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International Trade With Quality Differentiated Goods

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INTERNATIONAL TRADE

WITH QUALITY DIFFERENTIATED GOODS

by

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of the requirements for the degree of

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ABSTRACT

This thesis is primarily concerned with the role of both scale economies and demand differences between countries producing quality differentiated goods, in determining the variety of qualities produced both in autarky, and once trade opens, and on the pattern of trade. Attention is also paid to the effects of trade on individual consumers in each country.

Chapter 3 focuses on the role of demand differences within a country in determining the variety of qualities produced by both price discriminating and non-price discriminating monopolists, in a very flexible two consumer-type version of the basic model of quality differentiated goods production developed in Chapter 2. It is shown for both price discriminating and non-price discriminating monopolists, that the number and level of qualities produced will depend on the number of consumers in each group, and their respective preferences for quality.

If one thinks of the consumer types as residing in distinct regions of an economy, the no price discrimination equilibria could be viewed as the result of allowing arbitrage or free trade between the two groups. Comparing the two equilibrium configurations, shows that freeing trade between the two groups reduces either the number of qualities produced, or the average level of quality produced. High willingness to pay consumers are shown to gain consumer surplus but only in the presence of a substantial population of low willingness to pay consumers who warrant being served.

Chapter 4 examines the consequences of opening up trade between two economies characterized by the basic model of quality differentiated goods production, but where consumers in each country are assumed to be uniformly distributed over their willingness to pay for quality. For the special case where the distribution of willingnesses to pay is the same in each country, trade causes the two firms to push their qualities apart from their autarky levels, to minimize the price competition between them. This contrasts sharply with horizontally differentiated goods models where firms hold their varieties together to attract consumers from the other, which is the standard Hotelling result.

All consumers are shown to gain consumer surplus as a result of the competition brought about by trade. These gains are shown to be larger between countries with narrow distributions of willingnesses to pay for quality, as the scope for firms to differentiate their qualities to avoid price competition is reduced. Differences between the two countries which allow the firms to further differentiate their qualities reduces the gains from trade.

Chapter 5 considers a rather special variation of the model of trade in quality differentiated goods developed in Chapter 4, which nicely highlights the role of demand differences in determining the pattern of trade, and its effects on individual consumers in each country. The distribution of willingnesses to pay for quality assumed for each country is characterized by a right triangular distribution, with the home country having a majority of high willingness to pay consumers, and the foreign country having a majority of low willingness to pay consumers. The pattern of trade which emerges is very much in the spirit of the Linder hypothesis as each country is producing a level of quality in keeping

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with its representative demand, which it exports to consumers who are on the fringe of the representative demand in the other country.

Chapter 6 reexamines the model of trade in quality differentiated goods developed in Chapter 4, under the alternative production technology that the burden of quality improvement falls on fixed costs such as R & D expenditure rather than on marginal costs as assumed throughout this thesis. This provides a very useful comparison between the effects of trade modelled here, and those found by Shaked and Sutton (1984).

ACKNOWLEDGEMENTS

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CHAPTER 1

INTRODUCTION AND OVERVIEW

As early as 1961, Staffan Linder pointed out that although the standard factor proportions model was a useful explanation of the pattern of trade in primary products, it was not a satisfactory explanation of the volume of trade in similar manufactured goods.

Linder's hypothesis was that countries tended to produce manufactured goods of the quality that were representative of the domestic demand, which he asserted was a function of the average domestic income. Scale economies implicitly restrict the variety of qualities that could be produced within a country, especially for consumers who are on the fringes of the representative demand.

Trade was viewed simply as an extension of the domestic market, and was expected to be most intense between economies with identical income and tastes. Linder apparently had a monopolistic competition type of model in mind, where the increased extent of the market brought about by trade, resulted in an in-filling of the variety of qualities available. Trade between economies with somewhat different income and tastes was expected to result in each economy exporting qualities which are in keeping with its representative demand, to consumers who are on the fringe of the representative demand in the other country.

While various authors have examined the effects of scale economies and taste differences in the context of the factor proportions model, little attention was paid to constructing models of trade in similar

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manufactured goods which capture these effects until very recently. Much of the resurgence in interest has stemmed from the rapidly developing industrial organization literature on product differentiation, which lends itself quite naturally to this task.

The most well known example of this research is the work of Krugman (1979). Krugman constructs a simple general equilibrium model of trade in horizontally differentiated goods, based on the Spence (1976)-Dixit and Stiglitz (1977) model of Chamberlinian monopolistic competition. Scale economies limit the number of varieties available to identical consumers in autarky, who are assumed to have a preference for variety. By increasing the extent of the market, trade helps capture scale economies, with the result that more varieties are produced, making consumers better off.

Related papers by Lancaster (1980) and Helpman (1981) draw on the perfect monopolistic competition work of Lancaster (1979). As in Krugman (1979), scale economies limit the number of varieties available to consumers in autarky who are assumed to be uniformly distributed over their most preferred variety. Trade helps capture these scale economies with the result that more varieties are produced, which leaves consumers closer to their most preferred variety and correspondingly better off. While both of these models are embedded within a factor proportions model, very little can be said about the pattern of trade in similar manufactured goods.

The symmetries which are inherent in these models also leave very little scope to explore the role of demand differences in determining the pattern of trade, and its effects on individual consumers in each country.

Recent papers by Eaton and Kierzkowski (1984) and Dinopoulos (1984) both construct more flexible models of trade in horizontally differentiated goods which begin to address some of these issues.

Considerably less attention has been paid to modelling the pattern of trade in quality differentiated goods. A casual examination of the empirical findings of Grubel and Lloyd (1975) would suggest that a significant volume of the trade in final goods is differentiated with respect to quality, which is not captured in these horizontally differentiated goods models.

To date, the only model of trade in quality differentiated goods is the partial equilibrium model of Jaskold-Gabszewicz, Shaked, Sutton and Thisse (1981), and Shaked and Sutton (1984). Their model is based on the notion of a 'natural oligopoly' equilibrium developed in a series of papers by Jaskold-Gabszewicz and Thisse (1980), and Shaked and Sutton (1982, 1983).

Their model centers around a production technology in which the burden of quality improvement falls on fixed costs such as R & D expenditure, rather than perhaps a more conventional technology, where the burden of quality improvement falls on marginal costs. Consumers are assumed to have identical preferences for quality, but are uniformly distributed over income, and hence differ in their willingness to pay for a unit of quality.

The constant marginal cost assumption in conjunction with the Bertrand-Nash price competition generates an equilibrium where exactly two firms survive independent of the size of the economy, which they refer to as a natural oligopoly. Thus the nature of the price competition

rather than scale economies determines the number of varieties that can be produced, and hence market structure. While population size has no effect on market structure it has an important effect on quality as larger populations allow firms to spread the fixed costs of quality improvements over a larger number of consumers, causing quality to increase.

Trade between two such economies is simply modelled as an increase in population size with the result that the levels of quality produced are upgraded, while the number of qualities available to consumers is unchanged. All consumers gain consumer surplus as a result of the quality upgrading effects of trade, as consumers' willingness to pay increases more than prices.

The location of production and hence the pattern of trade is indeterminate. The role of demand differences in determining the pattern of trade, and its important effects on individual consumers in each country are not addressed.

The main purpose of this thesis is to construct a very flexible model of trade in quality differentiated goods, which captures the effects of both scale economies and demand differences between countries, in determining the variety of qualities produced both in autarky and once trade opens, and on the pattern of trade. While the analysis is primarily positive in nature, attention is paid to the effects of trade on individual consumers in each country. As with Shaked and Sutton (1984), the more complicated issue of aggregate welfare is not addressed.

The model of quality differentiated goods production which underlies much of the analysis in this thesis is very much in the spirit of Mussa and Rosen's (1978) monopoly model of quality differentiated goods production,

where the burden of quality improvements falls on marginal costs rather than fixed costs. This provides a role for scale economies to determine the variety of qualities produced which contrasts with the Shaked and Sutton (1984) analysis, although that technology is also considered.

Chapter 2 sets out the basic model of quality differentiated goods production for an economy in autarky, which underlies much of the analysis in the chapters that follow. For expositional simplicity, all consumers are assumed to have identical willingness to pay for quality. In this simple one consumer-type model, a monopolist is shown to produce a socially efficient level of quality, extracting all consumers' net benefit (consumer surplus), which is defined as the difference between a consumer's willingness to pay for a quality differentiated good and its price. It should be pointed out that although the model is set in a general equilibrium framework, the welfare implications for individual consumers are rather partial equilibrium in nature, focusing on consumer surplus.

Chapter 3 focuses on the role of demand differences within a country in determining the variety of qualities produced by both price discriminating and non-price discriminating monopolists, in a very flexible two consumer-type version of the basic model of quality differentiated goods production developed in Chapter 2. It is shown for both price discriminating and non-price discriminating monopolists, that the number and level of qualities produced will depend on the number of consumers in each group, and their respective preferences for quality.

In the price discrimination equilibrium configuration, it is shown that increases in the population of either consumer types, helps overcome fixed costs, with the result that two distinct levels of quality are more likely to be produced, thereby increasing variety.

In the no price discrimination equilibrium configuration, it is shown that while balanced growth in the population of both consumer types also makes producing two distinct levels of quality more likely, growth in the population of low willingness to pay consumers alone, may lead to less variety as eventually only the low quality good is produced. High willingness to pay consumers are shown to derive positive net benefit in the presence of a significant population of low willingness to pay consumers who warrant being served. Their gain is greater, the larger is the population of low willingness to pay consumers, and the lower is their preference for quality.

If one thinks of the two consumer types as residing in distinct regions of an economy, the no price discrimination equilibria could be viewed as the result of allowing arbitrage or free trade between the two groups. Comparing the two equilibrium configurations, shows that freeing trade between the two groups reduces either the number of qualities produced, or the average level of quality produced. High willingness to pay consumers are shown to gain net benefit, but only in the presence of a substantial population of low willingness to pay consumers who warrant being served.

The simple two consumer-type model of quality differentiated goods production developed in this chapter provides a very flexible model to explore the effects of demand differences on a variety of important trade

issues should trade open between two such economies. The appendix to
this chapter begins to address some of these issues by examining the
consequences of opening up trade between two such economies. It is shown
however that with this simple two consumer-type model, some rather restrictive
assumptions on firm pricing behaviour are required for an equilibrium to exist.
Eaton and Kierzkowski (1984) run into the same difficulties in a simple two
consumer-type model of horizontally differentiated goods production.

Chapter 4 examines the consequences of opening up trade between two
economies characterized by the basic model of quality differentiated goods
production, but where consumers in each country are assumed to be uniformly
distributed over their willingness to pay for quality. While this leaves
little scope to examine the effects of trade on minority and majority taste
groups in each country, the non-existence problems associated with opening
trade between two economies characterized by the more flexible two-point
distribution, as discussed in the appendix to Chapter 3, are avoided.
Chapter 5 returns to the effects of trade on majority and minority tastes.

To simplify the analysis, fixed costs are assumed sufficiently
high such that each economy produces a single level of quality both in
autarky and once trade opens. This allows for a very clear examination of
the pattern of trade and its effects on individual consumers in each
country.

In each country in autarky, the equilibrium price and quality
combination offered to consumers by a monopolist is shown to depend on
the distribution of willingnesses to pay for quality. Consumers benefit
from wider distributions in which lower willingness to pay consumers are
served, which is analogous to the results obtained for the two-point
distribution examined in Chapter 3.

Once trade opens the two monopolists are assumed to compete as duopolists in price and quality for consumers in each country. The equilibrium is solved as a two stage game. In the first stage a Bertrand-Nash pricing equilibrium is derived for given (autarky) levels of quality. The second stage involves using the equilibrium pricing rules to solve for a quality setting equilibrium assuming that each firm conjectures that the quality of the other is held fixed. For the special case where the distribution of willingnesses to pay is the same in each country, trade causes the two firms to push their qualities apart from their autarky levels, to minimize the price competition between them. Small differences in the distribution of willingnesses to pay between the two countries are shown to push their qualities slightly farther apart, which further reduces the price competition between them.

This contrasts sharply with horizontally differentiated goods models where with perfectly inelastic demands, firms would keep their varieties at their autarky levels giving the standard Hotelling result. With quality differentiated goods, however, each firm attracts consumers by increasing its level of quality, which in conjunction with the severe Bertrand-Nash price competition, causes the firms to push their qualities apart.

All consumers are shown to gain net benefit as a result of the competition brought about by trade. These gains are shown to be larger between countries with narrow distributions of willingnesses to pay for quality, as the scope for firms to differentiate their qualities to avoid price competition is reduced. Differences between the two countries which allow the firms to further differentiate their qualities reduces the gains from trade.

The trading equilibrium described in this chapter is developed under the simplifying assumption that in equilibrium, all consumers in each country buy a quality differentiated good. This requires that the distribution of willingnesses to pay be sufficiently narrow such that in equilibrium, the lowest willingness to pay consumer in each country is served. The appendix to this chapter examines the complications which arise for wider distributions, in which the lowest willingness to pay consumers may not be served. Excluding the lowest willingness to pay consumer, however, is shown to cause kinked price reaction functions which greatly complicates the quality setting equilibrium. Computational methods are used to show that while trade still causes the two firms to push their qualities apart, discontinuities and multiple equilibria result.

Chapter 5 considers a rather special variation of the model of trade in quality differentiated goods developed in Chapter 4, which nicely highlights the role of demand differences in determining the pattern of trade, and its effects on individual consumers in each country.

The distribution of willingnesses to pay for quality assumed for each country is characterized by a right triangular distribution, with the home country having a majority of high willingness to pay consumers, and the foreign country having a majority of low willingness to pay consumers. The range of willingnesses to pay for quality and the total population in each country is assumed to be the same.

As a result of these differences, the equilibrium level of quality offered to consumers by a domestic monopolist in autarky, is lower in the foreign country, than in the home country, where the lowest willingness to pay consumers who are a minority, are never served. Consequently all consumers extract more net benefit in the foreign country where lower

willingness to pay consumers who are in a majority warrant being served, which is analogous to the results of Chapter 3. As in Chapter 4, all consumers in each country benefit more from wider distributions in which lower willingness to pay consumers are served.

Once trade opens, the two monopolists are assumed to compete as duopolists over price and quality for consumers in each country. As the two triangular distributions are assumed to sum to a uniform distribution, the competition between the two firms is modelled exactly as in Chapter 4. The competition brought about by trade causes the two firms to push their qualities even farther apart than the autarky levels found here, thereby reducing the price competition between them.

In equilibrium, the foreign firm specializes in the production of the low quality good which it exports to low willingness to pay consumers who are in a minority in the home country, while the home firm specializes in the production of the high quality good which it exports to high willingness to pay consumers who are a minority in the foreign country.

This pattern of trade is very much in the spirit of Linder hypothesis as each country is producing a level of quality in keeping with its representative demand, which it exports to consumers who are on the fringe of the representative demand in the other country.

While trade may actually harm lower willingness to pay consumers who are a majority in the foreign country, all consumers in each country are shown to gain net benefit from trade in narrower distributions where the scope for firms to differentiate their qualities and avoid price competition is reduced. These gains are shown to be greater for all home

country consumers than foreign country consumers over all distributions, which simply reflects the fact that they were able to extract less net benefit in autarky.

Chapter 6 reexamines the model of trade in quality differentiated goods developed in Chapter 4, under the alternative production technology that the burden of quality improvement falls on fixed costs such as R & D expenditure rather than on marginal costs as assumed to this point. This provides a very useful comparison between the effects of trade modelled here, and those found by Shaked and Sutton (1984).

In autarky, the equilibrium level of quality offered to consumers by a domestic monopolist is shown to depend not only on the distribution of willingnesses to pay, but also on the total population of consumers. Larger populations allow the firm to spread the fixed costs of a quality improvement over a larger number of consumers, which causes quality to rise.

Once trade opens, the two monopolists are assumed to compete as duopolists over price and quality for consumers in each country. As with Shaked and Sutton (1984), the Bertrand-Nash price competition in conjunction with a zero marginal cost assumption results in only one of the firms surviving for narrower distributions. For wider distributions, a trading equilibrium is reached, though some lower willingness to pay consumers may not be served.

In comparison to autarky, it is clear that for narrower distributions where only one of the firms survives, trade has the effect of upgrading the level of quality produced by the remaining firm which serves both economies. This is identical to the Shaked and Sutton results.

For wider distributions where a trading equilibrium exists, the two firms differentiate their qualities to minimize the price competition between them. This result is reinforced over the narrower distributions, where the price competition causes the share of consumers buying from each firm to be highly unequal. Thus for the majority of consumers, trade has the effect of upgrading the level of quality they buy from the home firm. As the distribution widens, however, each firm serves roughly the same number of consumers in the trading equilibrium as it did in autarky with the result that no quality upgrading takes place.

Thus in contrast to the Shaked and Sutton results, it is clear that for economies where set up costs or entry fees are sufficiently high such that only one firm is able to operate in autarky, the competition brought about by trade may well result more in quality differentiation than quality upgrading. This is especially true if the share of consumers served by each firm in the trading equilibrium is roughly the same as it was in autarky.

Chapter 7 follows with a brief summary and conclusions.

CHAPTER 2

THE BASIC MODEL

1. Introduction

The purpose of this chapter is simply to set out the 'basic model' of quality differentiated goods production which underlies much of the analysis in the chapters that follow. The model developed here follows closely on Mussa and Rosen's (1978) monopoly model of quality differentiated goods production. The general equilibrium structure of production is roughly based on Horstmann and Markusen (1984a).

Section 2 outlines the technology and costs of production. Section 3 examines preferences and the optimization behaviour for an individual consumer. Section 4 examines the monopolist's optimization problem and the overall equilibrium that results under the simplifying assumption that all consumers are identical. Section 5 concludes.

2. Technology and Production Costs

Consider an economy in which production is assumed to take place in two sectors, using a single factor of production, labour. Quality differentiated goods are produced by a monopolist, while all other production is devoted to a composite good in a perfectly competitive sector.

The notion of quality used follows from Rosen (1974), where goods are valued for the bundle of underlying characteristics which they embody. Higher quality goods of the same variety are assumed to have proportionally more of each attribute.

Quality differentiated goods are produced according to the production function:

$$1) \quad X_j = \begin{cases} 0 & G \geq L_j \geq 0 \\ a(k_j)[L_j - G] & L_j \geq G \end{cases}$$

where X_j is the quantity of good j produced, L_j is the variable amount of labour used in the production of good j, G is the fixed amount of labour required to produce a quality differentiated good, and $a(k_j)$ is the labour output coefficient indicating the amount of output produced by one unit of labour, where $a(k_j) > 0$ for all $k_j \in [0,\infty)$, $a'(k_j)$ and $a''(k_j) < 0$. For convenience, goods are ordered such that $k_j > k_{j-1}$.

The composite good is produced such that one unit of labour produces one unit of output, simply written as:

$$2) \quad Y = L_y$$

where L_y is the total amount of labour used in the production of Y.

Since labour is the only factor of production, it is a simple step to obtain the cost structure from the technology outlined above. With Y being produced in a perfectly competitive sector, and from (2), we have: $P_y Y = w L_y$. Making Y numeraire, the wage rate in terms of Y is equal to one. By solving for total labour input requirements, the cost functions for X_j and Y are written as:

$$3) \quad C(X_j) = c(k_j)X_j + G$$

$$C(Y) = Y$$

where, $c(k_j) = \dfrac{1}{a(k_j)} \geq 0$ for all $k_j \; \epsilon [\,0, \infty)$, $c'(k_j)$ and $c''(k_j) > 0$.

From (3) it is seen that the numeraire is produced with constant $MC = AC = 1$. Quality differentiated goods are produced with constant $MC = c(k_j)$ and decreasing $AC = c(k_j) + \dfrac{G}{X_j}$ in output. Notice that the burden of quality improvement falls on MC, which is increasing in quality. Alternatively, one could model the burden of quality improvement as falling on fixed costs, which is the subject of Chapter 6.

3. Consumer Behaviour

Consumers are assumed to have preferences defined over the composite good and one unit of a quality differentiated good which are represented by the utility function:

4) $\qquad U_i = Y_i + \theta_i k_j$

where, Y_i is a representative consumer i's consumption of the composite good, θ_i is consumer i's valuation of a unit of quality, and k_j is the quality of good j consumed by i. Thus $\theta_i k_j$ is consumer i's valuation or willingness to pay for good j. Consumers may differ only in their valuation of a unit of quality.[1]

All consumers are assumed to have identical income I, composed of labour income, and a per capita share of the monopolist's profit. The budget constraint facing consumer i is:

5) $\qquad I = Y_i + P_j$

where Y_i is consumer i's expenditure on the numeraire, and P_j is the price paid by i for one unit of a quality differentiated good j.

Consumer i's problem is to choose the level of quality from the price and quality combinations available, which maximizes utility (4), subject to the budget constraint (5). Expenditure on the numeraire is simply determined as a residual. Substituting the budget constraint into the utility function, and eliminating the numeraire allows consumer i's problem to be written as:

6) $$\text{Max}_{k_j} U_i = Y + [\theta_i k_j - P_j].$$

Defining $Z_i(P_j) = [\theta_i k_j - P_j]$ as the net benefit derived by i from consuming good j at price P_j, then consumer i's problem may be simply viewed as choosing the level of quality which provides the largest net benefit to be had from consuming a quality differentiated good[2]. Should no quality provide consumer i with non-negative net benefit, he would simply spend all of his income on the numeraire.

Consumer i's problem is represented diagrammatically in Figure 2.1. Consumers' valuations of quality θk, and prices P are drawn in the negative direction, which allows net benefit $Z(P)$, to be represented in the positive direction. Quality is measured along the horizontal axis. The ray $\theta_i k$ represents consumer i's valuation, or willingness to pay for quality, over all qualities. Notice that the ray $\theta_i k$ also serves as an upper bound on prices, as consumer i will consider only those qualities whose price does not exceed $\theta_i k$. Thus if offered a single
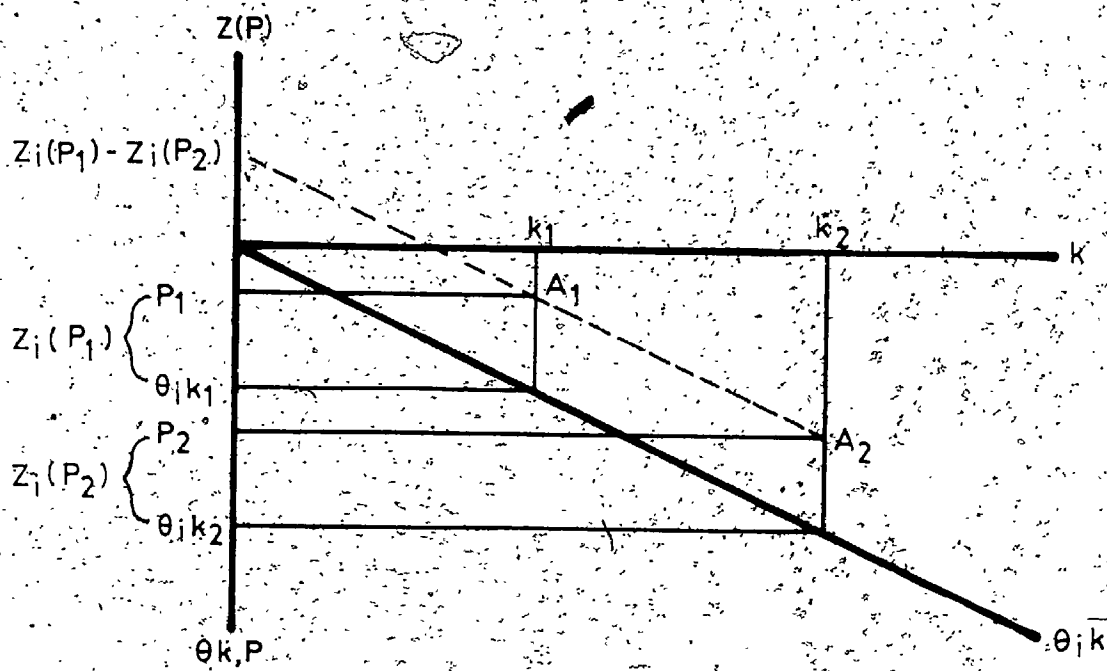
FIGURE 2.1
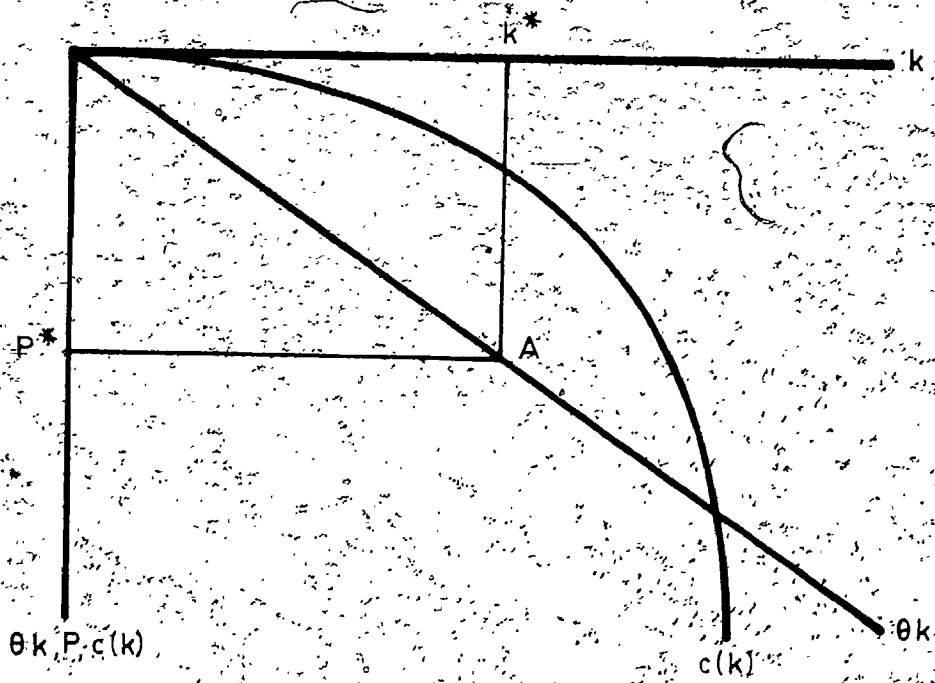
FIGURE 2.2

level of quality, say k_1, consumer 1 will buy so long as

$Z_1(P_1) = \theta_1 k_1 - P_1 \geq 0$, which is the case at point A_1 in Figure 2.1.

When choosing between two or more levels of quality, consumer 1 will

be indifferent between any pair of qualities, say k_1 and k_2, if

$Z_1(P_1) = Z_1(P_2)$, which is the case at points A_1 and A_2 in Figure 2.1.

Market demand for any particular level of quality will depend

on the variety of price and quality combinations available and on the

distribution of willingnesses to pay for quality.

4. Autarky Equilibrium - One Consumer Type

For expositional simplicity, it is assumed that the economy is

populated by M consumers with identical preferences for quality. In

this very simple one consumer-type model, market demand is given by

the behaviour of an individual consumer.

The monopolist's problem is to choose the profit maximizing

price and quality combination to offer consumers, given market demand

and the costs of production.[3] The decision problem is modelled as

a two stage game. In the first stage, the monopolist chooses an

optimal pricing rule for some fixed level of quality. This optimal

pricing rule is then used in the second stage to re-solve the monopolist's

problem for the optimal level of quality.

Using the cost function (3), the first part of the monopolist's

problem in a one consumer-type model is written as:

7) $\underset{P}{\text{Max}} \quad \Pi(P) = [P - c(k)]M - G$

$$\text{S.T.} \quad Z(P) = \theta k - P \geq 0$$

where the constraint requires that consumers must receive non-negative net benefit to buy. The optimal pricing rule is simply to set $P^* = \theta k$, extracting all consumers' net benefit. In terms of Figure 2.2, the monopolist sets price along the willingness to pay ray, θk.

Substituting the pricing rule into (7), the second part of the firm's problem is written as:

8) $\underset{k}{\text{Max}} \quad \Pi(k) = [\theta k - c(k)]M - G.$

Differentiating (8) w.r.t. k gives the first order condition

9) $\theta - c'(k^*) = 0$

where the profit maximizing level of quality k^*, just equates the value consumers place on an additional unit of quality with the cost of producing it. This equilibrium price and quality combination is shown at point A in Figure 2.2. For convenience, the cost function is drawn such that $c(k) = 0$ at $k = 0$. Notice also, that k^* maximizes the net social benefit to be had from producing a quality differentiated good, defined as the difference between consumers' willingness to pay ray θk, and the marginal cost schedule.

Differentiating (9) w.r.t. k yields the second order condition: $-c''(k) < 0$, which ensures that the profit function is concave and hence, k^* is a global maximum.

For the monopolist to offer k^* at price $P^* = \theta k^*$, the non-negative profit condition must be satisfied which is given by:

10) $\Pi(k^*) = [\theta k^* - c(k^*)]M - G \geq 0.$

Thus if θ and M are sufficiently large relative to $c(.)$ and G such that (10) holds, then an overall autarky equilibrium exists in which M units of the quality differentiated good k^* are traded at price $P^* = \theta k^*$. Notice that in this very simple one consumer type model, the monopolist is able to extract all consumers' net benefit, leaving them just indifferent between consuming k^* and the numeraire.

While the role of the numeraire has not been made explicit, it serves the important function of tying up the general equilibrium aspects of the model by absorbing any income effects that result from changes in the monopolist's profits, without feeding back into the quality differentiated goods equilibrium.

5.   Summary and Conclusions

This chapter develops the 'basic model' of quality differentiated goods production which underlies much of the analysis in the chapters that follow. Presenting the basic model at this early stage will hopefully help the exposition, and avoid needless repetition in what follows.

Endnotes

1. Consumers' valuation of quality is exogenous in this model. One could, however, make θ_i a function of consumer i's income, and have consumers differ in income. This type of formulation would be along the lines of Shaked and Sutton (1982, 1984) and would be more within the spirit of Linder (1961).

2. Since consumers have perfectly inelastic demand, the net benefit derived by a consumer from a quality differentiated good and consumer surplus are identical.

3. It is easy to show that in a simple one consumer type model, the monopolist would never offer more than a single level of quality.

CHAPTER 3

DEMAND DIFFERENCES, FIXED COSTS, AND PRODUCT VARIETY

1.   Introduction

This chapter focuses on the role of demand differences in determining the variety of qualities produced by a domestic monopolist, by introducing a second consumer type into the basic model of Chapter 2. It is shown for both price discriminating and non price discriminating monopolists, that the number and level of qualities produced will depend on the number of consumers in each group, and their respective preferences for quality.

If one thinks of the two consumer types as residing in distinct regions of an economy, the no price discrimination equilibria could be viewed as the result of allowing arbitrage or free trade between the two groups. Comparing the two equilibrium configurations shows that while freeing trade tends to reduce either the number of qualities produced, or the average level of quality produced, high willingness to pay consumers are shown to gain, but only in the presence of a significant population of low willingness to pay consumers who warrant being served.

The chapter is organized as follows. Section 2 briefly reviews the basic model of quality differentiated goods production developed in Chapter 2. Section 3 examines the various autarky equilibria that can result as a function of the underlying parameters, under the assumption that a domestic monopolist can price discriminate between the two consumer types. Section 4 repeats the analysis of Section 3 under the assumption that the monopolist can no longer price discriminate between the two groups. In Section 5, the two equilibrium configurations

are compared to examine the effects of freeing trade between the two consumer types as a function of the underlying parameters. Section 6 follows with some concluding remarks.

Finally, the simple two consumer-type model of quality differentiated goods production developed in this chapter, provides a very flexible framework to explore the effects of demand differences, on a variety of important trade issues.

The appendix to this chapter begins to address some of these issues by examining the consequences of opening trade between two such economies. It is shown, however, that for a trading equilibrium to exist in this simple two-consumer type model, some rather restrictive assumptions about firm behaviour are required.

2. The Basic Model

Consider an economy in which production is carried out in two sectors using a single factor of production, labour. Quality differentiated goods are produced by a monopolist with decreasing AC in output (i.e., fixed costs and constant MC), and increasing MC in quality, given by:

1) $C(X_j) = c(k_j)X_j + G$

where X_j is the quantity of good j produced, k_j is the quality of good j, where goods are ordered such that $k_j > k_{j-1}$, and $c(k_j)$ is the marginal cost of producing X_j with $c(k_j) \geq 0$ for all $k_j \in [0,\infty)$, $c'(k_j)$ and $c''(k_j) > 0$, and for convenience, $c(k_j) = 0$ at $k_j = 0$. G is the fixed cost required to produce a quality differentiated good.

All other production is devoted to a composite good Y, in a perfectly competitive sector, which is numeraire.

The economy is populated by two types of consumers; M_2 consumers with high willingness to pay for quality and M_1 consumers with lower willingness to pay for quality. All consumers are assumed to have identical income composed of labour income, and a per capita share of the monopolist's profits, which they must allocate between expenditure on the numeraire, and on one unit of a quality differentiated good. Consumers choose the level of quality which provides the largest net benefit to be had from consuming a quality differentiated good, leaving their expenditure on the numeraire to be determined as a residual. Net benefit is defined as the difference between a consumer's willingness to pay for a quality differentiated good, and its price which is written as:

2). $\quad Z_i(P_j) = \theta_i k_j - P_j \qquad i = 1,2$

where θ_i is a representative consumer i's valuation of a unit of quality, hence $\theta_i k_j$ is consumer i's valuation, or willingness to pay for good j, and P_j is the price of good j. Should no quality provide consumer i with non-negative net benefit, he would simply spend all of his income on the numeraire.

3. Price Discrimination Equilibria

Having described consumer behaviour and the costs of production, the monopolist's problem is now to choose the profit maximizing price and quality combinations to offer consumers, under the assumption that

he can price discriminate between the two groups. Depending on the level of fixed costs, the number of consumers in each group, and on their respective preferences for quality, four possible equilibrium configurations may result, which are; (1) both high and low willingness to pay consumers are offered distinct levels of quality, (2) both high and low willingness to pay consumers are offered a single intermediate level of quality, (3) high willingness to pay consumers receive a high quality good, while low willingness to pay consumers are not served, and (4) no quality differentiated good is produced. Fixed costs are assumed never to be so high, that the last uninteresting possibility occurs. It is also worth noting, that even if fixed costs were zero, it would never pay for the monopolist to offer a third level of quality in this simple two consumer-type model.

The analysis proceeds by examining each of the three equilibrium configurations and comparing their profitability to determine the parameter values under which each dominates.

The monopolist's price and quality selection problem is modelled as a two stage game. In the first stage, optimal pricing rules are derived for a given level(s) of quality. Since the monopolist can price discriminate between the two groups, prices are set to extract all of the net benefit consumers derive from whatever quality they consume. Using (2), the optimal pricing rules for good j are simply written as:

3) $P_j = \theta_i k_j$ $i = 1,2$

These pricing rules are then used in the second stage to re-solve the monopolist's problem for the optimal level(s) of quality.

