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PORTFOLIO BEHAVIOUR OF THE TRUST AND MORTGAGE LOAN COMPANIES OF CANADA, 1967 - 1972, A THEORETICAL AND ECONOMETRIC ANALYSIS

by

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ABSTRACT

Using portfolio theory a model of behaviour is specified and applied to monthly balance-sheet data for the trust and mortgage loan companies over the period January 1967 - March 1972. An outline of the particular institutional context, including an analysis of the recent history of the trust and mortgage loan company sector is presented; and the model is specified to suit this context. The structure of the model is estimated by consistent techniques, and where necessary "adding-up" constraints are imposed on the estimates. The most important asset in the portfolio, mortgage lending, responds to the variables which determine it with long lags and it is therefore treated as a predetermined variable at the time when other balance-sheet components are decided. Term deposits are treated as endogenous, responding to interest rates and the requirements of mortgage financing. Overall, the results indicate that interest rates play a very important role on both the asset and liability sides of the sector's balance sheet. The estimated interest rate responses are, with few exceptions, numerically large and statistically significant.

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On a number of mathematical and grammatical points I followed the advice of my wife, Gillian. I thank her for her help, and thank her also for putting up with the variety of inconveniences caused by the thesis.

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Chapter I

Portfolio Theory and Application

1. Portfolio theory

The theory of portfolio behaviour, as I shall use the term, is that body of theory deriving from the work of Markowitz (1952). The original emphasis of this theory was on the normative issues of optimal behaviour for an individual investor, but it inspired also an extensive inquiry into the positive issues of capital market theory.[1] The positive implications of the theory for an individual investor, which an economist usually thinks of in terms of comparative static experiments, have until recently been ignored.

Markowitz's hypothesis is that an investor seeks to maximize an objective function whose sole arguments are the first two moments of the return on his investment portfolio. Assets are held not so much for their intrinsic virtues as for the contribution which they make to the mean (E) and the variance (V) of the return on the whole portfolio. Writing the objective function as

$$U = U(E,V), \qquad (1)$$

and writing subscripts for partial derivatives, the mean-variance

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[1] An account of both developments, with bibliography, is to be found in Sharpe (1970).

(or EV) hypothesis contends that $U_E \geq 0$ and that $U_V \leq 0$. The investor is therefore supposed to want a portfolio with a high mean return and a low variance of return. The rationale for shunning variance is that variance of return is a measure of risk and that investors will usually be risk averse. The function (1) is to be maximized subject to certain constraints. A portfolio which satisfies these constraints is a feasible portfolio. Those feasible portfolios which provide minimal variance for a given expected return are said to be EV efficient. A rational EV investor would limit his choice to the set of EV efficient portfolios, choosing the particular efficient portfolio which best suits his tastes with respect to return and risk.

The assumptions of the EV model are by no means innocuous. The fundamental theoretical difficulty is that the EV model is consistent with the axioms of the theory of decisions under uncertainty only in special cases.[2] This difficulty is intuitively apparent

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[2] Borch (1968), Chapter IV, argues that either a quadratic von Neumann - Morgenstern utility function, or a normal distribution of returns is required. However, I am not sure that his argument is conclusive. Just recently a number of papers have appeared (most of them unpublished as yet) which provide a rigorous justification for the EV model using neither the quadratic utility function nor normality. Ross (1972), Tsiang (1972) and Morishima (in some unpublished research at the London School of Economics) seem to have reopened this issue.

in the implausibility of the identification of variance with risk, since variance treats upward and downward movements in return symmetrically. Notwithstanding these limitations, the EV hypothesis has not been supplanted. Its attraction lies in its simplicity as an operational hypothesis, whether for normative or positive purposes. The alternatives are either analytically intractable or are even more at odds with the theory of decisions under uncertainty[3] -- they are, in short, sufficiently repulsive that, to get anywhere, one is forced to work with the EV model.

The central purpose of the theoretical section which comprises the first four chapters of this thesis is the exploration of the implications of the EV theory for the portfolio behaviour of a financial intermediary. The resulting model is a complete set of mutually consistent security demand functions,[4] the structure of which is estimated in Chapter VIII by efficient

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[3] E.g. Porter (1967) assumes that banks maximize "expected additions to net worth" (p.22) rather than the utility of these additions.

[4] It might be noted that the assumption of the EV model is sufficient, but not necessary, for derivation of the restrictions on the demand functions which I use. The same restrictions can be obtained from quite different assumptions.

and consistent techniques. Readers wishing to go directly from theory to empirical test should read Chapter VIII immediately after Chapter IV, as Chapters V, VI and VII bear on the theory only tangentially

Three recent contributions have gone far in filling the lacuna which previously existed regarding the demand curves which are implicit in portfolio theory. Bierwag and Grove (1968) have derived equations analogous to the Slutsky equations of consumer theory. Their comparative statics results are then used to derive some (rather weak) restrictions on the asset demand functions in the form of certain elasticity dependencies and homogeneity relations. Royama and Hamada (1967) specialize the EV hypothesis by requiring that an investor have a quadratic utility function of the von Neumann-Morgenstern type -- a strong requirement considering the peculiarities of quadratic utility functions, and one which does little to strengthen the implications of the general EV utility function of equation (1). Parkin (1970), following Freund (1956), derives a different EV specialization by assuming that the investor perceives the stochastic yield prospects to be normally distributed, and that the investor has a von Neumann-Morgenstern utility function with constant absolute risk aversion.[5] The implications of these assumptions for the asset demand functions are very strong.[6]

The more restrictive are the assumptions made, then the greater is the usefulness of the theory in terms of the a priori knowledge it brings to the empirical estimation procedure and in terms of yielding implications which the data can conceivably refute. Conversely, if the data can bear the more restrictive assumptions there is a gain in the form of the more specific structural knowledge so obtained. The danger, of course, is that the more restrictive assumptions might involve too great a distortion of reality. My preference is to use as general a framework

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5. An investor has constant absolute risk aversion if his preference ordering is independent of his wealth. Pratt (1964) discovered the importance of this property.

6. The theoretical assertions in this paragraph will be substantiated in Chapters II and III.

as is consistent with well behaved statistical estimates. Parkin and others have found that collinearity of interest rates requires rigid patterns to be imposed a priori to obtain reasonable estimates but I have not found this with my data base.

A number of the results obtained in the following theoretical development can be found in the writings of Bierwag and Grove, Royama and Hamada and Parkin, though the derivations and interpretations may be novel -- particularly so the diagrammatic technique explained in Chapter III. The emphasis of this development is on those features of the theory which pertain to security demand functions, and to consolidate and systematize the existing results where they may be brought to bear on this aspect. Furthermore, the inquiry is directed towards questions of particular importance to financial intermediaries.

2. Application of theory

Explicit tests of the theory of Chapters II, III and IV are in Chapter VIII. From the point of view of testing this theory Chapters V, VI and VII can be regarded as background material, but the topics covered in these chapters are important and certain readers will find their content more interesting than Chapter VIII. Chapter V is an institutional description of the trust and mortgage loan company sector of the Canadian financial system as it has operated in the past decade. Chapter VI presents quantitative estimates of the public's demand functions for deposits in trust and mortgage loan companies. Chapter VII contains a model to explain mortgage lending by these companies. (Because of the time lags between the decision to make a mortgage loan and the acquisition of the loan, mortgages are not treated on the same basis as other assets.)

The empirical pieces are formally consistent with one another, fitting into the framework described with flowcharts in Chapter V and Chapter VIII. But these pieces have not been blended into a numerical structure which would be useful for simulation purposes. This deficiency arises from two sources: 1. The equations of the present structure determine the interest rates on trust and mortgage loan term deposits implicitly, through supply and demand equations. Complete system solutions with variables determined in this way are apt to track historical data poorly and to give unrealistic simulation results. 2. More work is needed on the causal links between mortgage lending and the supply of liabilities from the trust and mortgage loan industry, especially with regard to their dynamic aspects.

Construction of a cohesive simulation model is the obvious next step in the empirical investigation of the trust and mortgage loan companies (the job is currently under way). Using conventional single-equation criteria the present empirical results are consistently satisfactory, so there is reason to believe that they provide a useful foundation for further work.

Chapter II

Positive Implications of the EV Hypothesis

1. Some preliminaries

The precise characterization of the EV model has differed from one author to another with respect to whether the quantity of a component asset of a portfolio should be measured as a number of securities or as a dollar value.[1] The distinction has important implications and is based on what is to be regarded as the basic choice unit about which the investor holds expectations. It may be formalized using the following notations: x_i, the number of securities of type i held (this number will be negative for liability items); e_i, the expected price per unit of i at the end of the decision period; p_i, the price now per unit of i (so that the expected return on this security is $e_i - p_i$); and ν_{ij}, the covariance of returns so defined on i and j. A zero superscript denotes initial holdings or endowments. There are n assets of which at least n-1 are risky in that the return is a stochastic variable.

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[1] Compare Royama and Hamada (1967) with Bierwag and Grove (1968).

An investor who thinks in terms of the price per unit of each security, and whose estimates of the future price are independent of the current price, will attempt to maximize

$$U = U\left(\sum_{i=1}^{n} e_i x_i,\ \sum_{i=1}^{n}\sum_{j=1}^{n} \nu_{ij}\, x_i x_j\right), \tag{1}$$

subject to the balance sheet constraint

$$\sum_{i=1}^{n} p_i\,(x_i - x_i^0) = 0. \tag{2}$$

Compare this with the maximizing problem faced by an investor who thinks in terms of dollar values invested; an investor, that is, who expects the ratio of the future price to the current price to be a constant. The difference between the two types of problem is that the former investor's elasticity of expectation for future price with respect to current price is zero, whilst the latter's is unity. Where y_i is the dollar value of the investment in i, r_i is the expected return on a dollar's worth of i, and σ_{ij} is the covariance of returns as just defined between i and j, the latter investor is to maximize

$$U = U\left(\sum_{i}^{n} r_i y_i,\ \sum_{i}^{n}\sum_{j}^{n} \sigma_{ij}\, y_i y_j\right), \tag{3}$$

subject to (2). Clearly for the first type of investor r_i is not constant with respect to a change in p_i, neither is σ_{ij}: for him

$\partial r_i/\partial p_i = e_i/p_i^2$ and $\partial \sigma_{ij}/\partial p_i = -\nu_{ij}/p_i^2 p_j$. The second type of investor regards these moments as data not changing as asset prices change. It follows that the two types of investor have different optimizing responses to changes in the asset prices.

For the investor who holds future price expectations independent of current prices, an increase in current prices will exert two conceptually distinct effects on the portfolio composition: consider an increase in p_i. First, there is a wealth effect proportional to the initial holding of x_i, x_i^0. Second, the trade off between risk and return is altered, causing a complex pattern of substitution effects. For the other type of investor there is the wealth effect only, since the change in price does not change his views as to the relevant moments. This will be made clear in the comparative statics exercises to follow.

Which, if either, of the two characterizations is to be used depends on the context. It is obviously necessary to take the security unit measure of asset volume if one constructs a general equilibrium model designed to explain the formation of security prices (see, e.g. Mossin (1966)). On the other hand, if the model is designed to explain the behaviour of an investor whose wealth is initially entirely in cash then it is natural to use dollar unit measures. For the present sort of investigation the information available on asset returns is in the form of interest rates, and the form in which balance sheets are reported is in dollar values for their various components. These considerations force us to use the investors characterized by (2) and (3) as our model for portfolio choice.

2. The general EV model: the optimizing decision

The foregoing considerations suggest that the investor's objective function be written

$$U = U(E,V)$$

where $E = \sum_{i}^{n} r_i y_i$

and $V = \sum_{i}^{n} \sum_{j}^{n} \sigma_{ij} y_i y_i$,

so that

$$U = U\left(\sum_{i}^{n} r_i y_i, \sum_{i}^{n} \sum_{j}^{n} \sigma_{ij} y_i y_j\right). \tag{3}$$

(3) is to be maximized subject to the constraint (2) which we can write as

$$\sum_{i}^{n} y_i = N \tag{4}$$

where $N \equiv \sum_i^n p_i x_i^o$. This rewriting is possible, of course, because $p_i x_i = y_i$. The constraint requires total assets to sum to total liabilities plus net wealth (N). Liabilities appear as assets sold short and thus have negative signs in the summation. Important in many circumstances, yet ignored here, are non-negativity constraints on certain of the assets. If these constraints are incorporated the problem to be solved is one of quadratic programming.[2]

The appropriate Lagrangean expression for the present case, to be maximized with respect to the y_i, is

$$L = U\left(\sum_i^n r_i y_i, \sum_i^n \sum_j^n \sigma_{ij} y_i y_j\right) - \lambda\left(\sum_i^n y_i - N\right), \qquad (5)$$

where λ is a Lagrangean multiplier. The first order conditions necessary for a maximum are

$$U_E r_i + U_V 2\sum_j^n \sigma_{ij} y_j - \lambda = 0\,, \qquad i = 1, 2, \ldots, n \qquad (6)$$

$$-\sum_i^n y_i = -N\,. \qquad (7)$$

Define $U_i \equiv U_E r_i + U_V 2\sum_j^n \sigma_{ij} y_j$, and the corresponding second order derivatives as U_{ij}. The second order conditions for a maximum are ensured if the matrix

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[2] The y_i are sometimes interpreted as ratios to net worth so that $\sum_i y_i = 1$, e.g. Burgess (1971).

$$H = \begin{bmatrix} U_{11} & U_{12} & \cdots & U_{1n} & -1 \\ U_{21} & U_{22} & \cdots & U_{2n} & -1 \\ \vdots & \vdots & & \vdots & \vdots \\ U_{n1} & U_{n2} & \cdots & U_{nn} & -1 \\ -1 & -1 & \cdots & -1 & 0 \end{bmatrix}$$

is negative definite.

Without further knowledge of the form of the utility function it is not possible to derive explicit expressions for the demand functions implied by these conditions. However, certain properties of these functions can be derived using the results from some comparative statics exercises. Before doing this, it is worth considering the economic interpretation of the first order conditions.

At the optimum, U_i equals U_k for all i and k. The investor holds such quantities of securities as to equalize the marginal utility of each. The utility function has the property that its units of measurement can be chosen arbitrarily, so that $U_E/2$ may be set at unity in the region of the maximum. This transformation allows one to write

$$r_i + U_v \sum_j \sigma_{ij} y_j = r_k + U_v \sum_j \sigma_{kj} y_j$$

or
$$r_i - r_k = U_v \sum_j (\sigma_{kj} - \sigma_{ij})\, y_j.$$

Securities are selected so that the security-weighted sum of the differences in variances is made equal to the difference in expected return. If $\sigma_{ij} \geq \sigma_{kj}$ for all j, and all asset holdings non-negative, then, since U_v is negative r_i must exceed r_k. Clearly a premium in the expected return on an asset is required if it is to be held when it is unambiguously more risky than some other asset. The required premium is greater the greater is the degree of risk aversion (i.e. the greater is the absolute value of U_v).

The Lagrangean multiplier has its usual meaning as a shadow price. Consider what would be the effect of a small relaxation in the constraint.

$$\frac{\partial U}{\partial N} = \sum_j^n U_j \frac{\partial y_j}{\partial N}$$

and $$\sum_{j}^{n} \frac{\partial y_j}{\partial y} = 1.$$

$$\therefore \quad \sum_{j}^{n} \lambda \frac{\partial y_j}{\partial N} = \lambda,$$

$$\therefore \quad \frac{\partial U}{\partial N} - \lambda = \sum_{j}^{n} (U_j - \lambda) \frac{\partial y_j}{\partial N}$$

but under the first order conditions, $U_j = \lambda$ for all j,

$$\therefore \quad \frac{\partial U}{\partial N} = \lambda .$$

The multiplier is therefore the marginal utility of net worth. An intermediary will expand to the point where the (common) marginal utility of the component assets is equal to the marginal utility to be derived from a further expansion of the whole portfolio. I.e. λ is the price the intermediary would be prepared to pay, at the margin, to obtain risk free deposits for investment into the risky assets which it holds.

Therefore, even when an intermediary is left to determine its own size, regardless of legal constraints imposed by reserve requirements, owned capital requirements and so on, there is a natural economic limit imposed by its own attitudes to risk and return on its scale of operation.[3] It is usually asserted in empirical monetary economics that the volume of deposits in an intermediary is determined by the preferences of the public -- that the size of its portfolio is, from the point of view of the institution, determined exogenously. But it seems clear that financial intermediaries do, to a significant extent, manage the liability side of their balance sheets as well as the asset side. The present study does not treat the size of the balance sheet as a matter of assumption but leaves it as a variable to be determined by the analysis: surely a more fruitful approach.[4]

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3 This observation is not new -- cf. Tobin (1967) -- but its implications have been largely ignored by financial model builders, as Kareken (1967) has shown.

4 These opinions, I would argue, are supported by the empirical results which are presented in later chapters of this thesis.

3. Comparative statics of the general system

A Change in Net Worth

The expression for $\partial y_i/\partial N$ is sought. In so doing the derivative $\partial y_i/\partial p_j$ is also found, given by

$$\frac{\partial y_i}{\partial p_j} = \frac{\partial y_i}{\partial N}\frac{\partial N}{\partial p_j} = \frac{\partial y_i}{\partial N}\frac{\partial y_j}{\partial p_j} = \frac{\partial y_i}{\partial N}\frac{\partial y_j}{\partial p_j} .$$

To find the desired derivative take the total differential of equations (6) and (7), holding all parameters except N constant:

$$\sum_j^n U_{ij}\frac{\partial y_j}{\partial N} - \frac{\partial\lambda}{\partial N} = 0 , \qquad i = 1,2, \ldots , n \qquad (8)$$

$$-\sum_j^n \frac{\partial y_j}{\partial N} = -1. \qquad (9)$$

Defining det $H \equiv D$ and the (i,j)th cofactor of D by D_{ij}, and using Cramer's rule:

$$\frac{\partial y_i}{\partial N} = \frac{-D_{n+1,\ i}}{D} .$$

Asset i can be "normal" or "inferior" in the sense that an increase in N may cause an increase or a decrease in the holding of i. However, normal assets must predominate since an increase in wealth must be completely allocated into one of its components: summing the effect of the increase in wealth over all assets.

$$\sum_{i}^{n} \frac{\partial y_i}{\partial N} = - \sum_{i}^{n} \frac{D_{n+1,\ i}}{D} = 1,$$

since $\sum_{i}^{n} - D_{n+1,\ i}$ is simply the expansion of D along its (n+1)th row.

These derivations apply whether an increase in wealth is brought about by an increase in the number of securities initially held by an investor or by an appreciation of the values of these securities. The results can also be applied to the case of a financial intermediary, where the rate of interest paid on its deposits is known with certainty (the decision period being sufficiently short that uncertainty attends only lending) and where the levels of these deposits are determined exogenously by the asset allocations of the public: N then represents total deposit liabilities plus net worth.

An Increase in Expected Return

Again totally differentiating (6) and (7), but this time varying only r_k:

$$\sum_{j}^{n} U_{ij} \frac{\partial y_j}{\partial r_k} - \frac{\partial \lambda}{\partial r_k} = \frac{-\partial U_i}{\partial r_k}, \qquad i = 1, 2, \ldots, n \tag{10}$$

$$-\sum_{j=1}^{n} \frac{\partial y_j}{\partial r_k} = 0 . \tag{11}$$

Solving these equations for $\partial y_m/\partial r_k$,

$$\frac{\partial y_m}{\partial r_k} = -\sum_{i}^{n} \frac{\partial U_i}{\partial r_k} \frac{D_{im}}{D} . \tag{12}$$

The expression can be negative or positive corresponding to (gross) substitute or complement relationships between the assets m and k. A useful restriction on the demand functions is implicit in (11) which asserts that the sum of all the responses to a change in r_k must be zero.

It is worth examining the expression for $\partial y_m/\partial r_k$ in some detail. Before doing so the following definitions are made

$$S_{ik} = -D_{ik}/D \ ,$$

$$U^r_{ik} = \frac{\partial U_E}{\partial r_k} r_i + \frac{\partial U_V}{\partial r_k} 2\sum_j \sigma_{ij} y_j + U_V 2\sum_j \sigma_{ij} \frac{\partial y_j}{\partial r_k} \ ,$$

$$K_{km} = \sum_i S_{im} U^r_{ik} \ .$$

Then one may write (12) as

$$\frac{\partial y_m}{\partial r_k} = \sum_{i}^{n} S_{im} U^r_{ik} - U_E \frac{D_{km}}{D} \ , \tag{13}$$

or as

$$\frac{\partial y_m}{\partial r_k} = K_{km} + U_E S_{km} \ . \tag{14}$$

The term S_{km} is a "net" substitution term, clearly bearing an affinity to the substitution effect of the Slutsky equation of consumer theory. K_{km} is a weighted sum of such terms with the U^r_{ik} as weights. A sufficient condition for the term K_{km} to be zero is that U^r_{ik} be constant across i. The result for $\partial y_m/\partial r_m$ is

$$\frac{\partial y_m}{\partial r_m} = K_{mm} + U_E S_{mm} . \tag{15}$$

Now, K_{mm} can take either sign but S_{mm} is positive from the second order conditions for a maximum. The net substitution effect for an increase in the expected yield on an asset is to increase demand for that asset. It is <u>sufficient</u> for the gross own substitution effect to be positive if <u>either</u> (i) K_{mm} is positive, <u>or</u> (ii) U^r_{ik} is constant across i. Otherwise it is still possible that the gross effect is positive as long as K_{mm} is small.[5]

The sum of all net substitution effects is

$$\sum_i^n S_{im} = 0,$$

which, since $S_{mm} > 0$, implies that

$$\sum_{i \neq m}^n S_{im} < 0,$$

so that substitution dominates complementarity in net inter-asset relationships.

The effect on the holding of the mth asset of an equal increase in all expected returns is given by

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[5] Cf. Hicks (1939) p. 35 in the context of consumer theory.

$$\sum_{k}^{n} \frac{\partial y_m}{\partial r_k} = -\sum_{k}^{n}\sum_{i}^{n} \frac{\partial u_i}{\partial r_k} \frac{D_{im}}{D} = \sum_{i}^{n} S_{im} \sum_{k}^{n} \frac{\partial u_i}{\partial r_k} , \qquad (16)$$

which is a weighted sum of the substitution terms, and not necessarily equal to zero. An equal increase in all expected rates of return will in general cause a reallocation of the portfolio. These allocations are not determined by interest rate spreads only. The commonly adopted procedure of using interest rate differentials between different securities as arguments in their demand functions, rather than the levels of these rates, implies that the model builder has adopted assumptions stronger than those of the general EV model. (Some special EV models do require only the differentials by making $\sum_{k}^{n} (\partial U_i/\partial r_k)$ constant across i so that $\sum_{k}^{n} (\partial y_m/\partial r_k)$ is zero.)

The Variance-Covariance Matrix

Tobin (1963) has conjectured that assets whose returns are highly correlated will be substitutes, whilst those whose returns are negatively correlated will tend to be complements. Interpreting these concepts in a net sense the conjecture can be shown correct. The expression for $\partial S_{km}/\partial \sigma_{km}$ is required.

The inverse of the bordered Hessian matrix H may be written as

$$H^{-1} = \begin{bmatrix} -S_{ij} & v \\ v' & 0 \end{bmatrix} \tag{17}$$

where the S_{ij} $(i, j = 1, 2, \ldots, n)$ are the substitution terms defined above and v is an n-element vector. Using the chain rule for differentiation, and recalling that the elements of the matrix of second-order derivatives H are denoted U_{ij}, the required expression is

$$-\frac{\partial S_{km}}{\partial \sigma_{km}} = -\sum_{i}^{n} \sum_{j}^{n} \frac{\partial S_{km}}{\partial U_{ij}} \frac{\partial U_{ij}}{\partial \sigma_{km}} . \tag{18}$$

Using the "no U-turn" rule[6] for the derivative of an inverse of a matrix this becomes

$$-\frac{\partial S_{km}}{\partial \sigma_{km}} = \sum_{i} \sum_{j} S_{ki} S_{jm} \frac{\partial U_{ij}}{\partial \sigma_{km}} \tag{19}$$

Ignoring derivatives of third order and above, $(\partial U_{km}/\partial \sigma_{km}) = (\partial U_{mk}/\partial \sigma_{km}) = U_v$, and all other $\partial U_{ij}/\partial \sigma_{km} = 0$. Thus

$$\frac{\partial S_{km}}{\partial \sigma_{km}} = (S_{kk} S_{mm} + S_{km}^2) U_v, \tag{20}$$

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[6] This rule was so named by Theil (1971), p. 33.

which is negative. As the covariance between two assets increases the cross substitution terms diminish - the assets become "less complementary".

4. The general EV model: conclusions

A number of empirically useful restrictions on security demand functions are implicit in the theory of portfolio selection but the definite predictions which can be derived are distressingly few. This, after all, should not be surprising. Neo-classical consumer theory, to which portfolio theory bears many formal analogies, is ambivalent on many of the fundamental questions put to it. It does not, for example, prove that demand curves slope downwards -- the simple law of demand is in the end an empirical judgement. So in the present context one might expect a positive response in the demand for a security if the expected yield on that security increases. The notion can be believed but it cannot be proved.

The ambiguities manifest the fact that if the assumptions are weak so too will be the implications. Definiteness in implications can be purchased at the cost of reducing the generality of the assumptions. By doing just this -- by assuming, for example, that consumer preference functions are separable -- Frisch (1959), Barten (1964) and others have been able to specify consumer demand functions with interesting properties for empirical purposes. The same sort of thing occurs with security demand functions when the EV assumptions are specialized.

Chapter III

Two Specializations of the EV Hypothesis

1. The models

Two specializations of the EV hypothesis have attracted sufficient attention in the literature[1] that they merit explicit consideration in the context of the results for the general model which have been derived in Chapter II. These two leading cases involve respectively: (a) a quadratic von Neumann-Morgenstern utility of income function; (b) a utility function of constant absolute risk aversion (CARA) with normally distributed returns.

(a) The quadratic utility function may be written

$$W = R - \frac{\beta}{2} R^2, \tag{1}$$

where R is the return on the portfolio, a stochastic variable with mean E and variance V, and β is a positive parameter measuring risk aversion. (Note a restriction to $R < 1/\beta$.) Letting ε represent the expectation operator,

$$\begin{aligned} \varepsilon(W) \equiv U &= \varepsilon(R) - \frac{\beta}{2} \varepsilon(R^2) \\ &= E - \frac{\beta}{2} (V + E^2). \end{aligned} \tag{2}$$

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[1] E.g. Quadratic utility functions have been invoked by Tobin (1958), Royama and Hamada (1967), and Borch (1968). Freund (1954), Parkin (1970) and Farrar (1962) have used assumptions equivalent to the alternative specialization.

Thus we have the mean-variance objective function

$$U(E,V) = \sum_i^n r_i y_i - \frac{\beta}{2} (\sum_i^n \sum_j^n \sigma_{ij} y_i y_j + \sum_i^n \sum_j^n r_i r_j y_i y_j)$$

$$U(E,V) = \sum_i^n r_i y_i - \frac{\beta}{2} \sum_i^n \sum_j^n (\sigma_{ij} + r_i r_j) y_i y_j. \qquad (3)$$

(b) The CARA utility function is written

$$W = 1 - e^{-\beta R} . \qquad (4)$$

Using the assumption of normality for R, one obtains

$$\varepsilon(W) = \int_{-\infty}^{\infty} (1 - e^{-\beta R}) (2\pi V)^{-\frac{1}{2}} \exp - \frac{1}{2} \frac{(R-E)^2}{V} dR \qquad (5)$$

Following Freund (1956), it can be shown that maximizing (5) is equivalent to maximizing

$$U(E,V) = E - \frac{\beta}{2} V \qquad (6)'$$

which is

$$U = \sum_i^n r_i y_i - \frac{\beta}{2} \sum_i^n \sum_j^n \sigma_{ij} y_i y_j . \qquad (7)$$

Thus both sets of assumptions reduce to simple EV models. Since returns in the real world cannot be normally distributed (due to bankruptcy laws etc.) the justification for (7) is weak. Farrar has attempted to extend the appeal of the objective function by deriving it as a quadratic Taylor series approximation to a concave utility function. The approximation is taken around expected returns: this unfortunately is meaningless in a comparative statics context when expected return itself changes. It must then be replaced by a quadratic approximation around an arbitrary level of returns which yields (2) -- i.e. it becomes equivalent to a quadratic utility function.

2. The properties of the demand functions for these special cases

(a) The quadratic

Consideration need be given here only to those features which do not hold generally. The implications of the quadratic utility function have been thoroughly explored by Royama and Hamada -- it turns out that this utility function does not get one much further than one can go with the completely general EV function. It is therefore unnecessary to repeat the derivations of Chapter II for this special case.

(b) The CARA

The implications of the CARA utility function with normality of returns are very interesting. Maximizing (7) with respect to the wealth constraint requires

$$r_i - \beta \sum_j^n \sigma_{ij} y_j - \lambda = 0, \qquad i = 1, 2, \ldots, n \tag{8}$$

$$- \sum_j^n y_j \qquad = -N. \tag{9}$$

This system of equations can be written in matrix form

$$Hy = x, \tag{10}$$

where

$$H = \begin{bmatrix} -\beta\sigma_{11} & -\beta\sigma_{12} & \cdots & -\beta\sigma_{1n} & -1 \\ -\beta\sigma_{21} & -\beta\sigma_{22} & \cdots & -\beta\sigma_{2n} & -1 \\ \vdots & \vdots & & \vdots & \vdots \\ -\beta\sigma_{n1} & -\beta\sigma_{n2} & \cdots & -\beta\sigma_{nn} & -1 \\ -1 & -1 & \cdots & -1 & 0 \end{bmatrix}$$

$$y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \\ \lambda \end{bmatrix} \quad \text{and } x = \begin{bmatrix} -r_1 \\ -r_2 \\ \vdots \\ -r_n \\ -N \end{bmatrix}.$$

H is negative definite to provide sufficient conditions for a maximum. The interesting features of this model are that:

1. The demand functions for securities are linear in expected returns and wealth, since (8) and (9) can be solved explicitly for the y_i in the form

$$y = H^{-1}x,$$

or, less cryptically,

$$y_i = -\sum_j^n \frac{D_{ji}}{D} r_j - \frac{D_{n+1,\, i}}{D} , \qquad i = 1, 2, \dots , n. \tag{11}$$

2. There is no distinction between the gross and net substitution effect for a change in expected returns since

$$\frac{\partial y_i}{\partial r_k} = \frac{-D_{ki}}{D} = S_{ki} . \tag{12}$$

(In the previous notation, $U^r_{ik} = 0$ for all i). An immediate corollary is that the "own rate" responses $\partial y_k / \partial r_k$ are unambiguously positive.

3. Since H is symmetric, $\partial y_i/\partial r_k = \partial y_k/\partial r_i$. This symmetry implication can be tested statistically -- and if the theory is good then the implication can be used to effect a substantial increase in the efficiency of the structural estimates.

4. Using the property of symmetry, since $\sum_i^n S_{ij} = 0$, so $\sum_j^n S_{ij} = 0$. I.e. an equal rise in all expected rates of return causes

no change in the composition of the portfolio: differentials must change if changes in expected rates of return are to cause portfolio reallocations.

3. Indifference curves for the leading EV specializations

The purpose of this section is to extend and clarify the idea of indifference curves in asset space for the EV model, using the theory of conic sections.[2] The argument is based on the two leading specializations of the EV hypothesis but the method is readily applicable to other EV formulations. An indifference map in asset space is an explicit depiction of the choice variables themselves and is therefore distinct from the more familiar curves drawn in EV space, although both sets of curves are loci of constant expected utility.[3]

Indifference curves for assets in asset space differ in crucial aspects from conventional indifference curves in that they embody, as parameters, market data as well as the preferences of the investors. They depend on the means, and on the variances, of the

---

[2] Bierwag and Grove (1966) have derived indifference curves for assets diagrammatically. The present treatment is considerably more explicit and more easily generalized.

