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Three Essays On Applied General Equilibrium Analysis

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THREE ESSAYS ON APPLIED GENERAL EQUILIBRIUM ANALYSIS

by
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ABSTRACT

The thesis develops three essays which involve diverse issues like the estimation of the parameters of a fully specified general equilibrium model, estimation of import and export demand elasticities with some general equilibrium (GE) closure rule and formulating a decomposition algorithm for GE models with proper structure. While the questions addressed are somewhat unrelated to each other, the essays involve issues related to models which are essentially Walrasian in nature. The three essays are:

(1) "On the Estimation of General Equilibrium Models";

(2) "Estimation of Import and Export Demand Elasticities and Elasticity Pessimism"; and

(3) "A Dantizig-Wolfe Type Decomposition Algorithm for General Equilibrium Models With Applications to International Trade Models".

The first essay discusses various econometric approaches to estimate simple and rleatively small GE systems. We discuss both system and subsystem estimation, reviewing some of the literature on demand and production functions and suggest two types of Nonlinear Instrumental Variable Methods. We report and compare the parameter estimates obtained from both stochastic and deterministic methods for a small GE tax model of the U.S. economy, and finally evaluate alternative approaches to parameter selection.

The second essay argues that the import and export demand functions should be estimated simultaneously along with trade balance conditions which close the system, a kind of specification recognized in pure trade theory literature and empirically oriented GE models. Ignorance of this simultaneously makes the estimates biased downwards for both small and large sample cases with bearing on the issue surrounding the controversy of "elasticity pessimism".

The third essay describes the computation of general equilibrium via a fixed point decomposition procedure similar in spirit to the Danzig and

Wolfe (1961) decomposition algorithm for the solution of linear programming problems. We show that for a GE model of block diagonal structures, one can compute equilibria of the full dimensional problem by solving sequences of smaller dimensional "master" and subproblems". Information passes between master and subproblems and the procedure is guaranteed to terminate at an approximate equilibrium without cycling. We discuss the existence and computation of such equilibria, potential applications and computational efficiency compared to the full dimensional methods.

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CHAPTER I

INTRODUCTION

General equilibrium theory and the associated computational procedures developed in the recent years have opened a wide range of potential applications to various economic issues. While application to real world problems is the ultimate objective of most economic theories, many methodological issues need to be addressed at the same time. Keeping this in mind, this thesis develops three essays which involve diverse issues like the estimation of the parameters of a fully specified general equilibrium model, estimation of import and export demand elasticities with some general equilibrium closure rule and formulating a decomposition algorithm for general equilibrium models with proper structure. Although the questions addressed are somewhat unrelated to each other, the essays involve issues related to models which are essentially Walrasian in nature. The three essays are:

1. On the estimation of general equilibrium models;
2. Estimation of import and export demand elasticities and elasticity pessimism; and
3. A Dantzig-Wolfe type decomposition algorithm for general equilibrium models with applications to international trade models.

In the following sections we summarize the three essays separately, mainly focusing on the central issues involved along with the key findings.

1. Chapter Two (First Essay)

The first essay discusses various econometric approaches to estimate simple and relatively small general equilibrium systems. We discuss both system and subsystem estimation, reviewing some of the literature on demand and production functions. We briefly discuss (Section II) calibration or deterministic methods to the specification of general equilibrium parameters and in view of their various limitations focus on the potential use of the literature on the estimation of demand and production systems. From a model decomposition described in the text, it appears that the simplest way to obtain the relevant parameters of a general equilibrium (GE) model is to separately estimate the commodity demand and production system embedded into the fully specified GE model. The vast literature on applied consumption and production analysis is clearly relevant to this effect and this essay briefly refers to some of those. We also emphasize the potential sources of bias which any straightforward application of existing methods would entail and highlight the overall significance of their ignorance.

The following section (Section III) outlines some methods to estimate general equilibrium models which take into account the underlying simultaneity from the GE point of view. Beginning with a simple GE model with production, we discuss some of the problems associated with Full Information Maximum Likelihood (FIML) estimation of the complete model. In view of its potentially large dimension we suggest methods to estimate smaller dimensional subsystems using some nonlinear instrumental variable method

(NLIV). The choice of subsystems would vary in accordance with differences in specifications, the general nature being described in terms of a model disaggregation outlined in Section II (Figure I).

The estimation methods have been applied to the evaluation of the loss to the U.S. economy due to differential taxation of income from capital (Section IV). We investigate the nature and the values of the estimates obtained from the "benchmark equilibrium data set" and those available from the methods suggested in Section III. We observe that for the same elasticity value, the benchmark method with averaged data tends to yield share parameters close to those of the stochastic ones. 1/ However, in the deterministic method the estimated share parameters are surprisingly sensitive to changes in the adopted elasticity values, and this is a matter of concern because of the limited consensus on the relevant elasticity values. Using our estimated parameters we compare various measures of distortions:

(i) those from Harberger's (1962, 1966) method using his parameter values vis-a-vis our estimators (NLIV); and

(ii) with Shoven and Whalley (1972) and Shoven (1976) algorithmic approach due to Scarf (1973); for this we compare the loss estimates from benchmark equilibrium parameters vis-a-vis the corresponding stochastically estimated parameters.

1/ It may be noted that benchmark equilibrium cannot be a complete substitute for econometric estimation as we need to specify elasticity values from literature search; when applied to average data deterministic methods may yield good estimates of share and scale parameters but elasticity parameters are not identified.

Sensitivity of loss estimates in different formulations due to the variations in elasticity values have been noted. It appears that the Shoven-Whalley formulation in general gives a lower loss estimate compared to Harberger's by a factor of 2-3 even with NLIV parameter estimates. For the same Shoven-Whalley method benchmark parameters yield figures that are slightly downward biased compared to the same from NLIV parameters.

2. Chapter Three (Second Essay)

In the second essay we focus on a potential source of bias in the traditional estimation of import and export demand elasticities. In none of the earlier works, some of which have been summarized in the main text, the specified import and export demand functions have been considered as part of a more complete foreign trade sector specification. We argue that import and export demand functions should be estimated simultaneously along with the trade balance condition which closes the system. This is a kind of specification that has been recognized in the pure trade theory literature and in the empirically oriented general equilibrium models. Ignorance of this simultaneity makes the estimates biased downwards and this has bearing on the issues surrounding the controversy of 'elasticity pessimism'. This conclusion holds under different specifications of the relevant functions, including one where import demand function is obtained from choice theoretic foundations.

Computational experience with Japanese data, as has been reported in the text, supports our theoretical assertions that the Least Squares estimators of elasticity parameters are substantially lower than those of unbiased estimators. In view of very frequent evidence of serial autocorrelation in the disturbance terms in various studies, the analysis is extended to determine the asymptotic bias of least square estimates of elasticity parameters when error terms are serially correlated. It has been found that if the underlying true model is simultaneous and due to misspecification we are applying least squares, so far as the bias is concerned, we may be better off by not making the transformation to correct the serial correlation.

3. Chapter Four (Third Essay)

The third essay (prepared jointly with Professor John Whalley (1980)), describes the computation of general equilibrium via a fixed point decomposition procedure similar in spirit to the Dantzig and Wolfe (1961) decomposition algorithm for the solution of linear programming problems. We show that for a general equilibrium model of block diagonal structures it is possible to compute equilibria of the full dimensional problem by solving sequences of smaller dimensional 'master' and 'sub-problems'. The method involves the generation of labels for the vertices on a master simplex through the separate solution of sub-equilibrium problems whose parameters are determined by the vertex on the master simplex. Information is passed between the 'master' and 'sub' problems and 'coefficent generation' for a master linear programming problem in the Dantig-Wolfe algorithm is replaced by 'label generation' for vertices on a master simplex associated with the general equilibrium problem.

The decomposition structure we consider is that common goods are both demanded and owned by all the agents while non-common goods are owned and demanded by only a subset of agents. Each subproblem takes the relative prices of common goods as given and determines an approximate equilibrium characterized by non-positive excess demands for each non-common good along with an equilibrium price for a composite common good satisfying a non-negative excess demand condition. We evaluate the excess demand functions for common goods using the solution for sub-problems together with the common good prices on the master simplex; this generates a label for a vertex on the master simplex. Variants of Gale-Nikaido mapping are utilized for label generation and a completely labelled master simplex along with associated sub-equilibria yield an approximation to an equilibrium for the whole model. The procedure is guaranteed to terminate at an approximate equilibrium without cycling by the same argument underlying Scarf's (1973) algorithm and the approximation becomes exact in the limit approached by a dense grid.

This decomposition procedure has been used to solve a number of hypothetical models and our initial experience suggests that if the decomposition structure is sufficiently strong, significant computational savings are possible through the exploitation of that structure. Computational costs increase less sharply for the overall problem with the decomposition method than under full solution.

In this introductory chapter we have summarized some of the issues considered in the three essays. The next three chapters would spell out all three essays separately in more detail and each of those would individually describe their respective issues along with possible extensions. Chapters are completely independent of each other and each one contains its own list of references.

CHAPTER II

On The Estimation of General Equilibrium Models

I. Introduction

This paper investigates and suggests some methods of estimating parameters in the context of applied general equilibrium models with potential applications. General Equilibrium (GE) models normally incorporate production and demand systems along with hypothesized market clearing conditions. All of these, because of their respective choice theoretic micro foundations, involve the same set of parameters appearing in different equations or having cross equation restrictions among the parameters. Hence the spirit of general equilibrium simultaneity requires us to estimate the specified models using some full-information method (applied to the full dimensional model or to its parts). However, full information estimation of GE models poses problems which are both theoretical and practical, and has so far been ignored in applied GE works. Available modern econometric techniques to deal with the large number of nonlinear interdependent relations has not been properly employed to obtain a set of consistently estimated parameters. This paper intends to investigate various approaches to estimating the parameters of applied GE models. In view of various limitations of the existing methods this paper suggests the use of nonlinear instrumental variable methods to the subsystems of full dimensional models with the intention of capturing cross equation restrictions.

Conceptualization of the workings of the economic system in a GE framework is quite old but most of the work has been done at the theoretical level. While recently some empirical models have been formulated in the GE tradition, almost all of these are concerned with computation; very little attention has been paid to obtain a set of parameters that are consistent not only from a statistical point of view but also with the adopted sectoral classification. The reasons for this are both practical and conceptual: at a practical level econometric work, particularly when applied to a general equilibrium model, requires observations on the multitude of individual agents over a long period of time and at a conceptual level the measurement of tatonnement stability requires observations or construction of hypothetical actions that are never observable. Moreover, efficient methods of estimating nonlinear models have been available only for the last few years; [see Goldfeld and Quandt (1972), G. Chow (1973), Eisenpress and Greensbadt (1966), Berndt et al. (1974), etc.]. Extensions of various single equation (or limited information) estimation procedures (as applied to linear systems) to nonlinear ones are either not very satisfactory 1/ or are at early stages of development; (see the passing comments of Eisenpress and Greenstadt (1966) and subsequent works by Kelejian (1971), Amemiya (1974, 1977), Hansman (1975), etc.). All these factors have impaired the potential flow of econometric applications to general equilibrium models.

1/ In the linear case 2SLS has the same asymptotic distribution as the LIML estimator and has the smallest asymptotic variance-covariance matrix in the class of instrumental variables estimators. In the case of general nonlinear models (i.e., nonlinear in both parameters and variables) no similar statement is valid.

It is well accepted that the GE model is an improvement over the simple input-output model (or its variants) in the sense that the market mechanism is allowed to play its proper role in the determination of production, exchange, and allocation. By specifying the model on choice theoretic foundations and restoring the inherent price mechanisms flexible GE models go a long way in the desired direction. However, this movement is not complete unless we have some satisfactory way to obtain the parameter set being used as an input in the computation algorithms. This is particularly important in view of our observations that different studies use different sectoral classification (sometimes different sub-sectors under the same broad name in different studies) and corresponding choice theoretic demand and production parameters would vary accordingly. The sensitivity of parameters to variations in sectoral classifications seems to be quite substantial and thus we can validly argue against the use of numbers obtained through a literature search (which is the most widely used practice in current applied works).

Estimation of GE parameters can be done in several ways with various degrees of computational ease and theoretical incompleteness. While deterministic methods tend to be widely used in various studies, different econometric (empirical) works can be directly or indirectly associated with estimation of general equilibrium parameters. All these, along with deterministic ones will be discussed in the next section (Section II). In this context we investigate how the existing methodologies dealing with estimation of commodity demand and production system can serve as a first step forward in the desired direction. In section III we outline the estimation of a stochastic GE model with time series data. In view of the dimensions of potential applied GE models and consequent data requirements we consider estimation of various subsystems of the full dimensional general equilibrium model. Nonlinear limited information methods are suggested to avoid the simultaneous equation bias and in view of its potential limitations we suggest a second method. This second method (as applied to subsystems) is an extension of method I, and utilizes GE solution algorithms to generate more appropriate sets of instruments. In section IV, the suggested methods are applied to a small dimensional GE model of the U.S. economy to measure the distortionary effects of the differential rates of taxation on income from capital. Using the same data set (described in Appendix I), comparisons have been made with the Harberger (1962), and Shoven and Whalley (1972) methods of measuring the costs due to resource misallocations. The differences in the outcomes, if any, can be attributed to the differences in methodologies resulting in different parameter estimates. Estimates of the relevant parameters have been obtained using different methods (i.e., those from subsystems of stochastic specification and by the deterministic method) and their deviations from each other and sensitivities are examined to some extent. These comparisons have been reported in Section IV and may also help us in the selection of appropriate methods of estimation in future studies.

Regarding data requirements, it can be said that we need observations on the relevant variables over a reasonable length of time. Multi-sector time series data on output or value added and primary factor use along with the relevant input-output matrix are prerequisites for these suggested methods to work. These are not beyond the reach of the econometricians for many countries.

II. Some Observations on Other Methods

In general, methods for the estimation of GE parameters can be classified into two broad categories:

a. deterministic or non-stochastic methods

b. stochastic or econometric methods.

Both of these are to be discussed in this section in succession. While discussing stochastic methods, among other things, we focus on the potential use of existing literature surrounding the estimation of sets of commodity demand and production system.

Deterministic method: As indicated earlier, parameters of production and demand relations tend to change significantly as the sectoral classification changes. Recognizing this variability of the parameter values due to different classification of sectors, a kind of deterministic method has been used to obtain the relevant parameters in some current works (Piggot and Whalley (1977), Mansur and Whalley (1981)). They construct a 'bench-mark equilibrium data set' by adjusting the blocks of data that are available from different sources but are not consistent with each other and with the equilibrium conditions. Examples of this method have been discussed in Piggot and Whalley (1977) where data is adjusted in such a way that the equilibrium conditions underlying the Walrasian model are satisfied. The economy is partitioned into demand and production relations where commodities are produced by industries and purchased by consumers. Taxes imposed on consumers (income and expenditure taxes) and on producers, finance public sector expenditure. Input-output transaction accounts provide most of the production side information. Since data is obtained from different sources there is no reason that consistency conditions would necessarily be satisfied. Some of these consistency conditions are:

1. demand equals supply for each product
2. each industry's total costs equal total sales
3. purchases of each consumer group equals disposable income for that group
4. endowments of consumers match factor usage
5. value of final demand equals the sum of value added
6. foreign trade balance and so on.

Using the 'RAS' method of Bacharach (1970), the 'bench-mark' approach in fact tries to capture or impose these restrictions that are implicit in general equilibrium theory on the constructed data set. This type of problem of constructing matrices consistent with known sets of column and row sums has received attention since the work of Deming and Stephens (1943) in the early 1940s and has been used in various contexts.

There is some arbitrariness in these adjustment procedures and there is wide scope for further works in devising appropriate matrix methods evaluated by some objectively measurable loss function. However, the most serious objection to this is its basic assumption that the economy in that particular year is in equilibrium satisfying all the conditions of the model under consideration. The parameters are then estimated utilizing the identifying restrictions of the general equilibrium model. To illustrate the issue let's take a Cobb-Douglas form of production function for the ith industry:

$$Q_i = A_i K_i^{a_i} L_i^{(1-a_i)}$$

where K_i and L_i are respectively capital and labor used in the ith industry to produce output Q_i. The value of A_i is determined from the first order conditions of cost minimization. By defining the units appropriately and using the benchmark equilibrium data set (i.e., adjusted data) the following condition is satisfied, from which we get the estimate of a_i:

$$(1-a_i)\ [Q^*_{it}/L^*_{it}] = a_i\ [Q^*_{it}/K^*_{it}] = 1$$

Here Q^*_{it} is the benchmark output in the ith sector and other variables with superscript *'s refer to the adjusted 'benchmark' values of the corresponding unadjusted variables (obtained from national income accounts and other sources). This indicates, how the estimated parameters a_i can change substantially from year to year due to changes in Q^*_{it}, L^*_{it}, and K^*_{it} while the underlying true a_i may have remained stable. Since variables are subject to random shocks in the real world, ratios of adjusted values or values of their combinations would tend to change from year to year. These changes may be quite substantial depending on the random shocks giving different estimates of the parameters (over the same sample period in different years) while in practice true value may be unchanged.

Because the consistency adjustments of the data is not based on any criterion function, various sets of combinations may satisfy the same consistency conditions. We may very well construct a large number of estimates for the same parameters, one corresponding to each combination of adjusted data set satisfying the marginal and consistency conditions. Above all, from a statistical point of view we do not know anything about the properties of the estimates obtained in this way and no hypothesis testing is possible.

This method, however, has the computational advantage of saving efforts and costs. This is a movement in the right direction in the sense that it is a conscious effort to get the parameter values which correspond to the adopted sectoral classification. While other crude methods adopt parameters from literature search and national income accounts, this benchmark method integrates the two to generate a set of mutually consistent parameters.

Stochastic estimation: An obvious problem with the calibration procedures outlined above is the need to adjust basic data to 'force' equilibrium conditions to hold before the deterministic calculation of parameter values can be made. The need to adjust data can be interpreted simply as measurement errors in the equilibrium conditions which in fact prevail and therefore are not so illegitimate. In line with traditional econometrics, an alternative view would be that the underlying process generating the data is stochastic rather than deterministic. This view would not prescribe the adjustment of basic data but instead suggest using a stochastic formulation of the general equilibrium model which would be estimated from unadjusted data. This would, if performed appropriately, also allow for statistical tests of model specifications against the data, which is not possible with the calibration procedures.

The simplest way to obtain the relevant parameters of a general equilibrium model is to separately estimate the commodity demand system and the production functions or appropriately formulated production system. The vast literature on applied consumption and production analysis is clearly relevant in this context and in this section we briefly refer to some of this literature.

A complete general equilibrium system can be disaggregated into three broad interrelated subsystems:

1. a commodity demand system;
2. a production system; and
3. the market equilibrium conditions.

A commodity demand system derived from the underlying (direct or indirect) utility function characterizes consumer behavior in the tradition of applied (micro) consumption analysis. The production system characterizing producer behavior, given the technology underlying the production function, forms the second subsystem. The market equilibrium conditions then coordinate the separate subsystems.

This model disaggregation is outlined in Figure 1. Within part A, A-1 describes the consumer's optimization process and A-2 the optimizing behavior of producers. The two subsystems of part A can be expressed in convenient notations as:

$$F_1 [g_1, Z_1, b_1] = e_1 \quad \ldots \text{(A-1)}$$

$$F_2 [g_2, Z_2, b_2] = e_2 \quad \ldots \text{(A-2)}$$

Here $g_s (s=1,2)$ are vectors of endogenous variables; prices and income are subsumed under the vector Z_s along with other predetermined variables. By construction, different subsystems in part A do not have common parameters and errors in one can be assumed to be independent of those of others. However, these two subsystems are linked to each other indirectly through the subsystem in part B containing market equilibrium conditions. This subsystem generally contains parameters common to both A-1 and A-2.

From this formulation it is clear that independent estimation of commodity demand and production system would provide estimates of all the relevant parameters that are needed to solve a complete general equilibrium model. If equilibrium conditions are ignored we could restrict ourselves to the separate estimations of complete demand and production systems. This is the case where the vast literature on applied econometric works developed over the years surrounding the key issues of estimating the

Figure 1

General Equilibrium Subsystem Decomposition: Consumer's and Producer's Optimization Separated from Market Equilibrium Conditions 1/

PART A

A-1: Consumer's Optimization Process

Consumer Demand System:

$F_1 (g_1, Z_1, b_1) = e_1$

Obtainable through consumers' utility maximization process.

g_1: a vector of endogenous variables

Z_1: a vector of variables exogenous to the consumer decisions including commodity prices and income.

b_1: parameter vector which includes all relevant parameters of the specified demand system

e_1: a vector of random error terms

A-2: Producer's Optimization Process

The equation system from producer cost minimization behavior is represented in a matrix function:

$F_2 (g_2, Z_2, b_2) = e_2$

g_2: a vector of endogenous choice variable

Z_2: a vector of variables exogenous to the producer's optimization activities

b_2: a vector containing all the relevant parameters of the specified system

e_2: a vector of random error terms

PART B

Market General Equilibrium Conditions

$Q_{it} = \Sigma a_{ij}Q_{jt} + g_{it}(Z_1, b_1)$

$\Sigma_i L_{it} = \overline{L}_t$

$\Sigma_i K_{it} = \overline{K}_t$

Conditions (1) and (2) together ensure that demand does not exceed supply for each of the products and inputs.

$P_i = \Sigma_j a_{ij}P_j + w \cdot l_i + r \cdot k_i$

or in vector notation,

$P = [I - A]^{-1}(wl + rk)$

This ensures zero profit conditions prevail in equilibrium for each activity operated at a positive level of intensity

1/ The decomposition given in this represents a class of general equilibrium models with two primary factors of production and fixed coefficient intermediate inputs.

production and demand systems can be utilized directly. In general, however, this simultaneity can not be completely ignored, and the issue facing users of subsystem estimates is to determine exactly what the degree of unreliability is.

