1972

A Two-sector Growth Model With An Intermediate Product In An Open Economy

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MONEY AND ECONOMIC GROWTH

by

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Submitted in partial fulfillment

of the requirements for the degree of

Doctor of Philosophy

Faculty of Graduate Studies

The University of Western Ontario

London, Canada

May 1972

ABSTRACT

The present study investigates the short- and long-run equilibrium behaviour of a neoclassical two-sector growth model when the social contrivance, money, is introduced.

Money enters into the model as a consumer's good. It acts as the means of payments as well as a store of wealth that competes with physical capital in asset portfolio considerations. Savings and investment, consequently, are affected both in the short and the long run by the stock of real balances.

We first establish the existence and uniqueness of the momentary equilibrium by determining the wage-rental ratio and the rate of inflation which will maintain equilibrium between savings and investment and asset portfolio equilibrium. Under the assumptions of perfect myopic foresight and a fixed monetary rule, money is shown to exert a destabilizing influence on the processes of accumulation of physical capital and real balances. The system can be stable, however, if the Pigou effect dominates the Wicksell effect.

Alternatively, stability of the system can be achieved if the monetary authorities adopt a flexible monetary rule which stabilizes the rate of inflation.

If we assume a fixed monetary rule and an adaptive price expectation function, stability of the system requires, among other conditions, a sluggish speed of adjustment of the expected to the actual rates of inflation.

Money is not neutral in the long run. Changes in the rate of monetary expansion, or the rate of inflation pegged by the monetary authorities, or the aggregate savings propensity will affect the balanced growth equilibrium stock of real balances and the capital intensity. The latter is necessarily lower in a monetary than a Solow-Swan-Uzawa type of nonmonetary economy.

ACKNOWLEDGMENT

The present study is an outgrowth of the seminar on money and growth conducted by Professor H. Herberg who not only initiated the author to this rapidly expanding field and suggested the present topic for a doctoral dissertation but also provided warm encouragement and expert guidance. His assistance is most gratefully acknowledged.

Appreciation must also be expressed to my thesis supervisors, Professor R. N. Batra, J. S. Fried, and P. C. Thanh, who have generously provided their time and expertise in guiding the preparation of the thesis. Professors R. S. Boyer, J. R. Melvin, C. G. Plourde, P. N. Roy and D. Scheffman have also made valuable comments on various parts of the dissertation.

Thanks also go to my colleagues S. Chand, C. Y. Kuo, B. Raj, P. Reinhardt and K. Singh who have provided the moral support and many helpful suggestions.

My deepest gratitude, however, is due to my parents who have nurtured and educated me with the greatest affection. Financial support was partly provided by the Ontario Government.

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CHAPTER I

INTRODUCTION

The purpose of the present study is to analyze the equilibrium growth behaviour of a neoclassical two-sector economy which employs the social contrivance, money.

The pioneering work in money and growth comes from Tobin (1955) who introduces money as an alternative store of wealth to physical capital. The processes of capital accumulation are seen to be influenced by the existence of this government debt. In particular he investigates the possibility of cyclical fluctuations in income and employment when wages are inflexible downward. Ten years later, he delivers his Fisher Lecture (1965) in which he discusses the role that money plays in bringing equilibrium between the warranted rate of growth and the natural rate of growth at a positive rate of interest. Due to the presence of monetary debt of the government and under the assumption that a constant proportion of the real disposable income is saved, the capital intensity at the steady state for his monetary economy is lower than that for the Solow-Swan type of barter economy. He finds that increasing the rate of monetary expansion will increase the balanced growth capital intensity because the higher the rate of monetary expansion, the higher the rate of inflation, the more costly it will be to hold the monetary asset and hence a larger proportion of savings is devoted to investment in physical capital. Money is not neutral in the long run in the sense that the steady-state capital stock is affected by the rate of money creation adopted by the monetary authorities.

The question of the neutrality of money in the long run is further examined by Johnson (1966, 1967). From considerations of intertemporal utility maximization by individuals over an infinite horizon, Sidrauski (1967a) finds that the stock of capital along the balanced growth path must be at the level such that its marginal product is equal to the sum of the subjective rate of time preference and the rate of growth of population, both of which are assumed to be constant and independent of monetary influences. Hence money has neutral effect on the capital intensity in the steady state. The system yields a saddle point solution. If a sufficiently long lag is introduced into the formation of price expectations, the system can be made stable.

Making use of Tobin's approach in which real balances affect consumption and saving behaviour, he (1967b) finds that money is not neutral in the long run. Changing the rate of monetary expansion has the same effect on the balanced growth equilibrium capital intensity as that found by Tobin (1965). The growth path is stable if the speed of adjustment between the expected and the actual rates of inflation is sufficiently low.

Money assumes one of two roles in the work of Levhari and Patinkin (1968). It can act either as a consumer's good in which case it affects consumption and saving behaviour or as a producer's good in which case it is treated as working capital and cooperates with physical capital and labour in the production processes. Money is not neutral in both cases. They point out that monetary neutrality will no longer hold in Sidrauski's (1967a) model should the subjective rate of time preference be affected by portfolio decisions as it would if it were determined by the ratio of wealth to disposable income or if money enters into the production function.

Rather than using Tobin's definition of real disposable income which includes real national product and increase in real government transfers, they redefine it to include imputed interest income from holding of real balances as well. They postulate a variable saving function. The average propensity to save is made an increasing function of the marginal product of capital and a decreasing function of the rate of inflation. Due to their definition of real disposable income, the steady-state per capita capital stock of this monetary economy is not necessarily lower than that of a barter world. Even if it is lower, it does not follow that people are worse-off,for people now derive utility from real balances as well as from physical consumption.

When money is considered as a consumer's good, the system is stable if the demand for real balances is inelastic with respect to the money rate of interest. For the case of money as a producer's good, the stability of the system depends on the effects of interactions of various factors of production on total output and on physical savings.

It is interesting to note that in the analysis of the stability properties of their models, they do not analyze the two differential equations for the accumulation of capital and money simultaneously using mathematical or the phase-diagram techniques. They assume that the money market is stable and proceed to analyze the stability of the processes of capital accumulation. But stability of the system depends on the interactions between the real and monetary factors. A more sophisticated treatment of the stability analysis of money and growth models is found in Stein (1966, 1969), Nagatani (1970), and Burmeister and Dobell (1970).

In the neoclassical growth models, all savings are automatically invested. There is no independent investment behaviour. To relax this assumption, Stein (1966) postulates that the rate of inflation is positively related to the difference between investment and savings. He terms it the Keynes-Wicksellian approach. He assumes that full employment always prevails and shows that money is not neutral in the long run. In his comment on Sidrauski's paper, Stein (1968) demonstrates that monetary neutrality in the latter's model in which utility is maximized over infinite horizons depends on the assumption of instantaneous adjustment to clear all markets. If either there is a real balance effect in consumption behaviour in the short run or markets do not clear instantaneously, then neutrality will not be obtained.

Of the literature cited above in connection with money and growth, they are all extensions of the Solow-Swan type of one-sector growth model. Very few attempts have been made to extend it to deal with two-sector or multi-sector growth models.

The one-sector model makes the rather strong assumptions that the relative prices of the composite good is completely fixed and that the technology of the economy can be represented by one production function. Analysis involving changes in the relative prices of, say, consumer goods to capital goods as a result of structural changes is ruled out. The introduction of a second production sector will allow us to analyze the more realistic case of increasing opportunity cost of production.

There are two notable exceptions to the one-sector approach. Uzawa (1966a) extends his earlier work on two-sector growth models to study the optimal fiscal and monetary policies that will maximize the discounted sum of utilities derived from the consumption of private and public goods in per capita terms over time using the classical calculus of variations technique.

He shows that "by a proper choice of dynamic fiscal policy, which consists of income tax rates and growth rates of money supply through time, it is possible to achieve an optimal growth path corresponding to any form of social utility function and any rate of discount." (Uzawa, 1966a, p. 113.) The solution to his system displays a saddle point. There is one and only one growth path which will lead the system towards the steady state.

Foley and Sidrauski (1970) also use the two-sector framework to analyze the combination of fiscal and monetary policy that will bring about stability in the price level.

There are two production sectors: an investment good and a consumption good sector. The economy possesses three assets: physical capital, money (of the outside variety), and government bonds.

The objective of their exercise is to achieve a stable consumer price level by varying either the composition of the debt by open market operations (monetary policy) or the levels of government expenditure and taxes by the "marginally balanced budget" (fiscal policy). In both cases the per capita stock of government debt is assumed to remain constant. The movement of the nominal money supply is not independent of the movement of the stock of government bonds outstanding. As a result, there are only two differential equations describing the dynamics of their system. They choose an adaptive price expectation function. A sufficiently long lag in the adjustment of expectations is a necessary condition for the system to be stable.

The instantaneous rate of investment in their model is determined by three types of decisions: a portfolio decision as to the amount of capital and other assets to possess, a production decision as to the amount of capital required, and a supply decision as to the amount of capital equipments to be produced. The separation of the supply and demand decisions implies that the flow demand and supply of assets may not be equal at the prices which equilibrate stock demand and supply. The asset prices and yields will have to change over time to maintain portfolio equilibrium.

In a later work (1971), Foley and Sidrauski elaborate and extend their work to explore the impacts of different experiments with fiscal and monetary policies, their optimality properties, and international capital flow based on their "fixings" in 1969.

The present study also employs the two-sector framework. We introduce money into our model as a consumer's good. Consumption and savings decisions are affected due to the real balance effect. By postulating the existence of an alternative store of wealth, investment decisions are affected by the relative rates of returns from the two assets: physical capital and money. The characteristics of the steady state as well as its stability are affected. In this thesis we attempt to analyze the conditions under which short-run and long-run equilibria are maintained for a simple monetary growth model cast in the neoclassical two-sector framework.

Our work differs from that of Foley and Sidrauski in many aspects. In the first place, the structure of our model is different from theirs. They have three assets: capital, money and bonds. We do not have bonds. As observed before, the movement of bond holding is not independent of that of money holding in their model. Though their work looks more complicated, both studies are still dealing basically with a two differential equation system. To introduce a third asset like bonds or a second capital good will make the analysis much more difficult and result in as many differential equations as there are assets.

Secondly, the main purpose of their study is to find the policy mix which will achieve a stable price level. In order to achieve a stable system, they start with the assumptions that the consumption good industry is more capital intensive than the investment good industry and that the expected rate of inflation adjusts sluggishly to the actual rate. In our analysis, we do not invoke Uzawa's factor-intensity hypothesis. We test the stability of our monetary two-sector system under different assumptions about the monetary rule pursued by the monetary authorities and the price expectation function that prevails. Our dynamic equilibrium analysis shows how the processes of capital accumulation and the accumulation of real balances interact with each other when the stocks of capital and real balances are changing over time.

Thirdly, the functional forms of our demand for money and other behavioural relationships are different from those of Foley and Sidrauski. We take the intuitively more appealing approach of deflating the GNP and nominal balances by a price index which is a weighted average of the two commodity prices. They use the price of consumption good as the deflator. The choice of the consumption good as the numeraire commodity not only violates the spirit of a monetary economy, it also implies that a change in the price of investment good alone will not affect their "real" income and "real" balances.[1]

Finally, the methods by which short-run and long-run equilibrium analysis are carried out differ in the two studies. We take the wage-rental ratio as the key variable in Uzawa's tradition. All the other variables can be expressed in terms of the wage-rental ratio when the momentary equilibrium is being established. Changes in the two state variables, the per capita stocks of capital and real balances, will affect their rates of growth directly as well as indirectly via their effects on the wage-rental ratio. In their model short-run equilibrium is established at the point at which the asset equilibrium locus intersects the consumption good market equilibrium locus. The capital

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[1]The shortcoming of deflating the GNP and money balances by the price of the consumption good is recognized by Foley and Sidrauski (1971, p. 4)"In our assets demand functions we include national output measured in consumption good units as a measure of transactions. This is also open to criticism, since choosing investment goods or money as numeraire would alter the functions. In these cases we can only make a plea that many will find doubly damning: we knew it was not exactly right, but we went ahead and did it anyway."

intensity and the expected rate of change of the relative price of capital good in terms of the consumption good are the two key variables in their long-run equilibrium analysis.

Owing to the different specification of our model from that of Foley and Sidrauski, our results are shown to be different from theirs. We hope by taking a different approach from theirs, we are able to throw more light on the role that money plays in a monetary growth model.

We first outline the structure of the two-sector monetary growth model for a closed economy and examine the meaning of some of the crucial assumptions employed in the model in Chapter II. We then proceed to establish the existence and uniqueness of the momentary equilibrium and solve the model in the short run in Chapter III. In Chapter IV we analyze the stability of the model in the long run under the alternative assumption of a fixed and a flexible monetary rule and under the alternative expectation hypothesis of perfect myopic foresight and adaptive expectations. It will be shown that Uzawa's barter model is a special case of the present model.

In Chapter V we study the question of whether money is neutral and serves only as a veil in the long run and the impact on the steady-state capital intensity when the values of certain parameters are altered. The per capita stock of capital in the balanced growth equilibrium is compared with that of a barter economy to find out the long-term effect of the existence of this financial asset.

In the concluding chapter, we summarize the crucial assumptions employed in this study and the theoretical findings based on this set of assumptions.

CHAPTER II

A MONETARY TWO-SECTOR MODEL OF ECONOMIC GROWTH

I. INTRODUCTION

The traditional two-sector growth model as developed by Uzawa and others portrays a pure credit world in which there exists no medium of exchange or financial assets. This would not be too unrealistic an assumption in the context of growth if the existence of a medium of exchange and a financial asset like money did not influence the values of the real variables in the long run. But from the work of Tobin (1955, 1965), Stein (1966) and others, we know that money plays a significant role in determining the stability of the system and the magnitudes of the key variables in the balanced growth equilibrium. Therefore, it would be interesting to find out how the barter world of Uzawa changes when we postulate the existence of a dead-weight government debt, money.

In this chapter we sketch the two-sector monetary growth model which forms the basis for future analysis under alternative assumptions about price expectations and the monetary rule pursued by the monetary authorities.

II. THE BASIC MODEL

We consider an economy which employs the two factors of production, capital (K) and labour (L) to produce two commodities, an investment good (I) and a consumption good (C). The two commodities have different production processes, both of which exhibit constant returns to scale and diminishing marginal rate of substitution. The production function for good j is

$$Y_j(t) = F_j(K_j(t), L_j(t)). \qquad (2.1)$$

Hereafter, the time subscript (t) is omitted for the sake of simplicity in notation. Expressing the production functions in intensive form, we have

$$f_j(k_j) = F_j(K_j, L_j)/L_j = F_j(k_j, 1) \qquad (2.2)$$

where $k_j = K_j/L_j$. We assume that they are twice differentiable and satisfy the following conditions:

$$f'_j(k_j) > 0, \qquad f''_j(k_j) < 0. \qquad (2.3)$$

We also make the following end-point conditions that the wage-rental ratio (v) varies between zero and infinity as k_j varies in the same range.

$$\lim_{k_j \to 0} v(k_j) = 0, \qquad \lim_{k_j \to \infty} v(k_j) = \infty. \qquad (2.4)$$

Condition (2.4) is satisfied by the class of constant elasticity of substitution (C. E. S.) production functions that take the form

$$f_j = (ak_j^{-c} + b)^{-1/c} \tag{2.5a}$$

where a and b are positive constants and the elasticity of factor substitution, $e = 1/(1 + c)$, where $-1 < c < \infty$. The Cobb-Douglas production function is a special case of a C.E.S. production function with $c = 0$ and $e = 1$ and it satisfies (2.4).

The outputs of the investment good (Y_I) and the consumption good (Y_C) are determined by the production processes and the allocation of factors to the two industries.

$$y_j = f_j(k_j)l_j \tag{2.6}$$

where $y_j = Y_j/L$, $l_j = L_j/L$.

We assume that there is perfect competition in the commodity, factor and money markets. All productive resources are fully employed.

$$k_I l_I + k_C l_C = k, \tag{2.7a}$$

$$l_I + l_C = 1, \tag{2.7b}$$

where $k = K/L$ is the aggregate capital-labour ratio. It is a weighted average of the capital intensities of the two industries.

Efficiency in production requires the factors of production be allocated between the two industries so that the values of the marginal product of each factor are the same in each employment.

$$p_I f_I'(k_I) = p_C f_C'(k_C), \tag{2.8a}$$

$$p_I(f_I(k_I) - k_I f_I'(k_I)) = p_C(f_C(k_C) - k_C f_C'(k_C)),$$

where p_j is the price of good j in terms of money.