[3] These curves are used extensively in portfolio theory. See e.g. Tobin (1958).

returns from the various assets. A change in the return expected from an asset changes the entire map. Geometrical comparisons between different points in asset space, to determine which is preferred, can be made using the indifference curves only when expected returns on all assets are held constant.

Where U(E,V) is held to a arbitrarily determined (but admissible) level, $\overline{U}$, the expression (3) for a quadratic utility function becomes

$$-\sum_{i}^{n} r_i y_i + \frac{\beta}{2} \sum_{i}^{n} \sum_{j}^{n} (\sigma_{ij} + r_i r_j)\, y_i y_j + \overline{U} = 0, \tag{13}$$

while expression (7) for a CARA function becomes

$$-\sum_{i}^{n} r_i y_i + \frac{\beta}{2} \sum_{i}^{n} \sum_{j}^{n} \sigma_{ij} y_i y_j + \overline{U} = 0 . \tag{14}$$

(13) and (17) are the desired loci of constant expected utility. These equations are hyperellipsoids, as will presently be made clear for the two dimensional case.

Some Theory of the Conic Section: A Digression[4]

The general equation for a conic is

$$ay_1^2 + 2hy_1y_2 + by_2^2 + 2gy_1 + 2fy_2 + \overline{U} = 0, \qquad (15)$$

where a, h, b, g, f and U are given. Let us return to equations (13) and (14) for n = 2 where neither asset is riskless. The following table shows the relationship of the general equation (15) with (13) and (14):

(15) General Equation	(13) Quadratic	(14) CARA
a	$\beta\,(\sigma_{11} + r_1^2)\,/2$	$\beta\sigma_{11}/2$
h	$\beta\,(\sigma_{12} + r_1r_2)/2$	$\beta\sigma_{12}/2$
b	$\beta\,(\sigma_{22} + r_2^2)\,/2$	$\beta\sigma_{22}/2$
$2g$	$-r_1$	$-r_1$
$2f$	$-r_2$	$-r_2$
$\overline{U}$	$\overline{U}$	$\overline{U}$

---

[4] A thorough treatment of this topic is to be found in the treatise by Smith (1882). I prove a number of propositions which are used in later sections. For those without interest in the proofs these propositions are summarized at the beginning of the next section.

The nature of the indifference curves is easily established once the term containing y_1y_2 is removed by the appropriate axial rotation. I need to rotate the axes through an angle θ such that the new coefficient of y_1y_2 is zero. Substituting for y_1 and y_2 respectively

$$(y_1 \cos\theta - y_2 \sin\theta)$$

and

$$(y_1 \sin\theta + y_2 \cos\theta)$$

(15) becomes

$$\begin{aligned} &a(y_1 \cos\theta - y_2 \sin\theta)^2 + b(y_1 \sin\theta + y_2 \cos\theta)^2 \\ &+ 2h\,(y_1 \cos\theta - y_2 \sin\theta)\,(y_1 \sin\theta + y_2 \cos\theta) \\ &+ 2g\,(y_1 \cos\theta - y_2 \sin\theta) + 2f\,(y_1 \sin\theta + y_2 \cos\theta) \\ &+ \bar{U} = 0. \end{aligned} \qquad (16)$$

The coefficient of y_1y_2 in (16) is

$$2(b-a) \sin\theta \cos\theta + 2h(\cos^2\theta - \sin^2\theta)$$

which is zero when $\tan 2\theta = 2h/(a-b)$. Thus the necessary angle of rotation is

$$\theta = 1/2 \tan^{-1} [2h/(a-b)]. \qquad (17)$$

This rotation allows (15) to be written as

$$Ay_1^2 + By_2^2 + 2Gy_1 + 2Fy_2 + \bar{U} = 0. \qquad (18)$$

By expanding (16) the coefficients A and B are found to be

$$\left.\begin{aligned} A &= a\cos^2\theta + 2\ h \cos\theta \sin\theta + h \sin^2\theta \\ B &= a \sin^2\theta - 2\ h \cos\theta \sin\theta + b \cos^2\theta \end{aligned}\right\} \qquad (19)$$

Now some facts about a, b and h can be used. These coefficients are essentially determined by the σ_{ij} and $r_i r_j$ in (13), and by the σ_{ij} alone in (14). Recalling that the variance-covariance matrix (σ_{ij}) is positive definite, observe that the matrix

$$\begin{bmatrix} a & h \\ h & b \end{bmatrix}$$

is positive definite also. Use of this observation in (19) establishes that both A and B are strictly positive.

Completing the squares on (18) gives

$$A(y_1 + G/A)^2 + B(y_2 + F/B)^2 = G^2/A + F^2/B - \overline{U} \equiv K$$

or, taking a new origin at the point (-G/A, - F/B),

$$Ay_1^2 + By_2^2 = K.$$

If K is not zero

$$\frac{y_1^2}{K/A} + \frac{y_2^2}{K/B} = 1. \qquad (20)$$

K/A and K/B take the same sign as K. If K is positive then (20) is

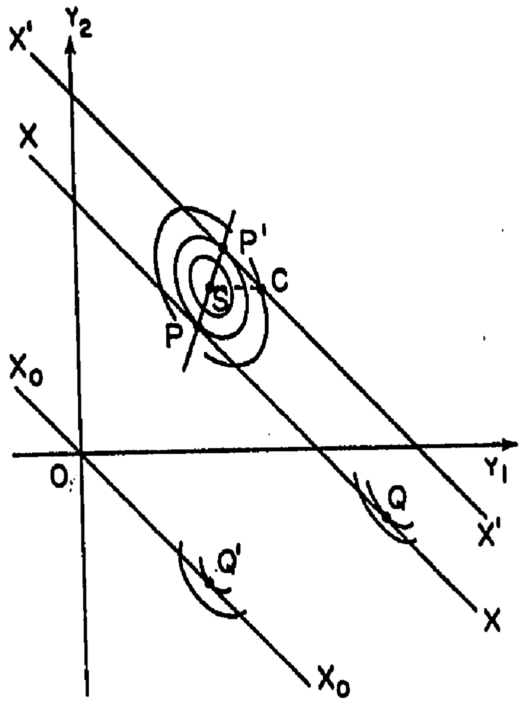

FIG. 1

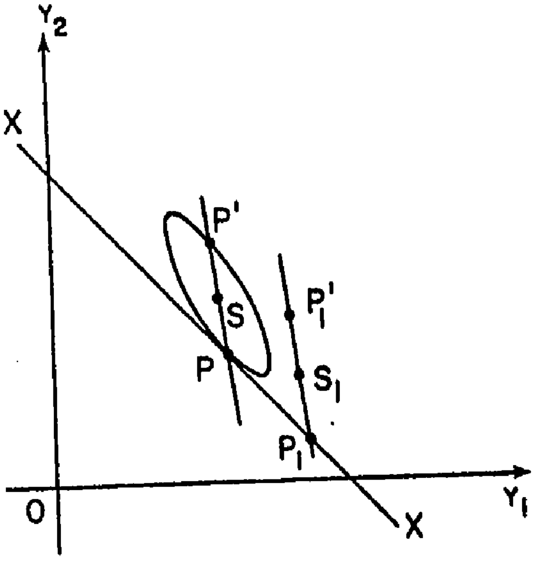

FIG. 2

an ellipse. If K is negative then (20)is an imaginary ellipse with no economic meaning. Thus I am only concerned with $\bar{U} \leq U^*$, where $U^* = G^2/A + F^2/B$ is an upper bound to expected utility. And the contour for $\bar{U} < U^*$ in (y_1, y_2) coordinate space is an ellipse.[5]

How does the contour change when $\bar{U}$ changes? In the first place the centre point of the ellipse does not move since this is at

$$(y_1^*, y_2^*) = \left\{ \frac{h\ f - bg}{ab - h^2}, \frac{gh - af}{ab - h^2} \right\} \qquad (21)$$

and does not depend on $\bar{U}$ -- the indifference ellipses are therefore concentric. Neither does the change in $\bar{U}$ affect the angle of the principal axes of the ellipse to the y_1 and y_2 coordinates, since θ does not depend on $\bar{U}$. Neither does the "shape" of the ellipse change since the ratio of the semi-axes is $\xi = B/A$ and B and A do not depend on $\bar{U}$. (The closer ξ is to unity the more distended is the ellipse. At $\xi = 1$ it is a circle.)

The lengths of both semi-axes do change. There are given by $\sqrt{\frac{U^* - \bar{U}}{A}}$ and $\sqrt{\frac{U^* - \bar{U}}{B}}$. These axes shorten towards zero as $\bar{U} \to U^*$, i.e. the degenerate ellipse for $\bar{U} = U^*$ is just the centre point of the system (y_1^*, y_2^*).

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[5] These ellipses are not to be confused with the isovariance ellipses drawn by Markowitz (1952) in the (x_1, x_2) space.

The Indifference Map

I have proved that an investor whose behaviour is determined by a quadratic utility function, or by a CARA utility function when R is normal, possesses indifference curves in asset space which are elliptical.

These ellipses are concentric and converge on their centre point as the level of expected utility is raised. Given the returns and the variance-covariance matrix, this centre point is the unconstrained maximum position for the investor - the point of asset satiation. Moreover, as expected utility is increased the concentric ellipses shrink without changing their eccentricity or their slope in (y_1, y_2) co-ordinate space. Such an indifference map is depicted in Figure 1 concentrated around S.

The point S is too prosaic to be called a _bliss_ point. It will always be achieved if the investor holds a riskless asset (as is shown below). In any case the investor can always be made better off by increasing the mean returns or by reducing the variances. Asset satiation arises because assets are tainted with risk. Roughly speaking, as asset levels are increased _ceteris paribus_, EV objective functions eventually turn downwards because total variance is _quadratic_ in the asset levels, the mean return but _linear_.

The Maximum Position

The investor's equilibrium position is given by the condition that the objective function be maximized subject to the wealth constraint.

$$y_1 + y_2 = N. \tag{22}$$

The constraint is represented in Figure 1 by the line XX, whose slope is minus one. As in consumer theory the optimal point is the point of tangency between XX and an indifference curve. The point P in the diagram has as ordinates the optimal bundle of y_1 and Y_2.

The investor's objective is to get as close as he can to the point of unconstrained maximum S. Thus, if the level of wealth were X'X' rather than XX, the equilibrium point is P' rather than P. It is rather interesting that the second order conditions for a maximum are satisfied at points like P': one is accustomed to convexity of indifference curves in the region of the optimum point.

It is also interesting that the equilibrium does not have to lie in the positive quadrant of the graph. An equilibrium point at Q (Figure 1) is permissible given the right family of ellipses. No non-negativity assumptions have been made: at Q the investor sells Y_2 short to finance a large holding of y_i.

The case of a financial intermediary involves just such a solution. For a pure intermediary net wealth is zero so that the constraint line goes through the origin (X_oX_o). If y_2 represents its deposit liabilities and y_1 represents its holdings of primary securities, then the equilibrium point Q' indicates the optimal size and asset holding of an intermediary.

Comparative Statics

(i) A Change in Wealth Whatever the tastes of the investor and whatever the returns on the assets, the tangency condition implies that the slope of the indifference curves at the maximum point is minus unity. The geommetrical derivation of an efficiency locus showing efficient combinations of y_1 and y_2 for any level of wealth is therefore extraordinarily simple. Pick out all of the points at which a member of the indifference ellipse family has the unit negative slope. This locus is linear.[6] The straight line PSP' in Figure 1 exemplifies an efficiency path: in Figure 1 an increase in wealth causes the investor to increase the holdings of both risky assets. Both assets, in other words, are "normal".

Although normal assets must predominate, inferiority is possible. The efficiency path of Figure 2, PSP', demonstrates the case in which y_1 is inferior. The slope of this efficiency path is less than -1.

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[6] A proof is left to the Appendix of this Chapter.

(ii) A Change in Expected Return For the quadratic utility function no strong implications can be derived.[7] Consider an increase in the expected return on the first asset ((r_1)): in general the eccentricity of the ellipses and the centre point are both changed. The net result is that y_1 can go up or go down ($(y_2$ has to move in the opposite direction to maintain the balance sheet). Figure 3 illustrates the ambiguity. The "usual" case is where the centre moves from S to, say, S' as a result of the increase in y_1, and the equilibrium moves from P to P' so that there

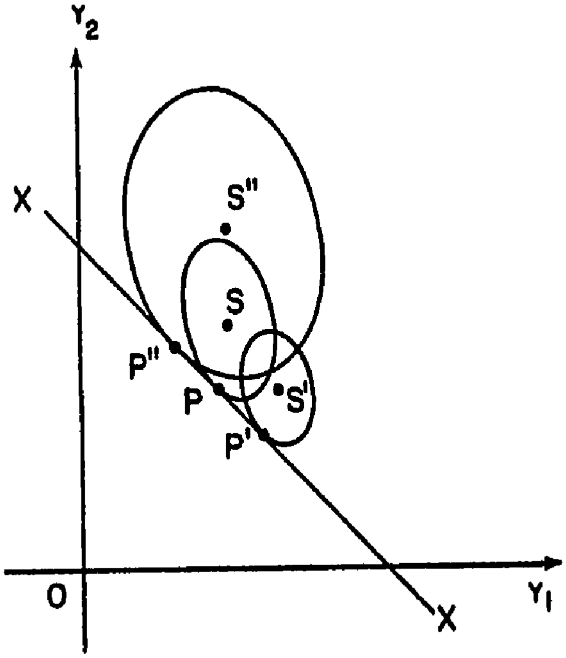

FIG. 3

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[7] Recall the results in Chapter II.

is a substitution of y_1 for y_2 in the portfolio. But one cannot rule out the movement from P to P" which involves a reduction in the level of the asset whose expected return has risen.

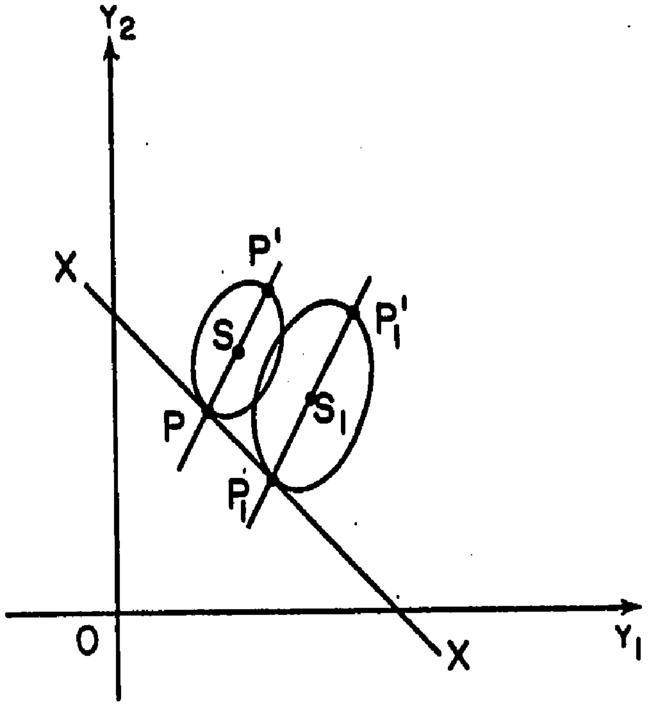

FIG. 4

The CARA case is more definite in its implications. The table on page** 33 shows that a change in r_1 does not change a, h or b; i.e. neither the angle of the ellipses to the y_1 axes nor their eccentricity is changed. The only change is that the family of ellipses is concentrated around a different point. Since the slope of the major axes is not changed, the new efficiency path is parallel with the original efficiency path. It can be shown that the new efficiency path lies entirely to the east of the old --whether y_1 is normal or inferior.[8] The normal case is in Figure 4 and the inferiority case in Figure 2. PP' is the initial efficiency path, cutting XX at P, the initial equilibrium. After the change in r_1 the new efficiency locus is P_1P_1' with equilibrium at P_1.

A Riskless Asset

The generalization to a third asset is straightforward when the third asset is riskless. This allows us to treat, with the diagrams, the reserve asset behaviour of an intermediary. The most illuminating way to pursue this generalization is to designate the third asset as cash with a zero yield: this gives us sharp results which have to be modified in details for the more general case ($r_3 > 0$).

Let λ be a Lagrangean multiplier for the wealth constraint,

$$y_1 + y_2 + y_3 = N. \tag{23}$$

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[8] A proof is left to the Appendix.

Under this constraint the first order maximum conditions are given by:

(a)
$$r_1 - \beta\ [(\sigma_{11} + r_1^2)\ y_1 + (\sigma_{12} - r_1 r_2)\ y_2] - \lambda = 0$$
$$r_2 - \beta\ [(\sigma_{21} + r_1 r_2)\ y_1 + (\sigma_{22} + r_2^2)\ y_2] - \lambda = 0$$
$$- \lambda = 0$$
$$-y_1 - y_2 - y_3 = -N$$

for the quadratic investor; or by

(b)
$$r_1 - \beta\ [\sigma_{11}\ y_1 + \sigma_{12}\ y_2] - \lambda = 0$$
$$r_2 - \beta\ [\sigma_{21}\ y_1 + \sigma_{22}\ y_2] - \lambda = 0$$
$$- \lambda = 0$$
$$-y_1 - y_2 - y_3 = -N$$

for the CARA investor. These conditions imply, respectively,

$$\left.\begin{aligned} r_1 - \beta[(\sigma_{11} + r_1^2)y_1 + (\sigma_{12} + r_1 r_2)y_2] = 0 \\ r_2 - \beta[(\sigma_{21} + r_1 r_2)y_1 + (\sigma_{22} + r_2^2)y_2] = 0 \end{aligned}\right\} \qquad (24)$$

and

$$\left.\begin{aligned} r_1 - \beta\ [\sigma_{11}y_1 + \sigma_{12}y_2] = 0 \\ r_2 - \beta\ [\sigma_{21}y_1 + \sigma_{22}y_2] = 0. \end{aligned}\right\} \qquad (24')$$

In neither case does the amount of wealth affect the optimal levels of the risky assets. Either pair of equations ((24)

or (24')) solves uniquely for y_1 and y_2. Only the holding of cash remains to be determined by the wealth constraint. It is significant that (24) and (24') are precisely the conditions which yield an unconstrained maximum if there is no riskless asset. The implication is that the investor goes to his risky asset satiation point (y_1^*, y_2^*) and holds any residual wealth (debt) in the form of cash reserves. The case to pursue further here is the case where satiation dictates a positive reserve.[9] This is illustrated in Figure 1 when X'X' is the constraint and S is the satiation point: the investor's holdings of y_1 and y_2 are given by the co-ordinates of S, and his holding of cash by SC.

This analysis gives a novel interpretation to Tobin's separation theorem.[10] The ratio of y_2 to y_1 at the satiation point is, from (21), $y_2^*/y_1^* = (gh-af)/ (hf-bg)$. This ratio is independent

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[9] Even though our treatment does not require it, it seems sensible to impose non-negativity on the reserve holding, i.e. to confine attention to satiation points below the constraint line. If satiation were to involve a negative reserve, then the intermediary holds no reserve and is in the two asset situation analyzed above.

[10] The theorem asserts that a mean-variance investor will hold risky assets in a constant ratio, regardless of his degree of risk aversion, if there is a riskless asset in his portfolio.

of β. The locus of satiation points in asset space obtained by varying β is therefore linear and through the origin. If some cash is held initially, then the efficient response to an increase in β is to move into cash, reducing y_1 and y_2 proportionately.

Comparative statics exercises are now very simple: one just has to keep track of the satiation point. Consider an increase in r_1. The satiation point shifts. For the quadratic the shift could be in any direction, depending on the parameter values. Consequently it is not possible to predict definite changes in the composition of the portfolio.

For the constantly absolute risk averse investor the satiation point shifts eastwards when r_1 increases. There is necessarily an increase in y_1 and a decrease in the sum $y_2 + y_3$. (Figures 2 and 4 illustrate this, if one imagines the constraint to go through P' rather than P: S shifts to S_1.) Moreover, if both r_1 and r_2 increase proportionately, the satiation point shifts towards the northeast, nearer the constraint line. A reduction in "demand for money" occurs when the expected returns on all alternative assets go up proportionately, but the EV model leaves open, as an empirical matter, whether money is a substitute for any particular risky asset.

Conclusions

The leading EV specifications have equations of the second degree in the asset levels. This implies that their indifference curves in asset space are conics. I have analyzed two cases -- the quadratic utility function, and the case of normally distributed returns with constant absolute risk aversion. The indifference curves turn out to be concentric ellipses. The regularity of these conics makes the notion of the indifference curve in asset space a useful heuristic device for the two risky asset case: I have demonstrated this with some comparative statics exercises.

Appendix to Chapter III

The efficiency path in asset space

a) The basic equation is

$$ay_1^2 + by_2^2 + 2hy_1y_2 + 2gy_1 + 2fy_2 + \bar{U} = 0 \qquad (15)$$

The efficiency path is defined by

$$\frac{dy_2}{dy_1} = -\frac{ay_1 + hy_2 + g}{by_2 + hy_1 + f} = -1,$$

so that the equation of the efficiency path is the linear function,

$$y_2 = \frac{h - a}{h - b} y_1 + \frac{f - g}{h - b} \qquad (25)$$

b) Now I deal with an increase in r_1 for specification (14). By referring to the table of page 24 it can be seen that

$$a, b \geq 0, \qquad (26)$$

$$ab - h^2 \geq 0 \text{ (implying } a \geq h \text{ and/or } b \geq h), \qquad (27)$$

and that

$$a + b \pm 2h \geq 0. \qquad (28)$$

It can also be seen that only g, of the parameters in (25), changes when r_1 changes, with

$$\frac{dg}{dr_1} = \frac{-1}{2} \qquad (29)$$

I want to prove that an increase in r_1 causes an eastward parallel shift in the efficiency locus. The slope of the efficiency locus is (h-a)/(h-b) which is invariant with respect to a change in r_1: the new efficiency locus must be parallel with the old.

Without loss of generality one may take y_2 as normal and consider the cases of y_1 normal and y_1 inferior. In the normal case the slope of the efficiency locus is positive (cf. Fig. 1). But if $(h-a)/(h-b) > 0$ then (26) implies a, $b > h$. Now consider $h > b$. From (28),

$$a - h > h - b$$

so that $(h-a)/(h-b) < -1$, which is the case of inferiority (cf. Fig. 2). I have shown that y_1 is normal or inferior as $h < b$ or $h > b$.

But the intercept on the y_2 axis decreases or increases as $h < b$ or $h > b$. I.e. if the slope of the efficiency locus is positive, the intercept shifts down; but if the slope of the locus is less than -1 the intercept shifts up. This proves that in both cases the efficiency locus shifts entirely to the east when r_1 increases.

Chapter IV

Inflexibilities in Portfolio Management

The argument of the preceding sections has been predicated on the assumption not only that the investor has complete latitude to determine the components of his balance sheet but also that the necessary adjustments take place without delay. To apply the model to the operations of a financial institution requires that these assumptions be relaxed to allow for unforeseen movements of deposits caused by the behaviour of depositors, for exogenous changes impinging on the asset side of the balance sheet, for costs of adjustment in responding to changes in data and for delayed responses.

Taking account of these inflexibilities in the management of the portfolio is the object of this chapter. In the first section of the chapter the implications for balance sheet behaviour of exogenously determined asset components are analyzed. In the next section the basic model is modified to incorporate brokerage costs -- the resulting demand functions incorporate lagged stocks of the securities as explanatory variables. In the third section the restrictions on the dynamic properties of a disequilibrium portfolio model posed by the wealth constraint -- a topic whose importance was first stressed by Brainard and Tobin (1968) and on which Ladenson (1971) has since elaborated -- are stated in a general form which includes these previous contributions as special cases. Last, sufficient conditions for maintenance of the balance sheet identity are given for models in which lagged effects are specified in terms of lagged values for the explanatory variables in the system.

1. Exogenous components of the balance sheet

The theory has been developed on the assumption that all items in the balance sheet, excepting net worth, are choice variables whose levels are determined by the investor. In order to cope with institutional realities it is necessary to introduce items with an uncertain yield whose levels are either determined exogenously (i.e. by the behaviour of the general public of by the legislative authorities) or are predetermined by past decisions of the investor (e.g. the commitment to make a mortgage loan is made some time before the asset appears on the lender's balance sheet). Since the behaviour of these items is not determined by the investor, their levels as well as their yields may be validly represented as stochastic variables.

The conventional treatment of financial intermediaries takes the composition of the asset side of the balance sheet as being determined by the intermediary itself, whilst the level of deposits is determined exogenously.[1] I have already argued that this does not seem to be the most fruitful approach to intermediary behaviour. Some asset items will not be, whilst some liability items will be, in the intermediary's choice set. For example, non-market clearing interest rates may cause lending to customers to be, at least in part, determined by the supply of debt from customers rather than demand for this asset by an intermediary -- this certainly happens when there are large unused overdraft facilities or credit line arrangements at the discretion of the customers. On the other side of the balance sheet financial institutions can take an active role in liability management, bidding for funds by offering debt instruments at attractive interest rates. They do not appear simply to accept whatever deposits happen their way. It is worth emphasizing that the present treatment considers the size of the intermediary (in terms of total liabilities) to be an endogenous variable -- determined, at least in some degree, by the optimizing behaviour of the institutions themselves.

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1 My comments apply here to theoretical models such as Porter's (1967) and also to empirical financial sector models such as De Leeuw's (1965) and Silber's (1970). The latter might treat intermediary deposits as endogenous to the whole model, but they are exogenous in so far as the intermediary is concerned.

The preceding argument suggests that the model be modified in the following respects:[2] partition the securities into a set whose levels are determined by the intermediary ($i = 1, 2, \ldots, d$) and a complement set ($i = d + 1, d + 2, \ldots, n$) whose levels are not known with certainty for the coming period, being determined by forces beyond its control. However the institution will have in mind some expectations about these levels.

Let the actual volume (unknown to the intermediary at the time at which it makes its decisions) of asset i in the coming period be $y_i + e_i$ for $d < i \leq n$, with expected value y_i. The stochastic terms e_i (whose mean is by definition zero) have a covariance matrix δ_{ij}, and are assumed to be distributed independently of the errors of forecast for expected returns. The EV investor caricatured by these assumptions will attempt to maximize

$$U(E,V) = U\left(\sum_i^n r_i y_i,\ \sum_i^n \sum_j^n \sigma_{ij} y_i y_j + \sum_{i,j=d+1}^{n} \delta_{ij} r_i r_j + \xi\right) - \lambda\left(\sum_i^n y_i - N + \sum_{i=d+1}^{n} e_i\right) \qquad (1)$$

where ξ is a constant, a complex of covariances. Necessary conditions for a maximum are given by

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[2] These modifications are an adaptation of a specification used by Parkin, Gray and Barrett (1970).

$$U_E r_i + U_V 2 \sum_{j=1}^{n} \sigma_{ij}\, y_j - \lambda = 0\,, \qquad i = 1, 2, \ldots, d \tag{2}$$

$$- \sum_{j=1}^{n} y_j = -N + \sum_{j=d+1}^{n} e_j. \tag{3}$$

This system can be rewritten

$$U_V 2 \sum_{j=1}^{d} \sigma_{ij} y_j - \lambda = -U_E\, r_i - U_V 2 \sum_{j=d+1}^{n} \sigma_{ij} y_j, \quad i=1,2,\ldots d \tag{4}$$

$$- \sum_{j=1}^{d} y_j = -N^* + \sum_{j=d+1}^{n} y_j. \tag{5}$$

It is not possible to solve this system of d+1 equations explicitly for d demand functions and for λ unless there is a specific form to the utility function. However it is clear that the demand functions will include, as independent variables, the expected levels of the exogenous assets. Observe that equation (3) contains the sum of the errors of prediction of the asset levels; i.e. that the random movements in these levels necessarily cause changes in the size of the balance sheet. This sum enters symmetrically with net wealth: these are therefore incorporated into the same variable, N^*, in equation (5) by a change of notation.

When the subject of the exercise is a financial intermediary with $N = 0$, N^* reduces to stochastic terms only.[3] Assume that of all the institution's balance sheet items only its deposits are prone to unavoidable and unforeseen fluctuations. Then N^* represents a swing in this item: if there is an unexpected influx of these liabilities (N^* is positive) there must be a corresponding increase in the assets of the institution over and above the allocations which the institution had expected to make. (In this sort of situation it is likely that some asset -- say day loans -- will act as a buffer stock. If so the dth choice asset becomes a stochastic variable and its level is determined by the intermediary only up to the <u>expectation</u> y_d'. Similarly, when an intermediary makes a decision on the quantity of liabilities which it will attract, it cannot usually determine a precise dollar sum but rather an expected level, net of random influences. No conceptual difficulties arise from treating an item of choice as the mean of a probability distribution rather than a certain quantity. There will, of course, be stochastic departures from the expectation as e_d varies to absorb the unforeseen movements in the exogenous items.)

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[3] This treatment of changes in deposit levels is one theoretical justification for the use of the term ΔN in equation (22) of this chapter.

The new question of interest which arises with this model is -- what are the effects of a change in the expected value of the level of the kth asset, $d < k < n$? To find the effects, take the total differential of equations (4) and (5):

$$\sum_{j}^{d} U_{ij} \frac{\partial y_j}{\partial y_k} - \frac{\partial \lambda}{\partial y_k} = - \frac{\partial U_i}{\partial y_k} \tag{6}$$

$$- \sum_{j}^{d} \frac{\partial y_j}{\partial y_k} = 1 . \tag{7}$$

Where D is the determinant of the Hessian matrix of the system, the solution for $\partial y_i / \partial y_k$ is seen to be

$$\frac{\partial y_i}{\partial y_k} = \sum_{j}^{d} \frac{\partial U_j}{\partial y_k} S_{ji} - \frac{\partial y_i}{\partial N} \tag{8}$$

If $\partial U_j / \partial y_k$ is constant for $j = 1, 2, \ldots , d$ then

$$\frac{\partial y_i}{\partial y_k} = - \frac{\partial y_i}{\partial N} , \tag{9}$$

and the effect, say, of an anticipated inflow of savings deposits (which amounts to an increase in the scale of the intermediary) is exactly the same as the effect of an increase in scale due to an increase in net wealth (the sign difference arising from my convention of measuring liabilities such as savings deposits in the negative direction).

When the proviso of the previous paragraph does not hold the weighted sum of the substitution terms $\sum_{j}^{d} (\partial U_j/\partial y_k)\, S_{ji}$ makes a contribution. This sum could be positive, in which case it is possible for $\partial y_i/\partial y_k$ to be positive even though i be normal. That is, continuing with the illustration, an anticipated inflow of savings deposits may cause the intermediary to reduce its holdings of bonds, even though bonds may be "normal" in the sense of Chapter II. The pattern of covariances may be such that a strong substitution relationship exists between savings deposits and bonds, so that when the former increases the latter is reduced, notwithstanding the normal scale effect.

However, because $\sum_{j}^{d} \partial y_j/\partial y_k = -1$, $\partial y_i/\partial y_k$ will be predominantly negative. The effects of the change will otherwise not add up properly across the portfolio. In the illustration, the extra funds must be alloted somewhere even if they do not go into bonds.