Demand systems: In the empirical analysis of demand systems, income (Y) and prices (P, in vector form) are taken to be exogenously given. The budget balance condition along with other restrictions taken from the theory of consumer demand act as constraints on the system, which in compact notation can be represented as:

$$X = f\,[Y, P] + U$$

where X is a n-vector of quantities of n-goods and services demanded. Although there are an infinite variety of possible functional forms for the demand system, these can be classified under three different approaches. All of these three functional forms, if they satisfy the restrictions of consumer theory, can potentially be used as component subsystems of general equilibrium models. Two of these functional forms originate from appropriately specified direct and indirect functions and thus capture almost all the relevant properties of demand system. 1/

1/ Stone (1954), Pollak and Wales (1969), Deaton (1972) are examples of papers dealing with demand systems originating from direct utility functions. Johansen (1969) formulates a general additive utility function which implies that the Direct Translog of Christensen et al. (1975), Stone-Greary and CobbDouglas type utility functions are the special cases. Houthakker (1960) introduced the indirect utility function and empirically estimated the derived demand system. Various forms of Leontief reciprocal indirect utility function are suggested and estimated by Diewert (1969, 1974), Gussman (1972), Darrough (1975); for translog utility functions, see Christensen et al. (1975), Jargenson and Lau (1975), and Christensen and Manser (1975).

The third approach starts with directly specified demand equations and imposes theoretical restrictions in the process of estimation (see Nasse (1973), and Barten (1977)).

The theoretical constraints (which also depend on the specification of utility functions) along with the budget constraint imply cross equation restrictions. Joint estimation is a necessity and full information methods such as FIML (full information maximum likelihood) or variants of GLS methods (e.g., Zellner (1962)) can be employed. One equation is conventionally dropped from the system to avoid the problem of singularity of the variance-covariance matrix (due to the budget constraint). All these are widely discussed topics in applied consumption analysis, an overall survey of which also reveals that variations in income and prices generally can explain only some variations in observed demand (see Barten (1977), Desai (1976), and Bridge (1971)). The importance of prices in econometric studies on demand seem to be less than what the theory would suggest and is important if a fine definition of commodities is used. Moreover, for broad aggregates, the price indices over time tend to move in the same direction following similar patterns making substitution possibilities insignificant. Thus for more aggregate models (like the one we intend to estimate in section IV), researchers may be inclined to adopt simpler specification with limited or restricted (price) substitution effect.

It is widely believed that consumer demand models (in the form these are being estimated) concentrate on one side of the market and additional strength (or explanatory power) might come from the simultaneous considerations of demand and supply (Barten (1977)). Although no systematic investigation has been made in this direction by explicitly introducing the supply side, the general equilibrium formulations can potentially take care of this. However, most of the applied consumption literature can be seen as independent exercises and very little attention has been paid to see this as a part or subsystem of an overall general equilibrium type model. Nevertheless, the theoretical understanding and consensus developed over the last three decades surrounding the specification and estimation of sets of demand equations would always be useful to the applied general equilibrium model builders.

Production relations

On the production side, once again one can utilize the methodologies suggested in a series of well-known papers on the estimation of production functions. However, our purpose is not simply to estimate the technological relations called production functions, but emphasize the Marschak-Andrew (1944) approach which utilizes the optimizing behavior of the producer. 1/

1/ Marschak and Andrew (1944) have shown that under perfect competition and profit maximizing conditions the Ordinary Least Squares (OLS) method does not yield consistent estimates of the parameters of Cobb-Douglas production function. This essentially stresses the simultaneous equation character of the estimation procedure and sometimes the associated problem of identification. For a competitive firm maximizing profit (or minimizing cost) the choice of inputs is governed by economic considerations, along with the technological relation characterizing the production function. Thus independent estimation of technological relation alone would not provide consistent estimates. Hoch (1958), Kmenta (1964), and Mundlak (1963) provided consistent estimators of the (Cobb-Douglas production function) parameters assuming that technical disturbances (associated with production function) and the 'economic' disturbance (in the profit maximizing equations) are uncorrelated. Further, in another classic paper, Zellner, Kmenta, and Dreze (1966) show that under the assumption of expected profit maximization and perfect competition, OLS does provide consistent and unbiased estimates of the parameters. Most of these discussions, although, cast in terms of Cobb-Douglas production functions, can very well be relevant for various other specifications as well.

The use of inputs is governed by economic considerations and hence the estimation of the parameters of any production system should be based on the optimum input combination or the derived input demand system. Since firms tend to choose that bundle of inputs which minimize the total cost of producing a given level of output, the derived demand for inputs depends on the level of output and substitution possibilities among inputs as permitted by the production technology, given relative input prices.

If we impose the condition that factors are fully employed in the conventional general equilibrium tradition, the endowment constraint (if relevant) would be binding and impose cross equation restrictions among the random errors of input demand equations. In existing econometric literature on input demands the focus is only on the input demand functions for particular sectors. Cost minimization conditions and various other cross equation restrictions (depending on the specification used), are frequently imposed. The input demand functions employed in this empirical literature are either obtained directly from the production function using cost minimization or indirectly from cost functions using Shephard's Lemma.

For both approaches a variety of functional forms have been used. Using the CES specification of Arrow, Chenery, Minhas, and Solow (1961) (under constant returns to scale) and cost minimization, the production system can be represented as:

$$Q_i = A_i \, [a_i \, K_i^{-\rho_i} + (1 - a_i) \, L_i^{-\rho_i}]^{-1/\rho_i}$$

$$A_i^{-\rho_i} \cdot a_i \cdot (Q_i/K_i)^{(1 + \rho_i)} = P_K$$

$$A_i^{-\rho_i} \cdot (1 - a_i) \cdot (Q_i/L_i)^{1 + \rho_i} = P_L$$

where, Q_i = Value added in the i-th sector,

K_i = Amount of capital used in the sector,

L_i = Amount of labor used in the sector,

a_i = The share parameter (of capital),

ρ_i = The elasticity parameter,

A_i = The scale parameters, and

P_k and P_L = Prices of capital and labor inputs.

This system in intensive form (i.e., per unit of labor terms) can be expressed as:

$$q_i = Q_i/L_i = A_i \, [a_i \cdot K_i^{-\rho_i} + (1 - a_i)]^{-1/\rho_i}$$

$$k_i = K_i/L_i = [(1 - a_i)/a_i]^{1/(1 + \rho_i)} \times [P_L/P_k]^{1/(1 + \rho_i)}$$

We can estimate this production system either in the direct form or in its intensive form (with or without logarithmic transformation).

Berndt and Wood (1975), and Christensen et al. (1971, 1973) use translog cost functions, while other forms such as generalized Cobb-Douglas cost functions are discussed by Diewert (1971 and 1973). In its general form the cost function can be expressed as:

$$C = C\,(Q, P_i, \cdots Pn)$$

where C is the total cost, P_i's (i = 1, ···, n) are the input prices and Q is the flow of gross output of j-th product. Using a translog cost function with symmetry and constant returns to scale imposed, the cost function can be expressed as:

$$\ln C = \ln a_o + \ln Q + \sum_i \alpha_i \ln P_i + 1/2 \sum_i \sum_j \gamma_{ij} \ln P_i \cdot \ln P_j$$

Linear homogeneity in prices impose restrictions:

$$\sum_i \alpha_i = 1$$

and $\sum_j \gamma_{ij} = 0$ for all i.

Using Shephards Lemma, input demand equations are obtained which can be expressed in cost share form as:

$$S_i = a_i + \sum_j \gamma_{ij} \ln P_j \; ; \; i,j = 1, \cdots, n$$

and $\sum_i S_i = 1$

where the S_i are the shares of inputs in the total cost of producing Q.

In another recent piece Jorgenson and Fraumeni (1980) employ translog price functions, impose homogeneity (of degree one), and derive sectoral value shares of factors. Since share elasticities with respect to prices are symmetric, further cross equation restrictions are imposed. As the cost or value shares of the factors of production for each sector sum to unity, in Berendt and Wood (1975) and Jorgenson-Fraumeni (1980), the sum of the disturbances across the input cost shares or value share equations is zero at each observation. To avoid singularity of the covariance matrix they drop one equation arbitrarily (for each sector) and assume the disturbances in remaining equations to be independently and identically normally distributed with zero mean and variancecovariance matrix Ω.

This literature on input demand systems emphasizes producer behavior and cost minimization, but ignores economy-wide restrictions which the general equilibrium approach suggests. This is perfectly consistent with the behavior of a single competitive price taking firm, but potential use of these methods for general equilibrium models incorporates analysis of broad aggregate sectors with economy-wide restrictions. At an aggregate industry level input prices are unlikely to be exogenous and exogenously given factor endowments introduce additional cross equation restrictions (among different sectors), particularly across the errors. 1/ Thus, in estimating production systems for use in multisector general equilibrium analysis it may be inappropriate to assume that prices are exogenous and that regressors in the input demand system are uncorrelated with the disturbances. If we are interested in obtaining consistent estimates of the parameters of the production subsystem of a general equilibrium model this simultaneity should be taken into account. 2/

1/ In terms of the production system specified earlier $\sum_i L_{it} = L_t$ and $\sum K_{it} = \bar{K}_t$ imply that all the errors are to be associated with Lit's or Kit's (for all $i = 1, \cdots, n$) are not independent of each other. In the intensive form $\sum_i \theta_i K_{it} = k_t$ where $\sum_i \theta_i = 1$ which implies that $\sum_i \theta_{it} \hat{U}_{it} = 0$ for all t.

2/ Berendt and Wood (1975) use the instrumental variable method to deal with the simultaneity. The instruments are formed by regressing each of the regressors on a set of variables considered to be exogenous to the aggregate sector.

Allingham's approach

Most econometric work on commodity and production (or input demand) systems proceeds independently and is not developed in the context of general equilibrium modelling where classifications of the two systems must match and restrictions from the general equilibrium model are fully incorporated. Using an alternative approach, Allingham (1973) attempts to capture the general equilibrium market clearing process while giving less attention to the specification of production and demand systems. Allingham estimates a form of general equilibrium model where he uses a variant of a 'Keynesian type' ad hoc consumption function not based on utility maximization. Income, or expected income, determine total consumption and its division among different categories then depends on relative prices (represented by the price for the category relative to the general consumer price level). Moreover, to avoid nonlinearities in both parameters and variables he uses a linear production function (linear in inputs). This has the two features that the marginal product of any factor is constant and the elasticity of substitution (between the factors) is infinite, implying perfect substitutability. Because of these simplifications and the limited treatment of commodity and input demand systems (along with underlying cross equation restrictions), this model reduces to a system of linear simultaneous equations. This system does not, however, satisfy the usual properties of production and consumer demand systems, such as homogeneity, Walras Law and other restrictions which are the cornerstones of Walrasian general equilibrium analysis.

All these simplifications enable Allingham to use conventional limited information (single equation) methods like Two Stage Least Squares (2SLS) or Limited Information Maximum Likelihood (LIML). Ignorance of restrictions (or information coming from economic theory, if those are true), necessarily makes his model less attractive on many counts. However, this is (to our knowledge) the first work which formally attempts to incorporate some features of general equilibrium modelling into a complete system approach and estimates the model parameters using some time series data.

Allingham examines the performance of this complete system approach compared to traditional macro models. He estimates the model with the U.K. data over the period 1956 through 1966. In the standard macroeconomic tradition three classes of agents are distinguished: consumers, producers, and the government. It distinguishes ten producing agents (or industries) based on the Standard Industrial Classification (SIC) for 1958. After estimating the unknown parameters by 2SLS, the estimated model is solved to obtain some quantitative static and dynamic properties of the system. After ensuring that the equation system $Y = F(X)$, typically has a solution for various values of X, Allingham studies the properties of the solution. He finds that general equilibrium predictions on the whole are significantly more accurate than the partial equilibrium prediction, typically by a factor of three. His final conclusion is that the complexities associated with general equilibrium modelling are justified in predicting equilibrium values of broad aggregates and individual components and also for investigating the process whereby the equilibrium positions are reached.

III. Outline of some methods to estimate general equilibrium models

In the current and following sections we visualize the working of the economy in a stochastic framework, different from the deterministic formulations discussed earlier. We begin with a simple general equilibrium model with production and discuss some of the problems with FIML. In view of its potentially large dimension, we suggest methods to estimate smaller dimensional subsystems using some nonlinear instrumental variables method. The choice of subsystems would vary in accordance with different specifications, the general nature being as described in Figure I.

Consider an economy with fixed endowments of labor ($\overline{L}_t$) and capital ($\overline{K}_t$) at time period t. Given the utility and production functions, the conventional behavioral assumption of utility maximization by consumers, cost minimization by producers and the relevant policy structure (if any), we can specify the complete stochastic model. The full system consists of subsets of equations representing commodity demand systems, a set of input demand functions, full employment conditions for inputs, demand-supply balance conditions along with zero profit conditions by sector and the flow of funds relation to define income. The complete model is represented in a convenient general form as:

$$F_t\ [Y_t,\ X_t,\ B] = e_t \qquad \text{(III-1)}$$

where Y_t is a (1XM) dimensional row vector of endogenous variables and X_t is a (1XK) vector of exogenous variables. F_t [*] is a (1XM) vector of functions $[f_{1t}, f_{2t}, \ldots, f_{M_1 t}]$, which in the neighborhood of R- dimensional space of true parameter values B, are assumed to be uniformly bounded and twice differentiable with uniformly bounded derivatives. The (1XM) vector of errors, e_t, is assumed to be multi-variate normal with expected valûe zero and variance-covariance matrix Σ.

The identifiability of the parameters of this system has to be ensured and is assumed to be satisfied. General equilibrium systems usually contain a large number of endogenous variables along with relatively fewer exogenous ones, which might cause potential identification problems. However, restrictions of specific general equilibrium formulations such as constant returns to scale, budget balance, homogeneity, etc., provide additional restrictions. We assume, for now, that these independent restrictions along with exclusion restrictions can provide us enough information to identify the complete model.

Equation (III-1) can be written in another convenient form, following Jorgenson and Laffont (1974), as:

$$f_{it} [Y_{1t}, \cdots, Y_{M,t}, X_{1t}, \cdots, X_{kt}, B_i] = e_{it}$$

where $i = 1, \cdots, M$ and $t = 1, \cdots, T$.

$[X_{kt}]$, $k = 1, \cdots, K$ are predetermined variables

$[Y_{mt}]$, $m = 1, \cdots, M$ are endogenous variables

$[B_i]$, is a R_i vector of parameters; $i = 1, \cdots, M$.

We assumed that the Jacobian of the transformation from the disturbances to the observed randam variables, Y_t, is nonvanishing. Given the multinormal distribution for the errors we can derive the likelihood function. The logarithm of the likelihood function is:

$$L = - [MT/2] \text{ Log } (2\Pi) + T/2 \text{ Log}|\det \Sigma^{im}| + \sum_t \text{Log}|\det J_t|$$

$$- 1/2 \sum_{i,m,t} f_{it} \Sigma^{im} f_{mt}$$

where f_{it} is as defined above;

$\Sigma^{im} = [\Sigma_{im}]^{-1}$, the inverse of the variance-covariance matrix

$J_{i,m,t} = (\delta f_i / \delta Y_m)_t$

and J_t is the matrix of such derivatives. The loglikelihood function is concentrated as:

$$\frac{\delta L}{\delta \Sigma^{im}} = T/2 \Sigma_{mi} - (1/2) \sum_t f_{it} f_{mt} = 0$$

$$\hat{\Sigma}_{mi} = \hat{\Sigma}_{im} = (1/T) \sum_t f_{it} f_{mt}$$

Substituting this expression into the loglikelihood function gives the concentrated loglikelihood function:

$$L^*(B) = \text{constant} + \sum_t \log|\det J_t| - (T/2)\log|\det \hat{\Sigma}|$$

Here we can assume that $E(e_{it}) = 0$; for all $i = 1, \cdots, M$; $t=1, \cdots,$

T; E $[e_t e_t] = \Sigma$ is positive definite and has full rank, while E $[e_t e_t'] = 0$ if $t \neq t'$.

At this stage we can use any of the algorithms suggested by Eisenpress and Greenstadt (1966) and G. Chow (1973) which use Newton's method of the variant or the gradient maximization method suggested by Berndt, Hall, Hall and Hausman (1974). The later algorithm by Berndt et al. has the advantage that convergence to a local maximum of the likelihood function is guaranteed and unlike the Newton methods as employed by Eisenpress and Greenstadt (1966) and Chow (1973) does not involve the evaluation of third derivatives. This requires the evaluation of the model up to second derivatives while third derivatives are eliminated by taking advantage of the fundamental statistical relation that the asymptotic variance-covariance matrix of a maximum likelihood estimator is equal to the variance-covariance matrix of the gradient of the likelihood function.

This full treatment, although most satisfactory from the statistical point of view, is of limited applicability to applied general equilibrium modellers for a number of reasons. One major problem with a wide class of general equilibrium models is that the likelihood function may not be well defined. For example, a complete general equilibrium model will have market clearing conditions requiring that labor and capital employed in all sectors together add up to the exogenously given factor endowments. Errors in the input demand functions are thus not independent of each other and we cannot define a likelihood function for the model since we do not have independently distributed error terms.

Another major problem with any full information estimation method for general equilibrium models is that the number of parameters to be estimated increases very rapidly with each subsequent increase in the number of sectors. With a moderate sample size many of the applied general equilibrium models would be of the type for which the number of independent parameters to be estimated may very well exceed the number of available data points. The degrees of freedom problem is very acute if the translog framework is used in modelling both production and consumer behavior. For example, in the thirty-six sector model of Jorgenson and Fraumeni (1980) there are thirty-eight relative share parameters for each sector (these are relative shares of inputs in the value of the output of each sector). Also there are relative shares parameters representing relative shares of commodity groups in the value of total consumer expenditure. Besides these share parameters, there are 'second-order' parameters that correspond to the measures of substitutability in production and consumption. Jorgenson-Fraumeni estimates these parameters subject to separability restrictions, leading to a hierarchy of submodels. In their empirical implementation about fifty such parameters are estimated for each sector. An additional fifty or so are needed to specify consumer behavior. The total number of second-order (or elasticity) parameters is approximately 2,000 for the thirty-six sector model considered by Jorgenson and Fraumeni. Thus for models of this type any full information method has little hope for implementation.

One conceivable solution to the problems is to apply full information methods to subsystems of the complete general equilibrium model by dividing it in such a way that structural equations whose coefficients or error variances are related to each other, are part of the same subsystem. This enables us to impose cross equation constraints for each of the subsystems. The most natural choice of subsystems are the demand system and production structure that are embodied in the complete general equilibrium formulation and we now turn to the separate estimation of these subsystems.

Estimation of general equilibrium subsystems

The specification of subsystems in general equilibrium systems varies from model to model and in general they may be linear or nonlinear in variables, nonlinear in parameters or nonlinear in both variables and parameters. Let us consider a subsystem of the general model (III-1) which consists of L (<M) equations and in normalized form can be expressed as: 1/

$$Y_{it} = f\,(G_{it}, B) + e_{it} \quad \cdots \qquad \text{(III-2)}$$

where Y_{it} is the t-th observation on the dependent variable in the i-th equation and e_{it} follows a normal distribution as described earlier. G_{it} is a g_i-component vector of observations at time t on endogenous functions appearing in the i-th equation. Each of these functions contains

1/ In fact subsystems of complete general equilibrium models very often appear in such forms as shown in (III-2), so that further normalization may not be essential.

endogenous variables (i.e., random variables correlated with e_{it}) and exogenous variables or combinations of the two. $G_{it} = [g_{1t}^{i}, \cdots, g_{g_i,t}^{i}]$, where in general $g_{j,t}^{i} = g_j^{i}[Y_t, X_t]$ and $Y_t = [Y_{1t}, \cdots, Y_{Mt}]$ is the complete set of endogenous variables of the system (III-1). X_t is the set of all exogenous variables.

It is implicit in the construction of the subsystems that some of the Y's are determined outside the subsystem but within the complete model. If, for example, we look at any demand or expenditure system, the income term is not endogenously determined within the subsystem while in a complete general equilibrium model income is determined endogenously. In general terms, if we treat $Y_{1,t}, \cdots, Y_{M,t}$ as endogenous variables, then $Y_{L+1}, \cdots, Y_{M,t}$ need careful treatment. One might hope that we could treat the variables $Y_{L+1,t}, \cdots, Y_{M,t}$ as exogenous or predetermined for this subsystem and go ahead with the usual NLFIML estimation procedure. It is worth noting that this would give us the conventional estimates of demand and production systems (as described earlier). However, strictly speaking this would give us consistent estimators only if the variables $Y_{L+1}, \cdots, Y_{M,t}$ are uncorrelated with e_{it}'s (of the relevant subsystem), a condition that would not generally be satisfied for general equilibrium models.

