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BEHAVIOUR OF QUARK AND STRANGE QUARK MATTER FOR HIGHER DIMENSIONAL BIANCHI TYPE - I UNIVERSE IN $f(R,T)$ GRAVITY

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This research paper delves into a thorough examination of the behaviour exhibited by higher dimensional Bianchi Type-I universes, incorporating the presence of quark and strange quark matter within the framework of $f(R,T)$ gravity. The solutions derived for the field equations encompass both exponential volumetric expansion and power law scenarios.Under the exponential expansion model, both the pressure (p_q) and energy density (ρ_q) associated with quark matter are initially finite at the inception of cosmic time, gradually diminishing to zero as time progresses towards infinity. Conversely, within the power law model, these parameters start off infinitely large at $t = 0$, subsequently decreasing to zero as time approaches infinity. Furthermore, an exploration of the physical and geometrical attributes of the model is conducted. Notably, in power law expansion models, the behaviour of strange quark matter mirrors that of quark matter concerning pressure (p) and energy density (ρ). But in exponential expansion model quark pressure and strange quark pressure behave differently. The bag constant emerges as a critical factor influencing the universe's expansion, with observations revealing that both pressure and energy density tend towards the bag constant at large time scales ($t \to \infty$). Specifically, the pressure $p \to -B_C$ and the energy density $\rho \to B_C$ as time approach infinity.The negative pressure sign denotes the universe's expansion during later epochs.

Keywords: *Quark and Strange quark matter; Bag constant; Higher dimensional Bianchi Type-I universe; $f(R,T)$ gravity*

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1. INTRODUCTION

Modern cosmology has attracted an enormous amount of attention due to its outstanding ability to explain the natural phenomenon of rapid expansion that takes place in the conclusive stages of the universe. As we endeavour to explore the universe this field is establishing itself as the one that is advancing the most quickly. An important discovery of accelerated expansion was made primarily through the analysis of type- $\mathrm{I_a}$ supernovae experiment performed by various researchers [1, 2, 3, 4, 5]. These investigations have produced strong evidence revealing the universe is presently going through an accelerated expansion phase. Notably, many scientists have made major attempts to find confirmation of dark energy an idea put forward in Einstein's theory permeating the universe. These researchers came to the conclusion after an exhaustive examination of observational evidence suggesting dark energy often regarded as the primary driving force shaping the universe is characterised by negative pressure.

Researchers in cosmology have a strong desire to learn more about how the universe functions. Albert Einstein stands out among them for his work on the general theory of relativity which attracted a lot of curiosity for its effectiveness in building cosmological models as well as offering insights into the development and the formation of the universe. However, it fails to tackle a significant issue associated with modern cosmology the late-time acceleration. As a result, several attempts to alter the theory of gravity have been established in order to explain the current accelerated phase. To solve this constraint and offer brief explanations for the universe's late-time rapid expansion cosmologists have created a number of alternative theories including $f(R)$ [6], $f(T)$ [7] and $f(R,T)$ [8] theories to the general theory of relativity.

One such alternative theory that has garnered interest and motivation is the $f(R,T)$ theory of gravity. This theory proposed by Harko et al. in 2011 incorporates the Ricci scalar (R) and the trace of the stress-energy tensor (T). The researchers derived the gravitational field equations in the metric formalism as well as the equation of motion for test particles based on the covariant divergence of the stress-energy tensor. Within this particular field, various forms of the $f(R,T)$ function have been extensively explored and discussed by researchers.This motivates the researchers [9, 10, 11, 12, 13] to construct various models in the context of $f(R,T)$ gravity.

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Considerable progress has been made in the field of $f(R,T)$ theory of gravity using various kinds of matter. Among these, two notable examples are quarks and strange quarks. Jokwani et al. [15] have explored locally rotationally symmetric Bianchi-I model filled with strange quark matter in $f(R,T)$ gravity and found that model is shear - free at late time but remains anisotropic throughout the evolution. Pawar et al. [16] have discussed LRS Bianchi type-V Cosmological model in $f(R,T)$ theory of gravity and they found universe has an initial singularity. Several authors [17, 18, 19] have explored evolution of cosmic universe by analysing Kalunza-Klien cosmological model with strange quark matter in different theories of gravitation. Pawar et al. [20] have obtained exact solutions of field equations with quark and strange quark matter for FRW universe in fractal gravity with the help of assumption a fractal parameter and fractal function in the form of power law.

According to the established standard model of physics quarks are the smallest known units found within the nuclei of atoms. However, isolating individual quarks proves challenging as they are perpetually bound in groups of three. The family of quarks comprises six members: up (u), down (d), charm (c), strange (s), top (t), and bottom (b) quarks, with up, down, and strange quarks being the primary types. Quarks serve as the fundamental building blocks of particles. During an early phase transition of the universe, when the cosmic temperature was around T $\sim$ 200MeV it is widely accepted that a state known as quark-gluon plasma existed. The possibility of quark matter's existence was initially proposed in the early 1970s by Itoh [21], Bodmer [22], and Witten [23], who suggested two pathways for its formation: the quark-hadron phase transition in the early universe and the conversion of neutron stars into strange stars under ultrahigh densities. In theories related to strong interaction, the concept of quark bag models assumes the occurrence of vacuum breaking within hadrons. Consequently, a notable distinction arises between the vacuum energy densities inside and outside a hadron leading to a significant difference in the pressure on the bag wall and the pressure exerted by the quarks. This equilibrium stabilizes the system. The equation of state for strange quark matter based on the phenomenological bag model of quark matter is given by $p = \frac{(\rho - 4B_c)}{3}$ within this equation the bag constant (B_C) represents the disparity between the energy density of the perturbative and non-perturbative QCD vacuum. Here, ρ and p denote the energy density and thermodynamic pressure of the quark matter respectively. In this model, quarks are treated as degenerate fermi gases existing within a region of space characterized by the vacuum energy density (B_C) known as the bag model. Within this framework, the quark matter is composed of massive s quarks and electrons, alongside massless u and d quarks. A simplified version of this bag model assumed that quarks are massless and noninteracting. Therefore, we have quark pressure $p_q = \frac{\rho_q}{3}$, where ρ_q is the quark energy density. The total energy density and pressure is $\rho = \rho_q + B_c$ and $p = p_q - B_c$ respectively.