Using the cost function (1) and the pricing rules (3), the second part of the monopolist's problem when producing two distinct levels of quality is given by:

4) $\underset{k_1, k_2}{\text{Max}} \quad \Pi(k_1, k_2) = [\theta_1 k_1 - c(k_1)]M_1 + [\theta_2 k_2 - c(k_2)]M_2 - 2G.$

Differentiating (4) w.r.t. k_1 and k_2 gives the first order conditions:

5) $\theta_1 - c'(k_1^*) = 0 \quad , \quad \theta_2 - c'(k_2^*) = 0$

where k_1^* and k_2^* just equates the value the two consumer types place on an additional unit of quality with the marginal cost of producing it. The equilibrium price and quality combinations offered to low and high willingness to pay consumers are shown at points A_1 and A_2 in Figure 3.1 respectively. Notice that k_1^* and k_2^* maximize the net social benefit to be had from producing quality differentiated goods, defined as the difference between consumers' willingness to pay ray, and the marginal cost schedule.

The profit from offering k_1^* and k_2^* at prices $P_1^* = \theta_1 k_1^*$ and $P_2^* = \theta_2 k_2^*$ respectively, is given by:

6) $\Pi(k_1^*, k_2^*) = [\theta_1 k_1^* - c(k_1^*)]M_1 + [\theta_2 k_2^* - c(k_2^*)]M_2 - 2G.$

Now consider the profitability of producing a single level of quality. Using the cost function (1), and the pricing rules (3),
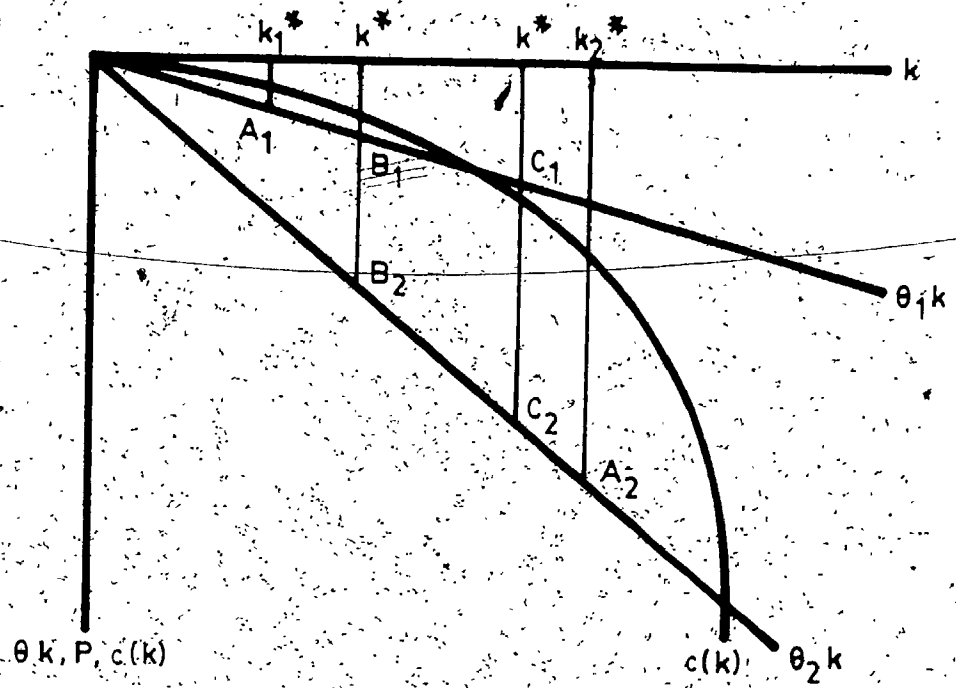
FIGURE 3.1
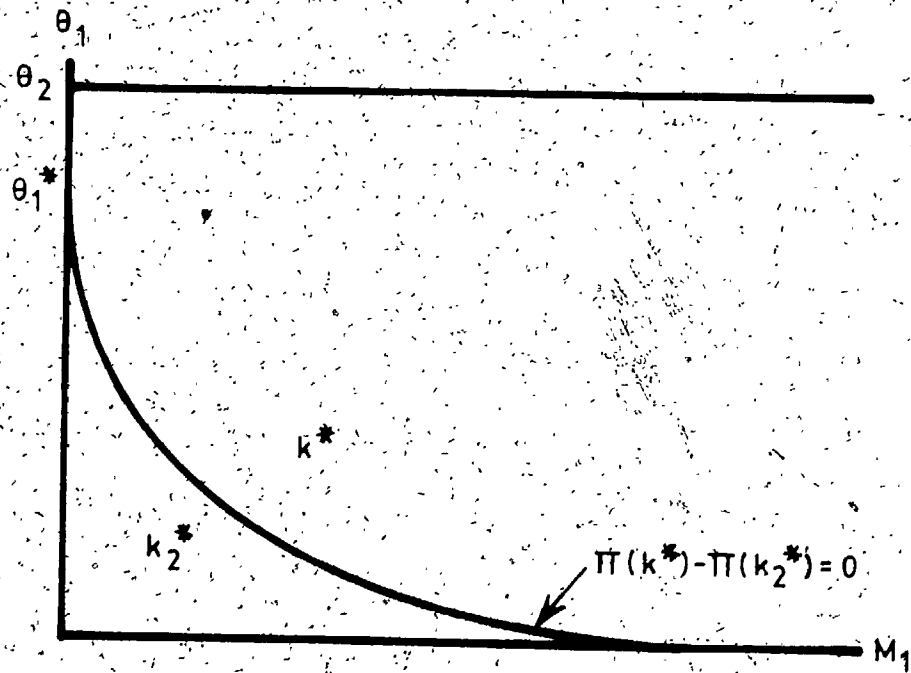
FIGURE 3.2

the second part of the monopolist's problem is given by:

7) $\underset{k}{\text{Max}}\ \Pi(k) = [\theta_1 k - c(k)]M_1 + [\theta_2 k - c(k)]M_2 - G.$

Differentiating (7) w.r.t. k gives the first order condition:

8) $[\theta_1 - c'(k^*)]M_1 + [\theta_2 - c'(k^*)]M_2 = 0.$

The profit maximizing level of quality k^* is simply a weighted average of the qualities k_1^* and k_2^* that would have been offered to each group separately. Totally differentiating (8) reveals that $\frac{dk^*}{d\theta_1}, \frac{dk^*}{d\theta_2}, \frac{dk^*}{dM_2} > 0$, while $\frac{dk^*}{dM_1} < 0$ which are all rather intuitive results. The equilibrium price and quality combinations offered to low and high willingness to pay consumers are shown at points B_1 and B_2 in Figure 3.1 respectively.

The profit from offering k^* to low willingness to pay consumers at price $P^* = \theta_1 k^*$ and to high willingness to pay consumers at price $P^* = \theta_2 k^*$ is given by:

9) $\Pi(k^*) = [\theta_1 k^* - c(k^*)]M_1 + [\theta_2 k^* - c(k^*)]M_2 - G.$

From (9), it is easy to see that if $[\theta_1 k^* - c(k^*)] < 0$, which is the case at points C_1 and C_2 in Figure 3.1, the monopolist would clearly increase profits by excluding low willingness to pay consumers.

This gives rise to the third equilibrium configuration in which high willingness to pay consumers are offered the high quality good k_2^* at price $P_2^* = \theta_2 k_2^*$ which gives profits:

$$10) \quad \Pi(k_2^*) = [\theta_2 k_2^* - c(k_2^*)]M_2 - G.$$

The equilibrium price and quality combination is shown at point A_2 in Figure 3.1.

Subtracting (10) from (9), reveals that when producing a single level of quality, it is more profitable to offer an intermediate level of quality k^* to both high and low willingness to pay consumers than to offer k_2^* to high willingness to pay consumers alone iff:

$$11) \quad \Pi(k^*) - \Pi(k_2^*) =$$
$$[\theta_1 k^* - c(k^*)]M_1 - \{[\theta_2 k_2^* - c(k_2^*)] - [\theta_2 k^* - c(k^*)]\}M_2 > 0$$

For (11) to hold, the net social benefit from offering k^* to low willingness to pay consumers must be sufficiently large to compensate for the reduction in net social benefit obtained from high willingness to pay consumers. Thus a necessary condition for (11) to hold is that $[\theta_1 k^* - c(k^*)] > 0$. A sufficient condition for (11) to hold is that $\theta_1 \geq \theta_1^*$, where θ_1^* is implicitly defined by $[\theta_1^* k_2^* - c(k_2^*)] = 0$. The intuition here is that if it pays to offer low willingness to pay consumers the high quality good, profits must further be increased by offering both groups an intermediate level of quality.

Condition (11) is more likely to hold the larger is θ_1 and M_1, and the smaller is θ_2 and M_2. These effects are conveniently summarized in Figure 3.2. Setting $\Pi(k^*) - \Pi(k_2^*) = 0$, an indifference locus is constructed in $(\theta_1\, M_1)$ space, with slope $\frac{d\theta_1}{dM_1} < 0$. Points above the indifference locus represents $(\theta_1\, M_1)$ combinations for which offering k^* to both consumer types dominate, while points below represent $(\theta_1\, M_1)$ combinations for which offering k_2^* to high willingness to pay consumers alone dominates. Increases in M_2 and θ_2 cause the indifference locus to shift out with θ_2 having the added effect of shifting θ_1^* up as well. Notice that changes in G have no effect.

The profitability of producing either an intermediate level of quality for both consumer types, or a high quality good for high willingness to pay consumers alone, must now be compared with the profitability of producing two distinct levels of quality.

Should (11) hold, then subtracting (9) from (6) reveals that offering consumers two distinct levels of quality dominates an intermediate level of quality iff:

12) $\Pi(k_1^*\, k_2^*) - \Pi(k^*) =$

$\{[\theta_1 k_1^* - c(k_1^*)] - [\theta_1 k^* - c(k^*)]\}M_1 +$

$\{[\theta_2 k_2^* - c(k_2^*)] - [\theta_2 k^* - c(k^*)]\}M_2 - G > 0.$

For (12) to hold, the gain in net social benefit from offering consumers distinct levels of quality over an intermediate level of quality, must be sufficiently large to cover the final cost of producing an additional unit of quality.

Condition (12) is more likely to hold, the smaller is θ_1 and G_1, and the larger is θ_2, M_1 and M_2. The direct effect of increasing θ_1 causes the net social benefit to increase more for k^* than k_1^*, as $k^* > k_1^*$ which makes (12) less likely to hold. The effects may also be represented in $(\theta_1 \, M_1)$ space by constructing the indifference locus which satisfies $\Pi(k_1^* \, k_2^*) - \Pi(k^*) = 0$ with slope $\dfrac{d\theta_1}{dM_1} > 0$, which is shown in Figure 3.3. Notice that below the $\Pi(k^*) - \Pi(k_2^*) = 0$ locus, the $\Pi(k_1^* \, k_2^*) - \Pi(k^*) = 0$ locus is not relevant, and has therefore been drawn in dashed lines. Points above the indifference locus indicate $(\theta_1 \, M_1)$ combinations for which offering k^* to both consumer types dominates, while points below represent $(\theta_1 \, M_1)$ combinations for which offering consumers two distinct levels of quality dominates. Increasing θ_2 and M_2, and reducing G cause the indifference locus to shift up making it more likely that two distinct levels of quality will be offered.

Should (11) fail, then subtracting (10) from (6) reveals that it is more profitable to offer consumers two distinct levels of quality than to offer high willingness to pay consumers the high quality good alone iff:

13) $\quad \Pi(k_1^* \, k_2^*) - \Pi(k_2^*) = [\theta_1 k_1^* - c(k_1^*)]M_1 - G > 0.$

Condition (13) simply states that unless the low quality good is profitable on its own, it will not be produced, and low willingness to pay consumers will not be served.

Condition (13) is more likely to hold the larger is θ_1 and M_1, and the smaller is G, while θ_2 and M_2 have no effect. All of these
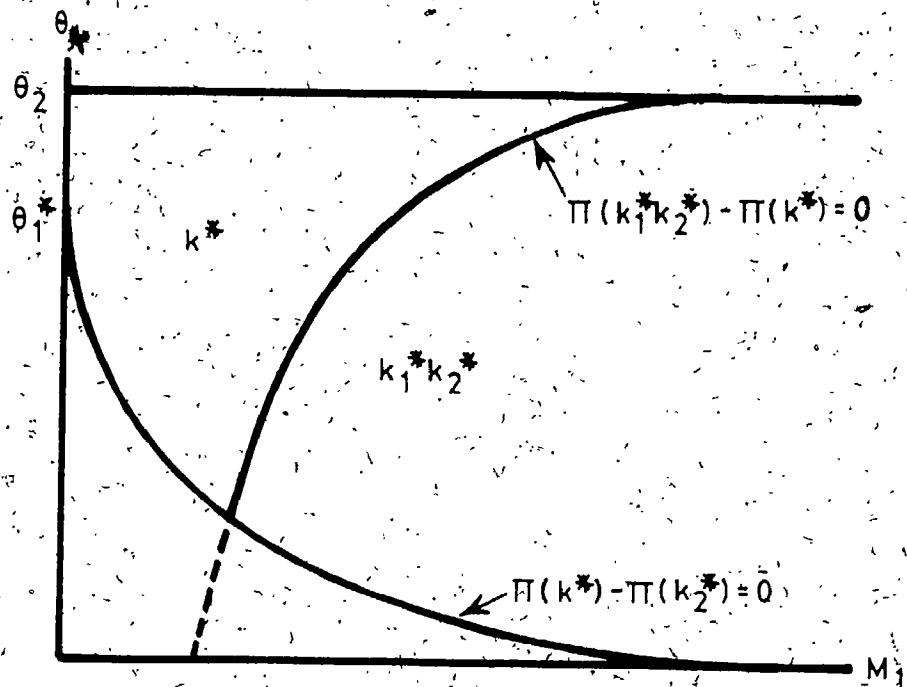
FIGURE 3.3
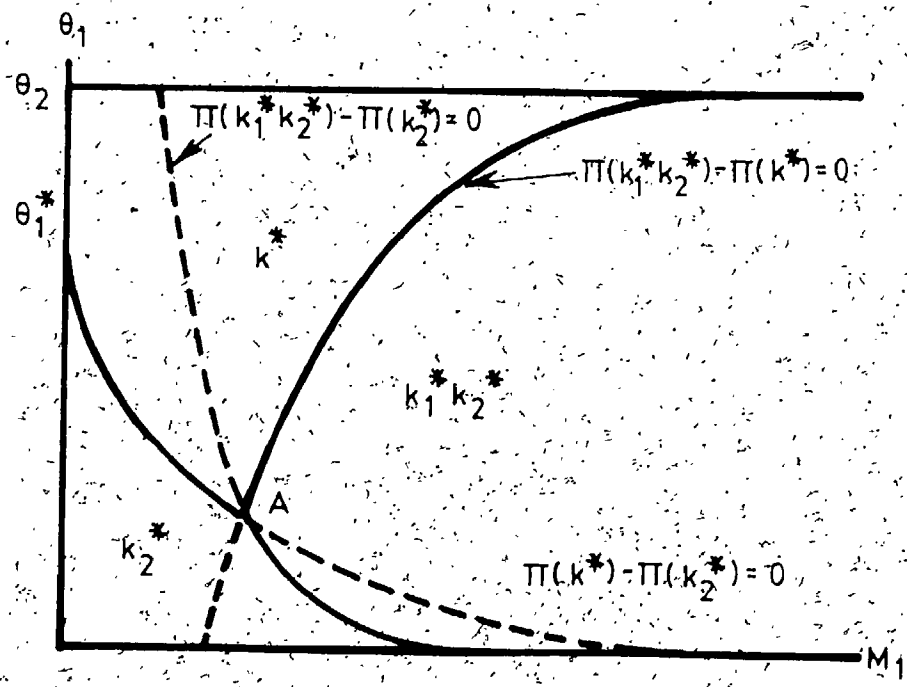
FIGURE 3.4

effects are summarized in $(\theta_1 \ M_1)$ space by constructing the indifference locus $\Pi(k_1^* \ k_2^*) - \Pi(k_2^*) = 0$, with slope $\frac{d\theta_1}{dM_1} < 0$, as shown in Figure 3.4. Points above the locus indicate $(\theta_1 \ M_1)$ combinations for which offering consumers two distinct levels of quality dominates, while points below, represent $(\theta_1 \ M_1)$ combinations for which offering k_2^* to high willingness to pay consumers alone dominates. Increasing G causes the locus to shift up making k_2^* more likely to dominate alone. Notice that above the $\Pi(k^*) - \Pi(k_2^*) = 0$ locus, the $\Pi(k_1^* \ k_2^*) - \Pi(k_2^*) = 0$ locus is not relevant, and is therefore drawn with a dashed line. Also notice that below the $\Pi(k_1^* \ k_2^*) - \Pi(k^*) = 0$ locus, and above the $\Pi(k_1^* \ k_2^*) - \Pi(k_2^*) = 0$ locus, offering two distinct levels of quality dominates, and hence the $\Pi(k^*) - \Pi(k_2^*) = 0$ locus is not relevant and is therefore drawn with a dashed line. Should any of the two indifference loci intersect at point A in Figure 3.4, the third must also by transitivity.

Thus, Figure 3.4 provides a complete description of the possible equilibria that may result as a function of the underlying parameters, in an economy where a domestic monopolist can price discriminate between two consumer types with different willingnesses to pay for quality. While the monopolist's ability to price discriminate between the two groups leaves the welfare implications rather uninteresting, the effects of taste and population differences between the two groups do lead to some useful insights.

Notice first from Figure 3.4, that if the population of low willingness to pay consumers is small, it is more likely that both groups will be offered an intermediate level of quality, the closer are their preferences for quality, otherwise low willingness to pay consumers

may not be served. The larger the population of low willingness to pay consumers is, the more likely it is that two distinct levels of quality will be offered. Thus, immigration or growth in the population of low willingness to pay consumers (not balanced by growth of high willingness to pay consumers) results in increased variety, as fixed costs are overcome.

The effect of increasing M_2 as shown in Figure 3.5, tends to shift both the $\Pi(k^*) - \Pi(k_2^*) = 0$ locus, and the $\Pi(k_1^* \; k_2^*) - \Pi(k^*) = 0$ locus up along the $\Pi(k_1^* \; k_2^*) - \Pi(k_2^*) = 0$ locus, intersecting at point A', which makes it more likely that either k_2^*, or k_1^* and k_2^* will be produced at the expense of the intermediate level of quality. Thus as the population of high willingness to pay consumers grows, the low willingness to pay consumers are served only if the low quality good is profitable on its own.

Finally, the effect of balanced growth in M_1 and M_2 is shown in Figure 3.6. Increasing M_1 and M_2 in proportion is analytically equivalent to reducing G by the same proportion, which shifts both the $\Pi(k_1^* \; k_2^*) - \Pi(k^*) = 0$ locus, and the $\Pi(k_1^* \; k_2^*) - \Pi(k_2^*) = 0$ locus, up along the $\Pi(k^*) - \Pi(k_2^*) = 0$ locus, intersecting at point A". Balanced growth makes offering two distinct levels of quality more likely, and thereby increases variety.

4. No Price Discrimination Equilibria

The monopolist must now choose the profit maximizing price and quality combinations to offer consumers, under the assumption that he can no longer price discriminate between the two groups. As before,

FIGURE 3.5
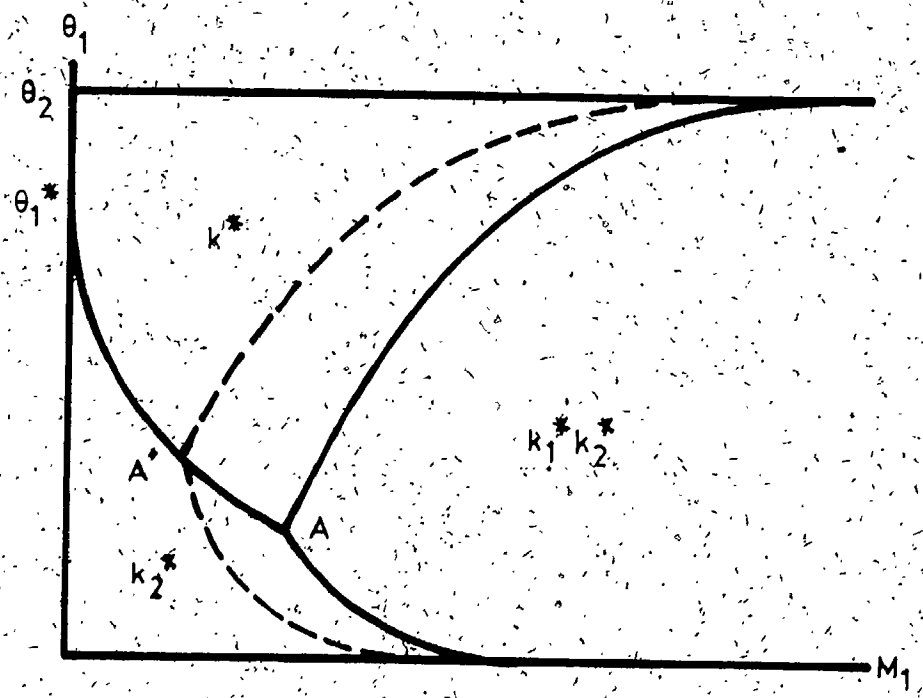
FIGURE 3.6

four possible equilibrium configurations may result, depending on the level of fixed costs, the number of consumers in each group, and on their respective preferences for quality, which are; (1) two distinct levels of quality are produced, with high willingness to pay consumers receiving a high quality good and low willingness to pay consumers receiving a reduced level of quality, (2) both high and low willingness to pay consumers receive a low quality good, (3) high willingness to pay consumers receive a high quality good alone, and (4) no quality differentiated good is produced, which is assumed not to occur.

The analysis proceeds by deriving each of the three equilibrium configurations and then comparing their profitability, to determine the parameter values under which each dominates.

When producing two distinct levels of quality, the monopolist's pricing problem is now complicated by the fact that he can no longer price discriminate between the two groups. The nature of the firm's pricing problem is most clearly illustrated in terms of a diagram, beginning at the price discrimination equilibrium points A_1 and A_2, shown in Figure 3.7. Recalling that consumers will choose the level of quality which provides the largest non-negative net benefit, for given qualities k_1^* and k_2^*, the monopolist must reduce the price of the high quality good to point D_2 to prevent the high willingness to pay consumers from jumping to the low quality good. Notice that low willingness to pay consumers will never jump to the high quality good, as they consider only those qualities whose price falls on or below their willingness to pay ray $\theta_1 k$. Thus when producing two distinct levels of quality, the optimal pricing rules are:
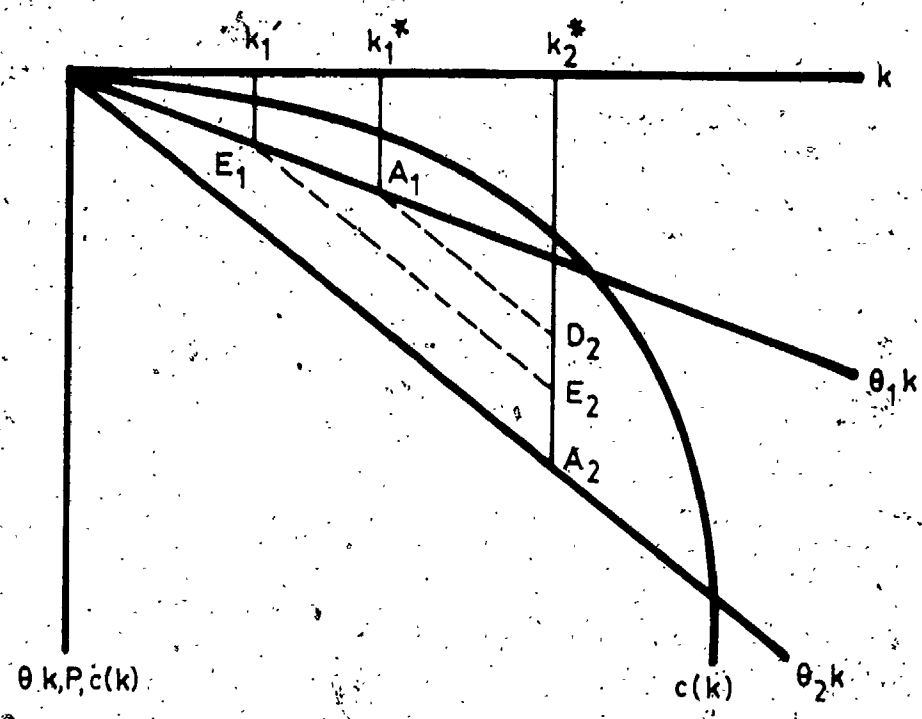
FIGURE 3.7
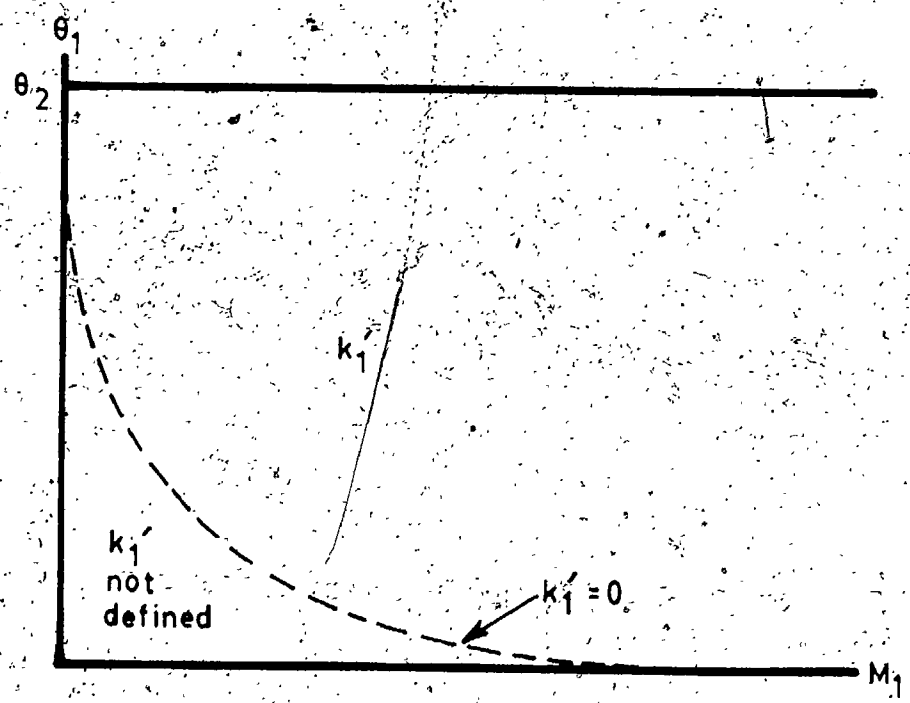
FIGURE 3.8

14) $P_1^* = \theta_1 k_1$, $P_2^* = \theta_2 k_2 - (\theta_2 - \theta_1)k_1$

where $(\theta_2 - \theta_1)k_1$ is the net benefit high willingness to pay consumers would derive from consuming the low quality good.

Using these pricing rules, and the cost function (1), the second part of the monopolist's problem when producing two distinct levels of quality is:

15) $\underset{k_1, k_2}{\text{Max}} \quad \Pi(k_1 \ k_2) = [\theta_1 k_1 - c(k_1)]M_1$

$$+ \{[\theta_2 k_2 - (\theta_2 - \theta_1)k_1 - c(k_2)]\}M_2 - 2G.$$

Differentiating (15) w.r.t. k_1 and k_2 gives the first order conditions:

16a) $[\theta_1 - c'(k_1')]M_1 - (\theta_2 - \theta_1)M_2 = 0$

16b) $\theta_2 - c'(k_2^*) = 0$

From (16b), it is easy to see that the level of quality offered to high willingness to pay consumers k_2^*, is unchanged from when the monopolist was able to price discriminate. From (16a), however, the level of quality offered to low willingness to pay consumers k_1', is reduced below the level of quality they were offered when he was able to price discriminate. The intuition here is that by reducing the level of quality offered to low willingness to pay consumers, the monopolist is able to increase the price of k_2^* and thereby capture back some of the net benefit lost to high willingness to pay consumers.

The monopolist may well reduce k_1 to zero depending on the underlying parameters. From (16a), a $k_1 = 0$ locus can be constructed in $(\theta_1 M_1)$ space which is shown in Figure 3.8. Below the $k_1 = 0$ locus, k_1 is not well defined. For $(\theta_1 M_1)$ combinations falling above the $k_1 = 0$ locus, the equilibrium price and quality combinations offered to low and high willingness to pay consumers are shown at points E_1 and E_2 in Figure 3.7 respectively.

The profit from producing k_1 and k_2^* is given by:

17) $\quad \Pi(k_1 \; k_2^*) = [\theta_1 k_1 - c(k_1)]M_1 + [\theta_2 k_2^* - (\theta_2 - \theta_1)k_1 - c(k_2^*)]M_2 - 2G.$

When producing a single level of quality, the monopolist's price and quality selection problem is actually simplified by the fact that he can no longer price discriminate between the two groups, and must therefore charge a single price.

If low willingness to pay consumers are to be served, price must be constrained to fall along their willingness to pay ray $\theta_1 k$, shown in Figure 3.7. Thus, quality is chosen as though all consumers were low willingness to pay consumers, which results in k_1 being optimal. The equilibrium is shown at point A_1 in Figure 3.7. Notice that in equilibrium, high willingness to pay consumers derive net benefit: $Z_2(\theta_1 k_1^*) = [\theta_2 - \theta_1]k_1^*$. The profit from offering k_1^* to both high and low willingness to pay consumers at price $P_1^* = \theta_1 k_1^*$ is given by:

18) $\quad \Pi(k_1^*) = [\theta_1 k_1^*) - c(k_1^*)](M_1 + M_2) - G.$

Alternatively, the monopolist could exclude low willingness to pay consumers, and offer high willingness to pay consumers k_2^* at price

$\theta_2 k_2^*$, which gives profits:

19) $\quad \Pi(k_2^*) = [\theta_2 k_2^* - c(k_2^*)]M_2 - G.$

Subtracting (19) from (18), reveals that when producing a single level of quality it is more profitable to offer k_1^* to both high and low willingness to pay consumers than to offer k_2^* to high willingness to pay consumers alone iff:

20) $\quad \Pi(k_1^*) - \Pi(k_2^*) =$

$[\theta_1 k_1^* - c(k_1^*)]M_1 - \{[\theta_2 k_2^* - c(k_2^*)]-[\theta_2 k_1^* - c(k_1^*)]\}M_2 > 0.$

For (20) to hold, the net social benefit obtained on low willingness to pay consumers must be sufficiently large to compensate for the reduction in net social benefit extracted from high willingness to pay consumers by offering them the low quality good.

Condition (20) is more likely to hold the larger is θ_1 and M_1, and the smaller is θ_2 and M_2. As before, these effects are conveniently summarized by constructing the indifference locus, $\Pi(k_1^*) - \Pi(k_2^*) = 0$, in $(\theta_1 \; M_1)$ space with slope $\frac{d\theta_1}{dM_1} < 0$, as shown in Figure 3.9. Points above the indifference locus represents $(\theta_1 \; M_1)$ combinations for which offering k_1^* to both high and low willingness to pay consumers dominates, while points below indicate $(\theta_1 \; M_1)$ combinations for which offering k_2^* to high willingness to pay consumers alone dominates. Increases in θ_2 and M_2 both cause the indifference locus to shift out, while changing G has no effect.

θ_1
θ_2

$\Pi(k_1^*) - \Pi(k_2^*) = 0$

k_1^*

k_2^*

$k_1' = 0$

M_1

FIGURE 3.9

θ_1
θ_2

$\Pi(k_1^*) - \Pi(k_2^*) = 0$

k_1^*

$\Pi(k_1' k_2^*) - \Pi(k_1^*) = 0$

$k_1' k_2^*$

$k_1' = 0$

M_1
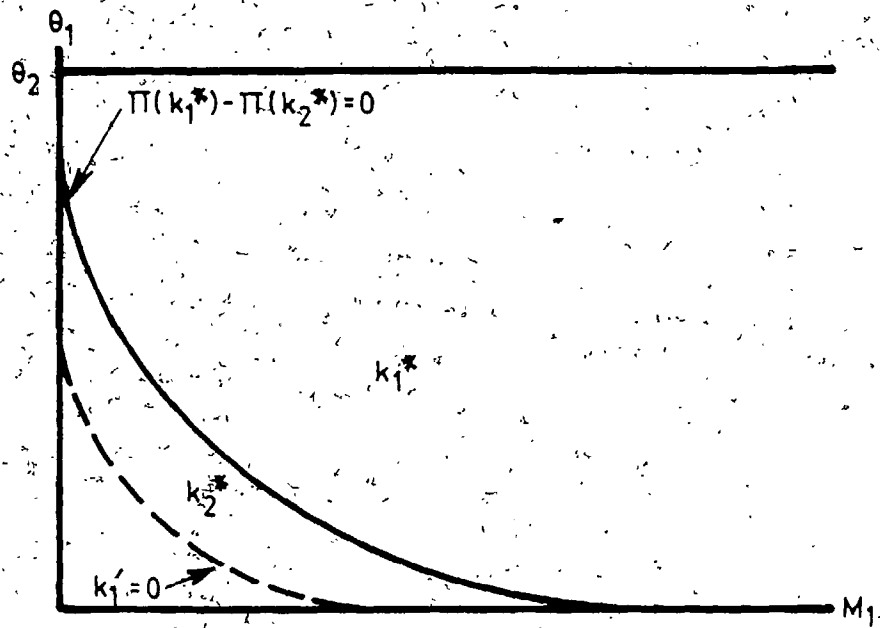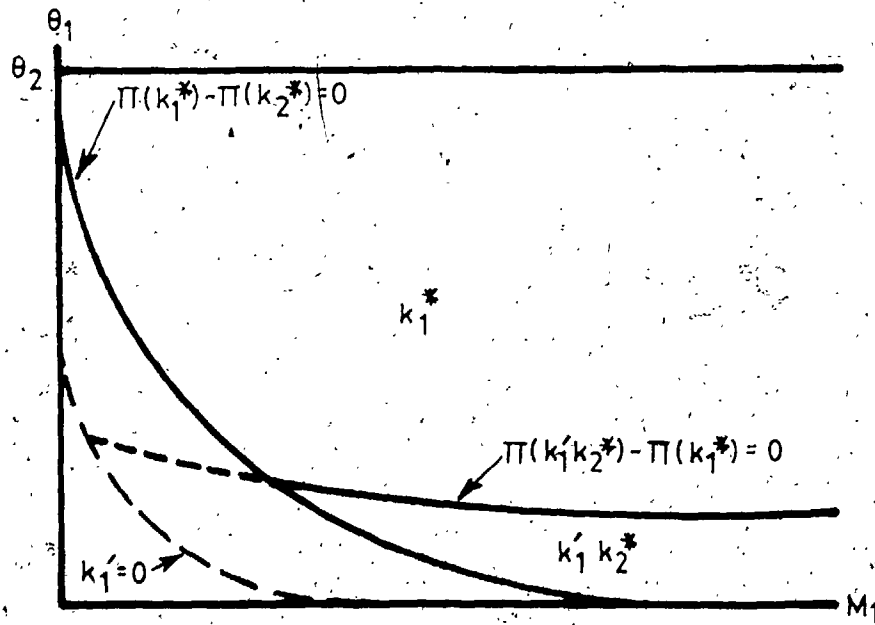
FIGURE 3.10

The profitability of producing either k_1^* for both consumer types, or k_2^* for high willingness to pay consumers alone, can now be compared to the profitability of producing two distinct levels of quality.

