information available at the end of the current period.

The superscript "e" accompanying a variable is used in Sections 4 and 5 to represent the expectations operator E.

The superscripts "s" and "d" refer to the quantity supplied or demanded respectively, of the attached variable. Variables written without a superscript denote the actual flow exchanged or the actual stock held.

APPENDIX II.2

DERIVATION OF THE OUTPUT SUPPLY FUNCTION WHEN EITHER THE FIRM'S REQUIREMENT TO FINANCE IN ADVANCE INPUTS OF PRODUCTION OR THE LIMITED CAPITAL MARKET ASSUMPTION IS RELAXED

The purpose of this Appendix is to prove that in this model, the presence of the real monetary base in the aggregate supply of output results from the assumption that firms need to finance inputs of production in advance to sales and the limited capital markets assumption taken together. In what follows, each assumption will be relaxed in turn and it will be shown that the real monetary base will no longer be an argument in the aggregate supply function.

1. Relaxing the assumption that firms need to finance inputs of production in advance in an economy with limited capital markets

In such an economy, the demand for labor will not be tied down to the demand for credit. However, the assumption of limited capital markets implies that households cannot engage in borrowing and lending activities and hence, the supply of labor will not be affected by the interest rate. In such an economy, the supply and demand for labor are:

$$\text{(II.2.1)} \quad l_t^d = \alpha_o - \alpha_1 \left(w_t - {}_{t}p_t^e - u_t\right)$$

$$\text{(II.2.2)} \quad l_t^s = \gamma_o + \gamma_1 \left(w_t - {}_{t}p_t^e\right) - \gamma_2 \left({}_{t}p_{t+1}^e - {}_{t}p_t^e\right)$$

Since the supply of output is a positive linear function of the employment level, it is clear that the real monetary base is not an argument in the supply of output.

2. Assuming that firms need to finance inputs of production in advance in an economy with developed capital markets

Assume now that a market for primary securities exists such that households can buy securities from firms. To simplify the exposition, it will be assumed that the young generation receives all monetary net transfers at the beginning of the period (this assumption has no effect on the model presented in the main text). Money is still needed for transactions, hence, the total amount of nominal securities that the household can buy from the firms is limited by the amount of transfers received. Households cannot buy securities out of their money wage because firms need to borrow before paying their wage bill. Assuming that the value of the parameters and exogenous variables (from the point of view of households) are such that the households' demand for securities is less than the amount of transfers, the households' supply of credit will depend positively on the expected real interest rate. Rather than work through the microeconomic foundations to derive the households' supply of credit, the macromodel of Section 4 will now be modified to allow for the introduction of such additional supply of credit. This short-cut procedure is justified given that the purpose of this Appendix is only to show the impact of an extended credit market.

This modified model is:

(II.2.3) $l_t^d = \alpha_o - \alpha_1 (w_t - {}_{at}p_t^e + i_t - u_t)$

(II.2.4) $l_t^s = \gamma_o + \gamma_1 (w_t - {}_{at}p_t^e) + \gamma_2 (i_t - {}_{at}p_{t+1}^e + {}_{at}p_t^e)$

(II.2.5) $c_t^d = \beta_o + \beta_1 ({}_{at}p_t^e - i_t + u_t) + (1 - \beta_1) w_t$

$$\text{(II.2.6)} \quad c_t^s = \tau_0 + \tau_1 h_t + \tau_2 (i_t - {}_{at}p^e_{t+1} + {}_{at}p^e_t) + {}_{at}p^e_t$$

where (to strictly follow the microfoundations of the model): $\alpha_1 = \beta_1$; $\gamma_1 = \gamma_2$.

The firm's behavior has not been changed, and so thus equations (II.2.3) and (II.2.4) are exactly the same as before. The supply of labor has been modified to allow for the real interest rate (rather than the inflation rate) to affect the households' leisure-consumption decision. In addition, the supply of credit (equation (II.2.6)) is no longer solely determined by the banking system. Households supply of real credit is now added and depends positively on the real interest rate. It should be noted that with perfect capital markets, banks now have to pay interest on demand deposits. In this Appendix, we will assume that the banks' supply of credit equals: $c_t^s = h_t + k$. (This is an assumption chosen for convenience only.)

Solving equations (II.2.3) through (II.2.6) for the equilibrium level of employment (and using $\alpha_1 = \beta_1$; $\gamma_1 = \gamma_2$) yields:

$$\text{(II.2.7)} \quad l_t = \text{constant} - \frac{\gamma_1 \alpha_1 (1+\tau_2)}{\gamma_1(1+\tau_2 + 2\alpha_1) + \alpha_1(1+\tau_2)} ({}_{at}p^e_{t+1} - {}_{at}p^e_t - u_t)$$

Once more the real monetary base will not be an argument in the aggregate supply function. Notice that because banks pay interest rates on demand deposits, the demand for money (under the assumption that economic agents hold their money in the form of demand deposits) will now depend positively on the real interest rate.

Notice that in the two cases presented in this Appendix, even though the real monetary base does not appear as an argument in the aggregate supply function, the expected inflation rate does. Hence, if an increase in the rate of growth of the money supply generates inflation, it will also have a negative effect on real output.

Only if both assumptions (the firm's requirement to finance inputs of production in advance, and the limited capital market assumptions) were relaxed at the same time, the supply of output would depend on the real interest rate. In that case, an increase in the rate of growth of the money supply that generates inflation would also rise the nominal interest rate proportionally, leaving the real interest unchanged, and hence, money would be superneutral.

APPENDIX II.3

PROOF THAT THE FORWARD-LOOKING SOLUTION OF THE PRICE LEVEL IS THE STABLE SOLUTION

Recalling that the microfoundations of the model require $a_1 = a_2$. Equation (2.56) in the main text can be rewritten as:

$$(II.3.1)\quad {}_{at}p^e_{t+1} = \frac{1 + b_2}{b_2 + b_1 a_1}\, {}_{at}p^e_t + (b_o + a_o b_1) - (1 - b_1 a_1) h_t + b_1 u_t - \varepsilon_t$$

or, applying the lag operator L:

$$(II.3.2)\quad [L^{-1} - \frac{(1+b_2)}{b_2 + b_1 a_1}]\, {}_{at}p^e_t = (b_o + a_o b_1) - (1 - b_1 a_1)\, h_t + b_1\, u_t - \varepsilon_t$$

where the operator L^{-1} applied to ${}_{at}p^e_t$ leads the expectations of the price level without changing the information set upon which expectations are formed.

Multiplying equation (II.3.2) by L:

$$(II.3.3)\quad [1 - \frac{(1+b_2)}{b_2 + b_1 a_1}]\, {}_{at}p^e_t = (b_o + a_o b_1) - (1 - b_1 a_1)\, h_t + b_1\, u_t - \varepsilon_t$$

Equation (II.3.3) has multiple solutions. Imposing the restriction that the solution to be chosen is a "stable" solution, the choice of the solution depends on the absolute value of the coefficient: $\frac{1 + b_2}{b_2 + b_1 a_1}$.

Since b_2, b_1, $a_1 > 0$, and $b_1 a_1 < 1$, it turns out that: $\frac{1 + b_2}{b_2 + b_1 a_1} > 1$.

Hence, the forward-looking solution is the one to be chosen for stability purposes.

APPENDIX II.4

SOLUTIONS FOR THE REAL INTEREST RATE AND THE REAL WAGE RATE IN THE INCOMPLETE CURRENT INFORMATION CASE

$$(\text{II.4.1})\quad i_t - {}_{a t}p^e_{t+1} - {}_{a t}p^e_t = \left[\frac{\gamma_o' - \gamma_o' b_1 a_1 - \gamma_1' b_o - \gamma_1' a_o b_1}{1 - b_1 a_1}\right]$$

$$= \left[\frac{\gamma_1' b_2 + \gamma_1' b_1 a_2 + \gamma_2' - \gamma_2' a_1 b_1 - 1 + b_1 a_1}{1 - b_1 a_1}\right] m$$

$$- g_1 \left[\frac{\gamma_1'(b_1 a_2 + b_2) + \gamma_2' - \gamma_2' b_1 a_1 + b_1 a_1 - 1}{1 + b_1 a_2 + b_2 - b_1 a_1}\right](v_t + x_t)$$

$$+ \left[\frac{1 + b_2 + \gamma_2' b_1 - \gamma_1' b_1 - b_1 a_1 + b_1 a_2 - b_1}{1 + b_2 - b_1 a_1 + b_1 a_2}\right] u_t$$

$$+ \left[\frac{\gamma_1' - \gamma_2' + 1}{1 + b_2 - b_1 a_1 + b_1 a_2}\right] \varepsilon_t$$

and:

$$(\text{II.4.2})\quad w_t - p_t = \left[\frac{\beta_o' - \beta_o' b_1 a_1 + \beta_2'(b_o + a_o b_1)}{1 - b_1 a_1}\right]$$

$$+ \left[\frac{\beta_1' - \beta_1' b_1 a_1 - \beta_2'(b_2 + b_1 a_2)}{1 - b_1 a_1}\right] m$$

$$- g_1 \left[\frac{\beta_1'(1 - b_1 a_1) - \beta_2'(b_1 a_2 + b_2)}{1 + b_2 - b_1 a_1 + b_1 a_2}\right] (v_t + x_t)$$

$$+ \left[\frac{b_1\ (\beta_2' + \beta_1')}{1 + b_2 - b_1 a_1 + b_1 a_2}\right] u_t$$

$$- \left[\frac{\beta_2' + \beta_1'}{1 + b_2 - b_1 a_1 + b_1 a_2}\right] \varepsilon_t$$

APPENDIX II.5

DERIVATION OF THE FINAL SOLUTION FOR THE PRICE LEVEL IN THE MODEL WITH INVESTMENT DEMAND

Substituting equation (2.86) into equation (2.89), and assuming full current information, equations (2.89) and (2.91) in the main text case can be rewritten in the following way:

(II.5.1) $p_t = \tilde{A}_o + \tilde{A}_1 h_t + \tilde{A}_2\ {}_{at}p^e_{t+1} - \tilde{A}_3 u_t - \tilde{A}_4 I_{t-1} + \tilde{\varepsilon}_t$

(II.5.2) $I_t = \tilde{B}_o + \tilde{B}_1 h_t + \tilde{B}_2\ {}_{at}p^e_{t+1} - \tilde{B}_3 u_t - (\tilde{B}_1 + \tilde{B}_2)\, p_t$

where:

$\tilde{A}_o = (\tilde{b}_o - \tilde{b}_1 \tilde{a}_o + \tilde{b}_2 I_o - \tilde{b}_2 \beta_2 \tilde{\gamma}'_o)\, \tilde{K}$

$\tilde{A}_1 = (1 - \tilde{b}_1 \tilde{a}_1 + \tilde{b}_2 \beta_2 \tilde{\gamma}'_1)\, \tilde{K}$

$\tilde{A}_2 = (\tilde{b}_1 \tilde{a}_2 + \tilde{b}_3 + \tilde{b}_2 \beta_2\, (1-\tilde{\gamma}'_2))\, \tilde{K}$

$\tilde{A}_3 = (\tilde{b}_2 \beta_2 \tilde{\gamma}'_3 + \tilde{b}_1 \tilde{a}_4)\, \tilde{K}$

$\tilde{A}_4 = (\tilde{b}_1 \tilde{a}_3)\, \tilde{K}$

$\tilde{B}_o = I_o - \beta_2 \tilde{\gamma}'_o$

$\tilde{B}_1 = \beta_2 \tilde{\gamma}'_1$

$\tilde{B}_2 = \beta_2\, (1-\tilde{\gamma}'_2)$

$\tilde{B}_3 = \beta_2 \tilde{\gamma}'_3$

$\tilde{K} = [1 + \tilde{b}_3 - \tilde{b}_1 \tilde{a}_1 + \tilde{b}_1 \tilde{a}_2 + \tilde{b}_2 \beta_2\, (1-\tilde{\gamma}'_1 - \tilde{\gamma}'_2)]^{-1}$

Equations (II.5.1) and (II.5.2) are not independent since the expected future price level depends on current investment demand.

Recalling (from Section 4) that

$$h_t = h_{t-1} + m + v_t + x_t - x_{t-1}$$

the method of undetermined coefficients suggests that the final solution for the price level takes the following form:

$$\text{(II.5.3)} \quad p_t = \tilde{\theta}_0 + \tilde{\theta}_1 h_{t-1} + \tilde{\theta}_2 m + \tilde{\theta}_3 v_t + \tilde{\theta}_4 x_t + \tilde{\theta}_5 x_{t-1} + \tilde{\theta}_6 u_t + \tilde{\theta}_7 \varepsilon_t + \tilde{\theta}_8 I_{t-1}$$

Updating equation (II.5.3) and taking beginning of period expectations:

$$\text{(II.5.4)} \quad {}_{at}p^e_{t+1} = \tilde{\theta}_o + \tilde{\theta}_1 h_{t-1} + (\tilde{\theta}_1 + \tilde{\theta}_2) m + \tilde{\theta}_1 v_t + (\tilde{\theta}_1 + \tilde{\theta}_5) x_t - \tilde{\theta}_1 x_{t-1} + \tilde{\theta}_8 I_t$$

Substituting equation (II.5.2) into equation (II.5.4) and substituting the resulting expression for ${}_{at}p^e_{t+1}$ into equation (II.5.1) we obtain:

$$(\text{II.5.5})\quad p_t = \frac{1}{1-\tilde{\theta}_8\tilde{B}_2}\Big\{[\tilde{A}_o - \tilde{A}_o\tilde{\theta}_8\tilde{B}_2 + \tilde{A}_2(\tilde{\theta}_o+\tilde{\theta}_8\tilde{B}_o-\tilde{\theta}_8\tilde{B}_1\tilde{\theta}_o-\tilde{\theta}_8\tilde{B}_2\tilde{\theta}_o)]$$

$$+ [\tilde{A}_1 - \tilde{A}_1\tilde{\theta}_8\tilde{B}_2 + \tilde{A}_2(\tilde{\theta}_1+\tilde{\theta}_8\tilde{B}_1-\tilde{\theta}_8\tilde{B}_1\tilde{\theta}_1-\tilde{\theta}_8\tilde{B}_2\tilde{\theta}_1)]\,(h_{t-1}+v_t-x_{t-1})$$

$$+ [\tilde{A}_1 - \tilde{A}_1\tilde{\theta}_8\tilde{B}_2 + \tilde{A}_2(\tilde{\theta}_1 + \tilde{\theta}_2 + \tilde{\theta}_8\tilde{B}_1 - \tilde{\theta}_8\tilde{B}_1\tilde{\theta}_2 - \tilde{\theta}_8\tilde{B}_2\tilde{\theta}_2)]\, m$$

$$+ [\tilde{A}_1 - \tilde{A}_1\tilde{\theta}_8\tilde{B}_2 + \tilde{A}_2(\tilde{\theta}_1 + \tilde{\theta}_5 + \tilde{\theta}_8\tilde{B}_1 - \tilde{\theta}_8\tilde{B}_1\tilde{\theta}_4 - \tilde{\theta}_8\tilde{B}_2\tilde{\theta}_4)]\, x_t$$

$$- [\tilde{A}_3 - \tilde{A}_3\tilde{\theta}_8\tilde{B}_2 + \tilde{A}_2(\tilde{\theta}_8\tilde{B}_3 + \tilde{\theta}_8\tilde{B}_1\tilde{\theta}_6 + \tilde{\theta}_8\tilde{B}_2\tilde{\theta}_6)]\, u_t$$

$$+ (1-\tilde{\theta}_8\tilde{B}_2 - \tilde{A}_2(\tilde{\theta}_8\tilde{B}_1\tilde{\theta}_7 + \tilde{\theta}_8\tilde{B}_2\tilde{\theta}_7))]\, \varepsilon_t$$

$$- (\tilde{A}_4-\tilde{A}_4\tilde{\theta}_8\tilde{B}_2+\tilde{A}_2\tilde{\theta}_8^2(\tilde{B}_1+\tilde{B}_2))\, I_{t-1}\Big\}$$

Now, since (from the parameters of the model) $\tilde{A}_1 + \tilde{A}_2 = 1$:

$$\tilde{\theta}_1 = \tilde{\theta}_3 = - \tilde{\theta}_5 = 1$$

$$\tilde{\theta}_2 = \frac{1 + \tilde{\theta}_8(\tilde{B}_1 - \tilde{A}_1\tilde{B}_1 - \tilde{A}_1\tilde{B}_2)}{\tilde{A}_1 + \tilde{\theta}_8(\tilde{B}_1-\tilde{A}_1\tilde{B}_1-\tilde{A}_1\tilde{B}_2)} > 1 \text{ since } \tilde{A}_1 < 1,\ \tilde{\theta}_8 < 0 \text{ and } (\tilde{B}_1+\tilde{B}_2) > 1$$

$$\tilde{\theta}_4 = \frac{\tilde{A}_1 + \tilde{\theta}_8(\tilde{B}_1-\tilde{A}_1\tilde{B}_1-\tilde{A}_1\tilde{B}_2)}{1 + \tilde{\theta}_8\,(\tilde{B}_1-\tilde{A}_1\tilde{B}_1-\tilde{A}_1\tilde{B}_2)} < 1 \text{ since } \tilde{\theta}_2 > 1$$

$$\tilde{\theta}_6 = \frac{-\tilde{A}_3 + \tilde{\theta}_8\,(\tilde{A}_3\tilde{B}_2-\tilde{A}_2\tilde{B}_3)}{1 - \tilde{\theta}_8\,(\tilde{B}_2-\tilde{A}_2\tilde{B}_1-\tilde{A}_2\tilde{B}_2)} < 0 \text{ since } \tilde{\theta}_8 < 0, \text{ and: } (\tilde{A}_3\tilde{B}_2-\tilde{A}_2\tilde{B}_3) > 0.$$

$$\tilde{\theta}_7 = \frac{1 - \tilde{\theta}_8\tilde{B}_2}{1 - \tilde{\theta}_8\,(\tilde{B}_2-\tilde{A}_2\tilde{B}_1-\tilde{A}_2\tilde{B}_2)} > 0$$

$$\tilde{\theta}_8 = \frac{-(1 - \tilde{A}_4\tilde{B}_2) + \sqrt{(1-\tilde{A}_4\tilde{B}_2)^2 - 4\tilde{A}_4\,(\tilde{A}_2(\tilde{B}_1+\tilde{B}_2) - \tilde{B}_2)}}{2\,(\tilde{A}_2\,(\tilde{B}_1+\tilde{B}_2) - \tilde{B}_2)}$$

and $\tilde{\theta}_8 < 0$ because the term inside the square root is positive and less than $(1 - \tilde{A}_4\tilde{B}_2)$.

Notice that the positive root is chosen as the solution for θ_8. The reason for that selection is that we have followed McCallum (1983) in imposing the requirement that the solution for θ_8 must be valid for all admisible values of the structural parameters. In particular, if $A_4 = 0$, we should expect $\tilde{\theta}_8 = 0$ and that result is obtained only by selecting the positive root (see Appendix V.3 for a more detailed explanation of McCallum's methodology).

Substituting equations (II.5.3) and (II.5.4) into equation (2.86) in the main text, the final solution for the output level is obtained.

2

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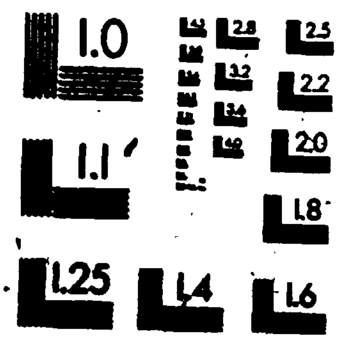

CHAPTER III

A MODEL FOR AN OPEN ECONOMY WITH LIMITED CAPITAL MARKETS

1. Introduction

This chapter extends the analysis of Chapter II to allow for international transactions in commodities. While zero capital mobility is assumed, the economy produces and consumes two kinds of goods: tradable and nontradable commodities. The economy is a price taker in the world market for tradable goods; however, the price of the nontradable good is determined by its domestic demand and supply; it is, therefore, an endogenous variable. All the assumptions made in Chapter II concerning the limitations of the capital market are maintained.

The purpose of this chapter is to highlight the differences in agents' optimal decision rules which arise from relaxing the "closed economy" assumption in the way described above. Optimal decision rules relevant to the open economy will then be used as the basis of a macroeconomic model which will be solved in the next two chapters under alternative assumptions about the exchange rate regime. Chapter IV will analyze the price and output solutions for the model implied by the assumption of flexible exchange rates, and Chapter V will deal with the same economy under fixed exchange rates.

The remainder of this chapter is organized as follows: Section 2 describes the organization of markets and the timing of transactions. Section 3 derives the optimal decision rules implied by the maximizing behavior of the agents in an open economy environment. Section 4 derives the domestic aggregate supply and demand functions for tradable and nontradable commodities and the aggregate demand for money. In this Section it will be shown that equilibrium in the market for tradable commodities depends on the choice of the exchange rate system. Finally, Section 5 sets out the complete macro model to be used in subsequent chapters.

The notation from Chapter II carries through this chapter. Hence, only new variables will be defined in what follows.

2. The structure of the economy

As in the model of Chapter II, all transactions are carried out with money. These transactions now involve four markets: tradable commodities, nontradable commodities, (domestic) labor, and (domestic) credit. The economy's current output consists of both tradable and non-tradable commodities produced by firms using domestic labor services provided by households as the only factor of production. Both types of commodities are treated solely as consumption goods. Domestic households buy the nontradable good from domestic firms and the tradable good from both domestic firms and the rest of the world (imports). Domestic firms sell the tradable good to domestic residents and to the rest of the world (exports). Furthermore, it is assumed that the goods in question are imperfect substitutes in consumption, so that they enter as separate arguments of the households' utility function.

The credit market is exactly as described in Chapter II. The banking system is the only supplier of credit and firms are the only agents demanding it. However, in the open economy, the central bank has an extra duty: it not only finances the government deficit through the issue of high powered money and imposes a required reserve ratio on the commercial banks; but it also acts as the foreign exchange authority. That is, it buys and sells foreign exchange in the amount necessary to keep the exchange rate constant in a regime of fixed exchange rates, or it allows the exchange rate to fluctuate to keep equilibrium in the money market in a system of flexible exchange rates. In either situation, the monetary base has two asset counterparts: the government's outstanding debt (GC_t) and the foreign exchange reserves (F_t).

It is still assumed that households and firms keep all their money in the form of demand deposits issued by the commercial banks, and that the entire monetary base is held as reserves by the banks. Domestic residents do not hold foreign exchange. It is assumed that they buy foreign money at the end of the period when they decide to import the tradable good. However, holdings of foreign money are not a constraint on buying tradable commodities, 1/ because the commercial banks sell it instantaneously on demand at the end of the period. Total holdings of demand deposits issued by commercial banks are the relevant constraint for consumption purchases (of both tradables and nontradables). Also, demand deposits are still the only (tradable) store of value for households. Note finally that only domestic residents hold domestic demand deposits.

Domestic purchases of the nontradable commodity involve a transfer of demand deposits from households to firms with no variation in the total amount of money in the economy. The purchase of an import requires a purchase of foreign money by the domestic household. This purchase will reduce the total amount of demand deposits (on the liability side of the balance sheet of a commercial bank) and the holdings of reserves (on the asset side of the commercial bank's balance sheet). An export will generate the reverse changes.

Trading is sequenced as follows: the labor and credit markets open and clear at the beginning of the period; the markets for tradable and nontradable commodities open and clear at the end of the period. The sequencing of decisions corresponds with this trading arrangement. At the beginning of the period firms decide on their demand for labor and credit, and households decide on their supply of labor. At the end of the period, households decide on the demand for consumption goods (tradable and nontradable) and money.

To highlight the differences with the closed economy case, the activities of the economy during any given period t will now be described. At the beginning of period t, most activities are the same as in the closed economy case: firms and banks distribute profits realized at the end of period t-1 to households (the old generation); firms pay back past loans from the commercial banks, and demand labor from households (the young generation) and loans from the banking system; and the government provides net transfers to the old generation. Under a flexible exchange rate regime, the monetary base is totally determined at the beginning of the period. However, under a system of fixed exchange rates, the beginning of period monetary base differs from the end of period monetary base if there is any net change in the economy's stock of foreign reserves. While the component of the monetary base corresponding to the government's credit outstanding is determined at the beginning of the period under either regime, changes in foreign exchange reserves, in a fixed exchange rate regime, occur at the end of the period when transactions in tradable commodities take place.

After the credit and labor markets clear, production takes place and no further transactions occur until the end of the period when the markets for tradable and nontradable commodities open. At this moment, the young generation decides on its current and future consumption plan and thus, on its demand for money. Both generations demand commodities, and the price of the nontradable good and the equilibrium level of the exchange rate (flexible exchange rate) or the equilibrium level of foreign reserves (fixed exchange rate) are determined.

3. The maximizing behavior of the economic agents

The problems to be solved by the representative bank, firm, and households are now considered. The information assumptions are similar to those used in the closed economy case. Agents do not observe the current period price levels of the commodities at the beginning of the period, nor do they know the future period prices of the commodities at the end of the period. Hence, they have to form expectations about these variables. In addition, agents have to form beginning of period expectations about the exchange rate.

3.1 The bank

The assumption of zero capital mobility implies that there is no foreign borrowing in this economy. In addition, assumptions about the imperfections of the domestic capital markets (banks do not engage in making loans to households, demand deposits are noninterest bearing while banking loans to firms are interest bearing) are retained here and imply that the representative bank's maximization problem may be written:

$$(3.1)\quad \underset{\{C_t^s\}}{\text{Max}}\ \frac{^b\pi_t}{_{at}E(PI_{t+1})} - \frac{i_t}{E(PI_{t+1})}\, C_t^s - 0$$

where: $_{at}E(PI_{t+1})$ = beginning of period expectations of the period t+1 domestic price index

The bank maximize "expected real profits" because it belongs to households who will only receive bank's profits at the beginning of the subsequent period. The only difference between equation (3.1) and the corresponding equation for the closed economy case (equation 2.2) is the choice of the deflator. In the closed economy case, households consumed only one type of good, a nontradable domestically produced output; however, in the open economy case, households' income from all sources (including profits) are used to purchase both tradable and nontradable goods; hence, the relevant deflator is a "consumer price index" (or "general" price level) that will prevail in period t+1. The consumer price index will be defined later on, when we deal with the households' decision problems.

