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Equilibrium In A Cash-in-advance Economy With Endogenous Labour Supply

Benoit Carmichael

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EQUILIBRIUM IN A CASH-IN-ADVANCE ECONOMY WITH ENDOGENOUS LABOUR SUPPLY

by
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Submitted in partial fulfilment
of the requirements for the degree of
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ABSTRACT

This thesis consists of three essays In each essay, the analysis is conducted within the framework of a choice-theoretic intertemporal general equilibrium model with money introduced by imposing a Clower cash-in-advance constraint on all transactions In this environment, inflation is not superneutral, it is a tax on trade, raising the price of goods purchased relative to those supplied The thesis investigates three aspects of the disruptive effects of this inflation tax on the allocation of resources

The first essay analyses the effects of fully anticipated inflation on stock market prices The essay demonstrates that a rise in the anticipated rate of inflation which is believed to be permanent has a depressing effect on the stock market There are two reasons for this result Firstly, the real value of a given stream of dividends is reduced when inflation rises because the inflation rate is shown to be a specific tax on dividends Secondly, the firms' profits are lowered, because the inflation rate distorts adversely the equilibrium level of employment

The second essay of the thesis analyses the effects of perfectly foreseen monetary policies on the time paths of output, prices, interest rates -- real and nominal -- and stock market prices Anticipated monetary policies are shown to have real effects because of the existence of a distortion in the labour market, introduced by the requirement to use money as a medium of exchange An increase in the anticipaped rate of inflation

reduces the return to labour and induces a substitution away from time spent in the labour market. Consequently, prospective monetary policies have real effects by influencing inflationary expectations. The following conclusions emerge from the analysis of the second essay. First, once a policy is anticipated, most of the required adjustment occurs before the policy is actually implemented. Second, the adjustment path between the announcement and the implementation of the new policy depends on the elasticity of marginal utility. Third, along the adjustment path, the real rate of interest and the anticipated rate of inflation are negatively correlated. Fourth, if a change in the level of the money supply is interpreted as a transitory change in the growth rate of the money supply, then it may be said that the time paths of the economy before the implementation of known permanent or transitory changes in the growth rate of the money supply are qualitatively the same.

Finally, the third essay of the thesis investigates the uniqueness of a monetary equilibrium in models that motivate money through a Clover cash-in-advance constraint. It is demonstrated that under standard conditions on preferences and technology, Clover-type models can have an infinite number of equilibrium paths with self-fulfilling expectations. These equilibria are seen to lead asymptotically to either a collapse of the monetary economy or the accumulation of an infinate level of real balances. A condition on the representative agent's utility function is specified that permits to rule out explosive price level paths This condition is shown to be formally identical to the one imposed in other monetary models

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CHAPTER 1: Money in Cash-in-Advance Economies: A Survey.

I. Introduction.

This thesis consists of three essays in monetary economics In each of these essays, the analysis is conducted within the framework of a choice theoretic intertemporal general equilibrium model that makes explicit the trading technology and the role of money as a medium of exchange This is done by motivating the use of money via a Clower cash-in-advance constraint The imposition of this type of constraint to motivate the existence of fiat money is not a new phenomenon in the money-macro literature It dates back at least to the work of Tsiang (1956) on the liquidity preference versus loanable funds controversy It also played an important role in Robertson's (1940) analysis of forced saving, see Kohn (1984a) Notwithstanding these early contributions, the cash-in-advance constraint is more generally associated with Clower's (1967) influential work

Clower (1967) criticized the monetary models derived from the neoclassical synthesis on the ground that they did not capture the essence of money, in particular its role as a medium of exchange For instance, he demonstrated that in Patinkin's model, whether someone holds money or not does not restrict in any meaningful way his trading possibilities Exchange can take place even when traders do not hold money This is because all goods and services have the same degree of liquidity in exchange Barter exchanges are as likely as monetary exchanges As a solution to this problem, Clower suggested to add to the standard budget constraint a set of liquidity constraints that require agents to finance consumption expenditures with money balances held over from previous periods This was to capture more accurately the fact that 'money buys goods, goods buy money but goods do not buy goods' With this alternative formulation, money is the only good that provides liquidity services. Each market transaction must have a monetary side associated with it, either the exchange of goods for money or the exchange of money for goods

Only recently have the implications of imposing a cash-in-advance constraint been investigated The early literature following the publication of Clower's paper has been more concerned with justifying this constraint than with deriving its implications This thesis is a contribution to the more recent literature that takes for granted the cash-in-advance constraint and attempts to derive its implications in various situations. The thesis analyses three separate issues. Specifically, the first essay analyses the effects of anticipated inflation on stock market prices in stationary equilibrium. The second essay investigates the effects of anticipated monetary policies on the time paths of output, prices and the interest rate. Finally, the third essay analyses the conditions that must be met in models that motivate money via a Clower constraint to obtain a unique monetary equilibrium

This thesis begins with a survey that relates the three essays to the existing literature This survey does not pretend to be a complete review of the literature on the cash-in-advance constraint. Instead, it has the more limited objective of putting into perspective the three essays of this thesis. The survey focusses exclusively on papers that address specifically the questions discussed in the thesis. A more complete review of the cash-in-advance literature may be found in Kohn (1984). The survey proceeds in the following manner. The first section is an introduction. The second section reviews the literature on the inflation stock-market price relationship. The third section reviews the literature on the effects of monetary policy outside the stationary state. Finally, the fourth section reviews the literature on the existence of self-fulfilling equilibrium paths in dynamic optimizing models.

II. The Inflation-Stock Market Prices Relationship.

The first essay of this thesis analyses the effects of anticipated inflation on stock market prices. Traditionnally, variations of the inflation rate which are believed to be permanent are viewed as having no real effect on the stock market. This is because equities are claims against assets whose real income is considered to be unaffected by variations of the inflation rate. Thus, as long as nominal profits evolve at the same pace as inflation, it is expected that the nominal value of equities will grow at the same rate as inflation. Consequently, the real value of equities is independent of the level of the inflation rate. This conclusion may be derived formally from the standard capital asset price formula of the theory of finance.

This formula, which is given by equation (1) below, defines stock market prices as the discounted present value of a stream of future dividend payments

$$q_t = \sum_{i=0}^{\infty}(1+r)^{-i}(D_{t+i}/P_{t+i}) \qquad (1)$$

Here, q_t, D_t, and P_t are respectively the relative price of equities, the nominal dividend, and the price level at time (t), and $[1/(1+r)]$ is the discount factor used to capitalize future equity earnings Equation (1) embodies the assumption often made in the theory of finance, that the discount factor is constant and independent of the inflation rate

Under these conditions, it is clear from the asset pricing function (1) that if the rate of growth of D parallels the rate of inflation (the growth rate of P), then q, the relative price of equities, is left unaffected by variations in the level of the inflation rate Alternatively, the nominal value of equities, which is given by the product of P and q, evolves at the same rate as the inflation rate Observe that the nominal value of equities may also be obtained by discounting the stream of nominal dividends at the nominal interest rate, i_t, prevailing between any two adjacent periods That is,

$$Q_t = P_t q_t = \sum_{k=0}^{\infty}\prod_{j=0}^{k}[1/(1+i_j)]D_{t+k}$$

According to this theory, the real value of equities is derived by capitalizing future real dividends at a real rate while the nominal value of equities is obtained by capitalising future nominal dividends at a nominal rate

Empirically this theory does not fit the facts well Specifically, many studies of the post war period have reported that stock market prices have moved consistently in the opposite direction to the inflation rate. This inverse relationship is most apparent between 1965 and 1976 when the market value of the U S nonfinancial corporate sector has fallen by almost fifty percent Modigliani and Cohn (1979) estimate that, for the United States, each percentage point increase in the inflation rate typically reduces the market value of equities by thirteen per cent Data for the Canadian economy unveiled a similar qualitative association between inflation and stock market prices For instance, the real value of the Toronto Stock Exchange 300 index has fallen by fourty percent between the years 1965 and 1975 while the level of inflation was multiplied by a factor of four, going from an annual rate of three percent in 1965 to well above ten percent in 1975 More recently, the resurgence of the stock market, observed both in Canada and in the United States, has occurred concurrently with a reduction in the long term inflation rate of the two countries Therefore, there is empirical evidence to suggest that the level of inflation is an important variable influencing the way capital asset prices are determined by the stock market in Canada and in the United States This is clearly in contradiction with the theory outlined above which predicts that inflation is neutral

The fundamental reasons behind this perceived non neutrality of inflation is still very much a subject of discussion This section of the paper reviews the three main alternative explanations suggested in the literature for the observed negative correlation between inflation and stock market prices These explanations may be found in Fama (1981), Feldstein (1980) and, Modigliani and Cohn (1979) They will be discussed here in the same chronological order that they appear in the literature

First, Modigliani and Cohn (1979) argue that the negative correlation between inflation and stock market prices arises because " investors, at least in the presence of unaccustomed and fluctuating inflation, are unable to free themselves from certain forms of 'money illusion' and, as a result price equities in a way that fails to reflect their true economic value " Modigliani and Cohn single out two distinct sources of money illusion

The first source of money illusion comes from the fact that stock market investors discount the stream of future real dividends using a discount rate that parallels the nominal interest rate instead of using, as would be dictated by equation (1), the more economically sound real interest rate This is an important modification to equation (1) because periods of rising inflation generally coincide with periods of rising nominal interest rates as inflationary expectations are gradually embodied into these nominal interest rates It is only if market participants expect a zero rate of inflation that the movements of the nominal rate reflect well the movements of the real rate Therefore, as long as inflation expectations eventually catch up with the actual inflation rate, and investors capitalise equity earnings with a rate that parallels the nominal interest rate, then equity prices will fall when the rate of inflation goes up

The second source of money illusion arises because in the case of levered firms, investors understate the true value of profits in periods of positive inflation Specifically, they argue that investors fail to realise that the inflation premium embodied in the nominal interest rate is really a use of profits rather than a cost of capital It is a use of profits because in periods of sustained inflation, the inflation premium is the amount just sufficient to compensate the firm's creditors for the depreciation of the firm's nominal liabilities Therefore if investors do not correct reported profits for the gains received by shareholder due to the reduction of the firm's real liabilities, then they consistently understate the true value of profits The underestimation of profits is directly proportional to the inflation premium of nominal interest rates As a result, an increase in the long term inflation rate depresses the real value of equities because of its negative effect on reported profits

The two types of money illusion discussed by Modigliani and Cohn are not completely independent of one another Investors confuse the nominal with the real interest rate Given this confusion, it seems perfectly 'rational' on their part to deduce from the firms' income the full amount of interest payments and to discount real earnings at the perceived real interest rate, which in this case is the nominal interest rate

Alternatively, Feldstein (1980) suggests that the reason for the negative correlation between inflation and stock prices can be found in the provisions of the United-States corporate tax laws Under current arrangements, the effective tax rates relevant for corporate source income are not independent of the level of the inflation rate This is because the tax tables are not completely indexed to the variations of the inflation rate For instance, depreciation allowances are based on historic cost figures instead of current replacement costs, and the nominal rather than the real capital gains are subject to taxation by the government. Under these conditions, an increase in the inflation rate reduces the price of equities because it raises firms' tax liabilities in real terms The depressing effect of inflation on equity prices emphasised by Feldstein may be seen from equation (1) by interpreting D/P as an after tax dividend payment

It should be clear from this discussion of Feldstein's argument that the effect of inflation on stock market prices is country specific as tax laws differ from one country to another In a country with fully indexed taxes, the level of asset prices would be unrelated to the level of inflation In addition, one should not associate incomplete indexation with money illusion It is quite possible that a society, through the actions of its elected government, decides to tax corporations at a rate that varies systematically with the inflation rate

Finally, a third possible explanation for the negative correlation between inflation and stock market prices can be derived from Fama's (1981) study of the inflation-stock returns relationship Even though Fama does not discuss specifically the inflation-stock-market price correlation, his argument may nevertheless be easily interpreted as such This can be done as follows

Fama makes two basic hypotheses, (i) he postulates that asset prices are fundamentally determined by real factors which are in turn positively correlated with the economy's future real income and, (ii) he presumes that inflation and real income are linked by a stable forward looking quantity theory type demand for money function Under this last assumption, a fall in the expected growth rate of income keeping constant the growth rate of the money supply, will raise the expected and actual rate of inflation

If Fama's hypotheses are true, and if market participants make rational forecasts of future economic activity, then inflation and stock market prices will tend to move systematically in opposite directions when money growth is independent of real income growth For example, higher future economic growth would, according to hypothesis (i), make stocks more valuable because it is a signal for higher future dividend payments while it would, according to hypothesis (ii), reduce the actual and expected inflation rate Therefore, even though money is superneutral in the model outlined above, it should still be expected that inflation and stock prices move in opposite directions

The force behind this adjustment is the variations in expected future economic growth The ensuing inflation-stock market price correlation is not causal, but is instead spurious It is only a reflection of the fact that inflation and stock prices are endogenous variables which may respond to common, exogeneous disturbances, such as changing expectations about future economic growth

The first essay of this thesis proposes another explanation for the anomalous inflation-stock-market price relationship that is an alternative to the explanations suggested by Modigliani and Cohn, by Feldstein and by Fama The essay is part of the growing literature that sees inflation as a tax on trade The view that inflation is a tax on trade comes out quite naturally from models that specify precisely the technology of exchange and the role of money as a medium of exchange. This is the case for example of models that motivate the existence and the use of money by imposing a Clover type cash-in-advance constraint on market transactions In a cash-in-advance environment, current purchases of goods and services have to be financed with money balances held over from previous periods This assumption is made to capture the notion that in an economy where money is a medium of exchange, purchases and sales are not always perfectly synchronized, so that the proceeds from sales are generally held in the form of cash balances for a certain time interval before they are either spent on consumption goods or converted into a different asset In the case of a cash-in-advance economy, the length of the interval between sales and purchases is fixed exogenously to one period.[1] This in effect precludes financing current purchases from the receipt of current sales Money income must be held in cash for one period before it can be spent on goods and services

In this institutional arrangement, the prices of goods purchased relative to those supplied increase with inflation as money income is taxed by inflation during the time interval between sales and purchases The disruptive effect of this inflation tax on the allocation of real resources has attracted greater attention lately Stockman (1981) has demonstrated that if gross investment in addition to consumption expenditures is subject to a cash-in-advance constraint, then in the steady-state, the capital stock is inversely related to the rate of inflation Stockman attributes this reverse Tobin effect to the fact that, in his model, inflation is a tax on investment expenditures Specifically, an increase in the inflation rate depresses the net return of investment because it raises the inflation tax levied on the stream of prospective investment income This clearly makes investment less desirable and consequently, it reduces investment expenditures and the steady-state capital stock Kohn (1984) extends Stockman's model to situations outside the steady state He shows that following a step increase in the rate of inflation, the real return on capital is depressed below its long run equilibrium value on the adjustment path to the new stationary state if real wages adjust slowly to variations in economic conditions

Wilson (1979), Aschauer (1981) and Aschauer and Greenwood (1983) have demonstrated how inflation disrupts individuals' consumption-leisure choice in cash-in-advance economies When money wage payments are made at the end of the period, labour suppliers evaluate real wage not in terms of current goods, but rather in terms of future goods The technology of exchange implied by a cash-in-advance constraint makes the labour supply an investment-like decision As a result, an increase in the inflation rate induces individuals to supply less labour because it reduces the consumption goods value of nominal wages Therefore, in a cash-in-advance framework, inflation is also perceived by labour suppliers as a specific tax on work effort

Stockman (1985) extends the model developed in Stockman (1981) to the situation of a small open economy He shows that through its effect on the supplies of labour and capital, variations in the inflation rate can also be expected to affect the quantity and the direction of international trade The sign of this effect is shown to depend on parameters of taste and technology

The first essay of this thesis uses this notion that inflation is a tax on trade to explain the empirical relationship between inflation and stock market prices Specifically, the essay shows that inflation, through the cost of holding money, levies a tax on corporate earnings and consequently, exerts a depressing effect on stock market prices The essay modifies the asset pricing model of Lucas (1978) by making production endogenous and by introducing money via a cash-in-advance constraint imposed on all market transactions, including those made in the stock market Production is made endogenous by allowing individuals to vary their labour supply in response to changing market conditions In this model economy, a share of equity is a claim to a stream of nominal payments This stream is subject to an inflation tax because firms have to keep earnings in cash for a time interval before they can pay out dividends to shareholders Therefore, when inflation goes up, the inflation tax levied on dividends increases This makes equity investment less desirable and for a given supply of equities it reduces their equilibrium price

The essay concludes that the inflation-stock price relationship may be explained without relying on money illusion or the incomplete indexation of corporate tax. It needs only to be recognized that money exists because it is an efficient means of economizing on transaction costs Under these conditions, an increase in the inflation rate induces a reallocation of real resources, as economic agents try to economize on their money holdings This reallocation of resources does not occur without having some adverse effects on the economy, however In particular, it should be expected to have a depressing effect on stock market prices

This concludes the review of the literature relevant to the question addressed in the first essay of this thesis The next section surveys the background literature to the second essay of the thesis

III. **Equilibrium outside the Steady State.**

The steady-state properties of cash-in-advance economies are now well known They have been investigated by Aschauer and Greenwood (1982), by Kohn (1985) and by Stockman (1981) among others These studies all established that in cash-constrained economies money is not superneutral In particular, it has been demonstrated that variations of the inflation rate change the relative prices of goods purchased in terms of those supplied In the stationary state, this means that a step increase in the growth rate of the money supply induces a variation of relative prices that makes work effort, capital accumulation, and investment in the stock market less desirable

Less is known however on the dynamic properties of these models outside the stationary state. The second essay of this thesis is an attempt to partly fill this gap The essay focusses on a specific question, namely the effects of perfectly foreseen exogenous shocks on the time paths of output, prices, interest rates -- real and nominal -- and stock market prices. The analysis puts more emphasis on the adjustment of the economy to known future changes in the growth rate and in the level of the money supply, but it also considers to a certain extent the effects of anticipated real shocks

Many recent studies have emphasised the need to distinguish sharply between anticipated and unanticipated disturbances when decisions are based on forward-looking elements, and expectations are formed rationally The reason for making this distinction is that private agents can absorb forseen shocks more easily by taking actions that partly offset the effects of unfavorable future events

Brock's (1974, 1975) papers were among the first studies to point out the importance of this distinction for the determination of the price level and money income Brock built a model that has the important characteristic that all demands and supplies are drawn up from the solutions to intertemporal utility maximization problems under the hypothesis of rational expectations In this economy the role of money is not modelled explicitly but is rather taken into account indirectly by including real balances as an argument of the utility function Brock demonstrates that within this framework, a known future increase (decrease) in the growth rate of the money supply produces an instantaneous jump (drop) in the price level followed by a continuous increase (decrease) in the rate of inflation that ends with the implementation of the new policy The anticipation of a rise in the growth rate of the money supply is sufficient to generate an increase in the rate of inflation well before money starts to grow at a faster rate Inflation may lead monetary growth even though the latter is the ultimate cause of inflation

Brock explains this result intuitively as follows The prospect of higher future money growth makes individuals feel wealthier, as government transfer payments are a source of income for private agents The desire for a smooth consumption path induces agents to spend part of this increase in wealth over all periods, including the current one. The rise in the level of spending puts pressure in the goods markets, and makes prices higher than what they would otherwise have been without the increase in the future growth rate of the money supply. As a result, prices and nominal income increase well before the new policy is implemented by the monetary authorities

One of the implications of this analysis is that the more distant in the future is the implementation of a new policy, the smaller is the unanticipated jump in the price level occuring at the moment the policy is announced. If the policy is implemented in the infinitely distant future, then there is no unanticipated jump in the price level. This result is also obtained in Sargent and Wallace's (1973) analysis of the effects of monetary growth on inflation in Cagan's model of the demand for money under rational expectations.

In Brock's analysis, as in Sidrauski's (1967) before him, money is superneutral when preferences are additively separable. There are three major reasons for this conclusion. Firstly, the marginal product of capital is independent of the level of real balances. Secondly, the economy's rate of time preference is independent of the level of wealth. And thirdly, the planning horizon of agents is the same as the planning horizon of the economy as a whole. Under these assumptions, inflation affects welfare only. A higher inflation rate induces individuals to lower their money holdings, which reduces welfare since the marginal utility of real balances is assumed positive.

If preferences are not additively separable, and if labour supply is endogenous, the superneutrality of money in the stationary state breaks down. This is because under these circumstances, variations in the growth rate of the money supply affect the marginal utility of leisure since the cross partial derivative between money and leisure is no longer zero. Thus, a variation in the growth rate of the money supply is not superneutral because it induces a shift of the labour supply schedule.