Let the ratio of the prices of the investment good to the consumption good be denoted by q.

$$q \equiv p_I/p_C = f_C'(k_C)/f_I'(k_I). \tag{2.8b}$$

We assume that the general price level (p) is a linear homogeneous function of p_I and p_C.

$$p = p(p_I, p_C), \qquad \partial p/\partial p_j > 0,$$

$$= p_C \cdot h(q), \qquad h'(q) > 0, \tag{2.9a}$$

$$p(1, 1) = 1. \tag{2.9b}$$

Since money is used to purchase both the consumption and the investment goods, the stock of nominal balances is deflated by the general price level, p, to obtain the stock of real balances. This procedure is preferred to deflating the nominal supply of money by either the price of the consumption good or the price of the investment good.

Equilibrium in production requires the real value of the marginal product of capital (r) and that of labour (w) to be the same in both industries.

$$r = p_j f_j'(k_j)/p, \tag{2.10}$$

$$w = p_j(f_j(k_j) - k_j f_j'(k_j))/p. \tag{2.11}$$

The wage-rental ratio (v) is given by

$$v \equiv w/r = f_j(k_j)/f_j'(k_j) - k_j. \qquad (2.12)$$

For the C.E.S. production function (2.5a),

$$v = (b/a)k_j^{1+c} \qquad (2.5b)$$

which satisfies (2.4) as alleged before.

The real GNP (Y) of the economy as measured by the total output is

$$y = (p_I y_I + p_C y_C)/p \qquad (2.13)$$

and as measured by the total factor income is

$$y = r(k + v) = p_j f_j'(k + v)/p = y(v, k) \qquad (2.14)$$

where $y = Y/L$.

The medium of exchange used by this economy is assumed to be a fiat currency issued by the monetary authorities in the form of transfer payments and is held for transaction purposes and as a store of wealth. "In accordance with the usual approach of monetary theory, we shall also assume that there exist "market frictions" and/or uncertainties with respect to the timing of payments which generally cause individuals to hold a portfolio which consists of both of these assets."[2] Money competes with tangible capital as a store of

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[2] D. Levhari and D. Patinkin, "The Roles of Money in a Simple Growth Model," American Economic Review, 55 (September, 1968), 715.

value. We assume that the government does not pay interest on its monetary obligations. The rate of return to holding money is the rate of deflation that prevails in the economy.

In general, the demand for per capita real balances ($m = M/pL$) can be treated as an increasing function of the real income per head (y) and the rate of return from holding money which is the expected rate of deflation ($-\hat{\pi}$). If the income elasticity of demand for real balances is unity, we can write the demand function for per capita real balances as follows:

$$\begin{aligned} m^d &= \lambda(\hat{\pi})y(v, k), \qquad \lambda > 0, \qquad \lambda' < 0, \\ &= m(\hat{\pi}, v, k). \end{aligned} \tag{2.15}$$

We assume that the money market is always in equilibrium.

$$M = M^d, \tag{2.16}$$

or in real per capita terms,

$$m = \lambda(\hat{\pi})y(v, k) = m(\hat{\pi}, v, k). \tag{2.17}$$

In the first instance, we adopt the perfect myopic foresight assumption about price expectations, i.e., the actual rate of inflation (π) is the same as the expected rate ($\hat{\pi}$).

$$\pi = \hat{\pi}. \tag{2.18}$$

This is usually justified on the ground that if everybody holds their expectations with conviction, the prophesy may be self-fulfilling. If people expected a 10% rise in prices in the future, they would demand an equiproportionate increase in wages to keep their real income constant. Firms would raise prices to defray the higher costs. The actual rate of inflation might turn out to be 10% as expected.

We make the saving hypothesis that a constant proportion, s, is saved out of the personal disposable income (Y_D) which is defined as the sum of the GNP and the increase in real balances.

$$y_D = y + (u - \pi)m \tag{2.19}$$

where $y_D = Y_D/L$, $u = \dot{M}/M$, $\pi = \dot{p}/p$ and the dot above a variable indicates the operation of taking the time derivative.

Total savings out of the real disposable income are allocated between investment on physical capital goods and the acquisition of real balances.

$$sy_D = p_I y_I/p + (u - \pi)m. \tag{2.20}$$

From (2.14), (2.15), (2.19) and (2.20), we get

$$\begin{aligned} y_I &= (s - (1 - s)(u - \pi)\lambda(\hat{\pi}))f_I'(k + v) \\ &= \gamma f_I'(k + v) \end{aligned} \tag{2.21}$$

where

$$\gamma = s - (1 - s)(u - \pi)\lambda(\hat{\pi}) < 1, \tag{2.22}$$

is the fraction of the GNP devoted to capital formation.

We assume that capital is not consumed but lasts forever and labour grows at the constant exponential rate, $n = \dot{L}/L$.[3]

To complete the dynamic model, we add the following differential equations which specify the growth path traversed by the economy over time:

$$\dot{p}/p = \pi, \tag{2.23}$$

$$\dot{k}/k = y_I/k - n, \tag{2.24}$$

$$\dot{m}/m = u - \pi - n. \tag{2.25}$$

We have sixteen equations: (2.6) to (2.9), (2.12), (2.13), (2.17), (2.18), (2.19), (2.20), (2.23) to (2.25) and sixteen endogenous variables: y_j, k_j, l_j, p_j, y, y_D, v, $\hat{\pi}$, π, k, m and p. The growth path is completely determined if the values for the parameters s, u and n and the initial values for $k(0)$, $m(0)$ and $p(0)$ are given. The solution of the model for momentary and long-run equilibria will be discussed in the next two chapters.

There are in effect three goods circulating in this economy, namely, the consumption and investment goods, and money. Money is the numeraire good by which the value of

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[3]The alternative assumption of capital depreciating at a constant rate, say δ, can be incorporated into the model simply by stipulating that net capital formation $\dot{K} = Y_I - \delta K$.

other goods are denominated. The characteristics of a monetary economy is that commodities are exchanged for money and vice versa, and money is exchanged for money. The cost of production of money is assumed to be nil.

III. THE DEFINITION OF REAL DISPOSABLE INCOME

The definition of real disposable income here follows that of Tobin (1965) rather than that of Johnson (1967) or Levhari and Patinkin (1968) who include the value of the services of money as part of real income. All three approaches, however, are not immune to criticisms.[4]

Johnson (1967) argues that money facilitates exchange. Therefore, its utility as measured by the integral under the demand curve for money should be included as part of real income.

$$y_D^J = y(k) + (u - \pi)m + U(m) \tag{2.26}$$

where y_D^J is disposable income as defined by Johnson, $y(k)$ is income from physical output and $U(m)$ is utility yield from the stock of real balances, m, all in per capita terms.

One objection raised by Stein (1970) about this approach is that the value of physical output, $y(k)$, is not measured by the integral under its demand curve. There is no economic justification for the asymmetrical treatment of money as against goods.

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[4] See Marty (1969), Shell, Sidrauski and Stiglitz (1969) and Stein (1970) for discussions on the implications of different definitions of real disposable income.

Levhari and Patinkin (1968) also define real income to include the value of the services of money which is measured by the opportunity cost of holding cash (i) which is taken to be the sum of the marginal product of capital (r) and the expected rate of inflation ($\hat{\pi}$).

$$\begin{aligned} y_D{}^P &= y(k) + (u - \pi)m + (r + \hat{\pi})m \\ &= y(k) + (u + r)m \end{aligned} \tag{2.27}$$

where $y_D{}^P$ is real per capita disposable income as defined by Levhari and Patinkin. Such a treatment has been criticized for many reasons.

Stein (1970) points out that while physical output, y(k), is measured at constant prices, the value of services of money is measured at current prices since its opportunity cost, $i = r + \hat{\pi}$, does not remain unchanged during the period of reference. Furthermore, if the demand for money is interest inelastic as is assumed, then an increase in m will actually decrease $y_D{}^P$, a rather paradoxical result:

$$d(im)/dm = i(1 - 1/\epsilon) \tag{2.28}$$

where $\epsilon = -(idm/mdi)$ is the interest elasticity of demand for money. If $\epsilon < 1$, then (2.28) is negative.

Both Johnson's and Levhari-Patinkin's approaches may give rise to negative real balance effect on physical consumption (c_p) which contradicts the traditional theory.

Using the concept of real disposable income formulated by Johnson (2.26), we have

$$c_p + U(m) = (1 - s)y_D^J,$$

or

$$c_p = (1 - s)y + (1 - s)(u - \pi)m - sU(m). \qquad (2.29)$$

Using Levhari-Patinkin's definition (2.27), we have

$$c_p + (r + \hat{\pi})m = (1 - s)y_D^P,$$

or

$$c_p = (1 - s)y + (u - \pi)m - s(u + r)m. \qquad (2.30)$$

At the steady state, $u - \pi = n$, the rate of growth of population. If $n = 0$, the negative effect of real balances on physical consumption is inevitable in both Johnson's and Levhari-Patinkin's approaches.

Marty (1969) also points out that when prices are changing the imputed income from real balances depends on the interest elasticity of demand for money. His main objection to the approaches of Johnson and Levhari and Patinkin, however, stems from the fact that the utility yield from real balances are treated as a substitute for material consumption which he considers to be economically implausible and unjustifiable. He proposes to define disposable income to include the utility yield from cash balances but keeps physical consumption as a constant fraction of disposable income excluding the imputed income from cash balances.

$$y_D = y(k) + (u - \pi)m + U(m). \tag{2.31}$$

But,

$$c_p = (1 - s)(y + (u - \pi)m), \tag{2.32}$$

$$\begin{aligned} i_p &= s(y + (u - \pi)m) - (u - \pi)m \\ &= sy - (1 - s)(u - \pi)m, \end{aligned} \tag{2.33}$$

where i_p is per capita real physical investment. The utility yield of real balances affects neither consumption nor investment behaviour. Such a consumption hypothesis displays precisely the same economic implications as that of Tobin's model in spite of the fact that a different definition of disposable income is used.

In addition to the above considerations, Tobin's approach is adopted here also because of its simplicity and because it conforms to the conventional national income accounting practices which are recognized as inappropriate theoretically. As pointed out by Levhari and Patinkin, the major difference between their analysis and that of Tobin does not lie in the definition of disposable income but rather in their assumption of a variable saving ratio which is made a function of the real rate of interest and the rate of deflation.

IV. THE SAVINGS HYPOTHESIS

As an alternative to a constant proportional saving hypothesis, one can assume that there are two constant saving ratios which are applied to wage and profit income separately. Though this is an interesting variant of a constant aggregate propensity to save, it does not add significantly to our theoretical underpinning or alter the basic character of the analysis but only make mathematical manipulations more complicated.

It seems that a theoretically more appealing alternative to a constant proportional saving hypothesis is to postulate, say, a life-time savings hypothesis. But even this approach is not free of serious objections. As pointed out by Hahn (1969), within a neoclassical framework this implies that people must form correct expectations about the future over their life-time. Any errors committed, though unwittingly, will throw the system off its equilibrium growth path. Only an infinite set of future markets could guarantee the correct anticipations of all the imponderables in the future. He, therefore, opts for the simple "rule of thumb" assumption that savings are a constant proportion of real disposable income which he defines to include imputed income from real balances.

V. THE EQUILIBRIUM DYNAMICS

The differential equation structure of the two-sector growth model describes the dynamic behaviour of the various variables over time. At any instant of time, we are given the stock of capital, labour and money and the price level which are inherited from the past. The savings hypothesis determines the future rate of consumption and the rate of capital formation. Asset portfolio equilibrium determines the relative rates of returns from holding physical capital and real balances. Given the economic structure, they determine the equilibrium values of the other variables and the new values of the stock variables for the next instant. The equilibrium growth path is completely determined by the differential equation system if the initial conditions of the stock variables are specified.

In comparative statics analysis, different stationary positions are compared and equilibrium in the money market determines the equilibrium price level. This approach is invalid when a differential equation structure is adopted. As observed by Burmeister and Dobell:

> ... much of the usual theorizing about money markets and price level determination is based on a Keynesian structure, with the time period short enough so that the capital stock may be treated as fixed, but long enough so that adjustments in employment and prices may be accomplished. In this context we study equilibrium configurations in which prices have adjusted

> fully to a stationary equilibrium value.... But since it fails to take capital gains and losses explicitly into account in the formal equation structure, the analysis is really quite difficult. It has to deal implicitly with qualifications arising from changing prices without having an explicit mechanism for incorporating these in the model studied.[5]

In the specification of the neoclassical growth models, we do not assume independent savings and investment behaviour. What is saved according to the saving hypothesis is automatically invested. The ex ante (desired) savings and investment plans are identical to the ex post (realized) savings and capital formation. The flow equilibrium of savings and investment is always satisfied.

Portfolio equilibrium between the demand and supply of the two assets, money and physical capital, is established by adjusting the expected rates of return from the two assets. The rate of return from the holding of money is the expected rate of deflation while that for capital is the real value of the marginal product of capital. If the two assets are perfect substitutes, the rate of return from the two assets must be the same. If they are imperfect substitutes

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[5]E. Burmeister and A. R. Dobell, Mathematical Theories of Economic Growth, New York: The Macmillan Co., 1970, p. 157.

because money is more liquid, there will be a whole range of differentials in yield between the two assets that will maintain portfolio balance.[6] Trading equilibrium is achieved in every period. The dynamic growth paths represent a succession of trading equilibrium positions over time.

The fact that real balances are considered as part of the real wealth of the economy and that capital gains and losses from the holding of money as a result of changes in prices are considered as part of personal income imply that people are subject to "an illusion, but only one of the many fallacies of composition which are basic to any economy or any society. The illusion can be maintained unimpaired so long as the society does not actually try to convert all of its paper wealth into goods." (Tobin, 1965, p. 676).

One of the crucial assumptions of the model is that money market equilibrium always prevails. The growth path over time is an equilibrium growth path. We do not allow disequilibrium situations to arise. One of the consequences of this assumption is that if there exists an excess supply of money, for example, the expected rate of inflation must fall to induce people to hold more money and restore

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[6]See Tobin (1965, 1968), and Levhari and Patinkin (1968) for further discussions on the substitutability between money and capital.

equilibrium in the money market. As observed by Hahn (1969, p. 183), "... the price of money was changing because this was required for asset equilibrium and not because any reason was adduced why in fact it should change." This "perverse" movement of the prices is a direct consequence of the requirement that portfolio balance must prevail at all time. One could, however, venture to explain this phenomenon by comparing the holding of excess supply of money by wealth holders to the holding of excess supply of grain by the farmers. The latter are persuaded to withhold the excess stock rather than rushing to the market and attempt to sell it because they know such an action would only drive the prices down and they would be hurt even more. Similarly, the excess stock of real balances will be willingly held if by withholding it they expect a reduction in commodity prices so that the return from holding money is increased. Since we assume that the expected rate of inflation is the same as the actual rate, the rate of inflation will indeed fall as expected.

Although we are only concerned with the equilibrium path, we can depict the behaviour of the system when it is out of equilibrium a la Tobin (1965). There are two opposing forces at work, as he claims, when the system is in disequilibrium.

Suppose there exists a general excess demand for real balances, i.e., an excess supply of goods. Prices tend to fall. The supply of real balances rises and via the real balance effect, the demand for consumption goods increases and the initial drop in prices may be arrested. This Pigou effect is stabilizing. On the other hand, the initial drop in the rate of inflation makes the holding of money more attractive and hence increase its demand. This Wicksell effect is destabilizing. "There is no a priori reason why one effect should be stronger than the other in the neighbourhood of equilibrium" (Tobin, 1965, p. 683).

VI. PRICE EXPECTATIONS AND KEYNES-WICKSELLIAN MODELS

The perfect myopic foresight assumption means that the actual rate of inflation in the next instant always agrees with the expected rate. We can envisage a situation where an auctioneer contracts for the prices in the next instant while the current prices are left at their current level. Since the demand for assets depends on their expected yields, portfolio equilibrium will determine the future rate of inflation.

Sidrauski (1967a, 1967b) adopts the adaptive expectation hypothesis that the rate of change of the expected rate of inflation is positively related to the differences between the actual and the expected rates of inflation. In spite of its obvious superiority over the myopic foresight assumption, Stein (1968) raises the objection that such an adaptive expectation model is "incongruous" to the neoclassical assumption of all markets being cleared instantaneously.

Stein (1966) postulates independent saving and investment functions and assumes that prices rise whenever planned investment exceeds planned savings. When aggregate real demand outstrips real output, neither planned investment nor planned savings are fully satisfied. He assumes that actual capital formation is a linear combination of the two. It is less than planned investment but greater than planned savings. Money is not neutral in the long run because the steady-state capital intensity is affected by "forced savings" caused by persistent inflationary tendencies. Since persistent deflation may be inconsistent with his assumption of full employment, he only considers the case in which inflation prevails. One special feature of the Keynes-Wicksellian models is that a positive rate of inflation in the steady state implies that there is permanent excess demand even along the balanced growth path.

We assume in the first place that the monetary authorities keep the rate of money creation constant and allow the rate of inflation to adjust to maintain equilibrium in the money market. We analyze the stability of the system under the alternative assumptions of perfect myopic foresight and adaptive expectations. In the second place we drop this Friedmanesque monetary rule and assume that the rate of inflation is kept constant by adjusting the rate of monetary expansion. Changes in the money stock are accomplished by government transfer payments as a result of the government's budget deficit. We assume that there is no distributional effect as a result.