In this case we can use a Nonlinear Three Stage Least Squares (NL3SLS) estimator -- a form of instrumental variable estimator often referred to as the minimum distance estimator. Here we first transform the normalized structural form of any given subsystem and then minimize the sum of squares of errors through an iterative procedure. We begin with the subsystem (III-2) represented in vector form as:

$$y - f(G, B) = e \quad \cdots\cdot (III-2')$$

Premultiplying each equation by a matrix $Z'_{(KXT)}$ such that: $E(Z' \cdot e) = 0$, ..., (III-3). The transformed system is:

$$(I_L \otimes Z')[y - f(G, B)] = [I_L \otimes Z'] e$$

Assume, $E[ee'] = \Omega \otimes I_T$; where $\Omega = \begin{bmatrix} W_{11} & \cdots & W_{1L} \\ \vdots & \ddots & \vdots \\ W_{L1} & \cdots & W_{LL} \end{bmatrix}_{(LXL)}$

$$E(e_{it} \cdot e_{jt}) = 0 \text{ for } i \neq j \text{ and/or } t \neq t'$$
$$= W_{ij} \text{ for } i = j \text{ and } t = t'$$

For the transformed system the variance-covariance matrix can be represented as:

$$E[I_L \otimes Z'] \cdot ee' \cdot [I_L \otimes Z]$$
$$= [I_L \otimes Z'][\Omega \otimes I_T][I_L \otimes Z]$$
$$= \Omega \otimes (Z'Z)$$

The minimum distance estimator can be obtained by minimizing:

$$J(B) = [y - f(G, B)]'[I_L \otimes Z][\Omega \otimes Z'Z]^{-1}$$
$$[I_L \otimes Z'][y - f(G, B)]$$
$$= (y - f)' S (y - f) \quad \cdots\cdot (III-4)$$

where, $f = f(G,B)$ and

$$S = [I_L \otimes Z]\ [\Omega \otimes Z'Z]^{-1}\ [I_L \otimes Z']$$

Under certain regularity conditions (following Ammeniya (1977, 1974), Jorgensen and Laffont (1974)) it can be shown that the estimated value $\hat{B}$ converges in probability to the corresponding true value B and $\sqrt{T}\ (\hat{B} - B)$ converges in distribution to:

$$N\ [0,\ (H'\ (\Omega \otimes M)^{-1}\ H)^{-1}]$$

where, $M = \lim_{T \to \infty} \frac{1}{T} Z'Z$, which is assumed to exist and be nonsingular.

$$H_i = \operatorname{Plim}_{T \to \infty} \frac{1}{T} Z' \frac{\delta Fi}{\delta B'}, \text{ uniformly in B}$$

$$\text{and so, } \operatorname{Plim}_{T \to \infty} \frac{1}{T} Z' \frac{\delta f}{\delta B'} = \begin{bmatrix} H_1 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ H_L \end{bmatrix} = H \text{ is of rank R}$$

uniformly in B, where,

$$\left|\frac{\delta fi}{\delta B'}\right| = \frac{\delta fi'}{\delta B} = \begin{bmatrix} \frac{\delta fi}{\delta B'}\ (Z_{i1},\ B),\ \cdots,\ \frac{\delta fi}{\delta B'},\ (Z_{iT}, B) \\ \cdots \\ \frac{\delta fi}{\delta B^K}\ (Z_{i1},\ 3),\ \cdots,\ \frac{\delta fi}{\delta B^K}\ (Z_{iT}),\ B) \end{bmatrix}$$

For the existence of such an estimator with this property further regularity assumptions are needed (see, Amemiya (1974, 1977), Jorgensen and Laffont (1974)). These are that e_t's are independently identically distributed random vectors and $\frac{1}{T} \frac{(\delta^2 F}{(\delta B_i \delta B_i)}$ .Z converges in probability to a constant matrix uniformily in B (for i=1, 2, ····, R), where B_i is the i-th element of B. Under these assumptions a minimum distance estimator exists with properties described above. Minimization of Equation (III-4) is to be done by an iterative procedure as it is highly nonlinear in both parameters and variables. Gauss-Newton or its different variants are normally employed to this end. The iterations take the form:

$$\hat{B}_{(n)} = \hat{B}_{(n-1)} + \left| \frac{\delta f'}{\delta B} \cdot S \cdot \frac{\delta f}{\delta B} \right| \cdot \frac{\delta f'}{\delta B} \cdot S \cdot (y - f) \quad \cdots\cdots \quad \text{(III-5)}$$

where f (*) and $\frac{\delta f}{\delta B}$ are evaluated at the estimated parameter values of the (n-1)th iteration, i.e., at $\hat{B}_{(n-1)}$. For more discussion of the properties of this estimator see Amemiya (1974, 1977), Berendt et al. (1974), and Jorgensen and Laffont (1974).

Selection of instruments: So far we have ignored the issue of selecting the components of the matrix Z, the matrix of instruments. Any set of variables that are uncorrelated with the errors (so as to satisfy condition III-3), but at the same time highly correlated with the endogenous functions G_{jt}'s should qualify as instruments. We suggest two methods of selecting the instruments. The first method is somewhat analagous to that suggested by Goldfeld and Quandt (1968), Kelejian (1971), and Amemiya (1974) which is to regress each G_{jt} on the elements of a polynomial of degree r in X_t. The underlying regression model can be expressed as:

$$G_{jt} = Z_{jt} + \eta_{jt}$$

where $Z_{jt} = \theta_0 + \theta_{1,1} X_{1t} + \cdots + \theta_{1,K} \cdot X_{K,t} + \cdots$

$$\theta_{r,1} \cdot X^r_{1t} + \cdots + \cdots + \theta_{r,K} \cdot X^r_{K,t}$$

$$j = 1, \cdots, g_1; \; t = 1, \cdots, T.$$

Here X's are the elements of X_t and η_{jt} are residuals which are not correlated with the elements of Z_{jt}. Defining,

$$Q_t = [1, X_{1,t}, \cdots, X_{K1,t}, \cdots X^r_{K1,t}]$$

and θ_j, the associated vecotr of parameters. The instruments we are considering are:

$$\hat{Z}_{jt} = Q_t \hat{\theta}_j$$

where, $\hat{\theta}_j = (Q'Q)^{-1} Q'G_j$; Q being the matrix formed with the vectors Q_t and G_j is the vector with corresponding elements G_{jt} for $t = 1, \cdots, T$. The matrix Z would then consist of low order polynomials of all the exogenous variables of the complete system.

Alternative method: In the method described above, the instruments are obtained by regressing the endogenous functions (G_{it}'s) on low order polynomials in the exogenous variables of the complete system. This takes care of the inherent simultaneity and thus the estimators are consistent. But this does not directly take care of the restrictions of general equilibrium model, such as full employment of factors. An alternative method is to impose these restrictions on the instruments to be used with the expectation that this would serve the role better.

Using the method described earlier, a set of consistent estimators are obtained for the subsytems of the model. Adopting the parameters from one set of subsystem estimates we can go on to impose the conditions specified in the other that would ensure the restrictions of the general equilibrium model. General equilibrium solution algorithms can be employed at this stage to produce appropriate set of instruments. Using these equilibrium magnitudes (e.g., prices, income, or various combinations of these) as instruments we can reestimate earlier subsystems repeating the methods outlined above. In applications solution algorithms apply to each year's data separately as the endowments of factors change from year to year. The process is repeated for each year's data with changed values of exogenous variables, yielding a time series for the instruments to be used in subsystem reestimation in the next round.

The vectors (in time series form) obtained in Part B (Fig. I) are in principle equivalent to what in the linear case for a model

$$Y\Gamma + XB = U$$

can be represented as, $\hat{Y} = X\hat{\pi}$ where $\hat{\pi} = -\hat{B}\hat{\Gamma}^{-1}$; $\hat{B}$ and $\hat{\Gamma}$ are the matrices obtained from the coefficients of 2SLS or any other consistent estimation of individual equations of the system. In the context of linear systems such estimates ($\hat{Y}$) of endogenous variables are often called derived reduced from forecasts. The derivbed reduced form estimates for the case of non-linear models of the general equilibrium type is not easy to obtain. Fortunately we can use fixed point or other algorithms to obtain the derived reduced form estimates (the deterministic part) of the endogenous variables or functions. In terms of our generalized notation for the complete model,

$$F_t (Y_t, X_t, B) = U_t$$

we first estimate B by some suitable method (e.g., the method outlined above) to produce consistent estimators. We then produce forecasts

$$\hat{Y}_t = S (X_t, \hat{B})$$

The estimates of the endogenous functions from this kind of solved reduced form can be used as instruments (or in constructing appropriate instruments) for each of the submodels in reestimation.

This alternative method is basically an extension and partial modification of the method described earlier. Reduced form estimates of endogenous functions (G_{it}'s) can be used in the formation of matrix Z and once again we can apply the algorithm of the minimum distance estimator based on this newly constructed matrix. Properties of the estimators would essentially remain the same as described in the context of the earlier method.

IV. An Application to the U.S. Economy

General equilibrium models, as we all know, have a wide range of applications and the estimation procedures outlined in the previous section can potentially be applied to many of those. However, as one can easily understand, at the present state of computer speed the computation costs would tend to be quite substantial, especially if the dimension of the model is large and we are applying the second method (Method II). Keeping this in mind the suggested methods have been applied to a two sector general equilibrium model of the U.S. economy to capture the general equilibrium effects of differential taxation of income from capital. Although we are trying to measure the costs of distortions in capital use which are due to the nonneutrality of the tax system in the tradition of Harberger (1962, 1966), its use in the present context is basically to serve three purposes:

a. as an illustration of how the suggested estimation procedures can be applied to a real world problem;

b. to investigate the nature and the values of the estimates obtained from the 'bench-mark equilibrium data set' and those available from the currently suggested methods; and

c. to make comparisons of the measures of distortions: (1) with those from harberger's method using his parameter values vis-a-vis our estimators; and (2) with Shoven-Whalley (1972) and Shoven (1976) algorithmic approach (due to Scarf (1973)) using bench mark parameters.

For all the measures of distortions the same data set is utilized and the differences come only from the use of different sets of parameters. While on the one hand estimates from methods I and II have been employed, parameters from bench mark data set are utilized for the Shoven-Whalley methodology and for Harberger some ad hoc values are adopted. More or less the same type of sectoral classification as followed in Harberger. and Shoven-Whalley has been maintained. We have two sectors, the predominantly corporate, heavily taxed sector, and the lightly taxed sector (incorporating agriculture, housing or real estate, crude oil, and gas). The corporate sector includes mining, manufacturing, transport, communication, contract construction, electric, gas, and sanitary services, whole sale and retail trade, finance and insurance (except real estate), and services. Note that personal and business services and whole sale and retail trade are not overwhelmingly corporate in structure but are taxed heavily. Rosenberg (1969) shows that without allowing for taxes on their dividends, approximately 30 per cent of the income from capital generated in these activities is taken away by taxation.

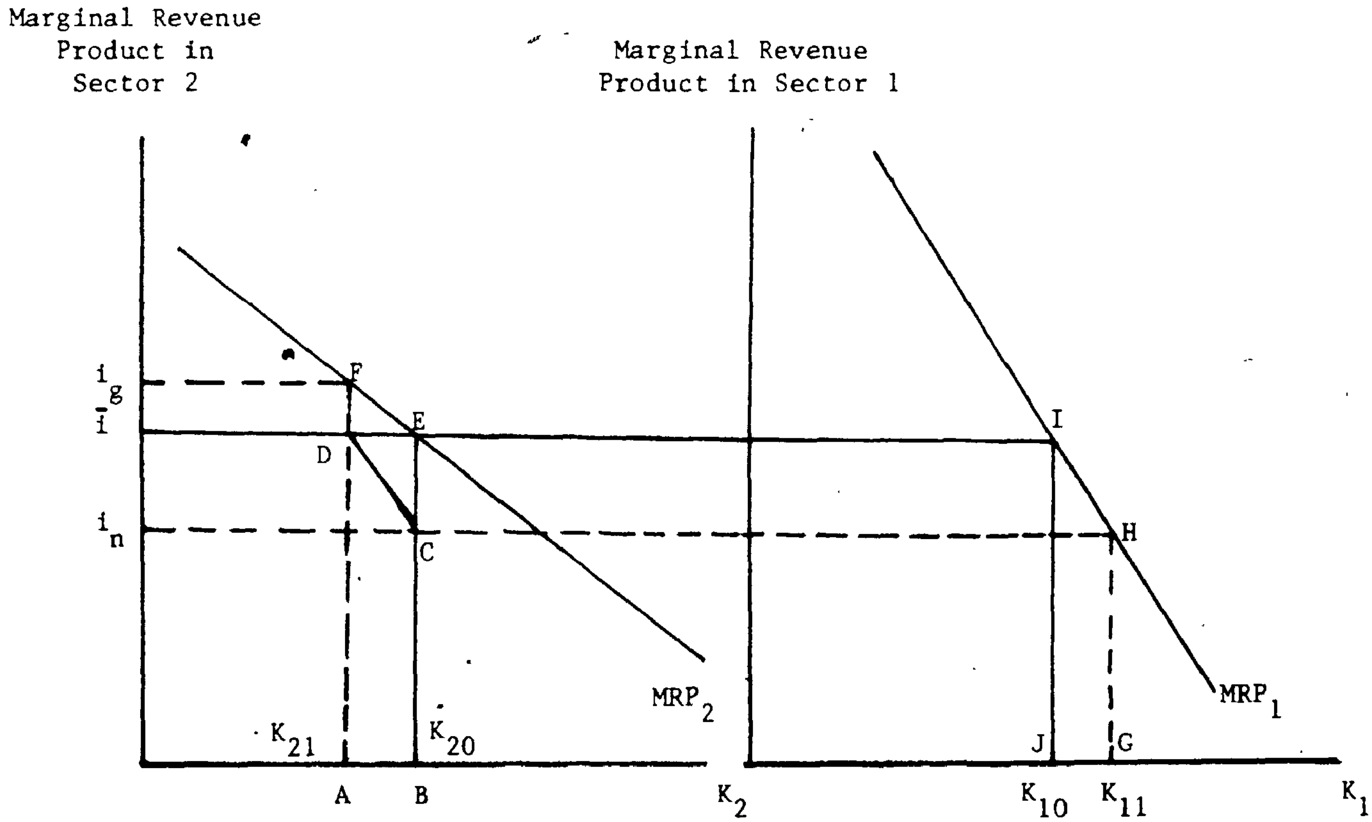
During the classification some deviations from the national income accounting has been made. The real estate industry as defined in the national income accounts includes all types of property. We follow the redefinition used by Rosenberg (1969) which include only nonfarm residential dwellings, rented, and owner occupied. The returns to capital for any other type of property has been assigned to the industry group which pays for the use of the property (using the proportions based on Rosenberg's study). For details on this reclassification see Appendix A-1.

Exposition of the Tax Probelm (Harberger Model)

The Harberger model as described in his (1962) article and subsequent follow ups distinguishes between a heavily and a lightly taxed sector, sometimes referred to as corporate and noncorporate sectors. This division does not exactly correspond to the legal distinction but due to the major role played by the corporation income tax in causing differential rates the heavily taxed industries are classified and summed as the corporate sector. Harberger employs the concept of the traditional Marshallian

Figure 2

Differential Taxes on Returns to Capital Corresponding Reallocation of Factors and Consequent Welfare Loss



producer surplus to evaluate the efficiency loss due to differential taxation. Assuming linear marginal product of capital schedule and commodity prices as unity by appropriate choice of units the marginal revenue product schedules of capital for both the sectors can be represented as downward sloping straight lines (as in Fig. 2). The total quantity of both labor and capital available in the economy at a particular year is assumed fixed and fully employed. In the absence of taxes the rate of return on capital, $\bar{i}$, would be equal for both the sectors with full employment. With tax on capital income (T_i) in the corporate sector (so that i = 1) net returns to capital ($i_n = i_{g_i} - T_i$; i = 1, 2; $T_2 = 0$; i_{g_i} being the gross return to capital in the i-th sector) would be equalized across the sectors, once again with full employment of capital.

In terms of Fig. 2, the area ABCD represents the reduction in output in sector 1, as K_2, the amount of capital employed in sector 2 decreases from K_{20} to K_{21} due to the imposition of the tax. Since capital is always fully employed so that $K_{20} - K_{21} = K_{11} - K_{10}$, output increases in sector 1, (the noncorporate sector) by the area GHIJ. Then it is simple to show that EFDC represents the social loss in the Marshallian sense. This loss being the sum of two triangles can be represented as:

$$(1/2)\cdot(i_g - \bar{i})(K_{20} - K_{21}) + 1/2\,(\bar{i} - i_n)(K_{11} - K_{10}) = (1/2)\cdot T\cdot \Delta K_1 \quad \text{(IV-1)}$$

Since, $(i_g - \bar{i}) + (\bar{i} - i_n) = (i_g - i_n) = T$, the tax rate on capital income, and $(K_{20} - K_{21}) = (K_{11} - K_{10}) = \Delta K_1$, the reduction in capital use in the corporate sector.

Harberger obtains the expression for ΔK_1 by solving the system representing static two sector model (of Johnson (1956) type). Without going into its deduction (see Harberger (1962)), the solution for the capital shift can be expressed as:

$$\Delta K_1 = K_1 \cdot T_1 \cdot \frac{-E\,[f_{2K} \cdot S_1 \cdot \frac{L_1}{L_2} + f_{1K} \cdot S_2] - S_1 \cdot S_2 \cdot f_{1L}}{E\,[f_{2K} - f_{1K}] \left[\frac{K_1}{K_2} - \frac{L_1}{L_2}\right] - S_2 - S_1 \left[\frac{f_{1L} \cdot K_1}{K_2} + \frac{f_{1K} \cdot L_1}{L_2}\right]} \quad \text{(IV-2)}$$

where E = price elasticity of demand for commodity 1;

S_i = elasticity of factor substitution in sector i;

f_{iL} = share of labor in the ith sector;

f_{iK} = share of capital in sector i; and

T_i = tax per unit of capital utilized in sector i.

While Harberger directly evaluates the expressions derived above, the Shoven-Whalley (1972) algorithmic approach applies the search procedure due to Scarf (1973) for the computation of general equilibrium prices to the problem of the type described above. This formulation is not based on differential calculus and does not explicitly evaluate a function of the types IV-1 and IV-2. This requires no linearity assumption and is well suited for studying large distortions. Thus we can avoid the major criticism of the Harberger approach that it is based on local analysis and approximations. Moreover, the algorithm is capable of handling any number of sectors, primary factors of production and consumer groups. This algorithm computes a general equilibrium price vector (with all input and output prices as its components). This price vector has the property that for nonzero prices supply equals demand and for commodities (inputs and outputs) with zero prices supply is greater than or equal to demand. For those techniques (or activities) of production which are utilized, profit equals zero at this equilibrium price vector and constant returns to scale in production is assumed. Since the demand functions are homogenous of degree zero in terms of prices, the prices can be normalized to sum to unity. The algorithm is then essentially a search procedure on this unit simplex for an approximate equilibrium price vector, P*, which would satisfy the conditions of equilibrium. The approximation may be made closer and closer to the true equilibrium price vector through continuous grid refinement while searching on the price simplex. For more details on this method, see Scarf (1973), and Shoven and Whalley (1972).

While estimating the parameters in the deterministic approach, a benchmark equilibrium data set is constructed for each year. Using the input-output flow matrix for the U.S. economy (appropriately reduced to the desired dimension as described in the Appendix), general equilibrium conditions are imposed. To ensure mutual consistency between the sets of data, the RAS' adjustment method is employed. These bench-mark data sets are usually constructed with one particular model specification in mind, while some different model specification may be consistent with the same bench-mark data set. In the specification of models, tractable functional forms are used to describe the behavior patterns of consumers and producers. In practice the choice of functional forms for demand and production functions in applied general equilibrium models is guided by the considerations of computational ease and simplicity. In many cases to determine the equilibrium values it is simpler to work directly with consumer demand functions for commodities and the per unit output factor demand functions of producers. In our present analysis to maintain the desired comparability we restrict ourselves to the same functional forms as adopted for stochastic estimation procedures. These are the simple systems corresponding to Cobb-Douglas and CES production and utility functions.

Given the selected functional forms and the associated bench-mark equilibrium data set, the parameter values are obtained by the 'calibration' procedure. The set of estimated parameters must be such that they are capable of reproducing the complete equilibrium data set as an equilibrium solution to the model. The 'calibration' procedure thus uses the equilibrium condition of the specified model and the bench-mark equilibrium data set to solve for parameter estimates. Before this we need to separate the bench-mark transaction data into separate price and quantity observations. This generally involves a unit convention which has been described in the Appendix. (For more discussion on this see Piggott and Whalley (1976), and Mansur and Whalley (1981)).

To illustrate the method we can start with a CES value added function for each of i-industries:

$$X_i = A_i \left[\delta_i K_i^{-\rho_i} + (1 - \delta_i) L_i^{-\rho_i}\right]^{-1/\rho_i}$$

where A_i is the scale parameter defining units of measurement, δ_i the share parameters and σ_i (= $1/1 + \rho_i$), is the elasticity of substitution. K_i and L_i are capital and labor service inputs and X_i is the industry scale of operation.

Units are so chosen that $P_L = P_K = 1$ (the net of tax factor prices) at the bench-mark equilibrium, K_i and L_i are directly obtained from bench-mark data set. Once the value of σ_i is selected for each industry the values of share parameters δ_i are given by,

$$\delta_i = \frac{\left[\dfrac{K_i^{1/\sigma i} \ (1+t_i^K)}{L_i^{1/\sigma i} \ (1+t_i^L)}\right]}{\left[1 + \dfrac{(K_i^{1/\sigma i} \ (1+t_i^K)}{(L_i^{(1/\sigma i)}(1+t_i^L)}\right]}$$

In this case of factor tax model t_i^K and t_i^L are the tax rates on income from capital and labor, respectively. Values for A_i are then derived from the zero profit conditions for each industry given the unit definitions for outputs.

V. Stochastic Parameter Estimates and Comparison with Calibration Procedures

This section reports the estimated parameters of the two sector general equilibrium tax model discussed earlier. We consider both Cobb-Douglas and CES specifications; beginning with a Cobb-Douglas production and demand system we go on to the estimation of a model characterized by CES production with Cobb-Douglas demand system. 1/ Table V-1 reports the benchmark equilibrium parameter estimates for the Cobb-Douglas case. Since we are using time series, for every year data has to be adjusted to ensure the conditions of general equilibrium accounting (as described above). The 'RAS' method has been applied to yearly data obtained from national income accounting to ensure the equilibrium conditions.