If (20) holds, subtracting (18) from (17) reveals that offering two distinct levels of quality dominates offering both consumer types the low quality good iff:

21) $\quad \Pi(k_1' \, k_2^*) - \Pi(k_1^*) =$

$$\{[\theta_2 k_2^* - (\theta_2 - \theta_1)k_1' - c(k_2^*)] - [\theta_1 k_1^* - c(k_1^*)]\}M_2$$

$$- \{[\theta_1 k_1^* - c(k_1^*)] - [\theta_1 k_1' - c(k_1')]\}M_1 - G > 0.$$

Thus, if the gain in net benefit from offering high willingness to pay consumers k_2^* at price $P_2^* = \theta_2 k_2^* - (\theta_2 - \theta_1)k_1'$ over offering them k_1^* as though they were low willingness to pay consumers, exceeds the loss in net benefit from reducing the level of quality offered to low willingness to pay consumers and the fixed cost of producing an additional level of quality, then the two distinct levels of quality will be produced.

Condition (21) is more likely to hold the larger is θ_2 and M_2 and the smaller is θ_1, M_1 and G. These effects are summarized in $(\theta_1 \, M_1)$ space by constructing the indifference locus $\Pi(k_1' \, k_2^*) - \Pi(k_1^*) = 0$, with slope $\frac{d\theta_1}{dM_1} < 0$, as shown in Figure 3.10. Notice that while the $\Pi(k_1' \, k_2^*) - \Pi(k_1^*) = 0$ locus is not well defined below the $k_1' = 0$ locus, that section falls below the $\Pi(k_1^*) - \Pi(k_2^*) = 0$ locus and is therefore not relevant to the analysis. Points above the indifference locus represent $(\theta_1 \, M_1)$ combinations for which offering both high and low willingness to pay

consumers k_1^* dominates, while points below indicate $(\theta_1 \; M_1)$ combinations for which offering the two distinct levels of quality k_1' and k_2^* dominates. Increasing θ_2 and M_2, and reducing G, cause the indifference locus to shift up such that the two distinct levels of quality are more likely to be produced.

Should (20) fail, then subtracting (19) from (17) reveals that offering two distinct levels of quality is more profitable than offering the high quality good to high willingness to pay consumers alone iff:

22) $\Pi(k_1' \; k_2^*) - \Pi(k_2^*) =$

$[\theta_1 k_1' - c(k_1')]M_1 - (\theta_2 - \theta_1)k_1' M_2 - G > 0.$

For k_1' to be produced, the net social benefit obtained on low willingness to pay consumers must be sufficiently large to compensate for the reduction in net benefit extracted from high willingness to pay consumers, in addition to the fixed cost of producing a second level of quality.

Condition (22) is more likely to hold, the larger is θ_1 and M_1, and the smaller is θ_2, M_2 and G. These effects are summarized in Figure 3.11 in $(\theta_1 \; M_1)$ space by constructing the indifference locus $\Pi(k_1' \; k_2^*) - \Pi(k_2^*) = 0$, also with slope $\frac{d\theta_1}{dM_1} < 0$. Points above this indifference locus represent $(\theta_1 \; M_1)$ combinations for which offering the two distinct levels of quality dominates, while points below represent $(\theta_1 \; M_1)$ combinations for which offering k_2^* to high willingness to pay consumers alone dominates. Increases in θ_2, M_2 and G all shift the indifference locus out, making it more likely that producing k_2^* alone will dominate.
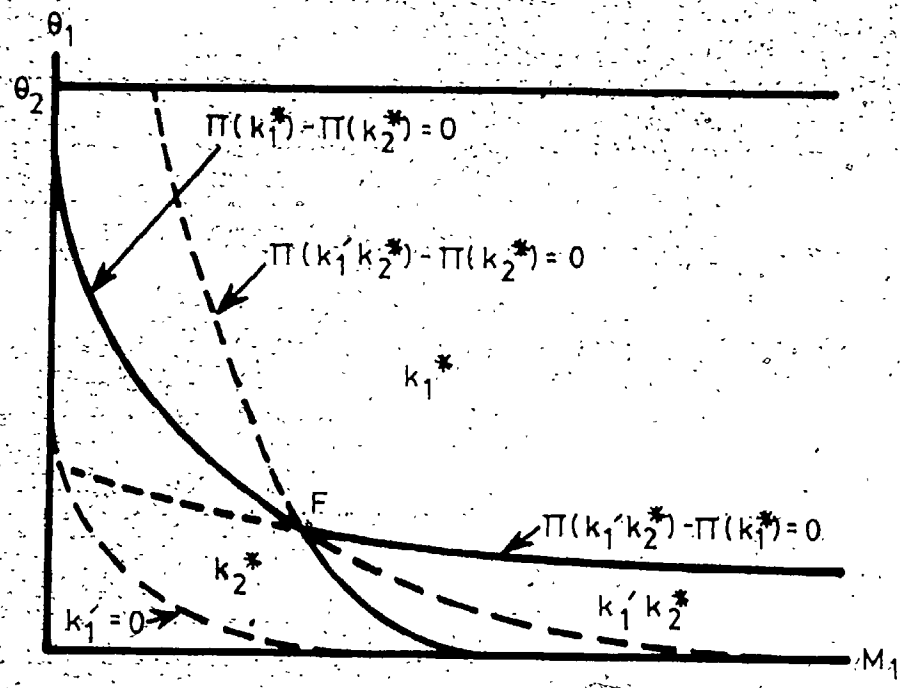
FIGURE 3.11
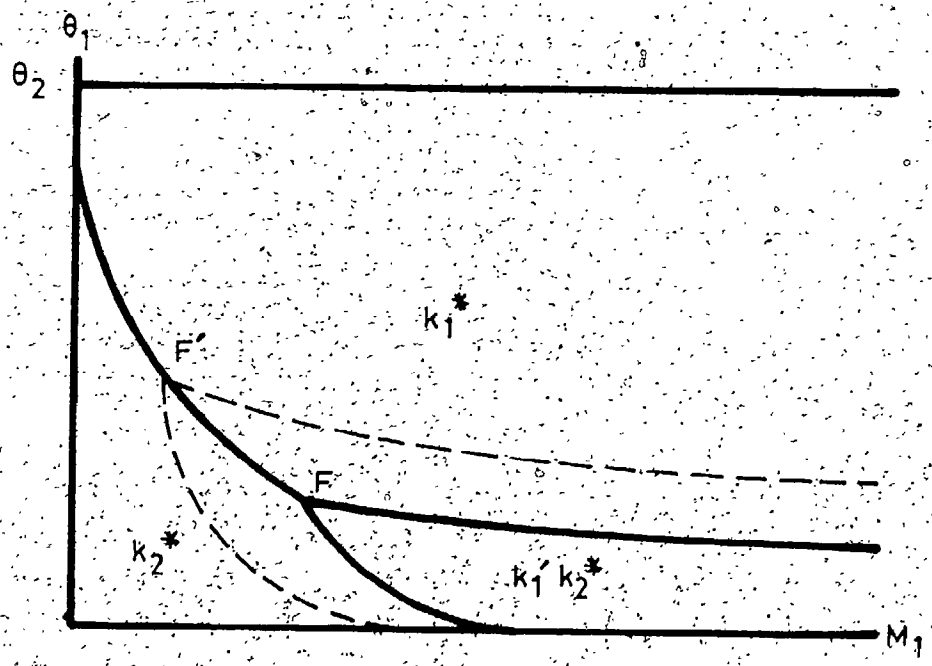
FIGURE 3.12

In Figure 3.11, notice that the $\Pi(k_1' \; k_2^*) - \Pi(k_2^*) = 0$ locus is not relevant above the $\Pi(k_1^*) - \Pi(k_2^*) = 0$ locus, and is therefore drawn with dashed lines. It is also important to note that the $\Pi(k_1' \; k_2^*) - \Pi(k_2^*) = 0$ locus must be bounded away from the $k_1' = 0$ locus for $G > 0$. Thus, if the three indifference loci intersect, it must occur above the $k_1' = 0$ locus at a point like F in Figure 3.11. In the region to S.E. of point F, below the $\Pi(k_1' \; k_2^*) - \Pi(k_1^*) = 0$ locus, and above the $\Pi(k_1' \; k_2^*) - \Pi(k_2^*) = 0$ locus, producing two distinct levels of quality dominates and hence the $\Pi(k_1^*) - \Pi(k_2^*) = 0$ locus is not relevant and is drawn with dashed lines.

Thus Figure 3.11 provides a complete description of the possible equilibria that may result as a function of the underlying parameters in an economy where a domestic monopolist cannot price discriminate between two consumer types with different willingnesses to pay for quality.

Notice first from Figure 3.11, that if the population of low willingness to pay consumers is small, it is more likely that both consumer types will be offered the low quality good the more similar are their preferences for quality, otherwise low willingness to pay consumers may not be served. The larger the population of low willingness to pay consumers is, the more likely it is that they will be served, first a reduced level of quality, then as M_1 gets big, the low quality good.

While low willingness to pay consumers remain indifferent between consuming quality differentiated goods and the numeraire, the welfare effects on high willingness to pay consumers are now quite significant. In the region where only the high quality good is produced, high willingness to pay consumers continue to extract no net benefit.

In the regions where low willingness to pay consumers are served as well, however, high willingness to pay consumers now extract positive net benefit. This surplus is greater, the larger is the population of low willingness to pay consumers, and the lower is their preference for quality. Notice that the gain is even greater from consuming the low quality good, than consuming the high quality good in the presence of a reduced level of quality: $(\theta_2 - \theta_1)k_1^* > (\theta_2 - \theta_1)k_1'$.

The implication is clear, high willingness to pay consumers benefit from having a significant population of low willingness to pay consumers who warrant being served, and should therefore favour the immigration of low willingness to pay consumers. Not suprisingly, the immigration of high willingness to pay consumers tends to have the opposite effect.

The effect of balanced growth in M_1 and M_2 which is modelled as a reduction in G, is shown in Figure 3.12. Reducing G causes the $\Pi(k_1' \; k_2^*) - \Pi(k_1^*) = 0$ locus to shift up, and the $\Pi(k_1' \; k_2^*) - \Pi(k_2^*) = 0$ locus to shift down which moves the intersection point up along the $\Pi(k_1^*) - \Pi(k_2^*) = 0$ locus to F'. While the region in which the two distinct levels of quality are produced widens, thereby increasing variety, the welfare implications are ambiguous as high willingness to pay consumers gain if the economy was previously producing k_2^*, but lose if the economy was previously consuming k_1^*.

5. **From Price Discrimination to No Price Discrimination**

Section 3 and 4 have examined the possible autarky equilibria that may result as a function of the underlying parameters, in a simple two

consumer-type version of the basic model of quality differentiated goods production outlined in Chapter 2, for price discriminating and non-price discriminating monopolists respectively.

If one thinks of the two consumer types as residing in distinct regions of an economy, the no price discrimination equilibria could be viewed as the result of allowing arbitrage or free trade between the two groups.[2] Thus, by comparing the two equilibrium configurations, the effects of freeing trade between the two groups can be examined as a function of the underlying parameters.

The price discrimination equilibrium configuration is shown in Figure 3.13 with the indifference loci intersecting at point A. Freeing trade between the two groups causes an adjustment to the non price discrimination equilibrium configuration, where the indifference loci are drawn with the dashed lines intersecting at point F. Since $\Pi(k^*) \geq \Pi(k_1^*)$ it is easy to see that the no price discrimination locus $\Pi(k_1^*) - \Pi(k_2^*) = 0$ must lie above the price discrimination locus $\Pi(k^*) - \Pi(k_2^*) = 0$, and since $\Pi(k_1^* \ k_2^*) \geq \Pi(k_1^* \ k_2^*)$ the no price discrimination locus, $\Pi(k_1^* \ k_2^*) - \Pi(k_2^*) = 0$ must fall above the price discrimination locus $\Pi(k_1^* \ k_2^*) - \Pi(k_2^*) = 0$. Thus, point F must lie somewhere above point A, though on which side of the upward sloping $\Pi(k_1^* \ k_2^*) - \Pi(k^*) = 0$ locus, F falls is not clear.

The effects of freeing trade between the two groups on the variety of qualities produced are summarized in Table 3.1. Notice first, that freeing trade may result in either more or less variety, though if F falls below the upward sloping $\Pi(k_1^* \ k_2^*) - \Pi(k^*) = 0$ locus, variety is never increased. Thus, there is a presumption that the overall effect of freeing trade either reduces the number of qualities produced, or
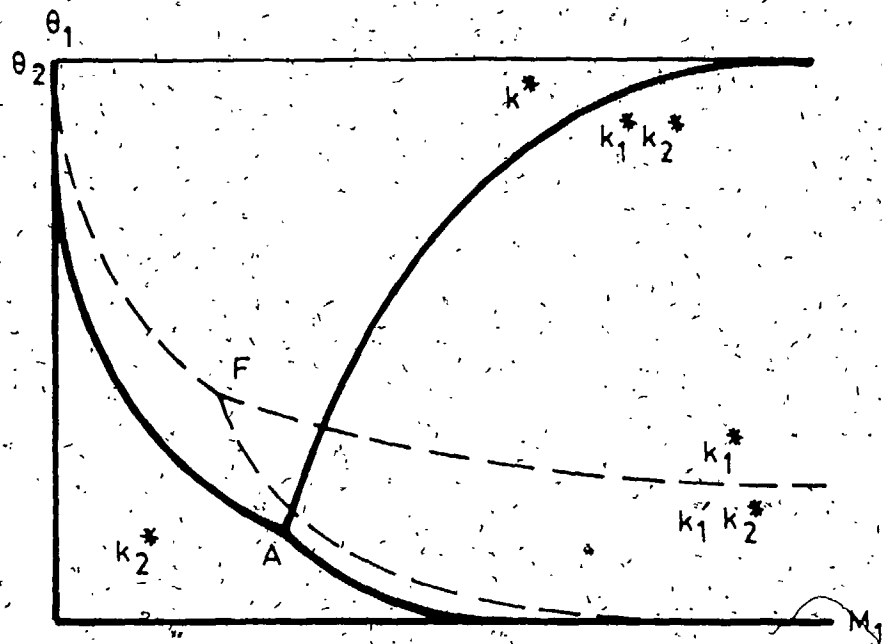
FIGURE 3.13

PRICE DISCRIMINATION		NO PRICE DISCRIMINATION
		$k_1^* k_2^*$
$k_1^* k_2^*$	$\rightarrow$	k_1^*
		k_2^*
		$[k_1 k_2^*]$
k^*	$\rightarrow$	k_1^*
		k_2^*
k_2^*	$\rightarrow$	k_2^*

TABLE 3.1

reduces the average level of quality produced or both, if the population of low willingness to pay consumers is large.

The welfare effects of freeing trade are simply those gains in net benefit derived by high willingness to pay consumers, in the presence of a significant population of low willingness to pay consumers who warrant being served, as described in Section 4. The gains are greater, the larger is the population of low willingness to pay consumers, and the lower is their willingness to pay for quality.

6. Summary and Conclusions

This chapter has focused on the role of demand differences in determining the variety of qualities produced by both price discriminating and non price discriminating monopolists, in a simple two consumer type version of the basic model of quality differentiated goods production outlined in Chapter 2.

In the price discrimination equilibrium configuration, it is shown that increases in the population of either consumer types, helps overcome fixed costs, with the result that two distinct levels of quality are more likely to be produced, thereby increasing variety.

In the no price discrimination equilibrium configuration, it is shown that while balanced growth in the population of both consumer types also makes producing two distinct levels of quality more likely, growth in the population of low willingness to pay consumers alone, may lead to less variety as eventually only the low quality good is produced. High willingness to pay consumers are shown to derive positive net benefit in the presence of a significant population of low willingness

to pay consumers who warrant being served. Their gain is greater, the
larger is the population of low willingness-to pay consumers, and the
lower is their preference for quality.

If one thinks of the two consumer types as residing in distinct
regions of an economy, the no price discrimination equilibria could be
viewed as the result of allowing arbitrage or free trade between the
two groups. Comparing the two equilibrium configurations, shows that
freeing trade between the two groups reduces either the number of
qualities produced, or the average level of quality produced. High
willingness to pay consumers are shown to gain, but only in the presence
of a substantial population of low willingness to pay consumers who
warrant being served.

Appendix - Trading Equilibrium

The simple two consumer-type model of quality differentiated goods
production developed in this chapter provides a very rich framework to
explore the effects of demand differences on a variety of important
trade issues.

This appendix begins to address some of these issues, by examining
the effects of opening trade between two such economies. It is shown,
however, that for a trading equilibrium to exist in this simple two
consumer-type model, some rather restrictive assumptions on firm
behaviour are required.

Consider two economies which are characterized by the simple
two consumer-type model of quality differentiated goods production
described in Section 4, where the domestic monopolist price discriminates
between the two groups.

The two economies are assumed to be identical in every respect, except for the population of consumers in each group. The foreign country is assumed to be populated by a majority of low willingness to pay consumers, such that in autarky, both low and high willingness to pay consumers receive the low quality good k_1^*, at price $P_1^* = \theta_1 k_1^*$ shown at point A^f in Figure 3.14. Notice that in autarky, high willingness to pay consumers in the foreign country derive net benefit: $Z_2(\theta_1 k_1^*) = (\theta_2 - \theta_1)k_1^*$. The home country is assumed to be populated by a majority of high willingness to pay consumers, such that in autarky, high willingness to pay consumers receive the high quality good k_2^*, at price $P_2^* = \theta_2 k_2^*$, shown at point A^h in Figure 3.14, while low willingness to pay consumers are not served.

Once trade opens between the two countries, the home and foreign monopolists are assumed to compete as duopolists for both consumer types in each country. As was the case in autarky, firms' optimal price and quality selection problem is modelled as a two stage game.

In the first stage for fixed autarky levels of quality, each firm chooses its optimal pricing rule given some conjecture about the pricing behaviour of the other. In Figure 3.14, it is easy to see that simple Bertrand-Nash pricing conjectures leads to cyclical price cutting and hence non existence problems in the simple two consumer-type model.[3] Beginning at the autarky equilibrium points A^h and A^f for the home and the foreign firms respectively, the home firm must clearly cut the price of k_2^* to point D^h just to keep his high willingness to pay consumers from jumping to k_1^*. Notice that low willingness to pay consumers in the home country now import k_1^* from the foreign firm. From points A^f and D^h, however, each firm has an incentive to marginally cut price and
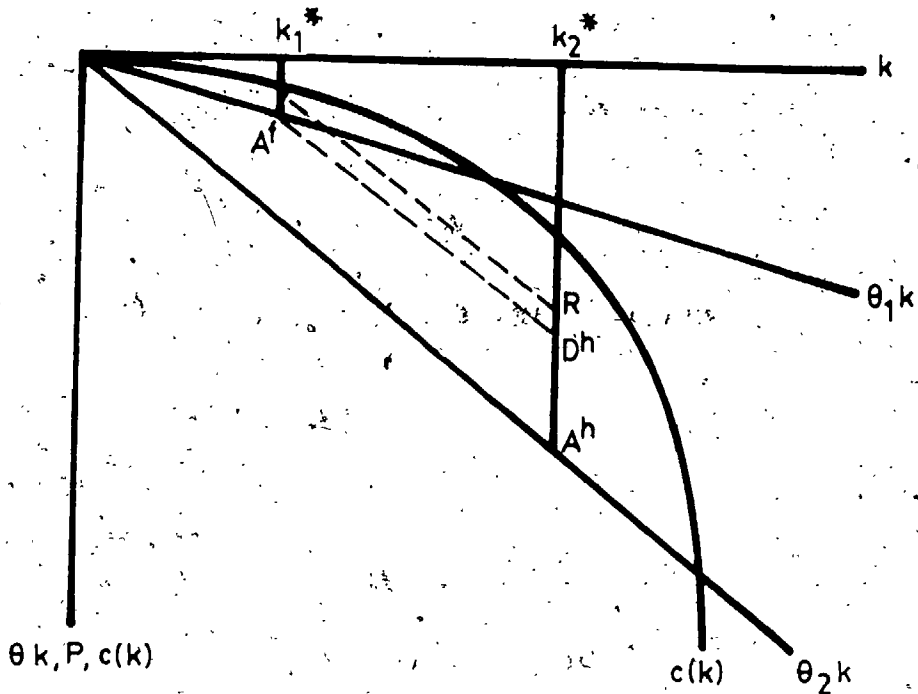
FIGURE 3.14
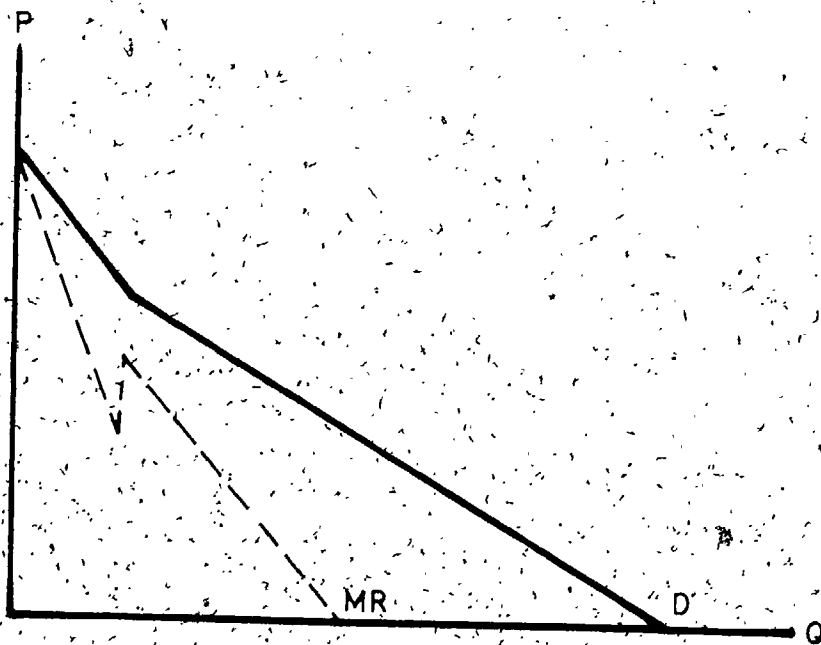
FIGURE 3.15

thereby capture all high willingness to pay consumers, conjecturing that the other firm's price will not change. Once the price of k_1^* reaches the marginal cost schedule, the Home firm is able to capture all high willingness to pay consumers by reducing the price of k_2^* to just below R. Having lost the battle for high willingness to pay consumers, the foreign firm raises the price of k_1^* back to A^f which allows the home firm to raise the price of k_2^* back to D^h, then the cycle repeats.

Alternatively, one could think of imposing what might be called a Modified Bertrand-Nash conjectural assumption under which firms continue to believe that the price of the firm is held fixed, except in the case of price reductions that cause consumers to jump, which firms believe will be met.[4] Thus beginning at the autarky equilibrium points A^f and A^h in Figure 3.14, the home firm must cut the price of k_2^* to point D^h, in order to retain its high willingness to pay consumers, but no further price reductions by either firm would take place.

While this behavioural assumption ensures that a pricing equilibrium exists, it severely restricts competition to the point where it effectively preserves the initial allocation of consumers between the two firms. Given this pricing behaviour, the second stage finds the two firms simply choosing to leave their qualities at their autarky levels.

Thus points A^f and D^h in Figure 3.14 represent the equilibrium price and quality combinations offered by the foreign and home firms respectively, once trade opens between the two economies. The foreign

firm continues to produce k_1^* for both of its low and high willingness
to pay consumers, but now exports k_1^* to low willingness to pay consumers
in the home country who were not served in autarky.[5] The home firm
continues to produce k_2^* for its high willingness to pay consumers, who
now derive positive net benefit as a result of the 'pro-competitive'
reduction in price. Thus in this very simple example, opening up
trade has the very important effect of increasing the variety of
qualities from which consumers choose. This allows high willingness to
pay consumers the leverage to demand a lower price, and minority consumers
who were not served in autarky, to buy.

While this analysis yields a very flexible model of trade in quality
differentiated goods, the restrictive assumptions on firm behaviour greatly
limit its scope for addressing a wide variety of interesting trade issues.

The underlying cause of the non existence problem in the pricing
equilibrium is that firms do not face smooth downward sloping market
demand curves in this very simple two consumer-type model. Thus,
marginal reductions in price cause all consumers of a particular type
to jump. The alternative to imposing restrictive assumptions on firm
behaviour to prevent cyclical price cutting, is to try and smooth out
the market demand curve.

One possible way to accomplish this within the simple two
consumer-type framework, is to allow individual consumers to have elastic
demands which results in downward sloping individual demand curves.[6]
Allowing the individual demand curves to differ in either slope of
intercept, leads to a kinked market demand curve with discontinuous

MR as shown in Figure 3.15, which causes a discontinuity in firms' price reaction functions, and hence non existence problems as well.

The alternative is to abandon the simple two-consumer-type framework in favour of a continuous distribution of consumer types differing in their willingness to pay for quality, which does lead to smooth market demand curves, and hence simple Bertrand-Nash pricing equilibria. This is the subject of Chapter 4.

Endnotes

1. Krishna (1984) and Donnenfeld (1984) both explore various aspects of commercial policies such as tariffs, quotas and minimum quality standards to control a foreign monopolist which exports quality differentiated goods, in a similar type of model.

2. Alternatively, one could think of the two consumer types as residing in different countries which are served by a single monopolist or multinational corporation. Preventing the monopolist from price discriminating between them could be viewed as opening up trade between the two countries.

3. Jaskold-Gabszewicz (1983) discovers this 'Edgeworth cycle' in a simple two consumer-type duopoly model of horizontally differentiated goods production.

4. Eaton and Kierzkowski (1984) modify Bertrand-Nash pricing behaviour in a somewhat analogous way to rule out price cycling in a two consumer-type model of horizontally differentiated goods production.

5. In addition to the important function of tying up the general equilibrium aspects of the model for individual consumers, the numeraire also serves as a medium of exchange to settle countries' international payments.

6. With elastic demands, quality differentiated goods must also be horizontally differentiated for two distinct levels of quality in equilibrium. If this was not the case, consumers would all choose the good providing the most quality per unit of the numeraire.

CHAPTER 4

DUOPOLY AND THE GAINS FROM TRADE

IN QUALITY DIFFERENTIATED GOODS

1.   Introduction

This chapter examines the consequences of opening up trade between two economies which are characterized by the basic model of quality differentiated goods production developed in Chapter 2. To simplify the analysis, each economy is assumed to produce a single level of quality both in autarky, and once trade opens. This allows for a very clear examination of the pattern of trade that emerges once trade opens, and the resulting effects on consumers in each country.

The basic model of Chapter 2 is extended by assuming that each country is populated by consumers who are uniformly distributed over their willingness to pay for quality. While this leaves little scope to examine the effects of trade on minority and majority taste groups in each country, the non-existence problems associated with opening trade between two economies characterized by the more flexible two-point distribution, as discussed in the appendix to Chapter 3, are avoided.[1]

In each country in autarky, the equilibrium price and quality combination offered to consumers by a monopolist is shown to depend on the distribution of willingnesses to pay for quality. Consumers benefit from wider distributions in which lower willingness to pay consumers are served, which is analogous to the results obtained for the two-point distribution examined in Chapter 3.

Once trade opens, the two monopolists are assumed to compete as duopolists over price and quality for consumers in each country. For the special case where the distribution of willingness to pay is the same in each country, trade causes the two firms to push their qualities apart from their autarky levels to minimize the price competition between them. Small differences in the distributions of willingnesses to pay between the two countries are shown to push their qualities slightly farther apart, which further reduces the price competition between them.

All consumers are shown to gain as a result of the competition brought about by trade. These gains are shown to increase as the distribution of willingness to pay narrows, as the scope for firms to differentiate their qualities to avoid price competition is reduced.

The chapter is organized as follows. Section 2 briefly outlines the basic model of quality differentiated goods production developed in Chapter 2. Section 3 examines the equilibrium price and quality combination offered to consumers by a monopolist for one of the economies in autarky. Section 4 examines the equilibrium that is reached when trade opens between the home and foreign economies. In Section 5, the trading equilibrium and autarky equilibrium are compared to examine the effects of trade on the variety of qualities, prices, and the welfare of consumers in each country. Section 6 provides some concluding remarks.

Finally, the trading equilibrium considered in Section 4, was developed under the simplifying assumption that in equilibrium, all consumers in each country buy a quality differentiated good. This requires that the distribution of willingnesses to pay be sufficiently narrow such that in equilibrium, the lowest willingness to pay consumer in each country is served.

The appendix to this chapter examines the complications which arise for wider distributions, in which the lowest willingness to pay consumers may not be served. Excluding the lowest willingness to pay consumer, however, is shown to cause kinked price reaction functions which greatly complicates the quality setting equilibrium. Computational methods are used to show that while trade still causes the two firms to push their qualities apart, discontinuities and multiple equilibria result.

2. The Basic Model

In each economy, production is carried out in two sectors using a single factor of production, labour. Quality differentiated goods are produced by a monopolist with decreasing AC in output (i.e., fixed costs and constant MC in output) and increasing MC in quality. To allow for explicit solutions, the cost function is assumed to take the form:

$$1) \quad C(X_j) = ck_j^2 X_j + G$$

where X_j is the quantity of good j produced, k_j is the quality of good j, where $k_j > k_{j-1}$, ck_j^2 is the marginal cost of producing good j, where $c > 0$, and G is the fixed cost required to produce a quality differentiated good.

All other production is devoted to a composite good Y in a perfectly competitive sector, which is used as numeraire. There are no cost or comparative advantage differences between the two countries.

Each country is populated by consumers who differ in their willingness to pay for quality. All consumers in each country are assumed to have identical income composed of labour income and a per

capita share of the domestic monopolist's profits. Consumers allocate their income between expenditure on the numeraire, and on one unit of a quality differentiated good, by choosing the level of quality which provides the largest net benefit to be had from consuming a quality differentiated good, leaving expenditure on the numeraire to be determined as a residual. Net benefit is defined as the difference between what a consumer is willing to pay for a quality differentiated good, and its price, written as:

$$2) \qquad Z_i(P_j) = \theta_i k_j - P_j$$

where θ_i is a representative consumer i's valuation of a unit of quality and thus $\theta_i k_j$ is consumer i's valuation or willingness to pay for good j, and P_j is the price of good j. Should no quality provide consumer i with non negative net benefit, he would simply spend all of his income on the numeraire.

The market demand for any particular level of quality depends on both the distribution of willingnesses to pay for quality, and on the variety of price and quality combinations available.

Consumers in each country are uniformly distributed in their valuation of a unit of quality. The home country is populated by M^h consumers distributed over the interval $[\theta_1^h, \theta_2^h]$ with density and cumulative density given by:

$$3) \qquad h(\theta) = \frac{1}{(\theta_2^h - \theta_1^h)} \;, \quad H(\theta) = \frac{\theta_2^h - \theta}{(\theta_2^h - \theta_1^h)}$$

where $\theta_2^h > \theta_1^h$, and $H(\theta_1^h) = 1$, $H(\theta_2^h) = 0$. Similarly, the foreign country
is populated by M^f consumers distributed over the interval $[\theta_1^f, \theta_2^f]$ with
density and cumulative density given by:

$$4) \quad f(\theta) = \frac{1}{(\theta_2^f - \theta_1^f)}, \quad F(\theta) = \frac{\theta_2^f - \theta}{(\theta_2^f - \theta_1^f)}$$

where $\theta_2^f > \theta_1^f$, and $F(\theta_1^f) = 1$, $F(\theta_2^f) = 0$.

The only differences that may arise between the two countries are
in total population, and the range of willingnesses to pay for quality.
Examples of density functions and their corresponding cumulative density
functions are shown in Figures 4.1 and 4.2 respectively.

3.   Autarky Equilibrium

Having described consumer behaviour and the costs of production
the monopolist's optimal price and quality selection problem can now be
examined for each country in autarky. The analysis focuses on the home
monopolist's problem as the foreign monopolist's problem is identical,
except for f's in place of h's where appropriate.

The monopolist's problem is modelled as a two stage game. In
the first stage an optimal pricing rule is derived for a given level of
quality. The optimal pricing rule is then used in the second stage to
re-solve the firms problem for the optimal level of quality. Fixed costs
are assumed sufficiently high to prevent the monopolist from producing
more than a single level of quality.
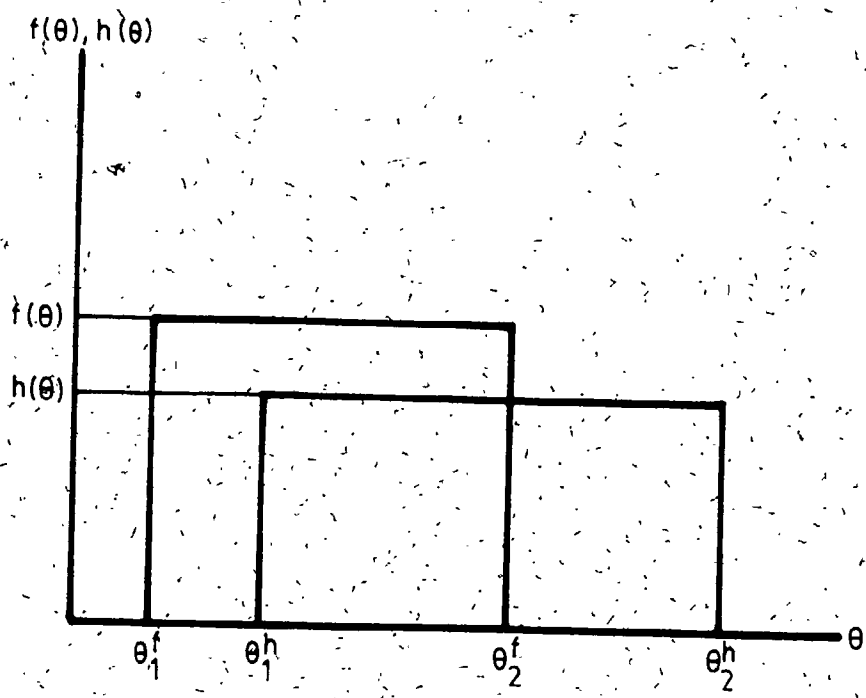
FIGURE 4.1
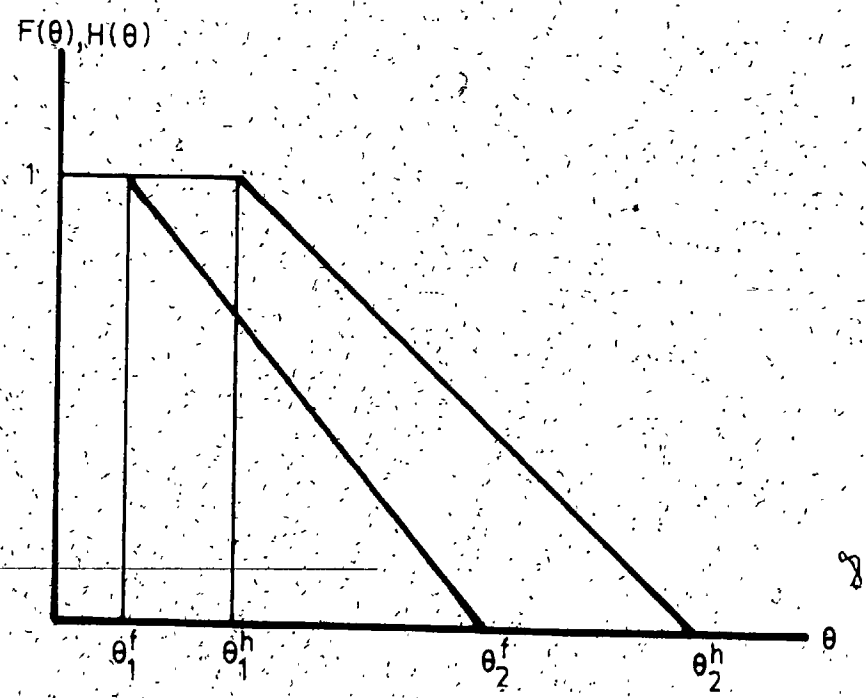
FIGURE 4.2

Recalling the consumers' decision rule (2), and the cumulative density function (3), market demand for a single level of quality in the home country is given by:

5) $\quad D^h(P) = \begin{cases} \dfrac{[\theta_2^h k - P]M^h}{(\theta_2^h - \theta_1^h)k} & \theta_2^h k \geq P \geq \theta_1^h k \\[4mm] M^h & \theta_1^h k \geq P \end{cases}$

which is shown in Figure 4.3. At $P = \theta_2^h k$, only the highest willingness-to pay consumers will buy. As price falls, lower willingness-to pay consumers are induced to buy which gives rise to a smooth, downward sloping, linear market demand curve. Once price reaches $\theta_1^h k$, all consumers are in the market buying k.