In addition, money may have real effects outside the stationary state even if it is superneutral in the stationary state. Specifically, Fischer (1979) has shown rigorously that when the utility function exhibits constant relative risk aversion, higher monetary growth speeds up the adjustment to the stationary state by inducing a more rapid rate of capital accumulation.[2]

One of the problems with this approach is that the role of money is not modelled explicitly but is rather taken into account indirectly by including real balances as an argument of the utility function With this assumption, the effects of money depend crucially on the cross partial derivatives between money and consumption goods (Fischer), as well as between money and leisure time (Brock) One can easily understand that the marginal utility of real balances increases with the level of consumption (i e the increased need for a means of transaction), it is however harder to justify that the marginal utility of consumption should vary with the level of cash balances. Thus, even if the results obtained by putting money into the utility function are useful to understand the way in which the economy works, there is still a need to investigate alternative formulations that specify more precisely the role played by money In the literature, there are two alternatives to introducing money into the utility function Money can be introduced as a store of value, as in overlapping generations models, or it can be introduced as a medium of exchange, as in cash-in-advance models All three essays of this thesis use the latter formulation to generate a demand for money This strategy has the advantage that the transaction role of money is modelled explicitly by the introduction of a sharp distinction between liquid and illiquid assets. This should permit to assess the validity and the generality of results obtained when money is used as an argument of the utility function The implications of modelling money as an intrument that permits inter-generational trades, as in overlapping generation models, are discussed extensively in Wallace (1980)

It is argued in Lucas and Stokey (1983), and in Fischer (1983), that imposing a cash-in-advance constraint may not be different from putting money into the utility function This is because in a cash constrained economy money yields indirect utility Consequently, there may exist a direct utility function that will have the same characteristics as the indirect utility function implied by the cash-in-advance constraint Svensson (1983) analyses this issue extensively and concludes that imposing a cash-in-advance constraint is not in general equivalent to putting money into the utility function More specifically, he demonstrates that the indirect utility function implied by the cash-in-advance constraint is not a structural relationship It should not be expected to stay invariant to structural changes This is especially important if one wants to study the dynamics outside the stationary state Therefore, the imposition of a cash-in-advance constraint is a genuine alternative that can be used to evaluate the generality of the results obtained when money is an argument of the utility function

Many of the papers that motivate money through a cash-in-advance constraint focus more or less exclusively on the analysis of the stationary state There are however a few important exceptions to this rule These are, Helpman and Razin (1982), Abel (1985) and, Rotemberg (1984)

Helpman and Razin study the effects of anticipated monetary policies on exchange rates (real and nominal) and consumption allocations in a cash-in-advance framework They show that anticipated changes in monetary policy have no real effects when production is exogenous and preferences are well described by a logarithmic utility function This means that an increase in the level or in the growth rate of the money supply impinges on nominal variables in the periods following the implementation of the new policy only Helpman and Razin obtain this result because the velocity of circulation implied by the cash-in-advance constraint is not interest elastic Therefore, there is no link between known future monetary shocks and the level or the rate of change of current nominal prices

In Helpman and Razin's framework, only unanticipated changes in the money supply have real effects This is because surprise changes in the price level redistribute wealth between countries through the capital gains and losses on nominal claims In addition, this means that if there are no international debts denominated in one of the currencies, then unanticipated changes in the outstanding stock of that currency have also no real effects In this particular situation, anticipated and unanticipated monetary policies have the same effects on nominal variables

The result obtained by Helpman and Razin that anticipated monetary policies have no real effects depends crucially on the assumptions that production is exogenous and preferences are logarithmic When production is endogenously determined, anticipated monetary policies can have real effects by changing the terms of trade between goods purchased and goods sold Abel (1985) and Rotemberg (1984) analyse the effects of money when capital accumulation is endogenously determined The second essay of this thesis considers the case where economic agents can adjust their labour supply in response to changing economic conditions

Abel (1985) studies the influence of monetary growth on capital accumulation outside the stationary state More specifically, he evaluates whether the results obtained in Fischer (1979) on the effect of monetary growth on the transition path to the stationary state characterises Stockman's cash-in-advance economy His analysis leads to the conclusion that the effect of monetary growth on the rate of capital accumulation depends on the structure of production and on whether the cash-in-advance constraint applies to gross investment Specifically, Abel found three important results Firstly, money is superneutral along the transition path as well as in the stationary state if the cash-in-advance constraint applies to consumption expenditures only Secondly, a higher rate of monetary growth increases the rate of capital accumulation if the cash-in-advance constraint applies to investment expenditures and if capital does not depreciate Thirdly, a higher money growth rate leads to a faster (slower) (the same) speed of adjustment if the elasticity of marginal utility is less than (greater than) (equal to) one when capital depreciates completely after one period

As emphasized in Abel (1985), these results differ from those obtained by Fischer (1979) in one important way Fischer's results do not depend on the specification of the production technology This is not the case in a cash-in-advance economy where the effects of monetary growth depend crucially on the rate of depreciation of capital

Rotemberg (1984) is concerned with the effects of open-market operations on the time path of the capital stock In his analysis, money and claims on capital are the only assets available People hold money because it is required in exchange for consumption goods, and purchase claims on capital because they yield a positive return It is assumed that the exchange of money for claims on capital is costly so that agents have an incentive to visit financial intermediaries only at discrete intervals For convenience, Rotemberg assumes the length of the interval between visits to the financial intermediaries to be exogenous and fixed at two periods Therefore, at each visit to the bank, agents withdraw just enough money to finance consumption expenditures for two periods. Between financial transactions, agents are subject to a Clower-type cash-in-advance constraint In addition, not everybody visits the financial intermediary at the same period Specifically, it is postulated that half of the households visit the financial intermediary during the even periods while the other half do the same during the odd periods. This assumption is made to reflect the fact that in reality only a subset of agents visit financial intermediaries on any given day

Rotemberg demonstrates that in this environment, the steady-state value of the capital stock is independent of the growth rate of the money supply. As in Brock (1974,1975) and Sidrauski (1967), the effect of inflation in the stationary state is on welfare only. Higher inflation reduces welfare by distorting intertemporal consumption decisions. It induces households to consume relatively more (less) than required to maximize welfare in periods when they (do not) visit financial intermediaries to withdraw cash balances.

The fact that money is superneutral in the stationary state does not mean that monetary policies have no real effects. Previously unexpected open-market purchases of claims on capital raise the capital stock in the short-run, while perfectly anticipated future open-market purchases reduce the stock of capital before monetary expansions take place and increase it afterward. These results are explained intuitively by Rotemberg as follows. A previously unexpected increase in the stock of money brought about by an open-market operation pushes the price level upward. This reduces the value of nominal balances held by households who visited the financial intermediary in the past, and therefore reduces their current consumption. This fall in consumption raises capital and output in the following period. In the case of a perfectly foreseen open-market purchase, agents expect the price level to rise during the periods preceeding the monetary expansion. During this interval, agents have an incentive to consume a larger portion of their cash withdrawals in the periods during which they visit financial intermediaries This makes the capital stock fall until the period the monetary expansion takes place, at which point, the stock of capital jumps

The assumption that households visit the financial intermediaries at different periods is crucial for these results If every household visited the bank every period, then open-market purchases of claims to capital would have no real effects whether anticipated or not

Abel (1985) and Rotemberg (1984) have demonstrated that anticipated as well as actual monetary policies may have real effects in a cash-in-advance framework by affecting investment decisions These papers do not deal with endogenous labour supply however The second essay of this thesis abstracts from investment decisions and considers instead an economy where agents choose their labour supply optimally The essay analyses the effects of perfectly foreseen exogenous shocks on the time paths of output, prices, interest rates --real and nominal-- and stock market prices The model is a closed economy version of Helpman and Razin's (1982) model with endogenous production In addition the analysis is not restricted, as in Helpman and Razin, to logarithmic utility functions

This concludes the review of the literature relevant for the second essay

IV. Equilibrium with Self-Fulfilling Expectations.

Many macroeconomic models exhibit saddle path stability In these models, the standard neoclassical solution in which the price level increases at the same rate as the growth rate of the money supply is only one of the possible equilibria The other possible equilibria implied by these models are characterized by self-fulfilling expectations. This results from two basic characteristics of these models Firstly, current market clearing prices are increasing functions of expected future prices Secondly, expectations are formed rationally Under these conditions, variations of prices may not be related to market fundamentals and may instead happen simply because market participants expect prices to change, i e expectations are constantly validated When the economy embarks on a self-fulfilling expectations trajectory, the price level increases (decreases) asymptotically to infinity (zero). This gives rise to either implosive or explosive paths for the level of real balances

Generally, the practice has been to disregard these self-fulfilling equilibrium paths on the ground that they have never been observed empirically. Others like Sargent and Wallace (1973) propose to eliminate these equilibria by imposing a terminal condition that precludes the existence of runaway inflation (deflation) in the absence of accomodating monetary growth

The elimination of self-fulfilling equilibrium paths is not based on any welfare maximizing criterion In fact, the aggregative models which suffer from this indeterminacy problem are not detailed enough to justify this action on the basis of optimizing behavior To evaluate more objectively if the solutions which exhibit self-fulfilling expectations are equilibria, it would be more appropriate to determine whether they are consistent with the postulate of utility maximization Presumably, economic agents will always choose the solution that maximizes utility This requires a more complete specification of the economic environment For instance, the preferences of economic agents and the constraints that limit their choices within a period as well as over time must be known In the case of a monetary economy, the role of money must also be specified precisely In fact, a complete description of the microeconomic structure of the economy is necessary

Brock (1974,1975) analysed the existence of self-fulfilling equilibrium paths in maximizing models in which money is introduced as an argument of the utility function He was able to conclude from his analysis that by imposing certain restrictions on preferences, it is possible to show that self-fulfilling equilibrium paths do not exist (i.e no extraneous equilibrium path exits) The restrictions on preferences are of two types Firstly, the preference for liquidity must be such that in the neighborhood of zero real balances, the marginal utility of money has to change faster than the level of real balances Secondly, the marginal utility of money must always be positive and must satisfy a certain inequality for large values of real balances The first restriction is required to rule out explosive price level paths, while the second restriction is needed to eliminate implosive paths These results have provided an important justification for ruling out the speculative equilibrium paths of many reduced form macroeconomic models

Obstfeld and Rogoff (1983) reexamines Brock's conclusions and finds the condition to rule out speculative hyperinflations very restrictive and economically unreasonable It requires that agents receive infinitely negative utility when they hold zero real balances Once this condition is relaxed, a large number of speculative paths can no longer be ruled out as equilibria when the economy is in a pure fiat monetary regime of the type studied by Brock In addition, Obstfeld and Rogoff demonstrate that speculative hyperinflations can be eliminated, without any restrictions on preferences, if the government is willing to back its currency with a promise to redeem each unit of money for a small share of some real asset

The third essay of this thesis investigates the existence of self-fulfilling expectations in a cash-in-advance economy It is well known (Farmer 1984, Kohn 1985) that in a cash-in-advance economy of the type studied in Lucas (1980, 1982) with exogenous production, self-fulfilling equilibrium paths unrelated to market fundamentals do not exist . This is because the demand for real balances is given by the simple quantity theory equation with constant unit velocity, so that it does not respond to fluctuations in the expected rate of inflation The current price level is determined uniquely by the current level of the money supply and by the current level of real income irrespective of the future values of these variables This result arises from two basic assumptions made in this model Firstly, consumption expenditures must be financed entirely with money balances held over from previous periods Secondly, production is exogenous

Scheinkman (1980) relaxes the first hypothesis to consider a more general exchange technology Specifically, he postulates that agents can acquire their preferred basket of goods in two distinct markets In the first market (the monetary market), agents can sell their endowment for money and purchase consumption goods in exchange for money carried over from the previous periods In the other market (the barter market), goods can be exchanged directly for other goods In this economy, agents are not subject to a strict cash-in-advance constraint because they are not formally required to hold money in order to buy consumption goods They can alway rely on barter exchanges if necessary Inflationary expectations have real effects by inducing agents to reallocate their expenditures between the two markets The higher is the anticipated inflation rate, the higher (lower) is the return of barter (monetary) exchanges, and consequently, the higher (lower) is the share of one's expenditures in the barter (monetary) market Thus, it is possible for the economy to embark on a trajectory which has the property that agents spend an increasingly larger share of their endowment in the barter market This leads to an increasingly higher price level and, at the limit, to a demonetisation of the economy Scheinkman argues that the only way this type of equilibrium can be ruled out is by making money essential This would require that consumption expenditures be financed entirely with money balances held over from previous periods

The third essay considers the alternative case with endogenous production by letting economic agents choose the time path of their labour supply optimally It demonstrates that under general conditions on preferences, technology and monetary growth, self-fulfilling equilibrium paths cannot be ruled out in a cash-in-advance economy with endogenous production There exists an infinite number of equilibrium paths that lead asymptotically either to an infinite level of real balances or a collapse of the monetary economy

The essay derives the conditions that must be imposed to make sure that the convergent path is the unique equilibrium path of the model These conditions are compared to those that must be imposed in overlapping generations models and in models that motivate the use of money by introducing real balances as an argument of the utility function

The essay also discusses another type of indeterminacy which arises when the economy follows Friedman's rule for the optimum quantity of money Specifically, it is shown that when the monetary authorities deflate the money stock at the discount rate, the economy is indifferent between any level of real balances higher than a certain critical value This is because there is no cost for agents to hold cash in excess of what they need for transaction purposes

Footnotes

[1]An exception to this general rule may be found in Grandmont and Younes (1972, 1973) where it is assumed that agents can always spend a constant fraction of their income in the period it is earned

[2]This result does not hold when preferences are described by a logarithmic utility function

CHAPTER 2: Anticipated Inflation and the Stock Market.

I. INTRODUCTION

This essay investigates the effect of fully anticipated inflation on the stock market It is shown within the construct of a choice-theoretic intertemporal general equilibrium model that inflation, through the cost of holding money, levies a tax on corporate earnings and consequently, exerts a depressing effect on the stock market The essay's main contribution is therefore to provide an explanation for the observed negative correlation between inflation and stock prices

The analysis proceeds by introducing money into the asset pricing model of Lucas (1978) via a cash-in-advance constraint This allows the solution for stock prices to take explicitly into account the monetary nature of the economy More specifically, stock prices that are the discounted value of real future corporate earnings net of inflation tax are shown to vary inversely with the inflation rate

The effect of the inflation tax on the stock market discussed here is independent of any features of corporate taxation emphasized in Feldstein (1980), especially the absence of complete indexation

The essay proceeds as follows The second section presents and solves the intertemporal choice problem faced by the representative consumer-worker and by the representative firm The third section considers the steady-state equilibrium of the model and discusses some comparative static results The conclusions are drawn in the fourth section

II. The Consumer–Worker and the Firm Optimization Problems.

Consider an economy with two representative agents, a consumer-worker and a firm, both assumed to be blessed with perfect foresight. The consumer's objective is to maximize his lifetime utility

$$\mathcal{U} = \sum_{t=1}^{\infty}\beta^{t-1}U(c_t,\ell_t), \quad 0 < \beta = 1/(1+\rho) \tag{1}$$

where ρ is the consumer's subjective rate of time preference, $U(\cdot)$ his momentary utility function, c_t his current consumption and ℓ_t his current work effort. The utility function is assumed to be strictly quasi-concave, with c_t and ℓ_t being normal and inferior goods respectively.

The consumer-worker has three main sources of income. He sells his labour services to the representative firm at the competitive nominal wage W_t. He can claim a share of the firm's real profits, D_t, by purchasing equity. This share corresponds to the fraction, z_t, of the firm's only equity which is assumed to be perfectly divisible. The consumer also receives from the government a lump-sum nominal transfer payment Γ_t. The government finances these transfers entirely by printing money.

Each period, the consumer-worker optimally allocates his resources between consumption, equities, labour supply and end-of-period real balances. In addition to a standard budget constraint, these choices must satisfy a cash-in-advance constraint which requires that his expenditures be financed with money balances held at the beginning of the period[2] This restriction has important implications for the sequence of transactions occurring within any given period, in the following way

The consumer enters period (t) holding M_t units of money balances He makes a labour supply decision, and he uses his money balances to finance his consumption and his net purchase of the firm's equities, $s_t - s_{t-1}$ These transactions are carried out at market prices P_t and $P_t q_t$ respectively, where P_t is the 'dollars' price of one unit of c_t and q_t is the price of s_t in terms of current consumption Thus $P_t q_t$ is the nominal value of one unit of s_t . Once these transactions are completed, the goods and stock markets close down and the consumer-worker receives his wage income, $W_t \ell_{t,s}$ and his dividend, $s_t P_t D_t$ His choices of c_t, $\ell_{t,s}$, and s_t implicitly determine $M_{t,d}$ his end-of-period money demand, which must be equal to his wage income plus his dividend, and the fraction of M_t he has not spent on c_t or s_t . Finally, $M_{t,d}$ together with Γ_{t+1} form his (t+1) beginning-of-period money holdings The important point is that a share bought during period (t) or a unit of labour supplied at (t) gets remunerated at the end of the period and therefore is useful only to increase consumption in period (t+1)

On the production side of the economy, the firm's objective is to choose labour inputs, $\ell_{t,d}$, to maximise its present value PV

$$PV = \sum_{t=1}^{\infty}\Big[\prod_{j=1}^{t}[1/(1+r_{j-1})]\Big]D_t \tag{2}$$

where,

$$D_t \equiv f(\ell_{t,d}) - (W_t/P_t)\ell_t \quad t = 1, \ldots, \infty \tag{3}$$

and r_t is the implicit real rate of interest in period (t) [3]

In solving this optimisation problem, the firm takes the real wage W_t/P_t and the real rate of interest r_t as given. The function $f(\cdot)$ embodies the firm's production process. It is assumed to be twice differentiable and to exhibit diminishing returns to scale.

It is clear that this stream of profit is maximized when the firm chooses $\ell_{t,d}$ such that

$$f'(\ell_{t,d}) = W_t/P_t \quad t = 1, \ldots, \infty \tag{4}$$

the marginal product of labour equals the real wage in each period. This obtains because labour inputs used in any one period do not affect the firm's profits in any other period.

The economy is in equilibrium when all markets clear. This situation is characterized by the following conditions.

$$c_t = f(\ell_{t,d}) \tag{5a}$$

$$\ell_{t,d} = \ell_{t,s} = \ell_t \tag{5b}$$

$$M_{t,d} = M_t = (1+\mu)M_{t-1} \quad (5c)$$

$$z_t = 1 \quad (5d)$$

Equations (5a)–(5d) imply that the goods, factor, money and equities market clear in each period. As may be seen from condition (5c), the money supply $M_{t,s}$, is assumed to grow (shrink) at the constant rate μ. This assumption is made for simplicity and does not affect the qualitative features of the conclusion reached in the third section.

Consideration of the general equilibrium properties of the economy requires a prior derivation of the solution to the maximization problem solved by the consumer. So, let us turn to this first task. The recursive equation of the consumer-worker's dynamic programming problem is given by (6), where the maximization is made with respect to c_t, $\ell_{t,s}$, z_t and $M_{t,d}$

$$V(\Omega_t) = \text{Max}\left[U(c_t, \ell_t) + \beta V(\Omega_{t+1})\right] \quad (6)$$

subject to

$$c_t + q_t z_t + M_{t,d}/P_t \leq M_{t-1}/P_t + \Gamma_t/P_t + (W_t/P_t)\ell_{t,s} + z_t D_t + q_t z_{t-1} \quad (7)$$

and

$$c_t + q_t(z_t - z_{t-1}) \leq M_t/P_t \quad (8)$$

and where,

$$\Omega_t \equiv \{s_j\} \quad \text{for} \quad j = t, \ldots, \infty$$

$$s_j \equiv (W_j/P_j, q_j, z_{j-1}, D_j, M_j/P_j)$$

$$M_t = M_{t-1,d} + \Gamma_t$$

Equation (7) is the standard budget constraint that equates total expenditures to total resources each period And equation (8) is the cash-in-advance constraint which formalizes the need to finance expenditures with previously accumulated money balances

It is interesting to note that there are two concepts of wealth in this model Total wealth as given by the right-hand side of (7) and liquid wealth as measured by the level of real balances held at the beginning of each period This distinction exists because a unit of wealth may be consumed only if it is held in real balances at the beginning of the period

Besides the budget constraint (7) and the liquidity constraint (8), the consumer's choices through time must satisfy the following necessary Euler equations [4]

$$U_c(\cdot_t) = \alpha_t + \gamma_t \tag{9}$$

$$-U_\ell(\cdot_t) = \beta U_c(\cdot_{t+1})(W_t/P_t)(P_t/P_{t+1}) \tag{10}$$

$$U_c(\cdot_t)q_t = \beta U_c(\cdot_{t+1})[(P_t/P_{t+1})D_t + q_{t+1}] \tag{11}$$

$$\alpha_t = \beta U_c(\cdot_{t+1})(P_t/P_{t+1}) \tag{12}$$

where α_t and γ_t are the Kuhn-Tucker multipliers associated with constraints (7) and (8) respectively, and must be greater than or equal to zero. Moreover, given the different wealth concepts, α_t and γ_t may also be interpreted as being the marginal indirect utility of wealth and the marginal indirect utility of liquidity.