2. Brokerage costs in the EV model

The implications of transactions costs in asset management can be comprehensively handled without the additional complication of exogenously determined balance sheet items. Therefore, a modification of the pristine model of Chapter II is sought which will bring costs of investment into the optimizing framework. "Brokerage costs" are to be interpreted in a broad sense to include information costs, inconvenience and so on, as well as any explicit brokerage fee. A simple representation for investment costs which has been found useful in inventory control analysis[4] is a quadratic function. Applying this specification to the present theory, brokerage costs are given by

$$\frac{1}{2}\sum_{i}^{n} c_i \, (y_i - y_i^o)^2 \tag{10}$$

where, as before, y_i^o is the initial holding of the ith asset, and c_i is an appropriate cost parameter. The problem now is to maximize

$$L = U(\sum_i r_i y_i - \frac{1}{2}\sum_i c_i (y_i - y_i^o)^2, \sum_i \sum_j \sigma_{ij} y_i y_j) - \lambda(\sum_i y_i - N). \tag{11}$$

The first order conditions are

$$U_E[r_i - c_i \, (y_i - y_i^o)] + U_V 2\sum_j \sigma_{ij} y_j - \lambda = 0, \; i = 1, 2, \ldots, n \tag{12}$$

$$-\sum_i y_i = -N. \tag{13}$$

The previously established results with respect to wealth and interest rate effects carry over in all germane respects, but now some additional questions can be put to the analysis, concerning the effects of <u>ceteris paribus</u> changes in the initial levels of the component assets, the y_i^o. From the viewpoint of the current

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[4] Mills' book (1962) provides an interesting example.

decision period, these stocks are given.

Differentiating the first order conditions with respect to y_k^o, where $\partial U_i / \partial y_k^o = U_{ik}^o$,

$$-U_E\, c_i \frac{\partial y_i}{\partial y_k^o} + U_v\, 2 \sum_j \sigma_{ij} \frac{\partial y_j}{\partial y_k^o} = -U_{ik}^o,\ i \neq k \tag{14}$$

$$-U_E\, c_k \frac{\partial y_k}{\partial y_k^o} + U_v\, 2 \sum_j \sigma_{kj} \frac{\partial y_j}{\partial y_k^o} = -U_{kk}^o + U_E,\ i = k \tag{15}$$

$$-\sum_j \frac{\partial y_j}{\partial y_k^o} = -\frac{\partial N}{\partial y_k^o}\,. \tag{16}$$

There are two distinct ways to view $\partial N/\partial y_k^o$ in (16), corresponding to distinct <u>ceteris paribus</u> experiments:

1. If net wealth is held constant through an equal reduction in other endowments, $\partial N/\partial y_k^o = 0$. Then,

$$\left\{\frac{\partial y_i}{\partial y_k^o}\right\}_{N\ \mathrm{const}} = -\sum_j U_{jk}^o \frac{D_{ji}}{D} + U_E \frac{D_{ki}}{D} \tag{17}$$

$$= \sum_j U_{jk}^o S_{ji} + U_E S_{ki}. \tag{18}$$

If assets i and k are substitutes $(\partial y_i/\partial y_k)_{N\ \mathrm{const}}$ will be negative. If assets i and k are complements, the presumption is that the derivative is positive. Setting i = k suggests, since S_{kk} is positive, that the greater is the endowment of an asset, the greater will be the optimal holding of that asset for the current period, total wealth remaining unchanged. These conjectures are transformed into certainties if U^o_{jk} is constant over j -- a condition in fact satisfied for the CARA case analyzed in Chapter III.

There is an empirically useful restriction on the security demand functions from (7) which is that, under present assumptions, $\sum_j (\partial y_j/\partial y^o_k)_{N\ \text{const}} = 0$. This enforces the balance sheet constraint. Also, of course, $\sum_j \partial y_j/\partial N = 1$.

2. If all other assets (rather than net wealth) are held constant, $\partial N/\partial y^o_k = 1$. Then

$$\left\{\frac{\partial y_i}{\partial y^o_k}\right\}_{y^o\text{const}} = \sum_j U^o_{jk}S_{ji} + U_E S_{ki} + \frac{\partial y_i}{\partial N}\,. \tag{19}$$

There is now a scale effect appended to the preceding result, so equation (9) is an expression similar to that for an "uncompensated" price change in consumer theory:

$$\left\{\frac{\partial y_i}{\partial y^o_k}\right\}_{y^o\text{const}} = \left\{\frac{\partial y_i}{\partial y^o_k}\right\}_{N\text{ const.}} + \frac{\partial y_i}{\partial N}\,. \tag{20}$$

The scale effect requires modification of the conjectures made above to the extent that, if y_i is normal, the presumption that an increase in y^o_k will cause a decrease in y_i when i and k are substitutes no longer exists. A net substitution relationship is consistent with a gross complementary relationship. Mutatis mutandis this modification applies to the other conjectures too.

The adding up requirement now emerges as $\sum_j \partial y_j/\partial y_k^o = 1$. Since any increase in N is attributed to the component y_i^o, a wealth effect per se has no independent existence in the present context. Arguments 1 and 2 imply that either the sum of derivatives with respect to y_k^o $(k = 1, 2, \ldots, n)$ be zero and the sum of the derivatives with respect to wealth be unity, or the sum of derivatives with respect to y_k^o be unity and wealth not appear independently as an explanatory variable. These sums, of course, are to be taken over the security demand equations. The alternative sets of restrictions are both sufficient for exact exhaustion of the balance sheet.

In concluding this section, it should be noted that in financial markets certain marginal transactions costs decline at the margin (especially brokers' fees); and this fact renders the assumption of quadratic transactions costs arbitrary, even perhaps unrealistic. However, presumably implicit costs of changing what one is currently doing do increase more rapidly as the size of the intended change increases. It is difficult to test the assumption. Security demand equations of the form implied by the existence of quadratic transactions costs (i.e. equations with lagged values of the securities as arguments) are also implied by quite different theories, of which the adaptive expectations hypothesis is the best known. Empirical support for the present hypothesis might instead be adduced as support for an expectations hypothesis.

3. Lagged dependent variables

The model just outlined yields security demand functions which have as arguments initial stocks of the relevant securities. It has been shown that there are restrictions imposed by the balance sheet on the nature of the adjustment responses. These restrictions from an optimizing model are comparable with the restrictions for the deterministic approach used by Brainard and Tobin (1968). They emphasize the necessity of specifying asset demand functions consistent with the wealth constraint both in and out of equilibrium. However their work -- even when supplemented by the recent contribution of Ladenson (1971) -- is incomplete in certain respects and misleading in others. The purpose of the rest of this chapter is to spell out, as fully as possible, the implications of the adding up requirements for disequilibrium asset demand functions.

Taking the customary linear approximation to the model derived in the previous section of this chapter, one may write the set of demand functions as

$$y_i = \sum_j \alpha_{ij} r_j + \beta_i N + \sum_j \gamma_{ij} y_j^0. \qquad i = 1, 2, \ldots , n \qquad (21)$$

The α_{ij} are interest rate coefficients, the β_i is a wealth coefficient, and the γ_{ij} are the coefficients of adjustment on the lagged asset stocks. The balance sheet constraint requires $\sum_i \alpha_{ij} = 0$ for all j. Now suppose that I modify this system slightly to allow for an asset disbursement effect; i.e. to allow <u>changes</u> in the level of wealth an independent effect on asset allocations due to, say, differences between costs of investment for new wealth and for existing wealth. (E.g. a financial intermediary might be deterred by transactions costs from switching out of one asset into another but be quite willing to put an inflow of new deposits directly into the alternative asset.) This asset disbursement effect gives rise to equations such as

$$y_i = \sum_j \alpha_{ij} r_j + \beta_i N + \sum_j \gamma_{ij} y_j^0 + \gamma_i \Delta N, \qquad 1 = 1, 2, \ldots n \qquad (22)$$

where $\Delta N = N - N_{-1}$ (the subscript -1 denoting a one period time lag). System (22) is precisely the system discussed in the writings of Brainard and Tobin and Ladenson.

Now sum the set of equations to obtain

$$\sum_i y_i = \sum_i \beta_i N + \sum_i \sum_j \gamma_{ij} y_j^o + \sum_i \gamma_i \Delta N. \tag{23}$$

Noting that $\sum_i y_i = N$, the sum is put into a form in which portfolio balance considerations are conveniently handled:

$$N = \sum_i \beta_i N + \sum_i \sum_j \gamma_{ij} y_j^o + \sum_i \gamma_i \Delta N . \tag{24}$$

There are various sets of conditions in the form of linear dependencies between the coefficients which suffice for the maintenance of the adding up constraint. The existing literature unnecessarily complicates this topic since each article contributes just one sufficient condition and since taken together these articles do not exhaust the possibilities. A general statement is possible which covers the contributions of Brainard and Tobin and Ladenson as special cases (due allowance being made for my specification differing in certain respects from theirs).

Replacing y_i^o with $y_{i,-1}$, (24) becomes

$$N = \sum_i \beta_i N + \sum_i \sum_j \gamma_{ij} y_{j,-1} + \sum_i \gamma_i \left(N - \sum_j y_{j,-1}\right). \tag{25}$$

Assuming that $\sum_i \gamma_{ij}$ is constant across j,

$$N = \sum_i (\beta_i + \gamma_i) N + \sum_i \left(\sum_j {}_{ij} - \gamma_i\right) N_{-1}. \tag{26}$$

In a steady state, with $N = N_{-1}$, (26) is written

$$N = \sum_i (\beta_i + \gamma_i + \gamma_{ij} - \gamma_i) N$$

$$= \sum_i (\beta_i + \gamma_{ij}) N$$

so that $\sum_i \beta_i = 1 - \sum_i \gamma_{ij}$. Substituting this restriction on the model into (26), the equation becomes

$$N = \sum_i (1 - \gamma_{ij} + \gamma_i) N + \sum_i (\gamma_{ij} - \gamma_i) N_{-1} \tag{27}$$

It is therefore sufficient [5] for balance that

$$\sum_i \gamma_i = \sum_i \gamma_{ij}, \text{ all } j. \tag{28}$$

The restriction asserted by Brainard and Tobin amounts to setting $\sum_i \gamma_i = 1$. The supplementary restriction derived by Ladenson sets $\sum_i \gamma_i = 0$. But clearly we are not confined to these suggestions. One rather neat way to impose (27) is to impose, for all j, the following equalities:

$$\text{i.e.} \quad \gamma_{ij} \begin{cases} = \gamma^* = \sum_i \gamma_i & \text{for } i = j \\ = 0 & \text{for } i \neq j \end{cases}$$

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[5] Brainard and Tobin refer to their conditions as being necessary (1968 p. 106). This is incorrect of course. As Ladenson points out, only sufficiency is at stake.

Then each demand equation has only the lagged value of the security to which it pertains on the right hand side. The single adjustment coefficient is common to each equation, and by (28) the asset disbursement effects sum to this common value, viz. $\sum_i \gamma_i = \gamma^*$.

In view of this last condition it is interesting to find that Brainard and Tobin write that "If no cross effects are allowed in the explicit equations of adjustment ... , then the counterparts of all the own adjustments would be loaded into the implicit adjustment equation for [the security ommitted using Walras' Law]".[6] In other words, if a lagged dependent variable appears in one equation then it should also appear in at least one other equation, to provide for exact exhaustion of the balance sheet. However I have just provided a counterexample to this assertion. Cross effects have all been set at zero ($\alpha_{ij} = 0$ for $i \neq j$) and there is no omitted equation.

4. Lagged independent variables

It is customary to view autoregressive models of the type of equation (22) as having both impact and long run solutions. The restrictions which I state ensure that the balance sheet is maintained in balance on immediate impact of an exogenous shock, in the final equilibrium and throughout the transition process.

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[6] P. 106. The bracketed expression is mine.

If the lag structure is specified in terms of independent variables, foregoing the lagged dependent variables, one may write

$$y_i = \sum_{t=0}^{c} \sum_{j=1}^{n} \alpha(t)_{ij} r_{j,-t} + \sum_{t=0}^{c'} \beta(t)_i N_{-t} \tag{29}$$

where $\alpha(t)_{ij}$ is the coefficient of r_j lagged t periods in the ith security equation. The requirements for balance are now

$$\sum_{i}^{n} \alpha(t)_{ij} = 0 \qquad \begin{array}{l} t = 0,1,2, \ldots ,c \\ j = 1,2, \ldots ,n \end{array} \tag{30}$$

$$\text{and} \quad \sum_{i}^{n} \beta(t)_i \begin{cases} = 1 & t = 0 \\ = 0 & t = 1,2, \ldots ,c' \end{cases} \tag{31}$$

These restrictions would be appropriate when lagged structures are estimated with only lagged independent variables on the right hand side of the demand equation -- e.g. as in the case when the Almon (1965) method for estimating distributed lags is used.[7]

5. Inflexibilities: conclusions

The types of frictions and limitations on choice which I have considered in this chapter have been varied and the results defy quick summary. If certain asset levels are determined exo-

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[7] The Almon method and other methods relevant to this section are explained in Johnston (1972), pp. 292 - 300.

genously, then these levels become explanatory variables in the security demand functions. If there are costs in asset transactions, then the lagged levels of the dependent variables become explanatory variables in the functions. The restrictions imposed by the balance sheet identity on the relevant coefficients are in both cases interesting.

The list of theoretical restrictions which has been compiled is now quite long and should prove its worth in terms of econometric efficiency in the empirical study which follows.

CHAPTER V

Institutional Description of the Trust and Mortgage Loan Companies

1. Introduction

There has not been an analytical study of trust and mortgage loan (TML) company portfolio behaviour since the University of Western Ontario monograph of 1962. It is pertinent therefore to review some institutional developments as background for the econometric analysis of the data. What follows is in no sense an updating of the 1962 monograph, rather it is my perception of the way the TMLs have worked in the 1960s. The differences in view between this thesis and the University of Western Ontario study can be largely resolved by taking account of the changed circumstances of the TML companies in the 1960s, particularly during the period after 1967 with which this study is principally concerned.

The intermediary business of the TML companies is by virtue of size an important part of the Canadian financial system. Their major assets totalled almost $11 billion in early 1972, compared to a chartered bank total of $35 billion, and their share of all institutional mortgage lending approached forty per cent.[1] Trust

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[1] Table V1 gives a percentage breakdown of the industry balance sheet as of end 1971 which will give the reader some feel for the structure of the industry.

Table V1

BALANCE SHEET AT END 1971

Percentage distribution

Major Assets		Major Liabilities	
Mortgages	70	Over 1 yr. term deposits	67
Cash and liquid assets	12	Under 1 yr. term deposits	11
Gov't. of Canada bonds	6	Savings deposits	22
Corporates	6		
Municipals & provincials	6		
Total	100		100

and mortgage loan companies are interesting for another reason. Compared with other financial institutions in North America, particularly institutions in the mortgage business, the TML companies are relatively unencumbered with regulations or the putative requirements of social objectives. Subject to the legal requirements described below, the TML companies have been free to arrange their portfolios much as they have wanted them.

The TML sector's size and its relative freedom in portfolio choice makes it an interesting application for the theory developed in chapters I through IV. Also, one is well supplied with recent data. The Dominion Bureau of Statistics (Statistics Canada) and the Bank of Canada have compiled monthly time series of the major balance sheet items since September 1967 for the mortgage loan companies and since January 1963 for the trust companies (these data have been published only quarterly). By interpolating the quarterly figures for the mortgage loan companies it is possible to construct a series for the complete TML sector from 1963 onwards.[2] For a number of reasons I confine my econometric attention to the post 1967 period. First, the reliance on interpolated data is reduced to eight months in 1967.[3] Second, some of the series used

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[2] The trust companies account for about seventy per cent of the total industry assets so that interpolation is required only for the minor part of the industry.

[3] A programme of quadratic interpolations was used. The programme was written by G.R. Sparks and M.A. Walker.

are not available in earlier years -- one of these series gives a breakdown between less-than-one-year and over-one-year term liabilities that is essential for econometric model building of the TML sector.[4] Third, the rapid evolution of the financial system in the 1960s requires either that econometric investigation be restricted to a fairly short time in which the industry's experience is relatively settled or that in the investigation account be somehow taken of structural changes. It seems to me -- and I argue the point at greater length later in this chapter -- that the environment in which the TML companies live underwent a significant change around 1966 and that the Bank Act revisions of 1967 mark a good starting point for a new era. Where I have felt that I could go back further for data without losing too much precision I have done so.[5]

2. The Company and guaranteed accounts

The "company and guaranteed accounts" is the name given to

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[4] The data available on this breakdown are in any case very rough and ready. Separate over and under one year figures are available only quarterly so I have interpolated between quarters, splicing the interpolated data with the actual data on total term deposits to ensure consistency.

[5] The data are discussed in all their tedious detail in the appendix to this chapter.

the intermediary business of the trust companies. The major liabilities are savings deposits (chequable and non-chequable) and fixed-term deposits (there is also some equity financing -- the "own funds" of the companies). The trust companies' term deposits with more-than-one-year original term to maturity[6] are known as "guaranteed investment certificates" (GICs) but there are a variety of names differing from company to company for the less-than-one-year deposits which I call "guaranteed investment certificates of less-than-one-year maturity". Mortgage loan companies issue debentures rather than fixed-term deposits but I do not think that there is a secondary market for them. They are equivalent to the over-one-year GICs and I draw no distinction between the two from this point onwards. Likewise I use the terms GIC and term deposit interchangeably. The money brought in through savings and term deposits is invested in a variety of assets: mortgages, personal loans, corporate bonds and equities, government bonds (federal, provincial and municipal), chartered bank deposits, commercial paper, finance company paper and so on.

Chart VI shows the movements of the percentage distribution of the major balance sheet components from 1967 to 1972 (the sum of

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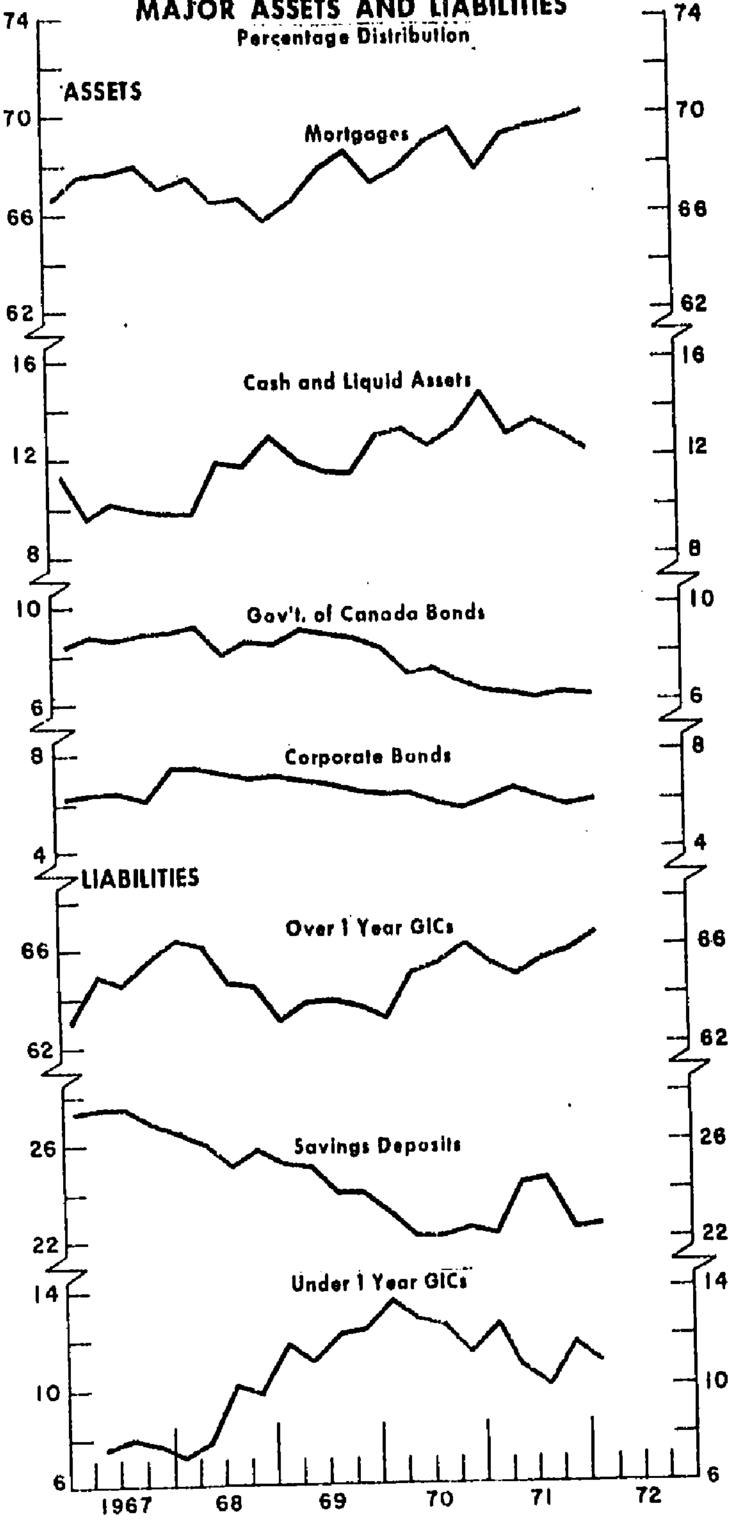
[6] In all cases I refer to original term to maturity rather than existing term to maturity because the data are collected this way. Also the data are collected using historical cost to value bonds rather than market price.

CHART V 1

TRUST AND MORTGAGE LOAN COS. MAJOR ASSETS AND LIABILITIES

Percentage Distribution



municipal and provincial bonds is omitted from the chart but can be determined residually). I have consolidated a number of things. The category cash and liquid assets is the sum of miscellaneous foreign currency assets, Canadian dollar deposits in the chartered banks, finance company and commercial paper, collateral loans and a small quantity of intrasectoral deposits (there is an element of double counting in the industry data). In the item corporates I combine bond and equity holdings.

The chart gives useful preliminary insight into the magnitudes involved and the extent to which portfolio reallocations have taken place. The ordering of these balance sheet classifications by size has not changed since 1967. Far and away the biggest asset is mortgages. Bonds are divided roughly equally between corporate, federal and (combined) municipal and provincial issues. (Federal government bond holdings are concentrated in less-than-five-year-maturities.) The chart shows that the total of over-one-year term deposits and GICs is almost as great as outstanding mortgages. There is a similar correspondence between less-than-one-year items. The matching by term to maturity of assets and liabilities is not fortuitous.

A few trust companies (e.g. Guaranty, Montreal and Royal) have a continuously active money market operation intermediating between less-than-one year GICs and commercial and finance company paper. Other companies use the less-than-one-year GIC as an adjustment item.[7] The lenders are usually large corporate or institutional investors. There is a profit to be made because the holders of short term GICs are deterred from direct investment in the paper market. They are deterred because (a) there is a large minimum transaction size in the paper market; (b) investment requires special skills; (c) the quality street paper is sometimes dubious with concomitantly high risks; and (d) certain investors are forbidden by law to purchase commercial and finance company notes (charitable institutions, municipalities, Crown corporations, etc.). Money market operations are a small part of the overall TML scene, and a minor part of the business even of the participating firms.

Chart V2 puts the TML sector into an economy-wide perspective. The percentage of new mortgage loans approved by the sector since 1967 has fluctuated around fifty per cent of all institutional mortgage approvals. Because the TML share of outstanding mortgages has been less than fifty per cent, its share has risen. TML

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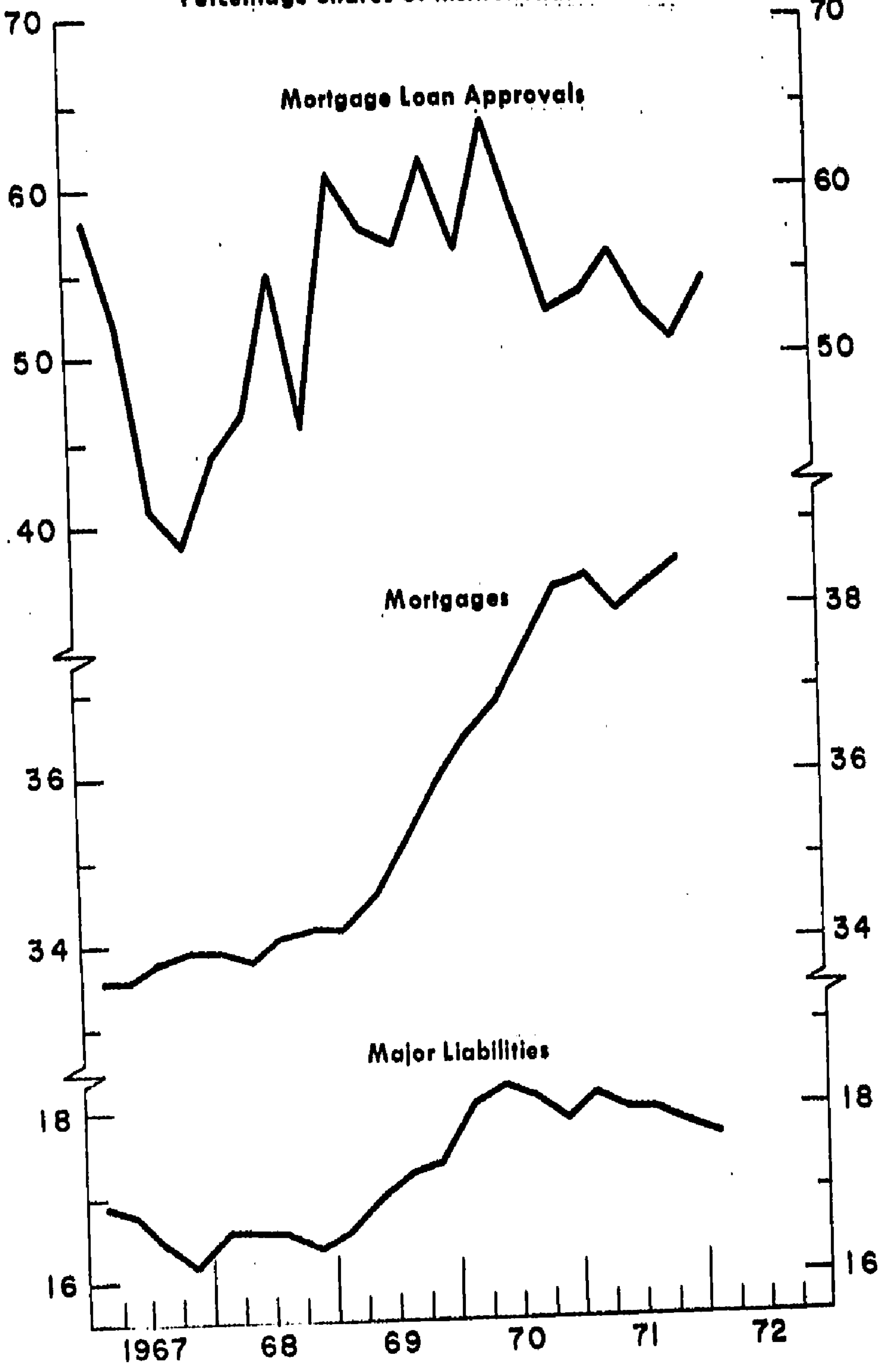
[7] A noticeable "mirror-image" pattern between savings deposits and less-than-one-year GICs (Chart V1) manifests the use of short-term deposits as an adjustment item. Savings deposits are determined largely by exogenous factors and are relatively interest-inelastic (cf. Chapter VI), so e.g. the TMLs respond to a loss of savings deposits by attracting more short term deposits.

CHART V 2

TRUST AND MORTGAGE LOANS COS.

Percentage Shares of Institutional Markets



major liabilities are shown as a proportion of private sector deposit wealth[8] (a more instructive comparison perhaps is with chartered bank liabilities alone as in Section 6). The TML share of total deposit wealth rose from 1967 to 1970 since when it has been static.

As is generally the case with aggregate figures for an industry, the picture presented by summing over all firms conceals significant heterogeneity. Some trust companies (e.g. Montreal Trust and Royal Trust) came into the personal intermediary business on a large scale only within the past twenty years and this is reflected by a relatively low proportion of mortgages in their portfolios. At the other end are the chartered bank affiliated mortgage loan companies (e.g. RoyMor, TorDom, Kinross and Central Covenants) which have over ninety per cent of their assets in mortgages. These companies would not be included as part of the TML sector if the criteria for data collection were economic rather than legal; they would then be counted as part of the banking system. The TML statistics also include companies such as Investors Syndicate that offer contractual savings schemes resembling life insurance. The presence of the bank affiliates and of the invest-

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[8] This variable, which is later given the mnemonic DEPOSITS, is the sum of privately held money supply, chartered bank CDs, swapped deposits, major liabilities of the TMLs and outstanding Canada Savings Bonds.

ment-contract firms in the data brings in unnecessary noise and it is to be hoped that in the near future these will be excluded.

Given the data problems attending the mortgage loan part of the industry it might seem better to deal with the trust companies separately. The drawback then is that a division between trust companies and mortgage loan companies does not make economic sense. Such a division would reflect legal and accounting artifacts having nothing to do with the functions of the companies as financial intermediaries. The larger firms have both "trust" and "mortgage loan" companies under their auspices but where the one ends and the other begins is a matter only of accounting convenience; it is less misleading to consider the two as a composite industry than to try to keep them apart. The industry so defined has firms with the common characteristic that their activities are dominated by mortgages and by the financing of mortgages, and the recognition of this is the key to the way the industry works.

3. The legal framework

Many thousands of words could be used in elucidating the law as it pertains to the TML companies. Moreover they can be registered either provincially or federally so that there are provincial regulations (those of Ontario and Quebec are the important ones) as well as federal. And the laws do not always mean what they appear to mean. For instance, by law trust companies are forbidden to accept deposits and sell debentures when this is just what their business involves. The legal fiction is that they accept money on deposit in trust and do not borrow money on deposit.

Instead of sorting out the exquisite legal details I will simply outline the regulations that have put real constraints on TML portfolio behaviour in the recent past in the sense that they have prevented the TML companies from doing what they might otherwise have done. There seem to be three such constraints:

1. The TML companies are prohibited from borrowing more than twenty times their own funds, i.e. their capital must be at least five per cent of their intermediary business as measured by total deposits. Before 1970 the limit was fifteen times capital. The most rapidly growing companies have seen to it that regular infusions of new equity increase the borrowing ceiling. Other companies have been reluctant to go to the stock market regularly for a variety of reasons. The borrowing limit has not in the event been very restrictive for it has proved to be an elastic straightjacket: every so often the limit is raised and it seems that combined industry pressure to raise the limit when it looks as though it might bite is a more convenient way of overcoming the

ceiling than going to the stock market.[9] The companies which have been inhibited by the borrowing limits have typically reacted by reducing the least profitable of their activities.

2. Until 1970 TML companies felt unable to enter the unsecured personal loan field. There is no explicit provision for personal loans in the statutes setting out eligible assets for TML purchase but since 1960 the so-called "basket clause" has allowed a company to invest up to fifteen per cent of its own funds in ways not specifically mentioned in the statutes. The TML companies ignored the opportunity to get into personal loans probably because the sums allowed under the basket clause were not sufficient to make the venture profitable. In 1970 the basket clause was widened to fifteen per cent of own funds plus seven per cent of guaranteed funds (i.e., the maximum is now about one and one-half times own funds). The TML companies have not yet made a big plunge into personal loans.

3. Corporates are eligible assets only if a corporation has a five-year record of earnings at least equal to ten times its total debt interest. Recently formed corporations are therefore excluded and this regulation may have caused TML companies to

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[9] There is no good reason, in my opinion, for the existence of the limit in view of the supervisory powers of the Superintendent of Insurance and the Ontario Registrar, and the deposit insurance provided by the Canada Deposit Insurance Corporation since 1967.

miss out on assets they would have liked to acquire.