Table V-2 presents the estimated parameters obtained by employing nonlinear instrumental variable methods as suggested above in section III. We have also shown the estimates obtained by applying FIML method (to the subsystems of the complete model ignoring the simultaneity).

1/ On the demand side we tried other variations like Linear Expenditure System and CES demand system, but the estimates either did not converge or were not of proper sign. Cross price effects are not found to be significant or of proper magnitude and this can possibly be due to the high level of aggregation followed in defining the sectors. Price indices for the sectors move very closely following the general trend in Consumer Price Index and this tends to make potential cross-price effects insignificant. LES is generally found to be appropriate and statistically satisfactory at commodity level studies which have not been followed here. The Cobb-Douglas demand system which is a special case of LES is, however, found to be good at this level of aggregation.

Table V-1: Parameters of the Cobb-Douglas Production and Demand System 1/ Over Time Under Bench-Mark Equilibrium Method

Year \ Parameters	a_1	a_2	b_1	b_2
1948	0.516	0.330	0.160	0.840
1950	0.564	0.333	0.147	0.857
1955	0.600	0.276	0.126	0.874
1960	0.633	0.298	0.109	0.891
1962	0.636	0.271	0.115	0.885
1965	0.687	0.280	0.115	0.885

1/ $Q_i = A_i K_i^{ai} L_i^{1 - ai}$: Production function.

$U = \prod_{i=1}^{2} X_i^{bi}$: Utility function.

Table V-2: Comparative Computational Experience with Different Methods of Estimation with the U.S. Data (Cobb-Douglas Production and Demand Systems)

Parameters / Method	FIML 1/ (Ignoring simultaneity)	NLIV 1/ (Method I)	NLIV 1/ (Method II)	Benchmark Equilibrium Parameters 2/
a_1	0.606	0.627	0.647	0.516-0.678 (0.609)
a_2	0.369	0.307	0.325	0.332-0.272 (0.292)
b_1	0.122	0.124	0.124	0.160-0.109 (0.124)
b_2	0.878	0.876	0.876	0.890-0.840 (0.876)

1/ All coefficients are statistically significant at 1 per cent level of significance.

2/ The numbers shown side by side in each box represent the range of estimated values over different years as obtained under Bench mark Equilibrium method. The numbers within the paramthesis represent the arithmatic mean of the whole range of estimated values.

Comparison of the estimates as presenteed in Tables V-1 and V-2 indicates that different methods give somewhat similar values for the parameters. Benchmark equilibrium parameters show some degree of variation from year to year but, if we take the average of those yearly values, the mean is once again not very different from the estimate obtained by the other methods. Thus we can possibly argue that if we are to apply the benchmark method, we should apply the consistency adjustments to the average data of the relevant variables, the average being taken over some reasonable number of years. This should tend to minimize the effects of random shocks on individual years' outcomes.

However, as the earlier discussion emphasizes, the benchmark equilibrium method can not be a complete substitute for the econometric estimation because of the need to specify elasticity values for functions more complex than Cobb-Douglas. The benchmark method when applied to average data may give reasonably good estimates of the share and scale parameters but can not estimate the elasticity parameters. Conventionally the elasticity parameters are obtained from a literature search and are then used in the process of estimating other parameters in the benchmark method (as described in part 1). The numbers used as elasticity of substitution parameters will influence the estimated values of share parameters along with the scale parameters. In Table V-1 this issue does not arise because we use Cobb-Douglas utility and production functions which automatically ensures that elasticity of substitution in both demand and production is unity. If we take a CES specificaion, the estimated elasticity parameters would not generally be the same as those from literature search and consequently we would expect the parameters of benchmark method to differ from those corresponding to econometric estimations. Table V-3 shows how the share and scale parameters are influenced by the variations in adopted elasticity values.

We note that the estimated share parameters are surprisingly sensitive to changes in the adopted values of substitution elasticities. This is important because of the limited consensus on the relevant elasticity values; opinions vary widely for many important elasticity parameters. Moreover, different studies use different sectoral classificaions (sometimes different subsectors under the same broad name in different studies) and corresponding choice theoretic parameters (including elasticities) vary accordingly. This sensitivity of parameters to variations in sectoral classifications seems to be significant and thus one can validly argue against the use of numbers obtained through literature search. The benchmark method, by using only the elasticity values from the literature search tends to reduce these problems. However, the fact remains that the errors committed at this stage tend to be propagated on to the other estimated parameters.

Table V-4 reports the estimated parameters for a model with CES production functions under different methods of estimation. The NLFIML method when applied to the subsystems without the consideration of simultaneity the estimators would generally be biased and the values appear to be biased downwards. Moreover, for the corporate sector (sector 2),

Table V-3. Sensitivity of Share and Scale Parameters Due to Variations in Elasticity Values under Benchmark Method 1/

Other Parameters / Elasticity values 2/	a_1	a_2	A_1	A_2
0.333	0.547	0.048	4.0988	3.6383
0.500	0.531	0.136	4.1008	3.9338
1.000	0.516	0.329	4.1028	4.4412

1/ This is based on the benchmark equilibrium data set constructed for the year 1948. The elasticity values are arbitrarily chosen to see the impact of its variation on other parameters.

2/ For both sectors the same values are taken as elasticities.

Table V-4. Stochastic Estimates of the Elasticity and Share Parameters Under Various Methods of Estimation

Stochastic Methods \ Parameters	Elasticity (Production) σ_1	Elasticity (Production) σ_2	Share Parameters (Production) a_1	Share Parameters (Production) a_2	Share Parameters (Demand) b_1	Share Parameters (Demand) b_2
NLFIML 1/	.2419	.2465	.8648	.0015 2/	0.122	0.878
NLIV (Method I)	1.45	0.485	0.623	0.18	0.124	0.876
NLIV (Method II)	1.489	0.535	0.620	.16	0.124	0.876
Calibration 3/ (with elasticities obtained by NLIV method I)	1.45	0.485	0.574	0.15	0.124	0.876
Calibration 3/ (with elasticities obtained by NLIV method II)	1.489	0.535	0.574	0.12	0.124	0.876

1/ NLFIML applied to the subsystems ignoring the underlying simultaneity.
2/ The parameter is not statistically significant.
3/ The share parameters are the average figures.

both the elasticity of substitution and share parameters tend to be statistically insignificant. For the NLIV methods the sets of estimated parameters are not very different from one another and on the basis of this we might suggest that NLIV method I could be a good and dependable way of estimating the parameters in applied general equilibrium works. Method I (NLIV) is computationally much simpler and involves significantly less work than the NLIV method II. However, this statement is only tentative and much more quantitative experience is needed before making any general statement of this kind. This table once again lends its support to the observation that when averaged over time, the share parameters from benchmark method (with some elasticity value) tend to be not much different from those of NLIV methods.

Next we intend to see the sensitivity of the estimated welfare loss measures due to variations in the estimated parameter values as obtained under different methods. Table V-5 reports welfare costs due to differential rates of taxation in the U.S. economy using a Cobb-Douglas production and utility function.

This table shows that the measures of loss in the Shoven-Whalley algorithmic approach tend to be lower with benchmark equilibrium parameters than those with NLIV estimators. The extent of underestimation increases rapidly in both absolute and relative terms over time. While for 1949 the benchmark parameters underestimates the loss by 25 per cent by 1965 this figure (for underestimation) increases to 62 per cent. With Harberger's formulation of measuring the costs we use some ad hoc sets of

Table V-5. Comparison of the Estimates of Loss Due to Nonneutrality of Taxes on Income from Capital in the U.S. Economy with Cobb-Douglas Production and Demand System 1/

Methods	HARBERGER FORMULATION				SHOVEN-WHALLEY FORMULATION		
	NLIV Estimators (Method II)		Harberger Parameters	Col. 1 as % of col. 3	NLIV Parameters	Benchmark Equilibrium Parameters	% variation between col. 5 and 6 5-6 x 100
Selected years	Loss (m) (1)	% Loss (2)	Loss (m) (3)	(4)	% loss (FI) 2/ (5)	% loss (FI) 2/ (6)	(7)
1949	5,451	2.29	5175	105.33	0.558	0.446	25.11
1952	6,159	2.19	5,874	104.86	0.558	0.387	44.19
1955	6,501	2.15	6,246	104.08	0.569	0.340	67.35
1958	6,563	2.05	6,328	103.72	0.558	0.368	51.63
1962	7,841	2.09	7,577	103.49	0.598	0.340	74.41
1965	9,573	2.14	9,292	103.04	0.900	0.343	62.32

1/ The estimates are based on the parameters reported in Table III-2.
2/ FI is the Fisher's Index (the geometric mean of Laspayer's and Paasche's index) with no-tax situation as the base.

parameter values along with NLIV estimators. These are represented in column 3 and column 1, respectively, of Table V-5. The difference between the two outcomes is not very significant and while the ad hoc parameters tend to underestimate the measured costs, the difference tends to be reduced over time (as indicated in column 4). The surprising degree of closeness in the measured costs is due to the fact that Harberger's ad hoc method of assigning the parameter values, for this CobbDouglas case yields parameters very close to the NLIV estimates. Moreover, Harberger's method is based on a local approximation which may not be suitable for large distortions that we are considering here. Thus, even if the parameters are different, the outcomes may not be very responsive to these changes.

For CES production functions the welfare costs are reported in Table V-6 in a manner similar to those of Table V-5. The figures in Table V-6 are based on the parameters reported in Table V-4. In the first nine columns we report Harberger's measures of costs under three different sets of parameter values. It appears that loss estimates are very sensitive to the price elasticity of demand for corporate sector products (see columns 5 and 8 of Table V-6). NLIV Method I parameters slightly but systematically underestimates the welfare loss compared to those from NLIV Method II parameters. While the Harberger parameters with E2 = 1/7 (price elasticity of corporate sector products) underestimates the welfare loss compared to NLIV methods, the figures with E2 = -1.0 are significantly higher than those of NLIV counterparts.

Table V-6

COMPARISON OF THE ESTIMATES OF WELFARE LOSS DUE TO NON-NEUTRALITY OF TAXES ON INCOME FROM CAPITAL IN THE U.S. ECONOMY, USING CES PRODUCTION FUNCTIONS*1

Methods / Years	HARBINGER FORMULATION: NLIV Estimates (Method I) Loss (m) (1)	% Loss (2)	NLIV Estimates (Method II) Loss (m) (3)	% Loss (4)	Col. 1 as % of Col. 3 (5)	Harberger Parameters*2 Loss (m) $E2=-\frac{1}{7}$ (6)	% Loss $E2=-\frac{1}{7}$ (7)	Loss (m) E2 = -1.0 (8)	% Loss E2 = -1.0 (9)	(10)
1948	3611.44	1.48	3838.27	1.55	94.1	1962.09	.79	5216.27	2.11	.359
1949	3531.64	1.48	3711.11	1.56	95.2	1890.75	.80	5175.26	2.17	.390
1950	3967.91	1.52	4169.16	1.60	95.2	2094.46	.80	5804.53	2.22	.392
1951	3922.97	1.45	4126.11	1.52	95.1	2094.95	.77	5777.59	2.13	.401
1952	3937.96	1.40	4144.30	1.47	95.0	2102.93	.75	5873.89	2.09	.417
1953	3942.48	1.35	4155.58	1.43	94.9	2103.11	.72	5984.36	2.06	.441
1954	3834.36	1.35	4043.25	1.42	94.8	2062.12	.72	5850.98	2.05	.447
1955	4071.10	1.35	4294.92	1.42	94.8	2157.76	.72	6245.83	2.07	.454
1956	4353.95	1.35	4594.33	1.42	94.8	2283.38	.71	6693.53	2.07	.457
1957	4301.81	1.32	4542.51	1.39	94.7	2284.21	.70	6672.29	2.04	.469
1958	4075.13	1.28	4302.97	1.35	94.7	2230.31	.70	6327.46	1.98	.470
1959	4854.15	1.41	5124.64	1.49	94.7	2530.25	.73	7486.32	2.17	.461
1960	4455.74	1.27	4708.80	1.34	94.6	2398.35	.68	6980.91	1.99	.483
1961	4536.70	1.27	4794.30	1.34	94.6	2464.10	.69	7109.92	1.99	.483
1962	4830.37	1.29	5113.96	1.36	94.6	2603.04	.69	7577.06	2.01	.482
1963	5025.98	1.29	5311.17	1.36	94.6	2692.84	.69	7867.23	2.01	.481
1964	5508.85	1.31	5829.79	1.39	94.5	3025.36	.72	8779.90	2.09	.506
1965	5846.61	1.30	6185.27	1.34	94.5	3229.28	.73	9291.49	2.07	.502

Years	SHOVEN - WHALLEY FORMULATION: NLIV Estimate (Method I) Loss $(FI)^3$ (11)	% Loss $(FI)^{*3}$ (12)	NLIV Estimate (Method II) Loss $(FI)^{*3}$ (13)	% Loss $(FI)^{*3}$ (14)	Benchmark Parameters Loss $(FI)^{*3}$ (15)	% Loss $(FI)^{*3}$ (16)	% Variation $\frac{(13)-(15)}{(13)} \times 100$ (17)
1948	1522.60	.515	1397.8	.491	1392.8	.50	.004
1949	1649.6	.567	1359.3	.49	1280.2	.67	.038
1950	2354.0	.74	2247.5	.74	1331.5	456	.408
1951	1870.8	.53	2046.3	.61	1893.6	.49	.075
1952	2101.8	.56	2162.0	.62	1907.4	.55	.118
1953	2299.3	.57	1998.2	.54	1605.1	.45	.197
1954	1844.7	.42	2254.5	.62	1592.8	.44	.494
1955	2637.7	.58	2094.8	.55	1780.0	.49	.150
1956	2680.8	.61	2264.1	.56	1676.1	.43	.253
1957	2958.4	.72	4280.3	1.8	1733.7	.43	.395
1958	2461.7	.54	2254.6	.56	1765.0	.45	.217
1959	2498.1	.54	2409.4	.56	1700.4	.42	.294
1960	2617.5	.56	2409.4	.52	1920.4	.44	.203
1961	3405.5	.63	2525.4	.54	2017.1	.44	.201
1962	3447.5	.58	2838.9	.56	2368.5	.47	.173
1963	3517.2	.57	2844.3	.551	2138.4	.41	.248
1964	3773.3	.60	5026.6	.939	2445.1	.48	.514
1965	3393.8	.58	8289.1	1.36	2395.1	.43	.685

*1 The parameters used are based on the estimates reported in Table 9. The same figures as shown in the footnote of Table 10 have been used as 'Harberger' parameters; the elasticities of substitution are also the same, i.e., unity.

*2 We consider two situations, the original Harberger assumption that demand elasticity for this corporate sector is only $-(\frac{1}{7})$ and the other more reasonable assumption that demand elasticity is -1.0 for both sectors.

*3 FI is the Fisher's Index (the geometric mean of Laspeyres and Paasche's Index), with no-tax situation as the base.

With the Shoven-Whalley algorithmic approach the expressions for loss estimates are somewhat similar to each other. Under all three sets of parameters the values of welfare loss tend to increase over time following the pattern observed with Harberger formulations. The absolute measures of loss are not significantly different from the others, though the benchmark method yields figures that are slightly downward biased compared to other methods. This lack of differences may once again be due to the fact that for the benchmark case we use elasticities from NLIV (method II) estimation procedure. As has been indicated above in the context of the Cobb-Douglas case that as long as the elasticities are the same, the benchmark estimation method in general tend to produce share parameters close to those from stochastic methods.

VI. Conclusions

With the natural progress in computation methods during the late sixties and early seventies it appears that the 'usefulness' of recently formulated general equilibrium models crucially depends on the numerical specifications used. It seems natural that more attention should be given to the issue of specifying a general equilibrium model in a 'reasonable' manner, independent of the solution procedure. We argue that as a method for selecting parameter values, calibration appears to have two weaknesses of requiring pre-selection of elasticities and not providing any basis for a test of specification since it fits perfectly the single data point. Stochastic estimation also appears to have problems; even for moderately elaborate models, prohibitively long time series are needed for full system estimation. In view of these limitations we devise subsystem estimation using a nonlinear instrumental variable method and suggest two ways of selecting the instruments. Applying this to a two sector general equilibrium model of the U.S. economy we report on some comparisons with the corresponding calibration parameters. Using both types of parameters we measure the costs of distortions in capital use (in the U.S. economy) due to the non-neutrality of the tax system in the tradition of Harberger. This serves as an illustration of how the suggested estimation procedures can be applied to a real world problem. However, this piece is only a small step in the desired direction and we believe that there is enormous potential and need for further refinement.

Data Base

This appendix presents a summary of the data used in the estimation of the two sector general equilibrium models specified for the U.S. economy. Most of the basic data is collected from the various issues of the Survey of Current Business, published by the U.S. Department of Commerce. The sample used in this study covers 18 years from 1948 to 1965. The two sectors, corporate and noncorporate (as we refer to them), are based on 11 (aggregated) original sectors based on national income accounting reported in Survey of Current Business.

Description of the 11 original sectors

1. Agriculture, forestry and fisheries
2. Mining
3. Contract construction
4. Manufacturing
5. Wholesale and retail trade
6. Finance and insurance (excluding real estate)
7. Real estate
8. Transportation
9. Communication and public utilities
10. Services
11. Government and government enterprises

The aggregation into two broad sectors is based on somewhat similar classification as followed in Harberger (1966) and Shoven and Whalley (1972). The aggregation can be described as:

Sector 1: noncorporate or lightly taxed sector

1. Agriculture, forestry, and fisheries

7. Real Estate

11. Government

Sector 2: Corporate or heavily taxed sectors

2. Mining

3. Contract construction

4. Manufacturing

5. Wholesale and retail trade

6. Finance and insurance (excluding real estate)

8. Transporation

9. Communication and public utilities

10. Services

Note that personal and business services, wholesale and retail trade are not overwhelmingly corporate in nature but are taxes heavily. Rosenberg (1969) shows that without allowing for taxes on their dividends, approximately 30 per cent of the income from capital generated in these activities is taken away by taxation. During the classification the real estate industry has been redefined following Rosenberg (1969). The real estate industry, as defined in the national income accounts, includes all types of property. The redefinition used by Rosenberg includes only nonfarm residential dwellings, rented and owner occupied. The return to capital for any other type of property would be assigned to the industry group which would pay for the property. For example, the net rent paid on farm land to nonfarm landlords is included by Rosenberg as part of income from agriculture, but comes under real estate in national income accounting. Using the proportions (average over the years 1953-1959) of capital income transferred from real estate to other sectors from Rosenberg's study, we adjust the figures (for real estate and other industries) in national income accounts. In our present classification, capital income in real estate or agriculture is not differentiated as both belonging to the same noncorporate sector.

Following Harberger we take as a unit of capital that amount which generates one (constant 1958) dollar of net income in either sector. Analogously, we define a unit of labor as that amount which generates one dollar (constant 1958), return to labor. Thus, the amount of labor in a sector is the same as the wage bill (in constant 1958 dollar) for the corresponding sector. Because of this unit convention, the net-of-tax interest payment for each sector would give us the expression for capital stock for that sector. The assumption that marginal revenue products of factors are equalized in all uses in equilibrium enables us to use factor payments data by industry as observations on physical quantities of factor. While the physical dimensions remain undefined as there are no conventional weights on volume measures to be associated with the underlying physical units of measurement given by such a procedure.

We use the modified tax rate figures from Shoven (1976) because of some errors in the determination of tax rates used by Harberger. Harberger notes that total taxes on net income in the "noncorporate" and "corporate" sectors average respectively 45 per cent and 168 per cent. Thus, according to Harberger the taxation of income from capital (during the period) may be divided into a general tax of 45 per cent on all net income from capital and an 85 per cent surtax on the net income from capital originating in the heavily taxed sector (1.45 x 1.85 = 2.68). Harberger's actual evaluation is wrong since the surtax is not applied to the net capital income of the heavily taxed sector, but to the capital income in that sector gross of the neutral 45 per cent tax. Thus, Shoven (1976) points out that Harberger evaluates a smaller tax than his data reveals and the social marginal product of each of Harbeger's units of capital is $1.45 rather than one dollar as recognized by him. To rectify this problem, following Shoven one either has to evaluate both the 45 per cent and 168 per cent taxes simultaneously, or to evaluate only the surtax and take as a unit of capital that amount which earns one dollar net of the surtax but gross of the neutral tax. Shoven also notes that Harberger's figures contain another arithmetic mistake. The surtax rate, according to Shoven's calculation is 53 per cent instead of Harberger's 85 per cent. In our evaluations we keep this tax rate as constant over the sample period and use this 53 per cent surtax rate to evaluate the figures for capital stock.

Table AI-1 presents the adjusted data set that we have used in our analysis in terms of the units defined earlier. Labor in Sector 1, because of the adopted unit convention, corresponds to the compensation to the employees in Sector 1 and the same for Sector 2. According to national income accounts, compensation includes wages, salaries, and supplements to wages and salaries. Payment to capital is determined residually out of the value added in that sector. Net of tax interest payment to capital would give the use of capital in each sector. Total net of tax interest payments to capital represents the endowment of capital in the economy each year.