Briefly consider the intuition behind the Euler equations. Condition (9) states that the consumer makes his consumption decision by equating the marginal utility of c_t with the marginal indirect utility of M_t/P_t, the latter being the sum of the marginal utilities of wealth and liquidity. Condition (10) determines the consumer's labour supply by equating the marginal disutility of work, $-U_{\ell}(\cdot_t)$ with the marginal benefit of work. By offering one more unit of labour at time (t), the worker raises his end-of-period income by W_t "dollars", which may be consumed at time (t+1) when it has a consumption good value of W_t/P_{t+1}. This extra consumption is expressed in current utility when weighted by the discounted marginal utility of period (t+1) consumption. Condition (11) dictates consumer choices in the equities market. It says that the consumer cannot gain at the margin from buying an extra share at time t, consuming its return and converting it back into consumption at time (t+1). Note that in (11) P_t/P_{t+1} multiplies D_t because it is available for consumption at time (t+1) only. Condition (12) indicates that the consumer chooses $M_{t,d}$ such that the marginal indirect utility of money balances held at the end of (t) equals the discounted marginal utility of the extra consumption the money balances will allow in the next period

Three points should be made before concluding this section First, the rate of inflation, π_t, influences the consumer's choices through the appearances of $P_t/P_{t+1} = 1/(1+\pi_t)$ in equilibrium conditions (10)–(12) Hence money is not superneutral in this model Second, when the consumer-worker makes his labour supply decision, he does not take into account the effect of his choice on D_t This follows because he is representative of many identical individuals. Therefore it is assumed that he perceives his *individual* actions as having no effect on the firm's profits Third, it is postulated that the government makes lump-sum transfer payments to the private sector, so that consumers' decisions with respect to money holdings are independent of transfers Of course, when the consumer is representative, *it looks as if* the distribution of money is made in proportion to existing holdings so that money *appears* to be earning interest This occurs only because of the simplifying assumption mentioned above, and it would not arise if the economy were modeled explicitly as having many identical consumers

III. Steady-State Equilibrium

The economy is in equilibrium when the goods, factor, money and equities markets clear in each period. This prevails when choices satisfy (5a)–(5d) in addition to (7)–(12). Moreover, the analysis is restricted to the steady-state when all real variables c_t, ℓ_t, M_t/P_t, W_t/P_t, q_t, α_t and γ_t are constant through time.[5] Therefore, upon making these substitutions, the conditions characterizing the model's steady-state equilibrium are

$$c = f(\ell) \tag{13}$$

$$M/P \geq c \tag{14}$$

$$U_c(\cdot) = \alpha + \gamma \tag{15}$$

$$-U_\ell(\cdot) = \beta U_c(\cdot)[1/(1+\pi)]f'(\cdot) \tag{16}$$

$$U_c(\cdot)q = \beta U_c(\cdot)\Big[[1/(1+\pi)]D + q\Big] \tag{17}$$

where equation (13) is the economy's steady-state budget constraint,[6] which says that consumption equals production each period. Condition (14) is the economy's liquidity constraint which becomes binding under the assumption that μ, the growth rate of the money supply, is strictly greater than $-[\rho/(1+\rho)]$. From now on this assumption will be made to ensure that[7]

$$M/P = c \tag{14'}$$

Therefore the economy obeys a simple version of the quantity theory of money which says that the steady-state rate of inflation equals the growth rate of the money supply

Using the definition of M, (14') may be rearranged to yield

$$c = (1 - \pi/(1+\pi))((W/P)\ell + D) + \Gamma/P \tag{18}$$

which is an alternative way of writing the economy's steady-state budget constraint This reformulation highlights the tax that inflation levies on the representative consumer-worker Wage income and dividends must be detained in money balances before they can be spent on consumption goods When the inflation rate is positive, the real value of these balances available for consumption is reduced, in fact, each real dollar received either as wage income or as dividend is taxed by a factor of $\pi/(1+\pi)$ However, in the model's general equilibrium, the real value of the government's transfer payment always equals the loss of real income caused by the inflation tax

$$(\pi/(1+\pi))((W/P)\ell + D) = \Gamma/P$$

This last expression is found by substituting (3) into (18) The inflation tax levied on the individual as worker and as shareholder is used to subsidize him as consumer

In this set-up, inflation is a specific tax on work effort and on dividends This distortion influences resource allocation and relative prices Hence it is important to consider how the economy adjusts to a change in the steady-state rate of inflation

The effect of inflation on the labour-leisure choice is investigated in Aschauer and Greenwood (1983) Their conclusion that the steady-state values of c and ℓ are negatively related to inflation holds in the present set-up Market activities use fiat money more intensively than do non-market activities like leisure As a result, when inflation goes up, it becomes advantageous to substitute out of the former into the latter Specifically, it can be shown that[8]

$$d\ell/d\pi < 0$$

and,

$$dc/d\pi = f'(\ell)(d\ell/d\pi) < 0.$$

This result has important implications for the steady-state value of the firm's dividend Under the assumption that $f(\cdot)$ is strictly concave,

$$dD/d\pi = -f''(\ell)\ell(d\ell/d\pi) < 0 \tag{19}$$

a higher rate of inflation is associated with a lower level of profit

Once the model's equilibrium values of c and ℓ are determined, condition (17) may be used to price the firm's outstanding equity. Rearranging (17) reveals that in general, q_t adjusts such that the dividend price ratio plus $((q_{t+1}-q_t)/q_t)$, the proportional rate of change of q_t (the capital gain), equals the real rate of interest, r_t, in each period

$$U_c(\cdot_t)/\beta U_c(\cdot_{t+1}) - 1 = r_t = (D_t/(1+\pi))/q_t + (q_{t+1} - q_t)/q_t \tag{20}$$

In steady-state equilibrium, the rate of capital gain is zero and the marginal utility of consumption is constant. Therefore, the steady-state real rate of interest equals ρ, the consumer's constant rate of time preference, and is consequently invariant across different inflationary regimes,

$$r = \rho = (D/(1+\pi))/q \tag{21}$$

Rearranging this condition, it can be observed that the steady-state value of q is the discounted present value of the firm's dividend stream, net of inflation tax

$$q = (1 - \pi/(1+\pi))D/\rho$$

Expression (21) clearly identifies the channels through which variations in the inflation rate affect the steady-state value of the firm's equity. A higher rate of inflation caused by a previously unanticipated step increase in the growth rate of the money supply has two effects on the stock market. First, it lowers the consumption goods value attached to a given stream of dividends because it raises the inflation tax levied on dividends Second, it reduces the firm's profits (dividends) because it distorts the labour-leisure choice in favour of non-market activities Consequently, to maintain equality in (21) between ρ and the dividend price ratio, a permanent reduction in q following a rise in π is necessary

Inflation affects the stock market because the equity is a claim on the firm's nominal earnings Profits may be spent on consumption goods only after money has been detained for one period If inflation rises, the consumption goods value of the dividends is affected adversely for the reason mentioned above This clearly reduces the attractiveness of equities and leads to a fall in the demand for them The stock market must however clear, so the price of equities has to fall

It is important to realise that the channels through which inflation affects q are independent of one another. For instance, even if the government were to subsidize labour suppliers with newly printed money so that their labour-leisure choices were not disturbed by variations in the inflation rate, q would still be affected through the tax that inflation levies on dividends [9]

Whether the same result applies for the impact effect on the nominal value of the firm's equity, $Q_t = P_t q$, is uncertain The reason is that an unexpected step increase in the inflation rate -- growth rate of the money supply -- by reducing the economy's level of output, makes the price level higher than it would be otherwise. This additional effect works in the opposite direction to the fall in q, and may result in a higher Q_t. Figure I below describes on a logarithmic scale two potential adjustments following an increase in the steady-state inflation rate occuring at t^*.

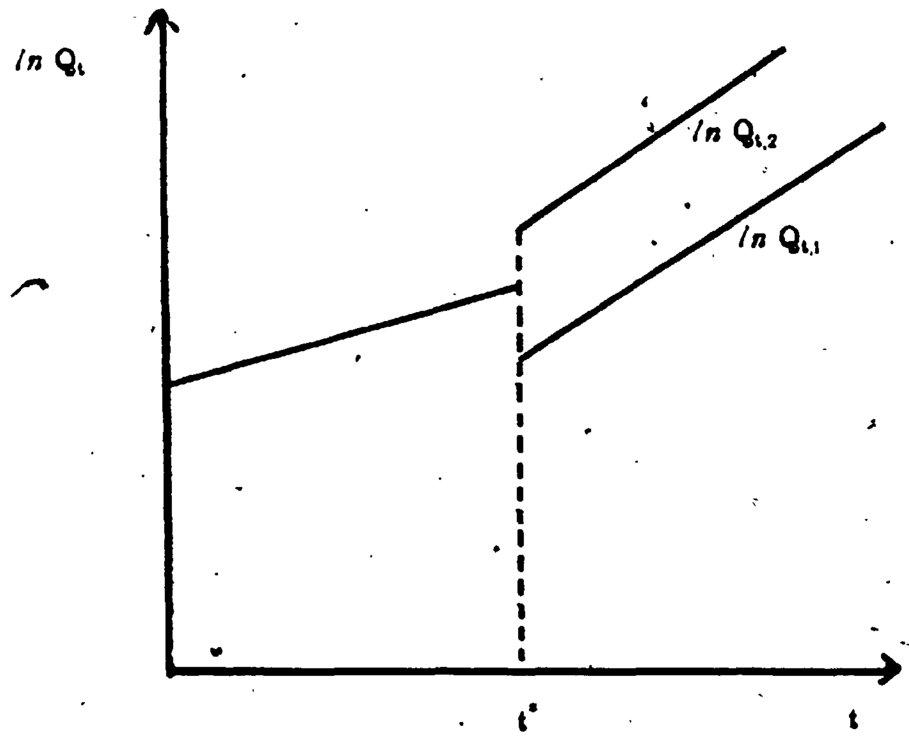
The curve labelled $ln\ Q_{t,1}$ is drawn under the assumption that the fall in q dominates the rise in P_t, and $ln\ Q_{t,2}$ shows what happens under the opposite assumption. It is understood that these two curves have a slope equal to the inflation rate.

The fact that Q_t may move in a different direction than q clearly undermines its usefulness as an indicator of the firm's value. Movements in the stock market are preferably assessed by using q.

IV. Conclusion.

One of the most puzzling empirical facts that has emerged in recent decades is the apparent negative correlation between inflation and stock prices. This essay has presented a simple theoritical model that can account for this phenomenon. It has been shown that if transactions are subject to a cash-in-advance constraint then inflation levies a tax on market activities and therefore exerts a depressing effect on the stock market.

FIGURE I



FOOTNOTES

[1] Imposing this type of constraint is now standard practice in monetary economics. See for example, Lucas (1980, 1982), Wilson (1979), Helpman (1981), Helpman and Razin (1984), Kohn (1981), and Stockman (1981).

[2] Kohn (1981) thoroughly discusses alternative formulations of this constraint.

[3] It is assumed that the world starts at the beginning of period one. For this reason, r_0 is defined as zero.

[4] Stockman (1981) explains in a closely related set-up how the Euler equations are derived from the first-order conditions of the consumer choice problem.

[5] The reader is warned that speculative bubbles equilibria are being ignored. For a discussion of speculative bubbles see Obstfeld and Rogoff (1983).

[6] For convenience, time subscripts will be dropped when the context is clear.

[7] To prove that the cash-in-advance is binding when μ is greater than $-[\rho/(1+\rho)]$ one can proceed as follows. Observe that in a steady-state, $U_c(\cdot)$ is constant. From (9) this means that $\alpha_t + \gamma_t$ has to be constant through time. Suppose γ_t equals zero (i.e., the liquidity constraint is not binding). Then (12)

implies that at varies through time, which is a contradiction to (9) Consequently, the cash-in-advance constraint has to be binding in the steady-state

[8]The first of these results is found by differentiating (16),

$$d\ell/d\pi=-(1+\rho)U_\ell/[f'(\ell)^2(U_{cc}-U_c/U_\ell U_{c\ell})-f'(\ell)(U_c/U_\ell U_{\ell\ell}-U_{c\ell})+f''(\ell)U_c]<0$$

under the assumption that labour is inferior and consumption normal

[9]Alternatively, if equity holders were subsidized with newly printed money, the inflation tax would disappear, but the labour-leisure choice would still be disturbed

CHAPTER 3: Anticipated Monetary Policy in a Clower Cash-in-Advance Economy.

I. Introduction

It is generally agreed that government policies may have a significant influence on the allocation of resources In a world of perfect price flexibility and rational expectations, the extent and nature of these effects depend on whether the government's actions are perceived as permanent, transitory, and/or if they are perceived by the private sector before they are instituted This essay will emphasize particularly the effects of anticipated monetary policy by analysing the responses of an economy to perfectly foreseen changes in the growth rate and in the level of its money supply The analysis is conducted within the construct of an intertemporal general equilibrium model that makes explicit the technology of exchange and the role of money as a medium of exchange This is done by introducing money via a Clower type cash-in-advance constraint Stockman (1981), Aschauer and Greenwood (1983) have demonstrated that in this type of model money is not superneutral and inflation is a tax on market activity This tax that inflation levies is the key element that permits anticipated monetary policy to have any real effects in this economy The prospect of a higher monetary growth generates inflationary expectations that reduce the expected return of labour supply This induces a substitution of leisure time for consumption The essay analyses the implication of this substitution for the time path of output, employment, the real and the nominal interest rates, and the stock market prices

The essay is organised as follows. The first section is the introduction. The second section states the choice problems faced by the different economic agents and gives the definition of equilibrium. The third section analyses the conditions for optimality and discusses the properties of the stationary equilibrium. The fourth and fifth sections study the economy's adjustment to perfectly foreseen changes in the growth rate and in the level of the money supply. The sixth section highlights how the model can be used to analyse the effects of prospective changes which are real rather than monetary in nature. It also discusses the implications of relaxing some of the previously made assumptions. Finally, the conclusions are drawn in the seventh section.

II. The Model

Consider an economy with two distinct infinitely lived representative agents. There is a consumer/worker who maximizes his lifetime utility and there is a firm that maximizes its present value. Every period, the consumer makes a consumption decision, a labour supply decision, and a portfolio allocation decision between money, bonds and equities. Here, a bond is a claim to one unit of money to be paid next period while equities entitle the bearer to a fraction of the firm's profits earned during the period. For analytical simplicity, it is assumed that the representative firm has only one unit of equity outstanding. This unit is perfectly divisible however. Labour services supplied by the consumer is the only factor of production. Goods, factors and assets are traded in perfectly competitive markets. All agents have perfect foresight.

In this economy, the demand for money is motivated through a cash-in-advance constraint which forces net transactions in the goods and assets markets to be financed with money balances held at the beginning of the period.[1] This requirement restricts the circulation of money in the model in a way that is crucial for the analysis conducted below. For this reason, it is useful to review the timing of transactions occuring within any given period.

At the beginning of every period, all financial markets open and operate. The representative consumer allocates his portfolio between bonds, money and equities under the restriction that his net acquisition of assets be financed with the stock of money balances he held at the beginning of the period. The latter is the sum of the stock of money he carried over from the previous period plus a lump-sum transfer payment he received from the government at the beginning of the period. After the clearance of all financial markets, the consumer moves to the factor and goods markets where he supplies labour services and purchases consumption goods for a value not exceeding his holdings of money balances at the closing of financial markets. As a result of these transactions, the firm accumulates cash balances throughout the period. At the end of the period, it distributes these balances back to the consumer as factor and dividend payments.

Therefore, the flow of money payments through the economy implies that the money outflow of one agent is exactly matched by the money inflow of the other. More importantly, money income earned during one period can finance expenditures in later periods only [2]

Formally, the representative consumer/worker solves the following dynamic optimisation problem

Choose

$$\{ c_t, \ell_{t,s}, z_t, B_t, M_{t,d} \} \quad \text{for} \quad t = 0, \ldots, \infty$$

to maximise

$$\mathcal{U} = \sum_{t=0}^{\infty} \beta^t [U(c_t) + V(\ell_{t,s})] \quad \text{with} \quad 0 < \beta \equiv [1/(1+\rho)] < 1 \tag{1}$$

subject to

$$c_t + q_t z_t + R_t(B_t/P_t) + M_{t,d}/P_t \leq M_t/P_t + B_{t-1}/P_t + (W_t/P_t)\ell_{t,s} + z_t D_t + q_t z_{t-1} \tag{2}$$

$$c_t + q_t(z_t - z_{t-1}) + R_t(B_t/P_t) - B_{t-1}/P_t \leq M_t/P_t \tag{3}$$

$$c_t \geqq 0, \ \ell_{t,s} \geqq 0, \ z_t \geqq 0, \ M_{t,d} \geqq 0.$$

where c_t is consumption in period (t), $\ell_{t,s}$ is labour supply in period (t), β is a subjective discount factor which is equal to one over one plus the rate of time preference, P_t is the price level in period (t), W_t is the nominal wage in period (t), B_t is the number of bonds purchased in period (t), R_t is the nominal price of bonds in period (t), z_t is the fraction of the firm's equity held in period (t), q_t is the price of the equity expressed in terms of period (t)'s consumption goods, D_t is the dividend distributed by the firm in period (t), and finally, M_t is the money stock the consumer/worker detains at the beginning of period (t), it is composed of $M_{t-1,d}$, the money stock held over from period (t-1) and of Γ_t, a lump-sum transfer payment made by the monetary authorities before markets open, i e $M_t = M_{t-1,d} + \Gamma_t$

Equation (2) is period (t)'s budget constraint while equation (3) is the liquidity constraint formalizing the need to finance net transactions with money balances held at the beginning of the period [3]

Finally, $U(\cdot)$ and $V(\cdot)$ are the momentary utility functions They are assumed to satisfy the following standard restrictions [4]

$$U'(\cdot) \geq 0, \quad U''(\cdot) \leq 0, \quad U'(0) = \infty, \quad U'(\infty) = 0$$

$$V'(\cdot) \leq 0, \quad V''(\cdot) \leq 0, \quad V'(0) = 0,$$

$$V'(\ell_m) = -\infty \text{ for some positive level } \ell_m$$

On the production side of the economy, the representative firm chooses labour inputs to maximise its present value PV Formally, the firm solves the following problem

Choose

$$\{\ell_{t,d}\} \quad \text{for} \quad t = 0, \ldots, \infty$$

to maximize

$$PV = \sum_{t=0}^{\infty}\left[\prod_{j=0}^{t}[1/(1+r_{j-1})]\right]D_t \tag{4}$$

subject to

$$D_t = f(\ell_{t,d}) - (W_t/P_t)\ell_{t,d} \tag{5}$$

$$r_{-1} = 0$$

where $r_t = [(1/R_t)(P_t/P_{t+1}) - 1]$ is the real rate of interest prevailing in the bonds market during period (t). The function $f(\cdot)$ embodies the firm's production process. It is taken to be twice continuously differentiable, to exhibit diminishing returns to scale and to satisfy $f(0) = 0$

The government's intervention in this economy is made through lump-sum nominal transfer payments to the consumer/worker. This expenditure is financed entirely by printing new money. Thus, the government's budget constraint is equal to

$$\Gamma_t = M_{t,s} - M_{t-1,s}$$

where $M_{t,s}$ is the economy's outstanding money stock in period(t). To simplify the analysis further it is assumed, unless otherwise specified, that the money stock grows at a constant rate μ. This assumption permits to rewrite the government's budget constraint as,

$$\Gamma_t = \mu M_{t-1,s} \tag{6}$$

The economy is in perfect foresight equilibrium when the plans made by the representative consumer/worker are consistent with those of the firm This situation prevails when the following conditions are satisfied

(II a) The representative consumer selects a plan $\{ c_t, \ell_{t,s}, z_t, B_t, M_{t,d} \}$ which solves (1) subject to (2) and (3), taking $\{ P_t, W_t, q_t, R_t, D_t, \Gamma_t=\mu M_{t-1} \}$ as given

(II b) The representative firm chooses a plan $\{ \ell_{t,d} \}$ which maximizes (4), taking $\{ W_t/P_t \}$ as given

(II c) Given these individual choices, prices $\{ P_t, W_t, q_t, R_t \}$ are such that all markets clear

$$c_t = f(\ell_{t,d}) \qquad t = 1, \ldots, \infty$$

$$\ell_{t,d} = \ell_{t,s} = \ell_t \qquad t = 1, \ldots, \infty$$

$$M_{t,d} = M_t = M_{t,s} = (1+\mu)M_{t-1} \qquad t = 1, \ldots, \infty$$

$$B_t = 0 \qquad t = 1, \ldots, \infty$$

$$z_t = 1 \qquad t = 1, \ldots, \infty$$

This definition of equilibrium is standard in general equilibrium analysis It means that if the representative consumer/worker and the representative firm were to individually optimise taking equilibrium prices as given, then the resulting 'market' demand and supply quantities would just clear the markets and would yield market prices indentical to those taken as given.

This concludes the description of the economic environment under study The next section of the essay states the equilibrium conditions and analyses their implications for the model's endogeneous variables. This prepares the ground for sections IV and V where the economy's response to anticipated monetary policy is analysed

III. Equilibrium Conditions and Stationarity.

This section of the essay begins with a discussion of the conditions that must be satisfied by the consumer/worker and by the firm in order for them to behave efficiently through time. The implications of these conditions for the general equilibrium of the economy are analysed subsequently.