Mak and Harko [24] have conducted an investigation on spherically symmetric space-time in the presence of charged strange quark matter considering conformal motion. Dixit et al. [25] have derived deterministic solution of Kontowski-Sachs space-time with strange quark matter in $f(R)$ gravity and they noticed that the function $f(R)$ satisfies the cosmological viability constraint. Sahoo and Mishra [26] have confined their work to strange quark matter attached to string cloud in general relativity for higher dimensional Bianchi type - III universe. Katore [27] has discussed the FRW cosmological model incorporating strange quark matter attached to a string cloud. Santhikumar et al. [28] have discussed the properties of axially symmetric cosmological models with strange quark matter attached to a string cloud. Katore and Shaikh [29] have discussed the properties of axially symmetric space-time incorporate strange quark matter attached to a string cloud within the framework of general relativity. Yilmaz et al. [30, 31] have explored the implications of quark and strange quark matter in Bianchi type-I and V space-times within the context of $f(R)$ theory of gravity. Additionally, they have also investigated the presence of strange quark matter within a Robertson-Walker cosmological model using the general theory of relativity. Adhav et al. [32] have investigated the behaviour of quark and strange quark matter for Kantowski-Sachs cosmological model within the context of $f(R)$ theory of gravity. Chirde and Sheikh [33] investigated plane symmetric cosmological model with the distribution of quark and strange quark matter in deformations of the Einstein's theory of General Relativity. Hatkar et al. [34] have studied Bianchi-I universe incorporating quark and strange quark matter in $f(G)$ theory of gravity and they observed that quark matter is transformed into strange quark matter for power law and exponential law model. Aygün et al. [35] have explored FRW cosmological model with quark and strange quark matter in creation field cosmology.

Furthermore, Pawar and Agrawal [36] have examined the behaviour of quark and strange quark matter within the context of $f(R,T)$ gravity for a plane symmetric cosmological model and they found that the mean anisotropy parameter remains constant throughout the evolution. Pawar and Mapari [37] have explored magnetized strange quark matter within the Lyra geometry for a plane symmetric cosmological model, revealing that the model remains anisotropic throughout its evolution except for the case where $n = 1$. Sahoo et al. [38] have discussed magnetized strange quark matter distribution for LRS Bianchi type-I with cosmological constant Λ in $f(R,T)$ gravity. Kumbhare and Khadekar [39] have investigated higher dimensional spherically symmetric space time with magnetized quark and strange quark matter admitting conformal motion. Nagpal et al. [40] have explored the FLRW cosmological model incorporating magnetized quark matter and strange quark matter within the framework of $f(R,T)$ theory of gravity.

Chirde and Shekh [41] have investigated a plane symmetric dark energy model represented by a wet dark fluid incorporating $f(R,T)$ gravity. The precise solution of LRS Bianchi type-I spacetime with strange quark matter and variable cosmological term Λ in the context of $f(R,T)$ theory of gravity has discussed by Singh and Beesham [42]. Khadekar and Shelote [43] have analysed Kalunza-Klein cosmological model with Quark and Strange Quark matter. Aygün et al. [44] have investigated higher dimensional FRW universe in presence of quark and strange quark matter for cloud string with perfect fluid in Lyra geometry. They obtained that cloud of string with perfect fluid is non-existent for higher dimensional FRW universe. Aygün et al. [45] examined Marder's universe with strange quark matter in $f(R,T)$ gravity. Pawar et al. [46] investigated interacting field model for plane symmetric universe with cosmological constant in the framework of $f(R,T)$ theory. Krishna et al. [47]have studied plane symmetric cosmological model with bulk viscous and cosmic strings in Lyra's geometry and they observed inflation phase. Mete et al. [48] have delved into higher dimensional plane symmetric cosmological models featuring two fluid sources within the realm of general relativity. Thakre et al. [49] analysed higher dimensional plane symmetric cosmological model with quadratic equation of state in $f(R,T)$ gravity. Plane symmetric inflationary models play a crucial role in the formation of the universe's structure and are of significant astrophysical interest. While the present state of the universe exhibits overall spherical symmetry and isotropy its early stages of evolution did not possess such a smoothed-out characteristic. Hence, we consider the less restrictive plane symmetry, allowing for deviations from isotropy.

Furthermore, the objective of this study is to explore a higher dimensional Bianchi type I cosmological model incorporating quark and strange quark matter within the framework of $f(R,T)$ theory of gravity. The paper is structured as follows: Section 1 provides an introduction to the research topic, while Section 2 presents the general framework of $f(R,T)$ gravity. In Section 3, we have studied the metric and the field equations for quark and strange quark matter within the context of $f(R,T)$ gravity. Furthermore, in Section 4, we obtained the solutions of field equation by considering power law model and exponential expansion model. In section 5, we explored the results that were obtained in the preceding section. It is important to note that our investigation builds upon the previous works conducted by [36].

2. GRAVITATIONAL FIELD EQUATIONS OF $f(R,T)$ GRAVITY

The $f(R,T)$ theory of gravity [8] is proposed by Harko et al. (2011) which is the modification of General Relativity. In this theory, the gravitational action is given by the following equation:

$$s = \frac{1}{16\pi G}\int f(R,T)\sqrt{-g}d^5x + \int L_m\sqrt{-g}d^5x \tag{1}$$

where $f(R,T)$ is an arbitrary function of Ricci scalar R and trace T of energy momentum tensor of matter T_{ij}. L_m is the matter Lagrangian density. The energy momentum tensor T_{ij} can be stated as

$$T_{\kappa\beta} = -\frac{2}{\sqrt{-g}}\frac{\partial\left(\sqrt{-g}L_m\right)}{\partial g^{\kappa\beta}} \tag{2}$$

In simpler terms, the $f(R,T)$ theory proposes a modified version of General Relativity that considers additional terms involving the Ricci scalar, trace of the energy-momentum tensor, and matter Lagrangian density. These modifications are incorporated into the gravitational action to describe the behavior of gravity in a different manner than predicted by General Relativity. On varying the action with respect to metric tensor g_{ij}, the field equations of $f(R,T)$ gravity are obtained as

$$f_{R(R,T)}R_{\kappa\beta} - \frac{1}{2}f(R,T)g_{\kappa\beta} - f_R(R,T)\left(\nabla_\kappa\nabla_\beta - g_{\kappa\beta}\Box\right) = 8\pi T_{\kappa\beta} - f_T(R,T)\left(T_{\kappa\beta} + \theta_{\kappa\beta}\right) \tag{3}$$

where,

$$\theta_{\kappa\beta} = -2T_{\kappa\beta} + g_{\kappa\beta}L_m - 2g^{lk}\frac{\partial^2 L_m}{\partial g^{\kappa\beta}\partial g^{l\alpha}} \tag{4}$$