The constraints faced by the representative commercial bank are also similar to the closed economy case, but a specific point, particular to the open economy case, deserves some attention: at the beginning of the period, the central bank has two options with respect to the required reserve ratio. It can be imposed with respect to the monetary base at the beginning of the period.

$$(3.2) \quad {}_aH_t = {}_bH_{t-1} + \Delta GC_t = GC_t + F_{t-1}$$

or with respect to the expected level of the monetary base at the end of the period.

$$(3.3) \quad {}_bH_t = {}_aH_t + \Delta F_t = GC_t + F_t$$

where: $_aH_t$ = beginning of period nominal monetary base

$_bH_t$ = end of period nominal monetary base

GC_t = government's outstanding debt in period t

F_t = stock of foreign exchange reserves in units of domestic currency at the end of period t

In a flexible exchange rate system, equations (3.2) and (3.3) coincides, but in a fixed exchange rate system there may be a difference between $_aH_t$ and $_bH_t$. It is assumed here that the central bank imposes a beginning of period reserve requirement ratio and hence, the commercial bank face the following set of constraints:

$$(3.4) \quad {}_aR_t \geq K\, {}_aD_t$$

$$(3.5) \quad {}_aR_t + C_t^s = {}_aD_t$$

$$(3.6) \quad {}_aR_t \equiv RR_t + RE_t$$

$$(3.7) \quad {}_aR_t = GC_t + F_{t-1} = {}_aH_t$$

where: $_aR_t$ = beginning of period reserves in the vault of the commercial bank

$_aD_t$ = beginning of period level of demand deposits

The maximization procedure will result in a supply of credit function identical to the corresponding equation derived in Chapter II (equation (2.12)):

$$(3.8) \quad C_t^s = \frac{1-K}{K}\, {}_aH_t$$

This result implies that the choice of deflator is irrelevant to the supply of credit function. In addition, notice that the assumption of a beginning of period required reserve ratio is defensible given the fact that the bank supply credit at the beginning of the period. The supply of bank loans would involve a more complicated decision if the bank first had to form expectations about the end of period level of foreign reserves before being able to supply credit, and so this assumption also lends simplicity to our model.

3.2 The firm

The representative domestic firm is assumed to produce both tradable (Y_t^*) and nontradable (Y_t) commodities. The firm engages in a production process that yields joint products. In addition, a single input, domestic labor supply (L_t) is used in the production of the two outputs. In implicit form, the production function faced by the firms is:

$$(3.9) \quad F(Y_t, Y_t^*, L_t) = 0$$

Specifically, the following function is assumed:

$$(3.10) \quad L_t = \frac{1}{2}(Y_t^2 + (Y^*)_t^2)\frac{1}{\phi_t}$$

Equation (3.10), the product transformation curve states costs of production in terms of labor units. 2/ ϕ_t is a productivity shock (as in Chapter II) and is assumed to affect both outputs equally.

The maximization problem for the representative firm becomes:

$$(3.11)\quad \underset{\{Y^s_t,(Y^*)^s_t\}}{\text{Max}} \; {}_tE\left[\frac{{}^f\pi_t}{PI_{t+1}}\right] =$$

$$\underset{\{Y^s_t,(Y^*)^s_t\}}{\text{Max}} \; {}_tE \left\{\frac{P_t}{PI_{t+1}}Y^s_t + \frac{P^*_tS_t}{PI_{t+1}}(Y^*)^s_t - \frac{W_t}{PI_{t+1}}L^d_t - \frac{i_t}{PI_{t+1}}C^d_t\right\}$$

where: Y^s = domestic supply of the nontradable good

$(Y^*)^s_t$ = domestic supply of the tradable good

P_t = price level of the nontradable good

P^*_t = price level of the tradable good

S_t = exchange rate: the domestic currency price of the foreign exchange

The firm maximizes expected real profits. Once more, the expected price index of period t+1 is the relevant deflator because the firm's profits realized during period t are distributed to its owners (households) at the beginning of period t+1.

As in Chapter II, the total cost faced by the firm is the total wage bill plus the interest payments made on bank loans needed to pay labor in advance.

Equation (3.11) is maximized subject to the product transformation curve (equation (3.10)) and to the firms' financial constraint:

$$(3.12)\quad C^d_t = W_tL^d_t$$

which is identical to its closed economy counterpart (equation (2.16)). Given that the only reason for a firm to demand loans is to finance labor, this constraint has to be satisfied as an equality every period. (Notice that the assumption of joint production allows us to treat the demand for credit as a single variable, without having to decompose it according to its uses (the production of tradable or nontradable goods).

In this chapter, the information set that the firm uses at the beginning of the period to form its expectations is assumed to be the same as the information set for the closed economy case:

$$(3.13)\quad \Omega_{at} = \Omega_{t-1}, W_t, i_t, \phi_t, \mu_1$$

That is, it is assumed here that under either exchange rate regime, the price level of the tradable good and the exchange rate are not observable at the beginning of the period. However, an alternative information set will also be considered in the next two chapters.

The maximization procedure leads to the following supply function for the nontradable good:

$$(3.14)\quad Y_t^s = \frac{a_t E(P_t)}{W_t(1+i_t)} \phi_t$$

The supply function for the tradable commodity is:

$$(3.15)\quad (Y^*)_t^s = \frac{a_t E(P_t^* S_t)}{W_t(1+i_t)} \phi_t$$

Several results emerge from these decisions rules: first, as in the commercial bank maximization problem, the choice of deflator does not affect the results. Second, the supply of each commodity depends positively on its own price and on a "common" productivity shock, and negatively on the "effective" cost of labor: $W_t(1+i_t)$

Substituting equations (3.14) and (3.15) into equation (3.10) yields the demand for labor:

$$(3.16)\quad L_t^d = \frac{1}{2}\left[\left(\frac{a_t E(P_t)}{W_t(1+i_t)}\phi_t\right)^2 + \left(\frac{a_t E(P_t^* S_t)}{W_t(1+i_t)}\phi_t\right)^2\right]\frac{1}{\phi_t}$$

$$= \frac{1}{2}\phi_t\left[\frac{(a_t E(P_t))^2 + (a_t E(P_t^* S_t))^2}{(W_t(1+i_t))^2}\right]$$

The demand for labor depends positively on the expected money prices of the final outputs and on the productivity shock, and negatively on the effective cost of labor.

Finally, equation (3.12) can be used to derive the firms' demand for credit:

$$(3.17)\quad C_t^d = \frac{W_t\,\phi_t\left[(a_t E(P_t))^2 + (a_t E(P_t^* S_t))^2\right]}{2[W_t(1+i_t)]^2}$$

3.3 Households

As in Chapter II, households live two periods and maximize expected utility. The representative agent of the old generation spends his total money holdings in exchange for consumption goods at the end of any period, while the representative agent of the young generation chooses his supply of labor at the beginning of that period and his demand for commodities (current and future) at its end. In contrast to the closed economy case, in the open economy the young agent decides not only on his total level of consumption, but also on the composition between tradables and non-tradables.

When young, the agent maximizes the following expected utility function:

$$(3.18) \quad U = \mu_1 \ln [Y_1^{\tau}(Y^*)_1^{(1-\tau)}] + \ln(\bar{L}-L_1) + \beta Y_2^{\tau} (Y^*)_2^{(1-\tau)}$$

where the subscript 1 is used to indicate the agent's first period of life and 2 to indicate his second and last period. μ_1 refers to a random shock affecting the utility derived from current consumption, β is the rate of time preference ($0<\beta<1$) and $(\bar{L}-L_1)$ is the amount of leisure available to the agent after he has supplied labor.

The utility function (3.18) is additive separable in consumption and leisure and shows constant marginal utility of future consumption. In addition, the separability property makes it possible to rewrite (3.18) as:

$$(3.18') \quad U = \mu_1 \; f_a[(Y_1^d),(Y^*)_1^d] + f_b[\bar{L}-L_1] + f_c[(Y_2^d), (Y^*)_2^d]$$

where f_a, f_b, and f_c are the subutility functions associated with current consumption, leisure and future consumption, respectively. The use of Cobb-Douglas functions to specify f_a implies that the derived elasticities of demand for current consumption of both tradable and nontradable commodities with respect to <u>total</u> expenditure on <u>current</u> consumption is equal to one. By the same token, the elasticities of demand for future consumption of both tradable and nontradable commodities with respect to <u>total</u> expenditure on <u>future</u> consumption is equal to one.

The representative agent of the young generation maximizes the expected value of (3.18) subject to his lifetime budget constraint:

$$(3.19) \quad P_2Y_2^d + (P_2^*S_2)(Y^*)_2^d = W_1L_1^s + Tr_2 + {}^h\pi_2 - P_1Y_1^d - (P_1^*S_1)(Y)_1^d$$

where: Tr = net government transfers

${}^h\pi_2$ = income from profits received by the household in period 2,

and to the condition that the young agent's demand for money be nonnegative:

$$(3.20) \quad W_1L_1^s - P_1Y_1^d - P_1^*S_1(Y^*)_1^d > 0$$

The inequality constraint (3.20) is necessary to satisfy the assumption that all transactions are carried out with money, given that the young generation starts life with no money. As in the closed economy case, it is assumed that (3.20) is always positive. The justification for this assumption is provided in Chapter II, and hence no further comments are necessary here.

The representative young agent faces a two-stage decision problem: at the beginning of the period, he chooses his labor supply, taking into account the available and relevant information at his disposal at that moment. At the end of the period, the agent decides on his demand for commodities and money taking into account the information available at that time. To illustrate the agent's maximization problem, let us first consider the end of the period when he already knows his labor supply.

At the end of period 1, the agent will choose his current and future consumption levels of tradable and nontradable goods. He knows that in period 2, he will have a nominal wealth: "A_2" and will have to choose Y_2 and Y_2^* in order to:

(3.21) $\text{Max } Y_2^{\tau} Y_2^{*(1-\tau)}$

subject to:

(3.22) $A_2 = P_2 Y_2^d + (P_2^* S_2)(Y^*)_2^d$

From (3.21) and (3.22) the demand functions for the agent's second period of life are obtained:

(3.23) $Y_2^d = \tau A_2/P_2$

(3.24) $(Y^*)_2^d = (1-\tau)A_2/P_2^* S_2$

As stated above, these demand functions show a unitary elasticity of demand with respect to expenditure on future consumption (A).

Given (3.23) and (3.24), the maximized value of $Y_2^{\tau} (Y^*)_2{}^{(1-\tau)}$ is

(3.25) $(Y_2^{\tau} (Y^*)_2{}^{(1-\tau)})_{max} = A_2/\tau^{-\tau} (1-\tau)^{-(1-\tau)} P_2^{\tau}(P_2^* S_2)^{1-\tau}$

$\equiv A_2/j' PI_2$

where: $j' \equiv \tau^{-\tau}(1-\tau)^{-(1-\tau)}$

$PI_2 \equiv$ price index in period 2

$\equiv P_2^{\tau}(P_2^* S_2)^{1-\tau}$

The price index PI is homogenous of degree one, due to the Cobb-Douglas specification of the utility function. Such a price index is the relevant deflator for the maximization problem of all the agents in this economy. Substituting (3.25) into (3.18) and noticing that the budget constraint (3.19) requires:

(3.26) $A_2 = W_1L_1 + Tr_2 + {}^h\pi_2 - P_1Y_1^d - (P_1^*S_1)(Y^*)_1^d,$

the first period problem is obtained. This requires the agent to choose Y_1^d, $(Y^*)_1^d$ to:

$$(3.27)\quad \underset{\{Y_1^d,(Y^*)_1^d,\lambda_3\}}{\text{Max}} = \mu_1 \ln\,[Y_1^\tau (Y^*)_1^{(1-\tau)}] + \ln(\overline{L}-L_1)$$

$$+ \frac{\beta}{j'}\, {}_{b1}E\left[\frac{W_1L_1+Tr_2+{}^h\pi_2-P_1Y_1^d-(P_1^*S_1)(Y^*)_1^d}{PI_2}\right]$$

$$+ \lambda_3[W_1L_1^s - P_1Y_1^d - P_1^*S_1(Y^*)_1^d]$$

where λ_3 is the Kuhn-Tucker multiplier associated with the constraint (3.20).

The information set available to the agent at the end of period 1 is independent of the exchange rate regime, because under either regime it is assumed that the current exchange rate and the price level of the tradable good are known at the end of the period. That is:

$$(3.28)\quad \Omega_{b1} = \Omega_{a1},\ P_1,\ S_1/P_1^*$$

where the information set at the beginning of period 1 (Ω_{a1}) is identical to the set faced by firms and is given by equation (3.13).

The maximization procedure, under the assumption of $\lambda_3 = 0$ (see Chapter II, Section 3.3), leads to the following decisions rules:

$$(3.29)\quad Y_1^d = \frac{j'\tau}{\beta}\ \mu_1\ \frac{{}_{b1}E(PI_2)}{P_1}$$

$$(3.30)\quad (Y^*)_1^d = \frac{j'(1-\tau)}{\beta}\,\mu_1\,\frac{b_1E(PI_2)}{P_1^*S_1}$$

That is, current consumption of tradable and nontradable commodities depend positively on expected inflation, where the relevant future price level is the "general" price level or "price index", while the relevant current price level is the own commodity price. More specifically, current consumption of each commodity depends positively on expected inflation of the general price index, and negatively on its own price relative to the general price index. 3/ The decisions rules (3.29) and (3.30) also depend positively on a random term (μ_t) which affects the marginal utility of (total) current consumption. 4/

Now, we can turn to the young agent's maximization problem at the beginning of the period. At that time, the individual's demands for commodities are stochastic variables. Defining:

$$(3.31)\quad \tilde{Y}_1^d = \frac{j'\tau}{\beta}\,\mu_1\,\frac{b_1E(PI_2)}{P_1}$$

and:

$$(3.32)\quad (\tilde{Y}^*)_1^d = \frac{j'(1-\tau)}{\beta}\,\mu_1\,\frac{b_1E(PI_2)}{P_1^*S_1}$$

the agent's problem is:

$$(3.33)\quad \underset{\{L_1^s\}}{\text{Max:}}\ \mu_1\ {}_{a1}E\,[\ln((\tilde{Y})_1^{\tau}(\tilde{Y}^*)_1^{(1-\tau)})] + \ln(\bar{L}-L_1)$$

$$+\ \frac{\beta}{j'}\ {}_{a1}E\left(\frac{W_1L_1+Tr_2+{}^h\pi_2-P_1\tilde{Y}_1-P_1^*S_1\tilde{Y}_1^*}{PI_2}\right)$$

and the following labor supply decision rule is obtained:

$$(3.34)\quad L_1^s = \bar{L} - \frac{1'}{\beta}\ \frac{a_1 E(PI_2)}{W_1}$$

That is, labor supply depends positively on the expected future real wage where the relevant deflator is the expected future price index.

Equations (3.29) and (3.30) represent the consumption demands of the young generation. To obtain the total market demands for consumption goods we have to add to these the old generation demands. Such demands have already been obtained in equations (3.23) and (3.24) where the solution to the agent's second period problem was presented.

It can easily be proved that: 5/

$$(3.35)\quad A_t \equiv W_{t-1}L_{t-1} + Tr_t + {}^h\pi_t - P_{t-1}Y^d_{t-1} - (P^*_{t-1}S_{t-1})\ (Y^*)^d_{t-1}$$

$$\equiv {}_aH_t$$

Hence, the old generation demands can be written as:

$$(3.36)\quad {}^oY^d_t = \tau\,\frac{{}_aH_t}{P_t}$$

$$(3.37)\quad {}^o(Y^*)^d_t = (1-\tau)\frac{{}_aH_t}{P^*_t S_t}$$

where the "o" superscript refers to the "old" generation. That is, the agent's second period consumption of each commodity is a constant fraction of the beginning-of-period real monetary base, where the fractions are equal to the corresponding "weights" used in the utility function.

Adding equations (3.29) and (3.36) and equations (3.30) and (3.37), the total current demand for nontradables (Z_t^d) and tradables ($(Z^*)_t^d$), respectively, will be obtained.

$$(3.38) \quad Z_t^d = \frac{j'\tau}{\beta} \mu_t \frac{b_t E(PI_{t+1})}{P_t} + \tau \frac{a H_t}{P_t}$$

$$(3.39) \quad (Z^*)_t^d = \frac{j'(1-\tau)}{\beta} \mu_t \frac{b_t E(PI_{t+1})}{P_t^* S_t} + (1-\tau)\frac{a H_t}{P_t^* S_t}$$

Finally, the household's demand for money (${}^hD_t^d$) can be derived by substituting equations (3.29) and (3.30) into the individual's first period budget constraint:

$$(3.40) \quad {}^hD_t^d = W_t L_t - P_t Y_t^d - P_t^* S_t (Y^*)_t^d$$

$$= W_t L_t - \frac{j'\tau}{\beta} \mu_t \, b_t E(PI_{t+1}) - \frac{j'}{\beta}(1-\tau)\mu_t \, b_t E(PI_{t+1})$$

$$= W_t L_t - \frac{j'}{\beta} \mu_t \, b_t E(PI_{t+1})$$

or:
$$\frac{{}^hD_t^d}{PI_t} = \frac{W_t}{PI_t} L_t - \frac{j'}{\beta} \mu_t \frac{b_t E(PI_{t+1})}{PI_t}$$

That is, the household's real demand for money function is similar to that derived in the closed economy case. The only difference is that the general price index is now the relevant variable affecting the agent's decision to hold money.

4. The aggregate supply and demand functions

4.1 The aggregate supply functions

In Section 3, domestic supply equations for the tradable and non-tradable commodities were derived as functions of the wage rate and the interest rate. However, the equilibrium conditions in the labor and credit markets assumed to hold at the beginning of the period, determine both the nominal wage rate and the interest rate. This section will obtain the solutions for the wage rate and the interest rate, and thus the final form of the aggregate supply functions.

As in the closed economy case, the model will be cast in log-linear terms using lower case letters to represent the log of a variable (once more, the exception will be i_t which stands for the observed value of the interest rate).

Based on equation (3.16), a log-linear version of the demand for labor is:

$$(3.41)\quad l_t^d = \xi_o + \ln[({}_{at}E(P_t))^2 + ({}_{at}E(P_t^* S_t))^2] - 2(w_t + i_t) + u_t$$

where: $u_t = \log \phi_t \sim N(0,\sigma_u^2)$, and u_t is independent of the rest of disturbances in the model.

Now, the term: $\ln[({}_{at}E(P_t))^2 + ({}_{at}E(P_t^* S_t))^2]$ can be approximated as:

$$(3.42)\quad \ln[({}_{at}E(P_t))^2 + ({}_{at}E(P_t^* S_t))^2] = \xi_1 + \xi_2 \ln({}_{at}E(P_t))^2 + \xi_3 \ln({}_{at}E(P_t^* S_t))^2$$
$$= \xi_1 + 2\xi_2\, {}_{at}p_t^e + 2\xi_3({}_{at}(p^*)_t^e + {}_{at}s_t^e)$$
$$= {}^f pi_t$$

where the expectations operator E has been replaced by the superscript "e".

The expression represented by equation (3.42) can be interpreted as the relevant price index (${}^f pi_t$) for the firm's decision problem. However, to deal with a firm's price index that differs from the consumer's price index would unnecessarily complicate the model; hence, it will be assumed that:

$$\frac{\xi_2}{\xi_2+\xi_3} \equiv \tau \qquad \text{and:} \quad \xi_2 + \xi_3 \equiv 1$$

$$\frac{\xi_3}{\xi_2+\xi_3} \equiv (1-\tau)$$

where τ and $(1-\tau)$ are the "weights" used in the consumer price index. Therefore, (3.42) can be written as:

$$(3.43) \quad {}^{f}pi_t = \xi_1 + 2(\tau\,{}_{at}p_t^e + (1-\tau)({}_{at}(p^*)_t^e + {}_{at}s_t^e))$$

Substituting (3.43) into (3.41), the firm's demand of labor can be approximated as: 6/

$$(3.44) \quad l_t^d = \alpha_o + \alpha_1[{}_{at}pi_t^e - w_t - i_t] + u_t$$

where: $\alpha_o \equiv \xi_o + \xi_1$

$\alpha_1 \equiv 2$

$pi_t \equiv$ consumer's price index

$\equiv \tau\, p_t + (1-\tau)(p_t^* + s_t)$

Similarly, based on equation (3.12) a log-linear approximation for the demand for credit is:

$$(3.45) \quad c_t^d = \beta_o + \alpha_1[{}_{at}pi_t^e - i_t] + (1-\alpha_1)\, w_t + u_t$$

Log-linear versions for the supply of labor and credit are based on equations (3.34) and (3.8), respectively. Hence:

$$(3.46) \quad l_t^s = \gamma_o + \gamma_1(w_t - {}_{at}pi_{t+1}^e)$$

and:

$$(3.47) \quad c_t^s = k + {}_{a}h_t$$

The system of equations (3.44) through (3.47) can now be compared to its closed economy counterpart formed by equations (2.42) through (2.45). Recalling that in the closed economy model, $\alpha_1 = \beta_1$ and $\gamma_1 = \gamma_2$, the only differences between the two systems are: (a) the relevant price level in the open economy case is the general price index in contrast with the price level of the single output in the closed economy case; and (b) in the open economy case, a productivity term affects the demands for labor and credit with a unitary coefficient, while in the closed economy case, a productivity term affects such demands with a coefficient equal to the sensitivity of the demand for labor to its "effective" cost (α_1).

Equations (3.44) through (3.47) and the market equilibrium conditions can now be used to solve for the equilibrium levels of the nominal wage rate and the nominal interest rate as functions of the monetary base, the expectations about prices and the random term to productivity.

Equilibrium between the demand and supply of labor gives rise to the following equation for the wage rate:

$$(3.48)\quad w_t = \frac{\alpha_o+\gamma_o}{\alpha_1+\gamma_1} - \frac{\alpha_1}{\alpha_1+\gamma_1} i_t + \frac{\alpha_1}{\alpha_1+\gamma_1} {}_{at}pi^e_t + \frac{\gamma_1}{\alpha_1+\gamma_1} {}_{at}pi^e_{t+1} + \frac{1}{\alpha_1+\gamma_1} u_t$$

Also, equilibrium between the demand and supply of credit generates the following equation:

$$(3.49)\quad i_t = \frac{\beta_o-k}{\alpha_1} - \frac{1}{\alpha_1} h_t + {}_{at}pi^e_t + \frac{(1-\alpha_1)}{\alpha_1} w_t + \frac{1}{\alpha_1} u_t$$

Most features of equations (3.48) and (3.49) are similar to their closed economy counterpart of Chapter II and do not need further discussion. The only difference lies on the effect of the productivity term. In the closed economy case, both the productivity term and the effective nominal cost of labor (w_t+i_t) affected the demands for credit and labor in the same way. Hence, the impact effect of a positive productivity change was a proportional rise in the interest rate (given the inelastic supply of credit). However, in the open economy model, the choice of the product transformation function implies a unitary elasticity of the demands for labor and credit with respect to the productivity term. Hence, the impact effect of a positive random productivity change on the interest rate will now depend on the sensitivity of the demand for labor to the effective cost of labor (α_1).

Given the close interrelation between the labor and credit markets, the equilibrium levels of w_t and i_t require the simultaneous solution of both markets. 7/ Solving the model represented by equations (3.44) through (3.47) yields:

$$(3.50)\quad w_t = {}_{at}pi_t^e + \beta_0' + \beta_1'({}_{at}pi_{t+1}^e - {}_{at}pi_t^e) + \beta_2'({}_a h_t - {}_{at}pi_t^e)$$

and:

$$(3.51)\quad i_t = \gamma_0' - \gamma_1'({}_a h_t - {}_{at}pi_t^e) + \gamma_2'({}_{at}pi_{t+1}^e - {}_{at}pi_t^e) + \gamma_3' u_t$$

where:

$$\gamma_3' = \frac{1}{\alpha_1}$$

and β_0', β_1', β_2', γ_0', γ_1', and γ_2' are defined exactly equal as the corresponding coefficients of the closed economy case. 8/

As in Chapter II, $\beta_1' + \beta_2' = 1$, implying that, ceteris paribus, an increase in the current price index will have no effects on the nominal wage (the real wage will decrease proportionally). Also, as in the closed economy case, u_t does not affect the equilibrium level of the wage rate, while it affects the interest rate positively. However, in the closed economy case, a random change in productivity affected the interest rate with a unitary coefficient leaving the level of employment unchanged. This result arose because both the effective cost of labor and the productivity term entered the demand for labor with the same coefficient (α_1). In the open economy case, the particular specification chosen for the product transformation curve implies that the productivity term affects the demand for labor with a unitary coefficient. Thus, when a positive productivity shock occurs, the interest rate has to increase by ($1/\alpha_1$) in order to clear the labor and credit markets simultaneously.

The solutions for the wage rate and the interest rate can now be substituted into the following log-linear approximations of the supply functions for nontradable and tradable goods obtained in Section 3 (equations (3.14) and (3.15)).

(3.52) $y_t^s = - w_t - i_t + {}_{at}p_t^e + u_t$

(3.53) $(y^*)_t^s = - w_t - i_t + {}_{at}(p^*)_t^e + {}_{at}s_t^e + u_t$

The unitary coefficients (in absolute value terms) accompanying the arguments of y_t^s and $(y^*)_t^s$ are due to the specification of the production side of the model, which implies a unitary own price elasticity and a unitary factor price elasticity for the output supplies.

