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ANALYSIS OF MARDER'S SPACE-TIME TSALLIS HOLOGRAPHIC DARK ENERGY COSMOLOGICAL MODEL IN $f(R,T)$ THEORY OF GRAVITY

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In this paper, the investigation explores an anisotropic cosmological model based on Marder's space-time Tsallis holographic dark energy (THDE) within the framework of $f(R,T)$ theory of gravity, where R represents the Ricci scalar and T signifies the trace of the stress energy-momentum tensor. field equation have solved for class of $f(R,T)$ gravity i.e. $f(R,T) = R + f(T)$. To obtain the precise solution, we employed the density of the THDE model along with the volumetric expansion laws, namely the power law and exponential law. Also explores the physical and geometrical aspects of the model.

Keywords: *f(R,T) gravity; Marder's space-time; THDE; Volumetric expansion*

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1. INTRODUCTION

Based on the latest observations in astrophysics, there is strong evidence indicating that the universe is presently expanding at an accelerated rate, presenting intriguing opportunities for advancements in modern cosmological theories [1, 2, 3, 4]. The observed accelerated expansion of the universe is thought to be propelled by dark energy (DE). DE constitutes the dominant portion of the universe, making up 68% of the total energy in the observable universe at present. In contrast, dark matter (DM) and ordinary matter (baryonic matter) contribute 26% and 5% respectively [5]. The specific traits of DE continue to elude understanding, leading to the formulation of various theories and explanations. Indeed, within various theories and models, the cosmological constant model is often regarded as the most straightforward choice for DE, characterized by an equation of state (EoS) parameter $\omega = -1$. However, it is not without its challenges, including issues such as cosmic coincidence and fine-tuning problems [6, 7]. To address these challenges, the scientific literature has proposed various DE models, including quintessence, phantom, k-essence, tachyon, holographic dark energy (HDE), and others. In contemporary times, the exploration of HDE models has become a promising pathway for comprehending cosmic expansion, operating within the framework of the holographic principle (HP) [8]. The HP posits that the limit on the vacuum energy Λ of a system with size L should not surpass the threshold of the black hole mass with an equivalent size. This limitation arises from the potential formation of a black hole in the quantum field in a vacuum, and the infrared (IR) and ultraviolet cutoffs [10]. The energy density of HDE is defined as $\rho_{HDE} = 3d^2m_p^2L^{-2}$, where m_p is the reduced Plank mass and L represents the IR cutoff, describing the size of the universe in the context of the HP [11].

In recent times, various HDE models, including the Modified Ricci (MRHDE), THDE, Rényi HDE (RHDE), and Sharma-Mittal HDE (SMHDE), have been proposed and introduced. Certainly, within these models, RHDE stands out as it is founded on the absence of interactions between cosmic sectors. Notably, this model exhibits greater stability on its own [12]. M. Tavayef et al. [13] explored the Tsallis and Cirto entropy expressions while incorporating the HDE hypothesis. Their investigation led to the formulation of a novel type of DE called THDE. The study further delved into the dynamics of this THDE within the framework of a non-interacting flat Friedmann-Robertson-Walker (FRW) universe, examining the evolutionary aspects of the system. In the scenario of non-interacting cosmos, SMHDE is acknowledged for its classical stability [13, 14, 15]. M. Abdollahi Zadeh et al. [16] have delved into the repercussions of introducing various IR cutoffs, including the particle horizon, the Ricci horizon, and the Granda-Oliveros (GD) cutoffs, on the properties of the THDE. Spyros Basilakos et al. [17] demonstrated how Tsallis cosmology can effectively address both the Hubble constant (H_0) and the matter density fluctuation amplitude (σ_8) tensions simultaneously. This modified cosmological scenario is achieved by applying the gravity-thermodynamics conjecture with the use of non-additive Tsallis entropy instead of the standard Bekenstein-Hawking entropy. A. Mohammadi et al. [18] conducted a study exploring