Probably other regulations have from time to time proved irksome to particular companies if not pervasively. For example there is a liquidity requirement that guaranteed funds payable within one hundred days be backed by a twenty per cent reserve. Eligible reserve assets are bank deposits and government (federal and provincial) debt. TML holdings of these assets have been substantially in excess of the minimum demanded by the law and the requirement has not constrained TML behaviour.

4. Mortgage lending

Institutional mortgage lenders in Canada can buy conventional mortgages or National Housing Act (NHA) mortgages. The latter are insured by the federal government and until 1969 were subject to federal interest rate controls. Conventional mortgage loans have had several advantages to the lender offsetting the NHA insurance facilities: (a) lower administrative costs; (b) the conventional interest rate has been renegotiable after five years whereas until 1969 the term of the mortgage agreement and the amortization period of NHA mortgages had to coincide; and (c) the NHA borrower is allowed certain early repayment privileges. In retrospect, NHA mortgages in the mid-1960s were actually more risky than conventional mortgages because the twenty year term for which they were typically negotiated locked in the lenders at interest rates below the rates TML companies later had to pay to fund the loans. That is, a holder of a twenty year mortgage originating in 1965 had by 1970 incurred a substantial capital loss. The risk of default (covered by NHA insurance) has been small in comparison with the risk attending interest rate increases. (Five year conventional mortgages could be hedged with five year term deposits and the interest risk thereby removed.) By 1969 the advantages of the five year mortgage were sufficiently well established that the NHA regulations were changed to allow it.

The conditions attaching to NHA and conventional mortgages are clearly such that the instruments are imperfect substitutes. Until 1969 virtually no loans against existing homes were provided via the NHA route by the TML sector, NHA lending being almost exclusively for new residential construction. Over the period 1967-71 the dollar-value ratio of conventional loan approvals to NHA was 5:2.

The history of the NHA interest rate is a good case study of the defects of interest rate ceilings, even adjustable interest rate ceilings. Until 1969 a maximum NHA rate was set. Before November 1966 this rate was changed infrequently but as interest rates rose in the mid-1960s the actual rate became the maximum rate and sources of NHA mortgage money tended to dry up. After November 1966 the maximum was varied quarterly according to the formula "One and one-half per cent plus the average yield on long-term bonds of Canada . . ." where ". . .'average yield on long-term bonds of Canada' means the simple arithmetic mean of the Wednesday closing mid-market yields. . .calculated for the four consecutive Wednesdays immediately preceding the first day of [the relevant quarter beginning January 1, April 1, July 1 or October 1]."[10] With the ceiling rate for each quarter determined by market rates prevailing at the end of the previous quarter, upward or downward movements in the ceiling could be forecast quite confidently a few weeks before a new determination was due. An incipient downward movement in the ceiling would cause a rush of NHA loan approvals by lenders in these weeks to be followed by a month of famine. An incipient upward movement in the ceiling would cause a dearth of approvals in the weeks before the change followed by a month of plenty. The major social benefits of this intervention in the capital markets were the stimulus to the arithmetic of NHA approved lenders who were encouraged to forecast next quarter's NHA ceiling and some strange gyrations in monthly NHA approvals. In 1969 it

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[10] Statement of the Minister of Labour, November 23, 1966. The spread of one and one-half was later raised to two and one-quarter per cent. The phrase in brackets is mine.

was decided to forego these benefits and the ceiling was removed.

Mortgage lending has certain unique features. The acquisition of mortgages is peculiar in that it requires a considerable expenditure of resources to acquire suitable new assets[11] and in that there is a considerable average time lag between the approval of a loan and the subsequent disbursement of money. The necessary overhead implies a large fixed cost and economies of scale. Whereas bonds can be bought and sold at a moment's notice the volume of outstanding mortgages on a TML company's balance sheet is not well controlled in the short run. It is physically possible for a trust company to refuse to make new mortgage loans in a particular period and so reduce its disbursements over the coming months. But it is more difficult to ensure that borrowers will seek loans at a time when the company wants more mortgages. Customers for mortgage loans will be especially difficult to find if the company has in the recent past refused to lend to otherwise eligible borrowers because of its portfolio requirements at that time. This fact, together with the large overhead costs, makes the TML companies reluctant to effect a sudden cut-back in mortgage

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[11] The secondary mortgage market in Canada is undeveloped. The TMLs have been net sellers in this market, some companies using their comparative advantage in origination to earn profit: they sell blocks of "seasoned" mortgages e.g. to American life insurance companies.

loan approvals.

The lag between approvals and disbursements is another complicating feature in controlling mortgage lending. For the TML sector it is some six months before fifty per cent of approvals are taken down,[12] the lag varying from company to company and from time to time. Companies lending mainly for existing buildings experience short lags -- a lot of these loans will be disbursed within a month of approval. New apartment projects, sub-divisions and other large developments require a commitment well in advance of the time the money is made available -- in some cases the lag may be as long as two years and it is often greater than one year.

The lag in getting borrowed money out into the hands of the borrower is a point of concern for the TML companies. In the past they have made commitments at a particular interest rate and in the interim have seen the market interest rate rise. They are exposed to a risk that is difficult to hedge. Complete hedging of the total commitment to mortgage lending (i.e. outstanding mortgages plus the backlog of as yet undisbursed mortgage approvals) requires an equal quantity of over-one-year term deposits. To

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[12] This is an average estimate for the industry 1967-72 based on the statistical analysis of Chapter VII. The provisions for taking down individual loans vary. E.g. on an existing house the loan would be drawn down in one gulp. On new buildings there are usually successive advances as the various construction stages are finished.

illustrate -- if a dollar mortgage is approved today to be disbursed in one year at a rate of interest fixed for five years then the investment is approximately hedged by issuing a one-dollar, six-year term deposit today and investing the dollar in a one-year bill. Matching one hundred per cent like this is expensive because it will normally cost more to get the six-year money than the one-year bill will earn so that there is a loss on the carry for a year. The ideal solution is to charge the borrower a nonreturnable fee equal to this loss (two per cent per annum on the balance of the commitment that has not been drawn down is the sort of figure involved) and some trust companies have recently adopted the practice. The fee is known as a stand-by fee. The fee has the advantage that it encourages the borrower to take down the loan rapidly.

The other source of concern about the approval-disbursement lag is that the borrower is not committed to the deal at the time the lender is committed. A potential borrower may cancel the agreement before the money is disbursed. Construction firms involved in large projects have often cancelled agreements, having found cheaper finance from other sources. This is particularly apt to happen when interest rates are falling -- when rates are rising the borrower is only too happy to keep the arrangement made some months before. To discourage cancellations a commitment fee is now charged by a number of companies. This fee is refundable when the commitment is drawn down and forfeited if the borrower cancels. Even with the loss of the fee some builders cancelled during the recent period of falling interest rates (1971).

Mortgage lending is a balance sheet item that is difficult to control at the margin. The unwillingness of most mortgage lenders to fiddle with the mortgage approvals process is ultimately a result of their feeling that the effect on their profits in the long run would be deleterious.[13] In addition, instructions on mortgage approvals policy have to be communicated over a branch system and it would be costly to try to monitor the process closely (most firms operate in the context of an annual plan for mortgage lending). And even when mortgage approvals policy is changed the delayed response of disbursements is still to be taken into account. Control of total mortgage lending in the short run is therefore indirect and not precise. Indeed it is usually easier to change other things to suit mortgage lending than to change mortgage lending to suit the rest of the balance sheet.

5. Savings deposits and term deposits

It is generally felt within the industry that savings deposits

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[13] A typical reaction from trust company executives was that in the event of, say, a shortfall in savings deposits below expected levels, mortgage approvals would be almost the last thing to feel the squeeze.

are unresponsive to interest rates, particularly chequable savings deposits. Thus it is that the interest rate on chequable deposits is low[14] and rarely changed. The non-chequable rate is changed more often than the chequable rate but even it is hardly a market clearing rate, the concept of equilibrium in the savings deposit market being a hazy one. The savings deposits rates are probably best regarded as administratively determined prices, and the quantity of savings deposits as being determined by the demand of the public given those rates. Demand is itself determined largely by factors exogenous to the present study such as branching, advertising and public confidence in the TML sector.

Chequable deposits are a fairly stable quantity and firmly enough held so that an inflow of chequables would in the long run be allocated largely to mortgages. The same reaction is likely to follow an exogenous inflow of non-chequables, but to a lesser degree because non-chequables are more interest elastic. If interest rates elsewhere rise then the rate to be paid to retain the savings deposits increases and the TML companies could be

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[14] The posted rates are commonly applied to semi-annual minimum balances so that most customers the effective interest rate is near zero; some companies pay no interest at all. The TML sector's drawing card for chequing accounts is free chequing privileges not interest.

squeezed. This happened in the mid-1960s when bank competition for personal savings effectively pushed the TML companies into seeking new sources of funds.

In contrast with savings deposits, less-than-one-year term deposits tend to be held in large denominations by institutional and industrial investors who might otherwise hold bank certificates of deposit. The short-term deposits might even be placed through an investment dealer. It has happened that when the rate on non-chequable savings deposits has climbed above the rate on ninety-day GICs some of these large depositors have transferred substantial sums into savings deposits, to the dismay of the trust companies who discourage the practice by putting limits on the size of deposits they will accept in a savings account. The over-one-year deposits are held by a wide variety of investors with the typical deposit probably not exceeding ten thousand dollars. The over-one-year GIC is an instrument with a defined market of its own; given the rather thin bond markets of Canada, it offers yield, maturity and safety that no other instrument approximates.

There can be little doubt of the interest elasticity of demand for GICs and term deposits. The econometric results in Chapter VI confirm that the elasticities are high, particularly so for the less-than-one-year liabilities. The sensitivity of demand to changes in interest rates for the whole industry is such that, considering there are about one hundred firms in the industry, it is not an exaggeration to maintain that within the span of a couple of months each firm acts as a price taker. The whole industry probably faces a rising demand price for its term deposits, given the size it has reached and given the ostensibly well-circumscribed market for its over-one-year liabilities.

The matching of assets and liabilities in the TML balance sheet is largely due to liability management. Individual firms can contemplate quite precise quantity adjustments for term liabilities. Mortgages are matched by over-one-year deposits not so much because mortgages are made to fit the amount of GICs as because the GICs are raised to fit mortgage lending requirements. This statement is too strong because eventually the ease or difficulty with which long-term deposits are raised will ultimately affect the level of mortgage approvals. But given the loose control over mortgage approvals and the long lag between approvals and disbursements the statement is true in a significant sense. The mortgage lending of any particular month is a predetermined quantity and with monthly data it would be a grave distortion to write mortgage lending of that month as a function of current outstanding term deposits -- the chain of casuality is in the other direction.

That an asset item might be more difficult to control than liability items is something often overlooked by monetary economists who have become accustomed to think in terms of a model in which total liabilities are assumed to be given and in which choice is limited to determining the allocation of the total into various alternative asset categories.[15] The conventional treatment is not consistent with the experience as they perceive it of the executive officers of the TML sector with whom I have spoken.

6. Portfolio behaviour in the 1960s

Chart V3 shows the proportions of balance sheet components to total portfolio at a more aggregate level than Chart V1 and for a longer span of years so that some basic trends are emphasized. Bonds of all types are put into a single category. There is a discernible movement out of bonds and into mortgages in the first half of the 1960s but the trend slowed down after 1965. The increase in the proportion of mortgages to total assets from less than fifty-five per cent in 1961 to more than sixty-seven per cent in 1965 was fast indeed and probably a slowing down would have occurred even without the credit squeeze of 1966. The proportion

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[15] With the exception of Parkin's elegant Discount House piece, all the empirical work cited in this thesis takes intermediary deposits as either given or determined by public preferences.

of cash and liquid assets has shown significant variations but no distinct trend over the period. On the liabilities side the dominating feature is the decline in the proportion of savings deposits in the portfolio which only began to rise in 1970. As a matter of arithmetic the declining proportion of savings deposits is matched by an increasing proportion of GICs and term deposits.

Table V2 gives annual rates of growth for total major liabilities and mortgage lending of the TML sector from 1961 to 1971; the growth rate of banking system liabilities is also given. The rapid growth of the TML industry is evident. A dip in the rate of growth, 1966-68, began in a period of tight monetary policy, but tight money is not the whole story because TML deposits fell relative to chartered bank deposits. The TML companies did not respond vigorously to the competition of the banks for deposits. They presumably felt no incentive to struggle for deposits at unprecedentedly high interest rates when mortgage lending (their most profitable asset) was a declining proportion of total deposits.[16] It was not until mid-1968 that the TML sector began to regain its position relative to the banks. In 1969 total TML deposits amounted to about thirty per cent of total bank deposits --

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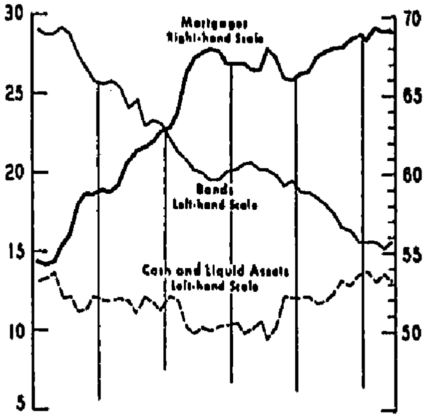
[16] Some of the slowdown was due to fears about the solvency of one company in the mid-1960s.

CHART V 3
TML ASSETS AND LIABILITIES
PER CENT DISTRIBUTION OF MAJOR ASSETS

ASSETS



LIABILITIES

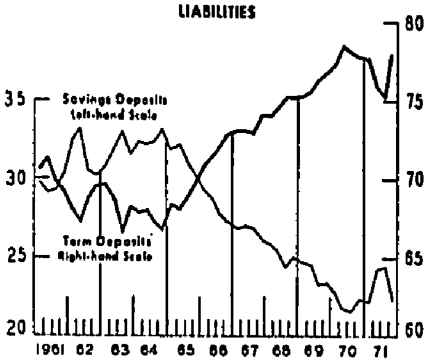


TABLE V 2 ANNUAL RATES OF GROWTH

	1951	1962	1963	1964	1965	1966	1967	1968	1969	1970	1971
Trust and mortgage loan companies											
Mortgages	23.9	25.1	24.9	28.4	29.3	8.3	9.0	10.4	16.7	15.6	14.0
Total deposits	21.4	18.6	25.8	21.9	18.4	11.0	10.4	12.0	13.5	15.0	14.4
Term deposits & GICs	18.6	18.9	23.4	19.1	23.6	15.8	12.2	13.7	16.7	16.6	14.5
Chartered banks											
Privately held Canadian dollar deposits	9.5	3.8	7.3	5.3	10.5	7.7	15.7	16.5	2.0	10.1	15.4

a position which has been maintained.

As I see it, the experience of the 1960s contains two distinct phases for the TML companies. In the period 1960-66 it is almost as though the sector is in the last phase of a long-run process of adjustment to post-1945 conditions: the movement out of bonds and into mortgages was the dominating feature of this adjustment, a movement that was pronounced until 1966.[17] Several coincidences conspired to end this movement: (a) Adjustment may have been near completion by 1966, given the prevailing interest rates and so on. (b) As interest rates rose deposit holders became more interest sensitive. (c) The increased competitiveness permitted to the banks by the Bank Act revisions enhanced the new interest rate consciousness. (d) Tight money in 1966 was a warning to the TML companies of the dangers of an unmatched portfolio -- the increased interest rates that had to be paid for deposits cut into profits since the bulk of the assets were long-term and fixed-interest.

The signal conditions of the mid-1960s amount to a watershed. In the early 1960s there was no reluctance to reduce the proportion of bonds in TML portfolios in order to increase that of mortgages. Deposits could fairly be assumed to grow at least as fast as

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[17] There is still an apparent trend in this direction but, as argued below, the nature of the trend has changed.

deposits in other institutions and the stable interest rates of the time gave rise to little concern about the matching of assets and liabilities. After 1966 the proportion of bonds was low enough that further reductions involved increasing marginal costs of illiquidity. Likewise after 1966 it was clear that deposits would not arise automatically from increased personal savings and that even existing deposits were susceptible to chartered bank competition. These things were compounded by the concurrent rise in interest rates which emphasized the risks of further movements out of short-term bonds with the existing structure of liabilities.

The TML companies have subsequently paid much more attention to liability management and to the opportunities for hedging offered by matching the term to maturity of assets and liabilities. The recent increase in the proportion of mortgages in the TML portfolio is intrinsically different from that of the early 1960s which was the culmination of a movement towards a preferred less liquid, higher risk, higher yield portfolio. Since 1966 there has been no inclination to be less liquid, more aversion to borrowing short and lending long and a rapid increase in the ratio of term deposits to total deposits. High interest rate elasticities in the deposits markets have contributed to the process by, on the one hand, allowing the TML companies to attract the liabilities that they require with small changes in yields and, on the other hand, by forcing the industry to abandon the idea (which used to exist) that there is a share of the deposit market which it can count on almost regardless of interest rates.

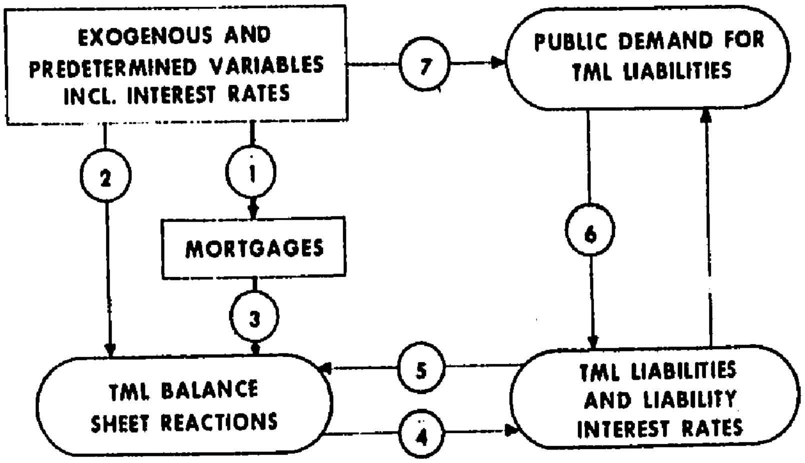
In the post-1966 era the response of mortgage lending to yield changes has probably undergone a fundamental change too. With a swollen bond portfolio the attraction of mortgage lending is obvious -- the wide spread between bond and mortgage yields. But once the slack in bonds is eliminated the remaining core of bonds (excepting corporates) provides the sort of liquid reserve that is complementary with mortgage lending. The relevant opportunity cost for mortgage lending is then not the bond rate but the rate that must be paid to attract the required term deposits. An increase in mortgage lending by the TML companies under prevailing circumstances will be financed only partly by selling bonds. The rest of the finance will come from supplying term deposits (risk aversion demands over-one-year term deposits). Of course the extent to which mortgages are funded from bond sales or from less-than-one-year deposits or from over-one-year deposits will depend on interest rates, but (unless the interest rate configuration is very unusual) the last source will normally predominate.

7. Condensation of the argument

The institutional history of the TML sector is fascinating but not the primary concern of this thesis. It is necessary to conclude this descriptive chapter by bringing together the several strands of argument. The stage has not yet been reached where it is convenient to graft the particular institutional devices of the trust and mortgage loans onto the theoretical foundations of chapters I through IV (this is done in detail in Chapter VIII). It is convenient though to set out the broad outlines of a useful framework.

FLOWCHART 1

A SKELETON FOR TML BALANCE SHEET ADJUSTMENTS



Flowchart 1 is what I have in mind. The numbered arrows represent channels of influence that are to be understood as embodying time lags. The major source of drive in the model is mortgages, the management of other assets and liabilities responding to the financing requirements of mortgage investment. Because of the delayed response of outstanding mortgage lending to the variables which motivate it, mortgage lending is a predetermined variable when other balance sheet items are decided. Mortgage lending, as determined via ①, therefore feeds into the balance sheet almost as an exogenous variable to which other items adjust (③). Mortgages can be financed by selling other assets or by supplying liabilities. The optimal reactions depend on interest rates and on the constraints imposed by other predetermined components of the balance sheet (②). The demand for the supply of TML liabilities determine their price and quantity simultaneously (④) and (⑥) and there is a feedback from these interest rates to the portfolio allocation decisions (⑤). The exogenous determinants of the public's demand for TML liabilities, through ⑦, include total private sector wealth, tastes and so on.

The framework is not fully recursive since interest rates on TML liabilities are endogenous. But it is an aid to understanding the model to pretend that link ④ is broken and that liability interest rates are exogenous. There is then a chain of causation running from interest rates and exogenous balance sheet items through a complicated lag structure to mortgages and thereby to the balance sheet. The interest rates affect other portfolio allocations directly as well as through mortgages.

As an example of how the model works, consider the case of an exogenous inflow of savings deposits. Mortgage approvals (and eventually mortgage loans) increase. As the loans hit the balance sheet financing decisions have to be made: in the considerable lapse of time between the inflow of savings and the ultimate effect on mortgages the inflow of deposits will, depending on interest rates, have been invested in marketable assets or (as is often overlooked) have been used to reduce other liabilities the retention of which requires payment of interest. When the loan is disbursed the marketable assets can be sold off and/or the pre-existing term deposits restored. This decision also will depend on interest rates, and in turn, if the decision is to raise more term deposits, the yields on term deposits will be raised. It is at this point that the causal chain is broken. The increase in rates paid on term deposits is itself a deterrent to that means of financing. The solution is therefore a simultaneous one with the quantities of term deposits and the yields on term deposits jointly determined.

The structure outlined has been intentionally oversimplified. Time lags have been neglected. The set of choice variables has not been explicitly distinguished from the predetermined set. The implications of non-market-clearing interest rates remain to be explored. Piece-meal these oversimplifications will be removed in later chapters. The movement towards realism is begun in the next chapter in which I analyze the demand for TML liabilities. Then, in Chapter VII, a model for mortgage lending is developed. In Chapter VIII material from all the preceding chapters -- theoretical and institutional and statistical -- is combined to specify an interdependent block of equations for the complete balance sheet which is estimated by constrained three-stage least squares.

Appendix to Chapter V

Data series: sources and mnemonics

The equations in Chapter VII involving mortgage approvals and mortgage lending are estimated over the period January 1965 to December 1971. All other equations are estimated over the period January 1967 to March 1972. In each case these were the longest periods available to me without an excessive amount of interpolation.

All dollar magnitudes are in millions and all interest rates are in percentage terms. All variables are measured at month-end.

The theory of portfolio preference requires the use of expected yields rather than current market interest rates as the arguments in security demand functions. In empirical economics expectations are usually represented by a more or less complicated weighted average of the variables whose expected values are deemed important. There seem to be no arguments of overriding persuasiveness in favour of these weighted averages and so I have simply used the month-end interest rates as proxies for expected interest rates.

Monthly balance sheet data

Quarter-end balance sheet figures are to be found in

Statistics Canada's Financial Institutions for trust companies and mortgage loan companies and mortgage loan companies separately. The data used in the regressions are monthly and not published. Terms to maturity refer to original, not existing, terms and bonds are valued at book value.

The aggregates which are used in this thesis are as follows:

PAPER	Cash plus foreign assets plus collateral loans plus holdings of term deposits in other institutions plus commercial and finance company paper.
GOVT	Canada treasury bills plus government of Canada bonds.
MP	Municipal plus provincial bonds.
CORP	Corporate bonds plus preferred and common shares.
MTG	Mortgage loans and sale agreements.
SAVDEP	Chequing (CHEQ) plus non-chequing (NOCHEQ) savings deposits.
GIC1	Deposits with a fixed term of less than one year.
GIC2	Deposits with a fixed term of more than one year.
NET	PAPER + GOVT + MP + CORP + MTG - SAVDEP - GIC1 - GIC2.
NET*	NET + SAVDEP - MTG.
LIAB	Total major liabilities, SAVDEP + GIC1 + GIC2.

To take account of reclassifications of certain items the balance sheet data are adjusted in two respects. The figures for

NOCHEQ are increased by 9.29 per cent, January 1967 to February 1969, and the same sum is subtracted each month from CHEQ. The figures for NOCHEQ are increased by 2.29 per cent of SAVDEP, January 1967 to December 1969, and the same sum is subtracted from GIC1.

The division between GIC1 and GIC2 is not available monthly. The series used in the regressions were obtained by interpolating the quarterly numbers to get monthly GIC1 and GIC2 numbers. These did not sum exactly to the total term deposit numbers which are available monthly. The difference between total term deposits and the sum of GIC1 and GIC2 was split 50:50 and allocated between the two.

Mortgage approvals (APRO).

Central Mortgage and Housing Corporation (CMHC) publishes quarterly mortgage approvals figures for the TML companies in its Canadian Housing Statistics. The monthly figures used in Chapter VII are not published.

Annual estimates of repayments of mortgage principal (REPAY) are also available in the CMHC publication.

Interest Rates.

Most of the interest rate series used are published in the Bank of Canada Review:

RGOVT	Weighted average of the rate on treasury bills and the rate on over-one-year Canadas. The two rates are weighted by the fractions of treasury bills and Canadas respectively in the TML portfolios (which means that the weight given the treasury bill rate is very small). The rate on over-one-year federal bonds averages the 1-to-3-year, the 3-to-5-year and the 5-to-10-year yields with weights .4, .4 and .2 respectively. This weighting scheme is postulated to reflect the fact that TML holdings of Canadas tend to be concentrated in the shorter maturities.
RMP	Average of yields on municipals and provincials, weighted 50:50.
RCORP	Industrials rate.
RMTG	Average, weighted by respective volume of approvals, of the rate on conventional mortgages and the rate on NHA mortgages. The NHA mortgage rate is defined as .5 times rental rate plus .5 times home-ownership rate.
RSDB	Rate on non-chequable savings deposits at chartered banks.
RCD	Rate on ninety-day deposit receipts at chartered banks.

The other interest rate series were derived using unpublished sources.

RPAPER	Average rate on ninety-day finance company paper and ninety-day commercial paper, weights equal to .5.
RSAVDEP	Rate on trust company non-chequable savings deposits.
TGIC1	Rate on ninety-day deposit receipts at trust companies.

RGIC2	Average rate on 1-to-2-year, 3-to-4-year and 5-year GICs, with weights .2, .2 and .6 respectively.

Other variables

CSBSALE	Net sales of Canada Savings Bonds, first differences of general-public holdings of CSBs as published in the *Bank of Canada Review*.
Q1, Q2, Q3	Quarterly dummy variables constructed so that seasonal influences sum to zero over the year.
QNHA	Dummy variable for the quarter-end effects of changes in the NHA ceiling rate, 1967-69. It is set at 1 for March 1967, June 1967, March 1968 and September 1968, and at -1 for April 1967, July 1967, April 1968 and October 1968, and at zero elsewhere.
C	Constant term.
BACKLOG	Weighted sum of past approvals to represent the backlog of undisbursed mortgage loan approvals.
BACKLOG*	BACKLOG plus approvals made this period.
DEPOSITS	General-public holdings of currency, chartered bank deposits, swapped deposits, CSBs and TML major liabilities. A measure of public deposit-type financial wealth.

First-stage regressors

The regressors used in the first stage of the two- and three-stage least squares regressions consist of variables considered as exogenous to the system and such lagged endogenous variables as entered the structural relations. The regressors are: RGOVT; RMP; RCORP; RMTG; RSDB; RPAPER; CSBSALE; NET; the seasonal dummies Q1, Q2, Q3 and a constant term; and the lagged values of PAPER, GOVT, MP and CORP.

CHAPTER VI

The Demand for Trust and Mortgage Loan Company Liabilities

1. Introduction

In this chapter I investigate the determinants of the public's demand for TML savings deposits and term deposits. Savings deposits are disaggregated into chequable and non-chequable components whilst term deposits are disaggregated into less-than-one-year and over-one-year categories. The disaggregation of term deposits is particularly important because the short-term and long-term business activities of the TML sector respond to quite different variables. The less-than-one-year deposits are used by the TMLs primarily for investment in commercial and finance company paper, and they compete closely with chartered bank CDs. The over-one-year deposits are used primarily for mortgage lending (the five-year term deposits are particularly important). It is difficult to identify precisely which assets are close substitutes for over-one-year GIC's since money seems to go into them from a wide variety of sources -- personal savings, institutional funds, and so on.

Of particular concern in this investigation is the interest-sensitivity of the public in its allocation of funds into the various media which the TMLs make available; but the investigation also sheds some light on the effects of Canadian Savings Bonds issues on TML deposits, and on the effects of changes in the public's total liquid asset holdings.

2. Savings deposits

The demand liabilities of the TML companies compete to some extent with the demand liabilities of the chartered banks. The division of savings deposits between the TML companies and the banks is largely influenced by factors such as branch locations, public confidence in the solvency of the institutions,[1] and so on. These are factors exogenous to the present inquiry. Not too much should be expected from econometric attempts to determine interest rate elasticities.

I tried a number of approaches to the specification of a demand function for TML savings deposits. The approach that worked best is based on the idea of a decision tree. Depositors are assumed to make their savings deposit decisions in two steps: Step (1) determines the quantity of deposits to be kept with the

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[1] Well publicised fears about the solvency of one company in 1965 (British Mortgage) caused confidence to weaken in the TML sector as a whole. Consequently TML savings deposits actually fell in 1966.

TML sector; Step (2) determines the division of this quantity between chequable and non-chequable deposits. The following specifications of the steps were made: (1) the volume of deposits placed with the TML sector (SAVDEP) depends on the spread between the savings deposit interest rates paid by the TML companies and the banks, on the intensity of the Canada Savings Bond campaign and on the total liquid wealth of the private sector. The interest rate spread is measured by the differential in non-chequable savings deposit rates, RSAVDEP - RSDB. Savings deposit rates tend to be "sticky"[2] and are probably not market clearing rates -- I take them to be administratively determined and consider them to be independent variables in the model. The CSB campaign cuts significantly into all savings media; a variable must be included to capture the intensity of the campaign although there is no obvious choice. One possible proxy is given by net CSB sales and this is what I use. The scale variable (DEPOSITS) is the sum of privately owned major liabilities of the TML companies and the chartered banks, currency, and outstanding CSBs -- a measure of the volume of private sector deposit-type wealth.[3] (2) The division

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[2] The rate on chequable deposits paid by most TML companies was constant between 1967 and 1972. It is because this rate has no variance that it is not used in regressions. For more details see Chapter V, section 5.

[3] Deposits at credit unions should be included but monthly data are not available.

of TML savings deposits between chequable and non-chequable deposits is a function of the interest rate paid on non-chequables (RSAVDEP).

The stock adjustment principle is used throughout this chapter as the model for asset demand.[4] The various hypotheses are tested against monthly balance sheet data running from January 1967 to March 1972. The regression results are given in Table VI 1.[5] Equation 1 is the regression for total TML savings deposits. Equations 2 and 3 respectively are the regressions for chequables and non-chequables. Long-run coefficients and elasticities (normalized at the sample means) have also been calculated, and the results are tabulated beneath the regression results. The derived parameters are given for complete intra-sectoral adjustments as well as for the intra-sectoral adjustments alone. These

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[4] The general model is presented in Chapter IV, section 3. The model can be illustrated by a simple one equation example.

$$y_t = a_1 x + a_2 y_{t-1}.$$

The effect of a change in x on y is a_1; the total equilibrium (or long run) effect is $a_1/(1-a_2)$. The mean lag is $a_2/(1-a_2)$ periods and $1 - a_2^j$ is the proportion of the total effect that is completed in j periods.

[5] The number in parentheses below each estimated coefficient is its t-ratio.

