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Monetary Versus Fiscal Policy: Three Essays

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MONETARY VERSUS FISCAL POLICY:
THREE ESSAYS

by

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ABSTRACT

The thesis consists of three essays which investigate the relative effectiveness of monetary and fiscal policy.

The first essay analyzes the effect on the real rate of interest of changes in the rate of price inflation and monetary growth which finance either fiscal policy (changes in the tax rate) or financial policy (changes in the bond/money ratio). The second essay investigates the relevance with respect to market and aggregate output of tax versus money financed government expenditure in a natural rate model. The third essay scrutinizes the prediction made by Turnovsky and Brock (1980) that if fiscal authorities do not have access to the rate of monetary expansion any optimal policy will be time inconsistent.

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TABLE OF CONTENTS

INTRODUCTION

The thesis consists of three essays which investigate the relative effectiveness of monetary and fiscal policy. In this introduction, the essays are summarized with their primary conclusions.

I. ESSAY I

The first essay, entitled "Inflation, Fiscal Policy and the Real Rate of Interest", addresses the issue of the responsiveness of the real rate of interest on nominal, interest bearing government debt to changes in the rate of inflation.

Recent research has re-examined the Mundell-Tobin prediction that an increase in inflation will cause a substitution away from non-interest bearing assets to interest bearing real and nominal assets, reducing their real rate of interest. This prediction (most often associated with Tobin (1965), Mundell (1963) and Fisher (1930)) has been addressed by a number of authors including Patinkin and Levhari (1972), Fried and Howitt (1983) and Stockman (1981), who argue that the impact of inflation on real rates will depend on how the liquidity services of assets are modeled. Fried and Howitt argue that while the real rate on capital may be tied to the rate of time preference (in a Sidrauski, steady state, context) the effect of inflation on the real rate on government bonds will depend on the transactions technology assumed. Stockman demonstrates in a cash-in-advance context, that if cash in advance of capital purchases is required, an increase in the inflation rate will reduce the net return on capital since the cost of holding cash for capital purchases rises. The effect will be to reduce the demand for capital and increase the steady state return on capital.

Other papers by Feldstein (1976, 1980), Martins (1980) and Sargent (1976) also demonstrate an ambiguous impact of inflation on the real rate. The empirical evidence, which includes papers by Fama (1975, 1977, 1979), Lucas (1980), Summers (1981), Nelson and Schwert (1977) and Mishkin (1981, 1984), tends to favour the Mundell-Tobin prediction. While Fama argues that the real rate is invariant to changes in the inflation rate (indicating a super-neutrality result), Mishkin (1981) and Nelson and Schwert (1977) show that there is a negative correlation between the real rate and inflation. Summers finds that, with the exception of a few subperiods, the real rate was negatively correlated with inflation in the U.S. Mishkin (1984), in a multi-country study finds in all cases a negative correlation between real rates and expected inflation. Thus, there seems to be an empirical consensus as to the qualitative impact of price inflation on the real rate. This conclusion is supported by recent real rate (ex-post) behaviour which appears to be negatively related to inflation.

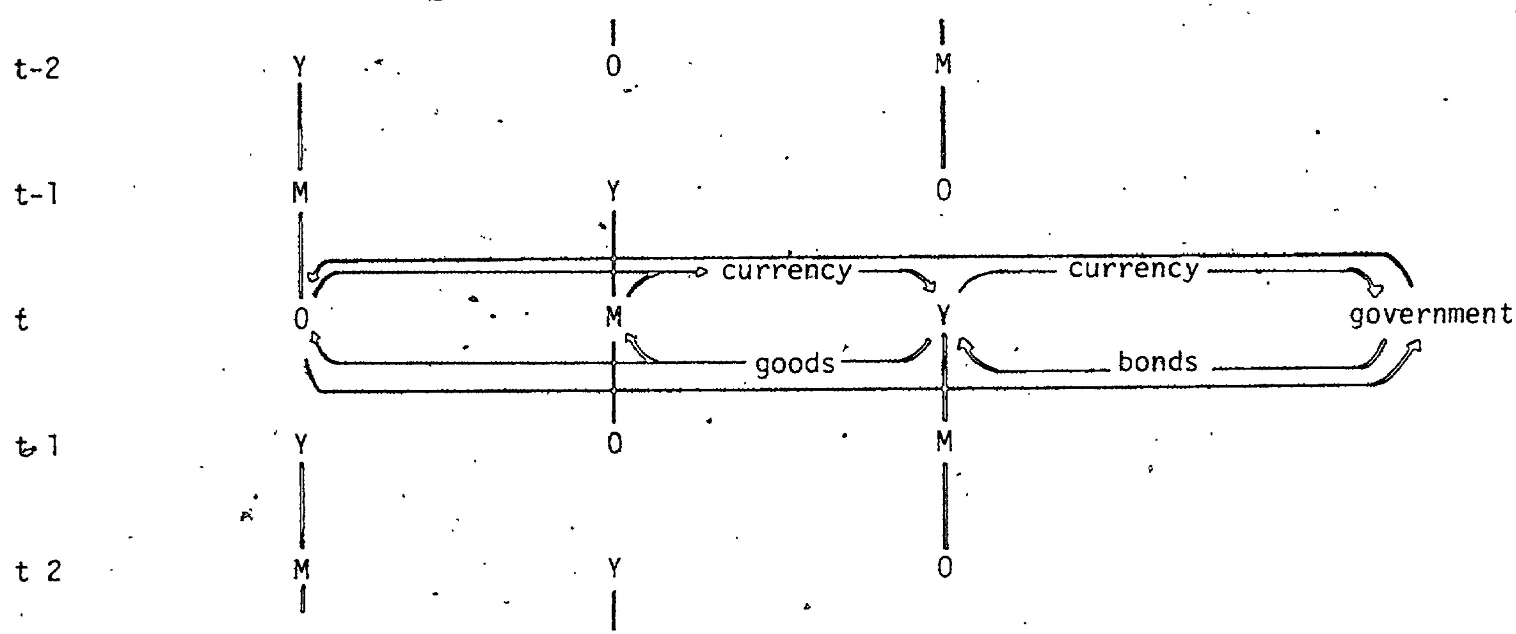
While, in general, real rates appear to decline in the face of an increase in the inflation rate, there can be little doubt that a single equation relating the two variables does not fully explain recent interest rate behaviour. In an attempt to reconcile the stylized fact that high inflation rates may be associated with either high or low real rates (or even negative real rates) a model is presented in which the Samuelsonian overlapping generations paradigm is adopted. Agents are assumed to live three periods and receive an exogenous endowment or income in their first period of life. Since the endowment is non-storable and agents receive utility from consumption in each period of life they will demand nominal money and bonds as stores of value. As suggested by Martins (1980), money and bonds may be differentiated by the assumption that bonds are infinitely illiquid if held for less than two periods. Thus, bonds will be held only as a store of value from the first period of life to the third period. In the steady state, if bonds bear a positive nominal interest, so that their return dominates that of money, bonds will be held to finance the total of third period consumption and real balances will finance second period consumption solely.

The economy is illustrated schematically below. At each point in discrete time, four economic agents operate; the government, a young agent, a middle-aged agent and an old agent.

As illustrated, at a given date t, the young person receives exogenously an endowment of a non-storable consumption good. The middle-aged and the old agents must trade storable assets with the young agent to finance their consumption as they receive no endowment.

There are two such assets, issued by the government (private debt is assumed not to exist); fiat currency and nominal bonds. Bonds are distinguished from currency through the assumption that they are non-transferable and require two periods to mature. Since they are illiquid if held for two periods, they will be purchased only by the young to finance third period consumption. It is also assumed that the supply of bonds relative to money is great enough to require that they bear a

FIGURE I



Y-young
M-middle-aged
O-old

positive nominal rate of interest. Thus, in a perfect foresight steady state, the young will purchase money to finance second period consumption and bonds to finance third.

As the diagram illustrates, at t the young receives its endowment, consumes part of it and trades the remainder for currency with the middle-aged and old agents (the old having redeemed its bonds for currency with the government). The young agent is taxed by the government and with part of its income purchases bonds from the government which will mature at t+2.

Formally, the young at t maximizes with perfect foresight, in its initial period of life, a utility function,

$$U = U_1[C_t^1] + U_2[C_t^2] + U_3[C_t^3]$$

whose arguments are first through third period consumption, C_t^i denotes ith period consumption by an agent who is young at time t.

The constraints the young agent faces are,

$$P_t C_t^1 = P_t - T_t - \frac{B_t}{(1+i_t)^2} - M_t$$

$$P_{t+1}\ C_t^2 = M_t$$

$$P_{t+2}\ C_t^3 = B_t$$

The first constraint limits nominal first period consumption to the level of nominal income (whose real value is normalized to unity) minus nominal government taxes T_t minus purchase of nominal bonds B_t and currency M_t. Note that the currency price of bonds is $1/(1+i_t)$ where i_t is the one period nominal rate of interest on bonds (in fact, strictly speaking, B_t are bills since they do not yield a coupon, however they are termed bonds for the sake of convenience).

The latter two constraints limit nominal consumption in the second and third period of life to be funded by currency and nominal bonds purchased in the first period respectively. These two constraints embody the assumption discussed above regarding the illiquidity of bonds relative to currency and the assumption that $i_t > 0$. Given that bonds cannot finance second period consumption (by assumption) if the supply of bonds relative to currency is increased to a great enough level agents will require a discount on bonds to clear the bond market and to induce the old to consume a relatively greater amount.

The model is closed by the assumption that markets clear. The goods market will clear when aggregate consumption demand equals aggregate supply. Aggregate supply is simply equal to the endowment the young agent receives while aggregate consumption is the sum of consumption demand by the young, middle-aged and the old agents. For the sake of convenience, it is assumed that government consumption is zero. Thus, when the goods market clears,

$$P_t = P_t\, C_t^1 + P_t\, C_{t-1}^2 + P_t\, C_{t-2}^3$$

It is assumed that the government follows a monetary growth rule.

If the money market clears, where μ is the rate of nominal money growth,

$$M_t = (1 + \mu)M_{t-1}$$

money demand equals money supply.

The government has several instruments, the growth of nominal bonds and money and the level of taxes levied on agents' endowment. The effect of changes in these instruments is examined in both a non-inflationary environment and an inflationary environment. In the case of a non-inflationary steady state equilibrium, the results of the comparative statics are similar to those derived by Patinkin (1975).

In an inflationary steady state, the government has at most two independent instruments; the rate of nominal monetary growth and the bond/money ratio or the real level of taxes. Inflationary policies are divided into two categories; inflationary fiscal policy, in which inflation increases while the bond/money ratio is held constant, and inflationary financial policy, in which the level of real taxes is held constant.

It is demonstrated that if the increase in inflation (and nominal monetary growth) is used to finance an increase in the deficit then the real rate must fall. In other words, an inflationary fiscal policy implies the Mundell-Tobin result, a decrease in the real rate.

On the other hand, if the government pursues an inflationary financial policy, increasing inflation, holding real taxes constant, the effect on the real rate will be ambiguous. If the degree of gross substitutability between second and third period consumption is low, or if money demand is not very responsive to the inflation rate, then it will be the case that the increase in inflation will increase the real rate. In other words, the bond to money ratio must rise by a sufficient degree to ensure that the real rate will fall.

The conclusion is that the impact of a change in the steady state rate of growth of nominal balances and inflation on the real rate depends on the type of inflationary policy pursued. Only if the government adopts an inflationary fiscal policy will the real rate unambiguously fall in response to an increase in the rate of monetary expansion.

II. ESSAY II

The second essay, "Tax Versus Money Financed Fiscal Policy in a Natural Rate Model", addresses the issue of how a change in the mode of finance affects aggregate and individual market output.

In recent years the relevance, with respect to output decisions of the private sector, of changes in the mode of finance has been investigated by several authors. Going beyond the monetarist-fiscalist debate, the relevance issue represents an attempt to determine the circumstances under which policy changes have no effect on output. In this fashion, it is often easier to determine how and why policy changes do have relevance.

An early example of this literature is Barro's (1974) discussion of the so-called Ricardian equivalence of bond and tax financed policy. Barro demonstrates that if agents perfectly discount the future tax burden represented by current bond purchases, the choice of tax versus bond financing is irrelevant to output and prices.

Recent papers by McCallum (1982), Chan (1983) and Gertler and Aiyazari (1983) re-examine the Ricardian equivalence proposition and show that if the deficit is improperly defined or if future tax burdens are distributed unevenly, equivalence between bond and tax financing is lost.

Wallace (1981) describes an economy in which government purchases of real goods, financed by fiat currency (or bonds) will have no relevance.

Bryant (1983) succinctly summarizes the relevance issue into two facets; relevance of government purchases and relevance of the mode of finance of those purchases. It is the latter issue to which this second essay is addressed. This issue is especially 'relevant' in the light of recent attempts by governments to reduce their dependence on monetary expansion as a mode of finance.

A model is introduced which is similar to that described by Lucas (1972). As in Lucas's original exposition, there are two types of stochastic shocks which cause output to vary, an aggregate and a distributional shock. In this case however, the aggregate shock represents a shock to government expenditure which is financed by a combination of taxes and money creation. The distributional shock is the distribution of government expenditure across markets.

At a point in discrete time two types of individuals exist, the young (normalized to unity) and the old. Generations overlap. There are two spatially and informationally separate markets or islands. The young are distributed equally between the two islands while prior to the beginning of each period the old migrate to equalize the expected return on their savings.

The young either consume leisure or produce a non-storable consumption good. Any of their production is traded with the old who inhabit the same market in exchange for fiat currency. The old supply their currency or savings inelastically in exchange for the consumption good which yields utility.

In turn, the currency the young receive is saved until the next period when it is used in exchange for the consumption good produced by next period's young.

The government intervenes by randomly distributing its demand for the consumption good across the two markets. It finances its purchases by taxing the savings (or the previous period's income) of the current old or by money creation.

The decision problem of the young person is to maximize a utility function,

$$E[u(c') - q(n)|I]$$

where c is consumption, n is labour input, I is current information and the prime denotes next period's value. Thus, the individual derives utility from next period's consumption and disutility from current labour. The constraints the individual faces are,

$$c' = \frac{py}{p'}(1 - \tau')$$

$$y = n$$

so that future consumption equals current nominal receipts, py, discounted by the next period's price level p' net of the tax rate τ' which is levied next period (or after production). Further, the production of the individual is a linear function of labour input.

The government finances its random demand for goods by taxation or money creation. The tax rate is assumed to be,

$$\tau = (1 - \mu) \frac{\gamma}{1+\gamma}$$

where γ is a random variable distributed across $[0, \bar{\gamma}]$ and $0 \leq \mu \leq 1$. The μ parameter measures the degree to which government expenditure is financed by money creation. Thus, as μ approaches unity the tax rate falls for a given μ. Note that μ is non-random.

The increase in the nominal money stock is assumed to be a random multiple of the previous period's nominal money stock (m) $\mu\gamma m$. Thus, since current tax revenue is levied on the old's previous period's income which will equal m the previous period's money stock, the government's budget constraint will be

$$(1-\mu) \frac{\gamma}{1+\gamma} m + \mu\gamma m = \frac{\gamma}{1+\gamma} (1+\mu\gamma)m$$

where the right-hand side represents the government's demand for goods.

Market equilibrium occurs when the supply of goods equals the demand on each island. Since the supply of goods equals the demand for money and vice versa, equilibrium in the goods market implies the same for the money market.

Since individual output is y and half the young are on each island (and the total normalized to unity), market supply is $y/2$. Private demand equals the after-tax supply of real balances offered by the current old. Since the old migrate to equalize the return on savings, market private demand is $m(1-\tau)/2p$. Public demand is a random fraction $\theta/2$, distributed symmetrically over [0,2], of the random aggregate level of government expenditure. Thus, market equilibrium occurs when,

$$y/2 = \frac{m}{2p}(1-\tau) + \frac{\theta}{2}\frac{(1+\mu\gamma)\gamma}{1+\gamma}$$

$$= \frac{m}{2p}\frac{(1+\mu\gamma)(1+\theta\gamma)}{(1+\gamma)} = \frac{mv}{2p}$$

Equilibrium occurs in the other market when output $\hat{y}/2$ equals demand $m\hat{v}/2p$, where $\hat{v}$ is v above with $2-\theta$ substituted for θ (since θ is distributed symmetrically).

In the second essay it is demonstrated that in the economy described above, the policy rule has the property that the ratio of private consumption to government consumption is constant with respect to changes in the mde of finance μ. Equilibrium output is examined, where aggregate output is defined as the sum of market outputs,

$$Y = \frac{1}{2}y + \frac{1}{2}\hat{y}$$

to determine the effect of changes in the mode of finance under the assumption that while μ is known with certainty, the exact realization of θ and γ may or may not be in agents' information sets at the time of production.

It is demonstrated that the policy rule derived above has the property that output is unaffected as the mode of finance of the stochastic government expenditure is varied under full information. Given the realization of the level and distribution of government expenditure, agents are indifferent between an inflation tax and an income tax. Producers are solely concerned with the share of nominal aggregate demand which is accruing to their market since it represents their ability to capture consumption next period. Thus, despite the asymmetric distribution of currency, agents are indifferent between modes of finance under full information. In effect, the mode of finance is irrelevant.

If there is less than full information, agents must make rational expectations of the random shocks they are subject to based on observation of their market price and the history of the economy. Under such circumstances, it can be demonstrated that the policy rule described above is no longer irrelevant with respect to the mode of finance. In fact, if the distributions of the random variables exhibit certain sufficient conditions, it can be shown that an increase in the degree to which government expenditures are financed by money creation will increase market and economy wide output for given realizations of the random variables.

Further, the response of market and aggregate output to expenditure shocks is affected by the mode of finance. If expenditures by government are financed solely by tax revenue, then prices offer sufficient information to predict the intertemporal rate of substitution. However, if any degree of money creation is adopted this will no longer be true unless agents have been given full information. With full information or complete tax financing, a shock to aggregate government expenditure will have a damped effect on aggregate output due to the negative correlation of market output responses. Since the aggregate expenditure is distributed unevenly, one market will increase its output, the other will decrease its output as their share of demand is increased and decreased respectively.

With less than full information and some degree of money finance, producers will not be able to predict accurately their market's share of demand and thus both markets may increase output in response to an increase in government expenditure. Thus, the variance of aggregate output may increase with respect to an increase in money finance under less than full information.

Thus, the second essay contains an examination of the relevance issue in the context of a Lucas natural rate model. It is demonstrated that while a policy rule may be derived which is irrelevant, such that market and aggregate output are unaffected by changes in the mode of finance, in the full information environment, if there is less than full information, output will increase with the degree of money finance. In this context, it is argued that the choice between tax and money finance is only relevant in the absence of full information.

III. ESSAY III

The third essay, "Time Inconsistency of Optimal Monetary and Fiscal Policy", explores the impact of different transactions technologies on the time consistency of optimal policies.

An optimal policy, which describes a sequence of strategies or controls which maximize an objective function, is defined as inconsistent if upon re-evaluation at some future date, the continuation of the sequence is no longer optimal.

This concept was first explicitly discussed in a macroeconomic context by Kydland and Prescott (1977). An example they describe yields a simple explanation of the concept of time inconsistency. Consider a government which wishes to maximize production. It might adopt a patent policy as optimal, at some initial date, giving monopoly rights to the fruits of innovation. However, at some future date that policy would not seem optimal since by making the property rights to all past innovation free, productive capacity would be increased. Thus, the optimal policy would be time inconsistent.

More recent investigations include those by Lucas and Stokey (1983), Turnovsky and Brock (1980) and Calvo (1978). In a continuous time context, Calvo argues that a government which wishes to set an optimal tax rate will always be faced with a time inconsistency. It will choose to forego a distortionary income tax for an inflation tax in the present, promising lower inflation in the future. Inevitably, Calvo argues, the government will have an incentive to renege on the promise of lower inflation and resort again to the inflation tax.

Turnovsky and Brock argue that if bonds are included as a mode of finance in a model in which liquidity preference is derived from inclusion of real balances in the utility function, then all optimal policies will be time inconsistent unless the rate of nominal monetary growth is included as an instrument. Thus, a k rule would promote time inconsistency.

Lucas and Stokey, in a discrete time model make a similar argument that when setting optimal tax rates, there is an incentive for government to issue nominal debt which it effectively reneges on by causing a surprise inflation in the future.

They point out that while they and the other authors discussed above derive the government's optimal policy under the assumption that neither the government nor the public conceive of the possibility of an inconsistency, the consistent policies have the property of being sub-game perfect Nash strategies. In other words, they are the same policies which would have been derived had the government taken into account the optimization procedure of future governments.

Papers which model a government which explicitly takes into account the consistency of an optimal policy derived at an initial date include those of Goldman (1980), Rotemberg (1983), Barro (1983) and Barro and Gordon (1983). These authors compare optimal (and possibly inconsistent policies) with those which are derived under the constraint that they be sub-game perfect strategies. As Lucas demonstrates, only the consistent optimal policy will correspond with the sub-game perfect policy or strategy.

The model introduced in the third essay is similar to that used by Turnovsky and Brock, it is assumed that the economy is deterministic and evolves in continuous time with no storage of goods. Its novelty lies in the adoption of a transactions technology suggested by Fried and Howitt (1983) and the examination of the effect of that technology on the time consistency issue.

Unlike Turnovsky and Brock, who assume real balances bear a utility yield, it is assumed that real consumption is less than real expenditures by an amount which may be minimized by holding real balances and bonds. In other words, there exist transactions costs which depreciate one's holdings of the consumption good.

The representative household maximizes, in continuous time and with perfect foresight,

$$\int e^{-\delta t} u(c,f,g)dt$$

where c, f and g represent the level of consumption, leisure and government production of a non-storable public good at each point of time (time subscripts have been omitted).

The household makes expenditures (x) on a non-storable consumption good such that,

$$c = x - h(x,m,b)$$
$$\equiv \ell(x,m,b)$$

In other words, expenditures depreciate by an amount h(.) which depends on the level of expenditures and the household's stocks of real balances m and bonds b. It is assumed that by varying those stocks the depreciation may be minimized. For example, for given levels of x and b, the marginal liquidity yield of money will be positive for $m < m^*$ but negative for $m > m^*$. In other words, an increase in real balances will increase consumption until a point is reached when real balances are too high to economize on transactions costs.

The household is constrained further by a budget constraint,

$$\mathring{m} + \mathring{b} + x = (1-\tau)[(1-f) + ib] - \pi(m + b)$$

which states that purchases of real balances, bonds and goods must equal after-tax income on labour (a linear technology is assumed) and interest income minus the depreciation of nominal assets due to inflation π.

The model is closed by imposing market clearing on two of the three markets, the goods market and the money market,

$$1-f = x + g$$

$$m(\mu-\pi) = \mathring{m} ,$$

where μ is defined as the rate of growth of nominal balances. Note that the transactions technology is 'real' such that expenditures by consumers do not equal consumption, real resources are used in transaction.

The government's instruments include τ, μ and g, all three of which may be independently varied. As Calvo points out, the government is constrained at all points of time except at t = 0 to effect continuous paths of the price level. However, at t = 0, the initial price level is 'free' to jump to any market clearing level discontinuously. Following Turnovsky and Brock, it is further assumed that the government is also able, at t = 0 to use open market operations to change the bond/money ratio. In subsequent periods, m and b evolve according to their equations of motion in a continuous fashion. Thus, it is demonstrated that at t = 0 the government may set the initial values of m and b at any level deemed optimal.

The essay is divided into a number of sections which derive the optimal policy from the standpoint of government setting its instruments to maximize the representative household's lifetime utility. The optimal policy is then examined under different assumptions regarding transactions technology to see how the time consistency of policy is affected.