Using the market demand function (5) and the cost function (1), the pricing stage of home monopolist's problem is written as:

6) $\quad \underset{P}{\text{Max}} \ \Pi^h(P) = \dfrac{[P - ck^2][\theta_2^h k - P]M^h}{(\theta_2^h - \theta_1^h)k} - G.$

Differentiating (6) w.r.t. P and solving gives the optimal pricing rule:

7) $\quad P^* = \dfrac{\theta_2^h k + ck^2}{2}$

where P^* just balances the savings in marginal cost from a price increase with the loss in marginal revenue. The pricing equilibrium is shown in
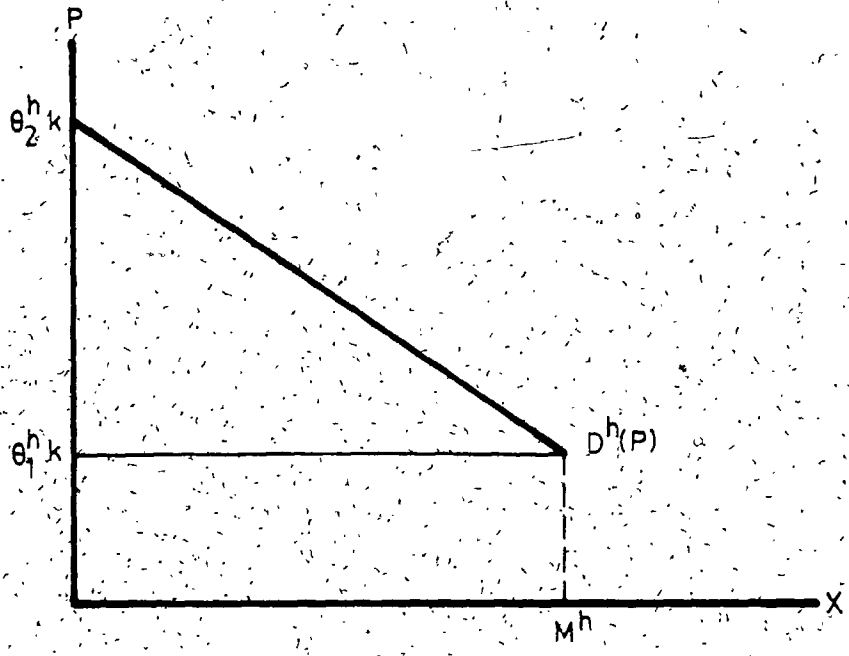
FIGURE 4.3
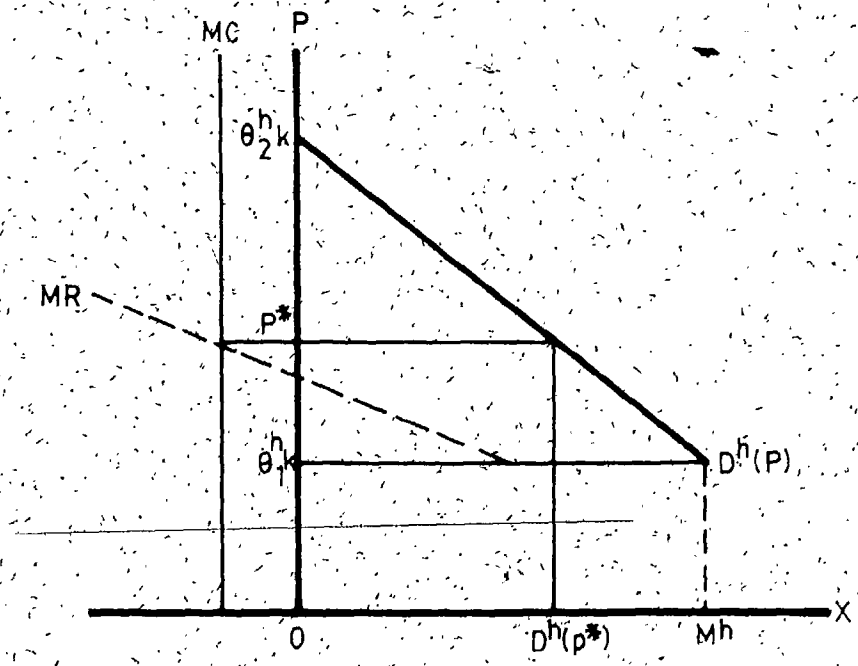
FIGURE 4.4

Figure 4.4. The second order conditions for a profit maximum are satisfied for all $P \geq 0$.

A more useful interpretation of the pricing equilibrium can be seen from Figure 4.5. The equilibrium pricing rule (7) is shown as the dashed line falling half way between the highest willingness to pay consumer's willingness to pay ray $\theta_2^h k$, and the marginal cost schedule. Suppose for the moment that $c = 0$. Then P^* just balances the gain in revenue from cutting price to attract the consumer with willingness to pay $\frac{\theta_2^h k}{2}$, with the loss in revenue suffered on all higher willingness to pay consumers. For $c > 0$, the net revenue gained on the marginal consumer is reduced, while the loss in revenue suffered on all higher willingness to pay consumers is unchanged, which causes P^* to rise. It is implicitly assumed here that the distribution of willingnesses to pay is sufficiently wide such that some low willingness to pay consumers are not served.

Substituting the optimal pricing rule (7) back into the monopolist's problem (6) gives the second part of the firm's problem which is written as:

8) $\quad \underset{k}{\text{Max}} \quad \Pi^h(k) = \frac{[\theta_2^h k - ck^2]^2 M^h}{(\theta_2^h - \theta_1^h)k} - G$

Differentiating (8) w.r.t. k and solving gives the optimal level of quality as:

9) $\quad k^* = \frac{\theta_2^h}{3c}$

The profit function is strictly concave for $k \leq \frac{2\theta_2^h}{3c}$, which ensures that k^* is a profit maximum.
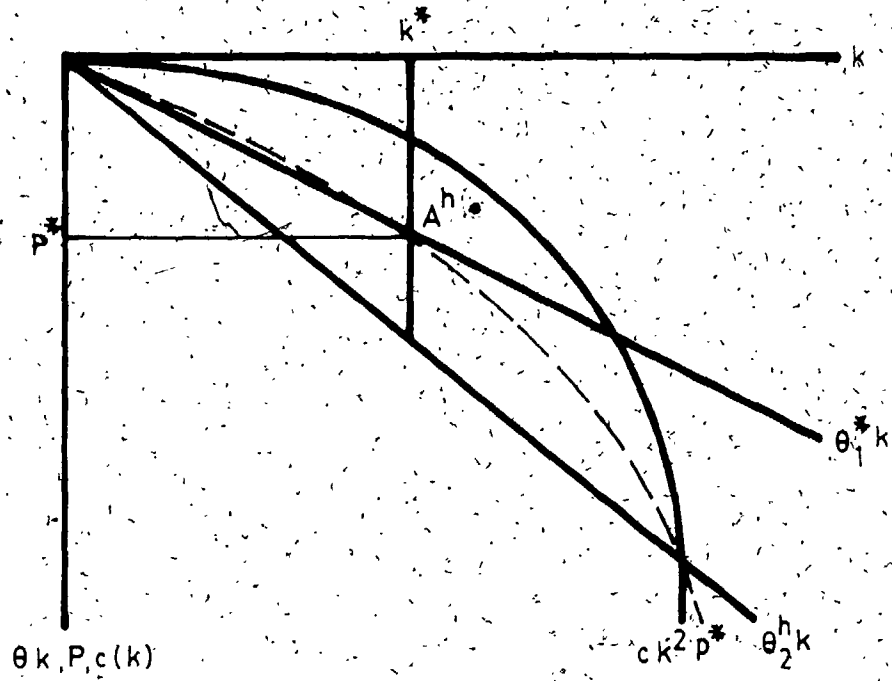
FIGURE 4.5
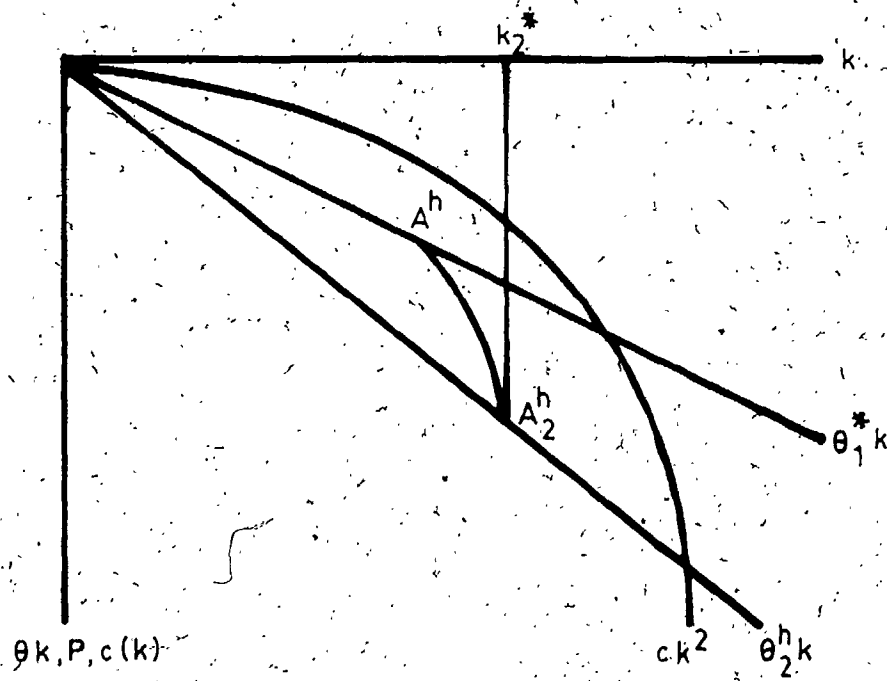
FIGURE 4.6

The equilibrium price charged for k^* is found by substituting (9) into the optimal pricing rule (7) giving:

$$10) \quad P^* = \frac{2(\theta_2^h)^2}{9c}$$

Let θ_1^* identify the consumer who is just induced to buy k^* at price P^* such that in equilibrium, $\theta_1^* k^* - P^* = 0$. Substituting in for k^* and P^* gives:

$$11) \quad \theta_1^* = \frac{2\theta_2^h}{3}$$

Thus assuming that the distribution of willingnesses to pay is sufficiently wide such that $\theta_1^h \leq \theta_1^*$, then conditions (9), (10), and (11) describe an autarky equilibrium in which $[\theta_1^*, \theta_2^h]$ is the set of consumers who buy k^* at price P^* while $[\theta_1^h, \theta_1^*)$ is the set of consumers who are not served. The equilibrium is shown at point A^h in Figure 4.5. Notice that the equilibrium level of quality k^*, maximizes the difference between the marginal consumer's willingness to pay ray $\theta_1^* k$, and the marginal cost schedule. Thus in equilibrium, quality is chosen as though all consumers were the marginal consumer.

For narrower distributions where $\theta_1^h > \theta_1^*$, the equilibrium price and quality combination offered at point A^h in Figure 4.5 is clearly not optimal. The monopolist could increase profits by raising the price to $\theta_1^h k^*$, and could further increase profits by altering quality.

When the distribution of willingnesses to pay is sufficiently narrow such that it pays to serve the lowest willingness to pay consumer, the optimal pricing rule is simply to set:

12) $\quad P^* = \theta_1^h k$

Substituting the pricing rule (12) into the monopolist's problem (6), gives the second part of the firm's problem which is written as:

13) $\quad \underset{k}{\text{Max}} \quad \pi^h(k) = [\theta_1^h k - ck^2]M^h - G$

Differentiating (13) w.r.t. k and solving gives the optimal level of quality:

14) $\quad k^* = \dfrac{\theta_1^h}{2c}$

The profit function is strictly concave for all $k > 0$ which ensures that k^* is at a profit maximum.

From (12), the price charged by the monopolist for k^* is:

15) $\quad P^* = \dfrac{(\theta_1^h)^2}{2c}$

Thus conditions (14) and (15) describe an autarky equilibrium in which all consumers in the home country are offered k^* at price P^*. The locus of equilibrium price and quality combinations is shown as the heavy line

between points A^h and A_2^h in Figure 4.6. Notice that along the locus, the equilibrium level of quality maximizes the difference between the lowest willingness to pay consumer's willingness to pay ray and the marginal cost schedule. Also notice that at point A_2^h where the distribution of willingness to pay has shrunk to a point, $k_2^* = \frac{\theta_2^h}{2c}$ and $P_2^* = \frac{(\theta_2^h)^2}{2c}$, which is exactly the outcome that was reached in the simple one-consumer type model described in Chapter 2.

The relationship between the equilibrium level of quality produced in the home country in autarky, and the distribution of willingnesses to pay is summarized in Figure 4.7. Thus for θ_1^h below θ_1^*, quality is fixed at $k^* = \frac{\theta_2^h}{3c}$. For θ_1^h above θ_1^*, k^* converges to $k_2^* = \frac{\theta_2^h}{2c}$ as θ_1^h approaches θ_2^h.

Using the equilibrium conditions for price and quality, and the definition of net benefit from (2), the welfare implications of the autarky equilibrium configuration are summarized as:

$$16) \quad Z(P^*) = \begin{cases} \left. \begin{array}{ll} \dfrac{\theta_2^h(3\theta - 2\theta_2^h)}{9c} & \theta_1^* \leq \theta \leq \theta_2^h \\[2ex] 0 & \theta \leq \theta_1^* \end{array} \right\} \theta_1^h \leq \theta_1^* \\[6ex] \dfrac{\theta_1^h(\theta - \theta_1^h)}{2c} \qquad\qquad\qquad \theta_1^h > \theta_1^* \end{cases}$$

These effects may also be summarized in terms of a diagram which relates consumer welfare to the distribution of willingness to pay for quality, which is shown in Figure 4.8. For distributions where $\theta_1^h < \theta_1^*$,
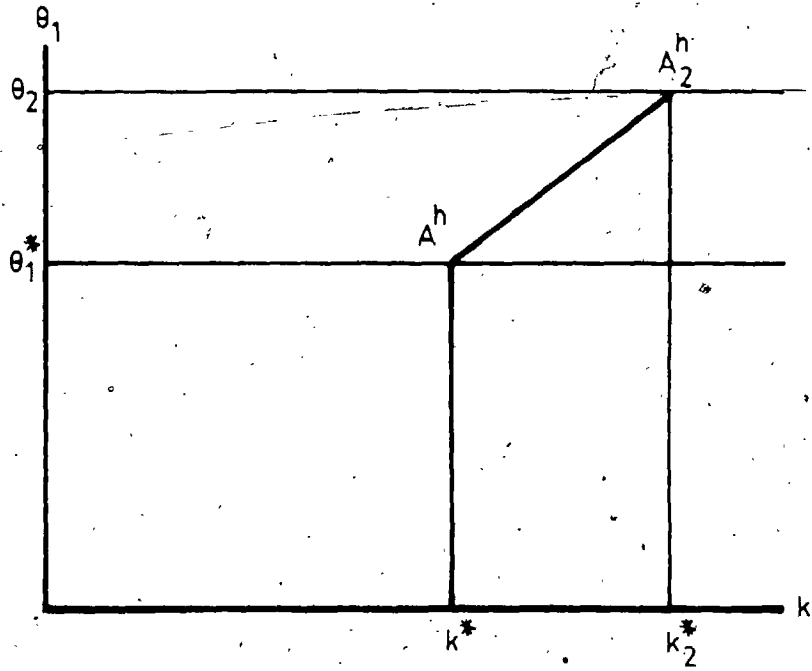
FIGURE 4.7
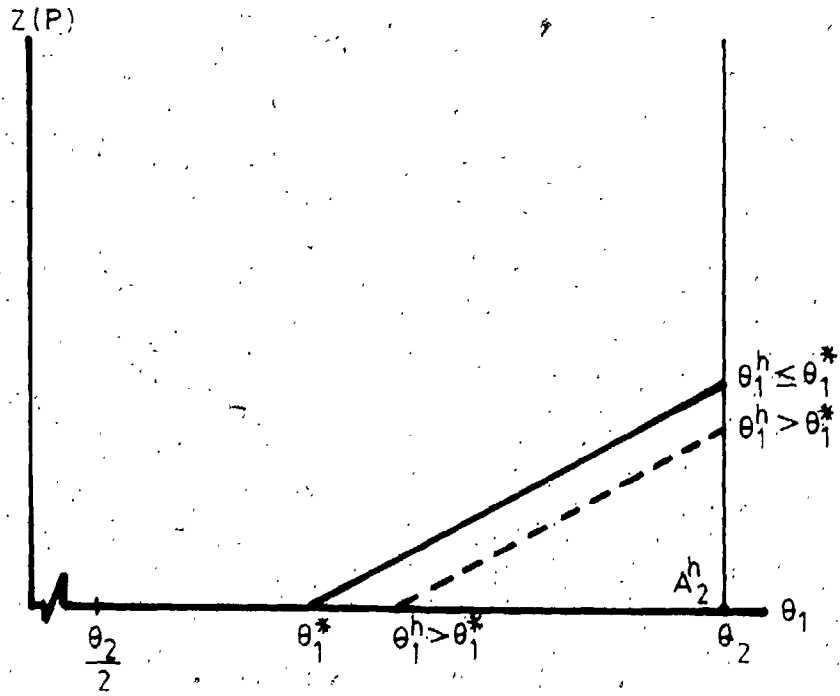
FIGURE 4.8

consumers with willingness to pay below θ_1^* derive no net benefit as they are not served, while consumers with willingness to pay above θ_1^*, derive positive net benefit shown along the upward sloping line labelled $\theta_1^h \leq \theta_1^*$. For $\theta_1^h > \theta_1^*$, the lowest willingness to pay consumer is just indifferent between k^* and the numeraire, while all higher willingness to pay consumers derive positive net benefit as shown along the $\theta_1^h > \theta_1^*$ line. Notice that as the distribution shrinks to a point, the monopolist is able to extract all consumers' net benefit. Thus wider distributions allow higher willingness to pay consumers to extract more net benefit from the monopolist in autarky, which is much the same as the result obtained for the two-point distribution examined in Chapter 3.

The analysis is identical for the foreign monopolist in autarky. Making different assumptions about the distribution of willingnesses to pay for quality will result in a different level of quality being produced in autarky. This also provides some scope to explore the effects of small differences between the two countries once trade opens.

4.   Trading Equilibrium

This section examines the consequences of opening up trade between the home and foreign economies which may differ only in total population, and the range of willingnesses to pay for quality.

Once trade opens, the two monopolists are assumed to compete as duopolists over price and quality for consumers in each country, in terms of a two stage game. In the first stage, a Bertrand-Nash pricing equilibrium is derived separately for each country, for given (autarky) levels of quality.[2] Treating each country separately avoids dealing

with the sum of two uniform distributions that may overlap and cause kinked market demand curves with non-existence problems of their own.

The second stage involves using the equilibrium pricing rules derived for each country to solve for a quality setting equilibrium, assuming that each firm conjectures that the quality of the other is held fixed. Fixed costs are again assumed sufficiently high to prevent either firm from producing a second level of quality.

The analysis presented here concentrates on an overall trading equilibrium in which all consumers in each country buy a quality differentiated good. The complications which arise when low willingness to pay consumers are not served are examined in the appendix to this chapter.

4.1 Pricing Equilibrium

This section examines the pricing equilibrium that is reached in the home country under the assumption that all consumers are served. The analysis is the same for the foreign country except for f's in place of h's where appropriate.

Each firm's quality is assumed to be held fixed at its autarky level. Let k_1 denote the quality produced by the foreign firm, and k_2 denote the quality produced by the home firm where the distributions differ such that in autarky, $k_2 > k_1$.

Home country consumers must now choose between the imported quality k_1, at price P_1^h, and the domestic quality k_2 at price P_2^h. Recalling from (2) that consumer will choose the quality providing the

largest non-negative, net benefit, let $\hat{\theta}$ identify the consumer who is just indifferent between k_1 and k_2 such that $\hat{\theta} k_1 - P_1^h = \hat{\theta} k_2 - P_2^h$ which gives:

17) $\quad \hat{\theta} = \dfrac{P_2 - P_1}{k_2 - k_1}$

Since all consumers are assumed to buy either k_1 or k_2, market demand functions for k_1 and k_2 in the home country are written as:[3]

18) $\quad D(P_1^h) = \dfrac{(\hat{\theta} - \theta_1^h)M^h}{(\theta_2^h - \theta_1^h)}$

19) $\quad D(P_2^h) = \dfrac{(\theta_2^h - \hat{\theta})M^h}{(\theta_2^h - \theta_1^h)}$

Substituting $\hat{\theta}$ from (17) into the demand functions (18) and (19), and using the cost function (1), allows the foreign and home firm's pricing problem in the home country to be written as:

20) $\quad \underset{P_1^h}{\text{Max}} \quad \Pi(P_1^h) = \dfrac{[P_1^h - ck_1^2][P_2^h - P_1^h - \theta_1^h(k_2 - k_1)]M^h}{(\theta_2^h - \theta_1^h)(k_2 - k_1)}$

21) $\quad \underset{P_2^h}{\text{Max}} \quad \Pi(P_2^h) = \dfrac{[P_2^h - ck_2^2][\theta_2^h(k_2 - k_1) - P_2^h + P_1^h]M^h}{(\theta_2^h - \theta_1^h)(k_2 - k_1)}$

Differentiating (20) w.r.t. P_1 assuming P_2 is held fixed, and differentiating (21) w.r.t. P_2 assuming P_1 is held fixed, gives the

price reaction functions for the foreign and home firms respectively:

22) $\quad P_1^h = \dfrac{P_2^h + ck_1^2 - \theta_1^h(k_2 - k_1)}{2}$

23) $\quad P_2^h = \dfrac{P_1^h + ck_2^2 + \theta_2^h(k_2 - k_1)}{2}$

Since both profit functions are strictly concave in their respective prices, the two price reaction functions (22) and (23) are easily solved to yield the equilibrium prices:

24) $\quad P_1^h = \frac{1}{3}[(\theta_2^h - 2\theta_1^h)(k_2 - k_1) + ck_2^2 + 2ck_1^2]$

25) $\quad P_2^h = \frac{1}{3}[(2\theta_2^h - \theta_1^h)(k_2 - k_1) + 2ck_2^2 + ck_1^2]$

The pricing equilibrium for the home country is shown in Figure 4.9. Notice from (24) and (25), that the closer the two qualities are together, the more severe the Bertrand-Nash price competition becomes. Should $k_1 = k_2$, then each firm would be forced to price at marginal cost, which is the standard Bertrand-Nash result.

4.2 Quality Setting Equilibrium

Using the equilibrium pricing rules (24) and (25) derived for the home country, and similarly derived rules for the foreign country, the quality selection problem for the foreign and home firms are written as:
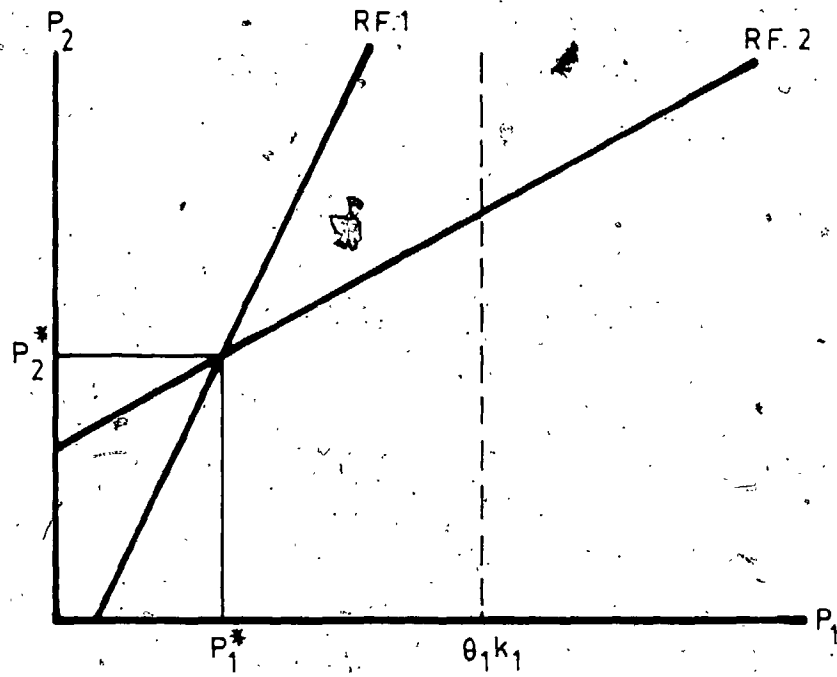
FIGURE 4.9
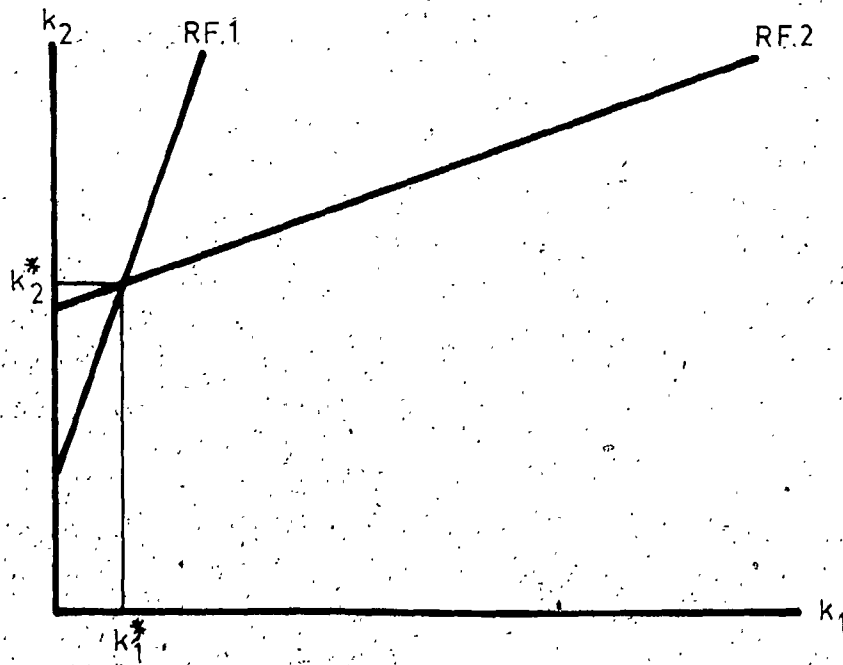
FIGURE 4.10

26) $\underset{k_1}{\text{Max}}$ $\Pi(k_1) = \dfrac{[(\theta_2^h - 2\theta_1^h)(k_2 - k_1) + ck_2^2 - ck_1^2]^2 M^h}{9(\theta_2^h - \theta_1^h)(k_2 - k_1)}$

$+ \dfrac{[(\theta_2^f - 2\theta_1^f)(k_2 - k_1) + ck_2^2 - ck_1^2]^2 M^f}{9(\theta_2^f - \theta_1^f)(k_2 - k_1)} - G$

27) $\underset{k_2}{\text{Max}}$ $\Pi(k_2) = \dfrac{[(2\theta_2^h - \theta_1^h)(k_2 - k_1) + ck_1^2 - ck_2^2]^2 M^h}{9(\theta_2^h - \theta_1^h)(k_2 - k_1)}$

$+ \dfrac{[(2\theta_2^f - \theta_1^f)(k_2 - k_1) + ck_1^2 - ck_2^2]^2 M^f}{9(\theta_2^f - \theta_1^f)(k_2 - k_1)} - G.$

Differentiating (26) w.r.t. k_1 assuming k_2 is held fixed and differentiating (27) w.r.t. k_2 assuming k_1 is held fixed gives the first order conditions:

28) $\Pi'(k_1) = \dfrac{[(\theta_2^h - 2\theta_1^h) + c(k_2 + k_1)][(\theta_2^h - 2\theta_1^h) - c(k_2 - 3k_1)]M^h}{9(\theta_2^h - \theta_1^h)}$

$+ \dfrac{[(\theta_2^f - 2\theta_1^f) + c(k_2 + k_1)][(\theta_2^f - 2\theta_1^f) - c(k_2 - 3k_1)]M^f}{9(\theta_2^f - \theta_1^f)} = 0$

29) $\Pi'(k_2) = \dfrac{[(2\theta_2^h - \theta_1^h) - c(k_2 + k_1)][(2\theta_2^h - \theta_1^h) - c(3k_2 - k_1)]M^h}{9(\theta_2^h - \theta_1^h)}$

$+ \dfrac{[(2\theta_2^f - \theta_1^f) - c(k_2 + k_1)][(2\theta_2^f - \theta_1^f) - c(3k_2 - k_1)]M^f}{9(\theta_2^f - \theta_1^f)} = 0$

For the moment consider the special case where the distribution of willingness to pay for quality in the two countries is the same such that $\theta_1^h = \theta_1^f = \theta_1$, and $\theta_2^h = \theta_2^f = \theta_2$. The two first order conditions reduce to linear quality reaction functions for the foreign and home firms which are:

30) $\qquad k_1 = \dfrac{ck_2 - (\theta_2 - 2\theta_1)}{3c}$

31) $\qquad k_2 = \dfrac{ck_1 + (2\theta_2 - \theta_1)}{3c}$

The two quality reaction functions (30) and (31) are easily solved for the equilibrium levels of quality:

32) $\qquad k_1^* = \dfrac{5\theta_1 - \theta_2}{8c}$

33) $\qquad k_2^* = \dfrac{5\theta_2 - \theta_1}{8c}$

The two quality reaction functions, and their intersection point is shown in Figure 4.10.

The equilibrium prices charged for k_1^* and k_2^* by the foreign and home firms respectively, are found by substituting (32) and (33) into the optimal pricing rules (24) and (25), giving:

34) $\qquad P_1^* = \dfrac{1}{64c} [25\theta_2^2 - 58\theta_2\theta_1 + 49\theta_1^2]$

35) $\quad P_2^* = \frac{1}{64c}[49\theta_2^2 - 58\theta_2\theta_1 + 25\theta_1^2]$

Thus conditions (32), (33) and (34), (35) describe the equilibrium qualities and prices that result once trade opens between the home and foreign economies, under the assumption that the distribution of willingnesses to pay is the same in each.

The relationship between the equilibrium levels of quality produced by the two firms and the distribution of willingnesses to pay assumed for the two countries is summarized in Figure 4.11, where (32) and (33) are represented as the heavy lines. It is easy to see that over all distributions of willingnesses to pay, the two firms push their qualities apart from their autarky levels to avoid the severe Bertrand-Nash price competition between them. Notice that as the distribution of willingnesses to pay shrinks to a point, the equilibrium levels of quality converge to k_2^*, which is exactly the outcome that was reached in autarky, though each firm is now pricing at marginal cost. With positive fixed costs, however, this outcome would clearly not be sustained.

To ensure that conditions (32) and (33) do in fact describe a quality setting equilibrium, the concavity of the respective profit functions must be checked. Assuming that the distribution of willingnesses to pay for quality is the same for the two countries, then differentiating (28) w.r.t. k_1, and evaluating at k_2^*, reveals that the foreign firm's profit function is concave in k_1 for:
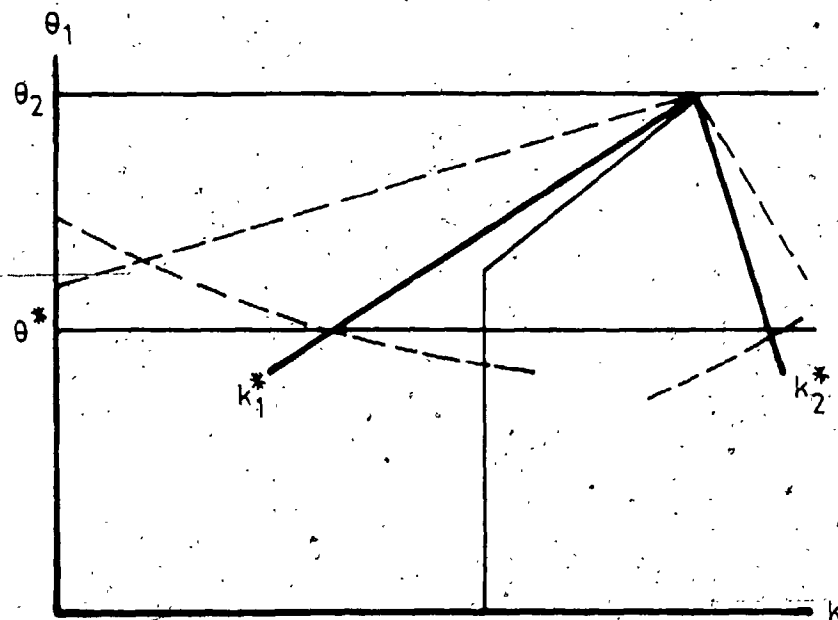
FIGURE 4.11
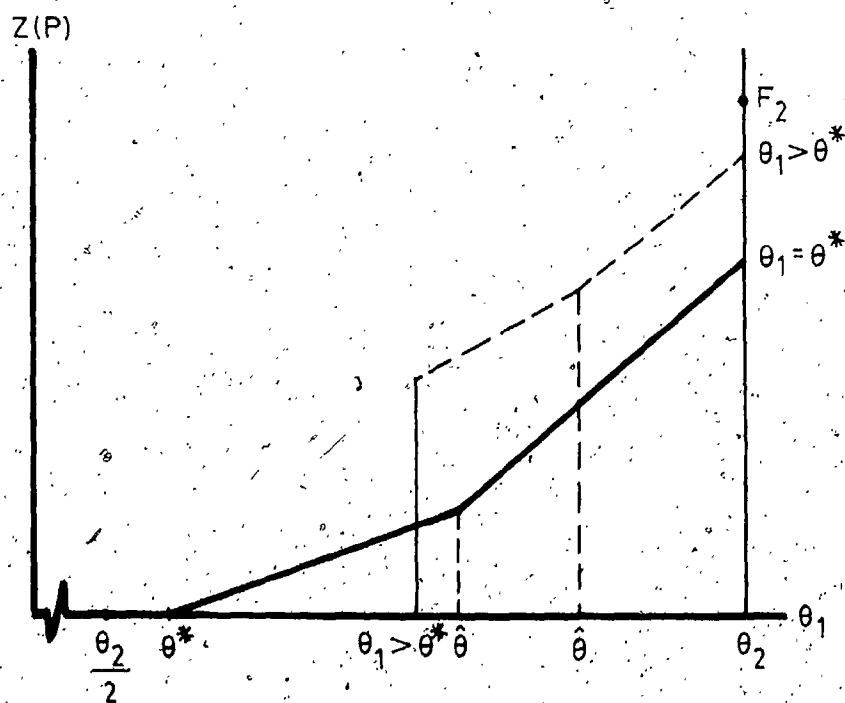
FIGURE 4.12

36) $\quad k_1 \geq \dfrac{11\theta_1 - 7\theta_2}{8c}$

which is represented as the upward sloping dashed line in Figure 4.11. Similarly, differentiating (29) w.r.t. k_2 and evaluating at k_1^*, reveals that the home firm's profit function is concave in k_2 for:

37). $\quad k_2 \leq \dfrac{11\theta_1 - 7\theta_1}{8c}$

which is shown as the downward sloping dashed line in Figure 4.11.