Table AI-1

Years	Variables			
	Labor in Sector 1 (noncorporate)	Labor in Sector 2 (corporate)	Capital in Sector 1 (noncorporate)	Capital in Sector 2 (corporate)
	(In millions)			
1948	19,209.4	139,501	20,451.8	44,726.1
1949	15,358.7	137,236	19,581.6	45,273.2
1950	16,117.2	149,410	21,107.7	48,612.7
1951	17,031.0	159,041	21,627.0	48,094.7
1952	16,311.1	169,202	21,577.9	48,345.6
1953	14,844.6	180,749	21,407.2	48,497.1
1954	14,392.1	176,954	21,377.1	47,191.0
1955	13,832.2	189,708	21,514.0	50,135.4
1956	14,028.0	205,119	22,095.4	53,633.5
1957	13,797.7	209,486	22,749.7	53,039.2
1958	15,047.3	203,454	24,135.4	50,242.4
1959	13,924.0	215,536	23,992.3	59,828.7
1960	14,285.0	227,430	24,739.2	54,993.4
1961	14,969.0	229,849	26,102.6	55,991.9
1962	15,297.1	242,586	26,800.8	59,724.3
1963	15,685.5	252,619	27,404.1	62,027.4
1964	15,157.2	267,470	33,183.1	68,110.7
1965	17,115.7	284,602	36,048.5	72,257.7

Table AI-2

	Value Added in Sector 1	Value Added in Sector 2	Wage Index (economy index) (1958 = 100)	Index of return to capital (1958 = 100)	Productivity of labor index (1958 = 100)	Productivity of capital index (1958 = 100)
	(In millions)					
1948	39,661.2	207,932	0.606	0.724	0.737	0.951
1949	34,940.2	203,444	0.620	0.721	0.760	0.956
1950	37,224.9	223,787	0.670	0.695	0.784	0.961
1951	38,658.1	232.625	0.723	0.772	0.808	0.966
1952	37,888.9	243,171	0.759	0.840	0.833	0.971
1953	36,251.8	254,950	0.811	0.922	0.858	0.975
1954	35,769.2	249,157	0.831	0.920	0.885	0.980
1955	35,346.2	266,415	0.874	0.946	0.912	0.986
1956	36,123.4	287,178	0.928	0.973	0.941	0.990
1957	36,547.4	290,636	0.981	0.989	0.968	0.995
1958	39,182.7	380,325	1.000	1.000	1.000	1.000
1959	37,916.2	307,074	1.059	0.940	1.031	1.005
1960	39,024.2	311,570	1.093	1.094	1.063	1.010
1961	41,071.6	315,517	1.123	1.029	1.095	1.015
1962	42,097.3	333,965	1.180	1.032	1.129	1.020
1963	43,089.6	347,521	1.224	1.038	1.164	1.026
1964	48,340.3	371,680	1.306	0.915	1.200	1.030
1965	53,164.1	395,160	1.360	0.935	1.237	1.035

The figures for labor and capital inputs have been adjusted for productivity changes over the sample period 1948-1965. The figures for average annual percentage rates of changes in productivity ratios for labor and capital inputs are collected from Kendrick (1973). Kendrick examines the compound rates of changes in productivities of labor and capital between 1948 and 1966 and then goes on to compare them with growth rates in earlier periods 1889-1919 and 1919-1948; (see Table 3.2 in Kendrick (1973)). Compound rates for labor input is 3.0 and that for capital is found to be 0.4 over the years 1948-1966.

While estimating the welfare loss following Harberger we also consider the case where the price elasticity of demand for corporate sector goods is unity. In his 1962 article, Harberger assumes that the demand elasticity for the noncorporate sector is (- 6/7), while the price elasticity of demand for the corporate sector is (- 1/7). The figures are derived from expenditure shares and are also used in his 1966 paper. It has been pointed out by Shoven and Whalley (1972) that the demand for the product of the "noncorporate" sector is six times as price elastic as all other products on the average is counterintuitive. According to them, a much more acceptable assumption seems to be to take as unity the price elasticity of demand for all products.

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CHAPTER III

On the Estimation of Import and Export Demand Elasticities and Elasticity Pessimism

I. Introduction

Controversy around the estimation of price elasticities in international trade models goes back to the empirical works of the inter-war period which indicate that the price elasticities for imports and exports are low. Since the pioneering article by Orcutt (1950), which indicates some potential sources of bias in the data used and estimation methods employed, various studies have attempted to explain this phenomenon and sometimes suggested alternative methods which could yield better estimates. However, in none of these works (some of which have been summarized in Section II) the specified import and export demand functions have been considered as a part of a more complete foreign trade sector specification. Import and export demand functions, along with the trade balance condition which closes the system, is a kind of specification that has been recognized in the pure trade theory literature and also in the empirically oriented general equilibrium models. Incorporation of trade balance condition endogenizes the exchange rate and price elasticities (for exportables and importables) take into account the simultaneous variations of foreign prices along with exchange rate variations (i.e., effective foreign prices) relative to corresponding domestic prices. Moreover, if we are interested in the estimation of long run import and export demand elasticities trade balance condition should be a binding the estimates biased towards zero. 1/ This bias may be quite significant giving substantially lower estimates of the parameters and consequent concern over the elasticity pessimism. From an analytical point of view the evaluation of the effectiveness of exchange rate policies or other policy actions in the form of tariffs or subsidies, is made on the basis of long run outcomes and it is in the long run estimates (of the elasticities) that we should be more interested.

In the existing literature some works indicate potential sources of bias and discuss some ways to remove those. However, no attention has been paid to the effects of trade balance condition on long run estimators and in most of those studies import demand (or supply) and export demand functions are specified and estimated independently ignoring the fact that these two are linked to each other through the trade balance condition. The role of the trade balance condition in this framework is somewhat analogous to the role of budget constraint in the demand analysis, endowment constraint in an exchange model and income identity in a macro model. In the short run there may be differences between export receipts and

1/ For a comprehensive bibliography of the price elasticities in international trade, see Stern, R. et.al. (1976). Survey of the literature indicates that most of the specifications of export demand functions have terms containing logarithm of index of export prices to the index of world export prices, the associated coefficients measuring the price elasticity. But in practice it is well recognized that variations of foreign prices relative to domestic prices can be offset or further reinforced by somewhat independent variation of exchange rate. What in fact matters is the variation of foreign prices relative to domestic prices of exportables net of exchange rate variation. This is also true for conventional import demand specifications.

import payments along with exogenously determined capital account adjustments, but that situation is temporary and cannot prevail in the long run. For any economy, imports are financed by export receipts and since surplus or deficit cannot exist indefinitely, in the long run, trade balance condition should act as a binding constraint for the long run elasticity estimators. Exchange rate movements would exert sufficient pressure to ensure that. 1/

This kind of specification is recognized in the pure trade theory literature and also in the applied general equilibrium models. In the pure theory of trade the foreign offer curve (or the import supply function) and the home country's offer curve (or the export demand function) intersect each other (under general conditions in a two good economy) and this ensures trade balance. In the recent applied general equilibrium models a system of import demand (or supply) and export demand functions along with a trade balance condition are introduced to specify the external sector. Most of these models are constructed for single economies sometimes using Armington (1969) type procedure (which distinguishes imports from domestic products of the same sectoral classification) simultaneously incorporating price taking behavior for imports, constant price

1/ If an economy enjoys transfer of foreign exchange (due to some unspecified reasons) so that imports exceed exports the trade balance condition can be modified accordingly to take that into account without changing any of the conclusions in the following sections.

elasticity for export demand and a zero trade balance condition. 1/

In this paper we would argue that:

(1) If we are interested in determining the long run effects, import and export demand elasticities should not be estimated independently because of the presence of trade balance condition. Least squares estimation of these elasticities ignoring the inherent simultaneity would give us biased and inconsistent estimators.

(2) The direction of bias can be determined under certain general conditions. If the sum of the true (absolute) values of export demand (a) and import demand or supply (b) elasticities is greater than 1, the estimated elasticities ($\hat{a}$ and $\hat{b}$) would both be biased towards zero. That is, if $|a| + |b| > 1$, the independent estimation of the equations without regard to the rest of the system would make,

$$|\hat{a}| < |a| \quad \text{and} \quad |\hat{b}| < |b|.$$

Here a and b are the true values of export demand and import demand elasticities and $\hat{a}$ and $\hat{b}$ are their respective least squares estimators.

(3) This result has obvious bearings on the issue surrounding the controversy of 'elasticity pessimism'. On the aggregate level policy makers may

1/ The exact specification of the external sector import and export demand functions in applied general equilibrium models vary from model to model but their broad characteristics are somewhat similar. The models presented by Adelman and Robinson (1977), Boadway and Treddnick (1978), Deardorff and Stern (1979), Dervis and Robinson (1978) etc. all belong to this category of external sector specifications.

need to determine what effects exchange rate realignments may have on the trade balance of different countries. Other issues like the effects of uniform changes in tariffs and in domestic indirect taxes on the composition of trade, level of income and employment, etc., depend upon the elasticity magnitudes. Our results indicate that most statistical estimates (obtained without the consideration of the inherent simultaneity) lower the absolute values of estimated elasticities, leading to considerable underestimation of the effectiveness of different policy actions.

(4) The basic conclusion holds under different specifications of the relevant functions, including one where the import demand funtion is obtained from choice theoretic foundations. Defining a commodity which is a CES aggregation of imports (M) and domestically produced and used commodities (D) and assuming cost minimization by the producers and consumers (in using the composite good), we can derive the import demand function. This specification has been used in different World Bank models but has never been estimated. This formulation (as described in Model II) has two desirable advantages in the sense that it does not assume separability between imports and domestic goods and can explain the use of imports as intermediate good.

(5) Computational experience with Japanese data also supports our theoretical assertion that the OLS estimators of a and b are substantially lower than those of asymptotically unbiased estimators like 2SLS, NL2SLS or FIML estimators of the corresponding parameters.

(6) In view of very frequent evidence of serial autocorrelation in the disturbance terms (in various studies), we extend our analysis to determine the asymptotic bias of OLS estimators of export demand elasticity with serially correlated error terms (w_t). It is straightforward to show that if $w_{1t} = p \cdot w_{1,t-1} + v_{1t}$, the asymptotic bias increases quadratically as the value of p, the autocorrelation coefficient increases. What is more important is that the bias gets much worse if $p > 0$ and the quasi first difference transformation of export demand function is made to correct the serial correlation problem. Thus if the underlying true model is simultaneous and due to misspecification we are applying OLS, so far as the bias is concerned we may be better off by not making the transformation to correct serial correlation (when $p > 0$ which is the most frequent case). This result is not necessarily true for only our specification but may very well be of relevance to other class of models as well.

The plan of the paper is as follows:

Section II summarizes some of the existing works on the estimation of trade elasticities and derivation of the directions of bias under various conditions. In Section III we introduce the basic models and demonstrate that single equation estimation by OLS would be asymptotically biased and inconsistent. Then we demonstrate and comment on the direction of bias. Estimates of various import and export demand systems would be presented and compared with the corresponding OLS counterparts in Section IV. This would be followed by a discussion on the sensitivity of the bias to the degree of autocorrelation and particularly on the possible adverse effects of correcting the autocorrelation without considering the simultaneous equation bias. Main conclusions are summarized in Section VI. Somewhat detailed mathematical deductions are given in the Appendix II.

II. Some Observations on the Existing Methods and Measurements of the Directions of Bias

Most of the empirical research on the estimation of import and export demand functions is based on multivariable regression analysis with time-series data. Investigators have estimated import demand equations by regressing the logarithm of a measure of imports on the national income and the ratio of imports to the price of domestic value added, all expressed in logarithms. Export demand functions are also expressed in analogous linear or log-linear forms. One striking feature of these estimates which has come to the surface is that the estimates for both import and export price elasticities appear to be surprisingly low. 1/ In an early attempt to address the issue Orcutt (1950) investigated the various sources of bias in the elasticity estimates in a non-simultaneous equation model framework. He emphasized on the bias due to shifts in the demand surface, the neglect of lagged prices and errors in the measurement of the price variable. Using a linear import demand function with measurement error

1/ It may also be noted that reported estimates of export price elasticities do not appear to follow the ranking we expect from the relative sizes of countries. Since it is believed that small economies are price takers, they should face higher export price elasticities than the same for large price-making countries. This does not correspond to the values of -1.41 for the U.S. versus -0.79 for Canada and -1.25 for Japan versus -0.70 for New Zealand.

in the price variable, Orcutt shows that least squares estimates of price elasticity would be biased towrds zero provided error in the quantity variable is not highly and negatively correlated with the error in the price variable.

In a follow-up paper, Kemp (1962) considered a modified version of Orcutt's model where the dependent variable is constructed by dividing the total money value with price index containing measurement errors. This model differs from the usual errors-in-variable model in the sense that the error in the dependent variable is highly and negatively correlated with the error in the independent variable. Assuming that the error in the price index is independent of the true price, Kemp demonstrates that large sample bias is not towards zero but towards minus one. As a continuation of the same issue, Kakwani (1972) proves that Kemp's conclusions are vald even for small sample and for a model with a stochastic import demand function. However, these results are concerned with errors in the measurement and are discussed in the context of a single equation model.

Some authors, Ball and Marwah (1962), Balassa and Kreinin (1967), argue for substantially higher elasticities than those obtained from time series estimation, which are to be taken as a king of lower limit. Because of the potential reasons for downward bias associated with the statistical methods applied, as upper bounds they suggest adding the corresponding standard errors or multiples of standard errors to the least squares estimates. This kind of upward adjustment has also been justified on the basis of so-called 'tariff elasticities', proposed by Kreinin (1961) and Krause (1962). Kreinin and Krause empirically demonstrate that the elasticitiy of the demand for imports with respect to tariff change is considerably higher than the elasticities calculated with respect to price. A possible intuitive explanation may be that tariff changes are regarded as more permanent while import price changes may be considered as temporary and a kind of 'ratchet' effect may also be operative in the latter case. These are, however, in addition to the downward bias in the least squares estimation of price elasticities, as has been referred to earlier.

In a simultaneous approach Goldstein and Khan (1978) estimate supply and demand functions for exports in a full-information framework. According to them, while the assumption of infinite price elasticity appears to be reasonable for a small country facing world supply, it is not equally applicable to the supply of exports of an individual country. They point out that an increase in the world demand for a country's exports can normally not be satisfied without an increase in the price of its exports and to ignore this simultaneous relationship between demand and supply of exports would bias the estimated export demand elasticites. Their empirical results indicate that export price elasticities are considerably larger than those reported by other researchers for the same group of countries. However, they do not investigate the direction of potential bias from an analytical point of view and no comparison has been reported vis-a-vis the least squares estimates from the same functional forms. Moreover, their specification has not been cast explicitly or implicitly in the form of an external sector closing system and hence is not so directly relevant to the present context.

III. External Sector Specifications

1. Model I

In this section we would deal with various forms of external sector specifications which can be used to estimate the (long run) elasticities and also have been used in various applied general equilibrium models. A simple system of external sector closure suggested by Boadway and Treddniok (1978) can be represented by the following 'simple' foreign export demand and import supply functions.

$$E_t/E_0 = C_1(PE_t/e_t)^a \; ; \quad -\infty < a < 0 \; ; \quad t = 1, .., T. \qquad \text{(III-1)}$$

$$M_t/M_0 = C_2(PM_t/e_t)^b \; ; \quad 0 < b < \infty \; ; \quad t = 1, .., T \qquad \text{(III-2)}$$

where E_t and M_t are exports and imports and E_0 and M_0 are 'base year' imports and exports respectively. PE_t and PM_t are the home country prices paid for exports and imports. a and b are the export demand and import supply price elasticities while e_t is the exchange rate between domestic and foreign currencies. Here (PE_t/e_t) is the price paid by the foreigners and (PM_t/e_t) is the price charged by the foreigners on home country imports and so enter the functions (III-1) and (III-2) respectively; C_1 and C_2 are some proportionality parameters.

Balance of payments condition,

$$PM_t \cdot M_t = PE_t \cdot E_t \qquad \text{(III-3)}$$

closes the system.

Given this specification it is convenient to treat E_t, M_t and e_t as endogenous variables; E_0 and M_0 as constants and PM_t, PE_t and any other terms (if appears) are exogenous variables. When it comes to estimation, the specified system is modified by the introduction of stochastic disturbances. This along with the logarithms of equations furnish the following system of equations:

$$\log E_t - \log E_0 = \log C_1 + a.\log PE_t - a.\log e_t + v_{1t} \qquad \text{(III-1')}$$

$$\log M_t - \log M_0 = \log C_2 + b.\log PM_t - b.\log e_t + v_{2t} \qquad \text{(III-2')}$$

$$\log M_t + \log PM_t = \log PE_t + \log E_t \qquad \text{(III-3')}$$

The additive error terms are assumed to be independent of each other and follow the conventional classical assumptions, v_i having mean zero and constant variance (σ_i^2 (for all i=1,2). It can be noted from the reduced form for e_t' (= log e_t) using all the equations of the system, that e_t' is correlated with v_{1t} and v_{2t}, the error terms of equations (III-1') and (III-2'). If ignoring this simultaneity we use OLS to estimate a and b from equations (III-1') and (III-2') independently, the estimators ($\hat{a}$ and $\hat{b}$) would be asymptotically biased and inconsistent.

This is a kind of result that we would generally expect but what is more important is the direction of bias. It has been shown in the Appendix II that the asymptotic bias for OLS estimators $\hat{a}$ and $\hat{b}$ (of a and b) are respectively positive and negative under very general conditions. As has been shown in the Appendix:

$$E(\hat{a} - a) = -\left[\frac{w_{22}^{*} \cdot a - w_{12}^{*}}{w_{22}^{*}}\right] \cdot e^{-\frac{\mu^2}{2}} \cdot {}_1F_1\left(\frac{T-1}{2}-1, \quad \frac{T-1}{2}, \quad \frac{\mu^2}{2}\right) \qquad \text{(III-4)}$$

where $w_{12}^{*} = \frac{a}{(a-b)^2} \cdot \sigma_2^2 + \frac{b}{(a-b)^2} \cdot \sigma_1^2$

$$w_{22}^{*} = \frac{1}{(a-b)^2} \cdot \sigma_2^2 + \frac{1}{(a-b)^2} \cdot \sigma_1^2$$

$$\mu^2 = \frac{(EY_2^{*}\cdot) \cdot M \cdot E(Y_2^{*})}{w_{22}^{*}} > 0$$

$$y_2^{*} = \log PE_t - \log e_t = (e_t'')$$

$$M = [I - \ell(\ell'\ell)^{-1} \ell']$$

$$\ell = [1, \ldots, 1]'$$

and ${}_1F_1\left[\frac{T-1}{2}-1, \frac{T-1}{2}, \frac{\mu^2}{2}\right]$ is known as confluent hybergeometric function.

The first expression within the parenthesis in (III-4) is the combination of var (y_2), cov (y_{1t}, y_{2t}^{*}) and the true elasticity value a, and its directions are well determined. We need to examine the remaining part ($_1F_1$ (*)) and its underlying properties. The function $_1F_1$ in its general form $_1F_1$ (α, β, x) is a Kummer's function and can be expressed as:

$$_1F_1\ [\alpha, \beta, x] \equiv 1 + \frac{\alpha}{\beta} x + \frac{\alpha\ (\alpha+1)x^2}{\beta(\beta+1)2!} + \frac{\alpha\ (\alpha+1)(\alpha+2)x^3}{\beta(\beta+1)(\beta+2)3!} + \ldots\ldots\ldots\ldots$$

The series is absolutely convergent for all values of α, β and x, real or complex excluding β being zero and negative integers. Elementary properties of this function was given by Kummer (1836), while one of its special cases had been discussed by Lagrange as early as 1762. The $_1F_1$ (*) function is an analytical function of x, i.e., are valued and differentiable for all values of x, real or complex. This is also an analytical function of α but not of β (for when β is zero or a negative integer, the function has simple poles at b = 0, -1, -2, ...). We can also determine its asymptotic approximations under various conditions. If both α and β are large (which would be the case under large sample assumption), with $\beta-\alpha$ and x bounded, Kummers' transformation gives:

$$_1F_1\ [\alpha, \beta, x] = e^x\ \{1 + 0\ (|b|)^{-1}\}$$

and $$_1F_1\ [\alpha, \beta, x] = \frac{\tau\ (\beta)}{\tau\ (\alpha)} . e^x . x^{\alpha-\beta}\ [1 + 0\ (x^{-1})]$$

as $x \to \infty$.

For more on $_1F_1$ function see Slater (1960) and for its uses in econometrics, see Sawa (1972), and Richardson (1968).

The expression,

$$e^{-x}\ \ _1F_1\ (\alpha, \beta, x) > 0 \text{ for } x > 0 \text{ and } \beta = T-1 > 0$$

and thus the direction of the exact bias as denoted in (III-4) can be determined by noting that:

$$w_{22}^{*} \cdot a - w_{12}^{*} = \frac{1}{a-b} \cdot \sigma_1^2 < 0$$

If $a < 0$ and $b > 0$, which we expect the coefficients to be according to theoretical justification, $E(\hat{a}-a) > 0$. Since algebraically a is negative, the expected value of $\hat{a}$, in absolute term would be smaller than the corresponding true (absolute) value of a, i.e., $E|\hat{a}| < |a|$.... (III-5).

It may be noted that the absolute exact bias is an increasing function of the absolute value of $\frac{w_{22}^{*} \cdot a - w_{12}^{*}}{w_{22}^{*}}$ and a decreasing function of concentration (noncentrality) parameter μ^2.