Substituting equations (3.50) and (3.51) into equations (3.52) and (3.53), the aggregate supply functions of outputs are obtained:

$$(3.54)\quad y_t^s = a_0 + a_1({}_a h_t - {}_{at}pi_{t+1}^e) + ({}_{at}p_t^e - {}_{at}pi_t^e) + a_3 u_t$$

$$(3.55)\quad (y^*)_t^s = a_0 + a_1({}_a h_t - {}_{at}pi_{t+1}^e) + ({}_{at}(p^*)_t^e + {}_{at}s_t^e - {}_{at}pi_t^e) + a_3 u_t$$

where: $a_0 = -(\beta_0 + \gamma_0')$

$$a_1 = \frac{\gamma_1}{\alpha_1 + \alpha_1 \gamma_1}$$

$$a_3 = 1 - \gamma_3'$$

The most important features of equations (3.54) and (3.55) are as follows. First, the only diffeteñce between the two functions lies in the third terms on the right-hand sides of the equations: the relative price terms where the own price of each commodity is related to the general price index. This difference is due to the fact that, although both commodities share the input markets indistinguishably, they face separate output markets. Each commodity supply has a unitary elasticity with respect to the corresponding relative price term due to: (a) the unitary own price elasticity of supply; (b) the unitary factor price elasticity of supply; and (c) the homogenity of degree one of the nominal wage rate with respect to the price index. Second, the supply of both types of commodities depend positively on the expected real monetary base evaluated at the future level of the price index. This result is identical to the one obtained in the closed economy case since in Chapter II, $\gamma_1 = \gamma_2$; hence, no further discussion is needed. Third, the random term affecting productivity affects the aggregate supplies with a positive coefficient less than one because in the open economy u_t has a nonproporrtional effect on the interest rate.

4.2 The aggregate demand functions

Equations (3.38) and (3.39) can be treated as the aggregate demand function for nontradables and tradables, respectively, because of the "representative individual" assumption. Equation (3.40) represents the households' aggregate demand for money. To obtain the total demand for money, the end of period demand for money by firms (equal to the value of final outputs) has to be added to equation (3.40). Thus:

$$(3.56)\quad D_t^d \equiv {}^hD_t + {}^fD_t$$

$$D_t^d \equiv W_tL_t - P_tY_t^d - P_t^*S_t(Y^*)_t^d + P_tY_t^s + P_t^*S_t(Y^*)_t^s$$

The three markets (the two commodities markets and the money market) clear at the end of every period. The eqilibrium conditions for the nontradable good and money markets are always:

$$(3.57)\quad Y_t^s = Z_t^d$$

and

$$(3.58)\quad D_t^s = D_t^d$$

However, the equilibrium condition for the tradable commodity market depends on the exchange rate regime in the following way:

$$(3.59) \quad (Y^*)^s_t = (Z^*)^d_t \qquad \text{under flexible exchange rate}$$

$$(3.60) \quad \frac{\Delta F_t}{P^*_t S_t} = (Y^*)^s_t - (Z^*)^d_t \qquad \text{under fixed exchange rate}$$

To prove that equations (3.59) and (3.60) effectively are the equilibrium condition in the market for tradable goods, consider the following: assuming equilibrium in the market for nontradable goods, Walras' law assures that equilibrium between the demand and supply of money implies equilibrium in the market for tradable commodities. Hence, the equilibrium characteristics of the money market under alternative exchange rate regimes will now be analyzed.

Under a flexible exchange rate system, the exchange rate will completely adjust to clear the money market and, therefore, an endogenous variation in the level of reserves will not occur. Under that circumstance, the end of period money supply will be:

$$(3.61) \quad D^s_t \equiv {}_aH_t + C^s_t$$

Equilibrium in the money market will be obtained by equating equation (3.56) to (3.61). Hence:

$$(3.62) \quad {}_aH_t + C^s_t = W_tL_t - P_tY^d_t - P^*_tS_t(Y^*)^d_t + P_tY^s_t + P^*_tS_t(Y^*)^s_t$$

Recalling that $C^s_t = W_tL_t$ and using equation (3.38) and the equilibrium condition (3.57), equation (3.62) can be written as:

$$(3.63)\quad (Y^*)^s_t = \frac{{}_aH_t}{P^*_tS_t} - \frac{P_t}{P^*_tS_t}\left(\frac{\tau_aH_t}{P_t}\right) + (Y^*)^d_t$$

or

$$(3.63')\quad (Y^*)^s_t = (1-\tau)\ \frac{{}_aH_t}{P^*_tS_t} + (Y^*)^d_t$$

The right-hand side of equation (3.63') is equal to the aggregate demand for tradable commodities; hence, the equilibrium conditions (3.57) and (3.58) imply $(Y^*)^s_t = (Z^*)^d_t$ in a system of flexible exchange rates.

Next, consider a fixed exchange rate regime. In such a state the level of foreign reserves will adjust to clear the money market; hence, the end of period money supply will be:

$$(3.64)\quad D^s_t = {}_aH_t + C^s_t + \Delta F_t$$

Equilibrium in the money market will now imply:

$$(3.65)\quad \Delta F_t = P_tY^s_t + P^*_tS_t(Y^*)^s_t - P_tY^d_t - P^*_tS_t(Y^*)^d_t - {}_aH_t$$

Using equations (3.38) and (3.39), equation (3.65) can be rewritten as:

$$(3.66)\quad \Delta F_t = P_tY^s_t + P^*_tS_t(Y^*)^s_t - P_t\left(Z^d_t - \frac{\tau_aH_t}{P_t}\right)$$

$$- P^*_tS_t\ \left((Z^*)^d_t - (1-\tau)\frac{{}_aH_t}{P^*_tS_t}\right) - {}_aH_t$$

The equilibrium condition (3.57) will then imply:

$$(3.67)\quad \frac{\Delta F_t}{P^*_t S_t} = (Y^*)^s_t - (Z^*)^d_t$$

The results obtained above have a straightforward explanation: under the assumption of zero capital mobility and fixed exchange rates, the overall balance of payments is identical to the balance of trade, which in this model is equal to the difference between the value of the supply and demand for tradable goods; that is, only net exports can be explained in this model, but not the composition between exports and imports. Such is the nature of equation (3.67). Under a flexible exchange rate system, the level of foreign reserves will not change ($\Delta F=0$) implying that $(Y^*)^s_t=(Z^*)^d_t$. Notice that this result does not mean zero trade; it means only that <u>net</u> exports equal zero; that is, the value of exports must be equal to the value of imports.

5. <u>The complete macro-model</u>

The results from the previous sections can now be consolidated into the macro-model to be analyzed in the next chapters. By Walras' law we can use only the tradable and nontradable commodity markets to represent the entire macro system. 9/ The complete model consists, then, of the following set of equations (expressed in log terms):

$$(3.68)\quad z^d_t = \delta_0 + \delta_1({}_{bt}pi^e_{t+1} - p_t) + \delta_2({}_ah_t - p_t) + \epsilon_t$$

$$(3.69)\quad (z^*)^d_t = b_0 + b_1({}_{bt}pi^e_{t+1} - p^*_t - s_t) + b_2({}_ah_t - p^*_t - s_t) + \epsilon_t$$

$$(3.70)\quad y^s_t = a_0 + a_1({}_ah_t - {}_{at}pi^e_{t+1}) + ({}_{at}p^e_t - {}_{at}pi^e_t) + a_3u_t$$

$$(3.71)\quad (y^*)^s_t = a_0 + a_1({}_ah_t - {}_{at}pi^e_{t+1}) + ({}_{at}(p^*)^e_t + {}_{at}s^e_t - {}_{at}pi^e_t) + a_3u_t$$

$$(3.72)\quad pi_t = \tau\, p_t + (1-\tau)(p_t^* + s_t)$$

$$(3.73)\quad {}_a h_t = \omega_0 + \omega_1\, gc_t + (1 - \omega_1) f_{t-1}$$

$$(3.74)\quad y_t^s = z_t^d$$

$$(3.75a)\quad (y^*)_t^s = (z^*)_t^d \qquad \text{under flexible exchange rate}$$

or:

$$(3.75b)\quad f_t = d_1 (y^*)_t^s - d_2 (z^*)_t^d + (1-d_3)(p_t^* + s_t) + d_3 f_{t-1}$$

under a fixed exchange rate

Equations (3.68) and (3.69) are log-linear approximations of equations (3.38) and (3.39), respectively, with ε_t being a serially independent log normal disturbance which is also independent of the rest of disturbances of the model, where:

$$\varepsilon_t \doteq \log \mu_t \sim N(0, \sigma_\varepsilon^2)$$

Equations (3.70) and (3.71) are identical to equations (3.54) and (3.55), respectively, and are repeated here only for convenience. These first four equations characterize the domestic markets for tradable and nontradable commodities.

Equation (3.72) is a log approximation of the price index already discussed above. Equation (3.73) is a log linear approximation (based on a Taylor expansion) of the identity for the beginning of period monetary base (equation 3.2). The equilibrium condition in the nontradable goods market is represented by equation (3.74), while the equilibrium in the market for tradable goods is expressed by equation (3.75a) under a flexible exchange rate or by (3.75b) under a fixed exchange rate system. Equation (3.75b) is a log approximation of equation (3.67) of the following form: 10/

$$(3.76) \quad q_1 \log F_t = \log((Y^*)^s_t (P^*_t S_t)) - (1-q_1-q_2) \log F_{t-1} - q_2 \log (Z^*)^d_t (P^*_t S_t)$$

or

$$(3.76') \quad \log F_t = d_1 \log(Y^*)^s_t - d_2 \log(Z^*)^d_t + (1-d_3) \log(P^*_t S_t) + d_3 \log F_{t-1}$$

where: $d_1 = 1/q_1$

$d_2 = q_2/q_1$

$d_3 = -(1-q_1 - q_2)$; $d_1, d_2, d_3 > 0$

and $(q_1 + q_2) > 1$. So $(1 - q_1 - q_2) < 0$.

Equation (3.75b) is identical to equation (3.76').

The model represented by equations (3.68) through (3.75) supplemented by the assumption of rational expectations can be solved and analysed for each exchange rate regime. This will be done in the next two chapters.

FOOTNOTES TO CHAPTER III

1/ This assumption contrasts with other models that highlight the transactions role of money in the open economy. For example, Stockman (1980) assumes that current imports are limited by initial holdings of foreign exchange.

2/ The productivity term in the denominator means that a positive random increase in productivity will result in a lower amount of labor required to produce a given amount of outputs.

3/ Notice that because of the use of a Cobb Douglas utility function, the demand for the tradable (nontradable) good does not depend on the current price level of the nontradable (tradable) good.

4/ Notice also that the ratio of the demand for the nontradable good to the demand for the tradable good equals:

$$\frac{Y_1^d}{(Y^*)_1^d} = \frac{\tau P_1 S_1^*}{(1-\tau)P_1}$$

which implies that the ratio of relative prices varies proportionally with the ratio of consumption demands by the young generation.

5/ Recall that:

(a) $W_{t-1}L_{t-1} - (P_{t-1}Y_{t-1}^d + P_{t-1}^* S_{t-1}(Y^*)_{t-1}^d) \equiv {}^yD_{t-1}$

(b) ${}^h\pi_t \equiv P_{t-1}Y_{t-1}^s + P_{t-1}^* S_{t-1}(Y^*)_{t-1}^s - W_{t-1}L_{t-1}$

$\equiv {}^fD_{t-1} - W_{t-1}L_{t-1}$

(c) $W_{t-1}L_{t-1} \equiv C_{t-1}^s$

(d) ${}_aH_t \equiv {}_bH_{t-1} + Tr_t$

(e) ${}_bH_{t-1} \equiv D_{t-1} - C_{t-1}$

(f) $D_{t-1} = {}^yD_{t-1} + {}^fD_{t-1}$

From (a) to (f):

$$_aH_t \equiv {}^yD_{t-1} + {}^fD_{t-1} - C_{t-1} + Tr_t$$

$$\equiv W_{t-1}L_{t-1} - P_{t-1}Y^d_{t-1} - P^*_{t-1}S_{t-1}(Y^*)^d_{t-1}$$

$$+ {}^h\pi_t + W_{t-1}L_{t-1} - C_{t-1} + Tr_t$$

$$\equiv W_{t-1}L_{t-1} + {}^h\pi_t + Tr_t - P_{t-1}Y^d_{t-1} - P^*_{t-1}S_{t-1}(Y^*)^d_{t-1}$$

$$\equiv A_t$$

6/ To save on the notation, the coefficients αs, βs, and γs used in the closed economy case will also be used in the corresponding equations of the open economy case.

7/ It should be recalled that these solutions are not final solutions because the price level (and its expectations) are taken as exogenous for the time being.

8/ Remember that in the closed economy case of Chapter II: $\gamma_1 = \gamma_2$ and $\alpha_1 = \beta_1$.

9/ Assuming equilibrium in the markets for tradable and nontradable commodities imply equilibrium in the money market.

10/ Equation (3.67) can be rewritten as:

$$(Y^*)^s_t\ (P^*_tS_t) = \Delta F_t + (Z^*)^d_t\ (P^*_tS_t)$$

By taking first differences and manipulating the above expression, we obtain:

$$\frac{\Delta((Y^*)^s_t\ (P^*_tS_t))}{(Y^{*s}_t)(P^*_tS_t)} = \frac{\Delta F_t}{F_t}\ \frac{F_t}{(Y^*)^s_t(P^*_tS_t)}$$

$$- \frac{\Delta F_{t-1}}{F_{t-1}}\ \frac{F_{t-1}}{(Y^*)^s_t\ (P^*_tS_t)}$$

$$+ \frac{\Delta((Z^*)^d_t\ (P^*_tS_t))}{(Z^*)^d_t\ (P^*_tS_t)}\ \frac{(Z^*)^d_t\ (P^*_tS_t)}{(Y^*)^s_t\ (P^*_tS_t)}$$

which equals equation (3.76), by defining:

$$q_1 = F_t/((Y^*)^s_t\ (P^*_t S_t))$$

$$q_2 = ((Z^*)^d_t\ (P^*_t S_t))/((Y^*)^s_t\ (P^*_t S_t))$$

$$(1-q_1-q_2) = -\ F_{t-1}/((Y^*)^s_t\ (P_t S_t))$$

CHAPTER IV

PRICE AND OUTPUT FLUCTUATIONS IN A FLEXIBLE EXCHANGE RATE REGIME

1. Introduction

This chapter solves the open economy model set out in Chapter III for the flexible exchange rate case in a flexible price-rational expectations framework. In contrast with most of the literature on flexible exchange rates which uses the rational expectations hypothesis (see for example Parkin, Bentley, and Fader (1979), Cox (1980) and Burton (1980)), we will find that the results do not depend on price misperceptions (a la Lucas) affecting the aggregate supply of output. Rather the central assumption will turn out to be that firms need to finance labor purchases in advance in an environment with limited capital markets, that is, the "financial constraint" assumption used throughout this thesis.

Even though it will be shown that a fully anticipated increase in the rate of growth of the monetary base, or a fully anticipated temporary monetary decrease, will result in a relative price effect favoring an increase in the supply of one commodity, and a decrease in the supply of the other, the negative financial constraint effect arising from such monetary changes will be found to dominate the relative price effect and the level of output of <u>both</u> commodities will decrease. In addition, under the assumption that agents cannot distinguish between permanent and temporary monetary shocks, even a permanent monetary decrease will have real effects and will cause the exchange rate to "overshoot" relative to its full current information level. For related reasons, money is also non-superneutral in cash-in-advance models that treat output as an endogenous variable. For example, Aschauer and Greenwood (1984) show that the economy's steady state levels of consumption, employment, and output are inversely related to its steady state-inflation rate. As it will be shown below, the model of this chapter proves that those results hold even in the short run.

Another feature of the model is that a change in the price of the tradable good (or in its rate of growth) anticipated or not, will only result in a proportional and inverse change in the exchange rate; it will leave the price level of the nontradable good and the output level of both commodities unchanged. Hence, the exchange rate will not overshoot in response to unanticipated changes in the price level of the tradable good. As is usual in rational expectations models, the insulation effect of a flexible exchange rate against changes in the foreign price level (the "insulation proposition") does not depend on the degree of capital mobility but on the assumptions regarding the information set on which economic agents base their decisions. In particular, in the model analyzed in this chapter, the price level of the tradable good and the exchange rate always enter together in both alternative information sets used in the solution of the model. If the exchange rate were assumed to be observable at the beginning of the period while the price level of the tradable good only became observable at the end of the period, the insulation proposition would not hold in this model, even with the assumption of zero capital mobility.

Some of the literature on flexible exchange rates under zero capital mobility and rational expectations has obtained a response of domestic variables to a change in the foreign price level by assuming that the relevant opportunity cost of holding money is the expected change in the exchange rate. For instance, Barro (1978) uses the assumption of purchasing power parity and Saidi (1980) assumes "currency substitution" in order to introduce the expected change in the exchange rate as an argument in the demand for money function. Both obtain the result that an anticipated temporary increase of the foreign price level (or a decrease in the rate of growth of world prices (in Barro's model) or an unanticipated permanent world price increase (in Saidi's model)), will increase the expected depreciation of the exchange rate and hence will decrease the real demand for money, and increase the domestic price level. By contrast, in the model of this chapter, the opportunity cost of holding money is the expected domestic inflation rate which is influenced by the domestic price level of both commodities. Since the price level of the tradable good and the exchange rate always enter together in the demand and supply functions of this model, changes in the price level of the tradable good will only result in inverse and proportional changes in the exchange rate, leaving the domestic inflation rate unchanged. It is true that, in this model, a decrease of the rate of growth of the price level of the tradable good or a temporary increase of the price level of that commodity (anticipated or not), will increase the expected depreciation of the exchange rate, but this variable plays no role in the model and hence cannot affect other endogenous variables.

Finally, the model of this chapter does not assume that purchasing parity power holds at all times. In fact, it will be shown that the maintenance of purchasing power parity following a monetary or a real change depends on the elasticities of demand for both commodities.

We can now turn to the flexible exchange rate version of the model formed by equations (3.68) through (3.72) and equations (3.74) and (3.75a). As stated in Chapter III under a flexible exchange rate system there is no difference between the beginning and end of period monetary base because the exchange rate adjusts to clear the money market. In what follows, it will be assumed that the level of foreign exchange reserves expressed in domestic currency is exogenously fixed at a level f_0. This assumption allows us to consider the level of domestic credit as the only source of variation of the monetary base; in addition, it evades the "valuation" problem; i.e., the change in the domestic value of the level of foreign reserves due to a change in the exchange rate.

The processes governing the behavior of the model's exogenous variables need to be specified. It is assumed that the monetary base follows the same process specified in Chapter II. That is:

$$(4.1) \quad h_t = h_{t-1} + m + v_t + x_t - x_{t-1}$$

where: m is a constant trend growth rate

v_t is a stochastic permanent shock

x_t is a stochastic temporary shock

and v_t and x_t are generated by white noise processes, that is:

$$v_t \sim N(0, \sigma_v^2)$$

$$x_t \sim N(0, \sigma_x^2)$$

with both v_t and x_t serially independent distributed.

The process governing the behavior of the price of the tradable good p_t^* is assumed to be similar to the process followed by h_t. This assumption allows for a high level of generality in the specification of the statistical processes and allows us to highlight different results of the model derived from different sets of information. Hence, p_t^* is generated in accordance with:

$$(4.2)\quad p_t^* = p_{t-1}^* + j^* + n_t + t_t - t_{t-1}$$

where: j^* is a constant trend growth rate

n_t is a stochastic permanent shock

t_t is a stochastic temporary shock

and n_t and t_t are white noise processes, that is:

$$n_t \sim N(0,\sigma_n^2)$$
$$t_t \sim N(0,\sigma_t^2)$$

and n_t and t_t are serially independent distributed.

The model will now be solved in a rational expectations framework under two alternative sets of information. Section 2 assumes full current information; hence, the effects of fully anticipated changes in the exogenous variables are analyzed. Here the emphasis of the analysis lies in the importance of the financial constraint in determining the nature of the solution for output levels. Section 3 relaxes the complete information assumption and allows for a confusion between permanent and temporary shocks to the monetary base and for a lack of beginning of period information about the price level of the tradable good. The consequences of such "lack of information" on the levels of output, the price of the nontradable good and the exchange rate are analyzed. In addition, differences between the results yielded here and those arising in the complete current information case are highlighted. Section 4 summarizes the results of this chapter.

2. <u>The full current information case</u>

Under the assumption that the beginning of period expected current price levels of both tradable and nontradable goods and the exchange rate are equal to their actual values, the macro-model of Chapter III yields the following set of semi-reduced forms for the price level of the nontradable good and the exchange rate:

$$(4.3) \quad p_t = \frac{1}{det}\{[(\delta_0-a_0)(\tau+b_1+b_2) + (b_0-a_0)(1-\tau)] + [(\delta_2-a_1)(\tau+b_1+b_2) + (b_2-a_1)(1-\tau)]h_t + [(\delta_1+a_1)(\tau+b_1+b_2) + (b_1+a_1)(1-\tau)]_{at}pi^e_{t+1} + [1+b_1+b_2](\varepsilon_t-a_3u_t)\}$$

and:

$$(4.4) \quad s_t = \frac{1}{det}\{[(b_0-a_0)(\delta_1+\delta_2+1-\tau) + (\delta_0-a_0)\tau] + [(b_2-a_1)(\delta_1+\delta_2+1-\tau) + (\delta_2-a_1)\tau]\, h_t + [(b_1+a_1)(\delta_1+\delta_2+1-\tau) + (\delta_1+a_1)\tau]\, {}_{at}pi^e_{t+1} - [(1-\tau)(b_1+b_2) + \tau(\delta_1+\delta_2) + (\delta_1+\delta_2)(b_1+b_2)]\, p^*_t - [a_3(\delta_1+\delta_2+1)]u_t + [\delta_1+\delta_2+1]\varepsilon_t\}$$

where: $det = (1-\tau)(b_1+b_2) + (\delta_1+\delta_2)\tau + (\delta_1+\delta_2)(b_1+b_2)$

Holding expectations constant for the meanwhile, several important results emerge from equations (4.3) and (4.4). First, the level of the monetary base positively affects both the price level of the nontradable good and the exchange rate if (1) the sensitivity of the demand for nontradables with respect to the real monetary base (δ_2) is greater than the sensitivity of the supply of nontradables with respect to the real monetary base (a_1); and (2) the sensitivity of the demand for tradables with respect to the real monetary base (b_2) is greater than the corresponding supply sensitivity (a_1). In the rest of this thesis it will be assumed that these conditions hold. The need to impose the restrictions $\delta_2 > a_1$ and $b_2 > a_1$ arises because of the assumption that the real monetary base affects <u>positively</u> the supply functions for tradables and nontradables (because of the financial constraint faced by the firms) and these restrictions are equivalent to the restriction: $b_1 a_1 < 1$ imposed in Chapter II for the closed economy case.

Second, the expected future price level affects both the price of the nontradable good and the exchange rate positively because it enters positively into the demand function for both commodities (intertemporal substitution effect in consumption) and negatively into the corresponding supply functions (because of the intertemporal substitution effect in the supply of labor).

Third, the price level of the tradable good affects the exchange rate negatively with a coefficient equal to one in absolute value, and has no effect on the price of the nontradable good or in the output of either commodity. This is so because p_t^* and s_t enter the model with the same coefficients; that is, the demand for nontradables and the supply of both kinds of goods depend on the domestic currency value of the price of the tradable good. Hence, when p_t^* increases, causing a "potential" substitution in production from nontradables toward tradable commodities and a "potential" decrease in the demand for tradables, the exchange rate decreases proportionally to keep constant the domestic price of the tradable good. With no change in the term $(p_t^* + s_t)$, the domestic price index remains unchanged, and hence the rest of the endogenous variables of the system remain unaffected.

Finally, a positive productivity change has a negative effect on both the price level of the nontradable good and the exchange rate while an exogenous change in preferences (a change in ϵ_t) has a positive impact on both prices.

Expectations are, of course, endogenous variables in the model, and equations (4.3) and (4.4) are not independent of each other. The term ${}_{at}pi^e_{t+1}$, involving the expectations of the future levels of both the price of the nontradable good and the exchange rate, is present in both equations. As a result, the exchange rate, for instance, depends on the expectations of the future exchange rate and on the expectations of the future price level of the nontradable good. To work towards a solution to the problem involved here, equation (3.72) can be used to rewrite equations (4.3) and (4.4) in the following way:

$$(4.5) \quad p_t = A_o + A_1 h_t + A_2 \tau\, {}_{at}p^e_{t+1} + A_2(1-\tau)\, {}_{at}s^e_{t+1} + A_2(1-\tau)\, {}_{at}(p^*)^e_{t+1} + A_3 u_t + A_4 \epsilon_t$$

$$(4.6)\quad s_t = B_o + B_1 h_t + B_2 \tau\ {}_{at}p^e_{t+1} + B_2(1-\tau)_{at}s^e_{t+1}$$

$$+ B_2(1-\tau)_{at}(p^*)^e_{t+1} + B_3 p^*_t + B_4 u_t + B_5 \varepsilon_t$$

where: $A_o = [(\delta_o - a_o)(\tau + b_1 + b_2) + (b_o - a_o)(1-\tau)]/\det$

$A_1 = [(\delta_2 - a_1)(\tau + b_1 + b_2) + (b_2 - a_1)(1-\tau)]/\det$

$A_2 = [(\delta_1 + a_1)(\tau + b_1 + b_2) + (b_1 + a_1)(1-\tau)]/\det$

$A_3 = [-a_3(1 + b_1 + b_2)]/\det$

$A_4 = -A_3/a_3$

$B_0 = [(b_o - a_o)(\delta_1 + \delta_2 + 1 - \tau) + (\delta_o - a_o)\tau]/\det$

$B_1 = [(b_2 - a_1)(\delta_1 + \delta_2 + 1 - \tau) + (\delta_2 - a_1)\tau]/\det$

$B_2 = [(b_1 + a_1)(\delta_1 + \delta_2 + 1 - \tau) + (\delta_1 + a_1)\tau]/\det$

$B_3 = -1$

$B_4 = [-a_3(1 + \delta_1 + \delta_2)]/\det$

$B_5 = -B_4/a_3$

Notice that $(A_1 + A_2) = (B_1 + B_2) = 1$, implying that, if future expectations of the general price index were to respond proportionally to an increase in the level of the monetary base, both the current price level of the nontradable good and the exchange rate would also rise proportionally. This is so because, in such a case, the expected real monetary base (deflated by ${}_{at}pi^{e}_{t+1}$) would remain constant and the price level of the nontradable good and the exchange rate would have to increase proportionally to clear simultaneously both commodity markets.