TABLE VI 1. DEMAND FOR TML SAVINGS DEPOSITS, (OLS)

Eq^n.		Explanatory variables					Seasonals				
		C	RSAVDEP -RSDB	DEPOSITS	CSBSALE	SAVDEP$_{-1}$	Q1	Q2	Q3	$\bar{R}^2$	DW
1.	SAVDEP	-38.21 (-1.32)	7.63 (1.74)	.0044 (2.31)	-.0849 (-7.79)	.9247 [a] (17.52)	-11.81 (-1.91)	1.35 (.21)	- 8.99 (-1.39)	.9870 [b]	1.40
			RSAVDEP	SAVDEP	CHEQ$_{-1}$	NOCHEQ-1					
2.	CHEQ	319.16 (2.93)	-21.03 (-2.91)	.0379 (.56)	.6272 (5.32)	-.0445 (-.65)	- 5.22 (-1.11)	4.16 (.89)	-.78 (-.16)	.8695	1.85
3.	NOCHEQ	-319.16 (-2.93)	21.03 (2.91)	.9621 (14.27)	-.6272 (-5.32)	.0445 (.65)	5.22 (1.11)	-4.16 (-.89)	.78 (.16)	.9937	1.85

IMPLICIT EQUILIBRIUM RESPONSES.

1. Total savings deposits

	RSAVDEP	RSD3	DEPOSITS
Coefficients	101.4	-101.4	.058
Elasticities	.300	-.276	1.387

(a) Intra-sector adjustment only

	2. Chequable		3. Non-chequable.	
	RSAVDEP	SAVDEP	RSAVDEP	SAVDEP
Coefficients	-64.1	.00	64.1	1.00
Elasticities	-.584	.00	.281	1.48

(b) Intra - and extra-sector adjustment

Chequable			Non-chequable		
RSAVDEP	RSDB	DEPOSITS	RSAVDEP	RSDB	DEPOSITS
-64.1	.00	.00	165.5	-101.4	.058
-.584	.00	.00	.495	-.409	2.041

a. This coefficient is not significantly different from unity at the .9 confidence level.

b. Using the first difference of the dependent variable $\bar{R}^2 = .5174$.

inferences can be drawn from the results. (1) The spread in interest rates between banks and TML companies has only a small influence on the allocation of savings deposits. The derived long-run elasticities with respect to both deposit rates are less than unity in absolute value. However TML savings deposits are highly wealth elastic in the long run. The estimated speed of adjustment is not significantly different from zero and according to the point estimate it is seven months before one half of total long-run adjustment to a desired position is achieved. The strong negative effect of CSB sales on savings deposits is apparent, and according to these estimates an increase in CSB sales of one dollar causes an immediate loss of savings deposits from the TML sector of eight cents. (2) Intra-TML deposit movements take place with more alacrity and interest sensitivity (Equations 2 and 3). Even so the estimated long-run interest elasticities are less than one. The meaning of these estimates can be illustrated by assuming that RSAVDEP, the rate on non-chequable deposits, increases by a hundred basis points with other things equal. The immediate effects are a switch of $21 million from chequables to non-chequables and an inflow of $7.6 million from outside the sector, the very small effect of a change in total savings deposits on chequables ensuring that the $7.6 million goes almost entirely into non-chequables. Thus in the short run some seventy per cent of the induced increase in non-chequable savings deposits is drawn from the TML's own chequable accounts. By consulting the tabulated equilibrium responses it can be seen that in the long run the shift from chequable to non-chequable is \$64.1 million and that the inflow from outside is \$101.4 million. Thus when both intra- and extra-sectoral adjustments are taken into account in the long run non-chequables increase by \$165.5 million of which forty per cent is from the TML's own chequables.[6]

I take the large and significant constant term (\$319.2 million) to reflect the fact that a large proportion of chequable deposits is held for transactions purposes and is thereby virtually immune to changes in interest rates.

The long-run intra-sectoral responses were calculated with the formula explained in Chapter VIII, Section 5, which in this case gives

$$\begin{bmatrix} 1 - .6272 & .0445 \\ .6272 & 1 - .0445 \end{bmatrix}^{-1} \begin{bmatrix} -21.03 & .0379 \\ 21.03 & .9621 \end{bmatrix}$$

The path to the long run equilibrium is such that equilibrium

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[6] The reader will recall (Chapter V, Section 5) that money sometimes finds its way from short-term GICs into savings deposits. The trust companies discourage the practice and it is not felt to be more than a transitory phenomenon -- I have set up the model accordingly. But the existence of the practice can be confirmed econometrically.

is approached monotonically and rapidly. Within one quarter all four response coefficients are no more than twenty per cent different from their equilibrium values.

These results taken as a whole are consistent with the kind of approach that has been adopted and with prior institutional knowledge. High interest elasticities are not to be expected and the estimated elasticities are low. The vulnerability of savings deposits to the CSB campaign is well captured. The sluggish adjustments to changes outside the TML sector which savings deposits are known to exhibit are also apparent in the estimates. Intra-sectoral movements are considerably more lively.

3. Less-than-one-year GICs and term deposits (GIC1)

Chartered bank CDs are in most respects good substitutes for short term GICs and compete for the same sources of money, particularly for less-than-one year money (probably the bulk is for ninety days).[7]

The stock adjustment equation for less-than-one-year liabilities is therefore written with private deposit wealth, the rate of interest on ninety-day GICs (RGIC1), and the rate of interest on

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[7] Commercial and finance company paper do not compete directly with less-than-one-year GICs to any great extent. See Chapter V, Section 2.

TABLE VI 2. DEMAND FOR GIC1 (2LS)

Eqn.	Explanatory variables					Seasonals			$\bar{R}^2$	DW
	C	RGIC1	RCD	DEPOSITS	$GIC1_{-1}$	Q1	Q2	Q3		
4.	-150.93 (-2.28)	51.18 (3.10)	-46.32 (-2.68)	.0055 (2.63)	.8557 (16.42)	11.75 (1.77)	-9.22 (-1.10)	6.46 (.94)	.9852	1.5

IMPLICIT EQUILIBRIUM RESPONSES.

	RGICI	RCD	DEPOSITS
Coefficients	354.7	-321.0	.038
Elasticities	2.94	-2.40	2.03

Using the first difference of the dependent variable $\bar{R}^2 = .5825$

ninety-day bank term deposits (RCD) as explanatory variables. The estimates are given in Table VI 2, Equation 4. Two-stage least squares (2LS) is used rather than ordinary least squares (OLS) because RGIC1 is endogenous to the model.[8] The table presents the estimated coefficients and the corresponding derived long-run coefficients and elasticities.

A lot of interesting information is carried in these estimates. Interest elasticities of demand for less-than-one-year term deposits appear to be high and adjustment appears to be quick (over fifty per cent of total adjustment occurring within four months). This is the sort of structure that is known to characterize short term money markets, rapid adjustment and fine interest rate sensitivity. The high wealth elasticity implies that GIC1 is a "luxury asset"; that its share in total financial wealth increases as the latter itself increases.

The interest rate responses are very nearly symmetric, so that the demand function could almost be written using the spread between the competing ninety-day rates as the independent variable rather than the levels of these rates. However, as might be expected, the measured response to a change in the own rate exceeds

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[8] The list of regressors for the first stage of the 2LS regressions is given in the Appendix to Chapter V.

the measured response to a change in the competing rate; this probably implies that an increase in RGIC1 draws money from other sources as well as from the banks' CDs.

The test statistics of Equation 2 show that the measured responses are statistically significant although the explanatory power is not high. There is evidence of residual serial correlation but in view of the high estimated speed of adjustment it does not seem to have caused serious bias (cf. Griliches (1961)).

4. Over-one-year GICs, term deposits and debentures (GIC2).

No single asset stands out as a competitor with GIC2). Canada Savings Bonds compete primarily with demand instruments such as savings deposits. The marketable government issues are very liquid with commensurately low yields and held by a different sort of investor to the over-one-year GICs'. Corporate bonds, on the other hand, tend to be of much longer maturity and to have a higher yield than GIC2. Neither governments nor corporates are well suited to the needs of the small savers who hold significant chunks of TML liabilities.

What interest rates can be put into an operational demand function? Of necessity the choice is reduced to TGIC2 (the rate on GIC2), RCORP (the rate on corporate bonds) and RGOVT (the rate

TABLE VI 3. DEMAND FOR GIC2 (2LS)

Eq^n.	Explanatory variables						Seasonals				
	C	RGIC2	RCORP	RGOVT	DEPOSITS	$GIC2_{-1}$	Q1	Q2	Q3	$\bar{R}^{2a}$	DW
5.	-108.13 (-.79)	124.05 (2.38)	-130.97 (-1.79)	-4.07 (-.18)	.0220 (2.68)	.8781 (13.42)	-11.19 (-.89)	26.34 (1.83)	-7.44 (-.59)	.9968	1.72
6.	-93.14 (-.85)	125.89 (2.45)	-139.13 (-2.28)		.0214 (2.87)	.8856 (17.96)	11.90 (-1.01)	26.79 (1.89)	-7.75 (-.63)	.9969	1.74
7.	-261.86 (-2.41)	42.44 (1.67)		-26.13 (-1.36)	.0200 (2.41)	.8857[b] (13.03)	-3.62 (-3.0)	13.56 (1.10)	-2.53 (-.20)	.9967	1.70

IMPLICIT EQUILIBRIUM RESPONSES

	Equation 5				Equation		
	RGIC2	RCORP	RGOVT	DEPOSITS	RGIC2	RCORP	DEPOSITS
Coefficients	1017.2	-1074.7	-33.4	.181	1100.5	-1215.9	.187
Elasticities	1.53	-1.82	-.08	1.601	1.65	-2.06	1.659

a. Using the first difference of the dependent variable the $\bar{R}^2$ for 5,6, and 7 respectively is .5237, .5399, and .3791.

b. This coefficient is not significantly different from unity at the .9 level.

on federal government bonds).[9] Table VI 3 gives 2LS estimates of an equation which includes all three rates (Equation 5), of an equation which excludes the government bond rate (Equation 6) and of an equation which excludes the corporate bond rate (Equation 7). The table also presents the equilibrium parameters implicit in the stock adjustment form for the preferred equations (Equations 5 and 6). As with the less-than-one-year liabilities the estimates are informative. In all cases the coefficients work in the expected direction, although the fit of the equations is not tight.

Equation 5 shows that when both RGOVT and RCORP are put into the same regression neither is statistically significant. But the tentative inference may be drawn that there is mild substitutability between governments and GIC2, and a stronger degree of substitutability between corporates and GIC2. This suggests that the regression be run excluding successively RGOVT and RCORP. The t-ratios and the sizes of the coefficients in Equations 6 and 7 respectively confirm that corporates and GIC2 are strong substitutes, and that governments and GIC2 are weak substitutes.

Now Equation 6 is not a bad equation. But the equation that

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[9] The rate on municipal and provincial debt was also tried, with discouraging results which are not reported.

best reflects the structure is to my mind Equation 5, despite the low t-ratios for the interest rates RCORP and RGOVT. Equation 5 otherwise embodies the information from Equations 6 and 7 and the estimated parameters are credible. There is something disquieting about both equations in that the coefficient of RCORP is greater than that of RGIC2. The excess is not statistically significant and is not precluded by economic theory but it does detract from the plausibility of the equations. It may be that RCORP acts as a good proxy for other variables as well as the yield on corporate bonds.

The demand for over-one-year GICs is not as interest elastic as the demand for less-than-one-year GICs. This accords with common sense, indeed it is surprising that the estimated elasticities for the longer maturities should be so high considering the apparent paucity of substitutes for GIC2. The long-run elasticities for both the corporate bond rate and the own rate exceed unity. So also does the wealth elasticity -- TML liabilities are palpably luxury goods.

The estimated speed of adjustment for the over-one-year liabilities is almost equal to the estimate for the less-than-one-year liabilities. This is certainly admissible for upward movements in the demand for GICs. But on downward movements the roll-over imposes a definite limit to the depletion of GICs. The total cannot decline more rapidly than the maturities in any month. The roll-over constraint is more important at the long end of the maturity spectrum and should cause downward adjustment in the GIC2 stock to be slow. As it happens that the period under study did not see a substantial decrease in GICs one should perhaps pragmatically ignore the constraint on the rate of decrease in choosing empirical specifications.

5. Conclusions

The demand for each of the liabilities of the TML sector has some interest elasticity. The long run elasticities are highest for less-than-one year term deposits. Chequables are the least elastic, being rather firmly held, but do respond to changes in the yield on non-chequables. An increase in the non-chequable yield draws money from the TMLs' own chequable accounts from extra-sectoral sources of which one, chartered bank personal deposits, has been identified.

CHAPTER VII

The Determinants of Mortgage Lending

1. Introduction

The lag between the approval of a new mortgage loan and the actual disbursement of money may be as long as two years. A quantitative understanding of this distributed lag relationship is essential to the explanation of mortgage approvals. In this chapter, monthly data on mortgage loan approvals by the TML sector is combined with the balance sheet data on outstanding mortgages to estimate the coefficients of the approvals-disbursement relationship. These estimates are then employed in the development of a model to explain mortgage loan approvals.

The substantive issues raised include the shape of the profile of the weights in the approvals-disbursement process, the responsiveness of approvals to changes in deposit flows, the interest-elasticity of mortgage investment, and the nature of the time delays due to lagged responses.

2. The lag between approvals and disbursements

The distributed lag relation between the granting of approval for a loan and the subsequent disbursement of money is a relatively straightforward relation to estimate. Disbursements may be defined as

$$DISB_t = MTG_t - MTG_{t-1} + REPAY_t \tag{1}$$

where the subscripts refer to time periods. The mnemonics DISB, MTG and REPAY are respectively disbursements, outstanding mortgage loans and repayments of principal. Equation (1) was used to construct a disbursement series. Since monthly data for repayments do not exist, annual CMHC figures were used to get an approximation: the ratio of annual repayments to total mortgages outstanding was calculated for each year and then divided by twelve to give monthly repayment ratios. The ratios were assumed constant within each year:

1965	1966	1967	1968	1969	1970,71
.0117	.0083	.0094	.0087	.0086	.0081

The disbursements series was then generated with the formula

$$DISB_t = MTG_t - (1 - \rho_t)\, MTG_{t-1} \tag{2}$$

where the repayments ratio ρ_t is set equal to .0117 for 1965 and so on.

The relation which is estimated using the derived disbursement series is

$$DISB_t = C + \text{seasonals} + \sum_{i=0}^{24} p_i \, APRO_{t-i} \qquad (3)$$

where p_i is the proportion of approvals granted in period t-i which are disbursed in period t and APRO is the volume of approvals. A constant term[1] is in (3) because of the sales of mortgages in the secondary market. If there were no secondary market and approvals were zero mortgage disbursements would eventually fall exactly to zero. The longest lag is set at twenty-four months although in rare cases there may be longer lags.

The sum of the weights in the distributed lag function, $\sum_{i=1}^{24} p_i$, would be unity if all loans approved were disbursed. The fact that some approved loans are cancelled by the intended borrower means that this sum is less than unity.[2] I have not treated the cancellation rate as an endogenous variable but I think the idea is worth pursuing -- there should be some functional relationship between interest rate changes and cancellations. The seasonal terms[3] allow the weights to vary between seasons, raising

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[1] C is used throughout to denote a constant term.

[2] Some mortgages are approved specifically for other mortgage lenders so that they are sold before they appear on the TML balance sheet; this too tends to make the sum less than unity.

[3] Quarterly seasonal variables were used (Q1, Q2 and Q3) and constructed so that the seasonal effects sum to zero over each year.

TABLE VII 1. DISBURSEMENTS AS A FUNCTION OF PRIOR APPROVALS (OLS)

		Almon variables[a]			Seasonals				
	C	APRO (DG1)	APRO (DG2)	APRO (DG3)	Q1	Q2	Q3	$\bar{R}^2$	DW
Regression coefficient	-7.848	1.535	-1.847	1.239	-6.68	-1.45	6.93	.5839	1.70
t-ratio	(-.55)	(.76)	(-.41)	(.48)	(-1.21)	(-.27)	(1.31)		

Unscrambled Almon weights

Lag	Weight	t-ratio
0	.0959	2.94
1	.0857	3.97
2	.0767	5.62
3	.0687	7.07
4	.0617	6.10
5	.0555	4.54
6	.0502	3.56
7	.0457	3.00
8	.0417	2.70
9	.0384	2.54
10	.0355	2.50
11	.0330	2.53
12	.0309	2.57

Lag	Weight	t-ratio
13	.0290	2.54
14	.0272	2.38
15	.0256	2.11
16	.0239	1.81
17	.0221	1.55
18	.0201	1.34
19	.0179	1.17
20	.0153	1.04
21	.0123	.94
22	.0088	.87
23	.0048	.81
24	.0000	.00

Sum of weights = .9265

Standard error of sum = .1343

Average lag = 7.639 months

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a. The programme constrains the weights to lie on a polynomial passing through zero at the end point (t-24). A third degree polynomial was used to represent the profile of weights. The three Almon variables are distinguished as to degree by the postfix in parentheses.

or lowering the weight profile by a constant. There is a good case for allowing the shape of the profile to vary between seasons, e.g. to allow the mean lag to be shorter in the summer than in the winter, but meaningful seasonal variations in the shape of the profile were not apparent in tests that I made.[4]

The estimates of equation (3) (Table VII 1) are consistent with the expectations about this relationship. (a) The negative constant reflects net mortgage sales.[5] (b) The weights are positive throughout the twenty-four month disbursement period. (c) The sum of weights, .9265, implies a "cancellation rate" of about seven per cent (notice that approvals made for other lenders are included in this seven per cent). (d) None of the seasonal terms is statistically significant, the greater part of the seasonal variation in disbursements being explained by the equally pronounced seasonal variation in approvals.

The weights from the disbursements - approvals regression can be used to measure the backlog of undisbursed commitments. This backlog is a distributed lag function of prior approvals,

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[4] Smith and Sparks (1971) and Pesando (1972) found significant and plausible differences in the weight profiles for different seasons.

[5] Net sales of NHA mortgages (which are known to be the bulk of sales in the secondary market even though figures for conventional mortgages are not available) averaged $4.18 million per month, 1965-70.

$$\text{BACKLOG} = \sum_{i=1}^{24} \Pi_i \text{APRO}_{t-i} \qquad (4)$$

where

$$\Pi_i = 1 - \sum_{j=1}^{i} p_j - p_0. \quad i = 1,2,\ldots,24 \qquad (5)$$

The constant p_0 is the cancellation rate. Since it is the anticipated cancellation rate which is crucial for the behaviour of the TMLs, the value given to p_0 depends on the assumptions made about anticipations. If p_0 is set at zero the assumption is that no cancellations are expected until the end of the twenty-third month when all cancellations occur at once. If p_0 is set at .0735 the assumption is that the TML companies automatically discount 7.35 per cent[6] of the approvals they make to allow for anticipated cancellations. The set of weights used to derive the backlog series used in the mortgage approvals equations is given in Table VII 2 and it is based on p_0 = .0735.

TABLE VII ^ DERIVED UNDISBURSED COMMITMENT WEIGHTS

LAG	WEIGHT	LAG	WEIGHT
1	.8303	13	.2066
2	.7446	14	.1776
3	.6679	15	.1504
4	.5992	16	.1248
5	.5376	17	.1010
6	.4821	18	.0789
7	.4318	19	.0588
8	.3862	20	.0410
9	.3444	21	.0256
10	.3606	22	.0133
11	.2706	23	.0045
12	.2375	24	.0000

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[6] The implied average rate of cancellations in Table VII 1.

3. Mortgage Approvals

Changes in TML mortgage lending are brought about primarily by changes in prior approvals for new loans. Rather than adopting an equation from the existing literature on mortgage investment I follow Jorgenson's well known model of business fixed investment (1963, 1965) which is well suited to mortgage investment. The approach which is used may be summarized as follows.

In any time period the investor has in mind a desired "mortgage commitment" by which I mean a desired level of outstanding mortgage loans plus a desired backlog of as yet undisbursed approved loans. New mortgage approvals are made in the period to approach the desired commitment from the actual commitment at the beginning of the period. In ideal circumstances the investor might be able to assess his desired mortgage commitment with precision. In such circumstances approvals might be made so that the total mortgage commitment is brought into exact equality with the desired total. But these are not the circumstances of a real world mortgage market. The model is therefore modified to allow for incomplete adjustment to a gap between actual and desired and for the inherent momentum of the mortgage approvals process.[7]

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[7] See Chapter V, Section 4.

The traditional partial-adjustment model is not satisfactory for mortgage lending. Over-adjustment is likely to occur in the mortgage market and over-adjustment (even complete adjustment) is precluded by the traditional model. Over-adjustment comes about because of the imprecise control of mortgage loan approvals. Once the machine has been cranked up to generate a large volume of approvals it is difficult to slow it down and vice versa.

Consider equation (6):

$$APRO_t - REPAY_t = \gamma_1(MTGB_t^* - MTGB_{t-1}) + \gamma_2(APRO_{t-1} - REPAY_{t-1}) \tag{6}$$

By equation (6) approvals in excess of repayments are made to close the gap between the desired commitment ($MTGB_t^*$) and the actual commitment ($MTGB_{t-1}$).[8]

Along the lines of the traditional model it is assumed that only the fraction γ_1 of the gap between desired and actual is made good in period t. The lagged dependent variable is included so that the momentum of previous high (or low) approvals in t-1 carries over into period t: it is this term which will cause overshooting to take place. For a stable process it is necessary that γ_2 be less

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[8] It should be obvious that $MTGB_t$ is the sum at time t of outstanding mortgage loans and undisbursed approvals.

than unity.[9] Equation (4) has the required property that in a steady state approvals in each period will be equal to repayments.

The Desired Mortgage Commitment

The desired mortgage commitment, it would be widely accepted, is a function of the scale of the investor and of the yield on mortgages relative to other yields. These other yields include the interest rate that has to be paid to attract the liabilities that are used to finance mortgage lending: mortgage intermediaries have a choice at the margin of financing mortgage lending by increasing liabilities or by reducing other assets. In the long run the ease or difficulty with which deposits are attracted will affect the volume of mortgage lending.

Therefore the interest rate on over-one-year term liabilities should be included as an explanatory variable for TML mortgage approvals. Another interest rate which should be included is the corporate bond rate: on the asset side of the TML balance sheet the instruments which are most likely to be good substitutes for mortgages (considering yield and marketability attributes) are corporate bonds.[10] The attributes of government bonds (particularly federal

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[9] γ_2 should be positive.

[10] See Chapter IX for a discussion of the relevant interest rate correlations. The correlation coefficient of RMTG with RCORP is .9685.

bonds) rather separate them from the higher yield assets.[11] Governments are held primarily as a liquidity reserve, complementing corporates and mortgages, not substituting for them.[12] So the interest rates which best represent the opportunity costs of making mortgages are those on over-one-year GICs and corporates. Of the two rates the GIC rate should perhaps have the larger impact since risk aversion compels the TMLs to match their mortgages with over-one-year deposits, a change in the cost of which has a direct impact on the profitability of mortgage lending.

Equation (7) has desired mortgages as a linear distributed lag function of the yield on mortgages (RMTG), the yield on over-one-year GICs (TGIC2), the yield on corporate bonds (RCORP) and the level of total major liabilities (LIAB).

$$\mathrm{MTGB}_t^* = \sum_{i=1}^{24} a_i \ \mathrm{RMTG}_{t-1} + \sum_{i=1}^{24} b_i \ \mathrm{RGIC2}_{t-i} + \sum_{i=1}^{24} c_i \ \mathrm{RCORP}_{t-i} + \sum_{i=1}^{24} d_i \ \mathrm{LIAB}_{t-i} + C \tag{7}$$

The function is written in distributed lag form to capture delays in the determination of desired mortgages due to inertia or expectations.[13] The coefficients of the lagged variables are denoted by

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[11] Over the period 1967-72 the mean for RGOVT was 6.3 per cent, the mean for RCORP was 8.3 per cent and the mean for RMTG was 9.3 per cent.

[12] This view is confirmed econometrically in Chapter VIII. The correlation coefficient between RGOVT and RMTG is .6180 which is certainly low enough to allow complementarity.

[13] The interpretation of Modigliani and Sutch (1966) is applicable.

a_i, b_i, c_i and d_i. A twenty-four month period seems sufficiently long to capture the delayed responses (the estimates are not sensitive to changes in this period).

The firm prior expectations are that the overall response from an increase in the mortgage rate should be positive, that the overall response from the other two rates should be negative and that total liabilities should make a positive contribution to the desired mortgage commitment. Less firmly, the long run response to a change in the own rate should exceed the long run response to an equal change in either of the competing rates. Less firmly still, the response to a change in the own rate should exceed the sum of the responses of the competing rates, and perhaps this should happen not just in the long run but throughout the period of adjustment to the long run. It is conceivable that some of the a_i be negative and that some of the b_i and c_i be positive -- following Modigliani and Sutch, this would manifest extrapolative interest rate expectations.

Since the quantity of major liabilities is largely determined by the decisions of the TML sector itself, the response of approvals to this variable is to be understood as a response only to exogenous changes in its level. That is, if there is an inflow of deposits greater than the TMLs had planned for then gradually over time there will be an increase in mortgage approvals. A planned it is less strongly believed that

$$\sum_{i=1}^{j} \alpha_i > - \sum_{i=1}^{j} \beta_i - \sum_{i=1}^{j} \xi_i . \quad j = 1,2, \ldots, 24 \qquad (12)$$

Assuming that each profile of distributed lag weights can be represented by a polynomial, the Almon procedure can be used to calculate them. OLS estimates of (8) in various guises were made and a representative selection is displayed in Table VII 3.

Equation 1 of Table VII 3 is an estimate which strikingly confirms the hypothesis. Various other combinations of Almon variables produced similar qualitative results but, as is bound to happen, some combinations of Almon variables do not put the theory in such a good light (e.g. Equation 2).[15] Taking the two equations together the maintained hypothesis is given solid empirical support. The level of explanatory power is high considering that the dependent variable has the dimensions of a first difference in a stock. All regressors make a meaningful contribution to the explained variance. There is no serial correlation in the residuals.

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[15] The negative weights on RMTG and the positive weights on RGIC2 for short-period lags in Equation 2 could be adduced as evidence of extrapolative short-run interest rate expectations but the adduction is vitiated by the uniformly negative weights on RCORP. If there are short-run extrapolative interest rate expectations they should apply to corporates too and the short-period lagged weights on RCORP should be positive.

inflow of deposits caused by the TMLs' own mortgage financing decisions will not affect future approvals because the source of funds has been preempted.

4. The mortgage approvals equation

To take account of seasonal variation and the peculiar end-of-greater effects of the NHA interest rate ceiling between 1967 and 1969 dummy variables can be used.[14] Then, substituting (7) into (6), the hypothesis becomes

$$\mathrm{APRO}_t - \mathrm{REPAY}_t = \sum_{i=1}^{24} \alpha_i \, \mathrm{RMTG}_{t-i} + \sum_{i=1}^{24} \beta_i \, \mathrm{RGIC2}_{t-i} + \sum_{i=1}^{24} \xi_i \, \mathrm{RCORP}_{t-i} + \sum_{i=1}^{24} \delta_i \mathrm{LIAB}_{t-i} - \gamma_1 \, \mathrm{MTGB}_{t-1} + \gamma_2 \, (\mathrm{APRO}_{t-1} - \mathrm{REPAY}_{t-1}) + C + \text{seasonals} + \mathrm{QNHA} \qquad (8)$$

The prior notions can be summarized by noting that it is strongly believed that

$$\sum_{i=1}^{24} \alpha_i > 0, \quad \sum_{i=1}^{24} \beta_i < 0, \quad \sum_{i=1}^{24} \xi_i < 0 \qquad (9)$$

and that

$$0 < \gamma_1, \gamma_2 < 1 \qquad (10)$$

and that

$$0 < \delta_i < 1 \text{ with all } \delta_i > 0; \qquad (11)$$

TABLE VII 3 MORTGAGE LOAN APPROVALS (OLS)

The dependent variable is approvals net of repayments, APRO-REPAY

Eqn.	C	Almon variables[a]						$MTGB_{t-1}$	$(APPRO\text{-}REPAY)_{t-1}$	QNHA	Seasonals		
		RMTG(DG1)	RMTG(DG2)	RMTG(DG3)	RGIC2(DG2)	RCORP(DG2)	LIAB(DG2)				Q1	Q2	Q3
1.	-277.4	361.05	-510.84	345.58	-62.9	-119.02	.0551	-.0611	.5719	54.53	-2.78	11.01	-2.92
	(-3.60)	(1.12)	(-.73)	(.85)	(-1.87)	(-2.50)	(2.04)	(-1.95)	(5.28)	(6.68)	(-.56)	(1.96)	(-.65)

	C	Almon variables[b]								$MTGB_{t-1}$	$(APRO\text{-}REPAY)_{t-1}$	QNHA	Seasonals		
		RMTG(DG1)	RMTG(DG2)	RMTG(DG3)	RGIC(DG2)	RGIC(DG3)	RCORP(DG2)	RCORP(DG3)	LIAB(DG3)				Q1	Q2	Q3
2.	-457.7	796.91	352.00	-745.87	-1243.09	1053.73	-272.51	796.92	.2809	-.2387	.5321	51.81	-6.51	2.55	4.10
	(-2.84)	(2.08)	(.51)	(-1.53)	(-3.62)	(3.27)	(-.71)	(2.08)	(4.60)	(-4.56)	(5.27)	(6.88)	(-1.40)	(.45)	(.92)

	C	Almon variables[c]					$MTGB_{t-1}$	$(APRO\text{-}REPAY)_{t-1}$	QNHA	Seasonals		
		RMTG(DG1)	RMTG(DG2)	RMTG(DG3)	RCORP(DG2)	LIAB(DG2)				Q1	Q2	Q3
3.	-264.41	622.21	-1065.15	590.44	-133.69	.0706	-.0888	.6580	56.58	-1.95	7.86	-2.50
	(-3.39)	(2.10)	(-1.65)	(1.50)	(-2.34)	(2.69)	(-3.18)	(6.60)	(6.89)	(-.39)	(1.44)	(-.55)

Equation 1 $\bar{R}^2 = .8515$ DW = 2.06
Equation 2 $\bar{R}^2 = .8749$ DW = 2.05
Equation 3 $\bar{R}^2 = .8463$ DW = 2.05

Notes. All the Almon weights were constrained to lie on a polynomial with a zero value at the end point (t-24).

a. The polynomial is of degree 3 for RMTG, and of degree 2 for RGIC2, RCORP and LIAB. The degree 2 polynomials were constrained to have zero slope at the end point of the lag distribution (t-24).

b. The polynomial is of degree 3 for all variables. The polynomials for RGIC2,RCORP and LIAB were constrained to have zero slope at the end point. The polynomial for LIAB was further constrained to have a zero coefficient at (t+1).

c. The degree is 3 for RMTG, and degree 2 (with zero slope constraints at the end points) for RCORP and LIAB.