If costs rise quickly in quality, k_2 will act as an upper bound on k_1, as the profitability of pushing k_1 past k_2 to become the high quality good is reduced.

Conditions on k_1 and k_2 must also be derived to satisfy $\theta_1 k_1 \geq P_1^*$, such that the lowest willingness to pay consumer is served, and thereby ensuring that the demand functions, and hence profit functions are well specified. Using the equilibrium pricing rule (24), the condition which ensures that the lowest willingness to pay consumer is served, is written as:

38). $\quad \theta_1 k_1 \geq \dfrac{1}{3}[(\theta_2 - 2\theta_1)(k_2 - k_1) + ck_2^2 + 2ck_1^2]$

The locus of k_1's satisfying (38) evaluated at k_2^* is shown as the downward sloping dot-dashed line in Figure 4.11. The locus k_2's satisfying (38) evaluated at k_1^* is shown as the upward sloping dot-dashed

line in Figure 4.11. Evaluating (38) at both k_1^* and k_2^* reveals that the quality setting equilibrium is well specified for $\theta_1 \geq \theta^*$, where θ^* is given by:

39) $\quad \theta^* = \frac{5}{9}\theta_2$.

Wider distributions will involve different demand functions and profit functions, which are examined in the appendix to this chapter.

Assuming that the conditions needed for a well specified trading equilibrium are satisfied, the effects of small differences in the distribution of willingnesses to pay between the two countries can be examined.

Beginning from a position where the two distributions are identical, the effect of sliding the distribution of willingnesses to pay along in the home country, while leaving the distribution of willingnesses to pay in the foreign country unchanged, is found by totally differentiating the first order conditions (28) and (29) and evaluating at $M^h = M^f$, and $d\theta_2^h = d\theta_1^h$, to preserve the same range, which gives:[4]

40) $\quad \dfrac{dk_2^*}{d\theta^h} \geq \dfrac{1}{4c} \geq \dfrac{dk_1^*}{d\theta^h}$

Thus small differences in the distribution of willingnesses to pay between the countries allows the two firms to push their qualities ever farther apart, helping them to further reduce the Bertrand-Nash competition between them.

4.3 Welfare Effects

In the trading equilibrium described by conditions (32), (33) and (34), (35), $\dfrac{(\hat{\theta} - \theta_1)(M^f + M^h)}{(\theta_2 - \theta_1)}$ is the share of consumers who buy k_1^* at price P_1^* from the foreign firm, while $\dfrac{(\theta_2 - \hat{\theta})(M^f + M^h)}{(\theta_2 - \theta_1)}$ is the share of consumers who buy k_2^* at price P_2^* from the home firm. Substituting the equilibrium conditions into (17) for $\hat{\theta}$ shows that:

$$41) \qquad \hat{\theta} = \frac{\theta_2 + \theta_1}{2}$$

which says that in equilibrium, each firm serves exactly half of the total number of consumers.

The net benefit derived by lower willingness to pay consumers from consuming k_1^* at price P_1^* is given by:

$$42) \qquad Z(P_1^*) = \frac{\theta(5\theta_1 - \theta_2)}{8c} - \frac{(25\theta_2^2 - 58\theta_2\theta_1 + 49\theta_1^2)}{64c}$$

where $\theta \in [\theta_1, \hat{\theta})$. Similarly, the net benefit derived by higher willingness to pay consumers from consuming k_2^* at price P_2^* is given by:

$$43) \qquad Z(P_2^*) = \frac{\theta(5\theta_2 - \theta_1)}{8c} - \frac{(49\theta_2^2 - 58\theta_2\theta_1 + 25\theta_1^2)}{64c}$$

where $\theta \in [\hat{\theta}, \theta_2]$.

The relationship between consumer welfare and the distribution of willingnesses to pay for quality assumed for the two countries is most easily seen by representing (42) and (43) in terms of a diagram as shown in Figure 4.12. For the widest possible distribution where $\theta_1 = \theta^*$, the lowest willingness to pay consumer is just indifferent between buying k_1^*, and the numeraire, while all higher willingness to pay consumers derive positive net benefit shown along the line labelled $\theta_1 = \theta^*$. As the distribution of willingnesses to pay narrows, the equilibrium levels of quality produced by the two firms are drawn closer together, which heightens the price competition between them, and thereby increases the net benefit derived by all consumers, as shown along the line labelled $\theta_1 > \theta^*$. Once the distribution has shrunk to a point, each firm is producing k_2^* and prices at marginal cost, providing, consumers with maximum net benefit

$$Z(P_2^*) = \frac{(\theta_2)^2}{4c}$$ shown at point F_2 in Figure 4.12.

Thus narrower distributions benefits all consumers in each country by limiting the scope for firms to differentiate their qualities, and thereby reduce price competition between them.

Small differences in the distributions of willingnesses to pay between the two countries which allows the two firms to push their qualities slightly farther apart, clearly reduces the net benefit derived by all consumers as the price competition between the firms is reduced.

5.   Autarky vs. Trade

Having described the autarky equilibrium for each country in Section 3, and the trading equilibrium that was reached between them in Section 4, the effects of opening up trade on the variety of qualities produced, prices and consumer welfare are now easily summarized.

The effects of trade on the variety of qualities produced are most easily seen in Figure 4.11. For all distributions of willingnesses to pay that are examined, the two firms push their qualities apart to avoid the price competition between them, brought about by trade. Only when the distribution has shrunk to a point, do the firms leave their qualities at their autarky levels, where they are forced to price at marginal cost.

The effects of trade on prices is summarized in Figure 4.13. The locus of equilibrium prices and quality combinations offered to consumers in each country in autarky is shown between points A and A_2 as before. The locus of equilibrium price and quality combinations offered to lower willingness to pay consumers in each country by the foreign firm once trade opens, is shown between points F^f and F_2. Similarly the locus of equilibrium price and quality combinations offered to higher willingness to pay consumers in each country by the home firm once trade opens, is shown between points F^h and F_2. Thus as the distribution of willingnesses to pay assumed for the two countries shrinks, the effects of the price competition brought about by trade, become more pronounced. Once the distribution has shrunk to a point, in autarky the monopolist is able to price at point A_2 extracting all consumers net benefit, while trade forces the two firms to price at marginal cost shown at point F_2.

Finally, the effects of trade on consumer welfare are summarized in Figure 4.14. The net benefit derived by consumers in the trading equilibrium, for the widest distribution of willingnesses to pay considered, is shown along the line labelled $\theta_1 = \theta^*$. In comparison,

FIGURE 4.13

Labels in figure: k, F^f, F_2, A, F^h, A_2, $\theta^* k$, $\theta_1^* k$, $\theta k, P, c(k)$, $c k^2$, $\theta_2 k$

FIGURE 4.14

Labels in figure: $Z(P)$, F_2, $\theta_1 > \theta_1^*$, $\theta_1 = \theta^*$, $\theta_1 \leq \theta_1^*$, $\theta_1 > \theta_1^*$, A_2, θ_1, $\frac{\theta_2}{2}$, θ^*, θ_1^*, θ_2

the net benefit derived by consumers in autarky for the same distribution is shown along the line labelled $\theta_1 < \theta_1^*$. Except for the lowest willingness to pay consumer who is indifferent, all consumers in each country benefit from the competition brought about by trade.

As the distribution of willingnesses to pay narrows, the benefits become even more pronounced. The line representing the net benefit derived by consumers in the trading equilibrium shifts up to say $\theta_1 > \theta_1^*$, as the scope for firms to differentiate their qualities to avoid price competition is reduced. The line representing the net benefit derived by consumers in autarky correspondingly shifts down to $\theta_1 > \theta_1^*$, as the monopolist is better able to extract consumers net benefit. In the extreme case where the distribution shrinks to a point, consumers derive maximum net benefit at point F_2 in the trading equilibrium, where in autarky, the monopolist is able to extract all of the net benefit leaving consumers at point A_2.

Small differences in the distribution of willingnesses to pay between the two countries which allows the two firms to push their qualities apart, and further avoid the price competition between them, would tend to reduce the benefits of the trading equilibrium.

Thus all consumers in each country gain as a result of the competition between the two firms brought about by trade. These gains are shown to be larger between countries with narrow distributions of willingnesses to pay for quality. Any differences in the distributions of willingnesses to pay between the two countries allows firms further scope to differentiate their qualities to avoid price competition, which reduces the gains from trade.
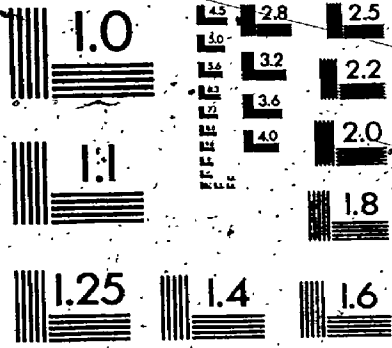
6. Summary and Conclusions

This chapter has examined the consequences of opening up trade between two economies characterized by the basic model of quality differentiated goods production, where consumers in each country are uniformly distributed over their willingness to pay for quality.

In each country in autarky, the equilibrium price and quality combination offered to consumers by a domestic monopolist is shown to depend on the distribution of willingnesses to pay for quality. Consumers benefit more from wider distributions in which lower willingness to pay consumers are served.

Once trade opens, the two monopolists are shown to compete as duopolists over price and quality for consumers in each country. For the special case where the distribution of willingnesses to pay is the same in each country, trade causes the two firms to push their qualities apart from their autarky levels, to minimize the price competition between them. Small differences in the distribution of willingnesses to pay between the two countries are shown to push their qualities slightly farther apart, which further reduced the price competition between them.

All consumers are shown to gain as a result of the competition brought about by trade. These gains are shown to be larger between countries with narrow distributions of willingnesses to pay for quality, as the scope for firms to differentiate their qualities to avoid price competition is reduced. Differences between the two countries which allows the firms to further differentiate their qualities reduces the gains from trade.

Appendix - Trading Equilibria For All Distributions

The trading equilibrium described in this chapter was developed under the assumption that the distribution of willingnesses to pay for quality in the two countries is sufficiently narrow such that in equilibrium, all consumers are served. This appendix explores the trading equilibria that result for wider distributions in which the lowest willingness to pay consumer may not be served.

Excluding the lowest willingness to pay consumer, however, is shown to cause kinked price reaction functions which greatly complicates the quality setting equilibrium. Computational methods are used to show that while trade still causes the two firms to differentiate their qualities to minimize the price competition between them, discontinuities and multiple equilibria now result.

The source of the difficulties stem from a kink in the market demand curve for the low quality good at $P_1 = \theta_1 k_1$, shown at point F in Figure 4.15. Below point F where all consumers are served, increases in P_1 cause higher willingness to pay consumers who were buying k_1 to defect to k_2. At point F_1 however, increases in P_1 now cause the lowest willingness to pay consumers who were buying k_1, to stop buying altogether, in addition to causing the defection of higher willingness to pay consumers to k_2. Thus the marginal revenue lost from an increase in P_1, jumps discontinuously at $P_1 = \theta_1 k_1$.

Below $P_1 = \theta_1 k_1$, where all consumers are served, recall that the optimal response of the foreign firm to increases in P_2 is given along the price reaction function derived earlier:
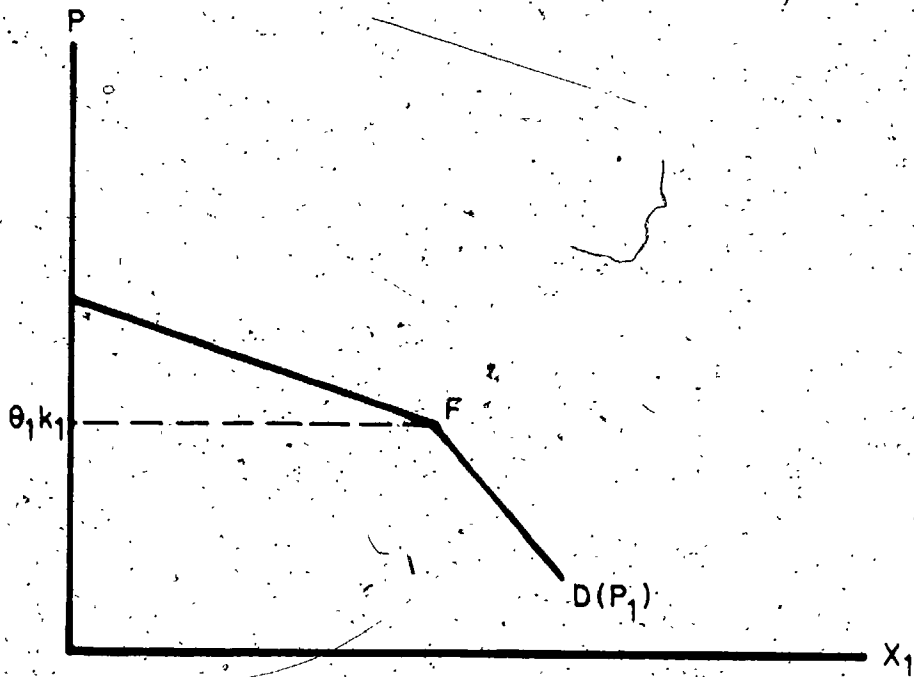
FIGURE 4.15
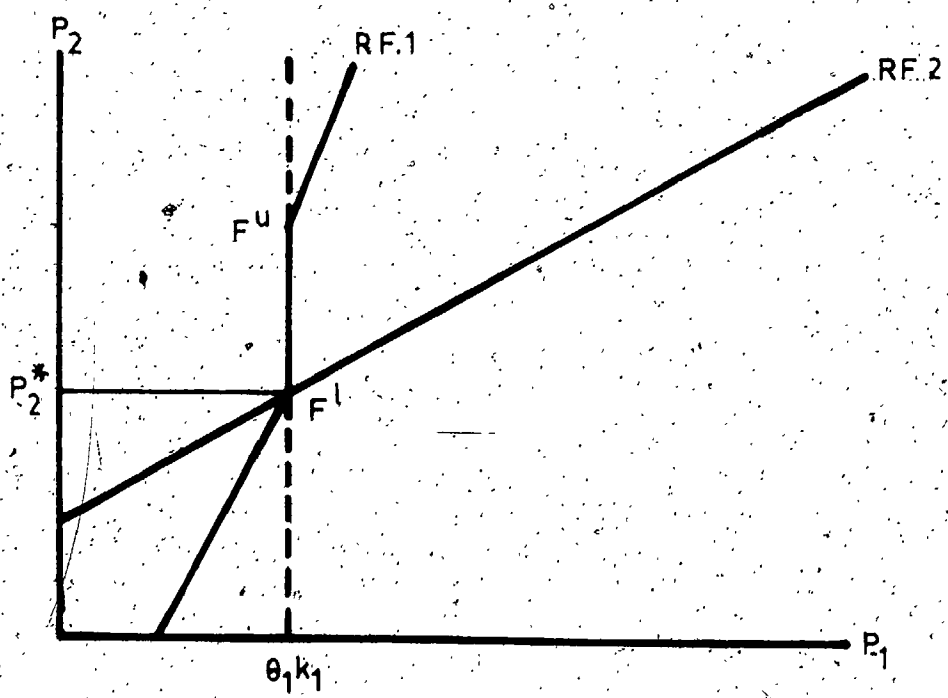
FIGURE 4.16

44) $$P_1 = \frac{P_2 + ck_1^2 - \theta_1(k_2 - k_1)}{2}$$

which is shown below point F^ℓ in Figure 4.16.

At $P_1 = \theta_1 k_1$, however, the discontinuity in the marginal revenue lost from an increase in P_1 generates a vertical section in the foreign firm's price reaction function, shown between points F^ℓ and F^u in Figure 4.16, where it pays to hold P_1 fixed over some range of P_2's, which is simply written as:[5]

45) $$P_1 = \theta_1 k_1$$

Above $P_1 = \theta_1 k_1$, where lower willingness to pay consumers are excluded, the foreign firm's price reaction function is derived as:

46) $$P_1 = \frac{P_2 k_1 + ck_1^2 k_2}{2k_2}$$

which is shown as the upper section beyond point F^u in Figure 4.16.

Excluding low willingness to pay consumers has no effect on the market demand for k_2, and consequently no effect on the home firm's price reaction function which was given earlier by:

47) $$P_2 = \frac{P_1 + ck_2^2 + \theta_2(k_2 - k_1)}{2}$$

It is relatively straightfoward to show that the home firm's price reaction function will cut one of the sections of the foreign firm's price reaction function to yield a unique price setting equilibrium for

given levels of quality. Where the intersection occurs will depend on both the distribution of willingnesses to pay, and the levels of quality chosen.

The condition which ensures that the price setting equilibrium falls along the lower section of the foreign firm's price reaction function is found by solving for the intersection between (44), (45) and (47), shown at point F^{ℓ} in Figure 4.16, which gives:

48) $0 \geq (\theta_2 - 2\theta_1)k_2 - (\theta_2 + \theta_1)k_1 + ck_2^2 + 2ck_1^2$

Similarly, the condition which ensures that the price setting equilibrium falls along the upper section of the foreign firm's price reaction function is found by solving for the intersection of (45), (46) and (47) shown at point F^u in Figure 4.16, giving:

49) $0 \leq (\theta_2 - 4\theta_1)k_2 - (\theta_2 - \theta_1)k_1 + ck_2^2 + 2ck_1k_2$

Should the distribution of willingnesses to pay and the levels of quality chosen be such that both conditions (48) and (49) fail to hold, then the pricing equilibrium falls along the vertical section of the foreign firm's price reaction function.

Solving for the quality setting equilibrium, however, is greatly complicated by the kinks in the foreign firm's price reaction function. Each section of the foreign firm's price reaction function must be used separately in conjunction with the home firm's price reaction function to solve for the quality setting equilibrium. The equilibria derived

for each section must then be checked against conditions (48) and (49) to determine the distributions over which they are well specified. The various equilibria may then be compared to determine over which distributions each dominates.

The analysis is further complicated by the fact that each firm's quality selection problem is highly non linear over the vertical and upper sections of the foreign firm's price reaction function. To allow for comparison between the equilibria that result over each of the three sections, a non-linear equation solving computer program is used to compute the equilibrium levels of quality for each of the sections, over various distributions of willingnesses to pay for quality.[6,7] To simplify the computations, it is assumed that $\theta_2 = c = 1$, and $G = 0$.

Beginning with the lower section of the foreign firm's price reaction function, recall that the relationship between the equilibrium levels of quality produced and the distribution of willingnesses to pay for quality which satisfy condition (48), is shown between points A and B_1, and A and B_2 in Figure 4.17, for k_1^* and k_2^* respectively. Distributions wider than $\theta_1 = .5555$ at points B_1 and B_2, cause the two price reaction functions to intersect beyond $P_1 = \theta_1 k_1$, violating condition (48). This causes the two firms to begin competing over the vertical section of the foreign firm's price reaction function.

The heavy dashed line in Figure 4.17, gives the relationship between the distribution of willingnesses to pay for quality assumed for the two countries, and the equilibrium level of quality produced in autarky.
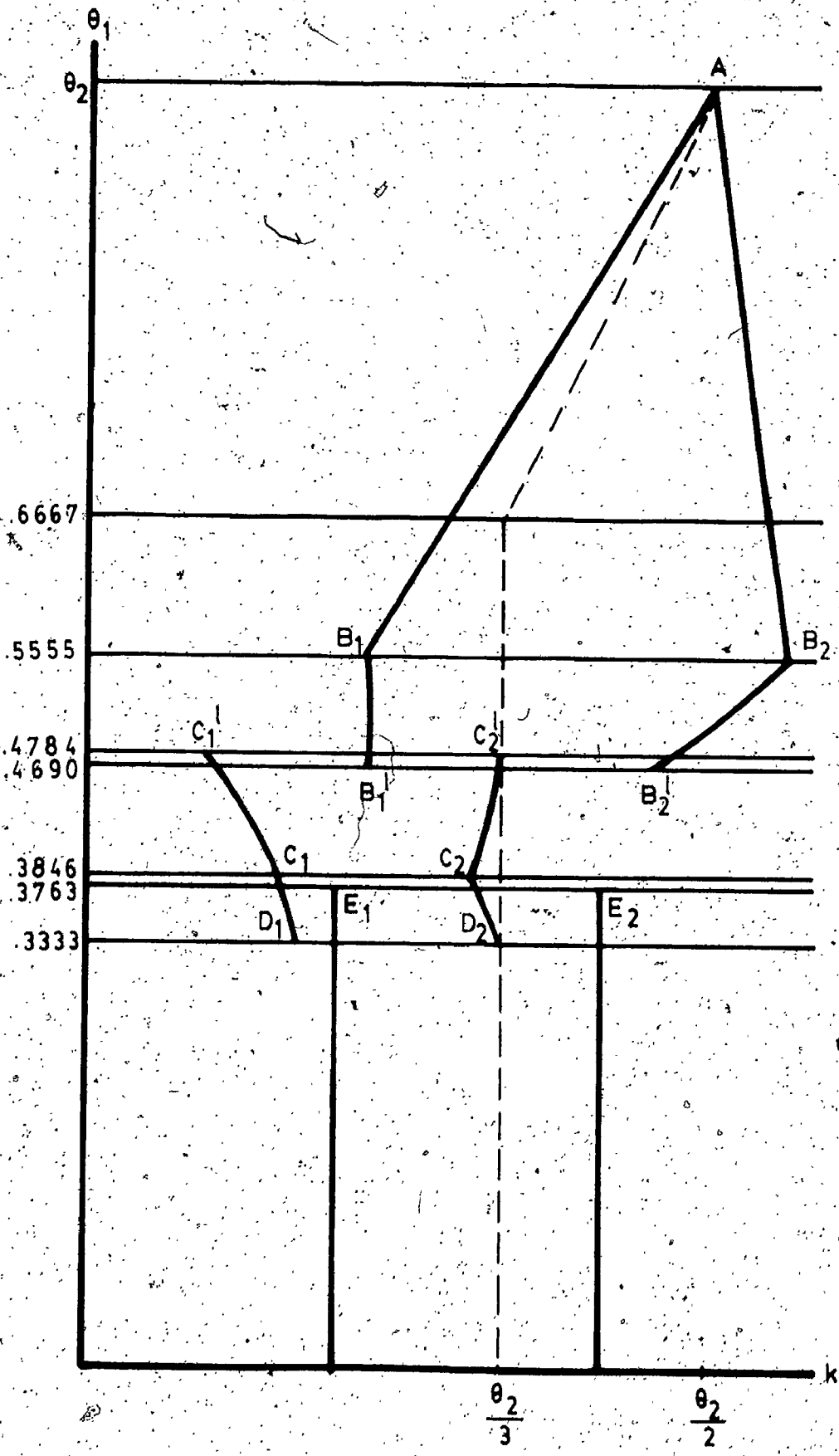
FIGURE 4.17

Now consider the vertical section of the foreign firm's price reaction function. The relationship between the equilibrium levels of quality and the distribution of willingnesses to pay for quality which cause both conditions (48) and (49) to fail is shown between points C_1 and D_1, and C_2 and D_2 in Figure 4.17 for k_1^* and k_2^* respectively. Distributions narrower than $\theta_1 = .3846$ at points C_1 and C_2 cause the two price reaction functions to intersect below the lower section of the foreign firm's price reaction function, satisfying condition (48). This causes the two firms to begin competing along the lower section of the foreign firm's price reaction function. On the other hand, distributions wider than $\theta_1 = .3333$ at points D_1 and D_2 cause the two price reaction functions to intersect above the upper section of the foreign firm's price reaction function, satisfying condition (49). This causes the two firms to begin competing along the upper section of the foreign firm's price reaction function.

Allowing the two firms to optimize over the upper section of the foreign firm's price reaction function where low willingness to pay consumers are excluded, results in the firms choosing qualities independent of the distribution of willingnesses to pay for quality. The equilibrium is well specified for all distributions wider than $\theta_1 = .3763$ shown at points E_1 and E_2 in Figure 4.17. Consumers with willingnesses to pay below $\theta = .3763$ are simply not served. Distributions narrower than $\theta_1 = .3763$ which cause the reaction functions to intersect below $P_1 = \theta_1 k_1$, such that condition (49) fails, cause the two firms to compete over the vertical section of the foreign firm's price reaction function.

Having determined the distributions over which the equilibria derived for each of the three sections are well specified, the various equilibria must now be compared to determine where each dominates. This involves plotting the profit functions of the two firms at various points and checking for concavity.

The foreign and home firm's profit functions for the equilibrium described at points A_1 and A_2 in Figure 4.17, which just satisfies condition (48) along the lower section of the foreign firm's price reaction function, are shown in Figure 4.18.[8] The foreign firm's profit function $\Pi_1^\ell(k_1)$, which is constructed for $k_2 = k_2^*$, is well specified for $k_1 \geq k_1^\ell$, which coincides with k_1^*. Similarly, the home firm's profit function $\Pi_2^\ell(k_2)$, which is constructed for $k_1 = k_1^*$, is well specified for $k_2 \leq k_2^\ell$, which coincides with k_2^*. Below k_1^ℓ, $\Pi_1^v(k_1)$ is the foreign firm's profit function defined on the vertical section of the foreign firm's price function constructed for $k_2 = k_2^*$, and above k_2^ℓ, $\Pi_2^v(k_2)$ is the home firm's profit function defined on the vertical section, constructed for $k_1 = k_1^*$. Since $\Pi_1^v(k_1^*) > 0$ and $\Pi_2^v < 0$, it is clear that the equilibrium described at points A_1 and A_2 is well defined.

Between $\theta_1 = .5555$ and $\theta_1 = .3846$, however, allowing the firms to optimize along the lower section of the foreign firm's price reaction pushes them on to the vertical section, and vice-versa. Rather than allowing the two firms to cycle back and forth, one might expect them to settle down at the corner of the foreign firm's price reaction function.

Accordingly, allowing the two firms to optimize along the lower section of the foreign firm's price reaction function subject to the constraint that condition (48) holds with equality, gives the equilibrium
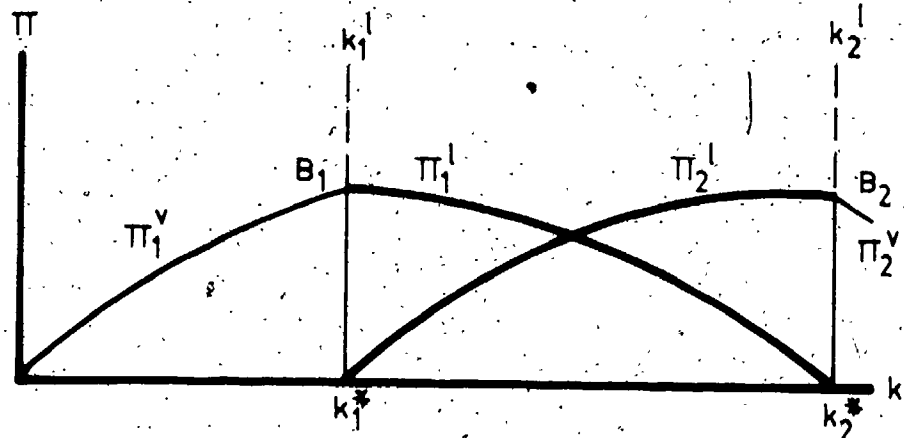
$\theta_1 = .5556$

$k_1^* = .2222$ $\qquad k_2^* = .5555$

$\pi_1'^{l}(k_1^*) = 0$ $\qquad \pi_2'^{l}(k_2^*) = 0$

$\pi_1'^{v}(k_1^{l}) > 0$ $\qquad \pi_2'^{v}(k_2^{l}) < 0$

FIGURE 4.18
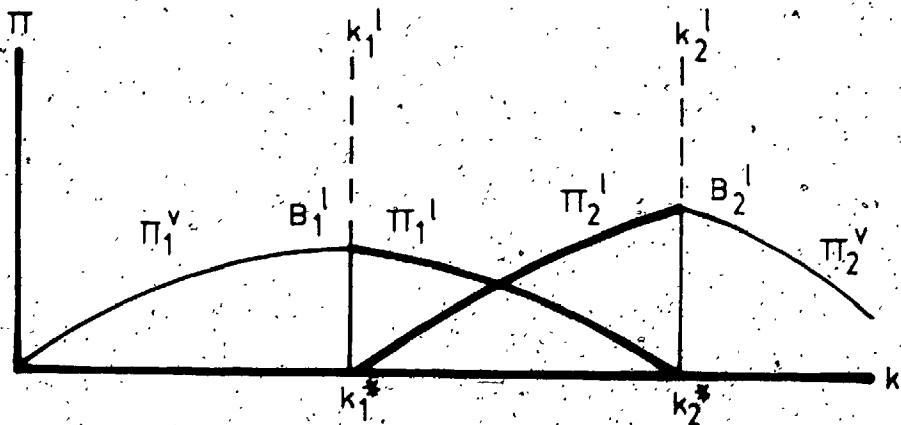
$\theta_1 = .46.90$

$k_1^* = .2242$ $\qquad k_2^* = .4483$

$\pi_1'^{l}(k_1^*) < 0$ $\qquad \pi_2'^{l}(k_2^*) > 0$

$\pi_1'^{v}(k_1^{l}) = 0$ $\qquad \pi_2'^{v}(k_2^{l}) < 0$

FIGURE 4.19

levels of quality as a function of the distribution of willingnesses to pay, shown between points B_1 and B_1^ℓ and B_2 and B_2^ℓ in Figure 4.17, for k_1^* and k_2^* respectively. For distributions wider than $\theta_1 = .4690$, the foreign firm's profit function is no longer concave at k_1^*, as shown in Figure 4.19. Notice that at point B_1^ℓ the slope of the foreign firm's profit function defined on the vertical section is zero. For distributions wider than $\theta_1 = .4690$, $\Pi_1^v(k_1^*) < 0$, which causes the firms to move away from k_1^* and k_2^*, onto the vertical section of the foreign firm's price reaction function.

Similarly, allowing the two firms to compete over the vertical section of the foreign firm's price reaction function subject to the constraint that condition (48) holds with equality, gives the equilibrium levels of quality as a function of the distribution of willingnesses to pay, shown between points C_1^ℓ and C_1, and C_2^ℓ and C_2 in Figure 4.17 for k_1^* and k_2^* respectively. For distributions narrower than $\theta_1 = .4784$, the foreign firm's profit function is not concave as shown in Figure 4.20. Notice that at point C_1^ℓ, the slope of the foreign firm's profit function defined on the lower section is zero. For distributions narrower than $\theta_1 = .4784$, $\Pi_1^\ell > 0$, which causes the two firms to move away from k_1^* and k_2^*, onto the lower section of the foreign firm's price reaction function.

In Figure 4.21, it is easy to see that both firm's profit functions are strictly concave at k_1^* and k_2^*, for the equilibrium described at points C_1 and C_2 in Figure 4.17, where condition (48) is just satisfied for the equilibria derived using the vertical section of the foreign firm's price

$\theta_1 = .4784$ $k_1^* = .0956$ $k_2^* = .3298$

$\pi_1'^v(k_1^*) > 0$ $\pi_2'^v(k_2^*) < 0$

$\pi_1'^l(k_1^l) = 0$ $\pi_2'^l(k_2^l) > 0$

FIGURE 4.20

$\theta_1 = .3846$ $k_1^* = .1539$ $k_2^* = .3077$

$\pi_1'^v(k_1^*) = 0$ $\pi_2'^v(k_2^*) = 0$

$\pi_1'^l(k_1^l) < 0$ $\pi_2'^l(k_2^l) > 0$
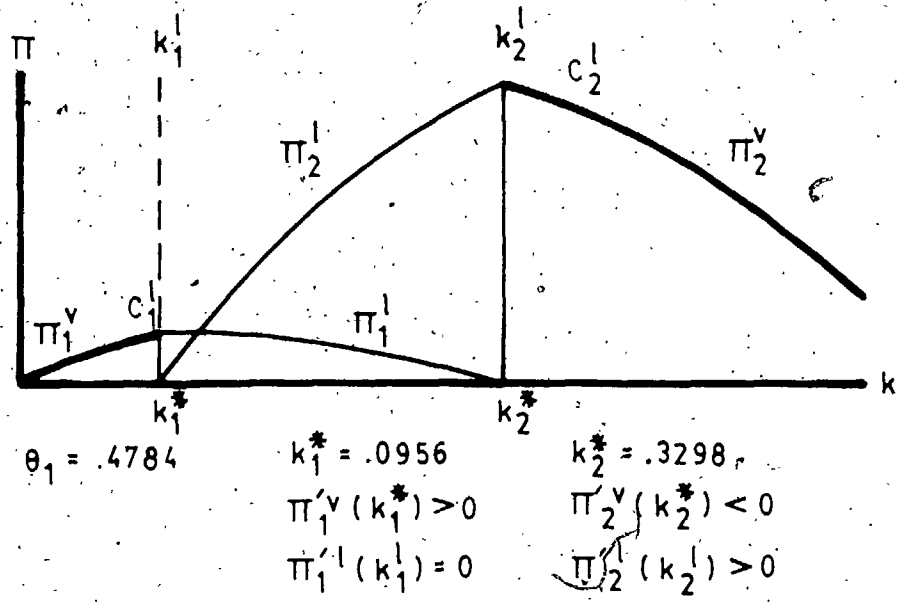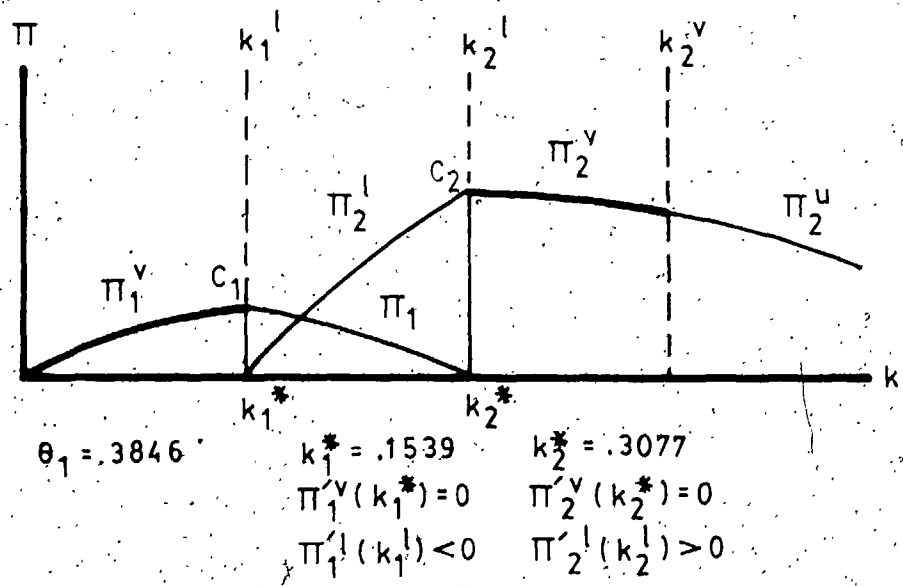
FIGURE 4.21

reaction function. Also notice that $\Pi_2^v(k_2)$ is only well specified for $k_2^\ell \leq k_2 \leq k_2^u$, where k_2^u just satisfies condition (49) evaluated at k_1^*. Above k_1^u, $\Pi_2^u(k_2)$ is the home firm's profit function defined on the upper section of the foreign firm's price reaction function, where $\Pi_2^u(k_2^u) < 0$. For $\theta_1 = .3846$, evaluating condition (49) at k_2^* shows that $k_1^u < 0$.