Here

$$f_R(R,T) = \frac{\partial f(R,T)}{\partial R}, f_T(R,T) = \frac{\partial f(R,T)}{\partial T}, \Box = \nabla^\kappa\nabla_\kappa \tag{5}$$

where ∇_κ is the co-variant derivative. Now Contraction of equation (4) gives

$$f_R(R,T)R + 3\Box f_R(R,T) - 2f(R,T) = 8\pi T - f_T(R,T)(T+\theta) \tag{6}$$

where $\theta = \theta_\kappa^\kappa$ equation (5) gives relation between Ricci Scalar R& the trace T of energy momentum tensor. In the present study, we assume that the stress energy tensor of matter is given by,

$$T_{\kappa\beta} = (\rho + p)u_\kappa u_\beta - pg_{\kappa\beta} \tag{7}$$

where five-velocity vector $u_\kappa = (1,0,0,0,0,0)$ and satisfies the conditions, $u_\kappa u^\kappa = 1$ and $u^\kappa \nabla_\beta u_\kappa = 0.\rho$ indicates energy density and p indicates pressure of the matter. Here the matter Lagrangian is assumed as $L_m = -p$. Therefore equation (4) becomes

$$\theta_{\kappa\beta} = -p g_{\kappa\beta} - 2T_{\kappa\beta} \tag{8}$$

The $f(R,T)$ theory of gravity takes into account the presence of matter fields, and as a result, various theoretical models can be formulated based on different types of matter. Three distinct functional forms of $f(R,T)$ gravity is described below

$$f(R,T) = \begin{cases} R + 2f(T) \\ f_1(R) + f_2(T) \\ f_1(R) + f_2(R) f_3(T) \end{cases} \tag{9}$$

In this paper we are going to focus on the case, $f(R,T) = R + 2f(T)$ where, $f(T)$ is an arbitrary function of stress energy tensor of matter and given by$f(T) = \lambda T$ where λ is a constant. In this particular case, the field equations take the form

$$R_{\kappa\beta} - \frac{1}{2} g_{\kappa\beta} R = 8\pi T_{\kappa\beta} + 2\dot{f}(T) T_{\kappa\beta} + [f(T) + 2P\dot{f}(T)] g_{\kappa\beta} \tag{10}$$

where, an overhead dot denotes differentiation with respect to the argument T.

3. METRIC AND FIELD EQUATIONS

Higher dimensional Bianchi type -I universe given by

$$ds^2 = dt^2 - A^2\left(dx^2 + dy^2\right) - B^2 dz^2 - C^2 d\omega^2 \tag{11}$$

where A, B, C are metric potentials which are functions of cosmic time t and fifth coordinate is taken as space-like. The energy momentum tensor for quark is defined as

$$T^{(quark)}_{\kappa\beta} = (\rho + p) u_\kappa u_\beta - p g_{\kappa\beta}$$

or

$$T^{(quark)}_{\kappa\beta} = \mathrm{dia}(\rho, -p, -p, -p, -p) \tag{12}$$

where $\rho = \rho_q + B_c$ is quark matter total energy density and $p = p_q - B_c$ is the quark matter total pressure and u_κ is the five velocity such that $u_\kappa u^\kappa = 1$ The EoS parameter for quark matter is defined as

$$p_q = \omega \rho_q \quad 0 \le \omega \le 1 \tag{13}$$

Also, the linear equation of state for strange quark matter is

$$p = \omega\left(\rho - \rho_o\right) \tag{14}$$

where ρ_0 is the energy density when pressure p is zero and ω is a constant. when $\omega = \frac{1}{3}$ and $\rho_o = 4B_c$ the above linear equation of state is reduced to the following equation

$$p = \frac{\rho - 4B_C}{3} \tag{15}$$

where B_c denotes the bag constant. In co-moving co-ordinate system, the field equation (10) for metric (11) with the help of equation (12) can be written as

$$\frac{\dot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{B}}{AB} + \frac{\dot{A}\dot{C}}{AC} + \frac{\dot{B}\dot{C}}{BC} = p_q - B_c - \lambda\left(\rho_q + 5B_c - 4p_q\right) \tag{16}$$

$$2\frac{\ddot{A}}{A} + 2\frac{\dot{A}\dot{C}}{AC} + \left(\frac{\dot{A}}{A}\right)^2 + \frac{\ddot{C}}{C} = p_q - B_c - \lambda\left(\rho_q + 5B_c - 4p_q\right) \tag{17}$$

$$2\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \left(\frac{\dot{A}}{A}\right)^2 + 2\frac{\dot{A}\dot{B}}{AB} = p_q - B_c - \lambda\left(\rho_q + 5B_c - 4p_q\right) \tag{18}$$

$$2\frac{\dot{A}\dot{B}}{AB} + 2\frac{\dot{A}\dot{C}}{AC} + \frac{\dot{B}\dot{C}}{BC} + \left(\frac{\dot{A}}{A}\right)^2 = -\rho_q - B_c - \lambda\left(3\rho_q + 5B_c - 2p_q\right) \tag{19}$$

Dynamical parameters for Bianchi type-I are defined as follows :
Average scale factor

$$a(t) = \left(A^2BC\right)^{\frac{1}{4}}$$

Spatial volume V is defined as

$$V = A^2BC$$

The directional Hubble parameters

$$H_x = H_y = \frac{\dot{A}}{A}, H_z = \frac{\dot{B}}{B}, H_\omega = \frac{\dot{C}}{C} \tag{20}$$

The generalized average Hubble's parameter H is defined as

$$H = \frac{1}{4}\left(2\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) \tag{21}$$

The mean anisotropic parameter is defined as

$$\Delta = \frac{1}{4}\sum_{i=1}^{4}\left(\frac{H_{i-H}}{H}\right)^2 \tag{22}$$

where $H_i(i = 1, 2, 3, 4)$ represent the directional parameters.
Dynamical scalar expansion θ is given by,

$$\theta = 4H = 2\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C} \tag{23}$$

The Shear Scalar is given by

$$\sigma^2 = \frac{1}{2}\left(\sum_{i=1}^{4} H_i^2 - 4H^2\right) = \frac{4}{2}AH^2 \tag{24}$$

The deceleration parameter q is defined by

$$q = \frac{d}{dt}\left(\frac{1}{H}\right) - 1 \tag{25}$$

The positive sign of q corresponds to standard decelerating model, whereas the negative sign accelerated expansion.