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the application of the HP within the framework of Bianchi type-III space-time. A. Pradhan and A. Dixit [20] explored a THDE model within a flat FRW space-time, considering the higher derivative theory of gravity. A. Al. Manon et al. [21] have investigated a cosmological scenario illustrating the ongoing acceleration of the universe, featuring the coexistence of DM and THDE. M. Vijaya Santhi and Y. Sobhanbabu [22] have explored the dynamics of THDE, utilizing the Hubble radius as the IR cutoff, in a homogenous and anisotropic Bianchi type-III universe. This investigation was conducted within the context of the Saez-Ballester (SB) theory of gravitation and by solving the field equations associated with SB theory they have developed both interacting and non-interacting DE models. Y. Sobhanbabu and M. Vijaya Santhi [23] have dedicated their efforts to examining THDE, incorporating the Hubble radius as the IR cutoff, within a homogenous and anisotropic Kantowski-Sachs universe. This investigation unfolds within the framework of SB theory of gravitation. They have formulated both non-interacting and interacting models for THDE by solving the field equations and employing the connection between the metric potentials. B. D. Pandey et al. [24] have developed HDE model incorporating Tsallis entropy, a one parameter extension of Boltzmann-Gibbs entropy. R. Saleem et al. [25] have investigated the dynamics of warm inflation within a modified cosmological framework in the context of Rastall gravity. Within this scenario, they altered the standard Friedmann equations by incorporating recently proposed Tsallis and Barrow HDE entropies. M. Vijaya Santhi and Y. Sobhanbabu [26] have formulated both interacting and non-interacting models for THDE in an anisotropic and homogenous Bianchi type- VI_0 space-time. This was accomplished within the context of a scalar-tensor theory proposed by SB. To achieve this, they employed the relationship between the metric potentials of the model and a varying deceleration parameter, resolving the SB field equations. M. Sharif and S. Saba [27] have explored the reconstruction paradigm for the THDE model by incorporating the generalized Tsallis entropy conjecture with the Hubble horizon. This investigation took place within the framework of $f(G,T)$ gravity. M. Zubir and L. Rukh Durrani [28] have investigated THDE in a flat FRW model, utilizing the framework of $f(R,T)$ gravity. Ayman A. Aly [29] has developed a novel $f(T)$ modified gravity model, incorporating a THDE model and a Hubble cutoff. A. Pradhan et al. [30] have explored the Tsallis holographic quintessence, k-essence, and tachyon models of DE within the context of modified $f(R,T)$ gravity, employing the GO cutoff. S. H. Shekh et al. [31] analyzed THDE, transitioning into HDE through a specific selection of the positive non-additivity parameter δ. This study was carried out within the framework of modified $f(T,B)$ gravity, examining the validity of thermodynamics and energy conditions for a homogenous and isotropic FRW universe.

Expanding on the constructive discussions and favorable results emphasized earlier, this article explores the THDE model within the context of $f(R,T)$ gravity. The inquiry focuses on validating both the power law and exponential law components of the model.

2. TSALLIS HOLOGRAPHIC DARK ENERGY

It is essential to remember that the establishment and derivation of the conventional HDE density ($\rho_{HDE} = 3d^2m_p^2L^{-2}$) are contingent upon the entropy area relationship $S \sim A \sim L^2$ of black holes, where $A = 4\pi L^2$ represents the area of the horizon [9]. Neverthless, the definition of HDE can be adjusted or revised in light of quantum considerations [32, 33]. Tsallis and Cirto illustrated that the horizon entropy of a black hole could potentially be modified according to mathematical expression of the form

$$S_\delta = \gamma A^\delta \tag{1}$$

where, γ is an unknown constant and δ dentoes the non-additivity parameter [14]. It is evident that the Bekestein entropy is registered when the appropiate limit of $\delta = 1$ and $\delta = (4G)^{-1}$. Certainly, at this limit, the power law distribution of probability becomes ineffective, and the system can be described by the conventional probabitilty distribution [14].

Following the HP, which asserts that the number of degrees of freedom of a physical system should scale with its bounding area rather than its volume [34]. Cohen et al. [9] suggested that system entropy (S) should be constrained by an IR (L) cutoff, leading to proposed relation involving the IR cutoff and UV (Λ) as

$$L^3\Lambda^3 \leq (S)^{3/4} \tag{2}$$

which combining with eqn. (1) leads to [9]

$$\Lambda^4 \leq (\gamma(4\pi)^\delta)L^{2\delta-4} \tag{3}$$

where, Λ^4 denotes the vacuum energy density. Employing the aforementioned inequality, we can suggest the THDE density as follows

$$\rho_t = BL^{2\delta-4} \tag{4}$$

where B is unknown parameter [35, 36, 37] and IR cutoff is taken as Hubble radius which leads to $L = H^{-1}$, where H is hubble parameter [10]. The density of THDE model along with its derivative by using eqn. (4) becomes

$$\rho_t = BH^{4-2\delta} \tag{5}$$

$$\dot{\rho}_t = B(4-2\delta)H^{3-2\delta}\dot{H} \tag{6}$$

where, $\dot{H}$ is the derivative of Hubble parameter w.r.t t [10].