Expression (III-4) can be expressed in an alternative form which has been derived in the Appendix to determine the large sample bias.

$$E(\hat{a}-a) = \left[\frac{w_{22}^{*} \cdot a - w_{12}^{*}}{w_{22}^{*}}\right] [1+\lambda]^{-1} \left[1 - \frac{2}{T-1} \cdot \frac{\lambda^2}{(1+\lambda)^2} + 0\left(\frac{1}{T^2}\right)\right] \quad \text{(III-6)}$$

where $\lambda = \frac{\mu^2}{T-1} = 0(1) \approx \frac{\mu^2}{T}$ when T is large.

$$\text{Then} \lim_{T \to \infty} E(\hat{a}-a) = \frac{[w_{22}^{*} \cdot a - w_{12}^{*}]}{w_{22}^{*} \cdot (1+\lambda)} = -\frac{\left(\frac{1}{a-b}\right) \cdot \sigma_1^2}{w_{22}^{*} (1+\lambda)} > 0.$$

the long run price elasticities of imports and exports (when exchange rate adjustment and its consequent effects are allowed to take place) would be greater (in absolute value) than the corresponding short run ones.

In another variation of the model we can incorporate the income term explicitly in the export demand function to take into account the variations due to income changes. This would make our export demand function somewhat similar to that of Houthakker and Magee (1969) except for the allowance for exchange rate changes. Given the export demand function of the form:

$$\log E_t = C + a.\log\left[\frac{\Pi_t \cdot e_t}{PD_t}\right] + c.\log YW_t + v_{1t}$$

where YW_t is the weighted index of the GNP of the countries which are importing the home country products.

Defining, $\theta_t = \log \Pi_t + \log e_t - \log PD_t$ and $\bar{\theta}_t$ the corresponding mean, the specified equation can be expressed as:

$$ex_t = C + a.\theta_t + c.y_t + v_{1t}$$

where $ex_t = \log E_t$ and $y_t = \log YW_t$. Then the OLS estimator for a is given by:

$$\hat{a} = \frac{m_{y,y} \cdot m_{\theta,ex} - m_{\theta,y} \cdot m_{y,ex}}{m_{\theta,\theta} \cdot m_{y,y} - m^2_{\theta,y}}$$

A similar demonstration for the import supply function shows that the exact bias for the OLS estimator $\hat{b}$ can be put as:

$$E(\hat{b}-b) = -\left[\frac{w_{22}^{**} \cdot b - w_{32}^{**}}{w_{22}^{**}}\right] \cdot e^{-\frac{\mu^2}{2}} \; {}_1F_1 \left[\frac{T-1}{2}-1, \frac{T-1}{2}, \frac{\mu^2}{2}\right]$$

where $\mu^2 = (E\, y_2^{**})'\, M(E\, y_2^{**})/w_{22}^{**} > 0$

$$y_2^{**} = [\log PM_t - \log e_t]$$

and other variables being defined in Appendix II.

$$w_{22}^{**}\, b - w_{32}^{**} = \frac{b\sigma_2^2}{(a-b)^2} + \frac{b\sigma_1^2}{(a-b)^2} - \frac{a}{(a-b)^2} \cdot \sigma_2^2 - \frac{b}{(a-b)^2} \cdot \sigma_1^2$$

$$= -\frac{1}{(a-b)}\sigma_2^2 > 0$$

Then $E(\hat{b}-b) < 0$.

The asymptotic bias derived as before can be expressed as:

$$\lim_{T\to\infty} E(\hat{b}-b) = \frac{\frac{1}{(a-b)} \cdot \sigma_2^2}{w_{22}^{**}(1+\lambda_1)}; \quad \lambda_1 = \frac{\mu^2}{T-1} \approx \frac{\mu^2}{T}$$

Once again, since $a < 0$ and $b > 0$, $\lim_{T\to\infty} (\hat{b}-b) < 0$ i.e., $\hat{b}$ underestimates the true value of b asymptotically.

2. Model II

Another formulation would be to consider the economy under investigation as facing fixed world prices for imports while simultaneously facing a constant elasticity demand function for exports. As a buyer of imports the country is a price taker (a typical small country assumption) while in the case of exports the demand depends on the export price charged relative to the world price and the corresponding export demand elasticity. In this price taking formulation (for imports) the investigators generally specify some kind of ad hoc import demand function which facilitates estimation (see Houthakker and Magee (1969), Leamer and Stern (1970)). Logarithm of the measure of import (quantity) is expressed as a linear function of the logarithm of income and logarithm of the ratio of import prices to the domestic price index. The estimated coefficients are then expressed as the measure of income and price elasticities of imports. 1/

This specification of import demand (along with its variants) is not well integrated with the behavioral relationships of the rest of the economy; neither do we have a satisfactory explanation for the import demand. One possible explanation may be that imports implicitly enter the utility function of the consumers as final goods. This, however, conflicts with the empirical evidence that most the import is used as

1/ There are some exceptions to this where import demand functions are obtained from microeconomic foundations, e.g., Gregory (1970), Burgess (1974a, 1974b).

intermediate goods. Also from the empirical point of view we need to drop the most unrealistic assumption of the conventional trade theory that foreign and domestic goods of the same sectoral classification are identical. This enables us to assert that domestic prices of tradables are not fully tied to the prices of imports and extreme sensitivity in the production and consumption structure due to slight variation in relative prices can be avoided. Following Armington (1969) we can define a 'composite' commodity that is a CES (we can also use a Cobb-Douglas form) aggregation of imports, M and the commodities produced and consumed domestically, D. The aggregation can be put as: 1/

$$Q_t = (d.M_t^{-p} + (1-d).D_t^{-p})^{-(1/p)}$$

Assuming that the producers and consumers minimize the cost of obtaining the 'composite good', Q_t, solving the corresponding first order conditions the import demand function can be expressed as:

$$M_t = (d/(1-d))^b. (PD_t/(e_t.PM_t))^b.D_t \ ; \quad b > 0$$

where b = (1/(1+p)) defines the elasticity of import substitution;

PD_t: domestic good price

PM_t: imported good price (fixed in foreign currency).

1/ In the context of imports this type of aggregation was originally suggested by Ahluwalia, Lysy and Pyatt (World Bank mimeo). It can also be shown that as the elasticity of substitution $b = 1/(1+p)^{+\infty}$, the equation tends to

$$Q_t = d. M_t + (1-d). D_t$$

In a somewhat similar spirit we distinguish between the world price of the product and the price received by an individual exporting country. The export demand function depends on the normal level of exports (E_0) and the export price of the country relative to the world price. This can be put as:

$$E_t/E_0 = C \cdot \left[\frac{\Pi_t \cdot e_t}{PE_t} \right]^a ; \qquad a > 0$$

where $PE_t = PD_t$, the domestic goods price (in the absence of export taxes and subsidies);

Π_t : price of exportables in the international market in foreign currency;

and C is a proportionality factor.

Since the ratio in the parenthesis on the right hand side is the price of exportables in the international (or world) market relative to the domestic price obtainable domestically, a, the price elasticity of export demand would be positive. Taxes and subsidies on imports and exports can also be captured. The system is closed with the trade balance condition:

$$PM_t \cdot M_t = (PE_t/e_t) \cdot E_t$$

Once again we can log-linearize the system and show that OLS estimate of a and b would be asymptotically biased and inconsistent. We can examine the direction of bias and its bearing on the estimated measure of elasticity.

$$E(\hat{a}-a) = -\left[\frac{w'_{22}\cdot a - w'_{12}}{w'_{22}}\right]\cdot e^{-\frac{\mu^{*2}}{2}} \cdot {}_1F_1\left[\frac{T-1}{2}-1, \frac{T-1}{2}; \frac{\mu^{*2}}{2}\right] \qquad \text{(III-7)}$$

where,

$$w'_{12} = \frac{a}{(a+b-1)^2}\cdot\sigma_2^2 + \frac{b}{(a+b-1)^2}\cdot\sigma_1^2$$

$$w'_{22} = \frac{1}{(a+b-1)^2}\cdot\sigma_1^2 + \frac{1}{(a+b-1)^2}\cdot\sigma_1^2$$

and other forms having similar interpretations as in the context of Model I. Then,

$$w'_{22}\, a - w'_{12} = \frac{1}{(a+b-1)}\cdot\sigma_1^2 > 0 \text{ if } a+b-1 > 0.$$

Thus, $E(\hat{a}-a) < 0$ and on analogous derivation,

$E(\hat{b}-b) < 0$ when $a + b - 1 > 0$.

From (III-7) we can derive the large sample bias in a manner similar to that for the Model I.

$$\lim_{T\to\infty} E(\hat{a}-a) = -\frac{(w'_{22}\cdot a - w'_{12})}{w'_{22}\cdot(1+\lambda'_1)} < 0 \text{ if } a+b-1 > 0$$

and,

$$\lim_{T\to\infty} E(\hat{b}-b) = -\frac{(w'_{22}\cdot b - w'_{32})}{w'_{22}\cdot(1+\lambda'_2)} < 0 \text{ if } a+b-1 > 0$$

where $\lambda'_i = \frac{(\mu_i')^2}{T-1} = \frac{(\mu_i')^2}{T}$ where μ_i' is the concentration (noncentrality) parameter of the corresponding confluent hybergeometric function. 1/ The bias is negative for both the estimators and this bias pulls down the estimated sum of the import and export demand elasticities

---

1/ Taking probability limit it can be established that

$$\underset{T\to\infty}{\text{Plim}}\ (\hat{a}-a) = \frac{-(1/(a+b-1)) \cdot \sigma_1^2}{Q}$$

where, $Q = \underset{T\to\infty}{\text{Plim}}\ (1/T). \sum_{t=1}^{T} (\Theta_t - \bar{\Theta})^2$ is assumed to exist and be positive.

$\Theta_t = (\log \Pi_t + \log e_t - \log PE_t)$

Similarly for the import demand elasticity:

$$\text{Plim}\ (\hat{b} - b) = \frac{-(1/(a+b-1))\sigma_2^2}{S}$$

where $S = \underset{T\to\infty}{\text{Plim}}\ (1/T). \sum_{t=1}^{T} (Z_t - \bar{Z})^2 > 0$

and $Z_t = \log PD_t - \log PM_t - \log e_t$ and $\bar{Z}$ is the corresponding mean value. Since a and b are both positive, if $a + b > 1$ (so that $a + b - 1 > 0$),

$$\underset{T\to\infty}{\text{Plim}}\ (\hat{a} - a) < 0$$

and

$$\underset{T\to\infty}{\text{Plim}}\ (\hat{b} - b) < 0$$

if the true value is greater than 1.

These demonstrations in terms of a somewhat simplified model have some bearing on the controversy of the elasticity pessimism. Conventionally, the export and import demand functions are thought to be independent and are estimated separately by OLS. As we have indicated in terms of Models I and II, this gives a lower absolute value of the estimated elasticities sometimes causing unnecessary concern regarding the effectiveness of various commercial policies. The qualifying restriction (for Model II) that $a + b > 1$ is important in determining the critical point. Bias would tend to reduce the absolute values of both $\hat{a}$ and $\hat{b}$ when the true sum of the absolute values of a and b (i.e., $|a| + |b|$) is greater than 1, the condition (so called Marshall-Lerner condition) under which devaluation would be effective in improving the trade balance of the depreciating countries.

Before going on to further extensions we probably need to make some comments on the models specified above. Due to the imposition of trade balance condition our models may be referred to as a kind of 'equilibrium model' and since no short run adjustment to equilibrium has been specified this essentially depicts a long run situation. Hence the elasticities a and b can be considered as long run export and import demand elasticities where full adjustment to price and exchange rate changes are taken care of. Analogously, the estimates obtained without the consideration of trade balance condition may be (somewhat crudely), called short run price elasticity estimates. Thus as one would expect, the long run price elasticities of imports and exports (when exchange rate adjustment and its consequent effects are allowed to take place) would be greater (in absolute value) than the corresponding short run ones.

In another variation of the model we can incorporate the income term explicitly in the export demand function to take into account the variations due to income changes. This would make our export demand function somewhat similar to that of Houthakker and Magee (1969) except for the allowance for exchange rate changes. Given the export demand function of the form:

$$\log E_t = C + a.\log \left[\frac{\Pi_t \cdot e_t}{PD_t}\right] + c.\log YW_t + v_{1t}$$

where YW_t is the weighted index of the GNP of the countries which are importing the home country products.

Defining, $\Theta_t = \log \Pi_t + \log e_t - \log PD_t$ and $\bar{\Theta}_t$ the corresponding mean, the specified equation can be expressed as:

$$ex_t = C + a.\Theta_t + c.y_t + v_{1t}$$

where $ex_t = \log E_t$ and $y_t = \log YW_t$. Then the OLS estimator for a is given by:

$$\hat{a} = \frac{m_{y,y} \cdot m_{\Theta,ex} - m_{\Theta,y} \cdot m_{y,ex}}{m_{\Theta,\Theta} \cdot m_{y,y} - m^2_{\Theta,y}}$$

where $m_{\Theta,ex} = \sum_{t=1}^{T} (\Theta_t - \bar{\Theta}).(ex_t - \overline{ex})$

$$m_{y,y} = \sum_{t=1}^{T} (y_t - \bar{y}).(y_t - \bar{y}) \text{ etc.}$$

Taking probability limit, on simplification we once again end up with the same condition that:

$$E\,(\hat{a}-a) < 0 \quad \text{and}$$

$$\lim_{T\to\infty} E\,(\hat{a}-a) < 0 \quad \text{if } a + b - 1 > 0$$

IV. Empirical Observations

The models outlined in Section III have been applied to the Japanese data to estimate the relevant parameters. Here we present the estimated parameters of the Model III which is a simple extension of Model II (due to the explicit incorporation of income term of the rest of the world). The estimators used are Two Stage Least Squares (2SLS), Nonlinear Two-Stage Lest Squares (NL2SLS), Full-Information Maximum Likelihood (FIML) and OLS.

Data is taken from International Financial Statistics, International Monetary Fund, for the period beginning 1972 to 1978. This is quarterly data adjusted for seasonal variations. The world price index Π_t has been calculated using the method employed by Houthakker and Magee. The weighted index incorporates export prices and trade shares of five other countries (e.g., Canada, France, Germany, U.K. and U.S.A.). Rest of the world real income is expressed as an index (following Houthakker-Magee), the weights coming from the relative shares of five other countries in their total GNP. Due to non-availability of GDP figures as a quarterly time series GNP values have been used as the proxy.

Tables 1 and 2 show the estimates of the parameters of the export and import demand respectively as obtained under different methods. For each of the estimated values of the parameters the corresponding T-statistics are presented in the parenthesis immediately below. Durbin-Watson statistics (D.W.) are presented for each of the methods in a separate column.

Table 1. Structural Equation Estimates of Export Demand Functon Quarterly Data: 1972-1978

$$\log E_t = a \cdot \log (\Pi_t \cdot e_t/PE_t) + c \cdot \log YW_t + v_{1t}$$

Parameters → / Methods ↓	a	c	D.W.
OLS	7.78 (1.85)	1.68 (20.37)	2.22
2SLS	12.96 (2.25)	1.68 (19.59)	2.02
NL2SLS	13.05 (2.31)	1.68 (19.55)	2.01
NLFIML	20.49 (4.83)	1.66 (16.72)	--

Table 2. Structural Estimation of the Parameters of the Import Demand Function: Quarterly Data: 1972-1978

$$\log (m_t/D_t) = C + b. \log (PD_t/e_t. PM_t) + v_{2t}$$

Parameters → / Methods ↓	C	b	D.W.
OLS	-4.01 (-88.38)	0.52 (3.11)	0.26
Adjusted for Autocorrelation	-4.11 (-26.12)	0.36 (1.30)	1.70
2SLS	-4.05 (-86.58)	0.56 (3.24)	0.26
Adjusted for Autocorelation	-4.09 (-19.39)	0.43 (1.93)	
NL2SLS	d = 0.01 (insignificant)	0.59 (3.36)	0.26
NLFIML	d = 0.05 (2.555)	1.30 (3.21)	

V. Serial Correlation and the Bias

In Section III we implicitly assumed that the errors were temporally independent which would not always be valid in our applications with time series data. Presence of autocorrelation (in the context of import and export demand estimations) have been tested in many of the applications that appear in the literature (see Stern, et. al. (1976), Houthakker and Magee (1969), etc.) and also in our applications presented in Section IV. Violation of the classicial assumption of no autocorrelation does not, however, change the qualitative conclusions of Section III; but worsens the magnitude of the bias as the value of p, the autocorrelation coefficient deviates from zero in either direction.

To illustrate this let us start with Model II of Section III, the export demand function being written as:

$$\log E_t = C + a.\ \log \Pi_t + a.\ \log e_t - a.\ \log PE_t + w_t \quad ..(V-1)$$

where $w_t = p.\ w_{t-1} + v_{1t}$ and the rest of the system being unchanged (i.e., as under Model II).

If we do not make any correction for the serial correlation and apply OLS to equation V-1; the asymptotic bias would be as before:

$$\operatorname*{Plim}_{T \to \infty} (\hat{a} - a) = \frac{-(1/(a + b \leq 1)).\ \sigma_w^2}{Q}$$

where $Q = \operatorname*{Plim}_{T \to \infty} (1/T).\ \Sigma(\theta_t - \bar{\theta})^2$ and once again,

$$\theta_t = \log \Pi_t + \log e_t - \log PE_t$$

However, due to the first order autoregressive scheme that we have assumed for the error term,

$$\sigma_w^2 = \frac{\sigma_{v_1}^2}{(1-p^2)}$$

On substitution,

$$\underset{T\to\infty}{\text{Plim}}\ (\hat{a} - a) = \frac{-\dfrac{1}{(a + b - 1)} \quad \dfrac{\sigma_{v_1}^2}{(1 - p^2)}}{Q} \quad \ldots.(IV-3)$$

If $p = 0$, the bias exactly equals that of Section III and as p gets away from zero in either the positive or negative directions the bias would get worse very rapidly. The bias tends to infinity asymptotically as $p \to 1$ (see the dotted curve in Figure I).

Now let us consider the situation where we take a quasi-first difference transformation of equation (V-1), so that we end up with:

$$ex_t - p \cdot ex_{t-1} = a \cdot (\theta_t - p \cdot \theta_{t-1}) + v_{1t}$$

or, $ex_t^* = a \cdot \theta_t^* + v_{1t}$; where v_{1t} is not autocorrelated and

$ex_t^* = ex_t - p \cdot ex_{t-1}$ and $\theta_t^* = (\theta_t - p \cdot \theta_{t-1})$.

Then once again we can apply OLS to this transformed system and derive the asymptotic bias of the OLS estimator a*.

$$\operatorname*{Plim}_{T\to\infty} (\hat{a}^* - a) = \frac{\operatorname*{Plim}_{T\to\infty} (1/T).\ \Sigma(\theta_t^* - \theta).(v_{1t} - v_1)}{\operatorname*{Plim}_{T\to\infty} (1/T).\ \Sigma(\theta_t^* - \bar{\theta})^{*2}}$$

Substituting the values of θ_t^* and $\bar{\theta}^*$ in the numerator and the underlying reduced form expression for log e_t we get:

$$\operatorname*{Plim}_{T\to\infty} (\hat{a}^* - a) = \frac{-(1/(a + b - 1))\ .\ \sigma^2_{v_1}}{D}$$

where $D = \operatorname*{Plim}_{T\to\infty} (1/T)\ \Sigma(\theta_t^* - \bar{\theta}_t^*)^2$

$$\text{Now, } D = \operatorname*{Plim}_{T\to\infty} (1/T).\ \ \Sigma[(\theta_t - p.\theta_{t-1}) - (\bar{\theta}_t - p.\bar{\theta}_{t-1})]^2$$

$$= \text{Plim } (1/T).\ \ \Sigma[(\theta_t - \bar{\theta}_t) - p.(\theta_{t-1} - \bar{\theta}_{t-1})]^2$$

On simplification this reduces to:

$$[(1+p^2) - 2p\ .\ \hat{d}_{\theta_t,\theta_{t-1}}]\ .\ \operatorname*{Plim}_{T\to\infty} (1/T)\ .\ \Sigma(\theta_t - \bar{\theta}_t)$$

$$\text{where } \hat{d}_{\theta_t,\theta_{t-1}} = \frac{\Sigma(\theta_t - \bar{\theta}_t)(\theta_{t-1} - \bar{\theta}_{t-1})}{\Sigma(\theta_t - \bar{\theta}_t)^2}$$

which is the least squares estimates of the slope coeffcients between θ_t and θ_{t-1}. Since θ_{t-1} is simply the one period lagged value of θ_t, $\hat{d}_{\theta_t,\theta_{t-1}}$ is normally not statistically significantly different from 1.

Thus for large samples:

$$D = (1-p)^2 \cdot \underset{T\to\alpha}{\text{Plim}} \left(\frac{1}{T}\right) \cdot \Sigma(\theta_t - \bar{\theta}_t)^2$$

$$= (1-p)^2 \cdot Q$$

Using this large sample approximation,

$$\underset{T\to\alpha}{\text{Plim}} (\hat{a}^* - a) = \frac{-\dfrac{1}{(\hat{a} + b - 1)} \cdot \dfrac{\sigma^2_{v_1}}{(1 - p)^2}}{Q} \qquad \ldots .(IV-4)$$

Comparing (IV-3) and (IV-4) we can make some interesting conclusions:

(i) If $-1 < p < 0$, then $(1 - p^2) < (1 - p)^2$ and the bias in the uncorrected autocorrelation case is greater than for the case where quasi first difference transformation has been made to correct the autocorrelation. Note that in both cases we are applying OLS.

(ii) If $1 > p > 0$, then $(1 - p^2) > (1 - p)^2$ and the bias in the uncorrected case is less than that for the corrected one. Of course, once again we are applying OLS to both cases and so simultaneous equation bias is unattended.