4. SOLUTION OF THE FIELD EQUATIONS

Subtracting equation (17) from equation (16), we get

$$\frac{d}{dt}\left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right) + \left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right)\frac{\dot{V}}{V} = 0$$

which on integration gives

$$\frac{A}{B} = c_1 \exp\left[d_1 \int \frac{dt}{V}\right] \tag{26}$$

Subtracting equation (18) from equation (17), we get

$$\frac{d}{dt}\left(\frac{\dot{B}}{B} - \frac{\dot{C}}{C}\right) + \left(\frac{\dot{B}}{B} - \frac{\dot{C}}{C}\right)\frac{\dot{V}}{V} = 0$$

which on integration gives

$$\frac{B}{C} = c_2 \exp\left[d_2 \int \frac{dt}{V}\right] \tag{27}$$

Subtracting equation (18) from equation (16)

$$\frac{d}{dt}\left(\frac{\dot{A}}{A} - \frac{\dot{C}}{C}\right) + \left(\frac{\dot{A}}{A} - \frac{\dot{C}}{C}\right)\frac{\dot{V}}{V} = 0$$

which on integration gives

$$\frac{A}{C} = c_3 \exp\left[d_3 \int \frac{dt}{V}\right] \tag{28}$$

where c_1, c_2, c_3 and d_1, d_2, d_3 are constant of integration which satisfies the relation $c_3 = c_1 c_2$ and $d_3 = d_2 + d_1$ In the view of $V = A^2BC$, we write metric potentials in explicit form

$$A = C_1 V^{\frac{1}{4}} \exp\left[D_1 \int \frac{dt}{V}\right] \tag{29}$$

$$B = C_2 V^{\frac{1}{4}} \exp\left[D_2 \int \frac{dt}{V}\right] \tag{30}$$

$$C = C_3 V^{\frac{1}{4}} \exp\left[D_3 \int \frac{dt}{V}\right] \tag{31}$$

where $C_i (i = 1, 2, 3)$ and $D_i (i = 1, 2, 3)$ which satisfies the relation ${C_1}^2 C_2 C_3 = 1$ and $2D_1 + D_2 + D_3 = 0$

Since we have set of four equations (16),(17),(18), (19) with five unknown which are highly nonlinear. Therefore, to solve the system completely we required additional condition. Here we used two different volumetric expansion law

$$V = \alpha_1 e^{4\beta_1 t} \text{ (Exponential Expansion)} \tag{32}$$

and

$$V = \alpha_1 t^{4n} \text{ (Power Law Expansion)} \tag{33}$$

where α_1, β_1, n are positive constants.

4.1. Model for Exponential Law

The exponential expansion of volume factor is

$$V = \alpha_1 e^{4\beta_1 t}$$

Using the equation (31) in (28)- (30), the scale factor obtained as follows:

$$A = C_1 \alpha_1 \frac{1}{4} e^{\beta_1 t} \exp\left(\frac{-D_1}{4\alpha_1 \beta_1} e^{-4\beta_1 t}\right) \tag{34}$$

$$B = C_2 \alpha^{\frac{1}{4}} e^{\beta_1 t} \exp\left(\frac{-D_2}{4\alpha_1 \beta_1} e^{-4\beta_1 t}\right) \tag{35}$$

$$C = C_3 \alpha_1^{\frac{1}{4}} e^{\beta_1 t} \exp\left(\frac{-D_3}{4\alpha_1 \beta_1} e^{-4\beta_1 t}\right) \tag{36}$$

where C_1, C_2, C_3 are the constant of integration.

It must be stated that, the metric potentials accept constant value at initial time, after which they evolve with time without a singularity and eventually diverge to infinity.

The directional Hubble parameter $H_x = H_y, H_z, H_\omega$ are given as

$$H_x = H_y = \beta_1 + \frac{D_1}{\alpha_1} e^{-4\beta_1 t} \tag{37}$$

$$H_z = \beta_1 + \frac{D_2}{\alpha_1} e^{-4\beta_1 t} \tag{38}$$

$$H_\omega = \beta_1 + \frac{D_3}{\alpha_1} e^{-4\beta_1 t} \tag{39}$$

Mean Hubble parameter H is given by

$$H = \beta_1 \tag{40}$$

Anisotropy parameter of the expansion is

$$\Delta = \frac{\mu^2}{4{\beta_1}^2 \left(V_1 e^{4\beta_1 t}\right)^2} \tag{41}$$

Where $\mu^2 = 2{D_1}^2 + {D_2}^2 + {D_3}^2$
Dynamical scalar is given by

$$\theta = 4H = 4\beta_1 \tag{42}$$

The Dynamical scalar is constant throughout the evolution Shear scalar

$$\sigma^2 = \frac{\mu^2}{2\left(V_1 e^{4\beta_1 t}\right)^2} \tag{43}$$

The deceleration parameter

$$q = \frac{d}{dt}\left(\frac{1}{H}\right) - 1 = -1 \tag{44}$$

In the context of the exponential expansion model when deceleration parameter $q = -1$ and $\frac{dH}{dt} = 0$ it signifies the most optimal value for the decelerating parameter. This optimal value indicates that the universe is undergoing acceleration, experiencing the fastest possible rate of expansion. The value of anisotropic parameter shows anisotropic universe, but for large time it approaches to isotropic Universe. Additionally, when the anisotropic parameter exhibits a particular value, it suggests an anisotropic universe meaning that the universe appears uniform and consistent in all directions. on subtracting equation (19) from (18), we get,

$$2\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} - 2\frac{\dot{A}\dot{C}}{AC} - \frac{\dot{B}\dot{C}}{BC} = (p_q + \rho_q)(1 + 2\lambda) \tag{45}$$