In other words, the optimal policy derived at t = 0 is examined from the vantage of a future date to determine whether the same policy is still optimal. Calvo's criteria for time consistency is adopted. He notes that since the government is free at an initial planning date to set nominal assets at any level, the marginal value of changes in those assets at t = 0 will be zero. However, for all t > 0 the government will no longer be free, when setting optimal policy at t = 0, to vary m and b discontinuously, so that the optimal plan will not, in general, set the marginal value of jumps in $\dot{m}$ and b equal to zero for $t > 0$.

If, on the other hand, the government re-evaluates the optimal plan at $t = t' > 0$ it will not be constrained anymore as to the values of m or b at $t = t'$. Thus, unless the optimal plan derived at $t = 0$ set the marginal value of jumps in m and b equal zero for all t, the optimal plan will be time inconsistent.

Thus, to determine whether or not the optimal plan is time inconsistent, the additional constraint is imposed, after maximization, that the marginal value of such jumps is zero for all t. If the plan is consistent with this constraint then it will be time consistent.

If bonds bear a real liquidity yield, so that $\ell_b \neq 0$ for all b, it is demonstrated that all optimal policies will be time inconsistent. The government will attempt to utilize a current inflation tax and promise lower inflation in the future, and will, in general, have an incentive to renege on that policy. This is in stark contrast to Turnovsky and Brock's result.

If, on the other hand, only real balances bear a liquidity yield (i.e., $\ell_b = 0$ for all b) the Brock and Turnovsky result will hold, that if the government's instruments include the rate of monetary growth the optimal policy will be time consistent. In this context, a time consistent optimal policy is one in which the economy moves immediately to a first best steady state in which the steady state level of real bonds is negative. This yields the government non-distortionary revenue in the form of interest payments from the private sector to the public. This first best is now feasible and optimal since bonds do not bear a liquidity yield which would prohibit the government from causing them to be negative.

These results are contrasted with an economy whose transactions technology mimics that of a cash-in-advance economy. In other words, the constraint that $c = \iota(\ )$ is replaced with $m \geq c$ and $c = x$. In this case, the transactions technology is solely pecuniary in that given the level of aggregate production and consumption, a change in real balances and/or bonds does not affect utility. This is contrasted with the transactions technology above for which a change in real assets would enable economizing on expenditures and an increase in public good production. In this economy, all optimal policies will be time consistent unless the government's instruments include only the level of public good production.

Thus, it is shown that the Brock and Turnovsky result is a special case, in which bonds do not bear a liquidity yield when transactions costs are real. It is also demonstrated that time consistency of optimal policy in a continuous time model is dependent on the type of transactions technology assumed.

An interpretation of these results may be thought of in terms of a similar experiment in the context of an optimal growth model with capital accumulation. Consider the problem of a consumer which desires to maximize utility, a separable function of consumption through the choice of savings rates which will affect capital accumulation. If the consumer is given an initial fixed stock of capital, the optimal choice of savings rates will imply an asymptotic approach to a 'modified golden rule' level of steady state consumption and capital stock. In general, this control problem will be consistent, a re-evaluation of the plan given the historically given capital stock will lead to no change in the optimal plan.

However, if the initial capital stock is 'free', to be chosen optimally instead of given to the consumer, the optimal plan will not, in general, be consistent. At the initial date the initial capital stock will be some 'bliss' value which maximizes utility. The optimal plan will imply, again, a gradual approach to the golden rule stock of capital. Re-evaluation of the plan will lead to a divergence from this path if the capital stock is free, and not governed by the differential equation which relates capital growth to savings. The consumer will, in general, desire to increase capital discontinuously to the bliss level and restart the plan which had been optimal initially. Thus, unless the bliss level of capital corresponds to the golden rule stock, the optimal plan will not be consistent.

This case is analogous to that discussed in the third essay. If the bond/money ratio is 'free', the government will cause it to jump discontinuously so as to maximize welfare. Unless that 'bliss' level of the bond/money ratio corresponds to the optimal steady state level of the bond/money ratio, the optimal plan will be time inconsistent.

TABLE 1

NOTATION

Essay 1	C_t^i	consumption at t of an individual 'born' at t+1-i (i = 1,2,3)
	P_t	price level at t
	T_t, τ_t	nominal and real taxes at t
	B_t, b_t	nominal and real bonds issued at t
	M_t, m_t	nominal and real balances at t
	i_t, r_t	nominal and real two period rate of interest at t
	μ	the rate of nominal monetary growth
Essay 2	c	current consumption in a given market
	n	individual labour supply
	y	individual output
	p	market price level
	τ	current tax on savings (past income)
	μ	index of the proportion of government expenditure financed by money creation
	γ	random expenditure shock
	θ	random distribution shock
Essay 3	c	consumption
	f	leisure
	g	government public good production
	x	real expenditures

Table 1 (cont'd)

m	:	real balances
b	:	real bonds
τ	:	income tax rate
i	:	nominal interest rate
π	:	inflation rate
μ	:	rate of nominal monetary growth

ESSAY I

INFLATION, FISCAL POLICY AND THE REAL RATE ON A NOMINAL ASSET

The issue to which this essay is addressed is the relationship between the real rate of interest and the rate of price inflation. It is often hypothesized that an increase in the rate of inflation will lead to an unambiguous change in the real rate. Most of such hypotheses have either predicted an invariant real rate (strict quantity theorists) or that the real rate declines in the face of price inflation (the Mundell-Tobin effect).

As can be seen in Table I.1, neither of the two orthodox views have been especially evident in recent years, either in the U.S., or in Canada. While there has been a slight negative relationship between the ex-post real rate and the inflation rate in Canada and the U.S. (correlation coefficients of -0.08 and -0.48 respectively), high rates of inflation have been associated with both high real rates (the 1980s) and with negative real rates (1974 and 1975). While no conclusions may be confidently derived from a casual observation of yearly data, ignoring the distinction between actual and expected rates, etc., it does appear that there may exist a relationship between real rates and inflation which is not captured by the two orthodox views mentioned above.

Recently, several authors have argued that the real rate may rise as inflation increases, due to tax distortions or to liquidity costs of

TABLE I.1: EX-POST REAL RATES ON CANADIAN AND U.S. TREASURY BILLS*

	Canada		U.S.	
	Inflation	Real Rate	Inflation	Real Rate
1983	5.8	3.5	3.2	5.4
1982	10.8	2.8	6.1	4.6
1981	12.4	5.3	10.4	3.7
1980	10.2	2.6	13.1	-1.5
1979	9.1	2.6	11.2	-1.2
1978	9.0	-0.3	7.6	-0.4
1977	8.0	-0.7	6.5	-1.2
1976	7.5	1.4	5.8	-0.8
1975	10.7	-3.3	9.2	-2.4
1974	10.9	-3.1	10.9	-3.0
1973	7.5	-2.0	6.3	-0.7
1972	4.9	-1.34	3.3	0.8
1971	2.9	0.7	3.6	0.8
1970	3.3	2.7	5.9	0.5
1969	4.5	2.7	5.4	1.3
1968	4.1	2.2	4.2	1.2

* The 'real rate' on T-Bills is equal to the yearly average nominal T-Bill rate minus the yearly average of the consumer price inflation rate.

Source: International Financial Statistics, I.M.F. June 1968-June 1983.

real assets. This paper is an attempt to lend further credence to this view, arguing instead that the real rate may rise due to the budgetary constraint of the government. The model adopted is a general equilibrium, full employment variant of Samuelson's consumption-loan model. The model uses the overlapping generations paradigm to generate an asset demand for currency and nominal bonds in a deterministic dynamic context.

Section I contains a brief survey of theoretical and empirical investigations into the above issues. Section II contains a presentation of the model, Sections III and IV contain the comparative static results in both a non-inflationary and inflationary context.

I. INTRODUCTION

The Mundell-Tobin effect may be defined as the theoretical proposition that an increase in monetary expansion and inflation will tend to lower the real rate. However, Fisher (1930) may have been the first to predict a decline in the real rate in the event of an increase in the inflation rate. His empirical findings show,

> evidence general and specific, from consulting p' (the rate of inflation) with both bond yields and short term interest rates, that price changes do, generally affect the interest rate in the direction indicated by a priori theory. But since forethought is imperfect, the effects are smaller than the theory requires and lag behind price movements, in some periods, very greatly.[1]

His argument was that asymmetric expectations between borrowers and lenders led to a decline in the real rate. Thus, while the increase in inflationary expectations will bid the nominal rate up in the loanable funds market, it will not fully adjust.

In a portfolio balance framework, Mundell (1963) argues that an increase in the anticipated rate of inflation causes the anticipated return on real balances to fall so that the demand for money falls. To jointly clear the asset and goods markets requires that the nominal rate rise by less than the anticipated rate of inflation so that the real rate falls. The only way for $di = d\pi^e$ is for the interest elasticity of money or the real balance effect on savings to equal zero.

This conclusion is echoed by several authors working within similar frameworks. Sargent (1972) appends an IS/LM model by considering a dynamic adjustment process for permanent income and investment to show that the nominal rate depends on the long-run rate and lagged inflation. Only in the case of zero interest elasticity of money demand will the adjustment of the nominal rate be immediate and complete. Levi and Makin (1978, 1979) isolate three different mechanisms by which inflation might affect the real rate; a) the Mundell effect, b) a Phillips effect, when real output increases due to money illusion, c) a Friedman effect, when uncertainty associated with increased inflation reduces investment. However, their empirical tests are only able to confirm their theoretical predictions during the 1950s and '60s. Summers (1981) performs a similar exercise in a dynamic context, and finds that while the short-run adjustment of i is ambiguous, in the long run the net of taxes real rate should fall.

Tobin (1965) examines the same question in a growth context, arguing that as the real return to money falls it is supplanted by capital as a mode of saving, driving the marginal product of capital down. Sidrauski (1967) shows that in the steady-state the real return to all assets must equal the (assumed constant) rate of time preference, fixing the real rate on capital. Ignoring non-pecuniary returns to other assets implies that an increase in [illegible] will be met by an equal increase in the nominal rate.

The Sidrauski result must be modified when the productive or liquidity services of nominal assets are considered vís-a-vis those of the real asset. Patinkin and Levhari (1972) argue that the Tobinesque experiment yields an ambiguous result on the capital stock if liquidity services of money are considered. Johnson (1967) makes a similar conclusion, arguing that if the utility yield on real balances is considered as part of income the net impact of inflation and growth of an increase in monetary expansion may be ambiguous. Fried and Howitt (1981) argue that if the liquidity services provided by money and a nominal interest bearing asset exhibit certain properties then in a Sidrauski type of model the real return on the nominal asset may exhibit the Mundell-Tobin effect. In general, they argue that the real rate on the nominal asset is not fixed since bonds and real assets are not perfect substitutes.

Sargent (1976) presents a neo-classical growth model with a Keynesian investment function, investment depending on the difference between the marginal product of capital and the real rate of interest. Comparing adaptive expectations with perfect foresight, it is argued that the real rate is constant with respect to changes in $\cdot\cdot$ only with perfect foresight since real variables are invariant to nominal shocks under that assumption.

Martins (1980), using an overlapping generations model with only two assets, money and bonds, argues that "the nominal rate of interest is determined by the relative supply of bonds with respect to money, and bears no relationship to the rate of inflation". This result appears to depend crucially on two assumptions, a) the demand for money is interest inelastic, b) and only the transition between changes in stocks of nominal assets are considered, not changes in their growth rates. The analysis presented in Section IV is an attempt to discover the implications of relaxing the two assumptions; by considering a more general utility function and comparing steady-state equilibria.

Feldstein (1976, 1980) presents a Tobin type growth model, examining its steady-state properties. Introducing nominal interest bearing government debt as a third asset allows him to examine the effects of different types of debt financing. He argues that the type of financing used to stimulate inflation will affect the sign of the change in capital stocks and the real rate on nominal assets.

Stockman (1981) also predicts that the real rate could rise with an increase in the rate of inflation. Using a neo-classical growth model with a cash-in-advance constraint, he shows that if nominal balances are required to purchase capital, the increase in the inflation rate may increase the opportunity cost of capital. Thus, as inflation increases, the steady-state stock of capital may fall, raising the real rate.

The empirical consensus tends to support the Mundell-Tobin hypothesis over the predictions of the Quantity Theory. Fama (1975, 1977) uses the maintained hypothesis that the expected real rate is independent of the rate of inflation, arguing that the lack of serial correlation of ex-post real rates supports this assumption. Several authors question his results with regards to methodology and the observation period that Fama restricts himself to, including Nelson and Schwert (1977), Carlson (1977) and Mishkin (1981). Fama (1976) himself cast doubts on his earlier conclusions by showing that expected future inflation variance will affect negatively the real rate on one period bonds.

Lucas (1980) argues that by filtering high frequency variation in the inflation rate and the Treasury Bill rate, a one to one relationship is evident. However, as discussed above, no statistical tests are performed to verify his observation. Summers (1981), working with filtered data, argues that, since his estimated coefficients on the inflation rate regressed on the real rate were generally less than unity, the real rate was negatively related to the rate of inflation. His attempts to verify this conclusion by regressing proxies for the real rate on the expected rate of inflation appeared to depend on the sample period and the proxy. He concludes that the inconclusive evidence is due to investor money and/or inflation illusion.

Nelson and Schwert (1977) point to a slight but significant correlation between changes in the nominal rate and changes in the rate of inflation as evidence of a Mundell-Tobin effect. Shiller (1980), by isolating two attempts to control real rates, the introduction of the Federal Reserve System and the Accord of 1951, argues that observation of ex-post real rates does not reveal a long term ability of monetary authorities to control the real rate. Using the Granger-Sims criterion of causality does yield the conclusion that changes in monetary growth will influence the real rate if it is unexpected. Mishkin (1981) argues that there is a significant negative correlation between the ex-post real rate and lagged inflation, confirming the Mundell-Tobin effect. He argues that Fama's results depend entirely on the sample period chosen, a period of relatively low inflation variance. Mishkin (1984) confirms the conclusion that there is a significant negative correlation between the real rate and inflation in a multi-country study.

In most general equilibrium models, the Mundell-Tobin effect appears to dominate. However, while the data may be said to reject the hypothesis of a constant expected real rate in favour of a negative relationship with inflation, they do not appear to offer conclusive evidence regarding the dependence of the real rate on changes in the rate of inflation. In the following sections, it is argued that no one-to-one relationship between the rate of inflation and the real rate should be expected. The rate of inflation does in many cases have an unambiguous effect on the real rate but that the direction of that effect will depend on the type of inflationary policy a government pursues.

II. THE MODEL

The analysis below is a slightly more general adaptation of Samuelson's (1958) exact consumption loan model of the rate of interest. Such models are predicated on the assumption that individuals are finitely lived, and that generations overlap. This yields the result that since generations will trade with each other only once, the issuance of private debt by the old is ruled out. Thus, in the non-growing economy, with a non-storable commodity, the issuance of public debt or fiat currency will be welfare improving as a costless means of transferring consumption from one period to the next. In a growth context, where there exists a storable asset (capital) it can be shown that the issuance of fiat money will increase welfare if the economy is over capitalized, since it allows individuals to economize on their holdings of real assets.

To allow for an interest-bearing asset distinct from money, Martins uses the assumption that individuals live for three periods. Bonds are only redeemable for goods after being held for two periods after issuance whereas money may be redeemed for goods within one period. The analysis below is essentially that of Martins except that the extreme assumption regarding the specification of the utility function is relaxed. Given the generality of the utility function, the analysis is confined to comparing steady-state equilibria.

There are two types of agents in the economy, private individuals, who rationally maximize their objective function, and the government. Each individual lives three periods, a generation being born each period.

Population growth is assumed zero so that without loss of generality, the number of individuals in each generation is normalized to unity.

The aggregate supply of the economy is exogenous, accruing to the generation just entering economic life, and is also normalized to unity. To an individual's first period income is added a transfer/tax from the government. The choice problem facing the individual is how to allocate consumption over his three periods of life given that storage of the consumption good is impossible. The sole assets available are fiat currency and bonds, issued by the government.

As stated above, it is assumed that the latter asset requires two periods to mature while the former is convertible to the consumption good within one period. This assumption regarding the maturity of the interest bearing debt may be motivated in terms of:

a) institutional restrictions regarding maturity and transfer of ownership,

b) a cash in advance constraint such that money is the only medium of exchange between bonds and the consumption good with the exchange between bonds and money requiring one period.[2]

The analysis is deterministic and thus in the absence of risk and other transactions costs, to generate a rate of return on bonds which is greater than the negative of the inflation rate requires that their real supply be greater than the real demand for third period consumption by individuals entering the economy at a zero nominal rate of interest.

In other words, for a given rate of inflation, at a zero nominal rate, the supply of assets available to transfer income to the third period must be greater than the demand for third period consumption. In such a case, to clear the market, the bonds will be sold at a discount so that the nominal rate of interest will rise above zero. The sole difference between money and bonds is their liquidity which implies that the least liquid asset will bear a positive nominal return given that its supply is great enough. The analysis below is confined to such a situation.[3]

a) The Individual Problem

The individual chooses his consumption path so as to maximize his utility function, which is assumed to have the properties necessary for an interior maximum for each period's consumption and is additively separable, such that,

$$U = U_1[C_t^1] + U_2[C_t^2] + U_3[C_t^3]$$

subject to the constraints that,

$$P_t C_t^1 = P_t + T_t - M_t - B_t/(1+i_t)^2$$

$$P_{t+1} C_t^2 = M_t$$

$$P_{t+2} C_t^3 = B_t$$

or more simply,[4]

(1) $$P_t C_t^1 = P_t + T_t - P_{t+1} C_t^2 - P_{t+2} C_t^3/(1+i_t)^2$$

where,

C_t^i : is planned consumption in the ith period of life by a newborn individual at t,

M_t : is nominal fiat currency purchased at t,

B_t : is the number of government bonds sold at t, promising one unit of fiat currency in period t+2,

T_t : is the government net transfer at period t,

$1/(1+i_t)^2$: is the discount or price of a government bond, so that i_t is the one period nominal interest rate.

Dividing (1) by P_t yields,

$$C_t^1 + (1 + \pi_t)C_t^2 + \frac{1}{(1+r_t)} C_t^3 = 1 + \tau_t$$

where,

$\tau_t = T_t/P_t$

$1 + \pi_t = P_{t+1}/P_t$

$1/(1+r_t) = (1+\pi_t)(1+\pi_{t+1})/(1+i_t)^2$

so that r_t is the two period real rate of return on a two period bond.

The first order conditions for an interior maximum are,

$$\frac{U_2'}{U_1'} = 1+\pi_t$$

$$\frac{U_1'}{U_3'} = 1+r_t$$

where the marginal rate of substitution between the first period and subsequent periods equals the available real rates of return. Between the first and second periods the individual may only store fiat currency so that the real rate of return is $1/(1+\pi_t)$. Between the first and third periods the real rate is $(1+r_t)$ which is assumed to be greater than $1/(1+\pi_t)(1+\pi_{t+1})$.

Notice, if the nominal rate falls to zero, the real rate on bonds equals that on money and the individual will be indifferent between money and bonds. As stated above, this type of situation is ignored by assuming that bond supply is so large as to require a positive nominal rate to clear the market.

The consumption decisions for individuals of all generations is as described above since the analysis is confined to the steady-state and it is assumed that individuals' decisions exhibit dynamic consistency.

b) The Government

The government is assumed to satisfy its budget constraint, that net revenues equal zero. Thus, dividing fiscal from financial revenues yields,

$$T_t = M_t - M_{t-1} + B_t/(1+i_t)^2 - B_{t-2}$$

so that the transfers/taxes collected equal sales of currency and bonds minus debt servicing. If,

$$B_t = (1+\beta)B_{t-1}$$

$$M_t = (1+\mu)M_{t-1}$$

then substituting for past values of B and M and dividing by the current price level yield the real government budget constraint,

$$(2) \qquad \tau_t = m_t \frac{\mu}{(1+\mu)} + b_t \left(\frac{1}{(1+i_t)^2} - \frac{1}{(1+\beta)^2}\right)$$

where m and b are the real stocks of money and bonds respectively.[5]

c) Market Clearing

To ensure market clearing, aggregate supply must equal the consumption demand by members of each generation,

$$(3) \qquad C_t^1 + C_{t-1}^2 + C_{t-2}^3 = 1$$

where, from the individual budget constraints,

$$(4) \qquad C_t^1 = 1+\tau_t - m_t - b_t/(1+i_t)^2$$

$$(5) \qquad C_{t-1}^2 = m_t/(1+\mu)$$

$$(6) \qquad C_{t-2}^3 = b_t/(1+\beta)^2$$

where using (4), (5) and (6) to solve out C^1, C^2 and C^3 in (3) yields the real government budget constraint, equation (2).

d) Steady-State

As stated above, the analysis will be confined to the comparison of steady-state equilibria. In the steady-state, real bond and money stocks are constant, implying that the rates of growth of nominal stocks of both assets equal the inflation rate. Dropping the time subscripts for the sake of convenience, the steady-state may be described by the following system,

$$(7) \qquad C^1 + (1+\pi)C^2 + C^3/(1+r) = 1+\tau$$

$$(8) \qquad C^2 = m/(1+\pi)$$

$$(9) \qquad C^3 = b/(1+\pi)^2$$

$$(10) \qquad U_1'[C^1]/U_2'[C^2] = 1/(1+\pi)$$

$$(11) \qquad U_1'[C^1]/U_3'[C^3] = 1+r$$

$$(12) \qquad C^1 + C^2 + C^3 = 1$$

$$(13) \qquad \tau = m\frac{\pi}{(1+\pi)} - \frac{b}{(1+\pi)^2}\frac{r}{1+r}$$

where $\beta = \mu = \pi$. The above is a system of seven equations, six of which are independent (since (7), (8), (9) and (12) imply (13)), in eight unknowns (C^1, C^2, C^3, r, μ, τ, m and b). It is assumed, given any two of the five possible policy variables (r, μ, τ, m and b) there exists a unique equilibrium such that C^1, C^2 and C^3 are strictly positive.

III. NON-INFLATIONARY STEADY-STATES

As a didactic exercise, the comparative statics of the model in a non-inflationary environment will be considered. The system from which the comparative static results are derived may be found by solving (7) through (13) for,

$$\frac{U_1'[1+\tau-m-b/(1+r)]}{U_2'[m]} = 1$$

$$\frac{U_1'[1+\tau-m-b/(1+r)]}{U_3'[b]} = 1+r$$

$$\tau = -b \frac{r}{1+r}$$

It will be assumed for the sake of simplicity that T is a tax and therefore less than zero. This implies, given the first order conditions, both b and r must be positive, and the first period transfer/tax finances the government's debt service. The comparative static results, described below, are derived by taking the total differential of the above system and solving for the relevant partial. The qualitative results of the experiments below are summarized in Table I.2. For the sake of brevity, the computations and expressions derived have been omitted from the text.[6]

a) Open Market Operation (dM = -dB)

Consider the standard open market operation, whereby the government seeks to increase the stock of nominal currency by way of a reduction in the stock of nominal bonds.

The excess supply of nominal balances drives the price level up as individuals attempt to divest themselves of currency. The excess demand for interest bearing assets causes their price to rise and the real rate of interest to fall. In the new steady-state equilibrium the level of real balances have increased while real taxes have decreased. Thus, the price level rises, but in a smaller proportion than the increase in nominal currency due to the commensurate increase in τ and therefore in first period income.

TABLE I.2: NON-INFLATIONARY STEADY-STATE COMPARATIVE STATICS

	dM = -dB	dT = 0		dB = 0	dM = 0
$\frac{dP}{dM}$	+	+	$\frac{dP}{dT}$	+	-
$\frac{dr}{dM}$	-	-	$\frac{dr}{dT}$	-	-
$\frac{dT}{dM}$	+	0	$\frac{dM}{dT}$	+	0
$\frac{dB}{dM}$	-	+	$\frac{dB}{dT}$	0	-
$\frac{dm}{dM}$	+	+	$\frac{dm}{dT}$	+	+
$\frac{d\tau}{dM}$	+	+	$\frac{d\tau}{dT}$	+	+
$\frac{db}{dM}$	-	-	$\frac{db}{dT}$	-	-

b) Helicopter Operation ($dT = 0$)

Consider now a helicopter operation, where the government increases the nominal units of non-interest bearing debt under the constraint of a constant nominal tax/transfer.