Leading equations (4.5) and (4.6) once and taking expectations conditional on information available during period t, we obtain: 1/

$$(4.7)\quad {}_{at}p^{e}_{t+1} = A_o + A_1\, {}_{at}h^{e}_{t+1} + A_2\tau\, {}_{at}p^{e}_{t+2} + A_2(1-\tau)\, {}_{at}s^{e}_{t+2}$$

$$+ A_2(1-\tau)_{at}(p^{*})^{e}_{t+2}$$

$$(4.8)\quad {}_{at}s^{e}_{t+1} = B_o + B_1\, {}_{at}h^{e}_{t+1} + B_2\tau\, {}_{at}p^{e}_{t+2} + B_2(1-\tau)_{at}s^{e}_{t+2}$$

$$+ B_2(1-\tau)\, {}_{at}(p^{*})^{e}_{t+2} + B_3\, {}_{at}(p^{*})^{e}_{t+1}$$

Solving for ${}_{at}p^{e}_{t+2}$ in equation (4.8), substituting into equation (4.7), and finally substituting the resulting expression for ${}_{at}p^{e}_{t+1}$ into equation (4.6), a first order difference equation for the expected level (equal to the actual level) of the current exchange rate is obtained. By the same token, solving for ${}_{at}s^{e}_{t+2}$ in equation (4.7), substituting into equation (4.8) and finally substituting the resulting expression for ${}_{at}s^{e}_{t+1}$ into equation (4.5), a first order difference equation for the price level of the nontradable good is obtained. These first order difference equations are:

$$(4.9)\quad s_t = {}_{at}s^{e}_{t} = [B_o+B_2A_o\tau-B_oA_2\tau]$$

$$+ [B_2(1-\tau)+A_2\tau]\, {}_{at}s^{e}_{t+1}$$

$$+ K_1\, {}_{at}V^{e}_{t}$$

$$(4.10)\quad p_t - {}_{at}p_t^e = [A_o + A_2(1-\tau)B_o - (1-\tau)A_oB_2]$$

$$+ [B_2(1-\tau) + A_2\tau]\ {}_{at}p_{t+1}^e$$

$$+ K_2\ {}_{at}V_t^e$$

where K_1, K_2, and ${}_{at}V_t^e$ are defined in Appendix IV.1.

To find the final solution for the current levels of the exchange rate and the price of the nontradable good, the method of undetermined coefficients will now be applied (see Appendix IV.2 at the end of this chapter, where it is shown that, for stability purposes, the forward solution implied by the use of the method of undetermined coefficients is, in fact, the solution to be chosen). Based on equation (4.9), given the information set and given our assumptions concerning the processes governing the exogenous variables (equations (4.1) and (4.2)), it can be conjectured that the solution for the full current information exchange rate is:

$$(4.11)\quad s_t = \pi_o^F + \pi_1^F h_{t-1} + \pi_2^F m + \pi_3^F v_t + \pi_4^F x_t$$

$$+ \pi_5^F x_{t-1} + \pi_6^F p_{t-1}^* + \pi_7^F j^* + \pi_8^F n_t$$

$$+ \pi_9^F t_t + \pi_{10}^F t_{t-1} + \pi_{11}^F u_t + \pi_{12}^F \varepsilon_t$$

where the π^Fs are the unknown coefficients for the flexible exchange rate case. The solution of those coefficients is presented in Appendix IV.3

The final solution for the exchange rate in terms of the parameters of the model is:

$$(4.12)\quad s_t = \{[(B_o + B_2A_o\tau - B_oA_2\tau)/(1-B_2(1-\tau) - A_2\tau)]$$
$$+ h_{t-1} + [(1 + B_2\tau - A_2\tau)/(1 - B_2(1-\tau) - A_2\tau]\ m$$
$$+ v_t + B_1\ x_t - x_{t-1}$$
$$- (p^*_{t-1} + j^* + n_t + t_t)$$
$$+ t_{t-1} + B_4\ u_t + B_5\ \epsilon_t\}$$

Before analyzing this equation, let us use the same procedure (the method of undetermined coefficients) to solve for the price level of the nontradable good. Based on equation (4.10), it can be conjectured that the solution to the full current information price level of the nontradable good is:

$$(4.13)\quad p_t = \theta^F_o + \theta^F_1\ h_{t-1} + \theta^F_2\ m + \theta^F_3\ v_t + \theta^F_4\ x_t + \theta^F_5\ x_{t-1}$$
$$+ \theta^F_6\ p^*_{t-1} + \theta^F_7\ j^* + \theta^F_8\ n_t + \theta^F_9\ t_t + \theta^F_{10}\ t_{t-1}$$
$$+ \theta^F_{11}\ u_t + \theta^F_{12}\ \epsilon_t$$

Appendix IV.4 presents the solution of the θ^F coefficients.

The final solution for the price level in terms of the parameters of the model is:

$$(4.14)\quad p_t = \{[(A_o + A_2(1-\tau)B_o - A_oB_2(1-\tau))/(1-B_2(1-\tau) - A_2\tau)]$$
$$+ h_{t-1}$$
$$+ [1 + (A_2/1-A_2\tau - B_2(1-\tau))]\ m$$
$$+ v_t + A_1\ x_t - x_{t-1}$$
$$+ A_3\ u_t + A_4\ \epsilon_t\}$$

The solution for the undetermined coefficients gives rise to the following final solutions for the levels of output of the ~~two~~ goods:

$$(4.15)\quad y_t = a_o - [\frac{a_1(\tau A_o + (1-\tau)B_o) - (1-\tau)(A_o - B_o + A_2B_o - B_2A_o)}{1-B_2(1-\tau)-A_2\tau}]$$

$$-[\frac{a_1 - (1-\tau)(A_2-B_2)}{1-B_2(1-\tau)-A_2\tau}]\, m$$

$$+ [a_1 + (1-\tau)(A_1-B_1)]\, x_t$$

$$+ [a_3 + (1-\tau)(A_3-B_4)]\, u_t$$

$$+ [(1-\tau)(A_4-B_5)]\, \varepsilon_t$$

$$(4.16)\quad y_t^* = a_0 - \frac{[a_1(\tau A_o + (1-\tau)B_o) + \tau\,(A_o - B_o + A_2B_o - B_2A_o)]}{1 - B_2(1-\tau) - A_2\tau}$$

$$- [\frac{a_1 + \tau\,(A_2-B_2)}{1-B_2(1-\tau)-A_2\tau}]\, m$$

$$+ [a_1 - \tau\,(A_1-B_1)]\, x_t$$

$$+ [a_3 - \tau\,(A_3-B_4)]\, u_t$$

$$- [\tau\,(A_4-B_5)]\, \varepsilon_t$$

Table 4.1 summarizes the main results derived from the output equations, and it can be seen that the effects of the exogenous variables on the levels of output may be decomposed into: (1) a financial constraint effect (a change in the real monetary base); and (2) a relative price effect.

2.1 An increase in the monetary base

As in the closed economy case, the past value of the monetary base (h_{t-1}) and the permanent monetary shock (v_t) have a positive and proportional effect on the domestic price level of the economy. Thus, the price level of the nontradable good and the exchange rate will change in the same proportion as the change in h_{t-1} or v_t, leaving the levels of outputs unchanged. The neutrality of fully anticipated changes in the level of the monetary base needs no further explanation.

The most interesting result derived in this Chapter concerns the effects of a change in the rate of growth of the monetary base. A rise in m will induce two effects: (1) it will increase the expected inflation rate and, hence, will lower the value of the real monetary base (a financial constraint effect); and (2) it will induce a change in the relative price term: $(p_t - p_t^* - s_t)$, but the direction of this change depends on the relative elasticities of the aggregate demands for tradable and nontradable commodities. The explanation of this result is straightforward: the impact effect of an increase in the rate of growth of the monetary base affects identically the aggregate supply of both commodities because the coefficient a_1 is common to both supply functions; however, the impact effect on the demand sides, although positive in both cases can differ in magnitude, depending on the value of the demand elasticity of the nontradable good with respect to the real monetary base (δ_2) relative to the corresponding demand elasticity of the tradable good (b_2) and on the value of the demand elasticity of the nontradable good with respect to the expected inflation term (δ_1) relative to the corresponding demand elasticity of the tradable commodity (b_1). Only if $\delta_1 = b_1$ and $\delta_2 = b_2$, the relative price effect would vanish.

Now, the particular specification of the utility function used in Chapter III implies that the value of the elasticity of demand with respect to the expected inflation rate is equal for both commodities (that is: $\delta_1 = b_1$) and that the value of the elasticity of demand with respect to the real monetary base is also equal for both commodities (that is: $\delta_2 = b_2$). As explained above, these restrictions imply that the relative price effect vanishes. However, since alternative utility functions might result in the elasticities of the demand for the tradable good being different from the corresponding elasticities of the demand for the nontradable good, the analysis that follows will also consider the possibility that $\delta_1 \neq b_1$ and $\delta_2 \neq b_2$. That is, we will concentrate on the general case where both the financial constraint effect and the relative price effect might be present. The particular case imposed by the specific utility function used in this thesis is the one in which only the financial constraint effect holds.

Taken together, the two above mentioned effects of an increase in the rate of growth of the monetary base result in a decrease of both levels of output. While the resulting decrease in the real monetary base decreases the firms' demand for labor and hence the output levels of both commodities (the financial constraint effect), the relative price effect induces firms to increase the production of the good whose relative price has risen and to decrease the production of the other commodity. However, it can be shown that when both the financial constraint and the relative price effects are present, the relative price effect is smaller (in absolute value) than the financial constraint effect. Hence, the net effect is a reduction in the level of output of both commodities, 2/ albeit in different magnitudes. Of course, in the particular case where only the financial constraint effect is present, the level of output of both commodities will decrease in the same magnitude (i.e., no relative price effect exists).

Finally, consider a positive temporary increase in the monetary base. The levels of output of both commodities will increase due to two effects: (1) a decrease in the expected inflation rate that will increase the real value of the monetary base; and (2) a relative price effect of undetermined sign but of smaller magnitude (in absolute terms) than the effect in (1). The effect of a positive temporary monetary shock is similar (but with opposite sign) to the effect of an increase in the rate of growth of the money supply and needs no further explanation.

2.2 A change in the price level of the tradable good

In contrast with a change in the monetary base, a change in the foreign price level of the tradable good causes only an inverse and proportional movement in the exchange rate, leaving the price level of the nontradable good and the output levels of both commodities unchanged. Hence, the classical insulation effect of a flexible exchange rate against foreign price level changes holds in this model. Notice that this insulation effect holds irrespective of the nature of the foreign price change; either a change in the level or in the rate of growth of p_t^* leaves the levels of domestic production unchanged. This result arises of course from the fact that in this model, both the foreign price level and the exchange rate always share the same coefficient. In particular, the insulation proposition occurs because <u>both</u> the price level of the tradable good and the exchange rate always enter as a sum in the supply and demand functions of the model and because both variables are treated equally in the specification of all information sets. In this context, the degree of capital mobility does not affect that result. Only if either variable were included in the information set while the other is not, the insulation proposition would not hold. It should be mentioned that an increase in the rate of growth of the price level of the tradable good or a temporary decrease in the price of that commodity, will result in a proportional decrease in the rate of exchange rate depreciation: $s_{t+1}^e - s_t$; however, this variable plays no role in the model and hence cannot affect other endogenous variables.

2.3 An increase in productivity or an exogenous change in preferences

An increase in productivity affects both the price level of the non-tradable good and the exchange rate negatively and has a positive impact on the supply of both commodities. A change in u_t does not affect the level of the real monetary base, because it has no role in the determination of the expected future price level ${}_{at}p_{t+1}^e$; hence, there is no financial constraint effect associated with productivity changes. However, there might be a relative price effect that, as in the case of m or

x_t, depends on the relative elasticities of demand for both commodities. If $(\delta_1 + \delta_2) > (b_1 + b_2)$, an increase in productivity will generate a relative price effect favoring the supply of the nontradable good. By the same token, if $(\delta_1 + \delta_2) < (b_1 + b_2)$, the relative price effect resulting from an increase in productivity will favor the supply of the tradable good. However, if the restrictions imposed by the particular utility function used in this thesis are strictly followed, no relative price effect will follow a change in productivity.

In addition to a possible relative price effect, a "direct" positive output effect resulting from the increased efficiency of labor will be common to both commodities. The parameters of the model imply that this latter effect will be greater than the relative price effect and hence, both outputs will increase but by different amounts. 3/ Of course in the absence of a relative price effect, both levels of output will increase in the same magnitude.

An exogenous change in preferences, that assigns a higher weight to current consumption relative to both, leisure and future consumption, affects both the price level of the nontradable good and the exchange rate positively, but implies an increase in the output level of one commodity and a decrease of the other if a relative price effect is present. This is the only exogenous variable in the model that, as a net effect, might result in the levels of outputs moving in opposite directions; but the explanation is straightforward: ϵ_t is also the only variable in the model that affects only the demand functions; hence, it will affect the final levels of output only in so far as it affects the relative price term. 4/ If $(\delta_1 + \delta_2) < (b_1 + b_2)$, the output level of the nontradable good increases and the output level of the tradable good decreases. 5/ Of course, if both commodities share the same elasticities of demand, an exogenous change in preferences would have no real effects, and p_t and s_t would increase in the same proportion.

2.4 The solutions for the interest rate and the real wage rate

Finally, the solutions for p_t and s_t can be substituted into equations (3.50) and (3.51) to obtain the final form solutions for the interest rate and the real wage rate. Appendix IV.5 shows those solutions. Neither a permanent change in the level of the monetary base nor a movement in the foreign price of the tradable good have an effect on the interest rate and the real wage rate, since the real monetary base and the expected inflation rate remains unchanged. Also, as in the closed economy case, a change in m or x_t affects the interest rate and the real wage through its impacts on: (1) the real monetary base; and (2) the expected inflation rate.

An increase in the rate of growth of the monetary base or a temporary decrease in the money stock cause an unambiguous increase in the interest rate because the resulting increase in the expected inflation rate and the associated decrease in the level of the real monetary base impinge on the interest rate positively.

As in the closed economy case, the direction of the real wage change caused by a movement in m or x_t remains undetermined in the open economy model. It depends on the sensitivity of the supply of labor to the real wage rate (η_1): the lower the value of η_1, the stronger the real monetary base effect will be relative to the expected inflation effect in the equation governing the determination of the real wage. 6/ If the real monetary base effect dominates, an increase in m or a decrease in x_t will cause the real wage rate to decrease.

2.5 Implications for the real exchange rate

The solutions of the flexible exchange rate model derived in this chapter have not assumed that purchasing power parity holds at all times. In this model, purchasing power parity would imply:

$$(4.17)\quad pi_t = p_t^* + s_t$$

or

$$(4.17')\quad p_t = p_t^* + s_t$$

Equation (4.17') represents not only purchasing power parity but also the inverse of the real exchange rate.

By substituting the solutions for the price level and the exchange rate into equation (4.17'), the following expression is obtained:

$$\begin{aligned}(4.18)\quad p_t - p_t^* - s_t = {} & (\theta_o - \pi_o) \\ & + (A_1 - B_1)\, x_t \\ & + (A_2 - B_2)/(1 - A_2\tau - B_2\,(1-\tau))\, m \\ & + (A_3 - B_4)\, u_t \\ & + (A_4 - B_5)\, \varepsilon_t\end{aligned}$$

That is, if the elasticities of demand for both commodities are not identical, the real exchange rate will be affected by variations in the anticipated rate of growth of the money supply, temporary monetary changes,

changes in productivity, and exogenous changes in preferences. Only changes in the foreign price level will leave the real exchange rate unchanged.

In this model, all the variables which affect output levels, also affect the real exchange rate; 7/ this is another manifestation of the nonsuperneutrality of money in this model. Notice that this absence of purchasing power parity holds in a rational expectations model with flexible prices and is obtained as a result and not by assumption. Finally, it should be noted that the real exchange rate would be a constant if: $\delta_1 + b_1$; $\delta_2 = b_2$. This result follows from the discussion in Section 2.1, that is, if the coefficients affecting the demand for tradables were equal to the coefficients in the demand for nontradables, purchasing power parity would hold at all times. Even in this case though, the maintenance of purchasing power parity would be a result rather than an assumption of the model. Movements in the exchange rate relative to changes in domestic prices in response to exogenous shocks also depend on parameters of tastes and technology in other equilibrium models that use a strict cash-in-advance constraint (see for example Aschauer and Greenwood (1983) and Stockman and Dellas (1984).

3. The incomplete current information case

In this section, agents are assumed to lack full current information. Specifically: (1) during the current period, they do not know the decomposition of a monetary shock between its temporary and permanent components; and (2) at the beginning of the period they do not observe the price level of the tradable commodity (in domestic terms) which will prevail at the end of the period. The beginning of period observation of the interest rate and wage rate allows agents, as in the closed economy case, to infer the current value of the monetary base but they cannot distinguish (during the current period) between permanent and temporary monetary shocks. In addition, knowledge of the monetary base does not convey any relevant information about the current price level of the tradable commodity. 8/

In this case, the model gives rise to the following semi-reduced form solutions for the incomplete current information price level of the nontradable good ($\hat{p}_t$) and the exchange rate ($\hat{s}_t$):

$$(4.19)\quad \hat{p}_t = C_0 + C_1\, {}_{at}\hat{p}^e_t + C_2\, {}_{at}\hat{s}^e_t + C_3\, {}_{at}\hat{p}^e_{t+1} + C_4\, {}_{at}\hat{s}^e_{t+1} + C_5\, {}_{bt}\hat{p}^e_{t+1} + C_6\, {}_{bt}\hat{s}^e_{t+1} + C_7\, h_t + C_8\, {}_{at}(p^*)^e_t + C_9\, {}_{at}(p^*)^e_{t+1} + C_{10}\, {}_{bt}(p^*)^e_{t+1} + C_{11}\, u_t + C_{12}\, \varepsilon_t$$

$$(4.20)\quad \hat{s}_t = D_0 + D_1\, {}_{at}\hat{p}^e_t + D_2\, {}_{at}\hat{s}^e_t + D_3\, {}_{at}\hat{p}^e_{t+1} + D_4\, {}_{at}\hat{s}^e_{t+1} + D_5\, {}_{bt}\hat{p}^e_{t+1} + D_6\, {}_{bt}\hat{s}^e_{t+1} + D_7\, h_t + D_8\, p^*_t + D_9\, {}_{at}(p^*)^e_t + D_{10}\, {}_{at}(p^*)^e_{t+1} + D_{11}\, {}_{bt}(p^*)^e_{t+1} + D_{12}\, u_t + D_{13}\, \varepsilon_t$$

Where: $C_0 = [\delta_0 - a_0]/[\delta_1+\delta_2]$

$C_1 = -C_2 = -C_8 = -[1-\tau]/[\delta_1+\delta_2]$

$C_3 = [a_1\tau]/[\delta_1+\delta_2]$

$C_4 = C_9 = [a_1(1-\tau)]/[\delta_1+\delta_2]$

$C_5 = [\delta_1\tau]/[\delta_1+\delta_2]$

$C_6 = C_{10} = [\delta_1(1-\tau)]/[\delta_1+\delta_2]$

$C_7 = [\delta_2 - a_1]/[\delta_1+\delta_2]$

$C_{11} = -C_{12} = -a_3/[\delta_1+\delta_2]$

$D_0 = [b_0 - a_0]/[b_1+b_2]$

$$D_1 = -D_2 = -D_9 = [\tau]/[b_1+b_2]$$

$$D_3 = [a_1\tau]/[b_1+b_2]$$

$$D_4 = D_{10} = [a_1(1-\tau)]/[b_1+b_2]$$

$$D_5 = [b_1\tau]/[b_1+b_2]$$

$$D_6 = D_{11} = [b_1(1-\tau)]/[b_1+b_2]$$

$$D_7 = [b_2-a_1]/[b_1+b_2]$$

$$D_8 = -1$$

$$D_{12} = -D_{13} = -a_3/[b_1+b_2]$$

The system formed by equations (4.19) and (4.20) differs from the corresponding system of the full current information case. It contains beginning of period expectations of the domestic price level of both commodities as well as distinguishing between beginning and end of period expectations of these variables. However, in the context of the present model, expectations of the current and future price levels of the nontradable good and the exchange rate are identical at both the beginning and end of period. The explanation for this feature is twofold. First, as in the closed economy case, knowledge of the price levels at the end of the period does not help agents to distinguish between the temporary and permanent monetary shocks; hence, ${}_a h_t^e = {}_b h_t^e = h_{t-1} + m + (v_t+x_t) - x_{t-1}$. Second, although the observation of the price level (in domestic terms) of the tradable good at the end of the period implies a difference between the beginning and end of period expectations of this variable, 9/ this difference has no implications for the solutions of the model because price expectational errors do not affect any equation of the system. That is, in this model all (perceived or not) foreign price changes impinge only on the exchange rate. They have no consequence for other variables; hence, the only additional consequence of a foreign price shock (either temporary or permanent) is to affect 10/ the term $(\hat{s}_t - {}_{at}\hat{s}_t^e)$ which plays no role in the model.

Notice that if it were assumed that the exchange rate becomes observable at the beginning of the period while the price level of the tradable good only becomes observable at the end of the period, the coefficient D_8 in equation (4.20) would be different from one (in absolute terms) and the "insulation proposition" obtained in the full current information case would no longer hold.

Given the properties of a foreign price shock, the only relevant distinction between the complete and incomplete current information cases lies in the agents' inability to distinguish clearly between permanent and temporary monetary shocks. This poses a signal extraction problem identical to the one which arises in the closed economy case and needs no further discussion (see Chapter 2, Section 4.3).

The method of solving the model is similar to the full current information case and hence will not be presented here. It will suffice to state that the conjectured solutions for p_t and s_t (based on the method of undetermined coefficients) will now be:

$$(4.21)\quad p_t = \hat{\theta}_0^F + \hat{\theta}_1^F h_{t-1} + \hat{\theta}_2^F m + \hat{\theta}_3^F (v_t + x_t) + \hat{\theta}_5^F x_{t-1} + \hat{\theta}_6^F p_{t-1}^* + \hat{\theta}_7^F j^* + \hat{\theta}_8^F n_t + \hat{\theta}_9^F \varepsilon_t + \hat{\theta}_{10}^F \varepsilon_{t-1} + \hat{\theta}_{11}^F u_t + \hat{\theta}_{12}^F \epsilon_t$$

$$(4.22)\quad \hat{s}_t = \hat{\pi}_0^F + \hat{\pi}_1^F h_{t-1} + \hat{\pi}_2^F m + \hat{\pi}_3^F (v_t+x_t) + \hat{\pi}_5^F x_{t-1}$$

$$+ \hat{\pi}_6^F p^*_{t-1} + \hat{\pi}_7^F j^* + \hat{\pi}_8^F n_t + \hat{\pi}_9^F t_t + \hat{\pi}_{10}^F t_{t-1}$$

$$+ \hat{\pi}_{11}^F u_t + \hat{\pi}_{12}^F \varepsilon_t$$

With the exception of $\hat{\theta}_3^F$ and $\hat{\pi}_3^F$, all the coefficients are identical to the corresponding θ^F's and π^F's derived for the full current information case. The only difference is that:

$$(4.23)\quad \hat{\theta}_3^F = A_2 (1-g_1) + A_1$$

$$(4.24)\quad \hat{\pi}_3^F = B_2 (1-g_1) + B_1$$

Where, it should be recalled, g_1 stands for the variance of the temporary shock relative to the total variance of (x_t+v_t).

The higher the value of g_1, the smaller will be the effect of a monetary shock on the price level of the nontradable good and on the exchange rate. Notice that if $g_1 = 0$, then $\hat{\theta}_3^F = \hat{\pi}_3^F = 1$, implying that a permanent shock will affect p_t and s_t with a unitary coefficient; this is so because if the variance of a temporary shock is zero, all monetary shocks will be viewed as permanent and $\hat{\theta}_3^F = \theta_3^F$; $\hat{\pi}_3^F = \pi_3^F$. If, on the other extreme, $g = 1$, then all monetary shocks would be perceived as temporary and $\hat{\theta}_3^F = \theta_4^F = A_1$; $\hat{\pi}_3^F = \pi_4^F = B_1$.

Since $(B_1 + B_2) = 1$ and $g_1 < 1$, then $\hat{\pi}_3^F < \pi_3^F$. Thus, a permanent increase in the level of the monetary base will cause the exchange rate to "undershoot" relative to its full current information solution; by the same token the exchange rate will "overshoot" if there is a permanent decrease in the monetary base. Also, since changes in the world price of the tradable good (anticipated or not) always cause a proportional and inverse change of the exchange rate, no overshooting effects will arise from the lack of beginning of period information on the price level of the tradable good. Hence, in this model, only permanent monetary shocks, not perceived as such, cause the exchange rate to overshoot relative to its full current information level.

The solution for the output levels will be:

$$(4.25)\quad y_t = a_o - \left[\frac{a_1(\tau A_o + (1-\tau)B_o) - (1-\tau)(A_o - B_o + A_2B_o - B_2A_o)}{1-B_2(1-\tau) - A_2\tau}\right]$$

$$- \left[\frac{a_1 - (1-\tau)(A_2-B_2)}{1-B_2(1-\tau)-A_2\tau}\right] m$$

$$+ [a_1g_1 + (1-\tau)\ (A_2(1-g_1) + A_1 - B_2(1-g_1) - B_1)]\ (v_t+x_t)$$

$$+ [a_3 + (1-\tau)(A_3-B_4)]\ u_t + [(1-\tau)(A_4-B_5)]\ \varepsilon_t$$

$$(4.26)\quad y_t^* = a_o - \left[\frac{a_1(\tau A_o + (1-\tau)B_o) + \tau(A_o - B_o + A_2B_o - B_2A_o)}{1 - B_2(1-\tau) - A_2\tau}\right]$$

$$- \left[\frac{a_1 + \tau(A_2-B_2)}{1-B_2(1-\tau)-A_2\tau}\right] m$$

$$+ [a_1g_1 - \tau\ (A_2(1-g_1) + A_1 - B_2(1-g_1) - B_1)](v_t+x_t)$$

$$+ [a_3 - \tau\ (A_3-B_4)]\ u_t - [\tau(A_4-B_5)]\ \varepsilon_t$$

As expected, the only difference between these solutions and those of the full information case arises from monetary shock confusions.