TABLE VII 3 (continued) UNSCRAMBLED ALMON WEIGHTS

	Equation 1							
	RMTG		RGIC2		RCORP		LIAB	
LAG	WEIGHT	t-RATIO	WEIGHT	t-RATIO	WEIGHT	t-RATIO	WEIGHT	t-RATIO
1	22.79	2.15	-7.73	-1.87	-14.56	-2.49	.0010	2.04
2	19.91	2.31	-7.04	•	-13.32	•	.0014	•
3	17.38	2.47	-6.42	•	-12.14	•	.0018	•
4	15.20	2.61	-5.82	•	-11.01	•	.0021	•
5	13.32	2.73	-5.25	•	- 9.94	•	.0024	•
6	11.73	2.81	-4.71	•	- 8.92	•	.0027	•
7	10.38	2.89	-4.20	•	- 7.95	•	.0029	•
8	9.27	3.00	-3.72	•	- 7.05	•	.0031	•
9	8.35	3.14	-3.27	•	- 6.19	•	.0032	•
10	7.61	3.41	-2.85	•	- 5.40	•	.0033	•
11	7.01	3.82	-2.46	•	- 4.65	•	.0033	•
12	6.52	4.31	-2.09	•	- 3.96	•	.0033	•
13	6.13	4.55	-1.76	•	- 3.33	•	.0033	•
14	5.80	4.16	-1.45	•	- 2.75	•	.0032	•
15	5.51	3.40	-1.18	•	- 2.23	•	.0031	•
16	5.23	2.72	- .93	•	- 1.76	•	.0029	•
17	4.92	2.23	- .71	•	- 1.35	•	.0027	•
18	4.58	1.89	- .52	•	- .99	•	.0024	•
19	4.15	1.65	- .36	•	- .69	•	.0021	•
20	3.63	1.48	- .23	•	- .44	•	.0018	•
21	2.98	1.35	- .13	•	- .25	•	.0014	•
22	2.18	1.25	- .06	•	- .11	•	.0010	•
23	1.19	1.18	- .01		- .03		.0005	
24	.00		.00		.00		.0000	
SUM	195.79		-62.91		-119.02		.0551	
Long-run								
Coefficient	3196.72		-1031.15		-1950.82		.9039	
Elasticity	5.363		-1.354		-2.890		1.399	

	Equation 2.							
	RMTG		RGIC2		RCORP		LIAB.	
LAG	WEIGHT	t-RATIO	WEIGHT	t-RATIO	WEIGHT	t-RATIO	WEIGHT	t-RATIO
1	-9.65	-.65	12.99	2.32	-13.84	-2.26	.0093	4.60
2	-1.33	-.11	14.60	2.16	-18.16	-2.48	.0128	•
3	5.86	.64	13.81	1.94	-21.05	-2.75	.0155	•
4	11.98	1.55	11.20	1.65	-22.71	-3.10	.0176	•
5	17.09	2.34	7.31	1.20	-23.35	-3.51	.0191	•
6	21.24	2.83	2.60	.50	-23.14	-3.94	.0200	•
7	24.51	3.06	-2.50	-.57	-22.26	-4.17	.0204	•
8	26.93	3.16	-7.62	-2.01	-20.85	-3.95	.0203	•
9	28.58	3.19	-12.45	-3.31	-19.07	-3.32	.0198	•
10	29.51	3.20	-16.73	-3.97	-17.03	-2.62	.0190	•
11	29.78	3.20	-20.26	-4.15	-14.85	-2.03	.0179	•
12	29.45	3.20	-22.87	-4.15	-12.63	-1.58	.0165	•
13	28.58	3.19	-24.46	-4.09	-10.45	-1.24	.0149	•
14	27.22	3.17	-25.00	-4.02	- 8.39	- .98	.0132	•
15	25.44	3.15	-24.49	-3.95	- 6.51	- .77	.0114	•
16	23.30	3.11	-22.98	-3.89	- 4.83	- .62	.0096	•
17	20.84	3.05	-20.59	-3.84	- 3.41	- .48	.0078	•
18	18.14	2.98	-17.49	-3.79	- 2.25	- .38	.0060	•
19	15.25	2.88	-13.90	-3.76	- 1.36	- .28	.0044	•
20	12.23	2.77	-10.09	-3.72	- .72	- .21	.0030	•
21	9.13	2.63	- 6.38	-3.69	- .31	- .14	.0017	•
22	6.02	2.46	- 3.17	-3.67	- .09	- .08	.008	•
23	2.96	2.28	- .88	-3.64	- .01	- .03	.0002	
24	.00		.00		.00		.0000	
SUM	403.04		-189.36		-267.29		.2809	

	Equation 3					
	RMTG		RCORP		LIAB	
LAG	WEIGHT	t-RATIO	WEIGHT	t-RATIO	WEIGHT	t-RATIO
1	15.85	1.57	-13.91	-2.34	.0013	2.69
2	12.90	1.63	-12.73	•	.0018	•
3	10.49	1.72	-11.59	•	.0023	•
4	8.56	1.83	-10.52	•	.0027	•
5	7.07	1.95	- 9.49	•	.0031	•
6	5.97	2.09	- 8.52	•	.0035	•
7	5.21	2.23	- 7.60	•	.0037	•
8	4.76	2.39	- 6.73	•	.0039	•
9	4.55	2.61	- 5.92	•	.0041	•
10	4.55	2.94	- 5.15	•	.0042	•
11	4.71	3.40	- 4.44	•	.0043	•
12	4.97	3.86	- 3.79	•	.0043	•
13	5.31	4.10	- 3.18	•	.0042	•
14	5.66	3.99	- 2.63	•	.0041	•
15	5.93	3.67	- 2.13	•	.0039	•
16	6.23	3.33	- 1.68	•	.0037	•
17	6.37	3.03	- 1.29	•	.0035	•
18	6.33	2.79	- .95	•	.0031	•
19	6.08	2.61	- .65	•	.0027	•
20	5.57	2.46	- .42	•	.0023	•
21	4.76	2.34	- .24	•	.0018	•
22	3.59	2.24	- .11	•	.0013	•
23	2.02	2.17	- .03		.0007	
24	.00		.00		.0000	
SUM	147.60		-113.69		.0706	

Equation 3, Table VII 3, is an equation with a more conventional specification.[16] It supresses the rate of interest on term liabilities as an explanatory variable. Under the usual statistical criteria this is another excellent equation and the estimated parameters are in every case consistent with the prior beliefs (9) - (12). The choice between the specifications embodied in Equations 1 and 3 cannot be resolved statistically because the data are consistent with both hypotheses. Both specifications have merit and it is a matter of judgement as to which should be preferred. My preference is for Equation 1 because this equation makes explicit that if it is more costly to obtain GIC financing for mortgages then there is a long-run inhibiting effect on TML mortgage lending. In Equation 3 the effect of the costs of attracting term deposits on mortgage loan approvals is captured only indirectly through the quantity of outstanding liabilities.

5. The long run

The equation which has been tested, equation (8), convolutes two lag structures. One lag structure is common to all explanatory variables -- the lag attributed to partial adjustment

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[16] Cf. Smith and Sparks (1971) or Silber (1970).

to desired asset stocks. This common lag structure is convoluted with the set of lag structures which pertain to each variable separately and which determine the desired stock of mortgages. It is easy enough to derive the implicit long-run (or equilibrium) coefficients from the regression coefficients (which are short-run coefficients). Comparing (6), (7) and (8) it can be seen that $\alpha_i = \gamma_1 a_i$, $\beta_i = \gamma_1 b_i$, $\xi_i = \gamma_1 c_i$ and $\delta_i = \gamma_1 d_i$. Therefore it follows that

$$\begin{aligned} a_i &= \Sigma\alpha_i/\gamma_1, \\ b_i &= \Sigma\beta_i/\gamma_1, \\ c_i &= \Sigma\xi_i/\gamma_1, \qquad (13) \\ d_i &= \Sigma\delta_i/\gamma_1. \end{aligned}$$

The summations on the left-hand side of the equalities in (13) are the equilibrium coefficients which are sought. The values of these coefficients for Equation 1 are

$$\Sigma a_i = 3196.72, \quad \Sigma b_i = -1031.15,$$

$$\Sigma c_i = 1950.82, \quad \Sigma d_i = .9039.$$

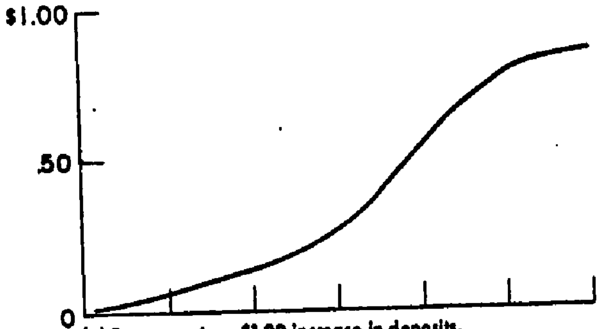
Some words of caution: these long-run coefficients refer to the *cumulative* effect on approvals of a persisting unit increment in one of the independent variables. They do not measure the equilibrium change in approvals following, say, an increase in RMTG: in full equilibrium the change is zero because approvals are equal to repayments regardless of interest rates. The coefficients indicate the long-run response of the desired stock not the long-run response of the flow of approvals (note that since there are cancellations of approved loans the cumulative sum of approvals net of repayments overstates the change in the total mortgage commitment).

The dynamic paths of cumulative response implicit in the lag schemes of Equation 1 are shown graphically in Chart VII 1. The paths for each of the explanatory variables differ from one to another but the profiles are similar in character. The long-run response is approached monotonically, and after about thirty months the paths converge asymptotically onto the limiting values. It is over fifteen months before one half of the total effect of a change in RCORP or RGIC is felt, over eighteen months before one half of the total effect of a change in RMTG is felt and over twenty-one months before one half of the total effect of a change in LIAB is felt. These are the sluggish responses which one expects to find. Bearing in mind that disbursements lag approvals the attainment of desired levels of mortgage lending is seen to be a very slow affair.

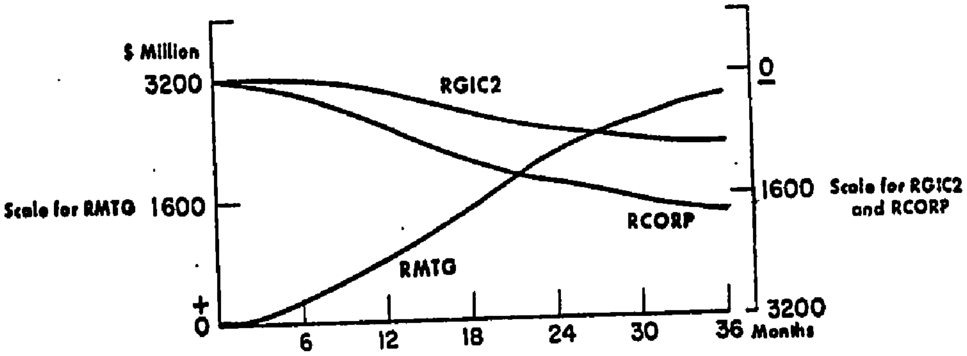
Despite the slow adjustment to long-run totals there is a significant short-run impact on mortgage approvals from a change in the variables which determine it because the total long-run effects are so large. The long-run elasticities of mortgage investment appear to be very high: the estimated elasticity with respect to RMTG is 5.363, with respect to RGIC2 it is -1.354 and with respect to RCORP it is -2.890 (Table VII 3). Thus, according to equation 1, if the yield on mortgages increases by a percentage point (everything else remaining the same) mortgage approvals net of repayments increase by $923.10 million within the year.

CHART VII 1

CUMULATIVE DYNAMIC RESPONSE OF MORTGAGE LOAN APPROVALS



(a) Response to a $1.00 increase in deposits.



(b) Response to a 100 basis point increase in interest rates on over one year GICs, corporate bonds and mortgages respectively.

These dynamic responses were derived from the coefficients in Equation 1.

CHAPTER VIII

Portfolio Balance Adjustments

1. Introduction

The purpose of this chapter is to explain the TML sector's demand for assets other than mortgages and its supply of term deposits, and at the same time to test the theory derived in Chapters II, III and IV. The model of portfolio behaviour that I have estimated statistically can be outlined with the aid of Flowchart 2. Mortgage lending and its finance are at the core of the system. In a monthly context mortgage lending is essentially predetermined:[1] the levels of six balance sheet components can be changed to accommodate mortgage lending. These six items are four assets (cash and short-term paper, federal government bonds, municipal and provincial bonds, and corporate bonds) and two categories of term deposit (GIC1 and GIC2). The major source of finance for mortgages will presumably be over-one-year term deposits (a movement along Arrow L) due to the hedging that they provide. But, depending on interest rates, other sources of finance might be used in some degree. Under-one-year deposits

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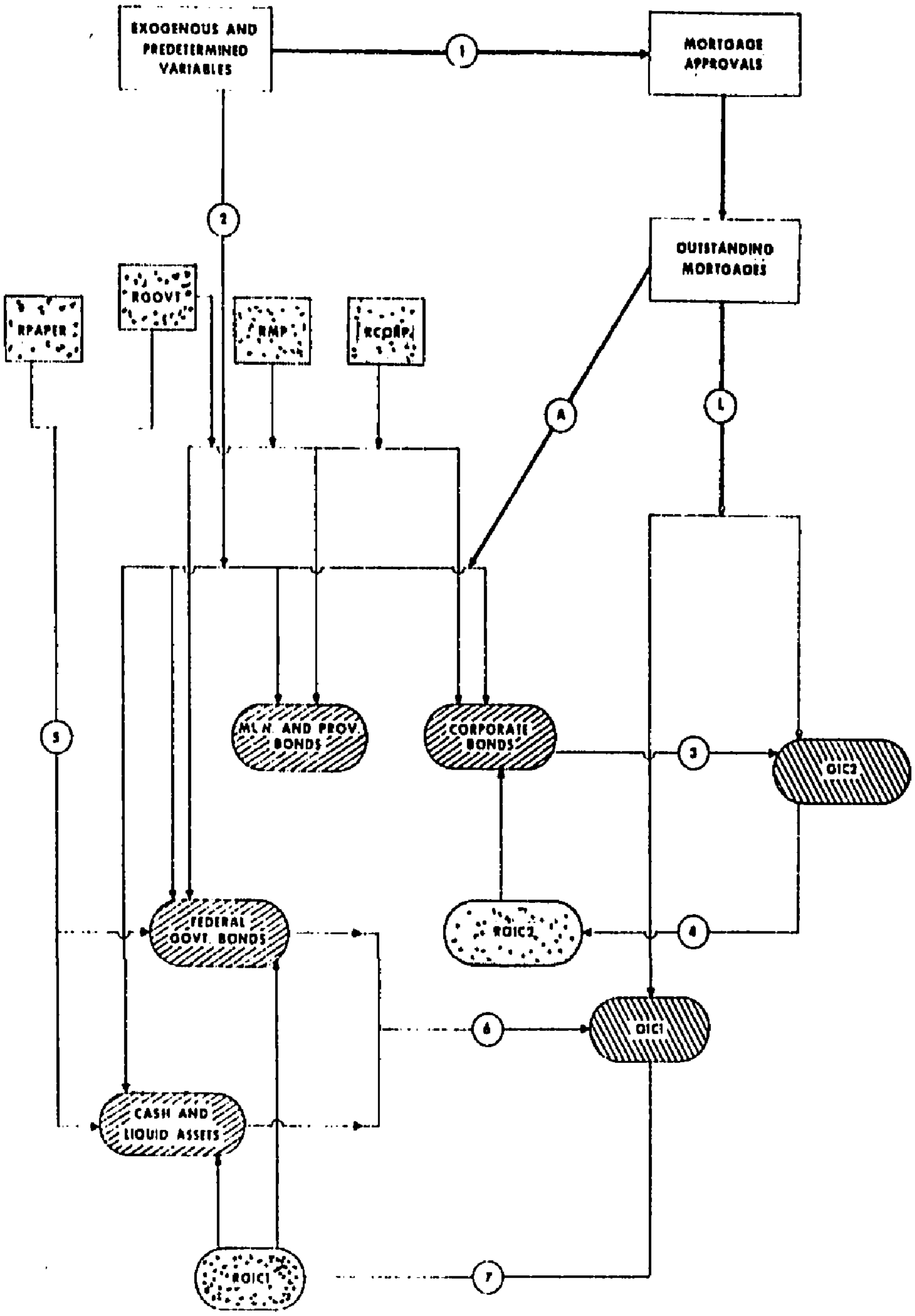
[1] The predetermined variables which explain mortgage lending include lagged endogenous variables such as the rate of interest paid on over-one-year term deposits. There is a "feedback" to mortgages but not simultaneously.

FLOWCHART 2

A SCHEME FOR TML BALANCE SHEET ADJUSTMENTS



Predetermined variables are depicted in rectangles, endogenous variables are in ovals.

▨ Assets
▧ Liabilities
▒ Interest rates

might be attracted (Arrow L also) or other assets might be sold (Arrow A).

Consider the case in which it is decided to fund mortgage disbursements with an increase in GIC2. The decision to raise GIC2 funds necessarily involves an increase in the GIC2 rate, since the industry has to attract the money from other sectors of the economy. Here the flowchart goes into a loop (arrows ③ and ④ since any bidding up of RGIC2 by the industry changes the circumstances facing each firm. A higher GIC2 rate will lead them to look for other sources of funds. Since the only asset that competes with mortgages as a use for GIC2 money is corporate bonds, I assume that GIC2 money may be freed by selling corporates <u>as a response to an increased GIC2 rate</u> (hence the loop through corporates in Flowchart 2). The direct response of non-mortgage assets (along arrow A) involves sales of bonds and liquid assets for mortgage finance at given interest rate levels. (Notice that corporates respond <u>directly</u> to an increase in mortgages at given interest rates and then <u>indirectly</u> to the higher interest rate on GICs caused by the increased mortgage lending.) There remains a further source of mortgage finance (apart from savings deposits which I take to be demand determined) in less-than-one-year GICs. This source is included in the model; but its dominating feature is the set of linkages running from mortgage approvals to over-one-year GICs.

More or less incidental to the central linkages is the short-term business -- largely a process of intermediation with less-than-one-year GICs as the source of funds and with finance company and commercial paper as the use. The size of this business naturally depends on its profitability, as summarized by interest rates on the relevant instruments. Consider, for instance, the effect of an increase in short-term paper rates. The TMLs, other things being equal, are stimulated to buy more short-term paper. To do so they supply more liabilities -- and these are necessarily short-term liabilities since the interest rate spread is against long-term deposits -- bidding up RGIC1 which ultimately chokes off the expansion. In the new equilibrium TML holdings of paper and GIC1 are greater than initially, as is RGIC1.

The short-term business linkages are represented by the loop through ⑥ and ⑦ , and arrow ⑤ . The model does not completely separate into short-term and long-term compartments because federal government bonds are represented in each. They are held as a liquidity reserve asset but they also compete with the less liquid bonds. (The short-term loop and the long-term loop are, of course, imposed upon simultaneously by mortgage lending requirements -- this in itself does not amount to a connecting link between the compartments since mortgages come in recursively.) Therefore the set of eight equations comprising the portfolio balance system provides a simultaneous determination of four asset levels (liquid assets, Canadas, municipals and provincials, and corporates), two liability levels (GIC1 and GIC2), and two interest rates (RGIC1 and RGIC2). A detailed algebraic account of the specification is given in the next section where the connection between the present model and portfolio theory is made clear.

2. Portfolio theory for the TMLs

This section takes as given two variables the behaviour of which has been analyzed in previous chapters as functions of pre-determined variables -- mortgage lending (MTG) and savings deposits (SAVDEP). The mnemonic notations adopted are PAPER (short-term paper, various foreign currency assets, cash, and chartered bank CDs) GOVT (government of Canada treasury bills and bonds) MP (municipal and provincial bonds) CORP (corporate bonds) and GIC1 and GIC2 (less-than-one-year and over-one-year term liabilities respectively): these items are endogenous. There is an additional residual variable, NET (total major assets minus total major liabilities).[2] Interest rate mnemonics have the prefix R.

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[2] NET contains a number of things for which there is no monthly information, primarily net worth. It does not play a great role in subsequent discussions, and is treated as exogenous. The conditions under which other items may be aggregated into net worth are worked out in Chapter IV, Section 1.

The four asset demand functions and the two liability supply functions of the TML sector are assumed to be adequately represented by linear equations. In principle each equation should contain all interest rates -- everything depends on everything else. In practice there is a pay-off from using institutional knowledge, casual theory and preliminary investigation of the data to omit certain interest rates from certain equations -- the omissions being subject to empirical verification. The pay-off is, as always, that zero restrictions enhance the efficiency of statistical estimation if they are correct and may result in estimates with a lower mean squared error even when they are incorrect.[3] The structure outlined in the flowchart is predicated on the zero restrictions embodying my assessment of TML behaviour. The estimates presented in this chapter[4] are essentially those of this structure.

The clearest way to set out the equations is the matrix form. The vector of TML decision variables is y, the vector of lagged endogenous variables is y_{-1} and the vector of "explanatory"

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[3] See Theil (1962), p. 333.

[4] The gain in efficiency in this application of constrained estimation appears considerable. Appendix 2 of this chapter gives unrestricted results which can be compared with the restricted estimates of this chapter. The increased t-ratios can be attributed to the imposition of the constraints.

variables[5] (some of them are endogenous to the system) is x where y and x are defined as

$$y = \begin{bmatrix} \text{PAPER} \\ \text{GOVT} \\ \text{MP} \\ \text{CORP} \\ \text{-GIC1} \\ \text{-GIC2} \end{bmatrix} \quad \text{and } x = \begin{bmatrix} \text{RPAPER} \\ \text{RGOVT} \\ \text{RMP} \\ \text{RCORP} \\ \text{RGIC1} \\ \text{RGIC2} \\ \text{NET} \\ \text{SAVDEP} \\ \text{MTG} \end{bmatrix}$$

The matrices $A = (\alpha_{ij})$, $B = (\beta_{jk})$ and $\Gamma = (\gamma_{ij})$ with $i,j = 1,2, \ldots, 6$ and $k = 1,2,3$ are coefficient matrices of the six equations. The matrix of interest rate coefficients is A, B is the matrix of coefficients for the exogenous balance sheet items and Γ is the matrix of coefficients for the lagged endogenous variables.

The presence of the lagged endogenous variables brings into the model the dynamic considerations that have been all but ignored in the flowcharts. The topic is extensively treated in Chapter IV where particular emphasis is given to the necessity of specifying asset demand functions obeying the balance sheet constraint in

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[5] RPAPER is the average rate on ninety-day commercial and finance company paper. RPAPER is supposed to be a representative yield for the assets in PAPER.

disequilibrium. Recall that there is a choice of economically meaningful restrictions[6] on the coefficients to guarantee the existence of the adding-up constraints. I make use of the following:

$$\sum_{i}^{6} \alpha_{ij} = \sum_{i}^{6} \gamma_{ij} = 0 \qquad j = 1,2, \ldots, 6 \qquad (1)$$

and

$$\sum_{i}^{6}\beta_{i1} = \sum_{i}^{6}\beta_{i2} = -\sum_{i}^{6}\beta_{i3} = 1. \qquad (2)$$

An immediate clarification is necessary. The β_{ik} apply to exogenous balance sheet components -- β_{i1} to NET, β_{i2} to SAVDEP, β_{i3} to MTG. Since NET and SAVDEP are liabilities their coefficients sum to unity (an increase in either NET or SAVDEP must be allocated somewhere) and since MTG is an asset its coefficients must sum to minus unity (if mortgages rise by one dollar then the sum of other assets must decrease by one dollar because of the balance sheet identity). One should also be aware that I have lapsed into the convention of treating liabilities as negative assets so that decreases in other liabilities could provide the entire required offset to an increase in NET of SAVDEP and increases in liabilities could provide the dollar required by a dollar increase in mortgages.

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[6] See Chapter IV,Sections 3 and r. I make no use of the initial disbursement concept according to which "new" wealth is treated differently to existing wealth.

An increase in a liability is formally equivalent to a decrease in an asset.

The four asset demand equations are conveniently written with the two liability supply equations appended beneath:

$$y = [A \quad B]\, x + \Gamma y_{-1}. \tag{3}$$

Two behavioural relations of the rest of the economy are closely associated with system (3). These are the demand functions for term liabilities[7] that, in conjunction with the supply equations in (3), determine the quantities and interest rates for term deposits and GICs. The demand functions have already been consistently estimated (Chapter VI) and it is the TMLs' behaviour on which I concentrate in this chapter.

What can reasonably be asserted a priori about the elements in the matrices A,B and Γ? There are the known adding up restrictions (1) and (2). These restrictions will be embodied into the estimating procedure. The other assertions are not so obvious and are left to the data to decide.[8] First there is a strong theoretical presumption that the diagonal elements of A and Γ are positive.

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[7] The specification for savings deposits demand has only predetermined variables as arguments. This may not be correct (see Chapter V, Section 5). Therefore in the constrained 3LS estimates of this chapter an instrument is used for SAVDEP so that the estimates are consistent whether the savings deposits equation is right or wrong.

[8] The theoretical case for these assertions is put in Chapters II and IV.

$$\alpha_{ii},\ \gamma_{ii} \gtrsim 0. \quad i = 1,2,\ldots,6 \qquad (4)$$

Second, there is a weaker presumption that gross substitution effects are symmetric (suggested by the symmetry of net substitution effects). This might be expressed in the weak form that on expects γ_{ij} and γ_{ji} to take the same sign (i.e. one expects either gross complementarity or gross substitutability, not both at once). Third there is a suggestion in the EV theory that γ_{ij} and γ_{ij} take the same sign. Finally one might expect a tendency for own rate responses to exceed cross responses (i.e. for α_{ii} typically to exceed in absolute value α_{ij} $i \neq j$). If the theory is to be believed these presumptions should be apparent to some degree in the empirical estimates.

3. Econometric Test

The system actually estimated is not quite that of system (3), in that (a) quarterly seasonal dummies are used to explain seasonal variation (the seasonal effects are made to sum to zero over each year and over the asset categories in the portfolio at any point of time; (b) a constant term is included and (c) a new variable NET* is constructed (NET* = NET + SAVDEP - MTG) and used in place of NET. Empirical estimates are presented in Table VIII 1. Equations are to be read column-wise. The sums of coefficients across the rows are zero for all explanatory variables except NET* for which the sum is unity. It may seem at odds with the adding up restrictions of the previous section to constrain the sum of coefficients of SAVDEP and the sum of coefficients of MTG to zero but these restrictions do in fact hold. Some simple arithmetic using

NET*'s coefficients unscrambles the restrictions and returns the regression equations to the form of equation (3)(which is the natural form for asset demand functions to be written):(a) subtract the coefficients of NET* from the coefficients for MTG to get MTG coefficients that sum to minus one and (b) add the coefficients of NET* to the coefficients for SAVDEP to get SAVDEP coefficients that sum to one.[9] One then has restored the natural form (3) with NET as the explanatory variable rather than NET*[10].

The six portfolio equations for the TML are estimated as a simultaneous block subject to restrictions (1) and (2) by three-stage least squares (3LS). In the final stages of the 3LS procedure instruments are used for RGIC1, RGIC2, and SAVDEP. The instruments are the predicted values of these variables from reduced form regressions on all predetermined variables in the system. The estimates are consistent if the

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[9] Consider the demand for any asset y_i where r_j are the relevant interest rates (the lagged variables are not relevant to this point):

$$y_i = \sum_{j}^{6} \alpha_{ij} r_j + \beta_{i1} \text{ NET*} + \beta^*_{i2} \text{ SAVDEP} + \beta^*_{i3} \text{ MTG},$$

so that $\beta_{i2} = \beta^*_{i2} + \beta_{i1}$ and $\beta_{i3} = \beta^*_{i3} - \beta_{i1}$. And whereas it is true that $\sum_{j}^{6} \beta^*_{i2} = \sum_{i}^{6} \beta_{i3} = 0$ so it is also true that $\sum_{i}^{6} \beta_{i2} = -\sum_{i}^{6} \beta_{i3} = 1$.

[10] The reason for the apparently futile substitutions forward and backward into NET* is that this is necessary to get the appropriate constraints from the FINPT 4 computer programme.

specification is correct.

Restricted Estimation: A Digression

The efficiency of the estimates can be improved taking advantage of economic theory. Since the adding up conditions are firm knowledge the estimates are constrained to obey them. The kind of gain available from embodying the constraints into the estimation procedure can be illustrated with a simple example. Assume that the sum of the two coefficients in a three variable model is unity so that the domain of admissable values is the line $\beta_1 + \beta_2 = 1$. Consider Fig. 5.

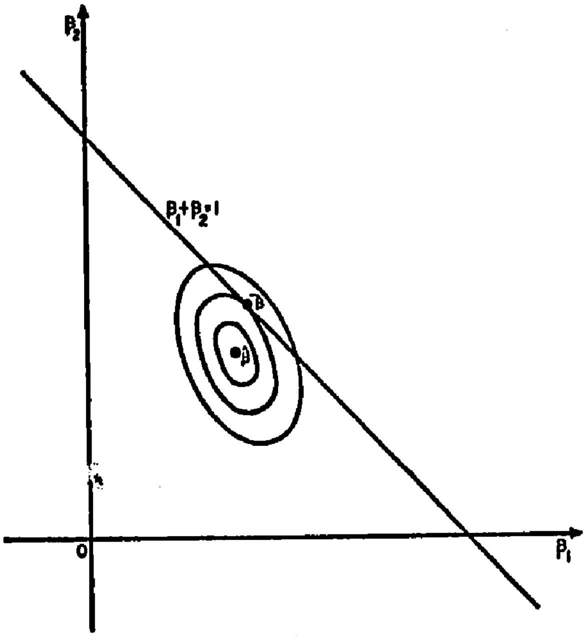

FIG. 5

The contours show constant sums of squared residuals. The minimum sum is at $\hat{\beta}$, the unconstrained least squares estimate. But the minimum sum consistent with prior knowledge is at $\tilde{\beta}$. In general $\hat{\beta}$ and $\tilde{\beta}$ do not coincide and in general $\hat{\beta}$ does not obey the theoretical restrictions. The R^2 will never be greater for $\tilde{\beta}$ than for $\hat{\beta}$ but the variance of $\tilde{\beta}$ will be less than that of $\hat{\beta}$. The reasoning in this paragraph is impressionistic and vague but the principle is sound and general.[11] Now for some equation counting. Six equations for the TML, two equations for the public and one balance sheet identity make nine equations. Only eight are linearly independent. Assuming consistency there are thus eight independent linear relations that can be solved fro the eight simultaneously dependent variables. The constrained 3LS estimates of the TML portfolio equations are presented in Table VIII 1. It should be noted that the residuals are not calculated using the actual values of the right-hand-side endogenous variables;they have instead been calculated using their instruments. The reader should also be aware that the liability supply functions have <u>negative</u> values of liabilities as their dependent variables. Similarly negative lagged values of GIC1 and GIC2 appear as independent variables in all the equations.

Empirical Evidence

Vacant elements in the cells of Table VIII 1 indicate that a variable is omitted from an equation. The decision to omit the particular combination of variables was made primarily on the basis of the theoretical and institutional considerations outlined in connection with the flowcharts. There are valid objections to constraining a coefficient to be zero just because preliminary regressions give estimates insignificantly different

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[11] Mathematical arguments are in Appendix 1.

TABLE VIII 1. CONSTRAINED 3LS ESTIMATES OF PORTFOLIO RESPONSES.