CHAPTER III

SHORT-RUN EQUILIBRIUM ANALYSIS

I. DETERMINATION OF THE INSTANTANEOUS EQUILIBRIUM

Uzawa's two-sector model is a miniature Walrasian general equilibrium model. The development of the system over time depends on the initial per capita stock of capital and the economic structure of the model. In our monetary two-sector growth model, we need to specify the initial conditions for the stock of capital, labour and money and the price level. We shall show that given the values for k, m and p at any instant of time, the momentary equilibrium exists and is uniquely determined.

From (2.12) and using the properties of the production functions (2.3), we have

$$k_j' = dk_j/dv = - f_j'^2/f_j f_j'' > 0. \tag{3.1}$$

The capital-labour ratio in each industry (k_j) is an increasing function of the wage-rental ratio (v).

The allocation of labour to the two industries can be obtained from (2.7).

$$l_j = (k-k_i)/(k_j-k_i) = l_j(v,k). \quad i \neq j \tag{3.2}$$

From (2.6) and (3.2), we get

$$y_j = f_j(k-k_i)/(k_j-k_i) = y_j(v,k). \quad i \neq j \tag{3.3}$$

To determine the effects of changes in v and k on the allocation of labour and the per capita output of the two industries, we differentiate (3.2) and (3.3) with respect to v and k, respectively.

$$\partial l_j/\partial v = -(l_i k_i' + l_j k_j')/(k_j-k_i), \tag{3.4}$$

$$\partial l_j/\partial k = 1/(k_j-k_i), \tag{3.5}$$

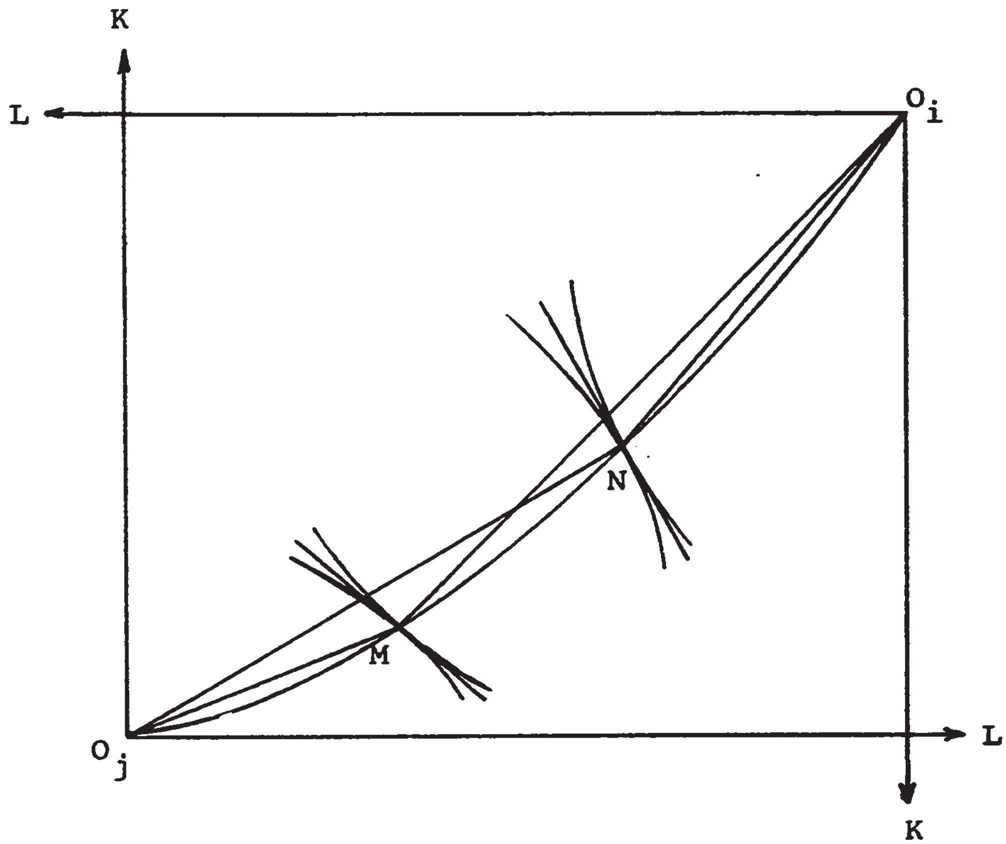
$$\partial y_j/\partial v = -f_j(l_j k_j'(k_i+v)/(k_j+v) + l_i k_i')/(k_j-k_i), \tag{3.6}$$

$$\partial y_j/\partial k = f_j/(k_j-k_i). \tag{3.7}$$

$$\begin{aligned} \text{sign } \partial l_j/\partial v &= \text{sign } \partial y_j/\partial v = \text{sign } (k - k_j) \\ &= -\text{ sign } \partial l_j/\partial k = -\text{ sign } \partial y_j/\partial k. \end{aligned} \tag{3.8}$$

Figure 1 shows the case of an increase in the wage-rental ratio while the aggregate capital-labour ratio is kept constant. The equilibrium position is shifted from point M to N on the efficiency locus O_jMNO_i. The capital-labour ratio in both industries are raised as indicated by (3.1). The per capita output and the proportion of labour employed by industry j which is labour-intensive increase

FIGURE 1



while those by industry i which is capital-intensive decrease.

Figure 2 shows the case of an increase in the aggregate capital-labour ratio while keeping wage-rental ratio constant. As a result of the latter assumption, the capital intensities of both industries are fixed. A rise in the overall capital-labour ratio will increase the distribution of labour and the per capita output of industry j which is capital-intensive and decrease that of industry i which is labour-intensive. The equilibrium position is shifted from P to Q.

This Rybczynski (1955) theorem can also be shown using Figure 3 for the case of $k_j > k > k_i$. A rise in k shifts the production-possibility frontier outward from TT' to DD'. The equilibrium production point changes from E to E'. The slope of the Rybczynski line (RR') is negative and constant because both k_j and k_i are constant. It can be derived from (3.7).

$$\partial y_j/\partial y_i = - f_j(k_j)/f_i(k_i). \tag{3.9}$$

The relationship between the relative price ratio (q) and the wage-rental ratio (v) can be determined by differentiating (2.8b) logarithmically with respect to v and using (2.12) and (3.1) to get

$$dq/qdv = (k_C - k_I)/(k_C + v)(k_I + v). \tag{3.10}$$

$$\text{sign } dq/dv = \text{sign } (k_C - k_I). \tag{3.11}$$

FIGURE 2

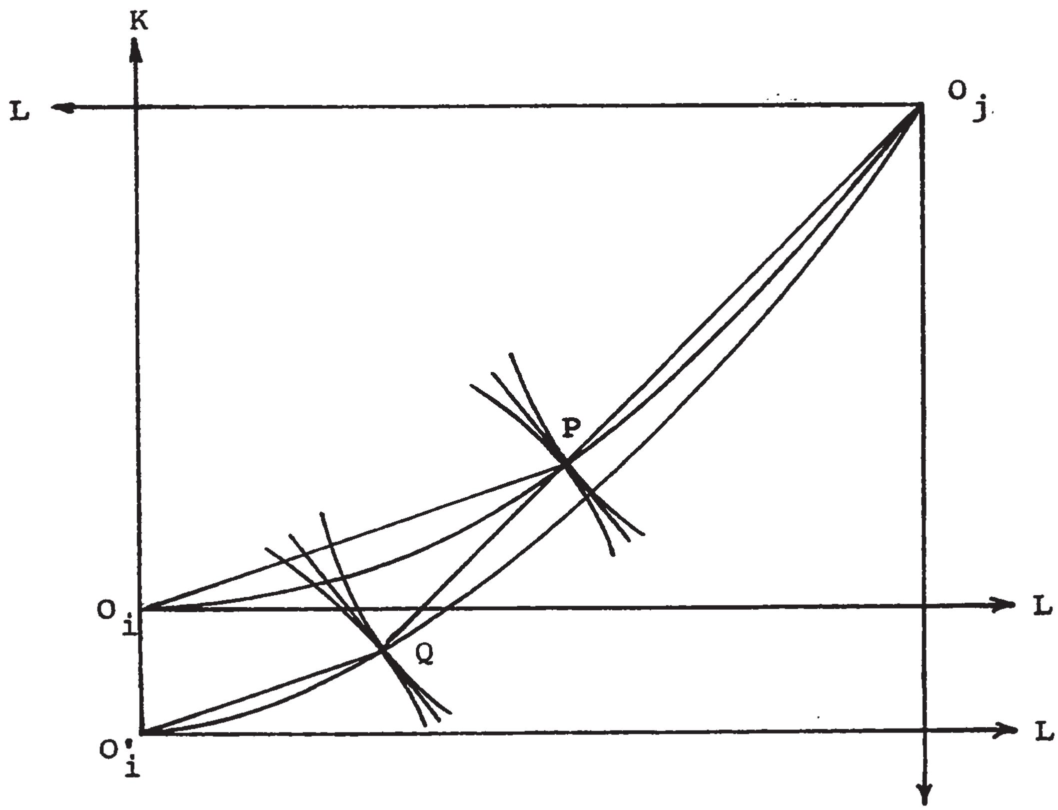
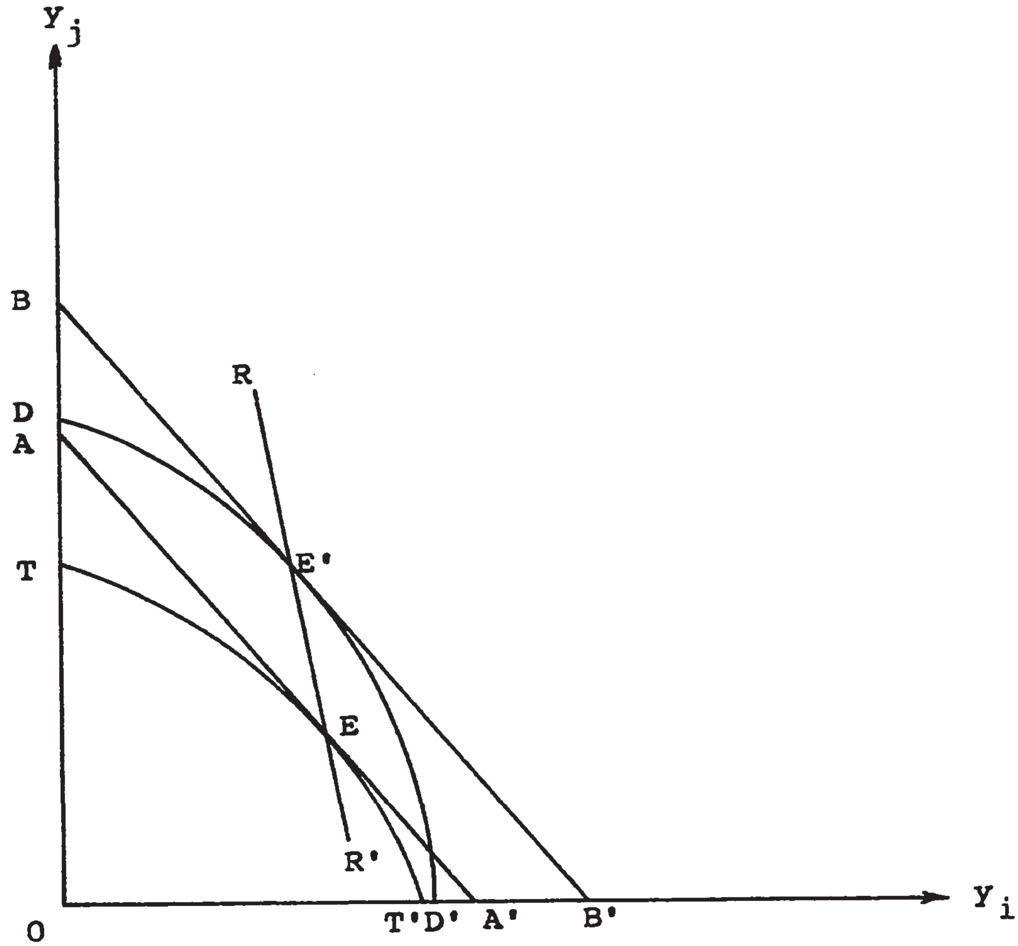


FIGURE 3



The relationships between k_j and v, and v and q are described in Figures 4 and 5 for the two cases of

$$k_C > k > k_I \tag{3.12}$$

and

$$k_I > k > k_C, \tag{3.13}$$

respectively. Factor intensity reversals are excluded to simplify the analysis.

From (3.6) and (3.10), we find that

$$\partial y_I/\partial q = (\partial y_I/\partial v)(dv/dq) > 0, \tag{3.14}$$

$$\partial y_C/\partial q = (\partial y_C/\partial v)(dv/dq) < 0, \tag{3.15}$$

the signs of both of which are unambiguous and do not depend on the factor-intensity hypothesis. An increase in the price of the investment good relative to the price of the consumption good will increase the output of the former relative to the latter. Suppose initial equilibrium is established at E in Figure 6. TT' is the production-possibility curve and the slope of AA' is the negative of the price ratio (q). An increase in q will move the equilibrium point from E to E'. The output of y_I is increased while that of y_C is reduced.

To determine how the real prices of investment and consumption goods are affected by changes in the wage-rental