Also, notice that the variance of the temporary shock affects the levels of output through both a financial constraint effect and a relative price effect. Moreover, the coefficient accompaning (v_t+x_t) in equation (4.25) can be rewritten as: 11/

$$g_1 [a_1 - (1-\tau)(B_1-A_1)]$$

and, the corresponding coefficient in equation (4.26) as:

$$g_1 [a_1 + \tau (B_1-A_1)]$$

Hence, the higher the value of g_1, the higher the impact of a monetary shock on both levels of output.

It is, of course, the case that the monetary shock confusion will also affect the values of the interest rate and the real wage. These effects are similar to those which arise in the closed economy case and hence do not need to be repeated (see Chapter 2, Section 4.4).

4. Summary

This chapter has solved the model set up in Chapter III for the flexible exchange rate case and, has analyzed the effects of changes in the monetary base, the price of tradable goods, and some real shocks to productivity and to marginal utility of current consumption on the output levels of the domestically produced commodities, the price level of the nontradable good and the exchange rate. It has been shown that the "financial constraint" assumption is crucial in determining the effects of monetary changes on output. In particular, even though an anticipated increase in the rate of growth of the monetary base, or an anticipated temporary monetary decrease, might result in a relative price effect favoring an increase in the supply of one commodity and a decrease in the supply of the other, the negative financial constraint effect resulting from such monetary changes dominates this relative price effect. The level of output of both domestically produced commodities will therefore decrease. Moreover, a confusion between permanent and temporary monetary shocks implies that even permanent changes in the level of the monetary base have real effects. In addition, such monetary confusion implies that a permanent change in the level of the monetary base causes the exchange rate to "overshoot" relative to its full current information level. This is the only case in which exchange overshooting arises in this model.

A change in the price level of the tradable good (or in its rate of growth) always results in an inverse and proportional change of the exchange rate (with no effects on the domestic endogenous variables). This result does not depend on assumptions about capital mobility but on the specification of the model and the information sets analyzed. Specifically, the price level of the tradable good and the exchange rate always enter together as a sum in the supply and demand functions of the model and in the two alternative information sets used to solve the model. Finally, note that in contrast with some flexible price-rational expectations models, the model of this chapter does not <u>assume</u> purchasing power parity. In fact, the maintenance of purchasing power parity following monetary and real changes depends on the parameters affecting the demand functions of both commodities.

Table 4.1. Decomposition of the Effects Impinging on the Domestic Levels of Output in the Full Current Information Case: The Flexible Rates Case

Exogenous Variables \ Endogenous Variables	y_t Financial Constraint Effect	y_t Relative Price Effect	y_t^* Financial Constraint Effect	y_t^* Relative Price Effect
Increase in h_{t-1}	0	0	0	0
Increase in m	$-\frac{a_1}{1-B_2(1-\tau)-A_2\tau}$	$\frac{(1-\tau)(A_2-B_2)}{1-B_2(1-\tau)-A_2\tau}$	$-\frac{a_1}{1-B_2(1-\tau)-A_2\tau}$	$-\frac{\tau(A_2-B_2)}{1-B_2(1-\tau)-A_2\tau}$
Increase in x_t	a_1	$(1-\tau)A_1-(1-\tau)B_1$	a_1	$-\tau(A_1-B_1)$
Increase in v_t	0	0	0	0
Increase in p_t^*	0	0	0	0
Increase in u_t 1/	0	$(1-\tau)(A_3-B_4)$	0	$-\tau(A_3-B_4)$
Increase in ϵ_t	0	$(1-\tau)(A_4-B_5)$	0	$-\tau(A_4-B_5)$

1/ A direct effect on the aggregate supply equal to a_3 has to be added to the relative price effect in order to get the total effect of a change in productivity.

FOOTNOTES TO CHAPTER IV

1/ Recall that:

$$a_t u^e_{t+1} = a_t \varepsilon^e_{t+1} = 0.$$

2/ From Table 4.1, it is clear that the financial constraint effect will dominate the relative price effect if $|A_2 - B_2| < a_1$.

$$\text{Now:} \quad A_2 - B_2 = \frac{a_1(b_2+b_1-\delta_1-\delta_2) + \delta_1 b_2 - \delta_2 b_1}{\det}$$

$$= a_1 \, (X)$$

Where the absolute value of X is less than one. Thus, the financial constraint effect dominates the relative price effect.

3/ The direct output effect dominates the relative price effect since: $a_3 > |A_3 - B_4|$.

$$\text{Proof:} \quad (A_3 - B_4) = \frac{a_3 \, (\delta_1 + \delta_2 - b_1 - b_2)}{\det} = a_3 \, (Y)$$

where the absolute value of Y is less than one.

4/ Due to the characteristics of the random variable affecting the level of utility and the assumption of full current information, ε_t does not affect the level of the real monetary base.

5/ The impact effect of a positive ε_t will rise both p_t and s_t in a magnitude according to the price elasticities of the supply functions, and the output levels will increase. But, as a second round effect, the aggregate supply of each commodity will decrease in response to the increased price of the other commodity. The lower the price elasticity of demand (given by $(\delta_1 + \delta_2)$ for the nontradable good, and $(b_1 + b_2)$ for the tradable good), the lower this negative second impact on the level of output.

6/ This is so because in that case $\beta_2' > \beta_1'$ in equation (3.50).

7/ Stockman (1980) used a cash-in-advance model to "show how a change in the terms of trade caused by relative suypply or demand shifts is divided between nominal price changes in each country and an exchange rate change, creating a correlation between the exchange rate and the terms of trade" (p. 674). Thus, in Stockman's model, deviations from purchasing power parity can also be generated as an equilibrium result.

The changes in real output creating deviations from purchasing power parity are assumed exogenous in Stockman's model. In contrast, output in this thesis is treated as an endogenous variable. However, it is interesting to note that those exogenous variables that effect the equilibrium level of output of both commodities and their relative supplies in this thesis are also the same variables that are capable to generate deviations from purchasing power parity. That is, in this thesis, deviations from purchasing power parity correlate with changes in relative supplies of both commodities.

8/ The beginning of period expectations of the price level of the tradable good is: ${}_a(p^*)^e_t = p^*_{t-1} + j^* - t_{t-1}$. There is no signal extraction problem here because knowledge of $\hat{n}_t$ does not help to infer $n_t + t_t$; this is due to the fact that v_t, x_t, n_t, and t_t are not correlated with each other.

9/ ${}_a p^*_t = p^*_{t-1} + j - t_{t-1}$

${}_b p^*_t = p^*_t = p^*_{t-1} + j + n_t + t_t - t_{t-1}$

10/ With a unitary coefficient.

11/ This is so because the structural parameters of the model imply that $(A_2 - B_2) \equiv (B_1 - A_1)$, and $A_1 + A_2 \equiv 1$.

APPENDIX IV.1

THE VECTORS K_1, K_2, and ${}_{at}V_t^e$

$$K_1' = \begin{bmatrix} B_1 \\ B_2A_1\tau - B_1A_2\tau \\ -1 \\ B_2(1-\tau)+A_2\tau \\ B_4 \\ B_5 \end{bmatrix} \qquad K_2' = \begin{bmatrix} A_1 \\ A_2B_1(1-\tau)-A_1B_2(1-\tau) \\ 0 \\ 0 \\ A_3 \\ A_4 \end{bmatrix}$$

$${}_{at}V_t^e = \begin{bmatrix} {}_{at}h_t^e \\ {}_{at}h_{t+1}^e \\ {}_{at}(p^*)_t^e \\ {}_{at}(p^*)_{t+1}^e \\ {}_{at}u_t^e \\ {}_{at}\varepsilon_t^e \end{bmatrix}$$

APPENDIX IV.2

PROOF THAT A STABLE SOLUTION FOR THE EXCHANGE RATE AND THE PRICE LEVEL OF THE NONTRADABLE GOOD IS PROVIDED BY THE FORWARD-LOOKING SOLUTION

1. The exchange-rate solution

Equation (4.9) in the main text can be rewritten as:

$$(IV.2.1)\quad {}_{at}s^e_{t+1} = \frac{1}{B_2(1-\tau) + A_2\tau}\{{}_{at}s^e_t - (B_o+B_2A_o\tau-B_oA_2\tau) - K_1\,{}_{at}V^e_t\}$$

or, applying the lag operator L:

$$(IV.2.2)\quad [L^{-1} - \frac{1}{B_2(1-\tau)+A_2\tau}]\,{}_{at}s^e_t = -\frac{1}{B_2(1-\tau)+A_2\tau}\{(B_o+B_2A_o\tau-B_oA_2\tau) + K_1\,{}_{at}V^e_t\}$$

where the operator L^{-1} applied to ${}_{at}s^e_t$ leads the expectations of the exchange rate without changing the information set upon which expectations are formed.

Multiplying equation (IV.2.2) by L:

$$(IV.2.3)\quad [1 - \frac{1}{B_2(1-\tau)+A_2\tau}L]\,{}_{at}s^e_t = \frac{-1}{B_2(1-\tau)+A_2\tau}\{(B_o+B_2A_o\tau-B_oA_2\tau) + K_1\,{}_{at}V^e_{t-1}\}$$

Equation (IV.2.3) has multiple solutions. Imposing the restriction of a "stable solution," the choice of the solution depends on the absolute value of the coefficient: $\frac{1}{B_2(1-\tau)+A_2\tau}$.

Expressing such a coefficient in terms of the structural coefficients of the model, it turns out that:

$$\frac{1}{B_2(1-\tau)+A_2\tau} > 1 \text{ if } \quad \delta_2 > a_1 \text{ and } b_2 > a_1$$

which are the two restrictions already imposed to the model. Hence, a forward-looking solution is a stable one.

2. The solution for the price level of the nontradable good

Equation (4.10) in the main text can be rewritten as:

$$\text{(IV.2.4)} \quad {}_{at}P^e_{t-1} = \frac{1}{B_2(1-\tau)+A_2\tau} \{ {}_{at}P^e_t - (A_o+A_2(1-\tau)B_o-(1-\tau)A_oB_2) - K_2 \, {}_{at}\bar{V}^e_t \}$$

applying the lag operator and lagging equation (IV.2.4) once:

$$\text{(IV.2.5)} \quad [1 - \frac{1}{B_2(1-\tau)+A_2\tau} L] \, {}_{at}P^e_t = - \frac{1}{B_2(1-\tau)+A_2\tau} \{(A_o+A_2(1-\tau)B_o-(1-\tau)A_oB_2) + K_2 \, {}_{at}V^e_{t-1}\}$$

As it has already been shown, the coefficient: $\frac{1}{B_2(1-\tau)+A_2\tau}$ is greater than one and hence the forward solution satisfies the stability criteria.

APPENDIX IV.3

SOLUTION FOR THE UNDETERMINED COEFFICIENTS IN THE EXCHANGE RATE EQUATION

Updating equation (4.11), in the main text, once and taking beginning of period expectations:

$$
\begin{aligned}
\text{(IV.3.1)} \quad {}_{a}s^{e}_{t+1} = {} & \pi^F_o + \pi^F_1 h_{t-1} + (\pi^F_1+\pi^F_2)\, m + \pi^F_1 v_t \\
& + (\pi^F_1+\pi^F_5)\, x_t - \pi^F_1 x_{t-1} + \pi^F_6 p^*_{t-1} \\
& + (\pi^F_6+\pi^F_7)\, j^* + \pi^F_6 n_t + (\pi^F_6+\pi^F_{10})\, t_t \\
& - \pi^F_6 t_{t-1}
\end{aligned}
$$

Substituting equation (IV.3.1) into equation (4.9) in the main text we obtain:

$$
\begin{aligned}
\text{(IV.3.2)} \quad s_t = {} & [B_o + B_2A_o\tau - B_oA_2\tau + \pi_oB_2(1-\tau) + \pi^F_oA_2\tau] \\
& + [\pi^F_1B_2\,(1-\tau) + \pi^F_1A_2\tau + B_1 + B_2A_1\tau - B_1A_2\tau]\, h_{t-1} \\
& + [(\pi^F_1+\pi^F_2)B_2\,(1-\tau) + (\pi^F_1+\pi^F_2)A_2\tau + B_1 + 2B_2A_1\tau - 2B_1A_2\tau]\, m \\
& + [\pi^F_1B_2\,(1-\tau) + \pi^F_1A_2\tau + B_1 + B_2A_1\tau - B_1A_2\tau]\, v_t \\
& + [(\pi^F_1 + \pi^F_5)\, B_2(1-\tau) + (\pi^F_1+\pi^F_5)A_2\tau + B_1]\, x_t \\
& - [\pi^F_1B_2\,(1-\tau) + \pi^F_1A_2\tau + B_1 + B_2A_1\tau - B_1A_2\tau]\, x_{t-1} \\
& + [\pi^F_6B_2(1-\tau) + \pi^F_6A_2\tau - 1 + B_2(1-\tau) + A_2\tau]\, p^*_{t-1} \\
& + [(\pi^F_6+\pi^F_7)B_2(1-\tau) + (\pi^F_6+\pi^F_7)A_2\tau - 1 + 2B_2(1-\tau) + 2A_2\tau]\, j^* \\
& + [\pi^F_6B_2(1-\tau) + \pi^F_6A_2\tau - 1 + B_2(1-\tau) + A_2\tau]\, n_t \\
& + [(\pi^F_6 + \pi^F_{10})B_2(1-\tau) + (\pi^F_6+\pi^F_{10})A_2\tau - 1]\, t_t \\
& - [\pi^F_6B_2(1-\tau) + \pi^F_6A_2\tau - 1 + B_2(1-\tau) + A_2\tau]\, t_{t-1} \\
& + B_4u_t \\
& + B_5\varepsilon_t
\end{aligned}
$$

Equating coefficients among equations (4.11) in the main text and (IV.3.2), the solutions for the π^F coefficients are obtained:

(IV.3.3) $\pi_0^F = [B_0+B_2A_0\tau-B_0A_2\tau]/[1-B_2(1-\tau)-A_2\tau]$

(IV.3.4) $\pi_1^F = \pi_3^F = \pi_{10}^F = 1$

(IV.3.5) $\pi_2^F = [1+B_2\tau-A_2\tau]/[1-B_2(1-\tau)-A_2\tau] > 1$

(IV.3.6) $\pi_4^F = B_1 < 1$

(IV.3.7) $\pi_5^F = \pi_6^F = \pi_7^F = \pi_8^F = \pi_9^F = -1$

(IV.3.8) $\pi_{11}^F = B_4 < 0$

(IV.3.9) $\pi_{12}^F = B_5 > 0$

SOLUTION FOR THE UNDETERMINED COEFFICIENTS IN THE PRICE LEVEL EQUATION

Updating equation (4.13), of the main text, once and taking beginning of period expectations:

$$(IV.4.1)\quad {}_{at}p^e_{t+1} = \theta^F_o + \theta^F_1 h_{t-1} + (\theta^F_1+\theta^F_2)m + \theta^F_1 v_t + (\theta^F_1+\theta^F_5)\, x_t - \theta^F_1 x_{t-1}$$

$$+ \theta^F_6 p^*_{t-1} + (\theta^F_6+\theta^F_7)\cdot j^* + \theta^F_6\, n_t + (\theta^F_6+\theta^F_{10})\, t_t$$

$$- \theta^F_6\, t_{t-1}$$

Substituting equation (IV.4.1) into equation (4.10) of the main text and solving for the undetermined coefficients, we obtain:

$$(IV.4.2)\quad \theta^F_o = [A_o + A_2(1-\tau)B_o - A_oB_2(1-\tau)]/[1 - B_2(1-\tau) - A_2\tau]$$

$$(IV.4.3)\quad \theta^F_1 = \theta^F_3 = -\theta^F_5 = 1$$

$$(IV.4.4)\quad \theta^F_2 = 1 + [A_2/1-A_2\tau-B_2(1-\tau)]$$

$$(IV.4.5)\quad \theta^F_4 = A_1$$

$$(IV.4.6)\quad \theta^F_6 = \theta^F_7 = \theta^F_8 = \theta^F_9 = \theta^F_{10} = 0$$

$$(IV.4.7)\quad \theta^F_{11} = A_3$$

$$(IV.4.8)\quad \theta^F_{12} = A_4$$

INTEREST RATE AND REAL WAGE SOLUTIONS UNDER THE ASSUMPTION OF FULL CURRENT INFORMATION

$$\text{(IV.5.1)} \quad i_t = \gamma_o' - \gamma_1' \frac{(\tau A_o + (1-\tau)B_o)}{1 - B_2(1-\tau) - A_2\tau}$$

$$- \left[1 - \frac{\gamma_1'}{1 - B_2(1-\tau) - A_2\tau}\right] m$$

$$+ \left[\tau A_1 + (1-\tau)B_1 - \gamma_1'\right] x_t$$

$$- \left[(\tau A_3 + (1-\tau)B_4)\,(\gamma_1' + \gamma_2')\right] u_t$$

$$- \left[(\tau A_4 + (1-\tau)B_5)\,(\gamma_1' + \gamma_2')\right] \varepsilon_t$$

$$\text{(IV.5.2)} \quad w_t - p_t = \left[\rho_o' + \rho_2' \left(\frac{\tau A_0 + (1-\tau)B_o}{1 - B_2(1-\tau) - A_2\tau}\right)\right]$$

$$+ \left[1 - \frac{\rho_2'}{1 - B_2(1-\tau) - A_2\tau}\right] m$$

$$+ \left[\rho_2' - (\tau A_1 + (1-\tau)B_1)\right] x_t$$

$$+ \left[(\tau A_3 + (1-\tau)B_4)\,(\rho_2' - \rho_1')\right] u_t$$

$$+ \left[(\tau A_4 + (1-\tau)\, B_5)\,(\rho_2' - \rho_1')\right] \varepsilon_t$$

CHAPTER V

PRICE AND OUTPUT FLUCTUATIONS IN A FIXED EXCHANGE RATE REGIME

1. Introduction

This chapter solves the model set out in Chapter III for the fixed exchange rate case. The behavior of prices and output in a small fixed rate open economy model that allows for the distinction between tradable and nontradable commodities, was first analyzed by Dornbusch (1973) under the assumption of zero capital mobility. In these respects, the Dornbusch model is similar to that analyzed in this chapter. However, as will be shown, several implications of the model dealt with here drastically differ from those of the Dornbusch model.

First, although both models predict the absence of short-run money neutrality, in the Dornbusch model an increase in the exogenous component of the money supply results in a short-run expansion of the output of nontradables and a reduction in that of tradables; in the model developed here, the response of output critically depends on the behavior of the equilibrium real monetary base. The output of both goods might increase if the increase in the real monetary base generated by a rise in the <u>level</u> of government debt outstanding more than offsets the relative price effect, which favors a decrease in the supply of tradables. However, if the <u>rate of growth</u> of government debt increases, the real monetary base might decrease by an amount enough to induce a decrease in the output of both commodities.

Second, in the Dornbusch model an increase in the (exogenously given) price of tradables results in a short-run increase in the domestic supply of tradables and a decrease in the supply of nontradables. In contrast, in the model of this chapter, both outputs might decrease if the decrease in the real monetary base generated by an increase in the price level of tradables is strong enough to more than offset the relative price effect which favors an increase in the supply of tradables. If instead, the inflation rate of the tradable good price increases, the output response of both commodities is ambiguous.

It is useful to point out in advance the major factors which lead to such differences in results. Leaving aside the fact that the Dornbusch model lacks any stochastic structure, the major differences lie in the specification of the supply side of the two models. In the Dornbusch model, only relative price effects are present in the determination of the supply of output, while in the model discussed here, both a relative price effect and a financial constraint effect influence output. Any variation in the exogenous variables which lead to an increase in domestic inflation causes a "tightening up" of the financial constraint. They therefore have a negative effect on both levels of output. Even in the presence of a favorable relative price effect, therefore, the supply of a component of output might decrease if the relative price effect is outweighted by a negative financial constraint effect. Once more it is important to recall that the financial constraint variable does not result from simply "expanding" wealth effects into the supply equations. Such an exercise would result in a negative partial derivative of supply of either commodity with respect to the real monetary base.

We can now turn to the fixed exchange rate version of the model formed by equations (3.68) to (3.74) and equation (3.75b). As discussed in Chapter III, under a fixed exchange rate, a distinction between the beginning and the end-of-period monetary base arises. The government decides on the outstanding government debt (gc_t) at the beginning of the period, but transactions leading to a change in the level of foreign reserves occur at the end of the period. This feature of the model implies that the end of previous period level of the foreign reserves ($_bf_{t-1}$) enters into the structural supply and demand equations in the same way as does the current level of government credit. 1/

The exchange rate is assumed to be an exogenous variable in this version of the model. It is assumed (as in Weber (1981)) that the monetary authority sets a target value ($\bar{s}_t$) for the exchange rate according to some known rule that is independent of the behavior of other variables in the model. The rule could be a linear feedback rule or could simply set the exchange rate constant for every period t. The important characteristic of this exchange rate system is not the particular form of the exchange rule, but the assumption that such a rule remains constant over time. For purposes of simplification, most of the analysis in this chapter will assume a constant exchange rate, leaving the analysis of a devaluation to the end of the chapter where a nonconstant feedback exchange rate rule will be allowed to hold.

Under a fixed exchange rate regime, the monetary authority must adjust its level of international reserves to achieve the exogenously determined level of the exchange rate. Since the level of foreign reserves and hence, the money supply, are now endogenous variables, the only exogenous monetary element in the domestic economy is the outstanding government debt. This will be assumed in this chapter to follow the same process as was followed by the monetary base in the flexible exchange rate regime. That is:

$$(5.1)\quad gc_t = gc_{t-1} + m + v_t + x_t - x_{t-1}$$

where the definitions for m, v_t, and x_t of Chapter IV carries through, but now refer, of course, to the outstanding government debt. Similarly, the process governing the behavior of the price of the tradable good will be assumed to be identical to the one specified in equation (4.2).

The remainder of this chapter follows the same basic organization of Chapter IV: Section 2 solves the model under the assumption of full current information in a rational expectations framework. While the solutions for the price level of the nontradable good and the level of foreign reserves are analyzed, implications for the levels of output of both commodities, and the economic intuition lying behind these results are emphasized. Section 3 discusses the implications of relaxing the assumption of full current information and allows for a confusion between permanent and temporary shocks to the government debt outstanding as well as for a lack of beginning-of-period information about the foreign price level. Devaluation will be the subject of Section 4. Finally, Section 5 summarizes the results of this chapter.

2. The full current information case

Assuming that the economic agents know, at the beginning of every period, the value of all the relevant variables that will prevail during that period, the macro-model of Chapter III can be solved to yield the following set of semi-reduced forms for the price level of the nontradable good and the end of period level of foreign exchange reserves. 2/ 3/

$$(5.2)\quad p_t = \frac{1}{1-\tau+\delta_1+\delta_2}\{[\delta_0 - a_0 - a_1\omega_0 + \delta_2\omega_0]$$
$$+ [\delta_2(1-\omega_1) - a_1(1-\omega_1)]f_{t-1}$$
$$+ [\delta_2\omega_1 - a_1\omega_1]gc_t$$
$$+ [\delta_1\tau + a_1\tau]_{at}p^e_{t+1}$$
$$+ [1 - \tau](p^*_t + \bar{s})$$
$$+ [\delta_1(1-\tau) + a_1(1-\tau)](_{at}(p^*)^e_{t+1} + \bar{s})$$
$$- a_3u_t + \varepsilon_t\}$$

and:

$$(5.3)\quad f_t = \frac{1}{1-\tau+\delta_1+\delta_2}\{[d_1\tau(a_0+a_1\omega_0-\delta_0-\delta_2\omega_0)$$
$$+ (1-\tau+\delta_1+\delta_2)(d_1a_0-d_2b_0+d_1a_1\omega_0-d_2b_2\omega_0)]$$
$$+ [d_1\tau(1-\omega_1)(a_1-\delta_2)$$
$$+ (1-\tau+\delta_1+\delta_2)\,(d_3+(1-\omega_1)(d_1a_1-d_2b_2))]f_{t-1}$$
$$+ [d_1\tau\omega_1(a_1-\delta_2) + (1-\tau+\delta_1+\delta_2)(\omega_1)(d_1a_1-d_2b_2)]gc_t$$
$$- \tau[d_1\tau(a_1+\delta_1) + (1-\tau+\delta_1+\delta_2)(d_1a_1+d_2b_1)]_{at}p^e_{t+1}$$
$$+ [d_1\tau(\delta_1+\delta_2) + (1-\tau+\delta_1+\delta_2)(1-d_3+d_2b_1+d_2b_2)](p^*_t + \bar{s})$$
$$- (1-\tau)[d_1\tau(a_1+\delta_1) + (1-\tau+\delta_1+\delta_2)(d_1a_1+d_2b_1)]$$
$$(_{at}(p^*)^e_{t+1} + \bar{s})$$
$$+ [d_1\tau a_3 + (1-\tau+\delta_1+\delta_2)(d_1a_3)]u_t$$
$$- [d_1\tau + (1-\tau+\delta_1+\delta_2)d_2]\varepsilon_t\}$$

Holding expectations constant for the time being, some important results emerge from equations (5.2) and (5.3). First, in contrast to the flexible exchange rate regime, the solution for the price level of the nontradable good involves only equations directly dealing with the market of that commodity. The current level of foreign reserves does not appear as an argument in the demand and supply of nontradables. Indeed, only the predetermined level of the previous period's reserves affects the current beginning-of-period monetary base and hence, the price level of nontradables. However, the model's solution for the level of foreign exchange reserves involves both markets, since any change in the price level of nontradables will induce a relative price change that will in turn affect the supply of the tradable good. The independence of equation (5.2) from the tradable market is a direct consequence of the financial constraint assumption and the sequencing of trade activities assumed in this thesis.