Substituting the values metric potentials, A, B, C from equation (34) (35) and (36) also by using equation of state (13) for $\omega = \frac{1}{3}$, we obtained quark pressure as follows,

$$p_q = \frac{\left(\frac{e^{-4\beta_1 t}}{\alpha_1}\right)^2 \left[2{D_1}^2 + {D_2}^2 - 2D_1D_3 - D_2D_3\right]}{4(1+2\lambda)} \tag{46}$$

The quark matter density is given as

$$\rho_q = \frac{\left(\frac{e^{-4B_1 t}}{\alpha_1}\right)^2 \left[2{D_1}^2 + {D_2}^2 - 2D_1D_3 - D_2D_3\right]}{12(1+2\lambda)} \tag{47}$$

Using equation (34), (35),(36) in equation(45) with the help of equation of state in equation (13) for $\omega = \frac{1}{3}$, the pressure and energy density of strange quark matter is found to be,

$$p = \frac{\left(\frac{e^{-4\beta_1 t}}{\alpha_1}\right)^2 \left[2{D_1}^2 + {D_2}^2 - 2D_1D_3 - D_2D_3\right]}{4(1+2\lambda)} - B_C \tag{48}$$

$$\rho = \frac{\left(\frac{e^{-4B_1 t}}{\alpha_1}\right)^2 \left[2{D_1}^2 + {D_2}^2 - 2D_1D_3 - D_2D_3\right]}{12(1+2\lambda)} + B_c \tag{49}$$

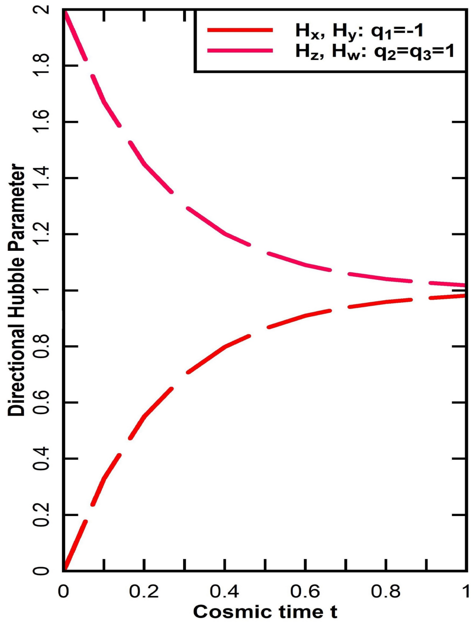

Figure 1. Directional Hubble Parameter Vs Cosmic time t for $\beta_1, D_2, D_3, \alpha_1 = 1, D_1 = -1$.

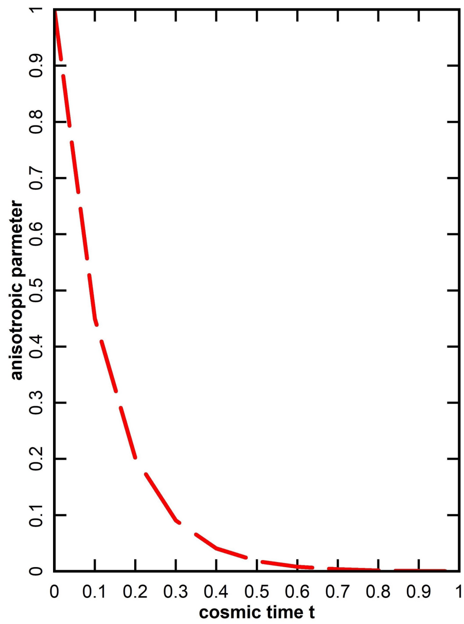

Figure 2. Anisotropic Parameter Vs Cosmic time t for $\beta_1, V_1, D_2, D_3 = 1, D_1 = -1$.

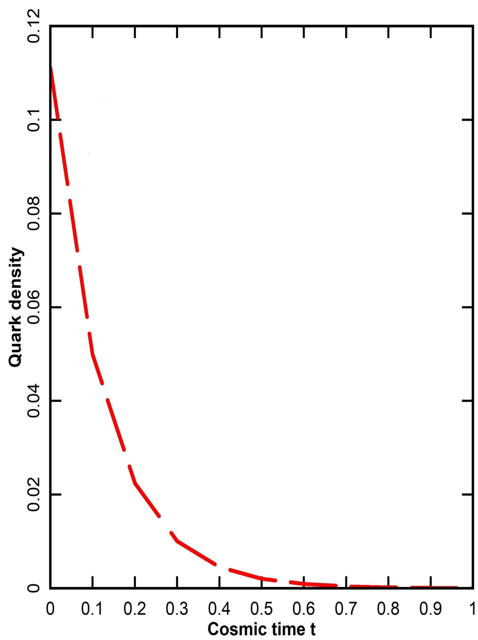

Figure 3. Quark density Vs Cosmic time t for $\beta_1, \alpha_1, D_2, D_3, \lambda = 1, D_1 = -1$

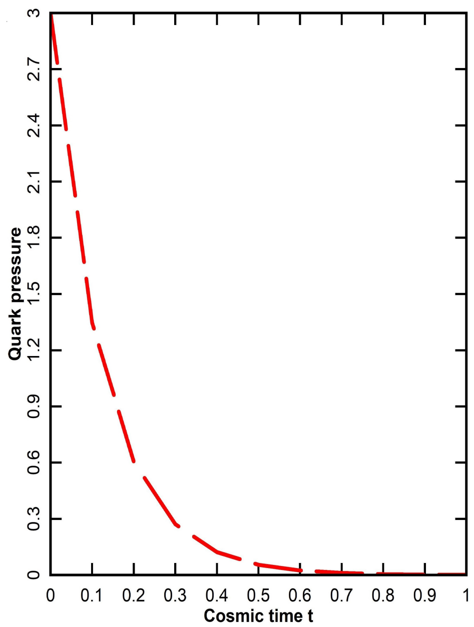

Figure 4. Quark Pressure Vs Cosmic time t for $\beta_1, \alpha_1, D_2, D_3, \lambda = 1, D_1 = -1$.