Consider first the consumer/worker's optimization problem The maximization of (1) subject to (2) and (4), taking all the prices as given, is a standard dynamic programming problem Necessary conditions for this type of problem are given by:

$$U'(\cdot_t) = \alpha_t + \gamma_t \tag{7}$$

$$-V'(\cdot_t) = \beta U'(\cdot_{t+1})(W_t/P_t)(P_t/P_{t+1}) \tag{8}$$

$$U'(\cdot_t)q_t = \beta U'(\cdot_{t+1})[(P_t/P_{t+1})D_t + q_{t+1}] \tag{9}$$

$$\beta U'(\cdot_{t+1})/U'(\cdot_t) = R_t(P_{t+1}/P_t) \tag{10}$$

$$\alpha_t(1/P_t) = \beta U'(\cdot_{t+1})(1/P_{t+1}) \tag{11}$$

$$M_t/P_t + B_{t-1}/P_t + (W_t/P_t)\ell_t + z_t D_t + q_t z_{t-1} - c_t - q_t z_t - R_t(B_t/P_t) - M_{t,d}/P_t \geq 0$$

$$[\cdot]\alpha_t = 0, \quad \alpha_t \geq 0 \tag{12}$$

$$M_t/P_t - c_t - q_t(z_t - z_{t-1}) - R_t(B_t/P_t) + B_{t-1}/P_t \geq 0$$

$$[M_t/P_t - c_t - q_t(z_t - z_{t-1}) - R_t(B_t/P_t) + B_{t-1}/P_t]\gamma_t = 0, \quad \gamma_t \geq 0 \tag{13}$$

Where α_t and γ_t are the Kuhn-Tucker multipliers associated with constraints (2) and (3). Conditions (7)-(11) are the Euler equations that guarantee that the representative household selects c_t, ℓ_t, z_t, B_t and $M_{t,d}$ efficiently through time. Conditions (12)-(13) state that the budget and the liquidity constraints are binding whenever α_t and γ_t are different from zero. Consumer equilibrium implies that α_t and γ_t may be interpreted as the marginal indirect utility of wealth and the marginal indirect utility of liquidity respectively. This is because they measure the gains in utility from relaxing the budget and the liquidity constraints by a small amount Observe that γ_t is only the liquidity component of the marginal utility of real balances Real balances are also part of wealth, so that the total marginal utility of real balances is the sum of α_t and γ_t

Euler equations (7)-(11) may be given the following intuitive interpretations Condition (7) states that the consumer/worker consumes until the marginal utility of consumption equals the marginal indirect utility of M_t/P_t The latter is the sum of the marginal utilities of wealth and of liquidity Condition (8) determines the worker's labour supply by equating the marginal disutility of work, $-V'(\cdot t)$, with the marginal benefit of work By offering one more unit of labour at time (t), the worker raises his end-of-period labour income by W_t units of money that he may exchange at time (t+1) for (W_t/P_{t+1}) units of consumption goods This increment in consumption is valued in terms of current utility when it is weighted by the discounted marginal utility of period (t+1)'s consumption. Expression (9) determines the consumer/worker's transactions in the equity market It says that at the optimum the consumer cannot gain at the margin from buying an extra share at time (t), consuming its return and converting it back into consumption at time (t+1) The firm's dividend is weighted by (P_t/P_{t+1}) because it is consumable only at time (t+1) Equation (10) is the condition for efficient borrowing It says that the consumer purchases (issues) bonds until the marginal rate of substitution of consumption in adjacent periods is equal to the currency price of bonds times one plus the anticipated rate of inflation The nominal and real interest rates, i_t and r_t, implicit in the bond's price, are given by $[(1/R_t)-1]$ and $[(1/R_t)(P_t/P_{t+1})-1]$ respectively Finally, condition (11) indicates that the consumer's choice of $M_{t,d}$ is such that the marginal indirect utility of an extra unit of money balances held at the end of (t) equals the discounted marginal utility of the extra consumption this unit of money will allow in the next period

Consider now the decision problem faced by the representative firm The formulation of this problem has no intertemporal dimension This is because the labour inputs used up in any one period do not influence the level of production in other periods Under these circumstances, the representative firm maximises its present value by hiring labour services, $\ell_{t,d}$, until

$$f'(\ell_{t,d}) = W_t/P_t \tag{14}$$

the marginal product of labour equals the real wage in every period

Having presented the conditions relevant for individual choices, it is now possible to look at behaviors in general equilibrium As mentioned above, the economy is in general equilibrium when (II a)-(II c) are jointly satisfied, so that prices and quantities are simultaneously determined. The links amongst the model's endogenous variables may be unveiled by manipulating conditions (7)-(14) evaluated in general equilibrium. For instance, solving equation (9) forward indicates that the value of the representative firm constantly adjusts to equate the discounted value of expected future dividends[5]

$$q_t = \sum_{j=t}^{\infty}\left[\prod_{i=t}^{j}[1/(1+r_i)]\right][1/(1+\pi_j)]D_j \qquad (15)$$

This asset pricing function is similar to Lucas' (1978) except for the appearance of the π_j's, the rates of inflation anticipated to prevail between all future adjacent periods. The anticipated rates of inflation appear in (15) because in this setup, contrary to Lucas', firms must raise cash balances before they can distribute dividends. Consequently, the rate at which money depreciates must be fully reflected in the valuation of the firm's stream of dividends.

Similarly, equation (11) may be solved forward to give,

$$1/P_t = (1/\alpha_t)\sum_{j=1}^{\infty}\beta^j(\gamma_{t+j}/P_{t+j}) \qquad (16)$$

That is, the price of money, $(1/P_t)$, equals the discounted sum of all future periods' liquidity services of money, divided by the present marginal utility of wealth. A similar expression for the price of money is also obtained in Svensson (1983). Expression

(16) shows that once the liquidity service of money, measured by the value of relaxing the cash-in-advance constraint, is properly identified as the return from holding money, then money is priced like any other asset, as a discounted sum of future returns

Turning attention to the bonds market, condition (7) in conjunction with (10) and (11) reveals that arbitrage requires that,

$$\gamma_t(1/P_t) = i_t \beta U'(\cdot_{t+1})(1/P_{t+1}) \tag{17}$$

The intuition behind (17) goes as follows Money provides liquidity services while bonds pay a pecuniary return Therefore, portfolio equilibrium between money and bonds necessitates that the discounted marginal utility of the extra consumption the interest payment will allow in the next period equals the marginal indirect utility of the liquidity foregone by investing one more dollar into bonds this period

Condition (17) is also useful to clarify two additional points Firstly, (17) shows that the nominal rate of interest cannot be negative given that one of the other equilibrium conditions requires that γ_t be greater than or equal to zero Basically, this says that the interest payment on money -- which here is zero, -- cannot be greater than the return on nominal bonds when money provides liquidity services and bonds do not[6] Secondly, condition (17) demonstrates that the economy's liquidity constraint is binding - γ_t positive - whenever the nominal rate of interest is greater than zero. Thus, the economy holds real balances in excess of what is required for transaction purposes only if money is as good a store of value as bonds[7]

Observe from (12) and (13), that in general equilibrium the budget and liquidity constraints reduce to the following simpler forms

$$c_t = f(\ell_t) \tag{12'}$$

$$c_t \leq M_t/P_t \tag{13'}$$

Substituting (10) into (8) shows that the economy's marginal rate of substitution between working and consuming in period (t) equals the marginal product of labour deflated by one plus the nominal interest rate[8]

$$-V'(\ell_t)/U'(\ell_t) = f'(\ell_t)/(1+i_t) \tag{18}$$

The presence of i_t in (18) indicates that working is analoguous to an investment decision. This is because in equation (18), future gains are weighted against current costs.

Equation (18) defines an equilibrium locus between ℓ_t and i_t. Under the assumptions made on $U(\cdot)$, $V(\cdot)$ and $f(\cdot)$, this locus is downward sloping with ℓ maximized when i_t equals zero. This link between i_t and ℓ_t also implies, through the production function, a similar relationship between i_t and c_t. Since the cash-in-advance constraint is binding whenever the nominal rate of interest is greater than zero, it must be the case that consumption and real balances are related through time. This may be expressed as follows,

$$c_t = \min(m_t, c^*) \qquad (19)$$

where m_t is the equilibrium level of real balances, M_t/P_t, and c^* is the level of consumption attained when the nominal rate of interest is zero. Similarly, by letting $h(c)$ be the inverse function of $f(\ell)$, the level of employment corresponding to (19) may be expressed as

$$\ell_t = h[\min(m_t, c^*)] \qquad (20)$$

This association between c_t, ℓ_t and m_t, in general equilibrium is useful because it permits to reduce the dynamics of the economy to a single expression.

Substituting (14), (19) and (20) into (8), multiplying the resulting expression by $M_{t,c}$ and evaluating in market equilibrium yields (21), an equation that is solely in terms of real balances at time (t) and (t-1)

$$A(m_t) = B(m_{t+1}) \qquad (21)$$

where,

$$A(m_t) = -\left[V'[h(\min(m_t, c^*))]/f'[h(\min(m_t, c^*))]\right]m_t \qquad (21a)$$

$$\tilde{B}(m_{t+1}) = [\beta/(1+\mu)]U'(\min(m_{t+1}, e^*))m_{t+1} \tag{21b}$$

Equation (21) is a first order non-linear difference equation. This equation summarises the dynamics of the economy in terms of real balances. The strategy employed below to analyse the dynamics of the economy consists of using (21) to determine the equilibrium path of real balances, and then to use this path of m_t in conjunction with the other equilibrium conditions to evaluate the time profile of the other endogenous variables. For instance, once the time path of real balances is known, the behaviors of e_t, i_t, q_t and P_t fall directly from (19), (10), (15) and the definition of m_t.

Given the importance of (21) in the determination of the economy's dynamics, it is worth spending some time examining the basic properties of functions $A(\cdot)$ and $B(\cdot)$.

Under the assumptions that $U(\cdot)$, $V(\cdot)$ and $f(\cdot)$ are concave, functions $A(\cdot)$ and $B(\cdot)$ have the following characteristics:

a) $A(0) = 0$

b) $A'(\cdot) \geq 0$

c) $Lim_{m\to 0}B(m)$ is positive and may be infinity

d) For $m < e^*$, $B'(\cdot) \gtreqless 0$ as $\eta(e) = -[U''(\cdot)/U'(\cdot)]e \lesseqgtr 1$

e) For $m \geq e^*$, $B'(\cdot)$ and $A'(\cdot)$ are constant and satisfy

$$A'(\cdot) = (1+\rho)(1+\mu)B'(\cdot)$$

Thus, function $A(\cdot)$ slopes upward and passes through the origine, while the shape of function $B(\cdot)$ depends on the value of $\eta(c)$, the elasticity of marginal utility. Specifically, function $B(\cdot)$ for $0 \leqq m \leqq c^*$ slopes upward or downward depending on whether $\eta(c)$ is smaller than or greater than one. In general the elasticity of marginal utility varies with c. However, for analytical convenience, it will be assumed throughout that $U(c)$ exhibits constant elasticity of marginal utility. With this simplifying assumption, the slope of $B(\cdot)$ is either positive, null or negative depending on whether η is smaller than, equal to, or larger than one. Therefore, (d) can be replaced by,

d') For $m \leqq c^*$, $B'(\cdot) = (\beta/(1+\mu))U'(c)[1-\eta] \gtreqless 0$ as $\eta \lesseqgtr 1$

It may be added that over the interval $[0, c^*]$, $B(\cdot)$ is strictly concave when $\eta < 1$, while it is strictly convex when $\eta > 1$. Finally, for m bigger than c^*, functions $A(\cdot)$ and $B(\cdot)$ are linear in m because consumption is constant at c^*. Figures I to IV illustrate graphically the different forms that $A(\cdot)$ and $B(\cdot)$ may take.

In general, (21) has an infinite number of solutions.[1] This indeterminacy is resolved here by choosing the time path of real balances leading to the steady-state as the equilibrium of the model. Given the forward looking nature of the model, this reduces to solving (21) forward under the (additional) restriction that the economy should be in stationary state at a pre-specified future period. Thus, equation (21) is best seen as determining the equilibrium value of m_t given the value of m_{t+1}.

It is fairly easy to establish the existence of a stationary equilibrium where the level of real balances is constant through time. Define $\Phi(m)$ as the ratio of $A(m)$ and $B(m)$

$$\Phi(m) = A(m)/B(m) = -[V'(\cdot)/\Gamma'(\cdot)U'(\cdot)](1+\rho)(1+\mu) \qquad (22)$$

Figure V illustrates (22) graphically. The properties of $A(\cdot)$ and $B(\cdot)$ stated above suggest that (22) satisfies the following restrictions:

f) $\Phi(0) = 0$

g) $\Phi'(m) \geq 0$ for $0 \leq m \leq e^*$

h) $\Phi'(m) = 0$ for $m \geq e^*$

i) $\Phi(m) = (1+\mu)(1+\rho)$ for $m \geq e^*$

A steady-state, $m(\mu)$, if it exists, is a solution to equation (23).

$$\Phi(m) = 1. \tag{23}$$

Restrictions (f)-(i) guarantee that there exists a $m(\mu)$, smaller than or equal to c^* which is a solution to (23) if

$$\mu > -\rho/(1+\rho) \tag{24}$$

Such a steady-state is represented by point a in figure V When (24) is violated, $\Phi(c^*)$ is smaller than one and the economy does not have a stationary equilibrium Hereafter, to avoid this difficulty, it is assumed that (24) is constantly satisfied

Graphically, it is easily verified that the higher is μ the lower is the steady-state level of real balances, $m(\mu)$ This result is also found by differentiating (23) with respect to μ [9]

$$dm/d\mu = -\left[(1+\mu)[(V''/V' - f''/f')h' - U''/U']\right]^{-1} < 0$$

In this model, the stationary equilibrium is uniquely determined by the growth rate of the money supply (the rate of inflation) The negative relationship between real balances and inflation is due to the effect of inflation on the labour-leisure choice Fiat money is not required to purchase leisure time Therefore, an increase in the steady-state rate of inflation (growth rate of the money supply) raises the price of consumption goods in terms of leisure time and induces individuals to substitute leisure time for consumption goods This substitution reduces the need for transaction balances

The rate of money growth that maximises welfare accross steady-state equilibria is given by Friedman's rule for the optimum quantity of money In the context of this model, this policy improves welfare because, by delivering a nominal rate of interest of zero, it removes the wedge between the private and social marginal rates of transformation of work effort for consumption

The steady-state equilibrium may also be represented as the intersection of $A(\cdot)$ and $B(\cdot)$ With this representation $m(\mu)$ falls with μ because $B(\cdot)$ rotates clockwise with higher value of μ See figure VI for example

Having determined the steady-state level of real balances, the values of the other endogenous variables fall from the arbitrage conditions as discussed above [10]

It is important to understand that the economy is in stationary equilibrium only when money has been growing at a constant rate and is expected to do so for the foreseeable future If μ is expected to vary over time, then the economy is not in a stationary equilibrium This is demonstrated in section IV and V by analysing the economy's adjustment to perfectly foreseen monetary policies Specifically, section IV studies the effects of a known future increase in the growth rate of the money supply, while section V looks at a known step increase in the level of the money supply

Before concluding this section, a few words should be said about the possibility for the model to have non convergent stationary cycling equilibria This may arise if the elasticity of marginal utility is greater than one, so that $B(\cdot)$ is downward sloping at the point where it crosses $A(\cdot)$ Under these circumstances, there may exist periodic equilibria of two or more periods For example, a periodic equilibrium of two periods would be characterised by two levels of m, let them be m' and m", such that

$$A(m') = B(m") \text{ and } A(m") = B(m')$$

If the economy was to start at either m' or m", then its level of income would forever alternate between these two values This particular situation is depicted in figure VII Equilibria with a number of periods higher than two are also possible and could be constructed similarly In the present essay, this type of equilibrium will be disregarded Interested readers are invited to consult Grandmont (1985) for a discussion of deterministic periodic equilibria in dynamic competitive economies

IV. Perfectly Foreseen Changes in the Growth Rate of the Money Supply.

Section III showed how unexpected and permanent changes in the rate of inflation (growth rate of the money supply) impact on the economy's stationary equilibrium by distorting the labour-leisure choice . In this and the next sections of the essay, the role of expected inflation is highlighted further by focusing on the effects of anticipated monetary policies on the economy's level of output, on the stock market, and on the nominal and real interest rates . A policy is anticipated if it is known to the private sector prior to its implementation. This will happen if for instance the Central Bank always announces new policies before putting them into effect. Accordingly, this section analyses the economy's adjustment after the announcement of a future change in the growth rate of the money supply, while section V studies the effects of the announcement of a prospective change in the level of the money supply.[11]

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Brock (1974, 1975) and Blanchard (1981) conduct similar experiments. The former studies the effects of a perfectly foreseen change in the growth rate of the money supply on the time path of the price level in a model where money is an argument of the representative agent's utility function. While the latter looks at the economy's adjustment to an anticipated change in the level of the money supply using a generalised IS-LM model. The analysis conducted here has much more in common with Brock's than Blanchard's because behaviors are also analysed within the construct of a flexible price general equilibrium model. The conclusion drawn differs however. The reason is that anticipated inflation impinges differently on the model's equilibrium.

The specific experiment undertaken in this section is the following Assume money has been growing at rate μ_0 and let the economy be in steady-state equilibrium Suddenly, at time (t), the government announces that beginning at time (T), the money supply will start to grow permanently at a faster rate μ_1, i e, each person's transfer payment is expected to grow at a faster rate after time period (T) This section studies the effects of this announcement on the economy

To evaluate the effects of this announcement on the equilibrium path of the economy, it is useful to separate conceptually the events according to whether they take place before or after the policy is instituted Firstly, from the perspective of time period (T), the cost of holding money is permanently higher because the money supply is expected to grow at a higher rate μ_1 Therefore, in time period (T), the economy should be in steady-state equilibrium with a lower level of real balances than before period (t) Secondly, over the interval running from time (t) to time (T), the consumer/worker is aware that at (T), the price level will be higher than otherwise, in order to avoid a capital loss on his money holdings, he will want to modify his spending decisions well before period (T) This puts pressure on the goods market and makes the price level higher than otherwise even before the implementation of the policy As a result, between the announcement and the implementation of the new policy real balances have to fall toward the new steady-state level

The exact nature of this adjustment path depends on η, the elasticity of marginal utility. Specifically, the economy converges monotonically to its new steady-state when η is smaller than one. It cycles around the old steady-state in reaching the new one when η is greater than one, and it reaches the new steady-state in one jump when η equals one.

This is demonstrated formally with the help of equilibrium condition (21). During the adjustment period, (21) must always be satisfied; this means that the equilibrium path of real balances must solve the following system of equations:

$$A(m_t) = [\beta/(1+\mu_0)]U'(m_{t+1})m_{t+1}$$

$$\cdots\cdots\cdots\cdots\cdots$$

$$A(m_{T-1}) = [\beta/(1+\mu_0)]U'(m_T)m_T \qquad (25)$$

$$A(m_T) = [\beta/(1+\mu_1)]U'(m_{T+1})m_{T+1}$$

$$m_s = m(\mu_1) < m(\mu_0) \qquad s \geq T$$

where $m(\mu_0)$ and $m(\mu_1)$ are the steady-state levels of real balances when money supply grows at rate μ_0 and μ_1 respectively. In accordance with the assumption of stability, observe that system (25) constrains real balances to converge to the new steady-state level in period (T).

The time path of real balances which satisfies (25) before the new policy is put into effect is found by working backward from period (T). Satisfaction of (25), requires that $A(m_T)$ be smaller than $A(m_{T-1})$ given that $\mu_1 > \mu_0$ and $m_{T+1} = m_T = m(\mu_1)$. This means that m_{T-1} has to be bigger than m_T since $A(\cdot)$ is an increasing function of m. Then proceeding by induction knowing that over the interval $[0, c^*]$, $B(\cdot)$ is an increasing or a decreasing function of m depending whether η is smaller or greater than one, it follows that

a) when $\eta < 1$, then

$$m_t > m_{t+1} > \ldots > m_{T-2} > m_{T-1} > m_T$$

b) when $\eta > 1$, then

$$m_T < m(\mu_0) < m_{T-1}$$

$$m_{T-1} > m(\mu_0) > m_{T-2}$$

. . . .

c) when $\eta = 1$, then

$$m_j = m(\mu_0) > m(\mu_1) \quad \text{for} \quad t \leq j \leq T-1$$

$$m_j = m(\mu_1) \quad \text{for} \quad j \geq T$$

It is important to understand that the only unexpected change in the price level, or equivalently in the level of real balances, occurs at the announcement of the new policy. At this time, m_t has to jump unexpectedly to a level consistent with (25). Thereafter, the behavior of the price level is perfectly foreseen by the consumer/worker, and the evolution of real balances involves no surprises to him. Figures VIII-XI illustrate the possible adjustment paths when (T) equals (t+2).

In each case, the economy is in steady-state -- point a -- until period (t-1). At the announcement of the new policy, in period (t), real balances jump to level m_t. This puts the economy right on the trajectory reaching the new steady-state -- point b -- at the time of the implementation of the policy.

Three characteristics of the adjustment path emerge from figures VIII-XI. (1) Most of the adjustment of real balances occurs before money starts growing at a faster rate. So, inflation may lead monetary growth even though the latter is the ultimate cause of inflation. This results from the intertemporal substitution of leisure for consumption created by the expectation of a higher inflation rate. Recall that in general equilibrium, the consumer/worker can lower his money holdings only by reducing his labour supply. (2) The proposition often made in monetary economics that the further away in the future is the implementation of a perfectly foreseen change in monetary growth, the smaller is the unanticipated jump occuring at the time the change in perception about future monetary policy takes place, will not hold in all cases here.[12] See for example Figure X. The validity of this proposition depends on whether the non-linear difference equation (21) is stable backward or forward The standard result applies when (21) is stable forward, while it is violated when (21) is stable backward In the latter case, any m_0 different from $m(\mu)$ puts the economy on a path that converges asymptotically to $m(\mu)$, so that the more distant is the implementation of the new policy, the bigger is the unanticipated jump occuring at time (t) Obviously, the stability of (21) depends on the characteristics of functions U(·), V(·) and f(·) Appendix A linearises equation (21) around the stationary state and shows that (21) is unstable whenever

$$\eta > 2 + (1/\sigma)(\lambda+\varepsilon)$$

the elasticity of marginal utility satisfies the above condition where σ, λ and ε are the labour elasticities of f(·), V'(·) and f'(·) respectively (3) As demonstrated in figure XI, if the elasticity of marginal utility equals one, then the economy's response to a perfectly foreseen change in the growth rate of the money supply is identical to its response when this policy is not anticipated In this situation, new policies affect the economy only after they have been put into effect This characteristic is interesting given that the logarithmic form for the utility function, U(e) = *log*(e) is widely used in both, theoretical and empirical studies.[18]

The effects of this policy on the model's endogenous variables fall directly from the behavior of real balances Here attention will center mainly on the time paths of r_t, π_t and q_t

Along the adjustment path, the real rate of interest is negatively correlated with the anticipated rate of inflation This is discovered by considering the following expressions for r_t and π_t

$$r_t = (i_t - \pi_t)/(1+\pi_t) = (1+\rho)U'(\cdot_t)/U'(\cdot_{t+1}) - 1 \tag{26}$$

$$\pi_t = (P_{t+1}/P_t) - 1 = (1+\mu)(e_t/e_{t+1}) - 1 \tag{27}$$

From (27) it is evident that the anticipated rate of inflation varies positively with e_t/e_{t+1}, while from (26) the real rate of interest varies negatively with e_t/e_{t+1} Consequently, along the adjustment path, when the growth rate of the money supply is still unchanged, upward movements in the anticipated rate of inflation are associated with downward movements in the real rate of interest.