Second, an increase in the level of government's debt outstanding will result in an increase in the price of the nontradable good 4/ and a decrease in the level of reserves. While the result for the price level of the nontradable good is obvious 5/ because it only involves the demand and supply of that commodity, the solution for the level of reserves requires some comments. The impact effect of an increase in government debt is a decrease in the level of foreign reserves, because (given the assumption that $b_2 > a_1$) the resulting increase in the demand for tradables is greater than the increase in their supply. In addition, as a second round effect, the increase in the price level of the nontradable good generates a relative price effect which reduces the supply of tradables. This causes an additional decrease in the level of reserves. The total decrease in the level of foreign exchange reserves is crucial when we assess the stability properties of the model. Indeed, if the total induced decrease in foreign reserves is greater than the initial increase in government's debt, the economy might evolve in explosive cycles. The key factor driving this result is the presence of the one period lagged foreign reserves in the model's structural equations. If an increase in gc_t results in a more than proportional decrease in f_t, the beginning-of-period monetary base in t+1 will, ceteris paribus, decrease relative to t, causing a decrease in the price level of nontradables relative to its initial value and an increase in foreign reserves. This pattern will again reverse itself in period t+2 with even larger magnitudes, generating a destabilizing element in the model.

From equation (5.3) it is clear that (leaving aside the coefficient d_3 corresponding to the lagged term in equation (3.75b)) the effects of gc_t and f_{t-1} on the level of foreign reserves are similar. This is, of course, due to the fact that both variables are components of the beginning-of-period monetary base. Hence, the usual requirement for the stability of equation (5.3):

$$\left| \frac{\delta f_t}{\delta f_{t-1}} \right| < 1 \text{ is related to the value of } \left| \frac{\delta f_t}{\delta gc_t} \right|$$

In general, in this model $|\delta f_t/\delta gc_t| < 1$ as consideration of the microfoundations of the model will reveal. In particular:

$$(5.4) \quad \frac{\delta f_t}{\delta gc_t} - \frac{d_1\tau\omega_1(a_1-\delta_2)}{\tau+\delta_1+\delta_2} + \omega_1(d_1a_1-d_2b_2)$$

In order to simplify equation (5.4), the restrictions imposed by the microfoundations in Chapter III will now be strictly followed. First, δ_2 and b_2 are respectively the elasticities of the demand for nontradables and tradables, with respect to the real monetary base. From equations (3.38) and (3.39) it is clear that both elasticities are less than one, 6/ and that $\delta_2 = b_2$. Second, δ_1 and b_1 are respectively the elasticities of the demand for nontradables and tradables with respect to the expected inflation rate, which again from equations (3.38) and (3.39) are less than one 7/ and $\delta_1 = b_1$. In addition, it is also clear that $(\delta_1 + \delta_2) = (b_1 + b_2) = 1$. Substituting those values into equation (5.4) yields:

$$(5.4') \quad \frac{\delta f_t}{\delta gc_t} = \frac{\omega_1[(d_1a_1-d_2\delta_2)(2) + \tau\delta_2(d_2-d_1)]}{2 - \tau}$$

which is less than one in absolute terms for most plausible values of the model's parameters. Leaving aside the parameters of linearization, i.e., assuming $d_1 = d_2$ reinforces this result. 8/

Now, from equation (5.3), it follows that:

$$(5.5) \quad \frac{\delta f_t}{\delta f_{t-1}} = \frac{(1-\omega_1)}{\omega_1}\frac{\delta f_t}{\delta gc_t} + d_3$$

Since, in general, $0 > [\frac{(1-\omega_1)}{\omega_1}\frac{\delta f_t}{\delta gc_t}] < 1$, the value of equation (5.5) depends on the value taken by d_3. In order to avoid a bias in the model arising from the constant of linearization, d_3, the remainder of this chapter will assume $d_3 = 1$. 9/ Under that assumption, the model will be stable. Since $1 > (\delta f_t/\delta f_{t-1}) > 0$, the model will converge smoothly. 10/ 11/

The effects of the additional arguments in equations (5.2) and (5.3) are straightforward: a rise in the expected future levels of prices of either commodity increases the expected inflation rate and hence the demand for both commodities. In addition, the equilibrium level of the real monetary base falls, and the supply of both commodities also decreases. The resulting shifts in supplies and demands unambiguously lead to an increase in the price level of the nontradable good and to a decrease in the level of foreign reserves.

An increase in the current foreign price level of tradables generates both a relative price effect against the supply of nontradables and a decrease in the demand for tradables. 12/ These unambiguously result in an increase in both the price level of the nontraded good and the level of foreign reserves. Finally, as expected, a positive productivity change or an exogenous change in preferences that assigns a lower weight to current consumption, decreases the price of nontradables, and rises the level of foreign reserves.

In order to proceed towards a final solution of the model, that is, taking into account the endogenous property of price expectations, it is important to realize that, as in the flexible exchange rate, equations (5.2) and (5.3) are not independent from each other, because the lagged variable f_{t-1} is present in equation (5.2) and the price expectation term ${}_{t}p^{e}_{t+1}$ is present in equation (5.3).

Following the same methods as the previous chapter, equations (5.2) and (5.3) can be rewritten in the following way: 13/

$$(5.6)\quad p_t = X_0 + X_1 f_{t-1} + X_2\, gc_t + X_3\, {}_{t}p^{e}_{t+1} + X_4\, p^{*}_{t} + X_5\, {}_{t}(p^{*})^{e}_{t+1} + X_6\, u_t + X_7\, \varepsilon_t$$

$$(5.7)\quad f_t = \gamma_0 + \gamma_1 f_{t-1} + \gamma_2 gc_t + \gamma_3\, {}_{at}p^e_{t+1} + \gamma_4 p^*_t + \gamma_5\, {}_{at}(p^*)^e_{t+1} + \gamma_6 u_t + \gamma_7 \varepsilon_t$$

Notice that since the exchange rate is treated as a constant in this section, it is contained in the terms X_0 and Y_0. In addition, $\sum_{i=1}^{5} X_i = \sum_{i=1}^{5} Y_i = 1$, implying that if future expectations of the general price level were to respond proportionally to a joint and equal increase in the total monetary base and the price level of the tradable good, leaving the real monetary base constant, both the price level of the nontradable good and the level of foreign reserves would also increase proportionally. This is so because, if the real monetary base (deflated by ${}_{at}pi^e_{t+1}$) remains constant after a joint increase in both ${}_ah_t$ and $\dot{p}^*_t$, a relative price effect favoring the supply of tradables and discouraging the supply of nontradables results. For both commodity markets to be in equilibrium, the price level of the nontradable commodity and the level of foreign reserves must increase proportionally. As a final outcome, the output levels of both commodities, as well as the level of real foreign reserves (f_t/pi_t) remain unchanged. Only nominal endogenous magnitudes change proportionally 14/ following a proportional increase in all nominal exogenous variables.

As in the previous chapters, the method of undetermined coefficients is used here to obtain the final solution of the model. Following McCallum (1983), notice that the solution for the price level of nontradables can be written as a linear function of the predetermined state variables f_{t-1}, gc_{t-1}, m, v_t, x_t, x_{t-1}, p^*_{t-1}, j^*, n_t, t_t, t_{t-1}, u_t, ε_t, and the constant 1. Hence:

$$(5.8)\quad p_t = \theta_o^N + \theta_1^N f_{t-1}\ \theta_2^N gc_{t-1} + \theta_3^N m + \theta_4^N v_t + \theta_5^N x_t$$
$$+ \theta_6^N x_{t-1} + \theta_7^N p_{t-1}^* + \theta_8^N j^* + \theta_9^N n_t + \theta_{10}^N t_t$$
$$+ \theta_{11}^N t_{t-1} + \theta_{12}^N u_t + \theta_{13}^N \varepsilon_t$$

When the θ^Ns are the unknown coefficients for the fixed exchange rate case. Appendix V.1 shows that the forward solution obtained from the method of undetermined coefficients is in fact the stable solution.

In addition, an equivalent proof of stability is also given in Appendix V.2 where the model formed by equations (5.6) and (5.7) is subject to the method proposed by Blanchard and Kahn (1980). To facilitate the exposition, the derivation of the final solutions for the price and output levels of the nontradable good, as well as the equilibrium solution for the θs is presented in Appendix V.3. The solution for p_t is:

$$(5.9)\quad p_t = \frac{1}{1-\theta_1^N Y_3} \{[X_0 - X_0\theta_1^N Y_3 + X_3\theta_0^N + X_3\theta_1^N Y_0]$$
$$+ [X_1 - X_1\theta_1^N Y_3 + X_3\theta_1^N Y_1]\ f_{t-1}$$
$$+ [X_2 - X_2\theta_1^N Y_3 + X_3\theta_1^N Y_2 + X_3\theta_2^N]\ (gc_{t-1}+v_t-x_{t-1})$$
$$+ [X_2 - X_2\theta_1^N Y_3 + X_3\theta_1^N Y_2 + X_3\theta_2^N + X_3\theta_3^N]\ m$$
$$+ [X_2-X_2\theta_1^N Y_3+X_3\theta_1^N Y_2+X_3\theta_2^N+X_3\theta_6^N]\ x_t$$
$$+ [(X_4+X_5)(1-\theta_1^N Y_3)+X_3\theta_1^N Y_4+X_3\theta_1^N Y_5+X_3\theta_7^N]\ (p_{t-1}^*+n_t-t_{t-1})$$
$$+ [(X_4+2X_5)(1-\theta_1^N Y_3)+X_3\theta_1^N Y_4+2X_3\theta_1^N Y_5+X_3(\theta_7^N+\theta_8^N)]\ j^*$$
$$+ [X_4-X_4\theta_1^N Y_3+X_3\theta_1^N Y_4+X_3(\theta_7^N+\theta_{11}^N)]\ t_t$$
$$+ [X_6-X_6\theta_1^N Y_3+X_3\theta_1^N Y_6]\ u_t$$
$$+ [X_7-X_7\theta_1^N Y_3+X_3\theta_1^N Y_7]\ \varepsilon_t\}$$

The solution for the level of foreign reserves can now be easily found by substituting equation (V.[illegible]2) of Appendix V.3 into equation (5.7):

$$
\begin{aligned}
(5.10)\quad f_t = \frac{1}{1-\theta_1^N\gamma_3}\{ & [\gamma_0+\gamma_3\theta_0^N] + [\gamma_1]\, f_{t-1} \\
& + [\gamma_2+\gamma_3\theta_2^N](gc_{t-1} + v_t - x_{t-1}) \\
& + [\gamma_2+\gamma_3(\theta_2^N + \theta_3^N)]\, m \\
& + [\gamma_2]\, x_t \\
& + [\gamma_4+\gamma_5+\gamma_3\theta_7^N](p^*_{t-1}+n_t-t_{t-1}) \\
& + [\gamma_4 + 2\gamma_5 + \gamma_3\,(\theta_7^N+\theta_8^N)]\, j^* \\
& + [\gamma_4]\, t_t \\
& + [\gamma_6]\, u_t + [\gamma_7]\epsilon_t \}
\end{aligned}
$$

Based on Appendix V.3, Table 5.1 distinguishes between the "financial constraint effect" and the "relative price effect" that impinge on the levels of output of both commodities following a change in the exogenous variables of the model.

The behavioral responses of the price level of nontradables, the level of foreign reserves and the levels of output, following a change in the exogenous variables of the model will now be analyzed.

2.1 <u>An increase in the level of government's outstanding debt</u>

In spite of the endogeneity of the money supply under a fixed exchange rate regime, changes in the exogenous component of the monetary base (gc_t) will have real effects because in a tradables-nontradables short-run framework, the level of reserves is not assumed to adjust to guarantee steady state equilibrium in the money market. Instead, it is assumed to adjust solely to guarantee short-run equilibrium in the money market. However, the specific way in which the absence of short-run money neutrality gets translated into price and output fluctuations depends critically on the nature of the monetary change.

Consider first an increase in the expected level of government debt caused either by a change in the past value of the government's outstanding debt (gc_{t-1}) or by a positive permanent monetary change (v_t). The price level of the nontradable good will increase and the level of foreign reserves will decrease, because there will be a net "potential" excess demand for each commodity. These changes in p_t and f_t, however, will be less than proportional to the increase in the government's debt. 15/

The response of outputs to the increase in gc_t, however, will depend (as in the flexible exchange rate case) on two effects: (a) a financial constraint effect; and (b) a relative price effect. While the increase in gc_t generates an unambiguous relative price change favoring an increase in the output of nontradables and a decrease in that of tradables, the response of the real monetary base (deflated by the expected future general price level) needs some explanation. Since only the predetermined level of foreign reserves (f_{t-1}) enters into the beginning-of-period monetary base, an increase in gc_t will unambiguously result in an increase in the nominal level of the monetary base.

However, the response of the real monetary base can be analyzed by considering the expected inflation rate. 16/ By using the relevant coefficients in equations (5.8) and (V.3.2). It is clear that:

$$(5.11)\quad \frac{\partial({}_{t}p^{e}_{t+1} - p_t)}{\partial gc_{t-1}} = \frac{\theta_2^N + \theta_1^N \gamma_2}{1 - \theta_1^N \gamma_3} - \theta_2^N = \frac{\theta_1^N(\gamma_2 + \theta_2^N \gamma_3)}{1 - \theta_1^N \gamma_3}$$

which is negative and less than one is absolute terms. Thus, although the current price level of the nontradable good will increase, the public will also expect a deflation in the subsequent period. The intuition behind this result is straightforward: the decrease in the level of reserves which follows an increase in v_t or gc_{t-1} does not affect the current general price level. This is affected solely by the predetermined value of f_{t-1}. However, the decrease in reserves will decrease the future level of the monetary base and hence will impinge negatively on the expected future general price level. Now, this decrease in the expected inflation rate generates an increase in the equilibrium level of the real monetary base 17/ (deflated by the future expected general price level) and hence an increase in the firms' demand for labor. Thus, it will generate a positive financial constraint effect tending to cause a short run increase in the output of both commodities. As a result, the response of nontradables output will always be positive following an increase in gc_{t-1} or v_t, but the output of tradables may either increase or decrease; it will increase if the positive financial constraint effect is greater than the negative relative price effect.

Next, consider an increase in the rate of growth of government debt. The price level of the nontradable good will increase and the level of foreign reserves will decrease, but the magnitude of these changes will be bigger than those corresponding to an increase in gc_{t-1} or v_t. 18/ While the relative price effect generated by an increase in m is similar (but bigger) than the one generated by an increase in the level of the debt, the financial constraint effect might be quite different and might even affect negatively the supply of both commodities. In fact, when the rate of growth of the government debt increases, the effect on the inflation rate is:

$$(5.12)\quad \frac{\delta({}_{at}p^e_{t+1} - p_t)}{\delta m} = \frac{\theta^N_2+\theta^N_3+\theta^N_1 Y_2}{1-\theta^N_1 Y_3} - \theta^N_3 = \frac{\theta^N_2+\theta^N_1(Y_2+\theta^N_3 Y_3)}{1-\theta^N_1 Y_3}$$

This effect might be positive or negative. 19/ If it is negative, the decrease in the expected inflation rate will generate a positive financial constraint effect by increasing the equilibrium level of the real monetary base. Hence, the effects on the level of outputs will be similar whether the increase in the government debt arises from a rise in either v_t or m. However, if equation (5.12) is positive, the real monetary base (deflated by either p_t or ${}_{at}p^e_{t+1}$) will decrease, generating a negative financial constraint effect on the aggregate supply of both commodities. In the latter situation, the level of output of tradable goods will unambiguously decrease and the level of output of nontradables might decrease if the negative financial constraint effect outweighs the positive relative price effect. Notice that in this case, output responses are opposite to those resulting from an increase in gc_{t-1} or v_t. This example serves to illustrate once more the importance of the nature of an observed increase in the government debt.

Even under fixed exchange rates, in the short run the general price level is an endogenous variable and hence in a model of the kind presented here, money is not superneutral. It might instead have perverse short-run effects on output levels.

Finally, consider a positive temporary monetary shock x_t. Once again the price level of nontradables will increase and the level of reserves will decrease, but this time the magnitude of changes will be smaller than those corresponding to an increase in v_t. 20/ The obvious reason for this result is that economic agents know that the shock is temporary and hence expect the level of government debt to decrease next period. This expectation will negatively affect their expectations of the future general price level.

Once more the relative price effect will be similar to that analyzed above, i.e., it will favor the supply of nontradable goods. It will, however, be of a smaller magnitude. The financial constraint effect will be unambiguously positive in this case because there will be an unambiguous reduction in the expected inflation rate. That is:

$$(5.13) \quad \frac{\delta({}_{at}P^e_{t+1} - P_t)}{\delta x_t} = \frac{\theta^N_1 Y_2}{1-\theta^N_1 Y_3} - \theta^N_5$$

which is negative. The intuition behind this result as follows. Not only will government debt decrease in period t+1, but the reduction in the current level of foreign reserves will further reduce the level of the monetary base in period t+1. Hence, the expected value of the future general price level will be lower.

2.2 An increase in the price level of the tradable good

An increase in the price level of the tradable good affects the expected general inflation rate through its effects on the price levels of both produced goods. In addition, while such a change does not affect the current level of the beginning of period monetary base, it does affect the future level of the monetary base through its impact on the level of reserves.

Consider first an increase in either p^*_{t-1} or n_t. Both the price level of the nontradable good and the level of foreign reserves will rise 21/ because this increase generates a "potential" excess demand for nontradables and an excess supply of tradables. The effect on the output levels of both commodities can, once more, be decomposed in a relative price effect and a "financial constraint" effect. From Table 5.1, it can be seen that the relative price effect favors an increase in the supply of tradables (since $\theta^N_7 < 1$) and a decrease in the supply of nontradables. Since the current nominal monetary base at the beginning of the period remains unchanged when there is a rise in the price of tradables, the financial constraint effect can be evaluated by analyzing the effect on the expected future general price level. 22/ That is:

$$(5.14) \quad \frac{\delta({}_{at}pi^e_{t+1})}{\delta p^*_{t-1}} = \frac{\tau\theta^N_1(\gamma_4+\gamma_5)+\tau\theta^N_7 + (1-\tau)}{1-\theta^N_1\gamma_3}$$

which is unambiguously positive because $|\gamma_4| > |\gamma_5|$ and $\theta^N_7 > 0$. This result is straightforward. A permanent rise in the current price level of the tradable good increases the current level of foreign reserves. This in turn increases the next period monetary base and hence generates an increase in the expected value of the future general price level. Since the change in reserves does not affect the current beginning-of-period monetary base, the increase in the expected value of the future general price level will be larger than the increase in the current general price level; that is, the expected inflation rate will increase. 23/ As a result, the current real monetary base (deflated by ${}_{at}pi^{e}_{t+1}$) will decrease, generating a negative financial constraint effect. Thus, the supply of the nontradable good will unambiguously decrease, while the response of the supply of tradables depends on the importance of the relative price effect compared to the financial constraint effect. If the latter outweighs the former, the supply of tradables also decreases. Notice that the increase in the expected inflation rate, following a rise in the price level of the tradable good, is a feature particular to the model discussed here. Indeed, it is a consequence of the trading sequence imposed in the model by which firms and households decisions are constrained by the available monetary base at the beginning of the period.

Next, consider an increase in the rate of growth of the price level of the tradable good. Although the price level of the nontradable good will increase (on a larger magnitude 24/ than the rise generated by an increase in n_t) this is the only case in which we have not been able to sign the effect on the level of reserves 25/ and thus, the responses of the inflation rate and the real monetary base are ambiguous. These indeterminacies arise because when j^* increases, there is a positive first round effect on the expected rate of inflation which is not present when n_t increases. The "impact" increase in ${}_{at}pi^{e}_{t+1}$ partially or totally offsets the "potential" excess supply of tradable goods generated by the increase in p^*_t. If the negative effect of a rise of ${}_{at}pi^{e}_{t+1}$ on f_t is large enough, the level of foreign reserves will decrease.

Similarly, if the initial excess supply of tradables is large enough, the level of reserves will increase. In the latter case, the future beginning-of-period monetary base will also increase, as will the expected inflation rate generating a negative financial constraint effect on the output levels of both commodities. However, it is also possible that the level of foreign reserves will decrease following a rise in j^*. In such a case, its effect on the expected inflation rate and hence on the financial constraint effect is indeterminate. The only unambiguous result inpinging on the supply of outputs is a relative price effect favoring the output level of the tradable good. 26/

Finally, consider a temporary rise in the price level of tradables. The price level of the tradable good and the level of foreign reserves will increase. This is a straightforward case. Because the public expect the change to be temporary, the expected future inflation rate of the tradable good decreases. Thus, the increase in the level of foreign reserves is larger when the rise in p_t^* is due to a temporary rather than to a permanent shock. 27/

The rise in the level of reserves increases the future beginning-of-period level of the monetary base, and this will result in a final increase in the expected future general price level. 28/ As a consequence, the equilibrium level of the current real monetary base (deflated by ${}_{at}pi^e_{t+1}$) will fall generating a negative financial constraint effect on the output levels of both commodities. Thus, the effects of a temporary change in the price level of the tradable good are qualitatively similar (but of a different magnitude) to those generated by a permanent change. 29/ The supply of the nontradable good will unambiguously decrease, while the response of the output of the tradable good depends on the importance of the relative price effect as compared to the financial constraint effect.

2.3 An increase in productivity or an exogenous change in preferences that assigns a lower weight to current consumption

In either case, the price level of the nontradable good will decrease and the level of foreign reserves will increase 30/ because both shocks generate a potential excess supply of both commodities. In addition, either an increase in u_t or a decrease in ϵ_t generates: (1) a relative price effect favoring the supply of tradables; and (2) a negative financial constraint effect on the supply of both commodities. Result (1) above is obvious, and result (2) emerges because of the positive impact of the rise in the current level of foreign reserves on the future expected general price level, which in turn lowers the equilibrium real monetary base.

Notice one interesting result which emerges from the above analysis: In the presence of a negative financial constraint effect, it is possible that the output response of the nontradable good to a productivity increase is negative. This will happen if the combined negative "financial constraint" and "relative price" effects outweigh the positive direct effect of a productivity increase on production. 31/ This result highlights once more the importance of the behavior of the expected future general price level in an economy constrained by a limited capital market. Any change in an exogenous variable leading to a rise in foreign reserves exerts an upward pressure on the expected future general price level, because of its impact on the future beginning-of-period monetary base.

3. The incomplete current information case

As we did in the flexible exchange rate case, we will now assume that agents lack full current information in the following ways: (1) they cannot distinguish between a temporary and a permanent shock to the government's outstanding debt; and (2) at the beginning of the period, they do not observe the price level of the tradable good that will prevail at the end of the period. In this case, the model gives rise to the following semi-reduced forms for p_t and f_t:

$$(5.15)\quad p_t = W_0 + W_1 f_{t-1} + W_2 gc_t + W_3\, {}_{at}p_t^e + W_4\, {}_{at}p_{t+1}^e + W_5\, {}_{bt}p_{t+1}^e + W_6\, {}_{at}(p^*)_t^e + W_7\, {}_{at}(p^*)_{t+1}^e + W_8\, {}_{bt}(p^*)_{t+1}^e + W_9 u_t + W_{10} \varepsilon_t$$

and

$$(5.16)\quad f_t = Z_0 + Z_1 f_{t-1} + Z_2 gc_t + Z_3\, {}_{at}p_t^e + Z_4\, {}_{at}p_{t+1}^e + Z_5\, {}_{bt}p_{t+1}^e + Z_6\, {}_{at}p_t^* + Z_7 p_t^* + Z_8\, {}_{at}(p^*)_{t+1}^e + Z_9\, {}_{bt}(p^*)_{t+1}^e + Z_{10} u_t + Z_{11} \varepsilon_t$$

where:

$$W_0 = X_0 + [X_0(1-\tau)/(\delta_1+\delta_2)]$$

$$W_1 = X_1 + [X_1(1-\tau)/(\delta_1+\delta_2)]$$

$$W_2 = X_2 + [X_2(1-\tau)/(\delta_1+\delta_2)]$$

$$W_3 = -(1-\tau)/(\delta_1+\delta_2)$$

$$W_4 = a_1\tau/(\delta_1+\delta_2)$$

$$W_5 = \delta_1\tau/(\delta_1+\delta_2)$$

$$W_6 = X_4 + [X_4(1-\tau)/(\delta_1+\delta_2)]$$

$$W_7 = a_1(1-\tau)/(\delta_1+\delta_2)$$

$$W_8 = \delta_1(1-\tau)/(\delta_1+\delta_2)$$

$$W_9 = X_6 + [X_6(1-\tau)/(\delta_1+\delta_2)]$$

$$W_{10} = X_7 + [X_7(1-\tau)/(\delta_1+\delta_2)]$$

$$Z_0 = [d_1a_0-d_2b_0+\omega_0(d_1a_1-d_2b_2) + (d_1+d_2b_2+d_2b_1+1-d_3)\bar{s} - (1-\tau)(d_1+d_1a_1+d_2b_1)\bar{s}]$$

$$Z_1 = d_3 + (1-\omega_1)(d_1a_1-d_2b_2)$$

$$Z_2 = \omega_1(d_1a_1-d_2b_2)$$

$$Z_3 = - d_1\tau$$

$$Z_4 = - d_1a_1\tau$$

$$Z_5 = - d_2b_1\tau$$

$$Z_6 = d_1\tau$$

$$Z_7 = d_2b_1 + d_2b_2 + 1 - d_3$$

$$Z_8 = - d_1a_1 (1-\tau)$$

$$Z_9 = -d_2b_1 (1-\tau)$$

$$Z_{10} = d_1a_3$$

$$Z_{11} = - d_2$$

Notice that the current price level of the tradable good does not appear in equation (5.15). At first sight this might seem to imply that an unanticipated temporary increase in the price level of tradables does not affect the price level of the nontradable good. 32/ However, the expectation of the future price level of the nontradable good, based on the current end-of-period information set (${}_{bt}\hat{p}^e_{t+1}$), is an argument in equation (5.15). If we forward equation (5.15) once, and take end-of-period expectations, it becomes clear that ${}_{bt}\hat{p}^e_{t+1}$ is a function of the current level of foreign reserves. This variable in turn depends on all the components of the process governing the behavior of the price level of the traded good. Thus, in the incomplete information case, it is appropriate to conjecture a solution for p_t of the following form:

$$(5.17)\ p_t = \theta_0^N + \theta_1^N f_{t-1} + \theta_2^N gc_{t-1} + \theta_3^N m + \theta_4^N (v_t+x_t) + \theta_5^N x_{t-1} + \theta_6^N p^*_{t-1} + \theta_7^N j^* + \theta_8^N n_t + \theta_9^N t_t + \theta_{10}^N t_{t-1} + \theta_{11}^N u_t + \theta_{12}^N \varepsilon_t$$

where θ^Ns are the undetermined coefficients.