Again, the excess liquidity increases the price level as individuals attempt to spread their increased wealth to other periods of life. By the same token, as individuals increase their demand for third period consumption, the price of bonds rises and the interest rate falls. However, this creates an increased need for government debt service since $dT = 0$, thus for market clearing, the stock of nominal bonds must increase.

In the new steady-state real tax receipts will fall ($d\tau > 0$, $\tau < 0$) and real debt service will also fall as r and b decline. As above, the level of real balances rises.

c) Decreased Nominal Taxes ($dB = 0$)

A money financed expansionary fiscal policy in this framework is an increase in T ($dT > 0$, $T < 0$) holding the nominal stock of bonds constant. Ultimately, given zero steady-state monetary growth, the reduced tax revenue will be reflected in a change in debt servicing. The transition to that new steady-state is financed, however, by an increase in nominal balances.

As taxes are reduced, individuals attempt to spread this increased income to later periods of life. Consequently, the price level rises and the real rate of return falls. The increased price level is not so high as to reduce the steady-state level of real balances. The real tax is reduced ($d\tau > 0$, $\tau < 0$) as are the stocks of real interest bearing debt outstanding.

d) Decreased Nominal Taxes (dM = 0)

An alternate mode of financing a decrease in first period taxes would be to use a change in the stock of nominal bonds, constraining the stock of nominal currency to be constant. As T rises and B falls, the price level begins to decrease as individuals attempt to transfer consumption to the second period by purchasing real balances. The real rate of return falls as individuals attempt to increase their holdings of real bonds. Note also that in this case, the bond financed fiscal policy (dM=0) will yield an aggregate demand effect. Since the present value to an individual of his tax liabilities and bond purchase and resale is

$$P.V. = \tau - \frac{b}{1+r} + \frac{b}{1+r}$$

the net change in wealth is the change in his tax liabilities. Since in example d) those tax liabilities fall ($d\tau > 0$, $\tau < 0$) aggregate demand increases as net wealth increases.

In the new steady-state equilibrium the real value of first period taxes falls ($d\tau > 0$, $\tau < 0$) while the real value of bonds and money rises as individuals increase their demand for second and third period consumption and therefore, for nominal assets.

The first two experiments correspond roughly to those performed by Patinkin (1965) in his full employment model. When comparing deficit financing to an open market operation he argued that the real rate of interest would fall by a greater amount in the latter case due to the wealth effect (assuming government bonds were net wealth). He argues that "the government's reacquisition of its own debt is accompanied by a corresponding reduction in taxes, so that its budget continues to be balanced."[7] This corresponds to the above finding that dT and $d\tau$ are positive given an open market operation ($T, \tau < 0$). Patinkin's conclusion of a larger interest rate effect for an open market purchase will necessarily follow.

In the latter two experiments, it is interesting to note that the price effect of a decrease in taxation depends entirely on the mode of financing. The increase in first period income will create a higher price level only if it is 'financed' by an increase in nominal balances. If it is financed by a change in nominal interest bearing debt, the demand for currency will increase without an increase in supply so that the price level will fall.

Martins argued that the nominal rate of interest was not directly affected by a change in the rate of inflation but was determined by the bond/money ($\gamma = b/m$) ratio. The latter conclusion is easily demonstrated in a non-inflationary environment by using the government budget constraint to solve out τ and substituting $m\gamma$ for b, yielding the following system,[8]

$$\frac{U_1'[1-m(1+\gamma)]}{U_2'[m]} = 1$$

$$\frac{U_1'[1-m(1+\gamma)]}{U_3'[m\gamma]} = 1+r$$

where the effect on r of an increase in γ is,

$$\frac{dr}{d\gamma} = \frac{-m[U_1''U_3''(1+r)(1+\gamma)+U_1''U_2''+U_3''U_2''(1+r)]}{U_3'[U_1''(1+\gamma)+U_2'']}.$$

Thus, $dr/d\gamma$ is positive, since to induce individuals to consume more in the later period of life requires that the return on bonds be increased. However, in the next section, it will be demonstrated that Martins's conclusions do not necessarily follow given a change in the steady-state inflation rate and a more general utility function.

IV. INFLATIONARY STEADY-STATES

In this section the comparative statics of the steady-state equilibria in an inflationary environment will be examined. The system, derived as before, is;

$$\frac{U_1'[1+\tau-m-b/(1+\mu)^2(1+r)]}{U_2'[m/(1+\mu)]} = \frac{1}{1+\mu}$$

$$\frac{U_1'[1+\tau-m-b/(1+\mu)^2(1+r)]}{U_3'[b/(1+\mu)^2]} = (1+r)$$

$$\tau = m\,\frac{\mu}{1+\mu} - \frac{b}{(1+\mu)^2}\;\frac{r}{(1+r)}$$

As discussed above, the purpose is to determine the effect on the steady-state real rate of interest of a given change in the rate of inflation. Since the argument of this essay is that the effect of a change in π on r depends on the source of inflation, the method adopted to allow strict comparisons of steady-state equilibria is to compare equilibria given an exogenous change in the rate of inflation under different policy constraints. This will allow the isolation of the interest rate implications of the different constraints imposed and yield implications as to the inflationary potential of different policy regimes.

The comparative statics have been divided into inflationary fiscal policy, where the level of τ is not held constant, and inflationary financial policy, where τ is held constant. It is assumed throughout that individuals exhibit gross substitutability between periods. With respect to the separable utility function assumed thus far, this is equivalent to the assumption that the degree of relative risk aversion (R_i) is,

$$R_i = -\frac{U_i'' C^i}{U_i'} \leq 1$$

contrary to the relatively restrictive assumption made by Martins, that $R_i = 1$, so that individuals exhibited no gross substitutability. It is easily shown that in the model above this is equivalent to assuming that the demand for real balances is not a function of the real rate.

As in the non-inflationary environment, the government has five policy variables at its disposition; r, π, τ, m and b, with only two degrees of freedom. If, as in the examples below, the government chooses to increase the rate of monetary expansion, it cannot simply accommodate changes in the other four policy variables but must offer an additional constraint on the move towards the new steady-state equilibrium.[9] As a taxonomical device, the comparative static exercises below are divided into; a) inflationary fiscal policy, where the bond/money ratio is held constant, b) inflationary financial policy, where the tax/transfer is held constant, and c) other inflationary policies, whereby γ and ι are not constrained to be constant.

The comparative static results are derived in the same manner as in section III, taking the total derivative of the above system and solving for the relevant partial, given the policy regime imposed. The qualitative results are summarized in Table I.3.

a) <u>Inflationary Financial Policy ($d\tau = 0$)</u>

An inflationary financial policy is one which raises the rate of inflation and rate of monetary growth, leaving the level of real transfers constant. Given the assumption on R and gross substitutability, the only unambiguous effect is to reduce the stock of real balances. The interest rate effect is ambiguous,

$$\left.\frac{dr}{d\pi}\right|_{d\tau=0} = \{[U_3''\pi \frac{(1+r)}{(1+\pi)^2} + \frac{U_1''\pi}{(1+\pi)^4(1+r)} + \frac{U_1''r}{(1+\pi)^3(1+r)}] (\frac{U_2''m}{1+\pi} + U_2')$$

$$+ U_1''U_3'' \frac{(1+r)}{(1+\pi)^3} m + \frac{U_2''U_1''m}{(1+\pi)^5(1+r)} + \frac{U_2''U_3''m(1+r)}{(1+\pi)^5}\} \Delta^{-1}.$$

commitment to the level of tax versus money finance through time.

Private island demand is equal to the supply of real balances by the current old. If m represents the total nominal balance carried over from the previous period, then $m/2p$ and $m/2\hat{p}$ is the total private demand for goods in each island prior to taxation (the 'hat' distinguishes the two islands' price levels).

The tax rate on the wealth of current old is,

$$\tau = \frac{\gamma}{1+\gamma}(1-\mu) \quad , \quad 0 \leq \mu \leq 1$$

where γ is a random variable distributed continuously over the interval $[0,\bar{\gamma}]$ by a known probability distribution $f(\gamma)$, and μ is a non-stochastic fraction representing the degree to which government expenditure is financed through money creation.

By the same token $\mu\gamma m$ is the addition to the current money stock, used by the government to finance its expenditures. Thus as μ approaches unity τ approaches 0 and government expenditures become financed solely through money creation. As μ approaches zero, public expense becomes financed solely by taxation.

Each period, government receipts equal its expenditures so that the addition to the money stock plus tax revenue levied on nominal balances held by the current old must equal nominal government expenditure. Thus,

$$\tau m + \gamma\mu m = \left[\frac{\gamma}{1+\gamma}(1-\mu) + \gamma\mu\right]m$$

$$= \frac{\gamma}{1+\gamma}(1+\mu\gamma)m$$

$$\Delta = \frac{U_1''U_3''b}{(1+\pi)^4(1+r)} + \frac{U_1''U_2''b}{(1+\pi)^5(1+r)^2} + \frac{U_2''U_3''b}{(1+\pi)^5(1+r)}$$

$$- \frac{U_3'r}{(1+\pi)^2(1+r)} [U_1''(1+\pi+\frac{\pi}{r}) + \frac{U_2''}{1+\pi}] > 0$$

As R_2 approaches unity, $(U_2''m/(1+\pi)+U_2')$ approaches zero and the more likely will be interest rate effect be positive. Equivalently, the less responsive is bond demand, and the demand for first period consumption, to changes in the inflation rate the less likely will a negative interest rate effect be needed to clear the goods and asset markets.

As R_2 approaches zero, implying a large degree of substitutability, the real rate may have to fall to reduce the resultant increase in demand for bonds. However, if the inflation effect on the demand for first and third period consumption is large enough, the income effect of the increase in π and a large decrease in r may reduce the steady-state stock of real bonds and further reduce the stock of real balances.

The steady-state bond/money ratio will unambiguously rise if $\tau \leq 0$. If the level of real transfers is very large, it may be the case that γ falls so as to maintain that initial τ.

This result is illustrated in Figure 1. For the sake of diagrammatic convenience it is assumed that U{.} is homothetic and the level of τ (constrained to be held constant) is equal to zero. Given the assumption regarding preferences, the initial steady-state equilibrium may be examined with respect to a typical indifference curve in (C^2,C^3).

The individual will choose C^2 and C^3 so as to satisfy his budget constraint so that in equilibrium the slope of the indifference curve will equal $-(1+r)(1+\pi)$ where the marginal rate of substitution equals the relative prices of second and third period consumption. Market equilibrium occurs when the goods market clears and the asset market clears. The latter condition occurs when the government's budget constraint in terms of C^2 and C^3 is satisfied. Thus, using 8) and 9) in 13) yields,

$$C^3 = \frac{\tau}{r}(1+r)\,C^2 - \bar{\tau}\,\frac{(1+r)}{r}$$

Point a in Figure I.1 represents a steady-state equilibrium, where both the individual's and the government's budget constraints are satisfied.

Consider an increase in τ. Prior to the real rate adjusting, the government budget constraint will rotate, as shown, intersecting the indifference curve at point d. Again prior to r adjusting, the individual's budget constraint may now tangent at a point above or below d.

At c there will exist an excess demand for third period consumption and bonds requiring the real rate to fall. As r falls the slope of the individual's constraint flattens while the slope of the government constraint steepens so that the equilibrium lies between c and d, with the real rate lower.

If, prior to r adjusting, the individual's budget constraint tangents at a point b, there will be an excess supply of third period consumption and bonds. The real rate must rise, steepening the slope of the

Figure I.1

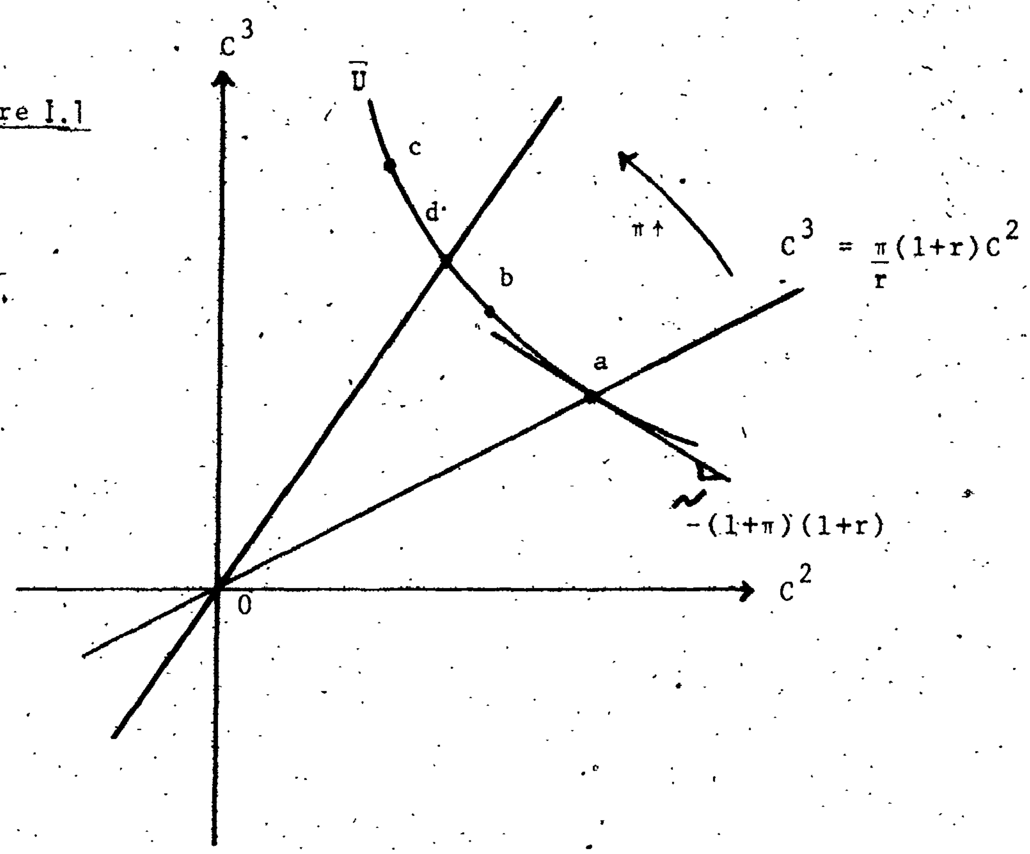
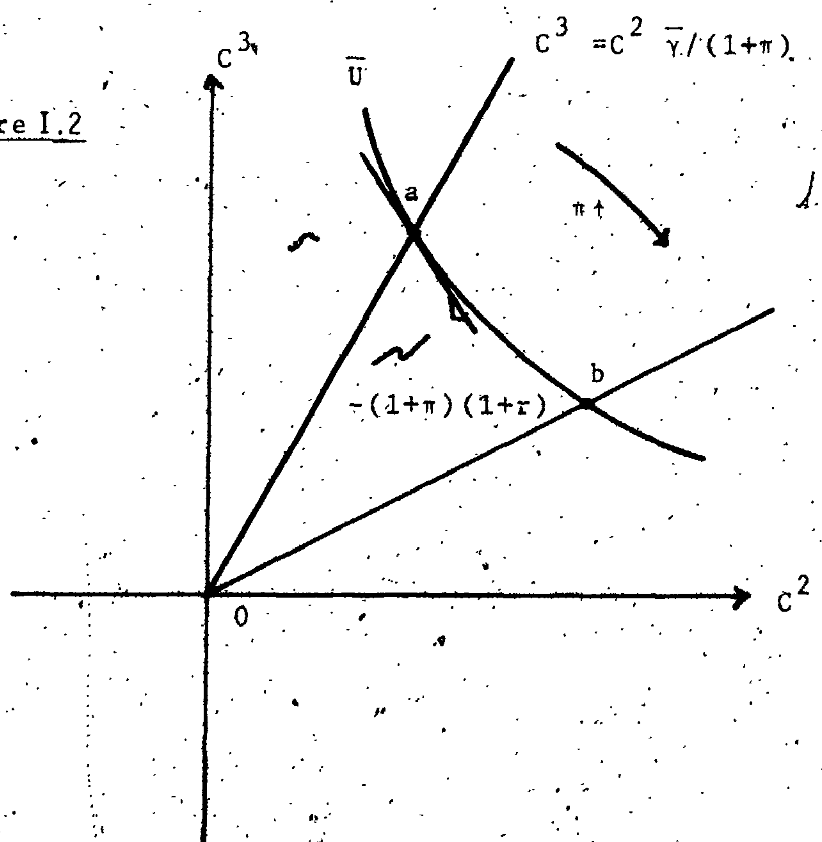


Figure I.2



individual's budget constraint, lowering the slope of the government's budget constraint. Thus the new equilibrium lies between b and d, at a higher real rate.

The fact that γ will rise if $\tau \leq 0$ is illustrated by noting that,

$$\frac{C^3}{C^2} = \frac{\gamma}{1+\pi}$$

which is the slope of the ray from the origin to point a, the initial equilibrium. If π increases, given γ the slope of the ray will fall. However, since the new equilibrium must lie between points c and b, the ray through the new equilibrium must have a greater slope than the original. Thus γ must increase so that the increase in inflation is used to finance a financial policy, an increase in γ.

While not a formal proof, it is obvious that the Mundell-Tobin effect depends on the degree of substitution between C^2 and C^3, or the steepness of the indifference map. If C^2 and C^3 are poor substitutes, the budgetary requirements of the government may induce the real rate to rise.

b). <u>Inflationary Fiscal Policy ($d\gamma=0$)</u>

In this case, the increased monetary expansion finances a change in the tax transfer under the policy constraint that γ, the bond/money ratio is constant. Under this policy regime, the Mundell-Tobin effect is unambiguous. As the opportunity cost of holding real balances rises faster than that of real bond holdings to maintain the initial γ, the real rate must fall,

$$\left.\frac{dr}{d\pi}\right|_{d\gamma=0} = \{-2b \frac{U_1''U_3''(1+r)}{(1+\pi)^3} - 2b \frac{U_3''U_2''(1+r)}{(1+\pi)^3} - (\frac{2b}{1+\pi} + m) \frac{U_1''U_2''}{(1+\pi)^3}$$

$$+ (U_2'' \frac{m}{(1+\pi)} + U_2') [\frac{U_1''}{(1+\pi)^2} (1 + \frac{\gamma}{1+\pi})$$

$$+ \gamma \frac{U_3''(1+r)}{(1+\pi)^3}]\} \Delta^{-1}$$

$$\Delta = -U_3'[U_1'' (1 + \frac{\gamma}{1+\pi}) + U_2''] > 0$$

As can be seen above, as R_2 falls, the interest rate effect is magnified. In other words, the greater degree of gross substitutability the greater the Mundell-Tobin effect. The net effect on equilibrium stocks of real balances and bonds also depends on R_2. If the degree of substitution from money to bonds is slight ($R_2 \simeq 1$), then m and b will both rise. If, however, individuals are extremely sensitive to changes in the inflation rate, both m and b will fall.

A sufficient condition for the level of real transfers to rise is that they be negative ($\tau \leq 0$) due to the fact that m will not vary in a relatively large manner given this policy constraint.

This result is illustrated in Figure I.2. For the sake of convenience it is assumed, as above, that U is homothetic. The initial equilibrium occurs at point a where the budget constraint is tangent to the typical indifference curve $\overline{U}$. Thus at a,

$$\frac{U_3'}{U_2'} = (1+r)(1+\pi)$$

the marginal rate of substitution equals the rate at which the consumer may transform C^2 and C^3. If $\bar{\gamma}$ is the fixed bond to money ratio, the ray from the origin to point a will have a slope,

$$\frac{C^3}{C^2} = \bar{\gamma}/(1+\pi)$$

If the monetary authorities increase the inflation rate to π', the consumer is constrained to consume C^3 and C^2 so that

$$\frac{C^3}{C^2} = \bar{\gamma}/(1+\pi')$$

and thus the new steady-state equilibrium will be at a point b where $r' < r$, or where the real rate has been reduced.

Note, if $\tau = 0$ initially, it can be demonstrated that an increase in π will imply an increase in τ. Since the government's budget constraint may be rewritten as,

$$C^3 = (C_2\pi - \tau)\,\frac{1+r}{r} = C_2\,\pi\left(\frac{1+r}{r}\right)$$

it is coincident with the ray through point a at the initial equilibrium. If the new equilibrium implies r is increasing as π increases then for $\tau = 0$, the slope of the government's budget constraint will increase. Thus, for it to intersect at point b, the new equilibrium point, requires that $\tau > 0$. Thus, the increase in inflation has financed an increase in the fiscal deficit.

c) Other Inflationary Policies

(i) $db = 0$

Consider an increase in monetary growth, where the government constrains the real stock of outstanding government interest bearing debt to be constant. In this case the standard Mundell-Tobin effect is evident, the real rate on interest bearing debt is reduced. However, the effect of an increase in π on τ, m and γ, the bond/money ratio is ambiguous given the assumption that R_1 is less than or equal to unity.

In fact, only if R_2 is significantly less than unity or as the degree of substitutability increases, will the effect on the equilibrium values of m and τ be negative. As the degree of substitutability rises, the more likely the individual will decide to consume in his first period of life, foregoing second period consumption. This would, as the substitution increased, require τ to fall (taxes increase) as financial revenues to the government started to decline.

(ii) $dm = -db$

Consider now an inflationary 'open market operation' where the inflation rate is financed by equal and opposite changes in real stocks of money and bonds. In other words, the sum of interest and non-interest bearing real government debt is held constant. Surprisingly, the effect on r of an increase in π will be ambiguous, depending on R_2 and the degree of gross substitutability between money and bonds.

If R_2 is relatively small then money and bonds will be close substitutes. In this case, the increase in π will increase the real rate to induce the public to hold bonds. By the same token, the equilibrium level of real balances will fall. If R_2 is close to unity, the opposite may occur. The increase in π will cause r to fall as individuals exchange bonds for money.

(iii) $d\tau = d\left(\frac{m\mu}{1+\mu}\right)$

Finally, consider a change in the level of real transfers that is constrained to be financed solely through the inflation tax. Given the constraint on the inflation tax, the revenue for interest bearing debt is also constrained to be constant. Thus, if π increases, b and r must move in opposite directions. Since bond demand increases with the increase in π, the real rate must be bid down.

The effect of the increase in π on τ and m will depend on the relevant elasticities of demand. If the cross elasticity of bond demand is great then the level of real balances will tend to fall as individuals substitute towards real bonds. The net effect on τ is ambiguous. By the same token, however, the larger is substitutability between assets, the more likely will it be the case that $d\tau < 0$, as the inflation tax falls.

V. CONCLUSION

The comparative static exercises summarized above imply that the relationship between the rate of inflation and the real rate of return on a nominal asset is sensitive to the type of policy which generates the inflation. For the most part, it would appear that the Mundell-Tobin effect dominates. That is to say that the substitution effect will tend to create an excess demand for bonds, bidding their price up and the real rate down. Furthermore, while the quantitative differences of the effects of the different policy regimes have not been discussed, the various constraints will also have a role on the magnitude of the interest rate effect.

Two sufficient conditions which guarantee a negative response of real rates to an increase in inflation and monetary growth: that the bond/money ratio be held constant or that there exist a large degree of substitutability between bonds and money. If either of these two conditions are not met, it is likely that the real rate will rise in response to an increase in monetary growth rates.

For example, it appears that R_i plays a major role in determining those quantitative effects, and in the case of several of the experiments, the qualitative effect on the interest rate. In particular, if R_2 is close to unity, implying a small degree of substitutability between assets as π increases, the effect of an increase in the inflation rate, given $d\tau = 0$ and therefore no first period income effect, is to induce individuals to hold real bonds, so to clear the asset and goods markets requires that the real rate increase.