Explanatory variables		Asset demand functions				Liability supply functions	
		CASH AND LIQUID ASSETS	Bonds				
			FEDERAL GOVT	MUN. AND PROV.	CORPORATE	-GIC1	-GIC2
Interest rates ^{a}	RPAPER	128.59 (5.22)				-128.59 (-5.22)	
	RGOVT		20.11 (2.19)	-12.98 (-2.40)	10.06 (1.35)	-17.19 (-1.69)	
	RMP		-40.50 (-3.15)	41.32 (4.02)	-.82 (-.06)		
	RCORP		62.23 (3.59)	-53.00 (-3.80)	83.02 (3.57)		-92.24 (-4.34)
	RGIC1	-144.81 (-5.70)	5.87 (1.00)			138.94 (5.37)	
	RGIC2				-70.75 (-3.65)		70.75 (3.65)
Predetermined balance sheet components	NET*^{b}			-.0242 (-.36)	.3907 (3.14)		.6335 (4.28)
	SAVDEP^a		.1080 (2.44)	.0415 (.49)	-.3324 (-2.07)	.2504 (3.33)	-.0675 (-.34)
	MTG^a	.0738 (4.09)	.0131 (.98)	-.0175 (-.27)	.3949 (3.20)	-.3134 (-5.41)	-.1509 (-.97)
Lagged dependent variables^a	PAPER$_{-1}$	.5856 (7.64)				-.5856 (-7.64)	
	GOVT$_{-1}$		.3517 (3.08)	.2214 (2.42)	-.2430 (-2.05)	-.3300 (-1.85)	
	MP$_{-1}$		-.2025 (-2.13)	.5712 (6.75)	-.3687 (-3.38)		
	CORP$_{-1}$		-.2207 (-3.70)		.2207 (3.70)		
	-GIC1$_{-1}$	-.0865 (-1.15)				.3377 (4.43)	-.2512 (-3.48)
	-GIC2$_{-1}$					-.3416 (-6.46)	.3416 (6.46)
	CONSTANT^a	5.37 (.09)	5.32 (.07)	204.71 (3.34)	76.99 (.83)	-1.81 (-.02)	-290.58 (-2.61)
Seasonals^a	Q1	5.59 (.66)	1.36 (.36)	5.09 (1.29)	-4.66 (-.68)	-1.05 (-.16)	-6.32 (-.60)
	Q2	.29 (.03)	7.58 (1.75)	2.96 (.91)	1.80 (.35)	19.15 (2.37)	-31.77 (-3.16)
	Q3	6.44 (.72)	-9.92 (-2.56)	-4.66 (-1.29)	11.69 (1.76)	-25.49 (-3.76)	21.93 (2.06)
	$\bar{R}^2$	.9815	.9166	.9222	.9354	.9882	.9982
	DW	1.62	1.48	1.41	1.39	1.59	1.37

a. The coefficients sum to zero across the rows.
b. The coefficients of NET* sum to one across its row.

from zero or just because the sign is "wrong" and I have refrained from doing this unless the t-ratios were very low (e.g. less then .5) or unless the value of a coefficient conflicted strongly with my prior notions. Now this sort of "cleaning up" makes the final estimates conform to the prior notions and the estimates are perhaps not a very strong test of the empirical validity of the notions. This is a perennial problem in applied econometrics.[12]

How does the model stand up to empirical test? Although the reported results are to some extent loaded in favour of the maintained hypothesis they nevertheless provide it with firm empirical support. The estimated own rate responses are all significantly positive as are the diagonal elements of the estimated Γ matrix. The pattern of signs is symmetric. There is a tendency for β_{ij} and γ_{ij} to take the same sign. In three cases (out of the possible six) the own-rate response is greater in absolute value than the response to any other interest rate. There is some evidence of positive serial correlation in the residuals of the equations but otherwise the results leave one satisfied with the specified model. The strong theoretical presumptions are clearly upheld and even the weaker presumptions tend to be supported by the statistical results.

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[12] The research strategy that led to these estimates is detailed in Appendix 2. The number of combinations of variables that could be "tried" in a system of the present size is almost infinite and I limited myself to less than ten -- the best of which is reported in Table VIII 1. As can be seen by looking at the unrestricted regressions in Appendix 2 which put the model in its worst light the zero restrictions basically improve on results that are in any case broadly consistent with the theory.

There is a question to be considered which was raised in Chapter III, the question of whether the TML companies behave as if they have constant absolute risk aversion and as if returns on their portfolios are normally distributed -- i.e. is the matrix of interest rate coefficients symmetric? This is not an easy question to deal with in a model containing lagged responses. Presumably symmetry is to hold for the long-run responses but not necessarily for the short-run responses.[13] The long-run matrix of interest rate coefficients is discussed in Section 5 of this chapter but, to anticipate a little, it does not look any more or less symmetric than the short-run matrix in Table VIII 1. These matrices both look sufficiently unsymmetric that imposition of symmetry conditions on the estimates would not be warranted. The same is true for all of the estimates of this block of equations which I have made so that, whilst I have not made a formal test of the symmetry hypothesis, it is my firm belief that it is unduly restrictive.[14]

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[13] If the reason for time lags is a quadratic brokerage cost function (as in Chapter IV, Section 2) then the assumptions of CARA and normality do imply symmetry in the short run.

[14] A formal test of symmetry would have involved a disproportionate computational burden. Parkin has advocated the use of symmetry restrictions primarily (it seems to me) because of multicollinearity problems with his data. Multicollinearity does not seem to have put the present estimates off the scent.

4. The constrained estimates.

The implications of the regression coefficients are more easily understood if the direct estimates of the short run structure are transformed by the method explained in Section 2. Point estimates of the transformed structure are displayed in Table VIII 2; the implicit short-run elasticities are also given. For the sake of brevity I discuss the point estimates as though they are certainly accurate.

The short run intermediation of the TMLs is, even within one month, strongly influenced by relative interest rates. Other things being equal, an increase of one per cent in the rate on short term paper causes the TMLs to increase their liquid asset holdings by \$128.59 million financed by issues of less than one year GICs.[15] Since federal bonds serve as a liquidity reserve I hoped to capture a substitution relationship between short-term paper and Canadas but the results do not show it. It may well be that Canadas and short-term paper have attributes sufficiently dissimilar that the TMLs do not regard them as substitutes; there was no measurable decrease in government bond holdings resulting from an increase in RPAPER. Now consider a one per cent increase in RGIC1. The extra expense involved in attracting less-than-one-year money reduces the TMLs' demand for it by \$138.94 million (remember that a positive response for

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[15] In a complete solution of the model RGIC1 would the bid up and the change in quantity outstanding would be less than \$128.59 million because of the deterrent of the rate increase. The coefficients refer to TML demand and supply curves and not to equilibrium quantities for the complete model.

TABLE VIII 2. POINT ESTIMATES OF SHORT RUN PORTFOLIO RESPONSES AND SHORT RUN INTEREST ELASTICITIES DERIVED FROM TABLE VIII 1.

Explanatory variables		Asset demand functions				Liability supply functions	
		CASH AND LIQUID ASSETS	Bonds			-GIC1	-GIC2
			FEDERAL GOVT.	MUN. AND PROVINCIAL	CORPORATE		
Interest rates[a]	RPAPER	128.59				-128.59	
	RGOVT		20.11	-12.98	10.06	-17.19	
	RMP		-40.50	41.32	-.82		
	RCORP		62.23	-53.00	83.02		-92.24
	RGIC1	-144.81	5.87			138.95	
	RGIC2				-70.75		70.75
Predetermined balance sheet components	NET[b]			-.0242	.3907		.6335
	SAVDEP[b]		.1080	.0173	.0584	.2504	.5660
	MTG[c]	.0738	.0131	.0067	.0042	-.3134	-.7844
Lagged dependent variables[a]	$PAPER_{-1}$	.5856				-.5856	
	$GOVT_{-1}$		.3516	.2214	-.2430	-.3300	
	MP_{-1}		-.2025	.5712	-.3687		
	$CORP_{-1}$		-.2207		.2207		
	$-GIC1_{-1}$	-.0865				.3377	-.2512
	$-GIC2_{-1}$					-.3416	.3416

a. The coefficients sum to zero across the rows. b. The coefficients sum to one across the rows. c. The coefficients of MTG sum to minus one across its row.

MATRIX OF SHORT RUN INTEREST ELASTICITIES

	Interest Rates					
Assets	RPAPER	RGOVT	RMP	RCORP	RGIC1	RGIC2
LIQUID ASSETS	.823	0	0	0	-.893	0
GOVT.	0	.201	-.510	.804	.057	0
MP	0	-.174	.708	-.918	0	0
CORPORATE	0	.120	-.012	1.281	0	-.813
-GIC1	-1.019	-.136	0	0	1.060	0
-GIC2	0	0	0	-.157	0	.106

a liability indicates a drop in its size since it is measured as a negative asset) and reduces the TMLs' demand for liquid assets by about the same amount ($144.81 million). The rate of interest on GIC1 has been left in the federal bond equation and the t-ratio of 1.0 demonstrates the apparent low degree of substitutability between governments and the less than one year items. The only significant estimated cross response between governments and short term items was the response of GIC1 to a change in RGOVT -- a small response confirming that if RGOVT increases there is some tendency for the TMLs to increase their short-term liabilities to provide part of the finance for the induced increase in federal bond holdings.

The regressions suggest that corporates and Canadas are complements and the suggestion is in both demand functions. In view of the nature of these instruments this is plausible, though the strength of the substitutability is probably overstated in the GOVT equation. Municipal and provincial issues are substitutes for both federal and corporate issues.

There is some intermediation between the over-one-year liabilities and corporate bonds that is measurably sensitive to yield changes, viz. the very significant coefficients for RCORP in the GIC2 equation and for RGIC2 in the CORP equation. An increase in RCORP encourages the TMLs to buy more corporates by reducing other bonds and by supplying more long-term liabilities. The results for the corporate bond equation therefore support the idea that corporates and mortgages are substitutes "competing" for GIC2 money. An increase in the yield on either encourages the TMLs to seek over-one-year funds.

The short-run elasticities of supply for over-one-year-term deposits with respect to RCORP and RGIC2 are not high. A suggestion in the unconstrained regressions of a pattern of substitutability between GIC1 and GIC2 [16] disappeared when lagged adjustments were allowed for.

The nice thing about the estimated interest rate responses is that all the cross relationships inferred are corroborated by the two reciprocal regressions. This backs up empirically the suggestive theoretical argument that there should be a tendency towards symmetry in interest rate responses.[17]

This brings the discussion to a variable only indirectly related to interest rates -- mortgage lending. The estimates suggest that if mortgage lending increases by one dollar then in the same month GIC2 is increased by just over seventy-eight cents. This response seems unbelievably fast but it should be recalled that mortgage lending decisions are made well in advance of the disbursement of funds and that the lenders can prepare their balance sheets in advance to receive mortgages. In fact it could even be that the over one year GICs are raised with some lead over mortgages -- an important question of timing arises here.[18]

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[16] See Appendix 2.

[17] See Chapter II, Section 3.

[18] This timing question has been discussed in Chapter V, Section 4.

The estimates, as indicated, show that the arrival of one dollar onto the balance sheet is accompanied, other things remaining the same, by an increase of $.7844 in over-one-year GICs in the same month. The rest of the finance is provided by an increase of $.3134 in less than one year GICs. Thus more than one dollar is raised to finance the dollar of mortgages and the reason is that the liquidity reserve complementary with mortgages is increased too and has to be financed. About nine cents is added to the sum of cash, liquid assets and Canadas for each additional mortgage dollar.

A ceteris paribus inflow of savings deposits of one dollar provides an interesting case study of liability and asset management within the model. The response to this change comes primarily on the liability side. The dollar inflow is used to reduce outstanding GICs by eighty-two cents and to purchase six cents worth of corporate bonds and twelve cents worth of government bonds. The response is most pronounced on the liability side and particularly for over-one-year liabilities. There is good reason for this. In the short run the inflow of deposits will not have an effect on mortgage lending;in the long run a substantial proportion of the inflow will be allocated to mortgages. The decision of where to allocate an exogenous inflow of savings largely boils down to choosing between increasing governments and liquid assets (which can be held until mortgages are made) and reducing other liabilities (which can be replenished when necessary). Since over-one-year GICs cost more in interest than the short-term assets earn the incentive to use the savings deposits inflow to defer GIC2 issues is strong.

TABLE VIII·3. POINT ESTIMATES OF LONG RUN RESPONSE COEFFICIENTS AND INTEREST ELASTICITIES DERIVED FROM TABLE VIII 2.

Explanatory variables		Asset demand functions: CASH AND LIQUID ASSETS	Bonds: FEDERAL GOVT.	Bonds: MUN. AND PROV.	Bonds: CORPORATE	Liability supply functions: -GIC1	Liability supply functions: -GIC2
Interest[a] rates	RPAPER	468.38	.00	.00	.00	-757.33	288.95
	RGOVT	14.16	32.11	-13.69	9.37	-67.82	25.87
	RMP	-13.23	-78.90	55.63	-2.76	63.55	-24.27
	RCORP	-10.82	80.68	-81.95	120.14	51.82	-159.87
	RGIC1	-522.92	9.31	4.81	-5.18	831.05	-317.07
	RGIC2	24.05	31.80	16.42	-108.47	-115.20	151.41
Predetermined balance sheet components	NET[b]	.1394	-.1668	-.1426	.6208	-.6678	1.217
	SAVDEP[b]	.0454	.1388	.1120	-.0214	-.2173	.9425
	MTG[c]	.1863	.0155	.0241	-.0112	-.0394	-1.1763

a. The coefficients sum to zero across the rows. b. The coefficients sum to one across the rows. c. The coefficients of MTG sum to minus one across its row.

MATRIX OF LONG RUN INTEREST ELASTICITIES

Assets	Interest Rates: RPAPER	RGOVT	RMP	RCORP	RGIC1	RGIC2
LIQUID ASSETS	3.000	.000	.000	.000	-6.001	.381
GOVT.	.089	.320	-.187	.107	-.538	.034
MP	-.107	-1.006	.944	-.045	.630	-.040
CORPORATE	.089	1.046	-1.421	1.853	.520	-.272
-GIC1	-3.223	.087	.062	-.059	6.340	-.404
-GIC2	.175	.354	.245	-1.483	-1.049	.229

5. Long-run responses.

In steady state equilibrium (3) becomes

$$y = \left[A \mid B\right]x + \Gamma y \tag{5}$$

so that

$$(I_6 - \Gamma)y = \left[A \mid B\right]x \tag{6}$$

or

$$y = (I_6 - \Gamma)^{-1}\left[A \mid B\right]x. \tag{7}$$

Matrices of long-run TML response coefficients corresponding to A and B are therefore given by

$$C = (I_6 - \Gamma)^{-1} A$$

and

$$D = (I_6 - \Gamma)^{-1} B \tag{8}$$

The long-run coefficients, C and D, involve a partial solution to the model only, since their derivation ignores the feedback that develops over time from mortgage lending which is in the long run responsive to interest rate changes.[19] Nevertheless the matrices C and D bring in an important part of the complete model and it is instructive to examine them. The C and D matrices derived from the estimates in Section 4 are in Table VIII 3 where long run elasticities are also given. Naturally the own rate elasticities are invariably greater in the long run than in the short run. Cross-rate responses tend to keep the same sign as in the short run and are larger than in the long run with

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[19] The long-run response coefficients are not at all similar to the long-run reduced form multipliers which are often used to summarize the effects of changes in exogenous variables in a simultaneous system. The long-run coefficients discussed here are quasi-structural parameters.

one exception -- the coefficient of RMP in the CORP equation is positive in the short run but negative in the long run. The coefficient is in neither case big but it is encouraging to see the sign change to its expected direction.

The interesting story about the long-run yield responses involves the coefficients that were set at zero in the short-run equations. In the long run it appears from the calculated elasticities that government bonds and short-term assets are mildly complementary and that corporates are complementary with both short term assets and governments. It is also apparent that the liquidity reserve assets, short term assets and governments, are complementary with GIC2. All of these complementarity inferences are credible in view of the asset classifications involved and are supported by the pairs of relevant regressions. There is also an important long-run substitution relation hidden in the short-run equations -- in the long run GIC1 and GIC2 are substitutes. As I have already pointed out, when lagged effects are not incorporated into the equations this pattern is identifiable in the results. Probably the static model is in a crude way imitating the long run dynamic model.

According to the estimates, if everything else remains constant,[20]

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[20] This is a very big <u>ceteris paribus</u> clause. The model is simultaneous and everything else <u>cannot</u> remain constant. An increase in RGI2 will cause a movement along the public's demand for GIC2 curve. The discussion in the text is to be read as referring to the TML sector's <u>desired supply</u> only, not as a prediction of what will happen to the <u>quantity</u> of GICs which involves both supply and demand. Also, remember that I have abstracted from the feedback of mortgage lending which is bound to follow the increase in RGIC2.

an exogenous increase in RGIC2 of one hundred basis points eventually reduces the desired supply of GIC2 by $151.41 million and increases the desired supply of GIC1 by $115.20 million. If it is the rate on less-than-one-year GICs that increases by a hundred basis points then the long-run decrease in the desired supply of GIC1 is $831.05 million and the concomitant increase in the desired supply of GIC2 is $317.07 million. If it is felt that the measured response of GIC1 to a change in RGIC1 is too high (the average level of less-than-one-year GICs over the period was just $806.5 million) it should be recognized that an increase in RGIC1 of one hundred basis points is an increase large enough to remove any profit from the short term intermediation process. If such an inconceivably large deterioration in interest rate spreads were to take place then the under-one-year intermediary function certainly would be dropped eventually -- just as the estimates imply.[21]

The long run sensitivity of TML portfolio management to interest rates is pronounced. Just how pronounced can be gauged by the absolute values of the measured elasticities. The items of less-than-one-year maturity respond to the relevant short-term interest rates with elasticities substantially greater than unity.
Only the demand for government bonds and the supply of over-one-

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[21] The long-run short-term asset equation shows that the one percent increase in RGIC1 would cause a drop in these items of $522.92 million. The average holding over the sample period was $998.7 million. Therefore even if the business of short term intermediation were dropped, and legal requirements ignored, the TMLs would continue to hold some short-term paper as a liquidity reserve and as an adjustment asset.

year deposits have own rate elasticities under unity. The complementarity of government bonds with mortgages, the legal requirements for government bonds and the subservience of over-one-year GICs to mortgage financing needs are possibly responsible for the low elasticities.

The long-run financing of mortgages by the TMLs implicit in the estimates involves raising over-one-year money in excess of mortgage lending. For every dollar of mortgages at the margin there is an eventual increment of \$1.1763 in GIC2, i.e. mortgages are overfunded. This degree of over-funding is a feature of the past five years (over which the estimates are made) that probably will not persist into the future. The TMLs have, over the period, been lengthening their liability structure not just to accommodate new mortgage lending but also to cover existing mortgage lending which had been financed by savings deposits in the mid-1960s. Additional finance is raised with GIC1 (\$.0394). The surplus twenty-one cents is used to increase liquid assets and governments <u>pari passu</u> -- portfolio balance factors at work in an easily understood fashion.

6. <u>The disequilibrium path</u>.

Between the short run and the long run there is a maze. If x_t is constant over time so that $x = x_t$ $t = \ldots, 0, 1, 2 \ldots$ one can write

$$y_{t+1} = [A \quad B]\, x + \Gamma y_t \qquad (9)$$

or

$$y_{t+2} = (I_6 + \Gamma)\, [A \quad B]\, x + \Gamma^2 y_t \qquad (10)$$

TABLE VIII 4 POINT ESTIMATES OF SIX MONTH DYNAMIC RESPONSE COEFFICIENTS

DERIVED FROM TABLE VIII 2.

Explanatory variables		Asset demand functions				Liability supply functions	
		CASH & LIQUID ASSETS	Bonds				
			FEDERAL GOVT.	MUN. & PROV.	CORPORATE	-GIC1	-GIC2
Interest rates	RPAPER	398.56	.00	.00	.00	-598.18	199.62
	RGOVT	9.66	32.04	-14.13	9.83	-57.17	19.77
	RMP	-7.66	-78.68	56.71	-3.91	49.94	-16.40
	RCORP	-6.26	80.75	-81.30	119.45	40.80	-153.44
	RGICI	-445.71	9.26	4.48	-4.83	655.22	-218.42
	RGIC2	13.86	31.43	14.16	-106.08	-90.47	137.10
Predetermined balance sheet items	NET	.0892	-.1647	-.1294	.6069	-.5506	1.1487
	SAVDEP	.0123	.1083	.1074	-.0165	-.1364	.8950
	MTG	.1936	.0164	.0233	-.0103	-.0626	-1.1603

and by induction it can be seen that

$$y_{t+\tau} = (I_6 + \Gamma + \Gamma^2 + \ldots + \Gamma^{\tau-1})[A \mid B]x + \Gamma^{\tau}y_t. \qquad (11)$$

The expression in (11) evaluated for successive periods yields

$$(I_6 + \Gamma + \Gamma^2 + \ldots + \Gamma^{\tau-1})[A \mid B] \qquad \tau=1,2,\ldots \qquad (12)$$

which are matrices of dynamic responses. As τ approaches infinity the dynamic responses approach the long run equilibrium responses if the system is stable (if the roots of Γ are not all within the unit circle the paths fly off to infinity). Undulations in the paths of a stable system are possible, occurring when Γ's roots are complex. In short, the freakiest disequilibrium paths are possible. Luckily Γ in the present case is friendly and plays no tricks. The paths go smoothly and monotonically from initial to ultimate response and there is no point in drawing the fifty-four charts that would show this. In Table VIII 4 the six-month dynamic response matrix is shown and it is not very different from the equilibrium response matrix, suggesting that within six months the bulk of any initial disequilibrium is eliminated.

7. The timing of liability management

As I have argued[22] complete hedging of the portfolio

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[22] Chapter V, Section 4.

TABLE VIII 5 CONSTRAINED 3LS ESTIMATES OF PORTFOLIO RESPONSES: INCLUDES BACKLOG[a]

Explanatory variables		Asset demand functions				Liability supply functions	
			Bonds				
		CASH AND LIQUID ASSETS	FEDERAL GOVT.	MUN. AND PROV.	CORPORATE	-GIC1	-GIC2
Interest rates[a]	RPAPER	133.53 (5.18)				-133.53 (-5.18)	
	RGOVT		17.55 (1.86)	-11.54 (-2.06)	11.91 (1.54)	-17.93 (-1.77)	
	RMP		-38.29 (-2.85)	40.10 (3.86)	-1.81 (-.13)		
	RCORP		62.30 (3.45)	-54.42 (-3.92)	78.66 (3.32)		-86.54 (-4.12)
	RGIC1	-149.60 (-5.61)	6.1000 (1.03)			143.50 (5.30)	
	RGIC2				-70.32 (-3.68)		70.32 (3.68)
Predetermined balance sheet components	NET*[b]			-.0197 (-.29)	.4011 (3.27)		.6186 (4.29)
	SAVDEP[a]		.0548 (1.02)	.0331 (.37)	-.3582 (-2.25)	.2419 (3.16)	.0285 (.14)
	MTG[a]	.0697 (3.08)	.0126 (.92)	-.0111 (-.17)	.4082 (3.35)	-.3205 (-5.40)	-.1589 (-1.03)
	BACKLOG*[a]	.0198 (.41)	.0504 (1.60)	-.0024 (-.10)			-.0677 (-1.25)
Lagged dependent variables[a]	PAPER$_{-1}$	.5911 (7.66)				-.5911 (-7.66)	
	GOVT$_{-1}$		.3172 (2.73)	.2229 (2.43)	-.2450 (-2.00)	-.2951 (-1.69)	
	MP$_{-1}$		-.2140 (-2.16)	.5772 (6.83)	-.3632 (-3.24)		
	CORP$_{-1}$		-.2411 (-3.74)		.2411 (3.74)		
	-GIC1$_{-1}$	-.0750 (-.99)				.3412 (4.39)	-.2662 (-3.57)
	-GIC2$_{-1}$					-.3533 (-6.44)	.3533 (6.44)
	CONSTANT[a]	11.54 (.17)	92.53 (.98)	207.76 (2.83)	97.13 (1.03)	-8.29 (-.11)	-400.68 (-3.13)
Seasonals[a]	Q1	6.25 (.71)	3.87 (.94)	4.46 (1.11)	-5.58 (-.83)	-.75 (-.12)	-8.25 (-.78)
	Q2	.31 (.03)	7.29 (1.70)	3.14 (.98)	2.000 (.40)	19.03 (2.34)	-31.76 (-3.14)
	Q3	5.91 (.66)	-11.92 (-2.99)	-4.367 (-1.21)	12.01 (1.85)	-25.78 (-3.78)	24.16 (2.24)
	$\bar{R}^2$	.9815	.9207	.9239	.9382	.9881	.9982
	DW	1.61	1.61	1.43	1.46	1.59	1.35

a. The coefficients sum to zero across the rows.

b. The coefficients of NET* sum to one across its row.

requires that undisbursed commitments be matched by liabilities of an appropriate maturity. This is a costly process and likely to be attempted only partially. The quantitative importance of pre-financing of mortgages can be measured in the context of the model by entering the backlog of undisbursed commitments plus commitments make this period (BACKLOG*) as an explanatory variable in the equations for GIC2, cash and short-term paper, and government bonds. If pre-financing takes place one expects to find a significant positive relationship between the backlog and outstanding deposits of over one year maturity. Since carrying the backlog itself involves no financing requirement the GIC funds are free to put into appropriately liquid assets such as governments and short-term paper.

The block of equations was re-estimated with BACKLOG* inserted. The estimates (Table VIII 5) do not support the hypothesis that pre-financing is significant. There is perhaps an indication that GIC2 money is raised and put into government bonds and liquid assets until it is disbursed but the sums involved are small (about seven cents on the undisbursed dollar) and not statistically significant. The long-run responses implicit in these estimates follow the same pattern. The estimates therefore suggest that the TMLs find pre-financing too expensive a means of reducing risk, preferring to time their liability management so that they do not start paying the relatively high rate on over-one-year GICs until they start receiving the yet higher rate from mortgages.

Appendix 1 to Chapter VIII

Regression with linear restrictions

1. OLS. The system of equations (3) is not stochastic. A vector of stochastic terms may be added to the system so that at time t one has

$$y_t = \left[A \mid B \mid \Gamma\right] \begin{bmatrix} x_t \\ y_{t-1} \end{bmatrix} + u_t \tag{13}$$

where u_t is the 6-by-1 vector of random elements. Transposing,

$$y_t' = \left[x_t' \mid y_{t-1}'\right] \begin{bmatrix} A' \\ B' \\ \Gamma' \end{bmatrix} + u_t'. \tag{14}$$

The sample size is T, i.e. t = 1,2 ..., T. A T-by-6 data matrix Y, a T-by-15 data matrix Z and a T-by-6 matrix of stochastic terms U can be assembled

$$Y = \begin{bmatrix} y_1' \\ y_2' \\ \cdot \\ \cdot \\ \cdot \\ y_T' \end{bmatrix}, \quad Z = \begin{bmatrix} x_1' & y_0' \\ x_2' & y_1' \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ x_T' & y_{T-1}' \end{bmatrix} \text{ and } U = \begin{bmatrix} u_1' \\ u_2' \\ \cdot \\ \cdot \\ \cdot \\ u_T' \end{bmatrix}$$

Setting $F' = \left[A \mid B \mid \Gamma\right]$, the equations in (14) become

$$Y = ZF + U \tag{15}$$

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23 It will be recalled that y_t and y_{t-1} are 6-by-1 vectors and x_t is 9 by 1. The coefficient matrices A, B and Γ are 6 by 6, 6 by 3 and 6 by 6 respectively.

Under the assumptions that Z and U are asympotically uncorrelated and that

$$\varepsilon(u_{it}) = 0 \qquad (16)$$

$$\varepsilon(u_{it}u_{jt'}) = \begin{cases} s_{ij} & t=t' \qquad (17) \\ 0 & t\neq t' \qquad (18) \end{cases}$$

then the OLS estimator of F,

$$\hat{F} = (Z'Z)^{-1}Z'Y\ , \qquad (19)$$

is asymptotically unbiased.

Now suppose that the variable NET* (net worth plus savings deposits minus mortgages) is appended to (15) as a dummy variable:

$$\left[Y \mid NET^*\right] = Z\left[F \mid r\right] + \left[U \mid \Theta_T\right] \qquad (20)$$

where

$$r = (0\ 0\ 0\ 0\ 0\ 0\ 1\ 1\ {-1}\ 0\ 0\ 0\ 0\ 0\ 0) \qquad (21)$$

$$\Theta_T = \text{T-by-1 null vector.} \qquad (22)$$

The equation which has been added asserts the tautology $NET^*_t = NET_t + SAVDEP_t - MTG_t$ for all t.

The budget constraint states that

$$\left[Y \mid NET^*\right]\phi = \Theta_T \qquad (23)$$

where $\phi' = (1\ 1\ 1\ 1\ 1\ 1\ {-1}\ {-1}\ 1)$.[24] (24)

The OLS estimator of $\left[F \mid r\right]$ in (20) is

$$\left[\hat{F} \mid r\right] = (Z'Z)^{-1}Z'\left[Y \mid NET^*\right] \qquad (25)$$

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[24] All I have written in (23) is
PAPER + GOVT + MP + CORP - GIC1 - GIC2 = NET + SAVDEP - MTG.

noting that from (23)

$$(Z'Z)^{-1}\,Z'\left[Y \mid NET^*\right]\emptyset = \Theta_{15} \qquad (26)$$

it follows that

$$\left[\hat{F} \mid r\right]\emptyset = \Theta_{15}. \qquad (27)$$

I have proved the adding up theorem of linear regression for my special case. Across equations, the coefficients of the interest rates and the coefficients of the lagged dependent variables sum to zero, the coefficients of NET and SAVDEP sum to one and the coefficients of MTG sum to minus one (as required by economic theory) if all variables are included in all regressions and if OLS is used on each equation separately.[25]

2. Zero restrictions. If the matrices A, B and Γ are not completely filled but contain elements set a priori to zero the adding up property of OLS does not apply. It is then necessary to impose constraints on the estimating procedure. This requires that equation (15) be rearranged so that the asset demand equations are stacked vertically into a single 6T-observation regression. To accomplish this the columns of Y and U are stacked into the 6T element vectors Ψ and μ,

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[25] My proof follows that of Prais and Houthakker (1955).

$$\psi = \begin{bmatrix} y_{11} \\ y_{12} \\ \cdot \\ \cdot \\ \cdot \\ y_{1T} \\ y_{21} \\ \cdot \\ \cdot \\ \cdot \\ y_{2T} \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ y_{61} \\ y_{62} \\ \cdot \\ \cdot \\ \cdot \\ y_{6T} \end{bmatrix} \qquad \mu = \begin{bmatrix} u_{11} \\ u_{12} \\ \cdot \\ \cdot \\ \cdot \\ u_{1T} \\ u_{21} \\ \cdot \\ \cdot \\ \cdot \\ u_{2T} \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ u_{61} \\ u_{62} \\ \cdot \\ \cdot \\ \cdot \\ u_{6T} \end{bmatrix}$$

and the explanatory variables are stacked into the 6T-by-90 matrix χ,

$$\chi = \begin{bmatrix} Z & 0 & 0 & \cdot & \cdot & 0 \\ 0 & Z & 0 & & & 0 \\ \cdot & & Z & & & \cdot \\ \cdot & & & Z & & \cdot \\ \cdot & & & & Z & 0 \\ 0 & \cdot & \cdot & \cdot & 0 & Z \end{bmatrix}. \tag{28}$$

The error term assumptions (16) and (17) become

$$E(\mu\mu') = S \otimes I_T \tag{29}$$

where $\otimes$ is the Kronecker product operator and S is the variance-covariance matrix of the contemperaneous error terms,

$$S = \begin{bmatrix} s_{11} & s_{12} & \cdots & s_{16} \\ s_{21} & s_{22} & & s_{26} \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ s_{61} & s_{62} & & s_{66} \end{bmatrix} \qquad (30)$$

A vectorization of $\left[A \,|\, B \,|\, \Gamma\right]$ into a 90-by-1 column completes the new arrangement. With zeros in the elements corresponding to omitted variables the vector of coefficients may be defined as

$$\Omega' = (\alpha_{11}\ \alpha_{12} \cdots \alpha_{16}\ \gamma_{11}\ \gamma_{12} \cdots \gamma_{16}\ \beta_{11}\ \beta_{12}\ \beta_{13}\ \alpha_{21} \cdots$$
$$\gamma_{61}\ \gamma_{62} \cdots \gamma_{66}\ \beta_{61}\ \beta_{62}\ \beta_{63})$$

The redefined equation whose coefficients are to be estimated is thus

$$\psi = \chi\,\Omega + \mu\,. \qquad (31)$$

The system (31) is to be estimated subject to the 15 linear restrictions

$$\sum_{i=1}^{6} \alpha_{ij} = \sum_{i=1}^{6} \gamma_{ij} = 0, \quad j = 1,2, \ldots 6$$
$$(32)$$
$$\sum_{i=1}^{6} \beta_{i1} = \sum_{i=1} \beta_{i2} = -\sum_{i=1} \beta_{i3} = 1.$$

These restrictions can be written in matrix form as

$$r - R\Omega = \Theta_{15} \qquad (33)$$

where R is a 15 by 90 restriction matrix whose elements are all either one or zero, and where r is the 15-element vector defined in (20). The restricted generalized least squares (RGLS) estimator of Ω is derived by minimizing the Lagrangean[26] expression

$$L = (\Psi - \chi\Omega)' \, \Sigma^{-1}(\Psi - \chi\Omega) - \lambda'(R\Omega - r) \qquad (34)$$

where $\Sigma = S \otimes I_\tau$. First order minimum conditions are provided by

$$\frac{\partial L}{\partial \Omega} = -2\chi' \, \Sigma^{-1}\Psi + 2\chi' \, \Sigma^{-1}\chi\hat{\Omega} - R'\lambda = \Theta_{15} \, , \qquad (35)$$

$$\frac{\partial L}{\partial \lambda} = R\hat{\Omega} - r = \Theta_{15}. \qquad (36)$$

The solution for the estimated parameters $\hat{\Omega}$ from (35) and (36) is

$$\hat{\Omega} = (\chi'\Sigma^{-1}\chi)^{-1}\chi'\Sigma^{-1}\Psi + (\chi'\Sigma^{-1}\chi)^{-1}R' \quad \{[R(\chi'\Sigma^{-1}\chi)^{-1}R']^{-1} \, [r - R(\chi'\Sigma^{-1}\chi)^{-1} \, \chi'\Sigma^{-1} \, \Psi]\} \, . \qquad (37)$$

The bias of the estimator is seen to be

$$\hat{\Omega} - \Omega = (I - P)(\chi'\Sigma^{-1}\chi)^{-1}\chi'\Sigma^{-1} \, \mu \, , \qquad (38)$$

where P is the positive definite matrix

$$P = (\chi'\Sigma^{-1}\chi)^{-1} \, R'[R(\chi'\Sigma^{-1}\chi)^{-1} \, R']^{-1} \, R. \qquad (39)$$

It follows at once that $\hat{\Omega}$ is an asymptotically unbiased estimator of Ω. The variance - covariance matrix of the estimated coefficients for the estimator in (37) is

$$\begin{aligned} \varepsilon(\hat{\Omega} - \Omega)(\hat{\Omega} - \Omega)' &= (I - P)(\chi'\Sigma^{-1}\chi)^{-1}(I - P)' \\ &= (I - P)(\chi'\Sigma^{-1}\chi)^{-1} \, . \end{aligned} \qquad (40)$$

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[26] λ is a column vector of 15 Lagrange multipliers. If the minimization were carried out without imposing the adding up constraints the derived estimator would be the seemingly unrelated regression estimator proposed by Zellner (1962).