FIGURE 4

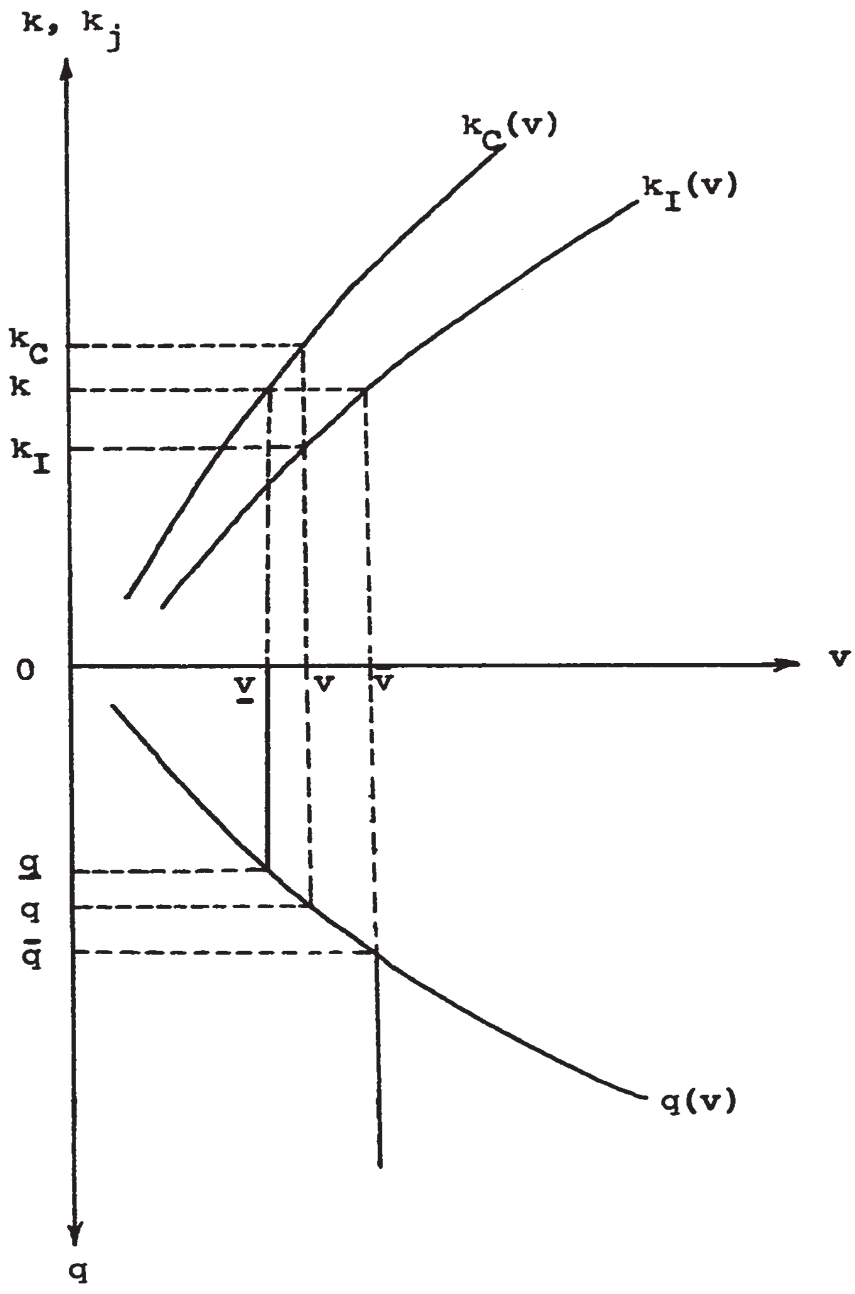


FIGURE 5

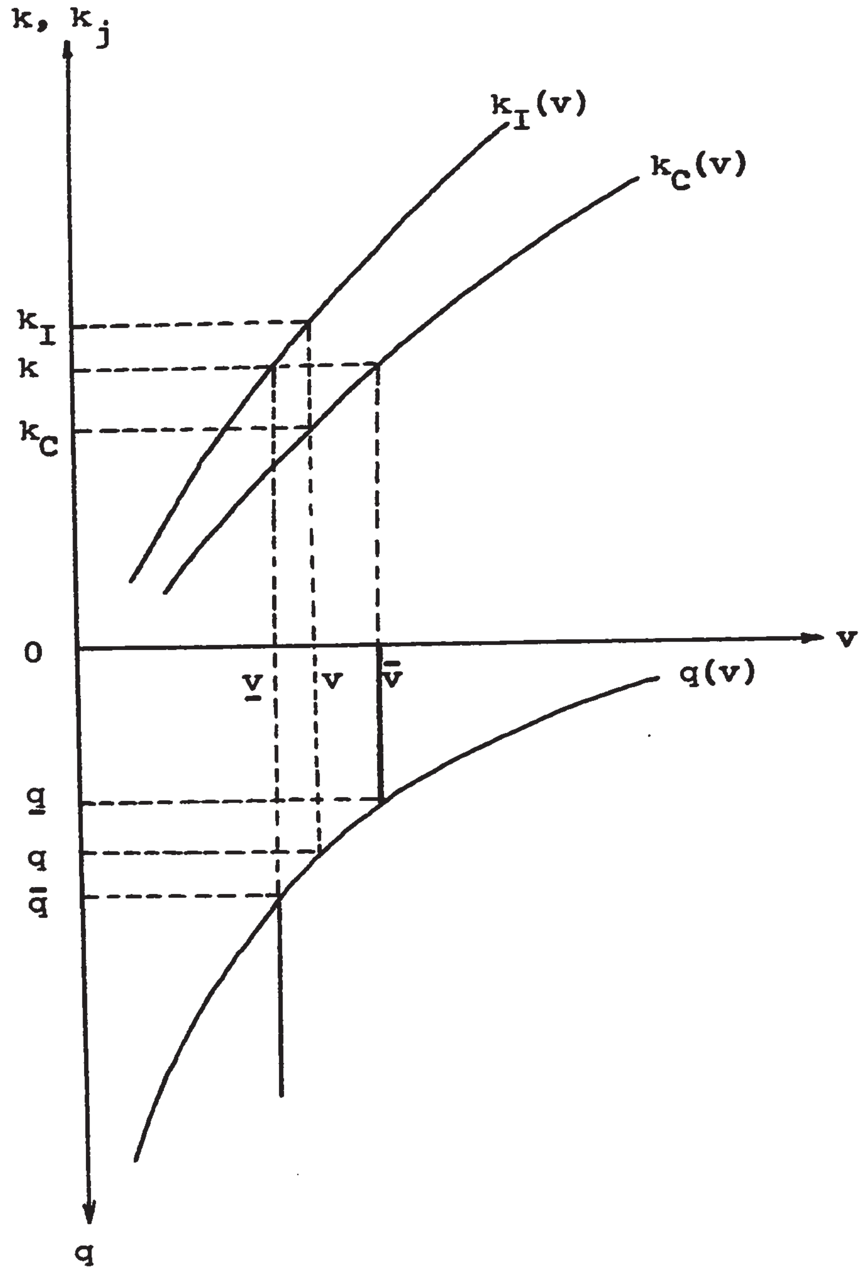
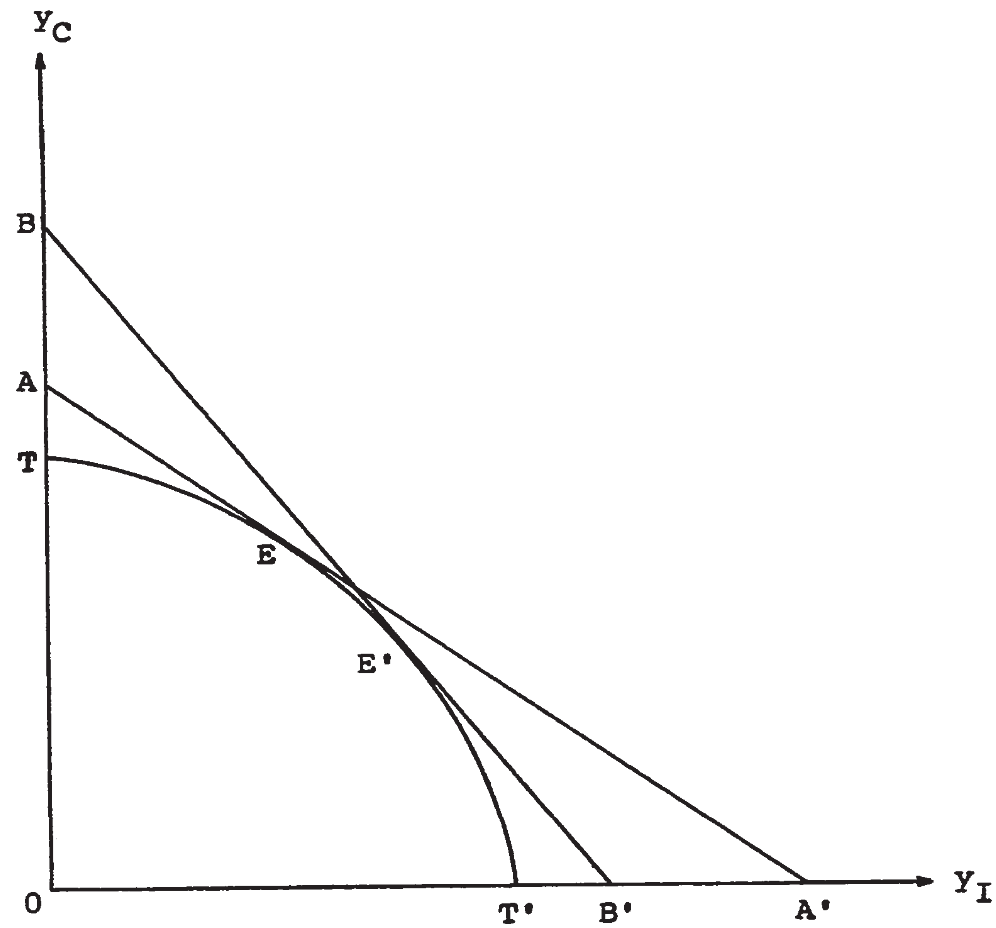


FIGURE 6



ratio, we obtain from (2.9a)

$$p_C/p = 1/h(q), \tag{3.16}$$

$$p_I/p = q/h(q). \tag{3.17}$$

Differentiating (3.16) and (3.17) with respect to v, we get

$$d(p_C/p)/dv = -h'dq/h^2dv, \tag{3.18}$$

$$d(p_I/p)/dv = (1 - \eta)dq/hdv, \tag{3.19}$$

where

$$0 < \eta = h'q/h = (\partial p/\partial p_I)(p_I/p) < 1, \tag{3.20a}$$

$$1 - \eta = (\partial p/\partial p_C)(p_C/p). \tag{3.20b}$$

η is the elasticity of the general price index (p) with respect to changes in the price of the investment good (p_I).

$$\text{sign } d(p_j/p)/dv = \text{sign } (k - k_j). \tag{3.21}$$

If (3.12) holds, an increase in the wage-rental ratio will reduce the real price of the consumption good (p_C/p) and increase the real price of the investment good (p_I/p). If (3.13) holds, the real price of the consumption good will be increased while that of the investment good will be reduced.

The real rental rate is defined as

$$r = p_C f_C'(k_C)/p = f_C'(k_C)/h(q) = r(v). \tag{3.22}$$

To find the effect of changes in the wage-rental ratio on the real rental rate, we take the logarithmic differentiation of (3.22) with respect to v.

$$dr/rdv = -(1 + \eta(k_C - k_I)/(k_I + v))/(k_C + v) < 0. \quad (3.23)$$

The real GNP per head (y) as defined by (2.13) and (2.14) is a function of v and k. Differentiating (2.14) partially with respect to k, we get

$$y_k = p_j f_j'(k_j)/p = r > 0. \quad (3.24)$$

When v is constant, relative prices q and the real prices of the two commodities will also be constant. An increase in k will shift the production-possibility curve TT' to DD' in Figure 3. The real income as measured by y_i will be increased from A' to B' and as measured by y_j from A to B.

From (2.21) and (3.3), we get

$$y_I = f_I(k_C - k)/(k_C - k_I) = \gamma f_I'(k + v). \quad (3.25)$$

Substituting (2.12) into (3.25), we get the following fundamental saving and investment equilibrium condition:

$$\gamma(\hat{r})(k + v)(k_C - k_I) = (k_C - k)(k_I + v). \quad (3.26)$$

Taking logarithmic differentiation of (2.14) partially with respect to v and using (3.23) and (3.26), we get

$$y_v/y = (\gamma - \eta)(k_C - k_I)/(k_C + v)(k_I + v). \quad (3.27)$$

$$\text{sign } y_v = \text{sign } (\gamma - \eta)(k_C - k_I). \quad (3.28)$$

Whether the real national income will be increased or otherwise depends on (1) the relative magnitudes of the physical savings ratio (γ) defined by (2.22) and the price elasticity

(η) defined by (3.20) and (2) the factor intensity hypothesis that prevails.

If the real income is expressed in terms of one of the commodities, then an increase in q from AA' to BB' in Figure 6 will reduce real income from OA' to OB' in terms of the investment good and increase real income from OA to OB in terms of the consumption good.

To determine the relationships between the amount of real balances and $\hat{\pi}$, v and k, we differentiate (2.15) with respect to its arguments.

$$m_{\pi} = y\lambda' < 0, \tag{3.29}$$

$$m_v = \lambda y_v, \tag{3.30}$$

$$m_k = \lambda y_k > 0, \tag{3.31}$$

$$\text{sign } m_v = \text{sign } y_v = \text{sign } (\gamma - \eta)(k_C - k_I). \tag{3.32}$$

Using the money market equilibrium condition (2.17), we can invert it and express $\hat{\pi}$ in terms of v, m and k.

$$\hat{\pi} = \pi(v, m, k), \tag{3.33}$$

$$\pi_v = - m_v/m_{\pi}, \tag{3.34}$$

$$\pi_m = 1/m_{\pi} < 0, \tag{3.35}$$

$$\pi_k = - m_k/m_{\pi} > 0, \tag{3.36}$$

$$\text{sign } \pi_v = \text{sign } m_v = \text{sign } y_v. \tag{3.37}$$

We can express the fundamental saving and investment equilibrium condition (3.26) as an implicit function of v, k and m.

$$\phi(v, k, m) = \gamma(\hat{\pi})(k+v)(k_C-k_I) - (k_I+v)(k_C-k) = 0. \tag{3.38}$$

Differentiating (3.38) with respect to its arguments and evaluating the partial derivatives at $\phi = 0$, we get

$$\phi_v\Big|_{\phi=0} = -((k_I+v)l_Ck_C' + (k_C+v)l_Ik_I' + (k_C-k)(k-k_I)/(k+v) - (k+v)(k_C-k_I)\gamma_\pi\pi_v), \tag{3.39}$$

$$\phi_k\Big|_{\phi=0} = (k_C+v)(k_I+v)/(k+v) + (k_I+v)(k_C-k)\gamma_\pi\pi_k/\gamma, \tag{3.40}$$

$$\phi_m\Big|_{\phi=0} = (k+v)(k_C-k_I)\gamma_\pi\pi_m, \tag{3.41}$$

where

$$\gamma_\pi = (1 - s)(\lambda - (u - \pi)\lambda'). \tag{3.42}$$

We can express the equilibrium wage-rental ratio (v) determined by (3.38) as a function of k and m.

$$v = v(k, m), \tag{3.43}$$

$$v_k = - \phi_k/\phi_v, \tag{3.44}$$

$$v_m = - \phi_m/\phi_v. \tag{3.45}$$

In order to show that instantaneous equilibrium exists and is unique, we must show that the fundamental saving and investment equilibrium condition (3.38) uniquely determines the equilibrium wage-rental ratio (v) for given k and m. As shown in Figure 4, let

$$k = k_C(\underline{v}), \qquad k = k_I(\bar{v}), \tag{3.46}$$

when (3.12) holds. From (3.38) and (3.46), we get

$$\phi(\underline{v}, k, m) = \gamma(k_C+\underline{v})(k_C-k_I), \tag{3.47}$$

$$\phi(\bar{v}, k, m) = (\gamma-1)(k_I+\bar{v})(k_C-k_I). \tag{3.48}$$

If (3.13) holds, as shown in Figure 5, we have

$$k = k_I(\underline{v}), \qquad k = k_C(\bar{v}). \tag{3.49}$$

Using (3.38) and (3.49), we get

$$\phi(\underline{v}, k, m) = (\gamma-1)(k_I+\underline{v})(k_C-k_I), \tag{3.50}$$

$$\phi(\bar{v}, k, m) = \gamma(k_C+\bar{v})(k_C-k_I). \tag{3.51}$$

In either case, we have shown that

$$\phi(\underline{v}, k, m) > 0, \tag{3.52}$$

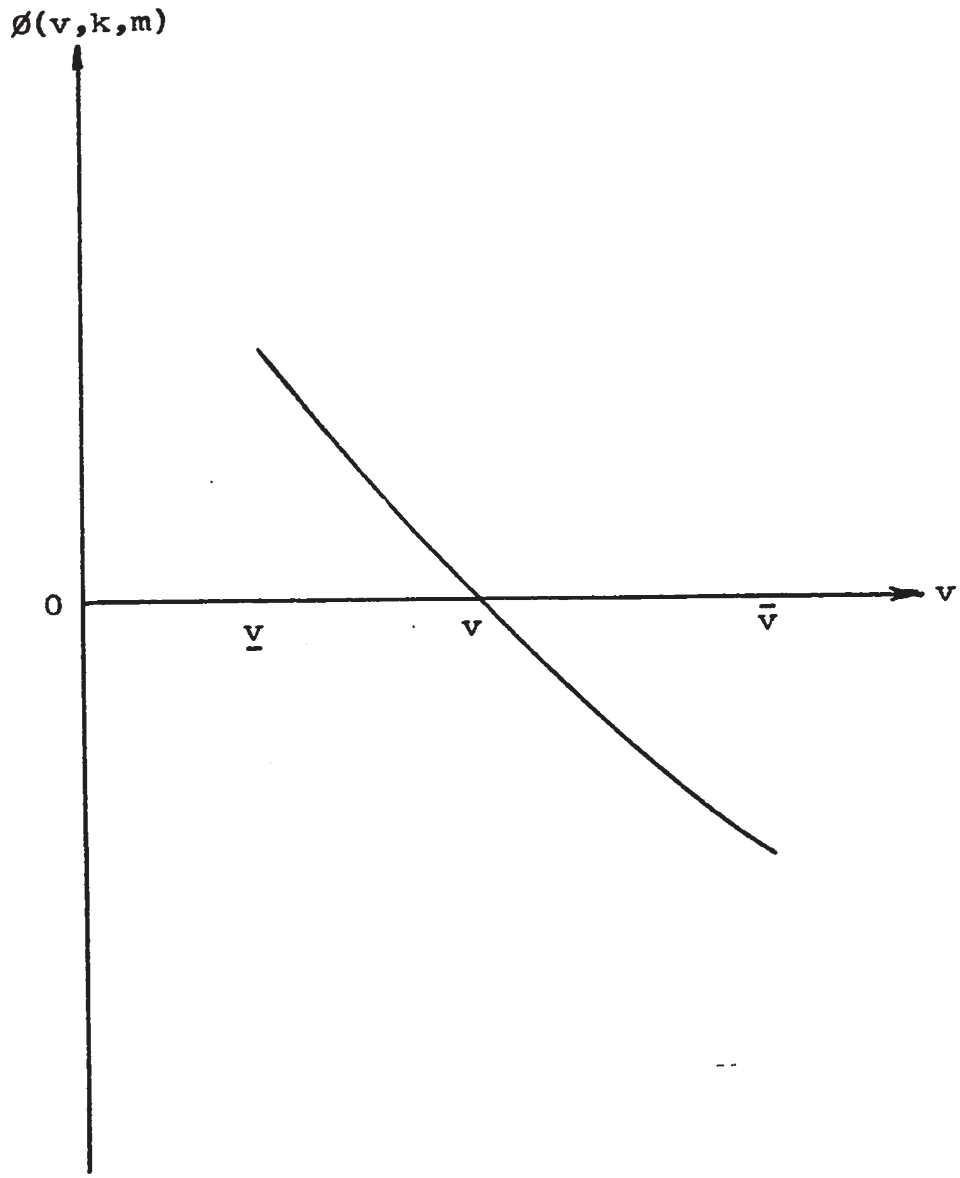
$$\phi(\bar{v}, k, m) < 0. \tag{3.53}$$

In Figure 7, we show the relationship between ϕ and v. Since the function ϕ changes sign from $\underline{v}$ to $\bar{v}$, the equilibrium wage-rental ratio, v, exists. It is uniquely determined by the saving and investment equilibrium condition (3.38) if the sign of (3.39) does not change. A sufficient condition for the function ϕ to be monotonically decreasing with respect to v is that

$$\text{sign } (k_C-k_I)\gamma_\pi \pi_v = \text{sign } \gamma_\pi(\gamma-\eta) \leq 0. \tag{3.54}$$

Once the equilibrium wage-rental ratio is uniquely determined, we can easily demonstrate the solution of the static system when k, m and p are given.