The intuition here goes as follows When tomorrow's level of consumption is low relative to today's consumption, the economy anticipates a higher rate of inflation than would be the case if consumption was the same in each period Because the economy prefers a smooth consumption path, consumers will try to purchase tradeable assets to substitute consumption away from today and toward tomorrow. However, this is not possible in the aggregate because the economy's output is a perishable good Therefore, the real rate of interest has to fall to curtail desired savings

This association between π and r holds only outside the steady-state It is easily seen from (26) that r is unaffected by variations in the steady-state anticipated rate of inflation Nonetheless, by allowing the economy to be outside of its steady-state, the model predicts that on average the economy's expected real return on equity will be negatively correlated with variations of the anticipated rate of inflation It should be noted that empirical work by Fama and others has established the existence of such a correlation between the return on equity and the anticipated rate of inflation[14]

Even if r_t and π_t move in opposite directions, it is still possible to determine what happens to the nominal rate of interest, i_t, during the adjustment period This is because (18)-(20) imply that i_t and m_t are inversely related to one another This permits to conclude that the nominal rate of interest must be rising (falling) whenever real balances are falling (rising)

The response of equity prices to the announcement of this new policy critically depends on the value of the elasticity of marginal utility On the one hand, if the elasticity of marginal utility is smaller than or equal to one, the announcement of the policy produces an unexpected drop in equity prices followed by a gradual depreciation which ends with the implementation of the new policy in period (T). The unexpected drop in q_t results from the re-evaluation of expectations about future inflation rates and about the value of future dividends, while the gradual depreciation of q_t occurs because during the adjustment period the economy moves slowly from a state of high dividend payments to a state of low dividend payments.

This may be shown formally as follows. Rewrite the asset pricing function (15)' as

$$q_t = [1/U'(\cdot_t)]\sum_{j=t}^{\infty}\beta^{j-t}s_j \qquad (15')$$

where,

$$s_j = \beta U'(\cdot_{j+1})(P_j/P_{j+1})D_j$$

Lead (15') one period, and substract the resulting expression from (18') to obtain[16]

$$q_t - q_{t+1} = [1/U'(\cdot t)]\sum_{j=t}^{\infty}\beta^{j-t}[s_j - s_{j+1}U'(\cdot_t)/U'(\cdot_{t+1})]$$

To determine whether the above expression is positive or negative, we have to evaluate the signs of $s_j - s_{j+1}U'(c_t)/U'(c_{t+1})$ for all j's. To do so, note that along a declining consumption path, $U'(c_t)/U'(c_{t+1})$ is smaller than or equal to one if $U''(\cdot_t) < 0$. Under those circumstances, a sufficient condition for the market value of the representative firm to be continuously falling during the adjustment period is that s_j be greater than s_{j+1} for $t \leq j \leq T-1$

Using (19) together with (21), (P_j/P_{j+1}) can be expressed as

$$(P_j/P_{j+1}) = [1/(1+\mu)]m_{j+1}/m_j = -(1/\beta)[V'(h(m_j))/f'(h(m_j))][1/U'(m_{j+1})] \qquad (28)$$

Substituting (5) and (28) into the above expression for s_j allows to rewrite s_j in terms of m_j only That is,

$$s_j = -V'[h(m_j)]/f'[h(m_j)]\Big[f[h(m_j)] - f'[h(m_j)]h(m_j)\Big]$$

The assumptions made on $V(\cdot)$ and $f(\cdot)$ guarantee that s is an increasing function of m Therefore, $s_j < s_{j+1}$ for $t \leq j \leq T-1$, given that m_j is falling over that interval

On the other hand, if the elasticity of marginal utility is greater than one, what happens to q_t at the announcement of the policy and during the adjustment period is uncertain This is because over this particular interval, some of the dividend payments are now higher than before and may make the representative firm more valuable For this reason, the time path of q_t cannot be ascertained precisely although it is still true that q_t has to fall to its lower steady-state level at some point during the adjustment period

Finally whether $Q_t = P_t q_t$, the nominal value of the firm's equity, moves in the same direction as q_t is also uncertain The reason is that during the adjustment period the price level is moving in the opposite direction to q_t and may easily outweight the fall in the latter

This concludes the analysis of the effects of a perfectly foreseen future change in the growth rate of the money supply The next section of the essay studies the economy's adjustment to a perfectly foreseen change in the level of the money supply

V. Perfectly Foreseen Changes in the Level of the Money Supply.

The analysis presented here proceeds as in section IV Let the economy be in stationary equilibrium Without loss of generality assume that the money stock is not growing, i e $\mu = 0$ Suppose that at time (t), the government announces that between (T-1) and (T), the money stock will be expanded by a factor of $(1+\mu)$ and will be maintained constant thereafter

A change in the level of the money supply does not affect the economy's stationary equilibrium This is because the stationary state, for given preferences and technology, is uniquely determined by the growth rate of the money supply For this reason, once the increase in the money supply has taken place, the economy should be in the same stationary equilibrium than before the announcement of the policy

However, this does not mean that the policy is without transitory real effects Before period (T), the consumer/worker knows that he will suffer at time (T) a capital loss on his money balances To minimise this cost, he will want to alter his previously planned money holdings, putting pressure in the goods market and making the price level higher than otherwise even before the money stock is increased This scenario may be unfolded using equilibrium condition (21) modified as in (29) for a step increase in the money supply occuring in period (T)

$$A(m_t) = \beta U'(m_{t+1})m_{t+1}$$

$$\cdots\cdots\cdots\cdots\cdots\cdots$$

$$A(m_{T-1}) = [\beta/(1+\mu)]U'(m_T) \tag{29}$$

$$A(m_T) = \beta U'(m_{T+1})m_{T+1}$$

$$m_j = m(0) \text{ for } j \geq T$$

The path of real balances which solves (29) is found by working backward in time from period (T) The fact that m_T equals $m(0)$ and that $A(\cdot)$ is an increasing function of m permits to certify that m_{T-1} must be smaller than $m(0)$ Prior to period (T-1), the characteristics of the equilibrium path depends, as in section IV, on η, the elasticity of marginal utility For this reason, the reader is invited to use the argument developed in section IV to verify that real balances fall monotonically toward m_{T-1} when η is smaller than one, cycle around $m(0)$ in attaining m_{T-1} when η is greater than one, and stay constant at $m(0)$ until period (T-1) when η equals one

An illustration of the economy's adjustment when $\eta < 1$ is described in figure XII. At the announcement of the policy, real balances fall to m_t putting the economy right on the trajectory reaching a level of real balances such that in period (T-1),

$$A(m_{T-1}) = [\beta/(1+\mu)]U'[m(0)]m(0)$$

At time (T), when the money supply is increased, the economy goes back to its original equilibrium. This is because the growth rate of the money supply is expected to stay constant thereafter.

As is clear by now, the economy's adjustment to a perfectly foreseen change in the level of M is very similar to its adjustment to a perfectly foreseen change in the growth rate of the money supply. Qualitatively, the economy's behavior before the implementation is the same under both circumstances. Therefore, most of what was said about the dynamics of the economy in section IV also applies here.

The major distinctions between these policies arise toward the end of the adjustment period and at the time of implementation of the new policy. In the case of a change in the level of the money supply, the economy bounces back to its original equilibrium at the time of the implementation of the new policy. Consequently, in period (T-1), the real rate of interest is above ρ, the consumer/worker's rate of time preference. This results from the unsuccessful attempt made by the consumer to substitute consumption away from period (T) The stock market may also behave differently It is possible, even when $\eta < 1$, that the value of the firm's equity starts to increase near the end of the adjustment period because market participants anticipate a capital gain on equity between periods (T-1) and (T) Nevertheless, during the adjustment, when $\eta < 1$, q_t must always be below its stationary value There are two reasons for this to be the case First, during the adjustment period, dividends are lower than their steady-state value Second, the dividends paid in the future, once the economy has returned to its stationary state, are discounted by a higher discount factor

Finally, it is important to realize that a change in the level of money has some short-run real effects only when it is anticipated in advance If it was unexpected instead, it would have no effect on the economy's equilibrium except for the fall of the ex-post real return on bonds, money and equity

VI. Discussion of Related Issues

The apparatus employed in sections IV and V is also useful to analyse the effects of other prospective changes in the economic environment For instance, it could be used to investigate the effects of future changes in (1) the marginal product of labour, (2) labour productivity that does not affect the marginal product of labour, (3) the tax rate on labour income, (4) the discount factor

As an example, consider the case of a future change in the marginal product of labour. To accommodate this situation, assume that the production function depends on time in the following manner,

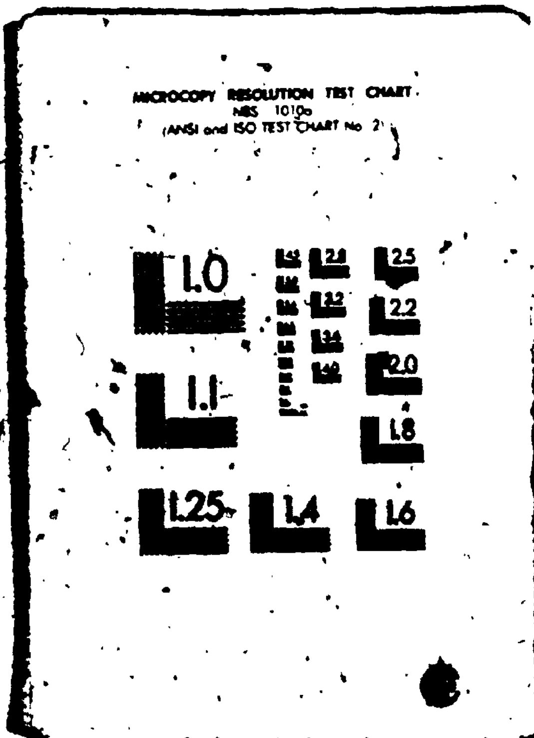
$$F_t(\ell_t) = k_t f(\ell_t)$$

where k_t is a shift parameter. To start with, suppose k_t is constant through time, i.e., the environment is the same as in sections IV and V. At time (t), it is discovered that k will be permanently higher after some future period (T).

This increase in the future marginal product of labour makes A(·) rotate clockwise around the origin and increases the steady-state level of real balances after period (T). The improvement in the future economic outlook also influences the equilibrium between periods (t) and (T) because the effective real wage varies with the anticipated rate of inflation. For instance, if η is smaller than one, employment and output will be rising until period (T). In addition, the rate of inflation and the nominal rate of interest will be lower than otherwise, while the real rate of interest will be higher. Figure XIII illustrates the adjustment of real balances to this future shock. Similar arguments could be applied to determine the impacts of (2), (3) and (4).[17]

This last example highlights an important characteristic of

2 OF/DE 2


MICROCOPY RESOLUTION TEST CHART
NBS 1010a
(ANSI and ISO TEST CHART No. 2)
1.0
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2.0
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2.5

the institution of money in this economy It is the mechanism through which future disturbances can have spillover effects on the current choices of work and production. Without the requirement that money be used as a medium of exchange, there would be no link between real activity in adjacent periods, and an anticipated future change in the economic environment could not affect the economy before it takes place

The analysis presented in sections IV and V is carried out under the assumptions that preferences are additively separable and that the elasticity of marginal utility is independent of the level of consumption The dynamics of the economy could be modified considerably by relaxing either of these assumptions. Without them, the B(·) function would not be necessarily monotonic, and the economy may have more than one steady-state level of real balances. With many steady-states, it is not possible to determine which one the economy will converge to after a modification in the perceived future economic environment. For the same reason, for some level of m_t, more than one level of m_{t+1} would solve (21) when the economy is not in stationary state. See Figure XIV

Under those circumstances, it becomes evident that it is not possible to determine the economy's response to a given anticipated future shock

VII. Conclusion.

This essay uses a simple intertemporal general equilibrium model with money introduced via a cash-in-advance constraint to analyse the effects of perfectly foreseen changes in the growth rate and in the level of the money supply The essay also indicates how the same model can be used to study anticipated future changes which are real rather than monetary in nature

Anticipated monetary policies have real effects in this model because of the existence of a distortion in the labour market that is introduced by the requirement to use money as a medium of exchange Basically, an increase in the anticipated rate of inflation reduces the return to labour and induces a substitution away from time spent in the labour market Consequently, prospective monetary policies have real effects by influencing inflationary expectations

The following conclusions emerge from this exercise First, once a policy is anticipated, most of the required adjustment occurs before the policy is actually implemented Second, the adjustment path between the announcement and the implementation of the new policy depends on the elasticity of marginal utility Third, along the adjustment path, the real rate of interest and the anticipated rate of inflation are negatively correlated Fourth, if a change in the level of the money supply is interpreted as a transitory change in the growth rate of the money supply, then it may be said that the time paths of the economy before the implementation of known permanent or transitory changes in the growth rate of the money supply are qualitatively the same.

These conclusions depend on the assumptions of constant elasticity of marginal utility and of additively separable preferences. Section VI briefly discusses what would happen if one was to relax either of these assumptions.