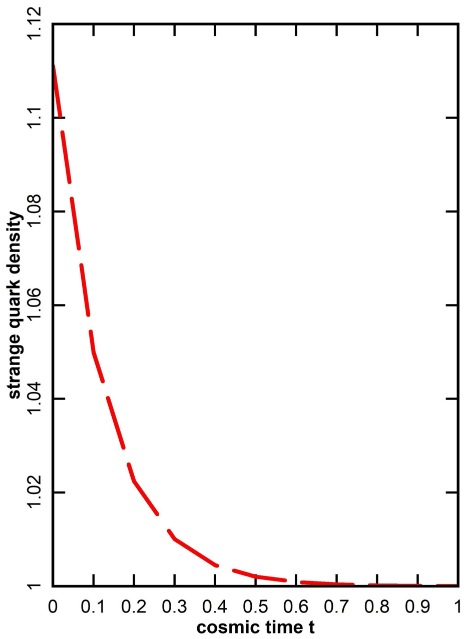

Figure 5. Strange Quark density Vs Cosmic time t for $\beta_1, \alpha_1, D_2, D_3, \lambda, B_c = 1,\ D_1 = -1$.

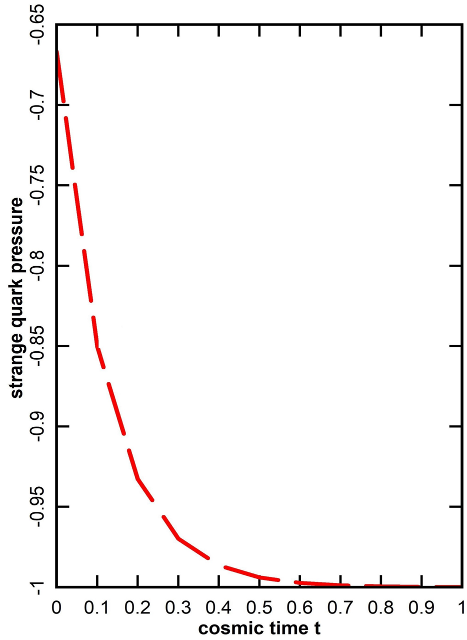

Figure 6. Strange Quark pressure Vs Cosmic time t for $\beta_1, \alpha_1, D_2, D_3, \lambda, B_c = 1,\ D_1 = -1$.

4.2. Model for Power Law Expansion

Here, a power law volumetric expansion is given by

$$V = \alpha_1 t^{4n}$$

Using the Equation (33)in (29)- (31), the scale factor obtained as follows:

$$A = C_1 \alpha_1^{\frac{1}{4}} t^n \exp\left[\frac{D_1}{\alpha_1}\frac{t^{1-4n}}{1-4n}\right] \tag{50}$$

$$B = C_2 \alpha_1^{\frac{1}{4}} t^n \exp\left[\frac{D_2}{\alpha_1}\frac{t^{1-4n}}{1-4n}\right] \tag{51}$$

$$C = C_3 \alpha_1^{\frac{1}{4}} t^n \exp\left[\frac{D_3}{\alpha_1}\frac{t^{1-4n}}{1-4n}\right] \tag{52}$$

At initial time $t = 0$, all the metric potentials are vanishing and finally they diverge to infinity as $t \to \infty$. Thus, the model compatible with a big bang model.
The directional Hubble parameter $H_x = H_y, H_z, H_\omega$ are given as

$$H_x = H_y = \frac{n}{t} + \frac{D_1}{\alpha_1 t^{4n}} \tag{53}$$

$$H_z = \frac{n}{t} + \frac{D_2}{\alpha_1 t^{4n}} \tag{54}$$

$$H_\omega = \frac{n}{t} + \frac{D_3}{\alpha_1 t^{4n}} \tag{55}$$

Mean Hubble parameter H is given by

$$H = \frac{n}{t} \tag{56}$$

Anisotropy parameter of the expansion is

$$\Delta = \left[\sum_{i=1}^{4}\left(\frac{H_i - H}{H}\right)^2\right] = \frac{\mu^2}{4n^2\alpha_1^2 t^{2(2n-1)}} \tag{57}$$

where $\mu^2 = 2D_1^2 + {D_2}^2 + {D_3}^2$
Dynamical scalar is given by

$$\theta = 4H = \frac{4n}{t} \tag{58}$$

Shear scalar is given by

$$\sigma^2 = \frac{\mu^2}{2\alpha_1^2 t^{4n}} \tag{59}$$

It is noticed that at initial time $t = 0$ the Hubble parameter H, dynamical scalar expansion θ starts with infinite value and finally tends to zero as $t \to \infty$. Deceleration parameter q is given by,

$$q = \frac{1}{n} - 1 \tag{60}$$

For $n > 1$ the deceleration parameter is always negative which represent the accelerating universe. Substituting the values metric potentials, A, B, C from equations(50) (51) and (52) in equation (45) also by using equation (13) for $\omega = \frac{1}{3}$, we obtained quark pressure as follows,

$$p_{\mathrm{q}} = \frac{\left(\frac{2\mathrm{D_1}^2 + \mathrm{D_2}^2 - 2\mathrm{D_1 D_3} - \mathrm{D_2 D_3}}{(\alpha_1 \mathrm{t}^{4n})^2}\right) - \frac{3n}{\mathrm{t}^2}}{4(1+2\lambda)} \tag{61}$$

The quark matter density is given as

$$\rho_q = \frac{\left(\frac{2{D_1}^2 + {D_2}^2 - 2D_1 D_3 - D_2 D_3}{(\alpha_1 t^{4n})^2}\right) - \frac{3n}{t^2}}{12(1+2\lambda)} \tag{62}$$

Using equation (50),(51), (52) in equation (45) with the help of equation of state in equation (13) for $\omega = \frac{1}{3}$, the pressure and energy density of strange quark matter as follows,

$$\rho = \frac{\left(\frac{2{D_1}^2 + {D_2}^2 - 2D_1 D_3 - D_2 D_3}{(\alpha_1 t^{4})^2}\right) - \frac{3n}{t^2}}{12(1+2\lambda)} + B_c \tag{63}$$

$$p = \frac{\left(\frac{2{D_1}^2 + {D_2}^2 - 2D_1 D_3 - D_2 D_3}{(\alpha_1 t^{2})^2}\right) - \frac{3n}{t^2}}{4(1+2\lambda)} - B_c \tag{64}$$