However, an inflationary open market operation ($dm = -db > 0$), will tend to increase the real rate if R_2 is close to zero or if there exists a large degree of substitution as π increases. As the inflation rate increases, the demand for real balances decreases and the demand for first and second period consumption increases. However, given the constraint on m and b, the supply of real bonds will increase faster than the demand, therefore, to induce their purchase, the real rate must rise.

The above conclusions appear not to depend on the relatively simple utility function. For example, to abstract from functional form one could replace the first order conditions with explicit money and bond demand functions (eg. $m = m(r,-,-)$ and $b = b(r,-,-)$). Performing the above comparative static exercises would yield qualitatively equivalent results, that the effect on r of a given change in π will depend on the respective magnitudes of the cross price elasticities of bond and money demand and the policy constraint imposed by the government. Furthermore, there does not exist a monotonic relationship between π and the real rate.

In conclusion, although it appears that the steady-state change in the real rate on a nominal asset, with respect to an increase in monetary expansion, is frequently ambiguous, there can be no doubt that it is not independent of π. The qualitative and quantitative effect on r depends both on the degree of gross substitutability between periods, and therefore on interest elasticity of money demand and the inflation elasticity of bond demand, and on the policy constraints imposed by the government. Since the latter are not, in general, constant and the former may also vary with changes in income and prices, this may in part serve to explain the difficulty of researchers to confirm the Mundell-Tobin effect.

FOOTNOTES

1. Fisher (1930), p. 451.

2. Strictly speaking, the analysis below is predicated on the former assumption. A cash-in-advance constraint would change the budget constraints described below. Note further, that private debt is assumed not to exist. This should not affect any of the major conclusions discussed below.

3. Note that this requires the assumption that in the steady-state the nominal rate is strictly positive.

4. A more general specification would allow individuals to carry money balances between the second and third period of life. However, since the analysis is limited to steady-state equilibria where real bond holdings are positive ($i > 0$) the distinction is irrelevant. Only if the dynamic path to a steady-state were to be considered would such an option be relevant.

5. The government, as presented, apparently has no real role in the economy beyond balancing its books. For those to whom such a description is abhorrent, an alternative representation could include a flow of real services (x) from a public good which only the government may provide. Thus, the real budget constraint could be amended to,

$$\tau + x = m \frac{\mu}{1+\mu} - \frac{b}{(1+\mu)^2} \frac{r}{1+r}$$

while the market clearing condition would be,

$$C^1 + C^2 + C^3 + x = 1$$

and the utility function would then be hypothesized to be,

$$U = U_1(C^1) + U_2(C^2) + U_3(C^3) + U(x)$$

However, given a constant x, it is unlikely that the comparative static results above would be appreciably changed.

6. See the appendix.

7. Patinkin (1965), p. 293, fn. 21.

8. Note that, strictly speaking, the bond/money ratio is not b/m in the steady-state since b is only the current new issue and does not include the previous period's issue whose current real value is $b/(1+\pi)$.

9. In fact, the government's instruments are all nominal; the nominal level of transfers, the nominal level of money and bonds, and their growth rates. However, by adjusting these instruments the price-level also varies so that the government has control over their real values.

APPENDIX

In the non-inflationary steady-state the comparative static results in Table 2 are derived from the total differential of equations (7) to (12). For instance, considering the effect of an increase in nominal balances to finance an increase in the tax/subsidy by an open market operation, if

$$dM = -dB$$

then

$$\frac{dP}{dM} = U_2'' U_3'(1+r)P/\Delta_1$$

$$\frac{dr}{dM} = [U_1'' U_3''(1+r)^2(m + \frac{b}{(1+r)} - \tau) + U_2'' U_3''(1+r)^2(m+b) + U_1'' U_2''(1+r)(m + \frac{b}{1+r} - \tau)]/\Delta_1$$

$$\frac{dG}{dM} = [-U_2'' U_3''(1+r)(B+T)(m+b) + P U_1'' U_3'(m+b) + U_2'' U_3' rM - U_1'' U_2''((b + \frac{m}{1+r}) \frac{B}{1+r} + \frac{rBm}{(1+r)^2})]/\Delta_1$$

$$\Delta_1 = [U_1''(m + \frac{b}{1+r} - \tau)(1+r) + U_2''(1+r)m]P U_3'$$

if

$$dT = 0$$

then

$$\frac{dP}{dM} = [U_1'' U_3'' B + U_3'' U_2'' B + U_2'' U_1'' \frac{B}{1+r}$$

$$- r U_1'' U_3' P - U_2'' U_3' rP]/\Delta_2$$

$$\frac{dr}{dM} = -r[U_1'' U_3'' b(1+r) + U_1'' U_2'' (\frac{b}{1+r} - \text{[illegible]})$$

$$+ U_2'' U_3'' b(1+r)]/\Delta_2$$

$$\frac{dB}{dM} = - \frac{B}{r(1+r)} \frac{dr}{dM}$$

$$\Delta_2 = m B U_3'' U_1'' + m B U_3'' U_2'' + \frac{mB}{1+r} U_1'' U_2''$$

$$-rP U_1'' U_3'(m+b) - r U_2'' U_3' mP$$

Alternatively, if the government increases the level of nominal taxes under the constraint that

$$dB = 0$$

then

$$\frac{dP}{dT} = P[-U_1'' U_3'(1+r) - U_2'' U_3'(1+r)]/\Delta_3$$

$$\frac{dr}{dT} = -(\frac{1+r}{B+T})$$

$$\frac{dM}{dT} = -(1+r)P U_3'(U_1''(m+b) + U_2'' m)/\Delta_3$$

$$\Delta_3 = U_1'' U_3'' bB + U_2'' U_3'' bB$$

or under the constraint that

$$dM = 0$$

then

$$\frac{dP}{dT} = -(1+r)\ U_1''\ U_3'\ P/\Delta_4$$

$$\frac{dr}{dT} = [U_3''\ U_1''\ m(1+r)^2 + U_3''\ U_2''\ m(1+r)^2$$

$$+ U_1''\ U_2''\ m(1+r)]/\Delta_4$$

$$\frac{dB}{dT} = -\frac{dr}{dT}\ \frac{B}{(1+r)r} - \frac{1+r}{r}$$

$$\Delta_4 = -m\ B\ U_1''\ U_3'' - U_2''\ U_3''\ mB - U_1''\ U_2''\ \frac{Bm}{1+r}$$

$$+ r\ U_3'[U_1'(m+b) + U_2''\ m]$$

In the inflationary steady-state the equilibrium values of r, m and b are determined by,

$$\frac{U_1'[1+\pi-m-b/(1+\pi)^2(1+r)]}{U_2'[m/(1+\pi)]} = \frac{1}{1+\pi}$$

$$\frac{U_1'[1+\pi-m-b/(1+\pi)^2(1+r)]}{U_3'[b/(1+\pi)^2]} = 1+r$$

$$= m\ \frac{\pi}{1+\pi} - \frac{b}{(1+\pi)^2}\ \frac{r}{1+r}$$

The comparative static results in Section IV are derived by solving the system below under the appropriate policy constraint.

(A.1)
$$dm[U_1'' + \frac{U_2''}{(1+\mu)^2}] + db\,[\frac{U_1''}{(1+\mu)^2(1+r)}]$$

$$+ d\tau[-U_1''] + dr[-\frac{U_1''b}{(1+\mu)^2(1+r)}]$$

$$= d\mu[\frac{U_1''2b}{(1+\mu)^2(1+r)} + \frac{U_2''m}{(1+\mu)^3} + \frac{U_2'}{(1+\mu)^2}]$$

(A.2)
$$dm[U_1''] + db[\frac{U_1''}{(1+\mu)^2(1+r)} + U_3''\,\frac{(1+r)}{(1+\mu)^2}]$$

$$+ d\tau[-U_1''] + dr[-\frac{U_1''b}{(1+\mu)^2(1+r)^2} + U_3']$$

$$= d\mu[\frac{2b\;U_1''}{(1+\mu)^3(1+r)} + 2b\,\frac{U_3''(1+r)}{(1+\mu)^3}]$$

(A.3)
$$dm[-\mu] + db[\frac{r}{(1+\mu)(1+r)}] + d\tau[1+\mu]$$

$$dr[-\frac{b}{(1+\mu)(1+r)^2}] = d\mu[\frac{2br}{(1+\mu)^2(1+r)} + \frac{m}{1+\mu}]$$

ESSAY II

TAX VERSUS MONEY FINANCED FISCAL POLICY IN A NATURAL RATE MODEL

I. INTRODUCTION

The importance of the way in which governments finance their expenditure has grown in both the academic and political spheres. In recent years, in both the U.S. and Canada, attempts have been made to reduce the rate of nominal monetary growth to reduce inflation, making this issue of practical interest.

Examples of recent research addressing this issue include that by Sargent and Wallace (1981) who argue that given exogenous fiscal policy an attempt to reduce monetary growth in the short run may increase current and/or future inflation when the debt used to finance the deficit is monetized. While this result is hardly surprising given their assumptions it does imply that the link between monetary growth and inflation is in part dependent on the fiscal rules that the government adopts and the long term mix between money and debt.

Barro (1974) has noted that the real and nominal impact between tax and bond financing may be zero under certain restrictions. Essentially, if the current population discounts future tax liabilities such that the future tax burden in present value terms is effectively equal to the current tax burden, then a marginal increase in bond financing of government expenditure will mean that individuals will substitute savings, in the form of government debt, for savings, in the form of taxes.

McCallum (1982) has argued that unless the deficit which government is financing is appropriately defined, even in a Ricardian economy, in which taxes are perfect substitutes for bond financing, a temporary increase in the deficit will not be neutral with respect to prices even when bond financed.

Similarly, Chan (1983) has argued that Barro's result does not hold if future tax burdens are distributed randomly across the population. In this case, current taxes are not perfect substitutes for current bonds since the household is no longer indifferent between certain current taxes versus uncertain future taxes. Gertler and Aiyazari (1983) note that if current bonds are to be monetized in the future, Barro's result will not hold with respect to nominal values, thus, given nominal rigidities, real effects may result from a change in the mode of finance. They also note, as does Chan, that if future tax burdens are not distributed equally across the population, real effects of switching from tax to bond financing will occur.

Most of the above discussion may be described in terms of the so-called relevance of government policy. As Wallace (1981) describes, policy rules may be derived in which the nominal money stock is irrelevant to the market equilibrium price and output level. Simply, this result follows when the government exchanges fiat currency for private savings. If the government promises to return the real value of those savings for the fiat currency it originally offered, there is no need for prices or quantities to change. This interpretation is confirmed by Bryant (1983) who, in a simple two-period context, decomposes 'relevance' theorems into two categories. The first argue that any government policy is irrelevant while the latter argue that the mode of finance of government policy is irrelevant.

In what follows, the latter hypothesis is examined in the context of a 'natural rate model' proposed by Lucas (1972). Specifically, the distinction, if possible, between tax and money financing of a stochastic shock to the distribution of producers, as in Lucas's original exposition, it is the distribution of government expenditures which is the source of confusion between aggregate and relative disturbances.

Specifically, a policy rule is derived which, given full current information to current producers, leaves current output unchanged with a change in the degree to which current government expenditures are financed by taxes versus money creation. However, if current information regarding current shocks is obscured, it is shown that the mode of finance is no longer irrelevant to the determination of current output.

These results are interesting for a number of reasons. Firstly, they yield an insight with respect to the importance of information to irrelevance and equivalence theorems. This is similar in spirit to Leijonhufvud's (1981) description of the costs and consequences of inflation. While, given full information, a change in the mode of finance is irrelevant to the private sector's decision process, if a lack of information means that prices no longer yield the same information, private agents must 'hedge' and change their behaviour. The results are also interesting for pedagogic reasons given the prevalence of 'Lucas style' aggregate supply curves in ad-hoc macro-models and under-graduate texts (Parkin (1982)).

No other claims are made as to the aptness of the model chosen to investigate the role of information and government finance. Nonetheless, the results below are intended to serve as a benchmark for subsequent research.

Section II describes the economy and the government's expenditure rule, Section III defines the equilibrium. Sections IV and V explore the effects of changes in the mode of finance on output given full and less than full information. Section VI discusses the optimal finance mode.

II. THE MODEL

The model is adapted from Lucas (1972), Polemarchakis and Weiss (1977) and Azariadis (1981). As in the above, confusion between aggregate and relative price shocks leads to nominal disturbances having real output effects. However, in the version below, the nature of the shocks is amended so that the relative shock is the proportion of total government expenditure allocated to a given island while the aggregate shock is the size of that total level of government expenditure, which is financed with either taxes or money creation. It is shown that under such circumstances money and tax financing are equivalent in their output impact except when there is a signal extraction problem regarding the size of the monetary shock.

The economy is characterized by two islands which are informationally isolated within a period. It is assumed that the population is static, each individual is economically active (or graduated) for two periods. The two periods are distinguished by the assumption that during youth, in the first period of life, agents work, in perfect competition, but do not consume, saving the fruits of their labour in the form of fiat currency, for consumption in old age, the second period of life.

Generations overlap in the Samuelsonian (1958) fashion, the young being distributed evenly between the two islands. Given that the islands are assumed identical, ex-ante, the young migrate after their first period of life, prior to the realization of any stochastic shocks in the second period, so as to equalize the expected return on their savings. This implies that the private supply of cash balances is equal in both islands at the start of each period.[1]

The islands are distinguished only by the intervention of a government, whose workings are stochastic as far as the young are concerned. It is assumed that the government has a stochastic demand for goods produced each period and that it distributes its demand for goods in a random fashion between the two islands. Thus, not only is the size of government expenditure stochastic, so is its composition. This is analogous to an uncertainty of the government's choice of guns versus butter (defence versus public education). Alternatively, this uncertainty may be viewed as a stochastic regional distribution of expenditures. Therefore, the uncertainty regarding market or regional distribution makes the exact identification of the total level of government expenditure based on observations of market or regional expenditure impossible, confronting the agent with the need to infer government policy from available information.

The government finances its expenditure through a combination of tax and money finance. Tax revenue is based on the proportion of wealth that the current old bring to the current period. This may also be viewed as a tax on their nominal income last period, since in this simple framework, the two are equal. An important assumption is that the tax is stochastic from the vantage of last period. Thus, the current young also face a tax on their current earnings which is not revealed until after they have made their work/leisure choice.

What is known with certainty is the extent to which a given period's government expenditure is financed by tax versus money finance. However, this will not, in general, be required to be the same across time.

The young must choose the optimal level of labour given the relevant history of the economy; the money supply last period, the distribution of the government's intervention, and the realization of the island specific level of demand given the government's pre-announced commitment to the level of tax versus money finance through time.

Private island demand is equal to the supply of real balances by the current old. If m represents the total nominal balance carried over from the previous period, then $m/2p$ and $m/2\hat{p}$ is the total private demand for goods in each island prior to taxation (the 'hat' distinguishes the two islands' price levels).

The tax rate on the wealth of current old is,

$$\tau = \frac{\gamma}{1+\gamma}(1-\mu) \quad , \quad 0 \leq \mu \leq 1$$

where γ is a random variable distributed continuously over the interval $[0,\bar{\gamma}]$ by a known probability distribution $f(\gamma)$, and μ is a non-stochastic fraction representing the degree to which government expenditure is financed through money creation.

By the same token $\mu\gamma m$ is the addition to the current money stock, used by the government to finance its expenditures. Thus as μ approaches unity τ approaches 0 and government expenditures become financed solely through money creation. As μ approaches zero, public expense becomes financed solely by taxation.

Each period, government receipts equal its expenditures so that the addition to the money stock plus tax revenue levied on nominal balances held by the current old must equal nominal government expenditure. Thus,

$$\tau m + \gamma\mu m = [\frac{\gamma}{1+\gamma}(1-\mu) + \gamma\mu]m$$

$$= \frac{\gamma}{1+\gamma}(1+\mu\gamma)m$$

equals the level of nominal government expenditure, which is a stochastic fraction of nominal balances.

This expenditure is distributed between the two islands by a stochastic proportion $\theta/2$ and $(1-\theta/2)$ respectively where θ is distributed symmetrically and continuously over the interval [0,2] by a known probability distribution $g(\cdot)$.

It is assumed that both γ and θ are serially uncorrelated and independent. Note that the expected value of θ is unity so that prior to the realization of γ and θ the islands are identical.

Island demand is the sum of private and public demand. In terms of the typical island,

$$(1) \qquad \frac{m}{2p}(1-\ldots) + \frac{\theta\gamma}{2(1+\gamma)}(1+\phi\gamma)\frac{m}{p}$$

$$= \frac{m}{2p}\frac{(1+\phi\gamma)(1+\theta\gamma)}{(1+\gamma)}$$

$$= \frac{m}{2p} v$$

A special feature of this policy is that the ratio of private to total demand is invariant to changes in ϕ. From the above, the ratio is,

$$\frac{\theta\gamma}{1+\gamma}(1+\phi\gamma)/\frac{(1+\phi\gamma)(1+\theta\gamma)}{(1+\gamma)}$$

$$= \frac{\theta\gamma}{1+\theta\gamma}$$

The policy described above has the property that changes in ϕ do not affect incentives to substitute current leisure for future consumption in a direct manner, by changing the above ratio. In other words, for a given supply of output, a change in the mode of finance does not affect the distribution of output between the public and private sector. Thus, the effect on current output of a change in μ will be due to the informational effect that changes will cause.[2]

Current output follows from a decision by the current young to forego current leisure for future consumption. In their youth, individuals maximize a common utility function, choosing their labour supply (n) and future consumption (c', the 'prime' denotes next period's value) so as to maximize the expected value of,

$$u(c') - q(n)$$

subject to,

$$c' = \frac{yp}{p'}(1-\tau') \,, \; y = y(n)$$

The first constraint restricts savings to the nominal balances acquired through sales of current production. Next period's consumption equals the real revenue from current sales in terms of future prices (p') net of next period's tax (τ'). The second constraint states that individual output is subject to a production technology.

It is assumed that u and q are increasing and twice continuously differentiable in c' and n respectively. Further, it is assumed that output is linear in labour supply. Specifically,

$$y(n) = n$$

Assume preferences satisfy,

$$u'' < 0, \; q'' > 0$$

and,

(2) $$\frac{u''c'}{u'} \leq -a < 0; \; u''c' + u' > 0$$

where,

$$0 < a < 1$$

and,

$$\lim_{c' \to 0} u'(c') = \infty, \; \lim_{c' \to \infty} u'(c') = 0$$

Substituting the constraints into the utility function and maximizing with respect to y, the output of the representative agent, yields the first order condition that,

(3) $$E[u'\left(\frac{yp}{p'}(1-\tau')\right)\frac{p(1-\tau')}{p'} \mid p,m]$$
$$= q'(y)$$
$$\equiv Q(y)$$

where the individual's expectation is conditional on the current price level observed and the last period's nominal money stock.

Given the assumptions on u, q and f made above, it can be shown that the second order condition is met by differentiating (3) above. Further, it is also the case that $Q'(y)$ is positive. Differentiating (2) with respect to y and p', yields the result that individual output is increasing in $p(1-\tau')/p'$, the rate of return on current production. This implies that c' is a normal good.

In equilibrium, island production equals island demand for goods. Letting y represent individual output and assuming that the total number of young are normalized to unity means that since the young are divided equally across the two islands,

$$(4) \qquad y = \frac{mv}{p}$$

in equilibrium. Substituting (4) into (3) yields the equilibrium condition for the typical island,

$$(5) \qquad Q\left(\frac{mv}{p}\right) = E\left[u'\left(\frac{mv}{p'}\frac{(1+\mu'\gamma')}{(1+\gamma')}, \frac{p}{p'}\frac{(1+\mu'\gamma')}{(1+\gamma')}\right) \Big| p,m\right]$$

which describes the current equilibrium price level in terms of current m, v and the unknown values of p' and γ'.

Since the other island is identical except for the distribution of government expenditure, its equilibrium condition will be the same except that $\hat{p}$ and v are substituted for $\hat{p}$ and v in (5), the 'hat' denoting the other island. The composite island shock will be, given the symmetric distribution of θ,

$$\hat{v} = \frac{(1+(2-\theta)\gamma)(1+\mu\gamma)}{(1+\gamma)}$$

Note that, as described in footnote 2, except when there is less than full information, tax revenue is equivalent to that from money creation, both represent a tax on the nominal balances held by the private sector. This is due to the assumption that the only asset is fiat currency and is model specific. However, the results below yield useful insights to the issue of policy relevance which may point to important applications to richer models.

III. EQUILIBRIUM

The output effect of tax versus money finance will be examined under two scenarios regarding the information available to current producers: (i) only current p and m are observed by current young, and (ii) current realizations of θ and γ are announced prior to production. These two scenarios represent a consecutive increase in the information available to current producers, consequently, examining the output response to changes in μ under these assumptions will indicate how tax versus money financing is affected by a change in information.

It remains to determine the properties of the equilibrium price function and output function with respect to changes in μ under the assumptions above. Given the lack of serial dependence of γ and θ it is reasonable to suppose that the current price level depends in some fashion on the current state of the economy, $(m,\theta,\gamma;\mu)$. Thus the current price level is some function $p(m,\theta,\gamma;\mu)$ of those state variables. By the same token current output will also be some function of these state variables. In general, each p will also be a function of the entire sequence of μ's the government has announced. This implicit relationship will be examined in subsequent sections.

Specifically, an equilibrium price will be defined as a continuous non-negative function $p(m,\theta,\gamma;\mu)$ which satisfies,

$$(6) \quad Q\left(\frac{mv}{p(m,\theta,\gamma;\mu)}\right) \frac{1}{p(m,\theta,\gamma;\mu)}$$

$$= E\left[u'\left(\frac{mv}{p(m',\theta',\gamma';\mu')} \frac{1+\mu'\gamma'}{1+\gamma'}\right) \frac{1}{p(m',\theta',\gamma';\mu')} \frac{1+\mu'\gamma'}{1+\gamma'} \,\middle|\, p(m,\theta,\gamma;\mu)\right]$$

where (6) is (5) with the price function substituted for p and p'. The expectation is now conditional on p(.) which is a function of two random variables with known probability distributions.

Following Lucas (1972) it will be demonstrated that the solution to (6) may have particular properties then demonstrating there is a unique solution to (6) which must therefore have those properties. This will be repeated for both the two scenarios mentioned above.

(i) Under the assumption that current values of γ and θ are unknown it can be demonstrated that for given m and μ,

$$v_0 = \frac{(1+\theta_0\gamma_0)(1+\mu\gamma_0)}{(1+\gamma_0)} \quad \frac{(1+\theta_1\gamma_1)(1+\mu\gamma_1)}{(1+\gamma_1)} = v_1$$

implies,

$$p(m,\theta_0,\gamma_0;\mu) \neq p(m,\theta_1,\gamma_1;\mu)$$

In fact it is convenient to limit the discussion of equilibrium price functions to those which are monotonic in v (though this is not guaranteed by the above property). As Lucas (1983) notes, although there may exist other price functions which satisfy (6), these will be ignored.

Further, to guarantee monotonicity it should be noted that the distributions of γ and θ will have to be restricted. The particulars of that restriction will be discussed in section IV below.