3. METRIC AND FIELD EQUATIONS

We consider the Marder's space-time in the form [38]

$$ds^2 = M_1^2(dx^2 - dt^2) + M_2^2 dy^2 + M_3^2 dz^2 \tag{7}$$

where, M_1^2, M_2^2, M_3^2 are functions of cosmic time t. In recent years, M. Vijaya Santhi et al. [39] have conducted research on the dynamics of a Marder's space-time cosmological model, which is grounded in the concept of bulk viscous strings within the framework of $f(R)$ gravity. Sezgin AYGÜN [40] has explored a homogenous and anisotropic Marder space-time model within the framework of (R,T) gravity, where the space-time is filled with a bulk viscous string matter distribution. D. D. Pawar and S. P. Shahre [38] have explored Marder's space-time within the framework of (R,T) gravity, integrating a perfect fluid under a titled congruence.

In this work we study the Marder's space-time in (R,T) gravity with THDE. Here, the energy-momentum tensor for matter (T'_{ij}) and THDE $(\overline{T}_{ij})$ are given as follows:

$$T'_{ij} = diag[1,0,0,0]\rho_m \tag{8}$$

$$\overline{T}_{ij} = diag[1,-\omega_t,-\omega_t,-\omega_t]\rho_t \tag{9}$$

it can be parameterized as

$$\overline{T}_{ij} = diag[1,-\omega_t,-\omega_t,-(\omega_t+\alpha)]\rho_t \tag{10}$$

where ρ_t and ρ_m are energy densities of THDE, and matter respectively and p_t and $_m$ is pressure of THDE and matter respectively. $\omega_t = \frac{p_t}{\rho_t}$ is an EoS parameter. Here, α the deviation from the EoS parameter in the z-direction, commonly referred to as the skewness parameter. We have an energy conservation equation as

$$(T'_{ij} + \overline{T}_{ij})_{;j} = 0 \tag{11}$$

The exploration of diverse cosmological models within the framework of (R,T) theory of gravity depends on the characteristics of the matter source under consideration. Harko et al. [41] introduced following class of $f(R,T)$ gravity:

$$f(R,T) = \begin{cases} R + 2f(T) \\ f_1(R) + f_2(T) \\ f_1(R) + f_2(R)f_3(T) \end{cases} \tag{12}$$

In this study, we have adopted a specific functional form, expressed as $f(R,T) = R + 2f(T)$. Here, $f(T)$ is a function of the trace of the energy-momentum tensor. By using this functional, the field equation can be rewritten as

$$R_{ij} - \frac{1}{2}Rg_{ij} = (T'_{ij} + \overline{T}_{ij})_{;j} + 2f_T(T'_{ij} + \overline{T}_{ij})_{;j} + [f(T) + 2pf_T]g_{ij} \tag{13}$$

where f_T is a partial derivative of f w.r.t T. We designate the function $f(T)$ to be contingent upon the trace of the energy-momentum tensor of matter, specifically as

$$f(T) = \lambda T \tag{14}$$

where λ is arbitrary constant. So $f_T = \lambda$. In metric (7), the Ricci scalar R can be represented in terms of metric potentials as [38]

$$R = -2\left(\frac{\ddot{M}_1}{M_1^3} + \frac{\ddot{M}_2}{M_1^2 M_2} + \frac{\ddot{M}_3}{M_1^2 M_3} - \frac{\dot{M}_1^2}{M_1^4} + \frac{\dot{M}_2\dot{M}_3}{M_1^2 M_2 M_3}\right) \tag{15}$$

In this analysis, we investigate the cosmological implications of the arbitrary function suggested by Harko et al. [41], which is represented by the expression

$$f(R,T) = R + 2f(T) \tag{16}$$

where R is the Ricci scalar and T is the trace of the energy-momentum tensor.