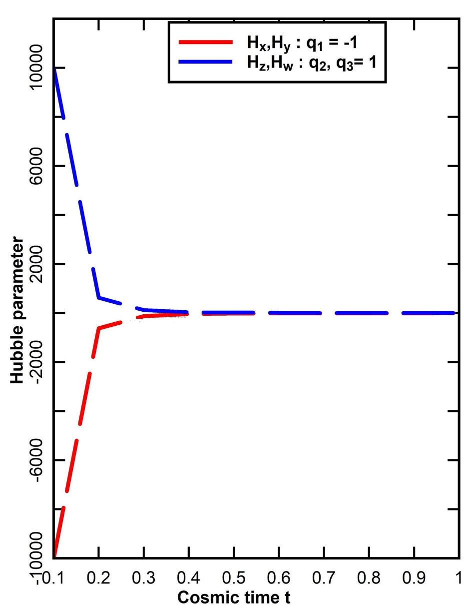

Figure 7. Directional Hubble Parameter Vs Cosmic time t t for $\beta_1, V_1, D_2, D_3 = 1, D_1 = -1$

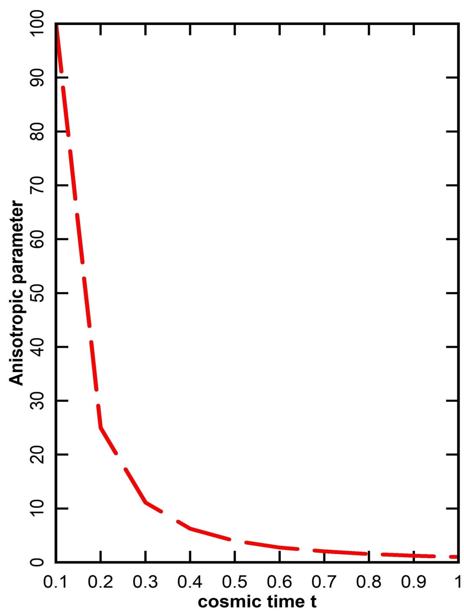

Figure 8. Anisotropic Parameter Vs Cosmic time t for $D_1 = -1, n, \alpha_1 D_2, D_3 = 1$

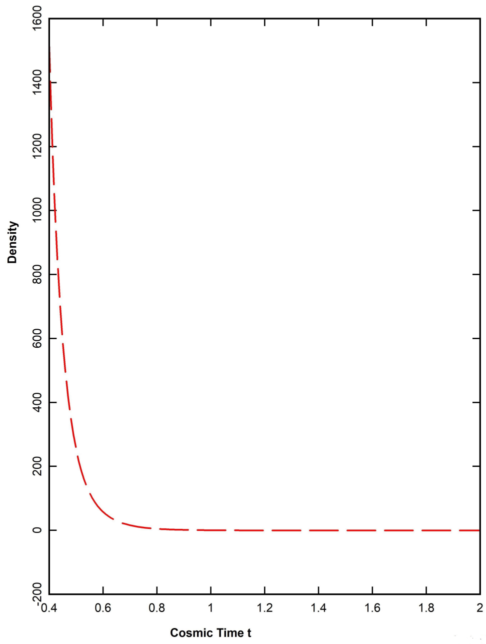

Figure 9. Quark density Vs Cosmic time t for $\beta_1, V_1, D_2, D_3, \lambda = 1, D_1 = -1$.

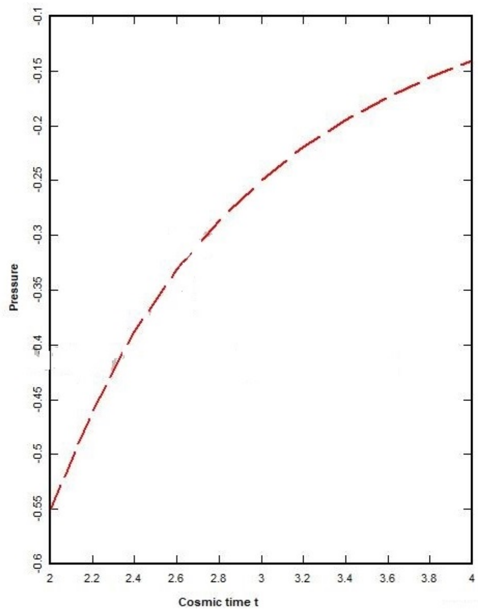

Figure 10. Quark Pressure Vs Cosmic time t for $\beta_1, V_1, D_2, D_3, \lambda = 1, D_1 = -1$.

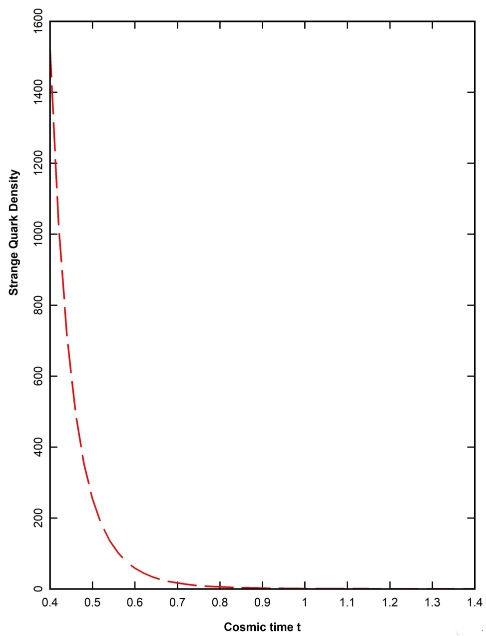

Figure 11. Strange Quark density Vs Cosmic time t for $\beta_1, V_1, D_2, D_3, \lambda, B_c = 1, D_1 =$ -1 .

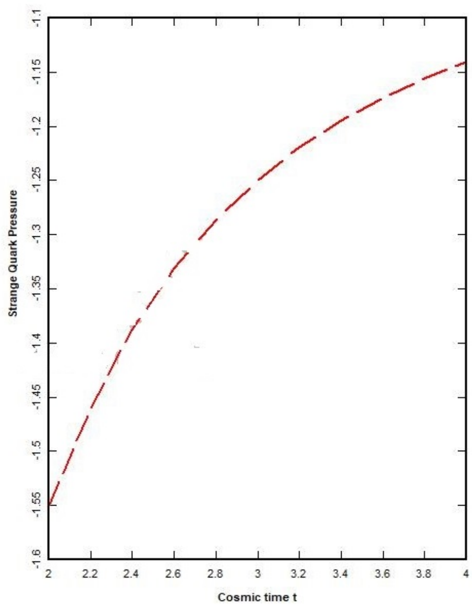

Figure 12. Strange Quark Pressure Vs Cosmic time t for $\beta_1, V_1, D_2, D_3, \lambda, B_c = 1, D_1 = -1$.