Thus, given p( ) is monotonic in v an expectation conditioned on p(.) is equivalent to an expectation conditional on v. Following Lucas, a plausible conjecture for the price function is,

$$(7) \quad p(m,\theta,\gamma;\mu) = m \cdot \phi_1(v;\mu)$$

since current θ and γ are unobserved so that only the composite random variable v will enter the price function with the known index of the degree to which the government expenditure is financed by money creation. The last period's stock of nominal balances enters homogeneously to degree one since one would expect that output would be homogeneous to degree zero in m,

$$(8) \quad y = \frac{mv}{p} = \frac{v}{\phi_1(v;\mu)}$$

Substituting for p( ) and noting that $m' = (1+\mu\gamma)m$ yields the equilibrium condition:

(9) $$Q\left(\frac{v}{\phi_1(v;\mu)}\right)\frac{v}{\phi_1(v;\mu)}$$

$$= E\left[u'\left(\frac{v}{1+\mu\gamma}\,\frac{1}{\phi_1(v';\mu')}\,\frac{1+\mu'\gamma'}{1+\gamma'}\right)\frac{v}{1+\mu\gamma}\,\frac{1}{\phi_1(v';\mu')}\,\frac{1+\mu'\gamma'}{1+\gamma'}\,\middle|\, v\right]$$

or, defining $f(\gamma',v')$ as the joint density function of γ' and v' given μ', and $f(\gamma|v)$ as the density of γ conditional on v given μ, the equilibrium condition may be expressed as,

(10) $$Q\left(\frac{v}{\phi_1(v;\mu)}\right)\frac{v}{\phi_1(v;\mu)}$$

$$= \int G_2\left[\frac{v}{1+\mu\gamma}\,\frac{1}{\phi_1(v';\mu')}\,\frac{1+\mu'\gamma'}{1+\gamma'}\right]f(\gamma',v')f(\gamma|v)\,d\gamma'\;dv'\;d\gamma$$

where,

(11) $$G_2[x] \equiv u'(x)x\ ,\ G_2' > 0$$

If it is assumed that $\mu = \mu'$, that μ is identical for each period, then the existence of ϕ_1 can be shown as follows;

Theorem 1. Equation (10) has as a unique, strictly positive, continuous solution $\phi_1(v)$ on $(0,\infty)$ such that $v/\phi_1(v)$ is bounded.[3]

Proof. See Appendix 1.

In general, ϕ_1, for non-constant μ sequences, is assumed to exist and to depend on v and the μ sequence as discussed in section IV. However a brief examination of equation (10) lends some insight as to how the equilibrium level of island output is determined. Remembering that $y = v/\phi_1$ and that G_2' and $Q'(x)x + Q(x)$ are both positive, it is apparent that equilibrium output is increasing in $v/(1+\mu\gamma)$ (ignoring changes in $f(\gamma|v)$).

While this is not a legitimate comparative static exercise it yields insight as to the economic variables which drive this economy. The ratio $v/(1+\mu\gamma)$ represents the share of nominal demand accruing to the typical island or, in other words, the proportion of nominal balances the island receives relative to the other island. The total stock of nominal balances in the current period is $(1+\mu\gamma)m$ while the level of nominal demand on the typical island is $mv/2$, which also equals the nominal balances the current young acquire through production. Thus, the ratio $v/(1+\mu\gamma)$ represents the degree to which the young benefit relative to the other island by producing. If,

$$\frac{v}{1+\mu\gamma} = \frac{1+\theta\gamma}{1+\gamma}$$

is less than unity then $\theta < 1$ and the typical island is receiving a less than proportional share of government expenditure. This in turn implies that the current young will be relatively disadvantaged, relative to the young from the other island, when it is their turn to consume next period. For a given level of output next period they will have a relatively smaller level of wealth to trade with.

The other arguments of G_2 represent the current young's estimate of their tax burden and next period's price level, both having an effect on their future consumption. Thus, equilibrium output depends on the current young's estimate of the degree to which they may transform current output to future consumption. An important aspect of this is their estimate of $v/(1+\mu\gamma)$ where v is observed and γ is inferred from its known probability distribution, conditioned on v.

(ii) If current information includes the current realization of θ and γ then as before the definition of an equilibrium price function will be a continuous non-negative $p(m,\theta,\gamma;\mu)$ which satisfies, suppressing the arguments of p(.),

$$(12) \qquad Q\left(\frac{mv}{p}\right)\frac{mv}{p} = E\left[G_2\left[\frac{mv}{p'}\,\frac{1+\mu'\gamma'}{1+\gamma'}\right] \Big| p,\gamma,\theta\right]$$

A conjecture regarding the form of the p( ) function is,

$$p(m,\theta,\gamma;\mu) = m\,\phi_2(\theta,\gamma;\mu)$$

so that the equilibrium condition becomes, suppressing the arguments of ϕ_2,

$$(13) \qquad Q\left(\frac{1+\theta\gamma}{1+\gamma}\,\frac{1+\mu\gamma}{\phi_2}\right)\frac{1+\theta\gamma}{1+\gamma}\,\frac{1+\mu\gamma}{\phi_2} = \int G_2\left[\frac{1+\theta\gamma}{1+\gamma}\,\frac{1}{\phi_2'}\,\frac{1+\mu'\gamma'}{1+\gamma'}\right]f(\gamma')g(\theta')d\gamma' d\theta'$$

where $\phi_2' = \phi_2(\theta',\gamma';\mu')$.

If it is assumed that $\mu = \mu'$, that μ is constant over time, then

Theorem 2. Under assumption (ii), equation (13) has the unique continuous solution function $\phi_2(\theta,\gamma)$ on $(0,\infty)$ and $m\phi_2$ is the unique price function where v/ϕ_2 is bounded.[4]

Proof. The proof is identical to that for Theorem 1 and is left to the reader. Essentially, it consists of arguing that equation (13) describes a contraction mapping and so the sequence of functions it describes has a fixed point. Since this is unique and $m.\phi_2$ is a solution, it is the unique solution with the above properties.

As in (i) above, equation (13) describes equilibrium output v/ϕ_2 as an implicit function of the expected tax rate and price next period and $(1+\theta\gamma)/(1+\gamma)$, the realization of the typical island's proportional share of the current money stock.

IV. TAX VERSUS MONEY FINANCE

It is now possible, given the description of the price functions and the equilibrium conditions they satisfy, under the two scenarios, to derive the output response of the representative individual to changes in μ, the finance mode index. Since the total number of young is normalized to unity, half going to each island, aggregate output is defined as

(14) $$y_i = \frac{1}{2} y_i + \frac{1}{2} \hat{y}_i \quad , \quad i = 1,2$$

where y_i and $\hat{y}_i$ are the individual outputs on each island, given the relevant information assumption.

(a) Under the assumption that agents have full current information, it can be demonstrated that,

Proposition 1. For given realizations of θ and γ, individual, island

and aggregate output is invariant to announced changes in the past, current or future values of the finance mode index μ under the assumption that agents have full current information.

Proof. By inspection of equation (13), which defines ϕ_2, it is clear that a change in any μ prior to the current value will leave the equilibrium values of ϕ_2 and ϕ_2' unchanged. Thus, defining current output as,

$$(15) \qquad y_2 = \frac{mv}{p} = \frac{v}{\phi_2}$$

island output will be unaffected, as will aggregate output.

Thus, since ϕ_2 is unaffected by prior μ's, ϕ_2' will be invariant to current μ (since the ϕ_2' function is the same function as ϕ_2). To determine the current output response to a change in current μ, differentiate (13) with respect to μ. Given the above discussion, the right-hand side of (13) is invariant to μ so,

$$\frac{\partial(v/\phi_2)}{\partial\mu} [Q' \frac{v}{\phi_2} + Q] = 0$$

so that

$$\frac{\partial(v/\phi_2)}{\partial\mu} = \frac{\partial y_2}{\partial\mu} = 0$$

and the output response to a change in μ on either island (just replace θ by $2-\theta$ in (13)) will be zero. Thus, any change in μ, which increases the demand shock v, will be met by a fully offsetting increase in the price level.

Thus since,

$$\frac{\partial(v/\phi_2)}{\partial\mu} = \frac{1+\theta\gamma}{1+\gamma}\,\frac{\partial((1+\mu_{+1})/\phi_2)}{\partial\mu} = 0$$

then,

$$\frac{\partial((1+\mu'\gamma')/\phi_2')}{\partial\mu'} = 0$$

so that the right-hand side of (13) is invariant to changes in μ'. Thus, clearly current prices and output are invariant with respect to μ'. The current young have no incentive to change their output despite the increase in the next period inflation tax, since the decrease in their tax rate is proportional to the increase in the next period price level.

Note, that current output is fixed so long as $(1+\theta\gamma)/(1+\gamma)$ is unchanged. As discussed above, this is the relevant variable determining the work-leisure choice. Thus, it is apparent that,

$$\frac{v}{\phi_2(\theta,\gamma;\mu)} = y_2\left(\frac{1+\theta\gamma}{1+\gamma}\right)$$

from inspection of equation (13).

This result is very similar to the standard neutrality proposition with respect to perfectly anticipated changes in the money stock. Both modes of finance act as a tax to finance the government's consumption. A change in μ, if perfectly anticipated, does not change the taxes' incidence since the price level responds to the increase in μ. For a given γ, and therefore a given level of expenditure, relative to total output, there is no incentive effect of a change in mode on current producers.

Note that the aggregate effect of a γ shock, an unexpected but announced increase in government expenditure, will be ambiguous. Recalling that aggregate income is,

$$Y_2 \quad \frac{1}{2} y_2 + \frac{1}{2} y_2$$

where y and y are positive functions of $(1+\theta\gamma)/(1+\gamma)$ and $(1+(2-\theta)\gamma)/(1+\gamma)$ respectively (from the equilibrium conditions), then

$$\frac{\partial(\frac{1+\theta\gamma}{1+\gamma})}{\partial\gamma} = \frac{\theta-1}{(1+\gamma)^2}$$

and

$$\frac{\partial(\frac{1+(2-\theta)\gamma}{1+\gamma})}{\partial\gamma} = \frac{1-\theta}{(1+\gamma)^2}$$

If output rises on one island as γ increases, the output declines on the other island. If $\theta > 1$, the increase in γ implies that the proportional share of nominal demand accruing to the island also rises, creating an incentive to substitute from current leisure to future consumption. By the same token, $\theta > 1$ implies that the other island's share of wealth declines with the increase in γ, reducing its equilibrium output.

(b) In this section the output response to a change in μ will be examined under less than full information, when current values of θ and y are unknown to current young. As discussed above, the equilibrium level of output is,

$$y_1 = \frac{v}{\phi_1(v;\mu)}$$

so that if two equilibria are compared, both with the same (θ,γ),

(16) $$\frac{dy_1}{d\mu} = \frac{\partial v}{\partial \mu} \frac{1}{\phi_1} [1 - \frac{\partial \phi_1}{\partial v} \frac{v}{\phi_1}] - \frac{v}{\phi_1^2} \frac{\partial \phi_1}{\partial \mu}$$

Thus, the change in output in response to a change in the finance mode index can be decomposed into two components, the direct effect of a change in μ on output for a given observation of v, and the indirect effect on output of the increase in v caused by μ, where,

$$\frac{\partial v}{\partial \mu} = (\frac{1+\theta\gamma}{1+\gamma})\gamma.$$

Note, the above discussion is with respect to a comparative static exercise, agents do not observe the change in v since this information, coupled with the observation of v itself, would allow perfect inference of current θ and γ. Recall that the p( ) function and, therefore, the ϕ_1 function, are implicitly functions of the entire sequence of μ's the government has announced. The experiment is then to compare equilibrium values of island output for given, unknown, values of θ and γ as μ changes.

To determine the output effect of a change in current μ, note that the equilibrium condition under the assumption of no current information except for v, is

(17) $$Q(\frac{v}{\phi_1}) \frac{v}{\phi_1}$$

$$= \int m(\frac{v}{1+\mu\gamma}) f(\gamma | v) d\gamma$$

where,

$$m(\frac{v}{1+\mu\gamma}) \equiv \int G_2[\frac{v}{1+\mu\gamma}\frac{1}{\phi_1'}\frac{1+\mu'\gamma'}{1+\gamma'}]f(\gamma',v')d\gamma'dv'$$

so that $m'(.)$ is positive. The output response to a change in v will depend both on the utility derived from increased future consumption for a given realization of γ and how the probability distribution of γ is affected by the increase in v. Thus,

$$(18) \quad \frac{1}{\phi_1}[1 - \frac{\partial\phi_1}{\partial v}\frac{v}{\phi_1}][Q'\frac{v}{\phi_1} + Q]$$

$$= \int m'(\frac{v}{1+\mu\gamma})\frac{1}{1+\mu\gamma}f(\gamma|v)d\gamma$$

$$+ \int m(\frac{v}{1+\mu\gamma})\frac{\partial f(\gamma|v)}{\partial v}d\gamma$$

where

$$(19) \quad \frac{\partial y_1}{\partial v} = \frac{1}{\phi_1}[1 - \frac{\partial\phi_1}{\partial v}\frac{v}{\phi_1}]$$

Since, by assumption, $Q'(x)x + Q(x)$ is positive, if the right hand side of (19) is positive, then $\partial y_1/\partial v$ is positive.

Since $m' > 0$, ignoring the effect on the probability distribution of γ conditional on v, the increase in v implies that $v/(1+\mu\gamma)$ is greater for all γ implying that the island receives a greater proportion of nominal demand. Thus, the first integral on the right hand side of (18) is positive. If the increase in v has the effect of decreasing the probability of γ then it is possible that the second integral will also be positive.

More precisely, if the cumulative density of γ conditional on v, defined as, for a given γ^*, $Pr\{\gamma \leq \gamma^*|v\}$ is an increasing function of v, this will be sufficient to guarantee that $\partial y_1/\partial v \geq 0$. The cumulative density function is,

$$(20) \quad F(\gamma^*|v) = \int_0^{\gamma^*} f(\gamma|v)d\gamma$$

with the assumed restriction,

$$\frac{\partial F}{\partial v}(\gamma^*|v) = F_v(\gamma^*|v) \geq 0$$

This has the effect of bunching the distribution to lower values of γ, i.e. reducing its expected value.

By noting that,

$$(21) \quad F_v(\gamma^*|v) = \int_0^{\gamma^*} \frac{\partial f(\gamma|v)}{\partial v} d\gamma$$

and integrating the second integral in (18) by parts equation (18) becomes,

$$(22) \quad \frac{1}{\phi_1}[1 - \frac{\partial \phi_1}{\partial v}\frac{v}{\phi_1}][Q'\frac{v}{\phi_1} + Q]$$

$$= \int m'(\frac{v}{1+\mu\gamma})[f(\gamma|v)\frac{1}{1+\mu\gamma}$$

$$+ F_v(\gamma|v)\frac{v\mu}{(1+\mu\gamma)^2}] d\gamma$$

where $F_v(0|v)$ equals zero by assumption and $F_v(\bar{\gamma}|v)$ is zero given that $F(\bar{\gamma}|v)$ is unity for all v.[5] Thus individual and island output rises as the observed v increases, if, Proposition 2. A sufficient condition for the relevancy of government finance is that the right-hand-side of equations (22) and (24) be non-negative.

For example, an increase in μ will increase island output if $F_v(\gamma|v) \geq 0$ and $F_\mu(\gamma|v)\mu \geq f(\gamma|v)\gamma$ for a given v. While these conditions are unnecessarily strict, they offer an insight as to the informational impact of a change in the mode of finance on production decisions.

The first condition ($F_v(\gamma|v) \geq 0$) is relatively transparent. As discussed above, the young's work/leisure choice is based on the degree to which the young may transform current labour into next period consumption. That rate of transformation is,

$$\frac{yp}{p'}(1-\tau') = \frac{v}{1+\mu\gamma}\,\frac{1}{\phi_1'}\,\frac{1+\mu'\gamma'}{1+\gamma'}$$

which is composed of current and future variables. The current variables represent, as argued above, the relative share of current demand that the island has captured. Since there is less than full information the young must conjecture the ratio $v/(1+\mu\gamma)$ given the observed composite shock v. A sufficient condition for an increase in v to increase the conjectured rate of transformation of current labour to future consumption and to increase the conjectured value of $x = (1+\theta\gamma)/(1+\gamma) = v/(1+\mu\gamma)$ is $F_v(\gamma|v) \geq 0$. Since $F_v(\gamma|v) \geq 0$ is sufficient to reduce the expected value of γ given v, by shifting the cumulative distribution upward to the left, then it is also sufficient to increase the expected value of x.[7]

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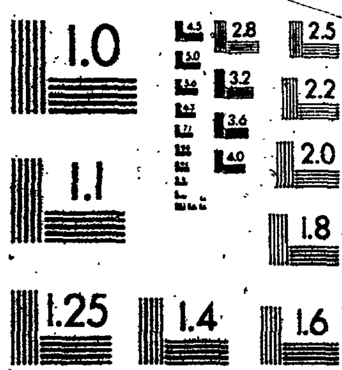

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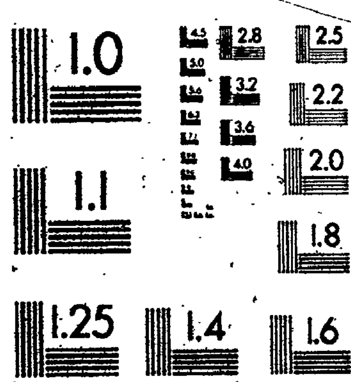

$$f(\gamma|v) + F_v(\gamma|v)\,\frac{v\mu}{1+\mu\gamma} > 0$$

for all v and μ.[6]

Individual output response to a change in μ, for a given v, is found by differentiating equation (17) with respect to μ,

$$(23)\quad -\frac{v}{\phi_1^2}\frac{\partial\phi_1}{\partial\mu}\left[Q'\,\frac{v}{\phi_1} + Q\right]$$

$$= -\int m'\left(\frac{v}{1+\mu\gamma}\right)\frac{v\gamma}{(1+\mu\gamma)^2}\,f(\gamma|v)d\gamma$$

$$+\int m\left(\frac{v}{1+\mu\gamma}\right)\frac{\partial f(\gamma|v)}{\partial\mu}\,d\gamma$$

Again, output will rise with the increase in μ if the right hand side of (23) is positive. Integrating the second integral by parts yields the result that,

$$(24)\quad -\frac{v}{\phi_1^2}\frac{\partial\phi_1}{\partial\mu}\left[Q'\,\frac{v}{\phi_1} + Q\right]$$

$$= m\left(\frac{v}{1+\mu\bar{\gamma}}\right)F_\mu(\bar{\gamma}|v)$$

$$+\int m'\left(\frac{v}{1+\mu\gamma}\right)\frac{v}{(1+\mu\gamma)^2}\left[F_\mu(\gamma|v)\mu - f(\gamma|v)\gamma\right]d\gamma$$

where again $F_\mu(0|v)$ and $F_\mu(\bar{\gamma}|v) = 0$. A sufficient condition for both sides of the equality to be positive is,

$$F_\mu(\gamma|v)\mu \geq f(\gamma|v)\gamma \geq 0$$

In general, however,

Proposition 2. A sufficient condition for the relevancy of government finance is that the right-hand-side of equations (22) and (24) be non-negative.

For example, an increase in μ will increase island output if $F_v(\gamma|v) \geq 0$ and $F_\mu(\gamma|v)\mu \geq f(\gamma|v)\gamma$ for a given v. While these conditions are unnecessarily strict, they offer an insight as to the informational impact of a change in the mode of finance on production decisions.

The first condition ($F_v(\gamma|v) \geq 0$) is relatively transparent. As discussed above, the young's work/leisure choice is based on the degree to which the young may transform current labour into next period consumption. That rate of transformation is,

$$\frac{yp}{p'}(1-\tau') = \frac{v}{1+\mu\gamma}\,\frac{1}{\phi_1'}\,\frac{1+\mu'\gamma'}{1+\gamma'}$$

which is composed of current and future variables. The current variables represent, as argued above, the relative share of current demand that the island has captured. Since there is less than full information the young must conjecture the ratio $v/(1+\mu\gamma)$ given the observed composite shock v. A sufficient condition for an increase in v to increase the conjectured rate of transformation of current labour to future consumption and to increase the conjectured value of $x = (1+\theta\gamma)/(1+\gamma) = v/(1+\mu\gamma)$ is $F_v(\gamma|v) \geq 0$. Since $F_v(\gamma|v) \geq 0$ is sufficient to reduce the expected value of γ given v, by shifting the cumulative distribution upward to the left, then it is also sufficient to increase the expected value of x.[7]

Thus, an increase in demand will increase output ($\partial y/\partial v > 0$) if the young believe that their share of current demand relative to the rest of the economy has increased.

The second condition, that $F_\mu(\gamma|v)\mu \geq f(\gamma|v)\gamma$, is sufficient to ensure that the expected value of x given v will increase in response to an increase in μ given v (the proof is identical to that in footnote 7). The two conditions, therefore, imply that the young will increase output, given θ and γ, if the probability distributions of θ and γ are such that the increase in μ increases their expectation of the value of x, their island's relative share of current nominal balances. Note that this result depends also on the assumption that m( )' is positive or that the index of the young's relative risk aversion is less than unity.

V. AGGREGATE DEMAND SHOCKS

If current producers have full current information then the effect of an aggregate demand shock, an increase in y or government expenditure, will have an ambiguous aggregate output effect. Since current producers will increase output if $x \equiv (1+\theta\gamma)/(1+\gamma)$ increases, island output will rise only if $\theta > 1$ in response to an increase in γ. Thus, in general, an increase in γ will increase output on one island while, due to the symmetric distribution of θ, decreases output on the other island.

This is due to the uneven distribution of government demand. If $\theta < 0$, an increase in γ will only serve to decrease the relative share of nominal demand the young are able to exchange output for.

In fact, it is apparent that

Proposition 3. If island output (y_2) defined by the relevant equilibrium condition, is a strictly concave function of $x \equiv (1+\theta\gamma)/(1+\gamma)$, then aggregate output is a decreasing function of γ.

Proof. If y_2 is strictly concave then $y_2' > 0$ and $y_2'' < 0$. Aggregate output is defined as,

$$Y_2 = \frac{1}{2}\left[y_2\left(\frac{1+\theta\gamma}{1+\gamma}\right) + y_2\left(\frac{1+(2-\theta)\gamma}{1+\gamma}\right)\right]$$

so that,

$$\frac{\partial Y_2}{\partial \gamma} = \frac{\theta-1}{2(1+\gamma)^2}\left[y_2'\left(\frac{1+\theta\gamma}{1+\gamma}\right) - y_2'\left(\frac{1+(2-\theta)\gamma}{1+\gamma}\right)\right]$$

If $\theta-1 > 0$ then

$$\frac{1+\theta\gamma}{1+\gamma} > \frac{1+(2-\theta)\gamma}{1+\gamma}$$

and if $\theta-1 < 0$;

$$\frac{1+\theta\gamma}{1+\gamma} < \frac{1+(2-\theta)\gamma}{1+\gamma}$$

so that, given $y_2'' < 0$,

$$\frac{\partial Y_2}{\partial \gamma} < 0$$

This result predicts that an increase in government expenditure, if announced, will reduce aggregate output, regardless of the mode of finance. By the same token as θ diverges further from unity, or as the government distributes its expenditure more unevenly, aggregate output declines.

This is similar to the observation that, given a convex Phillips Curve, if market inflation rates diverge from the average rate the average level of unemployment will rise. In the above, since $x + \hat{x} = 2$, the average x ratio is unity, it is the dispersion that varies with changes in γ and θ.

Note that this result is not predicted from the assumption made in Section II. If y_2 were convex, which is not ruled out by the assumptions made regarding preferences and technology, the above prediction would be reversed, that aggregate output would increase with y. However, y_2 will be concave if the youngs' labour supply curve has a positive and decreasing slope.

Note further, that this result is independent of the form of finance. Thus, under full information, the mode of finance is irrelevant for determining the multiplier. A tax financed versus a money financed increase in government expenditure will be identical in their output impact. Thus μ has no relevance regarding the amplitude of business cycles in this context.