As the distribution widens beyond $\theta_1 = .3846$, the foreign and home firm's profit functions remain locally concave at k_1^* and k_2^* for the equilibria derived along the vertical section of the foreign firm's price reaction function, until $\theta_1 = .3333$ at points D_1 and D_2 in Figure 4.17, where condition (49) is just satisfied. The profit functions for the firms at points D_1 and D_2 are shown in Figure 4.22. Since k_2^u now coincides with k_2^*, and the slope of the home firm's profit function defined on the upper section at k_2^u is positive, the two firms would begin to compete over the upper section of the foreign firm's price reaction function. For slightly narrower distributions where $k_2^* < k_2^u$, and $\Pi_2^u(k_2^u) > 0$, the home firm's profit function $\Pi_2^v(k_2)$ is concave as far as k_2^u, and thus the equilibrium at k_2^* is well defined, at least locally.

Finally, the foreign and home firm's profit functions for the equilibria described at points E_1 and E_2 in Figure 4.17, which just satisfy condition (49) for the upper section of the foreign firm's price reaction function are shown in Figure 4.23. It is easy to see that at $\theta_1 = .3763$, where k_1^u coincides with k_1^*, and k_2^u coincides with k_2^*, neither profit function is concave as $\Pi_1^v(k_1^*) < 0$, and $\Pi_1^v(k_2^*) < 0$, causing the two firms to begin competing over the vertical section of the foreign firm's

$\theta_1 = .3333$ $k_1^* = .1667$ $k_2^* = .3333$

$\pi_1'^v(k_1^*) = 0$ $\pi_2'^v(k_2^*) = 0$

$\pi_2'^u(k_2^u) > 0$

FIGURE 4.22

$\theta_1 = .3763$ $k_1^* = .1994$ $k_2^* = .4098$

$\pi_1'^u(k_1^*) = 0$ $\pi_2'^u(k_2^*) = 0$

$\pi_1'^v(k_1^u) > 0$ $\pi_2'^v(k_2^u) > 0$
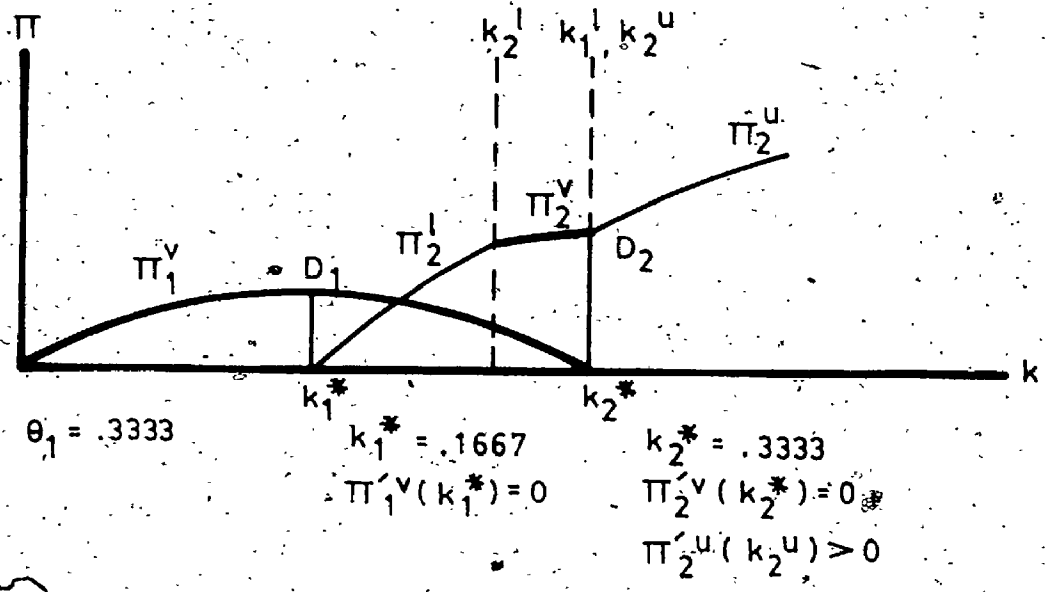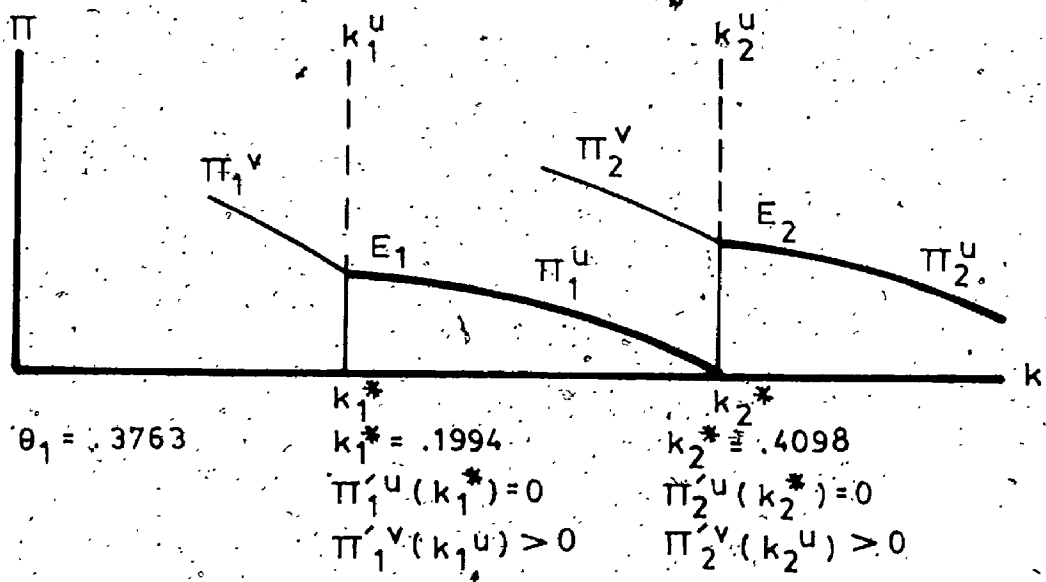
FIGURE 4.23

price reaction function. As the distribution widens, both k_1^u and k_2^u fall below k_1^* and k_2^* respectively, with the result that both $\Pi_1^u(k_1)$ and $\Pi_2^u(k_2)$ are concave over a wider region of k_1's and k_2's respectively.

Thus it is possible to show that a quality setting equilibrium exists over all distributions of willingnesses to pay for quality. From Figure 4.17, it is easy to see that while trade still causes the two firms to push their qualities apart to minimize the price competition between them, the kink in the market demand for the low quality good caused by excluding the lowest willingness to pay consumer, results in discontinuities and multiple equilibria, which greatly complicates the analysis.

Endnotes

1. Chapter 5 returns to the effects of trade on majority and minority tastes as the distribution of willingnesses to pay assumed for each country is characterized by a right triangular distribution, with one country having a majority of high willingness to pay consumers, and the other having a majority of low willingness to pay consumers. Once trade opens, however, the two triangular distributions are assumed to sum to a uniform distribution, which is the subject of this chapter.

2. Proceeding in this way requires that no arbitrage take place between the two countries. Accordingly, each firm could be thought of as holding an import licence, not available to consumers.

3. Notice that since $\hat{\theta}$ is increasing in k, and decreasing in k_2, each firm attracts consumers by producing a higher quality good for given prices. This contrasts sharply with horizontally differentiated goods models where firms attract consumers by pushing their varieties closer together. As will become clear in a moment, pushing their qualities close together intensifies the price competition between the two firms and reduces profits.

4. This is a local result evaluated at $\theta_1^h = \theta_1^f$ and $\theta_2^h \neq \theta_2^f$. While this should still hold for larger differences between the two countries, the second order conditions to ensure existence become very complicated.
   It is interesting to note that sliding both distributions along in proportion causes the qualities to increase by $\frac{1}{2}c$.

5. Notice that if the kink in the market demand curve had gone the other way, such that the marginal revenue lost from a price increase jumped up at $P_1 = \theta_1 k_1$, the result would have been a hole in the foreign firm's price reaction function, and non-existence problems.

6. The program used is a very simple Fortran based subroutine called ZSPOW, found in the IMSL software library. Implementing ZSPOW was rather straight forward using the clear examples provided in the IMSL USERS MANUAL.

7. The derivations of each firm's quality setting first order conditions, for the vertical and upper sections are a bit too tedious to reproduce even in an appendix. The analytical equations and the computer estimation of them are all readily available from the author upon request.

8. Notice that not only does each firm serve exactly half of the total number of consumers, their profit is also identical for all distributions along the lower section of the foreign firm's price reaction function.

CHAPTER 5

DEMAND DIFFERENCES, THE LINDER HYPOTHESIS, AND

TRADE IN QUALITY DIFFERENTIATED GOODS

1. Introduction

This chapter considers a rather special variation of the model of trade in quality differentiated goods developed in Chapter 4, which nicely highlights the role of demand differences in determining the pattern of trade, and its effects on individual consumers in each country.

The distribution of willingnesses to pay for quality assumed for each country is now characterized by a right triangular distribution, with the home country having a majority of high willingness to pay consumers, and the foreign country having a majority of low willingness to pay consumers. The range of willingnesses to pay for quality and the total population in each country is assumed to be the same.[1]

As a result of these differences, the equilibrium level of quality offered to consumers by a domestic monopolist in autarky, is lower in the foreign country, than in the home country, where the lowest willingness to pay consumers who are in minority, are never served. Consequently all consumers extract more net benefit in the foreign country where lower willingness to pay consumers who are in a majority warrant being served, which is analogous to the results of Chapter 3. As in Chapter 4, all consumers in each country benefit more from wider distributions in which lower willingness to pay consumers are served.

Once trade opens, the two triangular distributions are assumed to sum to a uniform distribution, and the analysis proceeds exactly as in Chapter 4. Trade causes the two firms to push their qualities even farther apart than the autarky levels found here, thereby minimizing the price competition between them.

In equilibrium, the foreign firm specializes in the production of the low quality good which it exports to low willingness to pay consumers who are in a minority in the home country, while the home firm specializes in the production of the high quality good which it exports to high willingness to pay consumers who are a minority in the foreign country.

This pattern of trade is very much in the spirit of the Linder hypothesis as each country is producing a level of quality in keeping with its representative demand, which it exports to consumers who are on the fringe of the representative demand in the other country.

In contrast to Chapter 4, however, it is shown that for the widest distribution of willingnesses to pay considered, low willingness to pay consumers who were able to extract positive net benefit as a majority from the foreign monopolist in autarky, derive less net benefit from the reduced level of quality they buy in the trading equilibrium. On the other hand, higher willingness to pay consumers who are a majority in the home country greatly benefit from the competition brought about by trade. Thus the majority of low willingness to pay consumers in the foreign country stand to lose the most from trade, while the majority of high willingness to pay consumers in the home country stand to gain the most.

As the distribution of willingnesses to pay for quality narrows, all consumers gain as a result of the competition brought about by trade. These gains are greater for all home country consumers than for foreign country consumers over all distributions, which simply reflects the fact that they were able to extract less net benefit in autarky.

The chapter is organized as follows. In Section 2, the basic model of quality differentiated goods production developed in Chapter 2 is briefly outlined. Section 3 examines the equilibrium price and quality combination offered to consumers in each country by a domestic monopolist in autarky. Section 4 briefly reviews the equilibrium that is reached once trade opens between the home and foreign economies. In Section 5, the trading equilibrium and autarky equilibria are compared to examine the pattern of trade and its effects on individual consumers in each country. Section 6 provides a brief summary and conclusions.

2. The Basic Model

In each country, production is carried out using a single factor of production, labour. Quality differentiated goods are produced by a monopolist with decreasing AC in output (i.e., fixed costs and constant MC in output) and increasing MC in quality. To allow for explicit solutions, the cost function is assumed to take the form:

1) $$C(X_j) = ck_j^2 X_j + G,$$

where X_j is the quantity of good j produced, k_j is the quality of good j,

where $k_j > k_{j-1}$, ck_j^2 is the marginal cost of producing good j, where c > 0, and G is the fixed cost required to produce a quality differentiated good.

All other production is devoted to a composite good Y in a perfectly competitive sector, which is used as numeraire. There are no cost or comparative advantage differences between the two countries.

Each country is populated by M consumers who differ in their willingness to pay for quality. All consumers in each country are assumed to have identical income composed of labour income and a per capita share of the domestic monopolist's profits. Consumers allocate their income between expenditure on the numeraire, and on one unit of a quality differentiated good, by choosing the level of quality which provides the largest net benefit to be had from consuming a quality differentiated good, leaving expenditure on the numeraire to be determined as a residual. Net benefit is defined as the difference between what a consumer is willing to pay for a quality differentiated good and its price, written as:

2) $\quad Z_i(P_i) = \theta_i k_j - P_j$,

where θ_i is a representative consumer i's valuation of a unit of quality, and thus $\theta_i k_j$ is consumer i's valuation or willingness to pay for good j. Should no quality provide consumer i with non-negative net benefit, then he would simply spend all of his income on the numeraire.

The market demand for any particular level of quality depends on both the distribution of willingnesses to pay for quality, and on the variety of price and quality combinations available.

The distribution of willingnesses to pay for quality assumed for each country is characterized by a right triangular distribution with the home country having a majority of high willingness to pay consumers and the foreign country having a majority of low willingness to pay consumers, where the range of willingnesses to pay is the same in each.

In the home country the density and cumulative density functions are given by:

3) $h(\theta) = \dfrac{2(\theta - \theta_1)}{(\theta_2 - \theta_1)^2}$; $h(\theta_2) = \dfrac{2}{(\theta_2 - \theta_1)}$, $h(\theta_1) = 0$

$$H(\theta) = \frac{(\theta_2 - 2\theta_1 + \theta)(\theta_2 - \theta)}{(\theta_2 - \theta_1)^2} \; ; \quad H(\theta_2) = 0, \quad H(\theta_1) = 1$$

where $\theta_2 > \theta_1$, is the highest and lowest valuations of a unit of quality in either country respectively. In the foreign country, the density and cumulative density functions are given by:

4) $f(\theta) = \dfrac{2(\theta_2 - \theta)}{(\theta_2 - \theta_1)^2}$, $f(\theta_2) = 0$, $f(\theta_1) = \dfrac{2}{(\theta_2 - \theta_1)}$

$F(\theta) = \dfrac{(\theta_2 - \theta)^2}{(\theta_2 - \theta_1)^2}$, $F(\theta_2) = 0$, $F(\theta_1) = 1$
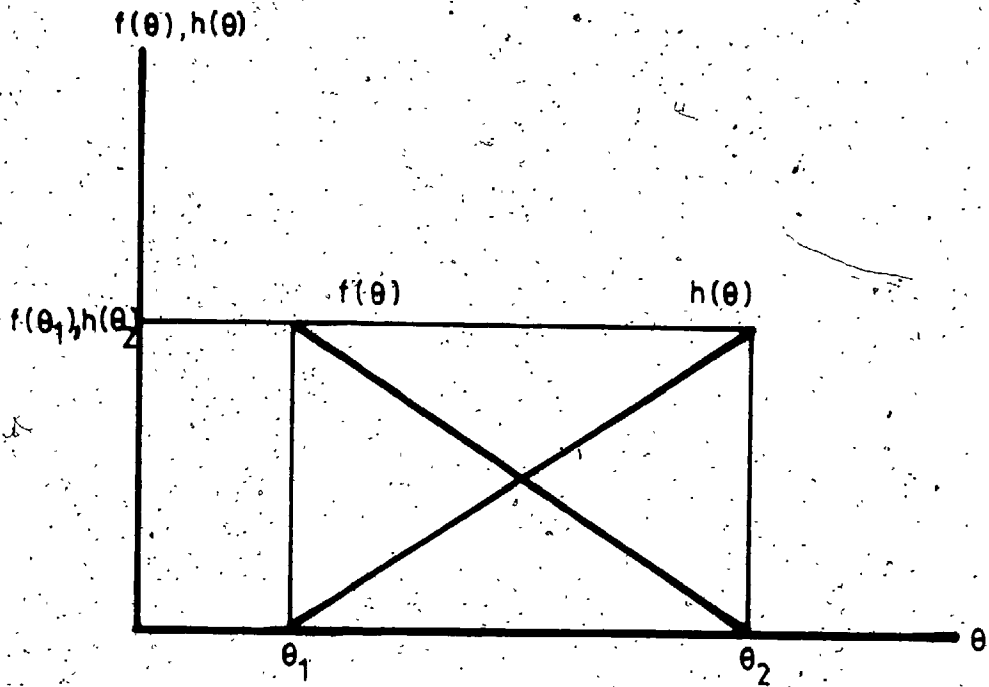
FIGURE 5.1
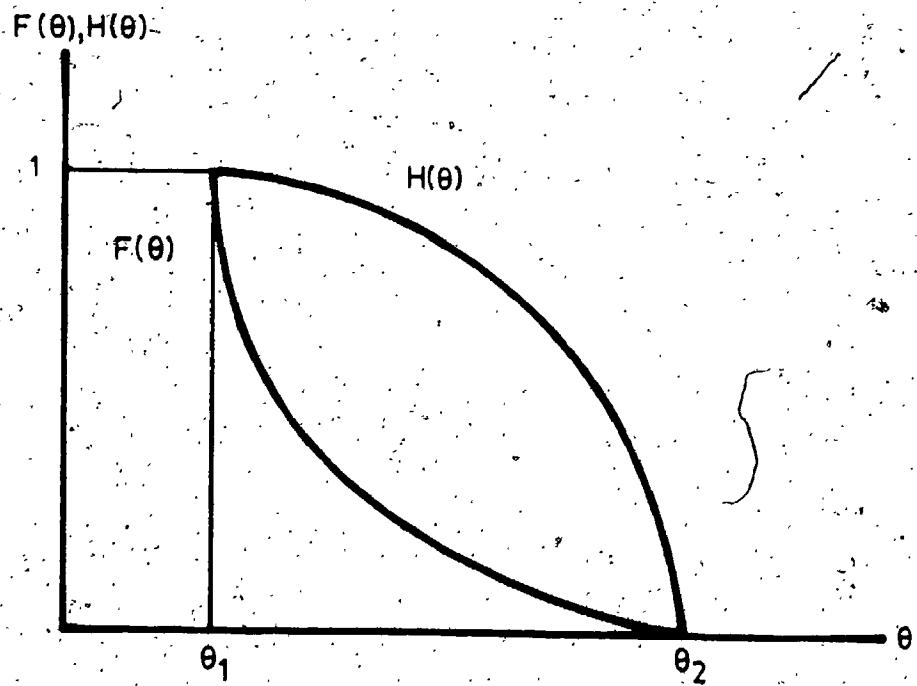
FIGURE 5.2

No other differences may arise between the two countries. The density and cumulative functions for the home and foreign countries are shown in Figures 5.1 and 5.2 respectively.

3. Autarky Equilibrium

Having described consumer behaviour and the costs of production, the monopolist's optimal price and quality selection problem can now be solved for each country in autarky. For convenience, denote k_1 and k_2, as the qualities produced by the foreign and home monopolist's respectively.

3.1 Foreign Country

The monopolist's problem is solved as a two stage game. In the first stage an optimal pricing rule is derived for a given level of quality. The optimal pricing rule is then used in the second stage to re-solve the firm's problem for the optimal level of quality. Fixed costs are assumed sufficiently high to prevent the monopolist from producing a second level of quality.

Recalling the consumers' decision rule (2) and the cumulative density function from (4), market demand for a single level of quality in the foreign country is given by:

$$5) \quad D^f(P) = \begin{cases} \dfrac{[\theta_2 k_1 - P_1]^2 M}{(\theta_2 - \theta_1)^2 k_1^2} & \theta_2 k_1 \geq P_1 \geq \theta_1 k_1 \\ \\ M & \theta_1 k_1 \geq P_1 \end{cases}$$

Using the demand function (5) and the cost function (1), the pricing stage of the foreign monopolist's problem for a given level of quality is written as:

6) $\quad \underset{P_1}{\text{Max}} \quad \Pi(P_1) = \dfrac{[P_1 - ck_1^2][\theta_2 k_1 - P_1]^2 M}{(\theta_2 - \theta_1)^2 k_1^2} - G$

Differentiating (6) w.r.t. P_1 and solving gives the foreign firm's optimal pricing rule:

7) $\quad P_1^* = \dfrac{\theta_2 k_1 + 2ck_1^2}{3}$

where P_1^* just balances the marginal revenue lost from a price increase with the savings in marginal cost. The profit function is concave in P_1 for $P_1 \leq \dfrac{2\theta_2 k_1 + ck_1^2}{3}$ which is satisfied at P_1^*. The equilibrium (7) is shown as the dashed line falling between the highest willingness to pay consumers willingness to pay ray, and the marginal cost schedule in Figure 5.3. It is implicitly assumed that the distribution of willingness to pay is sufficiently wide such that some low willingness to pay consumers are excluded.

The second part of the foreign monopolist's problem is solved by substituting (7) back into the profit function (6), and differentiating w.r.t. k_1 which gives:

8) $\quad k_1^* = \dfrac{\theta_2}{4c}$
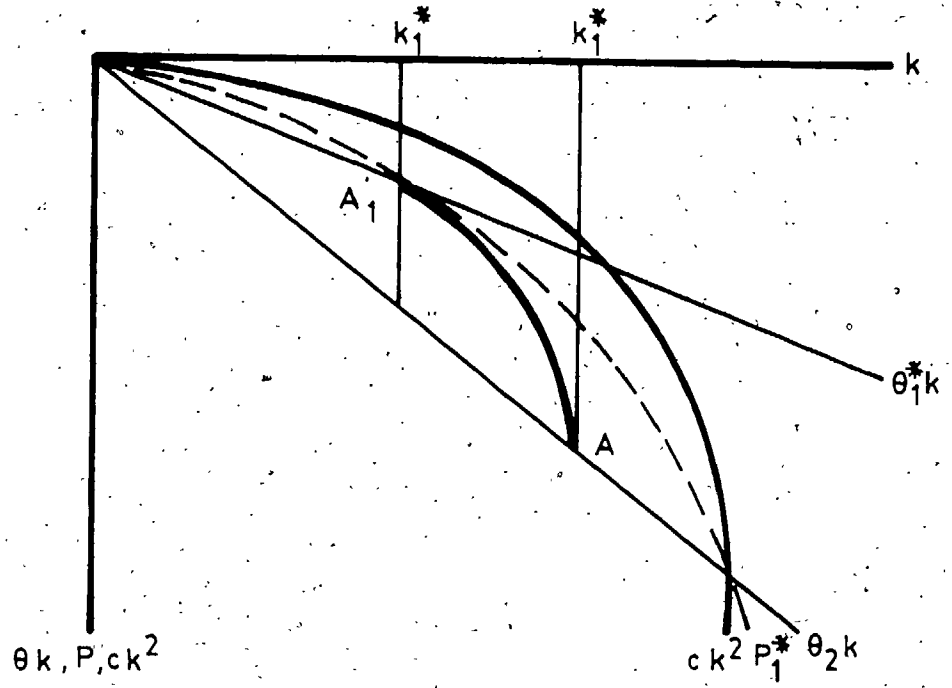
FIGURE 5.3
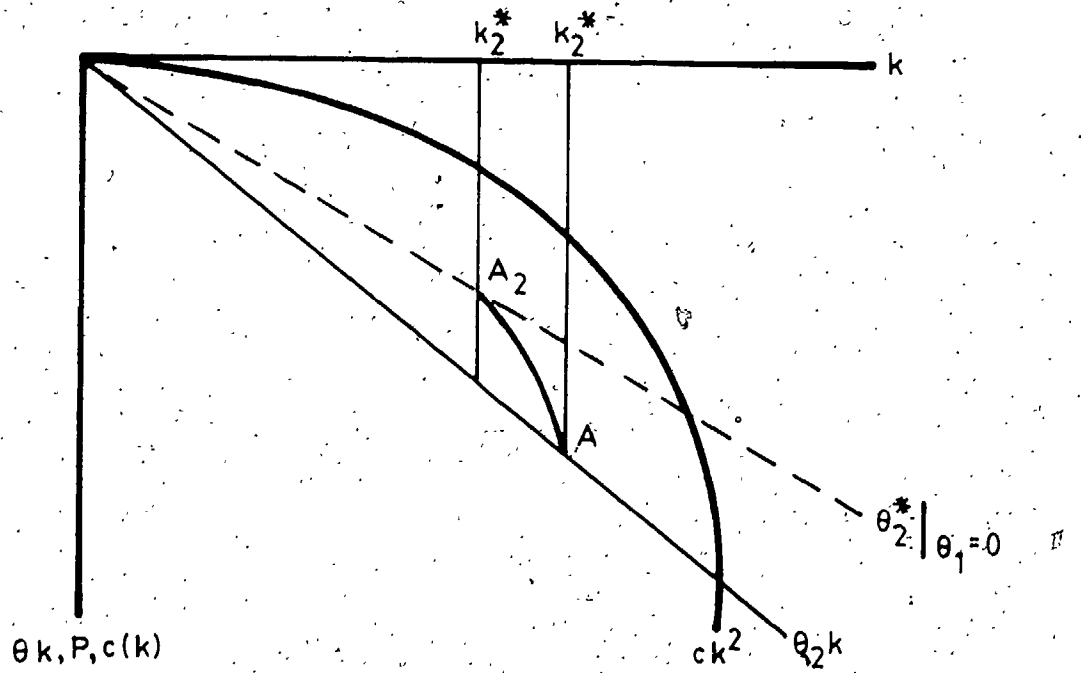
FIGURE 5.4

The profit is concave for $k_1 \leq \frac{\theta_2}{2c}$ which ensures that k_1^* is a maximum.

The equilibrium price charged for k_1^* is found by substituting (8) into the optimal pricing rule (7) which gives:

9) $\qquad P_1^* = \frac{(\theta_2)^2}{8c}$

If θ_1^* identifies the foreign country consumer who is just served in autarky such that $\theta^* k_1^* - P_1^* = 0$, then using (8) and (9), θ_1^* is given by:

10) $\qquad \theta_1^* = \frac{\theta_2}{2}$

Thus assuming that the distribution of willingnesses to pay is sufficiently wide such that $\theta_1 \leq \theta_1^*$, then conditions (8), (9) and (10) describe an autarky equilibrium in the foreign country where $[\theta_1^*, \theta_2]$ is the set of consumers who buy k_1^* at price P_1^*, while $[\theta_1, \theta_1^*)$ is the set of consumers who are not served. The equilibrium is shown at point A_1 in Figure 5.3. Notice that as θ_1^* is the marginal consumer who is just served in autarky, $k_1^* = \frac{\theta_2}{4c} = \frac{\theta_1^*}{2c}$, maximizes the difference between the marginal consumers willingness to pay ray, and the marginal cost schedule. Thus in equilibrium, quality is chosen as though all consumers were the marginal consumer.

For narrower distributions where $\theta_1 > \theta_1^*$, the equilibrium price and quality combination offered at point A_1 in Figure 5.3 is clearly not optimal, as profits could be increased by raising price to $\theta_1 k_1^*$, and further increased by altering quality.

When serving the lowest willingness to pay consumer, the pricing rule is simply given by:

11) $\quad P_1^* = \theta_1 k_1$

Since the pricing rule constrains P_1^* to fall along the lowest willingness to pay consumers willingness to pay ray, the optimal level of quality maximizes the difference between his willingness to pay ray and the marginal cost schedule giving:

12) $\quad k_1^* = \dfrac{\theta_1}{2c}$

When all consumers are served, the profit function is strictly concave in k_1 ensuring that k_1^* is a maximum.

From (11) it is easy to see that the equilibrium price charged for k_1^* is:

13) $\quad P_1^* = \dfrac{(\theta_1)^2}{2c}$

Thus conditions (12) and (13) describe an autarky equilibrium in which all consumers in the foreign country by k_1^* at price P_1^*. The locus of equilibrium price and quality combinations is shown as the heavy line between points A_1 and A in Figure 5.3.

The relationship between the equilibrium level of quality produced in the foreign country in autarky and the distribution of willingnesses to pay for quality is summarized in Figure 5.5. Thus for $\theta_1 \leq \theta_1^*$, quality is
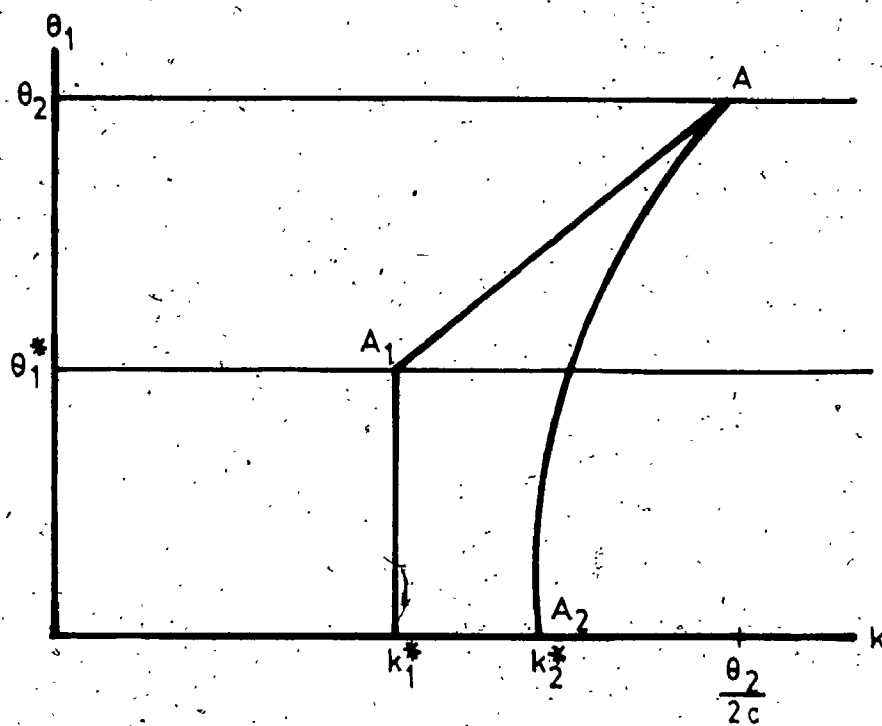
FIGURE 5.5
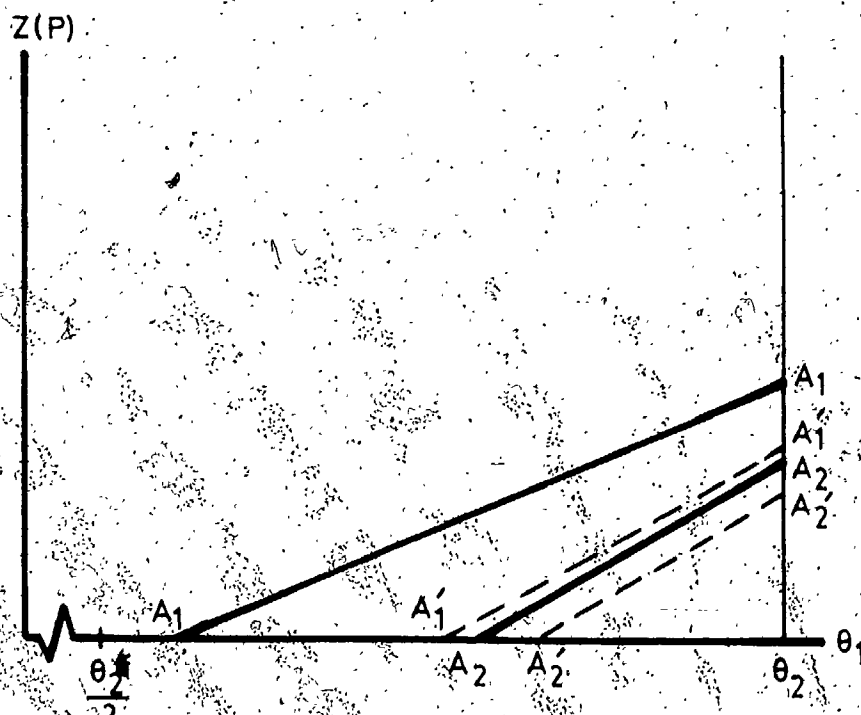
FIGURE 5.6

fixed at $k_1^* = \dfrac{\theta_2}{4c}$. For $\theta_1 \geq \theta_1^*$, k_1^* converges to $k^* = \dfrac{\theta_2}{2c}$ at point A.

Using the equilibrium conditions for price and quality, and the definition of net benefit (2), the welfare implications of the autarky equilibrium for foreign country consumers are summarized as:

$$14) \qquad Z(P_1^*) = \begin{cases} \dfrac{\theta_2(2\theta - \theta_2)}{8c} & \theta_1^* \leq \theta \leq \theta_2 \\[4mm] 0 & \theta \leq \theta_1^* \end{cases} \left. \right\} \quad \theta_1 \leq \theta_1^* \\[1cm] \dfrac{\theta_1(\theta - \theta_1)}{2c} \qquad\qquad\qquad\qquad \theta_1 \geq \theta_1^*$$

These effects are also summarized in Figure 5.6, where the solid line labelled A_1A_1, represents the net benefit derived by all consumers for $\theta_1 = \dfrac{5}{9}\,\theta_2$, and the dashed line labelled $A_1'A_1'$ represents the net benefit derived by consumers for $\theta_1 = \dfrac{3}{4}\,\theta_2$. Thus it is easy to see that all consumers derive more net benefit in wider distributions where the lowest willingness to pay consumer is served.