FIGURE 7



With the equilibrium wage-rental ratio determined by (3.38) for given $\underset{\sim}{k}$ and m, we can determine $\hat{\pi}$ using (3.33) and π by the perfect myopic foresight assumption (2.18). The wage-rental ratio determines the capital-labour ratio in each industry (k_j) by (2.12), the relative price ratio (q) by (2.8) and the allocation of labour to the two industries (l_j) by (2.7). By (2.9), the absolute prices of the two commodities are also determined. The per capita output in each industry is determined by (2.6). The real per capita GNP (y) and disposable income (y_D) are determined by (2.13) and (2.19), respectively. The entire system is solved in the short run.

II. AN IS-LM ANALYSIS

Short-run equilibrium can also be determined using the IS-LM type of analysis. Savings and investment or flow equilibrium is determined by

$$\phi(\hat{\pi}, v, k, m) = \gamma(\hat{\pi})(k+v)(k_C-k_I) - (k_C-k)(k_I+v)=0 \tag{3.55}$$

Equilibrium in the money market or asset portfolio equilibrium is determined by

$$\psi(\hat{\pi}, v, k,m) = m - \lambda(\hat{\pi})y(v, k). \tag{3.56}$$

To find combinations of $\hat{\pi}$ and v which will maintain flow and stock equilibrium for given k and m, we have to find the following partial derivatives:

$$\phi_{\hat{\pi}}\Big|_{\phi=0} = \gamma_{\hat{\pi}}(k+v)(k_C-k_I), \tag{3.57}$$

$$\phi_v\Big|_{\phi=0} = -((k_I+v)l_C k_C' + (k_C+v)l_I k_I' + (k_C-k)(k-k_I)/(k+v)) < 0, \tag{3.58}$$

$$\psi_{\hat{\pi}}\Big|_{\psi=0} = - \lambda' y > 0, \tag{3.59}$$

$$\psi_v\Big|_{\psi=0} = - \lambda y_v. \tag{3.60}$$

From (3.57) to (3.60), we get

$$\hat{\pi}_v\Big|_{\phi=0} = - \phi_v/\phi_{\hat{\pi}}, \tag{3.61}$$

$$\hat{\pi}_v\Big|_{\psi=0} = - \psi_v/\psi_{\hat{\pi}}. \tag{3.62}$$

If $\gamma < \eta$, then (3.61) and (3.62) have opposite signs. The equilibrium values of $\hat{\pi}$ and v are uniquely determined as found by the previous analysis. If $k_C > k_I$, then both (3.57) and (3.60) are positive. (3.61) is positive and (3.62) negative, as shown in Figure 8. If $k_I > k_C$, then both (3.57) and (3.60) are negative. The IS (ϕ=0) curve will have a negative slope while the LM (ψ=0) curve will have a positive slope, as shown in Figure 9.

The condition that $\gamma < \eta$ and $k_C > k_I$ implies that y_v is negative by (3.28) and dy_I/dv is positive by (3.6). An increase in v reduces y and hence reduces the demand for real balances. To maintain equilibrium in the money market, the rate of inflation must be reduced so that the cost of holding money is lowered and the excess supply of money is absorbed into people's portfolio.

The increase in the supply of investment goods as a result of an increase in v must be counterbalanced by an increase in the physical savings ratio (γ). An increase in $\hat{\pi}$ will increase the cost of holding money and induce substitution of physical capital for real balances. The demand for investment good is increased. Hence the IS curve has a positive slope and the LM curve a negative slope as illustrated in Figure 8.

FIGURE 8

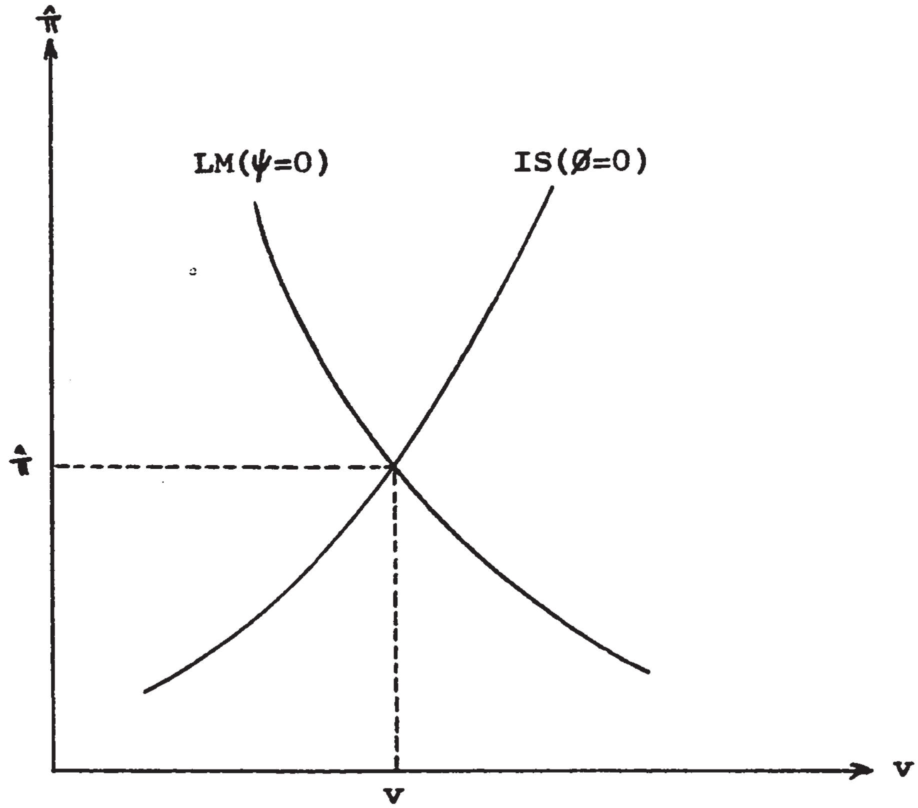
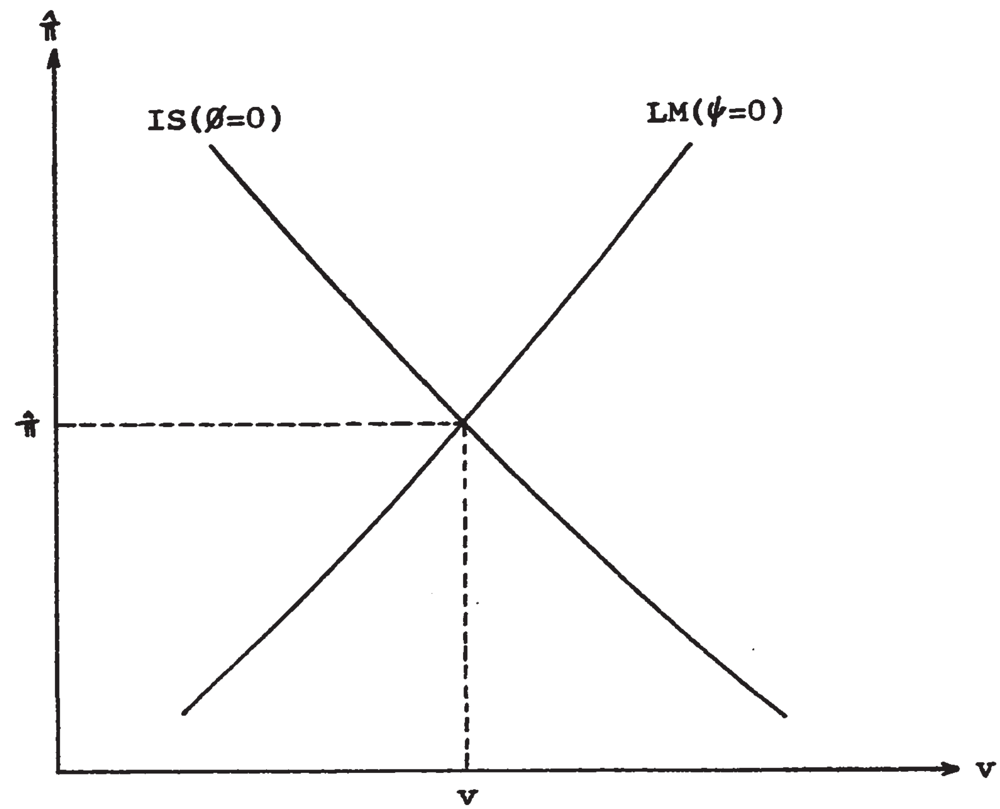


FIGURE 9



If we retain the condition $\gamma < \eta$ but instead assume that $k_C < k_I$, then y_v is positive by (3.28) and dy_I/dv is negative by (3.6). An increase in v increases y and hence increases the demand for real balances. Portfolio equilibrium can be regained if the rate of inflation increases. It becomes more costly to hold real balances and hence the excess demand created as a result of the increase in income will be eliminated.

An increase in v under the factor intensity hypothesis (3.13) reduces the output of investment goods. Flow equilibrium is reestablished if the rate of inflation is lowered which reduces the physical savings ratio (γ). The case for $\gamma < \eta$ and $k_C < k_I$ is shown in Figure 9.

III. THE ELASTICITY OF FACTOR SUBSTITUTION

Since the aggregate elasticity of factor substitution features prominantly in the stability analysis in the next chapter, we shall briefly describe its derivation.

In Uzawa's barter two-sector model, the wage-rental ratio (v) is only a function of the capital-labour ratio (k). The physical saving ratio (γ) is the same as the over-all saving ratio (s). Hence the last terms on the right-hand side of (3.39) and (3.40) are absent. The aggregate elasticity of factor substitution ($e = vdk/kdv$) can be expressed as a function of the elasticities of factor substitution of the two industries ($e_j = vdk_j/k_j dv$). From (3.39) and (3.40) and after simplification, we get

$$e = \frac{k+v}{k}\left(\frac{k_I l_I e_I}{k_I+v} + \frac{k_C l_C e_C}{k_C+v}\right) + \frac{(k_C-k_I)^2 l_I l_C v}{(k_C+v)(k_I+v)k} > 0. \tag{3.63}$$

We note that even if elasticities in both industries approach zero, the aggregate elasticity remains positive.

In our monetary model, the aggregate elasticity of factor substitution depends also on monetary influences. From (3.39) and (3.40), we get

$$e = v\partial k/k\partial v = - v\phi_v/k\phi_k. \tag{3.64}$$

Using condition (3.54), we find that a sufficient condition for the aggregate elasticity of factor substitution to be positive is that

$$\gamma < \eta \text{ and } k_C > k_I. \tag{3.65}$$

If we assume that $\gamma < \eta$, then ϕ_v is unambiguously negative. From (3.45) and (3.41),

$$\text{sign } v_m = \text{sign } \phi_m = - \text{ sign } (k_C - k_I). \tag{3.66}$$

Since (3.11) and (3.66) have opposite signs, we get

$$q_m = q_v v_m < 0. \tag{3.67}$$

If the consumption (investment) good industry is more capital intensive, then an increase in the real balances reduces (increases) the wage-rental ratio which reduces the relative price of investment to consumption goods. The net result is that an increase in m changes the price line from BB' to AA' and the equilibrium point shifts from E' to E in Figure 6.

CHAPTER IV

LONG-RUN EQUILIBRIUM ANALYSIS

I. INTRODUCTION

After we have shown that the short-run equilibrium exists and is unique, we shall determine the manner by which our monetary economy grows over time. In particular, we are interested in the questions of whether the system will lead to the steady state and achieve balanced growth equilibrium and whether the system is stable around the steady state if it is subject to exogenous disturbances.

Clearly a system is undesirable if a slight perturbation results in further divergence from the balanced growth path. If such a situation prevails, we want to explain the causes of instability. Furthermore, we want to determine whether stability can be achieved by making a different set of assumptions.

The stability of the system depends on whether the monetary authorities peg the rate of monetary expansion and let the rate of inflation find its own level or peg the rate of inflation by adjusting the rate of money creation. It also depends on the type of expectation hypothesis that prevails.

We shall first assume that the monetary authorities fix the rate of monetary expansion and the actual rate of inflation coincides with the expected rate. Secondly, the fixed monetary rule will be replaced by the assumption that the rate of inflation is being pegged by the monetary authorities by manipulating the rate of change of money supply. Finally, we experiment with an adaptive expectation function in conjunction with a fixed monetary rule.

II. A FIXED MONETARY RULE AND PERFECT MYOPIC FORESIGHT

Given the initial conditions for k(0) and m(0) and the values of the parameters s, n and u, the growth path of the model described in Chapter III is uniquely determined by the pair of differential equations:

$$\dot{k}/k = \dot{K}/K - \dot{L}/L$$

$$= y_I/k - n$$

$$= \gamma f_I'(k + v)/k - n = H(k, m) \qquad (4.1)$$

$$\dot{m}/m = u - \pi(v, k, m) - n = G(k, m) \qquad (4.2)$$

We want to determine whether the steady-state solution of the dynamic system is unique and stable.

The steady state of the system of differential equations (4.1) and (4.2) is the critical point (k*, m*) at which

$$H(k^*, m^*) = G(k^*, m^*) = 0 \qquad (4.3)$$

where the star above a variable indicates its steady-state value. To solve the pair of differential equations, we take the linear approximation of the system by Taylor's series of expansion about the critical point (k*, m*).

Differentiating (4.1) and (4.2) with respect to k and m, respectively, we get

$$H_k\Big|_{H=0} = \frac{\gamma f_I' v_k}{k}\left(\frac{k_I-k}{k_I+v} - e\right) + \frac{n\gamma_\pi}{\gamma}(\pi_v v_k + \pi_k), \quad (4.4)$$

$$H_m\Big|_{H=0} = \frac{\gamma f_I' v_m}{k}\left(\frac{k_I-k}{k_I+v}\right) + \frac{n\gamma_\pi}{\gamma}(\pi_v v_m + \pi_m), \quad (4.5)$$

$$G_k\Big|_{G=0} = -(\pi_v v_k + \pi_k), \quad (4.6)$$

$$G_m\Big|_{G=0} = -(\pi_v v_m + \pi_m). \quad (4.7)$$

The rate of capital accumulation is increased (decreased) when k is raised above its steady-state value (k^*) if (4.4) is positive (negative). There are two forces at work. A change in k changes the wage-rental ratio and hence the capital intensities in the two industries. It also affects the physical saving ratio (γ) via the direct effect (π_k) and an indirect effect ($\pi_v v_k$) on the rate of inflation. The sign of (4.4) depends on the elasticity condition

$$v_k\left(\frac{k_I - k}{k_I + v} - e\right) \quad (4.8)$$

and the condition

$$\pi_v v_k + \pi_k. \quad (4.9)$$

If both (4.8) and (4.9) are negative, then the system is stable in the k direction.

The effect of changes in m on the process of capital accumulation is shown by (4.5). Its sign is determined by the relative capital intensities in the two industries,

$$k_C - k_I, \tag{4.10}$$

the relative magnitudes of γ and η,

$$\gamma - \eta, \tag{4.11}$$

and the sign of

$$\pi_v v_m + \pi_m. \tag{4.12}$$

If (4.11) is negative, then $\pi_v v_m$ is positive. The real balance effect on the physical saving ratio ($\gamma_\pi \pi_m$) is stabilizing. An increase in real balances reduces the rate of inflation and hence the physical saving ratio. The indirect income effect ($\gamma_\pi \pi_v v_m$) is destabilizing. Via the effect on v and π, an increase in m increases the physical savings ratio and hence the rate of capital accumulation.

The effects of changes in k and m on the rate of accumulation of real balances also depend on the signs of (4.9) to (4.12). The direct real balance effect is destabilizing. An increase in m reduces π and hence increases the rate of increase of real balances. If (4.11) is negative, the indirect income effect is stabilizing. An increase in m raises π via its effect on v. The rate of accumulation of real balances is hence reduced.