When the composition of v is obscured from producers then it is much more difficult to determine the aggregate effect of an increase in government expenditure. As shown above, equilibrium output, when current shocks are unknown, is

$$y_1 = v/\phi_1(v;\mu)$$

so that,

$$\frac{\partial y_1}{\partial \gamma} = \frac{\partial v/\phi_1}{\partial v}\frac{\partial v}{\partial \gamma}$$

Even if $\partial(v/\phi_1)/\partial v$ is known,

$$\frac{\partial v}{\partial \gamma} = \frac{\theta+\mu+2\mu\gamma\theta - \overline{1+\mu\gamma^2\theta}}{(1+\gamma)^2} \gtrless 0$$

so that the island output effect is ambiguous, depending on μ. If $\partial(v/\phi_1)/\partial v > 0$ as above, if $\mu = 1$ then $\partial y/\partial\gamma > 0$ on both islands. If $\mu = 0$, and $\partial(v/\phi_1)/\partial v > 0$, the output effect will be opposite in sign on each island since if $\theta < 1$, $\partial v/\partial\mu > 0$ and $\partial\hat{v}/\partial\mu < 0$. This latter case is similar to the full information example above.

The two informational scenarios may now be examined to determine what distinguishes tax from money finance. As shown above, equilibrium output is invariant to changes in μ if there is full current information regarding the components of μ. Thus, in this simple framework it is the lack of information in the first scenario which creates the distinction between the two modes. More specifically, it is the uncertainty regarding the current nominal money stock which creates the distinction.

This hypothesis is demonstrated by noting that,

$$\lim_{\mu\to 0}\phi_1 = \lim_{\mu\to 0}\phi_2$$

As μ tends to zero, suppressing the second argument of ϕ_1 and ϕ_2,

$$\phi_1 = \phi_1(v) \quad , v = \frac{1+\theta\gamma}{1+\gamma}$$

and

$$\phi_2 = \phi_2 \frac{(1+\theta\gamma)}{1+\gamma}$$

since the equilibrium conditions (10) and (13) are identical as $\mu \to 0$ and ϕ_1 solves (10) uniquely and ϕ_2 solves (13) uniquely, then $\phi_1 = \phi_2$ when $\mu = 0$ for a given level of $(1+\theta\gamma)/(1+\gamma)$.

Thus, when the nominal money stock is non-stochastic, i.e. $\mu = 0$, and government expenditure is financed by taxation solely, then obscuring the components of v has no relevance to equilibrium output. This result relies on the importance of the share of nominal demand accruing to a given island in the determination of equilibrium output. If the nominal money stock is constant ($\mu = 0$) then an observation on v can be used to infer perfectly $\hat{v}$ since

$$\lim_{\mu \to 0} (v + \hat{v}) = 2.$$

The government's policy simply shifts nominal demand between islands by distributing its expenditure randomly. Since the total level of nominal demand is constant, if $v > 1$ this implies $\theta > 1$ and the island is receiving a greater share of the nominal money stock. Since island young will receive, given a level of output and taxes next period, a greater share of next period's production in exchange for their cash there is an incentive to increase current production.

When $\mu > 0$ and current young are ignorant of current shocks, then an observation of v is insufficient to predict the island's share of nominal wealth. The producers must form conjectures of $v/(1+\mu\gamma)$ based on the known probability distributions of γ and θ. In this context then, an increase in γ may increase output on both islands, whereas given full information, output would rise on one island and fall on the other, possibly reducing aggregate output.

It is simple to show that an increase in μ, the degree to which government expenditures are financed by money creation, will increase the variance of aggregate output under less than full information if the expected value of output is increasing in μ.

Since,

$$Y = \frac{1}{2} y_1 + \frac{1}{2} \hat{y}_1 \; ; \; y_1 = y_1(v) \; , \; \hat{y}_1 = y_1(\hat{v})$$

the '^' denoting the other island, then adopting the expectations operator notation,

$$\underset{v}{E} [y_1] = \underset{\hat{v}}{E} [\hat{y}_1]$$

since the distributions of v and $\hat{v}$ are identical. Thus,

$$\text{var } Y_1 = \frac{1}{2} \underset{v}{E} \, y_1^2 + \frac{1}{2} [\underset{v\hat{v}}{E} y_1 \hat{y}_1]^2$$

which, recalling that

$$v = \frac{(1+\theta\gamma)(1+\mu\gamma)}{(1+\gamma)} , \; \hat{v} = \frac{(1+(2-\theta)\gamma)(1+\mu\gamma)}{1+\gamma}$$

may be expressed, after substituting for v and $\hat{v}$ above,

$$\text{var } Y_1 = \frac{1}{2} \underset{\theta\gamma}{E} \, y_1^2 + \frac{1}{2} \underset{\theta\gamma}{E} \, y_1 \hat{y}_1$$

Thus, since the distributions of γ and θ are independent of μ and, if E y_1 is increasing in μ, it is clear that

$$\frac{d\,\mathrm{var}\,Y_1}{d\mu} = \frac{1}{2} E \frac{dy_1^2}{d\mu} + \frac{1}{2} E(\hat{y}_1 \frac{dy_1}{d\mu} + y_1 \frac{d\hat{y}_1}{d\mu}) \geq 0$$

Since, as argued above, y_2 is invariant to μ for all θ and γ then,

$$\frac{\partial\,\mathrm{var}\,Y_2}{\partial\mu} = 0,$$

and it is clear that since $\lim_{\mu\to 0} \phi_1 = \phi_2$ that,

$$\mathrm{var}\,Y_2 \leq \mathrm{var}\,Y_1$$

or that aggregate output exhibits more fluctuation as the degree to which exogenous shocks to government policy are financed by money creation.

The μ parameter affects aggregate output variance in two fashions. Firstly, it may increase the variance of individual output by increasing the output response for a given v. It also increases v for given unknown realizations of θ and γ. Secondly, aggregate output variance will depend on the covariance of island output. In the full information example either a γ increase or a divergence of θ from unity would increase the dispersion between market outputs. This negative correlation was reduced by the lack of full information as it was shown that an increase in γ which could have the same qualitative impact on both islands' demand could also increase both islands' output. Thus the negative correlation between island output is reduced.

VI. OPTIMALITY

An index of social welfare, used by Polemarchakis and Weiss (1977) and Azariadis (1981) in a similar context, is the unconditional expected utility of the representative agent. The appropriateness lies in the fact that it is state independent and is therefore aptly suited to a discussion of models in which the relevant distributions are assumed to be stationary.

However, in the above context, when the μ sequence is non-constant, it is less suitable. Nonetheless, if agents make production decisions based on full information, the following must be true;

Proposition 4. Given full information regarding current (θ,γ), any $\mu^* \epsilon [0,1]$ is optimal with respect to the unconditional expected value of the representative agent's utility.

Since output is constant, for all (θ,γ), as μ changes, the young and old will be indifferent to the mode of finance.

If agents have less than full information prior to production, no Pareto improvement in welfare is possible in general. If the entire μ sequence increases marginally, the expected utility of the representative agent may or may not increase, depending on the marginal rate of substitution between consumption and leisure and the effect on production. For example, if the increase in money finance increases output for all (θ,γ), the young will be worse off, since their leisure decreases, while the old will be better off, since consumption rises.

Given that $\lim_{\mu\to 0} t_1 = \lim_{\mu\to 0} t_2$ it is generally true that an economy where there is not full current information can achieve at least the level of output and welfare as an economy where there is full current information by setting $\mu = 0$. If a lack of information regarding current government expenditure etc. reduces welfare, it does so only if the current nominal money stock is currently unknown.

Note that any possible dynamic inconsistency resulting from the government setting μ' to maximize Eu has been ignored.

VII. CONCLUSION

A natural rate model was examined to determine the effect of tax versus money financed government expenditure. Output and price fluctuations occurred because of two types of random shocks; an aggregate shock to taxes and/or money creation required to finance government expenditure and a distributional shock to the share of government expenditure accruing to different markets.

The policy rule that the government adopted was to finance its expenditures in such a way so as to leave the government's share of output in either island or market constant with respect to the mode of finance. Given the specific policy structure assumed, the following conclusions regarding the difference between tax and money finance were made:

(i) If the current money stock is known prior to production, the mode of finance of current and future government expenditure is irrelevant to aggregate and island output.

(ii) It is the uncertainty regarding the current money stock which distinguishes tax and money finance. Tax financing ($\mu = 0$) does not obscure the return to current production and thus the current young, by observing v have sufficient information to predict prices and quantities on both islands. There is no signal extraction problem under full tax finance, no relevant information is obscured by a lack of information regarding the composition of v when $\mu = 0$.

(iii) If $\mu > 0$ and the government does not announce (θ,γ) then the current relative share of nominal demand to each island must be conjectured. It is this need to form a conjecture of γ which distinguishes tax and money finance.

(iv) If individual island output is a concave function of the share of current nominal demand accruing to that island, then aggregate output falls as government expenditure increases if current producers are cognizant of the level of government expenditure.

If current producers are unaware of current γ, the island and aggregate effect of an increase in γ is ambiguous, depending on the mode of finance.

(v) If current shocks are currently known, the mode of finance is irrelevant with respect to the social welfare index, the expected utility of the representative agent. If current (γ,θ) are unknown prior to production, the government may always achieve the social welfare level of the full information scenario by setting $\mu^* = 0$, by financing government expenditure solely by taxes.

(vi) It is apparent that the mode of finance of a permanent shock to government expenditure is irrelevant with respect to equilibrium output and welfare (e.g., scenario (ii) with γ non-random). In this context, the mode of finance is only relevant with respect to transitory shocks to government expenditure, and only when those shocks are obscured from current producers.

Thus, in conclusion, attempting to distinguish between tax and money finance with respect to such issues as: optimal rules, optimal finance of temporary and/or permanent expenditures and the determination of the natural rate is only possible in the context of the informational advantage of tax finance in the above context. As shown above, under less than full information, even policies which might otherwise be irrelevant (i.e., a change in the mode of finance with full information) will be relevant in determining the natural rate, equilibrium output and the output response to stochastic government shocks.

Finally, it should again be stressed that the results of the model discussed above are specific to the model itself and to the policy rule chosen. Therefore, the equivalence between money and tax finance, in the full information scenario, should be interpreted not as a general proposition but as a benchmark for comparison with models better suited to the examination of the informational impact of government finance.

FOOTNOTES

1. One could generate this result explicitly by assuming that procreation is a linear function of nominal wealth.

2. An important assumption in what follows is that the old do not, or cannot, inform the young of their tax burden since given the value of τ, the value of γ can be inferred with certainty. This in turn would enable the young to decompose v. It may be argued that there is scope for a market for that information to develop, the value of γ that the old observe being sold to the young. However, it is plausible that such a market would fail since, given perfect competition among the old, there would be an incentive to 'product differentiate' with respect to that information. The young could not enforce the veracity of the information bought and so would have no incentive to buy. The purpose of this assumption is to allow the investigation of the effect of a conventional signal extraction problem on the distinction between tax and money finance of government expenditure.

3. The second argument of ϕ_1 has been suppressed.

4. The second argument of ϕ_2 has been suppressed.

5. Note that $F_v(\gamma=0|v)>0$ will only be true for $v>1$ since, $F(\gamma=0|v=1)=1$ and $F(\gamma=0|v>1)=0$. This proviso is avoided if $f(\gamma=0)$ is assumed to be zero.

6. Further, as noted in Section III(i) it is necessary to restrict the distributions of y and θ so as to guarantee that ϕ_1 is monotonic in v, to maintain informational equivalence between the price function and v.

It will be assumed that,

$$0 \leq f(\gamma|v)+F_v(\gamma|v)\,\frac{v\mu}{1+\mu\gamma} < 1$$

for all v and y which guarantees that

$$0 < v\,\frac{\phi'(v)}{\phi(v)} < 1$$

so that an increase in v will cause both output and price to rise. The proof is similar to that given by Lucas (1972) for Theorem 4 and is left to the reader.

7. Since

$$E[\frac{v}{1+\mu\gamma}|v] = \int \frac{v}{1+\mu\gamma}\, f(\gamma|v)d\gamma$$

$$\frac{\partial E}{\partial v}[\frac{v}{1+\mu\gamma}|v] = \int \frac{1}{1+\mu\gamma}\, f(\gamma|v)d\gamma + \int \frac{v}{1+\mu\gamma}\, f_v(\gamma|v)d\gamma$$

which equals, integrating the second integral by parts,

$$= \int \frac{1}{1+\mu\gamma}\,[f(\gamma|v) + \frac{\mu}{1+\mu\gamma}\, F_v(\gamma|v)]d\gamma$$

which is greater than zero if $F_v(\gamma|v) \geq 0$. Thus $F_v(\gamma|v) \geq 0$ implies that the expected value of $v/(1+\mu\gamma)$ rises.

APPENDIX

The proof of Theorem 1 is similar to that used by Lucas (1972). It is sufficient to show that (10) is a contraction mapping to prove that the sequence of functions it defines has a limiting fixed point solution.

Define the inverse of $Q(y)y$ as $G_1(x)$. Given the assumed properties of Q and the definition of an inverse function it can be shown that since,

$$G_1(x) > 0 \quad \text{for } x > 0$$

and

$$\lim_{x \to 0} G_1(x) = 0 ,$$

then

$$\text{(A.1)} \qquad 0 < \frac{G_1'(x)x}{G_1(x)} < 1$$

Define $G_2(x) = u'(x)x$, then $G_2(x) > 0$ for all $x > 0$ and

$$\text{(A.2)} \qquad 0 < \frac{G_2'(x)x}{G_2} \leq 1-a < 1$$

which follows from the assumed elasticity of u'.

Equation (10) may now be expressed in terms of G_1 and G_2 and

$$j(v) = Q\left(\frac{v}{\phi_1(v)}\right) \frac{v}{\phi_1(v)}$$

so that,

$$\text{(A.3)} \qquad j(v) = \int G_2\left[\frac{v}{1+\mu\gamma}\,\frac{G_1(e^{f(v')})}{v'}\,\frac{(1+\mu\gamma')}{(1+\gamma')}\right] f(\gamma',v')f(\gamma|v)\,d\gamma'\,dv'\,d\gamma$$

Since j(.) is a monotonic function it is sufficient to show that there exists a unique j(.) function which satisfies A.3 to prove Theorem 1.

Let S denote the space of bounded, continuous functions on $(-\infty,\infty)$, normed by:

$$||f|| = \sup_{v} |f(v)|$$

Define the operator T on S by

$$\text{(A.4)} \qquad T\,f = \ln \int G_2\left[G_1\,\frac{(e^{f(v')})}{v'}\,\frac{v}{1+\mu\gamma}\,\frac{1+\mu\gamma'}{1+\gamma'}\right] f(\gamma',v')f(\gamma|v)\,d\gamma'\,dv'\,d\gamma$$

which implies that, in terms of T, A.3 becomes,

$$\ln j = T \ln j$$

It is sufficient to show that T is a contraction mapping such that,

$$||T\,f - T\,g|| \leq (1-a)||f-g|| \;, \; a < 1$$

to prove that the sequence of functions that the mapping defines has a unique solution

$$f^* = T\,f^*$$

by the Banach fixed point theorem.

To show that T is a contraction mapping note that,

$$\text{(A.5)} \qquad ||Tf - Tg|| = \sup_{v'} \left| \ln \int \omega(\gamma,v,\gamma',v') \frac{G_2[\frac{G_1(e^{f(v')})}{v'} \frac{v}{1+\mu\gamma} \frac{1+\mu\gamma'}{1+\gamma}] d\gamma' dv' d\gamma}{G_2[\frac{G_1(e^{g(v')})}{v'} \frac{v}{1+\mu\gamma} \frac{1+\mu\gamma'}{1+\gamma}]} \right|$$

since $\omega(.)$ is defined by,

$$\omega(\gamma,v,\gamma',v') \equiv [\int G_2[\frac{G_1(e^{g(.)})}{v'} \frac{v}{1+\mu\gamma} \frac{1+\mu\gamma'}{1+\gamma'}] f(\gamma',v') f(\gamma|v) d\gamma' dv' d\gamma]^{-1}$$

$$\cdot [G_2[\frac{G_1(e^{g(.)})}{v'} \frac{v}{1+\mu\gamma} \frac{1+\mu\gamma'}{1+\gamma'}] f(\gamma',v') f(\gamma|v)]$$

where $\omega(.) > 0$ and $\int \omega(.) d\gamma' dv' d\gamma = 1$.

Since,

$$\int f(x)g(x)dx \leq \int [\sup_x f(x)] g(x)dx$$
$$= \sup_x f(x) \int g(x)dx$$

thenit follows that;

$$||Tf - Tg|| \leq \sup_{\gamma'v'\gamma} \left| \ln G_2[\frac{G_1(e^{f(v')})}{v'} \frac{v}{1+\mu\gamma} \frac{1+\mu\gamma'}{1+\gamma'}] - \ln G_2[\frac{G_1(e^{g(v')})}{v'} \frac{v}{1+\mu\gamma} \frac{1+\mu\gamma'}{1+\gamma'}] \right|$$

Since, by the mean value theorem,

$$f(b)-f(a) = f'(c)(b-a) \ , \ c \in [a,b]$$

and,

$$\partial G_2[\frac{G_1(e^x)}{v'}\frac{v}{1+\mu\gamma}\frac{1+\mu\gamma'}{1+\gamma'}]/\partial x$$

$$= [\frac{G_1'e^x}{G_1}][\frac{G_2'}{G_2}\frac{G_1 v}{v'(1+\mu\gamma)}\frac{1+\mu\gamma'}{1+\gamma'}]$$

$$\leq (1-a)$$

where the inequality follows from the assumption regarding the elasticities of the G_1 and G_2 function.

Thus from the definition of the $||.||$ operator,

$$||Tf - Tg|| \leq (1-a)\ ||f-g||$$

and the T mapping is a contraction mapping.

Since T is a contraction mapping there exists a unique continuous function f^* such that $f^* = Tf^*$. Letting $j^*(v) = e^{f^*(v)}$, it is clear that j^* is that function. Thus $m\phi_1(v)$ is the price function which satisfies Theorem 1.

ESSAY III

TRANSACTIONS TECHNOLOGIES AND THE TIME CONSISTENCY OF OPTIMAL MONETARY AND FISCAL POLICY

I. INTRODUCTION

One of the first authors to formalize and define time inconsistency was Strotz (1955) who noted that unless the utility function of the consumer exhibits certain characteristics (the discount factor equal $e^{-\delta t}$) a sequence of events, which would seem optimal at the present would, upon re-evaluation in the future, no longer be optimal.

This concept was extended to consider the appropriateness of optimal control theory to deal with many macro-economic issues by Kydland and Prescott (1977). It is argued that a government when deciding on an optimal path or sequence of policies subject to market clearing constraints is further constrained by the possibility that announced future policies at $t = \tau$ $(\tau > 0)$ will affect the optimal value of current policy at $t = 0$. Thus, the value of policy at τ will optimally take into account its effect on the government's objective function at $t < \tau$ when announced at $t = 0$.

However, ~~at~~ $t = \tau$, if the government is free to re-evaluate and replan the optimal path of policy into the future, it is the case that since events between $t = 0$ and $t = \tau$ have already occurred and are fixed, the re-evaluation of policy for date τ may not be the same. If it is not, the optimal plan at $t = 0$ is called time inconsistent.

This type of analysis is often couched in game theoretic terms where the government acts as a Stackelberg leader, taking into account the reaction functions of the private sector when setting its optimum policy. Announced policy in the next period must take into account the reaction of the private sector today as well as in the future when its optimum is derived. However, when the future date occurs and re-evaluation takes place, only the reaction of the private sector at that date must be considered. Thus, the optimal value of policy may not be the same.

The issue of time inconsistency of optimal policy is especially relevant with respect to rational expectations models (including those assuming perfect foresight). From a practical or methodological point of view, there may be a logical inconsistency in assuming that agents are 'rational' to derive their reaction functions and the concomitant optimal policy and then arguing that optimal policies are time inconsistent without examining why agents were not able to include this possibility in their rational expectations. Thus, optimal policies, derived from a model in which agents do not in some fashion take into account the possibility of policy re-evaluation at some later date, which are time inconsistent are inconsistent with rational expectations unless the government is committed to the optimal path.

In fact, this notion of rationality provides a convenient benchmark with which to compare the relevant literature. It may be divided into two groups, those which consider the issue explicitly in game theoretic terms and those which do not. Explicit game theory modelling implies that the government take into account that future governments may renege and set current policy appropriately or that the private sector take into account that government may renege.

For example, Goldman (1980) explores the Strotz-Pollak notion of a Nash equilibrium in which it is shown that a strategy exists such that if the current agent believes future agents will use it, it will be optimal for the current agent to also adopt it. Rotemberg (1983) adopts this notion in a monetary model in which prices are costly to adjust. Given exogenous and random demand/velocity shocks it is shown that a Strotz-Pollak strategy exists and may be optimum depending on the government's objective function. On the other hand, an optimal rule without a legislated commitment will not be consistent.

Thus, in this context, when successive governments play a Nash game between each other it can be demonstrated that consistent and equilibrium strategies exist. This is distinct from a rule which may or may not be consistent.

On the other hand, the relationship between consumers and government may be explored as Barro and Gordon (1983) and Barro (1983) have done. Barro notes that inflation policy will be less efficient under discretionary policy since at each period in time inflationary expectations, based on past information, will be exogenous to the government. However, if the government adopts an optimal rule it may 'internalize' the inflationary expectations impact of its behaviour. Barro and Gordon take this notion one step further by considering the incentives that government then has to cheat on rules. They argue that if consumers 'punish' government with higher inflationary expectations in the future there will exist a best enforceable rule for which cheating costs equal the gains from cheating. This rule will imply a lower social welfare than the second best rule in which cheating cannot be prevented. The first best is the same rule when the government is allowed to cheat without punishment.

In the second area of research, optimal policies are explored in a context in which households and government are naive; they do not consider the possibility of the government reneging on its plan. Economies are described in which the issue of time consistency may or may not be relevant.[1]

Calvo (1978) argues that, in a perfect foresight context, attempts by the government to maximize social welfare by adjusting tax rates will lead to an inconsistent optimal policy. If the optimal policy does not jump to a steady state but approaches it asymptotically, there will be an incentive for the government, at a future date, to jump back to the initial period's optimal values of relevant economic variables.

Calvo argues that this is endemic to rational expectations models which constrain the government to continuous paths of policy variables when announcing their future paths. However, in the future that constraint is no longer binding on the price level etc., which, unless the economy is optimally at a steady state, will imply that a discontinuous jump will be optimal.

Heuristically, the government has the opportunity to use a jump in the price level as a non-distortionary tax on nominal government debt only if it is unanticipated. The government will promise a smooth price path in the future but currently will cause a jump in the price level for revenue purposes. Unless the system jumps immediately to a steady state, in the future it will again be optimal to use the same mechanism to generate revenue.

Turnovsky and Brock (1980) (T-B), extending Calvo's results by considering bond finance and government production of a public good, find that if the government maximizes with respect to any or all of its fiscal instruments taking its monetary instruments as given, the optimal policy will be time inconsistent.

Lucas and Stokey (1983) re-examine T-B's results in a discrete time model where the demand for money is generated from a cash-in-advance constraint. They argue that the source of time inconsistency in Calvo's problem is due to the fact that nominal bonds do not represent a real commitment. Thus, the government has an incentive to use an inflation tax (unannounced) as a non-distortionary tax. Any attempt to optimally set tax rates will be time inconsistent.

Lucas and Stokey note that when the optimal plan, formulated by the myopic government, without consideration that its future incarnation may or may not renege, is consistent, the "time-consistent optimal policy corresponds to a set of subgame perfect Nash equilibrium strategies" (p. 64)[2]. In other words, the time consistent plan formulated by the myopic government is equivalent to an equilibrium strategy which would be derived from a game played by the current government with the future government. Thus while their and others' research does not explicitly deal with the interaction between successive governments, it does identify policies which would result from such a model.