It is worth noting that because of their majority, lower willingness to pay consumers are served over a wider range of distributions than was the case for the uniform distribution. All consumers are correspondingly better off over the wider range of distributions where lower willingness to pay consumers warrant being served.

3.2 Home Country

The home monopolist's optimal price and quality selection problem is similarly solved as a two stage game.

Recalling the consumers' decision rule (2) and the cumulative density function from (3), market demand for a single level of quality in the home country is given by:

$$
15) \quad D^h(P_2) = \begin{cases} \dfrac{[\theta_2 k_2 - 2\theta_1 k_2 + P_2][\theta_2 k_2 - P_2]M}{(\theta_2 - \theta_1)^2 k_2^2} & \theta_2 k_2 \geq P_2 \geq \theta_1 k_2 \\[4mm] M & \theta_1 k_2 \geq P_2 \end{cases}
$$

Using the demand function (15) and the cost function (1), the pricing stage of the home monopolist's problem for a given level of quality is written as:

$$
16) \quad \underset{P_2}{\text{Max}} \quad \Pi(P_2) = \frac{[P_2 - ck_2^2][\theta_2 k_2 - 2\theta_1 k_2 + P_2][\theta_2 k_2 - P_2]M}{(\theta_2 - \theta_1)^2 k_2^2} - G
$$

Differentiating (16) w.r.t. P_2 and solving yields the optimal pricing rule:

$$
17) \quad P_2^* = \frac{k_2}{3} \{2\theta_1 + ck_2 + [3\theta_2^2 - 6\theta_2\theta_1 + 4\theta_1^2 - 2\theta_1 ck_2 + c^2 k_2^2]^{\frac{1}{2}} \}
$$

The profit function is concave in P_2 for $P_2 \geq \dfrac{2\theta_1 k_2 + ck_2^2}{3}$ which is satisfied at P_2^*.

It is readily apparent from (17) that there is no simple pricing rule independent of θ_1 as was the case in the foreign country. This follows because there is no clear cut-off point where lower willingness to pay consumers are served, as it never pays the home monopolist to serve the lowest willingness to pay consumers who are in a minority.

Rather than trying to substitute (17) back into the profit function and differentiating to solve for k_2^*, recall that in equilibrium, k_2^* maximizes the difference between the marginal consumer's willingness to pay ray and the marginal cost schedule. If θ_2^* identifies the home country consumer who is just served in autarky where $\theta_2^* k_2^* = P_2^*$, then k_2^* is given by:

$$18) \quad k_2^* = \frac{\theta_2^*}{2c}$$

Substituting into $\theta_2^* k_2^* - P_2^* = 0$ from (18) and the pricing rule (17), and solving identifies the marginal consumer as:

$$19) \quad \theta_2^* = \frac{3\theta_1 + [8\theta_2^2 - 16\theta_2\theta_1 + 9\theta_1^2]^{\frac{1}{2}}}{4}$$

The equilibrium price charged for k_2^* is simply:

$$20) \quad P_2^* = \frac{(\theta_2^*)^2}{2c}$$

Thus conditions (18), (19) and (20), describe the autarky equilibrium in the home country where $[\theta_2^*, \theta_2]$ is the set of consumers

who buy k_2^* at price P_2^*, while $[\theta_1, \theta_2^*)$ is the set of consumers who are not served. The equilibrium locus is shown between points A_2 and A in Figure 5.4, where point A_2 is the equilibrium price and quality pair for $\theta_1 = 0$.

The relationship between the equilibrium level of quality produced in the home country in autarky, and the distribution of willingnesses to pay for quality, is summarized in Figure 5.5, between points A_2 and A. At point A_2 where $\theta_1 = 0$, $k_2^* = \frac{\sqrt{2}}{4} \frac{\theta_2}{c}$, which is greater than k_1^*. Thus in the Linder sense, each economy is producing a level of quality in keeping with its representative demand.

Using the equilibrium conditions and the definition of net benefit, the welfare implications of the autarky equilibrium for consumers in the home country are summarized as:

$$21) \qquad Z(P_2^*) = \begin{cases} \dfrac{\theta_2^*(\theta - \theta_2^*)}{2c} & \theta_2^* \le \theta \le \theta_2 \\[3ex] 0 & \theta \le \theta_2^* \end{cases}$$

These effects are also summarized in Figure 5.6. The solid line labelled A_2A_2 represents the net benefit derived by home country consumers for $\theta_1 = \frac{5}{9}\theta_2$, and the dashed line labelled $A_2'A_2'$ represents the net benefit derived by consumers for $\theta_1 = \frac{3}{4}\theta_2$. As before consumers clearly benefit from wider distributions where the lowest willingness to pay consumer is served.

The benefit derived by all consumers from having a significant population of low willingness to pay consumers is clearly evident from Figure 5.6, where foreign country consumers are clearly able to extract more net benefit from their monopolist than in the home country where low willingness to pay consumers are in a minority.

4. Trading Equilibrium

Once trade opens, the two monopolists are assumed to compete as duopolists over price and quality for consumers in each country. As the two triangular distributions are assumed to sum to a uniform distribution, the analysis follows directly from Chapter 4.

The equilibrium is solved as a two stage game. In the first stage a Bertrand-Nash pricing equilibrium is derived for given (autarky) levels of quality. In the second stage, the equilibrium pricing rules are used to solve for the quality setting equilibrium assuming that each firm conjectures that the quality of the other firm is held fixed.

In the overall trading equilibrium outlined here, the distribution of willingnesses to pay for quality is assumed to be sufficiently narrow such that in equilibrium all consumers in each country buy a quality differentiated good. The complications which arise for wider distributions are examined in the Appendix to Chapter 4.

Consumers must now choose between the foreign quality k_1 at price P_1, and the home quality k_2 at price P_2. Recalling from (2) that consumers choose the level of quality providing the largest non-negative net benefit, let $\hat{\theta}$ identify the consumer who is just indifferent between k_1 and k_2 such

that $\hat{\theta} k_1 - P_1 = \hat{\theta} k_2 - P_2$, which gives:

22) $\qquad \hat{\theta} = \dfrac{P_2 - P_1}{k_2 - k_1}$

Since all consumers buy either k_1 or k_2, the market demand functions for each are written as:

23) $\qquad D(P_1) = \dfrac{(\hat{\theta} - \theta_1)2M}{(\theta_2 - \theta_1)}$

24) $\qquad D(P_2) = \dfrac{(\theta_2 - \hat{\theta})2M}{(\theta_2 - \theta_1)}$

Notice that each firm attracts consumers by increasing its quality, for given prices, which is in sharp contrast to horizontally differentiated goods models where firms attract consumers by pushing their varieties closer together.

Substituting $\hat{\theta}$ from (22) into (23) and (24), and using the cost function (1), the first part of the foreign and home firms' problem is written as:

25) $\qquad \underset{P_1}{\text{Max}}\ \Pi(P_1) = \dfrac{[P_1 - ck_1^2][P_2 - P_1 - \theta_1(k_2 - k_1)]2M}{(\theta_2 - \theta_1)(k_2 - k_1)} - G$

26) $\qquad \underset{P_2}{\text{Max}}\ \Pi(P_2) = \dfrac{[P_2 - ck_2^2][\theta_2(k_2 - k_1) - P_2 + P_1]2M}{(\theta_2 - \theta_1)(k_2 - k_1)} - G$

Differentiating (25) w.r.t. P_1 assuming P_2 is held fixed and differentiating (26) w.r.t. P_2 assuming P_1 is held fixed and solving, gives the equilibrium prices:

27.) $$P_1^* = \frac{1}{3}[(\theta_2 - 2\theta_1)(k_2 - k_1) + ck_2^2 + 2ck_1^2]$$

28) $$P_2^* = \frac{1}{3}[(2\theta_2 - \theta_1)(k_2 - k_1) + 2ck_2^2 + ck_1^2]$$

Notice from (27) and (28), that the closer the two qualities are together, the more severe the Bertrand-Nash price competition between them becomes. Should $k_1 = k_2$, then each firm would be forced to price at marginal cost, which is the standard Bertrand-Nash result.

The quality setting equilibrium is found by first substituting the equilibrium pricing rules (27) and (28) into the foreign and home firm's profit functions (25) and (26) respectively. Then differentiating the foreign firm's problem w.r.t. k_1 assuming k_2 is held fixed, and differentiating the home firm's problem w.r.t. k_2 assuming k_1 is held fixed and solving gives the equilibrium levels of quality:

29) $$k_1^* = \frac{5\theta_1 - \theta_2}{8c}$$

30) $$k_2^* = \frac{5\theta_2 - \theta_1}{8c}$$

The equilibrium prices charged for k_1^* and k_2^* by the foreign and home firms respectively are found by substituting (29) and (30) into the optimal pricing rules (27) and (28) giving:

31) $\quad P_1^* = \frac{1}{64c}[25\theta_2^2 - 58\theta_2\theta_1 + 49\theta_1^2]$

32) $\quad P_2^* = \frac{1}{64c}[49\theta_2^2 - 58\theta_2\theta_1 + 25\theta_1^2]$

Thus conditions (29), (30) and (31), (32) describe the equilibrium qualities and prices that result once trade opens between the home and foreign economies.

The relationship between the equilibrium levels of quality produced by the two firms and the distribution of willingnesses to pay is summarized in Figure 5.7, where (29) and (30) are represented by the heavy lines. It is easy to see that over all distributions, trade causes the two firms to push their qualities even farther apart than they were in autarky to reduce the severe Bertrand-Nash price competition between them. Notice that as the distribution of willingnesses to pay narrows, the equilibrium levels of quality converge, as the scope for firms to differentiate their qualities and avoid price competition is reduced. Also notice that $\theta^* = \frac{5}{9}\theta_2$, is the widest possible distribution considered in which the lowest willingness to pay consumer is served. Wider distributions involve different demand functions and profit functions as was examined in the Appendix to Chapter 4.

In the trading equilibrium described by conditions (29), (30) and (31), (32), $\frac{(\hat{\theta} - \theta_1)2M}{(\theta_2 - \theta_1)}$ is the share of consumers buying k_1^* at price P_1^* from the foreign firm, while $\frac{(\theta_2 - \hat{\theta})2M}{(\theta_2 - \theta_1)}$ is the share of consumers buying
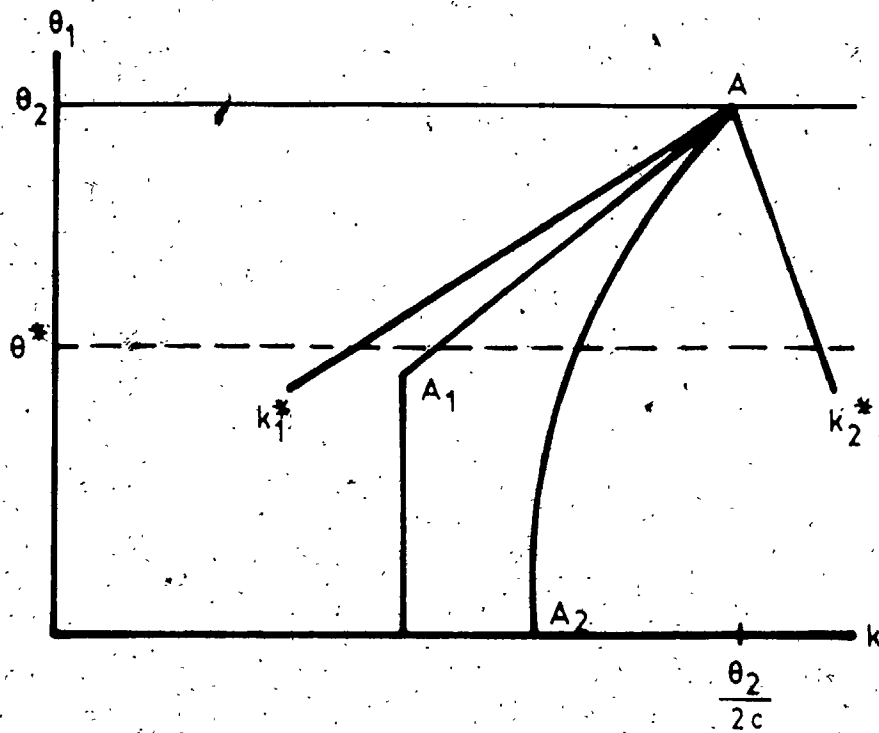
FIGURE 5.7
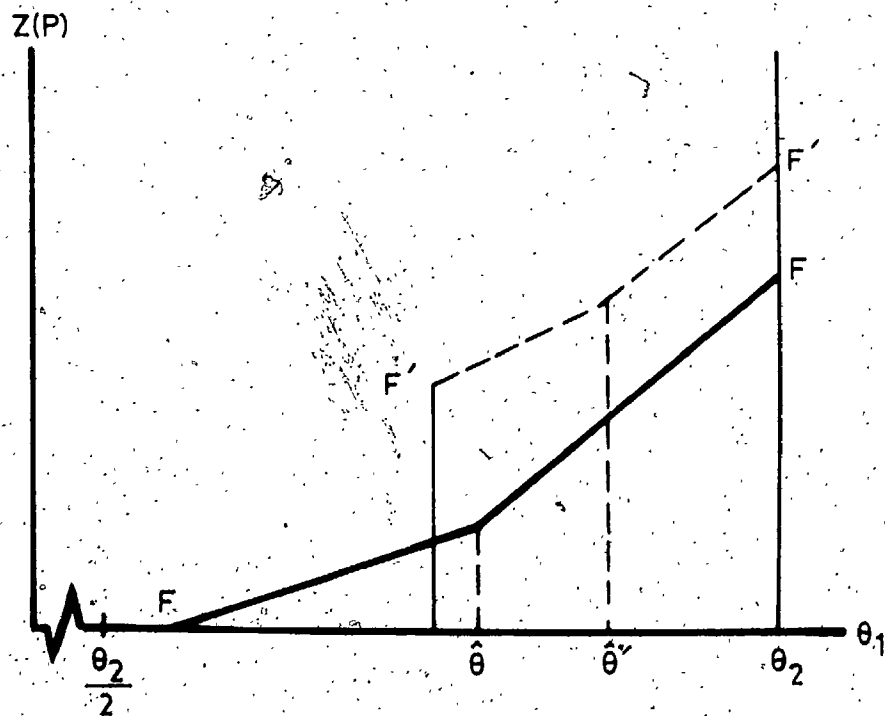
FIGURE 5.8

k_2^* at price P_2^* from the home firm. Substituting the equilibrium conditions into (22) for $\hat{\theta}$ gives:

33) $\quad \hat{\theta} = \dfrac{\theta_2 + \theta_1}{2}$,

which says that in equilibrium each firm serves exactly half of the total number of consumers.

The net benefit derived by lower willingness to pay consumers from consuming k_1^* at price P_1^* is given by:

34) $\quad Z(P_1^*) = \dfrac{\theta(5\theta_1 - \theta_2)}{8c} - \dfrac{[25\theta_2^2 - 58\theta_2\theta_1 + 49\theta_1^2]}{64c}$,

where $\theta \in [\theta_1, \hat{\theta})$. Similarly, the net benefit derived by higher willingness to pay consumers from consuming k_2^* at price P_2^* is given by:

35) $\quad Z(P_2^*) = \dfrac{(5\theta_2 - \theta_1)}{8c} - \dfrac{[49\theta_2^2 - 58\theta_2\theta_1 + 25\theta_1^2]}{64c}$

where $\theta \in [\hat{\theta}, \theta_2]$.

The relationship between consumer welfare and the distribution of willingnesses to pay for quality is easily seen by representing (34) and (35) in terms of a diagram as shown in Figure 5.8. For the widest possible distribution where $\theta_1 = \theta^*$, the lowest willingness to pay consumer is just indifferent between consuming k_1^*, and the numeraire, while all higher willingness to pay consumers derive positive net benefit shown along the line labelled FF. As the distribution of willingnesses to pay narrows,

the equilibrium levels of quality are drawn closer together, which heightens the price competition and thereby increases the net benefit derived by all consumers as shown along the dashed line labelled F'F'.

Thus narrower distributions benefit all consumers more as the scope for firms to differentiate their qualities and avoid price competition is reduced.

5.   Autarky vs Trade

Having described the autarky equilibrium for each country in Section 3, and the trading equilibrium that was reached between them in Section 4, the role of demand differences in determining the pattern of trade and its effects on individual consumers in each country are now easily examined.

It is clear from Figure 5.7, that the competition brought about by trade causes the two firms to push their qualities even farther apart than they were in autarky. The foreign firm specializes in the production of the low quality good which it exports to low willingness to pay consumers who are a minority in the home country, while the home firm specializes in the production of the high quality good which it exports to high willingness to pay consumers who are a minority in the foreign country.[2]

The pattern of trade which emerges is very much in the spirit of the Linder hypothesis as each country is producing a level of quality in keeping with its representative demand, which it exports to consumers who are on the fringe of the representative demand in the other country.

The effects of trade on individual consumers in each country are summarized in Figures 5.9 and 5.10. In Figure 5.9, the solid line labelled FF, represents the net benefit derived by all consumers in the trading equilibrium for the widest distribution of willingnesses to pay considered, while the solid lines labelled A_1A_1; and A_2A_2 represent the net benefit derived by foreign country consumers and home country consumers in autarky for the same distribution. The dashed lines labelled $F'F'$, $A_1'A_1'$, and $A_2'A_2'$, represent the net benefit derived by; all consumers in the trading equilibrium, foreign country consumers in autarky, and home country consumers in autarky, respectively, for a narrower distribution of willingnesses to pay for quality.

While it is clear that the net benefit derived by all consumers in the trading equilibrium increases relative to autarky as the distribution of willingnesses to pay narrows, the effects on individual consumers in each country are more easily seen in Figure 5.10, which represents the difference between the net benefit derived by consumers in the trading equilibrium and in autarky. The solid lines labelled $F-A_1$, and $F-A_2$ represent the difference in net benefit derived by consumers in the trading equilibrium and autarky for the foreign and home countries respectively, for the widest distribution considered. The dashed lines labelled $F'-A_1'$, and $F'-A_2'$ represent the difference in net benefit derived by consumers in the trading equilibrium over autarky in the foreign and home countries for a narrower distribution.

Notice first, that for the widest distribution of willingnesses to pay considered, low willingness to pay consumers who were able to
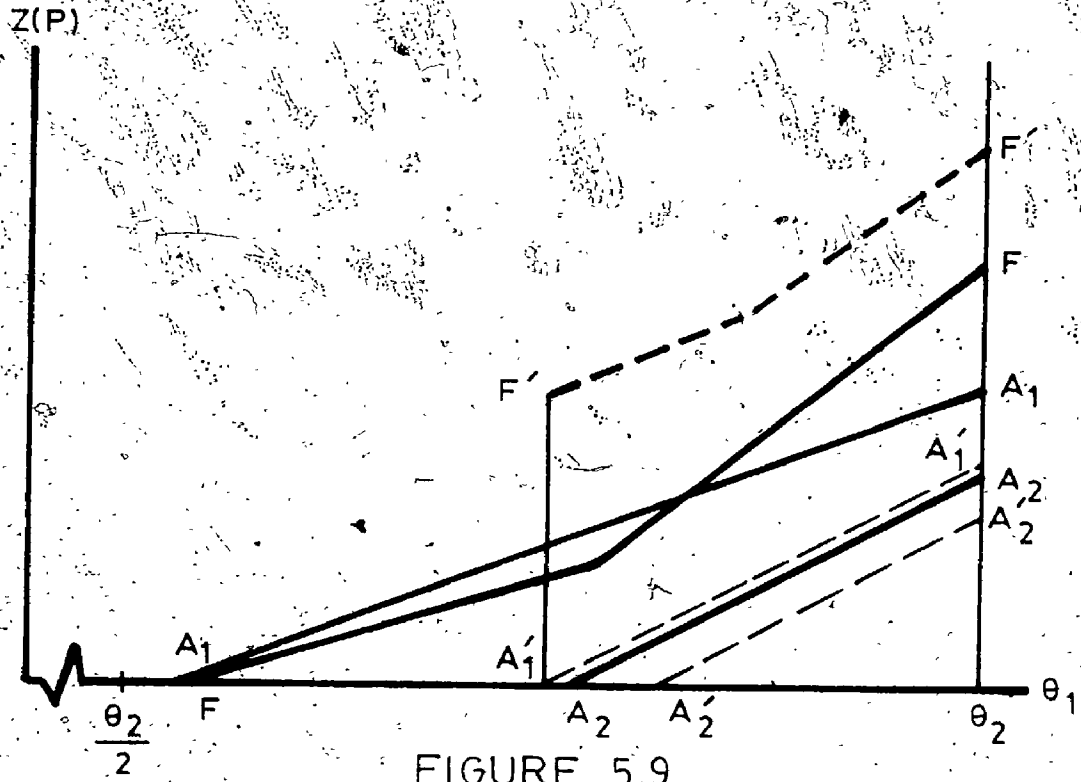
FIGURE 5.9
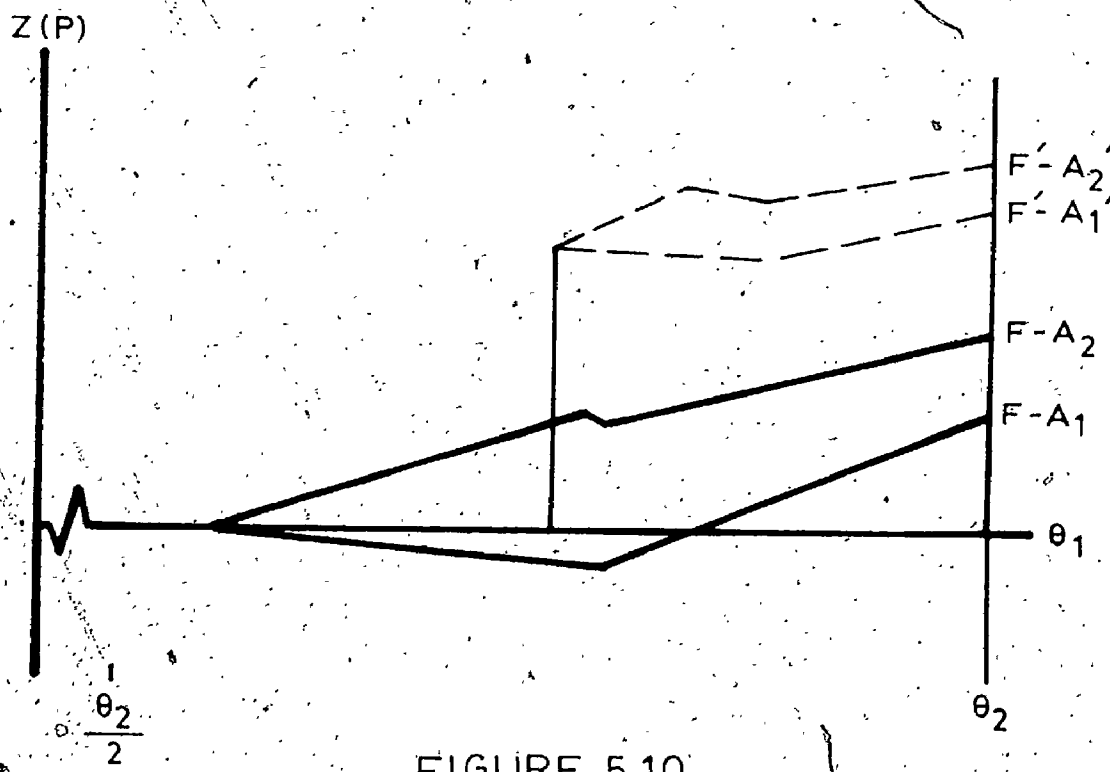
FIGURE 5.10

extract positive net benefit as a majority from the foreign monopolist
in autarky, derive less net benefit from the reduced level of quality
they consume in the trading equilibrium. On the other hand, higher
willingness to pay consumers who were not able to extract as much net
benefit as a majority from the home monopolist in autarky, greatly
benefit from the addition of low willingness to pay consumers, and the
price competition brought about by trade. Thus it would seem that the
majority of low willingness to pay consumers in the foreign country stand
to lose the most from trade, while the majority of high willingness to
pay consumers in the home country stand to gain the most from trade.

For narrower distributions, all consumers gain as a result of
the competition brought about by trade. These gains are greater for
all home country consumers than for foreign country consumers, over all
distributions, which simply reflects the fact that they were able to
extract less net benefit in autarky.

6.   Summary and Conclusions

This chapter has considered a rather special variation of the
model of trade in quality differentiated goods developed in Chapter 4,
which nicely highlights the role of demand differences in determining
the pattern of trade, and its effects on individual consumers in each
country.

The distribution of willingnesses to pay for quality assumed for
each country is characterized by a right triangular distribution with
the home country having a majority of high willingness to pay consumers,
and the foreign country having a majority of low willingness to pay consumers.

It is not surprising that in autarky, these differences cause the equilibrium level of quality produced in the home country to be greater than in the foreign country. As a result, all consumers in the foreign country are able to extract more net benefit from the monopolist, where lower willingness to pay consumers are a majority and warrant being served.

Once trade opens, the two monopolists are assumed to compete as duopolists over price and quality for consumers in each country. As the two triangular distributions are assumed to sum to a uniform distribution the competition between the two firms modelled exactly as in Chapter 4. The competition brought about by trade causes the two firms to push their qualities even farther apart than the autarky levels found here, thereby reducing the price competition between them.

The pattern of trade which emerges is very much in the spirit of the Linder hypothesis as each country is producing a level of quality in keeping with its representative demand, which it exports to consumers who are on the fringe of the representative demand in the other country.

While trade may actually harm lower willingness to pay consumers who are a majority in the foreign country, all consumers in each country are shown to gain from trade in narrower distributions where the scope for firms to differentiate their qualities and avoid price competition is reduced. These gains are shown to be greater for all home country consumers than foreign country consumers over all distributions, which simply reflects the fact that they were able to extract less net benefit in autarky.

Endnotes

1. Dinopoulos (1984) also uses triangular distributions to explore the effects of demand differences in a model of trade in horizontally differentiated goods.

2. A very small transportation cost is sufficient to show that the foreign firm would specialize in the production of the low quality good, and the home firm in the production of a high quality good.

CHAPTER 6

QUALITY UPGRADING OR QUALITY DIFFERENTIATION:

THE EFFECTS OF INTERNATIONAL TRADE

1.   Introduction

This chapter reexamines the model of trade in quality differentiated goods developed in Chapter 4, under the alternative production technology that the burden of quality improvement falls on fixed costs such as R & D expenditure, rather than on marginal costs as assumed to this point.

This provides a very useful comparison between the effects of trade modelled here, and those found by Shaked and Sutton (1984). There they show using a similar demand structure that with zero marginal costs, the price competition between firms puts severe restrictions on the number of firms that can survive in equilibrium, which depend on the distribution of willingnesses to pay for quality, but are independent of the population size. For the narrowest distributions only one firm survives, while over wider distributions exactly two firms survive, which they refer to as a 'natural oligopoly'. While population size has no effect on the number of firms, it has an important effect on quality, as larger populations allow firms to spread the fixed costs of quality improvements over a larger number of consumers, with the result that the level of quality produced increases. Beginning in autarky with either a monopoly for narrower distributions, or a duopoly for wider distributions, opening up trade between two such economies is modelled as an increase in population size which gives the quality upgrading effects of trade.

In the model developed here, the equilibrium level of quality offered to consumers in each country by a domestic monopolist in autarky is shown to depend not only on the distribution of willingnesses to pay for quality, but also on the total population of consumers as in Shaked and Sutton.

Once trade opens, the two monopolists are assumed to compete as duopolists over price and quality for all consumers in each country. As with Shaked and Sutton, the Bertrand-Nash price competition in conjunction with a zero marginal cost assumption results in only one of the firms surviving for narrower distributions. For wider distributions a trading equilibrium is reached though some low willingness to pay consumers may not be served for the widest distributions, which is analogous to the results obtained in the appendix to Chapter 4.

In comparison to autarky, it is clear that for narrower distributions where only one of the firms survives, trade has the effect of upgrading the level of quality produced by the remaining firm which serves both economies. This is identical to the Shaked and Sutton results.

For wider distributions where a trading equilibrium exists, the two firms differentiate their qualities to minimize the price competition between them, as was the case in Chapter 4. This result is reinforced over the narrower distributions, where the price competition causes the share of consumers buying from each firm to be highly unequal. Thus for the majority of consumers, trade has the effect of upgrading the level of quality that they buy from the home firm. As the distributions widens, however, each firm serves roughly the same number of consumers in the

trading equilibrium as it did as a monopolist in autarky, with the result that no quality upgrading takes place.

Thus in contrast to the Shaked and Sutton results, it is clear that for economies where set up costs or entry fees are sufficiently high such that only one firm is able to operate in autarky, the competition brought about by trade may well result more in quality differentiation than quality upgrading. This is especially true if the share of consumers served by each firm in the trading equilibrium is roughly the same as it was in autarky.

The chapter is organized as follows. Section 2 briefly outlines the production technology and consumer behaviour, and then examines the equilibrium price and quality combination offered to consumers by a monopolist for one of the economies in autarky. Section 3 first looks at the equilibrium that is reached once trade opens between the home and foreign economies, for narrower distributions where only one of the firms survives. The analysis then turns to wider distributions where a trading equilibrium between the two firms is shown to exist. Section 4 closes with a brief summary and conclusions.

2. Autarky Equilibrium

This section briefly outlines the production technology and consumer behaviour for an economy producing quality differentiated goods where the burden of quality improvement falls on fixed costs. The equilibrium price and quality combination offered to consumers by a domestic monopolist is examined for one of the economies in autarky.

2.1   Production Technology and Consumer Behaviour

In each country, production is carried out in two sectors using a single factor of production, labour. Quality differentiated goods are produced by a monopolist with decreasing average cost in output (fixed costs and zero marginal costs), where fixed costs are increasing in quality. To allow for explicit solutions, the cost function is assumed to take the form:

$$1) \qquad C(X_j) = gk_j^2 + G$$

where X_j is the quantity of good j produced, k_j is the quality of good j, where $k_j > k_{j-1}$, gk_j^2 is the component of fixed costs which is increasing in quality where $g > 0$, and G is the fixed cost independent of quality required to produce a quality differentiated good.

All other production is devoted to a composite good Y in a perfectly competitive sector, which is used as numeraire. There are no cost or comparative advantage differences between the two countries.

Each country is populated by M consumers who differ in their willingness to pay for quality. All consumers are assumed to have identical income composed of labour income and a per-capita share of the domestic monopolist's profits. Consumers allocate their income between expenditure on the numeraire, and on one unit of a quality differentiated good, by choosing the level of quality which provides the largest net benefit to be had from consuming a quality differentiated good, leaving expenditure on the numeraire to be determined as a residual. Net benefit is defined as the difference between what a consumer is willing to pay for

a quality differentiated good and its price, written as:

2) $Z_i(P_j) = \theta_i k_j - P_j$

where θ_i is a representative consumer i's valuation of a unit of quality
and thus $\theta_i k_j$ is consumer i's valuation or willingness to pay for good j.
Should no quality provide consumer i with non-negative net benefit, he
would simply spend all of his income on the numeraire.

The market demand for any particular level of quality depends on
both the distribution of willingnesses to pay for quality, and on the
variety of price and quality combinations available.

Consumers in each country are uniformly distributed over the
interval $[\theta_1, \theta_2]$, where θ_1 and θ_2 are the lowest and highest valuations
for quality in either country respectively, with density and cumulative
density given by:

3) $h(\theta) = \dfrac{1}{(\theta_2 - \theta_1)}$, $H(\theta) = \dfrac{(\theta_2 - \theta)}{(\theta_2 - \theta_1)}$

where $H(\theta_2) = 0$, and $H(\theta_1) = 1$. Thus no differences need arise betwen the
two economies for the purposes of this chapter.[1]

2.2   Monopoly Behaviour

Having described consumer behaviour and the costs of production,
the monopolist's optimal price and quality selection problem can now be
examined for one of the economies in autarky.

The monopolist's problem is modelled as a two stage game. In the first stage an optimal pricing rule is derived for a given level of quality. The optimal pricing rule is then used in the second stage to re-solve the firm's problem for the optimal level of quality. Fixed costs are assumed sufficiently high to prevent the monopolist from producing more than a single level of quality.

Recalling the consumers' decision rule (2), and the cumulative density function $H(\theta)$ from (3), market demand for a single level of quality is:

4)
$$
D(P) = \begin{cases} \dfrac{[\theta_2 k - P]M}{(\theta_2 - \theta_1)k} & \theta_2 k \geq P \geq \theta_1 k \\[3mm] M & \theta_1 k \geq P \end{cases}
$$

Using the demand function (4) and the cost function (1), the pricing stage of the monopolist's problem for a given level of quality is written as:

5)
$$
\underset{P}{\text{Max}} \quad \Pi(P) = \frac{P[\theta_2 k - P]M}{(\theta_2 - \theta_1)k} - gk^2 - G
$$

Differentiating (5) w.r.t. P and solving gives the optimal pricing rule:

6)
$$
P^* = \frac{\theta_2 k}{2}
$$

where P^* equates the marginal revenue lost from a price increase to zero. Alternatively one could think of P^* as just balancing the gain in revenue from attracting the consumer with willingness to pay $\frac{\theta_2 k}{2}$, with the loss in revenue suffered on all higher willingness to pay consumers. It is implicitly assumed that the distribution of willingnesses to pay is sufficiently wide such that some low willingness to pay consumers are not served.

Substituting the optimal pricing rule (6) back into the monopolist's problem (5) gives the second part of the firm's problem:

7) $\quad \underset{k}{\text{Max}} \ \ \Pi(k) = \dfrac{(\theta_2)^2 M k}{4(\theta_2 - \theta_1)} - g k_2^2 - G$

Differentiating (7) w.r.t. k and solving gives the optimal level of quality:

8) $\quad k^* = \dfrac{(\theta_2)^2 M}{8g(\theta_2 - \theta_1)}$

The profit function is concave for all k ensuring that k^* is a maximum. Notice that k^* is increasing in M, and increasing in θ_1, as a larger proportion of the consumers are served as the range narrows.

The equilibrium price charged for k^* is found by substituting (8) into the optimal pricing rule (6) which gives:

9) $\quad P^* = \dfrac{(\theta_2)^3 M}{16g(\theta_2 - \theta_1)}$

If θ^* identifies the consumer who is just served in autarky, such that $\theta^* k^* - P^* = 0$, then using (8) and (9), θ^* is given by:

10) $\theta^* = \dfrac{\theta_2}{2}$

Thus assuming that the distribution of willingnesses to pay for quality is sufficiently wide such that $\theta_1 \leq \theta^*$, then conditions (8), (9) and (10) describe an autarky equilibrium where $[\theta^*, \theta_2]$ is the set of consumers who buy k^* at price P^*, while $[\theta_1, \theta^*)$ is the set of consumers who are not served. Notice that when $\theta_1 = \theta^*$, $k^* = \dfrac{\theta_2 M}{4g} = \dfrac{\theta^* M}{2g}$.