Figure I shows how the asymptotic bias varies as a function of p, where corrections for serial correlation have or have not been made. This is represented for some arbitrary value of:

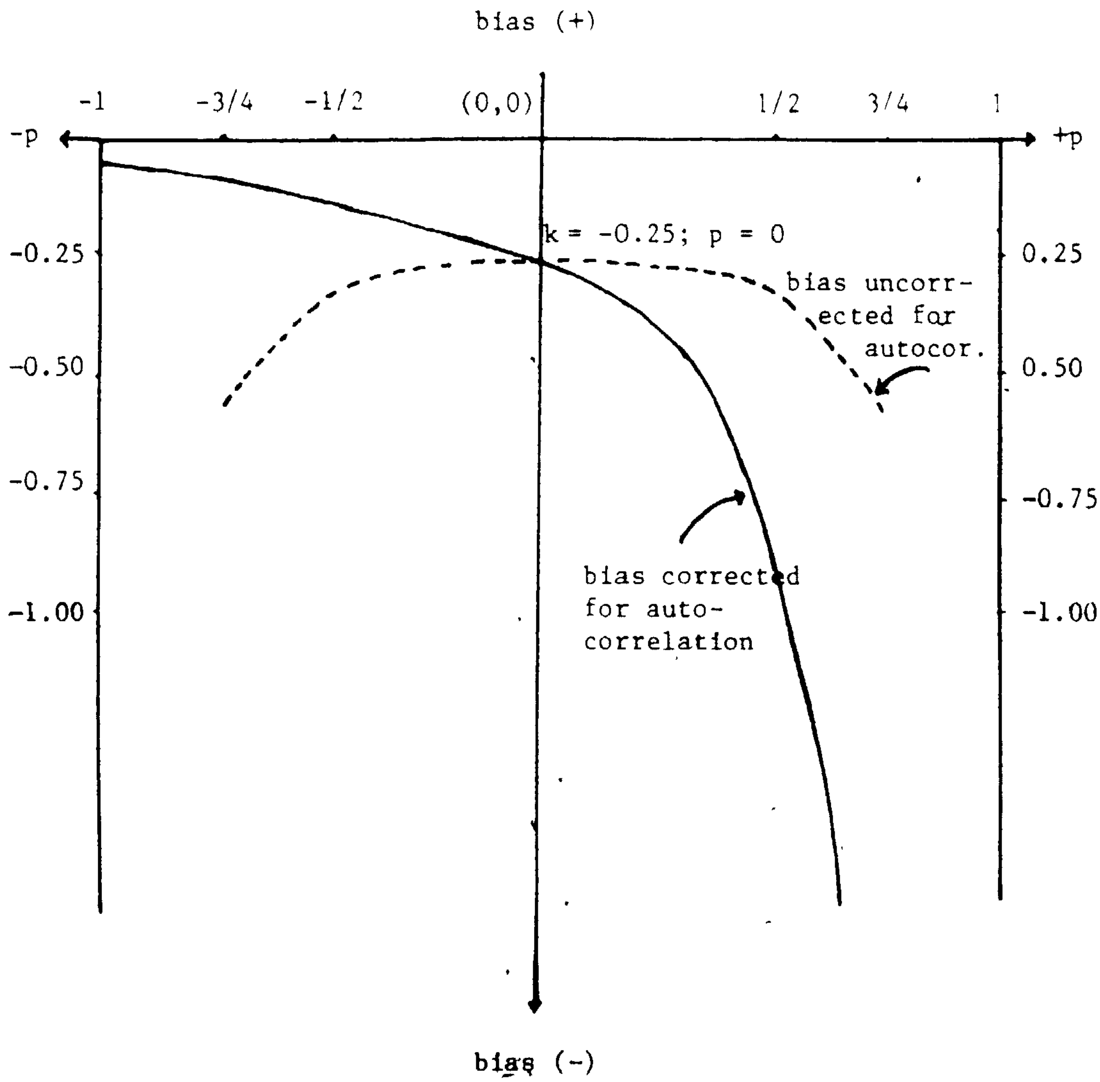
$$k = \frac{-(1/(a\ b-1)).\sigma^2_{v_1}}{Q} = 0.25 \text{ (say)}.$$

The dramatic increase of bias for the range $1 > p > 0$ indicates how serious could be the potential bias. Very often in econometric studies with time series data we face the existence of potential positive autocorrelation. Also in many of those (like the studies on international trade elasticities) we recognize the existence of potential simultaneity but without characterizing its exact nature and to make things simple apply OLS to get the parameter estimators. If we find evidence of autocorrelation (say $p > 0$) of first order normally we make necessary transformations to correct the situation. It is evident from Figure I that this could have potentially disastrous effects on the overall asymptotic bias.

VI. Conclusions

In this paper we have discussed the direction of bias in the conventional estimates of the import and export demand functions. This possibly could have caused underestimation of the values of the trade elasticity parameters which have also been supported by Japanese data. Although most of the arguments as specified in Section I, have been demonstrted in terms of specific models, most of these would possibly be valid under various other formulations. If we have multi-sectoral models, once again the trade balance condition would act as a binding constraint and the arguments should apply equally (although the algebra gets somewhat intractible). The observation regarding the effects of autocorrelation is valid for other models of this type. Its implications go beyond the context of estimating the international trade elasticity parameters and is of relevance to a wide class of potential applications to other studies.

Figure I: The Direction of Bias in the Presence of Autocorrelation for Uncorrected (dotted line) and corrected cases



APPENDIX II

Model I:

This model can be written as in general conventional notation as:

$$y_{1t} = \gamma + a\,(x_{1t} - y_{2t}) + v_{1t}$$

$$y_{3t} = \delta + b\,(x_{2t} - y_{2t}) + v_{2t}$$

$$y_{1t} + x_{2t} = x_{1t} + y_{1t}$$

where,

$$y_{1t} = \log\,(E_t/E_0)\ ; \quad x_{1t} - y_{2t} = \log\,(PE_t/e_t),$$

$$y_{3t} = \log\,(M_t/M_0)\ ; \quad x_{2t} - y_{2t} = \log\,(PM_t/e_t).$$

The OLS estimator of export demand elasticity:

$$\hat{a} = \frac{\Sigma\,(y_{1t}-y_1)\,(y^{*}_{2t}-y^{*}_{2})}{\Sigma\,(y^{*}_{2t}-y^{*}_{2})^2}$$

$$= \frac{y_1' M y_2}{y_2^{*\prime} M y_2^{*}}$$

where $y^{*}_{2t} = x_{1t} - y_{2t}$

y_1, y^{*}_{2} are Tx1 vectors and

$M = [I - \ell(\ell'\ell)^{-1}\ell'] = [I - \frac{\ell\ell'}{T}]$,

$\ell = [1, \ldots, 1]'$ is a Tx1 vector of unit elements.

M is an idempotent matrix of rank T-1.

The reduced form of this model can be expressed in generalized notation as:

$$y_{1t} = \Pi_{10} + \Pi_{11} x_{1t} + \Pi_{12} x_{2t} + \frac{a}{a-b} v_{2t} + \frac{b}{a-b} v_{1t}$$

$$y_{2t} = \Pi_{20} + \Pi_{21} x_{1t} + \Pi_{22} x_{2t} + \frac{a}{(a-b)} v_{1t} - \frac{1}{(a-b)} v_{2t}$$

$$y_{3t} = \Pi_{30} + \Pi_{31} x_{1t} + \Pi_{32} x_{2t} + \frac{a}{a-b} v_{2t} - \frac{b}{a-b} v_{1t}$$

where Π's are the appropriate reduced form parameters. In an alternative form, as a by product we can write,

$$y^{*}_{2t} = x_{1t} - y_{2t} = -\Pi_{20} + (1-\Pi_{21})x_{1t} - \Pi_{22} x_{2t} + \frac{1}{(a-b)} v_{2t} - \frac{1}{(a-b)} v_{1t}$$

$$y^{*}_{2t} = x_{2t} - y_{2t} = -\Pi_{20} + \Pi_{21} x_{1t} + (1-\Pi_{22})x_{2t} + \frac{1}{(a-b)} v_{2t} - \frac{1}{(a-b)} v_{1t}$$

Under the assumption that the errors are independently and normally distributed, from the reduced form expressions we can establish that:

$$y_{1t} \sim N(\theta_{1t}, w_{11}) ; \quad w_{11} = \frac{a^2}{(a-b)^2} . \sigma^2_2 + \frac{b^2}{(a-b)^2} . \sigma^2_1$$

$$y_{2t} \sim N(\theta_{2t}, w_{22}) ; \quad w_{22} = \frac{1}{(a-b)^2} . \sigma^2_2 + \frac{1}{(a-b)^2} . \sigma^2_1$$

$$y_{3t} \sim N(\theta_{3t}, w_{33}) ; \quad w_{33} = \frac{a^2}{(a-b)^2}.\sigma_1^2 + \frac{b^2}{(a-b)^2}.\sigma_2^2$$

$$y^*_{2t} \sim N(\theta^*_{2t}, w^*_{22}) ; \quad w^*_{22} = \frac{1}{(a-b)^2}.\sigma_2^2 + \frac{1}{(a-b)^2}.\sigma_1^2$$

$$y^{**}_{2t} \sim N(\theta^{**}_{22}, w^{**}_{22}) ; \quad w^{**}_{22} = \frac{1}{(a-b)^2}.\sigma_2^2 + \frac{1}{(a-b)^2}.\sigma_1^2$$

$$\text{cov}\,(y_{1t}, y^*_{2t}) = w^*_{12} = \frac{a}{(a-b)^2}.\sigma_2^2 + \frac{b}{(a-b)^2}.\sigma_1^2$$

$$\text{cov}\,(y_{3t}, y^{**}_{2t}) = w^{**}_{32} = \frac{a}{(a-b)^2}.\sigma_2^2 + \frac{b}{(a-b)^2}.\sigma_1^2$$

Here θ's are the means of the distributions which by construction are the systematic or nonstochastic part of the reduced form.

Now following Richardson and Wu (1971, pp. 976) we can deduce the bias of the OLS estimator $\hat{a}$ as

$$E\,(\hat{a}-a) = -\left[\frac{w^*_{22}.a - w^*_{12}}{w^*_{22}}\right].e^{-\frac{\mu^2}{2}} \cdot {}_1F_1\left[\frac{T-1}{2} -1, \frac{T-1}{2}, \frac{\mu^2}{2}\right] \quad \ldots(A-1)$$

where $\mu^2 = \dfrac{(E\,y^*_2)'\,M\,(Wy^*_2)}{w^*_{22}} = \dfrac{\theta^{*'}_2\,M\,\theta^*_2}{w^*_{22}}$

and ${}_1F_1\,(*)$ is a confluent hypergeometric function. 1/

1/ For more on confluent hypergeometric functions like this, see Slater (1960), and for its uses in econometrics see Sawa (1972), and Richardson (1968).

Since $\mu^2 > 0$, the confluent hypergeometric function ${}_1F_1$ (*) in the above expression is positive and the magnitude and direction of the exact bias is dependent on the elements within the first parenthesis of (A-1). It is easy to note that

$$w^*_{22} \cdot a - w^*_{12} = \frac{1}{(a-b)} \sigma^2_1 < 0 \text{ and}$$

so the algebraic value of the bias is positive as shown in equation (III-5) of the text.

Derivations for other models can be made analogously.

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Chapter IV

A Decomposition Algorithm for General Equilibrium Computation With Application to International Trade Models*

I. Introduction

In this paper we describe the computation of general equilibrium via a fixed point decomposition procedure similar in spirit to the Dantzig-Wolfe decomposition algorithm for the solution of linear programming problems [Dantzig and Wolfe (1961)]. We show that for a general equilibrium model of a particular structure it is possible to compute equilibria using 'master' and 'sub' simplices each of a dimensionality smaller than that of the total problem. The analogues to the common constraints in the Dantzig-Wolfe procedure are common commodities with common prices, and the block diagonal structure on non-common constraints is replaced by an analogous block diagonal pattern of demands and endowments of agents over non-common goods. The procedure is guaranteed to terminate at an approximate equilibrium without cycling by the same argument underlying Scarf's algorithm. In the case of a pure trade equilibrium we use variations on the traditional Gale Nikaido mapping for label generation but use dimensions smaller than that of the whole problem. A natural application of the method is to international trade models with 'traded' and 'non-traded' goods. We use this example for illustration but regional an intertemporal models would also appear to offer fruitful areas of application.

* This paper was originally prepared jointly with Professor John Whalley and a revised version is forthcoming in Econometrica (See Mansur, A. and J. Whalley (1980)).

The method involves the generation of labels for vertices on a master simplex through the separate solution of sub-equilibrium problems whose parameters are determined by the vertex on the master simplex. Information is passed between a master problem and sub problem as in the Dantzig-Wolfe algorithm; 'coefficient generation' for a master linear programming problem in the Dantzig-Wolfe algorithm is replaced by 'label generation' for vertices on a 'master' simplex associated with the general equilibrium problem. We have performed computation with this method for some numerical examples using Merrill's algorithm for solution of both full dimensional problems and the same problems by the decomposition procedure. A quicker procedure for large models would be to use the recent methods of vander Laan and Talman (1979), but for comparative purposes the consistent application of Merrill's algorithm would appear to be adequate. Where a number of blocs of consumer groups with only partially overlapping excess demand functions are involved significant computational gains are indicated, suggesting eventual possible application to empirically oriented large-scale general equilibrium models of world trade. At least two models in current use have the same structure of common and non-common goods as examined here; referred to as 'traded' and 'non-traded' goods in the international trade literature. In one [Deardorff and Stern (1979)] 18 separate countries appear and in the other [Whalley (1979)] 4 major trading areas are identified. Our discussion is exclusively in terms of integer labelling problems; we believe the extension to vector labelling problems follows naturally but have not extensively explored the issue.

The economic interpretation of our procedure is that we decompose the list of commodities in a general equilibrium problem into 'common' goods traded among all agents and 'non-common' goods traded only among a subset of agents. The allocation of non-common goods to agents can be represented in a bloc diagonal partition of demands and asset ownership by agent. We use a master simplex containing information on the prices of common goods. The sub-equilibrium problems use information on the relative prices of common goods from the master simplex to form a vector of prices for both common goods and non-common goods in the bloc. Each sub-equilibrium problem takes the relative prices of common goods as given and determines an approximate equilibrium characterized by non-positive excess demands for each non-common good along with an equilibrium price for a composite common good meeting a non-negative excess demand condition. The solution for subproblems together with the common good prices on the master simplex yield an evaluation of excess demand functions for common goods which generates a label for a vertex on the master simplex. A completely labelled master simplex along with associated sub-equilibria yield an approximation to an equilibrium for the whole model, which becomes exact in the limit approached by a dense grid.

Each subproblem may be interpreted as a equilibrium problem for a small open economy with non-traded goods, which takes relative prices of traded goods as given by world markets and determines an equilibrium in which demand supply equalities hold for non-traded goods along with an external sector balance condition. The demonstration of existence of

By reducing the dimensionality of each problem in this way, the hope is that by determining subsets of equilibrium prices with equilibrium holding for a composite of the common goods a complete equilibrium can be determined and yield computational gains over full solution. Our method shows that this is possible and initial computational experience suggests that computational gains of some potential significance are possible.

Sketch of the procedure

Taking a simplex of full dimensionality containing all N commodity prices π_i,(i=1,...,N), we reduce the dimensionality of this simplex to n_0 by retaining only the prices of common goods. We term this the master simplex.

Using a regular grid to represent the subdivision of the master simplex, we separately construct a simplex on which we search for a sub-equilibrium for each group where simplices for subproblems are all associated with the vertex of the subdivision of the master simplex from which they are derived. These associated simplices are each of dimension equal to the number of non-common goods for the group concerned plus one. For convenience, all the coordinate sums for vertices on associated simplices are taken to be equal to that of vertices on the master simplex.

We compute 'sub-equilibria' for each group using these simplices and assuming the relative prices of common goods from the vertex on the master simplex to be fixed. The equilibrium conditions in these problems are limited to the non-positive excess demand for the composite of common goods (bloc 2),..., and so on. The model has a total of $N = n_0 + \sum_{i=1}^{K} (n_i-(n_{i-1} + 1))$ goods. We consider a price vector π of dimensionality N which contains prices of all goods. w_i^k represents the endowment of good i by group k. We assume $w_i^k \geq 0$ and > 0 for some i for each k; in addition $w_i^k = 0$ for $i=n_{\ell-1} + 1,\ldots,n_\ell$ and $\ell \neq k$ (zero endowments in group k of non-common goods of other groups of agents).

The market demands by group k for the common goods, $1,\ldots,n_0$ and non-common goods relevant to the group, $n_{k-1} + 1,\ldots,n_k$, are functions only of the corresponding commodity prices $(\pi_1,\ldots,\pi_{n_0},\ldots,\pi_{n_{k-1}+1},\ldots,\pi_{n_k},\ldots)$. We will use this property later but for convenience write these functions as $\xi_i^k(\pi)$; the demands for good i by group k. The demand functions for each group are assumed to be non-negative, continuous, homogenous of degree zero in all prices, and to satisfy a version of Walras' Law defined for that group alone. This implies that total demand functions satisfy a model wide version of Walras' Law. The zero homogeneity of demands allows prices of all common and non-common goods in the economy to be normalized to sum to any non-negative constant; we work with a price simplex all of whose vectors contains coordinate with sum D; $\sum_{i=1}^{N} \pi_i = D$.

An equilibrium in this model is vector of prices π^* such that all excess demands are non-positive with zero prices prevailing for any commodity with strictly negative excess demands. These conditions may be written out explicitly as

$$
\begin{array}{llcc}
 & & \underline{\text{Demand by}} & \underline{\text{Endowments of}} \\
 & & \text{group } 1 \quad \text{group } 2 \quad \ldots\ldots \text{ group } K & \text{group } 1 \ \ldots\ldots \text{ group } K \\
\text{common goods} & \left(\begin{array}{l} \xi_1^1(\pi^*) + \xi_1^2(\pi^*) + \ldots\ldots + \xi_1^K(\pi^*) \leq w_1^1 + \ldots\ldots + w_1^K \\ \xi_{n_0}^1(\pi^*) + \xi_{n_0}^2(\pi^*) + \ldots\ldots + \xi_{n_0}^K(\pi^*) \leq w_{n_0}^1 + \ldots\ldots + w_{n_0}^K \end{array} \right. & & \\
\text{non-common goods of group 1} & \left(\begin{array}{l} \xi_{n_0+1}^1(\pi^*) + 0 \ \ldots\ldots + 0 \leq w_{n_0+1}^1 \\ \xi_{n_1}^1(\pi^*) + 0 \ \ldots\ldots + 0 \leq w_{n_1}^1 \end{array} \right. & & \\
 & \quad \vdots & & \vdots \\
\text{non-common goods of group K} & \left(\begin{array}{l} 0 \ \ldots\ldots\ldots\ldots \ 0 + \xi_{n_{K-1}+1}^K(\pi^*) \leq w_{n_{K-1}+1}^K \\ 0 \ \ldots\ldots\ldots\ldots \ 0 + \xi_{n_K}^K(\pi^*) \leq w_{n_K}^K \end{array} \right. & &
\end{array}
$$

with the corresponding π_i^* equalling zero if any inequality holds strictly.

The statement of equilibrium conditions above is visually similar to the constraint matrix in the Dantzig-Wolfe decomposition algorithm where a set of common constraints prevails with the remainder of the constraint matrix being written in block diagonal form. Here a similar structure applies to both demands and initial endowments when subscripted by group. One can clearly use Scarf's (1973) algorithm (applying one of the recent refinements to reduce execution times such as vander Laan and Talman (1979) and Shamir (1979)) directly to this N-dimensional model ignoring its special structure. Our interest is in a computational method using a lower dimensionality than N which takes advantage of this special structure while still preserving the Lemke-Howson no-cycling argument used by Scarf's algorithm.

III. A Decomposition Algorithm for Equilibrium Computation

In the Dantzig-Wolfe decomposition procedure a full dimensional linear programming problem with common constraints and a bloc diagonal structure for other constraints is rewritten as an equivalent problem requiring only one constraint for each of the diagonal blocs in the constraint matrix of the original problem. This problem involves all the (unknown) vertices associated with the constraint set for each of the diagonal blocs in the non-common portion of the constraint set. Columns are generated for the simplex tableau of the reduced dimension equivalent problem by solving sub LP problems generating a vertex for each bloc with the smallest entry for the objective function in the tableau. Vertices are successively added through the column generating procedure which subproblem solutions provide. Pivot steps in the master tableau in turn provide coefficients for the objective function in subproblems. When all subproblems produce non-positive solutions, all tableau entries for coefficients in the objective function associated with vertices of the constraint sets are non-negative and a maximum to the linear programming problem must have been determined. The method allows a large dimensional linear programming problem to be solved through a sequence of smaller dimensional problems without any necessary requirement that all vertices describing constraint sets for diagonal blocs be evaluated. The no-cycling argument in the traditional form of the simplex method is preserved.

In our fixed point decomposition procedure, we first rewrite the general equilibrium conditions above as two interdependent sets of conditions which if they jointly hold at the same set of prices ensure equilibrium. Our procedure involves passing information between a master simplex to sub simplices until both sets of conditions are satisfied. The labelling rule used for vertices on the master simplex guarantees that in the limit associated with a dense grid being used to solve both master and subproblems the same price vectors must occur at the two solution sets.

Our equivalent statement of equilibrium conditions involves rewriting the price vector π as $(\pi^c, \pi^1, \ldots, \pi^K)$ where π^c refers to prices of common goods and $\pi^1, \ldots, \pi^K$ are the prices of non-common goods for each group. We note that the demand functions $\xi_i^k(\pi)$ of group k can be rewritten as $\xi_i^k(\pi^c, \pi^k)$.