The field eqn. (13) for the metric (7), utilizing eqns. (8), (10), and (14) from modified $f(R,T)$ gravity, results in the following system of eqns:

$$\frac{1}{M_1^2}\left(\frac{\ddot{M}_2}{M_2}+\frac{\ddot{M}_3}{M_3}+\frac{\dot{M}_2\dot{M}_3}{M_2M_3}-\frac{\dot{M}_1\dot{M}_2}{M_1M_2}-\frac{\dot{M}_1\dot{M}_3}{M_1M_3}\right)=-(8\pi+2\lambda)(\omega_t+\alpha)\rho_t+[2p_t-3\omega_t\rho_t-\alpha\rho_t+\rho_m+\rho_t]\alpha \quad (17)$$

$$\frac{1}{M_1^2}\left(\frac{\ddot{M}_1}{M_1}+\frac{\ddot{M}_3}{M_3}-\frac{\dot{M}_1^2}{M_1^2}\right)=-(8\pi+2\lambda)\omega_t\rho_t+[2p_t-3\omega_t\rho_t-\alpha\rho_t+\rho_m+\rho_t]\alpha \quad (18)$$

$$\frac{1}{M_1^2}\left(\frac{\ddot{M}_1}{M_1}+\frac{\ddot{M}_2}{M_2}-\frac{\dot{M}_1^2}{M_1^2}\right)=-(8\pi+2\lambda)\omega_t\rho_t+[2p_t-3\omega_t\rho_t-\alpha\rho_t+\rho_m+\rho_t]\alpha \quad (19)$$

$$\frac{1}{M_1^2}\left(\frac{\dot{M}_1\dot{M}_2}{M_1M_2}+\frac{\dot{M}_1\dot{M}_3}{M_1M_3}+\frac{\dot{M}_2\dot{M}_3}{M_2M_3}\right)=-(8\pi+2\lambda)(\rho_m+\rho_t)+[2p_t-3\omega_t\rho_t-\alpha\rho_t+\rho_m+\rho_t]\alpha \quad (20)$$

We express the energy conservation eqn. (11) for both matter and THDE as follows,

$$(\dot{\rho}_m+\dot{\rho}_t)+\left(\frac{\dot{M}_1}{M_1}+\frac{\dot{M}_2}{M_2}+\frac{\dot{M}_3}{M_3}\right)[\rho_m+(1+\omega_t)\rho_t]+\frac{\dot{M}_1}{M_1}\alpha\rho_t=0 \quad (21)$$

where overhead (.) denotes for ordinary differentiation w.r.t t.

4. SOLUTION OF FIELD EQUATIONS AND COSMOLOGICAL MODELS

The set of field eqns. (17)-(20) represents a set of four independent eqns. with seven unknowns $M_1, M_2, M_3, \rho_m, \rho_t, \omega_t, \alpha$. From eqns. (18) and (19), we get

$$\frac{\dot{M}_2}{M_2}-\frac{\dot{M}_3}{M_3}=0 \quad (22)$$

on integration gives,

$$\frac{M_2}{M_3}=c_2\exp(c_1\int dt) \quad (23)$$

To simplify matters, we decide that $M_1 = M_2$. The dynamical parameters for Marder's space-time cosmological model are delineated as follows: The spatial volume of the metric is

$$V=a^3(t)=M_2^2M_3 \quad (24)$$

The directional Hubble parameters

$$H_x=H_y=\frac{\dot{M}_2}{M_2},\qquad H_z=\frac{\dot{M}_3}{M_3} \quad (25)$$

The generalized mean Hubble's parameter H is expressed as

$$H=\frac{1}{3}(H_x+H_y+H_z)=\frac{1}{3}\left(\frac{2\dot{M}_2}{M_2}+\frac{\dot{M}_3}{M_3}\right) \quad (26)$$

The expansion scalar

$$\theta=3H=2\frac{\dot{M}_2}{M_2}+\frac{\dot{M}_3}{M_3} \quad (27)$$

The mean anisotropic parameter

$$A_m=\frac{1}{3}\sum_{i=1}^{4}\left(\frac{H_i-H}{H}\right)^2 \quad (28)$$

The Shear scalar

$$\sigma^2=\frac{3}{2}A_mH^2 \quad (29)$$

The deceleration parameter

$$q = -1 + \frac{d}{dt}\left(\frac{1}{H}\right) \tag{30}$$

From eqns. (23) and (24), we get

$$M_2 = V^{1/3} c_2^{1/3} \exp\left(\frac{c_1}{3} t\right) \tag{31}$$

$$M_3 = V^{1/3} c_2^{-2/3} \exp\left(-\frac{2}{3} c_1 t\right) \tag{32}$$

where c_1 and c_2 are integrating constants. From eqns. (31) and (32), metric (7) becomes

$$ds^2 = \left[V^{1/3} c_2^{1/3} \exp\left(\frac{c_1}{3} t\right)\right]^2 (dx^2 + dy^2 - dt^2) + \left[V^{1/3} c_2^{-2/3} \exp\left(-\frac{2}{3} c_1 t\right)\right]^2 dz^2 \tag{33}$$

To obtain the complete solution, we require two different volumetric expansion laws, both the power law expansion and exponential law expansion i.e. $V = t^m$ and $V = e^{4H_0 t}$ respectively [42].