FIGURE I

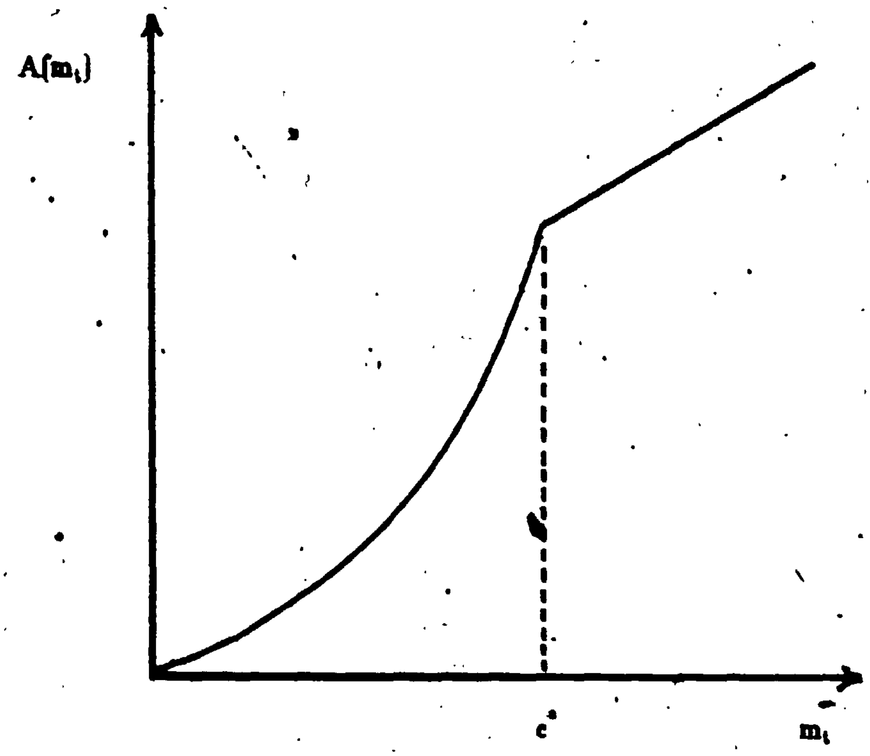


FIGURE II

$\eta < 1$

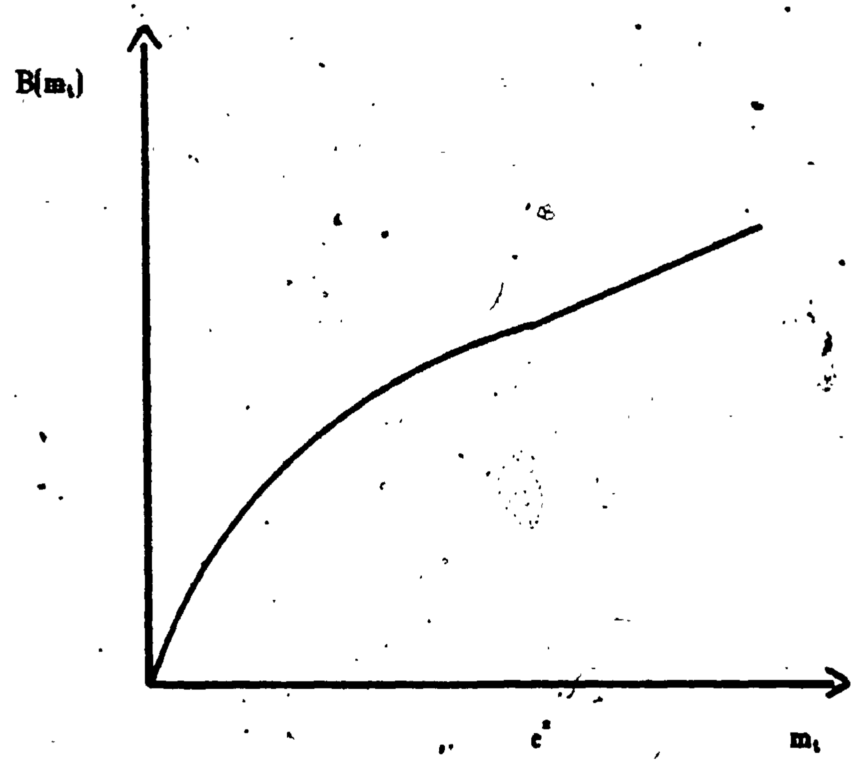


FIGURE III

$\eta > 1$

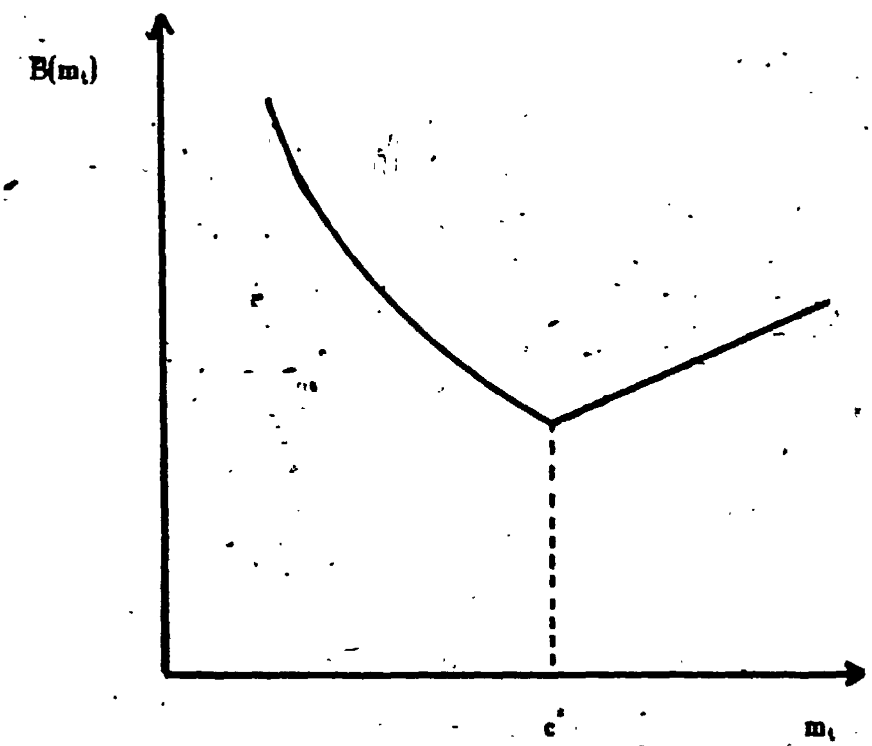


FIGURE IV

$\eta = 1$

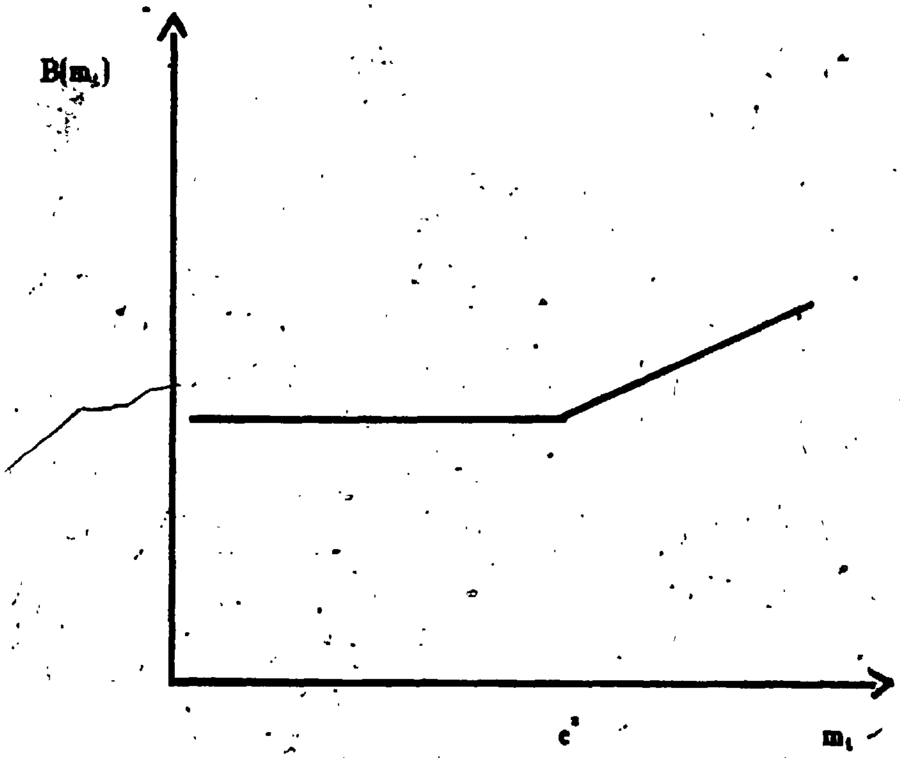


FIGURE V

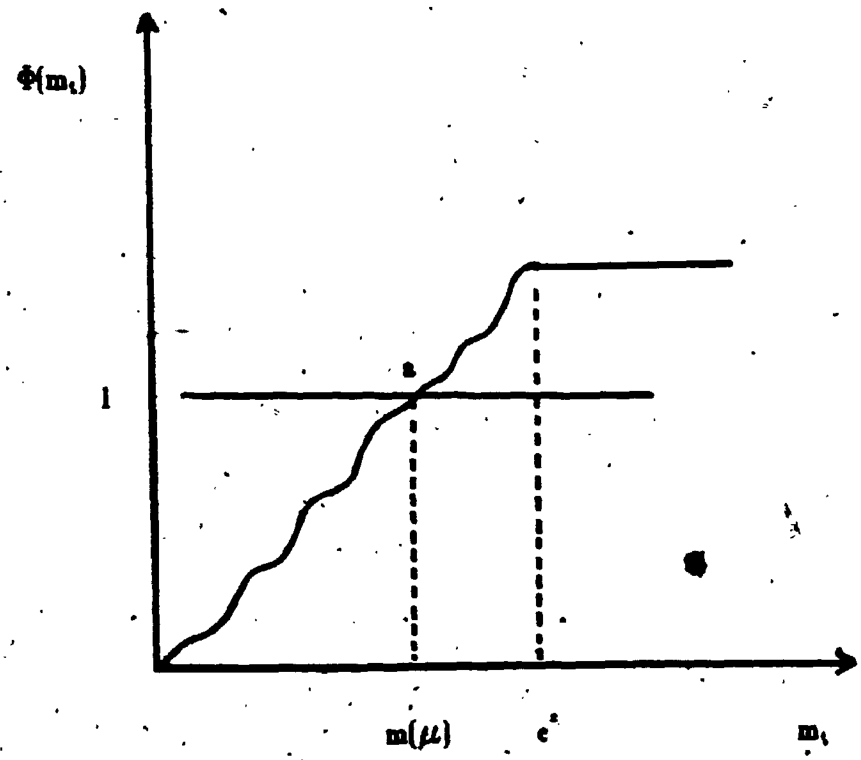


FIGURE VI

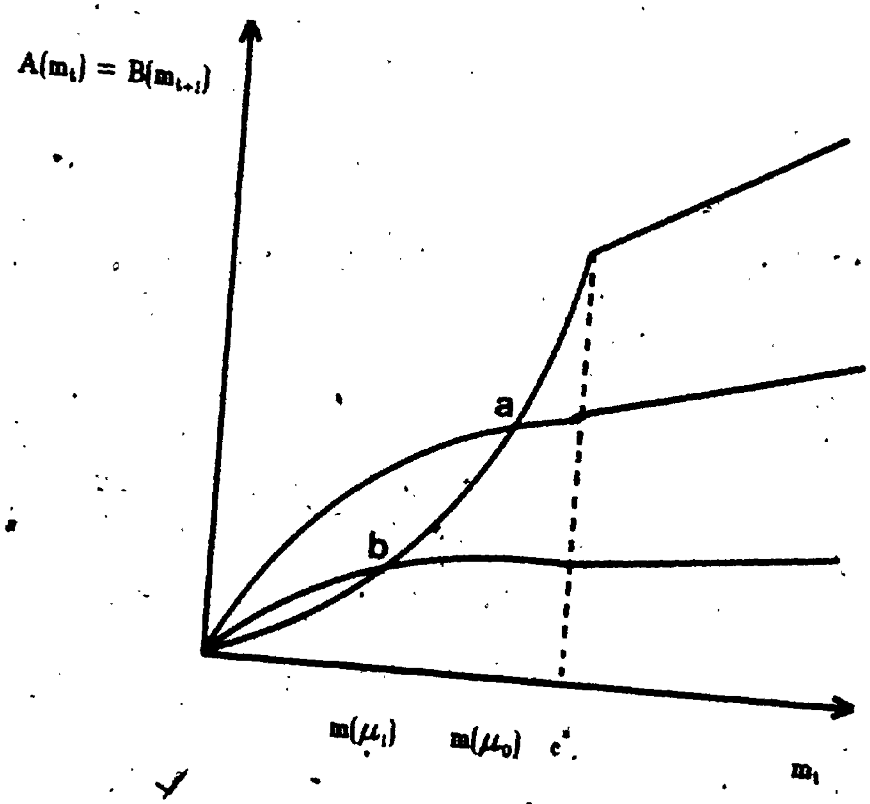


FIGURE VII

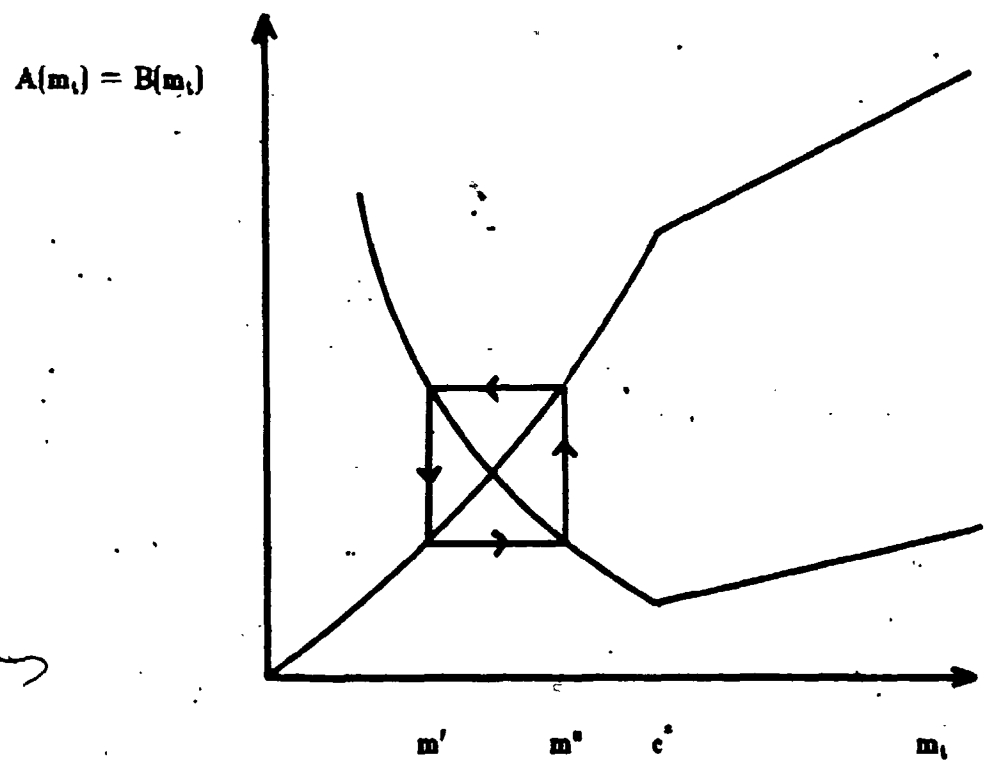


FIGURE VIII

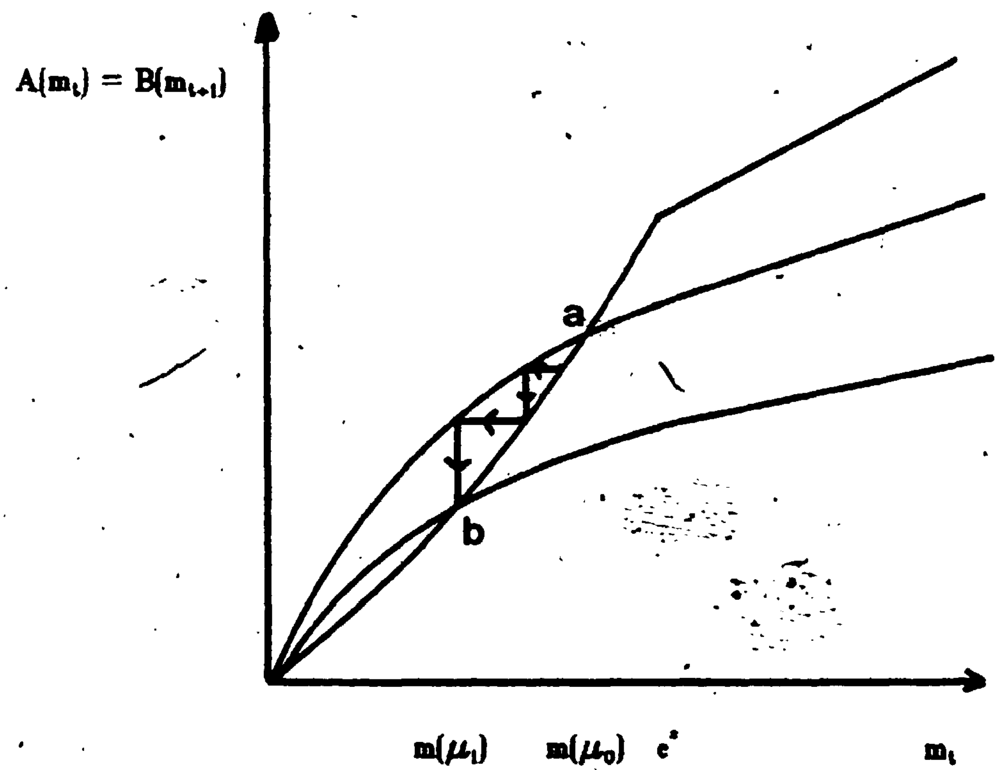


FIGURE IX

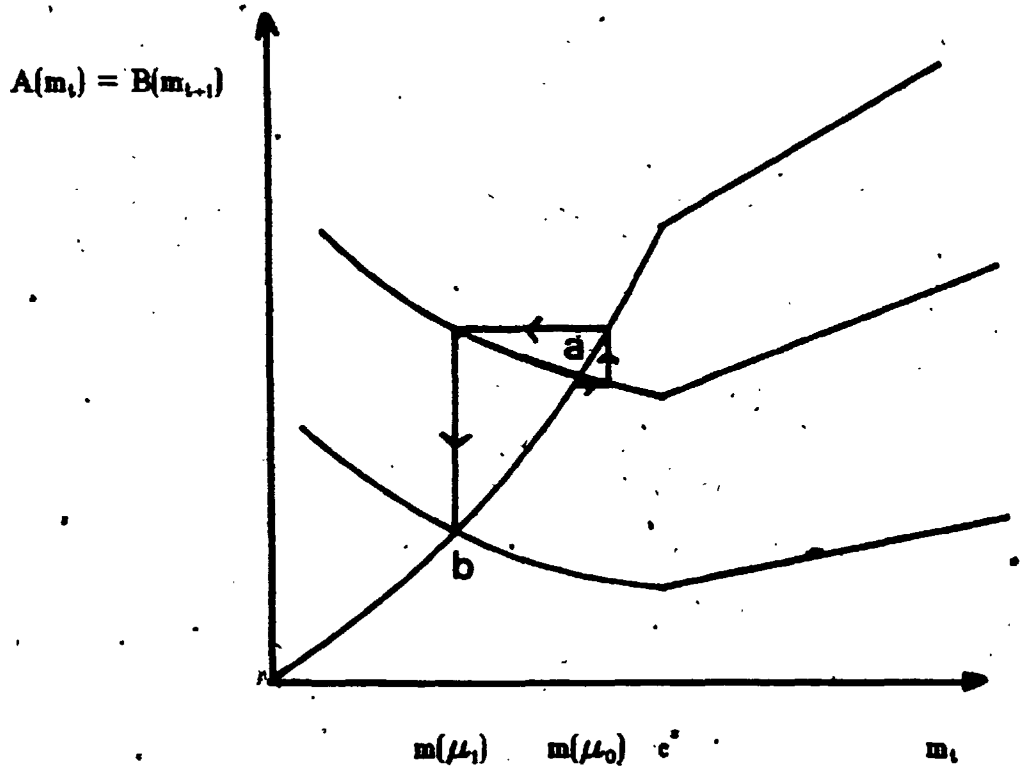


FIGURE X

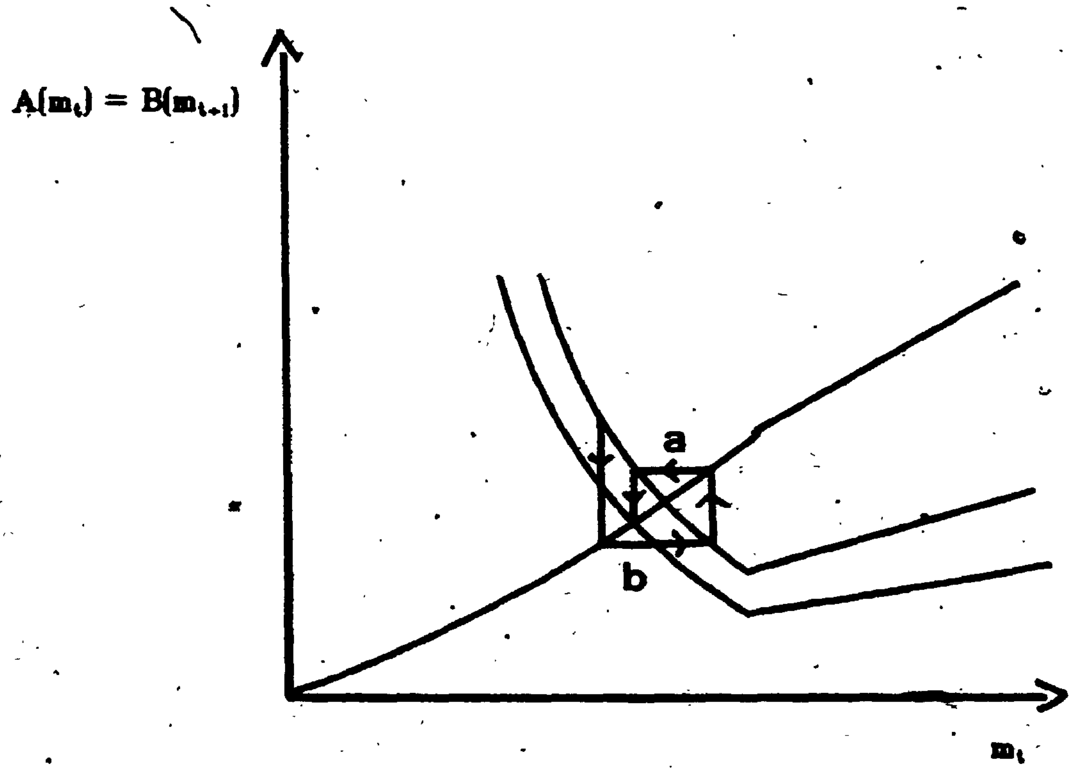


FIGURE XI

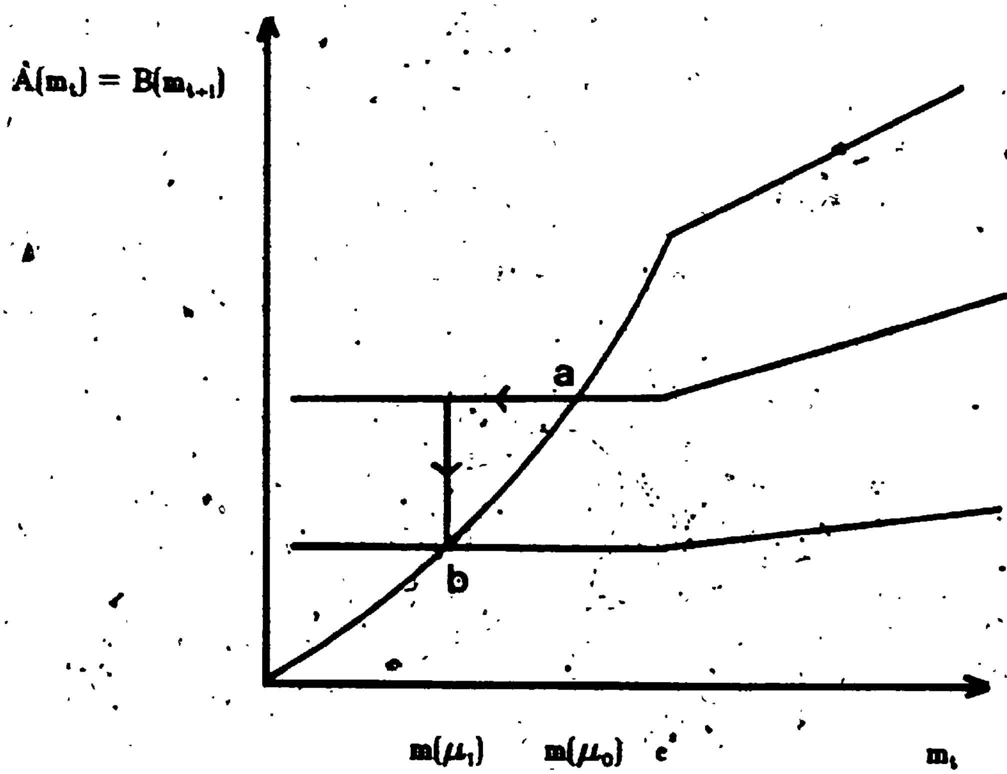


FIGURE XII

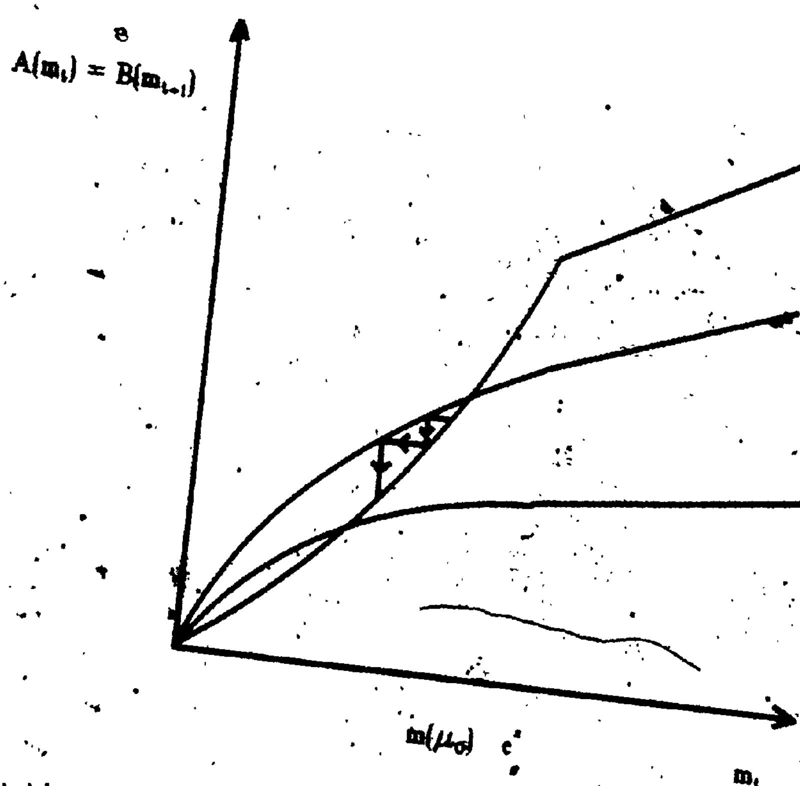


FIGURE XIII

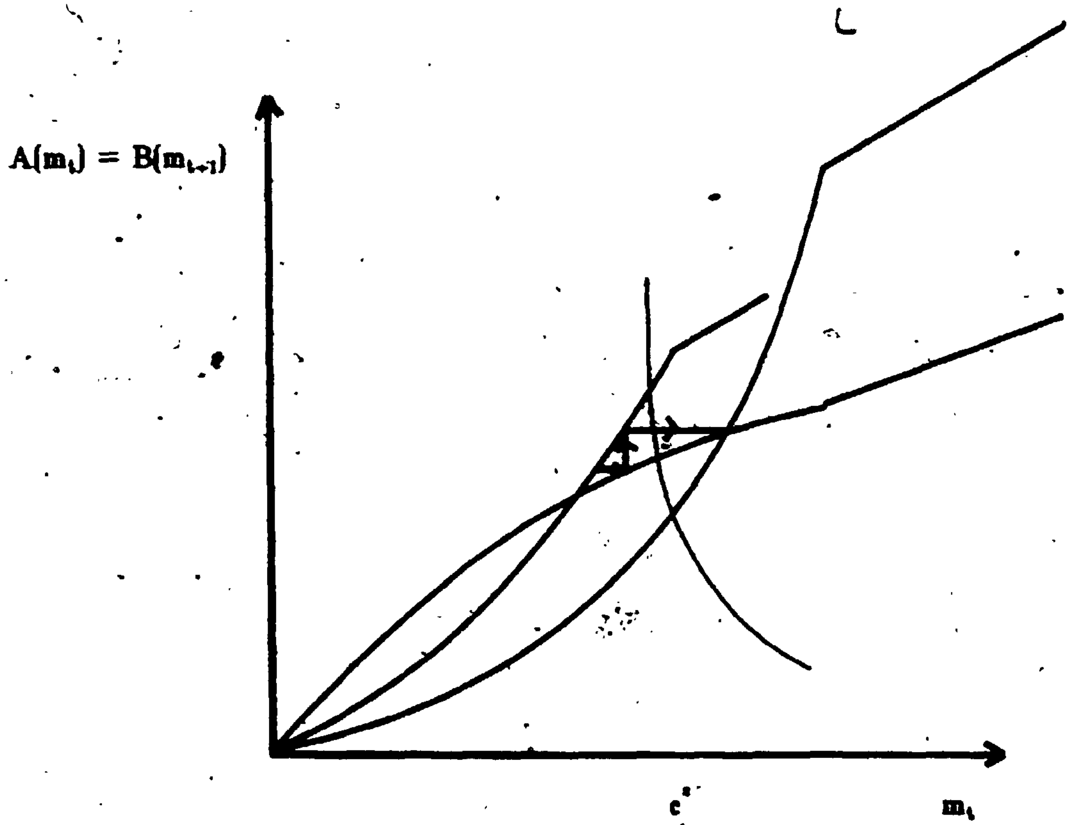
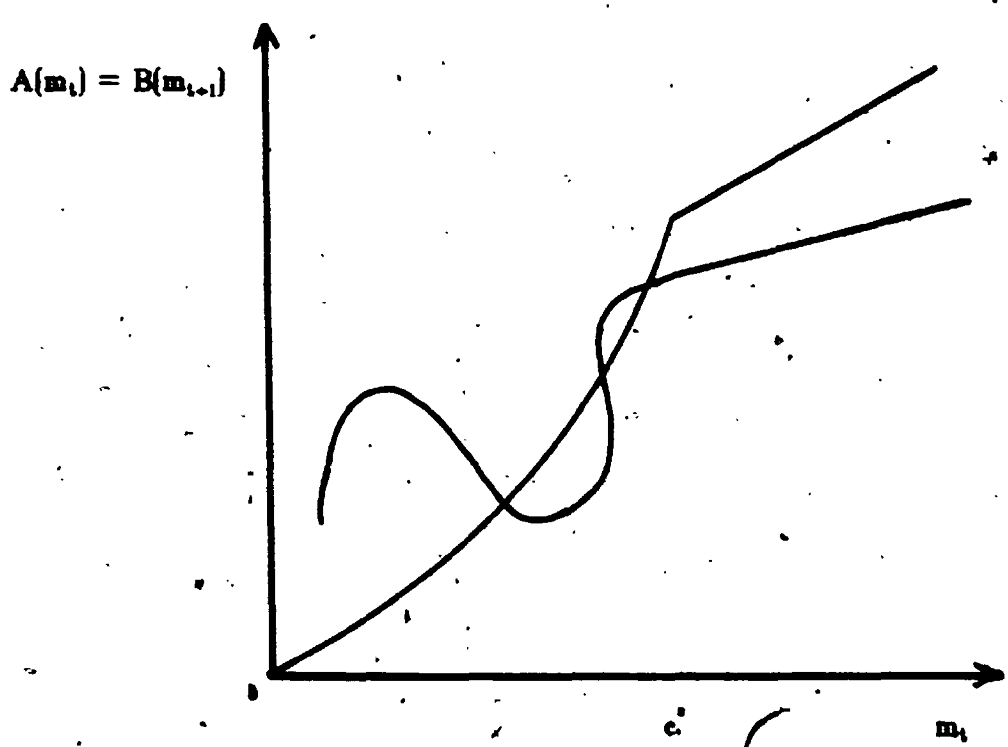