We partition the equilibrium conditions, written in full above, and characterize a 'common good' equilibrium (Master Equilibrium) as a vector $\pi^* = (\pi^{c*}, \pi^{1*}, \ldots, \pi^{K*})$ such that excess demands are non-positive for each common good, i.e.,

$$\Sigma(\xi_i^k(\pi^{c*}, \pi^{k*}) - w_i^k < 0 \quad (= 0 \text{ if } \pi_i^{c*} > 0) \quad i=1, \ldots, n_0$$

'Non-common' good equilibria (Sub Equilibria) are characterized in terms of a vector (π^{c*}, π^{k*}) for any group k such that excess demands are non-positive for each non-common good, i.e.,

$$\xi_i^k(\pi^{c*}, \pi^{k*}) - w_i^k < 0 \quad (= 0 \text{ if } \pi_i^{k*} > 0) \quad i=n_{k-1} + 1, \ldots, n_k.$$

We note that from Walras' Law and the property that non-common goods cannot be traded across groups, the value of excess demands for common goods equals zero for each group, i.e.,

$$\sum_{i=1}^{n_0} \pi_i^{c*}(\xi_i^k(\pi^{c*},\pi^{k*}) - w_i^k) = 0$$

If each of these sets of equilibrium conditions simultaneously holds at the same sets of prices then full equilibrium must prevail. While it may seem an indirect procedure to partition the equilibrium conditions in order to characterize two 'partial' equilibria which jointly imply the general equilibrium conditions a potential computational saving is suggested by this device. If, in some way, the common goods prices can be assumed to be fixed to solve for sub-equilibria and the non-common goods prices calculated in this way used in determining master equilibria, a procedure which passes information between alternative equilibrium problems offers a possibility of a solution method which exploits the special bloc diagonal structure of excess demand functions. The Hicks-Leontief composite commodity theorem suggests that aggregating quantities within a partition yields a composite commodity which can be analytically treated as equivalent to a single commodity. We exploit this by arguing that given any fixed relative prices for common goods, there exist prices for the composite commodity and non-common goods for the bloc that satisfy non-positive excess demand conditions. We thus solve sub-equilibria using the composite commodity and communicate prices of non-common goods back to the master simplex for an evaluation of excess demands of common goods.

By reducing the dimensionality of each problem in this way, the hope is that by determining subsets of equilibrium prices with equilibrium holding for a composite of the common goods a complete equilibrium can be determined and yield computational gains over full solution. Our method shows that this is possible and initial computational experience suggests that computational gains of some potential significance are possible.

Sketch of the procedure

Taking a simplex of full dimensionality containing all N commodity prices π_i,(i=1,...,N), we reduce the dimensionality of this simplex to n_0 by retaining only the prices of common goods. We term this the master simplex.

Using a regular grid to represent the subdivision of the master simplex, we separately construct a simplex on which we search for a sub-equilibrium for each group where simplices for subproblems are all associated with the vertex of the subdivision of the master simplex from which they are derived. These associated simplices are each of dimension equal to the number of non-common goods for the group concerned plus one. For convenience, all the coordinate sums for vertices on associated simplices are taken to be equal to that of vertices on the master simplex.

We compute 'sub-equilibria' for each group using these simplices and assuming the relative prices of common goods from the vertex on the master simplex to be fixed. The equilibrium conditions in these problems are limited to the non-positive excess demand for the composite of common goods constructed using the relative common goods prices from the master simplex, rather than requiring each excess demand to be non-positive as in full equilibrium. In any sub-equilibrium we thus determine market clearing prices of non-common goods and a scalar multiple for all common goods prices giving non-positive excess demands for the composite common good using the fixed relative prices from the vertex on the master simplex as weights.

We return our sub-solution to the vertex on the master simplex and list the prices of non-common goods along with the common goods prices. We use the derived price vector to evaluate excess demand functions, and our labelling rule for any vertex on the master simplex involves the excess demands for common goods. A completely labelled simplex of dimension n_0 in the subdivision of the master simplex (along with the associated approximate sub-equilibria) will characterize an approximation to an equilibrium for the entire general equilibrium system.

Description of 'master' and 'sub' simplices

We define the master simplex of dimension n_0 as S_0 and consider a subdivision of this simplex to be represented by the vertices S_0^j. Each vertex S_0^j will be written as

$$S_0^j = \begin{bmatrix} \pi_1^j \\ \cdot \\ \cdot \\ \cdot \\ \pi_{n_0}^j \end{bmatrix} ; \quad \sum_{i=1}^{n_0} \pi_i^j = D, \text{ for all } j.$$

For convenience, we will restrict ourselves to subdivisions of S_0 characterized by a regular grid with mesh size given by the integer D.

Corresponding to each vertex S_0^j, we also define K-simplices $S_1^j, S_2^j, \ldots, S_K^j$ (one corresponding to each group) such that the sums of coordinates on those simplices are also given by D. We consider simplicial subdivisions of each simplex $S_1^j, \ldots, S_K^j$ in addition to the subdivision of S_0 represented by the vertices S_0^j. The h^{th} vertex of the subdivision of the k^{th} simplex S_k^{jh} is written as

$$S_k^{jh} = \begin{pmatrix} \pi^{jh}_{n_{k-1}+1} \\ \cdot \\ \cdot \\ \cdot \\ \pi^{jh}_{n_k} \\ C_k^{jh} \end{pmatrix}$$

where $k=1,2,\ldots,K$ and the superscript j corresponds to the j^{th} vertex S_0^j.

Characterization of sub-equilibrium problems

Corresponding to any vertex S_k^{jh} in a subdivision of S_k^j we determine a vector of prices of common and non-common goods using the relative prices of common goods $\pi_1^j, \ldots \pi_{n_0}^j$ from S_0^j, the sum of common goods prices from S_k^{jh}, and the prices of non-common goods $\pi^{jh}_{n_{k-1}+1}, \ldots, \pi^{jh}_{n_k}$ from S_k^{jh}. We represent this by the function $\tilde{\pi}_k^{jh}(S_0^j, S_k^{jh})$ where

$$\pi_k^{jh}(S_0^j, S_k^{jh}) = \left(\frac{\pi_1^j . C_k^{jh}}{D}, \frac{\pi_2^j . C_k^{jh}}{D}, \ldots, \frac{\pi_{n_0}^j . C_k^{jh}}{D}, \pi_{n_{k-1}+1}^{jh}, \ldots, \pi_{n_k}^{jh} \right).$$

The function $\tilde{\pi}_k^{jh}(S_0^j, S_k^{jh})$ determines prices of both common and non-common goods for group k where the relative prices of common goods are consistent with corresponding values from the vertex S_0^j on the master simplex. The relative prices of the non-common goods are given by the vertex S_k^{jh} on the simplex associated with the subproblem. The sum of the prices of common goods is constrained to equal C_k^{jh}, the last element of the vector defining the vertex S_k^{jh}. C_k^{jh} can be zero in which case the terms

$$\frac{\pi_1^j . C_k^{jh}}{D}, \ldots, \frac{\pi_{n_0}^j . C_k^{jh}}{D} \quad \text{are all zero.}$$

For any group k, a 'sub-equilibrium problem' is constructed whose equilibrium solution is defined as a vector $\tilde{\pi}_k^{j*}(S_0^j, S_k^{j*})$ associated with a vector S_k^{j*} on the simplex S_k^j such that

$$\text{(i)} \quad \xi_i^k(\tilde{\pi}_k^{j*}) - w_i^k \leq 0 \qquad (= 0 \text{ if } \pi_i^{j*} > 0) \qquad (i = n_{k-1} + 1, \ldots, n_k)$$

$$\text{(ii)} \quad \sum_{i=1}^{n_0} \frac{\pi_i^j}{D} (\xi_i^k(\tilde{\pi}_k^{j*}) - w_i^k) \leq 0 \quad (= 0 \text{ if } C_k^{j*} > 0)$$

In this problem common goods are aggregated at the fixed prices π_i^j. Equilibrium prevails when both non-positive excess demands summed across all consumers in group k occur for the composite of common goods and for each non-common good. C_k^{j*} can be interpreted as the equilibrium price for the composite of common goods (constructed using the fixed relative prices $\pi_1^j,\ldots,\pi_{n_0}^j$ as weights) relative to each of the non-common goods for group k.

Solution of sub-equilibrium problems

Using the vector function $\tilde{\pi}_k^{jh}(S_0^j,S_k^{j*})$ for any vertex S_k^{jh} we can apply Scarf's algorithm (or one of the recent extensions mentioned earlier) to compute an approximation to a sub-equilibrium $S_k^{j*} = (\pi_{n_{k-1}+1}^{j*},\ldots,\pi_{n_k}^{j*},C_k^{j*})$. This provides an approximation to an equilibrium to the k^{th} subproblem which is used in the labelling procedure for the vertex S_0^j on the master simplex in a way to be described later.

The equilibrium computation differs in the subproblem from the application of Scarf's algorithm to a standard exchange economy only in a small modification in the use of the Gale-Nikaido mapping to determine labels for vertices on each simplex S_k^j. The conventional labelling procedure for an N commodity exchange equilibrium problem defined on an N dimensional price simplex calls for selecting as the label to any vertex π^j the index of the first coordinate for which $f_i(\pi^j) - \pi_i^j > 0$ $(i=1,\ldots,N)$, where f_i define the well known Gale-Nikaido transformation of the excess demand functions which produce a continuous mapping of the unit simplex into itself.

In the solution of each subproblem we use a related mapping of a $(n_k - n_{k-1})$ dimensional simplex into itself given by

$$g_i^k(\tilde{\pi}_k^{jh}(S_0^j, S_k^{jh})) = \frac{\pi_i^{jh}/D + \max[0, (\xi_i^k(\tilde{\pi}_k^{jh}) - w_i^k)]}{1 + \sum_{i=n_{k-1}+1}^{n_k} \max[0, (\xi_i^k(\tilde{\pi}_k^{jh}) - w_i^k)] + \max[0, \sum_{i=1}^{n_0} \frac{\pi_i^j}{D}(\xi_i^k(\tilde{\pi}_k^j) - w_i^k)]}$$

for $i = n_{k-1}+1, \ldots, n_k$

$$g_{n_{k+1}}^k(\tilde{\pi}_k^{jh}(S_0^j, S_k^{jh})) = \frac{C_k^{jh}/D + \max[0, \sum_{i=1}^{n_0} \frac{\pi_i^j}{D}(\xi_i^k(\tilde{\pi}_k^j) - w_i^k)]}{1 + \sum_{i=n_{k-1}+1}^{n_k} \max[0, (\xi_i^k(\tilde{\pi}_k^{jh}) - w_i^k)] + \max[0, \sum_{i=1}^{n_0} \frac{\pi_i^j}{D}(\xi_i^k(\tilde{\pi}_k^j) - w_i^k)]}$$

The label of vertex S_k^{jh} in the k^{th} subproblem is taken as the first i for which the image $g_i^k(\tilde{\pi}_k^{jh}(S_o^j, S_k^{jh}))$ exceeds one over D multiplied by its corresponding coordinate on S_k^{jh}. A completely labelled simplex will have all the labels $1, \ldots, (n_k - n_{k-1})$ including a label derived from the transformation of the excess demand for the composite good. The modification to the Gale-Nikaido transformation incorporates the feature that a composite of excess demands is involved for the common goods. The starting and termination procedures are as in any other application of Scarf's algorithm.

Labelling rules for vertices on the master simplex

While solutons to sub-equilibrium problems corresponding to a vertex S_0^j on the master simplex will yield approximations to the equilibrium conditions required for all non-common goods, they will not typically result in market excess demands for each of the common goods being less than or equal to zero. We therefore construct a labelling rule for any vertex S_0^j on the master simplex such that for a completely labelled simplex in a subdivision of S_0 this will be the case. The labels are selected from integers $1,\ldots,n_0$ and for a completely labelled simplex an approximation to an equilibrium for the entire general equilibrium problem will be obtained. This approximation becomes exact in the limit as the mesh of vertices defining the subdivision becomes everywhere dense on the simplex S_0.

We begin with the solution to the first sub-equilibrium problem and construct the vector

$$\hat{\pi}^{1j} = (\lambda_1^j \pi_{n_0+1}^{j*},\ldots,\lambda_1^j \pi_{n_1}^{j*})$$

where λ_1^j is defined as $\dfrac{\sum_{i=1}^{n_0} \pi_i^j}{C_1^{j*}}$, and $\pi_{n_0+1}^{j*},\ldots,\pi_{n_1}^{j*}$ define the sub-equilibrium non-common goods prices from the solution of the first sub-equilibrium problem.

Where a non-dense grid is used for a sub-equilibrium problem the non-common goods prices $\pi_{n_0+1}^{j*},\ldots,\pi_{n_1}^{j*}$ together with C_1^{j*} may be taken from the mid point of the completely labelled simplex in the subdivision of the simplex associated with the subproblem. In the finite grid case C_1^{j*} cannot equal zero, in the limiting case of a dense grid λ_1^j may approach infinity if C_1^{j*} approaches zero. A construction we use in our labelling rule below guarantees that even if C_k^{j*} approaches zero for all K subproblems, a limit vector on the master simplex with zero common goods prices but some non-zero non-common goods prices will be approached through a convergent subsequence.

Additional vectors $\hat{\pi}^{2j},\ldots,\hat{\pi}^{Kj}$ can be constructed in a manner similar to that for $\hat{\pi}^{1j}$ using solutions to the other sub-equilibrium problems. The vectors $\hat{\pi}^{1j},\ldots,\hat{\pi}^{Kj}$ together with the common goods prices $\pi_1^j,\ldots,\pi_{n_0}^j$ from S_0^j provide the analogue of the vector functions $\tilde{\pi}_k^{jh}$ from subproblems. The common goods prices are taken from the master simplex, prices of non-common goods come from sub-equilibrium solutions but are multiplied by a scalar λ_k^j for each group k such that prices of common goods are the same on the master simplex and at each sub-equilibrium problem. We write the vector $(\pi_1^j,\ldots,\pi_{n_0}^j,\hat{\pi}^{1j},\ldots,\hat{\pi}^{Kj})$ as $\hat{\pi}^j$ which we in turn normalize to sum to D. We note that the vector $\hat{\pi}^j$ contains all sub-equilibrium solution vectors for both common and non-common goods transformed by a scalar for each group; the relative prices of common and non-common goods for each group remain unaltered between sub-equilibria and the master problem. If a limit is approached for the case of a dense grid for which C_k^{j*} approaches zero for all k, the first n_0 entries of the normalized vector $\hat{\pi}^j$ will approach zero.

For the n_0 dimensional master simplex we use the traditional Gale-Nikaido mapping to produce a continuous mapping of the simplex into itself whose fixed points along with associated sub-equilibria meet the required equilibrium conditions. We note that by Walras' Law and the features of non-common goods, the value of excess demands for all common goods must equal zero. The mapping is given by the well known transformation of the excess demand functions for common goods evaluated at the vector $\hat{\pi}^j$ and summed over all groups

$$g_i(\pi_1^j,\ldots,\pi_{n_0}^j) = \frac{\pi_i^j/D + \max(0, \sum_{k=1}^{K} (\xi_i^k(\hat{\pi}^j) - w_i^k))}{\hat{1} + \sum_{i=1}^{n_0} \max(0, \sum_{k=1}^{K} (\xi_i^k(\hat{\pi}^j) - w_i^k))} \quad (i=1,\ldots,n_0)$$

The label $\ell(S_0^j)$ for vertices on the master simplex is determined as follows:

(i) If any element of S_0^j is zero, $\ell(S_0^j)$ is the index value of the first zero entry.

(ii) If S_0^j is strictly positive, the label is determined as the first index i for which the image $g_i(\pi_1^j,\ldots,\pi_{n_0}^j)$ exceeds $\frac{1}{D}$ multiplied by the corresponding coordinate on S_0^j, (for $i=1,\ldots,n_0$).

Demonstration of equilibrium conditions

It remains to argue that a completely labelled simplex in the subdivision of S_0 will be an approximation to an equilibrium of the entire general equilibrium model by a similar argument to that given in Scarf (1973) for a traditional general equilibrium model. There is a unique way to start the computational procedure on the master simplex as in Scarf's original algorithm and subsequenty on each simplex in any subproblem, and the Lemke-Howson no-cycling argumnt applies in exactly the same manner.

In order to argue that an approximation to an equilibrium for the entire problem will be found which becomes exact as the mesh corresponding to subdivisions of S_0 (and the corresponding simplices for subequilibrium problems) becomes finer and finer it remains to show that a completely labelled simplex in the subdivision of the master simplex will imply that all equilibrium conditions must hold.

The demonstration of this uses the same procedure as with a traditional Gale-Nikaido mapping where a fixed point under this mapping can be shown to imply the equilibrium conditions for a pure trade economy. A completely labelled simplex in the subdivision of the master simplex in the limiting case of a dense grid yields a price vector at which all excess demands for common goods are non-positive, common goods prices are zero if any excess demand is strictly negative. A completely labelled simplex in the subdivision of each associated simplex for subproblems yields a price vector at which excess demands for non-common goods are non-positive with non-common goods prices being zero if any excess demand is strictly negative. Both of these arguments use the property mentioned earlier that Walras Law together with the restriction that non-common goods cannot be traded across groups of consumers implies that the value of excess demands for common goods must equal zero, and similarly equals zero for each bloc of non-common goods. The equilibrium price vectors at master and sub-equilibria are related by a scalar for each group and from the zero homogeneity of demand functions the prices of common and non-common goods at sub-equilibria can be transformed to be identical to those characterizing a master equilibrium. Thus both master and sub-equilibrium conditions hold at the same set of prices and an equilibrium to the entire model must have been determined.

IV. A Numerical Example and Some Initial Computational Experience

Although we have described our procedure in terms of Scarf's original algorithm, the procedure can be easily adapted to extensions of Scarf's algorithm due to Merrill (1971), vander Laan and Talman (1979) and others. We have adapted Merrill's algorithm to this procedure and solved some numerical examples by this method. A computational point of some significance with the application of a decomposition procedure in such circumstances is the ability through a restart method to refine the grids for master and subsimplices at different speeds, an option not available with solution of the full problem.

We have programmed the decomposition procedure described above and solved a number of numerical examples both by this method and by solution of the corresponding full dimensional problem. While further improvements in efficiency of coding may be possible, our initial experience suggests that if the decomposition structure is sufficiently strong, significant computational savings are possible if that structure is exploited.

We consider a sequence of economies each with two common goods and an increasing number of blocs of non-common goods. Each bloc of non-common goods is of dimensionality 2 and each group of consumers corresponding to a bloc is of size 2. We consider each consumer in each group to have a CES preference function defined over common goods and non-common goods for that group. As we add blocs the general equilibrium problem increases by 2 goods and 2 consumer groups in dimensionality. Our parameter sets involve preference weightings which differ between consumers (although not markedly), and endowments which in aggregate are similar for each group.

We solve the same problem by both a full solution method and the decomposition method described above. We use Merrill's algorithm for full solution. We then adapt Merrill's algorithm to the decomposition procedure by using an artificial layer for both master and subsimplices. The same initial guess is used in both procedures. The recent method developed by vander Laan and Talman could also be used and would almost certainly be quicker for both methods, probably by a common factor approximated by the range vander Laan and Talman suggest of 3-5. For comparative purposes the consistent use of Merrill's algorithm for full and decomposition solution seems to provide the indications we seek that the potential gains from a decomposition approach increase as the decomposition structure becomes more and more evident.

Table 1 reports execution times as the sequence of economies extends by the addition of further consumer blocs. Increasingly improved relative performance by the decomposition procedure is evident as more blocs are added. We have no exploited the use of differential speeds of grid refinement between master and subproblems which could yield further computational gains. Computational costs increase less sharply with the overall problem dimension with the decomposition method than under full solution, an approximation which does not seem too far off is that execution times under full solution are proportional to the third power of the number of blocs while the decomposition execution time is proportional to the third power of the total number of goods (which increases more slowly). Comparisons in terms of the numbe of replacement operations required are also possible although we have not reported them. While this experience is only suggestive and may not hold for other examples, it does indicate a potential computational gain through exploitation of decomposition structure.

V. Conclusions

In this paper we present a decomposition procedure for general equilibrium computation in models where a partition of the commodities into common and non-common goods is possible. The similarity of the computational method outlined in this paper to the Dantzig-Wolfe decomposition algorithm for the solution of linear programming is the abilty to solve a large-scale general equilibrium problem through the separate solution

Table 1

Comparative Computational Experience for Solution of Numerical Examples Outlined in Text by Decomposition and Full Solution

No. of Common Goods	No. of Blocs (No. of Non-Common Goods in each Bloc in parenthesis)	Total No. of Goods	Execution Time 1/ for Full Solution by Merrill's Algorithm (Seconds)	Execution Time 1/ for Decomposition Solution Using Merrill's Algorithm (Seconds)	Ratio of Decomposition to Full Solution
2	2 (2)	6	11.68	6.67	.57
2	3 (2)	8	64.72	24.13	.37
2	4 (2)	10	119.73	41.93	.35
2	5 (2)	12	> 500	70.14	.14

1/ Times reported are for a CDC Cyber 73 which is slower than an IBM 370/168 by a factor of around 8. Both methods involve the same initial starting value and stopping criterion on excess demands.

of a sequence of smaller dimensional problems. Revisions to the master problem in Dantzig-Wolfe through coefficient generation are replaced by label generation for vertices on a master simplex.

Initial indications are that differences in computational cost between solution by the procedure described and full dimensional solution become increasingly significant as the decomposition structure becomes more pronounced and we report our experience in a comparative table. We suggest eventual application of this procedure may be possible to 'empirically' oriented large-scale general equilibrium models of world trade where the required structure is present. At least two such models incorporating traded and non-traded goods [Deardorff and Stern (1979), and Whalley (1979)] are in current use and others may follow in future developments.

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