The sum of the characteristic roots (ρ_1 and ρ_2) of the system of differential equations (4.1) and (4.2) is given by

$$\rho_1 + \rho_2 = H_k + G_m. \tag{4.13}$$

The product of the characteristic roots is given by

$$\rho_1 \rho_2 = H_k G_m - H_m G_k. \tag{4.14}$$

Substituting (4.4) to (4.7) into (4.14), we get

$$\rho_1 \rho_2 = \frac{\gamma f_I'}{k}\left(\frac{k_I - k}{k_I + v}\right)(v_m \pi_k - v_k \pi_m) + \frac{\gamma f_I' v}{k^2}(\pi_v v_m + \pi_m). \tag{4.15}$$

The system is stable if

$$H_k < 0, \quad G_m < 0, \text{ and } \quad H_k G_m - H_m G_k > 0. \tag{4.16}$$

Since the signs of (4.4) to (4.7) are ambiguous, the Jacobian determinant of the system may change signs. We cannot determine a priori whether the system is stable or otherwise. Also the possibility of multiple solutions of the system is not excluded.

In order to ascertain the sources of instability in our monetary model, we shall first make certain observations about the non-monetary barter model of Uzawa (1963). In the absence of money, (4.5) to (4.7) do not appear and the second term on the right-hand side of (4.4) also vanishes. The stability of his system depends crucially on the value of the aggregate elasticity of factor substitution (e). Since both v_k and e are unambiguously positive by (3.63), his system is stable if and only if

$$e > (k_I - k)/(k_I + v) \tag{4.17}$$

If Uzawa's factor-intensity hypothesis (3.12) holds, (4.17) will certainly be satisfied.

If both production functions are Cobb-Douglas then $e = e_I = e_C = 1$. (4.17) is satisfied no matter which factor intensity hypothesis holds.

If both e_I and e_C approach zero, in the limit,

$$e = (k_C - k_I)^2 l_I l_C v/(k_C + v)(k_I + v) > 0. \tag{4.18}$$

Substituting (4.18) into (4.17), we find that (4.17) holds if (3.12) holds.

The instability of Uzawa's two-sector growth model is due to the fact that the effects of partial increases in v are opposite from those of k on the allocation of labour (l_j) and the per capita output (y_j) of the two industries as indicated by (3.8). If (3.12) holds, the direct effects of an increase in k reduce y_I and l_I while its indirect effects via v raise them. The final outcome in the case of (3.12) is to reduce the rate of capital accumulation. The net result in the case of (3.13) depends on the elasticities of factor substitution in the two industries and the aggregate elasticity of factor substitution.

In our monetary model, in addition to the destabilizing factors inherent in two-sector models, we have the destabilizing influences of two assets competing for a place in investors' portfolios. There are three sources of instability in the monetary sector. The first one is the assumption that the expected rate of inflation adjusts so as to maintain portfolio equilibrium while the rate of monetary expansion is kept constant. The second one is the assumption that the money market is always in equilibrium. The third source of instability is due to the perfect myopic foresight assumption.

Suppose there is a chance increase in the supply of real balances above its equilibrium value. In order to induce people to absorb the excess supply of real balances into the asset portfolio, the expected and hence the actual rate of inflation must fall which will lower the cost of holding cash balances. A reduction in the rate of inflation will increase the excess supply of real balances. This Wicksellian effect clearly is destabilizing.

On the other hand, an increase in real balances will increase the real disposable income and the demand for goods. Other things being equal, this will increase the rate of inflation and slow down the rate of increase of real balances. This Pigou effect is stabilizing. In terms of equation (4.7), if $\gamma < \eta$, then $\pi_v v_m$ is positive. If this Pigou effect outweighs the Wicksell effect ($\pi_m < 0$), the money market will be stable. If the opposite holds, the money market will be unstable.

A high η is shown to be conducive to stability. An increase in p_I above its equilibrium will create an excess supply of investment goods. If p is responsive to changes in p_I, the demand for real balances will decline as the rate of inflation increases. More savings will be channelled into capital formation. Hence the system is stable.

A Cobb-Douglas type of p index function ($p = p_I{}^a p_C{}^{1-a}$) will give rise to a constant $\eta = a$. p is the geometric mean of the two money prices if $a = \frac{1}{2}$, in which case $\gamma < \eta$ will most likely be satisfied.

A linear price index function with constant weights ($p = a_I p_I + a_C p_C$) will yield an $\eta = a_I p_I / p$ and

$$\text{sign } (d\eta/dv) = \text{sign } d(p_I/p)/dv = \text{sign } (k - k_I).$$

The sign of (4.11) may change as k and v change.

It is well-known that a lagged price expectation function with sufficiently low speed of adjustment may bring about stability in such a system. (See Sidrauski (1967a, 1967b), Stein (1966, 1969, 1970), Foley and Sidrauski (1970).)

III. A FLEXIBLE MONETARY RULE AND PERFECT MYOPIC FORESIGHT

It has been shown that our monetary system is likely to be unstable if the rate of inflation is allowed to fluctuate freely to bring about asset portfolio equilibrium. In this section we shall adopt the alternative assumption that the monetary authorities stabilize the rate of inflation by adjusting the rate of money creation. Under the assumption of perfect myopic foresight, the expected rate of inflation also becomes fixed. We have, in effect, a static expectation function. Both the actual and the expected rates of inflation become parameters of the system while the rate of monetary expansion becomes an endogenous variable.

Treating the rate of inflation as a parameter of the system, equilibrium in the money market now determines the equilibrium wage-rental ratio as a function of k and m.

$$m = \lambda(\hat{\pi})y(v, k) = m(v, k), \tag{4.19}$$

$$m_v = \lambda y_v, \tag{4.20}$$

$$m_k = \lambda y_k. \tag{4.21}$$

Inverting (4.19), we get

$$v = v(k, m), \tag{4.22}$$

$$v_k = - m_k/m_v = - y_k/y_v, \tag{4.23a}$$

$$v_m = 1/m_v = 1/\lambda y_v, \tag{4.23b}$$

$$\text{sign } v_m = \text{sign } y_v = \text{sign } (\gamma-\eta)(k_C-k_I) = - \text{sign } v_k. \tag{4.24}$$

With the wage-rental ratio determined by the money market equilibrium condition (4.19), the fundamental savings and investment equilibrium condition now determines the equilibrium rate of money creation.

$$\begin{aligned}\phi(u, v, k) &= \gamma(u)(k + v)(k_C - k_I) - (k_I + v)(k_C - k) \\ &= 0. \end{aligned} \qquad (4.25)$$

Differentiating (4.25) with respect to its arguments, we get

$$\phi_u\Big|_{\phi=0} = \gamma_u(k+v)(k_C-k_I), \qquad (4.26)$$

$$\begin{aligned}\phi_v\Big|_{\phi=0} &= -\left((k_I+v)l_Ck_C' + (k_C+v)l_Ik_I' + (k_C-k)(k-k_I)\right/ \\ &\quad (k+v)) < 0, \end{aligned} \qquad (4.27)$$

$$\phi_k\Big|_{\phi=0} = (k_I+v)(k_C+v)/(k+v) > 0, \qquad (4.28)$$

where

$$\gamma_u = -(1 - s)\lambda < 0. \qquad (4.29)$$

Applying the implicit function rule, we can express u as a function of v and k.

$$u = u(v, k), \qquad (4.30)$$

$$u_v = -\phi_v/\phi_u, \qquad (4.31)$$

$$u_k = -\phi_k/\phi_u, \qquad (4.32)$$

$$\text{sign } u_k = -\text{ sign } u_v = -\text{ sign } \phi_u = \text{sign }(k_C - k_I) \qquad (4.33)$$

In order to understand the economic reasoning behind our mathematical derivation, we shall assume for the moment that

$$\gamma < \eta \text{ and } k_C > k_I. \tag{4.34}$$

We have $y_v < 0$, $v_k > 0$, $v_m < 0$, $\phi_u < 0$, $u_k > 0$, $u_v < 0$, $u_v v_m > 0$ and $u_v v_k < 0$. Starting from an equilibrium position, an increase in k raises the wage-rental ratio (v) which raises the output of investment goods relative to that of consumption goods. To restore equilibrium between savings and investment, the rate of monetary expansion must be lowered which increases the physical savings ratio (γ). More savings are devoted to capital formation and less on acquiring real balances. The indirect effect of changes in k on u via v, however, is counterbalanced by its direct effect. An increase in k increases the real income y and hence total savings. The increase in demand for investment goods can be removed by an increase in the rate of money creation which lowers the physical savings ratio.

An increase in m lowers the wage-rental ratio which reduces the output of investment goods. To regain savings and investment equilibrium, the rate of monetary expansion has to be increased so that the physical saving ratio is lowered.

In terms of the IS-LM analysis, savings and investment equilibrium is represented by a negative sloping IS curve in the (u, v) space while the LM curve is vertical as shown in Figure 10.

The growth path of the present system is determined by the pair of differential equations:

$$\dot{k}/k = \gamma f_I'(k + v)/k - n = H(k, m), \tag{4.35}$$

$$\dot{m}/m = u(v, k) - \tau - n = G(k, m). \tag{4.36}$$

Differentiating (4.35) and (4.36) with respect to their arguments, we get

$$H_k\Big|_{H=0} = \frac{\gamma f_I' v_k}{k}\left(\frac{k_I - k}{k_I + v} - e\right) + \frac{n\gamma_u}{\gamma}(u_v v_k + u_k), \tag{4.37}$$

$$H_m\Big|_{H=0} = \frac{\gamma f_I' v_m}{k}\left(\frac{k_I - k}{k_I + v}\right) + \frac{n}{\gamma}\gamma_u u_v v_m, \tag{4.38}$$

$$G_k\Big|_{G=0} = u_v v_k + u_k, \tag{4.39}$$

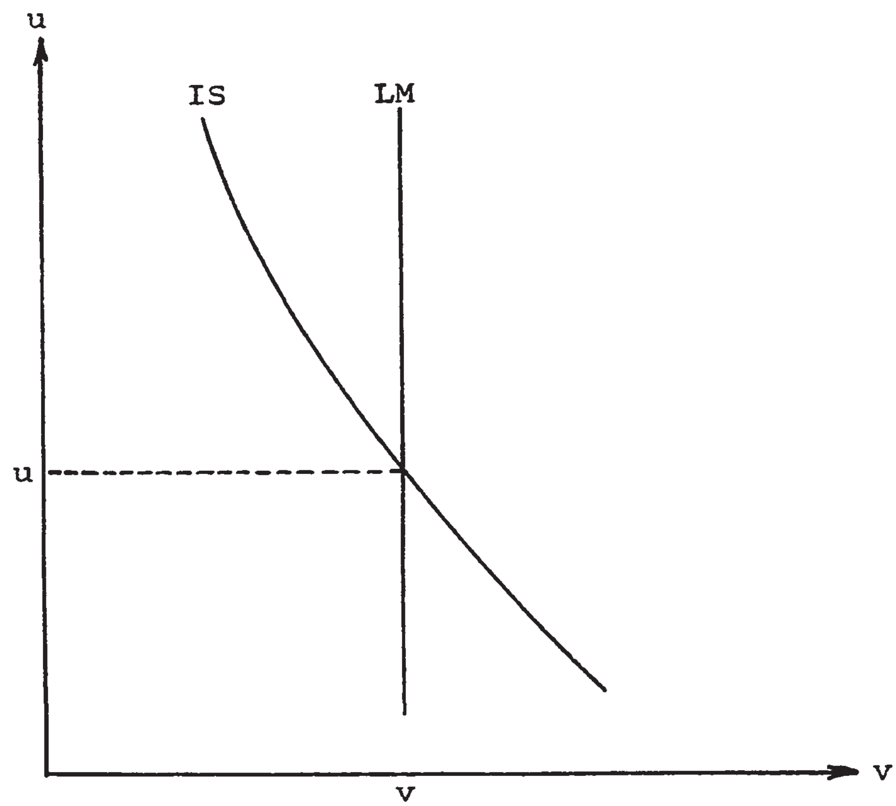
$$G_m\Big|_{G=0} = u_v v_m. \tag{4.40}$$

The Jacobian determinant of the system is given by

$$H_k G_m - H_m G_k = -\frac{\gamma f_I' v_m}{k}\left(\frac{k_I - k}{k_I + v}u_k + \frac{v u_v}{k}\right). \tag{4.41}$$

Stability of the system again depends on the signs of

FIGURE 10



(4.8), (4.10) and (4.11). We find that

$$\text{sign } (H_k G_m - H_m G_k) = - \text{ sign } G_m = \text{ sign } (\gamma - \eta). \quad (4.42)$$

The slopes of the two lines $H(k, m) = 0$ and $G(k, m) = 0$ are given by

$$m_k \Big|_{H=0} = - H_k/H_m, \tag{4.43}$$

$$m_k \Big|_{G=0} = - G_k/G_m, \tag{4.44}$$

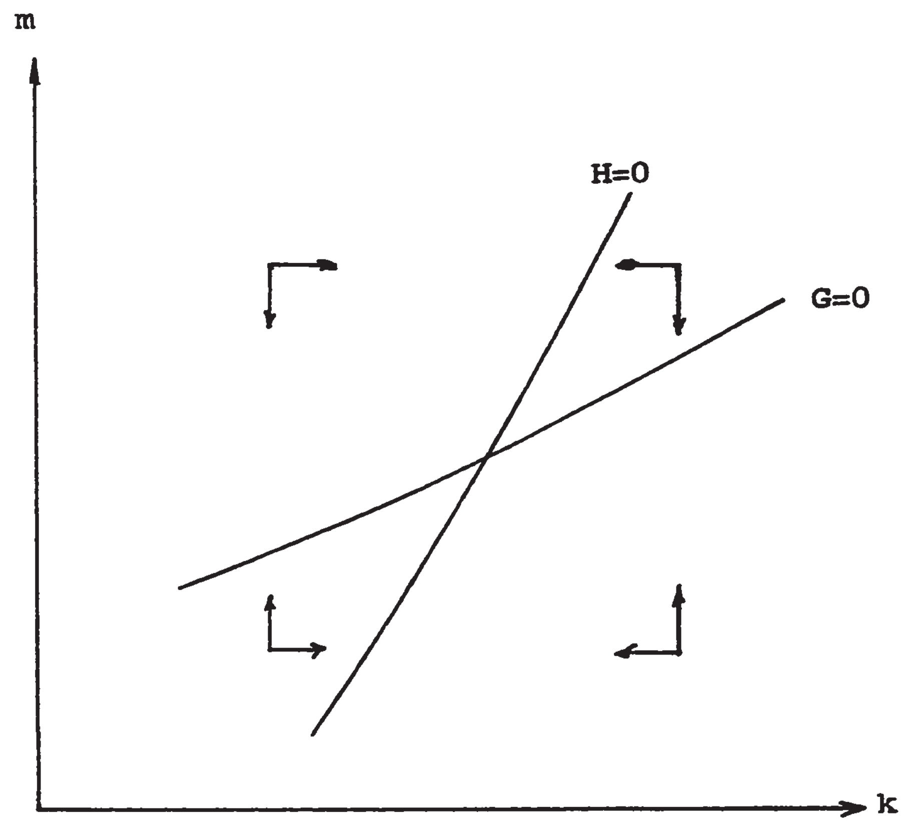
respectively. Comparing the slopes of the two curves we have

$$m_k \Big|_{H=0} - m_k \Big|_{G=0} = - (H_k G_m - H_m G_k)/H_m G_m. \tag{4.45}$$

The signs of (4.37) to (4.39) are in general indeterminate. If (4.8) is negative and both (4.10) and (4.11) are positive, then (4.37) and (4.40) are negative and (4.41) is positive. Condition (4.16) is satisfied and the system is dynamically stable. The phase diagram, Figure 11, is drawn on the assumption that (4.38) is positive.

We have shown that fixing the rate of inflation by adjusting the rate of monetary expansion is more conducive to stability. Changes in k and m affect the values of u and v and hence the demand for money and the processes of saving and investment. There will be structural shifts between the investment and consumption good sectors. Whether the impact of the initial shock will dwindle over time and the system

FIGURE 11



regains equilibrium or the initial disturbance brings forth further changes depend on complicated interactions between the real and monetary factors.

IV. A FIXED MONETARY RULE AND ADAPTIVE EXPECTATION

Instead of the perfect myopic foresight assumption (2.18), we postulate that the expected rate of inflation is adjusted to the actual rate of inflation with certain time lag. The change in the expected rate of inflation is a function of the difference between the actual and the expected rates.

$$\dot{\hat{\pi}} = b(\pi - \hat{\pi}) \tag{4.46}$$

where b is a positive constant, denoting the speed of adjustment. A static expectation function is equivalent to $b = 0$. Instantaneous price adjustment is equivalent to $b = \infty$.

We shall now treat $\hat{\pi}$ as historically given at any instant of time. The two variables $\hat{\pi}$ and k together determine the equilibrium demand and supply of real balances and the wage-rental ratio which will maintain equilibrium between savings and investment.