For narrower distributions where $\theta_1 > \theta^*$, offering consumers k^* at price $P^* = \theta^* k^*$ is clearly not optimal as profits could be increased by raising price to $\theta_1 k^*$, and further increased by altering quality.

When serving the lowest willingness to pay consumer, the pricing rule is simply given by:

11) $P^* = \theta_1 k_1$

Substituting (11) into the monopolist's problem (5) gives the second part of the firm's problem as:

12) $\underset{k}{\text{Max}} \quad \Pi(k) = \theta_1 kM - gk^2 - G$

Differentiating (12) w.r.t. k and solving gives the optimal level of quality:

13) $\quad k^* = \dfrac{\theta_1 M}{2g}$

From (11) it is easy to see that the equilibrium price charged for k^* is:

14) $\quad P^* = \dfrac{(\theta_1)^2 M}{2g}$

Thus conditions (13) and (14) describe the autarky equilibrium in which all consumers buy k^* at price P^*.

The relationship between the equilibrium level of quality produced in each country in autarky, and the distribution of willingnesses to pay is summarized in Figure 6.1. At point A_1 where $\theta_1 = 0$, $k^* = \dfrac{\theta_2 M}{8g}$.

For $\theta_1 = \theta^*$ at point A_2, where all consumers are served, $k^* = \dfrac{\theta_2 M}{4g}$, and for $\theta_1 > \theta^*$, k^* converges to $k^* = \dfrac{\theta_2 M}{2g}$ at point A_3, where $\theta_1 = \theta_2$.

3. Trading Equilibrium

This section examines the consequences of opening up trade between two identical economies producing quality differentiated goods, as developed in Section 2. For convenience denote k_1 and k_2 as the qualities produced by the foreign and home economies respectively.

Once trade opens, the two monopolists are assumed to compete as duopolists over price and quality for consumers in each country. The equilibrium is solved as a two stage game. In the first stage a
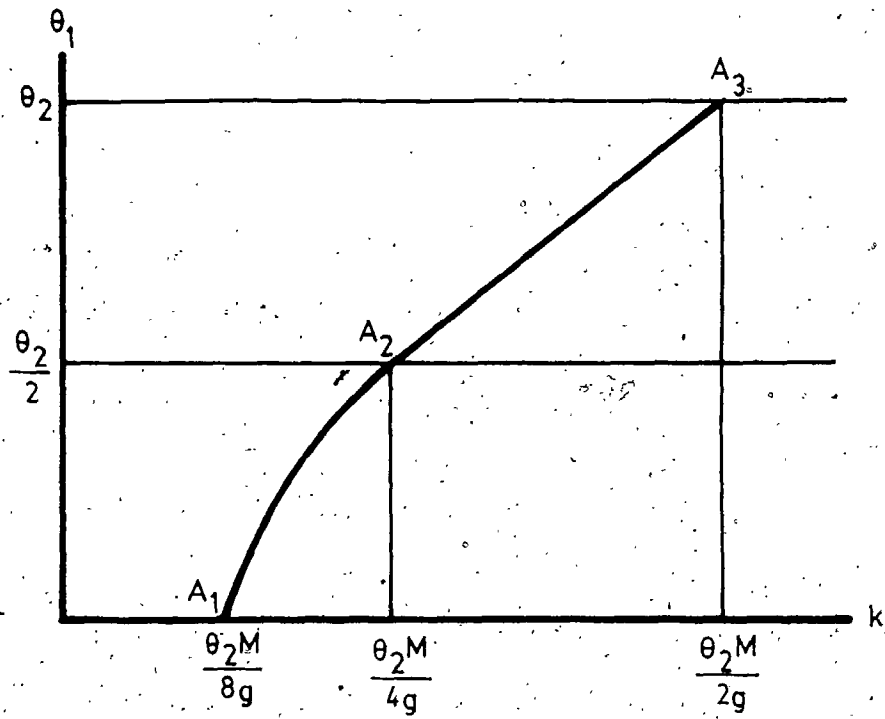
FIGURE 6.1

Bertrand-Nash pricing equilibrium is derived assuming each firm's quality is held fixed. In the second stage, the equilibrium pricing rules are then used to solve for the quality setting equilibrium, assuming that each firm conjectures that the quality of the other is held fixed.

This analysis proceeds by first examining the equilibrium that is reached for narrower distributions in which only one of the firms survive. Computational methods are then used to solve for the trading equilibrium that results for wider distributions, as the possibility of excluding low willingness to pay consumers complicates the analysis as it did in the appendix to Chapter 4.

3.1 Narrower Distributions - Natural Monopoly

Once trade opens, consumers in each country must choose between the foreign quality k_1 at price P_1, and the home quality k_2 at price P_2. Recalling from (2) that consumers choose the level of quality providing the largest non-negative net benefit, let $\hat{\theta}$ identify the consumer who is just indifferent between k_1 and k_2 such that $\hat{\theta}k_1 - P_1 = \hat{\theta}k_2 - P_2$ which gives:

$$15) \quad \hat{\theta} = \frac{P_2 - P_2}{k_2 - k_1}$$

Assuming that all consumers are served, the market demand functions for k_1 and k_2 are given by:

$$16) \quad D(P_1) = \frac{(\hat{\theta} - \theta_1)2M}{(\theta_2 - \theta_1)}$$

17) $\quad D(P_2) = \dfrac{(\theta_2 - \hat{\theta})2M}{(\theta_2 - \theta_1)}$

Notice that each firm attracts consumers by increasing its quality for given prices, which is in sharp contrast to horizontally differentiated goods models where firms attract consumers by pushing their varieties closer together.

Substituting $\hat{\theta}$ from (15) into (16) and (17) and using the cost function (1), the first part of the foreign and home firms' problem is written as:

18) $\quad \underset{P_1}{\text{Max}} \quad \Pi(P_1) = \dfrac{P_1[P_2 - P_1 - \theta_1(k_2 - k_1)]2M}{(\theta_2 - \theta_1)(k_2 - k_1)} - gk_1^2 - G$

19) $\quad \underset{P_2}{\text{Max}} \quad \Pi(P_2) = \dfrac{P_2[\theta_2(k_2 - k_1) - P_2 + P_1]2M}{(\theta_2 - \theta_1)(k_2 - k_1)} - gk_2^2 - G$

Differentiating (18) w.r.t. P_1 assuming P_2 is held fixed and differentiating (19) w.r.t. P_2 assuming P_1 is held fixed gives the price reaction function for the foreign and home firms respectively:

20) $\quad P_1^* = \dfrac{P_2 - \theta_1(k_2 - k_1)}{2}$

21) $\quad P_2^* = \dfrac{P_1 + \theta_1(k_2 - k_1)}{2}$

Since both profit functions are strictly concave in their respective prices, the two price reaction functions (20) and (21) are easily solved to yield the equilibrium prices:

22) $\quad P_1^* = \dfrac{(\theta_2 - 2\theta_1)(k_2 - k_1)}{3}$

23) $\quad P_2^* = \dfrac{(2\theta_2 - \theta_1)(k_2 - k_1)}{3}$

Notice immediately from (22) that for $P_1^* > 0$, $\theta_1 < \dfrac{\theta_2}{2}$, which says that for distributions narrower than $\dfrac{\theta_2}{2}$, the Bertrand-Nash pricing equilibrium is not well defined.

It is also useful to substitute P_1^* and P_2^* into (15) for $\hat{\theta}$ which gives:

24) $\quad \hat{\theta} = \dfrac{\theta_2 + \theta_1}{3}$

Thus the Bertrand-Nash price competition in conjunction with the zero marginal cost assumption results in the foreign firm having a zero market share for distributions narrower than $\dfrac{\theta_2}{2}$, which gives rise to the natural monopoly outcome. Also notice that the market share is independent of the levels of quality chosen.

For distributions wider than $\dfrac{\theta_2}{2}$, the foreign and home firm's quality setting problem is found by substituting (22) and (23) into the profit

functions (18) and (19) which gives:

25) $\underset{k_1}{\text{Max}} \quad \Pi(k_1) = \dfrac{(\theta_2 - 2\theta_1)^2(k_2 - k_1)2M}{9(\theta_2 - \theta_1)} - gk_1^2 - G$

26) $\underset{k_2}{\text{Max}} \quad \Pi(k_2) = \dfrac{(2\theta_2 - \theta_1)^2(k_2 - k_1)2M}{9(\theta_2 - \theta_1)} - gk_2^2 - G$

Differentiating (25) w.r.t. k_1 assuming k_2 is held fixed and differentiating (26) w.r.t. k_2 assuming k_1 is held fixed and solving yields the equilibrium levels of quality:

27) $k_1^* = 0$

28) $k_2^* = \dfrac{(2\theta_2 - \theta_1)^2 M}{9g(\theta_2 - \theta_1)}$

The foreign firm chooses to reduce its quality to zero as changes in k_1 have no effect on the allocation of consumers, while reductions in k_1 away from k_2 minimize the Bertrand-Nash price competition and increase profits.

The equilibrium described by conditions (27) and (28), however, is only well specified at $\theta_1 = \dfrac{\theta_2}{2}$ where $k_2^* = \dfrac{\theta_2 M}{2g}$. For distributions wider than $\dfrac{\theta_2}{2}$, lower willingness to pay consumers are excluded as $\theta_2 k_2^* - P_2^* < 0$, with the result that the demand functions and hence profit functions are not well specified. The trading equilibrium reached for distributions wider

than $\frac{\theta_2}{2}$, is examined in the next section.

For distributions narrower than $\frac{\theta_2}{2}$, only one of the firms survives to serve both economies as a monopolist in the trading equilibrium.

The relationship between the equilibrium level of quality offered to consumers in each country by the surviving firm and the distribution of willingnesses to pay is shown between points B_2 and B_3 in Figure 6.2.

Thus in contrast to the equilibrium level of quality produced in each country in autarky which is shown between points A_2 and A_3, trade clearly causes the surviving firm to upgrade the level of quality produced over all distributions, in response to the increased extent of the market.

3.2   Wider Distributions - Natural Oligopoly

For distributions wider than $\frac{\theta_2}{2}$ it is possible to show that a trading equilibrium exists in which the foreign firms produces a non-zero level of quality.

The equilibrium, however, will be at or above the kink in the foreign firm's market demand curve shown at point B in Figure 6.3, where lower willingness to pay consumers are excluded. This also corresponds to equilibria which fall along the vertical or upper sections of the foreign firm's price reaction function above point B as shown in Figure 6.4. Recall that along the lower section of the foreign firm's price reaction function, distributions narrower than $\frac{\theta_2}{2}$ cause the foreign firm to have a zero market share, while distributions wider than $\frac{\theta_2}{2}$, cause the
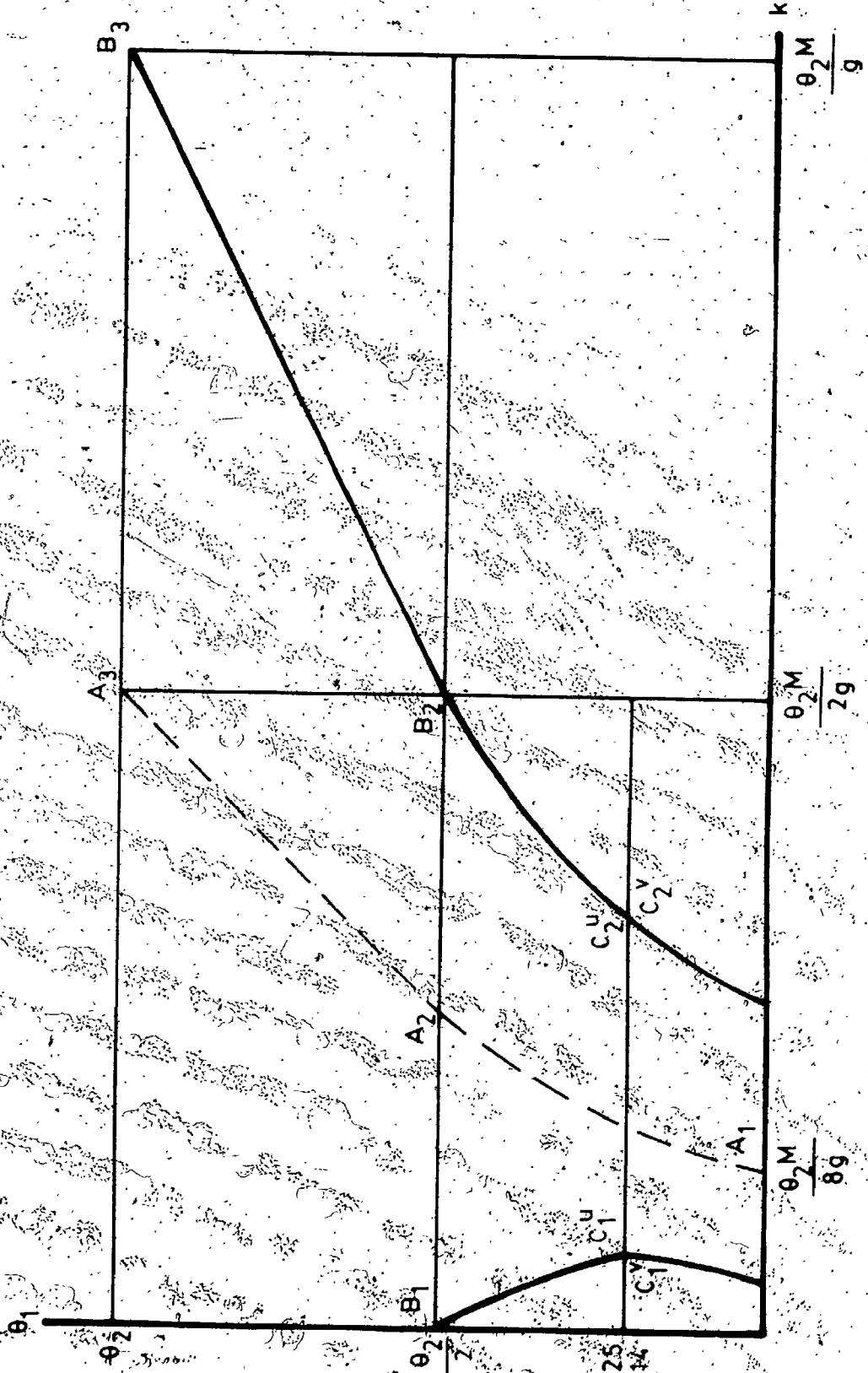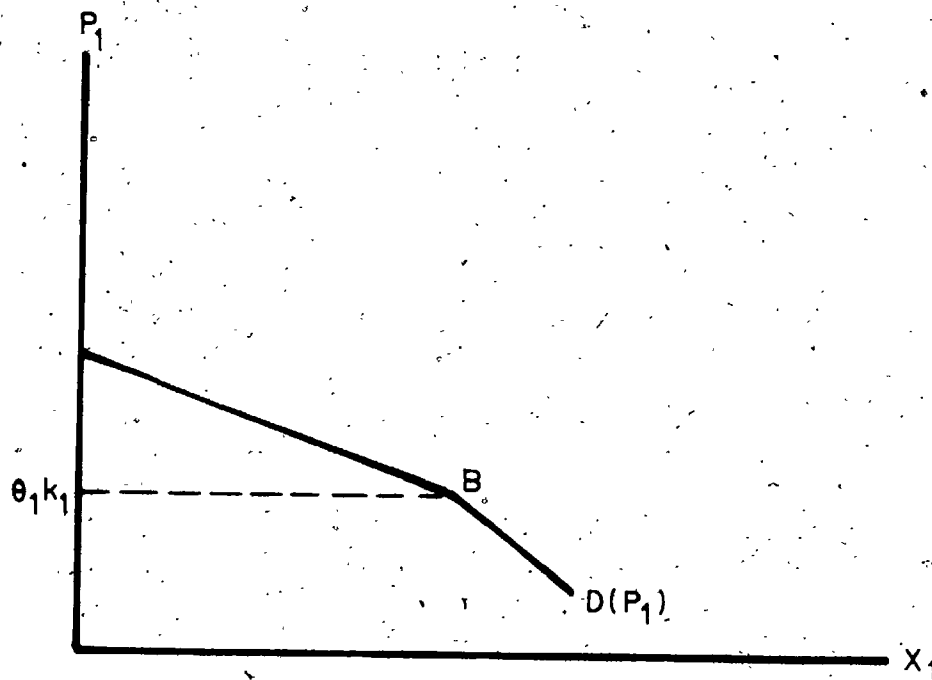
FIGURE 6.2

FIGURE 6.3
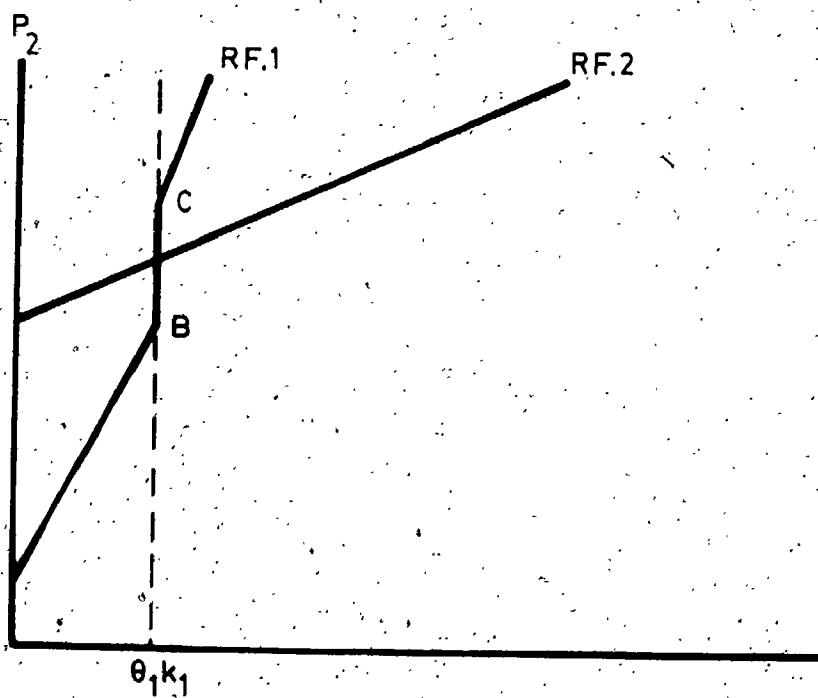
FIGURE 6.4

intersection to take place beyond point B, with the result that firms begin competing over the vertical section of the foreign firm's price reaction function.

Recall from section 3.2 that the lower section of the foreign firm's price reaction function is given by:

29) $$P_1 = \frac{P_2 - \theta_1(k_2 - k_1)}{2}$$

At $P_1 = \theta_1 k_1$, however, the kink in the foreign firm's market demand curve generates a vertical section in the foreign firm's price reaction function simply written as:

30) $$P_1 = \theta_1 k_1$$

Above $P_1 = \theta_1 k_1$, where lower willingness to pay consumers are excluded, the upper section of the foreign firm's price reaction function is easily found to be:

31) $$P_1 = \frac{k_1 P_2}{2k_2}$$

Excluding low willingness to pay consumers has no effect on the market demand for k_2, and consequently no effect on the home firm's price reaction function which was given by:

32) $$P_2 = \frac{P_1 + \theta_2(k_2 - k_1)}{2}$$

The condition which ensures that the price setting equilibrium falls above point B on the vertical section of the foreign firm's price reaction function is found by solving for the intersection between conditions (29), (30) and (32) shown at point B, which gives:

33) $\quad 0 \leq (\theta_2 - 2\theta_1)k_2 - (\theta_2 + \theta_1)k_1$

Thus it is clear that condition (33) can only hold for distributions wider than $\frac{\theta_2}{2}$.

Similarly, the condition which ensures that the pricing equilibrium does not fall above point C on the upper section of the foreign firm's price reaction function is found by solving for the intersection between conditions (30), (31) and (32) shown at point C, which gives:

34) $\quad 0 \geq (\theta_2 - 4\theta_1)k_2 - (\theta_2 - \theta_1)k_1$

Thus condition (34) will clearly hold for all distributions narrower than $\frac{\theta_2}{4}$.

Thus assuming conditions (33) and (34) both hold, the overall equilibrium will fall along the vertical section of the foreign firm's price reaction function, while if (34) fails, the intersection falls along the upper section.

Solving for the quality setting equilibrium is complicated by the kinks in the foreign firm's price reaction function, as each section

must be used separately in conjunction with the home firm's price reaction

function to solve for the equilibrium. The equilibria derived for each

section must then be checked against conditions (33) and (34) to determine

the distributions over which they are well specified, and then compared to

each other to determine the distributions over which each dominates.

As in the appendix to Chapter 4, each firm's quality selection

problem becomes highly non linear over the vertical and upper sections of

the foreign firm's price reaction function. Accordingly the equilibria

are computed using the non-linear equation solving computer program used

there.[2] For simplicity and without loss of generality (as there isn't

much left to lose!), it is assumed that $\theta_2 = f = 1$, $G = 0$, and $M = 2$.

The relationship between the equilibrium levels of quality

produced along the vertical section of the foreign firm's price reaction

function satisfying conditions (33) and (34), and the distribution of

willingnesses to pay for quality is shown between points B_1, and C_1^v, and

B_2 and C_2^v in Figure 6.2 for k_1^* and k_2^* respectively. The home and foreign

firm's profit functions are strictly concave for distributions narrower

than $\frac{\theta_2}{4}$ and at least locally concave at k_1^* and k_2^* until $\theta_1 = .2114$ at

points C_1^v and C_2^v, where condition (34) is just satisfied.

The profit function for the two firms at points C_1^v and C_2^v are

shown in Figure 6.5. Notice that the foreign firm's profit function

defined on the vertical section of the foreign firm's price reaction

function $\Pi_1^v(k_1)$, evaluated at k_2^*, is only well specified for $k_1 \geq k_1^u$,

where k_1^u just satisfies (34) evaluated at k_2^*. Similarly, the home

firm's profit function defined on the vertical section $\Pi_2^v(k_2)$, evaluated

$\theta_1 = .2114$ $k_1^* = .0619$ $k_2^* = .3162$

$\pi_1'^v = 0$ $\pi_2'^v = 0$

$\pi_1'^u < 0$ $\pi_2'^u > 0$

FIGURE 6.5

$\theta_1 = .2125$ $k_1^* = .0613$ $k_2^* = .3217$

$\pi_1'^u = 0$ $\pi_2'^u = 0$

$\pi_1'^v > 0$ $\pi_2'^v < 0$
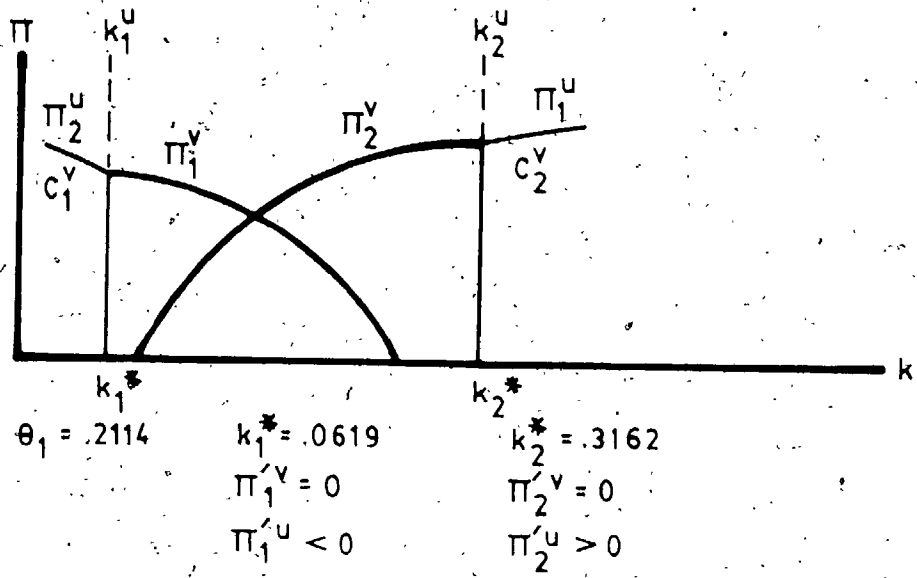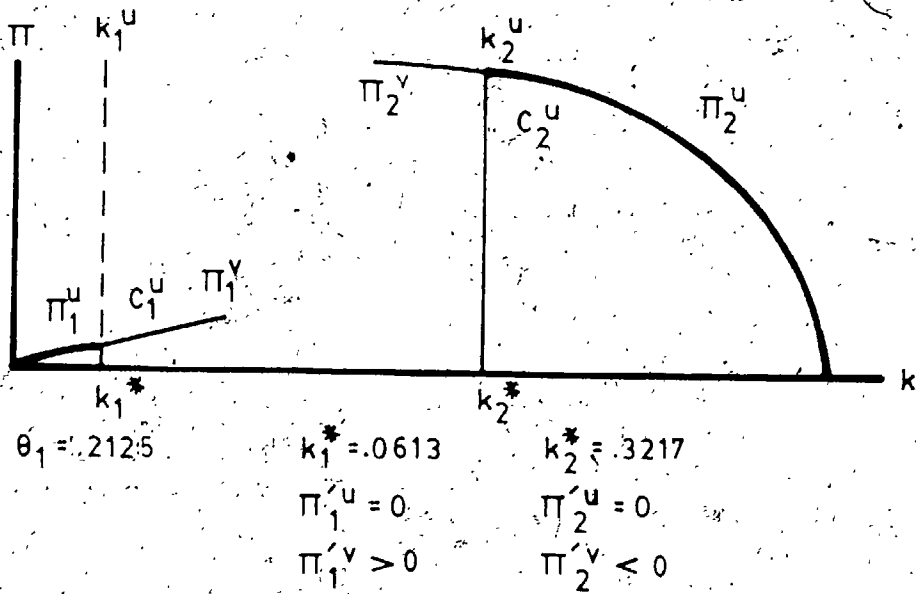
FIGURE 6.6

at k_1^* is only well specified for $k_2 < k_2^u$ where k_2^u just satisfies condition (34) evaluated at k_1^*. Since k_1^* and k_2^* coincide with k_1^u and k_2^u respectively at points C_1^v and C_2^v, and since $\Pi_1^u(k_1^u) < 0$ and $\Pi_2^u(k_2^u) > 0$, the two firms would clearly begin to compete over the vertical section of the foreign firm's price reaction function.

Turning to the upper section of the foreign firm's price reaction function, the relationship between the equilibrium levels of quality produced, and the distribution of willingnesses to pay for quality which cause condition (34) to fail, is shown in Figure 6.2 below points C_1^u and C_2^u for k_1^* and k_2^* respectively.

The profit functions for the two firms at points C_1^u and C_2^u where $\theta_1 = .2125$, and k_1^* and k_2^* just coincide with k_1^u and k_2^u to satisfy condition (34) are shown in Figure 6.6. Since $\Pi_1^v(k_1^u) > 0$, and $\Pi_2^v(k_2^u) < 0$, the two firms would begin to compete along the vertical section of the foreign firms price reaction function. For distributions wider than $\theta_1 = .2125$, the profit functions are at least locally concave at k_1^* and k_2^* to yield a trading equilibrium in which the lowest willingness to pay consumers are excluded.

Thus from Figure 6.2, it is clear that in comparison to the equilibrium level of quality produced in each country in autarky, trade causes the two firms to differentiate their qualities to minimize the price competition between them. This result is reinforced over the narrower distributions where the price competition causes the share of consumers buying from each firm to be highly unequal. Thus for the majority of consumers, trade has the effect of upgrading the level of

quality that they buy from the home firm. As the distribution widens, however, each firm serves roughly the same number of consumers in the trading equilibrium as it did in autarky, with the result that no quality upgrading takes place.

4. Summary and Conclusions

This chapter reexamines the model of trade in quality differentiated goods developed in Chapter 4, under the alternative production technology that the burden of quality improvement falls on fixed costs such as R & D expenditure rather than on marginal costs as assumed to this point. This provides a very useful comparison between the effects of trade modelled here, and those found by Shaked and Sutton (1984).

In autarky, the equilibrium level of quality offered to consumers by a domestic monopolist is shown to depend not only on the distribution of willingnesses to pay, but also on the total population of consumers. Larger populations allow the firm to spread the fixed costs of a quality improvement over a larger number of consumers, which causes quality to rise.

Once trade opens, the two monopolists are assumed to compete as duopolists over price and quality for consumers in each country. As with Shaked and Sutton (1984), the Bertrand-Nash price competition in conjunction with a zero marginal cost assumption results in only one of the firms surviving for narrower distributions. For wider distributions, a trading equilibrium is reached, though some lower willingness to pay consumers may not be served.

In comparison to autarky, it is clear that for narrower distributions where only one of the firms survives, trade has the effect of upgrading the level of quality produced by the remaining firm, which serves both economies. This is identical to the Shaked and Sutton results.

For wider distributions where a trading equilibrium exists, the two firms differentiate their qualities to minimize the price competition between them. This result is reinforced over the narrower distributions, where the price competition causes the share of consumers buying from each firm to be highly unequal. Thus for the majority of consumers, trade has the effect of upgrading the level of quality they buy from the home firm. As the distribution widens, however, each firm serves roughly the same number of consumers in the trading equilibrium as it did in autarky with the result that no quality upgrading takes place.

Thus in contrast to the Shaked and Sutton results, it is clear that for economies where set up costs or entry fees are sufficiently high such that only one firm is able to operate in autarky, the competition brought about by trade may well result more in quality differentiation than quality upgrading. This is especially true if the share of consumers served by each firm in the trading equilibrium is roughly the same as it was in autarky.

Endnotes

1. The autarky equilibrium has also been derived using the triangular distributions described in Chapter 5, with much the same results.

2. Each firm's quality setting problem for the upper and vertical sections of the foreign firm's price reaction function, and the computations are available from the author upon request.

CHAPTER 7

SUMMARY AND CONCLUSIONS

This thesis is primarily concerned with the role of both scale economies and demand differences between countries producing quality differentiated goods, in determining the variety of qualities produced both in autarky and once trade opens, and on the pattern of trade that results between two such economies. Attention is also paid to the effects of trade on individual consumers in each country.

Chapter 3 focuses on the role of demand differences within a country in determining the variety of qualities produced by both price discriminating and non-price discriminating monopolists, in a very flexible two consumer-type version of the basic model of quality differentiated goods production developed in Chapter 2. It is shown for both price discriminating and non-price discriminating monopolists, that the number and level of qualities produced will depend on the number of consumers in each group, and their respective preferences for quality.

If one thinks of the two consumer types as residing in distinct regions of an economy, the no price discrimination equilibria could be viewed as the result of allowing arbitrage or free trade between the two groups. Comparing the two equilibrium configurations, shows that freeing trade between the two groups reduces either the number of qualities produced, or the average level of quality produced. High willingness to pay consumers are shown to gain net benefit, but only in the presence of a substantial population of low willingness to pay consumers who warrant being served.

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The simple two consumer-type model of quality differentiated goods-production developed in this chapter provides a very flexible model to explore the effects of demand differences on a variety of important trade issues should trade open between two such economies. The appendix to this chapter begins to address some of these issues by examining the consequences of opening up trade between two such economies. It is shown however that with this simple two consumer-type model, some rather restrictive assumptions on firm pricing behaviour is required for an equilibrium to-exist.

Chapter 4 examines the consequences of opening up trade between two economies characterized by the basic model of quality differentiated goods production, but where consumers in each country are assumed to be uniformly distributed over their willingness to pay for quality. While this leaves little scope to examine the effects of trade on minority and majority taste groups in each country, the non-existence problems associated with opening trade between two economies characterized by the more flexible two-point distribution, are avoided.

In each country in autarky, the equilibrium price and quality combination offered to consumers by a monopolist is shown to depend on the distribution of willingnesses to pay for quality. Consumers benefit from wider distributions in which lower willingness to pay consumers are served, which is analogous to the results obtained for the two-point distribution examined in Chapter 3.

Once trade opens the two monopolists are assumed to compete as duopolists in price and quality for consumers in each country. For the special case where the distribution of willingnesses to pay is the same

in each country, trade causes the two firms to push their qualities apart from their autarky levels, to minimize the price competition between them. This contrasts sharply with horizontally differentiated goods models where firms hold their varieties together to attract consumers from the other, which is the standard Hotelling result.

All consumers are shown to gain net benefit as a result of the competition brought about by trade. These gains are shown to be larger between countries with narrow distributions of willingness to pay for quality, as the scope for firms to differentiate their qualities to avoid price competition is reduced. Differences between the two countries which allow the firms to further differentiate their qualities, reduces the gains from trade.

Chapter 5 considers a rather special variation of the model of trade in quality differentiated goods developed in Chapter 4, which nicely highlights the role of demand differences in determining the pattern of trade, and its effects on individual consumers in each country.

The distribution of willingnesses to pay for quality assumed for each country is characterized by a right triangular distribution, with the home country having a majority of high willingness to pay consumers, and the foreign country having a majority of low willingness to pay consumers. The range of willingnesses to pay for quality and the total population in each country is assumed to be the same.

The pattern of trade which emerges is very much in the spirit of the Linder hypothesis as each country is producing a level of quality in keeping with its representative demand, which it exports to consumers who are on the fringe of the representative demand in the other country.

Chapter 6 reexamines the model of trade in quality differentiated goods developed in Chapter 4, under the alternative production technology that the burden of quality improvement falls on fixed costs such as R & D expenditure rather than on marginal costs as assumed to this point. This provides a very useful comparison between the effects of trade modelled here, and those found by Shaked and Sutton (1984).

In autarky, the equilibrium level of quality offered to consumers by a domestic monopolist is shown to depend not only on the distribution of willingnesses to pay, but also on the total population of consumers. Larger populations allow the firm to spread the fixed costs of a quality improvement over a larger number of consumers, which causes quality to rise.

Once trade opens, the two monopolists are assumed to compete as duopolists over price and quality for consumers in each country. As with Shaked and Sutton (1984), the Bertrand-Nash price competition in conjunction with a zero marginal cost assumption results in only one of the firms surviving for narrower distributions. For wider distributions, a trading equilibrium is reached, though some lower willingness to pay consumers may not be served.

In comparison to autarky, it is clear that for narrower distributions where only one of the firms survives, trade has the effect of upgrading the level of quality produced by the remaining firm which serves both economies. This is identical to the Shaked and Sutton results.

For wider distributions where a trading equilibrium exists, the two firms differentiate their qualities to minimize the price competition between them. This result is reinforced over the narrower distributions,

where the price competition causes the share of consumers buying from each firm to be highly unequal. Thus for the majority of consumers, trade has the effect of upgrading the level of quality they buy from the home firm. As the distribution widens, however, each firm serves roughly the same number of consumers in the trading equilibrium as it did in autarky with the result that no quality upgrading takes place.

Thus in contrast to the Shaked and Sutton results, it is clear that for economies where set up costs or entry fees are sufficiently high such that only one firm is able to operate in autarky, the competition brought about by trade may well result more in quality differentiation than quality upgrading. This is especially true if the share of consumers served by each firm in the trading equilibrium is roughly the same as it was in autarky.

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