5. MODEL FOR POWER LAW EXPANSION

We are contemplating a volumetric expansion by a power law relation as

$$V = t^m \tag{34}$$

where m is a positive constant. The positive value of the exponent m aligns with observational evidence that anticipates the universe.

The metric potentials (31) and (32) becomes

$$M_2 = t^{m/3} c_2^{1/3} \exp\left(\frac{c_1}{3} t\right) \tag{35}$$

$$M_3 = t^{m/3} c_2^{-2/3} \exp\left(-\frac{2}{3} c_1 t\right) \tag{36}$$

As the time t approaches zero, the analysis suggests that metric potentials (35) and (36) tend toward zero. Consequently, the model exhibits an initial singularity. Eqn. (33) with the help of eqns. (35) and (36) can be written as

$$ds^2 = \left[t^{m/3} c_2^{1/3} \exp\left(\frac{c_1}{3} t\right)\right]^2 (dx^2 + dy^2 - dt^2) + \left[t^{m/3} c_2^{-2/3} \exp\left(-\frac{2}{3} c_1 t\right)\right]^2 dz^2 \tag{37}$$

From eqns. (25), (35), and (36), the directional Hubble parameters are

$$H_x = H_y = \frac{m}{3t} + \frac{c_1}{3} \tag{38}$$

$$H_z = \frac{m}{3t} - \frac{2c_1}{3} \tag{39}$$

From eqns. (26), (35), and (36), the mean Hubble parameter is given by

$$H = \frac{m}{3t} \tag{40}$$

From eqns. (27) and (40), the expansion scalar is given by

$$\theta = \frac{m}{t} \tag{41}$$

From eqns. (28), (38), (39), and (40), the mean anisotropic parameter is given by

$$A_m = \frac{2c_1^2 t^2}{m^2} \tag{42}$$

From eqns. (29), (40), and (42), the Shear scalar is

$$\sigma^2 = \frac{c_1^2}{3} \tag{43}$$

From eqns. (30) and (40), the deceleration parameter is

$$q = \frac{3}{m} - 1 \tag{44}$$

The energy conservation eqn. (21) results in the derivation of the subsequent separate conservation eqn. as

$$\dot{\rho}_m + 3H\rho_m = 0 \tag{45}$$

$$\dot{\rho}_t + 3H(1+\omega_t)\rho_t + \frac{\dot{M}_2}{M_2}\alpha\rho_t = 0 \tag{46}$$

On integrating eqn. (45), we get matter density as

$$\rho_m = \frac{c_3}{t^m} \tag{47}$$

where c_3 is an integrating constant.

The DM is pressure less [29] i.e.

$$p_m = 0 \tag{48}$$

From eqns. (5), (6), and (40), the density of THDE is given as

$$\rho_t = B\left(\frac{m}{3t}\right)^{4-2\delta} \tag{49}$$

$$\dot{\rho}_t = -B(4-2\delta)\left(\frac{m}{3t}\right)^{3-2\delta}\left(\frac{m}{3t^2}\right) \tag{50}$$

For Λ-CDM model, the DE EoS is

$$\omega_t = -1 \tag{51}$$

we get THDE pressure as

$$p_t = -B\left(\frac{m}{3t}\right)^{4-2\delta} \tag{52}$$

From eqns. (40), (46), (49), (50), and (51), the skewness parameter is given as

$$\alpha = \left(\frac{4-2\delta}{t}\right)\left(\frac{m}{3t} + \frac{c_1}{3}\right)^{-1} \tag{53}$$

The density parameter for THDE and the energy density parameter for matter are defined and calculated as follows:

$$\Omega_t = B\left(\frac{m}{3t}\right)^{4-2\delta}\left(\frac{m^2}{3t^2}\right)^{-1} \tag{54}$$

$$\Omega_m = \left(\frac{c_3}{t^m}\right)\left(\frac{m^2}{3t^2}\right)^{-1} \tag{55}$$