Notice that because of the assumption that economic agents cannot distinguish between permanent and temporary shocks to the government outstanding debt, there is a common coefficient for v_t and x_t in equation (5.17).

In contrast with the closed economy and the flexible exchange rate cases, the beginning and end-of-period price level expectations are not equal under fixed rates because it is assumed that n_t and t_t are only known at the end of each period. 33/ This feature of our model makes its solution particularly cumbersome. Hence, rather than present specific solutions of the θs, the remainder of this section will discuss the effects on the model's endogenous variables of removing the assumption of full current information.

3.1 The confusion between permanent and temporary monetary shocks.

Under incomplete current information, the direction of the responses of the price level of the nontradable good and the level of foreign reserves to an increase in the government debt, will be the same as in the full current information case. This is so because, in the full current information case, the effects of both a temporary or a permanent monetary shock were qualitatively similar. They were of different magnitudes, however. Under full current information, the (positive) response of the price level of the nontradable good and the (negative) response of the level of foreign reserves were larger if they followed a permanent rather than a temporary change in the government debt. It follows from this that if a temporary shock is mistakenly interpreted as permanent, the increase in p_t caused by that shock will be larger than in the full current information case. 34/

In addition, under full current information, both shocks generated an increase in the real monetary base (a positive financial constraint effect) and a relative price effect favoring the supply of the nontradable good. However, while the relative price effect was larger after a permanent monetary shock, the financial constraint effect was larger after a temporary shock. 35/ Thus, while both shocks generated a definite increase in the supply of the nontradable good, it could not be determined which shock generated the greater response. Moreover, although the response of the supply of the tradable good is ambiguous, a temporary monetary shock might lead to an output expansion of the tradable good, even if a permanent shock leads to an output contraction of this good. This result obviously follows because of the larger financial constraint effect and the smaller relative price effect generated by a temporary shock.

From the above considerations, it follows that, under incomplete current information, the confusion between permanent and temporary monetary shocks cannot change the direction of the response of the supply of the nontradable good relative to the full current information case; only the magnitude of the output increase will differ between the two alternative information sets. However, the confusion between monetary shocks might change the direction of response of the supply of tradables relative to the full current information case. In particular, if under full current information, a temporary shock leads to an output expansion while a permanent shock leads to an output contraction of the tradable good, a permanent shock mistakenly viewed as temporary (when lack of complete current information is assumed) might increase the output level of the tradable commodity.

To sum up, confusion between permanent and temporary monetary shocks can only affect the magnitude of the responses of the price and output levels of the nontradable good, as well as the level of foreign reserves relative to the corresponding responses under full current information. However, such a confusion can imply an increase in the output level of the tradable good following a permanent shock, even in situations where a fully anticipated permanent change would lead to an output contraction.

3.2 The lack of beginning-of-period observation of the current price level of the tradable good

In our model, output decisions are made at the beginning of every period based on the information set available at the time. If the current period price level of the tradable good is not known at the beginning of the period, agents must form expectations about it (which in the present case are equal to: ${}_{at}(p^*)^e_t = p^*_{t-1} + j^* - t_{t-1}$). In this fixed exchange rate case, agents cannot infer the value of the price level of the nontradable good by observing the beginning of period current level of the monetary base. In addition, p_t^* is an argument in the determination of $\hat{p}_t$. Thus, the short-run supplies of both commodities no longer depend on their actual price levels; instead, they depend on their <u>expected</u> price levels. <u>36</u>/ Hence, the current output levels of both commodities will remain <u>unchanged</u> following either a permanent or a temporary shock to the price level of the tradable good if that shock is unanticipated.

Now, since the outputs remain unchanged following an unanticipated shock to p_t^*, the price level of the nontradable good and the level of foreign reserves have to bear all the adjustment. In particular, an unanticipated permanent shock will raise the demand for nontradables and will decrease the demand for tradables <u>37</u>/ implying that (given $\hat{y}_t$ and $\hat{y}_t^*$ constant) the increase in both $\hat{p}_t$ and $\hat{f}_t$ will be larger than in the complete current information case. If the unanticipated shock is temporary, only the demand for tradable goods will decrease as a first round effect, because a temporary shock does not affect ${}_{bt}(p^*)^e_{t+1}$, which is the foreign price variable impinging on the demand for nontradable goods. However, in subsequent rounds, the increased level of reserves will increase the expected future general price level, leading to a rise in the demand for nontradables. Thus, the price level of nontradables will increase following an unanticipated temporary increase in the price of tradables, but by less than in the case of an unanticipated permanent increase in the price of tradables.

To sum up: relative to the full information case, incomplete beginning-of-period knowledge about the current period price level of the tradable good results in a larger response of both $\hat{p}_t$ and $\hat{f}_t$ to an

3 3

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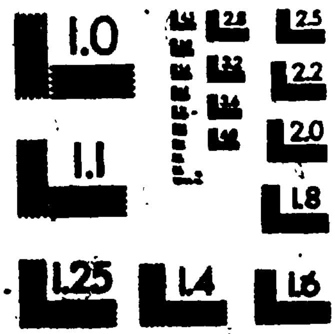

unanticipated change in p_t^*. Also, the increase in the price level of the nontradable good will be larger if the unanticipated increase in the price of the tradable good is permanent. In contrast with the full current information case, no output response will follow an unanticipated foreign price shock.

4. The effects of a devaluation

As stated in the introduction to this chapter, the exchange rate is treated here as an exogenous variable. Hence, in order to analyze the effects of a devaluation, it is necessary to specify the process governing the behavior of that variable. In particular, it will be assumed here that the exchange rate follows a random walk, that is:

$$(5.18)\quad s_t = s_{t-1} + \xi_t$$

where ξ_t is a white noise disturbance independently distributed from the other disturbances in the model.

As was also stated in the introduction, the rule described by (5.18) is assumed to be known to the public. But even if the rule is known, the current value of ξ_t may not be observable. Thus, there are two cases to be considered.

4.1 The full current information case

In this case, both s_{t-1} and ξ_t are known variables at the beginning of every period. Since s_t enters into the system of structural equations in the same way as p_t^* does, the economic effects of an anticipated permanent devaluation, i.e., a rise in ξ_t, are identical to the effects generated by a permanent rise in the price level of the tradable good.

That is, the price level of the nontradable good and the level of foreign reserves will rise, the output level of the nontradable good will decrease, and the output level of the tradable good will decrease if the financial constraint effect outweighs the relative price effect. Hence, a fully anticipated devaluation may be contractionary in the short run. This result contrast with that obtained from traditional tradable-nontradable frameworks of the Dornbusch kind where the short-run effect of a devaluation only reallocates resources from the supply of nontradables to the supply of tradables. 38/

4.2 The incomplete current information case

Assume now that, although economic agents know the rule (5.18), they do not know the value of ξ_t at the beginning of period t. In this case, the effects of an unanticipated permanent devaluation, i.e., an unanticipated rise in ξ_t, are identical to those effects generated by an unanticipated permanent rise in the price level of the tradable good. That is, the rise in the price level of the nontradable good and the level of foreign reserves will be larger if the devaluation is unanticipated than if it is anticipated. In addition, an unanticipated devaluation will have no current effects in the output of either commodity. This result contrasts with that obtained from models which stress the effects of price misperceptions on aggregate supply. In those models an anticipated devaluation has no effect on the output level, while an unanticipated devaluation does. 39/ The opposite result arises here. This is an interesting conclusion because, to the extent that real world economies are properly described by the features of this model, we may conclude that an unanticipated current devaluation will increase the level of foreign reserves without hurting the current levels of output and employment of those economies. However, notice that in period t+1, ξ_t will be part of the information set and hence the unanticipated devaluation of period t might then have the contractionary effects which an anticipated devaluation would have had in period t. That is, the output effects of an unanticipated devaluation will not be eliminated, but only postponed.

5. Summary

This chapter has solved the model set up in Chapter III for the fixed exchange rate case, and has considered the effects of changes in the level of government debt, the price of the tradable goods, and the exchange rate (a devaluation) on the output levels of the domestically produced commodities, the price level of the nontradable goods, and the level of foreign reserves. It has been shown that the domestic firms' need to finance working capital in advance from a domestic banking sector (the financial constraint) has important implications for the economy's response to such exogenous shocks.

In particular, if the financial constraint is strong enough, an anticipated rise in the level of the government debt will be expansionary, while an anticipated rise in the price level of the tradable good or an anticipated devaluation will be contractionary in the short run. These results contrast with the results generated by models of the Dornbusch type, where only short-run allocation effects are generated.

In addition, in the model of this chapter, a permanent monetary shock might generate an increase in the output of both commodities if it is mistakenly viewed as temporary even in situations where a fully anticipated permanent monetary change would lead to a reduction in the output of the tradable good.

Finally, an unanticipated devaluation or an unanticipated rise in the price level of the tradable good will have no current output effects, but might lead to a contraction of the output of both goods in subsequent periods. This result differs from that arising in models where price misperceptions enter the supply functions, because in those models, an unanticipated devaluation leads to an immediate, albeit short-run output expansion.

Table 5.1. Decomposition of the Effects Impinging on the Domestic Levels of Output in the Full Current Information Case: The Fixed Rates Case

Exogenous Variables \ Endogenous Variables	Y_t: Financial constraint effect	Y_t: Relative price effect	Y_t^*: Financial constraint effect	Y_t^*: Relative price effect
Increase in gc_{t-1} or v_t	$a_1[\eta_1 - \frac{\tau(\phi_1^N\gamma_2+\phi_2^N)}{1-\phi_1^N\gamma_3}]$	$(1-\tau)\phi_2^N$	$a_1[\eta_1 - \frac{\tau(\phi_1^N\gamma_2+\phi_2^N)}{1-\phi_1^N\gamma_3}]$	$-\tau\phi_2^N$
Increase in m	$a_1[\eta_1 - \frac{\tau(\phi_1^N\gamma_2+\phi_2^N+\phi_3^N)}{1-\phi_1^N\gamma_3}]$	$(1-\tau)\phi_3^N$	$a_1[\eta_1 - \frac{\tau(\phi_1^N\gamma_2+\phi_2^N+\phi_3^N)}{1-\phi_1^N\gamma_3}]$	$-\tau\phi_3^N$
Increase in x_t	$a_1[\eta_1 - \frac{\tau\phi_1^N\gamma_2}{1-\phi_1^N\gamma_3}]$	$(1-\tau)\phi_5^N$	$a_1[\eta_1 - \frac{\tau\phi_1^N\gamma_2}{1-\phi_1^N\gamma_3}]$	$-\tau\phi_5^N$
Increase in p_{t-1} or n_t	$-a_1[\frac{\tau\phi_1^N(\gamma_4+\gamma_5)+\tau\phi_7^N+(1-\tau)}{1-\phi_1^N\gamma_3}]$	$(1-\tau)(\phi_7^N-1)$	$-a_1[\frac{\tau\phi_1^N(\gamma_4+\gamma_5)+\tau\phi_7^N+(1-\tau)}{1-\phi_1^N\gamma_3}]$	$-\tau(\phi_7^N-1)$
Increase in j^*	$-a_1[\frac{\tau\phi_1^N(\gamma_4+2\gamma_5)+\tau(\phi_7^N+\phi_8^N)+2(1-\tau)}{1-\phi_1^N\gamma_3}]$	$(1-\tau)(\phi_8^N-1)$	$-a_1[\frac{\tau\phi_1^N(\gamma_4+2\gamma_5)+\tau(\phi_7^N+\phi_8^N)+2(1-\tau)}{1-\phi_1^N\gamma_3}]$	$-\tau(\phi_8^N-1)$
Increase in ε_t	$\frac{-a_1\tau\phi_1^N\gamma_4}{1-\phi_1^N\gamma_3}$	$(1-\tau)(\phi_{10}^N-1)$	$\frac{-a_1\tau\phi_1^N\gamma_4}{1-\phi_1^N\gamma_3}$	$-\tau(\phi_{10}^N-1)$
Increase 1/ in u_t	$\frac{-a_1\tau\phi_1^N\gamma_6}{1-\phi_1^N\gamma_3}$	$(1-\tau)\phi_{12}^N$	$\frac{-a_1\tau\phi_1^N\gamma_6}{1-\phi_1^N\gamma_3}$	$-\tau\phi_{12}^N$
Increase in e_t	$\frac{-a_1\tau\phi_1^N\gamma_7}{1-\phi_1^N\gamma_3}$	$(1-\tau)\phi_{13}^N$	$\frac{-a_1\tau\phi_1^N\gamma_7}{1-\phi_1^N\gamma_3}$	$-\tau\phi_{13}^N$

1/ A direct effect on the aggregate supply equal to a_1 should be added in order to obtain the total output effect of a productivity shock.

FOOTNOTES TO CHAPTER V

1/ With the obvious allowance made for the constant of linearization: ω_1.

2/ Notice that the assumption of fixed exchange rates implies that the distinction between the domestic and foreign values of the foreign exchange reserves can be ignored.

3/ In the remainder of this chapter f_t will always stand for the end of period level of foreign reserves since that is the relevant endogenous variable. In addition, the predetermined level of reserves at the beginning of every period: ${}_{a t}f_t$ equals the end of the previous period level of reserves: f_{t-1}.

4/ The assumptions that $\delta_2 > a_1'$ and $b_2 > a_1$ established in Chapter IV to assure stability are maintained in this chapter.

5/ In the tradable-nontradable short run framework, foreign reserves are not constrained to adjust such as to obtain steady state equilibrium.

6/
$$\frac{\delta Z_t^d}{\delta({}_aH_t/P_t)} \cdot \frac{({}_aH_t/P_t)}{Z_t^d} = \frac{\delta (Z^*)_t^d}{\delta({}_aH_t/P_t^* S_t)} \cdot \frac{({}_aH_t/P_t^* S_t)}{(Z^*)_t^d}$$
$$= \frac{{}_aH_t}{\frac{f'}{\beta}\mu_t E(PI_{t+1}) + {}_aH_t} < 1$$

7/
$$\frac{\delta Z_t^d}{\delta(E(PI_{t+1})/P_t)} \cdot \frac{(E(PI_{t+1})/P_t)}{Z_t^d} = \frac{\delta (Z^*)_t^d}{\delta(E(PI_{t+1})/P_t^* S_t)} \cdot \frac{(E(PI_{t+1})/P_t^* S_t)}{(Z^*)_t^d}$$
$$= \frac{\frac{f'}{\beta}\mu_t E(PI_{t+1})}{\frac{f'}{\beta}\mu_t E(PI_{t+1}) + {}_aH_t} < 1$$

8/ It might be interesting to consider the case where the financial constraint effect derived from the increase in gc_t: ${}_{\cdot}a_1\omega_1$ equals the relative price effect caused by such an increase: $\tau(\delta\omega_1 - a_1\omega_1)/(1-\tau+\delta_1+\delta_2)$. In that case, the supply of tradable goods will remain unchanged and foreign reserves will decrease in the same proportion to the increase in the demand for tradables multiplied by the constant d_2; that is, reserves will decrease by $d_2 b_2 \omega_1$, and hence: $\delta f_t/\delta gc_t < 1$ if $b_2 < 1$, which is the case in this model. However, it will be true, in general,

that: $a_1\omega_1 > \frac{\tau(\delta_2\omega_1 - a_1\omega_1)}{1-\tau+\delta_1+\delta_2}$, which implies that the supply of tradables

will experience a net increase, that is, the financial constraint effect will be bigger than the relative price effect dampening the decrease in reserves and, hence, contributing to the stability of the model.

9/ Assuming $d_3 = 1$ implies also $d_1 = d_2$ which is a convenient result (see equation (5.4')).

Proof: from Chapter III, we have that:

$$d_1 = 1/q_1$$

$$d_2 = q_2/q_1$$

$$d_3 = -(1-q_1-q_2)/q_1$$

This implies that:

$$d_3 = -(d_1-d_2-1)$$

For $d_3 = 1$, it is necessary that: $d_1 = d_2$

10/ In a model of a Dornbusch type where f_t enters the demand for output equations, the only impact effect of a change in f_{t-1} is to increase f_t proportionally. However, "second round" effects will increase the demand for both commodities, rising the price level of the nontradable good and causing reserves to decrease less than proportionally to the original change in f_{t-1}. The net effect (adding the first and consecutive rounds) will then be a less than proportional increase in the level of reserves which implies that the Dornbusch model does not generate cycles. In the model of this chapter, a change in f_{t-1} involves both a direct positive increase in reserves and a monetary base-induced decrease in reserves. Both effects are "first round" effects.

11/ Notice that if $d_3 < 1$ the model will also be stable; but then $(\delta f_t/\delta f_{t-1})$ might be less than zero, which would imply that the convergence pattern would exhibit dampened cycles.

12/ The demand for tradables decreases because of a reduction in both the expected inflation rate and the real monetary base evaluated in terms of the tradable good. Notice that the demand for nontradables will not be affected since the relevant deflator is the price of nontradables.

13/ Where:

$X_0 = [\delta_0 - a_0 - a_1\omega_0 + \delta_2\omega_0 + (1-\tau)(1+\delta_1+a_1)\bar{s}]/(1-\tau+\delta_1+\delta_2)$

$X_1 = [(1-\omega_1)(\delta_2 - a_1)]/(1-\tau+\delta_1+\delta_2)$

$X_2 = [\omega_1(\delta_2 - a_1)]/(1-\tau+\delta_1+\delta_2)$

$X_3 = [\tau(\delta_1 + a_1)]/(1-\tau+\delta_1+\delta_2)$

$X_4 = [1-\tau]/(1-\tau+\delta_1+\delta_2)$

$X_5 = [(1-\tau)(\delta_1 + a_1)]/(1-\tau+\delta_1+\delta_2)$

$X_6 = [-a_3]/(1-\tau+\delta_1+\delta_2)$

$X_7 = 1/(1-\tau+\delta_1+\delta_2)$

$Y_0 = [d_1\tau(a_0 + a_1\omega_0 - \delta_0 - \delta_2\omega_0 + (\delta_1+\delta_2)\bar{s} - (1-\tau)(a_1+\delta_1)\bar{s})$
$+ (1-\tau+\delta_1+\delta_2)(d_1a_0 - d_2b_0 + d_1a_1\omega_0 - d_2b_2\omega_0) + (1-d_3+d_2b_1+d_2b_2)\bar{s}$
$- (1-\tau)(d_1a_1+d_2b_1)\bar{s})]/(1-\tau+\delta_1+\delta_2)$

$Y_1 = [d_1\tau(1-\omega_1)(a_1-\delta_2) + (1-\tau+\delta_1+\delta_2)(d_3+(1-\omega_1)(d_1a_1-d_2b_2))]/$
$(1-\tau+\delta_1+\delta_2)$

$Y_2 = [d_1\tau\omega_1(a_1-\delta_2) + (1-\tau+\delta_1+\delta_2)\omega_1(d_1a_1-d_2b_2)]/(1-\tau+\delta_1+\delta_2)$

$Y_3 = -\tau[d_1\tau(a_1+\delta_1) + (1-\tau+\delta_1+\delta_2)(d_1a_1+d_2b_1)]/(1-\tau+\delta_1+\delta_2)$

$Y_4 = [d_1\tau(\delta_1+\delta_2) + (1-\tau+\delta_1+\delta_2)(1-d_3+d_2b_1+d_2b_2)]/(1-\tau+\delta_1+\delta_2)$

$Y_5 = -(1-\tau)[d_1\tau(a_1+\delta_1) + (1-\tau+\delta_1+\delta_2)(d_1a_1+d_2b_1)]/(1-\tau+\delta_1+\delta_2)$

$Y_6 = [d_1a_3(1+\delta_1+\delta_2)]/(1-\tau+\delta_1+\delta_2)$

$Y_7 = -[d_1\tau+(1-\tau+\delta_1+\delta_2)d_2]/(1-\tau+\delta_1+\delta_2)$

14/ Even when the demand and supply of tradables remain unchanged, its value increases proportionally, leading to a proportionate change in the level of nominal foreign reserves.

15/ This is so because $\theta_2^N < 1$ and (in equation (5.10)):

$|\delta f_t/\delta gc_{t-1}| < 1$ since $|Y_2| < 1$.

16/ Since p_t^* is an exogenous variable, the analysis of the expected general inflation rate can be conducted by solely considering the expected inflation rate of the nontradable good.

17/ Notice that although the expected inflation rate decreases, the net effect of an increase in gc_t is a less than proportionate change in at pi^e_{t+1} but the change might be positive or negative.

From equation (V.3.2) it can be seen that the effect of an increase in gc_t on the expected inflation rate can be decomposed into:

1. A direct effect = $\theta_2^N > 0$ (see equation (V.3.5))
2. An indirect effect caused by the reduction in the level of

foreign reserves: $\frac{\theta_1^N \delta f_t}{\delta gc_{t-1}} = \theta_1^N \frac{(\Upsilon_2+\Upsilon_3\theta_2^N)}{1-\theta_1^N\Upsilon_3} < 0$

Adding up both effects will give:

$$\theta_2^N + \theta_1^N \frac{(\Upsilon_2+\Upsilon_3\theta_2^N)}{1-\theta_1^N\Upsilon_3} = \frac{\theta_2^N+\theta_1^N\Upsilon_2}{1-\theta_1^N\Upsilon_3} < 0$$

which will always be less than one (in absolute terms) since $(1-\theta_1^N\Upsilon_3) > 0$.

A sufficient condition for the above expression to be positive is

$|\theta_2^N|>|\Upsilon_2|$. That is, the increase in the expected level of the future

price resulting from an increase in gc_t be greater than the resulting decrease in the level of foreign reserves.

In the case that $|\theta_2^N| < |\theta_1^N\Upsilon_2|$, the above expression will be negative implying that the expected future price level will decrease following an increase in the current government outstanding debt. In such a situation, the real monetary base will increase even further generating a stronger positive financial constraint effect favoring the short-run expansion of both commodities.

18/ $\theta_3^N > \theta_2^N$ and (in equation (5.10)) $\left|\frac{\delta f_t}{\delta_m}\right| > \left|\frac{\delta f_t}{\delta v_t}\right|$. This is so

because when m increases, there will be an additional increase in the next period level of the government's outstanding debt, which generates additional pressure on the "potential" current excess demands for both commodities through its affects on the expected future price level of the nontradable good.

19/ Notice that even if the effect on the expected inflation rate is negative that does not imply that p^e_{t+1} decreases. In fact,

$$\frac{\delta\, {}_{at}p^e_{t+1}}{\delta m} \text{ is positive in general.}$$

Proof:

$$\theta^N_2+\theta^N_3 = \theta^N_2 + \frac{\theta^N_2(1-\theta^N_1 Y_3)}{1-\theta^N_1 Y_3 - X_3} \qquad \text{(from equation (V.3.6))}$$

$$= \theta^N_2\left[\frac{2(1-\theta^N_1 Y_3)}{1-\theta^N_1 Y_3 - X_3} - \frac{X_3}{1-\theta^N_1 Y_3 - X_3}\right]$$

The first term inside the brackets is greater than 2 since $(1-\theta^N_1 Y_3)/(1-\theta^N_1 Y_3-X_3) > 1$. The second term inside the brackets is less than one since $X_3 < 1$. Hence, the expression in brackets is greater than one.

Thus, $\theta^N_2+\theta^N_3+\theta^N_1 Y_2$ is positive in general since $|Y_2| < 1$ and $\theta^N_1 < 1$. If the rise in ${}_{at}p^e_{t+1}$ is more than proportional to the increase in m, the expected inflation rate increases; otherwise it decreases.

20/ $\theta^N_5 < \theta^N_2$ (see equation (V.3.8)); and (in equation (5.10): $\left|\frac{\delta f_t}{\delta x_t}\right| < \left|\frac{\delta f_t}{\delta gc_{t-1}}\right|$.

21/ $\theta^N_7 > 0$ and (in equation (5.10)): $\dfrac{\delta f_t}{\delta p^*_{t-1}} = \dfrac{Y_4+Y_5+Y_3\theta^N_7}{1-\theta^N_1 Y_3} > 0$

because $|Y_4| > |Y_5+Y_3|$ and $\theta^N_7 < 1$.

Proof: $|Y_4| = \dfrac{d_1\tau(\delta_1+\delta_2) + (1-\tau+\delta_1+\delta_2)\,(d_2b_1+d_2b_2)}{1-\tau+\delta_1+\delta_2}$

$$|Y_3+Y_5| = \frac{d_1\tau(a_1+\delta_1) + (1-\tau+\delta_1+\delta_2)(d_1a_1+d_2b_1)}{1-\tau+\delta_1+\delta_2}$$

$|Y_4| > |Y_3+Y_5|$ since $\delta_2 > a_1$; $b_2 > a_1$; $d_1 = d_2$; $d_3 = 1$.

22/ Since p^*_{t+1} is a component of pi_{t+1}, it is no longer possible to solely concentrate on the effects of a rise in p^*_t on the inflation rate of the nontradable good.

23/ Proof:

$$\frac{\delta({}_{at}pi^e_{t+1}-pi_t)}{\delta p^*_{t-1}} = \frac{\tau\theta^N_1(Y_4+Y_5)+\tau\theta^N_7}{1-\theta^N_1Y_3} - \tau\theta^N_7 = \frac{\tau\theta^N_1(Y_4+Y_5+\theta^N_7Y_3)}{1-\theta^N_1Y_3}$$

which is positive (see footnote 21).

24/ $\theta^N_8 > \theta^N_7$ (from equation (V.3.10)).

25/ From equation (5.10): $\frac{\delta f_t}{\delta j^*} = \frac{Y_4 + 2Y_5 + Y_3(\theta^N_7+\theta^N_8)}{1-\theta^N_1Y_3}$

which will be negative if $|Y_4| < |2Y_5 + Y_3(\theta^N_7+\theta^N_8)|$

26/ $\theta^N_8 < 1$.