This paper will focus on a deterministic, continuous time model similar to that examined by T-B. However, their conclusions will be re-examined under the assumption that liquidity preference is generated by an exogenous transactions technology as opposed to assuming that real balances bear a utility yield. The primary experiment will be to examine the time consistency of optimal policy in an economy in which the households incur real transactions costs which may be mitigated through the use of stocks of real bonds and real balances. This assumption is identical to that used by Fried and Howitt (1983) in which the effect of such a transactions technology with respect to the responsiveness of real rates of interest to changes in inflation is examined.

It will be assumed that household expenditures on goods and services include that to finance consumption and transactions services. These transactions costs increase with total expenditures but may be reduced by the addition of real bonds and/or balances to the household's stocks.

In what follows, the time consistency of optimal policy, in such an economy, is found to depend on the liquidity of bonds. In other words, if the transactions technology is such that bonds bear a liquidity yield, no optimal policy is time consistent. However, if bonds do not bear a liquidity yield and do not reduce the real cost of transacting, any optimal policy is time consistent if and only if the policy instruments include the rate of nominal monetary expansion.

These results are then compared with an economy in which the transactions costs incurred by the household do not represent a real resource drain to the individual (a cash-in-advance constraint).

It is demonstrated that in such an economy, optimal policy will always be time consistent unless the only instrument is government public good expenditures. Further, the results are qualitatively unchanged if bonds are illiquid.

Finally, it is shown that restrictions to 'balance the budget' or to restrict the size of the deficit will, contrary to public perception, increase the time inconsistency of optimal policy.

Section II introduces the household's decision problem. Section III defines a perfect foresight equilibrium. Sections IV and V examine time consistency of optimal policies. Section VI introduces an alternate transactions cost technology which is solely pecuniary and Section VII examines the effect on time consistency of a legislated budget deficit.

II. THE HOUSEHOLD

The economy consists of only two agents, a representative, infinitely lived, household and the government. In this section the decision problem of the former will be discussed.

The household acts, at each point in time, to maximize its lifetime utility, a function of consumption (c), leisure (f) and the government's public good production (g),[3]

$$(1) \qquad \int_0^\infty e^{-\delta t}\, u(c,f,g)\,dt$$

where δ is the rate of time preference such that $0 < \delta$. The household's instantaneous utility function u(.) is assumed twice differentiable and concave in all of its arguments.

The household faces two constraints at each point in time. The first states that its rate of asset accumulation must equal the difference between income and expenditures,

$$(2) \qquad \dot{m} + \dot{b} = (1-\tau)[(1-f)+ib] - \pi(m+b) - x$$

where m and b are the household's holdings of real balances and bonds respectively, π is the rate of price inflation, τ is the interest and labour income tax rate, i is the nominal rate of interest on government bonds and x is the household's level of expenditures.

It is assumed that household labour income/production is a linear function of leisure such that labour income equals (1-f). The economy is assumed not to accumulate capital, the reason for which will become obvious in later sections.

The second constraint which the household must satisfy at each moment in time is that the level of expenditures equals the sum of consumption goods and transactions costs incurred. As Fried and Howitt (1983) assume, such transactions costs are increasing in the level of expenditures and depend on the level of real balances and bonds held. Thus,

$$(3) \qquad x = c + h(x;m,b)$$

so that,

$$(4) \qquad c = x - h(x;m,b)$$
$$\equiv \ell(x,m,b)$$

where h(.) is a transactions cost function which mimics the pecuniary costs associated with transacting.

It is assumed (as do Fried and Howitt) that $\ell(\ )$ is twice continuously differentiable and,

$$\ell_x > 0$$

and,

$$\ell_{mx}, \ell_{bx} \geq 0$$

It is assumed that an increase in expenditures increases transactions costs by a lesser amount so that $\ell_x > 0$ and a higher level of consumption may be financed. The latter two assumptions above imply that the marginal liquidity yields of real balances and bonds are increasing as the level of expenditures increases.

It will be assumed that the liquidity yields of real balances and bonds are increasing at a diminishing rate, reach a maximum and then decrease at an increasing rate. For example, for a given level of x and b, increasing real balances will increase the level of consumption as the household economizes on transactions costs. However, as real balances increase, a point will be reached at which $\ell_m = 0$ and an extra unit of real balances will reduce consumption and increase transactions costs. A specific example of a function which satisfies these properties is given by Fried and Howitt.

Finally, the household is constrained by the following initial conditions, that,

(5) $b(0) \equiv B(0)/P(0) = B_0/P_0$

and

(6) $m(0) = M(0)/P(0) = M_0/P_0$

where b(0) and m(0) are the initial levels of real balances and bonds held by the household.

Thus, the household maximizes (1) given the constraints above and the perfectly foreseen values of g, P_0 (the initial price level), π, τ and i. Lifetime utility is maximized when the current valued Hamiltonian

$$(7)\quad H \equiv e^{-\delta t}u(c,f,g)$$

$$+\ \beta e^{-\delta t}[c-\ell(x,m,b)]$$

$$+\ \alpha e^{-\delta t}[(1-\tau)[(1-f)+ib]-\pi(m+b)-x]$$

is maximized with respect to m, b, c, f and x. The Euler equations for an optimum are (after substituting out β and c),

$$(8)\quad U_c - \frac{\alpha}{\ell_x} = 0$$

$$(9)\quad U_f - \alpha(1-\tau) = 0$$

$$(10)\quad \frac{\ell_m}{\ell_x}\alpha - \alpha\pi = -\dot{\alpha} + \delta\alpha$$

$$(11)\quad \alpha\left(\frac{\ell_b}{\ell_x} - \pi + (1-\tau)i\right) = -\dot{\alpha} + \delta\alpha$$

and the transversality conditions,

$$(12)\quad \lim_{t\to\infty} \alpha m e^{-\delta t} = 0\ ,\ \lim_{t\to\infty} \alpha b e^{-\delta t} = 0$$

which state that the current value of planned holdings of real balances and real bonds at t as $t \to \infty$ must equal zero.

Note, in this derivation of a money demand based on the pecuniary liquidity services that real assets yield, the nominal, after tax, rate of interest will depend on the difference between the liquidity yields on real balances and real bonds. This is demonstrated by combining equations (10) and (11),

$$i(1-\tau) = \frac{\ell_m - \ell_b}{\ell_x}$$

so that only if the marginal liquidity services of money are greater than that of bonds will the nominal after tax yield on bonds be positive, compensating households for holding them.

III. PERFECT FORESIGHT EQUILIBRIUM (P.F.E.)

Equilibrium in the output market is satisfied when output supply equals expenditures. Thus, when

(13) $(1-f) = x + g$

supply equals demand. By Walras's law, if the money and the goods market are in equilibrium so will the bond market. To close the model, therefore, the equilibrium condition in the money market is that,

(14) $\dot{m} = (\mu(t)-\pi)m$

where μ is defined as,

$$\mu \equiv \dot{M}/M$$

an instrument of the government.

The government's budget constraint is found by combining equations (2) and (13), yielding,

(15) $\dot{m} + \dot{b} = g+(1-\tau)ib - \pi(m+b) - \tau(1-f)$

As discussed above, only perfect foresight equilibria will be examined. A perfect foresight equilibrium will be a sequence of x, f, c, m, b, α, i and π which satisfy equations (2), (4), (8) - (11), (13) and (14) at each point in time and (12), the transversality condition, given a pre-announced time path of π, g and μ, as well as the initial bond/money ratio. As discussed by Sargent and Wallace (1973) and Calvo (1978), a perfect foresight path of prices is constrained to be continuous except at the initial date t=0. Future discontinuities are precluded as rational agents would attempt to exploit resultant profits prior to the discontinuity, their actions would then tend to smooth the price path. However, there is no constraint on the initial price level, and it will in general 'jump' to that level necessary to adjust m(0) and b(0) to their market clearing values. Thus, in general, there will be a discontinuous jump in the real values of the household's state variables m and b at t=0.

IV. OPTIMAL POLICY

It will be assumed that the government seeks to maximize social welfare which is equivalent to the welfare of the representative household. As defined above, the P.F.E. must satisfy equations (2), (4), (8)-(11), (13) and (14) at each point in time, eight equations in m, g, c, x, b, α, i, π, g, τ and μ, eleven variables. Thus, the government has at most three instruments, while the initial price level P_0 adjusts to satisfy the transversality conditions.[5]

With respect to the economy described above, the government's decision problem is to maximize

$$\int e^{-\delta t} u(\ell(x,m,b), 1-g-x, g)\,dt$$

subject to,

(16) $u_c \ell_x - \alpha = 0$

(17) $u_f - \alpha(1-\tau) = 0$

(18) $\frac{\ell_m \alpha}{\ell_x} - \alpha(\pi+\delta) + z = 0$

(19) $\dot{m} = m(\mu-\pi)$

(20) $\dot{b} = (1-\tau)(g+x) + b(\delta - \frac{z}{\alpha} - \frac{\ell_b}{\ell_x}) - \mu m - x$

(21) $\dot{\alpha} = z$

where c, f and i have been eliminated from the problem through the use of equations (14), (13) and (11) respectively.

It is important for an understanding of the subsequent analysis to consider the optimization problem of the government in more detail.

The problem facing the government is, at t = 0, to announce a time path of its three independent instruments so as to maximize social welfare subject to the market clearing constraints. Given the time paths of μ, τ and g and the initial nominal stocks of money and bonds, the time paths of all the other variables will be determined.

It is important to note that at t = 0 the government has one other instrument, that may not be available at t > 0, namely the bond to money ratio. Unlike the household, M(0) and B(0) are not assumed given to the government. It is assumed that at t = 0 the government may avail itself of open market operations, swapping bonds for money, so as to set the bond to money ratio at will. Thus, given the initial bond to money ratio that the government chooses and the time path of its instruments it announces, the price level and interest rate will adjust so as to clear the goods and assets markets.

Thus, with respect to the government's optimal control problem, the initial real value of money and bonds, m(0) and b(0), are 'free' variables.[6] By changing the bond to money ratio and the time paths of its instruments m(0) and b(0) may discontinuously set to their optimal levels. In the standard optimal control problem discussed below, discontinuous jumps in the state variables (those variables whose time paths are governed by equations of motion) are ruled out since their behaviour is ruled by the relevant differential equation. For example, a jump in the controls will only cause a jump in the state's rate of change, not in its level.

The government's control problem may be modified to include jumps in its state variables (see Kamien and Schwartz, Section 18) however, as argued below, such modifications are irrelevant to the problem the government faces.

Note that the government is assumed to be as myopic as the private sector in that it does not consider future re-evaluation. However, as will be discussed in Section VIII below, it can be shown that if the optimal policy is time consistent then it is equivalent to a subgame perfect Nash equilibrium.

The Hamiltonian for the above optimal control problem is,

$$\begin{aligned}
H = {} & e^{-\delta t}u(\ell(x,m,b),1-g-x,g) \\
& + v_1 e^{-\delta t}[u_c\ell_x - \alpha] \\
& + v_2 e^{-\delta t}[u_f - \alpha(1-\tau)] \\
& + v_3 e^{-\delta t}[\frac{\ell_m \alpha}{\ell_x} - \alpha(\pi+\delta) + z] \\
& + q_1 e^{-\delta t}[m(\mu-\pi)] \\
& + q_2 e^{-\delta t}[(1-\tau)(g+x) + b(\delta - \frac{z}{\alpha} - \frac{\ell_b}{\ell_x}) - \mu m - x] \\
& + q_3 e^{-\delta t} z
\end{aligned}$$

which when maximized with respect to x, m, b, g, τ, μ, α, z and π yield the following necessary conditions for a maximum,

(22) $$\begin{aligned}
& u_c\ell_x - u_f + v_1[\ell_{xx}u_c + u_{cc}\ell_x^2 - u_{cf}\ell_x] \\
& + v_2[u_{fc}\ell_x - u_{ff}] + v_3(\frac{\ell_{mx}\ell_x - \ell_{xx}\ell_m}{\ell_x^2})\alpha \\
& + q_2[-\tau - b(\frac{\ell_{bx}\ell_x - \ell_{xx}\ell_b}{\ell_x^2})] = 0
\end{aligned}$$

$$(23)\quad u_c\ell_m + v_1[\ell_{xm}\ell_c + \ell_x\ell_m u_{cc}]$$

$$+ v_2\ell_m u_{fc} + v_3\left(\frac{\ell_{mm}\ell_x - \ell_{xm}\ell_m}{\ell_x^2}\right) + q_1(\mu-\pi)$$

$$- q_2\left[\mu + b\left(\frac{\ell_{bm}\ell_x - \ell_{xm}\ell_b}{\ell_x^2}\right)\right] = -\dot{q}_1 + \delta q_1$$

$$(24)\quad u_c\ell_b + v_1[\ell_{xb}u_c + \ell_x\ell_b u_{cc}] + v_2\ell_b u_{fc}$$

$$+ v_3\left(\frac{\ell_{mb}\ell_x - \ell_{xb}\ell_m}{\ell_x^2}\right) + q_2\left[\delta - \frac{z}{\alpha} - \frac{\ell_b}{\ell_x} - b\left(\frac{\ell_{bb}\ell_x - \ell_{xb}\ell_b}{\ell_x^2}\right)\right] = -\dot{q}_2 + \delta q_2$$

$$(25)\quad u_g - u_f + v_1[u_{cg} - u_{cf}]\ell_x + v_2[u_{fg} - u_{ff}]$$

$$+ q_2(1-\tau) = 0$$

$$(26)\quad v_2 - q_2(g+x) = 0$$

$$(27)\quad q_1 m - q_2 m = 0$$

$$(28)\quad -v_1 - v_2(1-\tau) - v_3(\pi+\delta) + q_2\frac{bz}{\alpha^2} = -\dot{q}_3 + \delta q_3$$

$$(29)\quad v_3 - q_2\frac{b}{\alpha} + q_3 = 0$$

$$(30)\quad -q_1 m - \alpha v_3 = 0$$

The optimal policy must also satisfy the government's transversality conditions, similar to those faced by the household,

$$(31a)\quad \lim_{t\to\infty} q_1 m e^{-\delta t} = 0$$

$$(31b)\quad \lim_{t\to\infty} q_2 b e^{-\delta t} = 0$$

(31c) $\lim_{t \to \infty} q_3 \alpha e^{-\delta t} = 0$

that the marginal value of m, b and α at the limit be zero.

As in the static nonlinear programming problem, the costate variables q_1, q_2, and q_3 have the interpretation of being the marginal valuation of the constraints they are associated with. Since, in the dynamic context above, the constraints are equal to $\dot{m}$, $\dot{b}$ and $\dot{\alpha}$ respectively, the costates measure the marginal value of an increment to m, b and α respectively (note that α was the costate associated with the asset accumulation problem of the household).

This interpretation of the costate variables leads to a final optimal condition. Since the government is unconstrained with respect to the initial values of m(0) and b(0); in other words, the government may vary the bond to money ratio and the initial price level through 'open market operations' so that they jump to their optimal values, then this implies that at an optimum the marginal valuation of an m or b increment at the initial period must be,

$$q_1(0) = q_2(0) = 0$$

Thus, since the costates measure the optimal value of a free variable, the optimal plan will be to set them equal to zero (see Kamien and Schwartz (1981), Chapter 11.4).

As discussed in the first section of this paper, a taxonomy of constraints will be imposed on the government and the resultant optimal plan will be examined for consistency. The above transversality condition and equations (13) - (25) represent the necessary conditions for an optimum if the government is unconstrained. The constraints which will be considered will be an imposition of exogeneity on one or two of the three instruments, g, τ and μ, and consideration of a legislated constraint regarding the size of the deficit.

With regards to consistency, Calvo's (1978) criterion will be adopted (used by T-B and Buiter [1983] among others). As described, the optimal P.F.E. will be derived by solving the government's optimal control problem. This will imply a path for the relevant variables through time. As Calvo has noted, however, unless that optimal plan sets the shadow price or costates of the free variables under government control equal to zero for all t, an inconsistency will result. For example, consider an optimal plan derived from the above problems. If that plan implies that $q_1(t)$ and/or $q_2(t)$ are not equal to zero at t_1, $t_1 > t = 0$, then at t_1, if the government is free to re-evaluate its plan, it will optimally cause a discontinuous jump in m and/or b at t, until their shadow prices are again zero; that is, $q_1 = q_2 = 0$. This is simply a consequence of the Pontryagin Maximum Principle and the fact that the system is autonomous. The methodology for testing whether or not the optimal plan is inconsistent is to derive the necessary conditions for the optimal plan, impose consistency on those conditions by setting $q_1 = q_2 = 0$ for all t, and examining the optimal solution to determine whether an optimal path still exists which will satisfy the necessary conditions and the consistency requirement.

a) Optimal Money Growth; g, τ Constant[7]

If the monetary authorities take g and τ as given by a fiscal authority (and constant) and set the rate of monetary growth to maximize social welfare, the relevant problem is as described above, with the relevant necessary conditions including the transversality conditions and equations (22) - (24), (27) - (30); in other words, excluding H_g and H_τ. Assuming that the system of equations defines a unique optimal path and imposing the consistency requirement that $q_1 = q_2 = 0$ for all t will either yield a unique solution to the government's optimal control problem or yield an inconsistent system. Only in the former case will the optimal path be time consistent.

If $q_1 = q_2 = 0$, equations (29) and (30) imply that,

$$v_3 = q_3 = 0$$

for all t. Given this result equations (22), (23), (24), (28) and, from equations (16) and (17),

$$(1-\tau)u_c\ell_x = u_f \;.$$

describe x, m, b, v_1 and v_2 in five equations in terms of g and τ. Thus, $\dot{m}$, $\dot{b}$ and $\dot{\alpha} = z$ must be zero. The optimal path is thus defined by

$$(22') \qquad u_c\ell_x - u_f + v_1[\ell_{xx}u_c+u_{cc}\ell_x^2-u_{cf}\ell_x]+v_2[u_{fc}\ell_x-u_{ff}] = 0$$

$$(23') \qquad u_c\ell_m+v_1[\ell_{xm}u_c + \ell_x\ell_m u_{cc}]+v_2\ell_m u_{fc} = 0$$

$$(24') \qquad u_c\ell_b+v_1[\ell_{xb}u_c+\ell_x\ell_b u_{cc}]+v_2\ell_b u_{fc} = 0$$

(28') $-v_1 - v_2(1-\tau) = 0$

and the five constraints, equations (16) - (20) where $\dot{\alpha} = z = \dot{m} = \dot{b} = 0$. These nine equations, which are in general independent, determine x, m, b, α, π, μ, v_1 and v_2, eight variables, in terms of g and τ. Thus, this optimal control problem is time inconsistent.

As discussed above, the solution to this problem, equations (16) - (21) and (22) - (30) must then imply an approach to the steady state levels of m and b as opposed to an immediate jump to their steady state values. Thus, if the monetary authorities were to re-evaluate its optimal strategy at a future t ($t < \infty$), the optimal plan would be to re-start the optimal plan exactly as before. Thus the optimal plan is time inconsistent.

This result is in stark contrast to the results of T-B who argue that a government's optimal policy will be time consistent if and only if its instruments include a monetary instrument such as μ, the rate of growth of nominal balances.

b) <u>Optimal Fiscal Policy; μ Constant</u>

The reciprocal problem to that discussed above is that faced by the fiscal authorities. The fiscal instruments in this context are the income tax rate and government public good expenditure, which are to be set optimally given a constant path for nominal money growth.

As above, the optimal control problem is equivalent to maximizing the Hamiltonian with respect to x, m, b, τ, g, α, z and π. Thus, the conditions which define the optimal path include equations (16) - (21) and (22) - (26) and (28) - (30) as well as the transversality conditions.

Imposing time consistency on this solution is equivalent to assuming that $q_1 = q_2 = 0$ for all t. Together, with equations (26) - (30), this implies

$$v_1 = v_2 = v_3 = q_1 = q_2 = q_3 = 0$$

so that, as before, $\dot{\alpha} = \dot{m} = \dot{b} = 0$, if the solution is time consistent.

The system which defines the optimal path will simplify to:

(32a) $\tau = 0$

(32b) $\ell_b = 0$

(32c) $\ell_m = 0$

(32d) $\delta = -\pi$

(32e) $\pi = \mu$

(32f) $u_c \ell_x = \alpha$

(32g) $u_c \ell_x = u_f$

(32h) $u_g = u_f$

(32i) $g + x + b\delta - \mu m - x = 0$

or nine equations in m, b, x, α, π, g and τ, seven variables. Thus, only by coincidence will the steady state solutions for the above variables defined by the first seven equations satisfy the final two equations.

c) Optimal Fiscal and Monetary Policy

If, in contrast to the above examples, the monetary and fiscal authorities are integrated, the optimization procedure is to maximize the Hamiltonian with respect to m, b, x, α, z, π, μ, g and τ. The conditions which define the optimal paths of these variables are equations (16) - (30) and the transversality conditions.

Again, imposing the restriction that the solution is time consistent implies that $q_1 = q_2 = 0$. Again, in this instance, it is simple to demonstrate that this also imposes an inconsistency on the system. Thus, since the necessary conditions are identical to that in Section (b) above with the addition of equation (27), the system collapses to that given by equations (32a) - (i). In this case, the nine equations determine eight variables so that an inconsistency results.

d) Other Optimal Policies

Table III.1 describes the range of optimal policies the government may adopt, optimizing over up to three policy instruments. T-B also examine a policy constraint they term a Phelps constraint in which the government is constrained to generate a fixed real amount of revenue from money

TABLE III.1: TIME CONSISTENCY OF OPTIMAL POLICY x GIVEN CONSTANT y $(x|y)$

	Real Transactions Cost Technology	
	$\ell(x,m,b)$	$\tilde{\ell}(x,m)$
$\mu\|g,\tau$	X	✓
$\tau\|g,\mu$	X	X
$g\|\tau,\mu$	X	X
$\mu,\tau\|g$	X	✓
$\mu,g\|\tau$	X	✓
$g,\tau\|\mu$	X	X
μ,g,τ	X	✓
Phelps	X	X

✓ time consistent optimal policy

X time inconsistent optimal policy

creation and tax revenue.

(33) $\quad \mu m + \tau(1-f) = \bar{K} \equiv g+(\delta - \frac{\dot{\alpha}}{\alpha})b-\dot{b}$

where the left hand side of (33) equals the inflation tax and labour income tax revenue. The Phelpsian problem is to maximize social welfare with respect to μ and τ, given a constant g under the constraint above (and the market clearing constraints).

As shown in Table III.1, under no circumstances will the optimal policy for the economy above be time consistent. This is again in contrast to T-B's results which purport to show, under the condition that money bears a utility yield, that any optimal policy is time consistent if one of the instruments is the rate of nominal monetary expansion.

This may be confirmed by examining a version of the reduced form that T-B use to conclude their arguments. The government's problem may be reduced, where v(.) is the indirect utility function, to,[8]

$$\max \int e^{-\delta t} v(m,b,\tau,g)dt$$

subject to,

$$\dot{m} = h^1(m,b;\tau,g,\mu)$$

$$\dot{b} = h^2(m,b;\tau,g,\mu)$$

Forming the requisite Hamiltonian and performing the same experiments above confirm the results in Table 1. The crucial difference between these results and those of T-B is that in this case the indirect utility function contains real bonds due to the transactions technology assumed versus the alternate specification in which real balances bear a utility yield.

For example, if the monetary authorities maximize social welfare with respect to μ, imposing time consistency implies the optimal paths of m, b and μ must satisfy,

$$v_m = v_b = h^1 = h^2 = 0$$

which, in general, will be impossible. However, if real bonds do not enter the indirect utility function, or if money demand is generated through the assumption that money bears only a utility yield then the time consistent optimal paths of m, b and μ need only satisfy

$$v_m = h^1 = h^2 = 0$$

which is sufficient to obtain a unique solution.

Thus, the transactions technology assumed above imposes an additional constraint which implies that any attempt by government to generate an optimal time consistent path will be impossible. The first-best policy would set the distortionary income tax τ equal to zero and m and b so as to minimize transactions costs. However, the government cannot achieve the optimal levels of m and b and finance exogenous or optimal government public good expenditure. The second-best policy will generally attempt to raise revenue with minimal distortions; utilizing a current inflation tax, postponing the income tax until the future. In this case there exists the incentive to renege at some future date.