The supply of real balances is a decreasing function of the rate of inflation while the rate of monetary expansion is kept constant. Equilibrium in the money market is given by

$$m(\pi) = \lambda(\hat{\pi})y(v, k) = m(\hat{\pi}, v, k). \tag{4.47}$$

Expressing π in terms of $\hat{\pi}$, v and k, we get

$$\pi = \pi(\hat{\pi}, v, k), \tag{4.48}$$

$$\pi_{\hat{\pi}} = \lambda'/m_{\pi} > 0, \tag{4.49}$$

$$\pi_{v} = \lambda y_{v}/m_{\pi}, \tag{4.50}$$

$$\pi_{k} = \lambda y_{k}/m_{\pi} < 0. \tag{4.51}$$

Substituting (4.48) into the fundamental savings and investment equilibrium condition, we get

$$\phi(v, \hat{\pi}, k) = \gamma(k+v)(k_C-k_I) - (k_I+v)(k_C-k) = 0. \tag{4.52}$$

Differentiating (4.52) with respect to its arguments, we get

$$\phi_v\Big|_{\phi=0} = -((k_I+v)l_C k_C' + (k_C+v)l_I k_I' + (k_C-k)(k-k_I)/(k+v) - (k+v)(k_C-k_I)\gamma_{\pi}\pi_v), \tag{4.53}$$

$$\phi_{\hat{\pi}}\Big|_{\phi=0} = \gamma_{\hat{\pi}}(k+v)(k_C-k_I), \tag{4.54}$$

$$\phi_k\Big|_{\phi=0} = (k_C+v)(k_I+v)/(k+v) + (k_I+v)(k_C-k)\gamma_{\pi}\pi_k/\gamma \tag{4.55}$$

where

$$\gamma_{\pi} = (1 - s)\lambda > 0, \tag{4.56}$$

$$\gamma_{\hat{\pi}} = -(1 - s)((u - \pi)\lambda_{\hat{\pi}} - \lambda\pi_{\hat{\pi}}) > 0. \tag{4.57}$$

An increase in the actual rate of inflation reduces the supply of real balances. Hence more savings are devoted to capital formation. An increase in the expected rate of inflation increases the actual rate and the cost of holding real balances. The demand for the latter is reduced. The physical savings ratio is increased as a result.

Using (4.52), we can express v in terms of k and $\hat{\pi}$.

$$v = v(k, \hat{\pi}), \tag{4.58}$$

$$v_k = -\phi_k/\phi_v, \tag{4.59}$$

$$v_{\hat{\pi}} = -\phi_{\hat{\pi}}/\phi_v. \tag{4.60}$$

Differentiating the money market equilibrium condition (4.47) logarithmically with respect to time and using (4.58), we get

$$u - \pi - n = (m_v\dot{v} + m_{\hat{\pi}}\dot{\hat{\pi}} + m_k\dot{k})/m$$

$$=((m_v v_{\hat{\pi}} + m_{\hat{\pi}})\dot{\hat{\pi}} + (m_v v_k + m_k)\dot{k})/m. \tag{4.61}$$

Substituting (4.61) into (2.21), we have

$$y_I = (s - (1-s)(n + ((m_v v_{\hat{\pi}} + m_{\hat{\pi}})\dot{\hat{\pi}} + (m_v v_k + m_k)\dot{k})/m)\lambda(\hat{\pi}))$$
$$f_I'(k+v). \tag{4.62}$$

Substituting (4.62) into (2.24) and after rearranging, we get

$$J_{11}\dot{k} + J_{12}\dot{\hat{\pi}} = C_1, \tag{4.63}$$

where

$$J_{11} = 1 + (1-s)(m_v v_k + m_k)\lambda f_I'(k+v)/m, \tag{4.64}$$

$$J_{12} = (1-s)(m_v v_{\hat{\pi}} + m_{\hat{\pi}})\lambda f_I'(k+v)/m, \tag{4.65}$$

$$C_1 = (s - (1-s)n\lambda)f_I'(k+v) - nk. \tag{4.66}$$

Substituting (4.61) into (4.46) and after rearranging, we get

$$J_{21}\dot{k} + J_{22}\dot{\hat{\pi}} = C_2, \tag{4.67}$$

where

$$J_{21} = b(m_v v_k + m_k)/m, \tag{4.68}$$

$$J_{22} = 1 + b(m_v v_{\hat{\pi}} + m_{\hat{\pi}})/m, \tag{4.69}$$

$$C_2 = b(u - n - \hat{\pi}). \tag{4.70}$$

We have a system of two differential equations in the two state variables k and $\hat{\pi}$. We define the steady state as the critical point $(k^*, \hat{\pi}^*)$ at which $\dot{k} = \dot{\hat{\pi}} = 0$, i.e.,

$$(s - (1-s)n\lambda)f_I'(k^*+v^*) - nk^* = 0, \tag{4.71}$$

$$u - n - \hat{\pi}^* = 0. \tag{4.72}$$

In the steady state the amount of savings devoted to capital formation is just sufficient to maintain the capital intensity constant. The expected rate of inflation is equal to the rate of growth of per capita nominal balances.

To investigate the stability properties of the system around the steady state, we solve the system of equations (4.63) and (4.67) for $\dot{k}$ and $\dot{\hat{\pi}}$ by Cramer's rule.

$$\dot{k} = A/J = H(k, \hat{\pi}), \tag{4.73}$$

$$\dot{\hat{\pi}} = B/J = G(k, \hat{\pi}), \tag{4.74}$$

where

$$A = \begin{vmatrix} C_1 & J_{12} \\ C_2 & J_{22} \end{vmatrix} , \tag{4.75}$$

$$B = \begin{vmatrix} J_{11} & C_1 \\ J_{21} & C_2 \end{vmatrix} , \tag{4.76}$$

$$J = \begin{vmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{vmatrix} = J_{11} + J_{22} - 1$$

$$= 1 + b(m_v v_{\hat{\pi}} + m_{\hat{\pi}})/m + (1-s)(m_v v_k + m_k)\lambda f_I'(k+v)/m. \tag{4.77}$$

Differentiating (4.73) and (4.74) with respect to k and $\hat{\pi}$, respectively, and making use of (4.71) and (4.72), we get

$$H_k \Big|_{H=0} = J_{22}(\partial C_1/\partial k)/J, \tag{4.78}$$

$$H_{\hat{\pi}} \Big|_{H=0} = (\ J_{22}(\partial C_1/\partial \hat{\pi}) - J_{12}(\partial C_2/\partial \hat{\pi})\ )/J, \tag{4.79}$$

$$G_k \Big|_{G=0} = -\ J_{21}(\partial C_1/\partial k)/J, \tag{4.80}$$

$$G_{\hat{\pi}} \Big|_{G=0} = (\ J_{11}(\partial C_2/\partial \hat{\pi}) - J_{21}(\partial C_1/\partial \hat{\pi})\ )/J, \tag{4.81}$$

where

$$\partial C_1/\partial k = ((k_I-k)/(k_I+v) - e)nkv_k/(k+v), \tag{4.82}$$

$$\partial C_1/\partial \hat{\pi} = nkv_{\hat{\pi}}(k_I-k)/(k_I+v)(k+v), \tag{4.83}$$

$$\partial C_2/\partial \hat{\pi} = -b < 0. \tag{4.84}$$

The Jacobian determinant matrix of the system of differential equations (4.63) and (4.67) is given by

$$H_k G_{\hat{\pi}} - H_{\hat{\pi}} G_k = (\partial C_1/\partial k)(\partial C_2/\partial \hat{\pi})/J. \tag{4.85}$$

The stability of the system in general depends on the signs of the elasticity condition (4.8), the factor-intensity condition (4.10), the relative magnitudes of γ and η (4.11) and the condition

$$1 + b(m_v v_{\hat{\pi}} + m_{\hat{\pi}})/m \tag{4.86}$$

which is positive if the speed of adjustment of the expected rate to the actual rate of inflation is sufficiently sluggish, i.e., if the value for b is small enough.

Rather than enumerating all the possible combinations of the different conditions, we shall discuss only the case where the following set of assumptions holds: (4.11) is negative and (4.10) and (4.86) are positive. Given the above assumptions, an increase in the wage-rental ratio will reduce real income (y), increase the rate of inflation (π) and lower the demand for real balances (m). An increase in the expected rate of inflation ($\hat{\pi}$) will increase the wage-rental ratio (v), increase the actual rate of inflation (π), lower the demand for real balances (m) and increase the physical savings ratio (γ). In this case the indirect effect on m via changes in v reinforces the direct effect on m from changes in $\hat{\pi}$. We have

$$m_v v_{\hat{\pi}} + m_{\hat{\pi}} < 0. \qquad (4.87)$$

An increase in the capital-labour ratio (k) decreases the rate of inflation (π), increases the real income (y) and hence increases the demand for real balances (m). If (4.53) is negative, then an increase in k will also increase v which reduces real income. Hence, the effect on the demand for real balances via v from changes in k is negative. The sign of

$$m_v v_k + m_k \qquad (4.88)$$

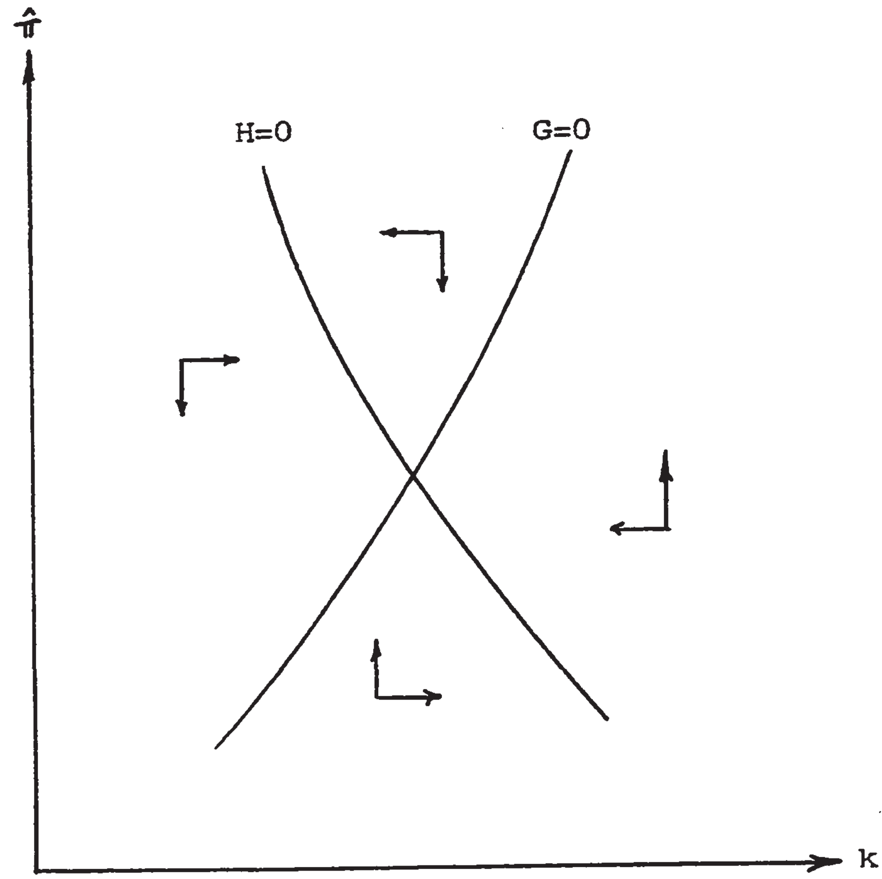
is ambiguous. Suppose we assume that the direct impact on m due to changes in k dominates the indirect effect via v, then (4.88) will be positive.

We find that J_{11}, J_{21} and J_{22} are positive while J_{12} is negative. An increase in k above its steady state value will change the equilibrium wage-rental ratio and cause structural changes in the two production sectors. We find that the net result is a reduction in the rate of capital accumulation. Hence the system is stable in the k direction. An increase in k, however, tends to increase the change in $\hat{\pi}$.

Given the above assumptions, the sign of (4.80) is positive while that of (4.81) is ambiguous. The product of the characteristic roots given by (4.85) is positive. If (4.81) is negative, the sum of the two roots will be negative. Condition (4.16) is satisfied. The system is locally stable. The phase diagram is drawn in Figure 12.

We have thus shown that with an adaptive expectation function and a fixed monetary rule, a stable system can be achieved if the speed of adjustment is sufficiently sluggish. Sidrauski (1967a, 1967b) and others have demonstrated similar results. Stability in our model, however, depends on many more conditions than those found by Sidrauski as a result of our two-sector approach.

FIGURE 12



CHAPTER V

A COMPARATIVE-DYNAMICS ANALYSIS

I. THE NON-NEUTRALITY OF MONEY

In this chapter, we shall investigate the effects that changes in certain parameters have on the processes of capital accumulation.

First, we examine the question of whether changes in the rate of monetary expansion, u, have a neutral effect in the long run. Money is defined to be neutral if

$$dk^*/du = 0, \tag{5.1}$$

i.e., the steady-state capital intensity (k^*) is not affected by the rate of change of the nominal money supply, u.

We shall first discuss the case of perfect myopic foresight and a fixed monetary rule. At the steady state, (4.3) holds. We have

$$\gamma^* f_I'(k_I^*)(k^* + v^*) = nk^*, \tag{5.2}$$

$$u^* - \pi^* = n, \tag{5.3}$$

where

$$\gamma^* = s - (1 - s)n\lambda(\hat{\pi}^*). \tag{5.4}$$

To find the effect that changes in the parameter, u, have on the steady-state capital intensity (k^*) and the stock of real balances per head (m^*), we differentiate (5.2) and (5.3) with respect to u. We have

$$B_{11}k_u + B_{12}m_u = 0, \tag{5.5}$$

$$B_{21}k_u + B_{22}m_u = 1, \tag{5.6}$$

where

$$B_{11} = \gamma f_I' v_k((k_I - k)/(k_I + v) - e)$$
$$- f_I'(k+v)(1-s)n\lambda'(\pi_v v_k + \pi_k), \tag{5.7}$$

$$B_{12} = \gamma f_I' v_m(k_I - k)/(k_I + v) - f_I'(k+v)(1-s)n\lambda'(\pi_v v_m + \pi_m), \tag{5.8}$$

$$B_{21} = \pi_v v_k + \pi_k, \tag{5.9}$$

$$B_{22} = \pi_v v_m + \pi_m. \tag{5.10}$$

The Jacobian determinant of the system is

$$J = B_{11}B_{22} - B_{12}B_{21}$$
$$= -\gamma f_I'(\pi_v v_m + \pi_m)v/k + \gamma f_I'(v_k\pi_m - v_m\pi_k)(k_I - k)/(k_I + v). \tag{5.11}$$

Solving for k_u and m_u from (5.5) and (5.6), we get

$$k_u^* = -B_{12}/J, \tag{5.12}$$

$$m_u^* = B_{11}/J. \tag{5.13}$$

Since B_{ij} and J are not equal to zero in general, a change in the rate of monetary expansion will change the steady-state capital intensity (k*) and the stock of real balances per head (m*). Money is not neutral in the sense of (5.1).

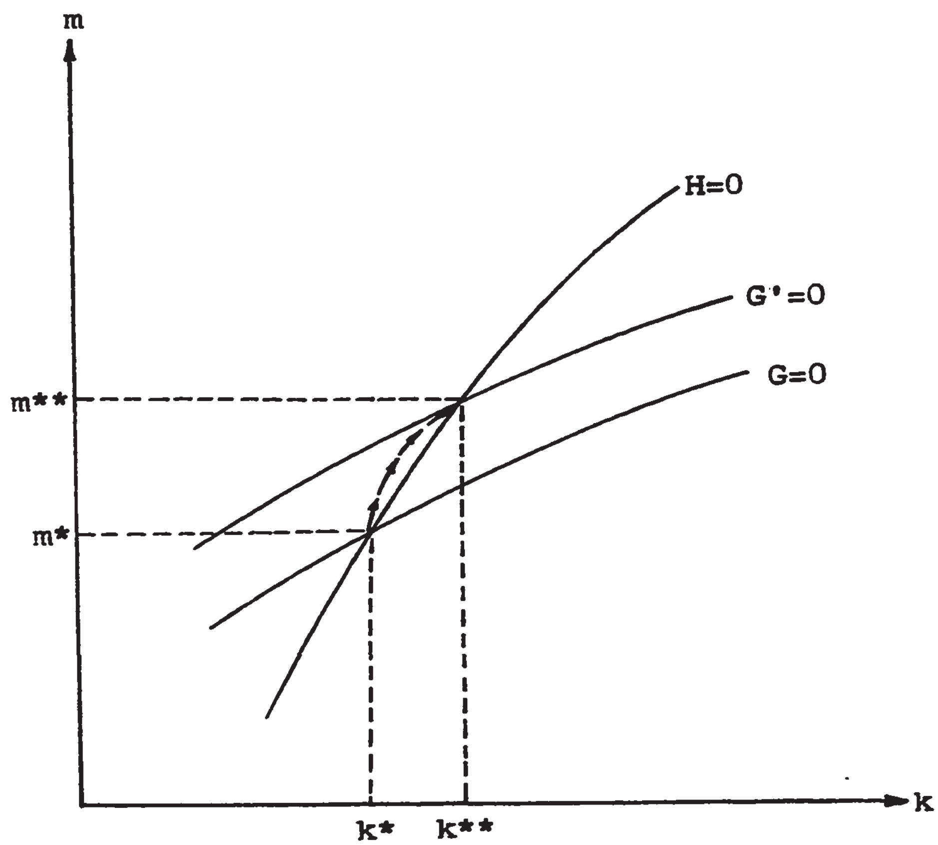
If (4.10) and (4.12) are positive and (4.11) and (4.9) negative, then the system is stable in both the k and m directions. (5.7) becomes negative and (5.8) positive. If, in addition,

$$v_k \pi_m - v_m \pi_k > 0, \tag{5.14}$$

then (4.15) is positive, (5.11) is negative, and both (5.12) and (5.13) are positive, i.e., if the system is locally stable about (k*, m*), then an increase in u will raise both the steady-state k* and m*. A possible description of the comparative dynamic growth path is shown in Figure 13.