FIGURE XIV



FOOTNOTES

[1]Kohn (1984) surveys the growing literature that motivates money demand with a cash-in-advance constraint

[2]This notion that current income cannot finance current expenditure can be traced back in the literature to Robertson and Tsiang (1956)

[3]Observe that before the goods market open, the individual can always issue new bonds to replenish his money holdings Thus, the bonds market may be seen as a place where cash-balances are redistributed efficiently among economic agents

[4]The assumption of additivity of preferences is crucial for some of the conclusions reached below Section VI discusses briefly what happens once it is relaxed

[5]The reader is warned that speculative equilibria are disregarded

[6]This point can be illustrated more clearly as follows Suppose that in addition to providing liquidity services, money earns a pecuniary return of a_t percent per period In this case condition (11) must be replaced by

$$\alpha_t(1/P_t) = (1+a_t)\beta U'(\cdot_{t+1})(1/P_{t+1}) \qquad (11')$$

which takes proper account of the fact that holding money pays interest Portfolio equilibrium between money and bonds now implies that

$$\gamma_t(1/P_t) = (i_t - s_t)\beta U'(\cdot_{t+1})(1/P_{t+1}) \quad (17')$$

Condition (17') demonstrates clearly that the restriction that γ_t be greater than zero means that $i_t > s_t$. Observe that theoritically, it is possible for the nominal rate of interest to be negative if s_t is smaller than zero.

[7]Under the condition illustrated in footnote six the liquidity constraint will be binding whenever $i_t > s_t$.

[8]The wedge $[R_t - 1]f'(\ell_t)$ between the private marginal rate of transformation of work effort for consumption, and the rate socially attainable, given by the marginal product of labour $f'(\ell_t)$, arises from the requirement to use money as a medium of exchange. See Wilson (1979) and Aschauer & Greenwood (1983) for a further discussion of this point.

[9]This result does not depend on the fact that $U(\cdot)$ is additively separable. See Aschauer & Greenwood.

[10]Carmichael (1985) analyses the effects of inflation on stock market prices in stationary equilibrium.

[11]Observe that because the analysis is in discrete time, there is no distinction between a change in the level of the money supply and a transitory change in the growth rate of the money supply.

[12]Alternatively, for (T-t) given, the higher is μ_1 relative to μ_0, the bigger is the unanticipated jump occuring at time (t).

[13]This means that contrary to what Helpman and Razin (1982) claim, anticipated monetary policy would not have any real effects in their model

[14]Fama also reported a negative correlation between stock returns and the unanticipated rate of inflation

[15]If the elasticity of marginal utility equals one, this expression reduces to,

$$q_t - q_{t+1} = [1/U'(\cdot_t)]\beta^{T-1-t}(s_{T-1} - s_T) < 0 \quad t < T-2$$

$$q_{T-1} - q_T = [1/U'(\cdot_{T-1})]\left[s_{T-1} - s_T[U'(\cdot_{T-1})/U'(\cdot_T)]\right] < 0.$$

[16]To accommodate a change in the future tax on labour income, it is necessary to modify the budget constraints of the consumer/worker and of the government This can be done by replacing the real wage in (2) by the after tax real wage, $[1-\tau][W_t/P_t]$, and by re-writing the government budget constraint as,

$$\Gamma_t = \tau W_{t-1} + \mu M_{t-1},$$

where Γ_t is still perceived as lump-sum.

APPENDIX A

The purpose of this appendix is to demonstrate under what condition equation (21) is stable forward This may be found by linearising (21) around its stationary state This gives rise to

$$(1+\mu)(V'/f')[1+(1/f'\ell)(V''\ell/V'-f''\ell/f')]\Delta m_t+(V'/f')m\Delta\mu_t+ \qquad \text{(A1)}$$

$$(1-\eta)\beta U'\Delta m_{t+1}+\beta U'm+(1+\mu)(V'/f')m$$

where all the functions are evaluated at their stationary equilibrium values and where,

$$\Delta m_t = m_t - m$$

$$\Delta\mu_t = \mu_t - \mu$$

Recall that in stationary equilibrium the following equations must always hold,

$$\beta U'm + (1+\mu)(V'/f')m = 0$$

$$\beta U' = -(V'/f')(1+\mu)$$

Consequently, equation (A1) can be simplified to,

$$(1-\eta)\Delta m_{t+1}-[1+(1/f'\ell)(V''\ell/V'-f''\ell/f')]\Delta m_t-(m/(1+\mu))\Delta\mu_t = 0$$

Now define σ, λ and ε as the labour elasticities of $f(\cdot)$, $V'(\cdot)$ and $f'(\cdot)$ evaluated at $\ell = h(m)$. That is,

$$\sigma = r\ell / r$$

$$\lambda = \underline{v\ell} / V'$$

$$\varepsilon = - r\ell / r$$

The dynamics of real balances can be simplified further to

$$(1-\eta)\Delta m_{t+1} - (1+(1/\sigma)(\lambda+\varepsilon))\Delta m_t - (\dot{m}/(1+\mu))\Delta\mu_t = 0. \qquad \text{(A2)}$$

Expression (A2) highlights that in the neighbourhood of the stationary state, the dynamics of real balances depend on η, μ, σ, λ, and ε. More importantly, (A2) shows that the economy is stable forward only if:

$$\eta < 2 + (1/\sigma)(\lambda+\varepsilon)$$

CHAPTER 4: Self–Fulfilling Expectations in a Clower Cash–in–advance Economy.

I. INTRODUCTION.

This essay is concerned with the uniqueness of a monetary equilibrium in a dynamic competitive economy where money is motivated through a Clower cash-in-advance constraint The essay analyses whether the practice of disregarding the unstable equilibrium paths emerging from these models can be rationalized in terms of optimizing behavior Scheinkman (1980) studies this problem using a variant of Clower's model Specifically, he shows that in an exchange economy where consumption goods can be acquired through barter or by holding money, unstable equilibrium paths fuelled by speculative anticipations can be the outcome of optimizing behavior. He concludes that unless money is made essential, self-fulfilling expectations will be supported by a model that motivates money through the Clower constraint. Here money is made essential in the sense that it is required in all market transactions. Contrary to Scheinkman, consumption goods cannot be acquired through barter It is shown that, under standard conditions, this more stringent version of Clower's model can also support equilibria with self-fulfillling expectations. These equilibria are seen to lead asymptotically to either a collapse of the monetary economy or the accumulation of an infinite level of real balances.

In addition, the essay states a restriction on the traders' utility function that can be used to rule out all the explosive price level paths This restriction is formally identical to the one proposed in Scheinkman (1980) for overlapping generation models and is analogous to the one that must hold in models where the liquidity services of money are modelled by putting real balances in the utility function.

The essay proceeds in the following manner An introduction is made in the first section. The second section describes the optimization problem solved by economic agents and gives the definition of a monetary perfect foresight equilibrium. The third section states the equilibrium conditions and derives an expression for the dynamics of the economy The fourth section shows that, under standard restrictions on preferences, the model can support an infinite number of equilibrium paths. The fifth section discusses a condition on preferences that permits to eliminate the equilibrium paths leading to a collapse of the monetary economy Finally, the conclusions are drawn in section six

II. The Model.

Consider an economy inhabited by one representative household blessed with perfect foresight that maximizes its lifetime utility:

$$\mathcal{U} = \sum_{t=0}^{\infty}\beta^{t}[U(c_t) + V(\ell_t)] \tag{1}$$

Here β is a subjective discount factor, c_t and ℓ_t are period (t)'s consumption and work effort. The within-period utility function is assumed additively separable and obeys the following conditions $U'(\cdot) \geq 0$, $U''(\cdot) \leq 0$, $V'(\cdot) \leq 0$, $V''(\cdot) \leq 0$, $Lim_{c\to 0}U'(c) = \infty$, $Lim_{c\to\infty}U'(c) = 0$, $Lim_{\ell\to 0}V'(\ell) = 0$.

The representative household has two income sources. It collects the benefits from the operation of a firm that uses its labour services as a factor input. It also receives a lump-sum nominal transfer payment, Γ_t, from the monetary authorities. The technology available for production is described by $f(\ell)$, a production function that exhibits positive but decreasing marginal product of labour, with $f(0) = 0$, $f'(0) = \infty$ and $Lim_{\ell\to\infty}f'(\ell) = 0$

Each period, the representative household optimally allocates its resources between consumption, labour supply and end-of-period money holdings, M_t. These choices must satisfy a budget and a liquidity constraint. These are

$$M_t/P_t + c_t \leq [M_{t-1} + \Gamma_t]/P_t + f(\ell_t) \qquad t = 0, \ldots, \infty \qquad (2)$$

$$c_t \leq [M_{t-1} + \Gamma_t]/P_t \qquad t = 0, \ldots, \infty \qquad (3)$$

where P_t is the price level in time period (t). Budget constraint (2) highlights that for the individual, money is the only asset that permits to transfer resources accross different time periods, while liquidity constraint (3) stresses that consumption purchases are bounded by the post transfer level of cash balances. This modelling of money as a medium of exchange is also invoked in Aschauer & Greenwood (1983), Carmichael (1985), Helpman & Razin (1982, 1984), Lucas (1980, 1982) and Stockman (1981). Observe that contrary to Scheinkman (1980), here consumption goods may be acquired only in exchange of money balances.

Government intervention in the economy is made through lump-sum nominal transfer payments to the representative household. The government finances this expenditure entirely by printing new money. Furthermore, for analytical simplicity, the analysis is restricted to the case where transfer payments are such that the money stock, $M_{t,s}$, grows at a constant rate μ. With this assumption, the government budget constraint is simply

$$\Gamma_t = \mu M_{t-1,s} \qquad t = 0, \ldots, \infty \tag{4}$$

where $M_{t-1,s}$ is the money stock outstanding in period (t-1).

The derivation of the main results of the essay is greatly simplified when the consumer's problem is reformulated as follows:

Choose

$$\{c_t, \ell_t, N_t\}_{t=0}^{\infty}$$

to maximize

$$\mathcal{U} = \sum_{t=0}^{\infty}\beta^t[U(c_t) + V(\ell_t)] \tag{1}$$

subject to

$$c_t + N_t/P_t \leq N_{t-1}/P_t + \Gamma_t/P_t + (P_{t-1}/P_t)f(\ell_{t-1})$$

$$c_t \geq 0,\ \ell_t \geq 0,\ N_t \geq 0 \qquad (2')$$

Where N_t is period t's demand for money in excess of what is required for transaction purposes. This demand is defined in terms of the original notation by (3')

$$N_t \equiv M_{t-1} + \Gamma_t - P_t c_t \qquad (3')$$

In this environment, a monetary perfect foresight equilibrium is a sequence of positive prices $\{P_t\}$ for $t=0, \ldots, \infty$ such that:

(IIa) The representative household selects a plan $\{c_t, \ell_t, N_t\}$ which solves (1) subject to (2'), taking $\{P_t, \Gamma_t = \mu M_{t-1}\}$ as given

(IIb) Given these individual choices, all markets clear

$$c_t = f(\ell_t) \qquad (5.a)$$

$$M_t = M_{t,s} = (1+\mu)M_{t-1} \qquad (5.b)$$

This concludes the description of the economic environment under study.

III. The Equilibrium Conditions.

The maximization of (1), subject to (2') is a standard dynamic programming problem. Necessary and sufficient conditions for equilibrium in this type of problem are given by

$$U'(\cdot_t) - \lambda_t = 0 \tag{6a}$$

$$V'(\cdot_t) + \beta U'(\cdot_{t+1}) f'(\cdot_t)(P_t/P_{t+1}) = 0 \tag{6b}$$

$$\beta U'(\cdot_{t+1})(1/P_{t+1}) - U'(\cdot_t)(1/P_t) \leq 0$$

$$[\beta U'(\cdot_{t+1})(1/P_{t+1}) - U'(\cdot_t)(1/P_t)]N_t = 0, \quad N_t \geq 0 \tag{6c}$$

$$Lim_{t\to\infty} \beta^t U'(\cdot_t)(N_t/P_t) = 0 \tag{6d}$$

$$N_{t-1}/P_t + \Gamma_t/P_t + (P_{t-1}/P_t) f(\ell_{t-1}) - c_t - N_t/P_t \geq 0$$

$$[N_{t-1}/P_t + \Gamma_t/P_t + (P_{t-1}/P_t) f(\ell_{t-1}) - c_t - N_t/P_t]\lambda_t = 0, \quad \lambda_t \geq 0 \tag{6e}$$

where λ_t is the Kuhn-Tucker multiplier associated with constraint (2').

Conditions (6a)-(6c) are standard. They balance the costs and the benefits associated with the choices of c_t, ℓ_t and N_t. Condition (6d) is the infinite horizon transversality condition of the household choice problem. It can be interpreted as saying that in equilibrium, the discounted present value of real balances held as a store of value must eventually go to zero. Generally, the transversality condition is not necessary for equilibrium However, it is demonstrated in section IV that a cash constrained economy of the type studied here cannot be in equilibrium when (6d) is violated Thus, for the type of economy studied in this essay, condition (6d) is necessary for equilibrium

It should be pointed out that with the restrictions made on U(·), V(·) and f(·), the representative household will always choose strictly positive values for c_t and ℓ_t This is the reason why the first-order Euler conditions associated with these choices have been written with strict equality sign

The economy is in general equilibrium when (5a)-(5b) hold in addition to (6a)-(6e) An important issue here is whether in general equilibrium the economy holds real balances in excess of what is required for transaction purposes[*] To shed light on this question, note that (5a), (6b) and (6c) together imply that

$$V'(\ell_t) + U'[f(\ell_t)]f'(\ell_t) \geq 0 \tag{7a}$$

$$[V'(\ell_t) + U'[f(\ell_t)]f'(\ell_t)]N_t = 0, \quad N_t \geq 0 \tag{7b}$$

It is clear from (7b) that expression (7a) is an equality or an inequality depending on whether N_t is greater than or equal to zero The strict concavity of U(·), V(·) and f(·) guaranties that (7a) has a unique solution ℓ^* when it is an equality, while the same properties imply that the equilibrium level of employment must be smaller than ℓ^* when (7a) is an inequality.

Thus, the economy holds real balances in excess of transaction needs only when employment is equal to ℓ^*

In equilibrium, the association between consumption and real balances obtained from (7a)-(7b) may be expressed compactly as,

$$c_t = \min(m_t, c^*) \tag{8}$$

where m_t is the equilibrium level of real balances M_t/P_t and c^* is the level of consumption corresponding to $f(\ell^*)$ Similarly, by letting $h(c)$ be the inverse function of $f(\ell)$, the level of employment corresponding to (8) is given by

$$\ell_t = h[\min(m_t, c^*)] \tag{9}$$

This association between c_t, ℓ_t and m_t, in general equilibrium, is useful because it permits to reduce the dynamics of the economy to a single expression Substitution of (8) and (9) into (6b) and multiplication of the resulting expression by M_t gives (10), an equation that is solely in terms of real balances at times (t), and (t+1)

$$A(m_t) = B(m_{t+1}) \tag{10}$$

where,

$$A(m_t) = -\left[V'[h(\min(m_t, c^*))]/f'[h(\min(m_t, c^*))]\right]m_t \tag{10a}$$

$$B(m_{t+1}) = [\beta/(1+\mu)]U'[\min(m_{t+1}, c^*)]m_{t+1} \tag{10b}$$

Expression (10) is a first order nonlinear difference equation that can be solved for the time path of m_t. The connection between P_t and m_t, i.e. $P_t = M_t/m_t$, allows us to redefine a perfect foresight monetary equilibrium as a sequence $\{m_t\}$ that satisfies the following restrictions:

(III.a) $\{m_t\}$ for $t=0, \ldots, \infty$ is a solution to (10)

(III.b) The sequence of N_t/P_t corresponding to $\{m_t\}$ satisfies (6d)

(III.c) $\{m_t\}$ must correspond to feasible levels of consumption and labour supply.

It is demonstrated in the remaining sections of the essay that unless certain restrictions are imposed on preferences and on the rate of monetary growth, an infinite number of sequences $\{m_t\}$ will satisfy equilibrium conditions (III.a)–(III.c).

IV. Equilibria with Self-Fulfilling Expectations.

This section of the essay derives the solutions to nonlinear difference equation (10) with the help of the diagrammatic analysis developed in the third essay. In addition, it demonstrates that these solutions must satisfy transversality condition (6d) to be an equilibrium. This amounts to a rigourous proof of the necessity of the transversality condition at infinity.

To prevent unnecessary complications, it will be convenient to restrict preferences in the following way[1]

(IV.a) B(m) is increasing and strictly concave over the interval $[0, e^*]$ with $Lim_{m\to 0}B(m) = 0$

(IV.b) A(m) is increasing and stictly convex over the interval $[0, e^*]$

(IV.c) $\mu > \beta - 1 = -\rho/(1+\rho)$.

With these assumptions, it is guarantied that there exists a unique $m(\mu)$ which is smaller than or equal to e^* and such that $A(m) = B(m)$ This level of real balances corresponds to the stationary state of the economy Note that for m greater than e^*, consumption and employment are constant, so that, A(m) and B(m) are straight lines with slopes $U'(e^*)$ and $(\beta/(1+\mu))U'(e^*)$ respectively. In this situation, equilibrium condition (10) reduces to (10') below,

$$m_{t+1} = ((1+\mu)/\beta)m_t. \tag{10'}$$

Under assumptions (IV.a)–(IV.c), the solutions to the nonlinear difference equation (10) can be represented graphically as in figure I.

The sequence of real balances that solves (10) for any given m_0 is found in the following manner Take as given period zero's level of real balances and find out on $B(\cdot)$ the level m_1 such $A(m_0) = B(m_1)$. Then given m_1, determine the level of m_2 such that $A(m_1) = B(m_2)$. By repeating this procedure iteratively, one can obtain the sequence of m_t that solves (10) for any given m_0.[2] Two possible paths initiated at m_0 and m_0' are illustrated in figure I as example.

As is apparent from figure I, the solutions to (10) are numerous. They can be classified into three broad categories depending on whether m_0 is smaller than, greater than or equal to $m(\mu)$. Specifically, if m_0 is smaller than $m(\mu)$, the sequence of m_t that solves (10) has the characteristic that real balances converge asymptotically to zero. While if m_0 is greater than $m(\mu)$, m_t rises asymptotically to infinity. Only if m_0 happens to be equal to $m(\mu)$ is the sequence of m_t constant through time at its stationary level $m(\mu)$.

Observe that for all $m_0 > m(\mu)$, there exists a time period $s \geq 0$ such that for $t \geq s$, then $m_t \geq c^*$. In this particular case, money balances held as a store of value can be described by the following expression:

$$N_t/P_t = [(1+\mu)/\beta]^{t-s} m_s - c^* \qquad t \geq s \tag{11}$$

Since μ is greater than $\beta-1$, the above expression also implies that N_t/P_t is rising through time.

A perfect foresight monetary equilibrium is a solution to (10) such that the representative agent, taking equilibrium prices as given, has no incentive to modify his planned behavior. That is, there does not exist an alternative plan which gives higher utility. Specifically, if plan $\{\bar{c}_t, \bar{\ell}_t, \bar{N}_t\}$ is chosen when the price level path is $\{\bar{P}_t = M_t/\bar{m}_t\}$ then it must be the case that any $\{c_t, \ell_t, N_t\}$ such that

$$\bar{P}_t c_t + N_t \leq N_{t-1} + \Gamma_t + \bar{P}_{t-1} f(\ell_{t-1})$$
$$N_t \geq 0, c_t \geq 0, \ell_t \geq 0 \qquad (12)$$

will give a lower level of welfare. That is,

$$\sum_{t=0}^{\infty} \beta^t [U(c_t) + V(\ell_t)] \leq \sum_{t=0}^{\infty} \beta^t [U(\bar{c}_t) + V(\bar{\ell}_t)] \qquad (13)$$

It will now be shown that an equilibrium is a solution to (10) that satisfies transversality condition (6d). More formally.