Hence,

$$\Omega_t + \Omega_m = \left[B\left(\frac{m}{3t}\right)^{4-2\delta} + \frac{c_3}{t^m}\right]\left(\frac{m^2}{3t^2}\right)^{-1} \tag{56}$$

6. MODEL FOR EXPONENTIAL LAW EXPANSION

The exponential law expansion is

$$V = e^{4H_0 t} \tag{57}$$

Typically, this results in a universe resembling de Sitter space-time. In this scenario, H_0 represents the Hubble Parameter during the current epoch. We have delved into the dynamics of the universe within the framework of the $f(R,T)$ gravity, specifically emphasizing exponential law. This exploration is aimed at offering a thorough grasp of the dynamics of the model and contrasting it with those observed in the power law model.

The metric potentials (31) and (32) becomes

$$M_2 = \left(e^{4H_0 t}\right)^{1/3} c_2^{1/3} \exp\left(\frac{c_1}{3}t\right) \tag{58}$$

$$M_3 = \left(e^{4H_0t}\right)^{1/3} c_2^{-2/3} \exp\left(-\frac{2c_1}{3}t\right) \tag{59}$$

Eqn. (33) with the help of eqns. (58) and (59) can be written as

$$ds^2 = \left[\left(e^{4H_0t}\right)^{1/3} c_2^{1/3} \exp\left(\frac{c_1}{3}t\right)\right]^2 (dx^2 + dy^2 - dt^2) + \left[\left(e^{4H_0t}\right)^{1/3} c_2^{-2/3} \exp\left(-\frac{2c_1}{3}t\right)\right]^2 dz^2 \tag{60}$$

From eqns. (25), (58), and (59), the directional Hubble parameters are

$$H_x = H_y = \frac{4}{3}H_0 + \frac{c_1}{3} \tag{61}$$

$$H_z = \frac{4}{3}H_0 - \frac{2c_1}{3} \tag{62}$$

From eqns. (26), (58) and (59), the mean Hubble parameter is given by

$$H = \frac{4}{3}H_0 \tag{63}$$

From eqns. (27) and (63), the expansion scalar is given by

$$\theta = 4H_0 \tag{64}$$

From eqns. (28), (61), (62) and (63), the mean anisotropic parameter is given by

$$A_m = \frac{c_1^2}{8} \tag{65}$$

From eqns. (29), (64) and (65), the Shear scalar is

$$\sigma^2 = \frac{c_1^2 H_0^2}{3} \tag{66}$$

From eqns. (30) and (63), the deceleration parameter is

$$q = -1 \tag{67}$$

On integrating eqn. (45), we get the matter density for the model as

$$\rho_m = -c_4 e^{-4H_0t} \tag{68}$$

where c_4 is an integrating constant. The DM is pressure less [29] i.e.

$$p_m = 0 \tag{69}$$

From eqns. (5), (6), and (63), the density of THDE is given as

$$\rho_t = B\left(\frac{4}{3}H_0\right)^{4-2\delta} \tag{70}$$

$$\dot{\rho}_t = 0 \tag{71}$$

For Λ-CDM model, we get THDE pressure as

$$p_t = -B\left(\frac{4}{3}H_0\right)^{4-2\delta} \tag{72}$$

From eqns. (46), (63), (70) and (71), skewness parameter is becomes

$$\alpha = 0 \tag{73}$$

The density parameter for THDE and the energy density parameter for matter are defined and calculated as follows:

$$\Omega_t = B\left(\frac{4}{3}H_0\right)^{4-2\delta}\left(\frac{16}{9}H_0^2\right)^{-1} \tag{74}$$

$$\Omega_m = \left(-c_4 e^{-4H_0t}\right)\left(\frac{16}{9}H_0^2\right)^{-1} \tag{75}$$

Hence,

$$\Omega_t + \Omega_m = \left[B\left(\frac{4}{3}H_0\right)^{4-2\delta} + \left(-c_4 e^{-4H_0t}\right)\right]\left(\frac{16}{9}H_0^2\right)^{-1} \tag{76}$$

7. DISCUSSION

In the preceding section, we endeavored to unravel the precise solution of the THDE cosmological model within Marder's space-time framework. To do so, we postulated both a power law expansion and an exponential expansion law as plausible scenarios for the evolution of the universe. We have discovered that,

In section 5, for power law model.