5. CONCLUSION

In this work, we have explored the higher dimensional plane symmetric cosmological model with quark and strange quark matter in the context of $f(R,T)$ gravity theory. We have obtained the exact solutions of field equations by assuming two different volumetric expansion laws namely, exponential expansion and power-law expansion.

In exponential expansion model

- The metric potentials accept constant value at initial time, after which they evolve with time without a singularity and eventually diverge to infinity. This result compatible with big bang scenario bear resemblance to [41].

- From Figure 1 and equations ((37), (38), (39)) represent that at initial epoch the directional Hubble parameters are finite whereas gradually decreases to constant β_1 as time tends to infinity.
- From equation (41), the mean anisotropy parameter shows a constant value at initial epoch, while as time increases the anisotropy parameter exponentially to null. Thus, universe approaches isotropy in this model as shown in Figure. 2.
- From equation (42), Dynamical scalar θ exhibits constant value throughout the evolution which shows uniform exponential expansion i.e., universe expands homogeneously as time t increases from initial epoch to infinity.
- From equation (43), Shear scalar measures constant value at t $= 0$ while vanish as t $\to \infty$.
- From equation (44), The deceleration parameter q $= -1$ represents universe is accelerating with highest rate which is in good agreement with present day observation.
- From equation (46) and (47) the pressure p_q and energy density ρ_q for the quark matter are finite in beginning of cosmic time and gradually decrease to zero as time tends to infinity as shown in Figure 3 and Figure 4. This result agreed with [36].
- The energy density (ρ_q) of strange quark matter exhibits the same behavior as quark matter. However, the difference in ρ values compared to ρ_q is attributed to the inclusion of an extra term, the bag constant B_c, in equation (49). Notably, while the quark pressure (p_q) shows a positive value, the strange quark pressure (p) is observed to be negative for the same constant values (refer to Figure 5 and Figure 6).

In power law expansion model

- The metric potentials vanish at initial time $t = 0$ and eventually they diverge to infinity as $t \to \infty$. thus, the model compatible with a big bang model and has a initial singularity.
- The directional Hubble parameter are diverging at initial epoch and as the time tends to infinity, they approach to zero monotonically, from Figure 7.Also, Hubble parameter is decreasing as time increases and agreed with the results of [50].
- From equation (57) and Figure 8, mean anisotropic parameter decreases with time and tends to zero as time tends to infinity. Which shows that at early stage of evolution universe was anisotropic and at large time it approaches to isotropy.
- At initial epoch the directional Hubble parameter H, dynamical scalar expansion θ, mean anisotropic parameter, shear scalar starts with infinite value and finally tends to zero as $t \to \infty$. This suggest that in the initial phase of universe, the expansion of the model is notably rapid and progressively decreases over time. This observation indicates that universe evolution began with exceptionally rapid expansion and subsequently moderated as it continues to expand. i.e., it decreases with the expansion of universe.
- For the value $n > 1$, the deceleration parameter shows negative value which indicates that universe undergoes accelerated expansion while the positive value of decelerating parameter shows decelerating model from equation (60).
- From equation (61) and (62) the pressure p_q and energy density ρ_q for the quark matter are infinitely large as $t = 0$ and it gradually decrease to zero as $t \to \infty$ shown in Figure 9 and Figure 10. This result agreed with [51].
- The pressure (p) and energy density (ρ) of strange quark matter exhibit behaviour similar to quark matter. The difference in p and ρ values compared to p_q andρ_q is attributed to the inclusion of an extra term, the bag constant, in equation (63) and (64). Also, we have observed the shifting of graph in Figure 11 and Figure 12 because of additional term Bag constantB_c.

In both the model, the pressure p and energy density ρ of strange quark matter behave same as quark matter. The bag constant plays a vital role in the expansion of universe. We observed that the pressure and energy density approaches to bag constant for large time (t $\to \infty$) energy density. In particular, pressure $p \to -B_c$ and energy density $\rho \to B_c$ as t $\to \infty$. Negative sign for pressure indicates the expansion of the universe in late time [42].

Finally, exact solutions introduced in this section might be valuable for better comprehension of development of the universe.

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ПОВЕДІНКА КВАРКА ТА ДИВНОЇ КВАРКОВОЇ МАТЕРІЇ ДЛЯ ВСЕСВІТУ ВИЩОГО ВИМІРУ ТИПУ Б'ЯНКІ-I У $f(R,T)$ ГРАВІТАЦІЇ

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Ця дослідницька стаття присвячена ретельному дослідженню поведінки, яку демонструють всесвіти типу Б'янкі у вищих вимірах типу I, включаючи присутність кварка та дивної кваркової матерії в рамках гравітації $f(R,T)$. Рішення, отримані для рівняння поля охоплюють сценарії як експоненційного об'ємного розширення, так і сценарію степеневого закону. Відповідно до моделі експоненціального розширення як тиск (p_q), так і щільність енергії (ρ_q), пов'язані з кварковою матерією, початково скінченні на початку космічного часу, поступово зменшуючись до нуля, коли час просувається до нескінченності. І навпаки, у моделі степеневого закону ці параметри починаються нескінченно великими при $t = 0$. згодом зменшуються до нуля, коли час наближається до нескінченності. Крім того, проводиться дослідження фізичних і геометричних атрибутів моделі. Зокрема, у моделях розширення за степеневим законом поведінка дивної кваркової матерії відображає поведінку кваркової матерії щодо тиску (p) і густини енергії (ρ). Але в моделі експоненціального розширення тиск кварків і тиск дивних кварків поводиться по-різному. Константа bag стає критичним фактором, що впливає на розширення Всесвіту, і спостереження показують, що і тиск, і щільність енергії мають тенденцію до константи bag у великих часових масштабах $(t \to \infty)$. Зокрема, тиск $p \to -B_C$ і щільність енергії $\rho \to B_C$ у міру наближення часу до нескінченності. Знак негативного тиску вказує на розширення Всесвіту протягом пізніших епох.

Ключові слова: *кварк і дивна кваркова матерія; постійна bag; Всесвіт вищого виміру Бьянкі типу I; $f(R,T)$ гравітація*