If the monetary authorities raise the rate of money creation, both the steady-state rate of inflation and the equilibrium wage-rental ratio will be increased which will raise the physical savings ratio, the real income and the demand for real balances for transaction purposes. At the steady state both k* and m* are higher than before.

FIGURE 13



II. CHANGES IN THE AGGREGATE SAVING PROPENSITY

To determine the effect of changes in the savings ratio on the steady-state k^* and m^*, we differentiate (5.2) and (5.3) with respect to s and get

$$B_{11}k_s + B_{12}m_s = - f_I'(k+v)(1+n\lambda), \tag{5.15}$$

$$B_{21}k_s + B_{22}m_s = 0, \tag{5.16}$$

where the B_{ij}'s are given by (5.7) to (5.10). Solving for k_s and m_s using Cramer's rule, we get

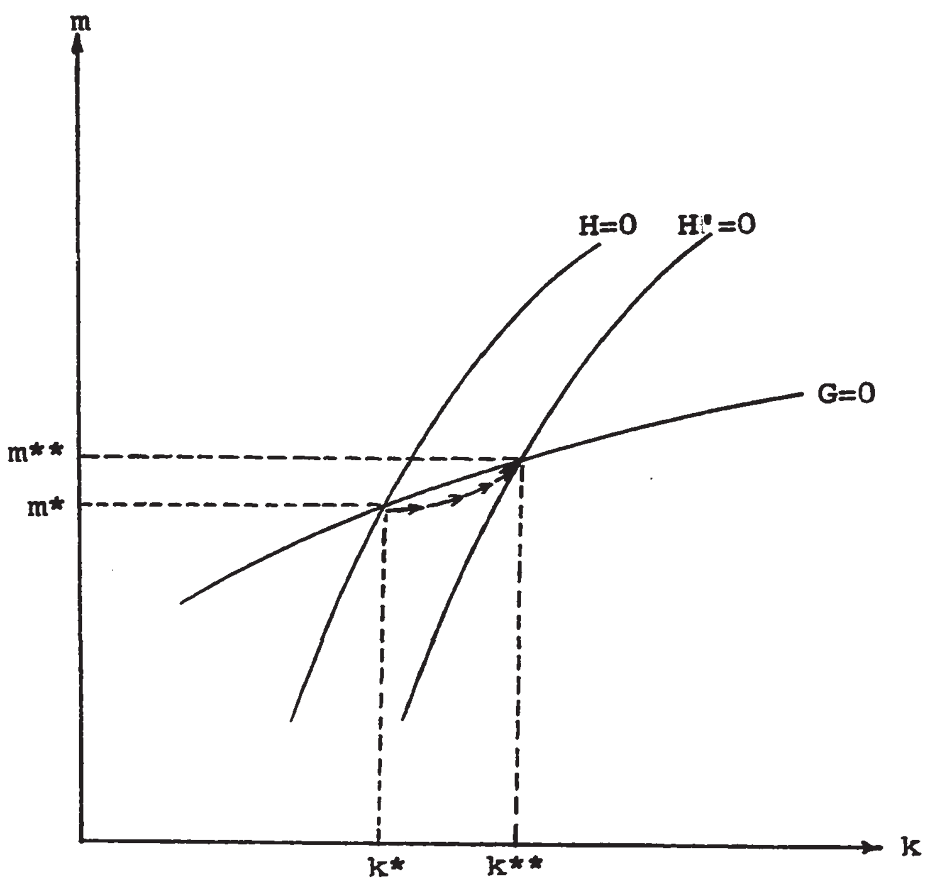
$$k_s^* = - f_I'(k+v)(1+n\lambda)B_{22}/J, \tag{5.17}$$

$$m_s^* = f_I'(k+v)(1+n\lambda)B_{21}/J, \tag{5.18}$$

where J is given by (5.11). If the situation described by Figure 13 holds, then both (5.17) and (5.18) are positive. The comparative dynamic growth path is illustrated by Figure 14 for an increase in s.

An increase in s increases the physical savings ratio (γ). A higher rate of investment shifts the H = 0 curve to the right. A larger per capita stock of capital increases the per capita real income and hence the demand for real balances. Changes in s, however, does not shift the G = 0 curve. The latter intersects the H' = 0 curve to yield higher steady-state values for both k and m.

FIGURE 14



III. CHANGES IN THE RATE OF INFLATION

Suppose the monetary authorities allow the rate of monetary expansion to adjust to maintain a fixed rate of inflation. To find out the effect of changes in the target rate of inflation on the steady-state values of k^* and m^*, we differentiate (5.2) and (5.3) with respect to π. We get

$$C_{11}k_{\pi} + C_{12}m_{\pi} = E, \tag{5.19}$$

$$C_{21}k_{\pi} + C_{22}m_{\pi} = 1, \tag{5.20}$$

where

$$C_{11} = \gamma f_I^{\prime} v_k((k_I - k)/(k_I + v) - e), \tag{5.21}$$

$$C_{12} = \gamma f_I^{\prime} v_m (k_I - k)/(k_I + v), \tag{5.22}$$

$$C_{21} = u_v v_k + u_k, \tag{5.23}$$

$$C_{22} = u_v v_m, \tag{5.24}$$

$$E = f_I^{\prime}(k+v)(1-s)n\lambda^{\prime} < 0. \tag{5.25}$$

The Jacobian determinant of the system is

$$J = C_{11}C_{22} - C_{12}C_{21}$$

$$= -\gamma f_I^{\prime} v_m (u_v v/k + u_k(k_I - k)/(k_I + v)). \tag{5.26}$$

If (4.8) is negative, (4.10), (4.11) and (4.38) positive so that the phase diagram Figure 11 describes the system around the steady state (k^*, m^*), then (5.21), (5.22) and (5.24) are negative, (5.23) and (5.26) are positive.

Solving (5.19) and (5.20) for k_π and m_π, we get

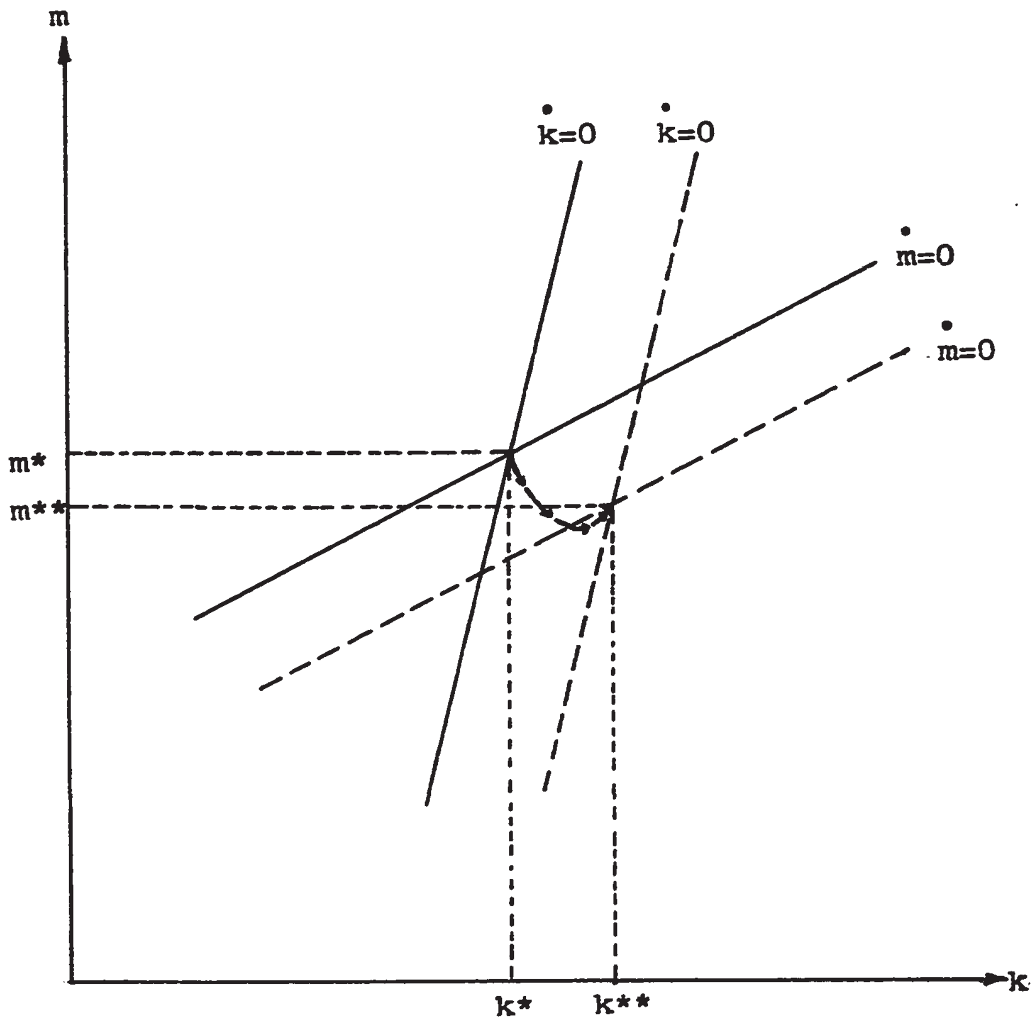
$$k_\pi = (EC_{22} - C_{12})/J, \tag{5.27}$$

$$m_\pi = (C_{11} - C_{21}E)/J. \tag{5.28}$$

(5.27) is positive and the sign of (5.28) is ambiguous. One possible description of the movement from the initial steady state (k*, m*) to the new steady state (k**, m**) is shown in Figure 15.

An increase in π raises the expected rate of inflation and hence the cost of holding money. A greater proportion of savings is channelled into capital formation. The steady-state capital intensity is increased as a result. The impact on the holding of real balances depends on the relative strength of substitution effect vs the income effect. An increase in π raises the cost of holding money. Less of the latter is held. But the increase in income raises the transaction demand for money. In Figure 15, the substitution effect dominates and the stock of real balances in the steady state is lowered.

FIGURE 15



IV. COMPARISON WITH A BARTER ECONOMY

We also observe that the steady-state capital intensity (k^*) in this monetary economy is necessarily less than that attained in a non-monetary world. Solving (5.2) for k^*, we get

$$k^* = \gamma^* v^* f_I' / (n - \gamma^* f_I'). \qquad (5.29)$$

Since $\gamma^* = s - (1 - s)n\lambda < s$, the steady-state capital intensity achieved by Uzawa's non-monetary world is necessarily higher than that achieved by our monetary model.

This result can be expected because in a monetary world, part of the savings which would have been devoted to capital formation in a barter world has been diverted to holding the monetary asset, money. This should not condemn the usefulness of money since the cost of transactions in a barter world is much higher than that in a monetary world and it has not been accounted for in the hypothetical barter models. Similar observations have been made by Tobin (1965, 1967) and Sidrauski (1967b) among others. It is argued by Burmeister and Dobell (1970, p. 177) that even though the capital-labour ratio in a monetary economy is less than that of a barter economy, if the latter is greater than the Golden Rule capital-labour ratio, a reduction in its value may actually increase per capita consumption in the steady state.

CHAPTER VI

CONCLUSION

We shall first list the assumptions that have been made in this study and then summarize the results that have been obtained.

The following set of assumptions is made in our dynamic model of a neoclassical monetary two-sector economy:

(1) The production functions are linear homogeneous in capital and labour, well-behaved and satisfy the concavity conditions (2.3) and the end-point conditions (2.4). There is no factor intensity reversal.

(2) There is perfect competition in the commodity, factor and money markets. Full employment of all resources always prevail.

(3) Money is treated as a consumer's good.

(4) Money is of the outside variety only. There is no bond or inside money.

(5) The supply and demand for money is always in equilibrium.

(6) There are no independent savings and investment decisions. All savings are allocated, either to capital formation or to acquisition of additional real balances.

(7) Either the expected and the actual rates of inflation coincide or the former adjusts to the latter with a time lag.

(8) Either the rate of monetary expansion or the rate of inflation is treated as a parameter of the system.

(9) A constant proportion of real disposable income is saved.

(10) The real personal disposable income includes only the real output and increase in real balances.

(11) The general price level is a linear homogeneous function of the money prices of the two commodities and is used to deflate the nominal supply of money, national and disposable income.

(12) There is no technical change.

(13) There is no trade with foreign countries. Complete specialization of production, as a result, is excluded.

(14) The government does not consume or invest. It controls either the rate of monetary expansion or the rate of inflation. Fiat currency is issued by the government as a transfer payment and does not bear any interest charges.

Based on the assumptions listed above, our analysis shows that:

(1) The equilibrium wage-rental ratio (v) and the expected rate of inflation ($\hat{\pi}$) are determined jointly by the flow equilibrium of savings and investment and the stock equilibrium of demand and supply of assets. The short-run momentary equilibrium for given k and m is shown to exist and to be unique.

(2) Under the perfect myopic foresight assumption about price expectations and a fixed monetary rule, the money market is stable if the Pigou effect which is stabilizing dominates the destabilizing Wicksell effect. On the other hand, the Wicksell effect is stabilizing while the Pigou effect is destabilizing to the processes of capital accumulation.

The steady state is not necessarily unique. The stability of the model depends on the signs of (1) the elasticity condition (4.8), (2) the factor-intensity condition (4.10), (3) the relative magnitudes of the physical savings ratio (γ) and the price elasticity (η) and (4) the relative strength of the direct vs the indirect effect via the wage-rental ratio (v) on the rate of inflation (π) from changes in k and m.

(3) The aggregate elasticity of factor substitution (e) is shown to be a function of the elasticities of factor substitution in the two industries (e_j). In Uzawa's (1963) barter two-sector model, the stability of the balanced growth equilibrium depends on the elasticity condition that $e > (k_I - k)/(k_I + v)$ which is satisfied if $k_C > k > k_I$, or if the production functions are Cobb-Douglas so that $e_I = e_C = e = 1$, or if they approach the Leontief-type of production functions with fixed coefficients and no factor substitution in which case $e_I = e_C = 0$ but $e > 0$ and Uzawa's factor intensity hypothesis (3.12) holds.

(4) Under a flexible monetary rule, the rate of inflation is stabilized by adjusting the rate of money creation. Equilibrium between savings and investment determines the equilibrium rate of monetary expansion. Stability is achievable if (4.8) is negative and both (4.10) and (4.11) are positive.

(5) If we retain the assumption of a fixed monetary rule but postulate an adaptive expectation function, we find that stability again depends on the signs of (4.8), (4.10) and, in addition, the speed of adjustment

of the expected to the actual rates of inflation. A sluggish response, in itself, is not sufficient to guarantee stability.

(6) Money is not neutral in the long run in the sense that a change in the rate of monetary expansion will alter the steady-state capital intensity. It will also change the stock of real balances demanded in the balanced growth equilibrium.

(7) Changes in the savings propensity or the target rate of inflation pegged by the monetary authorities will alter the equilibrium capital intensity and the stock of real balances in the steady state.

(8) The balanced growth equilibrium capital intensity of our monetary economy is necessarily less than that achieved by an Uzawa-type of nonmonetary economy.

The neoclassical monetary two-sector growth model that we have constructed in this study can be considered as an archetype of a miniature monetary general equilibrium model. By varying one or more of the assumptions employed in the model, we can seek the answers to a whole host of interesting questions in economics. For instance, due to its simplicity, the model presented here can be extended to investigate the dynamic behaviour of an international economy under the alternative regimes of fixed and flexible exchange rates.

Alternatively, we can assume that the government engages in either consumption activities as in Uzawa (1966a) and Foley and Sidrauski (1970, 1971) or in investment activities as in Hahn (1969). We can then study the effects that different fiscal and monetary policies have on the processes of capital accumulation.

The contributions of Solow (1956) and Uzawa (1961, 1963) in the theory of economic growth are important not because they have exhausted the possibilities of neoclassical one-sector and two-sector growth models but rather because they have laid down a logically consistent framework of analysis which can be used by later economists to investigate various economic hypotheses. In a similar vein, we can treat the basic model constructed in this dissertation as an analytical framework whereby modifications and extensions can be easily incorporated to examine relevant problems in economics.

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