- From Figure 1, both the Hubble parameter and the expansion scalar are approaching infinity as $t = 0$ signifies that at the inception of the universe, it was characterized by an infinitely dense and hot state. This characteristic behavior aligns with the concept of the Big Bang singularity, marking the beginning of cosmic expansion. As time progresses, both the Hubble parameter and the expansion scalar show a decrease. This pattern implies a diminishing rate of universal expansion over time. Despite the ongoing expansion, the momentum of this process gradually diminishes over time. As the cosmic time approaches infinity, both the Hubble parameter and the expansion scalar tend toward zero. This suggests that the rate of expansion of the universe is approaching a constant value. Such a scenario hints at a future phase where the rate of expansion of the universe becomes nearly constant, known as the de Sitter phase.

- Item From Figure 1, as cosmic time increases, the anisotropic parameter grows at an increasing rate. From eqn. (42), it is evident that the behavior of the anisotropic parameter is contingent upon the constants c_1 and m. Under this circumstance, the anisotropic parameter escalates as cosmic time t increases, yet diminishes with higher values of m.

- From eqn. (44),
  1. If $m = 3$ the deceleration parameter becomes zero. It indicates that the expansion of the universe is neither accerlerating, but rather proceeding at a constant rate. This scenario is consistent with a universe in which the gravitational effects of matter and energy are exactly balanced by the expansion itself.
  2. If $m > 3$ then the deceleration parameter is positive ($q > 0$), it indicates that the expansion of the universe is decelerating. In this scenario, the gravitational forces exerted by matter and energy within the universe are sufficiently strong to slow down the rate of expansion over time.
  3. If $m < 3$ then the deceleration parameter is negative ($q < 0$), it indicates that the expansion of the universe is accelerating. In this scenario, the gravitational effects of matter and energy are not sufficient to counteract the expansion, causing it to accelerate over time.

- From Figure 2(a), at the beginning of cosmic time $t = 0$ the density of THDE is significantly high, implying a phase of rapid expansion similar to cosmic inflation. As cosmic time progresses, the density of THDE decreases, indicating a corresponding reduction in the rate of expansion of the universe over time.

- From Figure 2(b), at the beginning of cosmic time ($t = 0$) the density goes to zero, which signifies a crucial epoch.

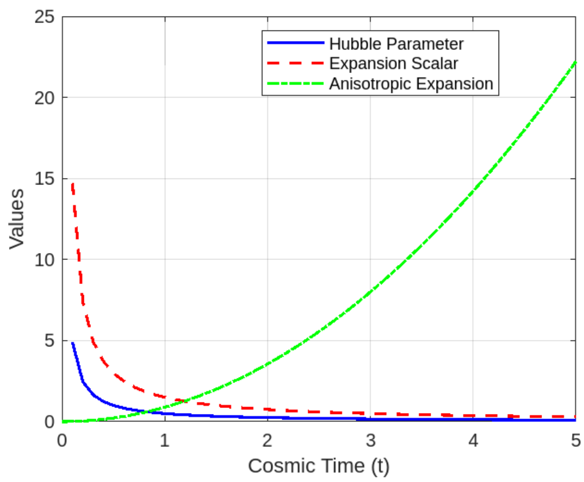
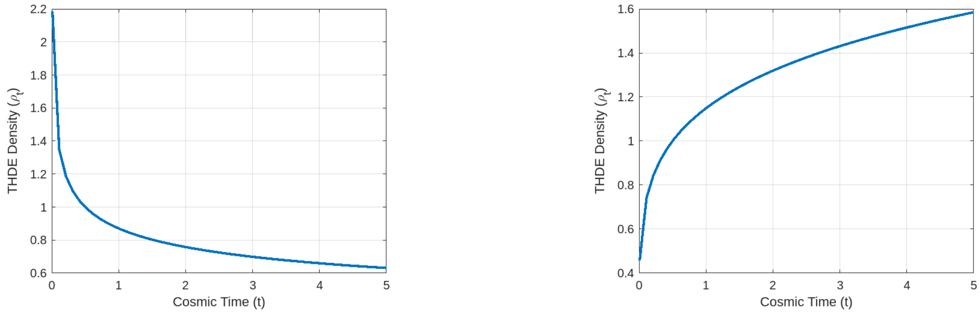