27/ Notice that $\left|\frac{\delta f_t}{\delta t_t}\right| > \left|\frac{\delta f_t}{\delta v_t}\right|$ (see equation (5.10)).

28/ $\frac{\delta({}_{at}pi^e_{t+1})}{\delta t_t} = \frac{\theta^N_1Y_4}{1-\theta^N_1Y_3}$ which is positive

29/ Notice however that although ${}_{at}pi^e_{t+1}$ increases following a positive temporary shock, the effect on the expected general inflation rate is ambiguous because of the negative impact of ${}_{at}p^*_{t+1}$ on the future general price level.

30/ $\theta^N_{12} < 0$; $\theta^N_{13} > 0$

and, in equation (5.10): $\frac{\delta f_t}{\delta u_t} = \frac{\gamma_6}{1-\theta_1^N \gamma_3} > 0$; and $\frac{\delta f_t}{\delta \varepsilon_t} = \frac{\gamma_7}{1-\theta_1^N \gamma_3}$

31/ That is, if: $|a_3| < \left| \frac{-a_1 \tau \theta_1^N \gamma_6}{1-\theta_1^N \gamma_3} + \theta_{12}^N (1-\tau) \right|$

32/ The terms involving the price of the tradable good in equation (5.15):

$$_{a}t(p^*)_t^e = p_{t-1}^* + j^* - t_{t-1}$$

$$_{a}t(p^*)_{t+1}^e = p_{t-1}^* + 2j^* - t_{t-1}$$

$$_{b}t(p^*)_{t+1}^e = p_{t-1}^* + 2j^* + n_t - t_{t-1}$$

do not include the variable t_t.

33/ The confusion between v_t and x_t does not generate a discrepancy between beginning and end of period expectations because the distinction between the shocks is only known with one period lag. In addition, in the flexible exchange rate case all the components of the price level of the tradable good (anticipated or not) have no effect on the price level of the nontradable good.

34/ As in the closed economy case, it is postulated that:

$$E(x_t) = g_1 (v_t + x_t) \text{ where } g_1 = \frac{\sigma_x^2}{\sigma_v^2 + \sigma_x^2}$$

The higher the relative variance of the temporary shock, the lower the response of the price level of the nontradable good to a monetary shock. This is so because the higher g_1, the more a permanent shock will be confused as transitory and hence the lower will be its effect on p_t.

35/ Comparing equations (5.11) and (5.13), it can be concluded that the decline in the expected inflation rate following a monetary shock was larger if the shock was temporary than if it was permanent and hence the increase in the real monetary base was larger under a temporary monetary shock.

36/ It is important to recall that here price misperceptions of the Lucas type are not an argument in the supply functions.

37/ The demand for y_t will increase because of a rise in ${}_{bt}(p^*)^e_{t+1}$. The demand for y_t^* will decrease because of a rise in p_t^*.

38/ See, for example, Jonson and Kierzkowski (1975).

39/ See, for example, Burton (1980), where a devaluation is taken as unanticipated if the current value of the exchange rate does not enter in the agent's information set.

APPENDIX V.1

PROOF THAT THE FORWARD SOLUTION IS THE STABLE SOLUTION FOR THE DETERMINATION OF THE PRICE LEVEL OF NONTRADABLE GOODS

Taking, beginning of period t, expectations from equation (5.6) and rearranging, we obtain:

$$ {}_{at}p^{e}_{t+1} - \frac{1}{X_3}\, {}_{at}p^{e}_{t} = \frac{-1}{X_3} [X_0 + X_1 f_{t-1} + X_2 gc_t + X_4 p^{*}_{t} + X_5\, {}_{at}(p^{*})^{e}_{t+1} + X_6 u_t + X_7 \varepsilon_t] $$

or, using the lag operator L:

$$ (L^{-1} - \frac{1}{X_3})\, {}_{at}p^{e}_{t} = \frac{-1}{X_3} [X_0 + X_1 f_{t-1} + X_2 gc_t + X_4 p^{*}_{t} + X_5\, {}_{at}(p^{*})^{e}_{t+1} + X_6 u_t + X_7 \varepsilon_t] $$

The forward solution will be the "stable" solution if $|X_3| < 1$; which is the case given the parameters of the model.

APPENDIX V.2

TEST OF THE STABILITY PROPERTIES OF THE MODEL IN TERMS OF THE BLANCHARD AND KAHN METHOD

Equations (5.6) and (5.7) can be recasted in matrix form of the kind suggested by Blanchard and Kahn (1980).

$$\begin{bmatrix} f_t \\ {}_{a t}p^e_{t+1} \end{bmatrix} = \begin{bmatrix} \frac{Y_1X_3 - X_1Y_3}{X_3} & \frac{Y_3}{X_3} \\ -\frac{X_1}{X_3} & \frac{1}{X_3} \end{bmatrix} \begin{bmatrix} f_{t-1} \\ p_t \end{bmatrix}$$

$$+ \begin{bmatrix} \frac{Y_oX_3 - X_oY_3}{X_3} & \frac{Y_2X_3 - X_2Y_3}{X_3} & \frac{Y_3(Y_4 - X_4)}{X_3} & \frac{Y_3(Y_5 - X_5)}{X_3} & \frac{Y_3(Y_6 - X_6)}{X_3} & \frac{Y_3(Y_7 - X_7)}{X_3} \\ -\frac{X_o}{X_3} & -\frac{X_2}{X_3} & -\frac{X_4}{X_3} & -\frac{X_5}{X_3} & -\frac{X_6}{X_3} & -\frac{X_7}{X_3} \end{bmatrix}$$

$$\begin{bmatrix} 1 & gc_t & p^*_t & {}_{a t}(p^*)^e_{t+1} & u_t & \epsilon_t \end{bmatrix}^{-1}$$

where f_{t-1} is a predetermined variable in the model and p_t is a nonpredetermined variable.

Following Blanchard and Kahn, the model will be stable if the 2x2 matrix multiplying the vector $\begin{bmatrix} f_{t-1} \\ Pt \end{bmatrix}$ (which we will call matrix A) has one root outside the unit circle and one inside.

The characteristic equation implied by matrix A is:

$$X_3 \overline{\lambda}^2 + (Y_3X_1 - Y_1X_3 - 1)\overline{\lambda} + Y_1 = 0$$

with roots:

$$\bar{\lambda}_1 = \frac{1}{2X_3}\{-(Y_3X_1 - Y_1X_3 - 1) + \sqrt{(Y_3X_1 - Y_1X_3 - 1)^2 - 4Y_1X_3}\}$$

$$\bar{\lambda}_2 = \frac{1}{2X_3}\{-(Y_3X_1 - Y_1X_3 - 1) - \sqrt{(Y_3X_1 - Y_1X_3 - 1)^2 - 4Y_1X_3}\}$$

Now, from Appendix V.3 the term: $(Y_3X_1 - Y_1X_3 - 1)$ is equal to:

$- \frac{\tau(\delta_1 + a_1)}{1 - \tau + \delta_1 + \delta_2} - 1$, which is negative and greater than one in absolute terms. In addition:

$$Y_1X_3 = \frac{\tau(\delta_1 + a_1)}{1 - \tau + \delta_1 + \delta_2} + X_1Y_3$$

so, the term under the square root equals:

$$(Y_3X_1 - Y_1X_3 - 1)^2 - 4Y_1X_3 = 1 + \frac{\tau^2(\delta_1 + a_1)^2}{(1-\tau + \delta_1 + \delta_2)^2} - \frac{4\tau(\delta_1 + a_1)}{1-\tau + \delta_1 + \delta_2}$$

$$+ \frac{4(1-\omega_1)(\delta_2 - a_1)(d_1\tau)(a_1 + \delta_1)}{1 - \tau + \delta_1 + \delta_2}$$

which is positive and greater than one, but less than:

$$1 + \frac{\tau(\delta_1 + a_1)}{(1 - \tau + \delta_1 + \delta_2)}$$

since $X_3 < 1$: $0 < \bar{\lambda}_1 > 1$

$0 < \bar{\lambda}_2 < 1$

and the model satisfies the stability conditions.

APPENDIX V.3

DERIVATION OF THE FINAL SOLUTION FOR THE PRICE AND OUTPUT LEVELS OF THE NONTRADABLE GOOD

Leading equation (5.8) in the main text once and taking beginning of period t expectations, we obtain:

$$\text{(V.3.1)}\quad {}_{at}p^{e}_{t+1} = \theta^N_0 + \theta^N_1 f_t + \theta^N_2 gc_{t-1} + (\theta^N_2+\theta^N_3)m + \theta^N_2 v_t + (\theta^N_2+\theta^N_6)x_t - \theta^N_2 x_{t-1} + \theta^N_7 p^{*}_{t-1} + (\theta^N_7+\theta^N_8)j^{*} + \theta^N_7 n_t + (\theta^N_7+\theta^N_{11})t_t - \theta^N_7 t_{t-1}$$

Now, equation (V.3.1) is not a reduced-form equation since it involves the endogenous variable f_t. To solve this problem, substitute equation (5.7) of the main text into equation (V.3.1) and solve for ${}_{at}p^{e}_{t+1}$ to obtain:

$$\text{(V.3.2)}\quad {}_{at}p^{e}_{t+1} = \frac{1}{1-\theta^N_1\gamma_3}\{(\theta^N_0+\theta^N_1\gamma_0) + (\theta^N_1\gamma_1)f_{t-1} + (\theta^N_1\gamma_2+\theta^N_2)(gc_{t-1}+v_t-x_{t-1}) + (\theta^N_1\gamma_2+\theta^N_2+\theta^N_3)m + (\theta^N_1\gamma_2+\theta^N_2+\theta^N_6)x_t + (\theta^N_1\gamma_4+\theta^N_1\gamma_5+\theta^N_7)(p^{*}_{t-1}+n_t-t_{t-1}) + (\theta^N_1\gamma_4+2\theta^N_1\gamma_5+\theta^N_7+\theta^N_8)j^{*} + (\theta^N_1\gamma_4+\theta^N_7+\theta^N_{11})t_t + (\theta^N_1\gamma_6)u_t + (\theta^N_1\gamma_7)\epsilon_t\}$$

Substituting equation (V.3.2) into equation (5.6) of the main text, the final solution for the price level of nontradables is obtained:

$$(\text{V.3.3})\quad p_t = \frac{1}{1-\theta_1^N Y_3}\{[X_0 - X_0\theta_1^N Y_3 + X_3\theta_0^N + X_3\theta_1^N Y_0]$$

$$+ [X_1 - X_1\theta_1^N Y_3 + X_3\theta_1^N Y_1]\, f_{t-1}$$

$$+ [X_2 - X_2\theta_1^N Y_3 + X_3\theta_1^N Y_2 + X_3\theta_2^N]\,(gc_{t-1}+v_t-x_{t-1})$$

$$+ [X_2 - X_2\theta_1^N Y_3 + X_3\theta_1^N Y_2 + X_3\theta_2^N + X_3\theta_3^N]\, m$$

$$+ [X_2-X_2\theta_1^N Y_3+X_3\theta_1^N Y_2+X_3\theta_2^N+X_3\theta_6^N]\, x_t$$

$$+ [(X_4+X_5)(1-\theta_1^N Y_3)+X_3\theta_1^N Y_4+X_3\theta_1^N Y_5+X_3\theta_7^N]\,(p_{t-1}^{*}+n_t-t_{t-1})$$

$$+ [(X_4+2X_5)(1-\theta_1^N Y_3)+X_3\theta_1^N Y_4+2X_3\theta_1^N Y_5+X_3(\theta_7^N+\theta_8^N)]\, \overset{*}{j}$$

$$+ [X_4-X_4\theta_1^N Y_3+X_3\theta_1^N Y_4+X_3(\theta_7^N+\theta_{11}^N)]\, t_t$$

$$+ [X_6-X_6\theta_1^N Y_3+X_3\theta_1^N Y_6]\, u_t$$

$$+ [X_7-X_7\theta_1^N Y_3+X_3\theta_1^N Y_7]\, \epsilon_t\}$$

Finally, equating coefficients among equations (5.8) and (V.3.3), the solution for the θ^Ns are obtained. In particular:

$$(\text{V.3.4})\quad \theta_1^N = \frac{-(X_3Y_1-X_1Y_3-1) \pm \sqrt{(X_3Y_1-X_1Y_3-1)^2 - 4Y_3X_1}}{2Y_3}$$

There are two possible solutions 1/ for θ_1^N. To choose between them, we will follow McCallum (1983) by imposing the requirement that the solution for θ_1^N must be valid for all admissible values of the structural parameters. In particular, f_{t-1} appears in the solution for p_t because it forms part of the system (equation (5.6)). In the special case in which $X_1=0$, f_{t-1} would not be an argument for p_t and hence, would not be included in the "minimal set of state variables." Thus, θ_1^N would be equal to zero. But from equation (V.3.4) it is clear that $\theta_1^N = 0$ would be obtained (under the assumption $X_1 = 0$) only if the negative root is used. 2/

The parameters of the model will imply θ_1^N to take a positive value 3/ and hence will allow us to sign the test of θs since they are functions of θ_1^N. Thus:

(V.3.5) $$\theta_2^N = \frac{\theta_1^N(X_3Y_2-X_2Y_3) + X_2}{1-\theta_1^NY_3-X_3} > 0 \text{ and less than one. } 4/$$

(V.3.6) $$\theta_3^N = \frac{\theta_2^N(1-\theta_1Y_3^N)}{1-\theta_1^NY_3-X_3} > 0 \text{ and } \theta_3^N > \theta_2^N$$

(V.3.7) $$\theta_4^N = - \theta_6^N = \theta_2^N$$

(V.3.8) $$\theta_5^N = \frac{\theta_2^N(1-\theta_1^NY_3-X_3)}{1-\theta_1^NY_3} > 0 \text{ and } \theta_5^N < \theta_2^N$$

(V.3.9) $$\theta_7^N = \frac{(1-\theta_1^NY_3)(X_4+X_5) + X_3\theta_1^N(Y_4+Y_5)}{1-\theta_1^NY_3-X_3} > 0 \text{ since } |Y_4|>|Y_5|;$$

and $\theta_7^N < 1$ 5/

$$(V.3.10)\quad \theta_8^N = \theta_7^N + \frac{X_5}{1-\theta_1^N\, Y_3 - X_3} > 0; \text{ and } \theta_8^N > \theta_7^N$$

$$(V.3.11)\quad \theta_9^N = -\theta_{11}^N = \theta_7^N$$

$$(V.3.12)\quad \theta_{10}^N = \frac{X_4\ (1-\theta_1^N Y_3) + X_3\theta_1^N Y_4}{1-\theta_1^N Y_3} > 0 \text{ since } Y_4 > 0; \text{ and } \theta_{10}^N < 1$$ 6/

$$(V.3.13)\quad \theta_{12}^N = \frac{X_6\ (1-\theta_1^N Y_3) + X_3\theta_1^N Y_6}{1-\theta_1^N Y_3} < 0 \text{ since } X_6 < 0$$

$$(V.3.14)\quad \theta_{13}^N = \frac{X_7(1-\theta_1^N Y_3) + X_3\theta_1^N Y_7}{1-\theta_1^N Y_3} > 0 \text{ since } X_7 > 0$$

$$(V.3.15)\quad \theta_0^N = \frac{X_0\ (1-\theta_1^N Y_3) + X_3\theta_1^N Y_0}{1-\theta_1^N Y_3 - X_3} > 0$$

The final solution for the levels of output of both commodities are found by substituting equation (5.8) and (V.3.2) into equations (3.70) and (3.71) of Chapter III. Since the aggregate supply functions of both commodities only differ in the constant accompanying the relative price term, it will be enough to present here the solution for the nontradable goods. Thus:

$$(V.3.16)\quad y_t^s = [a_0 + a_1\omega_o - \frac{a_1\tau(\theta_0^N + \theta_1^N\gamma_0)}{1-\theta_1^N\gamma_3} - a_1(1-\tau)\bar{s} + (1-\tau)(\theta_0^N - \bar{s})]$$

$$+ [a_1(1-\omega_1) - \frac{a_1\tau\theta_1^N\gamma_1}{1-\theta_1^N\gamma_3} + (1-\tau)\theta_1^N]\, f_{t-1}$$

$$+ [a_1\omega_1 - \frac{a_1\tau(\theta_1^N\gamma_2 + \theta_2^N)}{1-\theta_1^N\gamma_3} + (1-\tau)\theta_2^N](gc_{t-1} + v_t - x_{t-1})$$

$$+ [a_1\omega_1 - \frac{a_1\tau(\theta_1^N\gamma_2 + \theta_2^N + \theta_3^N)}{1-\theta_1^N\gamma_3} + (1-\tau)\theta_3^N]\, m$$

$$+ [a_1\omega_1 - \frac{a_1\tau\theta_1^N\gamma_2}{1-\theta_1^N\gamma_3} + (1-\tau)\theta_5^N]\, x_t$$

$$- [\frac{a_1\tau\theta_1^N(\gamma_4 + \gamma_5) + a_1\tau\theta_7^N}{1-\theta_1^N\gamma_3} + a_1(1-\tau) - (1-\tau)(\theta_7^N - 1)]\,(p_{t-1}^* + n_t - t_{t-1})$$

$$- [\frac{a_1\tau\theta_1^N(\gamma_4 + 2\gamma_5) + a_1\tau(\theta_7^N + \theta_8^N)}{1-\theta_1^N\gamma_3} + 2a_1(1-\tau) - (1-\tau)(\theta_8^N - 1)]\, j^*$$

$$- [\frac{a_1\tau\theta_1^N\gamma_4}{1-\theta_1^N\gamma_3} - (1-\tau)(\theta_{10}^N - 1)]\, t_t$$

$$- [\frac{a_1\tau\theta_1^N\gamma_6}{1-\theta_1^N\gamma_3} - \theta_{12}^N(1-\tau) - a_3]\, u_t$$

$$- [\frac{a_1\tau\theta_1^N\gamma_7}{1-\theta_1^N\gamma_3} - (1-\tau)\theta_{13}^N]\, \epsilon_t$$

FOOTNOTES TO APPENDIX V.3

1/ The roots from equation (V.3.4) are real since the term inside the square root is always positive. This is so because $Y_3 < 0$ and $X_1 > 0$. The fact that the roots are real is an indication that the model posses an economically sensible solution.

2/ $X_1 = 0$, then:

$$\theta_1^N = \frac{-(X_3Y_1-1) \pm \sqrt{(X_3Y_1-1)^2}}{2Y_3}$$

θ_1^N will be equal to zero only if the negative root is chosen because X_3 and Y_1 are both positive but less than one.

3/ $(X_3Y_1-X_1Y_3-1)$ will be negative under the simplifying assumptions: $d_1 = d_2$ and $d_3 = 1$. Recalling that a requirement from the microfoundations of the model is: $\delta_2 = b_2$; then:

$$(X_3Y_1-X_1Y_3-1) = \frac{\tau(\delta_1+a_1)}{1-\tau+\delta_1+\delta_2} - 1$$ which is negative and less than one

in absolute terms.

Next consider the term under the square root:

$$(X_3Y_1-X_1Y_3-1)^2-4Y_3X_1 = 1 + \frac{\tau^2(\delta_1+a_1)^2}{(1-\tau+\delta_1+\delta_2)^2} - \frac{2\tau(\delta_1+\alpha_1)}{1-\tau+\delta_1+\delta_2}$$

$$+ \frac{4(1-\omega_1)(\delta_2-a_1)(d_1\tau)(a_1+\delta_1)(1+\delta_1+\delta_2)}{(1-\tau+\delta_1+\delta_2)^2}$$

which is positive and greater than one. Since the relevant root in equation (V.3.4) is the negative root and since $Y_3 < 0$, θ_1^N will be positive.

4/ Under the restrictions imposed by the microfoundations, $X_3Y_2=X_2Y_3$. Thus, $\theta_2^N = \frac{X_2}{1-\theta_1^N Y_3-X_3}$, which is positive but less than one.

5/ Equation (V.3.9) can be rewritten as:

$$\theta_7^N = \frac{(X_4+X_5) + \theta_1^N (X_3Y_4 + X_3Y_5 - Y_3X_4 - Y_3X_5)}{1-\theta_1^N Y_3 - X_3}$$

Now $X_3Y_5 \equiv Y_3X_5$

Also, under the restrictions imposed by the microfoundations:

$$X_3Y_4 - Y_3X_4 \equiv -Y_3$$

so: $$\theta_7^N = \frac{(X_4+X_5) - \theta_1^N Y_3}{1-\theta_1^N Y_3 - X_3}$$

which is positive, since: $(1 - X_3) > 0$ and $Y_3 < 0$. In addition: $(X_4 + X_5) < (1 - X_3)$; hence, θ_7^N is less than one.

6/ Equation (V.3.12) can be rewritten as:

$$\theta_{10}^N = \frac{X_4 + \theta_1^N (X_3Y_4 - Y_3X_4)}{1 - \theta_1^N Y_3}$$

But $X_3Y_4 - Y_3X_4 \equiv -Y_3$

so: $$\theta_{10}^N = \frac{X_4 - \theta_1^N Y_3}{1 - \theta_1^N Y_3}$$

Hence, θ_{10}^N is less than one, since $X_4 < 1$ and $Y_3 < 0$.

CHAPTER VI

CONCLUSIONS

This thesis has set out a model of an economy in which firms are constrained, before engaging in production, to finance their advances to labor and their purchases of commodity inputs by borrowing from a domestic banking system, which in turn constitutes the entire financial system of the economy. The major implication of this "financial constraint" is that the supply of output comes to depend positively on the real value of the monetary base. This result contrasts with that obtained in models which introduce real money as a wealth variable that impinges positively on desired leisure, and hence negatively on supplies of labor and output.

In the closed economy version of the model, the most important result to arise is that a fully anticipated increase in the rate of growth of the monetary base or a fully anticipated temporary decrease in the level of the monetary base results in a reduction in the equilibrium level of output. In addition, the real wage rate and the real interest rate are affected by such monetary changes. Thus, money is not superneutral in this model and the Fisher effect does not hold. All of these results are a consequence of the "financial constraint" assumption. Under the assumption that agents cannot distinguish, in the short run, between permanent and temporary changes in the monetary base, even permanent monetary shocks have real effects. The lower the relative variance of the temporary shock, the lower the output effect of a permanent shock.

Real disturbances also have important effects in the model. While an increase in productivity unambiguously results in an increase in a output, an exogenous change in preferences in general has an undetermined effect on the level of output and might even have zero effects on output if certain restrictions on the model's parameters imposed by the micro-foundations are strictly imposed.

Velocity is an endogenous variable in this model and it depends on both nominal and real factors. While the velocity of money (which equals total demand deposits in this thesis) is bound to be less or equal to one, the velocity of the monetary base can be less or greater than one depending on the reserve ratio imposed by the monetary authorities.

Turning to the open economy version of the model, the assumption that the domestic economy produces and consumes both tradable and nontradable goods implies that the supply of each output depends on their relative price in addition to depending positively on the real monetary base. Thus, the effect of changes in the exogenous variables on the levels of output can be decomposed in (a) a "financial constraint effect," and (b) a "relative price effect."

In the flexible exchange rate case, it has been shown that the micro-foundations of the model imply that, when a monetary change occurs, the financial constraint effect always dominates the relative price effect. Hence, any monetary change that results in a decrease of output in the closed economy case, also results in a decrease of the levels of output of both goods in the flexible exchange rate case. The only new result to emerge in the flexible rate case concerns the exchange rate itself. A confusion between permanent and temporary monetary shocks implies that a permanent monetary decrease causes the exchange rate to "overshoot" relative to its full current information level.

In addition, a change in the price level (or in its rate of growth) of the tradable good, anticipated or not, only results in an inverse and proportional change in the exchange rate leaving the price level of the nontradable good and the output level of both commodities unchanged. It has been shown that this "insulation proposition" arises in this model because the price level of the tradable good and the exchange rate always enter together in the decision rules of the economic agents and in the alternative information sets assumed to solve the model. If the exchange rate were assumed to be observable at the beginning of the period, while the price level of the tradable good only became observable at the end of the period, the insulation proposition would not hold even under the assumption of zero capital mobility imposed in this thesis. As to purchasing power parity, it is not assumed to hold in this thesis. Instead, it is shown that the maintenance of purchasing power parity following a monetary or a real shock depends on the relative elasticities of demand for tradable and nontradable goods. An additional result is that while an increase in productivity always results in an increase in the output levels of both commodities, an exogenous change in preferences may result in an increase in the output level of one commodity and a decrease of the other, if the exogenous change in preferences generates a relative price effect.

In the fixed exchange rate case, the output effects of an anticipated change in government debt, or in the price level of the tradable good or of a devaluation, depend on the importance of the financial constraint effect as compared to relative price effect. Specifically, if the financial constraint effect is strong enough, an anticipated rise in the level of the government debt will increase the output of both commodities while an anticipated rise in the price level of the tradable good or an anticipated devaluation will reduce them. These results contrast with those generated by models of the Dornbusch (1973) kind where only the relative price effect is present. Moreover, the possibility that agents confuse permanent and temporary monetary shocks has important implications for the model's behavior. Specifically, a permanent monetary shock mistakenly viewed as temporary, might generate an output increase for both commodities, even in situations where a fully anticipated permanent change would lead to a reduction in the output of the tradable good.

Finally, neither unanticipated changes in the price level of the tradable good nor an unanticipated devaluation affect the current level of output of either commodity. Both might lead to a contraction of output of both commodities in subsequent periods, however. This result contrasts with those of models where price misperceptions are an argument in the aggregate supply function. In those models, an unanticipated devaluation results in a short-run output expansion.

Although this thesis has analyzed the effects of a variety of monetary and real shocks in an economy with limited capital markets, much work remains to be done. In particular, fiscal variables have not been considered here and the role of the government has not been investigated. This is an important omission since it is a well known feature of less developed countries, to which our financial constraint assumption might be particularly relevant, that governments play a significant role in the production process. In spite of these limitations, it is hoped that this thesis has made a contribution by highlighting the importance of financial limitations in the determination of price and output fluctuations.

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