An analogous problem would be in terms of a simple, non-monetary growth model in which the initial capital stock is a free variable. The optimal plan will set the initial capital stock to that level which finances a 'bliss' level of consumption. If that level is unsustainable, the optimal plan will generally imply an optimal path to the 'golden rule' level. However, if the planner is able to re-optimize at some future date, the plan will be re-started so as to set the capital stock to the bliss stock.' Thus, this type of problem would be time inconsistent unless the bliss stock of capital is sustainable.

V. LIQUIDITY OF BONDS

Since the novelty of the above discussion of time consistency is due to the transactions technology assumed it is of interest to determine the robustness of the above results with respect to the specification of the transactions technology.

As Fried and Howitt note, the specification of the liquidity function $\ell(\ )$ has important implications towards the comparative static (steady state) implications of the effect of changes in μ and π on the nominal and real rates of interest. It will be demonstrated below that there are also important implications with respect to the dynamics of the economy described above.

In fact, an important aspect of the discussion above is that in all cases the optimal policy does not imply an immediate jump to the new steady state, affording the government an incentive, upon re-evaluation of its policy, to jump to the initial, optimal, values of its instruments.

However, if the transactions technology is such that bonds do not have any impact on the real cost of transactions, i.e., ℓ_b and h_b are zero for all b, m and x, the results described above no longer hold. In fact, as summarized in Table III.1, any optimal policy will be time consistent so long as the set of instruments the government optimizes over includes μ, the rate of growth of nominal money (and there is no Phelps constraint).

For example, if the government optimizes over μ, τ and g, the optimal paths for m, b, x, α, μ, τ, π and g are described by,

(32a) $\tau = 0$

(32c) $\ell_m = 0$

(32d) $\delta = -\pi$

(32e) $\pi = \mu$

(32f) $u_c \ell_x = \alpha$

(32g) $u_c \ell_x = u_f$

(32h) $u_g = u_f$

(32i) $g + b\delta - \mu m = 0$

The optimal, and time consistent, policy sets the income tax to zero, minimizing labour supply distortions. Real balances are set so as to minimize transactions costs, setting the marginal liquidity yield to zero. The rate of inflation which generates the optimal quantity of money supply and demand is the negative of the rate of time preference. Since

$$i = \left(\frac{\ell_m}{\ell_x}\right)/(1-\tau)$$

the optimal nominal interest rate is zero. Unlike T-B, the result of the government's optimization is a Friedmanesque full liquidity equilibrium. The rate of return on all assets, including money, is equal to the rate of time preference and the marginal liquidity yield on real balances is zero. This is due to the fact that, in the transactions costs economy, the level of real balances does not directly affect the marginal rate of substitution between consumption and labour, as would be the case if money bore a utility yield. The level of real balances can be set at that level which minimizes transactions costs without 'distorting' the consumption/leisure choice.

Thus, T-B's results are duplicated only if bonds do not offer a real liquidity yield. If bonds are omitted from the liquidity function and the government has as an instrument μ, it will be optimal to move the economy immediately to a steady state, so that the optimal policy is time consistent.

Note that equations (32i), (32d) and (32e) imply that

$$g = -(m + b/\delta)$$

so that b, the optimal steady state value of real bonds is negative, given non-negativity of m and g.

This result, as T-B and Lucas and Stokey note, follows from the fact that in this context, real bonds bear neither a liquidity nor a utility yield which would improve social welfare when $b > 0$. In fact, the optimal level of b will be, as T-B argue, negative since this will afford the government the opportunity to extract revenue, in the form of interest payments from the private sector to the government, with which to finance the deflationary subsidy and government public good production.

Thus, in this case, at $t = 0$ the government purchases all outstanding interest bearing debt and further, purchases private interest bearing debt, so as to achieve an optimum. As noted above, this initial open market operation was included in the government's optimal control problem.

The optimal policy will be time consistent in this context since the government has enough instruments to achieve the first-best optimum. For example, if its instruments include μ, τ and g, the optimal values of μ and τ will be $-\delta$ and zero respectively. In other words, the optimal quantity of real balances and output will result if the rate of return on money is equal to the rate of time preference and there is no distortionary income tax.

However, to set g to its optimal value given the above values of μ and τ requires an instrument to generate the additional revenue required over that needed to finance the deflation. Fortunately, in the economy described in this section, bonds are a 'slack' variable such that, by an initial open market operation, b can be set to whichever level generates the appropriate revenue. In other words, the government is not constrained, as in the previous section, to set the level of b to minimize transactions cost and therefore can use the initial open market operation to set b to that level which generates the optimal interest income.

VI. OTHER TRANSACTIONS TECHNOLOGIES

If, as Fried and Howitt suggest, "each point in [continuous] time may be regarded as a discrete interval in meta-time" (p. 971) then assuming the interval of meta-time is of exogenous length, it may be that the household is constrained to hold cash in advance to finance purchases in the interval. Thus, the household's constraints may be amended so that

$$(2') \qquad \dot{m} + \dot{b} = (1-\tau)[(1-f)+ib]-\pi(m+b)-c$$

$$(3') \qquad m \geq c$$

where now expenditures equal consumption only, there is no real cost to transacting except for that embodied in (3') above.

Note, the use of a cash-in-advance constraint, a discrete concept, in a continuous framework is at best artificial. However, its adoption is of interest if only for didactive purposes. Thus, the results below are primarily illustrative.[9]

The first order conditions the household's lifetime utility to be at a maximum subject to equations (2') and (3') are,

(33a) $u_f(m,f,g) = \alpha(1-\tau)$

(33b) $u_c(m,f,g) = \alpha(1+\pi+\delta)+z$

(33c) $\dot{m} + \dot{b} = (1-\tau)(1-f)+b(\delta - \frac{z}{\alpha}) - (1+\pi)m$

(33d) $z = \dot{\alpha}$

where, given (3'), c has been replaced by real balances, given that the cash-in-advance constraint holds with equality, and from the household's first order conditions i has been replaced by,

(33e) $i = (\delta - \frac{z}{\alpha} + \pi)/(1-\tau)$

Again, α is the costate attached to the household's asset accumulation constraint and $z \equiv \dot{\alpha}$. The household's optimal plan also must satisfy the transversality conditions which are identical to those given by equation (12).

The government, as before, maximizes social welfare subject to equations (33a) - (d) and the market clearing conditions,

(34) $(1-f) = c + g = m + g$

and

$$\dot{m} = m(\mu-\pi)$$

Note, in the context of the cash-in-advance constraint, there is no direct real cost to liquidity. Now total expenditures equal consumption not the sum of consumption and liquidity services.

Thus, the optimal plan is found by maximizing,

$$\begin{aligned} H = {} & e^{-\delta t}u(m,1-m-g,g) \\ & + v_1 e^{-\delta t}[u_f-\alpha(1-\tau)]+v_2 e^{-\delta t}[u_c-\alpha(1+\pi+\delta)\ z] \\ & + q_1 e^{-\delta t}[m(\mu-\pi)]+q_2 e^{-\delta t}[(1-\tau)(m+g)+b(\delta - \tfrac{z}{\alpha})-m(1+\mu)]+q_3 e^{-\delta t}z \end{aligned}$$

The identical experiments as those performed in Sections IV and V may be replicated for the cash-in-advance economy by deriving the relevant first order conditions to the government's optimization problem and imposing the consistency criteria ($q_1=q_2=0$). As above, the policy will be consistent if the resultant system has a solution.

For example, if the government maximizes with respect to g and τ while the economy operates under a cash-in-advance constraint, the optimal time consistent paths for m, b, α, π, g and τ are given by,

(35a) $u_c = u_g$

(35b) $u_g = u_f$

(35c) $u_f = \alpha(1-\tau)$

(35d) $u_c = \alpha(1+\pi+\delta)$

(35e) $(1-\tau)(m+g)+b\delta-m(1+\mu) = 0$

(35f) $\mu = \pi$

so that the steady state nominal rate of interest, from equations (33e), (35c), (d) and (f) is

$$i = \frac{\delta+\mu}{1+\mu+\delta}$$

if μ is exogenous to the government. Thus, the optimal nominal rate, from the standpoint of the fiscal authorities, increases as μ increases. As one would expect from second best theory, the optimal tax rate given the exogenous distortion μ is not zero but,

$$\tau = -(\mu+\delta)$$

Only if the rate of nominal money growth is at its optimal first best level $\mu = -\delta$ will the tax rate equal zero.

Thus, if the government's instruments include g, μ and τ, the first best solution will be to set $\mu = -\delta$ and $\tau = 0$.

The consistency results of the cash-in-advance economy are summarized in Table III.2. As can be seen, the cash-in-advance economy is much less likely to suffer from an inconsistent optimal policy. Only if government expenditure g is the only instrument will the optimal policy be inconsistent.

Note, the consistency properties of the cash-in-advance properties are not affected by the liquidity of bonds as in the transactions cost

TABLE III.2: TIME CONSISTENCY OF GOVERNMENT POLICY x GIVEN CONSTANT y (x|y) CASH-IN-ADVANCE TECHNOLOGY

	Unconstrained	Balanced Budget
$\mu \mid g,\tau$	✓	X
$\tau \mid g,\mu$	✓	X
$g \mid \mu,\tau$	X	X
$\mu,\tau \mid g$	✓*	✓
$\mu,g \mid \tau$	✓	X
$\tau,g \mid \mu$	✓	X
$\mu,g \mid \tau$	✓*	✓
Phelps	✓	X

✓ time consistent

X time inconsistent

* time consistent policy non unique

economy examined above. For example, if bonds were as illiquid as consumption expenditures, cash-in-advance would be required to finance net expenditures on real bonds. Thus,

$$(3'') \qquad m \geq c + \dot{b} - (i - \pi) b$$

Replacing (3') with the above constraint does not affect the consistency results summarized in Table III.2.

The results of this section may, as for the previous sections, be represented in terms of the reduced form representation of the government's problem given in Section IV. The effect of the cash-in-advance constraint is to remove real bonds from the indirect utility function so that v_b and v_τ are equal to zero. The removal of the constraints that at the optimum $v_b = v_\tau = 0$ implies a greater degree of freedom so that with the consistency requirement the optimal path is consistent. In effect, by removing τ and b from the indirect utility function the cash-in-advance economy makes an immediate jump to a steady state optimal in cases in which such a jump would not be optimal for economies with other transactions technologies.

Thus, while as Lucas and Stokey argue "the imposition of a Clower constraint is not an alternative to Sidrauski's way [money in the utility function] of formulating the demand for money, but in fact is closely related to it", the alternate method of generating liquidity preference does have strong implications with respect to time consistency of optimal policy.

In fact, it is possible to duplicate the results in this and the previous section by amending the transactions technology introduced in Section II. If an economy could be conceived of for which a firm (or government) sells liquidity services such that,

$$c = x - \ell(x,m,b)$$

but that the firm has zero marginal resource costs so that market clearing implies,

$$1 - f = c + q$$

i.e., the liquidity services are pecuniary. The private sector's budget constraint would now include the profits of the firm selling liquidity services as would the government's budget constraint if those profits were taxed.

Thus, at least qualitatively, the cash-in-advance technology mimics a liquidity service technology for which those liquidity services are solely pecuniary. While this probably does not conform to reality, it offers an explanation for the inconsistency results in other economies, for which liquidity services are 'real'.

If bonds bear a real liquidity yield, the government cannot, optimally, drive $b < 0$ to acquire revenue in a non-distortionary fashion. Thus, it is optimal to use an inflation tax initially, promising a lower inflation in the future, to acquire revenue, without using distortionary labour income tax. This affords the government current revenue without reducing m and b since lower future inflation is promised.

Thus, if m and b offer real liquidity yields a non-stationary optimal policy is much more likely for the reasons discussed at the end of Section V. Essentially, since the first best policy is a stationary policy (which will be time consistent), a government will only desire a non-stationary (and time inconsistent) policy if it is constrained from achieving the first best equilibrium.

VII. BALANCED BUDGETS

It has been often argued that to constrain the government to either balance the budget or maintain a fixed deficit either instantaneously or over the cycle would promote consistency by reducing the government's ability to raise revenue at its discretion in the future. However, as the analysis above may suggest, in fact the imposition of such a constraint on government may indeed increase the propensity for time inconsistent optimal policies.

If the constraint takes the form that

$$(36) \qquad g - \;(1-f) - \; ib = d$$

where d is some fixed constant (possibly zero) representing the real deficit substituting this constraint into the government's decision problem examined in Section VI yields the Hamiltonian

$$H = e^{-\delta t}u(m,1-m-g,g)$$

$$+ v_1 e^{-\delta t}[u_f - \alpha(1-\tau)]$$

$$+ v_2 e^{-\delta t}[u_c - \alpha(1+\tau+\varepsilon)+z]$$

$$+ v_3 e^{-\delta t}[(1-\tau)g - \frac{\tau b}{1-\tau}(\varepsilon - \frac{z}{\alpha} + \pi) - \tau m - d]$$

$$+ q_1 e^{-\delta t}[(\mu-\pi)m]$$

$$+ q_2 e^{-\delta t}[(1-\tau)(m+g) + b(\varepsilon - \frac{z}{\alpha}) - m(1+\mu)]$$

$$+ q_3 e^{-\delta t}[z]$$

The necessary conditions for an optimal path are the six constraints, and

(37a) $u_c - u_f + v_1[u_{fc} - u_{ff}] + v_2[u_{cc} - u_{cf}] - v_3\tau + q_1(\mu-\pi) - q_2(\tau+\mu) = -\dot{q}_1 + \delta q_1$

(37b) $\frac{\tau}{1-\tau}v_3(\varepsilon - \frac{z}{\alpha} + \pi) + q_2\frac{z}{\alpha} = \dot{q}_2$

(37c) $v_1\alpha - v_3(g+m+\frac{b}{(1-\tau)^2}(\varepsilon + \frac{z}{\alpha} + \pi)) - q_2(g+m) = 0$

(37d) $v_2 + \frac{v_3\tau b}{(1-\tau)\alpha} - q_2\frac{b}{\alpha} + q_3 = 0$

(37e) $-v_2\alpha - \frac{v_3\tau b}{(1-\tau)} - q_1 = 0$

(37f) $-v_1(1-\tau) - v_2(1+\tau+\delta) - \frac{v_3\tau bz}{(1-\tau)\alpha^2} + q_2\frac{bz}{\alpha^2} = -\dot{q}_3 + \delta q_3$

(37g) $u_g - u_f - v_1[u_{ff} - u_{fg}] - v_2[u_{cg} - u_{cg}] + v_3(1-\tau) + q_2(1-\tau) = 0$

(37h) $q_1 m - q_2 m = 0$

which are derived by differentiating $\mathcal{H}$ with respect to m, b, τ, z, π, α, g and μ, respectively. The transversality conditions are, as before, equations (31a) - (31c).

Consistency implies, as above, that the optimal path from the vantage of t = 0 should imply no incentive for the government to deviate from that path at future points of time. There, the marginal valuation of the state variables m and b must be zero for all t for the optimal plan to be time consistent. This requires that $q_1 = q_2 = 0$ for all t.

a) Optimal μ Given τ and g Are Constant

Performing the same experiment as in Section IVa above yields the necessary conditions for an optimal path as equations (33a) - (33d), (36), (37a), (37b), (37d) - (37f) and (37h) and the relevant transversality conditions. Applying the constraint that the optimal path be time consistent implies from equation (43) that

$$v_3 \frac{\tau}{1-\tau} (\lambda - \frac{z}{\alpha} + \pi) = 0$$

If $v_3 = 0$, then by inspection of the necessary equations

$$v_1 = v_2 = v_3 = q_3 = 0$$

and the optimal paths for m, b, α, μ and π are found from

$$u_c = u_x$$

$$u_x = \alpha(1-\tau)$$

$$u_c = \mu(1+\pi+\delta)$$

$$(1-\tau)g - \frac{\tau b}{1-\tau}(\delta + \pi) - \tau m = d$$

$$\mu = \pi$$

which will in general be inconsistent with the final necessary condition that

$$(1-\tau)(g+m)+b\delta-m(1+\mu) = 0$$

By the same token, if $(\delta - \frac{z}{\iota} + \ -)$ is zero, the necessary conditions will again imply an inconsistency.

b) Other Optimal Policies

Table III.2 summarizes the possible permutations of the government's decision problem. As can be seen, the addition of the balanced budget constraint has a marked effect on the time consistency of optimal policies. Only if the government optimizes with respect to all its instruments or with respect to μ and τ alone will the optimal policy be time consistent.

In general, the addition of an extra constraint on the government will lead to a second-best solution to the optimizing process. Since such a constraint also tends to promote time inconsistency rather than consistent government behaviour, its value is nil in the present context.

Alternate measures of the deficit generate similar results. For instance, if the real deficit is defined as the real change in government liabilities

$$\dot{m} + \dot{b} = d'$$

If d' is held constant this constraint will be inconsistent with any steady state unless d' = 0, so that any optimal policy will be inconsistent.

VIII. DISCUSSION

As discussed in the introduction, the decision problem faced by the government analyzed above is relatively unsophisticated and unrealistic. The government sets current and future policy assuming that successive incarnations of itself will be committed to the original time path of policy.

A more realistic scenario would be one in which successive governments take into account the possibility that their successors will re-evaluate the time path of optimal policy when setting their own policies. Obviously there are a myriad of such 'games' that the sequence of governments could play, the simplest being the Nash type such that the current government set, optimally, current policy given the expectation that future governments will in turn set optimal policy in the same fashion.

This type of equilibrium is that which is explored by Goldman and Rotenberg. However, as Lucas and Stokey point out, a time consistent optimal policy, for which the optimal policy as formulated by the myopic government at the initial date will also be optimal from the perspective of future myopic governments, will correspond to a subgame perfect Nash equilibrium. That is to say it will correspond to the time path of policy which would result from successive governments setting policy optimally given future government behaviour.

A sketch of their proof of this statement is as follows (see Lucas and Stokey, Appendix B). If $\sigma_t(y_t)$ is a strategy belonging to the set of feasible strategies given state y_t, available to the t'th government, then the payoff to that government will be

$$\pi_t(\sigma_t^\infty, y_t)$$

where $\quad \sigma_t^\infty \equiv (\sigma_t, \sigma_{t+1}, \dots) \neq \hat{\sigma}_t^\infty \equiv (\hat{\sigma}_t, \hat{\sigma}_{t+1}, \dots)$

so that the current payoff will depend on current and successive strategies and the current state. Successive strategies are chosen by either the current government or future governments depending on the game played.

A time consistent sequence of strategies or policies σ_0^∞ formulated by the myopic government will have the property that

$$\pi_t(\sigma_t^\infty, y_t) \geq \pi_t(\hat{\sigma}_t^\infty, y_t)$$

for all t. In other words, the sequence of policies σ_0^∞ will maximize the payoff functions of all successive governments. Thus, even if the government is naive it will have no incentive to renege on past commitments.

A set of policies σ_0^∞ is a subgame perfect Nash equilibrium if

$$\pi_t(\sigma_t^\infty, y_t) \geq \pi_t(\hat{\sigma}_t, \sigma_{t+1}^\infty, y_t)$$

for all t. That is to say σ_0^∞ is a subgame perfect Nash equilibrium if each successive government has no incentive to deviate current policy from the σ_0^∞ sequence given future policy follows the sequence.

Since the former condition implies the latter the time consistent policy is also a subgame perfect Nash equilibrium policy. Thus, at least under certain circumstances, an optimal policy will be feasible under more complicated assumptions regarding intertemporal behaviour of government.

IX. CONCLUSION

The time consistency of optimal policy has been examined in the context of a continuous time, perfect foresight economy in which liquidity preference is generated from a transactions technology which imposes a cost on the consumer which may be reduced by appropriate acquisition of real balances and bonds.

A comparison was made of the consistency properties of such an economy under two different assumptions; that the transactions costs households incurred were real resource costs and, that the transactions costs were solely pecuniary. The results of that comparison were,

a) If bonds bear a liquidity yield in the transactions cost economy no optimal policy is time consistent. The optimal policy will always imply a slow adjustment to the steady state affording the government the incentive to re-start the optimal plan at a future date.

The government is constrained by the fact that bonds bear a real liquidity yield. This will mean that it is optimal to utilize an inflation tax and a high real rate on bonds to induce agents to hold a higher real quantity of bonds in the immediate future and avoid the distortionary income tax.

b) If bonds do not bear a liquidity yield, the optimal policy will be time consistent if and only if the government's instruments include the rate of nominal monetary growth. Thus, it would appear that the so-called 'k% rule', taking μ as exogenous, will in the case where a time consistent policy is feasible, tend to inhibit time consistency since it reduces the likelihood of an immediate jump to the steady state being optimal.

In fact, T-B's results are qualitatively duplicated when the liquidity yield of bonds is zero implying that with respect to time consistency of optimal policy, liquidity preference generated from a utility yield versus transactions costs are identical.[10]

c) If the transactions costs which generate liquidity preference are solely pecuniary and do not involve a resource cost, either in terms of goods or leisure, all optimal policies are time consistent except when g is the sole instrument. Unlike the above examples, the consistency results in this case are unaffected by the liquidity yield on bonds.

d) Legislated restrictions on the size of the deficit will tend to promote time inconsistency by constraining the government in such a manner so as to make an immediate jump to the steady state not optimal.

In conclusion, in the context of continuous time models, an optimal policy will not be time consistent if the optimal paths of those state variables which are free for the government to set upon re-evaluation are not immediate jumps to the steady state values. If those paths involve a slow adjustment towards the steady state the government will at some future date have an incentive to restart the optimal plan, causing an immediate discontinuous jump of the state variables back to their initial values.

As shown above, in a monetary model in continuous time, the type of transactions technology will be paramount in determining whether or not the optimal plan will be time consistent. If transactions require the expenditure of real resources the optimal plan will be more likely to involve a slow adjustment to the steady state and therefore offer an incentive to the government to renege on its policy, whereas if the transactions costs are solely pecuniary the government will be likely to find it optimal to jump immediately to a steady state.

The above conclusions are subject to the caveat mentioned in Section VIII and earlier in the text. The sophistication of the government and consumers with respect to their reaction to the ability for successive governments to renege on policy commitments has been limited. As argued, the optimal policy, if time consistent does conform to one derived under more complicated game theoretic behaviour. However, even more complicated behaviour is relatively naive.

FOOTNOTES

1. A paper by Sjaastad (1976) anticipates many of the issues and conclusions discussed below. Sjaastad notes that if inflation expectations are slow to adjust (adaptive) then there will be an incentive for governments to adopt a policy of unstable inflations.
2. See Lucas and Stokey, Appendix B, for a concise explanation of this point.
3. Time subscripts are ignored except where necessary.
4. It is assumed that the utility function and constraints exhibit the sufficient conditions for an equilibrium to exist.
5. Strictly speaking, the government's instruments are nominal; the growth of money, bonds, taxes etc. However, since a time path of nominal instruments implies a time path of prices, the government has effective control over real variables.
6. Since M(o) and B(o) may be adjusted by open market operations and P(o) adjusts in response to the time paths of M, B and the other government instruments, m(o) and b(o) are "free".
7. T-B consider briefly time varying exogenous paths of g and/or μ and find that consistency results are unaffected.
8. The system is derived by using equations (16), (21), (17) and (19) to solve out λ, z, x and π from the government's problem.
9. Strictly speaking, since m is a stock variable and c is a flow variable, the constraint should be that,

$$m \geq \int c\, dw$$

where consumption over the assumed interval is less than or equal to the stock held at a point in time (the beginning of the interval). In this case c may be defined as that integral.

10. In effect, T-B's results depend on Ricardian equivalence, which is no longer the case when bonds bear a liquidity yield.

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