Figure 1. Hubble parameter, expansion scalar and anisotropic parameter versus cosmic time (t) for the particular choice of constants $c_1 = 1, m = 1.5$ in power law model



(a) THDE density for δ=1.9 **(b)** THDE density for δ=2.1

Figure 2. THDE density versus cosmic time (t) for the particular choice of constant $m = 1.5$ in power law model

(a) THDE pressure for δ=1.9 **(b)** THDE pressure for δ=2.1

Figure 3. THDE pressure versus cosmic time (t) for the particular choice of constant $m = 1.5$ in power law model

- From Figure 2(a) (with $\delta = 1.9$), as cosmic time extends towards infinity the THDE density appears to stabilize at a fixed value. This suggests a scenario where, in the distant future, THDE could potentially dominate the energy density of the universe.
- From Figure 2(b) (with $\delta = 2.1$), as the cosmic time approaches infinity, the THDE density asymptotically approaches a non-zero value. In this case, the THDE density does not diminish to zero but approaches a finite value.

In section 6, for the exponential law model.

- The deceleration parameter $q = -1$ for this model signifies that the universe undergoes an accelerated expansion [42]. The expansion scalar maintains a constant value indicating that the rate of expansion of the universe remains constant over time [43]. The anisotropic parameter is constant, it implies that the universe exhibits isotropy.
- We found that the Hubble parameter, density, and pressure are constant.

8. CONCLUSION

We investigated Marder's space-time model incorporating THDE within the framework of $f(R,T)$ gravity. The volumetric expansion law is utilized to derive precise solutions for field equations. Both the power law model and exponential law model yield the following conclusions. We analyzed 2D graph of the parameter by using MATLAB.

In the power law model, many fascinating findings emerge from our investigation. When the Hubble parameter is positive (t), we observe a universe is expanding. As time progresses towards infinity, the expansion gradually decelerates and eventually approaches zero. Interestingly, we have obtained the value of the deceleration parameter that depends on constant $(m = [2, 4])$, i.e. whether the universe accelerating or decelerating. From Figure 2(a) and 2(b), $(\delta < 2, \delta > 2)$ the behavior of the universe suggests an evolving state where THDE plays a significant role in the expansion dynamics, potentially leading to an indefinite expansion. However, slight variations in the value of δ may influence the specific details of the expansion dynamics. Additionally, our examination highlights the existence of negative pressure, as depicted in eqn. (52). This negative pressure is a distinctive attribute of DE. The inclusion of DE within the model provides evidence supporting the idea that DE plays a crucial role in influencing the dynamics of our universe.

In the exponential law model, we have obtained all dynamics parameters such as Hubble parameter (H), expansion scalar (θ), Shear scalar (σ^2), anisotropic parameter (A_m) are constant. Hence, as the universe evolves over time, this model demonstrates behavior similar to that of a cosmological constant model in its later stages. Additionally, the negative sign of the deceleration parameter signifies the expansion of the universe is accelerating. The ratio of the Shear scalar to the expansion scalar is non-zero, indicating that the universe is anisotropic.

Our findings indicate that the universe exhibits anisotropy during its early stages, yet as time progresses, the anisotropic behavior diminishes, leading to an isotropic present day universe. This conclusion aligns with various observational data. The models proposed in this paper offer a suitable description of the evolutionary trajectory of the universe.

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АНАЛІЗ КОСМОЛОГІЧНОЇ МОДЕЛІ ГОЛОГРАФІЧНОЇ ТЕМНОЇ ЕНЕРГІЇ ЦАЛЛІСА У ПРОСТОРІ-ЧАСІ МАРДЕРА В $f(R,T)$ ТЕОРІЇ ГРАВІТАЦІЇ

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У цій роботі досліджується анізотропна космологічна модель, заснована на просторово-часовій голографічній темній енергії Цалліса (THDE) Мардера в рамках $f(R,T)$ теорії гравітації, де R представляє скаляр Річчі, а T означає слід тензора енергії-імпульсу напруги. рівняння поля розв'язано для класу гравітації $f(R,T)$, тобто $f(R,T) = R + f(T)$. Щоб отримати точне рішення, ми використали щільність моделі THDE разом із законами об'ємного розширення, а саме степеневим і експоненціальним законом. Також досліджуються фізичні та геометричні аспекти моделі.

Ключові слова: $f(R,T)$ *гравітація; простір-час Мардера; THDE; Об'ємне розширення*