The variance - covariance matrix of the unrestricted Zellner estimator is simply $(X'\Sigma^{-1}X)^{-1}$. Since P is positive definite the diagonal elements of the matrix in (40) are smaller than those of $(X'\Sigma^{-1}X)^{-1}$. Therefore RGLS is more efficient than unrestricted GLS which in turn is more efficient than unrestricted OLS applied to each equation separately. The gain in efficiency is possible only if not all variables are entered into all equations. The gain of Zellner's method over OLS even then exists only if not all covariances are zero. In most applications these conditions will be satisfied.

In the empirical work of Chapter VIII the assumption that Z and U are asymptotically uncorrelated does not hold. The interdependent system counterpart of the Zellner method, 3LS, is therefore used. There is a presumption that restricted 3LS has the same advantages over unrestricted 3LS and 2LS that RGLS has over unrestricted GLS and OLS.[27]

3. FINPT 4.

The estimator defined by (37) is the one which has been used for the constrained estimates of Chapter VIII. The subroutine of the MASSAGER system known as FINPT 4 which computes these estimators was written by G.R. Sparks. The programme works in two passes.[28] The first pass uses

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[27] The nature of the advantages of 3LS over 2LS is discussed by Zellner and Theil (1962).

[28] The two pass procedure was originally recommended by Zellner (1962) and Zellner and Theil (1962). For the present estimates the first pass is by constrained 2LS rather than OLS.

constrained OLS or constrained 2LS to estimate the residual variance matrix Σ. The estimate $\hat{\Sigma}$ replaces Σ in (37).

The balance sheet constraint is not a stochastic relation. Therefore the sum of the errors from the asset demand equations must be zero. And the matrix Σ as a result does not have full rank. Neither does the variance - covariance matrix of the constrained first pass residuals, $\hat{S}$, have full rank. The rank of S and $\hat{S}$ (in the present case) is five not six. Since the inverse of $\hat{\Sigma}$ is required this is a problem. The solution is to drop one equation: it does not matter which since the linear dependence of the system ensures that the estimates are invariant with respect to the choice of the omitted equation. The coefficients of the missing equation can be determined using the balance sheet constraint.

Appendix 2 to Chapter VIII

The search strategy

The automatic satisfaction of the adding up constraint when all independent variables are included in all asset demand functions allows preliminary examination of the data by OLS. The requirement that all variables be in all equations rather limits the usefulness of OLS in a system as large as the present one. When the lagged variables are included there are sixteen explanatory variables plus seasonals and only the most sanguine econometrician would expect to be able to find all the coefficients well determined. Therefore, in my first regressions I tested a static model excluding the lagged dependent variables.

The results (Table VIII 6) contain certain similarities to those from the constrained estimates which have already been discussed.[29] The diagonal elements of the interest rate coefficient matrix are significant and positive as they should be excepting only the cash and liquid assets equation. Corporates and Canadas are complements and both are substitutes for municipals and provincials. Short-term assets and federal bonds seem to be complements. Again there is definite evidence of interest sensitive intermediation between corporate bonds and over-one-year term deposits and between short-term assets and less-than-one-year term deposits. There is also the suggestion of substitutability between over and under-one-year

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[29] The appropriate comparison is with the implicit long-run estimates.

deposits. The response to a change in mortgages in these estimates is also similar to the long-run response discussed in Chapter VIII, as is the response to a change in savings deposits.

These estimates are remarkable. The literature on asset demand contains many despairing references to the problem of multicollinearity which allegedly precludes more than one interest rate from making a contribution in a multiple regression. Well here are six interest rates in each equation doing rather well (only one "perverse" coefficient). One of the outstanding findings of my research is that interest rates, although correlated, are not so correlated that sensible empirical results are impossible when several interest rates are put into the same regression. This is partly due to the use of end of month data which are less collinear than averages over a month. The imposition of theoretical restrictions is another contributing factor.[30]

Remarkable though these results may be in refuting an old myth, they are not in themselves impressive as structural estimates. The Durbin Watson statistics signal significant serial correlation in the residuals. If theory be any guide this is probably due to neglect of the delays in portfolio adjustment, delays which a better specification of the theory would allow for.

The next set of OLS estimates (Table VIII 7) includes all the lagged dependent variables. As is well known the satisfactory appearance

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[30] Interest rate correlations are presented and discussed in Chapter IX.

of the Durbin Watson statistics here is misleading.[31] Also, the inclusion of all the variables does put too much of a burden on OLS and the number of measurably significant interest rate responses drops. It can be seen from the t-ratios of the lagged dependent variables that there is less serial correlation with these data than one often finds with economic time series which gives rise to a conjecture. The asymptotic bias in estimates of autoregressive schemes due to improper aggregation over time is exacerbated by serial correlation in the dependent variable .[32] Since serial correlation is not great in the present case it is likely that any such bias will be small.

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[31] Nerlove and Wallis (1966).

[32] Mundlak (1961) has shown that positive serial correlation in the dependent variable will cause an upward bias in the regression coefficient on its lagged value if the data are aggregated over several decision periods.

TABLE VIII 6 UNCONSTRAINED OLS ESTIMATES OF PORTFOLIO RESPONSES: EXCLUDE LAGGED DEPENDENT VARIABLES

Explanatory variables		Asset demand functions				Liability supply functions	
			Bonds				
		CASH AND LIQUID ASSETS	FEDERAL GOVT	MUN. AND PROVINCIAL	CORPORATE	-GIC1	-GIC2
Interest rates ^{a}	RPAPER	-43.31	26.40	16.69	24.81	-37.70	-13.11
		(-1.37)	(2.44)	(1.45)	(1.26)	(-.77)	(.28)
	RGOVT	7.83	40.31	-.90	14.92	-29.30	-32.86
		(.28)	(4.25)	(-.09)	(.86)	(-.69)	(-.79)
	RMP	-61.54	-62.37	37.52	-28.53	- 9.17	124.09
		(-1.39)	(-4.12)	(2.32)	(-1.04)	(-.13)	(1.87)
	RCORP	32.44	55.60	-29.93	92.46	40.34	-190.91
		(.60)	(2.98)	(-1.51)	(2.73)	(.48)	(-2.34)
	RGIC1	-75.68	-15.26	-5.95	-17.61	97.54	16.96
		(-2.99)	(-1.76)	(-.64)	(-1.12)	(2.50)	(.45)
	RGIC2	201.16	- 8.60	-48.16	-60.40	-140.28	56.27
		(5.33)	(-.67)	(-3.49)	(-2.57)	(-2.41)	(.99)
Predetermined balance sheet components	NET* ^{b}	.4872	.1358	.0753	.2656	-.1895	.3762
		(2.76)	(2.25)	(-1.17)	(2.42)	(-.70)	(1.42)
	SAVDEP ^{a}	-.5613	-.1462	.0384	-.0893	.5650	.1934
		(-2.36)	(-1.79)	(.44)	(-.60)	(1.54)	(.54)
	MTG ^{a}	.6178	.1685	-.1087	.2640	-.4090	-.6226
		(3.65)	(2.90)	(-.30)	(2.50)	(-1.57)	(-2.45)
	CONSTANT ^{a}	-547.36	136.59	519.62	-230.13	496.40	-375.12
		(-3.57)	(2.60)	(9.27)	(-2.40)	(2.10)	(-1.63)
Seasonals ^{a}	Q1	19.76	-8.70	.78	-5.66	-9.669	3.488
		(1.71)	(-2.20)	(.19)	(-.79)	(-.54)	(.20)
	Q2	25.50	9.99	12.80	3.037	-31.27	-20.05
		(2.41)	(2.76)	(3.31)	(.46)	(-1.92)	(-1.26)
	Q3	-25.57	-6.27	-5.31	11.56	17.99	7.60
		(-2.31)	(-1.65)	(-1.31)	(1.68)	(1.05)	(.46)
	$\bar{R}^2$	.9833	.9501	.8830	.9203	.9492	.9967
	DW	1.57	1.44	.91	.98	.60	.85

a. The coefficients sum to zero across the rows.

b. The coefficients of NET* sum to one across its row.

TABLE VIII 7. UNCONSTRAINED OLS ESTIMATES OF PORTFOLIO RESPONSES

		Asset demand functions				Liability supply functions	
			Bonds				
Explanatory variables		CASH AND LIQUID ASSETS	FEDERAL GOVT.	MUN. AND PROV.	CORPORATE	-GIC1	-GIC2
Interest[a] rates	RPAPER	5.70 (.16)	22.04 (1.58)	20.84 (1.91)	19.15 (1.11)	-43.60 (-1.65)	-24.13 (-.62)
	RGOVT	-14.23 (-.54)	36.43 (3.55)	-5.28 (-.66)	12.37 (.98)	16.29 (.84)	-45.57 (-1.60)
	RMP	-8.47 (-.16)	-58.39 (-2.84)	21.33 (1.32)	4.11 (.16)	14.76 (.38)	26.67 (.47)
	RCORP	-53.52 (-.74)	50.59 (1.79)	-30.89 (-1.39)	8.39 (.24)	-11.84 (-.22)	37.28 (.48)
	RGIC1	-65.31 (-2.60)	-11.40 (-1.17)	-10.39 (-1.36)	-17.86 (-1.49)	34.77 (1.88)	70.19 (2.60)
	RGIC2	135.54 (2.84)	-15.16 (-.82)	-22.11 (-1.52)	-21.34 0.93)	18.52 (.53)	-95.44 (-1.86)
Predetermined balance sheet components	NET*[b]	.3357 (1.82)	.1381 (1.93)	.0067 (.12)	.0435 (.49)	-.2088 (-1.53)	.6849 (3.45)
	SAVDEP[a]	-.2387 (-.93)	-.1440 (-1.45)	-.0435 (.68)	-.2088 (-.58)	.6849 (1.95)	(-2.63)
	MTG[a]	.2447 (1.13)	.1758 (2.08)	-.0069 (-.10)	.0955 (.92)	-.3294 (-2.06)	-.1798 (-.77)
Lagged dependent variables	PAPER$_{-1}$	.3213 (1.69)	.0305 (.41)	-.0378 (-.65)	.0889 (.98)	-.2513 (-1.79)	-.1515 (-.74)
	GOVT$_{-1}$	.0577 (.16)	.2172 (1.51)	.0568 (.50)	-.0706 (-.40)	-.3149 (-1.16)	.0538 (.14)
	MP$_{-1}$	-.0649 (-.17)	.0558 (.37)	.5983 (5.06)	-.0649 (-.35)	.1652 (.58)	-.6894 (-1.65)
	CORP$_{-1}$	.2546 (1.11)	-.0389 (-.44)	-.0802 (-1.14)	.7443 (6.74)	-.2283 (-1.35)	-.6515 (-2.63)
	-GIC1$_{-1}$	.2221 (-1.46)	-.0218 (-.37)	-.0749 (-1.62)	.0952 (1.30)	.7565 (6.73)	-.5329 (-3.25)
	-GIC2$_{-1}$	-.1238 (-1.94)	.0268 (.58)	-.0258 (-.71)	.0367 (.64)	-.1629 (-1.86)	.2490 (1.95)
	CONSTANT[a]	-189.44 (-.78)	121.55 (1.29)	226.26 (3.06)	64.61 (.55)	-132.80 (-.74)	-90.17 (-.34)
Seasonals[a]	Q1	15.33 (1.39)	-9.33 (-2.18)	.28 (.09)	4.45 (.84)	-2.42 (-.30)	-8.314 (-.70)
	Q2	-1.082 (-.08)	4.16 (.82)	4.52 (1.14)	3.83 (.61)	26.47 (2.76)	-37.91 (-2.70)
	Q3	-7.43 (-.59)	-1.90 (-.39)	-1.96 (-.51)	-.09 (-.01)	-22.85 (-2.46)	34.22 (2.52)
	$\bar{R}^2$	.9886	.9547	.9445	.9659	.9921	.9988
	DW	2.00	1.87	1.97	2.35	1.91	1.89

[a] The coefficients sum to zero across the rows.

[b] The coefficients of NET* sum to one across its row.

Appendix 3 to Chapter VIII

The Interest Rate Correlation Matrix

Ordinarily econometricians have little use for simple correlation coefficients in and of themselves. This is not so in the econometrics of portfolio behaviour where the matrix of simple correlations between interest rates is important from the points of view of economic theory and of statistical estimation.

The theory of portfolio preference suggests that if the expected yields on two assets are highly correlated then they will be substitutes and if the expected yields are not so highly correlated then they may be complements. Hints of substitute and complement relations are therefore implicit in the interest correlation matrix (Table IX 1). Chart IX 1 conveys the same sort of information by graphing the movements in interest rates over the period 1967 - 72 and it shows in addition the magnitudes involved. The hints can be compared with the inferences drawn in Chapter VIII, Sections 4 and 5. These inferences cannot really be proved or refuted by the correlations, for the *ex post* evidence of simple contemporaneous correlations is by itself a poor indication of asset cross relations, but the plausibility of the inferences is at stake.

Assertions of asset substitutability are usually less controversial than assertions of complementarity so I consider first the cases where the coefficients in Table IX 1 indicate substitutability. For liquid assets

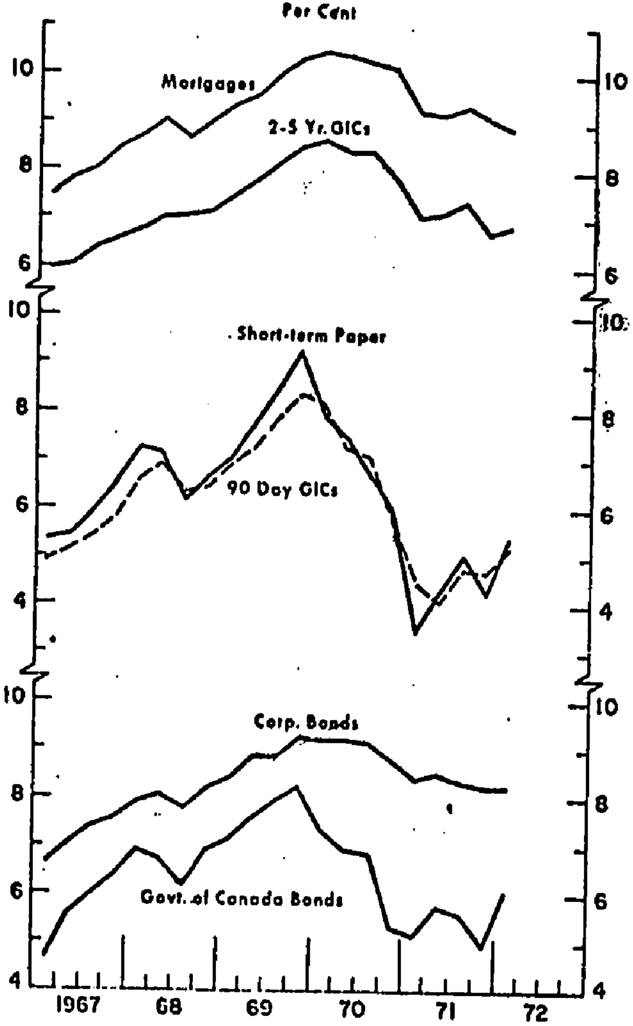
TABLE IX 1 INTEREST RATE CORRELATION MATRIX

	RPAPER	RGOVT	RMP	RCORP	RGIC1	RGIC2	RMTG
RPAPER	1.0000						
RGOVT	.9102	1.0000					
RMP	.4962	.6624	1.0000				
RCORP	.4656	.6284	.9840	1.0000			
RGIC1	.9713	.9040	.5646	.5489	1.0000		
RGIC2	.6402	.7057	.9415	.9417	.7178	1.0000	
RMTG	.5249	.6180	.9532	.9685	.6183	.9755	1.0000

CHART IX 1

INTEREST RATES

Per Cent



(whose yield is represented by RPAPER) it would appear that federal government bonds and less-than-one-year GICs are substitutes. The regression results[1] support the idea that liquid assets and GIC1 are substitutes[2] but do not support the idea that liquid assets and federal bonds are substitutes. Other than RPAPER the only yield very collinear with RGOVT is RGIC1. This supports the conjecture which I made in Chapter VIII that there is an opportunity for the TMLs to intermediate between Canadas and GIC1. One can now see that the arbitrage would be hedged because of the yield collinearity. As to whether the TMLs actually used the opportunity, the regressions are ambivalent. Probably the non-yield characteristics of federal government debt account for its tendency to disobey the implications of yield correlations, particularly its tendency to be complementary even with assets with very collinear yields.

The coefficient of .9840 between RMP and RCORP is the largest element in the matrix and the implied relation of substitutability is indeed borne out by the estimated equations. Substitutability is also

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[1] Table VIII 3 is used to summarize the estimates. It is a table of derived long run coefficients.

[2] An increase in RPAPER increases both the TMLs' demand for short term assets and the quantity of GIC1 which the TMLs desire to supply. Since both quantities increase it seems natural to call the relationship complementary. But an increase in the supply of a liability is equivalent to a <u>sale</u> of an asset. The terminology of consumer theory is admittedly not felicitous when two assets are on opposite sides of the balance sheet but it is formally applicable. In so far as the rest of the economy is concerned, on EV lines commercial and finance company paper seem to be good substitutes (in the ordinary sense of the word) for GIC1 but it will be recalled that this is not always the case (Chapter V, Section 2).

implied by the high correlations between RCORP and RGIC2, and RMP and RGIC2; i.e. the TMLs can conduct a well hedged arbitrage operation between GIC2 and both CORP and MP. The estimates establish a very clear pattern of arbitrage for corporates but not for municipals and provincials.

The mortgage interest rate is highly correlated with RMP, RCORP and RGIC2. Municipal and provincial bonds, and corporate bonds would therefore appear to be competitive with mortgages as an investment for the TMLs. The specification for the determination of mortgage lending in Chapter VII used RCORP to measure the interest opportunity cost of mortgage investment. The coefficient of .9685 between RMTG and RCORP suggests the measure was well chosen. Also the benefits to the TMLs of matching mortgages with over-one-year term liabilities are apparent in the pronounced collinearity of RMTG and RGIC2 which implies that the matching is a good hedge against interest rate changes.

I now consider the cases where complementarity seems plausible given the correlations. Liquid assets and Canadas could be complementary with all other assets excepting GIC1 but it looks unlikely that they could be mutually complementary. The regressions indicate that liquid assets are complementary with GIC2 and mortgages, and that Canadas are complementary with all assets excepting municipals and provincials. I have already noted that it is odd that Canadas and liquid assets appear to be complements but the other results are plausible.

By and large the estimated parameters are consistent with the demand relations implied by the correlation matrix. It is interesting that the correlation coefficients tend to agree with casual empiricism as to what asset pairs should be substitutes or complements. E.g. federal government bonds and liquid assets would usually be considered complements to corporate bonds and mortgages by market analysts who have no interest at all in correlation coefficients; the same analysts would usually consider corporates and mortgages as substitutes etc. The conformity of the analysts' views with the implications of the EV hypothesis is heartening.

A unique puzzle for the estimation of portfolio models is bound up with the interest rate correlation matrix. If the EV hypothesis is correct the most significant cross effects to changes in interest rates occur when a pair of rates is very collinear. It is in precisely these circumstances that statistical techniques are confounded -- the intractable problems of extreme multicollinearity amongst regressions arise -- and a regression to measure the cross effects is likely to come up with statistically insignificant results. What is economically significant becomes immeasurable and it is obvious that multicollinearity problems will often preclude the accurate estimation of portfolio balance equations. The period 1967-1972 with which this study is primarily concerned fortunately provides a set of interest rate series in which multicollinearity is not acute. In all the regressions which have been run it has proved possible to obtain well determined interest rate coefficients. Statistically significant results have been obtained with as many as six interest rates entered into one regression.

CHAPTER IX

Summary and Conclusions

In this thesis I attempt to specify a model of financial behaviour based on the mean-variance hypothesis, modified to fit the institutional devices of the trust and mortgage loan companies described in Chapter V. Implications of the mean-variance hypothesis for asset demand functions are explored in some detail, with particular emphasis on comparative-static derivations tracing optimizing responses to changes in parameters such as expected yields and exogenous balance-sheet components (Chapter II). Considering the strong behavioural assumptions implicit in the mean-variance hypothesis, its positive implications for individual investor behaviour are weak. Results of a suggestive nature, reminiscent of demand theory (cf. Hicks (1946)), are all that can be obtained. Of course, if the behavioural assumptions are strengthened still further, by imposing a special form on the mean-variance objective function, then definite qualitative comparative-static results can be derived (Chapter III). However, these special cases have certain peculiar properties[1] and, their theoretical

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[1] It is shown in Chapter III, Section 3 that the two leading mean-variance specializations have elliptical indifference curves in asset space, implying that as an investor's holdings of risky assets increase beyond a certain point he becomes worse off. For use of these specializations see e.g. Royama and Hamada (1967) and Parkin (1970).

neatness notwithstanding, I have purposely not constrained my empirical estimates to the pattern they dictate.

Any realistic model of financial behaviour has to recognize that there are inflexibilities in portfolio management: in the first place there are some components of a balance sheet whose level is determined largely by exogenous factors (in the traditional model of commercial bank behaviour, for instance, deposits are assumed to be determined by the preferences of the public); in the second place there are lagged adjustments of asset stocks due to brokerage costs, inertia and so on. These factors are analyzed in Chapter IV. Again, the results are suggestive rather than definite.

Such firm theoretical conclusions as can be reached derive mainly from the balance sheet identity rather than from any particular hypothesis of portfolio preference; as Brainard and Tobin (1968) have admonished, "adding-up" requirements apply to any portfolio hypothesis. These requirements can therefore be imposed on the empirical estimates as firm knowledge, serving not just the interests of theoretical consistency but statistical efficiency too. An interesting point is that the balance sheet identity must hold at all points in time so that the estimated coefficients in a complete set of asset demand equations must be consistent with exact balance-sheet exhaustion both in, and out of, equilibrium. The additional constraints that this requirement imposes on disequilibrium portfolio models are discussed in Chapter IV.

Explicit application of the model to the data is in Chapter VIII, within a general framework outlined at the beginning of that Chapter. The data are for the major assets and liabilities of the trust and mortgage loan companies, monthly, January 1967 - March 1972. In its tested form the model is a block of six generalized stock-adjustment equations, four demand equations for major assets (i.e., cash and liquid assets, federal government bonds, municipal and provincial bonds, and corporate bonds), and two supply equations for liabilities (i.e., under-one-year term deposits and over-one-year term deposits). Savings deposits and mortgages enter these equations as predetermined variables since in a monthly context this seems to correspond with institutional reality. Savings deposits and mortgages themselves are dealt with in Chapters VI and VII; and it appears that the former depend largely on factors exogenous to the sector whilst the latter respond to the variables which determine them with a considerable lag, therefore both can be represented as functions of predetermined variables only, which implies that they are themselves predetermined.

Judged by conventional single-equation criteria, the results in Chapter VIII are consistent with theory. Each asset responds positively to changes in its own yield, each liability negatively.

In terms of statistical significance and numerical magnitude the estimated interest-rate responses are large. The interest sensitivity is particularly marked for short-term assets and liabilities. Long-run responses to interest-rate changes implicit in the stock-adjustment equations are qualitatively the same as the directly estimated short-run responses. However there is an important difference in the short-run and long-run financing of mortgages. In the short-run a significant proportion of mortgage disbursements are covered by under-one-year term deposits, but in the long run the source of finance is shifted entirely to over-one-year deposits. Thus one observes over the long run a balance sheet well matched by term to maturity not just because exogenous inflows of liabilities are allocated into the asset classes that provide the best hedge, but also because the companies manage their liabilities to that end.

Trust-and-mortgage-loan-company liabilities are determined by interaction of supply and demand, so it is necessary to examine the public's demand for them. This is done in Chapter VI where liabilities are disaggregated into four components -- chequable savings deposits, non-chequable savings deposits, less-than-one-year term deposits, and more-than-one-year term deposits. Savings deposits respond only slightly to changes in relative yields, but seem strongly influenced by the annual Canada Savings Bond campaign. Term deposits -- particularly under-one-year term deposits -- are, on the other hand, very interest sensitive on the demand side. Under-one-year term deposits obviously compete closely with chartered bank certificates of deposit, but substitutes for over-one-year deposits are less easily identified. The present results indicate that corporate bonds provide the main source of competition for the longer term deposits of the trust and mortgage loan sector. A feature of all the equations in Chapter VI is that the implicit long-run wealth elasticities are greater than unity for all classes of the sector's liabilities.

Without doubt the most important relationships in a model of trust and mortgage loan company behaviour are those directly bearing on mortgage lending -- mortgages constitute seventy per cent of the industry's assets. The topic is investigated in Chapter VII. In Canada the secondary mortgage market is undeveloped and the trust and mortgage loan sector (being anyway a net seller in the secondary market) generates its mortgage lending through originating new mortgage loans. Once a new loan is approved it is usually some months -- on average seven months -- before the money is disbursed. Furthermore mortgage-loan approvals respond with a considerable delay to the variables which ultimately determine them. This is why, in a monthly model, it is appropriate to treat mortgage lending as a predetermined quantity when the other balance sheet items are decided. The model for mortgage-loan approvals in Chapter VII postulates that approvals are a function of the difference between desired and actual levels of mortgage lending. More specifically, it is assumed that new loans are approved in order to close the gap between actual mortgage lending plus backlog of undisbursed commitments and the desired stock of mortgages plus backlog. The desired stock is written as a function of interest rates and the total resources of the industry; and the model is specified so that in a stationary state new loan approvals are made only to cover repayments of existing loans. The estimates suggest that in the long-run mortgage-loan approvals are highly interest sensitive, but that adjustments are sluggish -- e.g. it is over eighteen months before one half of the total effect of a change in the mortgage interest rate is felt. Other interest rates to which mortgage-loan approvals respond are the corporate bond rate (the companies apparently regard corporates as being substitutes for mortgages), and the rate on the companies' own long-term deposit rates (over the long run a decrease in the spread between the mortgage rate and the over-one-year deposit rate leads the companies to scale down their activities in the mortgage market).

Lacking a full model of the financial sector, I hesitate to draw policy implications from my results. My tentative feeling is that the marked interest sensitivity that characterizes most of the operations of the trust and mortgage loan company sector renders it an effective channel for the transmission of monetary policy. Notwithstanding the lack of reserve requirements, the sector seems at least as responsive to monetary policy as the banking system simply as a result of market mechanisms. Consider the case of a tightening in monetary policy which induces the chartered banks to compete more vigorously for term deposits. The supply price of term deposits to non-bank financial institutions is thereby raised, and my results indicate that the concomitant squeeze of the interest-rate spread between deposits and earning assets causes the trust and mortgage loan companies to contract. This sort of phenomenon is, I think, observable in the 1960s.[2]

My suggestions for further research are mostly concerned with the empirical work in this thesis. Portfolio theory does not have a lot to contribute to the building of structural financial models -- the pay-off from the mean-variance model comes in general equilibrium models, where it is useful precisely because its aggregation theorems allow the by-passing of structural details. It does seem important to observe theory at least to the extent of

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[2] If the example is changed so that monetary tightness causes interest rates on earning assets to increase by more than those on deposits, the trust and mortgage loan sector is induced to expand. This is the case where non-bank financial intermediaries create changes in velocity which offset monetary policy. In the absence of a complete model of the financial sector this latter case cannot be ruled out, but the case discussed in the text seems to have more empirical relevance.

applying restrictions implied by the balance sheet constraint since, given the collinearity of interest rates, statistical efficiency is at a premium. The fact that I was able to include as many as six interest rate variables in a single regression and obtain well determined coefficients illustrates the point.

Future research into the operations of the trust and mortgage loan company sector should be able to improve on my efforts in a number of ways. The major ommission from this thesis is that the various empirical estimates have not been put together and simulated as a whole, and I do not believe that the structure as it stands would yield useful results in full system simulations, there being two reasons for this. (1) The interest rates paid by the sector on the two categories of term deposit (i.e. over-one-year, and under-one-year) are determined implicitly by supply and demand equations: models that do not have explicit equations for interest rates are apt to exhibit unrealistic solutions for interest rates and to track historical data poorly. (2) More attention needs to be given to the causal linkages between mortgage lending and long-term deposits; these relationships lie at the heart of the operations of the trust and mortgage loan companies. Perhaps the most efficacious method of modifying the present estimates to deal with these points would be the following: instead of estimating supply functions for term deposits by regressing quantity on price, estimate them by regressing price on quantity (both price and quantity are endogenous variables). This modification would give the desired equations for interest rates, and it seems (in retrospect) a more satisfactory representation of causality in the sector.

The trust and mortgage loan company balance sheets do provide an interesting and almost unexplored set of data. I am sure that there are fruitful approaches to these data that I have not even considered. As matters stand, the present investigation is the only comprehensive econometric study of the industry's accounts, and the statistical results are rather impressive.

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