Theorem A necessary and sufficient condition for a solution $\{\bar{m}_t\}$ of $A(m_t) = B(m_{t+1})$ to be a perfect foresight monetary equilibrium is that $Lim_{t\to\infty} \beta^t U'(\bar{c}_t)(\bar{N}_t/\bar{P}_t) = 0$

Proof (sufficiency) To prove that (6d) is sufficient to obtain equilibrium, it must be shown that all the alternative plans defined by (12) satisfy inequality (13) when (6d) is met. This may be done as follows. Define Q_T as the change in welfare that results from adopting an alternative plan from period (0) to period (T). Note that the concavity of objective function (1) restricts Q_T to satisfy the following inequality

$$Q_T = \sum_{t=0}^{T}\beta^t[U(c_t) + V(\ell_t) - U(\bar{c}_t) - V(\bar{\ell}_t)]$$
$$\leq \sum_{t=0}^{T}\beta^t[U'(\bar{c}_t)(c_t - \bar{c}_t) + V'(\bar{\ell}_t)(\ell_t - \bar{\ell}_t)] \qquad (14)$$

The alternative plans must satisfy budget constraint (12) so that it is possible to substitute out c_t in (14) to obtain

$$Q_T \leq \sum_{t=0}^{T}\beta^t[U'(\bar{c}_t)(\bar{N}_t - N_t)/\bar{P}_t + V'(\bar{\ell}_t)(\ell_t - \bar{\ell}_t)] -$$
$$\sum_{t=0}^{T}\beta^t U'(\bar{c}_t)[(\bar{N}_{t-1} - N_{t-1})/\bar{P}_t - f'(\bar{\ell}_{t-1})(\ell_{t-1} - \bar{\ell}_{t-1})(\bar{P}_{t-1}/\bar{P}_t)]$$

where the initial conditions that $N_{-1} = \bar{N}_{-1}$ and $\ell_{-1} = \bar{\ell}_{-1}$ have been imposed [8] Upon rearrangement, this is equal to

$$Q_T \leq \sum_{t=0}^{T-1}\beta^t[U'(\bar{c}_t)(1/\bar{P}_t) - \beta U'(\bar{c}_{t+1})(1/\bar{P}_{t+1})](\bar{N}_t - N_t) +$$
$$\sum_{t=0}^{T-1}\beta^t[V'(\bar{\ell}_t) + \beta U'(\bar{c}_{t+1})f'(\bar{\ell}_t)(\bar{P}_t/\bar{P}_{t+1})](\ell_t - \bar{\ell}_t) +$$
$$\beta^T U'(\bar{c}_T)(\bar{N}_T - N_T)/\bar{P}_T + \beta^T V'(\bar{\ell}_T)(\ell_T - \bar{\ell}_T).$$

The proposed equilibrium path satisfies (6b) and (6c).

Thus, the right-hand-side of the above expression can be simplified to obtain

$$Q_T \leq \beta^T U'(\bar{c}_T)(\bar{N}_T - N_T)/\bar{P}_T + \beta^T V'(\bar{\ell}_T)(\ell_T - \bar{\ell}_T) + X_{T-1} \qquad (14')$$

where,

$$X_{T-1} = \sum_{t=0}^{T-1} \beta^t [\beta U'(\bar{c}_{t+1})(1/\bar{P}_{t+1}) - U'(\bar{c}_t)(1/\bar{P}_t)] N_t$$

is bounded above by zero

From (7a), we know that $0 \leq \bar{\ell}_T \leq \ell^*$ Now, since the function $-V'(\ell)\ell$ is strictly increasing in ℓ, it must be the case that $0 \leq -V'(\bar{\ell}_T)\bar{\ell}_T \leq -V'(\ell^*)\ell^*$ Therefore Q_T must also satisfy this inequality,

$$Q_T \leq \beta^T U'(\bar{c}_T)(\bar{N}_T - N_T)/\bar{P}_T + \beta^T V'(\bar{\ell}_T)\ell_T - \beta^T V'(\ell^*)\ell^* + X_{T-1} \qquad (15)$$

Finally, by taking the limit of (15) as (T) tends to infinity,

$$Lim_{T\to\infty} Q_T \leq Lim_{T\to\infty} \beta^T U'(\bar{c}_T)(\bar{N}_T - N_T)(1/\bar{P}_T) + Lim_{T\to\infty} X_{T-1}$$

we obtain that the limit of Q_T has an upper bound of zero when (6d) is holding. Therefore satisfaction of (6d) is sufficient to guaranty that $\{\bar{c}_t, \bar{\ell}_t, \bar{N}_t\}$ is an equilibrium

Proof (necessity) To prove the necessity of the transversality condition, it is shown here that when (6d) does not hold, there exists an alternative plan among those defined by (12) which violates inequality (13) The proof proceeds in two steps

a) If $m_0 \leqq m(\mu)$, then (6d) is implied trivially by the other necessary conditions since $\overline{N}_t = 0$ for all t

b) If $m_0 > m(\mu)$, then according to the other necessary conditions we have

$$\text{(P1)}' \quad m_{t+1} = [(1+\mu)/\beta]m_t \quad t \geqq s$$

$$\text{(P2)} \quad P_{t+1} = \beta P_t \qquad t \geqq s$$

Using (P1) and (11), transversality condition (6d) reduces to

$$Lim_{T\to\infty}\beta^T U'(\bar{c}_T)(\overline{N}_T/\overline{P}_T) = \beta^s U'(c^*)\overline{m}_s Lim_{T\to\infty}(1+\mu)^{T-s}. \qquad (6d')$$

Clearly, the above limit depends on the growth rate of the money supply - Two cases are possible

i) If $\mu < 0$, once again the transversality condition is implied by the other necessary condition

ii) If $\mu \geqq 0$, the transversality condition is violated since $(1+\mu)^{T-s}$ is bounded below by one Suppose that the proposed path is nevertheless an equilibrium. This means that there should not exist an alternative plan as defined by (12) that violates inequality (13) Now consider the following plan which is constructed so as to satisfy (12).

Set $\ell_t = \bar{\ell}_t$ for all t, $N_t = \bar{N}_t$ for $t < s$, $N_t = \bar{N}_t - \varepsilon$ for $t \geqq s$, where $\varepsilon > 0$ is chosen so that $\bar{N}_t - \varepsilon > 0$, $c_t = \bar{c}_t$ for $t \neq s$, and $c_t = \bar{c}_t + \varepsilon/\bar{P}_t$ for $t = s$.

Note that this alternative plan should be preferred to the equilibrium plan because it gives rise to a higher consumption path than the equilibrium path

The necessity of (6d) is demonstrated if we can choose an ε so that the non-negativity constraints (6a)-(6c) are satisfied In particular we just need to show that an ε can be selected such that $\bar{N}_t - \varepsilon > 0$ for $t \geqq s$ This is the case if $Lim_{t\to\infty}\bar{N}_t > 0$ But this follows from (P2) and the fact that (6d) is violated That is,

$$Lim_{t\to\infty}\beta^t U'(\bar{c}_t)(\bar{N}_t/\bar{P}_t) > 0 \to \beta^s U'(c^s)(1/\bar{P}_s) Lim_{t\to\infty}\bar{N}_t > 0.$$

Therefore (6d) is a necessary equilibrium condition This completes the proof of the theorem.

It follows immediately from the above theorem, that any solution to (10) such that m_0 is smaller than or equal to $m(\mu)$ is an equilibrium. This is because along these paths (6d) is trivially satisfied given that the economy does not hold money balances in excess of what is required for transaction purposes.

Whether the sequences of real balances originating to the right of $m(\mu)$ are equilibria depends on the growth rate of the money supply. If μ is smaller than zero, then (6d') approaches zero as (t) tends to infinity, and consequently, the proposed paths of real balances are equilibrium paths. On the contrary, if μ is greater than or equal to zero, transversality condition (6d') is violated and the proposed paths of real balances can be ruled out as equilibria.

So we must come to the conclusion that under the assumptions made above, the model can support an infinite number of equilibrium paths. Specifically, for any m_0 smaller than $m(\mu)$, there exists a distinct perfect foresight monetary equilibrium that gives rise asymptotically to a collapse of the monetary economy. In addition, provided the nominal money stock is falling through time, the model can also support an infinite number of equilibrium paths leading asymptotically to a level of real balances equal to infinity. Only if m_0 equals $m(\mu)$ is it the case that the resulting sequence of m_t does not reflect speculative anticipations unrelated to market fundamentals.

Intuitively what happens along the explosive price level paths is the following. The requirement to use money as a medium of exchange introduces a distortion in the labour market that generates a wedge between the private and the social marginal rate of transformation of work effort into consumption. This wedge varies positively with the anticipated rate of inflation;

see Aschauer & Greenwood (1982), Carmichael (1985) and Wilson (1979) In this environment, an increase in the anticipated rate of inflation reduces the perceived return to labour and induces a substitution away from time spent in the labour market In equilibrium, the reduction in work effort lowers the equilibrium level of output and validates the higher inflationary expectations The situation is a little bit different when real balances increase to infinity Along these equilibrium paths, the cost of holding money is zero since the rate of deflation equals the economy's rate of time preference. Under these circumstances, economic agents are perfectly content to accumulate real balances in excess of their transaction needs as long as the discounted value of these balances eventually goes to zero. This is the case whenever the nominal money stock is shrinking through time, so that real balances grow at a rate smaller than the rate of time preference, c f eqs (10') and (11)

The result obtained above exemplifies Farmer's (1984) argument that the dependence of the demand for real balances on the anticipated rate of inflation is the crucial element in the appearance of self-fulfilling hyperinflations Here, this dependence arises through the elastic labour supply and can be explained intuitively as follows An increase in the anticipated rate of inflation reduces the need for transaction balances because it produces a substitution away from consumption in favour of leisure time As discussed in Farmer, a Clower cash-in-advance economy with exogeneous income could not support these extraneous equilibrium paths[4] This is because contrary to the model studied here, the demand for real balances in such a model is independent of the price level and is fixed exogeneously by the level of real income.

There is another type of nonuniqueness that has yet to be discussed It arises when the economy obeys Friedman's rule for the optimum quantity of money by continuously reducing the nominal money stock at the rate $-\rho/(1+\rho)$ In this particular case, for any m_0 greater than or equal to $m(\mu)$, the sequence of real balances that solves (10) is constant through time at m_0 This is easily seen from (10′) Graphically, functions $A(\cdot)$ and $B(\cdot)$ collapse together for the values of m greater than or equal to c^*

Friedman's original exposition requires for the optimum quantity of money to be determinate that the marginal utility of real balances becomes negative for large values of m Here the liquidity services provided by money is never negative In fact, it is constant at zero for m greater than or equal to c^* For this reason, economic agents are indifferent between any levels of m which are greater than c^* Thus, under Friedman's rule, the economy can support a continuum of equilibria with a constant real money stock Niehans (1980) makes a similar argument concerning the possible indeterminacy of the optimum quantity of money

This last observation concludes section IV, the next section

figure II

Looking at figure II, it is clear that the solutions to (10) originating to the left of $m(\mu)$ lead in a finite time to a negative level of real balances (or equivalently to a negative price level). This is not feasible, given that consumers can always freely dispose of money balances. Consequently, (IV a') permits to rule out as equilibria all the solutions to (10) that originate to the left of $m(\mu)$. Imposing (IV a') ensures the nonexistence of speculative hyperinflations in the model under consideration.

Scheinkman & Brock (1980) demonstrate that condition (16) is also sufficient to rule out self-fulfilling hyperinflations in a standard overlapping generation models,[5] while Brock (1978) shows that an analogous condition must be imposed in a model where real balances enter as an argument of the utility function. Specifically, by letting J(m) represent the preferences for liquidity, Brock's condition can be expressed as

$$Lim_{m \to 0} J'(m)m > 0. \qquad (17)$$

This last condition is discussed extensively in Obstfeld and Rogoff (1983). They show that (17) implies that the household receives a level of utility equal to minus infinity when it does not hold real balances (i.e. J(·) is unbounded below). Given the similarity of (16) with Brock's condition, it is evident that (16) implies that.

indicates how the equilibrium paths leading asymptotically to a collapse of the monetary economy can be ruled out by modifying one of the assumptions made earlier on preferences

V. Equilibrium Without Self-Fulfilling Hyperinflation.

In the last section, it has been established that in its actual format, the model can support an infinite numbers of self-fulfilling equilibrium paths This section introduces and discusses a restriction on the representative agent's utility function that permits to rule out the solutions of (10) initiated to the left of $m(\mu)$, and consequently to ensure the nonexistence of speculative hyperinflations It consists of replacing assumption (IV a) by the following,

(IV a') B(m) is increasing and strictly concave over the interval $[0, c^*]$ with $Lim_{m \to 0} B(m) > 0$

The crucial element is the assumption that $Lim_{m \to 0} B(m)$ is stricly positive. This is equivalent to postulating that in the neighborhood of c equal to zero, the marginal utility of consumption changes faster than c, so that, the following limit does hold

$$Lim_{c \to 0} U'(c)c > 0 \qquad (16)$$

With this assumption, figure I must be amended if one wants to characterize the dynamics of this economy This is done in

$$Lim_{c\to 0}U(c) = -\infty$$

Formulated in this manner, the restriction that must be imposed to rule out self-fulfilling hyperinflations in a cash constrained economy is more enlightning than (17) because it refers directly to the flow of benefits received from consumption instead of the flow of unspecified transaction services received from holding money

In addition, it has been argued in the literature by Scheinkman (1980) and Kingston (1982) that the imposition of (16) in overlapping generations models and of (17) in models with money in the utility function is equivalent to the assumption that the revenue from printing money in the stationary state is stricly positive even if μ tends to infinity In other words, money is so important to the economy that agents still hold some even if it is infinitely costly to do so

Scheinkman derives this conclusion using the following definition for Ω, the revenue raised from printing money,

$$\Omega = \mu m. \tag{18}$$

He demonstrates that satisfaction of (16) implies that the limit of Ω when μ tends to infinity and $m(\mu)$ tends to zero is strictly positive However, remark that (18) is not the standard definition of the revenue from printing money in a discrete time model From the government's budget constraint (4), the revenue from printing money in the stationary state is given by

$$\Omega^* = \tau = \mu(M_{t-1}/P_{t-1})(P_{t-1}/P_t) = [\mu/(1+\mu)]m \qquad (19)$$

Upon taking the limit of (19) when μ tends to infinity and $m(\mu)$ to zero, it is clear that the government's revenue from printing money, given by (19), tends to zero. Consequently, the conclusion reached by Scheinkman that (16) can be interpreted as a restriction on the government's revenue schedule from printing money critically depends on the choice of revenue schedule employed and does not hold under the standard definition Ω^*.

VI. Conclusion.

This essay has analysed the problem of uniqueness of a monetary equilibrium in a version of Clower's cash-in-advance model with endogeneous production. It has been shown that under standard restrictions on preferences and technology, this type of economy has an infinite number of equilibrium paths with all but one fuelled by self-fulfilling expectations. Along these extraneous paths, real balances (the price level) converge asymptotically to either zero (infinity) or infinity (zero). The essay also demonstrates that upon restricting the rate of money growth to be nondecreasing, it is possible to demonstrate the nonexistence of explosive paths for real balances. Finally, the essay introduces and discusses a restriction on preferences that permits to rule out all the implosive paths for real balances This condition is related to those that must be imposed in overlapping generations models and in models where money enters as an argument of the utility function

FIGURE I

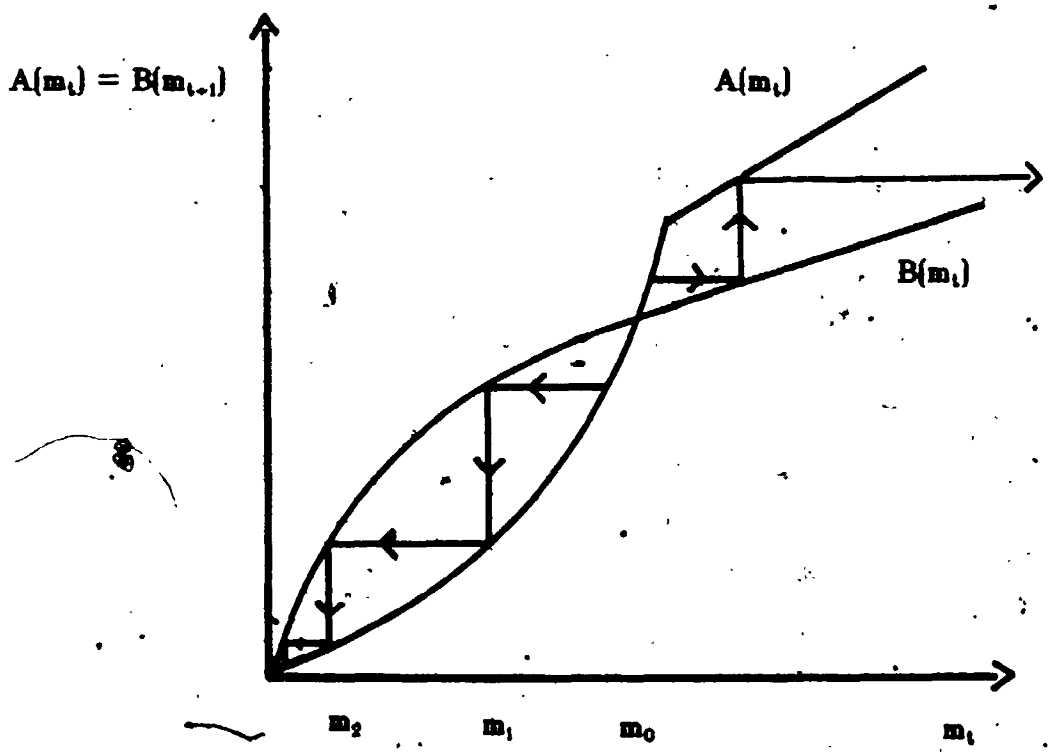
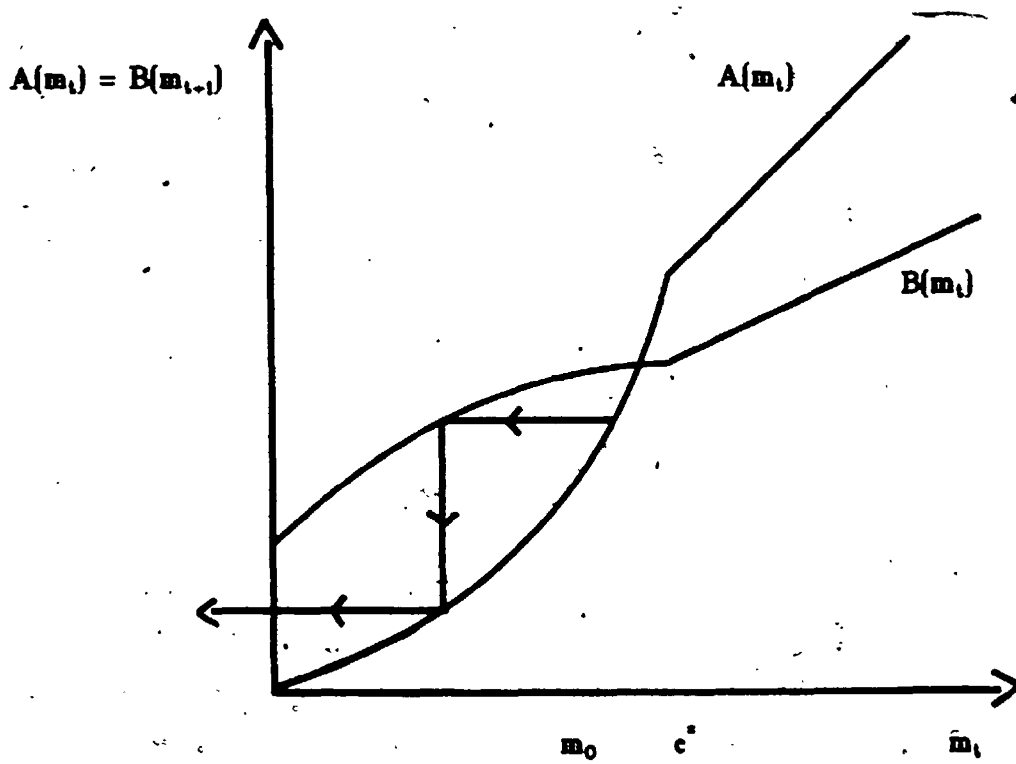


FIGURE II



FOOTNOTES

[1]The assumptions of strict concavity and convexity are not necessary for the analysis conducted below They are simply made here for convenience

[2]Alternatively, figure I can be seen as a method to solve the following equation,

$$m_{t+1} = D^{*}A(m_t)$$

where D is the inverse function of $B(\cdot)$

[3]This inequality is obtained by separating the income earned by the representative agent as the owner of the firm from his labour income This is done by rewriting budget constraint (2') as

$$c_t + N_t/P_t \leq N_{t-1}/P_t + (P_{t-1}/P_t)[f'(\cdot_{t-1})\ell_{t-1} + D_{t-1}]$$

where $f'(\cdot)\ell$ is labour income and D is dividend income defined as follows,

$$D_t = f(\cdot_t) - f'(\cdot_t)\ell_t.$$

[4]Lucas (1980) analyses such an economy.

[5]Their result applies to an economy populated with two-period-lived overlapping generations that have an endowment in the first period only.

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FIGURE I